Georgia Standards of Excellence Curriculum Frameworks

Mathematics

GSE Fifth Grade

Unit 3: Multiplying and Dividing with Decimals

Georgia Department of Education

Richard Woods, Georgia's School Superintendent

"Educating Georgia's Future"

Unit 3: Multiplying and Dividing with Decimals

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IF YOU HAVE NOT READ THE 5th GRADE CURRICULUM OVERVIEW IN ITS ENTIRETY PRIOR TO USE OF THIS UNIT, PLEASE **STOP** AND CLICK HERE:
https://www.georgiastandards.org/Georgia-Standards/Frameworks/5th-Math-Grade-Level-Overview.pdf
Return to the use of this unit once you've completed reading the Curriculum Overview. Thank you.

OVERVIEW

Perform operations with multi-digit whole numbers and with decimals to the hundredths.

General methods used for computing products of whole numbers extend to products of decimals. Because the expectations for decimals are limited to thousandths and expectations for factors are limited to hundredths at this grade level, students will multiply tenths with tenths and tenths with hundredths, but they need not multiply hundredths with hundredths. Before students consider decimal multiplication more generally, they can study the effect of multiplying by 0.1 and by 0.01 to explain why the product is ten or a hundred times as small as the multiplicand (moves one or two places to the right). They can then extend their reasoning to multipliers that are single-digit multiples of 0.1 and 0.01 (e.g., 0.2 and 0.02, etc.).

There are several lines of reasoning students can use to explain the placement of the decimal point in other products of decimals. Students can think about the product of the smallest base-ten units of each factor. For example, a tenth times a tenth is a hundredth, so 3.2 x 7.1 will have an entry in the hundredths place. Note, however, that students might place the decimal point incorrectly for 3.2 x 8.5 unless they take into account the 0 in the ones place in the product of 32 x 85. (Or they can think of 0.2 x 0.5 as 10 hundredths.) They can also think of decimals as fractions or as whole numbers divided by 10 or 100. When they place the decimal point in the product, they have to divide by a 10 from each factor or 100 from one factor. For example, to see that 0.6 x 0.8 = 0.48, students can use fractions: 6/10 x 8/10 = 48/100. Students can also reason that when they carry out the multiplication without the decimal point, they have multiplied each decimal factor by 10 or 100, so they will need to divide by those numbers in the end to get the correct answer. Also, students can use reasoning about the sizes of numbers to determine the placement of the decimal point. For example, 3.2 x 8.5 should be close to 3 x 9, so 27.2 is a more reasonable product for 3.2 x 8.5 than 2.72 or 272. This estimation based method is not reliable in all cases, however, especially in cases students will encounter in later grades. For example, it is not easy to decide where to place the decimal point in 0.023 x 0.0045 based on estimation. Students can summarize the results of their reasoning such as those above as specific numerical patterns and then as one general overall pattern such as "the number of decimal places in the product is the sum of the number of decimal places in each factor."

General methods used for computing quotients of whole numbers extend to decimals with the additional issue of placing the decimal point in the quotient. As with decimal multiplication, students can first examine the cases of dividing by 0.1 and 0.01 to see that the quotient becomes 10 times or 100 times as large as the dividend. For example, students can view 7 ÷ 0.1 = as asking how many tenths are in 7. Because it takes 10 tenths to make 1, it takes 7 times as many tenths to make 7, so 7 ÷ 0.1 = 7 x 10 = 70. Or students could note that 7 is 70 tenths, so asking how many tenths are in 7 is the same as asking how many tenths are in 70 tenths, which is 70. In other words, 7 ÷ 0.1 is the same as 70 ÷ 1. So, dividing by 0.1 moves the number 7 one place to the left, the quotient is ten times as big as the dividend. As with decimal multiplication, students can then proceed to more general cases. For example, to calculate 7 ÷ 0.2, students can reason that 0.2 is 2 tenths and 7 is 70 tenths, so asking how many 2 tenths are in 7 is the same as asking how many 2 tenths are in 70 tenths. In other words, 7 ÷ 0.2 is the same as 70 ÷ 2; multiplying both the 7 and the 0.2 by 10 results in the same quotient. Or students could calculate 7 ÷ 0.2 by viewing the 0.2 as 2 x 0.1, so they can first divide 7 by 2, which is 3.5, and then divide that result by 0.1, which makes 3.5 ten times as large, namely 35. Dividing by a decimal less than 1 results in a quotient larger than the dividend and moves the digits of the dividend one place to the left. Students can summarize the results of their reasoning as specific numerical patterns then as one

general overall pattern.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with these clusters are: **place value, patterns, multiplication/multiply, division/divide, decimal, decimal point, tenths, hundredths, products, quotients, dividends, rectangular arrays, area models, addition/add, subtraction/subtract, (properties)-rules about how numbers work, reasoning.**

These tasks are not intended to be the sole source of instruction. They are representative of the kinds of experiences students will need in order to master the content, as well as mathematical practices that lead to conceptual understanding. Teachers should NOT do every task in the unit; they should choose the tasks that fit their students' needs.

*For more detailed information about unpacking the content standards, unpacking a task, math routines and rituals, maintenance activities and more, please refer to the Grade Level Overview.

STANDARDS FOR MATHEMATICAL PRACTICE

This section provides examples of learning experiences for this unit that support the development of the proficiencies described in the Standards for Mathematical Practice. These proficiencies correspond to those developed through the Literacy Standards. The statements provided offer a few examples of connections between the Standards for Mathematical Practice and the Content Standards of this unit. The list is not exhaustive and will hopefully prompt further reflection and discussion.

1. **Make sense of problems and persevere in solving them.** Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.

2. **Reason abstractly and quantitatively.** Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.

3. **Construct viable arguments and critique the reasoning of others.** Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.

4. **Model with mathematics.** Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.

5. **Use appropriate tools strategically.** Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.

6. **Attend to precision.** Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.

7. **Look for and make use of structure.** Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.

8. **Look for and express regularity in repeated reasoning.** Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

*****Mathematical Practices 1 and 6 should be evident in EVERY lesson*****

STANDARDS FOR MATHEMATICAL CONTENT

Understand the place value system.

MGSE5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Perform operations with multi-digit whole numbers and with decimals to the hundredths.

MGSE5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

BIG IDEAS

- Students will understand that the placement of the decimal is determined by multiplying or dividing a number by 10 or a multiple of 10.
- Students will understand that multiplication and division are inverse operations of each other.
- Students will understand that rules for multiplication and division of whole numbers also apply to decimals.

ESSENTIAL QUESTIONS

- How can we use exponents to represent powers of 10?
- How does multiplying or dividing by a power of ten affect the product?
- How can we use models to help us multiply and divide decimals?
- How do the rules of multiplying whole numbers relate to multiplying decimals?
- How are multiplication and division related?
- How are factors and multiples related to multiplication and division?

- What are some patterns that occur when multiplying and dividing by decimals?
- How can we efficiently solve multiplication and division problems with decimals?
- What strategies are effective for finding a missing factor or divisor?
- How can we check for errors in multiplication or division of decimals?

CONCEPTS/SKILLS TO MAINTAIN

This standard requires students to extend the models and strategies they developed for whole numbers in grades 1-4 to decimal values. Before students are asked to give exact answers, they should estimate answers based on their understanding of operations and the value of the numbers.

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Number sense
- Whole number computation

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply "how to get the answer" and instead support students' ability to access concepts from a number of perspectives. Therefore, students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency and automaticity. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students:

- flexibly use a combination of deep understanding, number sense, and memorization.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.

For more about fluency, see: http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and: https://bhi61nm2cr3mkdgk1dtaov18-wpengine.netdna-ssl.com/wp-content/uploads/nctm-timed-tests.pdf

STRATEGIES FOR TEACHING AND LEARNING

As students developed efficient strategies to do whole number operations, they should also develop efficient strategies with decimal operations.
Students should learn to estimate decimal computations before they compute with pencil and paper. The focus on estimation should be on the meaning of the numbers and the operations, not on how many decimal places are involved. For example, to estimate the product of 32.84 × 4.6, the estimate would be more than 120, closer to 150. Students should consider that 32.84 is closer to 30 and 4.6 is closer to 5. The product of 30 and 5 is 150. Therefore, the product of 32.84 × 4.6 should be close to 150.
Have students use estimation to find the product by using exactly the same digits in one of the factors with the decimal point in a different position each time. For example, have students estimate the product of 275 × 3.8; 27.5 × 3.8 and 2.75 × 3.8, and discuss why the estimates should or should not be the same.
In addition to strategies specific to content standards, students should also practice the following throughout the unit:

- Students should be actively engaged by developing their own understanding.
- Mathematics should be represented in as many ways as possible by using graphs, tables, pictures, symbols, and words.
- Appropriate manipulatives and technology should be used to enhance student learning.
- Students should be given opportunities to revise their work based on teacher feedback, peer feedback, and metacognition which includes self-assessment and reflection.
- Students need to write in mathematics class to explain their thinking, talk about how they perceive topics, and justify their work to others.

Teachers need to provide instructional experiences so that students progress from the concrete level, to the pictorial level, then to the abstract level when learning mathematical concepts.

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The terms below are for teacher reference only and are not to be memorized by students.
Teachers should first present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or use them with words, models, pictures, or numbers.

- **array**
- **associative property of multiplication**
- **commutative property of multiplication**
- **distributive property**
- **dividend**
- **division**
- **divisor**
- **exponent**
- **factor**
- **hundred thousands**
- **hundreds**
- **hundredths**
- **identity property of multiplication**
- **measurement division (or repeated subtraction)**
- **millions**
- **multiple**
- **multiplier**
- **ones**
- **partial products**
- **partition/partitive division (or fair-sharing)**
- **place value**
- **power of ten**
- **product**
- **quotient**
- **remainder**
- **ten thousands**
- **tens**
- **tenths**
- **thousands**

TASKS

Scaffolding Task	Tasks that build up to the learning task.
Constructing Task	Constructing understanding through deep/rich contextualized problem solving tasks.
Practice Task	Tasks that provide students opportunities to practice skills and concepts.
Performance Task	Tasks which may be a formative or summative assessment that checks for student understanding/misunderstanding and or progress toward the standard/learning goals at different points during a unit of instruction.
Culminating Task	Designed to require students to use several concepts learned during the unit to answer a new or unique situation. Allows students to give evidence of their own understanding toward the mastery of the standard and requires them to extend their chain of mathematical reasoning.
Intervention Table	The Intervention Table provides links to interventions specific to this unit. The interventions support students and teachers in filling foundational gaps revealed as students work through the unit. All listed interventions are from New Zealand's Numeracy Project.
Formative Assessment Lesson (FAL)	Lessons that support teachers in formative assessment which both reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards.
CTE Classroom Tasks	Designed to demonstrate how the GSE and Career and Technical Education knowledge and skills can be integrated. The tasks provide teachers with realistic applications that combine mathematics and CTE content.
3-Act Task	A Three-Act Task is a whole-group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the ***Guide to Three-Act Tasks*** on georgiastandards.org.

Task Name	Task Type/Grouping Strategy	Content Addressed	Standard(s)	Task Description
Power-ful Exponents	Practice/Whole group, small group or individual	Whole number exponents	MGSE5.NBT.2	Expressing place value with exponents
What Comes Next?	Constructing/Whole group, small group	Multiplying or dividing by powers of ten	MGSE5.MBT.2	Exploring powers of ten with base ten blocks
Patterns-R-Us	Scaffolding-Constructing/Small group	Multiplying or dividing by powers of ten	MGSE5.NBT.2 MGSE5.NBT.7	Identifying patterns of multiplication and division
Base Ten Activity	Scaffolding-Constructing/Small group or pair	Modeling the multiplication of decimals	MGSE5.NBT.7	Creating rectangular arrays for multiplication and division with decimals
Missing Numbers	Constructing/Small group or individual	Multiplying decimals	MGSE5.NBT.7	Missing numbers in decimal multiplication
Multiplication Teasers	Constructing/Small group or individual	Multiplying decimals	MGSE5.NBT.7	Missing factors in multiplication
How Much Money?	Constructing/Individual or pair	Dividing decimals	MGSE5.NBT.7	Determining number of coins equal to a given value
Place the Point	Constructing/Small group or individual	Dividing decimals	MGSE5.NBT2 MGSE5.NBT.7	Using estimation to determine the place of the decimal
Super Slugger Award	Practice/Individual or pair	Dividing decimals	MGSE5.NBT.7	Determining baseball averages in a graph
Number Puzzles	Practice/Individual or pair	Multiplying & dividing decimals	MGSE5.NBT.7	Multiplying and dividing decimals for number puzzles
What's My Rule?	Practice/Whole group or small group	Multiplying & dividing decimals	MGSE5.NBT.7	Discovering rules with multiplication and division
Do You See an Error?	Practice/Individual or pair	Multiplying & dividing decimals	MGSE5.NBT.7	Finding errors in student work
Road Trip	Practice/Individual or pair	Multiplying & dividing decimals	MGSE5.NBT.7	Determining fuel costs

Multiplying and Dividing Decimals	Formative Assessment Lesson (FAL)	Multiplying & dividing decimals	MGSE.5.NBT.7	Formative Assessment on multiplying and dividing decimals
Penny Cube	Learning Task	Multiply and divide decimals to solve a problem	MGSE.5.NBT.7 MGSE5.MD.5	Wondering of a cube filling up with pennies
Teacher for a Day	Performance/Individual	Multiplying & dividing decimals	MGSE5.NBT.2 MGSE5.NBT.7	Explaining how to multiply and divide decimals
Bargain Shopping	Performance/Individual	Multiplying & dividing decimals	MGSE5.NBT.2 MGSE5.NBT.7	Determining the best price of objects
Hanging by a Hair	Culminating Task	Multiply and divide decimals to solve a problem	MGSE5.NBT.2 MGSE5.NBT.7	Wonderings of how much hair it would take to hold your body weight
Field Trip	Culminating/Individual	Multiplying &dividing decimals	MGSE5.NBT.2 MGSE5.NBT.7	Planning the most cost effective field trip

If you need further information about this unit visit the GaDOE website and reference the unit webinars.
https://www.georgiastandards.org/Archives/Pages/default.aspx

INTERVENTION TABLE

The Intervention Table below provides links to interventions specific to this unit. The interventions support students and teachers in filling foundational gaps revealed as students work through the unit. All listed interventions are from New Zealand's Numeracy Project.

Cluster of Standards	Name of Intervention	Snapshot of summary or Student I can statement. . .	Materials Master
Understand the place value system. **MGSE5.NBT.2**	Sherpa (Tensing)	Multiply by powers of 10.	
Perform operations with multi-digit whole numbers and with decimals to hundredths. **MGSE5.NBT.7**	Estimation in Decimal Multiplication and Division Problems	Estimate decimal products and quotients by rounding and using benchmark fractions.	
	Multiplication of Decimal Fractions	Reason about the location of decimal point for products and quotients using visual models.	
	Multiplication with Tenths	Solve multiplication and division problems by using place value.	
	When Big Gets Smaller	Reason about relative magnitude of decimal multiplication.	MM 8-15
	When Small Gets Bigger	Reason about relative magnitude of decimal division.	MM 8-16

Practice Task: Power-ful Exponents

Back to Task Table

Adapted from www.nzmaths.co.nz/resource/beyond-million-amazing-math-journey

In this activity, students will develop an understanding that place value can be expressed as a power of 10 (exponents). They will also explore multiplication as a very powerful operation that can create very large numbers (exponential multiplication).

STANDARDS FOR MATHEMATICAL CONTENT

Understand the place value system.

- **MGSE5.NBT.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

This lesson will extend students' previous experience with whole number place value. Before doing this activity, students should have an understanding of the place value names, the period names, and the values associated with them. They should also have prior experiences multiplying whole numbers by powers of ten in Unit 1. In this activity, they will extend that to multiply decimals by powers of ten.

COMMON MISCONCEPTIONS

- *Multiplication can increase or decrease a number*. From previous work with computing whole numbers, students understand that the product of multiplication is greater than the factors. However, multiplication can have a reducing effect when multiplying a positive number by a decimal less than one or multiplying two decimal numbers together. We need to put the term *multiplying* into a context with which we can identify and which will then make the situation meaningful. Also using the terms times and of interchangeably can assist with the contextual understanding.
- *Is a x a x a = 3a? Is a^3 = a x 3?* In mathematics each symbol has a uniquely defined meaning. a x 3 has been arbitrarily chosen as shorthand for a + a + a . It cannot mean anything else. a^3 has been, equally arbitrarily, chosen as shorthand for a x a x a. It means precisely this. Always consider the unique meanings of the mathematics you write.

