Georgia
Standards of Excellence
Curriculum Frameworks

Mathematics

GSE Grade 7

Unit 1: Operations with Rational Numbers

Georgia Department of Education

Richard Woods, Georgia's School Superintendent

"Educating Georgia's Future"

Unit 1
Operations with Rational Numbers

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OVERVIEW

The units in this instructional framework emphasize key standards that assist students to develop a deeper understanding of numbers. They learn to express different representations of rational numbers (e.g., fractions, decimals, and percents) and interpret negative numbers in everyday context (e.g., sea level change). The big ideas that are expressed in this unit are integrated with such previous knowledge as estimation, mental and basic computation. All of these concepts need to be reviewed throughout the year.

Take what you need from the tasks and modify as required. These tasks are suggestions, something that you can use as a resource for your classroom.

STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.
Students explain and demonstrate rational number operations by using symbols, visuals, words, and real life contexts. Students demonstrate perseverance while using a variety of strategies (number lines, manipulatives, drawings, etc.

2. Reason abstractly and quantitatively.
Students demonstrate quantitative reasoning by representing and solving real world situations using visuals, numbers, and symbols. They demonstrate abstract reasoning by translating numerical sentences into real world situations.

3. Construct viable arguments and critique the reasoning of others.
Students will discuss rules for operations with rational numbers using appropriate terminology and tools/visuals. Students apply properties to support their arguments and constructively critique the reasoning of others while supporting their own position.

4. Model with mathematics. Students model understanding of rational number operations using tools such as algebra tiles, counters, visuals, and number lines and connect these models to solve problems involving real-world situations.

5. Use appropriate tools strategically. Students demonstrate their ability to select and use the most appropriate tool (paper/pencil, manipulatives, and calculators) while solving problems with rational numbers.

6. Attend to precision. Students demonstrate precision by using correct terminology and symbols and labeling units correctly. Students use precision in calculation by checking the reasonableness of their answers and making adjustments accordingly.

7. Look for and make use of structure. Students look for structure in positive and negative rational numbers when they place them appropriately on the number line. They use this structure in calculation when considering the position of numbers on the number line. In addition, students recognize the problem solving structures of word problems and use this awareness to aid in solving them.

8. Look for and express regularity in repeated reasoning. Students will use manipulatives to explore the patterns of operations with rational numbers. Students will use these patterns to develop algorithms. They can use these algorithms to solve problems with a variety of problem solving structures.

(Adapted from Illinois' Curriculum Model)

STANDARDS FOR MATHEMATICAL CONTENT

Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers.

MGSE7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

MGSE7.NS.1a Show that a number and its opposite have a sum of 0 (are additive inverses). Describe situations in which opposite quantities combine to make 0. *For example, your bank account balance is -$25.00. You deposit $25.00 into your account. The net balance is $0.00.*

MGSE7.NS.1b Understand p + q as the number located a distance $|q||q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Interpret sums of rational numbers by describing real world contexts.

MGSE7.NS.1c Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (– q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

MGSE7.NS.1d Apply properties of operations as strategies to add and subtract rational numbers.

MGSE7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

MGSE7.NS.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (- 1)(– 1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

MGSE7.NS.2b Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers then $–(p/q) = (– p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real-world contexts.

MGSE7.NS.2c Apply properties of operations as strategies to multiply and divide rational numbers.

MGSE7.NS.2d Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

MGSE7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

BIG IDEAS

- Computation with positive and negative numbers is often necessary to determine relationships between quantities.
- Models, diagrams, manipulatives and patterns are useful in developing and remembering algorithms for computing with positive and negative numbers.
- Properties of real numbers hold for all rational numbers.
- Positive and negative numbers are often used to solve problems in everyday life.

ESSENTIAL QUESTIONS

- What strategies are most useful in helping develop algorithms for adding, subtracting, multiplying, and dividing positive and negative rational numbers?
- What are the steps to converting a rational number to a repeating or terminating decimal?

CONCEPTS AND SKILLS TO MAINTAIN

- Use of number lines to order whole number integers.
- Addition, subtraction, division and multiplication of whole numbers.

Addition, subtraction, division and multiplication of fractions.

FLUENCY

It is expected that students will continue to develop and practice strategies to build their capacity to become fluent in mathematics and mathematics computation. The eventual goal is automaticity with math facts. This automaticity is built within each student through strategy development and practice. The following section is presented in order to develop a common understanding of the ideas and terminology regarding fluency and automaticity in mathematics:

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply "how to get the answer" and instead support students' ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students:

- flexibly use a combination of deep understanding, number sense, and memorization.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.

For more about fluency, see: http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and: http://joboaler.com/timed-tests-and-the-development-of-math-anxiety/

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for middle school students. **Note – Different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.** The definitions below are from the CCSS glossary http://www.corestandards.org/Math/Content/mathematics-glossary/glossary, when applicable.

Visit http://intermath.coe.uga.edu or http://mathworld.wolfram.com to see additional definitions and specific examples of many terms and symbols used in grade 7 mathematics.

- **Additive Inverse**
- **Multiplicative Inverse**

- **Absolute Value**
- **Integers**
- **Long Division**
- **Natural Numbers**
- **Negative Numbers**
- **Opposite Numbers**
- **Positive Numbers**
- **Rational Numbers**
- **Repeating Decimal**
- **Terminating Decimal**
- **Zero Pair**

STRATEGIES FOR TEACHING AND LEARNING

This unit builds upon the understanding of rational numbers in Grade 6:
- quantities can be shown using + or – and having opposite directions or values
- points on a number line show distance and direction,
- opposite signs of numbers indicate locations on opposite sides of 0 on the number line,
- the opposite of an opposite is the number itself
- the absolute value of a rational number is its distance from 0 on the number line,
- the absolute value is the magnitude for a positive or negative quantity, and
- coordinates can be located and compared on a coordinate grid using negative and positive rational numbers
- order of operations should be applied to operations with rational numbers.

In Grade 7, learning now moves to exploring and ultimately formalizing rules for operations (addition, subtraction, multiplication and division) with integers.

Using both contextual and numerical problems, students should explore what happens when negative numbers and positive numbers are combined. The following strategies can be applied for students to explore combining negative and positive rational numbers:

- **Number lines present a visual image for students to explore and record addition an**d su**btraction results.**
- **Two-color counters or colored chips can be used as a physical and kinesthetic model for adding and subtracting integers. With one color designated to represent positive numbers and a second color for negative numbers, addition/subtraction can be represented by placing the appropriate numbers of chips for the addends and their signs on a board. Using the notion of opposites, the board is simplified by removing pairs of opposite colored chips. The answer is the total of the remaining chips with the sign representing the appropriate color.**
- Repeated opportunities over time will allow students to compare the results of adding and subtracting pairs of numbers, leading to the generalization of the rules. Fractional rational numbers and whole numbers should be used in computations and explorations. Students should be able to give contextual examples of integer operations, write and solve equations for real-world problems and explain how the properties of operations

apply. Real-world situations could include: profit/loss, money, weight, sea level, debit/credit, football yardage, etc.

- Using what students already know about positive and negative whole numbers and multiplication with its relationship to division, students should generalize rules for multiplying and dividing rational numbers. Multiply or divide the same as for positive numbers, the designate the sign according to the number of negative factors. Students should analyze and solve problems leading to the generalization of the rules for operations with integers.
  For example, beginning with known facts, students predict the answers for related facts, keeping in mind that the properties of operations apply (See Tables 1, 2, and 3 below).

Table 1	**Table 2**	**Table 3**
4 x 4 =16	4 x 4 = 16	-4 x -4 = 16
4 x 3 = 12	4 x 3 = 12	-4 x -3 = 12
4 x 2 = 8	4 x 2 = 8	-4 x -2 = 8
4 x 1 = 4	4 x 1 = 4	-4 x -1 = 4
4 x 0 = 0	4 x 0 = 0	-4 x 0 = 0
4 x -1 =	-4 x 1 =	-1 x -4 =
4 x -2 =	-4 x 2 =	-2 x -4 =
4 x -3 =	-4 x 3 =	-3 x -4 =
4 x -4 =	-4 x 4 =	-4 x -4 =

- Using the language of "the opposite of" helps some students understand the multiplication of negatively signed numbers (- 4 x - 4 = 16, the opposite of 4 groups of -4). Discussion about the tables should address the patterns in the products, the role of the signs in the products and commutativity of multiplication.

Then students should be asked to answer these questions and prove their responses.

- Is it always true that multiplying a negative factor by a positive factor results in a negative product?
- Does a positive factor times a positive factor always result in a positive product?
- What is the sign of the product of two negative factors?
- When three factors are multiplied, how is the sign of the product determined?
- How is the numerical value of the product of any two numbers found?

Students can use number lines with arrows and hops, groups of colored chips or logic to explain their reasoning. When using number lines, establishing which factor will represent length, number and direction of the hops will facilitate understanding. Through discussion, generalization of the rules for multiplying integers would be the result.
Division of integers is best understood by relating division to multiplication and applying the rules. In time, students will transfer the rules to division situations.
Ultimately, students should solve other mathematical and real-world problems requiring the application of these rules with fractions and decimal.

In Grade 7, the awareness of rational and irrational numbers is initiated by observing the result of changing fractions to decimals. Students should be provided with families of fractions, such as sevenths, ninths, thirds, etc. to convert to decimals using long division. The equivalents can be grouped and named (terminating or repeating). Students should begin to see why these patterns occur. Knowing the formal vocabulary *rational* and *irrational* is not expected.

FORMATIVE ASSESSMENT LESSONS (FAL)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Guide.

SPOTLIGHT TASKS

A Spotlight Task has been added to each MGSE mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Mathematics Georgia Standards of Excellence, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky.

3-ACT TASKS

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

Task Name	Task Type *Grouping Strategy*	Content Addressed	Standards Addressed
Show Me the Sign Part 1	Learning Task *Individual/ Partner*	Using models to add or subtract integers	MGSE7.NS.1 a-d
Show Me the Sign Part 2	Learning Task *Individual/ Partner*	Using models to add or subtract integers	MGSE7.NS.1 a-d
Subtracting Integers	Learning Task *Partner/Small Group*	Connect the idea of removing quantities to subtracting integers	MGSE7.NS.1 MGSE7.NS.1D MGSE7.NS.3
Deep Freeze	Spotlight Task *Individual/ Partner*	Using models to add or subtract integers	MGSE7.NS.3
Hot Air Balloon	Learning Task *Individual / Partner*	Using a number line to add and subtract integers	MGSE7.NS.1 a-d MGSE7.NS.3
Debits and Credits	Performance Task *Individual / Partner*	Sums to zero; operations with integers (sums and differences); debits and credits	MGSE7.NS.1 a-d MGSE7.NS.3
Integer Subitizing Cards	Practice Task *Partner/Small Group*	Identify zero pairs within quick images to determine the sum of integers	MGSE7.NS.1 a-b
Create Three	Practice Task *Partner/Small Group*	Adding and subtracting rational numbers to create 3 or -3	MGSE7.NS.1d
Using Positive and Negative Numbers in Context (FAL)	Formative Assessment Lesson *Partner/Small Group*	Extend understanding of adding and subtracting to include rational numbers	MGSE7.NS.1 MGSE7.NS.3
Multiplying Integers	Learning Task *Individual / Partner*	Multiplying integers using a number line and counters; properties of numbers	MGSE7.NS.2 a-c
Modeling the Multiplication of Integers	Learning Task *Individual / Partner*	Multiplying integers using a number line and counters; properties of numbers	MGSE7.NS.2 a-c

Patterns of Multiplication and Division	Learning Task *Individual / Partner*	Multiplying and dividing rational numbers; properties of numbers	MGSE7.NS.2 a-c
Do It Yourself Revisited	Practice Task *Individual / Partner*	Connecting division of integers to dividing rational numbers.	MGSE7.NS.2 a-c
What Does It Cost?	Performance Task *Individual/ Partner*	Finding part of a number, multiplying fractions and integers	MGSE7.NS.2d MGSE7.NS.3
Converting Fractions to Decimals	Learning Task *Partner/Small Group*	Changing fractions decimals using long division; looking at why	MGSE7.NS.2d
The Repeater vs. The Terminator	Learning Task *Individual / Partner*	Changing fractions decimals using long division; looking for patterns between terminating and repeating decimals	MGSE7.NS.2d
A Poster	Culminating Task *Individual / Partner*	Ties together comparing integers, absolute value, number problems; real world situations	MGSE7.NS.1 a-d MGSE7.NS.2 a-d
Whodunit? The Undoing of (-7)	Culminating Task *Individual / Partner*	Clues are given related to a number line model to find the guilty "number".	MGSE7.NS.1 a-d, MGSE7.NS.2

Show Me the Sign Part 1

Back to task table

In this task, students will find sums and differences of integers using tools (bead strings and modified rekenrek). Students will model and explain in words how they arrived at the sum or difference.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

MGSE7.NS.1a Show that a number and its opposite have a sum of 0 (are additive inverses). Describe situations in which opposite quantities combine to make 0. *For example, your bank account balance is -$25.00. You deposit $25.00 into your account. The net balance is $0.00.*

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MGSE7.NS.1c Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (– q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

STANDARDS FOR MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

COMMON MISCONCEPTIONS

- *Visual representations may be helpful as students begin this work. If they do not have a visual to illustrate what is happening when they are adding and subtracting integers, they will get lost in the symbols and will not know how to combine the absolute value of the integers.*
- *Ask students to create their own stories for integer operations and to answer the following three prompts:*
    - *Where did you start?*
    - *How far did you go?*
    - *Where are you now?*

- *Students want to subtract by just taking the counter off instead of bringing in a zero pair.*
- *Students do not always understand the value of a zero pair and how the value stays the same no matter how many zero pairs you bring.*

ESSENTIAL QUESTIONS

- What strategies are most useful in helping develop algorithms for adding and subtracting positive and negative rational numbers?

MATERIALS

- Bead strings
- Rekenrek
- Show Me Your Sign recording sheet

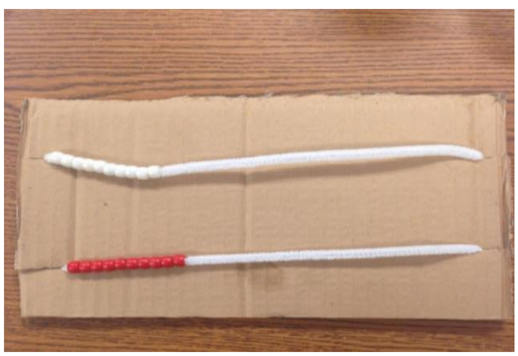
GROUPING

Individual or Partner

TASK DESCRIPTION

To combat the misconceptions students have when computing with integers, there are several tools that will aid in developing an understanding of operations with integers. Beginning with a bead string number line will help reinforce the quantity of the integers. The bead string can be created using pony beads and a pipe cleaner or string. Use a black bead to represent zero, white beads for each positive number and red beads for each negative number.

A modified rekenrek can help align zero pairs when adding and subtracting integers. A traditional rekenrek has two rows of 10 beads. Each row has five red beads and five white beads. They're useful when students are developing the ideas of unitizing, quantity of numbers and number strategies for addition and subtraction of whole numbers. The modified rekenrek for use with integers has two rows of ten beads. One row has ten white beads and the second row has ten red beads. It can be constructed using red and white pony beads, two pipe cleaners and cardboard or tag board.



It may prove useful to allow each student to construct a bead string and modified rekenrek, as through construction they will discover every positive integer has a matching negative integer. **Note: To do this with multiple classes, allow time at the end of this initial lesson for students to deconstruct the tools.**

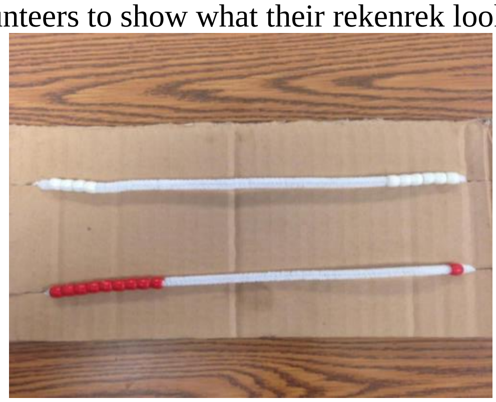
Activator:
Model for students how to use the bead string and modified rekenrek to add and subtract positive and negative integers.

Ensure each student has access to a bead string. Students may work with a partner and share a bead string. Using the bead string, instruct students to add 3 + 2 by moving their finger or clothes pin 3 beads from zero then 2 more beads. Ask several students at which number did they land. Record student responses on the board (*the correct response is 5*). Ask which integers did they add and record the information on the board (*3 and 2*).

Now instruct students to move 6 away from zero then back one. Circulate around the room to monitor students' movement along the bead string. Ask several students at which number did they land and record responses on the board. Ask students what integers did they add. Possible responses may be "6 and -1" or "We did not add, we subtracted". Use this as an opportunity to reinforce the idea that adding 6 + (-1) is the same as 6 -1. This can be done by discussing the movement on the bead string. When adding a negative integer you move in the direction of the negative integers, to the left. When subtracting positive integers you move in the direction of zero, to the left. *This is an idea you want students to derive through the use of the bead string.

Distribute the modified rekenrek to each student or pair of students. Rekenrek should be held in the starting position by moving all beads to the left. Instruct students to compute 3 + 2 by moving 3 white beads to the far right side of the rekenrek followed by 2 more white beads. Or students may move 5 white beads based on the understanding of 3 + 2 =5. Ask several students what quantity is represented on the right and record responses on the board. Clear the rack by returning beads to the starting position.

Instruct students to show 6 + (-1) on the rekenrek. Circulate to observe how students are moving the beads. Have student volunteers to show what their rekenrek looks like:



Discuss what students know about the matching white and red bead. Students should conclude it represents a zero pair. Ask several students what quantity is represented (*5*).

After the teacher teaches a mini-lesson about how to use models to show addition and subtraction of positive and negative integers, the students will complete the following learning task.

Use the bead string and rekenrek to complete the following.

1. Using positive integers, find all the combinations with a sum of 5. Explain what method you used to find the pairs.

Solution: Students should have an understanding of the commutative property of addition.

Integers	Sum	Equation	Method
0 and +5	*5*	*0 + 5 = 5*	***Explanation of how they used the tool to determine the answer.***
+5 and 0	*5*	*5 + 0 = 5*	***Explanation of how they used the tool to determine the answer.***
+3 and +2	*5*	*3 + 2 = 5*	***Explanation of how they used the tool to determine the answer.***
+2 and +3	*5*	*2 + 3 = 5*	***Explanation of how they used the tool to determine the answer.***
+4 and +1	*5*	*4 + 1 = 5*	***Explanation of how they used the tool to determine the answer.***
+1 and +4	*5*	*1 + 4 = 5*	***Explanation of how they used the tool to determine the answer.***

2. What conclusions can you make about the positive integers?
Solution: When positive numbers are added together it gives you a positive solution.

3. Using positive and negative integers, find four pairs of integers with a sum of 5. Explain what method you used to find the pairs.

Solution: There are several pairs of integers possible for this solution. The solutions below are examples of what students may model as the answer.

Integers	Sum	Equation	Method
7 and -2	*5*	*7 + (-2) = 5*	***Explanation of how they used the tool to determine the answer.***
-5 and +10	*5*	*-5 + 10 = 5*	***Explanation of how they used the tool to determine the answer.***
+6 and -1	*5*	*6 + (-1) = 5*	***Explanation of how they used the tool to determine the answer.***
+1and -6	*5*	*1 + (-6) = 5*	***Explanation of how they used the tool to determine the answer.***

4. What conclusions can you make about the positive and negative integers? Is there a pattern?
Solution: ***The absolute value for both integers needs to have a difference of five. The positive number has the bigger absolute.***

5. Find a pair of negative integers with a sum of 5. What do you notice about the result? Explain your findings.
Solution:
The sum of two negative integers will not result in a positive five or any positive number.

6. Find five pairs of integers with a sum of -6. What do you notice about each pairs of integers?
Solution: ***Answer may vary for five pairs. The difference between the absolute value of the two numbers should be 6. The negative number will have the bigger absolute value.***

7. What generalizations can you make about adding integers?
Summarizer:
For the following examples, write an equation (show numerically) and draw a model using a bead string or rekenrek to help explain your answer.
Comment:
The use of multiple representations will help students demonstrate their understanding of adding integers. The connection should be made between all illustrated representations.

a) Explain $a + b$ if both a and b are positive numbers.
Solution:

Equation ***a + b = positive number.***	***Students should see that the sum of two positive numbers can only produce a positive result.***

b) Explain (a) + (b) if a and b both represent negative numbers.
Solution:

Equation ***(a) + (b) = negative number***	***Students should see that the sum of two negative numbers can only produce a negative result.***

c) Explain a + (b) if a represents any positive number and (b) represents any negative number.
Comment: ***Students can use various numbers to illustrate the equation and explain reasoning.***

Solution:

Equation ***(a) + (b) has a positive result if the distance from 0 to a is greater than the distance from (b) to 0; |a| is greater than |b|.***
(a) + (b) has a negative result if the distance from 0 to a is less than the distance from (b) to 0; |a| is less than |b|.

d) Explain $b + (a)$ if b represents any positive number and (a) represents any negative number.
Solution:

Equation   ***b + (a) has a positive result if the distance from 0 to b is greater than the distance from (a) to 0; |b| is greater than |a|.***   ***b + (a) has a negative result if the distance from 0 to b is less than the distance from (a) to 0; |b| is less than |a|.***

e) Explain $a + b + c$ if a and b represent positive numbers and (c) represents a negative number.
Solution:
If the sum of a and b is a positive number, then the sum of (a + b) and (c) is positive if |a + b| is greater than |c|. (This should look similar to the illustration in problem #13.)
If the sum of a and b is a positive number, then the sum of (a + b) and (c) is negative if |a + b| is less than |c|. (See illustration below.)

