Georgia
Standards of Excellence Curriculum Frameworks

Mathematics

GSE Grade 7

Unit 4: Geometry

Georgia Department of Education

Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

Unit 4
Geometry

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OVERVIEW

The units in this instructional framework emphasize key standards that assist students to develop a deeper understanding of numbers. In this unit they will be engaged in using what they have previously learned about drawing geometric figures using rulers and protractor with an emphasis on triangles, students will also write and solve equations involving angle relationships, area, volume, and surface area of fundamental solid figures.
The challenges in this unit include understanding the geometric figures and solving equations involving geometric figures. The students also should be guided to realize how geometry works in real world situations. The Big Ideas that are expressed in this unit are integrated with such routine topics as estimation, mental and basic computation. All of these concepts need to be reviewed throughout the year.

Take what you need from the tasks and modify as required. These tasks are suggestions, something that you can use as a resource for your classroom.

STANDARDS ADDRESSED IN THIS UNIT

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of the problems involving geometric measurements (area, volume, surface area, etc.) through their understanding of the relationships between these measurements. They demonstrate this by choosing appropriate strategies for solving problems involving real-world and mathematical situations.

2. Reason abstractly and quantitatively. In grade 7, students represent a wide variety of real world contexts through the use of real numbers and variables when working with geometric figures. Students contextualize to understand the meaning of the number or variable as related to a geometric shape. Students must challenge themselves to think of three dimensional shapes with only two dimensional representations of them on paper in some cases.

3. Construct viable arguments and critique the reasoning of others. Students are able to construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.

4. Model with mathematics. Students are able to apply the geometry concepts they know to solve problems arising in everyday life, society and the workplace. This may include applying area and surface of 2-dimensional figures to solve interior design problems or surface area and volume of 3-dimensional figures to solve architectural problems.

5. Use appropriate tools strategically. Mathematically proficient students consider available tools that might include concrete models, a ruler, a protractor, or dynamic geometry software such as virtual manipulatives and simulations. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.

6. Attend to precision. In grade 7, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students determine quantities of side lengths represented with variables, specify units of measure, and label geometric figures accurately. Students use appropriate terminology when referring to geometric figures.

7. Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They can see complicated things as single objects or as being composed of several objects.

8. Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts.

STANDARDS FOR MATHEMATICAL CONTENT

Draw, construct, and describe geometrical figures and describe the relationships between them.

MGSE7.G.2 Explore various geometric shapes with given conditions. Focus on creating triangles from three measures of angles and/or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

MGSE7.G.3 Describe the two-dimensional figures (cross sections) that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms, right rectangular pyramids, cones, cylinders, and spheres.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

MGSE7.G.4 Given the formulas for the area and circumference of a circle, use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

MGSE7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

MGSE7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

RELATED STANDARDS

From Unit 3
MGSE7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

BIG IDEAS

Use freehand, ruler, protractor and technology to draw geometric shapes with give conditions. (7.G.2)
Construct triangles from 3 measures of angles or sides. (7.G.2)
Given conditions, determine what and how many type(s) of triangles are possible to construct. (7.G.2)
Describe the two-dimensional figures that result from slicing three-dimensional figures. (7.G.3)
Identify and describe supplementary, complementary, vertical, and adjacent angles. (7.G.5)
Use understandings of supplementary, complementary, vertical and adjacent angles to write and solve equations. (7.G.5)
Explain (verbally and in writing) the relationships between the angles formed by two intersecting lines. (7.G.5)
Solve mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (7.G.6)
Solve real-world problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (7.G.6)

ESSENTIAL QUESTIONS

- What are the characteristics of angles and sides that will create geometric shapes, especially triangles?
- How can attributes of specific shapes, symmetry, and angles be used to accurately describe the design of a mosaic pattern?
- How can angle and side measures help us to create and classify triangles?
- How can special angle relationships – supplementary, complementary, vertical, and adjacent – be used to write and solve equations for multi-step problems?
- How can the interior and exterior measures of polygons be used to write and solve equations for multi-step problems?
- How are angle relationships applied to similar polygons?
- How are the circumference, diameter, and pi related?
- How do we find the circumference of a circle?
- How are the areas of parallelograms and triangles related to the area of a rectangle?
- How can area be maximized when the perimeter is a fixed number?
- How is the formula for the area of a circle related to the formula for the area of a parallelogram?

- How do I apply the concepts of surface area and circumference to solve real-world problems?
- What two-dimensional figures can be made by slicing a cube by planes?
- What two-dimensional figures can be made by slicing: cones, prisms, cylinders, and pyramids by planes?
- How do you determine volume and surface area of a cube?
- How do you determine surface area of a cylinder? (Extension EQ)
- How can I use formulas to determine the volumes of fundamental solid figures?
- How can I estimate the surface area of simple geometric solids?
- How can I use surface areas of plane figures to derive formulas for the surface areas of solid figures?

CONCEPTS AND SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- number sense
- computation with whole numbers and decimals, including application of order of operations
- addition and subtraction of common fractions with like denominators
- measuring length and finding perimeter and area of rectangles and squares
- characteristics of 2-D and 3-D shapes
- angle measurement
- data usage and representations

FLUENCY

It is expected that students will continue to develop and practice strategies to build their capacity to become fluent in mathematics and mathematics computation. The eventual goal is automaticity with math facts. This automaticity is built within each student through strategy development and practice. The following section is presented in order to develop a common understanding of the ideas and terminology regarding fluency and automaticity in mathematics:

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.
Deep Understanding: Teachers teach more than simply “how to get the answer” and instead support students’ ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students:

- flexibly use a combination of deep understanding, number sense, and memorization.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.

For more about fluency, see: http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and: http://joboaler.com/timed-tests-and-the-development-of-math-anxiety/

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary. Note – At the elementary level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.

http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website.

Visit http://intermath.coe.uga.edu or http://mathworld.wolfram.com to see additional definitions and specific examples of many terms and symbols used in grade 7 mathematics.

- **Adjacent Angle**
- **Circumference**
- **Complementary Angle**

- **Congruent**
- **Cross- section**
- **Irregular Polygon**
- **Parallel Lines**
- **Pi**
- **Regular Polygon**
- **Supplementary Angle**
- **Vertical Angles**

FORMATIVE ASSESSMENT LESSONS (FAL)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.
More information on Formative Assessment Lessons may be found in the Comprehensive Course Guide.

SPOTLIGHT TASKS

A Spotlight Task has been added to each MGSE mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Georgia Standards for Excellence, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky.

3-ACT TASKS

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.
More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

TASKS

Task Name	Task Type Grouping Strategy	Content Addressed	Standards Addressed
Take the Ancient Greek Challenge	Performance Task *Individual/Partner*	Drawing geometric shapes with given conditions	MGSE.7.G.2
Building Bridges (FAL)	Formative Assessment Lesson *Partner/Small Group*	Determining whether triangles can be made with given conditions	MGSE.7.G.2
Roman Mosaic	Short Cycle Task	Utilizing attributes of specific shapes, symmetry, and angles to describe mosaic designs	MGSE.7.G.4 MGSE.7.G.5 MGSE.7.G.6
My Many Triangles Revisited	Scaffolding Task	Classify angles based on angle and side measures	MGSE.7.G.2
Thoughts About Triangles Revisited	Scaffolding Task	Investigate and explain properties of triangles	MGSE.7.G.2
Food Pyramid, Square, Circle	Learning Task *Individual/Group*	Using angle relationships to write/solve for missing angle	MGSE.7.G.5
I've Got A Secret Angle	Performance Task *Individual/Partner*	Using angle relationships to write/solve for missing angle	MGSE.7.G.5
Applying Angle Theorems (FAL)	Formative Assessment Lesson *Partner/Small Group*	Angle measurements of polygons	MGSE.7.G.4 MGSE.7.G.5 MGSE.7.G.6
Saving Sir Cumference	Learning Task *Indiv/Partner/SmGrp*	Proving the relationship between circumference and diameters as the value of pi. Deriving the formula for the circumference of a circle	MGSE.7.G.4
Area Beyond Squares and Rectangles	Scaffolding Task *Indiv/Partner/SmGrp*	Derive the formula for area of a parallelogram and triangle.	MGSE.7.G.6
Gold Rush (FAL)	Formative Assessment Lesson *Partner/Small Group*	Maximizing Area	MGSE.7.G.2

Circle Cover Up	Learning Task *Indiv/Partner/SmGrp*	Deriving the formula for the area of a circle	MGSE.7.G.4
Using Dimensions: Designing a Sports Bag (FAL)	Formative Assessment Lesson *Partner/Small Group*	Solve problems involving angle measure, area, surface area, and volume	MGSE7.G.4
Think Like a Fruit Ninja: Exploring Cross Sections of Solids	Learning Task *Indiv/Partner/SmGrp*	Describing/sketching polygons produced by cross-sections of cubes and pyramids	MGSE.7.G.3
Cool Cross-Sections	Performance Task *Indiv/Partner/SmGrp*	Solving application problem involving cross-sections	MGSE.7.G.3
What's My Solid? (only prisms & pyramids)	Performance Task *Indiv/Partner/SmGrp*	Identifying prisms/pyramids	MGSE.7.G.3 MGSE.7.G.6
Bigger and Bigger Cubes	Scaffolding Task *Indiv/Partner/SmGrp*	Determine volume and surface area of cubes and prisms.	MGSE.7.G.6
Storage Boxes	Scaffolding Task *Indiv/Partner/SmGrp*	Determine volume and surface area of cubes and prisms.	MGSE.7.G.6
Filling Boxes	Scaffolding Task *Indiv/Partner/SmGrp*	Ordering volume and surface area	MGSE.7.G.6
Discovering the Surface Area of a Cylinder (Extension Task)	Learning Task *Partner/Small Group*	Determine the surface area of a cylinder	Extension of MGSE.7.G.6
Three Little Pig Builders (only prisms)	Culminating Task *Indiv/Partner/SmGrp*	Solving application problem involving volume and surface area of prism	MGSE.7.G.6

Take the Ancient Greek Challenge

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In this task, students will draw geometric shapes using a ruler and a protractor. The focus of this task is constructing triangles.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.G.2 Explore various geometric shapes with given conditions. Focus on creating triangles from three measures of angles and/or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.

BACKGROUND KNOWLEDGE

In order for students to be successful, the following skills and concepts need to be maintained:
Knowledge of how to use compasses and straight edges
Knowledge of defining properties of polygons such as rectangles, trapezoids, and parallelograms
Ability to recognize and use congruent and similar
Knowledge of different types of triangles such as right, equilateral, and scalene

COMMON MISCONCEPTIONS

Conditions may involve points, line segments, angles, parallelism, congruence, angles, and perpendicularity. These concepts need to be reviewed or previewed.
Students may have misconceptions about how to correctly use and read a ruler, compass, and/or protractor.
Students may also confuse the ideas of perimeter and area.

ESSENTIAL QUESTION

- What are the characteristics of angles and sides that will create geometric shapes, especially triangles?

MATERIALS

- ruler
- protractor
- compass
- plain paper
- patty paper or tracing paper

GROUPING

- Individual/ Partner

TASK COMMENTS

Prior to beginning the task, students may need to "play" with the tools used in the activity. Give students the opportunity to familiarize themselves with a ruler, a compass, and a protractor and how they can be used to draw and measure different geometric figures. For example, ask students to draw a design of their choice. Use this as an opportunity to help students line up or hold the tools correctly on paper.

Constructions facilitate understanding of geometry. Provide opportunities for students to physically construct triangles with straws, sticks, or geometry apps prior to using rulers and protractors to discover and justify the side and angle conditions that will form triangles. Students should understand the characteristics of angles that create triangles. For example, can a triangle have more than one obtuse angle? Will three sides of any length create a triangle? Students recognize that the sum of the two smaller sides must be larger than the third side. Explorations should involve giving students: three side measures, three angle measures, two side measures and an included angle measure, and two angles and an included side measure to determine if a unique triangle, no triangle or an infinite set of triangles results. Through discussion of their exploration results, students should conclude that triangles cannot be formed by any three arbitrary side or angle measures. They may realize that for a triangle to result the sum of any two side lengths must be greater than the third side length, or the sum of the three angles must equal 180 degrees. Students should be able to transfer from these explorations to reviewing measures of three side lengths or three angle measures and determining if they are from a triangle justifying their conclusions with both sketches and reasoning.

Writing instructions for the constructions is a critical component of the task since it is a precursor to writing proofs. Just as students learn to write papers by creating and revising drafts, students need ample time to write, critique, and revise their instructions. By using peers to proofread instructions, students learn both how to write clear instructions and how to critique and provide feedback on how to refine the instructions. Teachers should allot sufficient instructional time for this writing process.

Constructions that utilize principles of similarity and congruence are found in high school MGSE. Teachers could incorporate construction activities possibly using technology such as, Geometer's Sketchpad or Geogebra (http://www.geogebra.org/cms/en/) to clarify the references to straight edge and compass work used in historical reference from activating reading passage.

TASK DESCRIPTION

When introducing the task, keep in mind important questions that you might ask students throughout the task.

Take the Ancient Greek Challenge

The study of Geometry was born in Ancient Greece, where mathematics was thought to be embedded in everything from music to art to the governing of the universe. Plato, an ancient philosopher and teacher, had the statement, "Let no man ignorant of geometry enter here," placed at the entrance of his school. This quote illustrates the importance of the study of shapes and logic during that era. Everyone who learned geometry was challenged to construct geometric objects using two simple tools, known as Euclidean tools:

- A straight edge without any markings
- A compass

The straight edge could be used to construct lines; the compass to construct circles. As geometry grew in popularity, math students and mathematicians would challenge each other to create constructions using only these two tools. Some constructions were fairly easy (Can you construct a square?), some more challenging, (Can you construct a regular pentagon?), and some impossible even for the greatest geometers (Can you trisect an angle? In other words, can you divide an angle into three equal angles?). Archimedes (287-212 B.C.E.) came close to solving the trisection problem, but his solution used a marked straight edge.

We will use a protractor and marked straight edge (you know it as a ruler) to draw some geometric figures.
What "constructions" can you create?

Your 1st Challenge:
Draw a regular quadrilateral.
Solution:
Check student drawings for specified attributes.
Your 2nd Challenge:
Draw a quadrilateral with no congruent sides.
Solution:
Check student drawings for specified attributes.
Your 3rd Challenge:
Draw a circle. Then draw an equilateral triangle and a square inside so that both figures have their vertices on the circle (inscribed).
Solution:
Check student drawings for specified attributes.
Your 4th Challenge:
Draw a regular hexagon. Then divide it into three congruent quadrilaterals
Solution:
Check student drawings for specified attributes.

Your 5th Challenge:
Draw a regular octagon. Then divide it into two congruent trapezoids and two congruent rectangles
Solution:
Check student drawings for specified attributes.
Your 6th Challenge:
Draw a triangle with side lengths of 5, 6, and 8 units.
Comment:

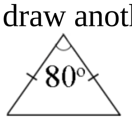
This construction will result in a unique triangle.
Solution:
Check student drawings for specified attributes.

Your 7th Challenge:
Draw a triangle with an obtuse angle.
Solution:
This construction will have more than one correct answer because more than one triangle can be drawn.

Your 8th Challenge:
Draw an equilateral right triangle.
Solution:
This construction is not possible.

Your 9th Challenge:
Create some challenges of your own and pose them to a classmate.
Solution:
Check student challenges and drawings for specified attributes.

Additional sample problems for students might include:
Is it possible to draw a triangle with a 90˚ angle and one leg that is 4 inches long and one leg that is 3 inches long? If so, draw one. Is there more than one such triangle?
Draw a triangle with angles that are 60 degrees. Is this a unique triangle? Why or why not?
Draw an isosceles triangle with only one 80 degree angle. Is this the only possibility or can you draw another triangle that will also meet these conditions?



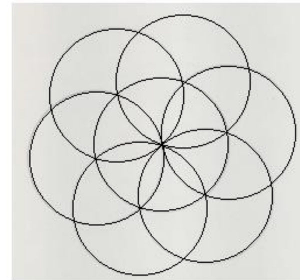
Can you draw a triangle with sides that are 13 cm, 5 cm and 6cm?
Draw a quadrilateral with one set of parallel sides and no right angles.

DIFFERENTIATION

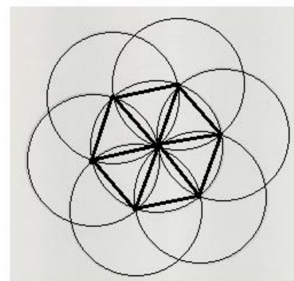
Extensions:
Teachers may wish to assign a mathematics history project exploring the contributions of ancient Greek mathematics and mathematicians.
The challenge is to see if they can form a triangular pyramid (tetrahedron) from the two pieces. The pieces could be made in advance by the teacher using constructions referenced in Challenges from Ancient Greece as a link to later constructions from HS MGSE.

Step 1:
Constructing equilateral triangles could be used to begin making the two-piece puzzle. One way to construct equilateral triangles is to construct a circle. Then using a point on the circle as the center, construct a congruent circle. Using the intersection of the two circles, construct another congruent circle. Connecting the intersections as demonstrated in the illustration to the right will produce an equilateral triangle because the sides are congruent radii. Two of these will be needed for each piece of the puzzle.

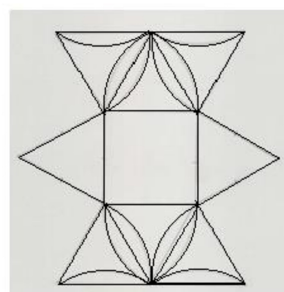
Step 2:
The next step in making a piece of the puzzle is to construct two isosceles trapezoids out of three equilateral triangles that are congruent to the one shown above. One method is to construct congruent circles beginning with the same directions as above. From the previous instructions, after drawing the three circles, continue to use the intersection of the most recently drawn circle and the original circle as the center for the next circle until you are back where you started.



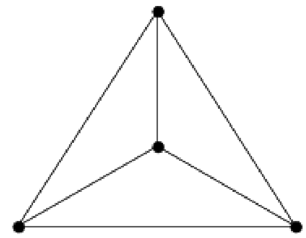
Step 3:
Connecting the centers as displayed in the construction to the right will form two congruent isosceles trapezoids out of six congruent equilateral triangles. For the puzzle to fit together, these six triangles must be congruent to the ones that were constructed in step 1.



Step 4:
Placing the two triangles and two trapezoids together around a square will produce a net for one of the two pieces of the puzzle. The other piece of the puzzle should be identical to the first piece. Folding along the sides of the square and taping the other sides together will complete the puzzle piece.



Step 5:
To solve the puzzle, place the two pieces together matching the squares and twist keeping the squares together. This will produce a triangular pyramid or tetrahedron.



A square represents one of the possible cross sections of the pyramid.

To complete the task, students could be divided into groups with each group exploring the question in a different way.
Constructing a regular octagon could also be used as a student extension – link to HS MGSE. (See the problem below.)
Constructing a regular octagon:
How would you construct a regular octagon? Discuss this with a partner and come up with a strategy. Think about what constructions might be needed and how they might be completed. Be prepared to share your ideas with the class.
Experiment to see if your strategy works. Write a justification of why your strategy works.

Solution:

Possible solutions to the octagon construction may include constructing a circle with a diameter, constructing a perpendicular bisector of the diameter and then bisecting each right angle. Alternatively, students may choose to begin with a line and construct a perpendicular bisector, bisect each of the right angles and then construct a circle to determine the vertices of the octagon.

Interventions:
Give students partial figures to help them get started.
Eliminate challenges that do not involve regular polygons.

Take the Ancient Greek Challenge

The study of Geometry was born in Ancient Greece, where mathematics was thought to be embedded in everything from music to art to the governing of the universe. Plato, an ancient philosopher and teacher, had the statement, “Let no man ignorant of geometry enter here,” placed at the entrance of his school. This illustrates the importance of the study of shapes and logic during that era. Everyone who learned geometry was challenged to construct geometric objects using two simple tools, known as Euclidean tools:

A straight edge without any markings
A compass

The straight edge could be used to construct lines; the compass to construct circles. As geometry grew in popularity, math students and mathematicians would challenge each other to create constructions using only these two tools. Some constructions were fairly easy (Can you construct a square?), some more challenging, (Can you construct a regular pentagon?), and some impossible even for the greatest geometers (Can you trisect an angle? In other words, can you divide an angle into three equal angles?). Archimedes (287-212 B.C.E.) came close to solving the trisection problem, but his solution used a marked straight edge.

We will use a protractor and marked straight edge (you know it as a ruler) to draw some geometric figures.
What “constructions” can you create?

Your 1st Challenge:
Draw a regular quadrilateral.

Your 2nd Challenge:
Draw a quadrilateral with no congruent sides.

Your 3rd Challenge:
Draw a circle. Then draw an equilateral triangle and a square inside so that both figures have their vertices on the circle (inscribed).

Your 4th Challenge:
Draw a regular hexagon. Then divide it into three congruent quadrilaterals

Your 5th Challenge:
Draw a regular octagon. The divide it into two congruent trapezoids and two congruent rectangles

Your 6th Challenge:
Draw a triangle with side lengths of 5, 6, and 8 units.

Your 7th Challenge:
Draw a triangle with an obtuse angle.

Your 8th Challenge:
Draw an equilateral right triangle.

Your 9th Challenge:
Create some challenges of your own and pose them to a classmate.

Building Bridges

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Source: Georgia Mathematics Design Collaborative

This lesson is intended to help you assess how well students are able to:

- Using lengths of sides to determine if a triangle can be formed
- Determining when angles and sides create multiple triangles
- Recognizing degenerate cases

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.G.2 Explore various geometric shapes with given conditions. Focus on creating triangles from three measures of angles and/or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

STANDARDS FOR MATHEMATICAL PRACTICE

This lesson uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Tasks and lessons from the Georgia Mathematics Design Collaborative are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding.

The task, *Building Bridges*, is a Formative Assessment Lesson (FAL) that can be found on the Georgia Mathematics online professional learning community, edWeb.net. Once logged in, the task can be found directly at: https://www.edweb.net/?14@@.5abfa3bd.

Roman Mosaic (Short Cycle Task)

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In this task, students decide how they would describe the design of a mosaic pattern over the telephone by utilizing the attributes of specific shapes, symmetry, and angles.

Source: Balanced Assessment Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=1163

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.G.4 Given the formulas for the area and circumference of a circle, use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

MGSE7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

MGSE7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTION

How can attributes of specific shapes, symmetry, and angles be used to accurately describe the design of a mosaic pattern?

TASK COMMENTS

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *Roman Mosaic*, is a Mathematics Assessment Project Assessment Task that can be found at the website:
http://www.map.mathshell.org/materials/tasks.php?taskid=391&subpage=expert

The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=1163

The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=1164

My Many Triangles revisited

Back to task table

Adapted from Van De Walle, J.A., Karp, K. S., & Bay-Williams, J. M. (2010). *Elementary and Middle School Mathematics: Teaching Developmentally* 7th Ed. Boston: Pearson Education, Inc., p. 413-414.
Students will classify triangles by their angles and length of sides.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.G.2 Explore various geometric shapes with given conditions. Focus on creating triangles from three measures of angles and/or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

Students should be able to identify triangles by the lengths of their sides (isosceles, equilateral, and scalene) as well as by the measure of their angles (right, obtuse, and acute).
The type of each triangle on the "My Many Triangles, Triangles to Cut and Sort" student sheet are shown below.

