Georgia
Standards of Excellence
Curriculum Frameworks

Mathematics

GSE Algebra I

Unit 1: Relationships between Quantities and Expressions

Unit 1
Relationships between Quantities and Expressions
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OVERVIEW

In this unit students will:
- Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems.
- Interpret units in the context of the problem.
- Convert units and rates using dimensional analysis (English–to–English and Metric–to–Metric without conversion factor provided and between English and Metric with conversion factor);
- Identify the different parts of the expression or formula and explain their meaning.
- Decompose expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts.
- Understand similarities between the system of polynomials and the system of integers.
- Understand that the basic properties of numbers continue to hold with polynomials.
- Draw on analogies between polynomial arithmetic and base–ten computation, focusing on properties of operations, particularly the distributive property.
- Operate with polynomials with an emphasis on expressions that simplify to linear or quadratic forms.
- Rewrite (simplify) expressions involving radicals.
- Use and explain properties of rational and irrational numbers.
- Explain why the sum or product of rational numbers is rational; why the sum of a rational number and an irrational number is irrational; and why the product of a nonzero rational number and an irrational number is irrational.

In this unit, students solve problems related to unit analysis and interpret the structure of expressions. In real–world situations, answers are usually represented by numbers associated with units. Units involve measurement and often require a conversion. Measurement involves both precision and accuracy. Estimation and approximation often precede more exact computations. Students need to develop sound mathematical reasoning skills and forms of argument to make reasonable judgments about their solutions. They should be able to decide whether a problem calls for an estimate, for an approximation, or for an exact answer. To accomplish this goal, teachers should provide students with a broad range of contextual problems that offer opportunities for performing operations with quantities involving units. These problems should be connected to science, engineering, economics, finance, medicine, or other career fields.

This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base–ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi–digit integers. In this unit, students also use and explain properties of rational and irrational numbers and rewrite (simplify) radical expressions. The current unit expands students' prior knowledge of radicals, differences between rational and irrational numbers, and rational approximations of irrational numbers. The properties of rational and irrational numbers and operations with polynomials have been included as a preparation for working with quadratic functions later in the course. This content will provide a solid foundation for all subsequent units.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. This unit provides much needed content information and excellent learning activities. However, the intent of the framework is not to provide a comprehensive resource for the implementation of all standards in the unit. A variety of resources should be utilized to supplement this unit. The tasks in this unit framework illustrate the types of learning activities that should be utilized from a variety of sources. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the **"Strategies for Teaching and Learning"** in the Comprehensive Course Overview and the tasks listed under **"Evidence of Learning"** be reviewed early in the planning process.

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

KEY STANDARDS

Extend the properties of exponents to rational exponents.
MGSE9–12.N.RN.2 Rewrite expressions involving radicals ~~and rational exponents using the properties of exponents.~~ (i.e., simplify and/or use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots).

Use properties of rational and irrational numbers.
MGSE9–12.N.RN.3 Explain why the sum or product of rational numbers is rational; why the sum of a rational number and an irrational number is irrational; and why the product of a nonzero rational number and an irrational number is irrational.

Reason quantitatively and use units to solve problems.
MGSE9–12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:
  a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
  b. Convert units and rates using dimensional analysis (English–to–English and Metric–to–Metric without conversion factor provided and between English and Metric with conversion factor);
  c. Use units within multi–step problems and formulas; interpret units of input and resulting units of output.

MGSE9–12.N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. Given a situation, context, or problem, students will determine, identify, and use appropriate quantities for representing the situation.

MGSE9–12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers' precision is limited to the precision of the data given.*

MGSE9–12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

    MGSE9–12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients, in context.

    MGSE9–12.A.SSE.1b Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.

Perform arithmetic operations on polynomials
MGSE9–12.A.APR.1 Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations. *(For the purpose of this course, operations with polynomials will be limited to the second degree. Higher degree polynomials will be addressed in future courses.)*

STANDARDS FOR MATHEMATICAL PRACTICE

Refer to the Comprehensive Course Overview for more detailed information about the Standards for Mathematical Practice.

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

ENDURING UNDERSTANDINGS

- The structure of expressions and the meaning of their parts in context.

- Appropriateness of units of measure within context.

- Similarities between the system of polynomials and the system of integers.

- Addition, Subtraction, and Multiplication of polynomials is closed.

- Properties of rational and irrational numbers.

- Simplify and/or use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots.

- Visual representation of radicals.

ESSENTIAL QUESTIONS

- How do I choose and interpret units of measure in context?

- How do I interpret parts of an expression in terms of context?

- How are polynomial operations related to operations in the real number system?

- How can polynomials be used to express realistic situations?

- How do I justify simplification of radicals using visual representations?

- Why is the sum or product of rational numbers rational?

- Why is the sum of a rational number and irrational number irrational?

- Why is the product of a nonzero rational number and an irrational number irrational?

CONCEPTS AND SKILLS TO MAINTAIN

Students may not realize the importance of unit conversion in conjunction with computation when solving problems involving measurement. Since today's calculating devices often display 8 to 10 decimal places, students frequently express answers to a much greater degree of precision than is required.

Measuring commonly used objects and choosing proper units for measurement are part of the mathematics curriculum prior to high school. In high school, students experience a broader variety of units through real–world situations and modeling, along with the exploration of the different levels of accuracy and precision of the answers.

An introduction to the use of variable expressions and their meaning, as well as the use of variables and expressions in real–life situations, is included in the Expressions and Equations Domain of Grade 7. Working with expressions and equations, including formulas, is an integral part of the curriculum in Grades 7 and 8.

During the school–age years, students must repeatedly extend their conception of numbers. From counting numbers to fractions, students are continually updating their use and knowledge of numbers. In Grade 8, students extend this system once more by differentiating between rational and irrational numbers.

Students are expected to have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre–assess to determine whether instructional time should be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Order of operations
- Algebraic properties
- Number sense
- Computation with whole numbers and integers
- Radicals
- Rational and irrational numbers
- Measuring length and finding perimeter and area of rectangles and squares
- Volume and capacity

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for high school children. **Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**
http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary.

http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

- **Algebra:** The branch of mathematics that deals with relationships between numbers, utilizing letters and other symbols to represent specific sets of numbers, or to describe a pattern of relationships between numbers.

- **Binomial Expression:** An algebraic expression with two unlike terms.

- **Capacity:** The greatest volume that a container can hold.

- **Circumference:** The distance around a circle.

- **Coefficient:** A number multiplied by a variable.

- **Constant Term:** A quantity that does not change its value.

- **Expression:** A mathematical phrase involving at least one variable and sometimes numbers and operation symbols.

- **Factor:** When two or more integers are multiplied, each integer is a factor of the product. "To factor" means to write the number or term as a product of its factors.

- **Integer:** The set of numbers ...,–3,–2,–1,0,1,2,3,…

- **Irrational Number:** A number whose decimal form is nonterminating and nonrepeating. Irrational numbers cannot be written in the form a/b, where a and b are integers (b cannot be zero). So all numbers that are not rational are irrational.

- **Monomial Expression:** An algebraic expression with one term.

- **Perimeter**: The sum of the lengths of the sides of a polygon.

- **Polynomial function:** A *polynomial function* is defined as a function,

  $$f(x) = a_o x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_{n-2} x^2 + a_{n-1} x^1 + a_n,$$

  where the coefficients are real numbers.

- **Pythagorean Theorem:** It is a theorem that states a relationship that exists in any right triangle. If the lengths of the legs in the right triangle are *a* and *b* and the length of the hypotenuse is *c*, we can write the theorem as the following equation: $a^2 + b^2 = c^2$

- **Radical:** The symbol, $\sqrt[b]{a}$, which is read "the bth root of a," is called a radical.