DIFFERENTIATION

Extension

- The generalization problems may be more appropriate in an advanced class.
- Have students create expressions with more than two integers.

Intervention

- This lesson could be used as a remediation activity for those students who are struggling understanding addition and subtraction rules.

Show Me the Sign Part 1

Use the bead string and rekenrek to complete the following.

1. Using positive integers, find all the combinations with a sum of 5. Explain what method you used to find the pairs.

Integers	Sum	Equation	Method

2. What conclusions can you make about the positive integers?

3. Using positive and negative integers, find four pairs of integers with a sum of 5. Explain what method you used to find the pairs.

Integers	Sum	Equation	Method

4. What conclusions can you make about the positive and negative integers? Is there a pattern?

5. Find a pair of negative integers with a sum of 5. What do you notice about the result? Explain your findings.

6. Find five pairs of integers with a sum of -6. What do you notice about each pairs of integers?

7. What generalizations can you make about adding integers?
   For the following examples, write an equation (show numerically) <u>and</u> draw a model using a bead string or rekenrek to help explain your answer.

   a) Explain $a + b$ if both a and b are positive numbers.

   b) Explain $(a) + (b)$ if a and b both represent negative numbers.

   c) Explain $a + (b)$ if a represents any positive number and (b) represents any negative number.

   d) Explain $b + (a)$ if b represents any positive number and (a) represents any negative number.

   e) Explain $a + b + c$ if a and b represent positive numbers and (c) represents a negative number.

Show Me the Sign Part 2

Back to task table

In this task, students will find sums and differences of integers using models (two color counters or number lines). Students will model and explain in words how they arrived at the sum or difference.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

MGSE7.NS.1a Show that a number and its opposite have a sum of 0 (are additive inverses). Describe situations in which opposite quantities combine to make 0. *For example, your bank account balance is -$25.00. You deposit $25.00 into your account. The net balance is $0.00.*

MGSE7.NS.1b Understand p + q as the number located a distance $|q||q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Interpret sums of rational numbers by describing real world contexts.

MGSE7.NS.1c Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (– q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

COMMON MISCONCEPTIONS

- *Visual representations may be helpful as students begin this work. If they do not have a visual to illustrate what is happening when they are adding and subtracting integers, they will get lost in the symbols and will not know how to combine the absolute value of the integers.*
- *Ask students to create their own stories for integer operations and to answer the following three prompts:*
    - *Where did you start?*
    - *How far did you go?*
    - *Where are you now?*

Students want to subtract by just taking the counter off instead of bringing in a zero pair. Students do not always understand the value of a zero pair and how the value stays the same no matter how many zero pairs you bring.

ESSENTIAL QUESTIONS

- What strategies are most useful in helping develop algorithms for adding and subtracting positive and negative rational numbers?

MATERIALS

- colored pencils (red and yellow)
- two – color counters

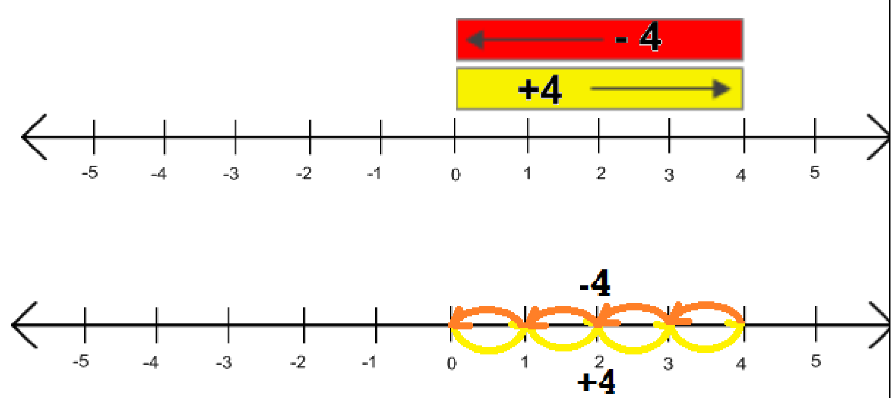
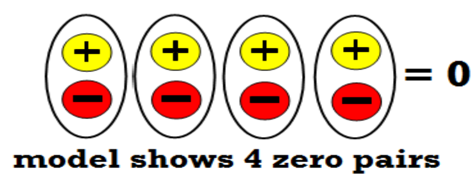
GROUPING

Individual or Partner

TASK DESCRIPTION

Number lines and counters are useful in demonstrating understanding of operations with integers.

Example: 4 + (-4)

<table>
<tr><td>Number line model</td><td>
</td></tr>
<tr><td>Two-color counters model</td><td>
</td></tr>
</table>

After the teacher teaches a mini-lesson about how to use models to show addition and subtraction of positive and negative integers, the students will complete the following learning task.

1. At sunrise, the outside temperature was 1° below zero. By lunch time, the temperature rose by 17° and then fell by 4° by night. What was the temperature at the end of the day?
   Solution: (-1) + 17 – 4=
   16 – 4= 12°
   Student explanations may vary.

2. Sheldon was doing a science experiment about temperature. He first measured the temperature of some water and found it was 17°C. Then he put the water in the freezer and recorded the temperature two hours later. It had fallen to -11°C. What was the change in temperature in two hours?
   Solution: 17° -(- 11°)= 28°
   Student explanations may vary, but should include that there was a drop in temperature from $17\circ$ *to* $-11\circ$ *of* $28\circ$.
3. During an electronics experiment in your laboratory, you measure the voltage at terminal A on your newly designed circuit. You measure -12 volts. You check the same terminal after making a small change to the circuit and this time you measure -18 volts. What was the voltage difference between the two readings?

   Solution: (-12) – (-18)=
   (-12) + 18= 6
   Student explanations may vary.

4. What is the change in temperature a customer in a grocery store experiences when they walk from the chilled vegetable section at 4 ºC to the frozen fish section which is set to −18 ºC?
   Solution: 4 - (-18) or 4 + 18
   22ºC
   Student explanations may vary, but should include that there was a drop in temperature from $4\circ$ *to* $-18\circ$ *of* $22\circ$.

<u>DIFFERENTIATION</u>

Extension

- Students can create their own story problems involving the addition and subtraction of integers. Students can switch problems and solve using the number line strategy or counters.
- Change the numbers so students work with larger numbers within the problems.

Intervention

- Change the numbers so students first begin working with smaller numbers before attempting these problems.

Show Me the Sign Part 2

Use a number line or counters to help you model with mathematics. Explain your answer using words, numbers and/or pictures.

1. At sunrise, the outside temperature was 1° below zero. By lunch time, the temperature rose by 17° and then fell by 4° by night. What was the temperature at the end of the day?

2. Sheldon was doing a science experiment about temperature. He first measured the temperature of some water and found it was 17°C. Then he put the water in the freezer and recorded the temperature two hours later. It had fallen to -11°C. What was the change in temperature in two hours?

3. During an electronics experiment in your laboratory, you measure the voltage at terminal A on your newly designed circuit. You measure -12 volts. You check the same terminal after making a small change to the circuit and this time you measure -18 volts. What was the voltage difference between the two readings?

4. What is the change in temperature a customer in a grocery store experiences when they walk from the chilled vegetable section at 4 ºC to the frozen fish section which is set to −18 ºC?

Subtracting Integers

Back to task table

This task will help students connect the idea of removing quantities to subtracting integers.

STANDARDS FOR MATHEMATICAL CONTENT

MCC7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

MCC7.NS.1c Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

MCC7.NS.1d Apply properties of operations as strategies to add and subtract rational numbers.

STANDARDS FOR MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

COMMON MISCONCEPTIONS

- *Visual representations may be helpful as students begin this work. If they do not have a visual to illustrate what is happening when they are adding and subtracting integers, they will get lost in the symbols and will not know how to combine the absolute value of the integers.*
- *Ask students to create their own stories for integer operations and to answer the following three prompts:*
  - *Where did you start?*
  - *How far did you go?*
  - *Where are you now?*

Students want to subtract by just taking the counter off instead of bringing in a zero pair.

Students do not always understand the value of a zero pair and how the value stays the same no matter how many zero pairs you bring.

BACKGROUND KNOWLEDGE

Often times, teachers use tricks to teach subtracting integers. These tricks undermine the concept and prevent students from connecting the concept of subtracting positive whole numbers to subtracting integers. Please refrain from teaching tricks in regards to the operations and signs. Allow students to derive what is happening through the exploration of this task.

The modified rekenerek has two rows, 10 white beads and 10 red beads. The white beads represent positive quantities while the reds represent negative quantities.

ESSENTIAL QUESTIONS

- What strategies are most useful in helping develop algorithms for adding and subtracting positive and negative rational numbers?

MATERIALS

- counters
- rekenrek

GROUPING

Individual or Partner

TASK DESCRIPTION

Mini-lesson: Using the rekenrek, ask students to model 6. Write 6-3 on the board. Ask students what action they will need to do in order to model the expression written. Students should conclude they will need to take away, remove, subtract, deduct, etc 3 white beads. Record the verbs used on the board. Students should model the expression using the rekenrek. Enccourgae students to record in their math notebook/journal an explanation of 6-3 and the equation. ***Possible explanation: we removed 3 positives from the 6 positives which left 3 positives.***

Instruct students to clear their racks then show -6 or 6 red beads. Record -6-(-3) on the board. Have students refer back to the verbs from the previous problem. Ask if the verbs still apply. Students should conclude the verbs still apply because the operation is still subtraction. students should model the expression by removing 3 red beads leaving 3 red beads. Encourage students to record an explanation of -6-(-3) along with the equation.

Students should clear their racks, then show positive 6. Record 6-(-3) on the board and ask students if the same verbs apply. Instruct students to discuss with a neighbor how to model the expression. Circulate and listen to the ideas shared. Students should conclude there aren't any negatives to remove. Ask students if the use of zero pairs would create negatives to remove. Ask students to determine how many zero pairs are needed ***(3).*** Students should model the zero pairs and check to ensure the quantity shown is still 6 ***(9 white beads and 3 red beads)***. Students should continue to model the expression removing 3 red beads leaving 9 white beads. Encourage students to record then share an explanation of 6-(-3) along with the equation.

Repeat previous step using -6-3 and -3 – (-6).

<u>TASK DIRECTIONS</u>:
Use your rekenrek to model the following problems. Determine if there are any patterns.

1. 4-5 2. 7-9 3. 2-6

What did you notice?

Possible solution: When subtracting a greater positive from a lesser positive, the difference is negative.

4. -6 - 4 5. -3 – 2 6. -5 – 1

What did you notice?

Possible solution: When subtracting a lesser positive from a greater negative, you must create positives using zero pairs. The difference will be negative.

7. 3 – (-4) 8. 4 – (-6) 9. 2 – (-8)

What do you notice?

Possible solution: When subtracting a greater negative from a lesser positive, you must create negatives using zero pairs. The difference is positive. It is just like adding 2 positives.

10. -3 – (-7) 11. – 1 – (-4) 12. – 4 – (-4)

What do you notice?

Possible solution: when subtracting a greater negative from a lesser negative, the answer is positive. It gives the same results as adding the negative to a positive.

13. -7 – (-3) 14. -4 – (-1) 15. – 5 – (-1)

What did you notice?

Possible solution: when subtracting a lesser negative from a greater negative, the difference is negative. You get the same result as adding the greater negative to a positive (opposite of the lesser negative).

<u>DIFFERENTIATION</u>

Extension

Have students apply the concept of subtracting integers using a number line. Students should view subtracting positives as removing positive spaces and subtracting negatives as removing negative spaces.

Intervention

Some students may not discover the patterns or rules with one experience. Providing multiple opportunities to construct their thinking using two color counters or algebra tiles as well as discussions with peers will aid in developing their understanding.

Subtracting Integers

Use your rekenrek to model the following problems. Determine if there are any patterns.

1. 4-5 2. 7-9 3. 2-6

What did you notice?

4. -6 - 4 5. -3 – 2 6. -5 – 1

What did you notice?

7. 3 – (-4) 8. 4 – (-6) 9. 2 – (-8)

What do you notice?

10. -3 – (-7) 11. – 1 – (-4) 12. – 4 – (-4)

What do you notice?

13. -7 – (-3) 14. -4 – (-1) 15. – 5 – (-1)

What did you notice?

Deep Freeze

Back to task table

Task adapted from www.gfletchy.wordpress.com

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTIONS

- What strategies are most useful in helping develop algorithms for adding and subtracting positive and negative rational numbers?

MATERIALS REQUIRED

- Videos for Deep Freeze – 3-Act task
- Recording sheet (attached)

TIME NEEDED

- 1 day

TEACHER NOTES

In this task, students will watch the video, then tell what they noticed. Ask students to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them

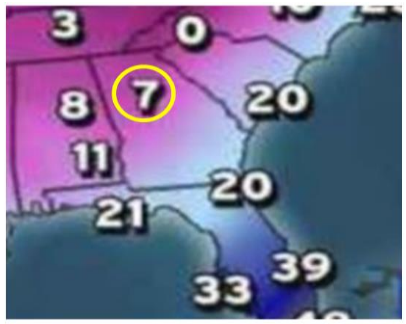
Task Description
The following 3-Act Task can be found at: http://vimeo.com/94462885
More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:
Watch the video:

- What's the temperature in Duluth, Minnesota? Estimate
- Write an estimate you know is too high. Write an estimate you know is too low.

ACT 2:

- It is 7 degrees in Atlanta, Georgia
- There is a 31 degree difference in the temperature between Atlanta and Duluth Minnesota.



There's a 31 degree difference between Atlanta, Georgia and Duluth, Minnesota

ACT 3
Students will compare and share solution strategies.

- Reveal the answer. Discuss the theoretical math versus the practical outcome.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.

ACT 4

- Have students identify the temperature difference between 5 different cities

Task Title: ________________________ Name:________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate the result of the main question? Explain?

Place an estimate that is too high and too low on the number line

Low estimate *Place an "x" where your estimate belongs* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:_______________________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Hot Air Balloon

Back to task table

Used with permission by Susan Mercer, www.supermathunits.com
In this task, students will use a concrete model to help them understand how to add and subtract integers. The vertical position of the hot air balloon is determined by adding or removing a number of helium bags (or gas bags), which represent positive integers, and sand bags, which represent negative integers. This activity provides a dynamic hand-made manipulative designed to help students understand addition and subtraction of positive and negative numbers.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

MGSE7.NS.1a Show that a number and its opposite have a sum of 0 (are additive inverses). Describe situations in which opposite quantities combine to make 0. *For example, your bank account balance is -$25.00. You deposit $25.00 into your account. The net balance is $0.00.*

MGSE7.NS.1b Understand p + q as the number located a distance $|q||q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Interpret sums of rational numbers by describing real world contexts.

MGSE7.NS.1c Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (– q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

MGSE7.NS.1d Apply properties of operations as strategies to add and subtract rational numbers.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

COMMON MISCONCEPTIONS

Visual representations may be helpful as students begin this work. If they do not have a visual to illustrate what is happening when they are adding and subtracting integers, they will get lost in the symbols and will not know how to combine the absolute value of the integers.

- *Students may struggle with the generalization section. It is very abstract and may be difficult because the algebra unit has not been done.*
- *In part III, students have been taught when they see parenthesis it represents multiplication. The signs shown in this part do not mean multiplication but are there for spacing purposes when there are multiple signs/operations.*

ESSENTIAL QUESTIONS

- What strategies are most useful in helping develop algorithms for adding and subtracting positive and negative rational numbers?

MATERIALS

- Balloon model (1 index card per student)
- Vertical number lines drawn on graph paper
- 3 colors per student (colored pencils, pens, or markers)
- rulers

GROUPING

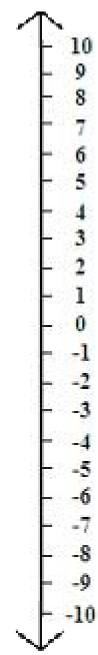
Individual / Partner

TASK DESCRIPTION

I. INTRODUCTION: Model with mathematics and use tools strategically (SMP 4 & 5)

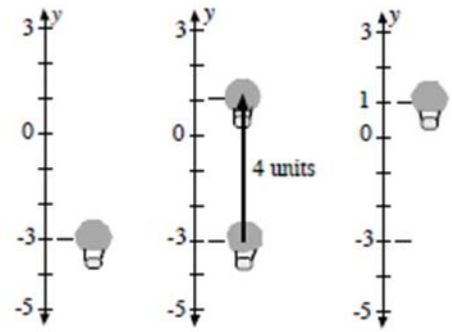
1. Have students draw a vertical number line on ¼ inch graph paper. Students need to use the lines of the graph paper and number the vertical number line using three different colors- one color for the positive numbers, one color for the negative numbers, and another color for zero.
2. Give each student an index card and tell them to draw a hot air balloon and an arrow.
3. Model and discuss with students what makes a balloon go up and what makes it go down.
   -Gas bags (hot air) make the balloon go up
   -Sand bags make the balloon go down

4. Tell students, "We are going to write bags of gas as positive numbers, for example 3 bags of gas will be expressed as +3 and 10 bags of gas will be expressed as +10. Bags of sand will be expressed as negative numbers, for example 5 bags of sand will be expressed as (-5) and 7 bags of sand will be expressed as (-7)."



The Hot Air Balloon Model

Sand bags *(negative integers)* and Hot Air bags *(positive integers)* can be used to illustrate operations with integers. Bags can be put on (added to) the balloon or taken off (subtracted).

<table>
<tr><td>Here is an example: -3 - (-4) = ?
● The balloon starts at -3 (think of the balloon being 3 feet below sea level or 3 feet below the level of a canyon) and you take off 4 sand bags.
● Now, think about what happens to a balloon if you remove sand bags, the balloon gets lighter. So, the balloon would go up 4 units.
If you think in terms of a vertical number line, it would start at -3 and end up at 1, so -3 - (-4) = 1. To help students make the connection between -3 - (-4) and -3 + (+4), present the addition and subtraction questions using the same numbers.</td><td></td></tr>
<tr><td colspan="2">Another example would include the first addition question as 9 + (-5) and the first subtraction question would then be 9 - (+5). The students see that putting on 5 sand bags (negative) produces the same result as taking off 5 hot air bags (positive).</td></tr>
</table>

Hand out the page "What happens when…" and complete it along with the students. Model each situation with a vertical number line and a hot air balloon (index card) or some other type of model.

II. Reason quantitatively (SMP 2): What happens to the balloon when…		**Mathematically**
Add bags of gas	**Balloon goes up**	3 bags of gas (+3) 10 bags of gas (+10)
Add bags of sand	**Balloon goes down**	3 bags of sand (-3) 10 bags of sand (-10)
Subtract bags of gas	**Balloon goes down**	Subtract 3 bags of gas – (+3) Subtract 10 bags of gas – (+10)
Subtract bags of sand	**Balloon goes up**	Subtract 3 bags of sand –(-3) Subtract 10 bags of sand – (-10)

TASK DIRECTIONS:

When using hot air balloons to add or subtract integers, there are several important things to remember:

- The first number indicates where the balloon starts.
- The sign tells you if you will be adding or subtracting something from the balloon. An addition sign tells you that you will be adding something to the hot air balloon and a subtraction sign tells you that you will be subtracting something from the balloon
- The second number tells you what you will add or subtract from the balloon (either bags of gas if the number is positive or bags of sand if the number is negative).

Use a number line and a model of a hot air balloon. Model each problem and answer the questions that follow; the first one gives you hints as to the types of answers you should give.

$-3 + 6$

1. Where does the balloon start? (#)________
2. Do you add or subtract something from the balloon? (operation)_________________
3. What do you add or subtract from the balloon? (# bags of?)_____________________
4. Where does the balloon end up? (#)___________

Solution:

1. -3
2. Add
3. 6 bags of gas
4. 3

$4 + (-7)$

5. Where does the balloon end up?_______________

Solution:

5. -3

-3 + -5

6. Where does the balloon end up?_______________

Solution:

6. -8

What do you think happens to the balloon if you take away sand instead of adding sand?

Solution:

The balloon will go up

$-3 - (-9)$

7. Where does the balloon start?________________

8. Do you add or subtract something from the balloon?________________________________

9. What do you add or subtract from the balloon?____________________________________

10. Where does the balloon end up?________________

Solution:
7. -3
8. Subtract
9. Nine bags of sand
10. 6

What do you think happens to the balloon if you take away gas bags instead of adding gas bags?

Solution:
The balloon will go down.

6 – 9

11. Where does the balloon end up?________________

Solution:
11. -3

−6 – 2

12. Where does the balloon end up?________________

Solution:
12.-8

For the following questions, look for patterns that you discovered in sections I-IV. Model the expressions with your balloon and number line to answer each. Describe what you did with the air balloon. Be sure to include where you began, whether you added or subtracted bags, what types of bags, and your ending point in relation to your starting point.