ESSENTIAL QUESTIONS

- How are exponents used to represent powers of 10?

MATERIALS

- Suggested literature: On Beyond a Million: An Amazing Math Journey by David M. Schwartz
- "Place Value Houses" recording sheets
- Six sided dice
- Calculator
- "Powers of 10 Yahtzee" recording sheet

GROUPING

Whole/individual/small group task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments: This task provides students with the opportunity to explore the different ways to express powers of 10 through a suggested literature connection. Instead of teaching this concept procedurally, allow students to discover the relationship between the powers of 10 and the number of zeros in a number with a 1 in the highest place and zeros in the rest.

TASK

Part 1

Prior to reading the story, ask: What is the largest number you can read? Record a number with many places such as 1,234,567,890,123,456,789 or 1,000,000,000,000 and explore the understanding of place value "houses" with your students. Use the place value houses and insert the digits in the places and practice reading the numbers, stressing to remember to name the house before you leave for the next one (example 42,509,670 read as 42 MILLION, 509 THOUSAND, 670).

The first time that you share the book with your students, start by sharing the story told in the middle sections of the 2-page spreads and focusing on the new vocabulary of the large numbers and the idea of infinity.

Next, or in a second session, read through the book again, this time focusing on the idea of exponents and the math being explored by the professor's dog on the sidebars. Have students record the numbers expressed as exponents and as ordinary notation.

Revisit the Place Value houses and in each section of the house record the place as an expression of a power of ten.

Explore this pattern with the rest of the standard place value houses. Support students to discover the link between powers of 10 and the number of zeros in any large number which has a 1 in the highest place and zeros in the rest.

Part 2

Students will play "Powers of 10 Yahtzee."

Directions:

- Students play against an opponent. The pair needs one die.

- Players take turns rolling the die until each has rolled the die 5 times. Each time they roll the die, they are rolling a power of 10. The base number is always 10. The object of the game is to have the greatest sum after rolling five numbers.
- Player 1 rolls the die, writes the number as 10 to whatever power is indicated on the die and finds the value for that expression. Both players write the exponential expression on their recording sheets and may check the solution with a calculator.
- It is then player 2's turn to roll the die, write the expression and find the value.
- The players continue taking turns until each has had 5 turns. Players record both the five turns for player one and the five turns for player two. At that point the players each find the sum of their answers. The player with the greatest sum wins.

FORMATIVE ASSESSMENT QUESTIONS

- Did you develop a shortcut to find your answers?
- Did you identify any patterns or rules? Explain!

DIFFERENTIATION:

Extension

- Students can explore writing large numbers in scientific notation.
- Students can research large numbers and the meaning of their names.

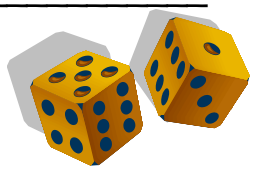
Intervention

- Most students, including students needing an intervention here, would benefit from the use of base ten materials. For example, 10^2 would mean taking ten sets of tens. Students would put these together to make another base ten material, in this case the 100 (flat). For larger exponents, students would still find a cube, rod, or flat, since that is the pattern found in the base ten materials.
    - For example, 10^5 would mean taking 10 sets of 10 **rods**, which as we found before, makes a 100 **flat**, then taking 10 sets of 100 flats to make a 1,000 **cube**, then ten thousands cubes to make a 10,000 **rod**, then ten 10,000 rods to make a 100,000 **flat**. It is likely that students won't be able to make some of these with actual materials, but it does provide students with an investigation into the order of magnitude of our base ten system.

Intervention Table

Name _____ Date _____

Powers of 10 Yahtzee



Materials: 1 die (6-sided); Recording Sheet
Number of Players: 2 or more
Directions:

- Take turns rolling the die until each has rolled 5 times. Each time you roll the die you are rolling a power of 10. The base number is always 10.
- Player 1 rolls the die, writes the number as 10 to whatever power is indicated on the die and finds the value for that expression. Both players write the exponential expression on their recording sheets and may check the solution with a calculator.
- It is then player 2's turn to roll the die, write the expression and find the value.
- Continue taking turns until each of you has had 5 turns. Record both the five turns for player one and the five turns for player two. At that point the players each find the sum of their answers. The player with the greatest sum wins.

Game 1:

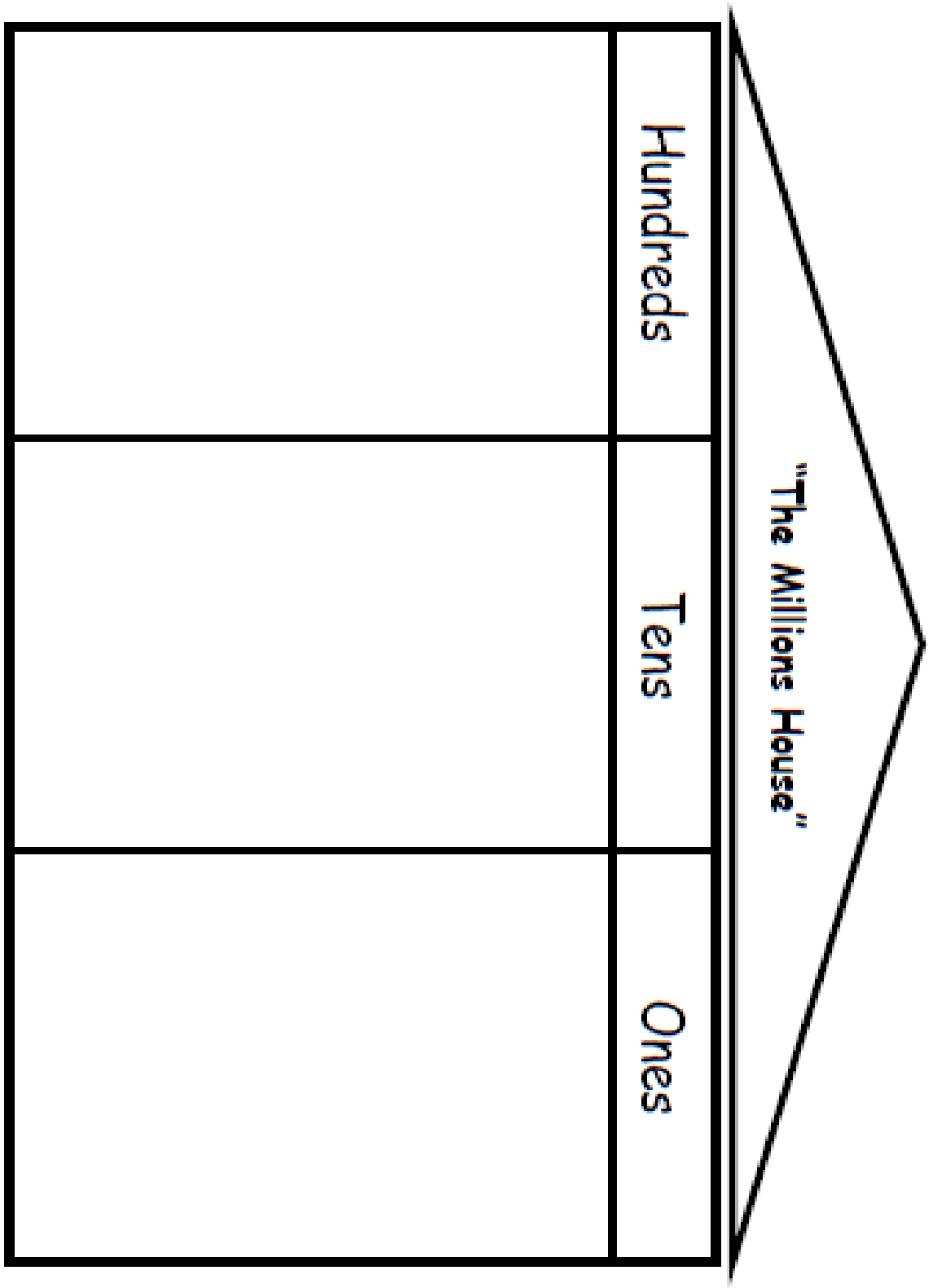
Player 1		Player 2	
Exponential Expression	Value	Exponential Expression	Value

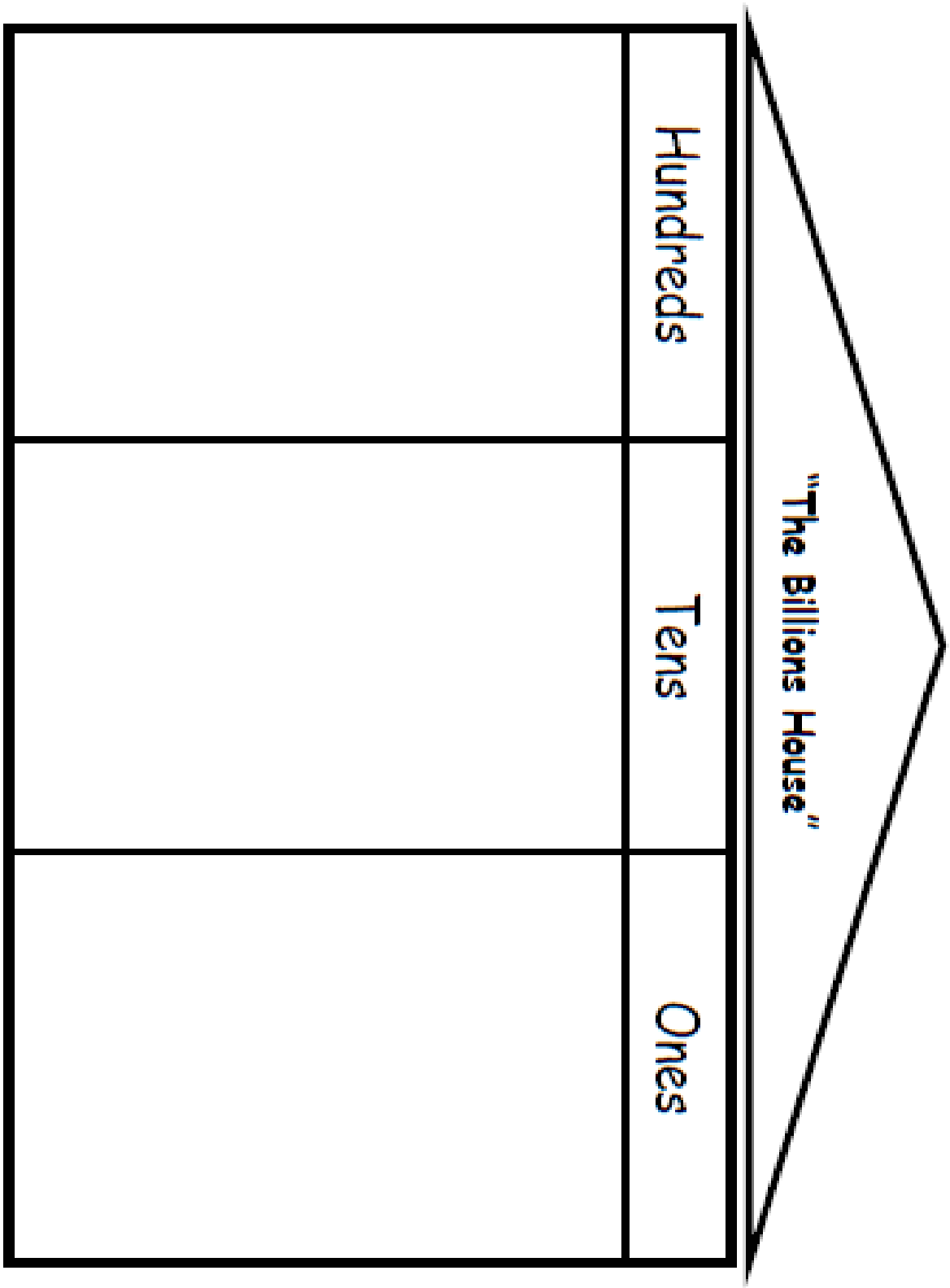
Player 1's Sum: _____ Player 2's Sum: _____

Game 2:

Player 1		Player 2	
Exponential Expression	Value	Exponential Expression	Value

Player 1's Sum: _____ Player 2's Sum: _____





Constructing Task: What Comes Next?

Back to Task Table

Adapted from "What Comes Next?" in Van de Walle's Teaching Student-Centered Mathematics

In this task, students explore the pattern of powers of 10 with base ten blocks.

STANDARDS FOR MATHEMATICAL CONTENT

Understand the place value system.

MGSE5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
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3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Two important ideas typically developed for three-digit numbers should be carefully extended to larger numbers. First, the grouping of ten ideas should be generalized. That is, 10 in any position make a single thing (group) in the next position, and vice versa. Second, the oral and written patterns for numbers in three digits are duplicated in a clever way for every three digits to the left. These two related ideas are not as easy for children to understand as adults seem to believe. Because models for larger numbers are so difficult to have or picture, textbooks must deal with these ideas in a predominately symbolic manner. That is not sufficient! Try the "What Comes Next?" discussion in the context of three-dimensional models through the use of base ten manipulatives. The first three shapes are distinct: a *cube*, a *long*, and a *flat*. What comes next? Stack 10 flats and they make a cube, the same shape as the first only 1000 times larger. What comes next? Ten *cubes* make another *long*. What comes next? Ten big *longs* make a big *flat*. The first three shapes have now repeated! Ten big flats will make an even bigger cube, and the triplet of shapes begins again. Each cube has a name. The first one is the *unit* cube, the next is a *thousand*, the next is a *million*, then a *billion*, and so on. Each long is 10 cubes: 10 units, 10 thousands, 10 millions. Similarly, each flat shape is 100 cubes. It is important for students to realize that the system does have a logical structure, is not totally arbitrary, and can be understood (Van de Walle, 2006). **Students will be using base ten strips and squares for this task which makes the modes two-dimensional versus three-dimensional. Because of this, the sequence will be square, strip, square, strip, etc. instead. A part of the problem-based task is to determine and continue this pattern.**

A difficult idea for students to comprehend is why any base number raised to the zero power has a value of one. Consider this example: 3^1 = **3**, 3^2 = **9**, 3^3 = **27**, 3^4 = **81**... Look at the pattern of the bold numbers. Every time you move to the right in the list, you multiply by 3. If you move to the left, you divide by 3. If we continued the sequence one more to the left you would have 3^0 = **1**. Examples such as this one can be explored with students to help clarify this concept.

COMMON MISCONCEPTIONS

- *Multiplication can increase or decrease a number*. From previous work with computing whole numbers, students understand that the product of multiplication is greater than the factors. However, multiplication can have a reducing effect when multiplying a positive number by a decimal less than one or multiplying two decimal numbers together. We need to put the term *multiplying* into a context with which we can identify and which will then make the situation meaningful. Also using the terms *times* and *groups of* interchangeably can assist with the contextual understanding.
- *Is a x a x a = 3a? Is a^3 = a x 3?* In mathematics each symbol has a uniquely defined meaning. a x 3 has been arbitrarily chosen as shorthand for a + a + a . It cannot mean anything else. a^3 has been, equally arbitrarily, chosen as shorthand for a x a x a. It means precisely this. Always consider the unique meanings of the mathematics you write.

ESSENTIAL QUESTIONS

- How can we use exponents to represent powers of 10?

MATERIALS

- "What Comes Next?" recording sheet
- Base ten blocks
- Base ten strips and squares
- Butcher paper
- Markers
- Calculators (optional)

GROUPING

Whole group/small group task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Comments: This task is designed to serve as a discovery opportunity for the students. Students should notice that a pattern is created when a number is multiplied or divided by a power of 10. While students may notice patterns in each individual part of the task, encourage them to look for a pattern when considering the overall task. Students should be able to explain and defend their solutions through multiple representations. Calculators can be used to assist students with their explanations. For example, students may want to calculate that 10 x 10 x 10 = 1000 with this tool rather than using a written method.