- **Radicand:** The number underneath the root symbol. So, in $\sqrt[b]{a}$, the *a* is called the radicand.

- **Rational Number:** A number expressible in the form *a/b* or – *a/b* for some fraction *a/b*. The rational numbers include the integers.

- **Standard Form of a Polynomial**: To express a polynomial by putting the terms in descending exponent order.

- **Term:** A number, a variable, or a product of numbers and variables.

- **Trinomial**: An algebraic expression with three unlike terms.

- **Variable:** A letter or symbol used to represent a number.

- **Volume:** The amount of space occupied by an object.

- **Whole numbers:** The numbers 0, 1, 2, 3, ….

The properties of operations. Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition $(a + b) + c = a + (b + c)$
Commutative property of addition $a + b = b + a$
Additive identity property of 0 $a + 0 = 0 + a = a$
Existence of additive inverses For every *a* there exists –*a* so that $a + (–a) = (–a) + a = 0$.
Associative property of multiplication $(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication $a \times b = b \times a$
Distributive property of multiplication over addition $a \times (b + c) = a \times b + a \times c$

Definitions and activities for these and other terms can be found on the Intermath website
http://intermath.coe.uga.edu/dictnary/homepg.asp

EVIDENCE OF LEARNING

By the conclusion of this unit, students should be able to demonstrate the following competencies:
.
- Interpret units of measure in context.

- Interpret parts of an expression in terms of context.

- Relate polynomial operations to the real number system.

- Use polynomials to express realistic situations.

- Simplify radicals and justify simplification of radicals using visual representations.

- Use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots.

- Understand why the sum or product of rational numbers is rational.

- Understand why the sum of a rational number and irrational number is irrational.

- Understand why the product of a nonzero rational number and an irrational number is irrational.

- Understand that results of operations performed between numbers from a particular number set does not always belong to the same set. For example, the sum of two irrational numbers $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$ is 4, which is a rational number; however, the sum of a rational number 2 and irrational number $\sqrt{3}$ is an irrational number $(2 + \sqrt{3})$

TEACHER RESOURCES

The following pages include teacher resources that teachers may wish to use for the purpose of instruction and assessment:
- http://brightstorm.com/search/?k=polynomials
- Illustrative Mathematics
  https://www.illustrativemathematics.org/
  Standards are illustrated with instructional and assessment tasks, lesson plans, and other curriculum resources.
- http://www.khanacademy.org/search?page_search_query=polynomials
- For mathematical applications
- http://www.thefutureschannel.com/

- The Georgia Online Formative Assessment Resource (GOFAR) accessible through SLDS contains test items related to content areas assessed by the Georgia Milestones Assessment System and NAEP. Teachers and administrators can utilize the GOFAR to develop formative and summative assessments, aligned to the state–adopted content standards, to assist in informing daily instruction.

  Students, staff, and classes are prepopulated and maintained through the State Longitudinal Data System (SLDS). Teachers and Administrators may view Exemplars and Rubrics in Item Preview. A scoring code may be distributed at a local level to help score constructed response items.

For GOFAR user guides and overview, please visit:
https://www.gadoe.org/Curriculum–Instruction–and–
Assessment/Assessment/Pages/Georgia–Online–Formative–Assessment–Resource.aspx

Web Resources

The following list is provided as a sample of available resources and is for informational purposes only. It is your responsibility to investigate them to determine their value and appropriateness for your district. GADOE does not endorse or recommend the purchase of or use of any particular resource.

- Scale of Universe
  http://scaleofuniverse.com/
  Fantastic applet exploring the magnitudes of the universe.

- Algebraic Expressions
  https://www.teachingchannel.org/videos/high–school–algebra–lesson?fd=1
  Teacher Resource: Video of a lesson showing implementation of a FAL activity.

- National Library of Virtual Manipulatives – Algebra Tiles. Visualize multiplying algebraic expressions using tiles.
  http://nlvm.usu.edu/en/nav/frames_asid_189_g_4_t_2.html?open=activities&from=category_g_4_t_2.html

- NASA Task
  http://www.nasa.gov/audience/foreducators/exploringmath/algebra1/Prob_SuitYourself_detail.html
  In–depth lesson plan relating linear equations to space suit production.

FORMATIVE ASSESSMENT LESSONS (FAL)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Overview.

SPOTLIGHT TASKS

A Spotlight Task has been added to each GSE mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit–level Georgia Standards of Excellence, and research–based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3–Act Tasks based on 3–Act Problems from Dan Meyer and Problem–Based Learning from Robert Kaplinsky.

3–ACT TASKS

A Three–Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

More information along with guidelines for 3–Act Tasks may be found in the Comprehensive Course Overview.

TASKS

The following tasks represent the level of depth, rigor, and complexity expected of all Algebra I students. These tasks, or tasks of similar depth and rigor, should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as a performance task, they may also be used for teaching and learning (learning/scaffolding task).

Task Name *Suggested Time*	Task Type *Grouping Strategy*	Content Addressed	Standards
Interpreting Algebraic Expressions *90 – 120 min*	Formative Assessment Lesson *Partner/Small Group*	Recognizing the order of algebraic operations, recognizing equivalent expressions, and understanding the distributive laws of multiplication and division over addition (expansion of parentheses).	A.APR.1 A.SSE.1a
Polynomial Patterns *30 – 45 min*	Scaffolding Task *Individual / Partner*	Add, subtract, and multiply polynomials	A.APR.1 A.SSE.1a
Modeling *30 min*	Performance Task *Small Group Task*	Applying Geometric Representations of Polynomials	A.APR.1 A.SSE.1a A.SSE.1b N.Q.1
Yogurt Packaging *45–60 minutes* PDF / Word	Achieve CCSS–CTE Classroom Task *Individual/Partner*	Convert units in order to solve problems.	N.Q.1 N.Q.2 N.Q.3
Corn & Oats *60 minutes* PDF / Word	Achieve CCSS–CTE Classroom Task *Small Group*	Making a scale model and make conversions.	N.Q.1 N.Q.2 N.Q.3
Leaky Faucet *30–45 min*	Spotlight Task *Individual / Partner*	Use units as a way to understand problems Define appropriate quantities for the purpose of descriptive modeling. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	N.Q.1 N.Q.2 N.Q.3
Visualizing Square Roots *50-60 min*	Learning Task *Individual/Partner*	To build the ideas of square and square root on their geometric interpretation. To justify simplification of radicals using geometric representations.	N.RN.2

Task Name *Suggested Time*	Task Type *Grouping Strategy*	Content Addressed	Standards
Rational and Irrational Numbers #1 *60–85 min*	Formative Assessment Lesson *Partner/Small Group*	Classifying numbers as rational or irrational and moving between different representations of rational and irrational numbers.	N.RN.2 N.RN.3 A.SSE.1a
Rational and Irrational Numbers #2 *60–85 min*	Formative Assessment Lesson *Partner/Small Group*	Finding irrational and rational numbers to exemplify general statements and reasoning with properties of rational and irrational numbers.	N.RN.2 N.RN.3 A.SSE.1a A.SSE.1b
Amusement Park *60–90 min*	Culminating Task *Partner/Small Group Task*	Reviewing of Unit Standards	N.RN.2 N.RN.3 A.APR.1 A.SSE.1a A.SSE.1b N.Q.1 N.Q.2 N.Q.3
Additional Task: **The Physics Professor**	Illustrative Mathematics Task Extension	Investigating a radical expression.	A.SSE.1 A.SSE.1a
Additional Task: **Delivery Trucks**	Illustrative Mathematics Task	Looking for structure in algebraic expressions related to a context.	A.SSE.1 A.SSE.1a
Additional Task: **Kitchen Floor Tiles**	Illustrative Mathematics Task	Practicing reading, analyzing, and constructing algebraic expressions arising from patterns.	A.SSE.1 A.SSE.1a

Interpreting Algebraic Expressions (Formative Assessment Lesson)
Back to Task Table

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=694

Essential Questions
- How do you recognize the order of algebraic operations?
- How do you recognize equivalent expressions?
- How do you understand the distributive laws of multiplication and division over addition (expansion of parentheses)?