Sample: If a and b are positive numbers, explain a+b

I will begin with my balloon above zero to represent a on the number line. Next, I would continue to move my balloon further up on the number line to represent the addition of a positive number (b). This models adding a gas bag to my balloon. My final destination would definitely be above zero on the number line, making my final answer positive.

1. If a and b are positive numbers, explain (-a) + (-b)
Solution: negative; I began below zero to represent (-a), then I added sand bags to represent the addition of (-b). My final position was below zero.

***The following questions are very abstract and require a high depth of knowledge, answers will vary.

2. If a and b are positive numbers, explain a + (-b)
Solution: I began above zero to represent a on the number line. Next, I will move down the number line to model the addition of sand bags. My final destination could be either above or below zero, it depends on the value of a and b. There is not enough information to determine whether or not the final destination of the balloon is above or below zero. ***Some students may be ready to make the connection between the fact that adding sand bags is the same as subtracting gas bags at this point. They will need to be able to see that the plus sign is not necessary in the expression a+ (-b) in order to describe this connection.

3. If a and b are positive numbers, explain a - (-b)
Solution: I began above zero to represent a on the number line. Next, I will move up the number line to model taking away sand bags. Taking away sand bags is essentially the same as adding gas bags. My final destination will definitely be above zero on the number line, making my final answer positive, regardless of the numerical value of a and b. ***Students hopefully will make the connection at this point that subtraction of a negative requires the same movement of the hot air balloon up the number line as addition of a positive.

4. If a and b are positive numbers explain a - b; describe this situation two ways: use the idea of the subtraction of gas bags *and* the addition of sand bags in your explanation
Solution: I began above zero to represent a on the number line. Next, I will move down the number line to model the subtraction of gas bags. My final destination could be either above or below zero, it depends on the value of a and b. There is not enough information to determine whether or not the final destination of the balloon is above or below zero.

5. If a and b are positive numbers explain (-a) - (-b)
Solution: I began below zero to represent -a on the number line. Next, I will move up the number line to model the subtraction of sand bags. My final destination could be either above or below zero, it depends on the value of a and b. There is not enough information to determine whether or not the final destination of the balloon is above or below zeroSome students may be ready to make the connection between the fact that subtracting sand bags is the same as adding gas bags at this point. This understanding will hopefully lead them to see that subtraction of a negative is essentially the same movement on the number line as adding a positive.***

6. If a and b are positive numbers explain (-a) + b
Solution: I began below zero to represent -a on the number line. Next, I will move up the number line to model the addition of gas bags. My final destination could be either above or below zero, it depends on the value of a and b. There is not enough information to determine whether or not the final destination of the balloon is above or below zeroThis question could lead to a discussion about absolute value and why it is an important concept when adding and subtracting integers.***

7. If a, b, and c are all positive numbers, explain a + b+ (-c)
Solution: I began above zero to represent a on the number line. Next, I will move up the number line to model the addition of gas bags (b). My last movement will be down the nuber

line to represent the addition of sand bags (or the subtraction of gas bags). My final destination could be either above or below zero, it depends on the value of a and b. There is not enough information to determine whether or not the final destination of the balloon is above or below zeroThis question can further emphasize the importance of the absolute values of the variables a, b, and c when determining the location of the final answer on the number line.***

Investigate these conjectures.

1. What happened when you were adding integers that had the same signs; (-a) + (-b) and a + b?
Solution: In both cases, wherever you begin on the number line, you continue moving in the direction of where you started in relation to zero on the number line. For example, if you start below zero and add a negative, you continue to move down the number line further away from zero. If you begin above zero, you continue to move above zero when adding another positive.

2. What rule can you make by your discovery? Explain.
Solution: answers will vary-when adding integers with the same signs, add the numbers and keep the sign the same. When adding integers of the same sign, you are either beginning below zero (in the negatives) and remaining in the negatives or beginning above zero (in the positive numbers) and remaining above zero.

3. What happened when you were adding integers that have different signs (a positive integer and a negative integer: (-a) + b and a + (-b))?
Solution: answers will vary-when adding integers with different signs, if you begin below zero, you will move the opposite direction; towards zero, up. If you begin above zero you will move down the number line when adding a negative.

4. What rule can you make by your discovery? Explain**.**
Solution: answers will vary-when adding integers with different signs, subtract and keep the sign of the number that was furthest away from zero (your starting point on the number line).

Hot Air Balloon

I. Make a balloon model and vertical number line.

When using hot air balloons to add or subtract integers, there are several important things to remember. They are:

- The **first number** indicates where the balloon starts.
- The **sign** tells you if you will be adding or subtracting something from the balloon. An addition sign tells you that you will be adding something to the hot air balloon and a subtraction sign tells you that you will be subtracting something from the balloon
- The **second number** tells you what you will add or subtract from the balloon (either bags of gas if the number is positive or bags of sand if the number is negative).

II. Reason quantitatively: What happens to the balloon when…		**Mathematically**
Add bags of gas	***Fill in the blanks: (up or down)*** **Balloon goes ________**	***Fill in the blanks: (+ or -)*** 3 bags of gas (___3) 10 bags of gas (___10)
Add bags of sand	**Balloon goes ________**	3 bags of sand (____3) 10 bags of sand (____10)
Subtract bags of gas	**Balloon goes ________**	Subtract 3 bags of gas___(___3) Subtract 10 bags of gas ___(__10)
Subtract bags of sand	**Balloon goes ________**	Subtract 3 bags of sand __(__3) Subtract 10 bags of sand ___(___10)

II. Use a number line and a model of a hot air balloon. Model each problem and answer the questions that follow; the first one gives you hints as to the types of answers you should give.

When using hot air balloons to add or subtract integers, there are several important things to remember:

- The first number indicates where the balloon starts.
- The sign tells you if you will be adding or subtracting something from the balloon. An addition sign tells you that you will be adding something to the hot air balloon and a subtraction sign tells you that you will be subtracting something from the balloon
- The second number tells you what you will add or subtract from the balloon (either bags of gas if the number is positive or bags of sand if the number is negative).

$\mathbf{-3+6}$

1. Where does the balloon start? (#)________
2. Do you add or subtract something from the balloon? (operation)_________________
3. What do you add or subtract from the balloon? (# bags of?) ___________________
4. Where does the balloon end up? (#)___________

$\mathbf{4+(-7)}$

5. Where does the balloon end up?_______________

-3 + -5

6. Where does the balloon end up?_______________

What do you think happens to the balloon if you take away sand instead of adding sand?

$\mathbf{-3-(-9)}$

7. Where does the balloon start?________________
8. Do you add or subtract something from the balloon?_________________________
9. What do you add or subtract from the balloon?_____________________________
10. Where does the balloon end up?_______________

What do you think happens to the balloon if you take away gas bags instead of adding gas bags?

$\mathbf{6-9}$

11. Where does the balloon end up?_______________

$-6 - 2$

12. Where does the balloon end up?________________

III. For the following questions, look for patterns that you discovered in sections I and II. Model the expressions with your balloon and number line to answer each. Describe what you did with the air balloon. Be sure to include where you began, whether you added or subtracted bags, what types of bags, and your ending point in relation to your starting point.

Sample: If a and b are positive numbers, explain a+b

I will begin with my balloon above zero to represent a on the number line. Next, I would continue to move my balloon further up on the number line to represent the addition of a positive number (b). This models adding a gas bag to my balloon. My final destination would definitely be above zero on the number line, making my final answer positive.

1. If a and b are positive numbers, explain (-a) + (-b)

2. If a and b are positive numbers, explain a + (-b)

3. If a and b are positive numbers, explain a - (-b)

4. If a and b are positive numbers explain a - b; describe this situation two ways: use the idea of the subtraction of gas bags *and* the addition of sand bags in your explanation

5. If a and b are positive numbers explain (-a) - (-b)

6. If a and b are positive numbers explain (-a) + b

7. If a, b, and c are all positive numbers, explain a + b+ (-c)

Investigate these conjectures.

1. What happened when you were adding integers that had the same signs; (-a) + (-b) and a + b?

2. What rule can you make by your discovery? Explain.

3. What happened when you were adding integers that have different signs (a positive integer and a negative integer: (-a) + b and a + (-b))?

4. What rule can you make by your discovery? Explain.

Debits and Credits

Back to task table

In this task, students will use a check register to record debits and credits and calculate a running total balance.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

MGSE7.NS.1a Show that a number and its opposite have a sum of 0 (are additive inverses). Describe situations in which opposite quantities combine to make 0. *For example, your bank account balance is -$25.00. You deposit $25.00 into your account. The net balance is $0.00.*

MGSE7.NS.1b Understand p + q as the number located a distance $|q||q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Interpret sums of rational numbers by describing real world contexts.

MGSE7.NS.1c Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

MGSE7.NS.1d Apply properties of operations as strategies to add and subtract rational numbers.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

COMMON MISCONCEPTIONS

Visual representations may be helpful as students begin this work. If they do not have a visual to illustrate what is happening when they are adding and subtracting integers, they will get lost in the symbols and will not know how to combine the absolute value of the integers.

- Students may struggle with the vocabulary since there are many different ways to debit your account.

ESSENTIAL QUESTIONS

- What strategies are most useful in helping develop algorithms for adding and subtracting positive and negative rational numbers?

MATERIALS

Student Ledger Sheet

GROUPING
Partner / Individual

TASK DESCRIPTION

Learning Task: DEBITS AND CREDITS

Suggestion- Bring an actual check register to show and demonstrate the situations in the task.

Suppose you have been given a checkbook. Your checkbook has a ledger for you to record your transactions. There are two types of transactions that may take place, (1) deposits (money placed in the account) and (2) debits/ payments (money which you spend and it comes out of the account). The difference between debits/payments and deposits tells the value of the account. If there are more credits than debits, the account is positive, or "in the black." If there are more debits than credits, the account is in debt, shows a negative cash value, or is "in the red."

Vocabulary key –
Transaction = debit or credit from an account
Debit (withdrawl) = Check or debit card usage written out of the checking account
Credit= Deposit of money put in the account

Situation #1:
Use the ledger to record the information and answer the questions.
Note: On August 12, your beginning balance is $0.00 (This will be the first line in the ledger.)

1. On August 16, you receive a check from your Grandmother for $40 for your birthday.
2. On August 16, you receive a check from your Parents for $100 for your birthday.
3. On August 17, you purchase a pair of pants from Old Navy for $23.42.
4. On August 18, you find $5.19 in change during the day.
5. On August 19, you purchase socks from Wal-Mart for $12.76.

Comment
The ledger below shows the transactions.

DATE	TRANSACTION	PAYMENT (-)	DEPOSIT (+)	BALANCE
8.12	***Beginning Balance***			***0.00***
8.16	***Money from Grandma***		***$40.00***	***$40.00***
8.16	***Money from Parents***		***$100.00***	***$140.00***
8.17	***Old Navy***	***$23.42***		***$116.58***
8.18	***Found Change***		***$5.19***	***$121.77***
8.19	***Wal-Mart***	***$12.76***		***$109.01***

A. What is your balance after five transactions?
Solution
$109.01

B. How much money did you deposit (show as a positive value)?
Solution
$145.19

C. How much money did you pay or withdraw (show as a negative value)?
Solution
$-36.18

Situation #2:
Use the ledger to record the information and answer the questions.
Note: On May 5, your beginning balance is $8.00
1. On May 6, you spent $4.38 on a gallon of ice cream at Marty's Ice Cream Parlor.
2. On May 7, you spent $3.37 on crackers, a candy bar, and a coke from Circle H convenience store.
3. On May 8, you received $10 for cutting the neighbor's grass.
4. On May 8, you spent $14.80 on a downloaded book for your Kindle.

Comment
The ledger below shows the transactions.

DATE	TRANSACTION	PAYMENT (-)	DEPOSIT (+)	BALANCE
5.5	*Beginning Balance*			*$8.00*
5.6	*Marty's Ice Cream Parlor*	*$4.38*		*$3.62*
5.7	*Circle H Convenience*	*$3.37*		*$.25*
5.8	*Cutting Grass*		*$10.00*	*$10.25*
5.8	*Book for Kindle*	*$14.80*		*$-4.55*

A. What is your balance after four transactions?
Solution
There is no money left. There is a negative balance of $4.55.

B. How much money did you deposit (show as a positive value)?
Solution
$10.00

C. How much money did you pay or withdraw (show as a negative value)?
Solution
$-22.55
D. Can you really afford to spend $14.80 on a book for your Kindle? If not, how much money do you need to earn to have an account balance of $0?

Solution
No. I need to deposit another $4.55 to have an account balance of $0.

Situation #3:
Use the ledger to record the information and answer the questions.

Note: On July 4, your beginning balance is (-$40).
Show, using at least eight transactions, a way you can have an ending account balance of more than $145. You must include debit and credit amounts that have cents in at least five of your transactions. Your ledger must show both credits and debits. Be sure to fill out the ledger as you go.
Comment

Answers will vary. An example is given below.

DATE	TRANSACTION	PAYMENT (-)	DEPOSIT (+)	BALANCE
7.4	*Beginning balance*			*$-40.00*
7.5	*Allowance*		*$20.50*	*$-19.50*
7.12	*Allowance*		*$20.50*	*$1.00*
7.14	*Cutting Grass*		*$10.00*	*$11.00*
7.15	*Yard Sale: Video Game*	*$8.25*		*$2.75*
7.17	*Birthday money*		*$100.00*	*$102.75*
7.19	*Wal-Mart*	*$8.43*		*$94.32*
7.20	*Allowance*		*$20.50*	*$114.82*
7.27	*Allowance*		*$20.50*	*$135.32*
7.31	*Money for Extra Chore*		*$15.50*	*$150.82*

DIFFERENTIATION

Extensions:

Situation #3 could be used as an extension activity. Also, you could extend the assignment to include a monthly budget. Use newspapers or technology so students find an apartment they can afford, a grocery budget, entertainment, and within a given income (ie. $1000 a month).

Intervention:

Only working through situations #1 and #2 would cover the standards.

Debits and Credits

Suppose you have been given a checkbook. Your checkbook has a ledger for you to record your transactions. There are two types of transactions that may take place, (1) deposits (money placed in the account) and (2) debits/ payments (money which you spend and it comes out of the account). The difference between debits and the deposits tells the value of the account. If there are more credits than debits, the account is positive, or "in the black". "in the black." If there are more debits than credits, the account is in debt, shows a negative cash value, or is "in the red."

Vocabulary key –
Transaction = debit or credit from an account
Debit (withdrawal) = Check or debit card usage written out of the checking account
Credit= Deposit of money put in the account

Situation #1:
Use the ledger to record the information and answer the questions.
Note: On August 12, your beginning balance is $0.00

1. On August 16, you receive a check from your Grandmother for $40 for your birthday.
2. On August 16, you receive a check from your Parents for $100 for your birthday.
3. On August 17, you purchase a pair of pants from Old Navy for $23.42.
4. On August 18, you find $5.19 in change during the day.
5. On August 19, you purchase socks from Wal-Mart for $12.76.

DATE	TRANSACTION	PAYMENT (-)	DEPOSIT (+)	BALANCE
8/12	Beginning balance			$0.00

A. What is your balance after five transactions?

B. How much money did you deposit (show as a positive value)?

C. How much money did you pay or withdraw (show as a negative value)?

Situation #2:
Use the ledger to record the information and answer the questions.

Note: On May 5, your beginning balance is $8.00

1. On May 6, you spent $4.38 on a gallon of ice cream at Marty's Ice Cream Parlor.
2. On May 7, you spent $3.37 on crackers, a candy bar, and a coke from Circle H convenience store.
3. On May 8, you received $10 for cutting the neighbor's grass.
4. On May 8, you spent $14.80 on a downloaded book for your Kindle.

DATE	TRANSACTION	PAYMENT (-)	DEPOSIT (+)	BALANCE

A. What is your balance after four transactions?

B. How much money did you deposit (show as a positive value)?

C. How much money did you pay or withdraw (show as a negative value)?

D. Can you really afford to spend $14.80 on a book for your Kindle? If not, how much money do you need to earn to have an account balance of $0?

<u>Situation #3:</u>

Use the ledger to record the information and answer the questions.

Note: On July 4, your beginning balance is (-$40).

Requirements:
- Use at least eight transactions, four of which are debits and four are credits.
- You must have an ending balance of $145.
- You must include debit and credit amounts that have cents in at least five of your transactions.

Be sure to fill out the ledger as you go.

DATE	TRANSACTION	PAYMENT (-)	DEPOSIT (+)	BALANCE

Integer Subitizing Cards

Back to task table

In this task, students will identify zero pairs within quick images to determine the sum of integers.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

MGSE7.NS.1a Show that a number and its opposite have a sum of 0 (are additive inverses). Describe situations in which opposite quantities combine to make 0. *For example, your bank account balance is -$25.00. You deposit $25.00 into your account. The net balance is $0.00.*

MGSE7.NS.1b Understand p + q as the number located a distance $|q||q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Interpret sums of rational numbers by describing real world contexts.

STANDARDS FOR MATHEMATICAL PRACTICE

2. Make sense of problems and persevere in solving them.
3. Reason abstractly and quantitatively.
4. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

Subitizing is the ability to instantly see "how many". It is often used in primary grades as students begin decomposing and recomposing quantities while associating them with the numerals they represent. This idea has been connected to positive and negative numbers and zero pairs.

ESSENTIAL QUESTIONS

- What strategies are most useful in helping develop algorithms for adding and subtracting positive and negative rational numbers?

MATERIALS

- Subitizing cards
- Numeral cards
- Whiteboard
- Dry erase markers

GROUPING

Individual or Partner

TASK DESCRIPTION

How to use the cards:

- Show a card to students for 3-5 seconds.
- Ask students what do you see and how do you see it? Encourage students to record what they see on a whiteboard. Call on various students to share how they "see" or determine the quantity on the card. Be sure to allot for time to have several students share their strategies. Listen for students to express how they used the concept of zero pairs to determine the sum.

- A matching game can be played with the dot cards and numeral cards.