#1, #11 – obtuse scalene
#2, #7 – right scalene
#4, #13 – acute scalene
#5, #10 – right isosceles
#8, #12 – acute equilateral
#3, #9 – acute isosceles
#6, #14 – obtuse isosceles

Allow students to struggle a little bit with this part of the task. Students may need to try out a few possibilities before finding that lengths of sides and measures of angles are two ways to sort these triangles so that each triangle belongs to exactly one group when sorted.

Sorted according to side lengths		Sorted according to angle measures
Equilateral triangles: ***8, 12***		Acute triangles: ***3, 4, 8, 9, 12,***
Isosceles triangles: ***2, 3, 5, 6, 9, 14***	or	Right triangles: ***2, 5, 7, 10***
Scalene triangles: ***1, 4, 7, 10, 11, 13***		Obtuse triangles: ***1, 6, 11, 14***

Students will need to be able measure the sides and angles.
Of the nine triangles, <u>two are not possible</u>.
An equilateral right triangle is not possible because an equilateral triangle also has equal angle measures (equiangular). A triangle can have no more than 180°, and $90° \times 3 = 270°$ which is more than 180°.
An equilateral obtuse triangle is not possible because an equilateral triangle has equal angle measures (equiangular).

ESSENTIAL QUESTION

- How can angle and side measures help us to create and classify triangles?

MATERIALS

- "My Many Triangles" student recording sheet
- "My Many Triangles, Triangles to Cut and Sort" student sheet
- White construction paper (one sheet per student or per pair of students)
- Colored construction paper cut into strips $\frac{1}{4}$" wide (each student will need approximately 10 strips of paper)

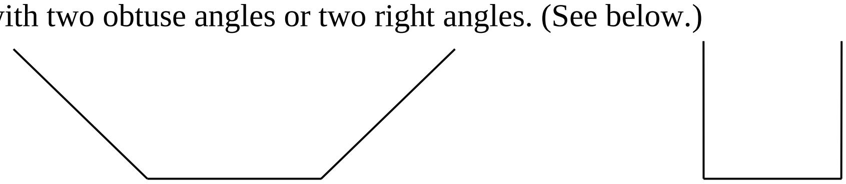
GROUPING

- Individual/Partner Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Part 1
Task Directions
Cut out the triangles below. Sort the triangles into groups where there are no triangles that do not fit into a group and there are no triangles that belong to more than one group. Then sort the triangles in a different way. Again, there should be no triangles that do not fit into a group and no triangles that belong to more than one group. Record how you sorted the triangles and the number of the triangles in each group. Be able to share how you sorted the triangles.
Part 2

Comments
Students may need some assistance using the chart to identify the triangles they need to create. Be sure students understand they need to attempt to make nine different types of triangles, two of which are not possible to create. Encourage students to try to make an equilateral obtuse angle and an equilateral right triangle so that they can see that it is not possible to create a three-sided closed figure with two obtuse angles or two right angles. (See below.)



TASK DIRECTIONS

Use the strips of construction paper to create the triangles described in each box below. Use the row label and the column label to identify the properties required for each triangle. For example, the box labeled "A" needs to be acute and isosceles because the row label is "Acute" and the column label is "Isosceles."

	Equilateral	Isosceles	Scalene
Acute		A	
Right			
Obtuse			

Two triangles are not possible; for those, explain why each triangle is not possible on the lines below. Glue each triangle onto the construction paper and label it.

DIFFERENTIATION

Extension

Challenge students to write directions for a triangle that they choose so that someone else could follow their directions and create the same triangle. Allow a partner to try these directions to see how successful they were at describing how to create their triangle.

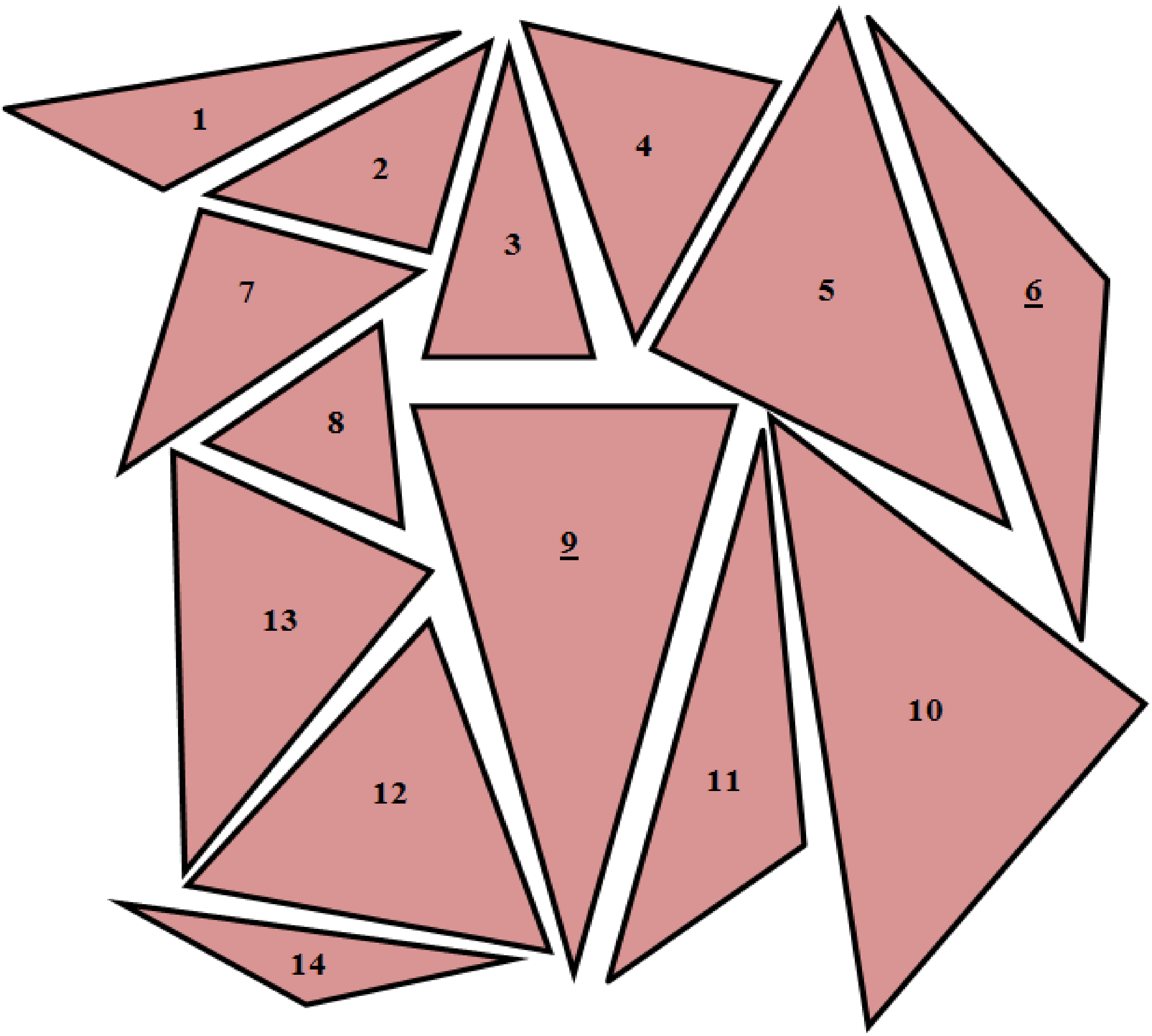
Intervention

Allow students to use a picture glossary or the triangles from Part 1 of this task to help them create the triangles for Part 2.

<u>My Many Triangles</u>

Triangles to Cut and Sort

Cut out the triangles below. Sort the triangles into groups where there are no triangles that do not fit into a group and there are no triangles that belong to more than one group. Then sort the triangles in a different way. Again, there should be no triangles that do not fit into a group and no triangles that belong to more than one group. Record how you sorted the triangles and the number of the triangles in each group. Be able to share how you sorted the triangles.



My Many Triangles

Use the strips of construction paper to create the triangles described in each box below. Use the row label and the column label to identify the properties required for each triangle. For example, the box labeled "A" needs to be acute and isosceles because the row label is "Acute" and the column label is "Isosceles."

Two triangles are not possible; for those, explain why each triangle is not possible on the lines below.

Glue each triangle onto the construction paper and label it.

	Equilateral	Isosceles	Scalene
Acute		A	
Right			
Obtuse			

Thoughts About Triangles Revisited

Back to task table

Adapted from a lesson in Navigating Through Geometry in Grades 3-5 by NCTM
Students will investigate and explain properties of triangles.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.G.2 Explore various geometric shapes with given conditions. Focus on creating triangles from three measures of angles and/or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

- Students should have the following background knowledge.
- Be able to use a straight edge or ruler to draw a straight line.
- Know how to use a ruler, and how to identify right angles (90 degrees), obtuse angles, and acute angles (using the corner of an index card or another object with a known angle of 90 degrees).
- Understand that the side across from an angle on a triangle can be described as an opposite side
- Know parallel means that lines will never intersect or cross over each other no matter how long they are extended.
- Understand that perpendicular means lines or segments intersect or cross forming a right angle. (Some students may use a known 90 degree angle to show an angle is a right angle.)
- Know that a property is an attribute of a shape that is always going to be true. It describes the shape.
- Be able to use a ruler to measure sides to verify they are the same length.
- Some properties of triangles that should be discussed are included below. As students draw conclusions about the relationships between different figures, be sure they are able to explain their thinking and defend their conclusions. Much of the information below may come out as a result of students' explorations. This is information to look for and highlight as they explore the triangles to pull out, not a list of understandings that you must teach them beforehand.
- A shape is a triangle when it has exactly 3 sides and is a polygon. (To be a polygon the figure must be a closed plane figure with at least three straight sides and having no curved lines.)

- A right triangle is a triangle with one angle that measures 90 degrees. A right triangle can be either scalene or isosceles, but never equilateral.
- An obtuse triangle has one angle that measures greater than 90 degrees. There can only be one obtuse angle in any triangle.
- An acute triangle has three angles that measure less than 90 degrees.
- An equilateral triangle has three equal angles and three sides of equal length.
- An isosceles triangle has two equal angles and two sides of equal length.
- A scalene triangle has three sides that are not equal and no angles that are equal.

ESSENTIAL QUESTIONS

- What are the characteristics of angles and sides that will create geometric shapes, especially triangles?

MATERIALS

- For Each Group:
- Geoboard with one rubber band for each student or Geoboard app
- A copy of "Geodot Paper for Geoboard"
- Paper
- Pencils

GROUPING

- Partner/Small Group Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Comments
Make sure that students complete this activity in partners or small groups to encourage mathematical discussion while they make their triangles and test conjectures. You may wish to have students explore some on their own and then come together to discuss their findings. Students can then explain and defend their conclusions as a group.
The purpose of this task is for students to become familiar with the properties of triangles. Working in pairs, students will create the following triangles: right triangles, obtuse triangles, acute triangles, isosceles triangles, scalene triangles, and equilateral triangles. They will identify the attributes of each triangle, then compare and contrast the attributes of different triangles. Though the standards only specifically state that students are to identify right triangles as a category for classification, the exploration of the attributes of all triangles is vital to students differentiating between right triangles and all other triangles.

TASK DIRECTIONS

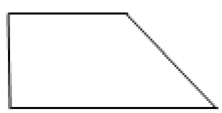
This task is a collection of investigations into triangles through the use of guiding questions. For each question students should (1) make a conjecture, (2) explore, using their geoboards, and (3) discuss their findings as a group. The class should come to a general consensus during their discussion. As students and the class come to a consensus about triangles, keep an anchor chart or running list of "true" ideas about triangles.

Make sure to guide discussion during explorations and discussion time through the use of questioning rather than intervening by answering their questions. For example, if students incorrectly identify a polygon as a right triangle, rather than telling them it's not a right triangle, ask them to explain how they know it is a right triangle and then discuss together the definition of a right triangle.

These questions lend themselves nicely to student reflection in math journals. The journal entries can be used as evidence of learning for the students. There is a sample journal entry question at the end of each exploration.

<u>Question #1: Is it possible to create a right triangle with all sides equal?</u>
Have students make their conjectures and record the conjectures as a group.
Have students explore answering and explaining their answer using their geoboards explorations.
At closing discussion, make a class list of all the properties of right triangles, including one 90 degrees angle, two sides can be of equal length or all sides unequal.
Journal Reflection Question: What have you learned about right triangles from this investigation?

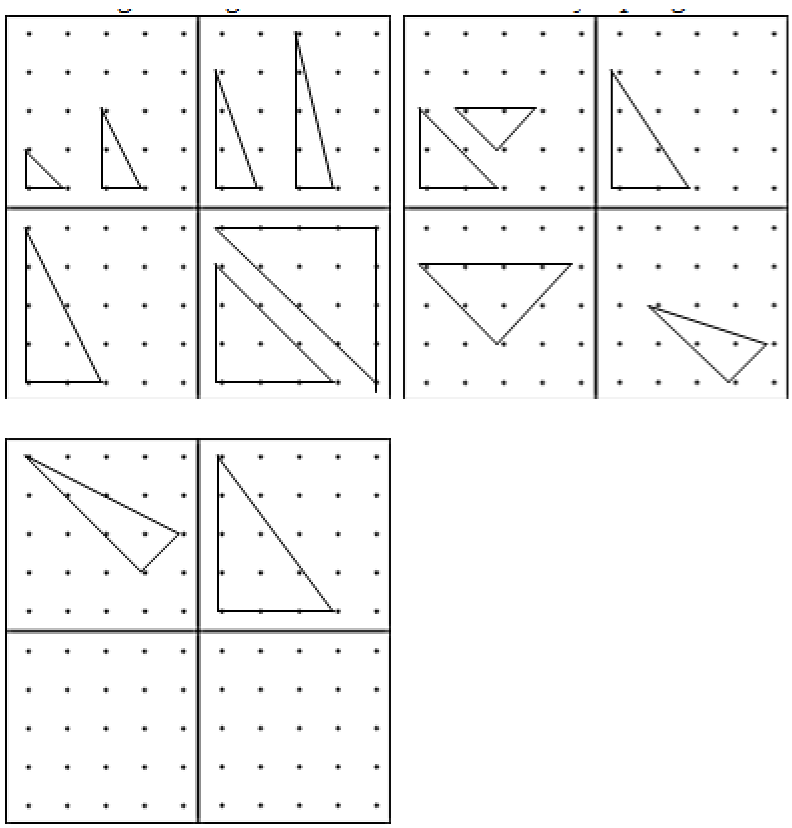
<u>Question #2: Is it possible for a triangle to have two right angles?</u>
Have students make their conjectures and record the conjectures as a group.
Have students explore answering and explaining their answer using their geoboards explorations.
Students may use the corner of an index card or another known right angle to tests for right angles.
If students create a figure like the one shown below that has 2 right angles, ask students if their figure has all the properties of a triangle.



At closing discussion, guide students to determine that there is a category of triangles referred to as right triangles because these have one right angle.
Journal Reflection Question: If you could make a triangle that was as large as you wanted, would you be able to make one that has two right angles? Explain your thinking.

<u>Question #3: How many different right triangles can be made on the geoboards with angles totaling 180°?</u>
Have students make their conjectures and record the conjectures as a group.
In the introduction of this exploration, discuss what different means. For the purposes of this exploration, if a triangle can be flipped or turned and matched up, it is not "different."
For this exploration, it would be helpful for students to record all their triangles on dot paper so that they can compare their right triangles.
Use guided questions to keep students on track during the exploration.
Have you found all of the right triangles that can be made? How do you know?
What is the measure of the angles in the triangles?
Journal Reflection Question: Write everything you know that is true about the angle measures of all right triangles.

The 14 right triangles that can made on a 5 by 5 pin geoboard are shown below.



<u>Question #4: How many different types of triangles can you find with at least one angle measuring 60°?</u>
Have students make their conjectures and record the conjectures as a group.
Show the students examples of a right triangle to review the definition of a right triangle. Show non-examples of a right triangle to stimulate discussion about differing length of sides and angle size. Encourage students to use a known right angle and rulers to differentiate between angle size and lengths of sides. Having students simply classify the angels as acute, right, or obtuse using a known right angle will be helpful for this exploration.
Have students record their triangles on dot paper. Using a protractor, students should measure the angles of each triangle.

NOTE: It is not possible to make an equilateral triangle on a geoboard. Some students may claim that some are, but if you measure the sides they will find them to have differing lengths.
Have students share the triangles with each other in a group. Have students cut out the triangles and sort them into piles that are the same and label them with their defining characteristic. In order the help guide students to grouping, beyond just having the exact same measurements, feel free to set restrictions on the sorting rules such as there must be at least 3 piles and at least 3 triangles in each pile.

Students should create posters with triangles displayed by category and should present and explain their groupings to the class.

Journal Reflection Question: What types of triangles have at least one angle measuring 60°?

Summary

After all explorations, have students complete the following journal entries with as many different answers as possible:
All triangles have….
Some triangles have…

DIFFERENTIATION

Extension

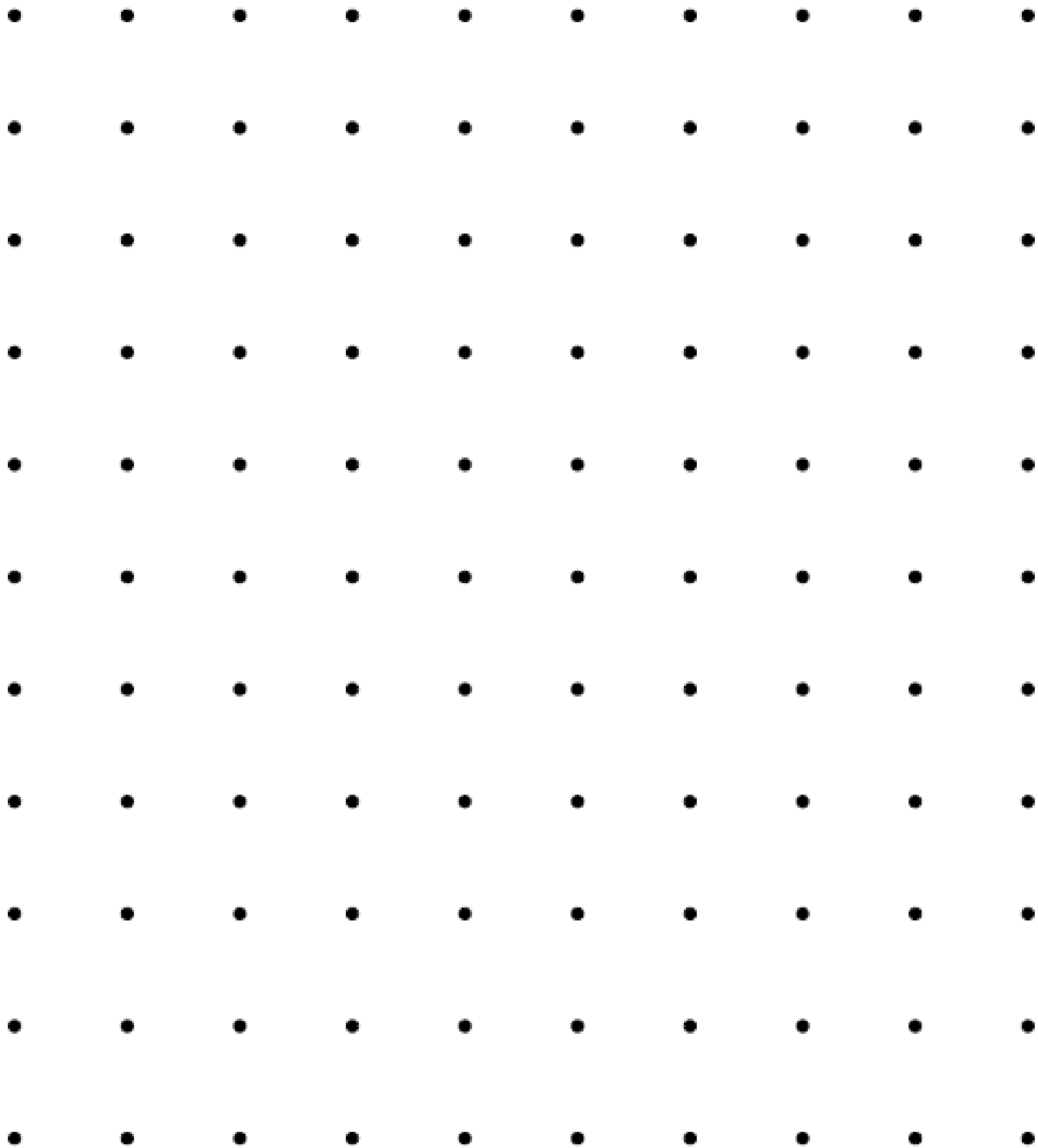
Using straws of different length or a computer geometry program such as The Geometer's Sketchpad, students can consider and explore the following questions:
Can a triangle be made with segments measuring five, six, and seven units? Can more than one triangle be made? Why or Why not?
If you are given any three lengths, can you always make a triangle? Why or why not?
Using several different sets of three lengths, try to make triangles. Can you make up a rule about the lengths of the sides of the triangles?

Intervention

Have students create the triangles using straws of different lengths rather than geoboards so they can more easily compare side lengths.
Angles are also a very useful tool for teaching students about the different attributes of triangles.

Thoughts About Triangles

Dot Paper



Food Pyramid, Square, Circle

Back to task table

Adapted from ETA/Cuisenaire Super Source task Food Pyramid, Square, Circle

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

COMMON MISCONCEPTIONS

Students have difficulties recognizing and justifying the relationships between supplementary, complementary, vertical, and adjacent angles.
One way to address these misconceptions and to reinforce vocabulary is for students to write their explanations and justifications in their journals.

ESSENTIAL QUESTION

- How can special angle relationships – supplementary, complementary, vertical, and adjacent – be used to write and solve equations for multi-step problems?

MATERIALS

- Tangrams, 1 set per student
- Circular Geoboards, 1 per student
- Rubber bands
- Tangram paper
- Circular geodot paper
- Protractors
- Activity Master

GROUPING

Individual/Partner

TASK COMMENTS

In previous grades, students have studied angles by type according to size: acute, obtuse and right, and their role as an attribute in polygons. Now angles are considered based upon the special relationships that exist among them: supplementary, complementary, vertical and adjacent angles. Provide students the opportunities to explore with manipulatives these relationships first through measuring and finding the patterns among the angles of intersecting lines or within polygons, then utilize the relationships to write and solve equations for multi-step problems.

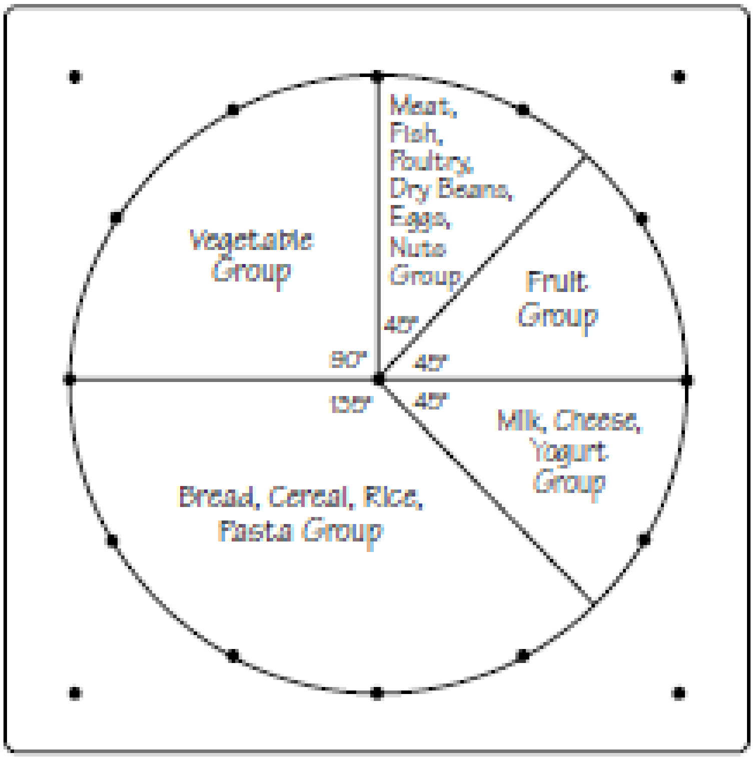
Angle relationships that can be explored include but are not limited to:
Same-side (consecutive) interior and same-side (consecutive) exterior angles are supplementary.