TASK

Part 1

- Hand out base ten blocks to each group. Ask each group to hold up the unit cube. Ask students to prove how many units they think it represents. Write 10^0 on the board and share that this is another way to represent the value of the unit cube. Discuss that mathematicians have agreed that no matter what the base number is, if it is raised to the zero power, its value is always one unit. (See Background Knowledge to support discussion with students)
- Have each group hold up a tens rod and ask them to justify how many units it represents. Write 10^1 on the board and share that this is another way to represent the value of the block. Have the students discuss why they think this could be another way to represent the value of 10. (There is one group of ten or one group of ten units)
- Have each group hold up a hundreds flat and ask them to justify how many units it represents. Write 10^2 on the board and ask why this could be another way to represent 100 units. (You can multiply 10 x 10 to get one hundred or you are multiplying ten by itself two times) *Optional*: Ask students why they think we call a base number that is raised to the second power, "squared". (Because it is shaped like a square)
- Have each group hold up a thousand's cube and ask them to justify how many units it represents. Ask the students to predict how to write this number with exponents and justify their conjectures. (You can multiply 10 x 10 x 10 to get one thousand or you are multiplying ten by itself three times) *Optional*: Ask students why they think we call a base number that is raised to the third power, "cubed". (Because it is shaped like a cube)

Part 2

- Use the "What Comes Next?" recording sheet to lead a discussion using the base ten strips and squares. Discuss that the unit is a 1-centimeter square. Ten units are represented using a 10 x 1 strip. The hundreds piece is a 10 x 10 square. Ask students what would come next. Allow time for groups to discuss their ideas. Ten hundreds would be a thousand, but what shape would it be? It could be a strip made by ten hundreds squares. Tape 10 hundreds together.
- After students have discovered how to represent the thousands piece, ask students what comes next. Allow groups the time to figure out the dimensions of the ten thousand's piece and the hundred thousand's piece.
- Students will fill out the "What Comes Next?" recording sheet as they progress through the task. Because of the discussion in part one, students will also be asked to note the whole number exponent that would be associated with each large number.

FORMATIVE ASSESSMENT QUESTIONS

- How do you know your answer is correct?
- What patterns are you noticing?

DIFFERENTIATION

Extension

Students may become interested in seeing the big pieces from the discussion. Have them draw the representations on butcher paper or using chalk on the blacktop. Create a pictorial representation of ten thousand and hundred thousand.

Intervention

If students are having difficulty transferring to the base ten strips, have them work in small groups with the base ten blocks instead.

Intervention Table

Name_____Date_____

What Comes Next?

1. Unit
    a. What is the numerical value?

    _____

    b. What shape is it?

    _____

    c. What are the dimensions?

    _____

    d. How would this be written using exponents?

    _____

2. Ten
    a. What is the numerical value?

    _____

    b. What shape is it?

    _____

    c. What are the dimensions?

    _____

    d. How would this be written using exponents?

    _____

3. Hundred
    a. What is the numerical value?

    _____

    b. What shape is it?

    _____

    c. What are the dimensions?

    _____

    d. How would this be written using exponents?

    _____

4. Thousand
    a. What is the numerical value?

    _____

    b. What shape is it?

    _____

c. What are the dimensions?

_____

d. How would this be written using exponents?

_____

5. Ten thousand
   a. What is the numerical value?

      _____

   b. What shape is it?

      _____

   c. What are the dimensions?

      _____

   d. How would this be written using exponents?

      _____

6. Hundred thousand
   a. What is the numerical value?

      _____

   b. What shape is it?

      _____

   c. What are the dimensions?

      _____

   d. How would this be written using exponents?

      _____

7. Do you notice a pattern in the sequence of shapes? If so, explain.

_____
_____
_____
_____

8. Describe the relationship between the exponent and the number of zeros in the numbers above.

_____
_____
_____
_____

9. Create a rule for using exponents with powers of ten.

_____
_____
_____
_____

Scaffolding/Constructing Task: Patterns-R-Us

Back to Task Table

In this task, students are asked to identify, describe, and explain any patterns they notice when multiplying or dividing numbers by 1000, 100, 10, 0.1, and 0.01.

STANDARDS FOR MATHEMATICAL CONTENT

Understand the place value system.

MGSE5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Perform operations with multi-digit whole numbers and with decimals to the hundredths.

MGSE5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students should develop an understanding that when a number is multiplied by a number less than 1, the product is less than the original number, and when a number is divided by a decimal number less than 1, the quotient will be greater than the dividend. This is important, yet often difficult for students to understand because it is counterintuitive based on students' previous experiences with multiplication and division.

Because students have not yet learned how to multiply or divide decimals, calculators are recommended for this investigation. Students will be more likely to explore a variety of numbers and be able to recognize patterns more efficiently with the use of a calculator. Require students to record what they put into the calculator and the result. If students have learned how to multiply and divide with decimals, they could benefit from solving the problems in part one without a calculator and allow students to use a calculator for the rest of the task.

COMMON MISCONCEPTIONS

- *Multiplication can increase or decrease a number*. From previous work with computing whole numbers, students understand that the product of multiplication is greater than the factors. However, multiplication can have a reducing effect when multiplying a positive number by a decimal less than one or multiplying two decimal numbers together. We need to put the term *multiplying* into a context with which we can identify and which will then make the situation meaningful. Also, using the terms *times* and *groups of* interchangeably can assist with the contextual understanding.

ESSENTIAL QUESTIONS

- How does multiplying or dividing by a power of ten affect the product?

MATERIALS

- "Patterns-R-Us" recording sheet
- Calculators

GROUPING

Small group task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments: This task is designed to serve as a discovery opportunity for the students. Students should notice that a pattern is created when a number is multiplied or divided by a power of 10. While students may notice patterns in each individual part of the task, encourage them to look for a pattern when considering the overall task. Students should be able to explain and defend their solutions through multiple representations. For example, students should try several numbers for each part to verify that each number follows the same pattern. This activity lends itself to working in pairs for reinforcement.
An introduction for this task could be a round of "What's My Rule?" The rule could be x1000, x100, x10, x0.1, or x0.01. Also, the rule could be ÷1000, ÷100, ÷10, ÷0.1, or ÷0.01.

TASK

Students will follow the directions below from the "Patterns-R-Us" Recording Sheet.

A statistician is interested in finding out what pattern is created, if any, under certain situations. Your mission is to help come up with concrete rules for certain mathematical situations. Record all of your work and explain your thinking in order to defend your answer. Good luck!

Part 1

1. Start with any whole number, for example 18.
2. Multiply that number by 1000, 100, 10, 0.1, and 0.01.
3. What is happening?
4. Is there a pattern?
5. What do you think would happen if you multiplied your number by 1,000,000? 0.00001?

Part 2

1. Pick any decimal as your number, for example 12.3.

2. Multiply that number by 1000, 100, 10, 0.1, and 0.01.
3. What is happening?
4. Is there a pattern?
5. What do you think would happen if you multiplied your number by 1,000,000? 0.00001?

Part 3

1. Start with any whole number, for example 18.
2. Divide that number by 1000, 100, 10, 0.1, and 0.01.
3. What is happening?
4. Is there a pattern?
5. What do you think would happen if you divided your number by 1,000,000? 0.00001?

Part 4

1. Pick any decimal as your number, for example 10.8.
2. Predict what will happen when you divide that number by 1000, 100, 10, 0.1, and 0.01.
3. After working out the problem, is your prediction correct? Why or why not?
4. Is there a similar pattern that you recognize?

FORMATIVE ASSESSMENT QUESTIONS

- How do you know your answer is correct?
- What would happen if you started with a different number?
- What patterns are you noticing?
- Can you predict what would come next in the pattern?

DIFFERENTIATION

Extension

- Have students multiply a number by 0.1. Now ask them to divide that same number by 10. What happened? Repeat this with several numbers. Can a conjecture be made based on the results? Have students write their conjecture. Now, share their conjecture with a partner. Are the two conjectures the same? (You may also use 0.01 and 100 as another example.)

Intervention

- Pair students who may need additional time so that they will have time needed to process this task.

Intervention Table

Name_____ Date_____

Patterns-R-Us

A statistician is interested in finding out what pattern is created, if any, under certain situations. Your mission is to help come up with concrete rules for certain mathematical situations. Record all of your work and explain your thinking in order to defend your answer. Good luck!

PART ONE

1. Start with any whole number, for example 18.
2. Multiply that number by 1000, 100, 10, 0.1, and 0.01.
3. What is happening?
4. Is there a pattern?
5. What do you think would happen if you multiplied your number by 1,000,000? 0.00001?

PART TWO

1. Pick any decimal as your number, for example 12.3.
2. Multiply that number by 1000, 100, 10, 0.1, and 0.01.
3. What is happening?
4. Is there a pattern?
5. What do you think would happen if you multiplied your number by 1,000,000? 0.00001?

PART THREE

1. Start with any whole number, for example 18.
2. Divide that number by 1000, 100, 10, 0.1, and 0.01.
3. What is happening?
4. Is there a pattern?
5. What do you think would happen if you divided your number by 1,000,000? 0.00001?

PART FOUR

1. Pick any decimal as your number, for example 10.8.
2. Predict what will happen when you divide that number by 1000, 100, 10, 0.1, and 0.01.
3. After working out the problem, is your prediction correct? Why or why not?
4. Is there a similar pattern that you recognize?

Scaffolding/Constructing Task: Base Ten Activity

Back to Task Table

This task was adapted from the following website http://argyll.epsb.ca/jreed/math7/strand1/1201.htm

In this task, students will create rectangular arrays as a representation of multiplication and division of decimals.

STANDARDS FOR MATHEMATICAL CONTENT

Perform operations with multi-digit whole numbers and with decimals to the hundredths.

MGSE5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

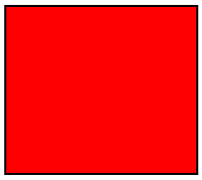
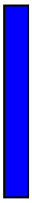
STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

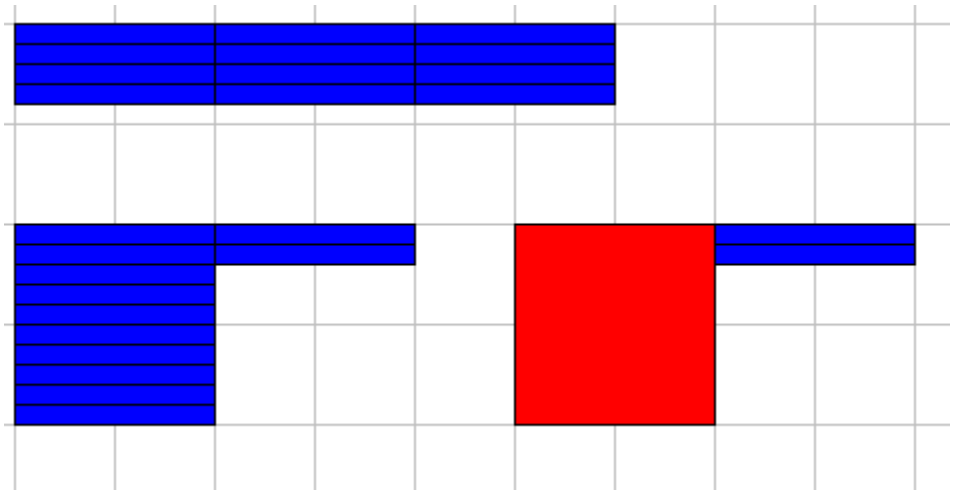
Representing decimal values with base ten blocks

In this activity you will use base ten blocks to represent decimals. In this case, the red block (flat) represents 1 unit, the blue (long or rod) represents one tenth of the unit or 0.1 units, and the green (unit) represents one hundredth or 0.01 units. You can use this system to introduce decimal numbers and the place value with decimal numbers. Also, students can explore addition and subtraction of decimals using base-ten blocks in a manner similar to adding and subtracting whole numbers using base ten blocks.

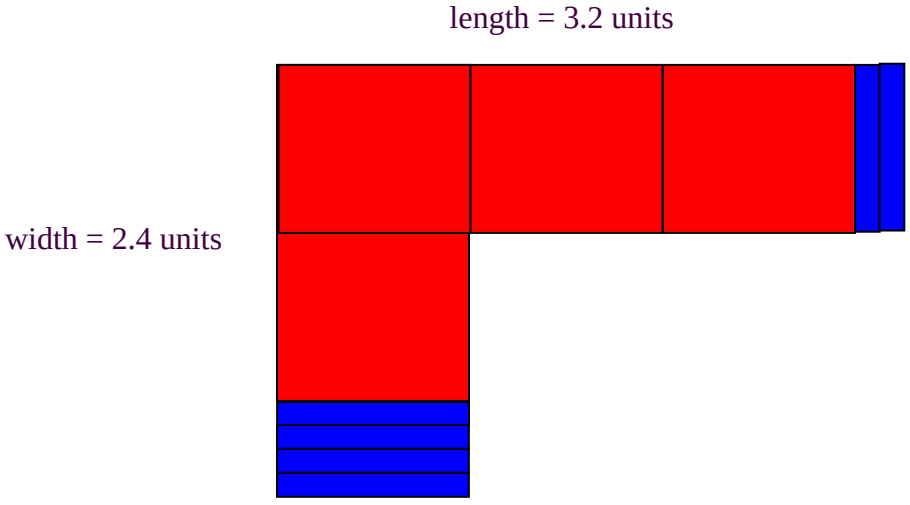
Decimal Block Values		
1 (flat)	0.1 (long)	0.01 (unit)
		

Representing decimal multiplication with base ten blocks

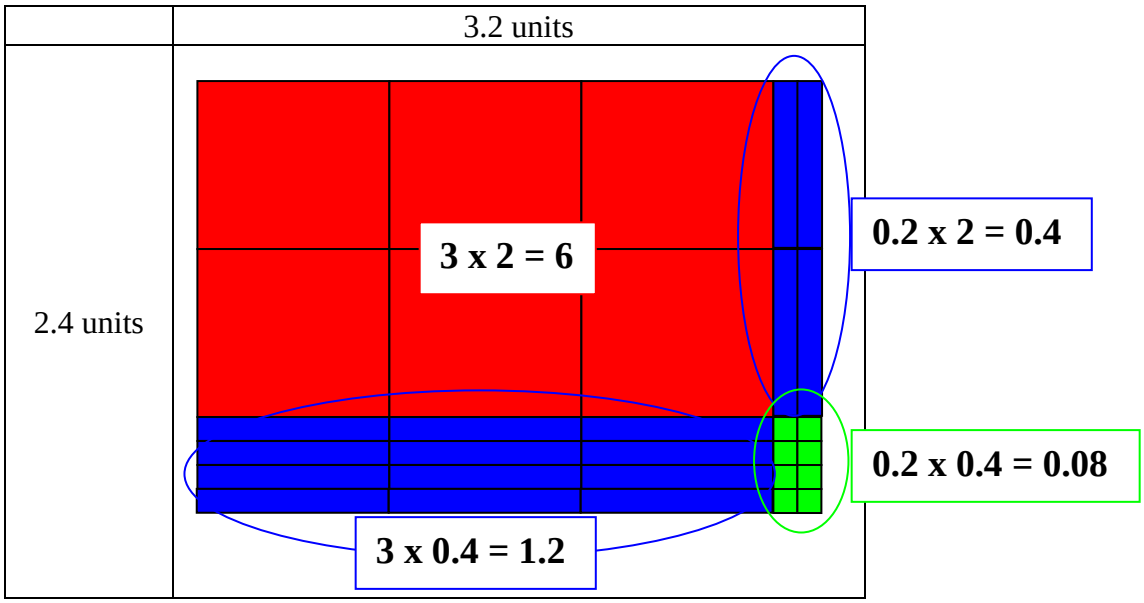
Decimal multiplication can be represented as an array or as repeated addition. For example, to multiply 3 x 0.4 you pull out 3 sets of 0.4, represented by 3 sets of 4 longs. Arrange them in a 3 by 0.4 array as shown below. There are a total of 12 longs. 10 longs can be traded for a flat, with 2 longs left over. Or you can think of 12 tenths as being one whole and 2 tenths, therefore, 3 x 0.4 = 1.2.



To illustrate the product of 3.2 x 2.4 you will need to combine blocks in a rectangular array. Start with the length and the width.

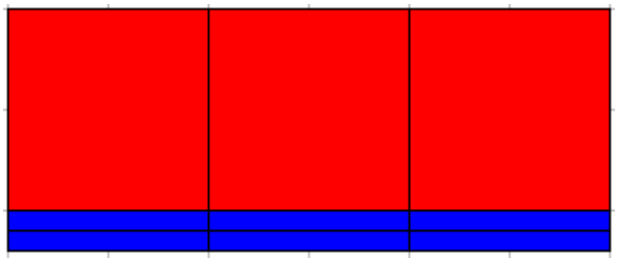


Then complete the rectangle.