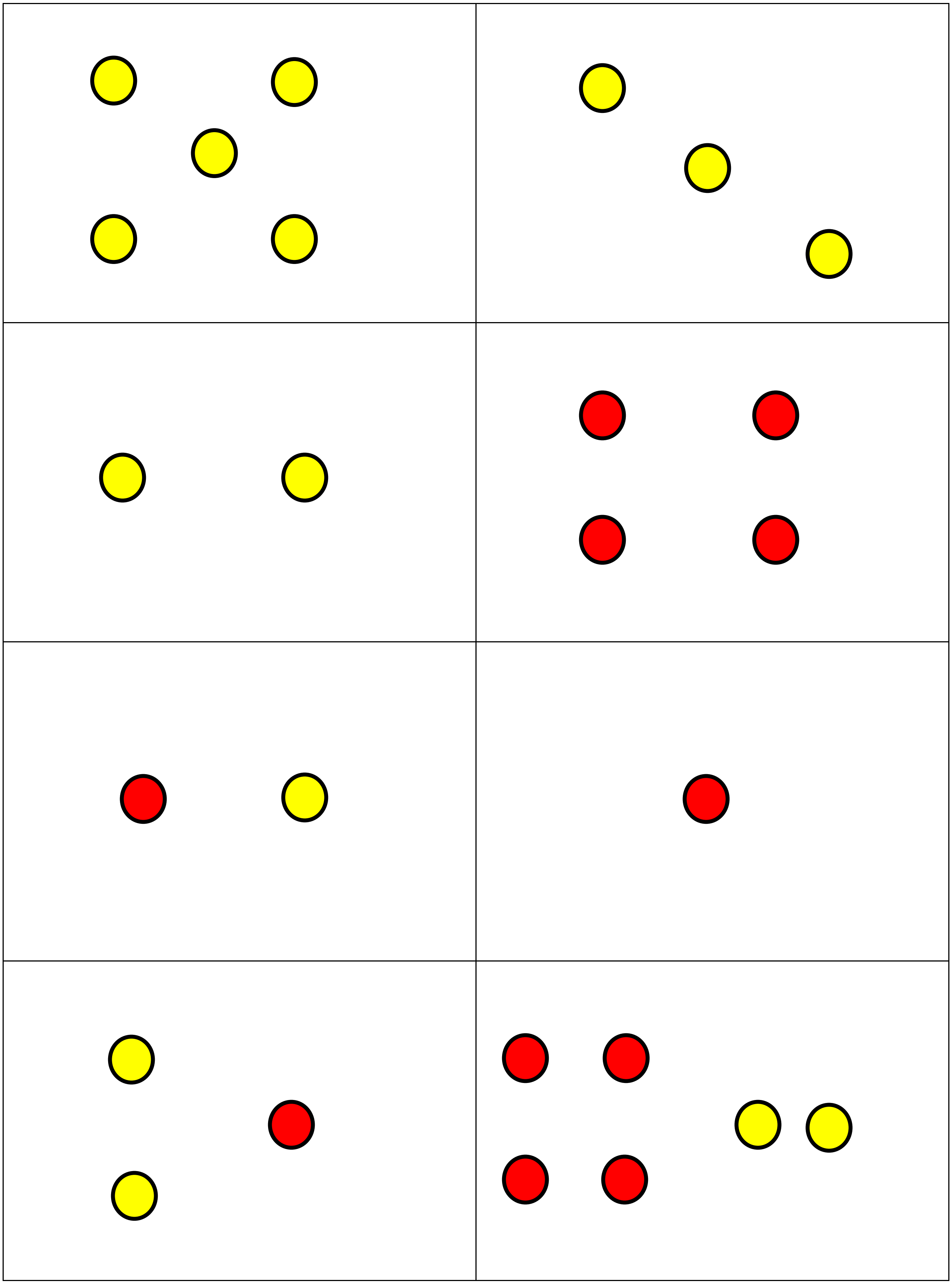
DIFFERENTIATION

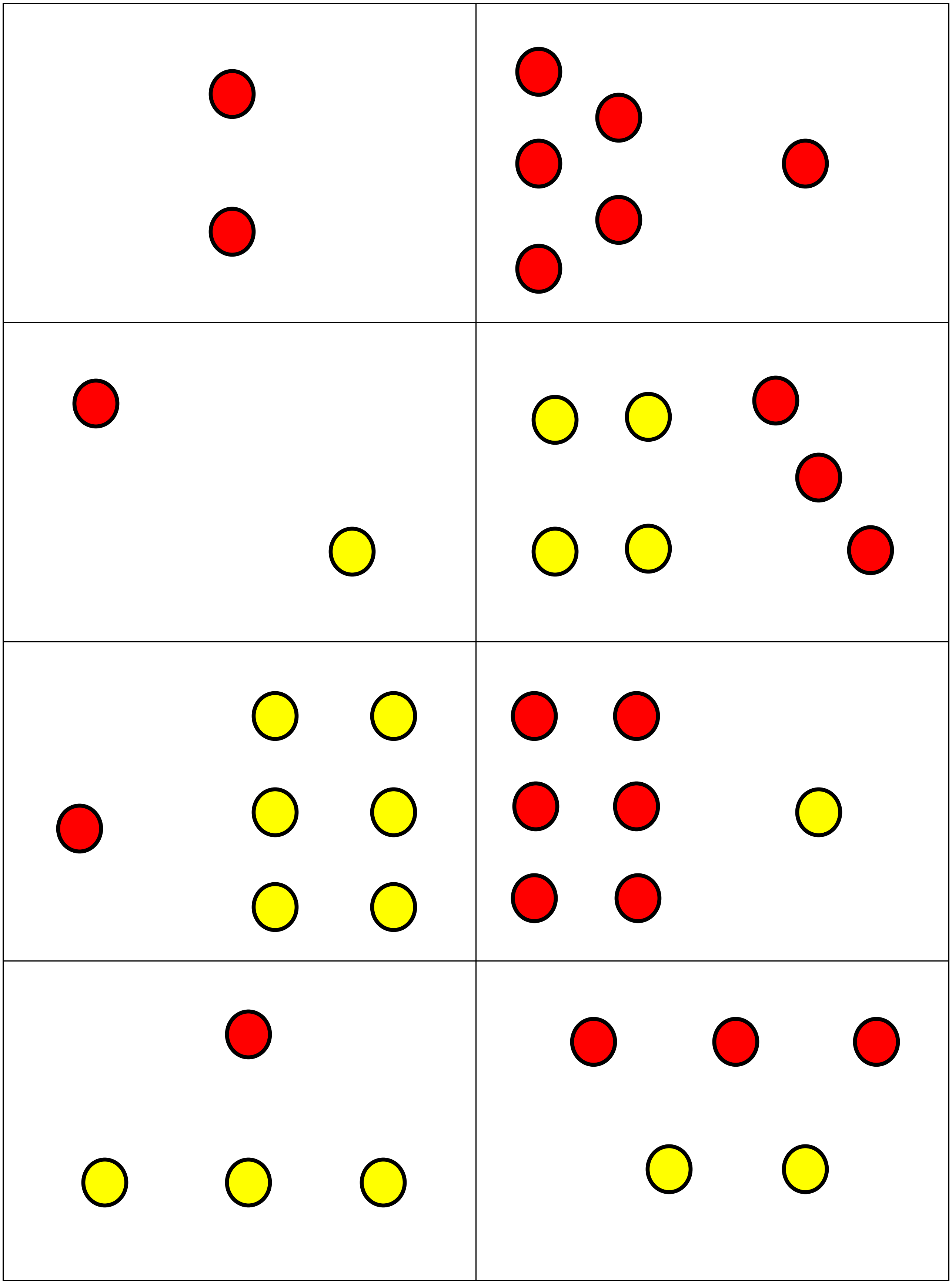
Extension

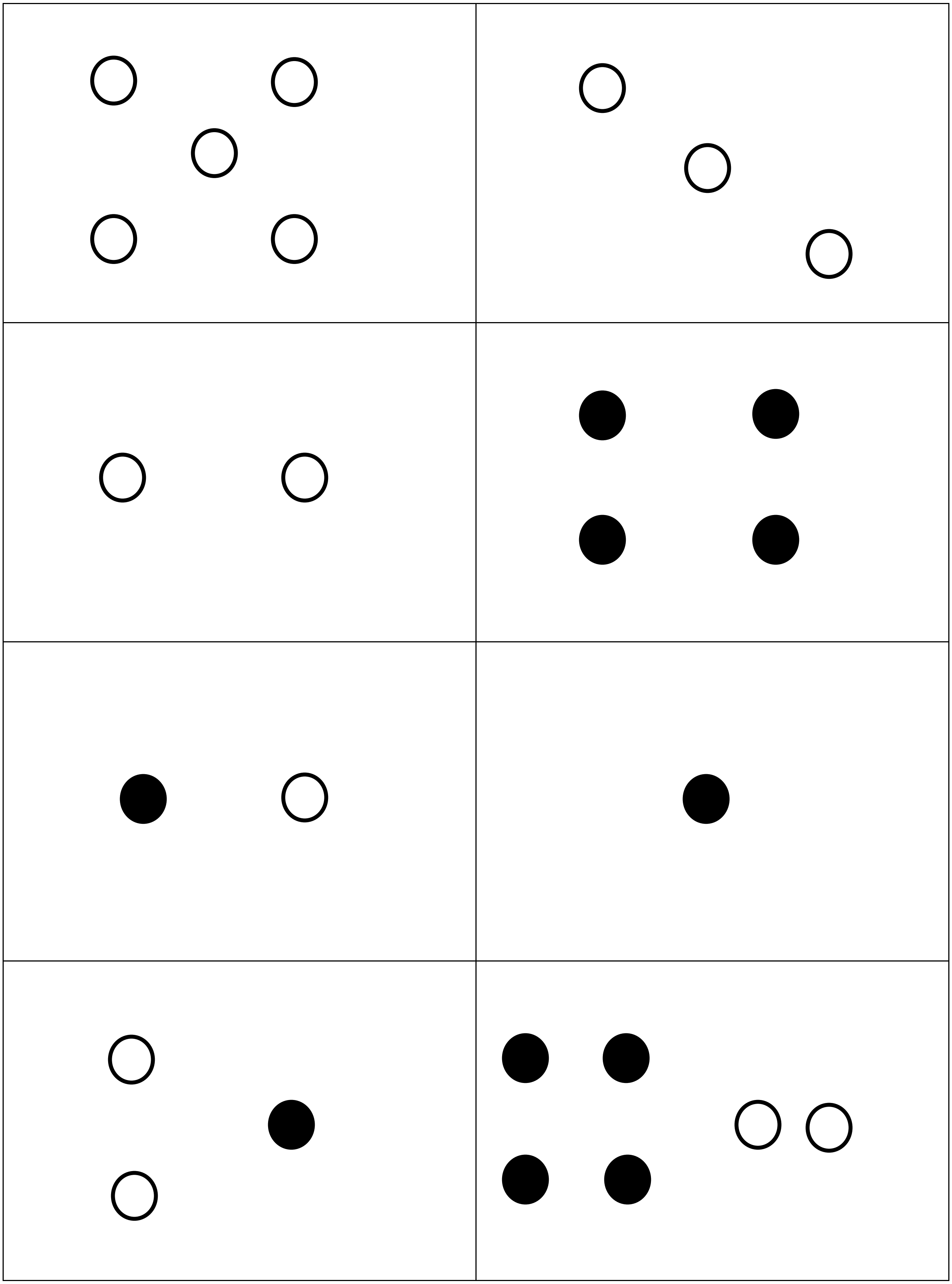
- Encourage students to create more complex integer subitizing cards with various arrangements. Students can work with a partner to practice quickly identifying zero pairs and determining the sum.

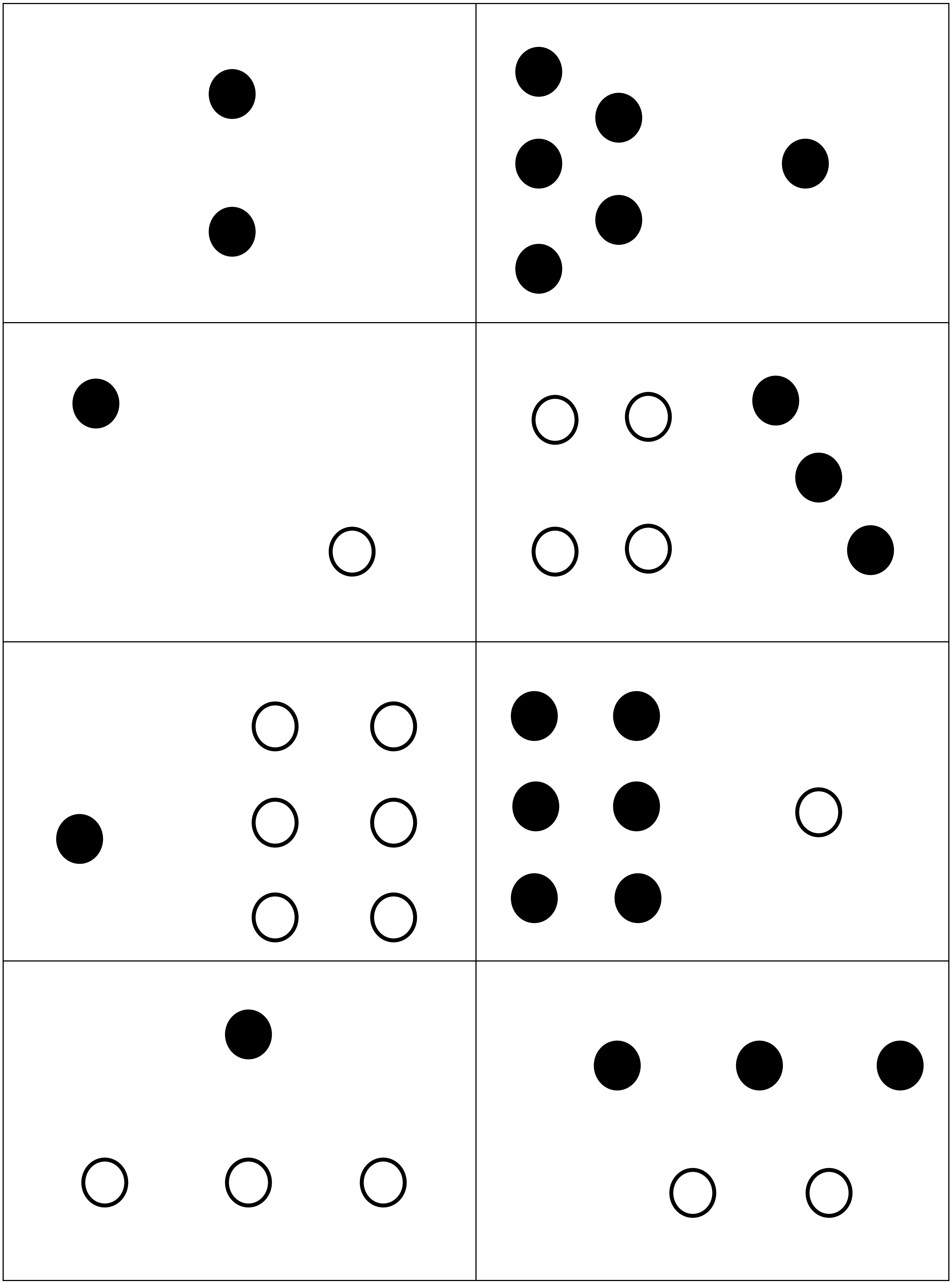
Intervention

- Have students use manipulatives to model the integer cards before discussing the quantity of the cards.
- Encourage students to identify zero pairs to help determine the sum.









5	3
2	-4
0	-1

1	-2
-2	-6
0	**1**
5	-5

2	-1

Create Three

Back to task table

In this game, students will add and/or subtract rational numbers in order to get a sum of 3 or -3.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

MGSE7.NS.1a Show that a number and its opposite have a sum of 0 (are additive inverses). Describe situations in which opposite quantities combine to make 0. *For example, your bank account balance is -$25.00. You deposit $25.00 into your account. The net balance is $0.00.*

MGSE7.NS.1b Understand p + q as the number located a distance $|q||q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Interpret sums of rational numbers by describing real world contexts.

MGSE7.NS.1c Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (– q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

MGSE7.NS.1d Apply properties of operations as strategies to add and subtract rational numbers.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason quantitatively and abstractly.
3. Critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

COMMON MISCONCEPTIONS

Visual representations may be helpful as students begin this work. If they do not have a visual to illustrate what is happening when they are adding and subtracting integers, they will get lost in the symbols and will not know how to combine the absolute value of the integers.

ESSENTIAL QUESTIONS

- What strategies are most useful in helping develop algorithms for adding and subtracting positive and negative rational numbers?

MATERIALS

- Create Three game board
- Different color counters (1 color for each player)

GROUPING
Partner / Small groups

TASK DESCRIPTION

Play with a partner. Each player chooses a fraction to place their counter on. Take turns moving your counter to another faction along the lines only. Add the new fraction to your total. The first player to make exactly three or negative three is the winner. Go over three or under negative three and you lose the game. Players use an additional counter to keep a running total along the number line.

DIFFERENTIATION

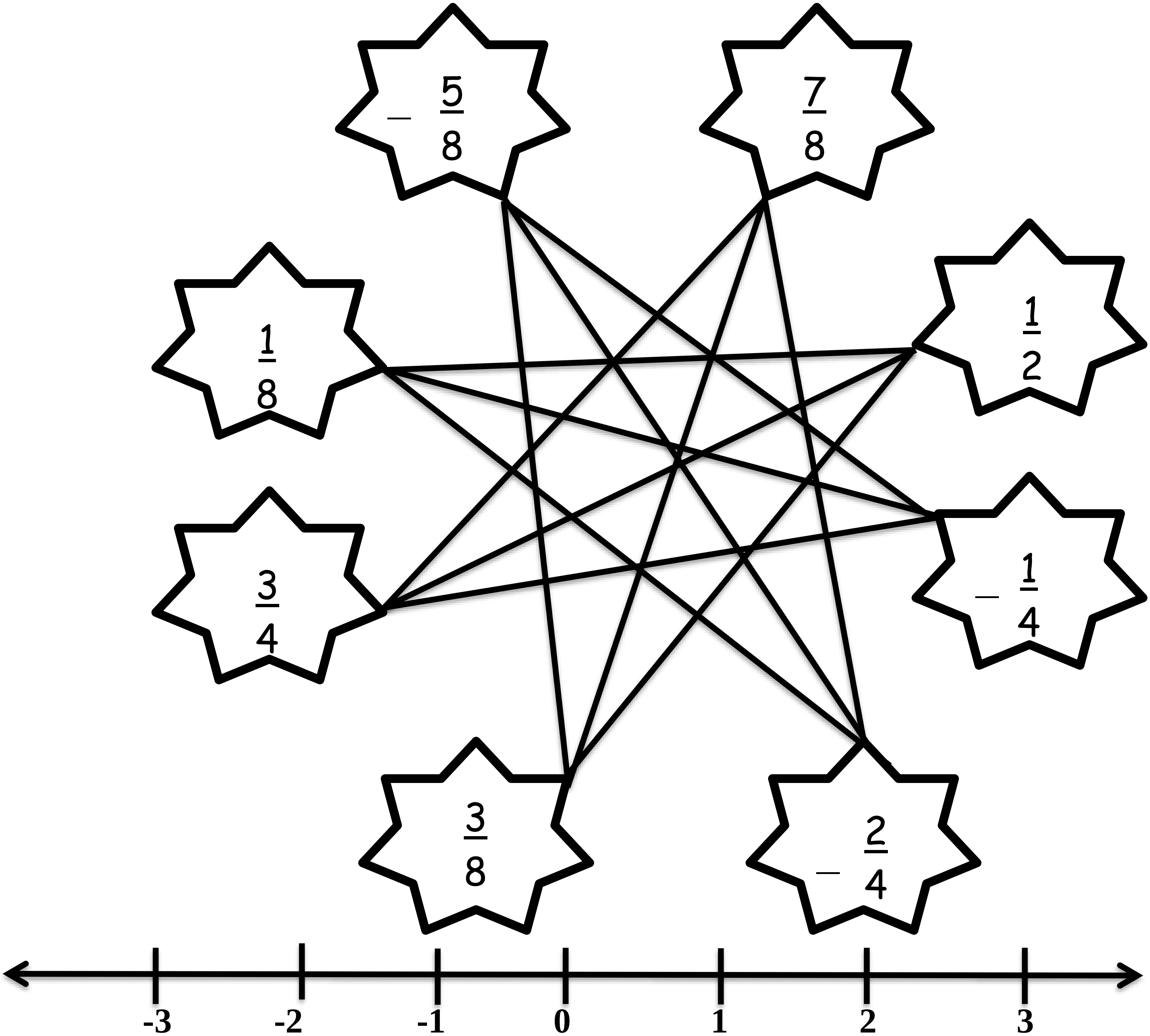
Extensions:

Record the strategy used to win the game. Explain if this strategy would work every time you played the game and why you believe it will.

Intervention:

Provide fraction strips or tiles for students to use as they add and subtract fractions.
Make an explicit connection between adding and subtracting whole numbers to adding and subtracting fractions. This can be done using a number line. It is important for students to see the movements occur on a number line to make the connection.

Create Three



Using Positive and Negative Numbers in Context

Back to task table

(Concept Development)
This lesson unit is intended to help you assess how well students are able to understand and use directed numbers in context. It is intended to help identify and aid students who have difficulties in ordering, comparing, adding, and subtracting positive and negative integers.

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1304

STANDARDS FOR MATHEMATICAL CONTENT

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

MGSE7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

MGSE7.NS.1a Show that a number and its opposite have a sum of 0 (are additive inverses). Describe situations in which opposite quantities combine to make 0. *For example, your bank account balance is -$25.00. You deposit $25.00 into your account. The net balance is $0.00.*

MGSE7.NS.1b Understand p + q as the number located a distance $|q||q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Interpret sums of rational numbers by describing real world contexts.

MGSE7.NS.1c Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (– q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

MGSE7.NS.1d Apply properties of operations as strategies to add and subtract rational numbers.

MCC7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.

ESSENTIAL QUESTIONS

- What strategies are most useful in helping develop algorithms for adding and subtracting positive and negative rational numbers?

TASK COMMENTS

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Using Positive and Negative Numbers in Context*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=453&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1304

Multiplying Integers

Back to task table

In this task, students will use a number line and counters to multiply positive and negative integers. Using patterns from multiplication of positive and negative integers, students will apply the same patterns to division of positive and negative integers. Prior to students completing the task, an introduction or mini-lesson showing students how to use a number line to show multiplication should be used. Students need experiences with this process prior to completing the task. The introduction below may be used to introduce the concept.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

MGSE7.NS.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (- 1)(– 1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

MGSE7.NS.2b Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

MGSE7.NS.2c Apply properties of operations as strategies to multiply and divide rational numbers.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable argues and critique the reasoning of others.-
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

COMMON MISCONCEPTIONS

- *Visualizing multiplication is a very difficult concept for students to comprehend. You may have some learners that find the task more confusing than helpful depending on their understanding of how concrete examples apply to abstract concepts.*

ESSENTIAL QUESTION

- What strategies are most useful in helping develop algorithms for adding, subtracting, multiplying, and dividing positive and negative rational numbers?

MATERIALS

- Two colored counters

GROUPING

- Individual / Partner

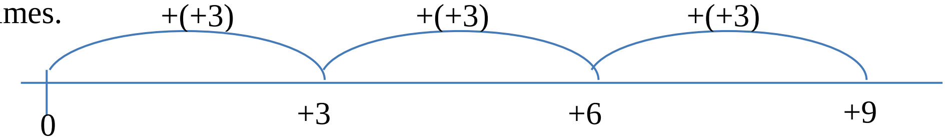
TASK DESCRIPTION

Activator

To introduce the concept of multiplying integers, help students make the connection between this and multiplying positive whole numbers.

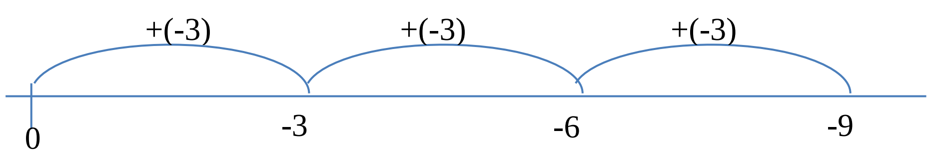
Let's begin with 3 x 3. Using counters, have students model 3 groups of positive 3. Ask a student to explain how they modeled the expression. Record their thinking on an empty number line. An empty number line is a line without designated numbers.