Task Comments
Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, Interpreting Algebraic Expressions, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=221&subpage=concept
The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=694

GEORGIA STANDARDS OF EXCELLENCE
Perform arithmetic operations on polynomials
MGSE9–12.A.APR.1 Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations.

Interpret the structure of expressions
MGSE9–12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients, in context.

STANDARDS FOR MATHEMATICAL PRACTICE
This lesson uses all of the practices with emphasis on:
2. Reason abstractly and quantitatively by requiring students to make sense of quantities and their relationships to one another in problem situations.
7. Look for and make use of structure by expecting students to apply rules, look for patterns and analyze structure.

Polynomial Patterns (Scaffolding Task)

GEORGIA STANDARDS OF EXCELLENCE

Perform arithmetic operations on polynomials
MGSE9–12.A.APR.1 Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations.

Interpret the structure of expressions
MGSE9–12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients, in context.

STANDARDS FOR MATHEMATICAL PRACTICE
2. **Reason abstractly and quantitatively** by requiring students to make sense of quantities and their relationships to one another in problem situations.
7. **Look for and make use of structure** by expecting students to apply rules, look for patterns and analyze structure.

Common Student Misconceptions:
1. Some students will apply the distributive property inappropriately. Emphasize that it is the distributive property of multiplication over addition. For example, the distributive property can be used to rewrite $2(x + y)$ as $2x + 2y$, because in this product the second factor is a sum (i.e., involving addition). But in the product $2(xy)$, the second factor,(xy),   is itself a product, not a sum.
2. Some students will still struggle with the arithmetic of negative numbers. Consider the expression $(-3)\bullet(2 + (-2))$. On the one hand, $(-3) \bullet(2 + (-2)) = (-3) \bullet(0) = 0$. But using the distributive property, $(-3)\bullet(2 + (-2)) = (-3)\bullet(2) + (-3)\bullet(-2)$. Because the first calculation gave 0, the two terms on the right in the second calculation must be opposite in sign. Thus, if we agree that $(-3)\bullet(2) =-6$, then it must follow that $(-3)\bullet(-2) = 6$.
3. Students often forget to distribute the subtraction to terms other than the first one. For example, students will write $(4x + 3) - (2x + 1) = 4x + 3 - 2x + 1 = 2x + 4$ rather than $4x + 3 - 2x - 1 = 2x+ 2$.
4. Students will change the degree of the variable when adding/subtracting like terms. For example, $2x + 3x = 5x^2$ rather than $5x$.
5. Students may not distribute the multiplication of polynomials correctly and only multiply like terms. For example, they will write $(x + 3)(x - 2) = x^2 - 6$ rather than $x^2 - 2x + 3x - 6$.

The following activity is a modification from NCTM's Illuminations Polynomial Puzzler:
http://illuminations.nctm.org/LessonDetail.aspx?id=L798

Comments:

When students have completed their puzzlers, allow them to share their answers and thinking with the class. Here are some ideas to help you structure this:

- *Do not simply put up the answer key. Have students write their solutions to the puzzlers on the board or fill them in on an overhead copy of the activity sheet. As they fill in the spaces, ask them to explain verbally or in writing how they approached the puzzle.*
- *If students worked in pairs, allow them to present the solutions in pairs.*
- *As students are reflecting, you may wish to ask them questions such as*

- ❖ *Did you use a traditional method to expand and factor, such as FOIL, or did you develop your own strategies as you worked?*
- ❖ *Were there certain paths to solving the polynomial puzzlers that were easier than others? Why?*

Assessment can be made by creating tables using the templates below. The question marks can be completed by the instructor, and then, the students will complete the tables.

	?	
?		*?*
	?	

		?
?		*?*
		?

1. Can you find the pattern to the number puzzle below? Explain the pattern.

2	–6	–12
4	1	4
8	–6	–48

2. Now, use the pattern to complete this table.

3	*–5*	–15
8	–2	–16
24	*10*	240

HINT: Start with the question marks.

3. This can be expanded to multiplication with polynomials by solving the following:

1	$x + 3$	*x + 3*
$–2x + 5$	2	*–4x + 10*
–2x + 5	*2x + 6*	*–4x²–2x + 30*

4. What about this one?

–5	*–2x + 3*	$10x – 15$
$3x – 2$	*4*	*12x – 8*
–15x + 10	$8x + 12$	*120x² –260x + 120*

5. Work the following on your own for 10 minutes, and then complete the tables with a partner.

a.

1	$x + 7$	*x + 7*
$–2x + 5$	2	*–4x + 10*
–2x + 5	*2x + 14*	*–4x² – 18x + 70*

b.

–2	$x – 3$	*–2x + 6*
3	$–5x + 1$	*–15x + 3*
–6	*–5x²+16x – 3*	$30x²–96x + 18$

c.

−4	2	−8
$x + 3$	$x - 3$	$x^2 - 9$
$-4x - 12$	$2x - 6$	$-8x^2 + 72$

d.

$x + 3$	3	$3x + 9$
2	$4x$	$8x$
$2x + 6$	$12x$	$24x^2 + 72x$

e.

2	$x + 5$	$2x + 10$
$x + 3$	7	$7x + 21$
$2x + 6$	$7x + 35$	$14x^2 + 112x + 210$

f.

6	$2x + 2$	$12x + 12$
3	$x + 3$	$3x + 9$
18	$2x^2 + 8x + 6$	$36x^2 + 144x + 108$

Polynomial Patterns (Scaffolding Task)   **Back to Task Table**

Name_____   Date_____

GEORGIA STANDARDS OF EXCELLENCE

Perform arithmetic operations on polynomials
MGSE9–12.A.APR.1 Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations. *(Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.)*

Interpret the structure of expressions
MGSE9–12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients, in context.

STANDARDS FOR MATHEMATICAL PRACTICE
2. **Reason abstractly and quantitatively** by requiring students to make sense of quantities and their relationships to one another in problem situations.
7. **Look for and make use of structure** by expecting students to apply rules, look for patterns and analyze structure.

The following activity is a modification from NCTM's Illuminations Polynomial Puzzler
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1. Can you find the pattern to the number puzzle below? Explain the pattern.

2	–6	–12
4	1	4
8	–6	–48

2. Now, use the pattern to complete this table.

3	?	–15
	–2	?
		240

  HINT: Start with the question marks.

3. This can be expanded to multiplication with polynomials by solving the following:

1	x + 3	
–2x + 5	2	

4. What about this one?

–5		$10x - 15$
$3x - 2$		
	$-8x + 12$	

5. Work the following on your own for 10 minutes, and then complete the tables with a partner.

a.

1	$x + 7$	
$-2x + 5$	2	

b.

	$x - 3$	
3	$-5x + 1$	
		$30x^2 - 96x + 18$

c.

–4		–8
	$2x - 6$	$-8x^2 + 72$

d.

$x + 3$		
2		$8x$
	$12x$	

e.

		$2x + 10$
$x + 3$	7	
$2x + 6$		

f.