Area = 3.2 x 2.4 = (3 x 2) + (3 x 0.4) + (0.2 x 2) + (0.2 x 0.4) = 6 + 1.2 + 0.4 + 0.08 = 7.68
Therefore, **3.2 units x 2.4 units = 7.68 units^2** as shown above.

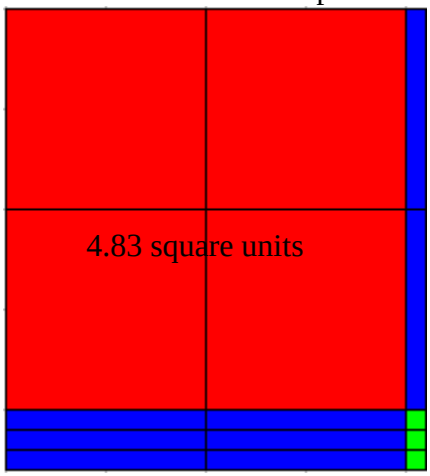
<u>Representing decimal division with base ten blocks</u>
Division can be represented by using the dividend as the total area and arranging the blocks in groups according to the divisor. The divisor would be built in columns, and columns would continue to be added until there are enough columns built so that the dividend is represented by the total area. The number of rows or groups created is the quotient. For the fifth problem, 3.6 ÷ 1.2, ask the students how many groups of 1 flat and 2 longs can be made with a group of 3 flats and 6 longs. The quotient can be thought of as the missing factor in a multiplication problem.



In the example below (4.83 ÷ 2.1), we are dividing a total of 4.83 blocks into equal groups of 2.1. In this example, the divisor is represented in rows. The quotient or number of groups is represented in columns, and it is 2.3. One dimension of this array is the divisor, or 2.1 given in the problem. The other dimension is 2.3 which is the quotient. It can also be thought of as the missing factor in a multiplication array.

2.1 units across the top

How many units along this side?



COMMON MISCONCEPTIONS

- *Multiplication can increase or decrease a number*. From previous work with computing whole numbers, students understand that the product of multiplication is greater than the factors. However, multiplication can have a reducing effect when multiplying a positive number by a decimal less than one or multiplying two decimal numbers together. We need to put the term *multiplying* into a context with which we can identify and which will then make the situation meaningful. Also using the terms *times* and *groups of* interchangeably can assist with the contextual understanding.

ESSENTIAL QUESTIONS

- How can we use models to help us multiply and divide decimals?

MATERIALS

- Base-ten recording sheet
- Base-ten blocks (or virtual base-ten blocks)
- Grid paper (or plain paper) to record work
- Colored pencils, crayons, or markers

GROUPING

Small group/Partner task

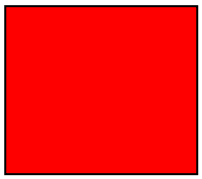
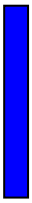
TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Comments: When making rectangles to represent decimal multiplication, you are actually using the length and the width of each block to represent the factors. Therefore, a flat is actually 1 unit by 1 unit, a long is 1 unit by 0.1 unit, and a unit block is 0.1 unit by 0.1 unit. (area model)

Task:

Students will follow the directions below from the "Base 10 Activity" Recording Sheet.

Your task is to use the base-ten blocks to represent multiplication and division with decimals. Use the decimal block values below to help you find the product or quotient of each decimal problem.

Decimal Block Values		
1 (flat)	0.1 (long)	0.01 (unit)
		

1. 4.8 times 3
2. 2.1 x 5.4
3. 0.6 x 1.9
4. 12 ÷ 0.3
5. 3.6 divided by 1.2

FORMATIVE ASSESSMENT QUESTIONS

- If the flat is 1 unit, what does a long represent? What does a unit block represent?
- How many groups do you need to represent? How many do you have in all?
- How can you create any array using multiplication? Where are the factors located? Where is the product? Can you identify the partial products?
- What properties of multiplication are you using?
- How can you create an array using the dividend? How do you represent the divisor in the array? Where is the quotient represented?

DIFFERENTIATION

Extension

- Have students create their own practice problems with solutions and then switch them with a partner. Have the partner work the problems using the base-ten blocks. When finished, students can compare solutions.

Intervention

- Have students work with a partner or with a teacher in small groups to help develop these concepts. Scaffold student understanding by initially providing arrays for students to use to find the product or quotient. Then provide a partially completed array or the outline of an array. Slowly remove scaffolding as students become more independent with finding a product or quotient using the base ten blocks.

Intervention Table

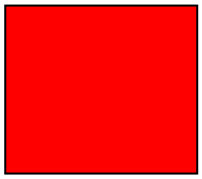
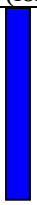
TECHNOLOGY CONNECTIONS

*https://www.desmos.com/calculator/pfpwczmore This is an interactive calculator from Desmos that models multiplication of decimals.

Name _____ Date _____

Base Ten Activity

Your task is to use the base-ten blocks to represent multiplication and division with decimals. Use the decimal block values below to help you find the product or quotient of each decimal problem.

Decimal Block Values		
1 (flat)	0.1 (long)	0.01 (unit)
		

1. 4.8 times 3
2. 2.1 x 5.4
3. 0.6 x 1.9
4. 12 ÷ 0.3
5. 3.6 divided by 1.2

Constructing Task: Missing Numbers

Back to Task Table

In this task, students are challenged to find all the possibilities for missing numbers in a decimal multiplication number sentence.

STANDARDS FOR MATHEMATICAL CONTENT

Perform operations with multi-digit whole numbers and with decimals to the hundredths.

MGSE5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

If students use their prior knowledge about whole number multiplication, they will realize that the products will have the exact same digits, but the decimal placement will change the value of the numbers. If students are methodical about their solution strategy, they will be able to find all of the possibilities systematically from 0.1 x 5 = 0.5 to 1.9 x 5 = 9.5.

COMMON MISCONCEPTIONS

- *Multiplication can increase or decrease a number*. From previous work with computing whole numbers, students understand that the product of multiplication is greater than the factors. However, multiplication can have a reducing effect when multiplying a positive number by a decimal less than one or multiplying two decimal numbers together. We need to put the term *multiplying* into a context with which we can identify and which will then make the situation meaningful. Also using the terms *times* and *groups of* interchangeably can assist with the contextual understanding.

ESSENTIAL QUESTIONS

- How can we multiply and divide decimals fluently?
- What strategies are effective for finding a missing factor or divisor?

MATERIALS

- Calculators (optional)

GROUPING

Small group/Individual task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Comments: This task will allow the teacher to see if students are able to use what they know about whole number multiplication and transfer it to decimal multiplication. Students will be able to come up with more solutions if they realize that whole numbers can be represented with decimals. There is no recording sheet provided with this task. Instead, encourage students to document their thinking in their math journals or notebooks.

TASK:

_____._____ *x 5* = _____._____

What might the missing numbers be? Try to find all the possibilities.

FORMATIVE ASSESSMENT QUESTIONS

- How did you get your answer?
- How do you know your answer is correct?
- What patterns are you noticing?

DIFFERENTIATION

Extension

- Students can explore problems like this with two factors that are decimal numbers.

Intervention

- Provide students with some whole number examples before incorporating decimals. Include some examples that only have one solution before moving to multiple solutions. Also, calculators can be used for this task.

Intervention Table

TECHNOLOGY CONNECTIONS

*https://www.desmos.com/calculator/pfpwczmore This is an interactive calculator from Desmos that models multiplication of decimals.

Constructing Task: Multiplication Teasers

Back to Task Table

Adapted from Marilyn Burns' lesson "Multiplication Puzzlers" in About Teaching Mathematics

In this task, students will be challenged to find the missing factor in a multiplication number sentence by first using estimation strategies, then refining their guesses by using a calculator.

STANDARDS FOR MATHEMATICAL CONTENT

Perform operations with multi-digit whole numbers and with decimals to the hundredths.

MGSE5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students often do not understand that if a whole number if multiplied by a decimal, the product will be less than the larger factor. This task can provide students with problems that will make this evident.

COMMON MISCONCEPTIONS

- *Multiplication can increase or decrease a number*. From previous work with computing whole numbers, students understand that the product of multiplication is greater than the factors. However, multiplication can have a reducing effect when multiplying a positive number by a decimal less than one or multiplying two decimal numbers together. We need to put the term *multiplying* into a context with which we can identify and which will then make the situation meaningful. Also using the terms times and of interchangeably can assist with the contextual understanding.

ESSENTIAL QUESTIONS

- What strategies are effective for finding a missing factor or divisor?
- How can we use estimation to assist in solving problems with decimal operations?

MATERIALS

- "Multiplication Teasers" recording sheet
- Calculators (optional)

GROUPING

Small group/Individual task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Comments: A great deal of emphasis has been put on paper-and-pencil algorithms when solving problems with decimals. The focus is often put on how to do the problem rather than on what makes sense. Students should be given many opportunities to calculate decimals mentally. The emphasis shifts from getting exact answers from paper-and-pencil calculations to arriving at estimates and being able to explain why they are reasonable.

TASK

For each problem, find the missing factor. Do not solve problems by dividing. Instead, use your calculator and the guess and check problem solving strategy. See how many guesses it takes for you to solve each one! For example, to solve 4 x _____ = 87, you might start with 23 and then adjust. See the solution steps below as an example of how to record your guesses.

4 x _____ = 87
4 x 23 = 92
4 x 22 = 88
4 x 21 = 84
4 x 21.5 = 86
4 x 21.6 = 86.4
4 x 21.7 = 86.8
4 x 21.8 = 87.2
4 x 21.74 = 86.96
4 x 21.75 = 87 ☆

It took 9 guesses!

Try the following:

8 x _____ = 108	6 x _____ = 99
4 x _____ = 97	5 x _____ = 66

FORMATIVE ASSESSMENT QUESTIONS

- How did you get your answer?
- How do you know your answer is correct?
- What patterns are you noticing?

DIFFERENTIATION

Extension

- Provide students with problems that incorporate factors to the thousands and repeating decimals to challenge their thinking.

Intervention

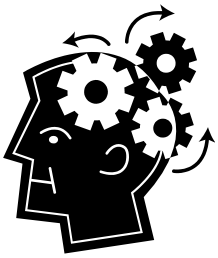
- Work with small groups of struggling students to model the problem solving process by doing a think-aloud to work through an example problem.
- Incorporate the use of manipulatives (grid paper, base ten blocks, etc.) to assist with student guesses.

Intervention Table

TECHNOLOGY CONNECTIONS

*https://www.desmos.com/calculator/pfpwczmore This is an interactive calculator from Desmos that models multiplication of decimals.

Name _____ Date _____



Multiplication Teasers

For each problem, find the missing factor. Do not solve problems by dividing. Instead, use your calculator and the guess and check problem solving strategy. See how many guesses it takes for you to solve each one! For example, to solve 4 x _____ = 87, you might start with 23 and then adjust. See the solution steps below as example of how to record your guesses.

4 x _____ = 87
4 x 23 = 92
4 x 22 = 88
4 x 21 = 84
4 x 21.5 = 86
4 x 21.6 = 86.4
4 x 21.7 = 86.8
4 x 21.8 = 87.2
4 x 21.74 = 86.96
4 x 21.75 = 87 ☆

It took 9 guesses!

Try the following:

8 x _____ = 108	6 x _____ = 99
4 x _____ = 97	5 x _____ = 66

Constructing Task: How Much Money?

Back to Task Table

In this task, students will use division or other basic operations to find the number of coins equal to a given dollar value.

STANDARDS FOR MATHEMATICAL CONTENT

Perform operations with multi-digit whole numbers and with decimals to the hundredths.
MGSE5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

We want students to understand that when they are trying to determine the number of coins of specific values, they are indeed performing division (measurement division). Then, when they are solving 34.50 divided by 0.25, one way to do this is to think of everything in terms of cents and determine how many groups of 25 cents can be made with 3450 cents. In this process, we changed both decimal numbers into whole numbers. Dividing to find the number of dimes might lead to an idea with efficiency. For example, instead of thinking about 3450 divided by 10, students can find the same answer by dividing 345.0 by 1.0.

NOTE: The purpose of the task is to give students practice in decimal division. They may be able to use reasoning such as "4 quarters in a dollar" so I can multiply dollars by 4 to get the answer. If they use multiplication or mental math to determine their answers, have them check their answers using division of decimals to see the relationship between the two methods and to check for accuracy.

Answers to the chart are shown below.

Final Daily Balance	*# of Pennies ($0.01)*	*# of nickels ($0.05)*	*# of dimes ($0.10)*	*# of quarters ($0.25)*
Example $34.50	*3450*	*690*	*345*	*138*
Day 1 -- $21.00	2100	420	210	84
Day 2 -- $35.50	3550	710	355	142
Day 3 -- $69.00	6900	1380	690	276
Day 4 -- $121.00	12100	2420	1210	484
Day 5 -- $234.50	23450	4690	2345	938

COMMON MISCONCEPTIONS

- *Multiplication can increase or decrease a number*. From previous work with computing whole numbers, students understand that the product of multiplication is greater than the factors. However, multiplication can have a reducing effect when multiplying a positive number by a decimal less than one or multiplying two decimal numbers together. We need to put the term *multiplying* into a context with which we can identify and which will then make the situation meaningful. Also using the terms *times* and *groups of* interchangeably can assist with the contextual understanding.

ESSENTIAL QUESTIONS

- What happens when we divide a decimal by a decimal?
- How do the rules of multiplying whole numbers relate to multiplying decimals?

MATERIALS

- "How Much Money?" recording sheet

GROUPING

Individual/Partner task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Students will follow the directions below from the "How Much Money?" recording sheet.

You have been asked by your school principal to help count the money at your school store. Your job is to determine how many pennies ($0.01), nickels ($0.05), dimes ($0.10), and quarters ($0.25) you have at the end of the day.

Complete the chart below and determine the maximum number of each type of coin that can be found in the final daily school store balance.

Final Daily Balance	# of Pennies ($0.01)	# of nickels ($0.05)	# of dimes ($0.10)	# of quarters ($0.25)
Example $34.50	3450	690	345	138
Day 1 -- $21.00				
Day 2 -- $35.50				
Day 3 -- $69.00				
Day 4 -- $121.00				
Day 5 -- $234.50				

After you have completed the chart, answer the questions below. Be prepared to share your answers with the class.

1. How can you be assured your answers are correct? *Students should be able to "prove" their solutions make sense and be able to show how they arrived at their solution.*
2. How did you find your individual solutions?
3. Can you think of another method to find the number of coins in your daily balance?
4. Do you see any patterns? If so, what are they? *Various responses may be noted here. One possible response could be that "dimes are half the number of nickels each time." Another response could be that the "number of nickels is five times the number if pennies."*

FORMATIVE ASSESSMENT QUESTIONS

- How did you think about this problem?
- What operation did you use to find the number of pennies? Nickels? Dimes? Quarters?
- Is there another way you could have found the number of pennies? Nickels? Dimes? Quarters? Which method is more efficient? Which method is easier to do in your head?
- Do you notice any patterns as you look down each column? As you look across each row?

DIFFERENTIATION

Extension

- Have the students determine how many ways they can create the final daily balance with different combinations of coins. (Example: $21.00 = 2100 pennies, or 80 quarters and 400 pennies, or…)

Intervention

- Use manipulatives and/or real coins to model smaller amounts of money before moving to the recording sheet.

Intervention Table

Name _____Date _____

How Much Money?

You have been asked by your school principal to help count the money at your school store. Your job is to determine how many pennies ($0.01), nickels ($0.05), dimes ($0.10), and quarters ($0.25) you have at the end of the day.

Complete the chart below and determine the maximum number of each type of coin that could be found in the final daily school store balance.

Final Daily Balance	# of Pennies ($0.01)	# of nickels ($0.05)	# of dimes ($0.10)	# of quarters ($0.25)
Example $34.50	3450	690	345	138
Day 1 -- $21.00				
Day 2 -- $35.50				
Day 3 -- $69.00				
Day 4 -- $121.00				
Day 5 -- $234.50				

Constructing Task: Place the Point!

Back to Task Table

Adapted from "Where Does the Decimal Go?" from Zeroing in on Number and Operations: Key Ideas and Common Misconceptions, Grades 5-6.