The student should explain they began with 0 counters and added 3 positive counters a total of 3 times.



Have students model 3 x -3 using the counters. Ask a student to explain how they modeled the expression. Record their thinking on an empty number line.

The student should explain they began with 0 counters and added 3 negative counters a total of 3 times.



Have students model -3 x 3 using the counters. Circulate around the room to see how students are grappling with the idea of having negative 3 groups of 3. Look for students who:

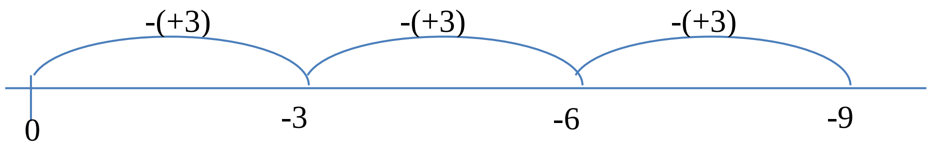
- Apply the commutative property to make sense of the expression.
- Attempt to make zero pairs to make sense of the expression but become stuck.

- Attempt to make zero pairs and remove 3 groups of positive 3.

Explain to students, with the previous examples, they began with 0 and repeatedly added the quantities, so they used repeated addition when creating the positive groups. Since the groups are no longer positive, they are the opposite; students should perform the opposite operation, repeated subtraction. With this model, students should remove 3 groups of positive 3 from 0.

Allow students the opportunity to discuss with a partner how to model removing from 0.
0 can be created through the use of zero pairs, +a + (-a) = 0. Instruct students create enough zero pairs in which 3 groups of positive 3 can be removed. Once this is modeled, instruct students to remove the 3 groups of +3. Ask students what quantity is left over. With a total of 9 positive counters removed, there should be -9 counters left over.

Record this thinking on an empty number line.

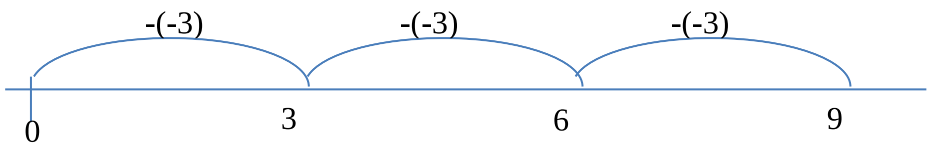


Have students model -3 x -3 using the counters. Circulate around the room to see how students are grappling with the idea of having negative 3 groups of -3. Look for students who:

- Attempt to apply a pattern determined from the previous models.
- Attempt to make zero pairs to make sense of the expression but become stuck.
- Attempt to make zero pairs and remove 3 groups of negative 3.

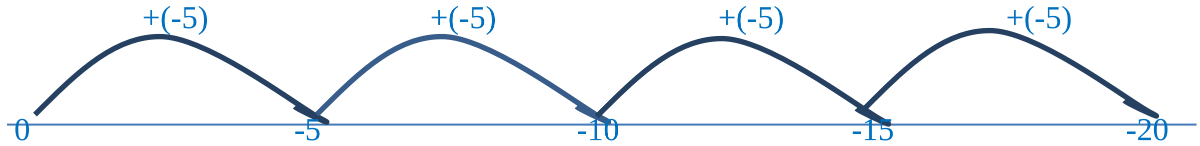
Allow students the opportunity to discuss with a partner how to model removing from 0.
0 can be created through the use of zero pairs, +a + (-a) = 0. Instruct students create enough zero pairs in which 3 groups of negative 3 can be removed. Once this is modeled, instruct students to remove the 3 groups of -3. Ask students what quantity is left over. With a total of 9 negative counters removed, there should be 9 counters left over.

Record this thinking on an empty number line.



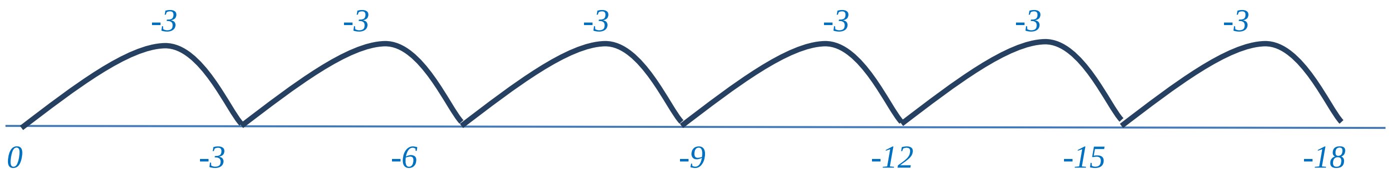
Apply the conventions within a context. <u>Try these problems on your own. Model each problem using counters or an empty number line. Record the equation and model on your paper.</u>

1. Mr. Fletcher's bank account is assessing a $5 withdrawal fee for everyday it is under $500. It has been a total of 4 days under $500. What amount will the bank withdraw from his account?



Mr. Fletcher has withdrawn $20 from his account. Students may also choose to model the problem using 4 groups of 5 red counters which represents 4 groups of -5.

2. Billy is participating in a biggest Loser competition. His goal is to lose 3 pounds a week. If Billy meets his goal every week for 6 weeks, how much weight will he lose?



Billy has lost 18 pounds. Students may also model the problem with 6 groups of 3 red counters to represent 6 groups of -3.

3. Mike's son has to pay his dad $7 for every pound he loses. Mike has lost 5 pounds. How much money does his son owe him?
Mike is owed $35.

<u>Summarize</u>
Students should be asked to answer these questions and prove their responses.
• Is it always true that multiplying a negative factor by a positive factor results in a negative product?
• Does a positive factor times a positive factor always result in a positive product?
• What is the sign of the product of two negative factors?
• How is the numerical value of the product of any two numbers found?

<u>DIFFERENTIATION</u>

Extension:
- Have students develop generalized conjectures about multiplying integers and explain them. For example, +a x –b = -c because a groups of –b added to 0 is –c.

Intervention:
- For students who struggle with the empty number line. Encourage them to continue modeling the problems using the two colored counters.

Multiplying Integers

Apply the conventions within a context. Try these problems on your own. Model each problem using counters or an empty number line. Record the equation and model on your paper.

1. Mr. Fletcher's bank account is assessing a $5 fee withdrawal for everyday it is under $500. It has been a total of 4 days under $500. What amount will the bank withdraw from his account?

2. Billy is participating in a biggest Loser competition. His goal is to lose 3 pounds a week. If Billy meets his goal every week for 6 weeks, how much weight will he lose?

3. Mike's son has to pay his dad $7 for every pound he loses. Mike has lost 5 pounds. How much money does his son owe him?

Modeling the Multiplication of Integers

Back to task table

In this task, students will use a number line and counters to multiply positive and negative integers. Using patterns from multiplication of positive and negative integers, students will apply the same patterns to division of positive and negative integers. Students need experiences with this process prior to completing this task. The **Multiplying Integers** task can serve as a great prerequisite for this task.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

MGSE7.NS.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (- 1)(– 1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

MGSE7.NS.2b Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers then – *(p/q) = (– p)/q = p/(–q)*. Interpret quotients of rational numbers by describing real-world contexts.

MGSE7.NS.2c Apply properties of operations as strategies to multiply and divide rational numbers.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable argues and critique the reasoning of others.
4. . Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for patterns and structure.

COMMON MISCONCEPTIONS

- *Visualizing multiplication is a very difficult concept for students to comprehend. You may have some learners that find the task more confusing than helpful depending on their understanding of how concrete examples apply to abstract concepts.*

ESSENTIAL QUESTION

- What strategies are most useful in helping develop algorithms for multiplying and dividing positive and negative rational numbers?

MATERIALS

- Two colored counters
- Task recording sheet

GROUPING

- Individual / Partner

TASK DESCRIPTION

Modeling the Multiplication of Integers

Try these problems on your own. Model each problem using counters or an empty number line. Record the equation and model on your paper.

1. Suppose the temperature outside is dropping 3 degrees each hour. How much will the temperature change in 8 hours?
2. A computer stock gained 2 points each hour for 6 hours. Describe the total change in the stock after 6 hours.
3. A drought can cause the level of the local water supply to drop by a few inches each week. Suppose the level of the water supply drops 2 inches each week. How much will it change in 4 weeks?
4. Mike's son has to pay his dad $8 for every pound he loses. Mike has lost 10 pounds. How much money does his son owe him?

5. What multiplication patterns can you see from each situation? Fill in the chart below according to the signs of the factors and products.

Solution:

Factor	Factor	Product
+ number	***+ number***	+ number
+ number	(-) number	(-) number
+ number	***(-) number***	(-) number
(-) number	(-) number	+ number

DIFFERENTIATION

Extension:

- Have students develop generalized conjectures about multiplying integers and explain them. For example, +a x –b = -c because a groups of –b added to 0 is –c.

Intervention:

- For students who struggle with the empty number line. Encourage them to continue modeling the problems using the two colored counters.

Modeling the Multiplication of Integers

Try these problems on your own. Model each problem using counters or an empty number line. Record the equation and model on your paper.

1. Suppose the temperature outside is dropping 3 degrees each hour. How much will the temperature change in 8 hours?

2. A computer stock gained 2 points each hour for 6 hours. Describe the total change in the stock after 6 hours.

3. A drought can cause the level of the local water supply to drop by a few inches each week. Suppose the level of the water supply drops 2 inches each week. How much will it change in 4 weeks?

4. Mike's son has to pay his dad $8 for every pound he loses. Mike has lost 10 pounds. How much money does his son owe him?

5. What multiplication patterns can you see from each situation? Fill in the chart below according to the signs of the factors and products.

Factor	Factor	Product
+ number		+ number
	(-) number	(-) number
+ number		(-) number
	(-) number	+ number

Patterns of Multiplication and Division

Back to task table

This task is designed to assist struggling students with the concepts of multiplication and division.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

MGSE7.NS.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

MGSE7.NS.2b Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

MGSE7.NS.2c Apply properties of operations as strategies to multiply and divide rational numbers.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Solve problems and persevere.
2. Reason abstractly and quantitatively.
5. Use tools strategically.
6. Attend to precision
8. Look for and express regularity in repeated reasoning.

COMMON MISCONCEPTION

The section for dividing using two color counters can be confusing upon first glance. Encourage students to persevere in working through the examples in order to gain understanding.

ESSENTIAL QUESTIONS

- What strategies are most useful in helping develop algorithms for multiplying and dividing positive and negative rational numbers?

MATERIALS

- Patterns of Multiplication and Division Task Sheet
- Colored pencils – red and yellow
- Extra blank number lines (optional)
- Two-color counters (red/yellow)

GROUPING

- Individual / Partner

TASK COMMENTS

This task uses the number line model to illustrate division of integers. When teaching this concept, it is important to revisit number line models of multiplication and addition. Revisiting patterns that are found in multiplying integers is also recommended.

TASK DESCRIPTION

To introduce the task, begin with student's knowledge of addition and subtraction facts and the relationship between the two processes. This same type of relationship occurs between multiplication and division. Point out that for any multiplication fact, we can write another multiplication fact and two different related division facts.

2 x 5 = 10 5 x 2 = 10 10 ÷ ÷ 5 = 2 10 ÷ ÷ 2 = 5

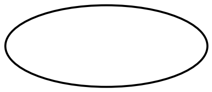
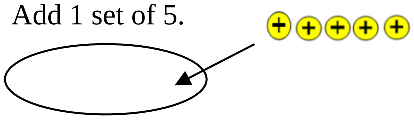
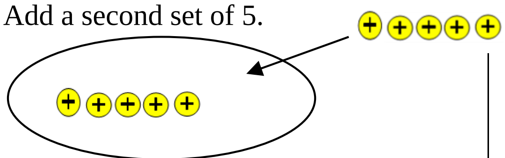
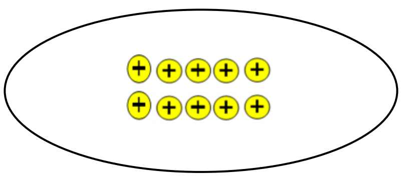
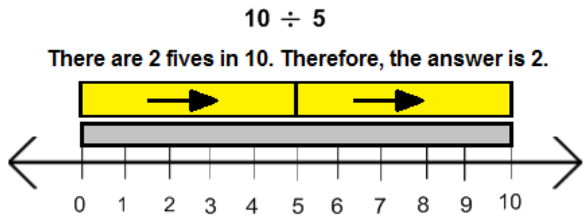
Revisit multiplication of positive and negative integers. You may use a number line model or you may want to revisit the chart developed from the Learning Task**, Multiplying Rational Numbers**, in this framework document.

Factor	Factor	Product
+ number		+ number
	(-) number	(-) number
+ number		(-) number
	(-) number	+ number

Tell your students that they will make models of integer division to prove whether or not they can apply their knowledge of multiplication to division. They will use their findings from the task to fill in a division chart similar to the one below.

Dividend	Divisor	Quotient
	+ number	+ number
(-) number		(-) number
+ number		(-) number
	(-) number	+ number

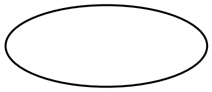
Use the example problems below to show the process of modeling division to the students. Be sure to how students the opportunity to use the counters and a number line to demonstrate their thinking about the division of integers. Be sure to discuss both models.
The following examples are adapted from Van de Walle, J., &Lovin, L.H. (2006). *Teaching student-centered mathematics: Grades 5-8*. Boston, MA: Pearson Education and Allyn& Bacon.
*******Show the examples with the color counters and the number line*******

Example 1: 10 ÷ 5 = 2	
How many sets of 5 will make a set of 10? Begin with zero.  Add 1 set of 5.  Add a second set of 5.  It took 2 sets of 5 to make 10. 	To arrive at the answer of +2, notice that on the number line we are moving forward. We move forward 2 times. 

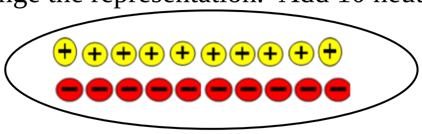
Example 2: (-10) ÷ 5 = (-2)

How many sets of +5 will make -10?

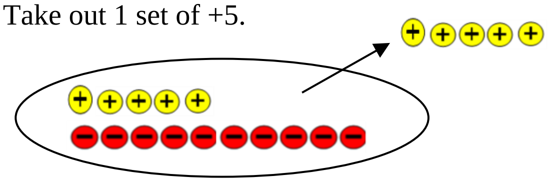
Begin with zero.



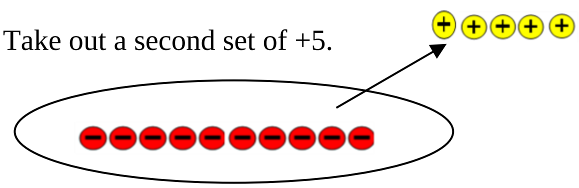
Change the representation. Add 10 neutral pairs.



Take out 1 set of +5.

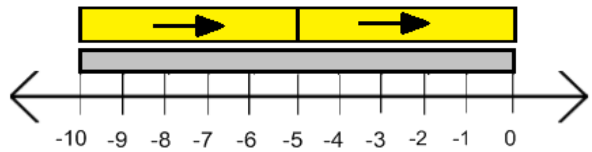


Take out a second set of +5.



2 sets of +5 were removed to make -10 or -2 sets of 5 were used to make -10.

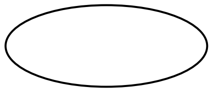
To arrive at the answer of (-2), notice that on the number line we are backing up. We back up 2 times.



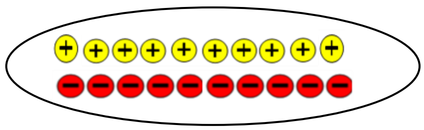
Example 3: 10 ÷ (-5) = (-2)

How many sets of -5 will make +10?

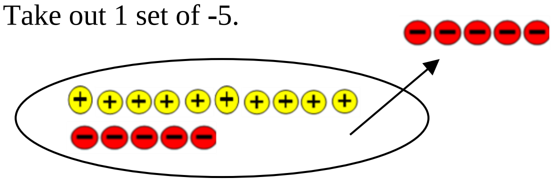
Begin with zero.



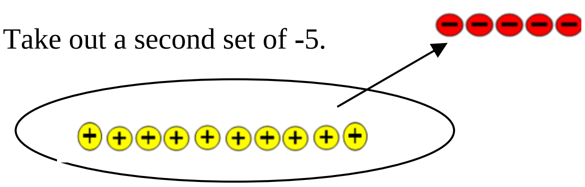
Change the representation. Add 10 neutral pairs.



Take out 1 set of -5.

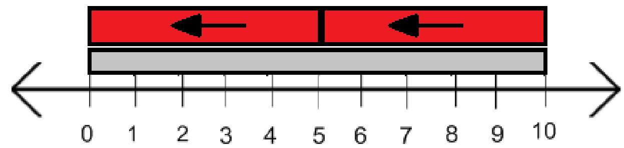


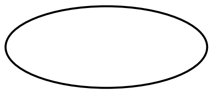
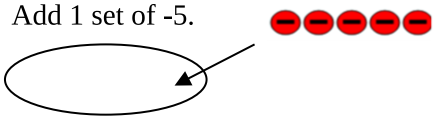
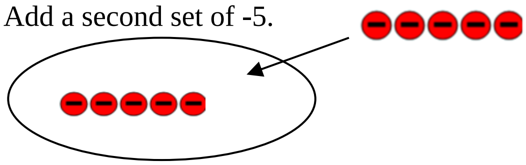
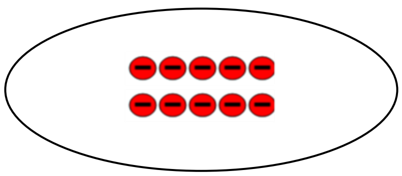
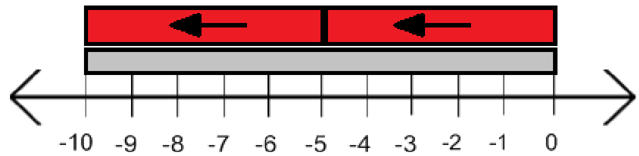
Take out a second set of -5.



2 sets of -5 were removed to make +10 or -2 sets of -5 were used to make +10.

To arrive at the answer of (-2), notice that on the number line we are backing up (-5). We back up 2 times.



Example 4: (-10) ÷ (-5) = 2	
How many sets of -5 will make a set of -10? Begin with zero.  Add 1 set of -5.  Add a second set of -5.  It took 2 sets of -5 to make -10. 	To arrive at the answer of +2, notice that on the number line we are moving forward. We move (-5) forward 2 times. 

As you teach the division model using counters, keep in mind these simple steps:

- Determine how many sets of the divisor are needed to make the dividend.
- Begin with zero or a representation of zero using neutral pairs.
- Remove or add sets of the divisor to make the dividend.
- The number of sets removed or added determines the answer.

As you teach the division model using a number line, keep in mind these simple steps:

- Identify the dividend on the number line.
- Look at the divisor, is it positive (yellow with right arrow) or negative (red with left arrow).
- Determine how many times the divisor will have to move forward (+) or backward (-) to equal the dividend.
- **The number of times it must move and the type of movement determine the answer.**

For further clarification of the division model using a number line, please watch the following video.
http://www.youtube.com/watch?v=Lh0tBKOTq8I

Have students model the problems. Once students complete the problem set, have them fill out the given chart and find the patterns. They will need to determine if the patterns are the same as the patterns for multiplication.

TASK DIRECTIONS

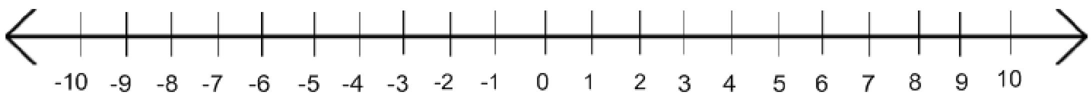
You have recently practiced multiplying positive and negative integers on a number line. It is now your turn to model how to divide. Below are "hints" to help you get started.

When you divide, keep in mind these simple steps:

- Identify the **dividend** on the number line.
- Look at the **divisor**, is it **positive (yellow with right arrow)** or **negative (red with left arrow)**.
- Determine how many times the **divisor** will have to **move forward (+)** or **backward (-) to equal the dividend**.
- The **number of times** it must move and the **type of movement determine the answer.**

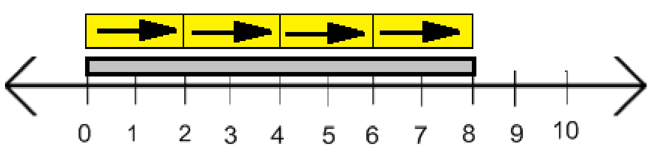
Model the following on the number line.

$8 \div 2$



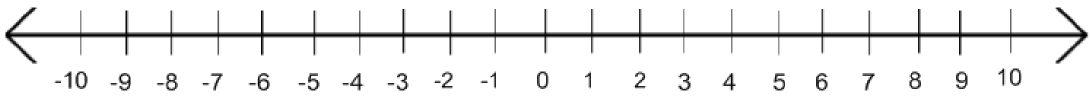
1. What is the dividend? ________
2. What is the divisor? ________
3. What is the solution and how did you find it?