Students may complete the task individually or with a partner.

To construct the Geoboard Food Circle and the corresponding graphic representation on circular geodot paper, students will rely on the fractions found in the first activity and the fact that the measure of an entire circle is 360°. To begin either of the circles (the Geoboard circle or the circle on geodot paper), students must calculate the number of degrees in the measure of the central angle for each food group, as shown below:

> 1⁄8 of 360° or 1⁄8 x 360° = 45° = measure of the central angle for milk, etc.
> 1⁄8 of 360° = 45° = measure of the central angle for meat, etc.
> 1⁄8 of 360° = 45° = measure of the central angle for fruit
> 1⁄4 of 360° or 1⁄4 x 360° = 90° = measure of the central angle for vegetables
> 3⁄8 of 360° or 3⁄8 x 360° = 135° = measure of the central angle for breads, etc.

In placing the rubber bands on the Circular Geoboard, students can use visual reasoning skills to form the central angles or the protractor to help measure their magnitudes. Students may choose to mark off these angles in order of decreasing measure as the larger measures are easier to identify on the Geoboard. Each new angle should be placed adjacent to one already in position. The rubber bands for each central angle can be placed with no difficulty since the Geoboard pegs are arranged at 30° and 45° intervals. The Food Circle graph can be completed as shown below.



Circular Geodot Paper

<u>TASK DESCRIPTION</u>
Part 1

Using all 7 Tangram pieces, form a triangle to represent the Food Pyramid.

This Food Pyramid is used to illustrate the 5 basic food groups.
The areas of specific Tangram pieces or combinations of them have been correlated with the proportions of the daily requirements.

Large Triangle Vegetable Group
Large Triangle- Grains Group A
Medium Triangle- Fruit Group
Small Triangle- Dairy Group A
Small Triangle- Dairy Group B
Square- Meat Group
Parallelogram- Grains Group B

Convert the Food Pyramid to a Food Square to determine the portion of each food group in relation to the daily nutritional requirement. Record the arrangement on Tangram paper and label each piece to represent the appropriate food group.

Find the fractional part of the whole square represented by each Tangram piece. Using this information, determine the fractional part of each of the 5 food groups in relation to the whole Food Square. Record your findings and strategy.

Part 2

What if... *a person wants to represent the Food Square as a Food Circle? Based on the data collected in the first activity, design a drawn-to-scale model representing the 5 basic food groups in a well-balanced diet.*

• Work with your partner and use the following mathematical concepts to help you:
◆ The measure of the entire circle is 360°.
◆ A central angle of a circle is formed by two radii.
◆ Angles are adjacent if they are in the same plane and share a common vertex and a common side lying between the other two sides.

• Using the fractional data from the first activity, calculate the number of degrees in each central angle that you would use to represent a specific basic food group.
• Place 2 rubber bands on the Circular Geoboard to form a central angle representing the measure of the Bread, Cereal, Rice, Pasta Group. Using a protractor, draw the corresponding angle on the circular geodot paper. Be sure to label your graph.
• Place another rubber band on the Geoboard to form an adjacent central angle representing the portion allocated for the Vegetable Group. Measure off and label the corresponding angle on the circle graph.
• Repeat the process for the remaining three food groups.

Reflection Questions
What food groups create complementary angles?
Answers will vary based on how students form there circles.

What food groups create supplementary angles?
Answers will vary based on how students form there circles.

What food groups are form adjacent angles?
Answers will vary based on how students form there circles.

Are there food groups which create vertical angles? If so, what are they?
Answers will vary based on how students form there circles.

Food Pyramid, Square, Circle

Part 1

Using all 7 Tangram pieces, form a triangle to represent the Food Pyramid.

This Food Pyramid is used to illustrate the 5 basic food groups.
The areas of specific Tangram pieces or combinations of them have been correlated with the proportions of the daily requirements.

Large Triangle Vegetable Group
Large Triangle- Grains Group A
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Square- Meat Group
Parallelogram- Grains Group B

Convert the Food Pyramid to a Food Square to determine the portion of each food group in relation to the daily nutritional requirement. Record the arrangement on Tangram paper and label each piece to represent the appropriate food group.

Find the fractional part of the whole square represented by each Tangram piece. Using this information, determine the fractional part of each of the 5 food groups in relation to the whole Food Square. Record your findings and strategy.

Part 2

What if... *a person wants to represent the Food Square as a Food Circle? Based on the data collected in the first activity, design a drawn-to-scale model representing the 5 basic food groups in a well-balanced diet.*

• Work with your partner and use the following mathematical concepts to help you:
◆ The measure of the entire circle is 360°.
◆ A central angle of a circle is formed by two radii.
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• Place another rubber band on the Geoboard to form an adjacent central angle representing the portion allocated for the Vegetable Group. Measure off and label the corresponding angle on the circle graph.

• Repeat the process for the remaining three food groups.

Reflection Questions

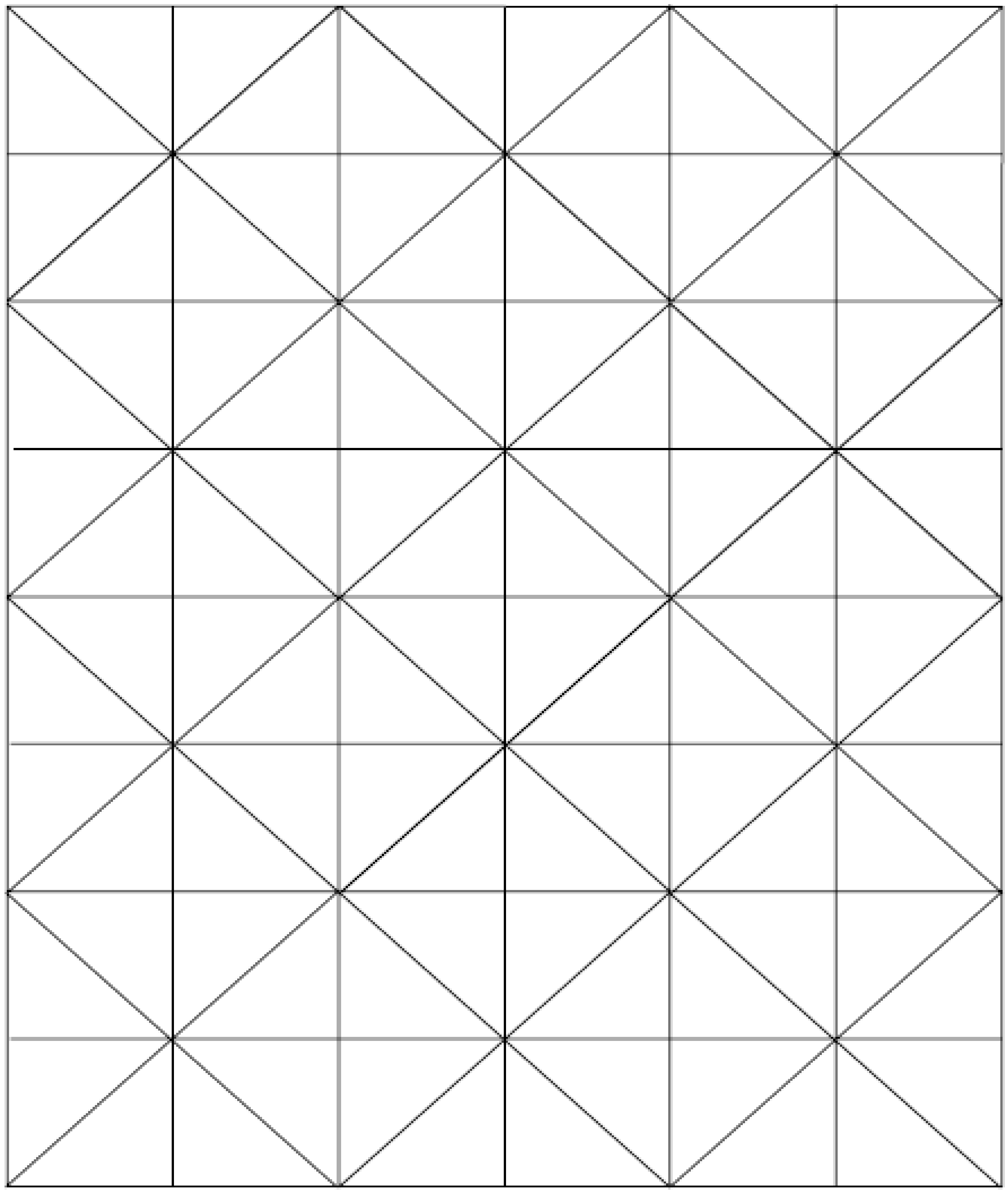
What food groups create complementary angles?

What food groups create supplementary angles?

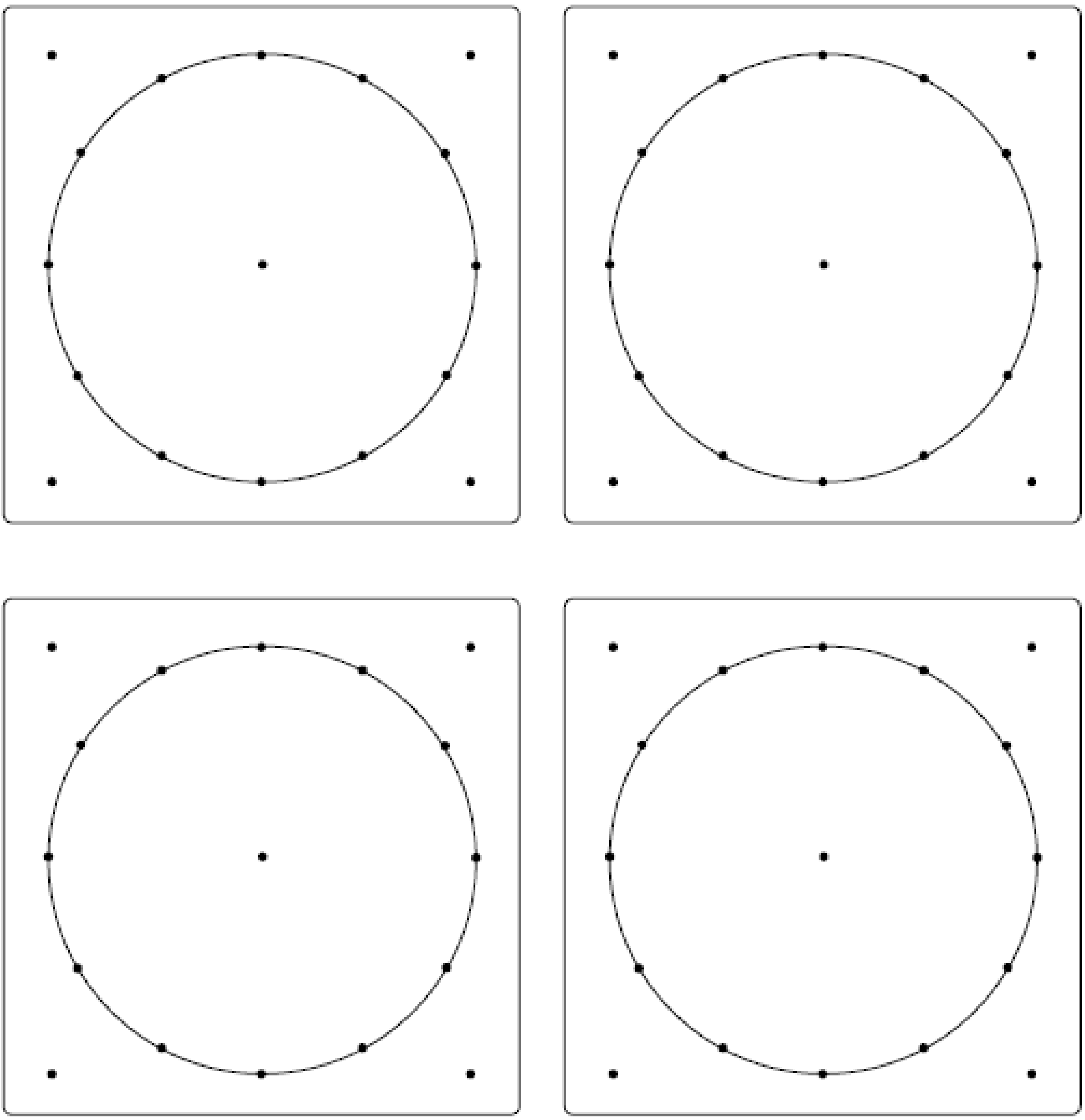
What food groups are form adjacent angles?

Are there food groups which create vertical angles? If so, what are they?

Tangram Paper



Circular Geodot Paper



I Have a Secret Angle

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STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.

COMMON MISCONCEPTIONS

Students have difficulties recognizing and justifying the relationships between supplementary, complementary, vertical, and adjacent angles.
One way to address these misconceptions and to reinforce vocabulary is for students to write their explanations and justifications in their journals.

ESSENTIAL QUESTION

- How can special angle relationships – supplementary, complementary, vertical, and adjacent – be used to write and solve equations for multi-step problems?

MATERIALS

- Craft sticks, straws, etc
- Ruler
- Protractor

GROUPING

- Individual/Partner

TASK COMMENTS

In previous grades, students have studied angles by type according to size: acute, obtuse and right, and their role as an attribute in polygons. Now angles are considered based upon the special relationships that exist among them: supplementary, complementary, vertical and adjacent angles. Provide students the opportunities to explore with manipulatives these relationships first through measuring and finding the patterns among the angles of intersecting lines or within polygons, then utilize the relationships to write and solve equations for multi-step problems.

Angle relationships that can be explored include but are not limited to:
Same-side (consecutive) interior and same-side (consecutive) exterior angles are supplementary.

Students may complete the task individually or with a partner.

TASK DESCRIPTION

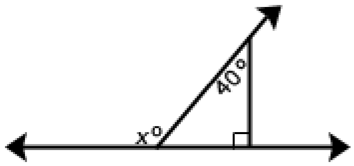
Before solving the two examples given in the task, allow students to use manipulatives to develop and/or refine concepts of supplementary, complementary, vertical, and adjacent angles. Guide students to solve the examples. Patty paper is also a great way for students to compare the relative sizes of angles by tracing the original figure. This allows students to test their hypothesis about sizes of angles (congruence) or about whether or not two angles are complementary or supplementary.

The task instructs students use manipulatives to develop concepts needed to solve exercises for a “Worksheet” that will be reviewed by a textbook publisher. Students will need to address the standard which states: use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
TE: I Have a Secret Angle

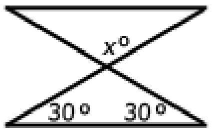
A 7th Grade Math Teacher saw the following job listing in a magazine:

Be a textbook writer!
We need teachers willing to develop exercises for our textbooks.
If you can write math problems that are appealing and motivating to students we want you.
Exercises should address the Georgia Common Core Math standards and be related to real-life situations/applications.
Call 1-800-123-4567 for more info.

The teacher decided to ask her 7th grade students for help, since they are *very creative*.
They will need to develop problems for the 7th grade standard:
Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
She gave them craft sticks, rulers, protractors, and other related supplies and instructed them to design exercises similar to exercises that they solved last year.
They designed worksheets about fashion design, cell phones, entertainment, etc. that contained problems with illustrations similar to the following examples:
Write and solve an equation to find the measure of angle *x*.



Write and solve an equation to find the measure of angle *x*.



We will use the information we have discovered about angle relationships to write and solve equations for the above examples.

Solutions:
Here are some possible solutions that students may use to solve each problem.

Use the sum of the angles of a triangle to begin the solving of the 1st example by writing the equation → 180 = "unknown angle" + 40 + 90.
The "unknown" interior angle will be 50 degrees.
Because x and the interior angle of the triangle are supplementary another equation can be written → 180 = x + 50.
Solve this equation to find the measure of angle x.
The missing "secret angle" value is 130 degrees.

Use the sum of the angles of a triangle to begin the solving of the 2nd example by writing the equation → 180 = "unknown angle" + 30 + 30.
The "unknown" interior angle will be 120 degrees.
Because x and the interior angle of the triangle are vertical angles the value of the vertical will be the same.
The missing "secret angle" value is also 120 degrees.

DIFFERENTIATION

Extension

Students can incorporate more complex angle relationship problems, including using circles and other closed figures to solve for unknown variables

Intervention

Give students "fill in the blank" to help solve the example problems

I Have a Secret Angle

A 7th Grade math teacher saw the following job listing in a magazine:

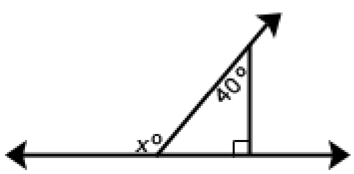
> **Be a textbook writer!**
> We need teachers willing to develop exercises for our textbooks.
> If you can write math problems that are appealing and motivating to students we want you.
> Exercises should address the Georgia Common Core Math standards and be related to real-life situations/applications.
> **Call 1-800-123-4567 for more info.**

The teacher decided to ask her 7th grade students for help, since they are *very creative*.
They will need to develop problems for the 7th grade standard:
Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

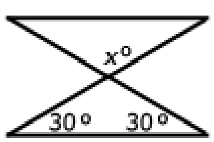
She gave them craft sticks, rulers, protractors, and other related supplies and instructed them to design exercises similar to exercises that they solved last year.

They designed worksheets about fashion design, cell phones, entertainment, etc. that contained problems with illustrations similar to the following examples:

Write and solve an equation to find the measure of angle *x*.



Write and solve an equation to find the measure of angle *x*.



We will use the information we have discovered about angle relationships to write and solve equations for the above examples

Applying Angle Theorems – (FAL)

Back to task table

(Concept Development Task)
This lesson is intended to help you assess how well students are able to solve problems relating to the measures of the interior and exterior angles of polygons.

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=680

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.G.4 Given the formulas for the area and circumference of a circle, use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

MGSE7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

MGSE7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.

ESSENTIAL QUESTIONS

- How can the interior and exterior measures of polygons?
- How are angle relationships applied to similar polygons?

TASK COMMENTS

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Applying Angle Theorems*, is a Formative Assessment Lesson (FAL) that can be found at the website: http://map.mathshell.org/materials/lessons.php?taskid=214&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=680

Saving Sir Cumference

Back to task table

Adapted from http://www.uen.org/Lessonplan/preview.cgi?LPid=15436

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.G.4 Given the formulas for the area and circumference of a circle, use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
3. Construct viable argument and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

COMMON MISCONCEPTIONS

Students sometimes believe Pi is an exact number rather than understanding that 3.14 is just an approximation of pi.

Many students are confused when dealing with circumference (linear measurement) and area. This confusion is about an attribute that is measured using linear units (surrounding) vs. an attribute that is measured using area units (covering).

ESSENTIAL QUESTIONS

- How are the circumference, diameter, and pi related?
- How do we find the circumference of a circle?

MATERIALS

For the class

- *Sir Cumference and the Dragon of Pi* by Cindy Neuschwander or similar book about the relationship between circumference and diameter
- Class graph to record the circumference and diameter data groups collect (See the Comments section)

For each group

- Several circular objects (e.g., soda cans, CD’s, wastebaskets, paper plates, coins, etc.).

For each student

- “Saving Sir Cumference” student recording sheet

- Measuring tape (metric)
- Narrow ribbon or string that doesn't stretch
- Centimeter grid paper

GROUPING

- Individual/Partner/Small Group

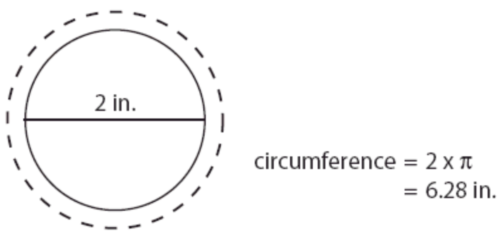
TASK COMMENT

In this task, students make a valuable connection between the circumference and diameter of circles and to derive the formula for the circumference of a circle. Students may work as partners or in small groups.

Content Background:

For any given diameter, the circumference of the object is the product of its diameter and pi (π). This relationship is often expressed in the formula,
Circumference (C) = Pi (π) x Diameter (d) or $C = \pi \times d$

Thus, if an object has a diameter of 2 in., the circumference of that object is approximately 6.28 in.



Pi (π) was probably discovered sometime after people started using the wheel. The people of Mesopotamia (now Iran and Iraq) certainly knew about the ratio of diameter to circumference. The Egyptians knew it as well. They gave it a value of 3.16. Later, the Babylonians figured it to 3.125. But it was the Greek mathematician Archimedes who figured that the ratio was less than 22/7, but greater than 221/77. But pi wasn't called "pi" until William Jones, an English mathematician, started referring to the ratio with the Greek letter π or "p" in 1706. Even so, pi really didn't catch on until the more famous Swiss mathematician, Leonhard Euler, used it in 1737. Thus, pi evolved through the contribution of several individuals and cultures.

Reference: http://www.uen.org/Lessonplan/preview.cgi?LPid=15436

TASK DESCRIPTION

Prior to the task, review how to measure the circumference and diameter of an object in centimeters and how to record the data in a table. (Ribbon or string may be used to wrap around circular objects and then held against a measuring tape.)

Allow students to build on their understanding of fact families to think about how they could find the circumference of a circle, given the diameter.

$$\begin{array}{lcl} C \div d = \pi & & \pi = C \div d \\ C \div \pi = d & \textbf{OR} & d = C \div \pi \\ \pi \times d = C & & C = \pi \times d \end{array}$$

Students will determine that the circumference (C) divided by the diameter (*d*) equals pi π (approximately 3.14). This equation and its "fact family" are shown. Provide opportunities for students to find the circumference of a circle given the diameter or radius, using the approximate value for pi as 3.14.

Pi is defined as the relationship between the circumference (*C*) and diameter (*d*);
$\frac{C}{d} \approx 3\frac{1}{7}$ or 3.14.

When graphing the data collected during the task, students create a collection of points that, if connected, should be very close to a straight line. This indicates that there is a direct/linear relationship between diameter and circumference. Students will study direct/linear relationships later, but this is a good preview of that topic. For teacher information only, the equation of the line would be $y = 3.14x$. Because it is a direct relationship, the graph of the equation would pass through the origin (0, 0) making the *y*-intercept 0 ($y = 3.14x + 0$).

During the task, the following questions/prompts will help you gauge student's understanding of the concepts.

Questions/Prompts for Formative Student Assessment
How are the diameter and circumference of a circle related?
How much bigger is the circumference than the diameter? How can you be more precise?
How can you organize the information you will collect?
What increments will you use on your graph's scale to allow all of the data to fit on your graph?
How do you know where to plot the points?
What do you notice about the points you plotted?
If you know that C ÷ D = π, how could you use this information to find the circumference of a circle given its diameter?
What is the formula for finding the circumference of a circle? How do you know?