In this task, students will use estimation to determine where to place the decimal point in several quotients. The digits remain the same; however, the value will change.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

MGSE5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Most students struggle with division of whole numbers and become even more confused once decimals are introduced. Make sure that students are proficient with whole number division first. Then, incorporate a lot of varied problems and computations with decimals as they build that proficiency. Some students will come to class having been told by parents or siblings that you just have to move the decimal over in the divisor to make it a whole number and move the decimal in the dividend the same number of places. These students may benefit from doing more estimation with decimals before engaging in formal computations that require an exact answer. Teachers should help students understand that decimals do not move. Instead, numbers move around the decimal as the magnitude changes. Students should be able to explain this concept using appropriate mathematical and quantitative thinking.

COMMON MISCONCEPTIONS

- *Multiplication can increase or decrease a number*. From previous work with computing whole numbers, students understand that the product of multiplication is greater than the factors. However, multiplication can have a reducing effect when multiplying a positive number by a decimal less than one or multiplying two decimal numbers together. We need to put the term *multiplying* into a context with which we can identify and which will then make

the situation meaningful. Also using the terms *times* and *groups of* interchangeably can assist with the contextual understanding.

ESSENTIAL QUESTIONS

- How can we efficiently solve multiplication and division problems with decimals?
- How can we use estimation to assist in solving problems with decimal operations?

MATERIALS

- "Place the Point" recording sheet
- Calculators (optional)

GROUPING

Small group/Individual task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Comments: A great deal of emphasis has been put on paper-and-pencil algorithms when solving problems with decimals. The focus is often put on how to do the problem rather than on what makes sense. Students should be given many opportunities to calculate decimals mentally. The emphasis shifts from getting exact answers from paper-and-pencil calculations to arriving at estimates and being able to explain why they are reasonable.

TASK:

The digits in the computation below are all correct, but the decimal point has been removed.

$$169 \div 4 = 4225$$

Use only estimation to find the quotients of the following. Justify each response, then check your answers using a calculator. If your solution is incorrect, fix the answer and your justification.

a. $169 \div 0.4 =$
b. $16.9 \div 4 =$
c. $16.9 \div 0.4 =$
d. $169 \div 40 =$

FORMATIVE ASSESSMENT QUESTIONS

- How did you get your answer?
- How do you know your answer is correct?
- What patterns are you noticing?

DIFFERENTIATION:

Extension

- Have students create problems of their own, not placing the decimal in the problem as done in the task, and swap with another student.

Intervention

- Use small numbers with only a decimal in the dividend.

Intervention Table

Name _____Date _____

Place the Point!

The digits in the computation below are all correct, but the decimal point has been removed.

169 ÷ 4 = 4225

Use only estimation to find the quotients of the following. Justify each response.

a. 169 ÷ 0.4 =

b. 1.69 ÷ 4 =

c. 16.9 ÷ 0.4 =

d. 169 ÷ 80 =

Practice Task: Super Slugger Award

Back to Task Table

In this task, students work to determine baseball players' batting averages, represent the data in a graph, and then determine the recipient of a "Super Slugger Award."

STANDARDS FOR MATHEMATICAL CONTENT

Perform operations with multi-digit whole numbers and with decimals to the hundredths.

MGSE5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Answers are shown below along with sample student responses.

1. Figure out the batting averages of each player on the team. Round each batting average to the thousandths decimal place. Remind students that in order to find a hitter's batting average you divide the number of hits (h) a player gets by the number of times they have been to bat (b); h ÷ b.

Player	Number of Hits	Number of Times at Bat	Batting Average
K. Smith	25	76	0.328
T. Rusch	33	80	0.412
A. Patrick	51	96	0.531
K. Waldrop	18	59	0.305
P. Corbett	29	62	0.467
J. Mark	29	64	0.453
C. Mudd	42	71	0.591
C. Cohen	38	67	0.567
D. Kirkland	37	61	0.606

2. Use the information that you collected to organize and display the data using the most appropriate graph.

3. Explain who should receive the *Super Slugger Award* from the team and why you feel the player deserves it. *Overall best batting average would go to D. Kirkland. Students should note mention that 0.606 is the largest number of all the batting averages.*

ESSENTIAL QUESTIONS

- How do we find the average?
- Why do our quotients include decimals?

MATERIALS

- "Super Slugger Award" recording sheet

GROUPING

Individual/Partner task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

TASK:

Students will follow the directions below from the "Super Slugger Award" Recording Sheet.

Congratulations! Your baseball team has made it to the World Series Little League Baseball Championship. The team has won a record-setting twelve games in a row, with an overall record of 23 wins and only 4 losses.

Your coach needs your help. He is having a hard time figuring out which player on the team has the best batting average and has asked for your assistance.

Your job has three parts.

1. Determine the batting averages of each player on the team. (Round each batting average to the thousandths decimal place.) *Remember that in order to find a hitter's batting average you divide the number of hits (h) a player gets by the number of times they have been to bat (b); h ÷ b.*
2. Use the information that you collected to organize and display the data using the most appropriate graph.
3. Explain who should receive the *Super Slugger Award* from the team and why you feel the player deserves it.

Player	Number of Hits (*h*)	Number of Times at Bat (*b*)	Batting Average
K. Smith	25	76	
T. Rusch	33	80	
A. Patrick	51	96	
K. Waldrop	18	59	
P. Corbett	29	62	
J. Mark	29	64	
C. Mudd	42	71	
C. Cohen	38	67	
D. Kirkland	37	61	

FORMATIVE ASSESSMENT QUESTIONS

- To how many place values do you need to divide? Why?
- What is an appropriate graph for this data? Why do you think so?
- What are the disadvantages of using this type of graph? What are the advantages?
- What criteria did you use to determine the student who deserves the Super Slugger Award? Who else could deserve this award?

DIFFERENTIATION

Extension

- Have the students look in the sports page for the batting averages of a minor or major league baseball team and construct graphs to represent the data.
- Have the students see if they can find the number of hits for a player, given only the batting average and the number of times at bat.

Intervention

- Teachers can create their own line-up with fewer players to meet the needs of their students.

Intervention Table

Name_____ Date _____

Super Slugger Award Recording Sheet

Congratulations! Your baseball team has made it to the World Series Little League Baseball Championship. The team has won a record-setting twelve games in a row, with an overall record of 23 wins and only 4 losses.

Your coach needs your help. He is having a hard time figuring out which player on the team has the best batting average and has asked for your assistance.

Your job has three parts.

1. Determine the batting averages of each player on the team. (Round each batting average to the thousandths decimal place.) *Remember that in order to find a hitter's batting average you divide the number of hits (h) a player gets by the number of times they have been to bat (b); h ÷ b.*
2. Use the information that you collected to organize and display the data using the most appropriate graph.
3. Explain who should receive the *Super Slugger Award* from the team and how you would justify your choice.

Player	Number of Hits (*h*)	Number of Times at Bat (*b*)	Batting Average
K. Smith	25	76	
T. Rusch	33	80	
A. Patrick	51	96	
K. Waldrop	18	59	
P. Corbett	29	62	
J. Mark	29	64	
C. Mudd	42	71	
C. Cohen	38	67	
D. Kirkland	37	61	

Practice Task: Number Puzzles

Back to Task Table

In this task, students solve problems using multiplication and division of decimals and find the answers in a number-search puzzle. After students solve the puzzle on the "Number Puzzle" Recording sheet, they create a number puzzle of their own using problems they make and solve. Students then trade papers allowing other students to find the solutions hidden within their number-search puzzle.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

STANDARDS FOR MATHEMATICAL PRACTICE

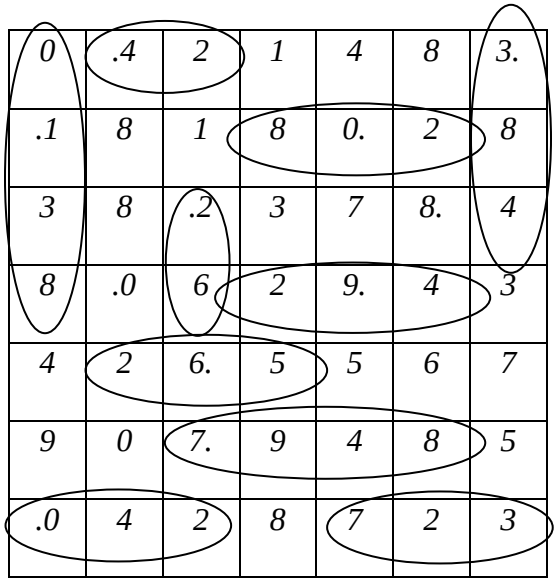
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

If your students are not familiar with this type of activity, you may want to share an example of a word search and/or number search with them prior to assigning the task.
The number puzzle and solutions are shown below.

1. 0.12 • 0.35
2. 0.2 • 1.3
3. 10.5 • 2.8
4. 0.69 • 0.2
5. 19.87 x 0.4
6. 12.288 ÷ 3.2
7. 200.5 ÷ 2.5
8. $\frac{1.26}{3}$
9. $\frac{7.23}{0.01}$
10. 15.9 ÷ 0.6

0	.4	2	1	4	8	3.
.1	8	1	8	0.	2	8
3	8	.2	3	7	8.	4
8	.0	6	2	9.	4	3
4	2	6.	5	5	6	7
9	0	7.	9	4	8	5
.0	4	2	8	7	2	3



0	.4	2	1	4	8	3.
.1	8	1	8	0.	2	8
3	8	.2	3	7	8.	4
8	.0	6	2	9.	4	3
4	2	6.	5	5	6	7
9	0	7.	9	4	8	5
.0	4	2	8	7	2	3

ESSENTIAL QUESTIONS

- How do we multiply decimals by decimals?
- How do we divide decimals by decimals?

MATERIALS

- "Number Puzzle" recording sheet
- "Create-a-Number Puzzle" recording sheet

GROUPING

Individual/Partner task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Comments:

Students may use a calculator to check their work before creating their puzzle to ensure accuracy.

TASK

Students will follow the directions below from the "Number Puzzle" and "Create-a-Number Puzzle" Recording Sheets.

Part I

Find the answers to the given problems hidden in the number search.

1. 0.12 • 0.35 =
2. 0.2 • 1.3 =
3. 10.5 • 2.8 =
4. 0.69 • 0.2 =
5. 19.87 x 0.4 =
6. 12.288 ÷ 3.2 =
7. 200.5 ÷ 2.5 =
8. $\frac{0.126}{3}$ =
9. $\frac{7.23}{0.01}$ =
10. 15.9 ÷ 0.6 =

0	.4	2	1	4	8	3.
.1	8	1	8	0.	2	8
3	8	.2	3	7	8.	4
8	.0	6	2	9.	4	3
4	2	6.	5	5	6	7
9	0	7.	9	4	8	5
.0	4	2	8	7	2	3

Encourage students to use estimation and mental math to find as many answers as they can.

Part II

Students will create their own number puzzles using the "Create-a-Puzzle" Recording Sheet. Directions are as follows:

1. Make up five multiplication problems and five division problems. All multiplication/division problems must include decimals.
2. Using the forty-nine squares on your game board, make a number search puzzle with the ten answers to your multiplication/division problems. The answers can be hidden horizontally or vertically in the grid. Fill in any unused spaces with random numbers.
3. Switch your number search puzzle with a partner and try to solve each other's number puzzles.

FORMATIVE ASSESSMENT QUESTIONS

- Can you explain your process for computing with decimals?
- What are your strategies for creating your own number puzzle?
- How can you use estimation to help you find the product? Quotient?

DIFFERENTIATION

Extension

Encourage students to try to create overlaps with their hidden solutions so that a given number appears in both a vertical and horizontal solution.

Intervention

Have students work with a partner or in small groups to solve and create the number puzzles.

Intervention Table

Name _____ Date _____

Number Puzzle

Find the answers to the given problems hidden in the number search.

1. 0.12 • 0.35 =
2. 0.2 • 1.3 =
3. 10.5 • 2.8 =
4. 0.69 • 0.2 =
5. 19.87 x 0.4 =
6. 12.288 ÷ 3.2 =
7. 200.5 ÷ 2.5 =
8. $\frac{0.126}{3}$ =
9. $\frac{7.23}{0.01}$ =
10. 15.9 ÷ 0.6 =

0	.4	2	1	4	8	3.
.1	8	1	8	0.	2	8
3	8	.2	3	7	8.	4
8	.0	6	2	9.	4	3
4	2	6.	5	5	6	7
9	0	7.	9	4	8	5
.0	4	2	8	7	2	3

Name _____ Date _____

Create - a - Number Puzzle

1. Make up five multiplication problems and five division problems. All multiplication/division problems must include decimals.
2. Using the forty-nine squares on your game board, make a number search puzzle with the ten answers to your multiplication/division problems. The answers can be hidden horizontally or vertically in the grid. Fill in any unused spaces with random numbers.
3. Switch your number search puzzle with a partner and try to solve each other's number puzzles.

Write your number problems below.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____

Create your number puzzle below.

Practice Task: What's My Rule?

Back to Task Table

In this task, students will work to discover an unstated rule that the teacher is using to find outcomes.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students may soon realize that by using 1 as their suggested number, the result will clearly show the rule. Since one is the multiplicative identity, any number times one will equal itself. For example, using the sample table above, if the rule is $n \bullet 0.25$ and students suggest the number 1, the rule is seen immediately in the result of $1 \bullet .25 = .25$. Once students start using this strategy for uncovering the rule, you may need to eliminate 1 as a number that students can suggest.

The second sample table gives the rule $n \div 0.01$. This rule gives the same result as $n \times 100$. This is a great problem to get students to rethink their understanding of the relationship between multiplication and division. If a student suggests $n \times 100$, be clear that this could be the rule, but isn't the one you had planned. Push students to determine another possibility to elicit the actual rule for the table. They should have had enough experience with dividing decimals that it will be apparent to them.

COMMON MISCONCEPTIONS

Students may not understand that multiplication and division of decimals are inverse operations. If they have never used an input/output table, they may not understand how it works. They may try a different rule with each entry in the table.

ESSENTIAL QUESTIONS

- What are some patterns that occur when multiplying and dividing by decimals?
- What strategies are effective for finding a missing factor or divisor?

MATERIALS

- Overhead projector, white/chalk board, or computer projector
- Scratch paper
- "Guess My Rule" problems (pre-determined)

GROUPING

Whole group/small group task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Comments:

When creating problems to present to the class, think about the level of difficulty of each problem and the order in which you present the problems.

Students will need time to consider number relationships after each entry, so encourage students to be patient and quiet during this time. When first doing this activity, you may want to begin with simpler rules such as adding or subtracting a number and then work toward using multiplication and division.

The teacher will want to decide on a predetermined rule for an input/output table. The students are asked to give numbers and the teacher will complete the chart using the predetermined rule. This process continues until several entries are in the table. When students think they've discovered the rule, they should keep it to themselves while other students are still trying to determine the rule. Here are some sample "rule tables."

NOTE: When recording the table for students to see, the rule is not to be displayed initially.

Rule: $x \cdot 0.25$	
In	**Out**
3	0.75
5	1.25
7	1.75
10	2.5
12	3

Rule: $n \div 0.01$	
3	300
6	600
10	1,000
15	1,500
0.1	10

TASK

- Draw an input/output table for students to see.
- Tell students you're thinking of a rule for the table and ask them to give you numbers to add to the table. You will then complete the chart for each number they give you.
- After several numbers are entered, students will likely begin to discover the rule. When students think they know the correct rule, give them a number and ask them to use the rule to give you the correct number to record in the table, without actually stating the rule. This will allow you to check their thinking and still allow other students to think independently.
- When students have discovered the rule, have a discussion about the strategies they used to determine the rule.

FORMATIVE ASSESSMENT QUESTIONS

- What strategies are you using to determine the rule?
- At what point did you know your prediction for the rule was correct?
- How did you know for sure your rule was correct?

DIFFERENTIATION

Extension

- Have students develop their own rule tables to share with partners or in small groups.

Intervention

- Begin with simpler rules and have students explain their thinking so any misconceptions can be addressed immediately.

Intervention Table

TECHNOLOGY LINK

http://www.mathplayground.com/functionmachine.html function machine activity

Practice Task: Do You See an Error?