6		
	$x + 3$	
18		$36x^2 + 144x + 108$

Modeling (Performance Task)

GEORGIA STANDARDS OF EXCELLENCE

Perform arithmetic operations on polynomials
MGSE9–12.A.APR.1 Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations.

Interpret the structure of expressions
MGSE9–12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients, in context.

MGSE9–12.A.SSE.1b Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.

Reason quantitatively and use units to solve problems.
MGSE9–12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:
   a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
   b. Convert units and rates using dimensional analysis (English–to–English and Metric–to–Metric without conversion factor provided and between English and Metric with conversion factor);
   c. Use units within multi–step problems and formulas; interpret units of input and resulting units of output.

STANDARDS FOR MATHEMATICAL PRACTICE
2. Reason abstractly and quantitatively by requiring students to make sense of quantities and their relationships to one another in problem situations.
7. Look for and make use of structure by expecting students to apply rules, look for patterns and analyze structure.

Common Student Misconceptions
1. Some students will apply the distributive property inappropriately. Emphasize that it is the distributive property of multiplication over addition. For example, the distributive property can be used to rewrite $2(x + y)$ as $2x + 2y$, because in this product the second factor is a sum (i.e., involving addition). But in the product $2(xy)$, the second factor,(xy), is itself a product, not a sum.
2. Some students will still struggle with the arithmetic of negative numbers. Consider the expression $(-3) \bullet (2 + (-2))$. On the one hand, $(-3) \bullet (2 + (-2)) = (-3) \bullet (0) = 0$. But using the distributive property, $(-3) \bullet (2 + (-2)) = (-3) \bullet (2) + (-3) \bullet (-2)$. Because the first calculation gave 0, the two terms on the right in the second calculation must be opposite in sign. Thus, if we agree that $(-3) \bullet (2) = -6$, then it must follow that $(-3) \bullet (-2) = 6$.
3. Students often forget to distribute the subtraction to terms other than the first one. For example, students will write $(4x + 3) - (2x + 1) = 4x + 3 - 2x + 1 = 2x + 4$ rather than $4x + 3 - 2x - 1 = 2x + 2$.

---

4. Students will change the degree of the variable when adding/subtracting like terms. For example, $2x + 3x = 5x^2$ rather than $5x$.
5. Students may not distribute the multiplication of polynomials correctly and only multiply like terms. For example, they will write $(x + 3)(x - 2) = x^2 - 6$ rather than $x^2 - 2x + 3x - 6$

The problems below will be placed on the walls around the room with large sheets of paper under each. Students will work in teams of four people to travel around the room and write their solutions under the papers. Each team should be given a letter name that corresponds to their starting problem. After each team is given about 3–4 minutes on a problem, the teacher should call time, and the teams move to the next station.

Review and practice: http://www.wiziq.com/tutorial/111186–7–5–Multiplying–a–Polynomial–by–a–Monomial

Having each time write in a different color can be beneficial as well as designating roles for each team member such as scribe, director, checker, and presenter. After all teams have rotated through all the problems, the teams can travel back through to check for differences in answers. This can lead to a discussion on which problems are correct or a discussion on the different methods used to arrive at the same answer.

Problem A (extension problem)

The volume in cubic units of the box is $a^3 + 8a^2 + 19a + 12$. Its length is $a + 4$ units and its width is $a + 3$ units. What is its height?
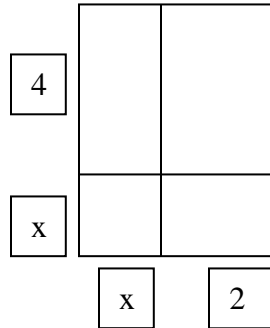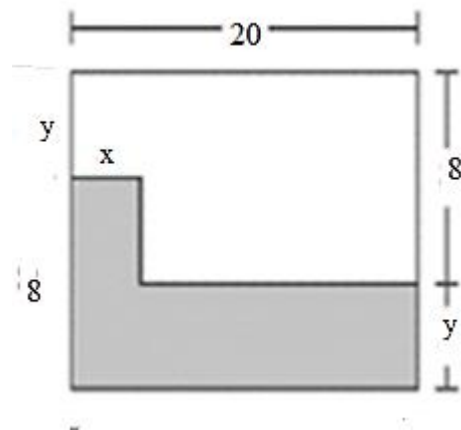

?

$a + 3$

$a + 4$
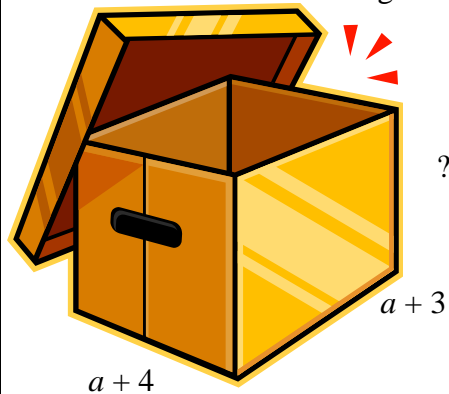
answer: a + 1

Problem B

What is an illustration of $(x + 2)(x + 4)$?

Possible answer:

Problem C: This rectangle shows the floor plan of an office. The shaded part of the plan is an area that is getting new tile. Write an algebraic expression that represents the area of the office that is getting new tile.
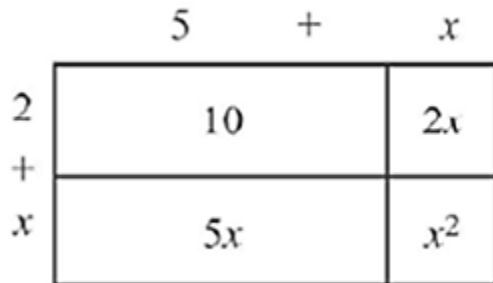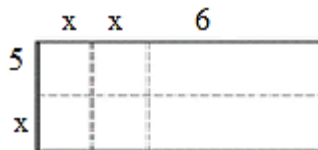

Possible Answer

8x −xy + 20y

Problem D

What is the rectangle modeling?

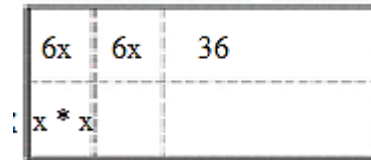*Answer: (x + 5)(x + 2)*

Problem E

What is the product of the expression represented by the model below?

Answer: $2x^2 + 16x + 30$

Problem F

Write the dimensions for the rectangle below.

6x	6x	36
x * x		

Answer: (x + 6) by (x + x + 6) or (x + 6) by (2x + 6)

Problem G

Find the area, including units, of the shape below.

x cm   x cm

8 cm

y cm

12 cm
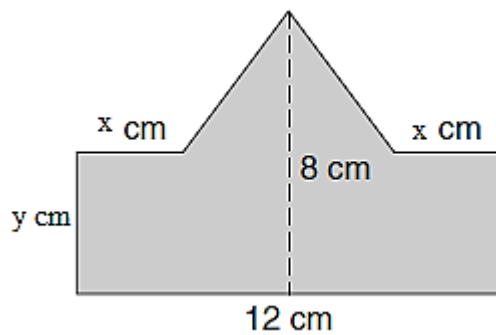
Answer: −8x + xy + 6y + 48

Modeling (Performance Task)

Name_____   Date_____

Perform arithmetic operations on polynomials
MGSE9–12.A.APR.1 Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations.

Interpret the structure of expressions
MGSE9–12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients, in context.

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   b. Convert units and rates using dimensional analysis (English–to–English and Metric–to–Metric without conversion factor provided and between English and Metric with conversion factor);
   c. Use units within multi–step problems and formulas; interpret units of input and resulting units of output.