Once students have completed the task, the following questions will help you further analyze student understanding.
Questions for Teacher Reflection
Do students understand the relationship between circumference and diameter?
Do students recognize that pi (π) is a constant and not a variable? Do students understand that π ≈ 3.14?
Are students able to explain how to derive the formula for the circumference of a circle?
Are students able to use the formula for circumference to find the circumference of a circle given the diameter?

As students move through the task and formative assessment takes place, consider the following suggestions for differentiating instruction. The extension is for those students who need enrichment and the intervention is for those students who need remediation.

TASK DIRECTIONS

One way to introduce this task is by reading the first part of the book, *Sir Cumference and the Dragon of Pi*, by Cindy Neuschwander. STOP after reading “The Circle’s Measure” on page 13. Tell the students that it is their job to solve the riddle and save Sir Cumference before the knights go to slay the dragon.

After giving each student the “Saving Sir Cumference” student recording sheet, students may work in small groups to complete Part A. It may be helpful to ask groups to use a “Think-Pair-Share” strategy. Individually think about the answers to part A (Think). Next, students share their ideas and thoughts with their group (Pair). Finally, allow 2-3 students to discuss their ideas with the class (Share). Time dedicated to class discussion is critical. If no student brings up the connection between “Measure the middle...” with the diameter of a circle and “...circle around” with the circumference of a circle, help students make this connection.

If using the book with this task, continue reading pages 14-18 after students finish number 1 of the student recording sheet. STOP after reading page 18. At this point the students should understand that there is a relationship between diameter and circumference and that the measure of the circumference is very close to 3 times the length of the diameter. If this is not generally understood, hold a brief discussion before continuing with the task.

Finish reading the story and then go on to step 4.

Use a measuring tape to find the diameter and circumference of at least 5 different sized circular objects. (You may want to measure the circumference with narrow ribbon and then measure the ribbon to find the measure of the circumference.) Discuss with your group how to record your data in a table, and then create the table below. Discuss with students why it is important to be extremely meticulous with your measurement. We are trying to be as exact as possible since the slightest “estimation” will skew your ratio. Students may work in groups to make the measurements, but should record the data individually. As students work, circulate around the room taking notes on the various strategies students are using.

Use the grid to make a coordinate graph. Use the horizontal axis for diameter and the vertical axis for circumference. Plot your data for each object your group measured on the graph.

OPTION:
If available, you may want each group or pair to measure five unique objects. If that is the case, you can create a class graph where you end up with more data points to analyze.

Once students have identified a more exact number (3.1 or 3.2) for pi, the symbol π (pi) may be introduced to represent the ratio of the circumference to the diameter of a circle. Note: For our purposes, 3.14 is a close enough approximation of pi, however, for the curious student, the value of π to nine decimal places is 3.141592654. This is still an approximation of the number whose decimal expansion has no end.

When students have finished the task and the findings have been discussed and clarified, the rest of the book may be read aloud to the students.

What do you notice about the points you plotted on your graph? How is your graph similar to/different from the class graph that is being created?

SAVING SIR CUMFERENCE

Sir Cumference has been turned into a dragon! Help Radius and Lady Di of Ameter break the spell and save Sir Cumference. The answer to this problem lies in this poem. Can you solve the riddle?

The Circle's Measure

Measure the middle and circle around,
Divide so a number can be found.
Every circle, great and small-
The number is the same for all.
It's also the dose, so be clever.
Or a dragon he will stay...
Forever.

Read "The Circle's Measure" to page 13. Help Radius interpret the potion.

What do you think is meant by "measure the middle and circle around"?
Measure through the center of the circle. Then, see how many times that length goes around the outside.

What two numbers should you divide? How do you know? How can you set up these numbers as a ratio?
Divide the circumference by the radius

How would you explain your emerging understanding of the relationship between circumference and diameter?
The diameter wraps around the circumference three and a little bit not matter the size of the circle.

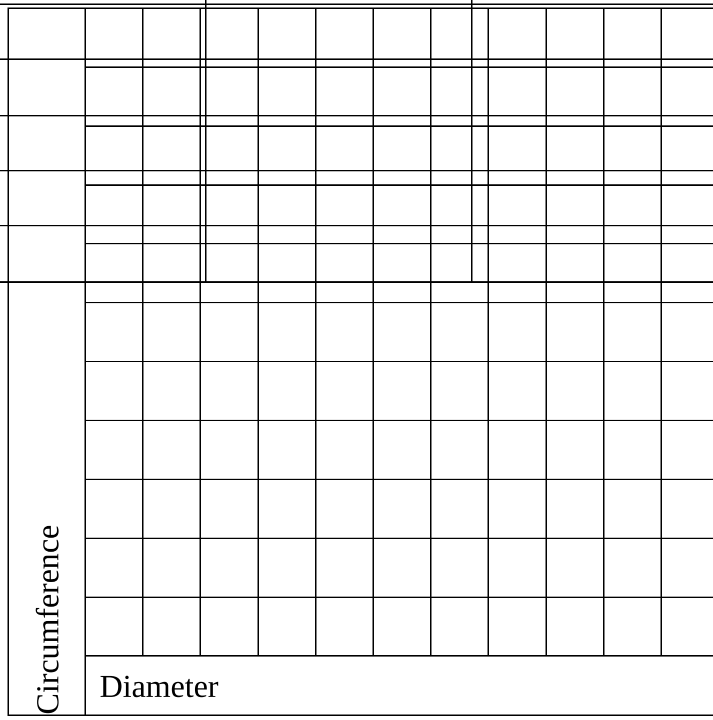
Upon completion of the book…
What does Radius use as the correct dosage? How did he come to this conclusion?
Answers may vary. Discuss how radius figures out that the solution is $\mathbf{3\frac{1}{7}}$ *and how this is related to pi.*

Proving the Dosage is Correct *Answers will vary based on the five objects chose.*

Use a measuring tape to find the diameter and circumference of at least 5 different sized circular objects. Use centimeters to measure. If you do not have measuring tape, you may use string and a ruler. Be as exact as possible. Record your results in the table.

OBJECT	CIRCUMFERENCE (MEASURE IN CENTIMETERS)	DIAMETER (MEASURE IN CENTIMETERS)	RATIO OF CIRCUMFERENCE TO DIAMETER	SIMPLIFIED RATIO

Use the grid below to make a coordinate graph. Use the horizontal axis for diameter and the vertical axis for circumference. Plot your data for each object your group measured on the graph.



The graph should show a straight line with the constant of proportionality being pi.

How does your graph reinforce the dosage amount that Radius chose to give his father?
No matter what object you choose to measure the constant of proportionality with remain the same. Since there is the same constant ratio, this is a direct variation.

The fraction $\frac{22}{7}$ is often used as an equivalent representation of pi. Using your knowledge of conversions between fractions and decimals, convert $\frac{22}{7}$ into a decimal. Why is this fraction an estimate for pi and not an exact value?

Solution: When converted to a decimal, 22/7 is a non-terminating and non-repeating decimal. Since pi is an irrational number it cannot be turned into a fraction.

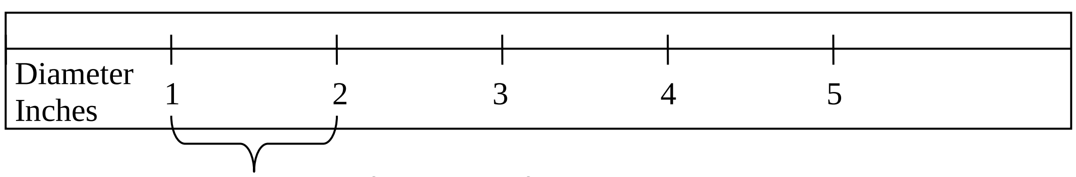
DIFFERENTIATION

Extension

Encourage students to explore a tape measure used in forestry management. Trees can be cut when they reach a certain diameter. However, it is impossible to measure the diameter of a tree using a traditional method with a tape measure without cutting it first. Therefore, a diameter ruler was created to measure the diameter using the relationship of diameter and circumference. More information on measuring trees can be found at the following web sites:
http://www.nycswcd.net/files/Forestry%20Measurements.pdf
http://phytosphere.com/treeord/measuringdbh.htm

Ask students to create a diameter tape measure. How would a tape that measures diameter when wrapped around the circumference be created? Once students have created the ruler, allow them to try it out on circular objects in the classroom. Alternatively, students could use their rulers outside to find the diameter of trees.

To create a diameter tape measure, students would need to identify the length of the circumference when the diameter is 1 inch, 2 inches, 3 inches, etc. To do so, students would need to multiply the diameter by 3.14, giving them the length of the circumference. Then they would need to mark the length on a ribbon or a roll of paper (such as adding machine tape). Instead of listing the actual measure, each interval would be labeled as the number of inches representing the diameter. See example below.

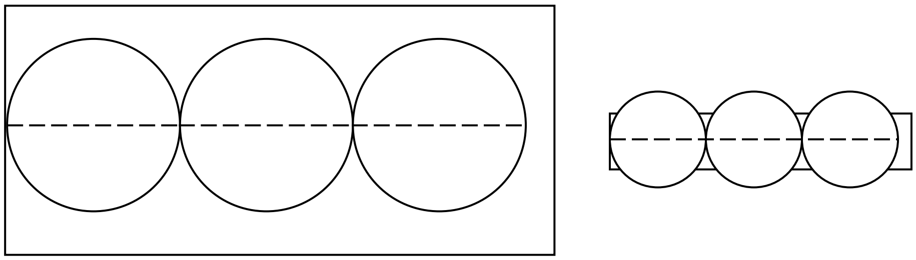


This length of the circumference represents a diameter measure of one inch. It is found by multiplying 1" by $\pi \approx 3.14$ (1" x 3.14 = 3.14"). Therefore, every 3.14" on the diameter measuring tape represents 1" of diameter. Students will need to approximate a measure of 3.14 which is approximately $\frac{1}{7}$. Using a traditional ruler, that would be a tiny bit more than $\frac{1}{8}$.

Another way to make this type of tape measure is to use a nonstandard unit of measure such as "soup can diameter". In this way, students can cut off the label of a can and lay it flat to mark off segments along a ribbon or roll of paper equal to the circumference of the can. However, the scale can be labeled as "can-diameter" units. The diameter of other objects can be measured using the tape measure and labeled in terms of the nonstandard unit "can-diameters."

An exploration of this type is an effective way for students to think more deeply about the relationship between π, diameter, and circumference. Also, it allows to students to measure units in terms of diameter (or radii) which is the basis for radians in trigonometry.

Intervention
Have students cut a strip of paper equal to the circumference of a circular object (i.e. a can). Then have the students place the can on the paper and trace it. Ask students to determine the number of times they can trace the can. Repeat with different sized cans. Ask students to write about what they notice and explain what their results mean regarding the relationship of diameter and circumference. As in the examples below, students should notice that a little more than three diameters fit on the circumference of a circle.



Hand out five index cards to each student or group of students. Write the words circumference, radius, pi, and diameter on the board. Ask the students to write one word on the top of each card. Encourage students to use a thesaurus or other reference material to write synonyms, definitions, and examples of each word on the back of the card. Students then arrange the cards in a manner that makes sense to them. (The students may arrange alphabetically, from least to greatest, or cluster the cards in groups.) Have several groups present and justify their arrangements.

http://www.eveandersson.com/pi/digits/ Digits of pi up to 1 million.

Name ____ Date ________________________

Saving Sir Cumference

Sir Cumference has been turned into a dragon! Help Radius and Lady Di of Ameter break the spell and save Sir Cumference. The answer to this problem lies in this poem. Can you solve the riddle?

Read "The Circle's Measure" to page 13. Help Radius interpret the potion.

What do you think is meant by "measure the middle and circle around"?

____

____

What two numbers should you divide? How do you know? How can you set up these numbers as a ratio?

____

____

The Circle's Measure

Measure the middle and circle around,
Divide so a number can be found.
Every circle, great and small-
The number is the same for all.
It's also the dose, so be clever.
Or a dragon he will stay...
Forever.

How would you explain your emerging understanding of the relationship between circumference and diameter?

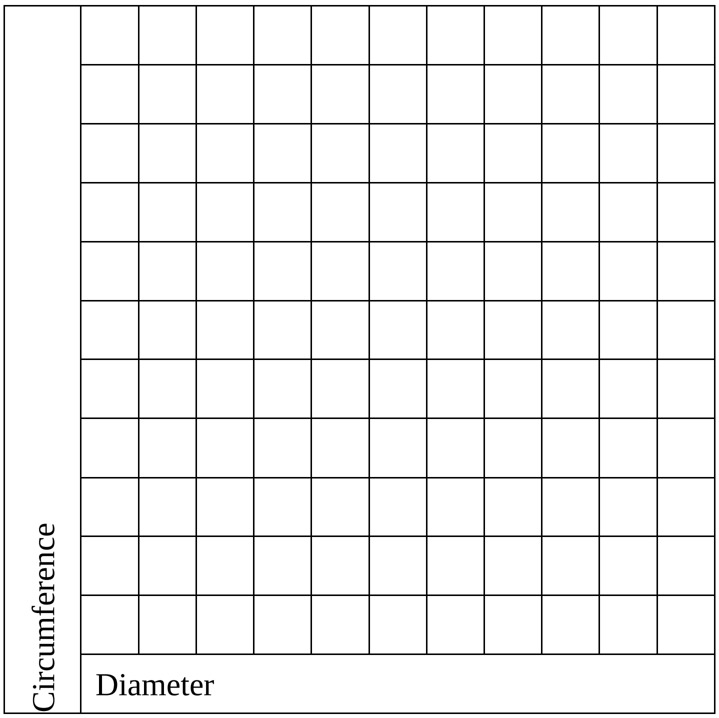
Upon completion of the book…
What does Radius use as the correct dosage? How did he come to this conclusion?

OBJECT	CIRCUMFERENCE (MEASURE IN CENTIMETERS)	DIAMETER (MEASURE IN CENTIMETERS)	RATIO OF CIRCUMFERENCE TO DIAMETER	SIMPLIFIED RATIO

Proving the Dosage is Correct

Use a measuring tape to find the diameter and circumference of at least 5 different sized circular objects. Use centimeters to measure. If you do not have measuring tape, you may use string and a ruler. Be as exact as possible. Record your results in the table.

Use the grid below to make a coordinate graph. Use the horizontal axis for diameter and the vertical axis for circumference. Plot your data for each object your group measured on the graph.



How does your graph reinforce the dosage amount that Radius chose to give his father?

The fraction $\frac{22}{7}$ is often used as an equivalent representation of pi. Using your knowledge of conversions between fractions and decimals, convert $\frac{22}{7}$ into a decimal. Why is this fraction an estimate for pi and not an exact value?

Area Beyond Squares and Rectangles

Back to task table

Adapted from MGSE 6th Grade Framework Task King Arthur's New Table

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make sure of structure

ESSENTIAL QUESTIONS

- How are the areas of parallelograms and triangles related to the area of a rectangle?

MATERIALS

- Centimeter grid paper
- Student direction sheet
- Colored pencils

GROUPING

- Individual/Partner/Small Group

TASK DESCRIPTION

Provide students with the directions for this task.

Did you know if you know the formula for finding the area of squares and rectangles, you can use this knowledge to find the formula for parallelograms and triangles?

Using centimeter grid paper cut out a rectangle of any size. Determine the area of your rectangle; be sure to record it in your journal.

2. Cut a right triangle from only one side of the rectangle and slide it to the opposite side. What shape was created? Can you change the shape back to a rectangle?

What is the area of this shape? Record it next to the area of the original rectangle.

How is the area of the new shape related to the area of the rectangle?

What formula can you use to determine the area of these shapes?

3. Using centimeter grid paper cut out another rectangle. Determine the area for this rectangle.

4. Draw a diagonal in the rectangle from one right angle to the opposite right angle.

What fraction of the rectangle is one triangle? Use this information about the triangle to determine the formula for the area of a triangle.

Use a parallelogram to determine the formula for area of a triangle. What did you conclude? In your journal, write about your discovery from today. How could you use this information in the future?

DIFFERENTIATION

Extension
Use the model to derive the area of a trapezoid.

Intervention
Provide instruction on a specific size of the rectangle in which students will cut out.

Area Beyond Squares and Rectangles

Did you know if you know the formula for finding the area of squares and rectangles, you can use this knowledge to find the formula for parallelograms and triangles?

1. Using centimeter grid paper cut out a rectangle of any size. Determine the area of your rectangle; be sure to record it in your journal.

2. Cut a right triangle from only one side of the rectangle and slide it to the opposite side. What shape was created? Can you change the shape back to a rectangle?

   What is the area of this shape? Record it next to the area of the original rectangle.

   How is the area of the new shape related to the area of the rectangle?

   What formula can you use to determine the area of these shapes?

3. Using centimeter grid paper cut out another rectangle. Determine the area for this rectangle.

4. Draw a diagonal in the rectangle from one right angle to the opposite right angle.

   What fraction of the rectangle is one triangle? Use this information about the triangle to determine the formula for the area of a triangle.

Use a parallelogram to determine the formula for area of a triangle. What did you conclude? In your journal, write about your discovery from today. How could you use this information in the future?

<u>Maximizing Area: Gold Rush</u> – (FAL)

Back to task table

(Problem Solving Task)
This lesson is intended to help you assess how well students are able to:
Explore the effects on the area of a rectangle of systematically varying the dimensions while keeping the perimeter constant.
Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1226

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.G.2 Explore various geometric shapes with given conditions. Focus on creating triangles from three measures of angles and/or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.

ESSENTIAL QUESTIONS

- How can area be maximized when the perimeter is a fixed number?

TASK COMMENTS

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Maximizing Area: Gold Rush*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=415&subpage=problem

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1226

Circle Cover-Up

Back to task table

Adapted from a task created by Michelle Parker, Gordon County Schools, Georgia

Students will extend their understanding of area and derive the formula for the area of a circle by rearranging the area of a square and by adapting the formula for the area of a rectangle.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.G.4 Given the formulas for the area and circumference of a circle, use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
5. Use appropriate tools strategically.
7. Look for and make use of structure.

COMMON MISCONCEPTIONS

Real-world and mathematical multi-step problems that require finding area, perimeter, volume, surface area of figures composed of triangles, quadrilaterals, polygons, cubes and right prisms should reflect situations relevant to seventh graders. The computations should make use of formulas and involve whole numbers, fractions, decimals, ratios and various units of measure with same system conversions.

Students may believe pi is an exact number rather than understanding that 3.14 is just an approximation of pi. Many students are confused when dealing with circumference (linear measurement) and area. This confusion is about an attribute that is measured using linear units (surrounding) vs. an attribute that is measured using area units (covering).

ESSENTIAL QUESTIONS

- How is the formula for the area of a circle related to the formula for the area of a parallelogram?

MATERIALS

- “Circle Cover-Up” student recording sheet
- “Circle Cover-Up, Cut and Cover” student recording sheet
- “Circle Cover-Up, Circles and Parallelograms” student recording sheet
- “Circle Cover-Up, Circles to Cut” student sheet (One per group of four)
- Scissors
- Crayons or colored pencils
- Tape or glue stick

GROUPING

- Individual/Partner Task

BACKGROUND KNOWLEDGE

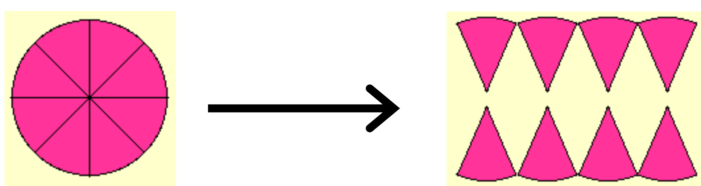
Students need to bring to this task their understanding of area of rectangles, including the ability to determine area by tiling. Also, students need to recognize an approximate value of π (3.14) and understand that it represents the relationship between the diameter and area of a circle.

Part 2

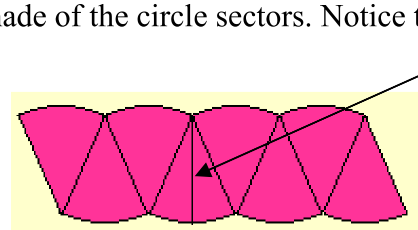
BACKGROUND KNOWLEDGE

Students need to know how to find the area of a parallelograms using the formula b x h before completing this task.

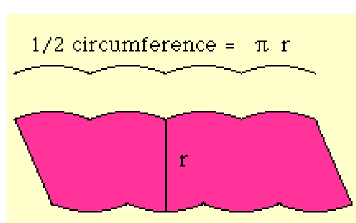
The images below are screen shots from http://curvebank.calstatela.edu/circle/circle.htm.
By cutting a circle into 8 (or more) equal sectors, the individual pieces can be arranged so that they begin to resemble a parallelogram. The smaller the pieces, the more it looks like a parallelogram.

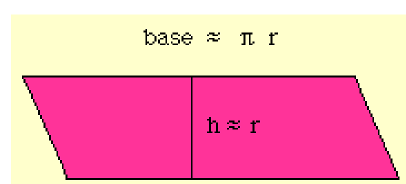


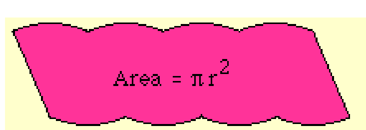
Below is our "parallelogram" made of the circle sectors. Notice the radius drawn in one of the middle sectors.



We know the formula for the area of a parallelogram is $A = b \times h$. So, now we need to use what we know about the characteristics of a circle (radius, circumference, and pi) to find the formula for the area of a circle.

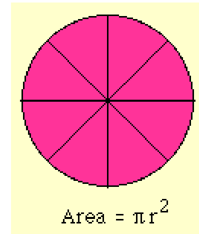






The radius(r) of a circle is any segment from the center of the circle to the circle's edge (circumference).

The circumference (C $= 2 \times \pi \times r$) is the distance around the circle. However, when the sections of the circle are arranged into a parallelogram, the circumference becomes the TWO bases of the parallelogram. Each base is $\frac{1}{2}$ the circumference or $\pi \times r$.



Since we know the formula for the area of a parallelogram, and the circle sectors now resemble a parallelogram, we can begin with the area formula for a parallelogram.

A* = *b* x *h *Formula for area of rectangle*	→	**$A = (\pi r) \times r$** *The base of our new "parallelogram" is $\frac{1}{2}$ the circumference of the circle or $\pi \times r$.* *The height of the new "parallelogram" is the radius of the circle r.*	→	**$A = \pi r^2$** *From $\pi \times r \times r$ we get πr^2.*

The formula for the area of rectangle can also be used to derive the formula for the area of a circle. As the sectors of the circle get smaller and smaller, the parallelogram gets closer and closer to the shape of a rectangle. For more information on using the area of a rectangle, go to the following link. http://www.worsleyschool.net/science/files/circle/area.html

TASK DESCRIPTION

Part 1

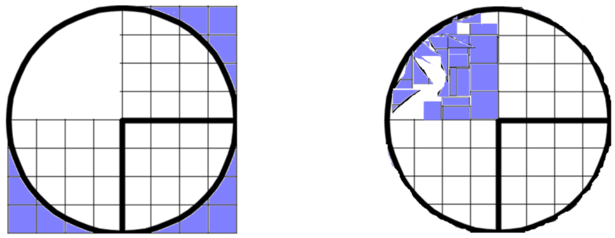
One way to introduce this task is to review the area formulas for squares, rectangles, triangles, and parallelograms.