Back to Task Table

In this task, students will look for errors in student work, find the correct solution, and write a letter to the students regarding the error and how it is corrected.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students should be able to check the placement of the decimal point quickly by estimating the product, 24 x 11 = 264. Therefore, students should find that the decimal is not placed correctly in the first example. For the second example, both the dividend and the divisor were multiplied by 100 to create an easier problem, 1468 ÷ 2. Again, the student placed the decimal incorrectly, because the estimate would be 1500 ÷ 2 = 750.

COMMON MISCONCEPTIONS

Students may line up the decimal points in a multiplication problem as in addition and subtraction. Students may fail to recognize the incorrect placement of the decimal point.

ESSENTIAL QUESTIONS

- What happens when we multiply a decimal by a decimal?
- How can we check for errors in multiplication or division of decimals?

MATERIALS

- "Do You See an Error?" recording sheet

GROUPING

Individual/partner task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Comments:

Finding and explaining errors is a powerful way for students to further develop their thinking about their own strategies for solving problems. It not only causes them to closely examine the solutions, but also may prevent them from making the same mistakes.

Using student work from your classroom as an "error analysis warm-up" on a regular basis can help students begin to find and appropriately address errors in their work with greater success. NOTE: When using actual student work, always conceal student names to protect their identity. Whenever possible, use work from a different class or a previous class with the student's identity concealed.

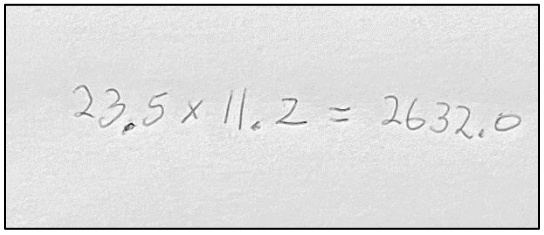
TASK:

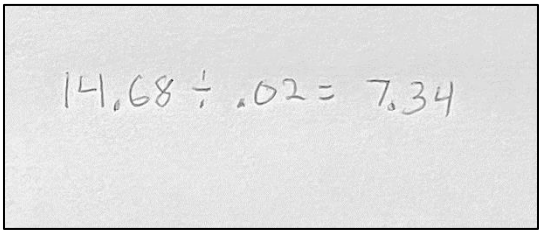
Students will follow the directions below from the "Do You See an Error?" recording sheet.

Below are samples of two students' work. Your job is to determine if they have the correct solution to the problem. If a student has the wrong solution, your next assignment is to:

1. Find the student error in each example.
2. Find the correct solution, using two different strategies.
3. Write a note to the student explaining where they went wrong and the steps they need to take to correct their mathematical thinking.

Sample student work:





FORMATIVE ASSESSMENT QUESTIONS

- How did you know that there is an error in the student work?
- How could estimation help you find an error?
- How will you explain the error and how to correct it?
- How can doing this task keep you from making the same errors?

DIFFERENTIATION

Extension

- Give students sample problems with multiple errors that involve not only decimal placement, but also computational errors. Have them explain all errors.
- http://illuminations.nctm.org/LessonDetail.aspx?id=L252 Activity 3 in the "Too Big or Too Small?" lesson gives a related exploration.

Intervention

- Revisit the "Patterns-R-Us" task of this unit. Encourage students to use the patterns found from that task to estimate answers for the students' work.

Intervention Table

Name_____ Date _____

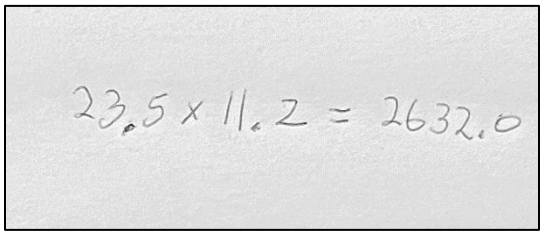
Do You See an Error? Recording Sheet

Below are samples of two students' work. Your job is to determine if they have the correct solution to the problem.

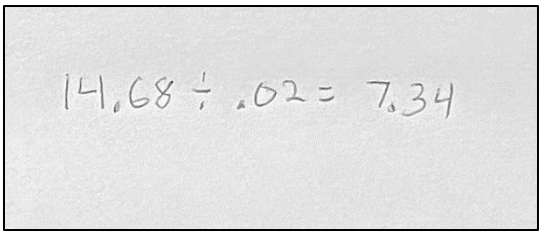
If a student has the wrong solution, your next assignment is to:

1. Find the student error in each example.
2. Find the correct solution, using two different strategies.
3. Write a note to the student explaining their error and the steps they need to take to correct their mathematical thinking.

Student #1 (Hannah)



Student #2 (Randy)



Practice Task: Road Trip

Back to Task Table

In this task, students determine fuel costs for a trip through the Southeast of the United States using a given cost of fuel and the number of miles per gallon the family car gets.

STANDARDS FOR MATHEMATICAL CONTENT

Perform operations with multi-digit whole numbers and with decimals to the hundredths.

MGSE5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

You may want to begin this task with a short class discussion about trips, costs of gasoline, and budgets for vacation.

How many miles is it from the starting point to the stopping point on each day?

TRAVELING DAYS	Starting Point	Stopping Point	Total Number of Miles
DAY ONE	Your Home Town	Birmingham, AL	will vary
DAY TWO	Birmingham, AL	Dallas, TX	641.75
DAY THREE	Dallas, TX	Memphis, TN	452.56
DAY FOUR	Memphis, TN	Chattanooga, TN	343.49
DAY FIVE	Chattanooga, TN	Atlanta, GA	118.39
DAY SIX	Atlanta, GA	Your Home Town	will vary

Make sure that students have access to maps, an atlas, or the Internet to determine the total number of miles between the cities. Also, be sure students are careful when they carry their daily miles over from one day to the next.

COMMON MISCONCEPTIONS

Students may multiply the number of miles by the cost of a gallon of gas, without taking into account how many miles the car can travel on each gallon of gas. They could use the wrong operation when figuring miles per tank of gas.

ESSENTIAL QUESTIONS

- What happens when we multiply a decimal by a decimal?
- What happens when we divide a decimal by a decimal?
- How do you know which operation to uses?

MATERIALS

- "Road Trip" recording sheet
- Maps, atlases, and/or internet access
- Markers or highlighters

GROUPING

Individual/partner task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Comments:

As a class, determine the price you will be using for gasoline. You may want to use the local price while working on the project. For consistency, it will be easier for students to use the same price for the entire trip.

If the primary goal of this task is using decimal numbers in a problem solving setting, you may consider having students use an online mapping service because they will give distances to the closest hundredth of a mile.

TASK:

Students will follow the directions below from the "Road Trip" Recording Sheet.

> You and your family want to take a road trip. However, before your family can begin this wonderful adventure, you must first determine how much your trip is going to cost. You are responsible for figuring out how much money you will need for gas.
>
> Using different resources in your classroom, determine how many miles it is to each day's planned stopping point from the starting point.
>
> Your family car can travel 21.2 miles on one gallon of gasoline.

DRIVING DAYS	Starting Point	Stopping Point
DAY ONE	Your Home Town	Birmingham, AL
DAY TWO	Birmingham, AL	Dallas, TX
DAY THREE	Dallas, TX	Memphis, TN
DAY FOUR	Memphis, TN	Chattanooga, TN
DAY FIVE	Chattanooga, TN	Atlanta, GA
DAY SIX	Atlanta, GA	Your Home Town

1. How many miles is it from each day's starting point to the stopping point?
2. How much will it cost your family to make the trip each day? What is the total cost of gasoline for the entire trip? Assume that when your family reaches the stopping point, the car is not driven until the next day.
3. Use the information you collected to organize and display the data using the most appropriate graph.
4. Show at least two of your computations and explain your thinking in words.
5. If your family car's gas tank holds 19.7 gallons of fuel, how far (in miles) can your family car travel on one tank of gasoline? Two tanks? Five tanks?
6. How long will your family spend traveling between cities if your average speed is 65 miles per hour?

FORMATIVE ASSESSMENT QUESTIONS

- Explain your strategy for determining daily mileage.
- How are you organizing your work for this task?
- How does this task help you understand the importance of budgeting for a trip?

DIFFERENTIATION

Extension

- Have students share other cities they could visit at the end of their road trip if they left Atlanta, GA before they had to fill up with gas again. Have them explain why they would/could visit those cities?
- You may want to allow students to use a calculator to check their work after they have completed the task to check for accuracy.
- Have students create a trip they would want to take and determine mileage and cost of gas.
- Have students create a budget for an entire trip that includes cost of gasoline, lodging, and an estimate for food and entertainment.

Intervention

- Give students one city at a time to compute the distance.
- Provide an organizer on which students can record the information required to find costs.

Intervention Table

Name _____ Date _____

Road Trip Recording Sheet

You and your family want to take a road trip. However, before your family can begin this wonderful adventure you must first determine how much your trip is going to cost. You are responsible for figuring out how much money you will need for gas.

Using different resources in your classroom, determine how many miles it is to each day's planned stopping point from the starting point.

Your family car can travel 21.2 miles on one gallon of gasoline.

DRIVING DAYS	Starting Point	Stopping Point
DAY ONE	Your Home Town	Birmingham, AL
DAY TWO	Birmingham, AL	Dallas, TX
DAY THREE	Dallas, TX	Memphis, TN
DAY FOUR	Memphis, TN	Chattanooga, TN
DAY FIVE	Chattanooga, TN	Atlanta, GA
DAY SIX	Atlanta, GA	Your Home Town

Be sure to answer all the questions below.

1. How many miles is it from each day's starting point to the stopping point?
2. How much will it cost your family to make the trip each day? What is the total cost of gasoline for the entire trip? Assume that when your family reaches the stopping point, the car is not driven until the next day.
3. Use the information you collected to organize and display the data using the most appropriate graph.
4. Show at least two of your computations and explain your thinking in words.
5. If your family car's gas tank holds 19.7 gallons of fuel, how far (in miles) can your family car travel on one tank of gasoline? Two tanks? Five tanks?
6. How long will your family spend traveling between cities if your average speed is 65 miles per hour?

MULTIPLYING AND DIVIDING DECIMALS

Formative Assessments Lessons (FALs)

Back to Task Table

What is a Formative Assessment Lesson (FAL)? The Formative Assessment Lesson is designed to be part of an instructional unit typically implemented approximately two-thirds of the way through the instructional unit. The results of the tasks should then be used to **inform** the instruction that will take place for the remainder of the unit. Formative Assessment Lessons are intended to support teachers in formative assessment. They both reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

What does a Formative Assessment Lesson look like in action? Videos of Georgia Teachers implementing FALs can be accessed HERE and a sample of a FAL lesson may be seen HERE

Where can I find more information on FALs? More information on types of Formative Assessment Lessons, their use, and their implementation may be found on the Math Assessment Project's guide for teachers.

Where can I find samples of FALs?
Formative Assessment Lessons can also be found at the following sites:

Mathematics Assessment Project

Kentucky Department of Education FALs

MARS Tasks by grade level

A **sample FAL** with extensive dialog and suggestions for teachers may be found HERE. This resource will help teachers understand the flow and purpose of a FAL.

Where can I find more training on the use of FALs?

All of our Georgia RESAs have had a math specialist trained to provide instruction on the use of formative assessment lessons in the classroom. The request for training should be made through the teacher's local RESA and can be referenced by asking for more information on the Mathematics Design Collaborative (MDC). Also, if done properly, these lessons should take about 120-150 minutes, 2-3 classroom periods.

Sources of Information: Vicki Mixon, Former MDC (Math Design Collaborative) trainer, http://www.reneeyates2math.com/ and from The Mathematics Assessment Project and https://education.ky.gov/curriculum/conpro/Math/Pages/ElemFormAssessLessons.aspx

3-ACT TASK: Penny Cube

Back to Task Table

Task adapted from Mike Wiernicki (http://mikewiernicki3act.wordpress.com/penny-cube/)

APPROXIMATE TIME: 1-2 class periods

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

MGSE5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non- overlapping right- rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will view the Act 1 video and discuss with their peers what they are mathematically curious about. After recording the main question to be investigated, students decide the best way in which to find a solution to the problem. If students find that the means they are using is not producing a reasonable result, they will make necessary changes to their method in order to get a more reasonable solution.

2. Reason abstractly and quantitatively. Since students haven't been formally introduced to the concept of volume, they will need to think about strategies that will help them solve the problem being investigated. Students will pay attention to the meaning of the quantities being worked with in the problem so that a reasonable solution can be determined. Students will pay particular attention to the relationships between the length, width and height of the cube and the diameter and thickness of the penny.

4. Model with mathematics. This problem will give students the chance to apply mathematical content learned about adding, subtracting, multiplying and dividing with decimals. As they work during Act 2, students will determine what quantities are important to look at and work with to determine the amount of pennies that will fill the cube. Students will use the quantities to represent the situation in video with mathematical equations.

5. Use appropriate tools strategically. Students will determine what information they have, as well as what information they need to be able to determine a solution to the problem being investigated through the video.

6. Attend to precision. As they work through the problem, students will use mathematical vocabulary and equations to show understanding of the problem. Students will effectively communicate conjectures and thinking with their peers using mathematical vocabulary and equations needed to solve the problem.

8. Look for and express regularity in repeated reasoning. In Act 4, students can take the information from the original problem and determine the weight of the cube or how many quarters would fill the cube. Extending their learning in this manner allows students to explain how the methods used in solving the penny cube problem would work in similar mathematical situations.

ESSENTIAL QUESTIONS

During Act 1, students view a cube being filled with one column of pennies. Students request necessary information to help find how many pennies it takes to fill the cube, as well as the total value of the money in the cube. It is imperative that teachers allow students to ask questions of each other and participate in discussion that will lead the students to infer that information during Act 2. The essential questions below can be shared at the beginning of Act 2 to define the emphasis of the problem solving opportunity being presented.

- How can we efficiently solve multiplication and division problems with decimals?
- What do the solutions of multiplication and division problems with decimals mean?

MATERIALS

Act 1 video "Penny Cube" http://mikewiernicki3act.wordpress.com/penny-cube/
Student recording sheet (attached)
Act 2 "Penny Cube Dimensions" and "Coin Specifications" documents (Use the link above to locate documents.)
Act 3 "Penny Cube" reveal video (Use the link above to locate the reveal video.)

GROUPING

Whole group, partners or small groups

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Students will view a video of a cube being filled with one column of pennies. Next, they will be asked to discuss what they wonder about or are curious about. These questions will be recorded on a class chart or on the board and on the student recording sheet. Students will then use mathematics to answer one of the questions generated on the chart. Students will be given information to solve the

problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them.

BACKGROUND KNOWLEDGE:

This task follows the 3-Act Math Task format originally developed by Dan Meyer. More information on this type of task may be found at http://blog.mrmeyer.com/category/3acts/. A Three-Act Task is a whole-group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the ***Guide to Three-Act Tasks*** on georgiastandards.org.

In fourth grade, students started writing a decimal equivalent for a fraction with a denominator of 10 or 100. They also compared two decimal numbers using visual models and reasoning about their size. Equipped with this knowledge, students are now making meaning of decimal quantities that are being used to add, subtract, multiply and divide in fifth grade. Visual models and drawings are still a crucial part of explaining the reasonableness of solutions that are produced as problems are solved.

COMMON MISCONCEPTIONS:

Students' prior experiences have led them to understand that when multiplying whole numbers the product is always larger than the factors. They also have experienced that dividing whole numbers always results in a quotient that is smaller than the dividend. They may experience disequilibrium when multiplying and dividing by decimal numbers because these beliefs will no longer always be true.

Task Directions:

Act 1 – Whole Group - Pose the conflict and introduce students to the scenario by showing Act I video or picture. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/**)**
"Introduce the central conflict of your story/task clearly, visually, viscerally, using as few words as possible."

- Show the Act 1 video "Penny Cube" to students. http://mikewiernicki3act.wordpress.com/penny-cube/
- Ask students what they noticed mathematically in the video, what they wonder about, and what questions they have about what they saw in the video.
- Give each student a copy of the Student Recording Sheet. Have students record their questions and curiosities in the Act 1 section that asks "What mathematical questions come to your mind?" Consider doing a think-pair-share so that students have an opportunity to talk with each other before sharing questions with the whole group. Students may need to view the video multiple times as they develop questions.
- Share and record students' questions. The teacher may need to guide students so that the questions generated are math-related.