Solution:
1. 8
2. 2
3. 4



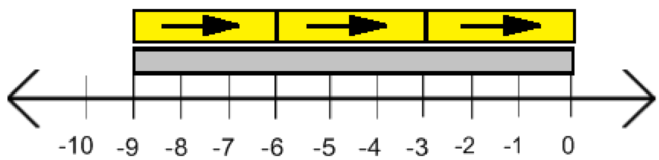
The dividend is 8, the divisor is 2, so I counted by two's.
I counted 4 times forward.

$(-9) \div 3$



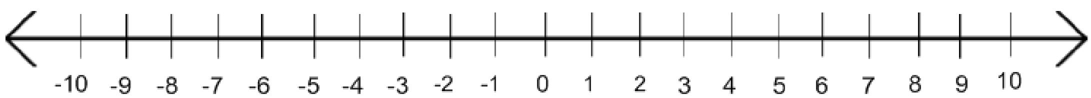
4. What is the dividend?________
5. What is the divisor?________
6. What is the solution and how did you find it?

Solution:
4. (-9)
5. 3
6. (-3)



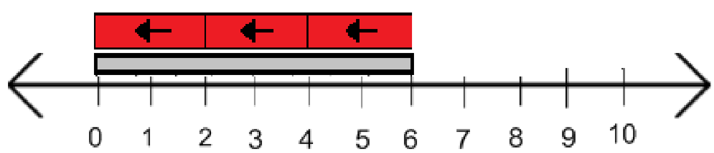
The dividend is (-9), the divisor is 3, so I counted by three's.
I counted 3 times backward.

$6 \div (-2)$



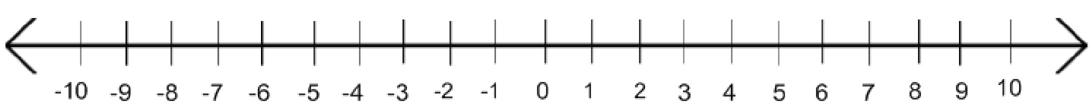
7. What is the dividend? ________
8. What is the divisor? ________
9. What is the solution and how did you find it?

Solution:
7. 6
8. (-2)
9. (-3)



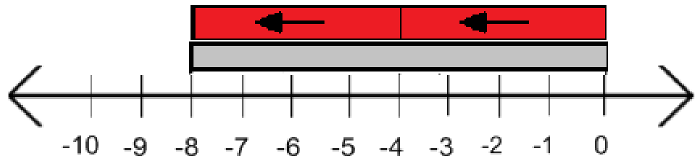
The dividend is 6, the divisor is (-2), so I counted by two's.
I counted 3 times backward.

$(-8) \div (-4)$



10. What is the dividend?________
11. What is the divisor?________
12. What is the solution and how did you find it?

Solution:
10. (-8)
11. (-4)
12. 2



The dividend is (-8), the divisor is (-4), so I counted by four's. I counted 2 times forward.

Let's look to see if there are any patterns.

1. When given a **positive integer** as the **dividend**…
   a. What was the result of **dividing** by a **positive integer**?
      Positive quotient
   b. What was the result of **dividing** by a **negative integer?**
      Negative quotient
2. When given a **negative integer** as the **dividend**…
   a. What was the result of **dividing** by a **positive integer**?
      Negative quotient
   b. What was the result of **dividing** by a **negative integer?**
      Positive quotient
3. Fill in each table below. Are the patterns the same as the multiplication patterns? Explain your findings.

	Factor	**Factor**	**Product**
Multiplication Patterns	+ number	***+ number***	+ number
	+ number	(-) number	(-) number
	+ number	***(-) number***	(-) number
	(-) number	(-) number	+ number

	Dividend	**Divisor**	**Quotient**
Division Patterns	***+ number***	+ number	+ number
	(-) number	***+ number***	(-) number
	+ number	***(-) number***	(-) number
	(-) number	(-) number	+ number

Model the following using two color counters.
8 ÷ 2

Drawing of Model

Students should model 8 yellow counters separated into 2 groups. The amount in each group is positive 4.

1. What is the dividend?__*8*________
2. What is the divisor? ___*2*________
3. What is the solution? *4*

$(-9) \div 3$

Drawing of Model

Students should draw 9 red counters separated into 3 groups resulting in 3 negative counters in each group.

4. What is the dividend? ___*-9*_________
5. What is the divisor? ____*3*_________
6. What is the solution? -3

$6 \div (-2)$

Drawing of Model

Students need to model the removal of 2 to make 6. The model should show 3 removals of 2 from the 6 zero pairs.

7. What is the dividend?_____*6*________
8. What is the divisor?___*-2*__________
9. What is the solution? *-3*

$(-8) \div (-4)$

Drawing of Model

Students need to model the two groups of -4 it takes to create -8.

10. What is the dividend?___*-8*__________
11. What is the divisor?_____*-4*________
12. What is the solution? _______*2*_

Let's look to see if there are any patterns.

13. When given a **positive integer** as the **dividend**…
   a. What was the result of **dividing** by a **positive integer**?
      Positive quotient
   b. What was the result of **dividing** by a **negative integer?**

Negative quotient

14. When given a **negative integer** as the **dividend**…
    a. What was the result of **dividing** by a **positive integer**?
    Negative quotient
    b. What was the result of **dividing** by a **negative integer?**
    Positive quotient

15. Fill in each table below. Are the patterns the same as the multiplication patterns? Explain your findings.

	Factor	Factor	Product
Multiplication Patterns	+ number	***+ number***	+ number
	+ number	(-) number	(-) number
	+ number	***(-) number***	(-) number
	(-) number	(-) number	+ number

	Dividend	Divisor	Quotient
Division Patterns	***+ number***	+ number	+ number
	(-) number	***+ number***	(-) number
	+ number	***(-) number***	(-) number
	(-) number	(-) number	+ number

DIFFERENTIATION

Extension:

- Have students apply the rules for multiplying and dividing integers to multiplying and dividing rational numbers.

Intervention:

- For students who struggle with using the number line, have them to complete the task using the two-color counters.

Patterns of Multiplication and Division with Number Lines

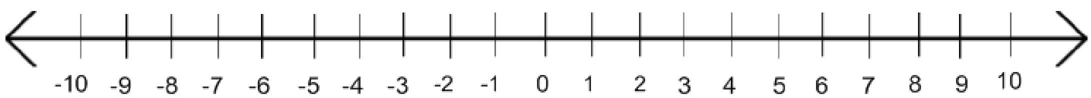
You have recently practiced dividing positive and negative integers on a number line. It is now your turn to model how to divide. Below are "hints" to help you get started.

When you divide, keep in mind these simple steps:

- Identify the **dividend** on the number line.
- Look at the **divisor**, is it **positive (yellow with right arrow)** or **negative (red with left arrow).**
- Determine how many times the **divisor** will have to **move forward (+)** or **backward (-) to equal the dividend.**
- The **number of times** it must move and the **type of movement determine the answer**.

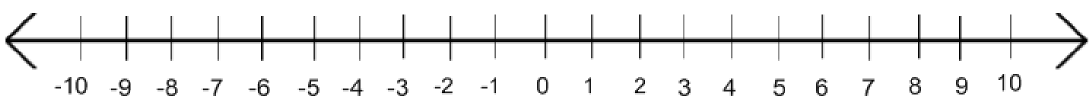
Model the following on the number line.

$8 \div 2$



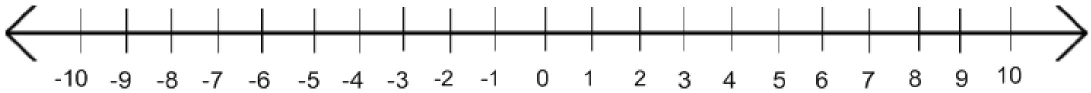
1. What is the dividend?____________
2. What is the divisor? ____________
3. What is the solution and how did you find it?

$(-9) \div 3$



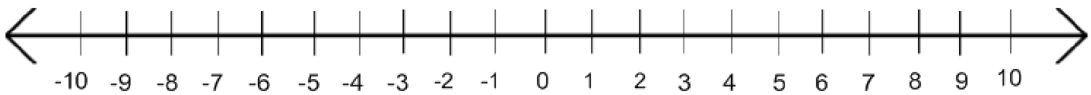
4. What is the dividend? ____________
5. What is the divisor? ____________
6. What is the solution and how did you find it?

$6 \div (-2)$



7. What is the dividend?____________
8. What is the divisor?____________
9. What is the solution and how did you find it?

$(-8) \div (-4)$



10. What is the dividend?____________
11. What is the divisor?____________
12. What is the solution and how did you find it?

Let's look to see if there are any patterns.

13. When given a **positive integer** as the **dividend**…
    - What was the result of **dividing** by a **positive integer**?
    - What was the result of **dividing** by a **negative integer**?

14. When given a **negative integer** as the **dividend**…
    - What was the result of **dividing** by a **positive integer**?
    - What was the result of **dividing** by a **negative integer**?

15. Fill in each table below. Are the patterns the same as the multiplication patterns? Explain your findings.

	Factor	Factor	Product
Multiplication Patterns	+ number		+ number
		(-) number	(-) number
	+ number		(-) number
		(-) number	+ number

	Dividend	Divisor	Quotient
Division Patterns		+ number	+ number
	(-) number		(-) number
	+ number		(-) number
		(-) number	+ number

Patterns of Multiplication and Division using Counters

Model the following using two color counters.

$8 \div 2$

Drawing of Model

- What is the dividend?____________
- What is the divisor? ____________
- What is the solution?

$(-9) \div 3$

Drawing of Model

- What is the dividend? ____________
- What is the divisor? ____________
- What is the solution?

$6 \div (-2)$

Drawing of Model

- What is the dividend?____________
- What is the divisor?____________
- What is the solution?

$(-8) \div (-4)$

Drawing of Model

- What is the dividend?___________
- What is the divisor?___________
- What is the solution?

Let's look to see if there are any patterns.

1. When given a **positive integer** as the **dividend**…
   a. What was the result of **dividing** by a **positive integer**?

   b. What was the result of **dividing** by a **negative integer**?

2. When given a **negative integer** as the **dividend**…
   a. What was the result of **dividing** by a **positive integer**?

   b. What was the result of **dividing** by a **negative integer**?

3. Fill in each table below. Are the patterns the same as the multiplication patterns? Explain your findings.

	Factor	Factor	Product
Multiplication Patterns	+ number		+ number
		(-) number	(-) number
	+ number		(-) number
		(-) number	+ number

	Dividend	Divisor	Quotient
Division Patterns		+ number	+ number
	(-) number		(-) number
	+ number		(-) number
		(-) number	+ number

Do It Yourself Revisited

Back to task table

This task is designed to allow students to make the connections between division of integers and division of rational numbers. The first "Do It Yourself" task appears in Grade 6, Unit 1, Number System Fluency, and addresses MGSE6.NS.1 – quotients of fractions.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.NS.2c Apply properties of operations as strategies to multiply and divide rational numbers.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
5. Use tools strategically.
6. Attend to precision
8. Look for and express regularity in repeated reasoning.

COMMON MISCONCEPTION

The section for dividing using two color counters can be confusing upon first glance. Encourage students to persevere in working through the examples in order to gain understanding.

ESSENTIAL QUESTIONS

- What strategies are most useful in helping develop algorithms for multiplying and dividing positive and negative rational numbers?

MATERIALS

- Recording sheet

GROUPING

- Individual / Partner

TASK DIRECTIONS

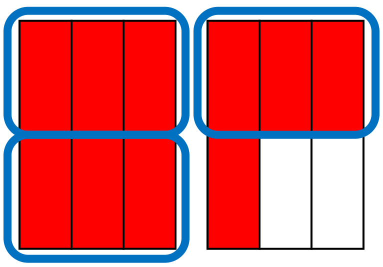
Students will write story problems to represent given numerical expressions.
Before the lesson, review strategies for dividing fractions by highlighting different ways of modeling and the meanings of the quotient, divisor, and dividend.

The Common-Denominator algorithm relies on the measurement or repeated subtraction concept of division.

$\frac{5}{3} \div \frac{1}{2}$ ***means "How many sets of*** $\frac{1}{2}$ ***are in*** $\frac{5}{3}$***?"***

Restated with common denominators: "How many sets of $\frac{3}{6}$ ***are in*** $\frac{10}{6}$***?"***

$\frac{5}{3} \div \frac{1}{2} = \frac{10}{6} \div \frac{3}{6} = 10 \div 3 \text{ or } 3\frac{1}{3}$



With your group write a story problem for each of the expressions shown below.

$2\frac{3}{5} \div \frac{1}{3}$ $\qquad\qquad$ $\frac{2}{3} \div \frac{3}{4}$

Students will work in groups on each of the problems. Then, form new groups by pulling one person from each of the first groups where they will meet with the other students to compare their solutions and plan their presentation to the class. They will need to provide an explanation, any discrepancies they had with group members, and share their strategies.

During the lesson, be sure that students are drawing pictures or using manipulatives to help them think about how to do the problems and explain their thinking.
Look for students who use different representations to think about the problems. Highlight those different ways in the Summary portion of the lesson.

Let each group present their solution to the problem.
Summarize
What does the quotient represent in each of the problems?
What does the divisor represent in each of the problems?
What does the dividend represent in each of the problems?
How would the solutions differ if the dividends were negative?
How would the solutions differ if both the divisor and dividend were negative?

Name___________________________

Do It Yourself Revisited

With your group write a story problem for each of the expressions shown below.

$$2\frac{3}{5} \div \frac{1}{3}$$

$$\frac{2}{3} \div \frac{3}{4}$$

Think about these questions.
What does the quotient represent in each of the problems?

What does the divisor represent in each of the problems?

What does the dividend represent in each of the problems?

How would the solutions differ if the dividends were negative?

How would the solutions differ if both the divisor and dividend were negative?

What Does It Cost?

Back to task table

(Adapted from Florida Center for Instructional Technology, College of Education, University of South Florida)

In this task, students will use multiplication of fractions and whole numbers to determine the change in price for given retail items over a period of time.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

MGSE7.NS.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (- 1)(– 1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

MGSE7.NS.2b Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers then – *(p/q) = (– p)/q = p/(–q)*. Interpret quotients of rational numbers by describing real-world contexts.

MGSE7.NS.2c Apply properties of operations as strategies to multiply and divide rational numbers.

MGSE7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.

BACKGROUND KNOWLEDGE

In order for students to be successful, the following skills and concepts need to be maintained:

- positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, debits/credits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
- rational numbers are points on the number line.
- numbers with opposite signs indicate locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$ $-(-3) = 3$, and that 0 is its own opposite

COMMON MISCONCEPTIONS

The phrase "Please Excuse My Dear Aunt Sally", or more simply PEMDAS, is sometimes used to help students remember the order of operations. Although this mnemonic may be helpful, it often leads students to think that addition is done before subtraction and that multiplication comes before division

Regularly ask students, "Are the parentheses required or optional in this equation?"

Remind students that multiplying a whole number by a fraction, is the same as repeatedly addition the fraction the whole number times (4.NF.4). for example, 5 x ¾ is the same as ¾ + ¾ + ¾ + ¾ + ¾. Students may also inquire about the order for setting up the problem and if it will affect the outcome. This is a good opportunity to review commutative property.

Some students may want to try and turn the decimal into a fraction in order to multiply the whole number. If you are evaluating student understanding of operations with fractions, converting the fraction to a decimal will not demonstrate any understanding of operations with fractions.

ESSENTIAL QUESTIONS

- What strategies are most useful in helping develop algorithms for adding, subtracting, multiplying, and dividing positive and negative rational numbers?

MATERIALS

- Student recording sheet
- Counting chips (optional)
- Calculator (optional)
- Paper and pencil

GROUPING

Individual/Partner

TASK DESCRIPTION

In this lesson, students will be asked to calculate the part of a number that is discounted off of an item over a given period of time. They must then determine the most cost effective way to make purchases of those items in order to get the most for their money.
Begin the lesson with a **Number Talk**. **Number Talks** are a great way for students to use mental math to solve and explain a variety of math problems. A **Number Talk** is a short, ongoing daily routine that provides students with meaningful ongoing practice with computation. **Number Talks** should be structured as short sessions alongside (but not necessarily directly related to) the ongoing math curriculum. It is important to keep **Number Talks** short, as they **are not intended to replace current curriculum or take up the majority of the time spent on mathematics**.

In fact, teachers need to spend only 5 to 15 minutes on Number Talks. **Number Talks** are most effective when done every day. As previously stated, the primary goal of **Number Talks** is computational fluency. Students develop computational fluency while thinking and reasoning like mathematicians. When they share their strategies with others, they learn to clarify and express their thinking, thereby developing mathematical language. This in turn serves them well when they are asked to express their mathematical processes in writing. In order for students to become computationally fluent, they need to know particular mathematical concepts that go beyond what is required to memorize basic facts or procedures.

All **Number Talks** follow a basic six-step format. The format is always the same, but the problems and models used will differ for each number talk.

1. **Teacher presents the problem.** Problems are presented in a word problem or a written algorithm.

2. **Students figure out the answer.** Students are given time to figure out the answer. To make sure students have the time they need, the teacher asks them to give a “thumbs-up” when they have determined their answer. The thumbs up signal, given at chest level, is unobtrusive- a message to the teacher, not the other students.

3. **Students share their answers**. Four or five students volunteer to share their answers and the teacher records them on the board.

4. **Students share their thinking.** Three or four students volunteer to share how they got their answers. (Occasionally, students are asked to share with the person(s) sitting next to them.) The teacher records the student's thinking.

5. **The class agrees on the "real" answer for the problem.** The answer that together the class determines is the right answer is presented as one would the results of an experiment. The answer a student comes up with initially is considered a conjecture. Models and/or the logic of the explanation may help a student see where their thinking went wrong, may help them identify a step they left out, or clarify a point of confusion. There should be a sense of confirmation or clarity rather than a feeling that each problem is a test to see who is right and who is wrong. A student who is still unconvinced of an answer should be encouraged to keep thinking and to keep trying to understand. For some students, it may take one more experience for them to understand what is happening with the numbers and for others it may be out of reach for some time. The mantra should be, "If you are not sure or it doesn't make sense yet, keep thinking."

6. **The steps are repeated for additional problems**.

   Similar to other procedures in your classroom, there are several elements that must be in place to ensure students get the most from their Number Talk experiences. These elements are:

   1. A safe environment
   2. Problems of various levels of difficulty that can be solved in a variety of way

3. Concrete models
4. Opportunities to think first and then check
5. Interaction
6. Self-correction

For this **Number Talk**, begin with the following problem, "1/4 of 24" Record the problem on the far left side of the board. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the "thumbs up" signal, call on several students (3-4) to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you. It is important to allow students ownership of their thinking.

Record, "1/3 of 24" on the board next to the previous problem. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the "thumbs up" signal, call on several students (3-4) to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you.

Record, "3/4 of 24" on the board next to the previous problem. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the "thumbs up" signal, call on several students (3-4) to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you.

Record, "2/3 of 24" on the board towards the far right. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the "thumbs up" signal, call on several students (3-4) to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you.

At the end of the Number Talk, discuss the strategies used to find the answers. Some of the strategies students may use are imaging a representation of 24 objects separated into groups, multiplying the numerator times the whole number and dividing by the denominator, finding the answer using a unit fraction and doubling or tripling the answer. Talk with the students about which strategy was most efficient (quick, easy and accurate).

Allow for a maximum of 15 minutes to conduct the **Number Talk** before moving into the lesson.

Solve this problem:
A warehouse sells clothing at a fraction of its original cost. The table below shows the fraction off the original price for clothing that remains in the warehouse after 10 days, 20 days, and 30 days.

WAREHOUSE PRICES				
Item	**Price**	**After 10 Days**	**After 20 Days**	**After 30 Days**
Jacket	$120	1/4 off	1/3 off	1/2 off
Shoes	$40	1/5 off	2/5 off	3/5 off
Shirt	$12	1/4 off	1/3 off	2/3 off

1. Find the price of the items after each 10-day period to complete the chart below. Write a numerical expression to show how you arrived at each solution.

WAREHOUSE PRICES

ITEM	PRICE	AFTER 10 DAYS	AFTER 20 DAYS	AFTER 30 DAYS
Jacket	$120	***120 - (1/4)(120) = $90.00***	***120 – (1/3)(120) =$80.00***	***120-(1/2)(120)=$60.00***
Shoes	$40	***40 – (1/5)(40) = $32.00***	***40 – (2/5)(120)=$24.00***	***40-(3/5)(40)=$16.00***
Shirt	$12	***12 – (1/4)(12) = $9.00***	***12-(1/3)(12)=$8.00***	***12-(2/3)(12)=$4.00***

Suppose you have $100 to spend…

2. Within the range of $90.00 to $100.00, what are all possible combinations of items you could buy after 10 days?
Solution: 1 jacket + 1 shirt, 11 shirts, 3 shoes, 2 shoes + 4 shirts

3. Would you have enough money to buy 2 jackets after 20 days? Justify your solution.
Solution: Two jackets would cost $160.00 at 20 days and you only have $100.