Students will need to recognize that if the length of the radius of the circle is represented by *r*, then the length of the large square would be $2r$ and the area of the large square would be $2 \cdot r \cdot 2 \cdot r$, or $4 \cdot r \cdot r$, (or $4r^2$). Therefore, the area of the three smaller squares in terms of the radius (*r*) of the circle can be written as shown.
$\frac{3}{4} \cdot 4 \cdot r \cdot r$ which is equal to $3 \cdot r \cdot r$

When students cut the shaded area and paste it inside the empty quadrant of the circle, they should notice that the area of three squares is not enough to fill the circle. There needs to be a little bit more shaded area to fill the blank quadrant, as shown in the example below.

The area of one square would be *(s)(s)*. But the length of each side is the same as the length of the radius of the circle, so it could be written as (*r*)(*r*). There needs to be three areas of the square plus a little bit more. Ask students what relationship in a circle is equal to three and a little bit more.

Students should remember that pi (π) is a little more than three. Therefore the area of a circle could be found by multiplying $\pi \times r \times r =$ Area of a circle. This is also written as $\pi \times r^2$ or simply πr^2.



Questions/Prompts for Formative Student Assessment

- What do the square and circle have in common?
- Why do you think the square (or the circle) has a larger area?
- How much of the blank quadrant did you fill with the shaded area? Did you have any left over? Did you have enough?
- Why do you think pi plays a role in the area of a circle?

Once students have completed the task, the following questions will help you further analyze student understanding.

Questions for Teacher Reflection

- Did students recognize that the area of the circle is a little more than the area of three squares with the square's side length equal to the circle's radius?
- Did the students recognize that pi is required to find the area of a circle?

Part 2

Part two of this task may be introduced by reading *Sir Cumference and the Isle of Immeter* by Cindy Neuschwander or a similar story about finding the perimeter and area of plane figures.

During the task, the following questions/prompts will help you gauge student's understanding of the concepts.

Questions/Prompts for Formative Student Assessment

- How can you arrange the sectors of the circle to create a shape that looks like a parallelogram?
- What is the measure of the radius in units?
- What is the measure of one of the bases of the "parallelogram"?
- What is the formula for the area of a parallelogram?
- What parts of the circle can be replaced in the parallelogram formula? How do you know?
- How do you find the length of the base of the "parallelogram" you created?
- How do you find the height of the "parallelogram" you created?
- How did you approximate the area of the "parallelogram" you created?

Once students have completed the task, the following questions will help you further analyze student understanding.

Questions for Teacher Reflection

- Which students were able to connect the base and height from the parallelogram formula with $\frac{1}{2}$ the length of the circumference of the circle ($\pi \times r$) and the radius (r) respectively?
- Which students were able to use the formula for a parallelogram and the grid paper to approximate the area of the "parallelogram" created with the circle sectors?

As students move through the task and formative assessment takes place, consider the following suggestions for differentiating instruction. The extension is for those students who need enrichment and the intervention is for those students who need remediation.

DIFFERENTIATION

Extension

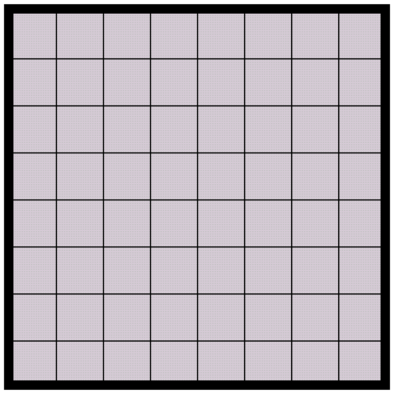
Ask students to explore and prepare an explanation of other ways to derive the formula for the area of a circle. Two web sites that show alternative methods are given below.
http://curvebank.calstatela.edu/circle2/circle2.htm Uses animation to derive the formula for the area of a circle based on the area of a triangle.
http://www.worsleyschool.net/science/files/circle/area.html Uses graphics to derive the formula for the area of a circle.

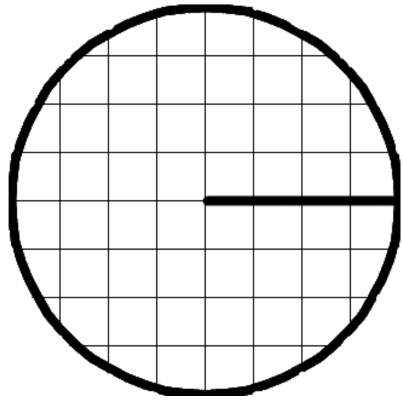
Intervention

Provide students with some scaffolding for the "Circle Cover-Up, Circles and Parallelograms" student recording sheet as shown in the examples below. These questions were added to "Circle Cover-Up, Circles and Parallelograms, Version 2" at the end of this task.

Part I: Circle Cover-Up

Compare the areas of the square and circle below.





Which one has the larger area? Write to explain how you know.

Answer may vary based on student perception of the figures. Most students will probably say the square has the larger area.

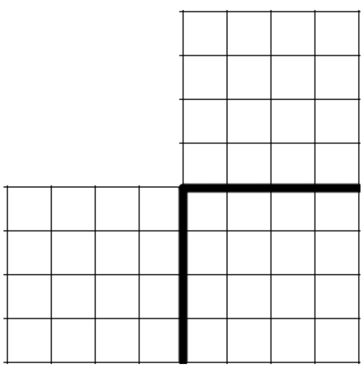
How is the radius of the circle related to the length of the square? Write your answer in terms of the example above and then make a generalization if the radius is *r*.

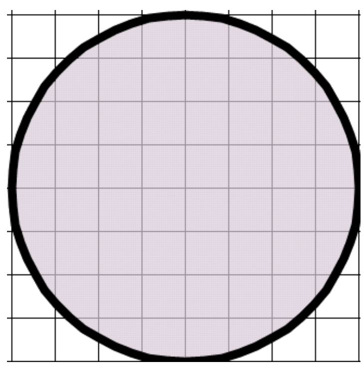
The square has a length of 8 units and the circle has a length of 4 units. If the radius is represented with an r then the length of the square is double that so would be 2r.

Based on the side length for the square you generalized from problem 2, what would be the area of the square?

Area of Square is side times side so the area = (2r)(2r) = $\mathbf{4r^2}$.

Compare the areas of the two figures below.





Do you think one of the areas is larger than the other? Write to explain your thinking.

Answers may vary.

What is the approximate area of the three smaller squares $\left(\frac{3}{4} \text{ of the large square}\right)$ in terms of *r*? (Use the area formula you created in problem 3 to get started.)

Area of the larger square = $\mathbf{4r^2}$*. The second figure is* $\frac{3}{4}(4r^2) = 3r^2$*.*

Follow the directions on the "Circle Cover-Up Cut and Cover" found on the next page.
Which figure has the larger area? Write below to explain your findings.
Give students a set amount of time in which to work. They should come to the conclusion that there will still be a small amount of space left in the circle.

How do you think finding the area of a circle is related to finding the area of a square?

The area of the square is larger than the area of the circle. The area of the square is $\mathbf{4r^2}$*. We discovered in part two the area of a circle is close to ¾ of this value.*

What role do you think pi plays in finding the area of a circle?
Pi is the ratio that relates the square to the circle.

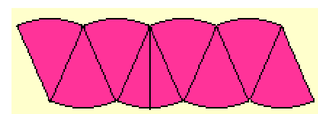
<u>Part II</u>
Task Directions

Students will follow the directions below from the "Circle Cover-Up, Circles and Parallelograms" student recording sheet.

*As an alternative, you may also give each student a small paper plate. Have students begin by cutting it into fourths and try to build a parallelogram. They can then cut the fourths to create eighths and try to build the quadrilateral. Students keep cutting the triangles and arranging the pieces. The smaller the triangles the more the shape begins to resemble a parallelogram proving the two areas are the same.

Cut out one circle from the "Circle Cover-Up, Circles to Cut" student sheet.

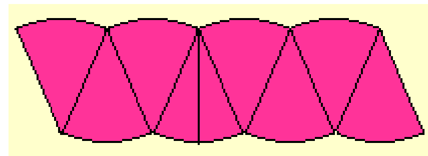
Cut the sectors of the circle apart and arrange them on the grid paper as shown to form a parallelogram.

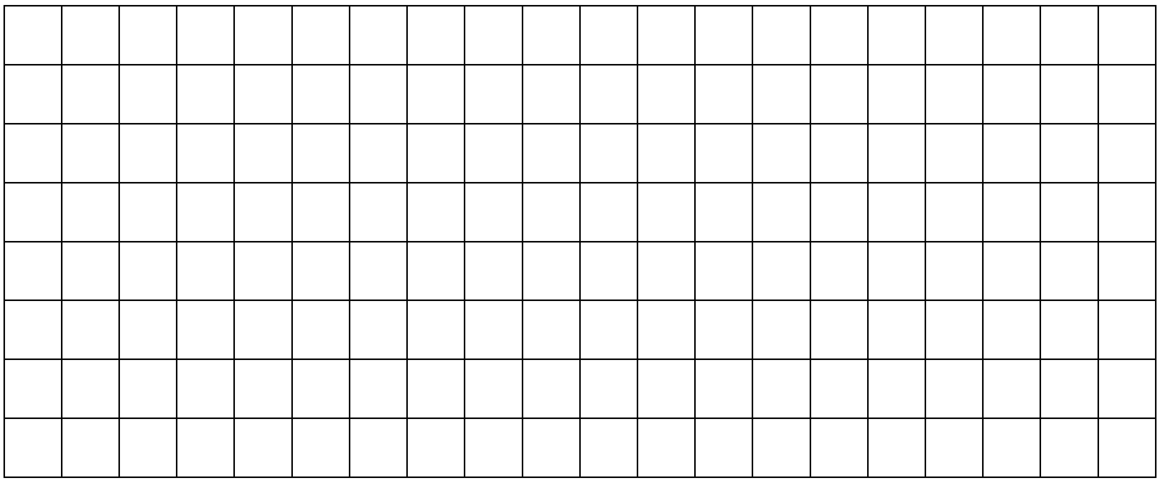


Use the grid paper to help you approximate the area of

Circle Cover-Up - Circles and Parallelograms

Cut out one circle from the "Circle Cover-Up, Circles to Cut" student sheet.
Cut the sectors of the circle apart and arrange them on the grid paper as shown to form a parallelogram.





Use the grid paper to help you approximate the area of the "parallelogram" formed. What is the approximate area of the "parallelogram"? __________ square units

- Write the formula for the area of a parallelogram. *Area = (base)(height)*

- How is the height of the parallelogram related to the original circle?
  The height of the rectangle is the radius of the circle.

- How is the base of the parallelogram related to the original circle?
  The base of the parallelogram is half the circumference.

- Rewrite the formula for the parallelogram in terms of the circle based on your observations from question 5 and 6. *Area of parallelogram = (base)(height)*
  Area = $(\frac{1}{2}\boldsymbol{C})(\boldsymbol{r})$
- What is the formula for the circumference of a circle? *C* = $\boldsymbol{\pi d}$ *or* 2 $\boldsymbol{\pi r}$

- Rewrite the formula from step 7 above. Replace C with the formula for circumference that uses the radius. *Area* =$(\frac{1}{2}\boldsymbol{C})(\boldsymbol{r})$
  Area = $(\frac{1}{2}\cdot \boldsymbol{2\pi r})(\boldsymbol{r})$
  Area = $\boldsymbol{\pi r^2}$

Why did we need to use the circumference formula that uses the radius instead of the diameter?
You already have the radius started in the formula. In order to be able to simplify the formula, you need to have the same variable

If the radius of a circle and the height of a parallelogram are the same, use what you discovered about how the circumference and base of a parallelogram are related in order to create a circle and parallelogram with the same area. Write the dimension for circumference and base in terms of pi.

Radius of circle = height of parallelogram
½ Circumference of circle = the base of the parallelogram.

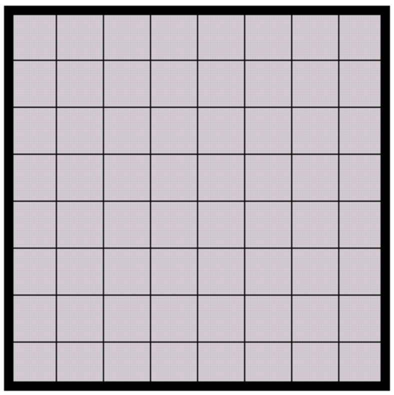
If the circle has a radius of 4, the circumference would be $2(4)\pi = 8\pi$.
The parallelogram would have a height of 4 and a base of (1/2)($8\pi) = 4\pi$

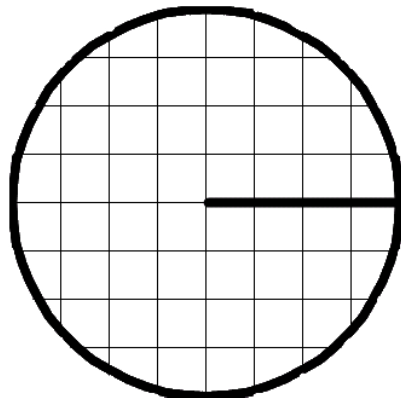
Area of the circle = $\pi(4)^2$ = 16π
Area of parallelogram = 4(4π)= 16π

Name ____ Date ________________________

Circle Cover-Up

Compare the areas of the square and circle below.



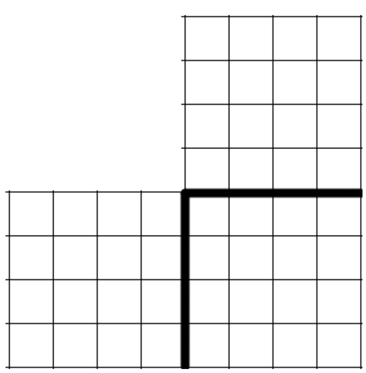


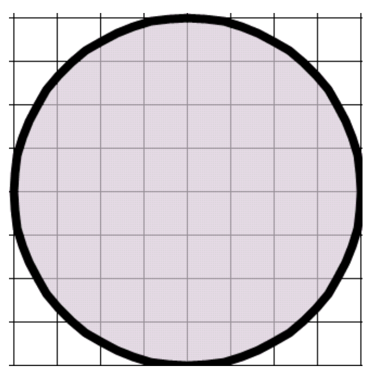
Which one has the larger area? Write to explain how you know.

How is the radius of the circle related to the length of the square? Write your answer in terms of the example above and then make a generalization if the radius is r.

Based on the side length for the square you generalized from problem 2, what would be the area of the square?

Compare the areas of the two figures below.





Do you think one of the areas is larger than the other? Explain your reasoning.

What is the approximate area of the three smaller squares $\left(\frac{3}{4}\text{ of the large square}\right)$ in terms of r? (Use the area formula you created in problem 3 to get started.)

Follow the directions on the “Circle Cover-Up Cut and Cover” found on the next page.
Which figure has the larger area? Write below to explain your findings.

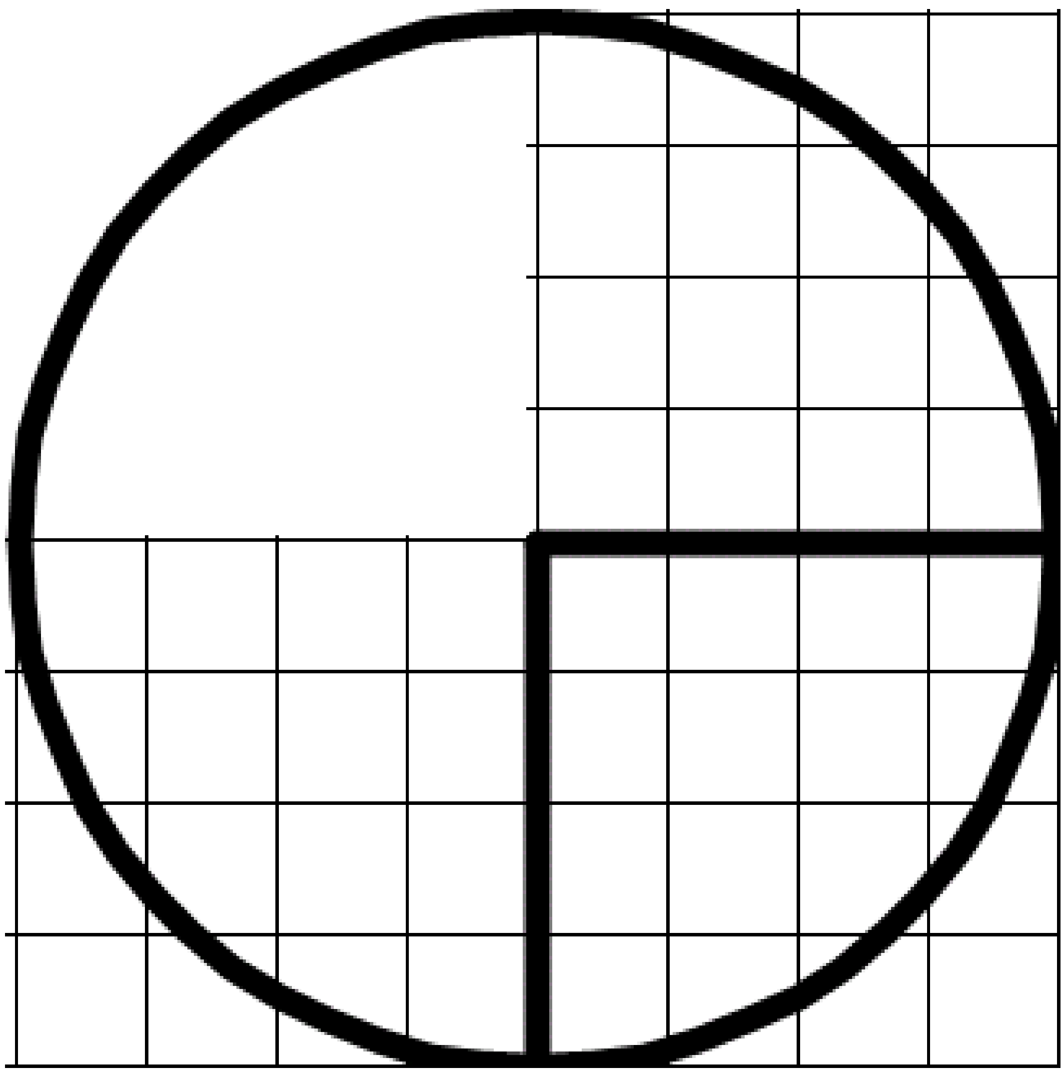
How do you think finding the area of a circle is related to finding the area of a square?

What role do you think pi plays in finding the area of a circle?

Name ____ Date ________________________

Circle Cover-Up - Cut and Cover

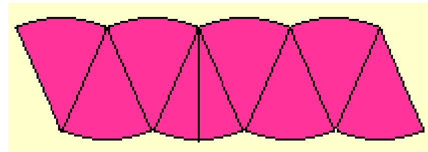
Color the area of the squares that are outside the circle.
Cut out the circle, but save the colored area that was not inside the circle.
Paste the area outside the circle into the blank quadrant in a mosaic design. Try not to overlap pieces. You may need to cut your colored pieces into smaller pieces so they will fit.

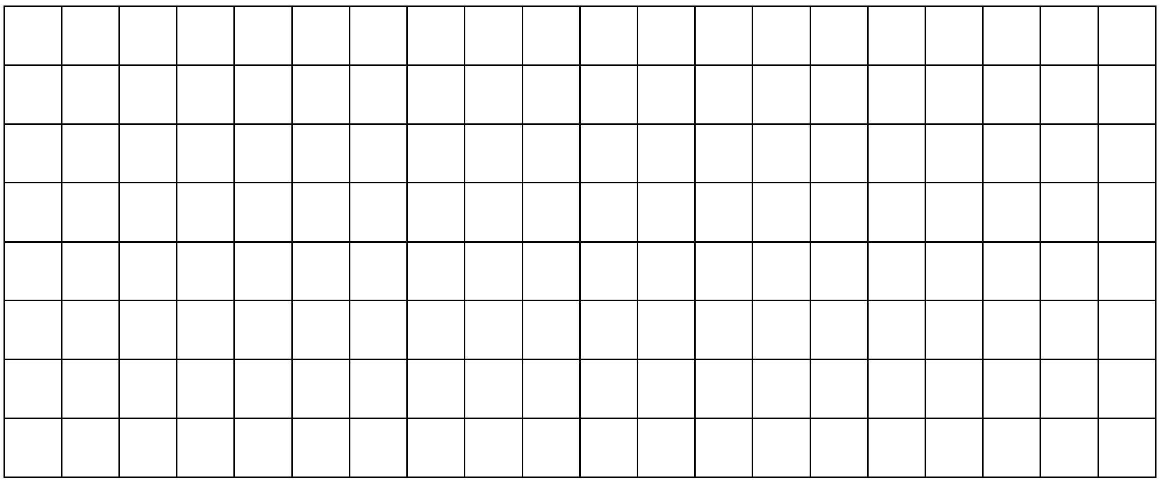


Name ______________________________________ Date ________________________

Circle Cover-Up - Circles and Parallelograms

Cut out one circle from the “Circle Cover-Up, Circles to Cut” student sheet.
Cut the sectors of the circle apart and arrange them on the grid paper as shown to form a parallelogram.





Use the grid paper to help you approximate the area of the “parallelogram” formed. What is the approximate area of the “parallelogram”? __________ square units
Write the formula for the area of a parallelogram.
How is the height of the parallelogram related to the original circle?

How is the base of the parallelogram related to the original circle?

Rewrite the formula for the parallelogram in terms of the circle based on your observations from question 5 and 6.

What is the formula for the circumference of a circle?

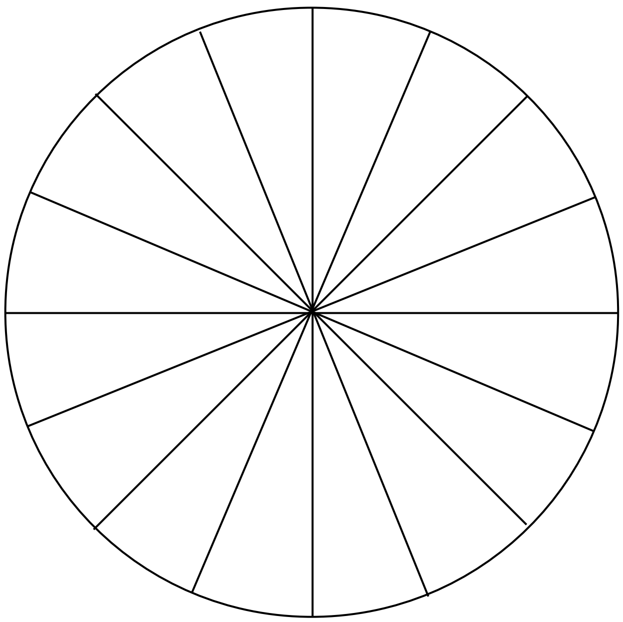
Rewrite the formula from step 7 above. Replace C with the formula for circumference that uses the radius.

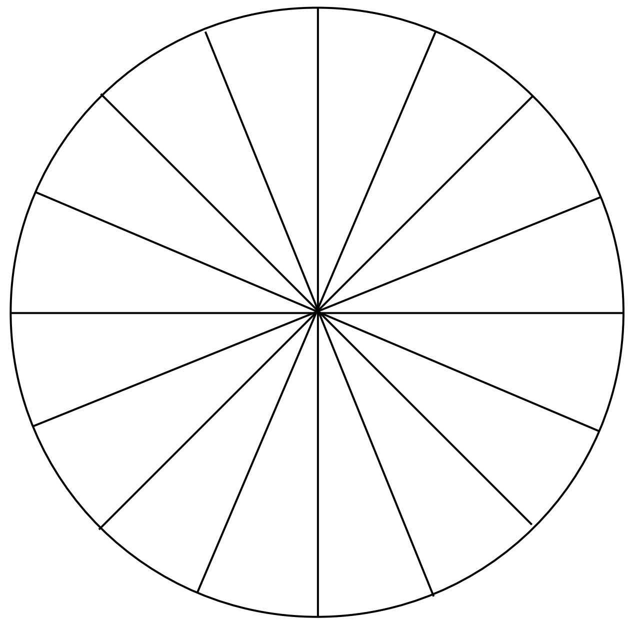
Why did we need to use the circumference formula that uses the radius instead of the diameter?