Problem A

The volume in cubic units of the box is $a^3 + 8a^2 + 19a + 12$. Its length is $a + 4$ units and its width is $a + 3$ units. What is its height?

?

$a + 3$

$a + 4$
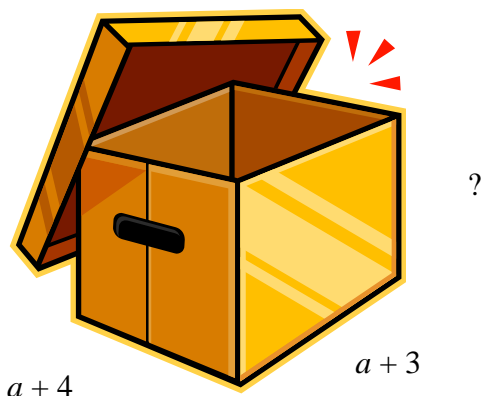
Problem B

What is an illustration of $(x + 2)(x + 4)$?

Problem C: This rectangle shows the floor plan of an office. The shaded part of the plan is an area that is getting new tile. Write an algebraic expression that represents the area of the office that is getting new tile.
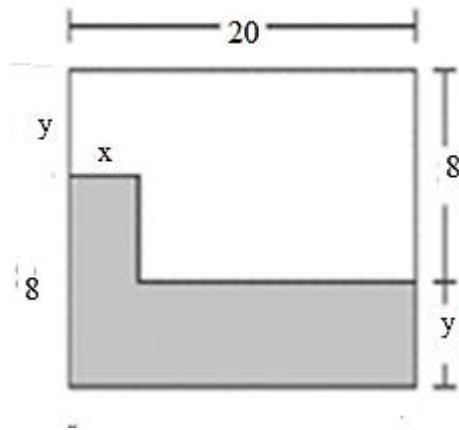
Problem D

What is the rectangle modeling?
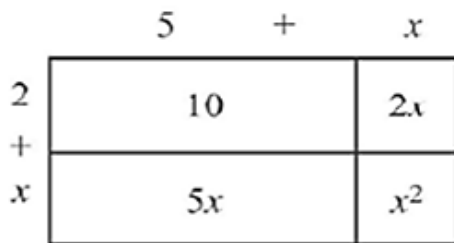
Problem E

What is the product of the expression represented by the model below?

Problem F

Write the dimensions for the rectangle below.

Problem G

Find the area, including units, of the shape below.

Yogurt Packaging (Career and Technical Education Task) **Back to Task Table**

Source: National Association of State Directors of Career Technical Education Consortium
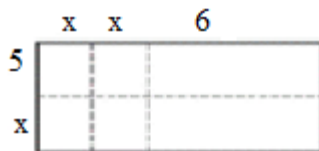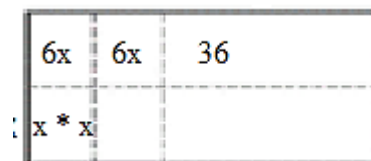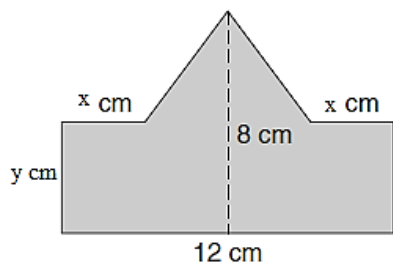PDF: http://www.achieve.org/files/CCSS–CTE–Yogurt–Packaging–FINAL.pdf
Word: http://www.achieve.org/files/CCSS–CTE–Yogurt–Packaging–FINAL.docx

Introduction
　　　Students use unit analysis to answer questions in the context of yogurt production.

Mathematical Goals
- Use unit analysis to answer questions.
- Students will use percent increase and decrease.
- Students will turn amounts (grams and fl. oz.) to unit rates (grams per fl. oz.)

Essential Questions
- How can I use unit analysis to answer questions in context?

GEORGIA STANDARDS OF EXCELLENCE

Reason quantitatively and use units to solve problems.
MGSE9–12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:
  a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
  b. Convert units and rates using dimensional analysis (English–to–English and Metric–to–Metric without conversion factor provided and between English and Metric with conversion factor);
  c. Use units within multi–step problems and formulas; interpret units of input and resulting units of output

MGSE9–12.N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. Given a situation, context, or problem, students will determine, identify, and use appropriate quantities for representing the situation.

MGSE9–12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers' precision is limited to the precision of the data given.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
   Students must make conjectures about the form and meaning of the solution pathway. The task requires multi–step problem solving.

2. Reason abstractly and quantitatively.
   Students make sense of quantities and their relationships in the problem situation.
6. Attend to precision.
   Students need to attend to units as they perform calculations. Rounding and estimation are a key part

Background Knowledge
- Students can work with percentages, including percent increase and percent decrease.
- Students can apply unit analysis to answer questions.

Common Misconceptions
- When converting units, students often divide when they should multiply (or vice–versa). Writing units throughout the problem, rather than only in the answer, can help with this issue, as students can ensure that units "cancel" appropriately.
- For percent increase and decrease, students may forget to add to 1 or subtract from 1. Remind students that they are multiplying by a fraction that compares the *entire* new amount to the *original* amount.

Materials
- For the first extension, students need paper with which to construct a yogurt tub.

Grouping
- Individual / partner

Differentiation
- See extensions in task.

Corn and Oats (Career and Technical Education Task)   **Back to Task Table**

Source: National Association of State Directors of Career Technical Education Consortium
PDF: **http://www.achieve.org/files/CCSS–CTE–Corn%26Oats–FINAL.pdf**
Word: **http://www.achieve.org/files/CCSS–CTE–Corn%26Oats–FINAL.docx**

Introduction

This task uses the process of planting corn and oats to help students convert units, calculate area, and work with percents to determine fertilization and planting needs for Producer Bob.

Mathematical Goals

- Convert and use appropriate units.
- Find the area of a geometric figure.
- Solve problems using percents.

Essential Questions

- How do I use appropriate units in real–life situations to determine area, fertilization, and planting needs?

GEORGIA STANDARDS OF EXCELLENCE

Reason quantitatively and use units to solve problems.

MGSE9–12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
b. Convert units and rates using dimensional analysis (English–to–English and Metric–to–Metric without conversion factor provided and between English and Metric with conversion factor);
c. Use units within multi–step problems and formulas; interpret units of input and resulting units of output

MGSE9–12.N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.

MGSE9–12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers' precision is limited to the precision of the data given.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
   This task requires multi–step problem solving, sense making, and understanding of relationships.
2. Reason abstractly and quantitatively.
   This task requires a great deal of quantitative reasoning.
4. Model with mathematics.
   Students employ mathematics and interpret their results in the context of the situation.
6. Attend to precision.
   The quantitative demands of this task are high, and students need to pay careful attention to units and unit conversions. They need to calculate accurately and express numerical answers with a degree of precision appropriate for the problem context.

Background Knowledge

- Students can convert units.
- Students can work with percentages.
- Students can find the area of triangles and rectangles.
- Students can use proportional reasoning to solve problems.

Common Misconceptions

- Students may forget how to convert from standard to metric units.
- Students must remember that all percents are out of 100 ("per cent") when setting up a proportion.

Materials

- None

Grouping

- Small group

Differentiation
Intervention:

- Many students are likely to need clarification on acreage and parceling land before beginning this task.