4. Could you buy all 3 items if you waited until after 30 days? Remember, you only have $100 to spend. Justify your answer.

   Solution: The three items would cost you a total of $80. Since you have $100 to spend you do have enough money to buy all three items after 30 days.

5. Jackets cost the retailer $75 a piece to purchase wholesale. If he has five to sell and doesn't sell them until 30 days have gone by, how much money does he lose? Remember a loss is shown with a negative rational number.

   Solution: Five jackets will be sold for a total of $300.00. The retailer paid $375 for the jackets. He will show a loss of -$75 if they don't sell until 30 days. He will lose -$15 per jacket.

FORMATIVE ASSESSMENT QUESTIONS

- How can we find part of a number? What does the phrase “part of a number” tell you about the process?
- How can you use the chart to justify the better deal for the seller and the buyer?

DIFFERENTIATION

Extension

- Have students create their own fraction chart that would save money for the seller and then a separate chart that would be a better buy for the buyer.
- After how many days will the seller break even for each retail item?

Intervention

- Change the fractions to quarters or fifths to make the mathematics easier but retain the integrity of the task.
- Use base ten blocks to model the quantities and show a fraction of the quantity.

What Does It Cost?

Solve this problem:
A warehouse sells clothing at a fraction of its original cost. The table below shows the fraction off the original price for clothing that remains in the warehouse after 10 days, 20 days, and 30 days.

WAREHOUSE PRICES				
Item	**Price**	**After 10 Days**	**After 20 Days**	**After 30 Days**
Jacket	$120	1/4 off	1/3 off	1/2 off
Shoes	$40	1/5 off	2/5 off	3/5 off
Shirt	$12	1/4 off	1/3 off	2/3 off

1. Find the price of the items after each 10-day period to complete the chart below. Show how you arrived at each answer.

WAREHOUSE PRICES				
ITEM	**PRICE**	**AFTER 10 DAYS**	**AFTER 20 DAYS**	**AFTER 30 DAYS**
Jacket	$120	$90.00		
Shoes	$40			
Shirt	$12			$4.00

Suppose you have $100 to spend…

2. Within the range of $90.00 to $100.00, what are all possible combinations of items you could buy after 10 days?

3. Would you have enough money to buy 2 jackets after 20 days? Justify your solution.

4. Could you buy all 3 items if you waited until after 30 days? Remember, you only have $100 to spend. Justify your answer.

5. Jackets cost the retailer $75 a piece to purchase wholesale. If he has five to sell and doesn't sell them until 30 days have gone by, how much money does he lose? Remember a loss is shown with a negative rational number.

Converting Fractions to Decimals

Back to task table

In this task, students will convert fractions to decimals.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

MGSE7.NS.2d Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

TASK MISCONCEPTIONS

When dividing a smaller dividend by a larger divisor, students will communicate they are adding a decimal point and zero in order to continue dividing. The purpose of this task is to demonstrate the decimal point and zero comes from exchanging the whole number for equivalent tenths. For example, 3 wholes are exchanged for 30 tenths, which is equivalent to 3.0.

ESSENTIAL QUESTIONS

- What are the steps to converting a rational number to a repeating or terminating decimal?

MATERIALS

- Base ten blocks
- Converting Fractions to Decimals recording

TASK DESCRIPTION

Level 1
Define.
Fraction- ***part of a whole***

Decimal- ***part of a whole***

Numerator- ***equal parts added to make the total, dividend***

Denominator- ***Total of equal parts, divisor***

Using the base ten blocks, model the following problems. Draw your models below. ***For each model, students can use squares to represent whole, sticks to represent tenths and dots to represent hundredths.***

$1 \div 2$ or $\frac{1}{2}$

$1 \div 5$ or $\frac{1}{5}$

$3 \div 4$ or $\frac{3}{4}$

$4 \div 5$ or $\frac{4}{5}$

What did you discover happened to the dividend each time?

The dividend must be exchanged for tenths in order to divide it into the given groups.

Use the base ten blocks to model

$1 \div 3$

$1 \div 6$

What do you notice?

Students should discover they will need to continuously make an exchange for smaller quantities in order to divide. They should conclude the same exchange will be made continuously resulting in a repeating decimal.

Level 2

Use long division to divide the following:

$1 \div 2$ or $\frac{1}{2}$

$1 \div 5$ or $\frac{1}{5}$

How is the abstract procedure connected to the models in level 1?

Students should conclude when adding a decimal point and the zero it represents the exchange of the whole to the tenths.

Level 3
In words explain why the "add a decimal and zero" "rule" works.
Students should be able to explain the rule works because it shows the whole number equivalent to the tenths. Therefore you are dividing tenths and not wholes.

In words explain how to convert 4/9 to a decimal.

4 wholes are exchanged for 40 tenths and represented as 4.0. 4 tenths are in each group and 4 tenths are left over. The 4 tenths are exchanged for 40 hundredths and 4 hundredths are in each group. This pattern will continue resulting in a repeating decimal.

Practice problems:

1. Convert 2/5 to a decimal
   0.4

2. Convert 6/8 to a decimal
   0.75

3. Convert 5/11 to a decimal.$0.\overline{45}$

4. Convert 9/10 to a decimal using long division.
   0.9

Converting Fractions to Decimals

Level 1

Define.

Fraction-

Decimal-

Numerator-

Denominator-

Using the base ten blocks, model the following problems. Draw your models below.

$1 \div 2$ or $\frac{1}{2}$

$1 \div 5$ or $\frac{1}{5}$

$3 \div 4$ or $\frac{3}{4}$

$4 \div 5$ or $\frac{4}{5}$

What did you discover happened to the dividend each time?

Use the base ten blocks to model

$1 \div 3$

$1 \div 6$

What do you notice?

Level 2

Use long division to divide the following:

$1 \div 2$ or $\frac{1}{2}$

$1 \div 5$ or $\frac{1}{5}$

How is the abstract procedure connected to the models in level 1?

Level 3

In words explain why the "add a decimal and zero" "rule" works.

In words explain how to convert 4/9 to a decimal.

Practice problems:

1. Convert 2/5 to a decimal
2. Convert 6/8 to a decimal
3. Convert 5/11 to a decimal
4. Convert 9/10 to a decimal using long division

The Repeater vs The Terminator

Back to task table

This activity was adapted from Van de Walle, J., &Lovin, L.H. (2006). *Teaching student-centered mathematics: Grades 5-8*. Boston, MA: Pearson Education and Allyn& Bacon, 148.

In this task, students will **convert fractions to decimals.** This activity also involves **prime factorization**. Prime and composite numbers as well as prime factorization of numbers may need to be revisited prior to doing this task.

Also, please be aware that calculators may round a repeating decimal and students may interpret this to mean that a fraction is terminating. Please address this with your students prior to beginning the task.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

MGSE7.NS.2d Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

TASK MISCONCEPTIONS

Students may struggle with finding the patterns in section one. There may need to be more of a class discussion in order to guide students through finding the patterns.

ESSENTIAL QUESTIONS

- What are the steps to converting a rational number to a repeating or terminating decimal?

MATERIALS

- **Calculator - ****Note:*** This task can be done by hand, but calculations are much more efficient via calculator.

TASK DESCRIPTION

Part One

The chart below includes 13 unit fractions, fractions with a numerator of 1. For this activity, first determine the prime factorization of the denominator of the unit fraction. Then, turn the fraction into a decimal and determine whether the fraction is repeating or terminating.

UNIT FRACTION	PRIME FACTORIZATION OF DENOMINATOR	DECIMAL FORM	TERMINATES OR REPEATS
$\frac{1}{2}$	PRIME	.5	Terminates
$\frac{1}{3}$	PRIME	$.\overline{3}$	Repeats
$\frac{1}{4}$	$2 \cdot 2$	*.25*	Terminates
$\frac{1}{5}$	***PRIME***	*.2*	***Terminates***
$\frac{1}{6}$	$2 \cdot 3$	$.1\overline{6}$	Repeats
$\frac{1}{7}$	***PRIME***	$.\overline{142857}$	***Repeats***
$\frac{1}{8}$	$2 \cdot 2 \cdot 2$	*.125*	***Terminates***
$\frac{1}{9}$	$3 \cdot 3$	$.\overline{1}\overline{1}$	***Repeats***
$\frac{1}{10}$	$2 \cdot 5$	*.1*	Terminates
$\frac{1}{11}$	***PRIME***	$.\overline{09}$	***Repeats***
$\frac{1}{12}$	$2 \cdot 2 \cdot 3$	$.08\overline{3}$	***Repeats***
$\frac{1}{13}$	PRIME	$.\overline{076923}$	***Repeats***

Answer the following questions based upon your results from the chart.

1. Which fractions terminate?
   $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}, \frac{1}{8}, \frac{1}{10}$

2. Give another example of a fraction that can be turned into a terminating decimal. Justify why this fraction is a terminating decimal.
   Answers may vary. Student justification should mention relationship of denominator to powers of 10 or provide a proof by long division to demonstrate that students understand the difference between terminating and repeating decimals

3. Consider the fractions $\frac{1}{3}, \frac{1}{7}$, and $\frac{1}{11} \frac{1}{3}, \frac{1}{7}$, and $\frac{1}{11}$. What do these fractions have in common?

   These are all repeating decimals and their denominators are prime numbers.

4. What do you hypothesize about rational numbers with denominators that are prime numbers?
   Students may recognize from the previous question that generally, fractions with denominators that are prime numbers are repeating decimals. There are two exceptions: ½ and 1/5, but both can be written as fractions that can be written as a power of 10.

Part Two
Convert the following fractions into decimals. (You have calculated a few of them before!)

Fraction	Decimal	Fraction	Decimal	Fraction	Decimal	Fraction	Decimal
$\frac{1}{4}$	*0.25*	$\frac{1}{8}$	*0.125*	$\frac{1}{9}$	$0.\bar{1}$	$\frac{1}{11}$	$0.\overline{09}$
$\frac{2}{4}$	*0.50*	$\frac{3}{8}$	*0.375*	$\frac{2}{9}$	$0.\bar{2}$	$\frac{2}{11}$	$0.\overline{18}$
$\frac{3}{4}$	*0.75*	$\frac{5}{8}$	*0.625*	$\frac{3}{9}$	$0.\bar{3}$	$\frac{3}{11}$	$0.\overline{27}$
$\frac{4}{4}$	*1*	$\frac{7}{8}$	*0.875*	$\frac{4}{9}$	$0.\bar{4}$	$\frac{4}{11}$	$0.\overline{36}$

Note: The first two columns are designed to help students recognize basic benchmark fractions. Teachers may add additional discussion questions about these fractions to help students make connections between fourths and eighths to their decimals.

1. Do you notice a pattern between the fractions with denominators of 9 and their decimals? If so, what is the pattern?

   Yes, there is a pattern. The numerator repeats in the tenths place.

2. **<u>Without using a calculator,</u>** what is the decimal form of $\frac{8}{9}$?
   $0.\overline{8}$

3. Do you notice a pattern between the fractions with denominators of 11 and their decimals? If so, what is the pattern?

   Yes, there is a pattern. If you multiply the numerator by 9, that is the repeating decimal.

4. **<u>Without using a calculator,</u>** what is the decimal form of $\frac{9}{11}$? $0.\overline{81}$

<u>DIFFERENTIATION</u>

Extensions:

- Students may learn the relationship between $0.\overline{9}$ and 1 and provide a proof or justification for why these two numbers are equal
- Students can begin to find relationships of rational numbers that are more complex than basic unit fractions or basic benchmark fractions

Intervention:

- For students who struggle with converting fractions and decimals, teachers may want to reverse the order of the two charts. By starting with the second chart, the first two columns can be used to review how to turn a fraction into a decimal, and they last two will allow students an opportunity to look for patterns.

The Repeater vs. The Terminator

Part One

The chart below includes 13 **unit fractions**, fractions with a numerator of 1. For this activity, first determine the prime factorization of the denominator of the unit fraction. Then, turn the fraction into a decimal and determine whether the fraction is repeating or terminating.

UNIT FRACTION	PRIME FACTORIZATION OF DENOMINATOR	DECIMAL FORM	TERMINATES OR REPEATS
$\frac{1}{2}$	PRIME	.5	Terminates
$\frac{1}{3}$	PRIME	$.\overline{3}$	Repeats
$\frac{1}{4}$	$2 \cdot 2$		Terminates
$\frac{1}{5}$			
$\frac{1}{6}$		$.\overline{16}$	Repeats
$\frac{1}{7}$			
$\frac{1}{8}$	$2 \cdot 2 \cdot 2$		
$\frac{1}{9}$		$.\overline{1}\overline{1}$	
$\frac{1}{10}$			Terminates
$\frac{1}{11}$			
$\frac{1}{12}$			
$\frac{1}{13}$	PRIME		

Answer the following questions based upon your results from the chart.

1. Which fractions terminate?

2. Give another example of a fraction that can be turned into a terminating decimal. Justify why this fraction is a terminating decimal.

3. Consider the fractions $\frac{1}{3}, \frac{1}{7}$, and $\frac{1}{11}$. What do these fractions have in common?

4. What do you hypothesize about rational numbers with denominators that are prime numbers?

Part Two
Convert the following fractions into decimals. (You have calculated a few of them before!)

Fraction	Decimal	Fraction	Decimal	Fraction	Decimal	Fraction	Decimal
$\frac{1}{4}$		$\frac{1}{8}$		$\frac{1}{9}$		$\frac{1}{11}$	
$\frac{2}{4}$		$\frac{3}{8}$		$\frac{2}{9}$		$\frac{2}{11}$	
$\frac{3}{4}$		$\frac{5}{8}$		$\frac{3}{9}$		$\frac{3}{11}$	
$\frac{4}{4}$		$\frac{7}{8}$		$\frac{4}{9}$		$\frac{4}{11}$	

1. Do you notice a pattern between the fractions with denominators of 9 and their decimals? If so, what is the pattern?

2. <u>**Without using a calculator,**</u> what is the decimal form of $\frac{8}{9}$?

3. Do you notice a pattern between the fractions with denominators of 11 and their decimals? If so, what is the pattern?

4. <u>**Without using a calculator,**</u> what is the decimal form of $\frac{9}{11}$?

A Poster

Back to task table

In this task, students will create a poster or electronic presentation that demonstrates their understanding of rational numbers including common misconceptions. This task reviews material from the 6^{th} and 7^{th} grade as it relates to integers, absolute value, comparing and ordering rational numbers, and real-life applications of integers.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

MGSE7.NS.1a Show that a number and its opposite have a sum of 0 (are additive inverses). Describe situations in which opposite quantities combine to make 0. *For example, your bank account balance is -$25.00. You deposit $25.00 into your account. The net balance is $0.00.*

MGSE7.NS.1b Understand p + q as the number located a distance $|q||q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Interpret sums of rational numbers by describing real world contexts.

MGSE7.NS.1c Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

MGSE7.NS.1d Apply properties of operations as strategies to add and subtract rational numbers.

MGSE7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

MGSE7.NS.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (- 1)(– 1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

MGSE7.NS.2b Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

MGSE7.NS.2c Apply properties of operations as strategies to multiply and divide rational numbers.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Solve problems and persevere.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
6. Attend to precision.
8. Look for and express regularity in repeated reasoning.

COMMON MISCONCEPTIONS

The phrase "Please Excuse My Dear Aunt Sally", or more simply PEMDAS, is sometimes used to help students remember the order of operations. Although this mnemonic may be helpful, it often leads students to think that addition is done before subtraction and that multiplication comes before division

This culminating task allows teachers to see where students are still struggling with operations of rational numbers. Misconceptions will become evident as students work their way through the different aspects of the task. In order to catch these and help students gain understanding, it is important students move through the class questioning and reviewing material as students create their posters.

ESSENTIAL QUESTIONS

- What strategies are most useful in helping develop algorithms for adding, subtracting, multiplying, and dividing positive and negative rational numbers?
- What are the steps to converting a rational number to a repeating or terminating decimal?

MATERIALS

- poster board
- computer

GROUPING

- Individual / Partner

TASK DESCRIPTION

You are to make and present a poster showing what you have learned from your study of positive and negative rational numbers. Choose a theme for your poster. Be creative!

Choose five rational numbers. Choose **one integer**, ***one negative fraction***, **one positive fraction, *one negative decimal*** and **one positive decimal**. **ALL** of your numbers MUST be between –1 and 1. Make your poster using the information below:

Comparing

- Use the >, <, and = to compare the two negative numbers.
- Use the >, <, and = to compare the two positive numbers.
- Graph all five numbers on a number line.

Absolute value

- Write the absolute value of **each** number and explain your reasoning.

Number problems

- You must choose from your five original numbers.

- **Create two addition problems;** one using numbers with like signs and the other using numbers with different signs.
- **Create two subtraction problems**; one using numbers with like signs and the other using numbers with different signs.
- **Create two multiplication problems**; one using numbers with like signs and the other using numbers with different signs.
- **Create two division problems**; one using numbers with like signs and the other using numbers with different signs.
- **Create two problems with <u>at least</u> two different operations**, using both positive and negative numbers

Three real-life problems with solutions

- **Write three real-life problems** involving rational numbers and solve to show their solutions. <u>Use a different operation in each problem.</u>

Rules and common misconceptions

- List any rules you have found for computing with positive and negative numbers
- Give examples of common misconceptions, or problems, students have when working with positive and negative numbers

<u>Comments:</u>
Throughout this culminating activity, or one similar in rigor and depth, the teacher should monitor students for understanding and help students correct any obvious weaknesses.

This task is extensive and students should not be expected to complete each part in one class period. Students should be allowed to represent their answers in a variety of ways to show their mastery of the concepts of this unit. Many teachers might also have students share their posters or presentations with verbal explanations to their peers during a classroom presentation.

For assessing this task, teachers should determine whether the skills listed on the poster rubric are performed accurately and with consistency. Models, rules stated in the students' own words, and explanations should all be included.

The following sections are sample solutions using the numbers –0.5, -1/3, +0.25 and +2/9.
Compare Rational Numbers: -1/3 = -.3333…
-0.5 < -1/3,

<u>Absolute value section</u>
Students should demonstrate an understanding that absolute value is the distance on the number line between the selected number and zero.
<u>Using the four basic operations</u>

Addition: Using the given numbers students could select –0.5 + -1/3 =-1/2 + -1/3 =-3/6 + - 2/6 = -5/6. In this case, students would also show they have mastered using decimals and fractions interchangeably.
Subtraction: Students showing 2/9 – (-0.5) = 4/18 + 9/18 = 13/18 would show mastery of subtraction operations with rational numbers.
Multiplication: 2/9 * (-1/3) = -2/27
Division: 2/9 (-1/3) = 2/9 * -3/1 = -6/9 = -2/3

At least TWO different operations: $\frac{1}{3}-(0.25)+-0.5=\frac{4}{12}-\frac{3}{12}+-\frac{6}{12}=-\frac{5}{12}$
****Note: This will be a good way to assess whether students understand order of operations***

Three real life problems with solutions

In this section, students will demonstrate how accurately they can apply the concepts of positive and negative rational numbers to real world situations. Teachers could expect to read problems involving game scores, temperatures, profit and loss, etc. Answers will vary.

Rules and Common Misconceptions

Students should state rules and properties in their own words. Having students identify common misunderstandings may be helpful in determining misunderstandings that they still have in dealing with rational numbers.

DIFFERENTIATION

Extension:
- Students can create different generalized scenarios of topics discussed within this task.
- Students can work together to develop formal rules and properties and provide justifications for why those rules and properties are applicable

Intervention:
- Manipulatives and other resources can help to support students who are struggling
- The task can also be modified by providing the 5 starting numbers to simplifying to only using three rational numbers.

A Poster

You are to make and present a poster or electronic presentation showing what you have learned from your study of positive and negative rational numbers. Choose a theme for your poster. Be creative!

Choose five rational numbers. Choose **one integer**, ***one negative fraction***, **one positive fraction,** ***one negative decimal*** and **one positive decimal**. **ALL** of your numbers MUST be between –1 and 1. Make your poster using the information below:

Comparing

- Use the >, <, and = to compare the two negative numbers.
- Use the >, <, and = to compare the two positive numbers.
- Graph all five numbers on a number line.

Absolute value

- Write the absolute value of **each** number and explain your reasoning.

Number problems

- You must choose from your five original numbers.

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- **Create two subtraction problems**; one using numbers with like signs and the other using numbers with different signs.
- **Create two multiplication problems**; one using numbers with like signs and the other using numbers with different signs.
- **Create two division problems**; one using numbers with like signs and the other using numbers with different signs.
- **Create two problems with at least two different operations**, using both positive and negative numbers

Three real-life problems with solutions

- **Write three real-life problems** involving rational numbers and solve to show their solutions. Use a different operation in each problem.

Rules and common misconceptions

- List any rules you have found for computing with positive and negative numbers
- Give examples of common misconceptions, or problems, students have when working with positive and negative numbers

Whodunit? The Undoing of (-7)

Back to task table

This task has been adapted from http://differentiatedlessons.blogspot.com/2005/06/integers-grade-7-math-assignment-raft.html.
Hartnett, K. & Thomas, W., ( ).Who dunit? The Undoing of (-7).Retrieved February 2012 from http://differentiatedlessons.blogspot.com/2005/06/integers-grade-7-math-assignment-raft.html.