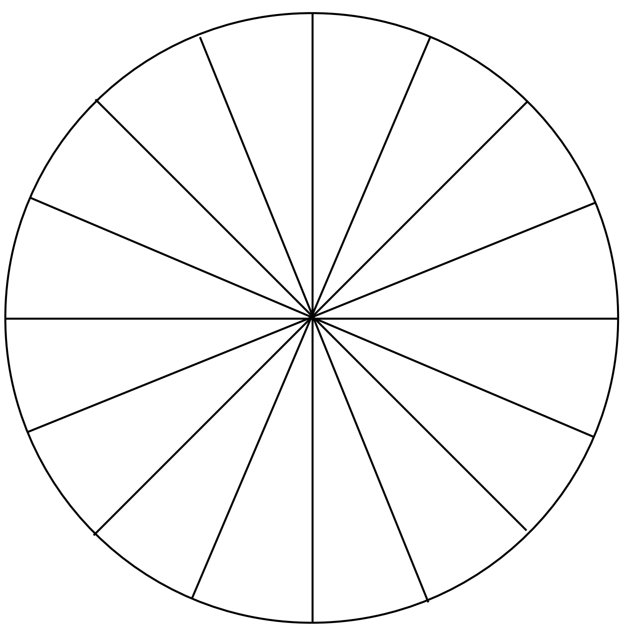
If the radius of a circle and the height of a parallelogram are the same, use what you discovered about how the circumference and base of a parallelogram are related in order to create a circle and parallelogram with the same area. Write the dimension for circumference and base in terms of pi.

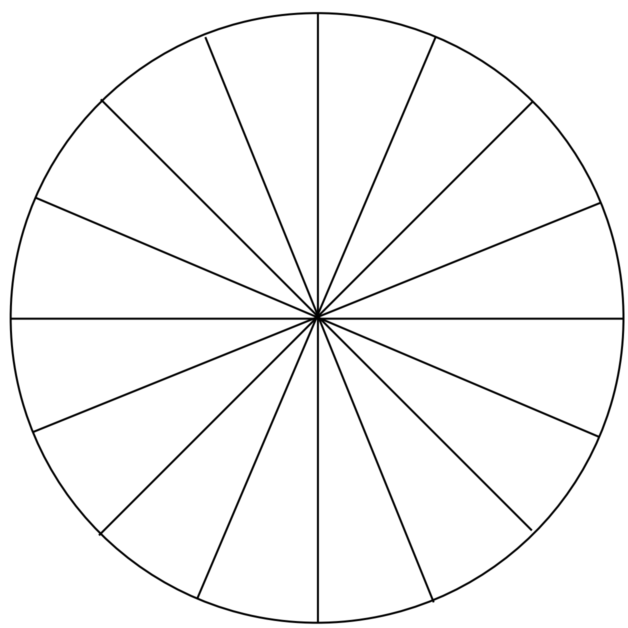
Name ____ Date ________________________

Circle Cover-Up - Circles to Cut









Using Dimensions: Designing a Sports Bag – (FAL)

(Problem Solving Task) **Back to task table**

This lesson is intended to help you assess how well students are able to: recognize and use common 2D representations of 3D objects; identify and use the appropriate formula for finding the circumference of a circle.
Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/lessons.php?taskid=416&subpage=problem

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.G.4 Given the formulas for the area and circumference of a circle, use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems of problems and persevere in solving them
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically.
6. Attend to precision

ESSENTIAL QUESTIONS

- How do I apply the concepts of surface area and circumference to solve real-world problems?

TASK COMMENTS

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Designing a Sports Bag*, is a Formative Assessment Lesson (FAL) that can be found at the website: http://map.mathshell.org/materials/lessons.php?taskid=416&subpage=problem

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:

http://map.mathshell.org/materials/download.php?fileid=1228

Think like a Fruit Ninja: Cross Sections of Solids

Back to task table

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.G.3 Describe the two-dimensional figures (cross sections) that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms, right rectangular pyramids, cones, cylinders, and spheres.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

COMMON MISCONCEPTIONS

Students usually think that slicing a sphere can result in a cross-section that is the shape of an ellipse. However, any way you slice a sphere, you will always get a cross section of a circle. It is common for students to have a hard time visualizing the difference between pyramids and prisms.

ESSENTIAL QUESTIONS

- What two-dimensional figures can be made by slicing a cube by planes?
- What two-dimensional figures can be made by slicing: cones, prisms, cylinders, and pyramids by planes?

MATERIALS

- dough or modeling clay
- fishing line or dental floss if using modeling clay
- Optional for demonstrations: power solids, geometric shapes with nets, paper, colored water, and rice
- http://www.learner.org/channel/courses/learningmath/geometry/session9/part_c/index.html

GROUPING

Individual/Partner/Small Group

TASK COMMENTS

In this task, students will discover what two-dimensional figures can be made when slicing a cube by planes. There are many ways to "hook" students and increase engagement in this topic. Choose one or use a combination of the examples suggested in the introduction so that students can manipulate and interact with cross sections in different ways. You will want to choose a method of demonstration or interaction based upon the technology and resources available at your school. Most schools have received both Power Solids and geometric shapes with nets. These will make it easy for the students to experiment with this task using colored water.
If some wish to use clay or play-dough, it is suggested that they cut the solids using something like fishing line or dental floss rather than a plastic knife.

TASK DESCRIPTION

Prior to beginning the activity, demonstrate with students the method or methods that will be used to explore the big idea of cross sections. Choose from the list below to demonstrate the general idea of cross sections and then more specifically the many cross sections of a cube:
The app for called "fruit ninja" is a great fun and visual way to introduce the idea of 3D shapes (fruit) being cut by a plane (sword). What you see once the fruit is cut is the cross section (2D shape)

There is also a Wii game called "sword play" on Wii sports resort that involves slicing many different objects with a sword. Objects include bamboo (cylinder), toaster (rectangular prism), orange (sphere), and many others. You may want to create a list ahead of time that tells students what some of the ambiguous shapes will be called for your game's purposes. For example, the cupcake is a bit of an odd-shape with some rounded sides, so you can choose to have students call it a hexagonal prism or rectangular prism. There will be less arguing amongst students if you establish what these imperfect 3D shapes will be considered before playing the game. Wii consoles can be hooked up to interactive white boards or classroom televisions and students can play this game individually or they can challenge each other two at a time. It can be a big event, and can even be called a "tournament" with a bracket of players. In order to win points, teachers should require students to say the name of the 2D cross section as they slice each object. This game changes the angle at which the object is sliced. Students can earn points for speed and accuracy (recorded by the game) and saying the correct cross section for each slice (recorded by the teacher).

Students can be given an opportunity to explore three-dimensional shapes with their hands, in a tactile way. Each student gets a small amount of modeling clay or dough, shapes it into a cube, and then cuts the cubes with fishing line or floss in different ways to see what cross-sections can be made.

Plastic, transparent models of cubes that have one open-face can be filled with rice, sand, or water, and tipping the cube in different ways, the students could demonstrate the different cross sections that can be made.

Students could access the website:
http://www.learner.org/channel/courses/learningmath/geometry/session9/part_c/index.html and use the interactive software to illustrate some of the different possible slices.

Show the following video called "sections of a cube":
http://www.youtube.com/watch?v=Rc8X1_1901Q.

Think like a Fruit Ninja: Cross Sections of Solids

Part I: Cross Sections of a Cube

Try to make each of the following cross sections by slicing a cube.
Record which of the shapes you were able to create and how you did it. If you can't make the shape, explain why not.

Solutions may vary. Here are some possible solutions that students may find.

2-D Cross Section	Possible?	Impossible?	Explanation why possible or why NOT possible?
Square	X		***A square cross section can be created by slicing the cube by a plane parallel to one of its square faces.***
Equilateral triangle	X		***An equilateral triangle cross-section can be obtained by slicing off a corner of the cube so that the three vertices of the triangle are at the same distance from the corner.***
Rectangle, not a square	X		***One way to obtain a rectangle that is not a square is by slicing the cube with a plane parallel to one of its edges, but not parallel to one of its square faces.***
Triangle, not equilateral	X		***If we slice off a corner of a cube so that the three vertices of the triangle are not at the same distance from the corner, the resulting triangle will not be equilateral.***
Pentagon	X		***To get a pentagon, slice with a plane going through five of the six faces of the cube.***
Regular hexagon	X		***To get a regular hexagon, slice with a plane going through the center of the cube and perpendicular to an interior diagonal.***
Hexagon, not regular	X		***Any other slice that goes through all six square faces of the cube gives a non-regular hexagon.***
Octagon		X	***It is not possible to create an octagonal cross-section of a cube because a cube has only six faces.***
Trapezoid, not a parallelogram	X		***To create a trapezoid that is not a parallelogram, slice with a plane going through one face near a vertex through the opposite face at a different distance from the opposite vertex.***
Parallelogram, not a rectangle	X		***To create a non-rectangular parallelogram, slice the cube by any plane that goes through two opposite corners of the cube but not containing any other vertex of the cube.***
circle		X	***It is not possible to create a circular cross-section of a cube because all slices are polygons with sides formed by slicing the square faces of the cube.***

<u>Part II: Cross Section of a Pyramid</u>

In the movie, Despicable Me, an inflatable model of The Great Pyramid of Giza in Egypt is created to trick people into thinking that the actual pyramid has not been stolen. When inflated, the false Great Pyramid was 225 m high and the base was square with each side 100 m in length. Construct a model of the pyramid, with a base that is 1 inch on each side. Be sure to make the height proportional to the base just as in the real pyramid.

What proportion can be used in order to determine the height of your model?
$\frac{225\ m}{100\ m} = \frac{2.25\ in.}{1\ in.}$

What is the height of your model in inches?
in.

Suppose the pyramid is sliced by a plane parallel to the base and halfway down from the top (you can cut your model to demonstrate this slice).
What will be the shape of the resulting cross section? *square*

Compare the dimensions of the base of the sliced off top in comparison to the base of the original un-sliced pyramid? How many inches is each side of base of the top? Justify your answer. *If you slice the pyramid halfway down from the top, you'll have a top with a base that is half the dimension of the base of the original inflatable pyramid. The sides of the base of the top will be .5 in.*

Next, the pyramid is put back together and then sliced by a plane parallel to the base and 25% of the way down from the top (you can cut your model to demonstrate this slice).

Compare the dimensions of the base of the new smaller sliced off top in comparison to the base of the original un-sliced pyramid? How many inches is each side of this new top? *If the slice is 25% of the way down from the top, you'll have a square base for the top with sides that are 25% of the original inflatable pyramid base. that is reduced in size from the base by 75%. Reducing 1 inch by 75% results in ¼ in. dimensions for the base of the new top*

What if the slicing plane is not parallel to the base? What will the shape of the cross section be under those conditions? *Trapezoid, not a parallelogram*

<u>DIFFERENTIATION</u>

Extension

Part I

Another version of the Wii sword play game would be to have students say the 3D shape of the object AND the 2D cross section after the slice.
Ask students to discuss and explain whether or not it is possible to cut a cube so that the resulting cross-section is a point or a line segment. Very advanced students may arrive at the conclusion that points and lines are not two-dimensional, therefore cannot be considered a two-dimensional

cross section. They can describe that slicing a cube so that only one edge would be removed would create a cross section that looks like a line, but in reality, it would be a very thin rectangle. Cutting a corner of a cube would produce a square or trapezoid so small that it will look like a point. Students can debate whether or not there is a point at which these shapes become a line segment or point and is no longer 2D. Research on exact definitions of zero-dimensional, one-dimensional, two-dimensional, three-dimensional, intersection, planes, points, line segments, and lines would help students gather evidence to defend their ideas.
Students should become aware of cross sections throughout the world in everyday situations such as nature, food, architecture, art, etc. Perhaps keeping a log of these could be helpful throughout the unit. If technology is available, students can also participate in a photo scavenger hunt where they try to take the most pictures of examples of cross-sections found in the classroom and around the school.

Part II
The Great Pyramid of Giza in Egypt is often called one of the Seven Ancient Wonders of the world. The monument was built by the Egyptian pharaoh Khufu of the Fourth Dynasty around the year 2560 BC to serve as a tomb when he died. When it was built, the Great pyramid was 145.75 m high. The base is square with each side 231 m in length.
Construct a model of the pyramid, with a base that is 6 inches on each side.
Be sure to make the height proportional to the base just as in the real pyramid.
Suppose the pyramid is sliced by a plane parallel to the base and halfway down from the top. What will be the shape of the top? What will the dimensions of the slice be? Justify your answer.

What if the slice is 15% of the way down from the top?

What if the slicing plane is not parallel to the base? What will the shape of the slice be under those conditions?
Solutions
The model of the pyramid should have a base that is 1 ft by 1 ft and .63 feet or about 7.5 inches high.
If you slice the pyramid halfway down from the top, you'll have a cross-section that is square and half the dimension of the base of the pyramid. If the slice is 15% of the way down from the top, you'll have a square that is reduced in size from the base by 85%.

Interventions
Have students use pre-cut Styrofoam shapes (found at craft store) as stamps with paint. Students can write the name of the 3D shape, stamp the 2D cross section and then name it as well on paper. Cubes, prisms, cylinders, and pyramids should be included.

<u>Think Like a Fruit Ninja: Cross-Sections of Solids</u>

Part I: Cross Section of a Cube

Try to make each of the following cross sections by slicing a cube.
Record which of the shapes you were able to create and how you did it. If you can't make the shape, explain why not.

2-D Cross Section	Possible?	Impossible?	Explanation why possible or why NOT possible?
Square			
Equilateral triangle			
Rectangle, not a square			
Triangle, not equilateral			
Pentagon			
Regular hexagon			
Hexagon, not regular			
Octagon			
Trapezoid, not a parallelogram			
Parallelogram, not a rectangle			
circle			

Part II: Cross Sections of a Pyramid

In the movie, Despicable Me, an inflatable model of The Great Pyramid of Giza in Egypt is created to trick people into thinking that the actual pyramid has not been stolen. When inflated, the false Great Pyramid was 225 m high and the base was square with each side 100 m in length.

Construct a model of the pyramid, with a base that is 1 inch on each side. Be sure to make the height proportional to the base just as in the real pyramid.

What proportion can be used in order to determine the height of your model?

What is the height of your model in inches?

Suppose the pyramid is sliced by a plane parallel to the base and halfway down from the top (you can cut your model to demonstrate this slice).
What will be the shape of the resulting cross section?

Compare the dimensions of the base of the sliced off top in comparison to the base of the original un-sliced pyramid? How many inches is each side of the top? Justify your answer.

Next, the pyramid is put back together and then sliced by a plane parallel to the base and 25% of the way down from the top (you can cut your model to demonstrate this slice).
Compare the dimensions of the base of the new smaller sliced off top in comparison to the base of the original un-sliced pyramid? How many inches is each side of this new top?

What if the slicing plane is not parallel to the base? What will the shape of the cross section be under those conditions?

Cool Cross-Sections

Back to task table

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.G.3 Describe the two-dimensional figures (cross sections) that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms, right rectangular pyramids, cones, cylinders, and spheres.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.

ESSENTIAL QUESTIONS

- What two-dimensional figures can be made by slicing a cube by planes?
- What two-dimensional figures can be made by slicing: cones, prisms, cylinders, and pyramids by planes?

MATERIALS

- power solids
- water or modeling clay
- fishing line or dental floss if using modeling clay

TASK COMMENTS

Students should become aware of cross sections throughout the world in everyday situations such as nature, food, architecture, art, etc. Perhaps keeping a log of these could be helpful throughout the unit.

While this task may serve as a summative assessment, it also may be used for teaching and learning. It is important that all elements of the task be addressed throughout the learning process so that students understand what is expected of them.

Peer Review
Display for parent night
Place in portfolio
Photographs

To get ideas for their cross-sections, you might have the students peruse the following:

Incredible Cross-Sections of Star Wars, Episodes IV, V, & VI: The Ultimate Guide to Star Wars Vehicles and Spacecraft *by David Reynolds, Richard Chasemore (Illustrator), and Hans Jenssen (Illustrator).*

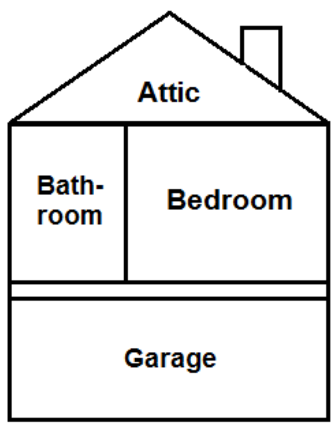
A series of books by Stephen Biesty and Richard Platt such as Coolest Cross-Sections Ever, Incredible Cross-Sections, Castles, and more.

Vist the following site to view a simple cross-section of the Mayflower:
http://www.mayflowerhistory.com/History/mflower5.php

TASK DESCRIPTION

Cool Cross-Sections

When an architect designs a house, he or she sketches not only views of the outside of the house, but also cross-sections of the house so that you can see the arrangement of the rooms in the house. For example, here's a cross-sectional view of a house, with the cross-section made by slicing the house with a plane perpendicular to the base and perpendicular to the front wall of the house, parallel and a few feet from the side of the house:



If you wanted to live in the house, you'd probably want to see more cross-sections, like a cross-section parallel to the front of the house, or a cross-section parallel to the foundation of the house (this would be called floor plans for each level of the house).

For your final project, design your own house, boat, or castle, and sketch some interesting cross-sections of your house, boat, or castle. Be sure to use all of the following shapes in your structure and show cross-sections: prism, cylinder, cone, and pyramid.

Also, include a section of your design that is formed by moving a two-dimensional figure through space to create a three-dimensional figure. Show the cross-section of this three-dimensional figure as well.

Comment:
Solutions could vary widely. You could have the students make sure their cross-sections are drawn to scale.

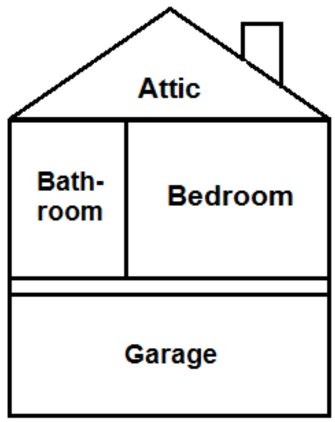
DIFFERENTIATION

Extension
Students can scale their model by a rational number scale factor and draw the resulting figure on graph paper

Intervention
Eliminate the garage and the bathroom from the floor plan

Cool Cross-Sections

When an architect designs a house, he or she sketches not only views of the outside of the house, but also cross-sections of the house so that you can see the arrangement of the rooms in the house. For example, here's a cross-sectional view of a house, with the cross-section made by slicing the house with a plane perpendicular to the base and perpendicular to the front wall of the house, parallel and a few feet from the side of the house:



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Also, include a section of your design that is formed by moving a two-dimensional figure through space to create a three-dimensional figure. Show the cross-section of this three-dimensional figure as well.

What's My Solid?

Back to task table

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.G.3 Describe the two-dimensional figures (cross sections) that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms, right rectangular pyramids, cones, cylinders, and spheres.

MGSE7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make sure of structure

COMMON MISCONCEPTIONS

Students believe that certain shapes are absolutely not possible, forgetting that there are many different possibilities for three-dimensional planes to cut a solid figure

Students do not have a clear idea of a plane, especially when using tools such as dental floss or fishing line that resemble lines

Students often look at the three-dimensional figure rather than the two-dimensional face

Students forget that the modeling clay can distort the shape and make it more rounded and can become confused about which two-dimensional cross-section was created

Real-world and mathematical multi-step problems that require finding area, perimeter, volume, surface area of figures composed of triangles, quadrilaterals, polygons, cubes and right prisms should reflect situations relevant to seventh graders. The computations should make use of formulas and involve whole numbers, fractions, decimals, ratios and various units of measure with same system conversions.

ESSENTIAL QUESTIONS

- What two-dimensional figures can be made by slicing a cube by planes?
- What two-dimensional figures can be made by slicing: cones, prisms, cylinders, and pyramids by planes?

MATERIALS

- power solids
- water or modeling clay
- plastic knife or dental floss if using modeling clay
- wax paper

GROUPING

Individual/Partner/Small Group

TASK COMMENTS

In this task, students will identify solid figures using clues about cross-sections, characteristics of solids when sliding (translating) or twisting (rotating) plane figures and knowledge of area and volume. Students can make physical models and explore solids through various means.

This activity could be set up as a "What's my line?" type of game, with one student knowing what the solid is and another student answering questions about it. The clues could be written on flash cards. Students could work in pairs taking turns drawing a card to read to his/her partner.

TASK DESCRIPTION

Before students begin the task, a mini-lesson about two-dimensional figures translating or rotating to make a solid figure should be presented. Simple ways to have students think about translating a plane figure could be to stack saltine crackers, pennies, or any other "flat" object that when stacked makes a three-dimensional like figure. To illustrate rotational movement of a two-dimensional figure, use a battery-operated drill with a shape attached to the end, turn on and let the shape "spin". This gives the illusion of a three-dimensional shape.

Performance Task: What's My Solid?

Each of the following descriptions fit one or more solids (prism, pyramid, cone, cube, a cylinder). For each clue, describe what solid it may be and your justification for selecting that solid. If the description fits more than one solid, name and provide justification for each solid. Sketch the solid, and illustrate the properties described.

Solutions:
Solutions may vary.
A set of my parallel cross sections are squares that are similar but not congruent.
This could describe a square pyramid with cross sections parallel to the base.
A set of my parallel cross sections are congruent rectangles.
This could describe a cylinder with cross sections perpendicular to the base, or a rectangular prism with cross sections parallel to a face, or a cube.
EXTENSION: A set of my parallel cross sections are circles that are similar but not congruent.
This could describe a cone with cross sections parallel to the base. It could also be a sphere.
A set of my parallel cross sections are congruent circles.
This could describe a cylinder.

A set of my parallel cross sections are parallelograms.
This could describe a prism or a cylinder.
One of my cross sections is a hexagon, and one cross section is an equilateral triangle.
This could be a cube.
I can be made by sliding (translating) a rectangle through space.
This could be a prism.
I can be made by twirling (rotating) a triangle through space.
This could be a cone.
My volume can be calculated using the area of a circle.
This could be a cylinder or a cone.
My volume can be calculated using the area of a rectangle.
This could be a prism or a pyramid.

<u>DIFFERENTIATION</u>

Extension
Students can create their own set of directions for cross sections and play a game of "Guess Who"

Intervention
Eliminate cross sections irregular polygons like letters f, g, and h

What's My Solid?

Each of the following descriptions fit one or more solids (prism, pyramid, cone, cube, a cylinder). For each clue, describe what solid it may be and your justification for selecting that solid. If the description fits more than one solid, name and provide justification for each solid. Sketch the solid, and illustrate the properties described.

a) A set of my parallel cross sections are squares that are similar but not congruent.
Example: *This could describe a square pyramid with cross sections parallel to the base.*

b) A set of my parallel cross sections are congruent rectangles.

c) EXTENSION: A set of my parallel cross sections are circles that are similar but not congruent.

d) A set of my parallel cross sections are congruent circles.

e) A set of my parallel cross sections are parallelograms.

f) One of my cross sections is a hexagon, and one cross section is an equilateral triangle.

g) I can be made by sliding a rectangle through space.

h) I can be made by twirling a triangle through space.

i) My volume can be calculated using the area of a circle.

j) My volume can be calculated using the area of a rectangle.