Leaky Faucet (Spotlight Task)　　　　　　　**Back to Task Table**

Task adapted from Dan Meyer 3 Acts Math　http://threeacts.mrmeyer.com/leakyfaucet/ and GSE Coordinate Algebra Unit 1 task **"Acting Out."**

GEORGIA STANDARDS OF EXCELLENCE

Reason quantitatively and use units to solve problems.
MGSE9–12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:
   a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
   b. Convert units and rates using dimensional analysis (English–to–English and Metric–to–Metric without conversion factor provided and between English and Metric with conversion factor);
   c. Use units within multi–step problems and formulas; interpret units of input and resulting units of output

MGSE9–12.N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. Given a situation, context, or problem, students will determine, identify, and use appropriate quantities for representing the situation.

MGSE9–12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers' precision is limited to the precision of the data given.*

STANDARDS FOR MATHEMATICAL PRACTICE
1. Make sense of problems and persevere in solving them*. In this task, students will have to interpret the given situation to determine what information is needed to solve the problem. It is possible, and even desirable, to have students decide on the problem this video is posing. Students should be allowed to think independently then work in small groups to decide what problem is being presented.*

5. Use appropriate tools strategically. *Students should decide what "tools" are needed to solve the problem they posed based on the video. What information is needed and what tools will allow them to access that information.*

6. Attend to precision. *Students will have to decide the level of precision needed to answer the question(s) posed after watching and discussing the video clip.*

Essential Questions
- How do I choose and interpret units consistently in solving application problems?
- How can I model constraints using mathematical notation?

Materials Required
- Video clip "leaky faucet" from 3 Acts Math
- Timing device (for first estimate; Act 2 provides exact amounts)
- Conversion factors for quantities mentioned in the problems posed after the video *(Act 2 will supply needed information but students could investigate independently before getting the information)*
- The facts about the dripping water in the video and the sink capacity, etc. *NOTE: the needed facts may vary based on the questions posed by students.*

Time Needed
- 30–45 minutes based on the depth of investigation

Teacher Notes
In this task, students will watch the video then discuss what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them

More information along with guidelines for 3–Act Tasks may be found in the Comprehensive Course Guide.

Task Description

Kim has a leaky faucet and asks Erik to come over and take a look at it.

ACT 1:
Watch the video: http://threeacts.mrmeyer.com/leakyfaucet/act1/act1.mov
Think and wonder: What do you notice? What do you want to know after watching the video? How can you come up with answers to your questions?
Guiding questions to consider if the students don't come up with them on their own might be:
- 1. How long will it take the sink to fill up?
- 2. Write down a guess.
- 3. Write down an answer you know is too high. Too low.

ACT 2:
What information would be useful to know here?
The links below are from Dan Meyer Leaky Faucet 3 Act Math site
- video — drops per second
- video — ml per second
- image — the capacity of the sink
- image — the cost of water

ACT 3
Students will compare and share solution strategies.
- Reveal the answer. Discuss the theoretical math versus the practical outcome.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.

- video — the sink fills up   Reveal the Actual Solution

ACT 4 A Sequel Option:
Describe two scenarios where a leaky faucet would take a week to fill something up.

Extension:
Suppose the sink in not plugged and the water leaks for a week before it is noticed. How much water would have leaked? How much would it cost?

Task Title:_____   Name:_____

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:_____

Estimate the result of the main question? Explain?

Place an estimate that is too high and too low on the number line

Low estimate   *Place an "x" where your estimate belongs*   High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:_____

Act 2 (con't)

Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Visualizing Square Roots (Learning Task)

Back to Task Table

Adapted from http://www.mathedpage.org/geometry-labs/gl/gl-9.pdf

Mathematical Goals:
- To build the ideas of square and square root on their geometric interpretation.
- To justify simplification of radicals using geometric representations.

Essential Questions:
- How do I represent radicals visually?
- What is the relationship between the radicand and the area of a square?
- How do I justify simplification of radicals using geometric representations?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.N.RN.2 Rewrite expressions involving radicals ~~and rational exponents using the properties of exponents.~~ (i.e., simplify and/or use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots).

STANDARDS FOR MATHEMATICAL PRACTICE
1. Make sense of problems and persevere in solving them
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure

Materials Needed:
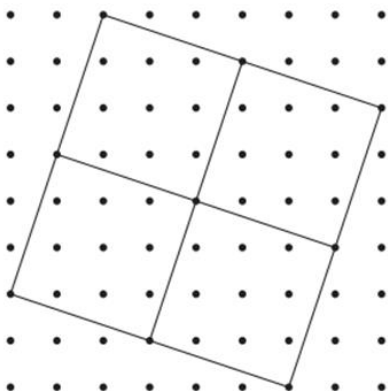- Dot Paper , Graph Paper, or Geoboard

Grouping:
- Individual/Partner

Time Needed:
- 50-60 minutes

Visualizing Square Roots (Learning Task)– Teacher Notes

Adapted from http://www.mathedpage.org/geometry-labs/gl/gl-9.pdf

In the above figure, what are the following measures?

  i.   The side of one of the small squares $\sqrt{10}$ units
  ii.   The area of one of the small squares $\sqrt{10} \cdot \sqrt{10} = 10$ units²
  iii. The perimeter of one of the small squares $\sqrt{10} + \sqrt{10} + \sqrt{10} + \sqrt{10} = 4\sqrt{10}$ units.
  iv.   The side of the large square $\sqrt{40}$ units
  v.   The area of the large square $\sqrt{40} \cdot \sqrt{40} = 40$ units²

Please note that the area of the large square could also be found by finding the area the outer square (the circumscribing square) and subtracting the areas of the triangles that would be "cut out" to form the large square (decomposition strategy).

The area of the outer square is $8 \cdot 8 = 64$ square units.
The outer square can be decomposed into the large square and four triangles.
The area of each triangle is $\left(\frac{1}{2}\right) 2 \cdot 6 = 6$ units²
The area of the large square is: $64 - 4(6) = 40$ units² (the area of the outer square minus the areas of four triangles surrounding the large square.)

  vi.   The perimeter of the large square $\sqrt{40} + \sqrt{40} + \sqrt{40} + \sqrt{40} = 4\sqrt{40}$ units.

1b. Explain, using the answers to Problem 1a, why $\sqrt{40} = 2\sqrt{10}$.
Both values represent the length of the side of the large square.
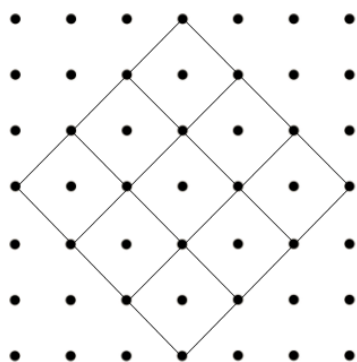
2a. In the above figure, what are the following measures?

   i.   The area of one of the small squares   2 units²

   ii.   The side of one of the small squares $\sqrt{2}$ units

   iii. The area of the large square   18 units²

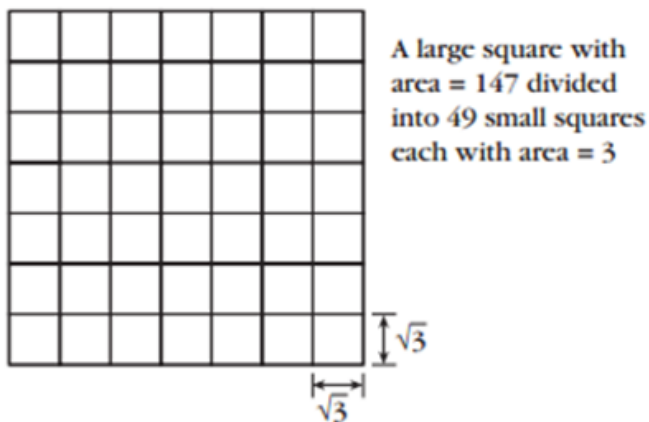   iv. The side of the large square $\sqrt{18}$ units

2b. Explain, using the answers to problem 2a, why $\sqrt{18} = 3\sqrt{2}$
Both values represent the length of the side of the large square.