- Share the main question that will be investigated during today's lesson. In the list below it is denoted with an asterisk. (*) Students will record the main question on their recording sheet.
- Ask students to estimate how many pennies they think it will take to fill the cube. Students will write their best estimate, then write two more estimates – one that is too low and one that is too high so that they establish a range in which the solution should occur. Students should plot their three estimates on a number line. Space is available on the recording sheet for students to record open number line with all three estimates.

Anticipated questions students may ask and wish to answer:

- How tall is the penny cube? How wide is it? How long is it?
- *How many pennies will it take to fill the cube?
- *What is the total value of the pennies that fill the cube?
- How many pennies are in a column?
- How many columns are needed to fill the cube?

**Main question(s) to be investigated*

Act 2 – **Student Exploration -** Provide additional information as students work toward solutions to their questions. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/**) "The protagonist/student overcomes obstacles, looks for resources, and develops new tools."**

- During Act 2, students review the main question from Act 1 and decide on the facts, tools, and other information needed to answer the question. The main question for this task is "How many pennies will it take to fill the cube? What is the total value of the pennies?" When students decide what they need to solve the problem, they should ask for those things. The Act 2 documents "Penny Cube Dimensions" and "Coin Specifications" contain information that students may request about the dimensions of the cube and the diameter, thickness and weight of a penny. Copies of the documents can be given to the students at their request. The documents can be found under "Act 2" using the link to the Act 1 video. It is pivotal to the problem solving process that students decide what is needed without being given the information up front.
- Students can record information that they need to solve the problem, given information, estimates and work on the student recording sheet under Act 2.
- The teacher provides guidance as needed during this phase. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin. Questioning is an effective strategy that can be used, with questions such as:
    - What is the problem you are trying to solve?
    - What do you think affects the situation?
    - Can you explain what you've done so far?
    - What strategies are you using?
    - What assumptions are you making?
    - What tools or models may help you?
    - Why is that true?
    - Does that make sense?

Additional Information for Act 2

"Penny Cube Dimensions" and "Coin Specifications" documents
http://mikewiernicki3act.wordpress.com/penny-cube/

Important note: Although students will only investigate the main question for this task, it is important for the teacher to not ignore student generated questions. Additional questions may be answered after they've found a solution to the main question, or as homework or extra projects.

Act 3 – Whole Group – Share solutions and strategies.

- Students present their solutions and strategies and compare them. Have students share the solutions they arrived at when determining how many pennies it takes to fill the cube, as well as the total value of the pennies in the cube. Record student solutions on the board, as you would in a Number Talk.
- Reveal the solution by showing the Act 3 Reveal video. The video can be found under "Act 3" using the link to the Act 1 video.
- After comparing the different solutions, have students discuss why their solutions were different from what was revealed in the Act 3 video.
- As the discussion progresses, students can discuss why different solutions were reached as they solved the problem. If students don't pick up on this idea through the course of the discussion, it is appropriate for the teacher to ask students to think about why various solutions were given.
- Have students record their results in the Act 3 section of the student recording sheet. Students can also plot their solution on the open number line on the student recording sheet.
- Lead discussion to compare these, asking questions such as:
    - How reasonable was your estimate?
    - Which strategy was most efficient?
    - Can you think of another method that might have worked?
    - What might you do differently next time?

Act 4, The Sequel - "The goals of the sequel task are to a) challenge students who finished quickly so b) I can help students who need my help. It can't feel like punishment for good work. It can't seem like drudgery. It has to entice and activate the imagination." Dan Meyer
http://blog.mrmeyer.com/2013/teaching-with-three-act-tasks-act-three-sequel/

Three different sequels are available at http://mikewiernicki3act.wordpress.com/penny-cube/.

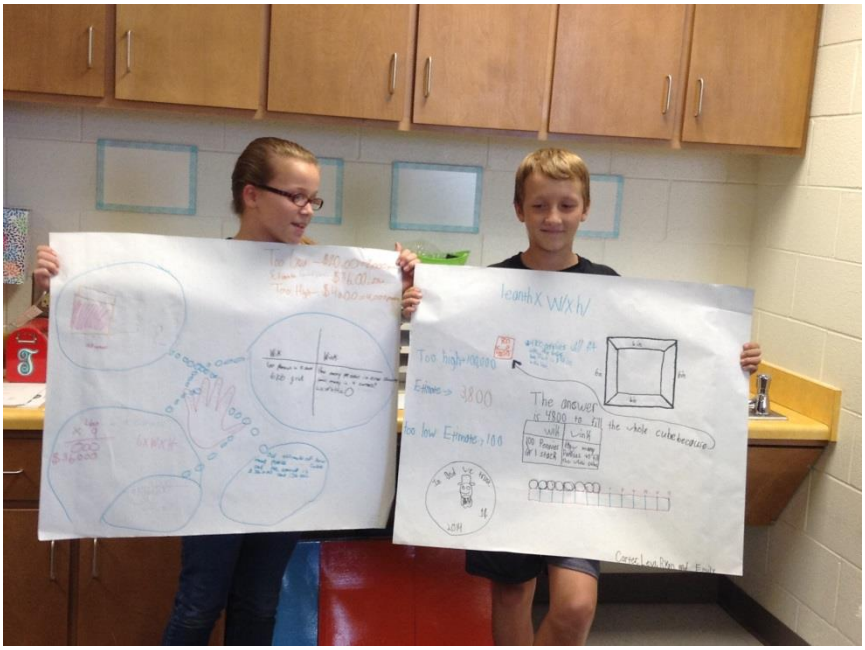
- Students can investigate the weight of the penny cube following similar strategies used to solve the original three act task.
- Students can investigate what would happen if quarters were used to fill the cube instead of pennies. The question on the website asks students to explain which cube they would rather have – a penny cube or a quarter cube. Students can continue the investigation by

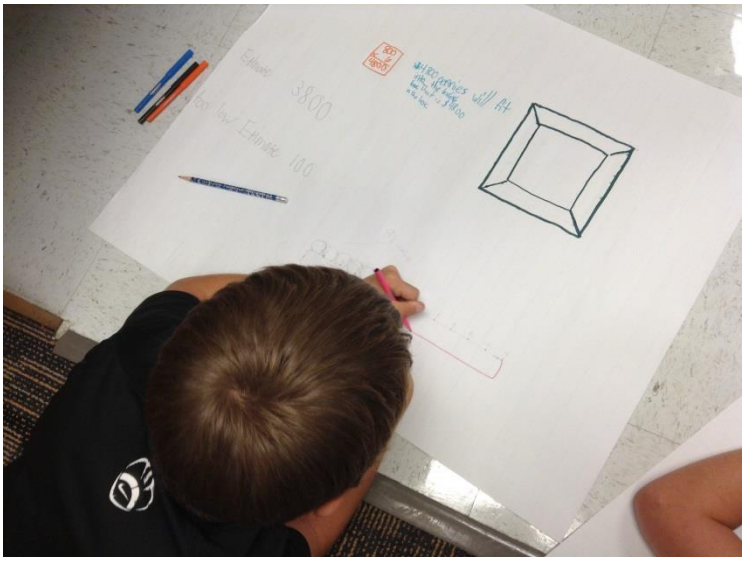
determining about how many quarters would fill the cube and their total value using the information in the "Coin Specifications" document.

- Students can investigate how many pennies it would take to fill a cube that has a volume of one cubic foot.

FORMATIVE ASSESSMENT QUESTIONS

- What models did you create?
- What organizational strategies did you use?
- How was your result different than other results in the class? Give examples that prove or disprove your results.
- What tools and resources were helpful in solving this problem?
- As you solved this problem, what connections did you make to other mathematical concepts you have learned?





DIFFERENTIATION

Extension
Students can extend their learning by investigating one or more of the problems in Act 4, The Sequel.

Intervention
Students can draw a six inch line using a ruler and lay pennies along the line to see how many pennies it take to equal the length of the cube and the width of the cube. This visual may help them see something that might be tough to visualize otherwise.

Intervention Table

TECHNOLOGY CONNECTIONS

The LearnZillion videos below show strategies for multiplying a decimal by a whole number as well as a decimal by another decimal.

- Multiply Decimals by Whole Numbers Using an Area Model http://learnzillion.com/lessons/556-multiply-decimals-by-whole-numbers-by-using-an-area-model
- Use Multiplication to Multiply Decimals by Decimals http://learnzillion.com/lessons/557-use-multiplication-to-multiply-decimals-by-decimals
- Multiplying Whole Numbers and Decimals Using an Area Model http://learnzillion.com/lessons/558-multiplying-whole-numbers-and-decimals-using-an-area-model

Three Act Task Student Recording Sheet Name _____

ACT 1

What questions come to your mind?

Main Question: _____

What is your first estimate and why?

Record an estimate that is too low and an estimate that is too high.

On an empty number line, record all three estimates made above.

ACT 2

What information would you like to know or need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimation with this information: _____

Act 2 (con't)

Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Performance Task: Teacher for a Day

Back to Task Table

This performance task represents the level of depth, rigor and complexity expected of all fifth grade students to demonstrate evidence of learning.

In this task, students represent and explain their understanding about how to multiply and divide decimals, their understanding of the place value system, and their knowledge of how decimals are used throughout life. Also, students are asked to explain and communicate why decimals are important. Finally, students are asked to teach the process of multiplying and dividing decimals. They can either create a lesson on PowerPoint or Prezi, or create visual displays with posters and markers.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

MGSE5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students should demonstrate a conceptual understanding of operations with decimals as opposed to a purely procedural knowledge. For example, students should understand that if they are multiplying tenths by tenths, the product must be expressed as hundredths. (i.e., 1/10 x 1/10 = 1/100). Students should also know to round to the nearest whole number and estimate to place the decimal, using the estimate to determine the reasonableness of an answer, rather than only knowing to count the digits after the decimal point to place the decimal point in the answer.

When multiplying or dividing decimals, students should demonstrate an understanding of place value and digit values rather than only showing the algorithmic steps one could use to divide. Students should show the operations using base ten blocks and/or partial products or quotients (rather than procedural steps), demonstrating an understanding of the operations modeled.

ESSENTIAL QUESTIONS

- What happens when we multiply and divide a decimal by a decimal?
- How do the rules of multiplying whole numbers relate to multiplying decimals?

MATERIALS

- Access to computer to create a PowerPoint, Prezi, or similar computer project; alternately, poster paper and markers to create visual displays

GROUPING

Individual/Partner/Small group task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Comments:

While this task may serve as a summative assessment, it may also be used for teaching and learning. It is important that all elements of the task be addressed throughout the learning process so that students understand what is expected of them.

TASK

Students will follow the directions below from the "Teacher for a Day" recording sheet.
Your fifth grade teacher has asked for your help. A teacher friend who teaches at Old Mill Elementary School has a fifth grade class that is having difficulty with multiplying and dividing decimals.
Your teacher knows that you are an expert at multiplying and dividing decimals. With your partner, create a presentation that addresses the following items:

1. Explanation of the place value system
2. How to multiply decimals
3. How to divide decimals
4. When good multiplication and division skills with decimals are needed and why they are important

Your presentation should include examples and visual models to help the fifth grade class at Old Mill Elementary School have a clear understanding of how to work with decimals. Remember, your presentation needs to have multiple representations using pictures, numbers, and/or words.

Be prepared to give a practice presentation to your classmates.

FORMATIVE ASSESSMENT QUESTIONS

- What models are you going to use to show examples of multiplying and dividing decimals?
- How do you know when to multiply and when to divide with decimals?

DIFFERENTIATION

Extension

- Have students include in their presentation an explanation of the relationship between multiplication and division of decimals.
- Have students include examples of when these skills are used in daily life.

Intervention

- Have students give their responses to the portions of the task verbally before asking students to put their explanations in writing.

Intervention Table

Name _____ Date _____

Teacher for a Day Task Directions

Your fifth grade teacher has asked for your help. A teacher friend who teaches at Old Mill Elementary School has a fifth grade class that is having difficulty with multiplying and dividing decimals.

Your teacher knows that you are an expert at multiplying and dividing decimals. With your partner, create a presentation that addresses the following items:

1. Explanation of the place value system
2. How to multiply decimals
3. How to divide decimals
4. When good multiplication and division skills with decimals are needed and why they are important

Your presentation should include examples and visual models to help the fifth grade class at Old Mill Elementary School have a clear understanding of how to work with decimals. Remember, your presentation needs to have multiple representations using pictures, numbers, and/or words.

Be prepared to give a practice presentation to your classmates.

Performance Task: Bargain Shopping

Back to Task Table

This culminating task represents the level of depth, rigor and complexity expected of all fifth grade students to demonstrate evidence of learning.

In this task, students will determine the best store to buy certain items on their list of needed school supplies. Students will determine how much money they will have left after making appropriate purchases. With the remaining money from their original purchases, students are asked whether or not they can purchase school supplies for their siblings.

STANDARDS FOR MATHEMATICAL CONTENT

Understand the place value system.
MGSE5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Perform operations with multi-digit whole numbers and with decimals to the hundredths.
MGSE5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

You may want to have some class discussions about what constitutes a good value before beginning this task. Students may have different answers for Question 3 based on prior experiences of what "better" means to them. Have students justify their answers.

ESSENTIAL QUESTIONS

- How do the rules of multiplying whole numbers relate to multiplying decimals?
- How can we multiply and divide decimals fluently?

MATERIALS

- "Bargain Shopping" Recording Sheet
- Sale Papers for Canton Supplies and Cherokee Discounts

GROUPING

Individual/Partner/Small group task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Comments:

While this task may serve as a summative assessment, it also may be used for teaching and learning. It is important that all elements of the task be addressed throughout the learning process so that students understand what is expected of them. Students may want to practice their estimation skills before participating in this activity.

You may want to have students bring in copies of local advertisements from the newspaper to use for a similar introductory activity. Also, you may want to collect advertisements during the first week of school and save them until you are ready to use them. This way, students could purchase supplies from the store they think offers a better buy.

TASK:

Students will follow the directions below from the "Bargain Shopping" Recording Sheet.

It is time to go shopping for school supplies for next school year. You have ads from two local stores indicating their prices (which include sales tax) for different school supplies. Your mother has given you $45.50 to spend, and wants you to find the best prices on things you will need.

School Supplies Needed

Eight folders
One calculator
Two packs of regular lined notebook paper (pack of 500 sheets)
Three rulers
Three bottles of glue
Thirty-three pencils
Ten pens
Five spiral notebooks
Two backpacks

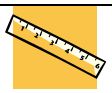
1. Determine the cheapest price for each item on your list of school supplies needed. From which store would you buy each school supply? How much money will you have to spend? How much of the $45.50 will you have left over?

2. With the remaining money, you decide to buy each of your three siblings the same school supply item. What item would you purchase?

3. Is one store better than the other? Why or why not?

4. If you could choose only one store, which would it be? Explain your reasoning by writing an overall summary of your findings.

See sale papers from two different stores shown below.

CHEROKEE DISCOUNTS		
Calculator **$6.99** 	**Lined Paper** ***500 sheets*** **93¢** 	**Folder** **15¢ each** 
Erasers – large **52¢ each** 	**Clipboard** **$2.15** 	**Book Covers** **4 for $2.00** 
Glue **2 oz. bottle** **95¢** 	**Backpack** **$8.97** 	**Pencils** **11¢each** **10 pack -- $1.01** 
Scissors **$1.75** 	**Pens** **29¢ each** **$3.56 – 15 pack** 	**Ruler** **$1.45 each** 
	Spiral Notebooks **99¢ each** 	

CANTON SUPPLIES		
Calculator **$12.98 – regular price** 	**Lined Paper** ***500***  ***sheets*** ***Regular Price*** **- 47¢**	**Folders** **11¢ each** 
Erasers – large **60¢ each** 	**Clipboard** **$1.99** 	**Book Covers** **$1.50** 
Glue **2 oz. bottle** **39¢** 	**Backpack** **$8.37** 	**Pencils** **69¢ each** **10 pack - - $2.02** 
Scissors **$2.13** 	**Pens** **35¢ each** **$4.08 – 15 pack** 	**Ruler** **$1.14 each** 
	Spiral Notebooks **75¢ each** 	

FORMATIVE ASSESSMENT QUESTIONS

- What is your strategy for completing this task?

- What factors influence your decision about which store has better values?
- How would you determine the store in which you would make your purchases if you had to choose only one store?

DIFFERENTIATION

Extension

- Have students research prices on websites such as Staples, Wal-Mart, Office Max, Office Depot, etc. and compare total costs with one another.
- Have students plan ways to earn the money necessary for school supplies and other back to school items such as clothes, shoes, computer, etc.
- Have students plan ways to earn money to provide school supplies for students in the community who are unable to purchase back-to-school supplies.