Bigger and Bigger Cubes

Back to task table

Adapted from ETA/Cuisenaire Super Source task Bigger and Bigger Cubes

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make sure of structure

ESSENTIAL QUESTIONS

- How do you determine volume and surface area of a cube?

MATERIALS

- Base Ten Blocks
- Student Recording Sheet

GROUPING

- Individual/Partner/Small Group

BACKGROUND KNOWLEDGE

7.G.6 Students continue work from 5th and 6th grade to work with area, volume and surface area of two-dimensional and three-dimensional objects (composite shapes). At this level, students determine the dimensions of the figures given the area or volume.

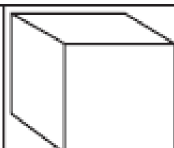
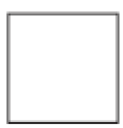
"**Know the formula" does not mean memorization of the formula**. To "know" means to have an understanding of *why* the formula works and how the formula relates to the measure (area and volume) and the figure. This understanding should be for *all* students.

Surface area formulas are not the expectation with this standard. Building on work with nets in the 6th grade, students should recognize that finding the area of each face of a three-dimensional figure and adding the areas will give the surface area. No nets will be given at this level; however, students could create nets to aid in surface area calculations.

TASK DESCRIPTION

Introduce the task by distributing as many thousand cubes as are available and instruct students to examine them by using the following questions:
How many units would be needed to build a cube of this size? *It should be established that 1,000 units would be needed.*
What is the volume of the cube? *and so 1,000 cubic units is the volume of the*
How many unit-sized squares would be needed to completely cover the outside of the cube? *Because a cube has six sides, each side is 10 x 10 or 100 square units, so a total of 600 square units would be needed to cover it.*
What is the surface area of the cube? *600 square units*

Distribute the student recording sheet for the task.
Using what you have determined about the thousand cube, explore the following:
What would the "next-bigger cube" look like?
Decide how you can build the next-bigger cube by adding the least number of Base 10 Blocks possible onto your thousands cube.
Predict the *volume* and the *surface area* of the next-bigger cube. Record your predictions.
Work together to build the next-bigger cube.
Decide together how to record the blocks you used and the dimensions, volume, and surface area of this next-bigger cube.
Predict the volume and surface area and build and record as many bigger and bigger cubes as you can.
Look for patterns in your work to help you figure out the blocks, dimensions, volumes, and surface areas of cubes that are too big for you to build.
Be ready to compare your cubes and tell about any patterns you found.
Use the chart below if needed.

Size of Cube					Volume (cm^3)	Surface Area (cm^2)
10 x 10 x 10	1				1,000	600
11 x 11 x 11	1	3	3	1	1,331	726
12 x 12 x 12	1	6	12	8	1,728	864
13 x 13 x 13	1	9	27	27	2,197	1,014
14 x 14 x 14	1	12	48	64	2,744	1,176
15 x 15 x 15	1	15	75	125	3,375	1,350

DIFFERENTIATION

Extension

Have students identify and record the blocks that they would need to build bigger and bigger cubes using only flats, only units, and only thousands cubes.

Intervention

Require students to use the chart to organize information. Inform them they will use exactly 1 thousand cube for each model.

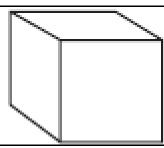
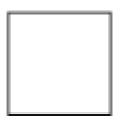
Building Task: Bigger and Bigger Cubes

Name:___________________________________ Date:___________________

Using what you have determined about the thousand cube, explore the following:

- What would the “next-bigger cube” look like?
- Decide how you can build the next-bigger cube by adding the least number of Base 10 Blocks possible onto your thousands cube.
- Predict the *volume* and the *surface area* of the next-bigger cube. Record your predictions.
- Work together to build the next-bigger cube.
- Decide together how to record the blocks you used and the dimensions, volume, and surface area of this next-bigger cube.
- Predict the volume and surface area and build and record as many bigger and bigger cubes as you can.
- Look for patterns in your work to help you figure out the blocks, dimensions, volumes, and surface areas of cubes that are too big for you to build.
- Be ready to compare your cubes and tell about any patterns you found.

Use the chart below if needed.

Size of Cube					Volume (cm^3)	Surface Area (cm^2)

Storage Boxes

Back to task table

Adapted from ETA/Cuisenaire Super Source task Storage Boxes

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make sure of structure

ESSENTIAL QUESTIONS

- How do you determine volume and surface area of a cube?

MATERIALS

- Cuisenaire rods
- Student Recording Sheet
- Isometric dot paper
- Metric ruler

GROUPING

- Individual/Partner/Small Group

BACKGROUND KNOWLEDGE

7.G.6 Students continue work from 5th and 6th grade to work with area, volume and surface area of two-dimensional and three-dimensional objects (composite shapes). At this level, students determine the dimensions of the figures given the area or volume.

“Know the formula” does not mean memorization of the formula. To “know” means to have an understanding of *why* the formula works and how the formula relates to the measure (area and volume) and the figure. This understanding should be for *all* students.
Surface area formulas are not the expectation with this standard. Building on work with nets in the 6th grade, students should recognize that finding the area of each face of a three-dimensional

figure and adding the areas will give the surface area. No nets will be given at this level; however, students could create nets to aid in surface area calculations.

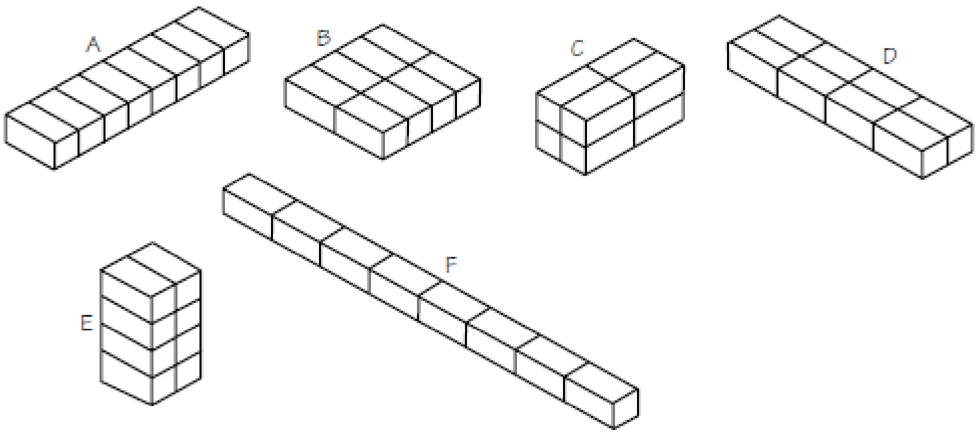
TASK DESCRIPTION:

Task comment

Using Cuisenaire Rods to model the shoe boxes makes it easy for students to make and compare an assortment of possible storage arrangements. They can also come to recognize that it is possible to make a variety of different-looking structures that all have the same volume.

Students should find that each red Cuisenaire Rod measures 2 cm x 1 cm x 1 cm. Using the given scale factor, they can calculate the actual dimensions of the shoe boxes, which are 30 cm x 15 cm x 15 cm.

There are a variety of ways in which the eight shoe boxes can be arranged to fit inside a rectangular prism-shaped storage box. Six possible arrangements are shown here.



Each of the models has a volume of 16 cm3. Students may find that several of their arrangements have the same dimensions. For example, of those pictured, A and D both measure 8 cm x 2 cm x1 cm, and C and E both measure 4 cm x 2 cm x 2 cm. The dimensions of some of the students' actual storage boxes may also be the same.

No matter how they arrange their shoe boxes, students should find that only four different-sized storage boxes are possible: a 240 cm x 15 cm x 15 cm storage box, a 120 cm x 30 cm x 15 cm storage box, a 60 cm x 60 cm x 15 cm storage box, and a 60 cm x 30 cm x 30 cm storage box.

Each of these has a volume of 54,000 cm3. Students can calculate the volumes of their storage boxes by first multiplying the dimensions of their models by 15 (the scale factor) and then finding their product (Volume = length x width x height), or by finding the volume of one shoe box (6750 cm3) and then multiplying by 8 (the number of shoe boxes). Still another method would be to find the volume of a model and multiply by 15 (length) x 15 (width) x 15 (height), or 3375.

To determine the amount of plywood needed to make the storage boxes, students need to calculate the surface area of each of their arrangements. The surface areas can be determined by finding the total of the areas of the six faces (front, back, left, right, top, and bottom) of

their arrangements. The surface areas of the different-shaped storage boxes are given in this table.

Storage Boxes	Area of each side in cm²						Total Surface Area
	Front	Back	Left	Right	Top	Bottom	
A	30 x 15 450	30 x 15 450	120 x 15 1800	120 x 15 1800	120 x 30 3600	120 x 30 3600	11,700 cm²
B	60 x 15 900	60 x 15 900	60 x 15 900	60 x 15 900	60 x 60 3600	60 x 60 3600	10,800 cm²
C	30 x 30 900	30 x 30 900	60 x 30 1800	60 x 30 1800	30 x 60 1800	30 x 60 1800	9,000 cm²
D	120 x 15 1800	120 x 15 1800	30 x 15 450	30 x 15 450	120 x 30 3600	120 x 30 3600	11,700 cm²
E	30 x 60 1800	30 x 60 1800	30 x 60 1800	30 x 60 1800	30 x 30 900	30 x 30 900	9,000 cm²
F	240 x 15 3600	240 x 15 3600	15 x 15 225	15 x 15 225	240 x 15 3600	240 x 15 3600	14,850 cm²

Part 1

Kathryne takes care of her shoes by keeping them in their original shoe boxes. She wants to find one large storage box that will hold 8 of her shoe boxes. Can you help Kathryne determine the dimensions of the storage boxes that would work?

- Work with a partner. Use red Cuisenaire Rods to represent the shoe boxes.
- Arrange 8 shoe boxes so that they could fit into a rectangular prism-shaped storage box. The box should be exactly the right size to hold the 8 shoe boxes with no extra space left over. Find as many different arrangements as possible.
- Record your models on isometric dot paper. Measure and record the dimensions and volumes of each model.
- Determine the *actual* dimensions of each of Kathryne's shoe boxes if 1 centimeter in your model represents 15 centimeters for the *actual* shoe boxes. Then calculate the actual dimensions and volumes of the storage boxes that you modeled.
- Be ready to discuss your findings.

Part 2

What if... *Kathryne decides to make her own storage box from sheets of plywood? What is the least amount of plywood she would need to create the box? What is the most?*

- Using your models from Part 1, determine the amount of plywood needed to construct each possible storage box. Record your measurements (in square centimeters) near your drawings.
- Determine which arrangement would need the least amount of plywood and which would need the most.
- Now use your observations to determine the least and greatest amounts of plywood needed to construct a storage box that would hold 12 shoe boxes.
- Try to come up with a general rule that could be used to predict what kinds of box arrangements will use the least amount of plywood.
- Be ready to explain your methods and discuss your findings.

DIFFERENTIATION

Extension

Imagine that the storage container is to be made of cardboard instead of plywood. Using your results from Part 2, make a scale drawing of a one-piece pattern that can be cut out and folded to form the storage box needing the least amount of cardboard. Use the same scale as you used for your models. Label your pattern with the actual measurements that would be needed to construct the storage box.

Intervention

Require students to use the chart to organize information.

Building Task: Storage Boxes

Name:___________________________ Date:____________________

Part 1

Kathryne takes care of her shoes by keeping them in their original shoe boxes. She wants to find one large storage box that will hold 8 of her shoe boxes. Can you help Kathryne determine the dimensions of the storage boxes that would work?

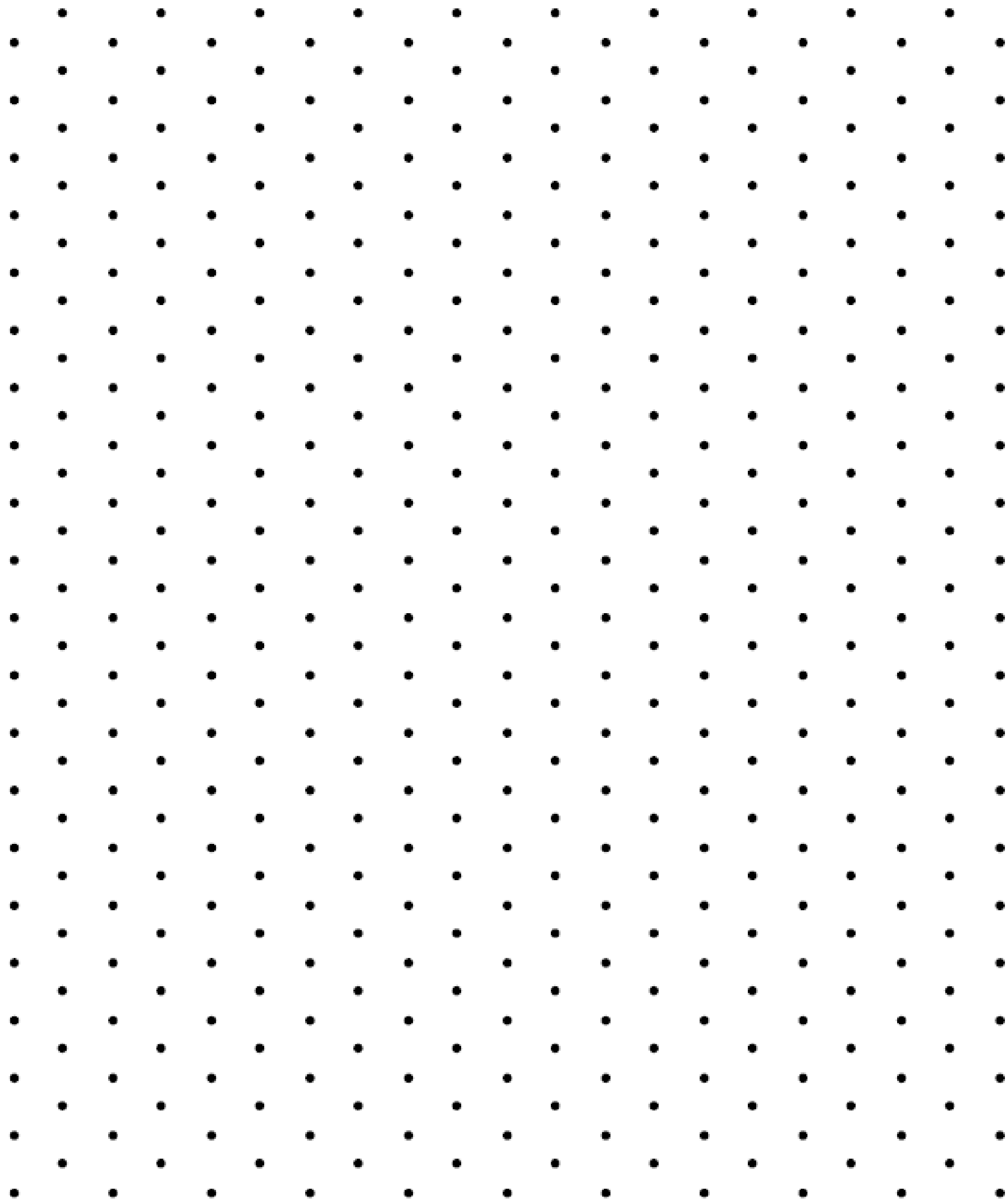
• Work with a partner. Use red Cuisenaire Rods to represent the shoe boxes.
• Arrange 8 shoe boxes so that they could fit into a rectangular prism-shaped storage box. The box should be exactly the right size to hold the 8 shoe boxes with no extra space left over. Find as many different arrangements as possible.
• Record your models on isometric dot paper. Measure and record the dimensions and volumes of each model.
• Determine the *actual* dimensions of each of Kathryne's shoe boxes if 1 centimeter in your model represents 15 centimeters for the *actual* shoe boxes. Then calculate the actual dimensions and volumes of the storage boxes that you modeled.
• Be ready to discuss your findings.

Part 2

What if... *Kathryne decides to make her own storage box from sheets of plywood? What is the least amount of plywood she would need to create the box? What is the most?*

• Using your models from Part 1, determine the amount of plywood needed to construct each possible storage box. Record your measurements (in square centimeters) near your drawings.
• Determine which arrangement would need the least amount of plywood and which would need the most.
• Now use your observations to determine the least and greatest amounts of plywood needed to construct a storage box that would hold 12 shoe boxes.
• Try to come up with a general rule that could be used to predict what kinds of box arrangements will use the least amount of plywood.
• Be ready to explain your methods and discuss your findings.

Isometric Dot Paper



Filling Boxes

Back to task table

Adapted from ETA/Cuisenaire Super Source task Filling Boxes

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make sure of structure.

ESSENTIAL QUESTIONS

- How do you determine volume and surface area of a cube?

MATERIALS

- Base Ten Blocks, units only
- Student Recording Sheet
- Filling Boxes sheets A-D, 1 for each group
- Tape
- Scissors

GROUPING

- Partner/Small Group

BACKGROUND KNOWLEDGE

7.G.6 Students continue work from 5th and 6th grade to work with area, volume and surface area of two-dimensional and three-dimensional objects (composite shapes). At this level, students determine the dimensions of the figures given the area or volume.

"Know the formula" does not mean memorization of the formula. To "know" means to have an understanding of *why* the formula works and how the formula relates to the measure (area and volume) and the figure. This understanding should be for *all* students.

Surface area formulas are not the expectation with this standard. Building on work with nets in the 6th grade, students should recognize that finding the area of each face of a three-dimensional figure and adding the areas will give the surface area. No nets will be given at this level; however, students could create nets to aid in surface area calculations.

TASK DESCRIPTION

Task comment

This activity helps children explore volume and surface area in a hands-on environment. Cutting out the shapes, then folding and taping them to form open-top boxes helps children to understand the relationships between two-dimensional and three-dimensional shapes. Seeing the boxes flattened, as they appear at first, may help them to understand the concept of surface area.

When children predict how many white rods each box can hold, they may arrive at an answer through a variety of techniques. Some may simply guess. Others may think that the tallest box (C) must have the greatest volume because it is the deepest. Still others may feel that box D has the greatest volume because the area of the opening is the greatest. Some children may be stumped by boxes A and B because they are the same height and because it is hard to tell which of their bases has the greater area. When children actually measure, they see that trying to guess the volumes just by looking at the boxes can be tricky because three factors—height, width, and length must be considered. Children will eventually find that boxes A and D both have volumes of 54 cubic centimeters, box B has a volume of 60 cubic centimeters, and box C has a volume of 64 cubic centimeters.

Children are apt to use a variety of techniques for finding the actual volume of the boxes with Cuisenaire Rods. Some may try to fill each box completely with white rods and, when they run out of white rods, select other rods and translate their volume into numbers of white rods. Children who use this method may keep a tally of the rods as they place them into the box while
other children will first fill the box, dump out its contents, and then count the rods.

How can you use Cuisenaire Rods to order a set of boxes according to volume?
• Work with a group to create a set of 4 open boxes. Cut out nets like these along the dotted lines. Fold them along the solid lines. Use tape to hold each box together.
• Once you have built the boxes, decide how they should be arranged, from the one with the least volume to the one with the greatest. Record the group's decision.
• Estimate how many unit cubes (cubic centimeters) each box can hold. Record the estimates.
• Use the unit cubes to find out how many cubic centimeters each box actually holds. Compare the actual volumes to your estimates.
• Use the rods to figure out the surface area of each box. Remember not to count the open side. Record the surface areas.

DIFFERENTIATION

Extension

Ask children to imagine that they own a toy factory and want to design a box that will hold 100 blocks, each the size of a unit cube. Have them try to design the box so that it uses as little cardboard as possible.

Intervention

Require students to use the chart to organize information.

Filling Boxes

How can you use Cuisenaire Rods to order a set of boxes according to volume?

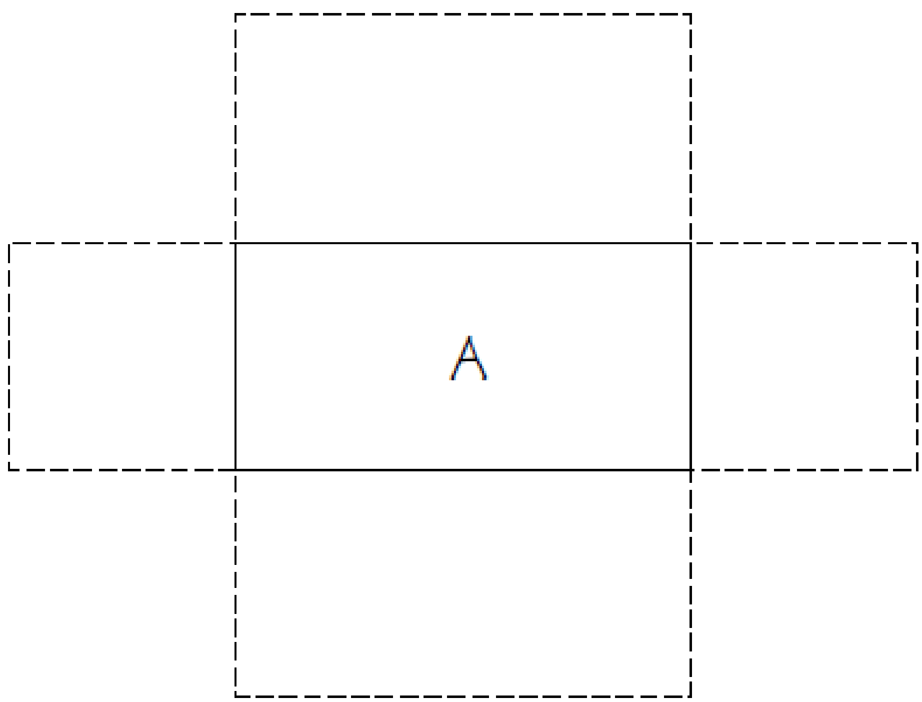
• Work with a group to create a set of 4 open boxes. Cut out nets like these along the dotted lines. Fold them along the solid lines. Use tape to hold each box together.

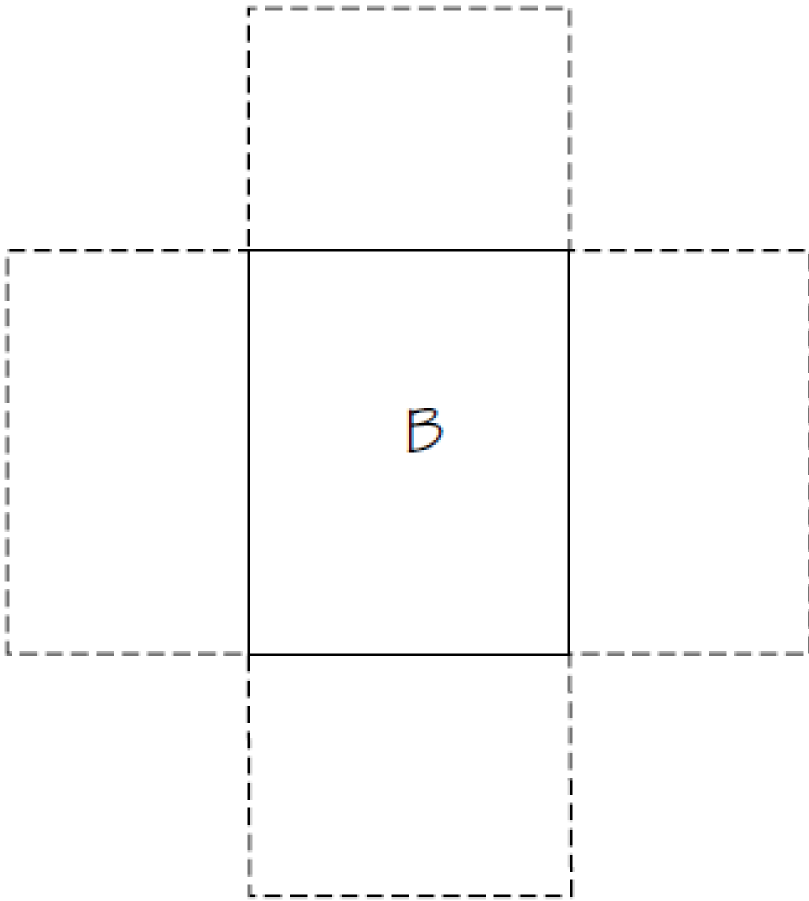
• Once you have built the boxes, decide how they should be arranged, from the one with the least volume to the one with the greatest. Record the group's decision.