3. On dot paper, create a figure to show that $\sqrt{8} = 2\sqrt{2}, \ \sqrt{18} = 3\sqrt{2}, \sqrt{32} = 4\sqrt{2}, and \ \sqrt{50} = 5\sqrt{2}.$

4. On dot paper, create a figure to show that $\sqrt{20} = 2\sqrt{5}$ and $\sqrt{45} = 3\sqrt{5}.$

In the figure on the previous page, and in the figures you made in Problems 3 and 4, a larger square is divided up into *a square number of squares.* This is the basic idea for writing square roots in *simple radical form.* The figure need not be made on dot paper. For example, consider $\sqrt{147}$. Since $147 = 3 \cdot 49$, and since 49 is a square number, we can divide a square of area 147 units ^{2} into 49 squares, each of area 3 untis ^{2}:

A large square with
area = 147 divided
into 49 small squares
each with area = 3

You will notice that the side of the larger square is $\sqrt{147} = 7\sqrt{3}$

5. Write the following in simple radical form.

    i. $\sqrt{12}$ $2\sqrt{3}$

    ii. $\sqrt{45}$ $3\sqrt{5}$

    iii. $\sqrt{24}$ $2\sqrt{6}$

    iv. $\sqrt{32}$ $4\sqrt{2}$

    v. $\sqrt{75}$ $5\sqrt{3}$

    vi. $\sqrt{98}$ $7\sqrt{2}$

Visualizing Square Roots (Learning Task)

Name_____   **Date**_____

Adapted from http://www.mathedpage.org/geometry-labs/gl/gl-9.pdf

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- To build the ideas of square and square root on their geometric interpretation.
- To justify simplification of radicals using geometric representations.

Essential Questions:
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- What is the relationship between the radicand and the area of a square?
- How do I justify simplification of radicals using geometric representations?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.N.RN.2 Rewrite expressions involving radicals ~~and rational exponents using the properties of exponents.~~ (i.e., simplify and/or use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots).

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3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure

1a. the above figure, what are the following measures?
   i.   The side of one of the small squares
   ii.   The area of one of the small squares
   iii.   The perimeter of one of the small squares
   iv.   The side of the large square
   v.   The area of the large square
   vi.   The perimeter of the large square

1b. Explain, using the answers to Problem 1a, why $\sqrt{40} = 2\sqrt{10}$.
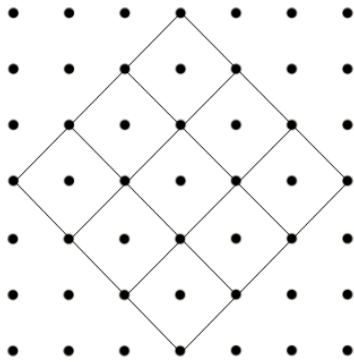
2a. In the above figure, what are the following measures?
   i.   The area of one of the small squares
   ii.   The side of one of the small squares
   iii.   The area of the large square
   iv.   The side of the large square

2b. Explain, using the answers to problem 2a, why $\sqrt{18} = 3\sqrt{2}$
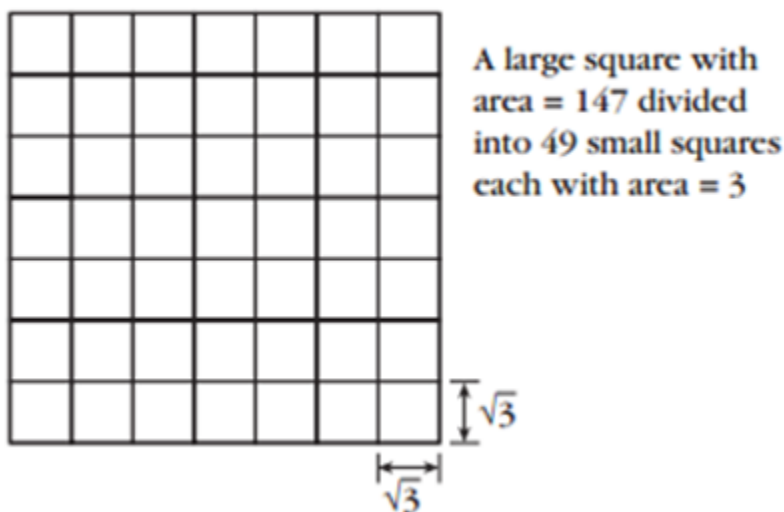
3. On dot paper, create a figure to show that $\sqrt{8} = 2\sqrt{2}, \ \sqrt{18} = 3\sqrt{2}, \sqrt{32} = 4\sqrt{2}, and \ \sqrt{50} = 5\sqrt{2}.$

4. On dot paper, create a figure to show that $\sqrt{20} = 2\sqrt{5}$ and $\sqrt{45} = 3\sqrt{5}$.

In the figure on the previous page, and in the figures you made in Problems 3 and 4, a larger square is divided up into *a square number of squares*. This is the basic idea for writing square roots in *simple radical form.* The figure need not be made on dot paper. For example, consider $\sqrt{147}$. Since $147 = 3 \cdot 49$, and since 49 is a square number, we can divide a square of area 147 units ^{2} into 49 squares, each of area 3 untis ^{2}:

A large square with area = 147 divided into 49 small squares each with area = 3

$\sqrt{3}$

$\sqrt{3}$

You will notice that the side of the larger square is $\sqrt{147} = 7\sqrt{3}$

5. Write the following in simple radical form.

   i. $\sqrt{12}$
   ii. $\sqrt{45}$
   iii. $\sqrt{24}$
   iv. $\sqrt{32}$
   v. $\sqrt{75}$
   vi. $\sqrt{98}$

Rational & Irrational Numbers – 1 (Formative Assessment Lesson)
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Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1245

Essential Questions
- How do you classify numbers as rational or irrational?
- How do you move between different representations of rational and irrational numbers?

Task Comments
Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Rational & Irrational Numbers – 1*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=424&subpage=concept
The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.
The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1245

GEORGIA STANDARDS OF EXCELLENCE

Extend the properties of exponents to rational exponents.
MGSE9–12.N.RN.2 Rewrite expressions involving radicals ~~and rational exponents using the properties of exponents.~~ (i.e., simplify and/or use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots).
Use properties of rational and irrational numbers.
MGSE9–12.N.RN.3 Explain why the sum or product of rational numbers is rational; why the sum of a rational number and an irrational number is irrational; and why the product of a nonzero rational number and an irrational number is irrational.
Interpret the structure of expressions
MGSE9–12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients, in context.

STANDARDS FOR MATHEMATICAL PRACTICE
3. Construct viable arguments and critique the reasoning of others by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Rational & Irrational Numbers - 2 (Formative Assessment Lesson)

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Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1267

Essential Questions
- How do you find irrational and rational numbers to exemplify general statements?
- How do you reason with properties of rational and irrational numbers?

Task Comments

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Rational & Irrational Numbers – 2*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=434&subpage=concept
The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.
The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1267

GEORGIA STANDARDS OF EXCELLENCE

Extend the properties of exponents to rational exponents.
MGSE9–12.N.RN.2 Rewrite expressions involving radicals ~~and rational exponents using the properties of exponents.~~ (i.e., simplify and/or use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots).
Use properties of rational and irrational numbers.
MGSE9–12.N.RN.3 Explain why the sum or product of rational numbers is rational; why the sum of a rational number and an irrational number is irrational; and why the product of a nonzero rational number and an irrational number is irrational.
Interpret the structure of expressions
MGSE9–12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients, in context.
MGSE9–12.A.SSE.1b Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.