Intervention

- Have students make an organized list of the items and the prices at both stores to help them make comparisons.

Intervention Table

Name _____ Date _____

Bargain Shopping Recording Sheet

It is time to go shopping for school supplies for next school year. You have ads from two local stores indicating their prices (which include sales tax) for different school supplies. Your mother has given you $45.50 to spend, and wants you to find the best prices on the things you will need.

School Supplies Needed

Eight folders
One calculator
Two packs of regular lined notebook paper (pack of 500 sheets)
Three rulers
Three bottles of glue
Thirty-three pencils
Ten pens
Five spiral notebooks
Two backpacks

1. Determine the cheapest price for each item on your list of school supplies needed. From which store would you buy each school supply? How much money will you have to spend? How much of the $45.50 will you have left over?

2. With the remaining money, you decide to buy each of your three siblings the same school supply item. What item would you purchase?

3. Is one store a better overall value than the other? Why or why not?

4. If you could choose only one store, which would it be? Explain your reasoning by writing an overall summary of your findings.

CHEROKEE DISCOUNTS		
Calculator **$6.99** 	**Lined Paper** ***500 sheets*** **93¢** 	**Folder** **15¢ each** 
Erasers – large **52¢ each** 	**Clipboard** **$2.15** 	**Book Covers** **4 for $2.00** 
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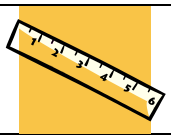
Scissors **$1.75** 	**Pens** **29¢ each** **$3.56 – 15 pack** 	**Ruler** **$1.45 each** 
	Spiral Notebooks **99¢ each** 	

CANTON SUPPLIES		
Calculator **$12.98 – regular price** 	**Lined Paper** ***500 sheets*** ***Regular*** ***Price* - 47¢** 	**Folders** **11¢ each** 
Erasers – large **60¢ each** 	**Clipboard** **$1.99** 	**Book Covers** **$1.50** 
Glue **2 oz. bottle** **39¢** 	**Backpack** **$8.37** 	**Pencils** **69¢ each** **10 pack -- $2.02** 
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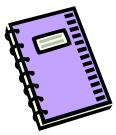
	Spiral Notebooks **75¢ each** 	

3-ACT TASK: Hanging by a Hair

Back to Task Table

Task created by Graham Fletcher (http://gfletchy.wordpress.com/3-act-lessons/)

APPROXIMATE TIME: 1 - 2 class periods

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

MGSE5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will make estimates about how many strands of hair it would take to support his/her body weight. This will encourage students to think about reasonable and unreasonable solutions. As they solve the problems, students will make decisions about what information they need as they try various ways to find a solution.

2. Reason abstractly and quantitatively. After collecting data about the number of pennies a strand of hair can hold without breaking, students will use the data to determine how many strands of hair are needed to support his/her body weight. This will require students to think about what the quantities mean as they perform operations to find a solution. For example, when students find the number of pennies a strand of hair can hold without breaking, they multiply that number by 0.08 ounces. The product of that calculation means that students have determined how many ounces one strand of hair can hold.

3. Construct viable arguments and critique the reasoning of others. After making estimates in Act 1, students will test their conjectures by testing how many pennies a strand of hair can hold in Act 2. Students will ask questions of peers and teachers as they collaborate to determine the information they need to request in order to find how many strands of hair will support his/her body weight.

6. Attend to precision. The data being gathered during Act 2 will contain decimal numbers that are very precise. Students may express frustration when working with such precise data. In addition, students will also include units of measurement used as data was gathered.

8. Look for and express regularity in repeated reasoning. In Act 3, students will determine a rule, or generalization that will allow any person to find how many strands of hair can support his/her body weight.

ESSENTIAL QUESTIONS

During Act 1, students view a video of a man that set a new record for the time spent hanging by his hair. Students will be asked to make high and low estimates about how many strands of hair it will take to support his/her body weight. In Act 2A, students will think and ask questions about the information needed to determine the solution. Teachers will have students test the strength of one strand of hair to compute how many strands it would take to support the child's body weight. The essential questions below can be shared at the beginning of Act 2 to define the emphasis of the problem solving opportunity being presented.

- How can we check for errors in multiplication and division of decimals?
- What strategies are effective for solving problems that involve multiplying and dividing decimals?

MATERIALS

Act 1 video "Hanging by a Hair" http://gfletchy3act.wordpress.com/hanging-by-a-hair/
A strand of hair
Books
Pencil
Tape
Supply of pennies
Table

GROUPING

Partners or Small Groups

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this task, students will watch a portion of a news video showing a man who set a new world record for the time spent hanging by his hair. Next, the teacher will pose the question "How many stands of hair are needed to support your body weight?" to the class. Students will then determine what information would be most useful to know as they begin to solve the problem posed. In Act 2A, students will be given instructions on how to test the strength of a single strand of hair. After gathering the data, students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them. The data will allow students to multiply and divide decimals, as well as relate multiplication and division of decimals to multiplication and division of whole numbers as they calculate how many strands of hair are needed to support their own body weight.

BACKGROUND KNOWLEDGE:

This task follows the 3-Act Math Task format originally developed by Dan Meyer. More information on this type of task may be found at http://blog.mrmeyer.com/category/3acts/. A Three-Act Task is a

whole-group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the ***Guide to Three-Act Tasks*** on georgiastandards.org.

In fourth grade, students learned how to multiply and divide larger numbers. In Unit 2 of the fifth grade frameworks, students are mastering the craft of multiplying and dividing whole numbers. During Unit 3, students relate whole number multiplication and division to decimal multiplication and division. Students also use their knowledge of powers of ten to make friendlier problems that will yield products and quotients that are similar or possibly even the same. Instead of learning a rote set of procedures to perform the multiplication or division algorithm, students are making meaning of the quantities they are working with by estimating to find out about how much the product or quotient will be of any given problem involving multiplying or dividing decimals.

COMMON MISCONCEPTIONS:

In second and third grade, students are introduced to whole number multiplication and division. Through their experiences, many children generalize that when quantities are multiplied that the product is always larger than either factor. Students also generalize that when a quantity is divided into equal groups that the quotient will always be less than the dividend. During this unit, students will experience disequilibrium as those ideas are challenged.

As students progress through their learning, they should think critically about what happens when a decimal less than one is multiplied by a whole number. For example, when multiplying 0.8 x 6, students should think that 0.8 is close to one and 1 x 6 = 6, so the product will be close to 6. However, 1 is more than 0.8, so the actual product will be less than six because 0.8 is less than 1. Since 8 x 6 = 48, the product can be divided by 10^1 to get 4.8, which is close to the estimate six.

Students should also think critically about division of decimals. When solving the problem 1 ÷ 0.2, students can use powers of ten to create a similar problem that will yield the same quotient. If both the dividend and the divisor are multiplied by 10^1, the problem 10 ÷ 2 is created, which is five. 10 ÷ 2 is equal to 1 ÷ 0.2, or 5. Students can also use repeated subtraction to find the quotient.

1 – 0.2 = 0.8
0.8 – 0.2 = 0.6
0.6 - 0.2 = 0.4
0.4 – 0.2 = 0.2
0.2 - 0.2 = 0

0.2 was repeatedly subtracted five times to get to zero. Therefore 1 ÷ 0.2 = 5.

If students are not making meaning of the quantities or relating their background knowledge of multiplying and dividing with whole numbers to performing the operation with decimal numbers, then they will most likely place the decimal point in an incorrect location when calculating products and quotients.

Task Directions:

Act 1 – Whole Group - Pose the conflict and introduce students to the scenario by showing Act I video or picture. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/**)**
"Introduce the central conflict of your story/task clearly, visually, viscerally, using as few words as possible."

- Show the Act 1 video "Hanging By a Hair" to students. http://gfletchy3act.wordpress.com/hanging-by-a-hair/.
- Present the following question to the students. "How many strands of hair would you need to support your body weight?"
- Ask students to estimate answers to their questions (think-pair-share). Students will write their best estimate, then write two more estimates – one that is too low and one that is too high so that they establish a range in which the solution should occur. Students should plot their three estimates on a number line.

Act 2 – **Student Exploration** - Provide additional information as students work toward solutions to their questions. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/**)**
"The protagonist/student overcomes obstacles, looks for resources, and develops new tools."

- During Act 2A, ask students to brainstorm what information they need to find a solution to the problem presented in Act 1. Consider doing a think-pair-share so that students have an opportunity to talk with each other before sharing ideas with the whole group. Students may need to watch the video again.
- Share and record students' ideas and questions. The teacher may need to guide students so that the questions generated are math-related.

Anticipated questions students may ask or information students may request:

- What was the man's weight?
- How tall was the man?
- How long was his hair?
- *How strong is one strand of hair?

**Main question(s) to be investigated*

- Students record the main question from Act 2A. Explain to students how to test the strength of a single strand of hair. Scroll down to Act 2A http://gfletchy3act.wordpress.com/hanging-by-a-hair/ and show students the steps. Students will take a single strand of hair and tape it to the end of a pencil. The pencil will then be inserted into a stack of books so that the hair is dangling from the pencil. Students will tape pennies to the strand of hair to see how much weight it can support. When the hair breaks, the test is complete.

- After the data is collected, students must decide on the facts, tools, and other information needed to apply the information learned and answer the question. The main question for this

task is "How many strands of hair would you need to support your body weight?" When students decide what they need to solve the problem, they should ask for those things. It is pivotal to the problem solving process that students decide what is needed without being given the information up front. Information that might be requested to solve this problem is the weight of the penny in ounces or grams and the conversion between ounces and pounds (16 ounces = 1 pound) or that 456 grams is about a pound.

- The teacher provides guidance as needed during this phase. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin. Questioning is an effective strategy that can be used, with questions such as:
    - What is the problem you are trying to solve?
    - What do you think affects the situation?
    - Can you explain what you've done so far?
    - What strategies are you using?
    - What assumptions are you making?
    - What tools or models may help you?
    - Why is that true?
    - Does that make sense?

Additional Information for Act 2

Act 2B information that may be requested by students http://gfletchy3act.wordpress.com/hanging-by-a-hair/
(Scroll down to Act 2B.)

Important note: Although students will only investigate the main question(s) for this task, it is important for the teacher to not ignore student generated questions. Additional questions may be answered after they've found a solution to the main question, or as homework or extra projects.

Act 3 – Whole Group – Share solutions and strategies.

- Students present their solutions and strategies and compare them to their estimates made during Act 1.
- Lead discussion to compare these, asking questions such as:
    - How reasonable was your estimate?
    - Which strategy was most efficient?
    - Can you think of another method that might have worked?
    - What might you do differently next time?

Act 4, The Sequel - "The goals of the sequel task are to a) challenge students who finished quickly so b) I can help students who need my help. It can't feel like punishment for good work. It can't seem like drudgery. It has to entice and activate the imagination." Dan Meyer
http://blog.mrmeyer.com/2013/teaching-with-three-act-tasks-act-three-sequel/

Students can determine a rule that would determine the how many strands of hair a person with any weight (x) would need. Students can write the generalization as an expression.

FORMATIVE ASSESSMENT QUESTIONS

- What models did you create?
- What organizational strategies did you use?
- How did you apply your knowledge of multiplying dividing decimals to solve this problem?
- How did your estimates help you find a solution?

DIFFERENTIATION

Extension

Students can collect the data from their peers and compare and contrast their peers' data to their own data. Students can complete the following tasks:

- Order the data from least to greatest or greatest to least.
- What is the range of the data? What is the difference between the most number of strands needed and the least number of strands needed? Explain what the range tells about the data collected.
- Are there pieces of data that appear more than once? If so, what are they?
- If you had a strand of hair from the man in the video, how many pennies could it hold?

Intervention

If a student needs assistance, allow them to round the numbers to the nearest whole number and perform the calculations. After using whole numbers to calculate an estimate, explain to students that a more precise solution is needed and work with the small group to show the parallels between whole number multiplication and division and decimal multiplication and division.

Intervention Table

TECHNOLOGY CONNECTIONS

- http://learnzillion.com/lessons/558-multiplying-whole-numbers-and-decimals-using-an-area-model
  Multiplying Whole Numbers and Decimals Using an Area model: The LearnZillion video will walk students through how to draw a model when multiplying a decimal with a tenths place by another name with the tenths place.

CULMINATING TASK: Field Trip

Back to Task Table

Adapted from Bowland Maths

This culminating task represents the level of depth, rigor, and complexity expected of all fifth grade students to demonstrate evidence of learning. Students will plan a field trip and analyze how to determine the most cost effective trip based on the given data. In order to do this, students will need to use reasoning to: determine which trip should be taken, calculate costs for one trip, and use repeated reasoning to explain how to compare costs on other trips.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE.5.NBT.4 Use place value understanding to round decimals up to the hundredths place.

MGSE5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used**.**

STANDARDS FOR MATHEMATICAL PRACTICE

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

BACKGROUND KNOWLEDGE

Students should have had many opportunities to identify, read, and illustrate decimal numbers. Students' work will represent some uniqueness based on how they approach the problem. Students will need to reflect on their own understanding of place value and decimal operations.

ESSENTIAL QUESTIONS

- What factors should be taken into consideration when deciding where to go on a field trip?
- How can decimal computation help people make decisions?

MATERIALS

- "Field Trip" task sheet.

GROUPING

Individual/Partner Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Comments

Students should be given opportunities to revise their work based on teacher feedback, peer feedback, and metacognition which includes self-assessment and reflection.

Suggestions for Classroom Use

While this task may serve as a summative assessment, it also may be used for formative assessment and/or as a project. It is important that all elements of the task be addressed throughout the learning process so that students understand what is expected of them.

Task Directions

Students will follow the directions below from the "Field Trip" students recording sheet.

Mr. Richards, a teacher at Crosstown Elementary School, plans to take 30 students on a school trip. The class voted on which place to visit.

The following questions can be asked to make sure the students understand the situation:

- Which place has the cheapest entrance fee?
- Which place is nearest?
- What are Lucy's first and second choices?
- How much do teachers have to pay?
- How much will the school pay towards the total cost of the trip?

FORMATIVE ASSESSMENT QUESTIONS

- How would you find the total cost for going on the _____ field trip?
- How does multiplication of decimals help you find the total cost of the field trip?
- How could you do this task without using multiplication?
- What stays the same no matter which field trip the class goes on?
- How is going on the _____ field trip different from the _____ field trip?
- How could you find out if one of the trips would be cheaper?
- Taking the total cost into account, which trip will be the cheapest/most expensive?
- How could an expression help you find the total cost of the different trips?

DIFFERENTIATION

Extension

- How much would it cost to go on all 3 trips?

- If the bus company charges $0.40 more per mile, how will this affect the total cost? How would this change affect your decision about which field trip is the best?
- Devise a fund raising plan to ensure that all students could go on all 3 trips.

Intervention

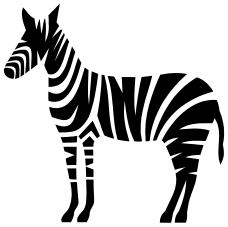
- Students who have difficulty organizing their work may benefit by working with a partner whose strength is organization, or may need a teacher's assistance in organizing their work before/during the task.

Intervention Table

Name _____ Date _____

Field Trip

Mr. Richards, a teacher at Crosstown Elementary School, plans to take 30 students on a school trip. Here are the places they could visit.

Big City Zoo	**Museum Of Aviation**	**Space Science Show**
		
36 miles from Crosstown Elementary School Entrance Fee: $2.50 per person	30 miles from Crosstown Elementary School Entrance Fee: $6.75 per person	10 miles from Crosstown Elementary School Entrance Fee: $11.25 per person

The class voted on which place to visit. Here are the results.

	Zoo	Museum of Aviation	Space Science Show
First Choice	12 Students	8 Students	10 Students
Second Choice	5 Students	14 Students	11 Students

1. Taking first and second choices into account, where do you think Mr. Richards should take his class? Explain how you decided.

Here are some more facts about the trip.

- **The bus company charges $6.29 per mile.**
- **The school will pay the first $200 of the trip.**
- **Teachers go free.**
- **Each student pays the same amount.**

2. How much will each student need to pay to go on the trip you have chosen? Explain your thinking.

3. Explain to Mr. Richards how he could figure out which trip would be the cheapest.