• Estimate how many unit cubes (cubic centimeters) each box can hold. Record the estimates.

• Use the unit cubes to find out how many cubic centimeters each box actually holds. Compare the actual volumes to your estimates.

• Use the rods to figure out the surface area of each box. Remember not to count the open side. Record the surface areas.

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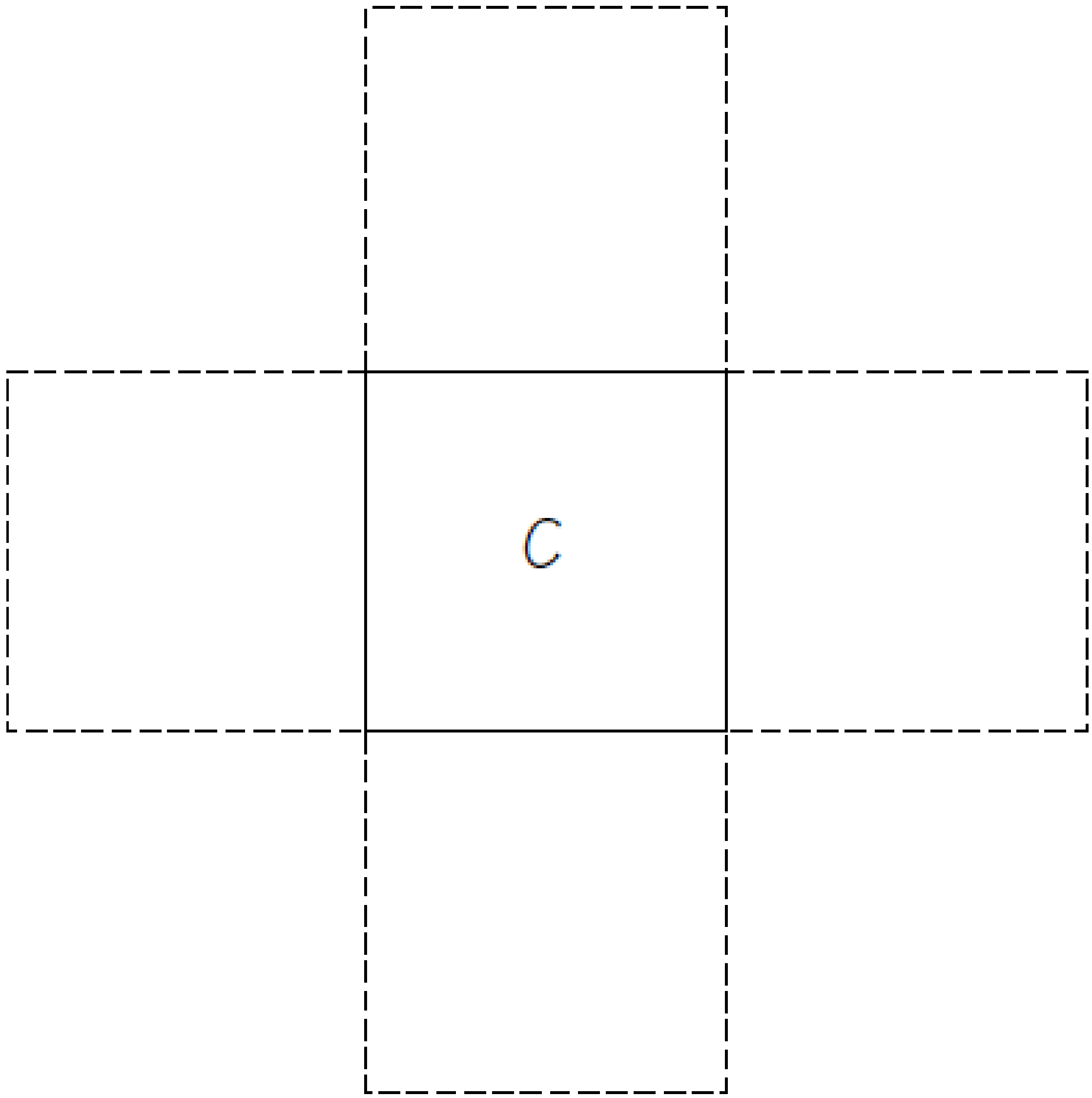
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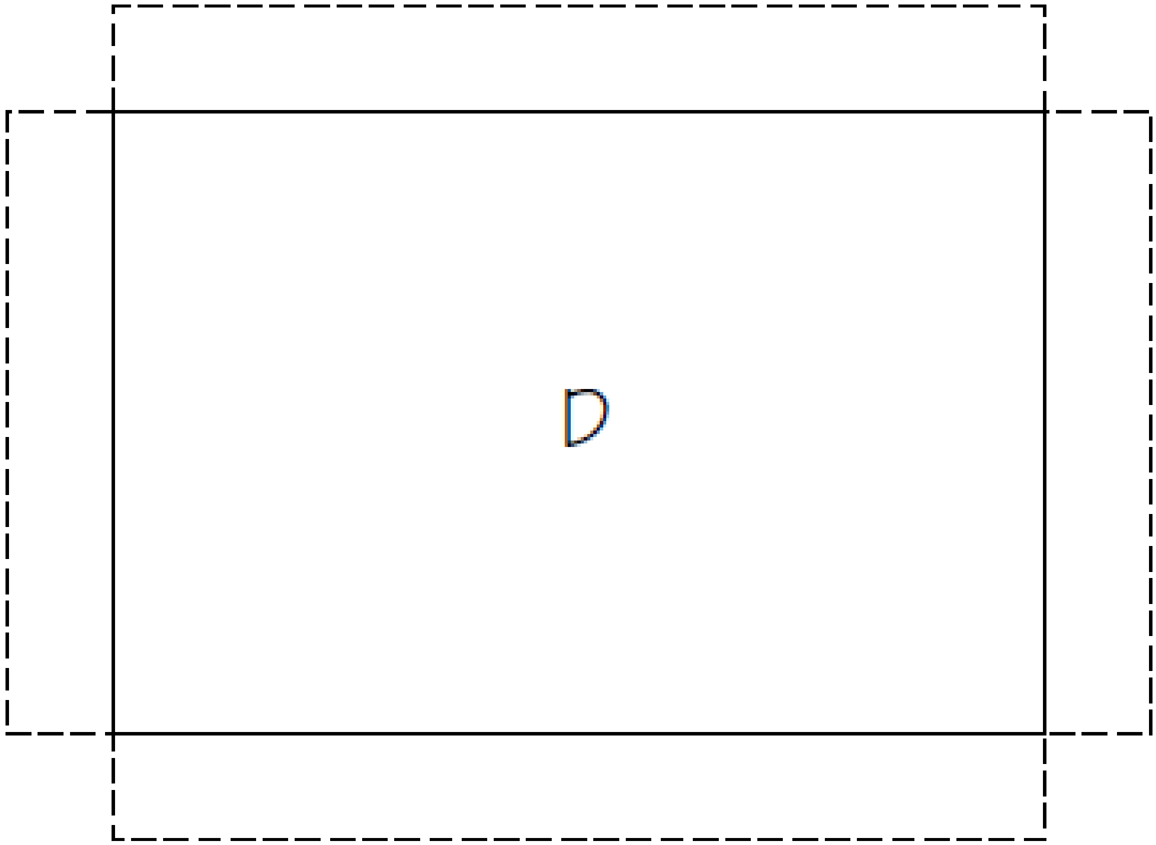

D

Discovering the Surface Area of a Cylinder (Extension Task)

Back to task table

Adapted from Teaching Channel lesson Discovering the Surface Area of a Cylinder
This task serves as an extension to the standard 7.G.6.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.G.6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make sure of structure

ESSENTIAL QUESTIONS

- How do you determine surface area of a cylinder?

MATERIALS

- 6 canned good products (two different sizes of cans, 3 of one size, 3 of another size)
- Ruler
- string

GROUPING

- Partner/Small Group

BACKGROUND KNOWLEDGE

7.G.6 Students continue work from 5th and 6th grade to work with area, volume and surface area of two-dimensional and three-dimensional objects (composite shapes). At this level, students determine the dimensions of the figures given the area or volume.

“Know the formula” does not mean memorization of the formula. To “know” means to have an understanding of *why* the formula works and how the formula relates to the measure (area and volume) and the figure. This understanding should be for *all* students.

Surface area formulas are not the expectation with this standard. Building on work with nets in the 6th grade, students should recognize that finding the area of each face of a three-dimensional figure and adding the areas will give the surface area. No nets will be given at this level; however, students could create nets to aid in surface area calculations.

TASK DESCRIPTION

This lesson can be viewed at https://www.teachingchannel.org/videos/surface-area-lesson on the Teaching Channel.

Introduce this task by review concepts about a circle. Record the following expressions on the board and ask students, "Which two expressions measure the exact same thing? Explain how you know."

πd $\qquad\qquad$ πr^2 $\qquad\qquad$ $2\pi r$

Students should conclude that the length of the diameter is equivalent to 2 of the radii, therefore πd and $2\pi r$ are equivalent expressions.

Now discuss the next problem: The "Ring of Fire" rollercoaster has a diameter of 60 ft. Allow students time to calculate the circumference and the area using 3.14 for π.

Circumference:	*Area:*
$C= \pi d$	$A= \pi r^2$
x 60	$3.14 \times (60)^2$
C= 188.4ft	$3.14 \times 3{,}600$
	A= 11,304ft

Part 1

Work together to find surface area of 1 of 2 canned goods.
How would you define surface area?
The sum of all the two dimensional shapes decomposed from the three dimensional shape.
Can this figure be decomposed? If so how, into what shape or shapes?
Yes, the figure can be decomposed into two circles and one rectangle.
Determine the surface area of your can. Explain your answer.
Answers may vary. Students should determine the area of the circle and rectangle and add the areas.
Discuss with students the need for decomposition of the cylinder to find surface area.

Part 2

Students should be instructed to take the label off the can they just calculated the Surface Area for and hold on to it. All 6 cylinders in the room will now have no label. Each table group will now trade cans with another group that has the other can they have yet to calculate surface area for. The task is the same:
Calculate the surface area of the can.

With no label to peel off, students will struggle to calculate the area of the rectangular face of the cylinder (they cannot flatten it out). The only way to calculate the area of this section is to realize that the length of the rectangular face is equal to the circumference of the circular bases. This is why the formula used to calculate the surface area of a cylinder is 2πrh + 2πr2. Students do not need to know the formula for finding the surface area of a cylinder.

Part 3

The supermarket around the corner used to sell Grandma's favorite Mandarin oranges. For reasons unknown, they no longer do and it really upsets Grandma.
Jackson, a thoughtful young seventh grader, remembers this fact as he's thinking of what to get his Grandmother for her 80th birthday party. Cans of Geisha Mandarin Oranges, of course!

There's one potential problem, however: Jackson has two cans to wrap but only has 550 cm2 of wrapping paper. Does he have enough paper to wrap the two cans?

Use the surface area of each can to complete the problem.

DIFFERENTIATION

Extension

Allow students to work on the following problem:
In an effort to fight back against pesky graffiti artists and corrosive weather, the Parks and Recreation Department of Pawnee, Indiana is spending money to paint a vandalized water tower and 15 local trash barrels. Since they are working within a budget, they need to estimate the cost of the paint job.

Information:

o 1 water tower (just cylinder) and 15 trash barrels (only parts that make sense) need painting.
o The hardware store estimates that paint will cost $0.75 per square foot.
o Two painters will be paid $500 each to do the job.

In total, how much will it cost the town of Pawnee for the renovations? Show all work.

Intervention

Discuss the relationship between the length of the label and the circumference of the base.

Discovering the Surface Area of a Cylinder (Extension Task)

Name:_______________________

Part 1
Work together to find surface area of 1 of 2 canned goods.
How would you define surface area?

Can this figure be decomposed? If so how, into what shape or shapes?

Determine the surface area of your can. Explain your answer.

Part 2
Calculate the surface area of the can of the can without the label.

Part 3
The supermarket around the corner used to sell Grandma's favorite Mandarin oranges. For reasons unknown, they no longer do and it really upsets Grandma.
Jackson, a thoughtful young seventh grader, remembers this fact as he's thinking of what to get his Grandmother for her 80th birthday party. Cans of Geisha Mandarin Oranges, of course!

There's one potential problem, however: Jackson has two cans to wrap but only has 550 cm2 of wrapping paper. Does he have enough paper to wrap the two cans?

Use the surface area of each can to complete the problem.

Culminating Task: Three Little Pig Builders – House B or D

Back to task table

STANDARDS FOR MATHEMATICAL CONTENT

MGSE7.G.6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.

COMMON MISCONCEPTIONS

Real-world and mathematical multi-step problems that require finding area, perimeter, volume, surface area of figures composed of triangles, quadrilaterals, polygons, cubes and right prisms should reflect situations relevant to seventh graders. The computations should make use of formulas and involve whole numbers, fractions, decimals, ratios and various units of measure with same system conversions.

ESSENTIAL QUESTIONS

- How can I use formulas to determine the volumes of fundamental solid figures?
- How can I estimate the surface area of simple geometric solids?
- How can I use surface areas of plane figures to derive formulas for the surface areas of solid figures?

GROUPING

- Students may work as an individuals or small groups of two or three.

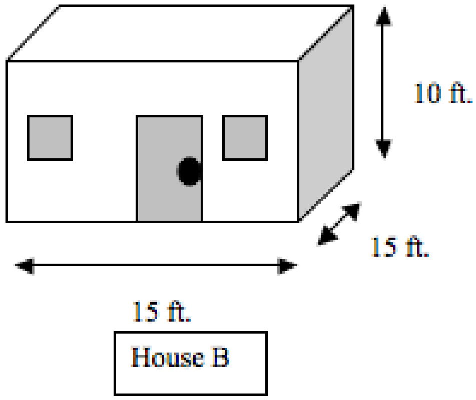
TASK COMMENTS

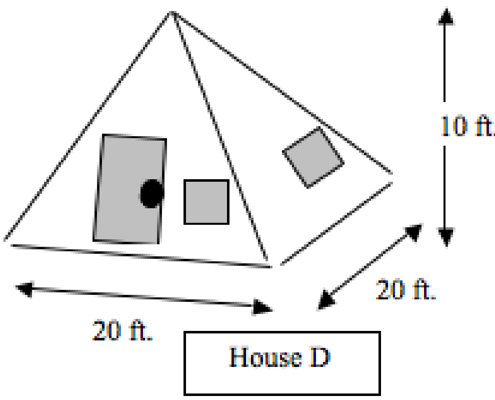
In this task, students will determine the sale price of four houses and the cost to paint the interior of two of the houses.

TASK DESCRIPTION

Task Directions:

Three Little Pig Builders have gone into business building the prefabricated homes shown below.





Give the geometric name for each house.
House B is a rectangular prism and House D is a square pyramid.
Estimate the surface area of House B and House D.

The inside of House B and House D are in need of painting before they can be sold. Each home has a 3 ft. by 5 ft. door and two 2 ft. by 2 ft. windows that do not need painting. All of the walls need painting including the floors and ceilings. A gallon of paint costs $25 and covers 300 ft^2. Three Little Pig Builders require that a work order be submitted for approval before making any purchases. Write a work order that explains how much paint needs to be purchased and the cost of the purchase. Be sure that your work order explains in detail how you know the amount of paint and money needed for the purchase.
House B has a surface area of (2)(lw) + (2)(wh) + (2)(lh) – (area of door and windows)

$$(2)(15)(15) + (2)(15)(10) + (15)(10) - (23)$$
$$450 \quad + \quad 300 \quad + \quad 300 \quad - 23$$

House B has a total surface area of 1027 ft^2.

House D has a surface area of $4(\frac{1}{2}bh)+lw-$ (area of door and windows)

$$4(\tfrac{1}{2}20x10)+20 \times 20 - (23)$$
$$400 + 400 - 23$$

House D has a total surface area of 777ft^2.

The total amount of surface are in the two houses is 1804 ft^2.

The number of gallons needed to paint both houses would be 6.01 which is about 6. Therefore, it would take more than 6 gallons of paint. We need to purchase 7 gallons of paint. At \$25 per gallon, this means that the cost would be (7)(25) = \$175.00.
The work order amount that is being requested to paint both House B and House D is \$175.00 which does not include sales tax.

<u>EXTENSION</u> – Estimate the volume of House B and House D.

Three Little Pig Builders have decided to charge \$30,000 for House B. What is the fair market value of each of the other homes assuming that the cost of each home is proportional to its volume?

Make a table to organize your data. Use words and symbols (including proportions and formulas) to explain your reasoning. <u>You may want to include a formula sheet for students who need additional reinforcement</u>.

House B an approximate volume of (200)(10) = 2000ft^3.

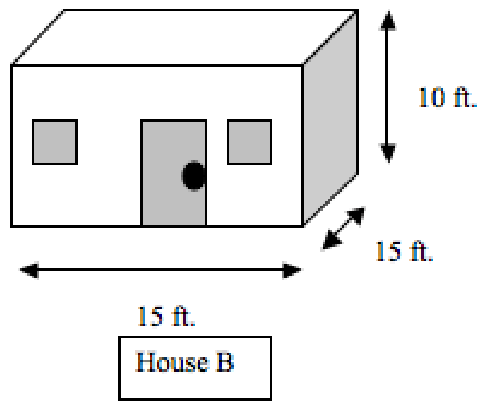
House D has an approximate volume of $4000\div3\approx3900\div3=1300$ft^3.

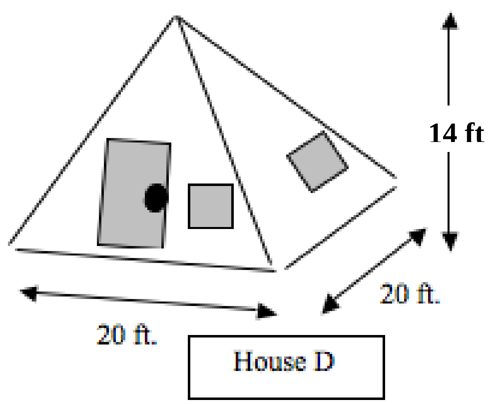
Three Little Pig Builders have decided to charge \$30,000 for House A, a cylinder shaped home not pictured. What is the fair market value of each of the other homes if they are based proportionately to the value of House A? Make a table to organize your data. Use words and symbols (including proportions and formulas) to explain your reasoning.

House	Volume	Proportion	Fair Market Value
B	$V = lwh$ $V = (15)(15)(10)$ $V = 2250ft^3$	$\frac{x}{2250} = \frac{16.99}{1}$ x = (2250)(16.99) x = 38227.5	\$38,227.50
D	$V = \frac{1}{3}lwh$ $V = \frac{1}{3}(20)(20)(10)$ V = 1333.3333	$\frac{x}{1333.3333} = \frac{16.99}{1}$ x = (1333.3333)(16.99) x = 22653.332	\$22,653.33

Three Little Pig Builders – House B or D

Three Little Pig Builders have gone into business building the prefabricated homes shown below.





Give the geometric name for each house.

Estimate the surface area of House B and House D.

The inside of House B and House D are in need of painting before they can be sold. Each home has a 3 ft. by 5 ft. door and two 2 ft. by 2 ft. windows that do not need painting. All of the walls need painting including the floors and ceilings. A gallon of paint costs $25 and covers 300 ft^2. Three Little Pig Builders require that a work order be submitted for approval before making any purchases. Write a work order that explains how much paint needs to be purchased and the cost of the purchase. Be sure that your work order explains in detail how you know the amount of paint and money needed for the purchase.

<u>EXTENSION</u> – Estimate the volume of House B and House D.

Three Little Pig Builders have decided to charge $30,000 for House B. What is the fair market value of each of the other homes assuming that the cost of each home is proportional to its volume?

Fill in the following table to organize your data. Use words and symbols (including proportions and formulas) to explain your reasoning.

House	Volume	Proportion	Fair Market Value
B			
D			

TECHNOLOGY RESOURCES

MGSE7.G.2 Explore various geometric shapes with given conditions. Focus on creating triangles from three measures of angles and/or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

http://illuminations.nctm.org/Lesson.aspx?id=2028

http://illuminations.nctm.org/Activity.aspx?id=3546

http://nzmaths.co.nz/resource/how-high-and-other-problems

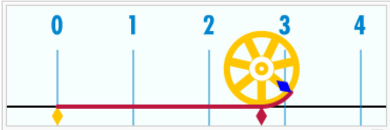
http://nzmaths.co.nz/resource/inside-irregular-polygons

http://nzmaths.co.nz/resource/red-october

MGSE7.G.3 Describe the two-dimensional figures (cross sections) that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms, right rectangular pyramids, cones, cylinders, and spheres.
https://www.illustrativemathematics.org/content-standards/7/G/A/3/tasks

MGSE7.G.4 Given the formulas for the area and circumference of a circle, use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

http://www.uen.org/Lessonplan/preview.cgi?LPid=15436 A link to the activity on which this task was based.
http://en.wikipedia.org/wiki/Pi The Pi entry provides a graphic for pi, a circle's circumference is measured on a ruler created using increments equal to the diameter of the circle.



When a circle's diameter is 1, its circumference is pi.

http://www.mathgoodies.com/lessons/vol2/circumference.html
This site provides background information on circles and allows students to practice finding the circumference of circles.
http://www.kathimitchell.com/pi.html A comprehensive list of sites about pi.
http://www.joyofpi.com/pi.html The First 10,000 digits of pi.
http://curvebank.calstatela.edu/circle/circle.htm Uses animation to derive the formula for the area of a circle based on the area of a parallelogram.
http://www.rkm.com.au/ANIMATIONS/animation-Circle-Area-Derivation.html Uses animation to derive the formula for the area of a circle based on the area of a parallelogram.
http://curvebank.calstatela.edu/circle2/circle2.htm Uses animation to derive the formula for the area of a circle based on the area of a triangle.
http://www.worsleyschool.net/science/files/circle/area.html Uses graphics to derive the formula for the area of a circle.
http://illuminations.nctm.org/Activity.aspx?id=3547

MGSE7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

https://www.illustrativemathematics.org/7.G.B

http://illuminations.nctm.org/Lesson.aspx?id=1992

MGSE7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
https://www.illustrativemathematics.org/7.G.B
http://illuminations.nctm.org/Lesson.aspx?id=2009
http://illuminations.nctm.org/Lesson.aspx?id=2232
http://illuminations.nctm.org/Lesson.aspx?id=2771