In this task, students will investigate a murder mystery whodunit. The teacher should tape off a silhouette of (-7) on the floor, decorate the room with caution tape and play sound bytes of screams or the theme song from CSI to engage student interest. This task also review 6^{th} grade standards in this area.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

MGSE7.NS.1a Show that a number and its opposite have a sum of 0 (are additive inverses). Describe situations in which opposite quantities combine to make 0. *For example, your bank account balance is -\$25.00. You deposit \$25.00 into your account. The net balance is \$0.00.*

MGSE7.NS.1b Understand p + q as the number located a distance $|q||q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Interpret sums of rational numbers by describing real world contexts.

MGSE7.NS.1c Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (- q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

MGSE7.NS.1d Apply properties of operations as strategies to add and subtract rational numbers.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.

COMMON MISCONCEPTIONS

Ideally, students need to work as individuals. If they only do part of the four roles, they are not addressing all the intended standards for the task.

ESSENTIAL QUESTIONS

- What strategies are most useful in helping develop algorithms for adding, subtracting, multiplying, and dividing positive and negative rational numbers?

MATERIALS

- Student Task sheet with descriptors of the characters

GROUPING

This is intended to be **RAFT** (Role, Audience, Format, Topic) assignment. Individually each student should work on all four roles. They are meant to be quick assignments and no more than thirty minutes each. Students should read each line of the chart to find out the requirements for that role. If students work in pairs or groups they are not being assessed on all four components of the culminating task.

On the first and second role, the students will have different answers depending on how the teacher facilitates the assignment. It is intentionally left open ended so students can create their own scenario for that role.

TASK DESCRIPTION

Prior to the task, review with students the rubric or process of evaluating the performance assessment. Explain to students that they may have to use different representation of addition/subtraction models to keep up with what is happening in the story.

Task Instructions

Culminating Task: WHODUNIT? THE UNDOING OF -7.

ROLE	AUDIENCE	FORMAT	TOPIC
Police Investigator	(-7)'s Family	Wanted Poster	Integer
Forensic Expert	CSI Team	Case Study	Whodunit
Funeral Director	Newspaper Readers	Obituary	Integers
Grave Digger	Plot Committee	Map (-7)'s grave	Plotting Points

Grading: You will be graded based on the four roles.

Role #1: **Police Investigator**
Neat/Complete /5
Use of details /10
Evidence of Learning /10

Role #2: **Forensic Expert**
Neat/Complete /5
Use of details /10
Evidence of learning /10

Role #3: **Funeral Director**
Neat/Complete /5
Use of details /10
Evidence of Learning /10

Role #4: **Grave Digger**
Neat/Complete /5
Use of details /10
Evidence of learning /10

Total Grade for the assignment: 100 points.

Role 1: Your goal is to act as police investigator to determine family members and obtain a composite sketch of the suspect to put on a wanted poster.

Your assignment as POLICE INSPECTOR is to interview the family of (-7) in order to narrow down the suspect list. The family consists of two numbers who can be added together to make (-7). All family members are currently considered suspects as (-7) was at a family reunion at the time of the unfortunate event.

The only two clues that the police actually know is that the unfortunate event was committed by two people, and that they were related to (-7).

Please make a list of relatives of (-7) and describe how you know that they are related.

Draw a wanted poster to notify people to be on the lookout for the list of possible suspects. The community of Pythagoras is very nervous and wants the perpetrator caught immediately. Please include reward, a picture, and a description of the suspects. You can draw different wanted posters for the different pairs of subjects.

Role 2: Your goal is to act as Forensic expert. You have to retrace (-7)'s steps on the night of the unfortunate event. Also, for each stop, you will draw a symbol to represent where he stopped, and trace the trail in the appropriate color. Also, for each stop, you will write down the mathematical equation that represents how many steps were taken between stops.

Draw a number line to retrace the steps. This will act as your map of Pythagoras.
Here were his steps the night of the unfortunate event.

- Negative Seven lives at (-7) Geometry Lane.
- He went to visit (-2) to borrow some cinnamon for a new tessellation tart recipe he wanted to try to bring to the reunion. How many steps did he have to take to get to -2's house? Please write an equation to show the distance he traveled. Color this part of the number line yellow for this part of the journey.

- According to (-2), while (-7) was standing on the doorstep with cinnamon in hand, he spied Mr. Positive Eight's dog sniffing around the fire hydrant at +1 Street. He ran from the house, cinnamon trailing behind him to catch the little dog. After all, he needs to be nice to Positive Eight as he is his boss at the Metric Factory. (-7) could really use a promotion. If (-7) started from (-2)'s house, how many houses did he pass in order to catch the dog at (+1)? Please write an equation to show how many steps he traveled. Also color the trail from (-2) to the dog on (+1) brown–oops that can of cinnamon was opened!

- The last -2 saw of (-7) was that he was running down the street with the dog under his arm. Oddly enough, when the experts were investigating, the cinnamon trail led to (+7)'s back yard!

- Negative Six, upon being asked, confirmed that (+7) was (-7)'s mortal enemy since birth! When asked where (+7) was the night of the unfortunate event, (+7) simply stated that he was watching a movie. There were no alibis. And weirdly enough, there were dog prints found in the mud in the front yard. Dog prints that went from (+2)'s house to (+7)'s house! Draw small black dog tracks from (+2) to (+7) and write an equation to represent the path of the dog. Also, Positive Six was sure that she heard dog barking noise coming from (+7)'s house even though she swears up and down that Positive Seven does not even own a dog!

- There were many clues, but the trail was getting cold. What happened to (-7) after he picked up the dog? Hmmm........

- Do some investigating and finish the trail of (-7) based on your own theory. You need to include at least 3 stops and three equations. Ultimately, who do you think did it and why? Be creative and use equations to support your findings. Also, keep in mind that (-7)'s unfortunate remains were found at the family reunion site at (-5)'s house.

- Your number line needs to be neat and easy to understand. Also you need to write a paragraph explaining what you believe happened to (-7) after the dog incident. Don’t forget your equations!

<u>Role 3</u>: Your goal is to act as funeral director and write an obituary, a tribute to the great life that (-7) led. Included in the obituary is the time and date of funeral, (-7)’s accomplishments, and his family and friends who are grieving his loss. Please include at least 10 grieving family members and/or friends. Many would be very offended if they were not mentioned! This obituary needs to be typed. Please use the newspaper obituaries as an example.

(-7) had many family members and friends. Any combination of integers added or subtracted together to equal (-7) were relatives and friends.

[An example of a family related to (-7) are the Centimeters. In the Centimeter family, there is (-5), the father, (-4) the mother, and (+2), the baby. We know that this family is related because when you add them together, they add up to (-7).]

<u>Role 4</u>: Your goal is to design a cemetery on the coordinate grid that will be the final resting place for (-7). His family wants to see all of the locations in the cemetery that add to or subtract to (-7). Remember, they will be walking, so each step counts as 1 unit.

Whodunit? The Undoing of (-7)

ROLE	AUDIENCE	FORMAT	TOPIC
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Evidence of Learning /10

Role #4: **Grave Digger**
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Total Grade for the assignment: 100 points.

Role 1: Your goal is to act as police investigator to determine family members and obtain a composite sketch of the suspect to put on a wanted poster.

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<u>**APPENDIX OF ADDITIONAL RESOURCES**</u> **Back to task table**

Models for Teaching Operations of Integers

These models have been adapted from http://teachers.henrico.k12.va.us/math/hcpsalgebra1/.

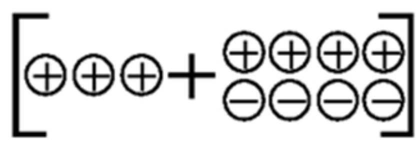
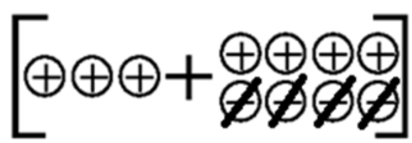
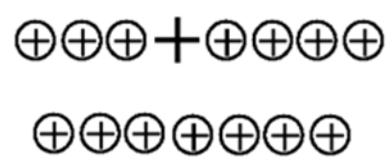
The following are some everyday events that can be used to help students develop a conceptual understanding of addition and subtraction of integers.

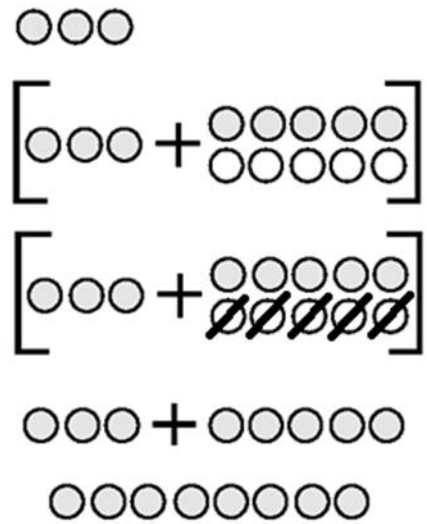
- Using a credit card example can make this subtraction concept clearer. If you have spent money you don't have (-5) and paid off part of it (+3), you still have a negative balance (-2) as a debt, or (-5) + 3 = (-2).

- Draw a picture of a mountain, the shore (sea level) and the bottom of the ocean. Label sea level as 0.

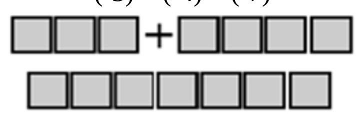
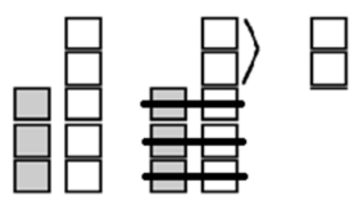
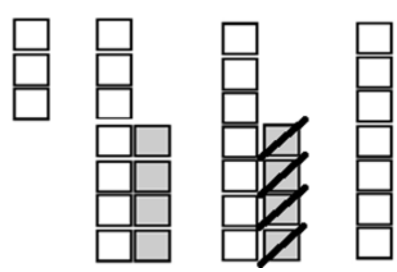
Any of the following models can be used to help students understand the process of adding or subtracting integers. If students have trouble understanding and using one model you can show students how to use another model.

1. The Charged Particles Model (same as using two-color counters)

When using charged particles to subtract, 3 – (-4) for example, you begin with a picture of 3 positive particles.	
Since there are no negative values to “take away”, you must use the Identity Property of Addition to rename positive 3 as 3 + 0. This is represented by 4 pairs of positive and negative particles that are equivalent to 4 zeros.	
Now that there are negative particles, you can “take away” 4 negative particles.	
The modeled problem shows that the result of subtracting 4 negative particles is actually like adding 4 positive particles. The result is 7 positive particles. This is a great way to show why 3 – (-4) = 3 + 4 = 7	

Two-Color Counters Method	
When using two-colored counters you would use the yellow side to represent positive integers and the red side to represent negative numbers. The problem represented is -3 – 5.	

2. The Stack or Row Model

To model positive and negative integers, use colored linking cubes and graph paper. Graph paper and colored pencils will allow students to record problems and results. Students should also write the problems and answers numerically.	
Create stacks or rows of numbers with the colored linking cubes and combine/compare the cubes. If the numbers have the same sign, then the cubes will be the same color. Stress that adding is like *combining*, so make a stack or row to show this.	(-3) + (-4) = (-7) 
If the numbers are not the same sign (color), for example -3 + 5, you compare the stacks of different colors.Using the concept of zero pairs, the result is the difference between the stacks or the result is based on the higher stack. This is easy to see and understand.	
For subtraction you create zeros by pairing one of each color. Then add as many zeros to the first number as needed so that you can take away what the problem calls for. Now physically take away the indicated amount and see what is left. The example problem shown is 3 – (-4).	

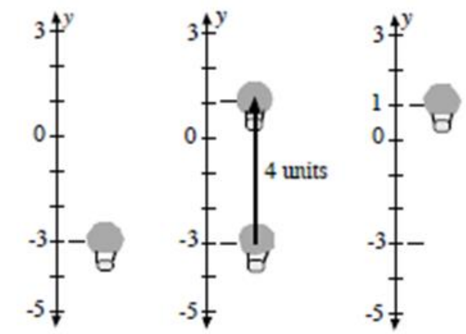
3. The Hot Air Balloon Model

Sand bags *(negative integers)* and Hot Air bags *(positive integers)* can be used to illustrate operations with integers. Bags can be put on (added to) the balloon or taken off (subtracted).

Here is an example: -3 - (-4) = ?

- The balloon starts at -3 (think of the balloon being 3 feet below sea level or 3 feet below the level of a canyon) and you take off 4 sand bags.
- Now, think about what happens to a balloon if you remove sand bags, the balloon gets lighter. So, the balloon would go up 4 units.

If you think in terms of a *vertical number line*, it would start at -3 and end up at 1, so -3 - (-4) = 1. To help students make the connection between -3 - (-4) and -3 + (+4), present the addition and subtraction questions using the same numbers.

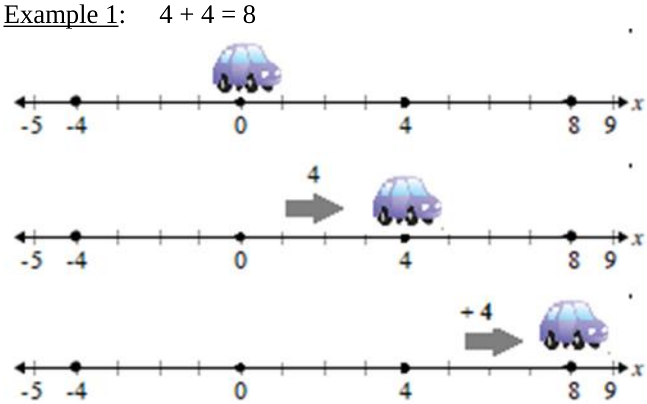


Another example would include the first addition question as 9 + (-5) and the first subtraction question would then be 9 - (+5). The students see that *putting on* 5 sand bags (negative) produces the same result as *taking off* 5 hot air bags (positive).

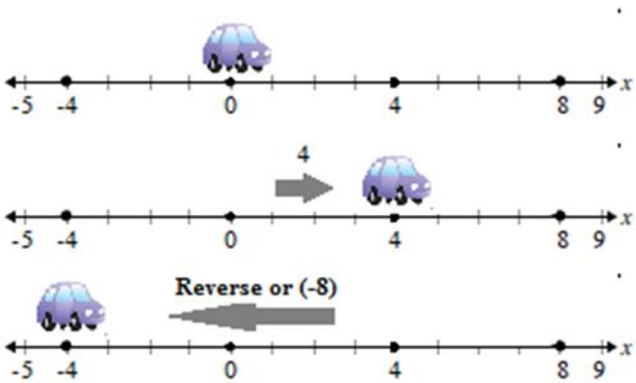
4. The Number Line Model

You can describe addition and subtraction of integers with a number line and a toy car. The car faces forward (to the right) to represent a positive direction. The car is moved forward to represent a positive integer. The car flips around backward (facing left) to represent a negative direction or subtraction. The car is moved backward (reverse) to represent a negative integer.

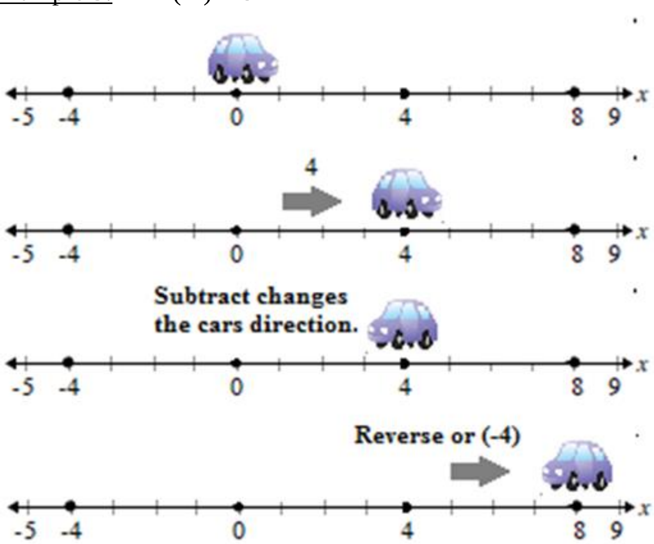
<u>Example 1</u>: 4 + 4 = 8



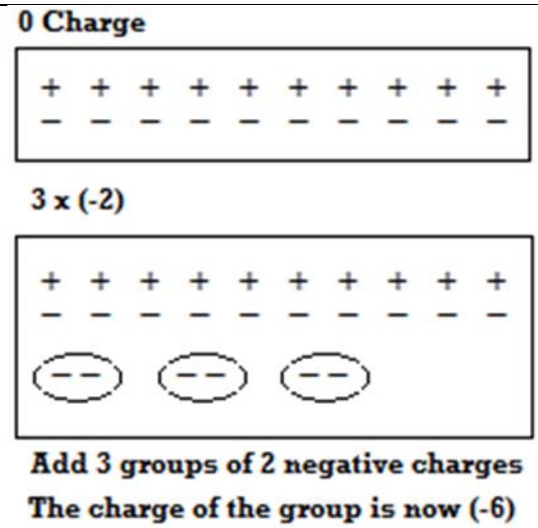
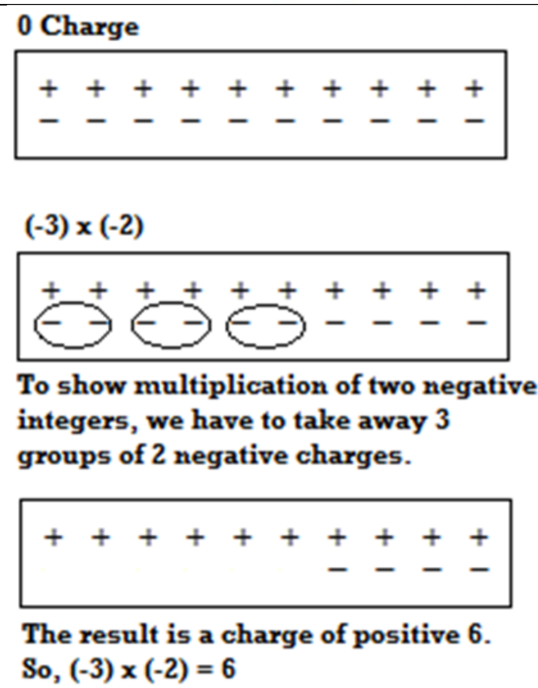
Example 2: $4 + (-8) = -4$



Example 3: $4 – (-4) = 8$



5. Charged Particle Model for Multiplication

The charged particle method can be used to illustrate multiplication of integers.	
To begin, a model with a 0 charge is illustrated. The 0 charge model will allow us to work with positive and negative integers. <u>Example 1</u>: In this problem, 3 x (-2), three groups of two negative charges is added to the 0 charged field. The result is (-6).	
<u>Example 2</u>: (-3) x (-2) = ?	

For more resources of multiplication and division of integers, see pages 144 – 146 of Van de Walle, J., & Lovin, L.H. (2006). *Teaching student-centered mathematics: Grades 5-8.* Boston, MA: Pearson Education and Allyn & Bacon

TECHNOLOGY RESOURCES

[Back to task table]

MGSE7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

https://www.illustrativemathematics.org/7.NS.A
http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132

MGSE7.NS.1a Show that a number and its opposite have a sum of 0 (are additive inverses). Describe situations in which opposite quantities combine to make 0. *For example, your bank account balance is -$25.00. You deposit $25.00 into your account. The net balance is $0.00.*

http://www.nzmaths.co.nz/search/node/integers

https://www.illustrativemathematics.org/7.NS.A

http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132

MGSE7.NS.1b Understand p + q as the number located a distance $|q||q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Interpret sums of rational numbers by describing real world contexts.

http://www.nzmaths.co.nz/search/node/integers
https://www.illustrativemathematics.org/7.NS.A
http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132

MGSE7.NS.1c Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

http://www.nzmaths.co.nz/search/node/integers

https://www.illustrativemathematics.org/7.NS.A

http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132

MGSE7.NS.1d Apply properties of operations as strategies to add and subtract rational numbers.

http://www.openmiddle.com/category/grade-7/the-number-system-grade-7/

http://mathmistakes.org/category/grade-7/the-number-system-grade-7/

https://www.illustrativemathematics.org/7.NS.A

http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132

http://fivetriangles.blogspot.com/2012/04/fractions.html

MGSE7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

http://www.openmiddle.com/category/grade-7/the-number-system-grade-7/

https://www.illustrativemathematics.org/7.NS.A.2

http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132

MGSE7.NS.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (- 1)(– 1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

http://www.nzmaths.co.nz/search/node/integers

https://www.illustrativemathematics.org/7.NS.A.2

http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132

MGSE7.NS.2b Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

http://www.nzmaths.co.nz/search/node/integers

https://www.illustrativemathematics.org/7.NS.A.2

http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132

MGSE7.NS.2c Apply properties of operations as strategies to multiply and divide rational numbers.

http://mathmistakes.org/category/grade-7/the-number-system-grade-7/

https://www.illustrativemathematics.org/7.NS.A.2

http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132

MGSE7.NS.2d Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

https://www.illustrativemathematics.org/7.NS.A

http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132

MGSE7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.
https://www.illustrativemathematics.org/7.NS.A
http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132