STANDARDS FOR MATHEMATICAL PRACTICE
3. Construct viable arguments and critique the reasoning of others by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Amusement Park Problem (Culminating Task)   **Back to Task Table**

GEORGIA STANDARDS OF EXCELLENCE

Extend the properties of exponents to rational exponents.
MGSE9–12.N.RN.2 Rewrite expressions involving radicals ~~and rational exponents using the properties of exponents.~~ (i.e., simplify and/or use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots).

Use properties of rational and irrational numbers.
MGSE9–12.N.RN.3 Explain why the sum or product of rational numbers is rational; why the sum of a rational number and an irrational number is irrational; and why the product of a nonzero rational number and an irrational number is irrational.

Perform arithmetic operations on polynomials
MGSE9–12.A.APR.1 Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations.

Interpret the structure of expressions
MGSE9–12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients, in context.

MGSE9–12.A.SSE.1b Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.

Reason quantitatively and use units to solve problems.
MGSE9–12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:
   a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
   b. Convert units and rates using dimensional analysis (English–to–English and Metric–to–Metric without conversion factor provided and between English and Metric with conversion factor);
   c. Use units within multi–step problems and formulas; interpret units of input and resulting units of output.

MGSE9–12.N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. Given a situation, context, or problem, students will determine, identify, and use appropriate quantities for representing the situation.

MGSE9–12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers' precision is limited to the precision of the data given.*

STANDARDS FOR MATHEMATICAL PRACTICE

2. Reason abstractly and quantitatively by requiring students to make sense of quantities and their relationships to one another in problem situations.

3. Construct viable arguments and critique the reasoning of others by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

6. Attend to precision by requiring students to calculate efficiently and accurately; and to communicate precisely with others by using clear mathematical language to discuss their reasoning.

7. Look for and make use of structure by expecting students to apply rules, look for patterns and analyze structure.

8. Look for and express regularity in repeated reasoning by expecting students to understand broader applications and look for structure and general methods in similar situations.
The Radical World of Math is reviewing the master plan of a proposed amusement park coming to your area. Your help is needed with the land space and with park signage.

First, the planners need help in designing the land space. The parameters are as follows:
- 15 rows of parking are required
- The rows will be the same length as the park
- The park size will be square with a length of X so that expansion is possible.
- "Green Space" for planting, sitting, or picnicking is a must.
- Parking will be adjacent to only two sides of the park

Your task is to choose 3 possible configurations of land use with 15 rows of parking. Find the area of the picnic (green space) for each configuration. There is more than one way to solve the problem. For your maximum picnic space, write an equation for the total AREA of the park.

As long as all the parameters are met, student designs will be correct. Maximum spaces are the closest to square designs. $(x + 7)(x + 8)$

Extension:

The park is expected to be successful and the planners decide to expand the parking lot by adding 11 more rows. Assume the new plan will add not only 11 rows of parking but will also triple the maximum original green space (approximately). Choose 1 of your park configurations (your best) to complete this section and redraw your park configuration. What is the percentage increase in area that was created by expanding to 26 rows of parking?

Second, signs have to be designed for the park. For one of the areas called "Radical Happenings", the signs must show conversions between radical expressions and exponential expressions. There must be at least 10 signs in all that reflect square roots, cube roots, and fourth roots. Create 10 unique signs for use in the park. Would there be appropriate areas for these values to be placed?

The signs can have values 1 through 10 and be used to designate ten different park areas.

You must create problems demonstrating each concept and have another team try to "get through" your obstacles.

Extension: Using fractals, create advertising for this park. This link may help:
http://mathworld.wolfram.com/Fractal.html

Amusement Park Problem (Culminating Task)

Name_____   Date_____

GEORGIA STANDARDS OF EXCELLENCE

Extend the properties of exponents to rational exponents.
MGSE9–12.N.RN.2 Rewrite expressions involving radicals ~~and rational exponents using the properties of exponents.~~ (i.e., simplify and/or use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots).

Use properties of rational and irrational numbers.
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MGSE9–12.A.APR.1 Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations.

Interpret the structure of expressions
MGSE9–12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients, in context.

MGSE9–12.A.SSE.1b Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.

Reason quantitatively and use units to solve problems.
MGSE9–12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:
   a.   Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
   b.   Convert units and rates using dimensional analysis (English–to–English and Metric–to–Metric without conversion factor provided and between English and Metric with conversion factor);
   c.   Use units within multi–step problems and formulas; interpret units of input and resulting units of output.

MGSE9–12.N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. Given a situation, context, or problem, students will determine, identify, and use appropriate quantities for representing the situation.

MGSE9–12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers' precision is limited to the precision of the data given.*

STANDARDS FOR MATHEMATICAL PRACTICE

2. Reason abstractly and quantitatively by requiring students to make sense of quantities and their relationships to one another in problem situations.

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7. Look for and make use of structure by expecting students to apply rules, look for patterns and analyze structure.

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Extension:
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You must create problems demonstrating each concept and have another team try to "get through"

your obstacles.

Extension: Using fractals, create advertising for this park. This link may help:
http://mathworld.wolfram.com/Fractal.html

ADDITIONAL TASKS

The Physics Professor (Illustrative Mathematics Task)   **Back to Task Table**

The task can be found at: https://tasks.illustrativemathematics.org/content-standards/HSA/SSE/A/1/tasks/23

GEORGIA STANDARDS OF EXCELLENCE

Interpret the structure of expressions
MGSE9–12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

> **MGSE9–12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients, in context.

The purpose of this task is to provide students practice in drawing conclusions about expressions they might encounter in classes outside mathematics, by parsing them in terms of their algebraic structure. Teachers might stress the subtle difference between "seeing" why the expression must be zero and the more mechanical process of algebraically simplifying the expression upon substituting $v=c$. Although part (b) might initially be thought of as an optimization problem in a calculus course, some elementary reasoning with the structure of the expression gives the same answer in a more fluid and conceptual fashion.

Delivery Trucks (Illustrative Mathematics Task)   **Back to Task Table**

The task can be found at: https://tasks.illustrativemathematics.org/content-standards/HSA/SSE/A/1/tasks/531

GEORGIA STANDARDS OF EXCELLENCE
Interpret the structure of expressions
MGSE9–12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

> **MGSE9–12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients, in context.

The primary purpose of this task is to assess students' knowledge of certain aspects of the mathematics described in the A.SSE.1a. The task has students look for structure in algebraic expressions related to a context, and asks them to relate that structure to the context.

Kitchen Floor Tiles (Illustrative Mathematics)   **Back to Task Table**

The task can be found at: https://tasks.illustrativemathematics.org/content-standards/HSA/SSE/A/1/tasks/215

GEORGIA STANDARDS OF EXCELLENCE
Interpret the structure of expressions
MGSE9–12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

> **MGSE9–12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients, in context.

The purpose of this task is to give students practice in reading, analyzing, and constructing algebraic expressions, attending to the relationship between the form of an expression and the context from which it arises. The context here is intentionally thin; the point is not to provide a practical application to kitchen floors, but to give a framework that imbues the expressions with an external meaning.

Analyzing and generalizing geometric patterns such as the one in this task may be familiar to students from work in previous grades, so part **a** may be a review of that process. It requires students to make use of the structure in the expression (MP7), to notice and express the regularity in the repeated geometric construction (MP8), and to explain and justify the reasoning of others (MP3). Part **b** requires a deeper analysis of the expression, identifying the referents for its various parts (MP7). Students may still need guidance in writing the formula for part **c** since it introduces a second variable.