Georgia Standards of Excellence

Mathematics

Glossary: K-12

Georgia Department of Education

Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

Glossary

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (– 3/4) = (– 3/4) + 3/4 = 0.

Additive vs multiplicative comparison. Additive comparisons focus on the difference between two quantities (e.g., Deb has 3 apples and Karen has 5 apples. How many more apples does Karen have?). A simple way to remember this is, "How many more?" Multiplicative comparisons focus on comparing two quantities by showing that one quantity is a specified number of times larger or smaller than the other (e.g., Deb ran 3 miles. Karen ran 5 times as many miles as Deb. How many miles did Karen run?). A simple way to remember this is "How many times as much?" or "How many times as many?"

Algorithm. *See:* Computation Algorithm.

Area model. A model for multiplication and/or division problems, in which the length and width of a rectangle represents the factors, or quotient and dividend. *For further examples, see:* http://www.learner.org/courses/learningmath/number/session4/part_b/multiplication.html and http://bit.ly/1J6Xey2

Assess reasonableness. Use of strategies (e.g. estimation) to ensure an answer makes sense or is reasonable in context of a given problem.

Assessment. The evaluation or estimation of the ability of someone. *For examples, see:* http://www.edutopia.org/assessment and http://www.gadoe.org/Curriculum-Instruction-and-Assessment/Assessment/Pages/GeorgiaFIP.aspx

Associative property of addition. *See Table 3 in this Glossary.*

Associative property of multiplication. *See Table 3 in this Glossary.*

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

Cardinality. The understanding that when you count items, the number word applied to the *last object counted* represents the total amount. *For further explanation, see:* http://bit.ly/1HOOnCg

Commutative property. *See Table 3 in this Glossary.*

Complex fraction. A fraction *A*/*B* where *A* and/or *B* are fractions (*B* nonzero).

Composition/Decomposition of number. A method of expressing a number in terms of its simpler components or of combining components of numbers in order to simplify computation. *For further explanation and examples, see:* http://mathcoachscorner.blogspot.com/2012/07/composing-and-decomposing-numbers.html

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[1] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Conceptual understanding. Understanding which ideas are critical to a mathematical concept, allowing for the use of ideas strategically and flexibly to solve problems (especially non-routine problems), thereby avoiding common misunderstandings.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Digit. A single symbol used to write numerals. (example: the digits 1 and 2 may be used to make the numeral 12) *See:* Number and Numeral.

Dot plot. *See:* line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

e.g. vs i.e. e.g means "for example" while i.e. means "that is" or "in other words". When e.g. appears in a standard followed by elaboration, the elaborations are examples, not what must be literally taught. When i.e. appears in a standard followed by elaboration, the elaboration serves as further clarification of the standard.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median *M*, the first quartile is the median of the data values less than *M*. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[2] *See also:* median, third quartile, interquartile range.

Fluency. Fluency includes three ideas: efficiency, accuracy, and flexibility:
Efficiency implies that the student does not get bogged down in too many steps or lose track of the logic of the strategy. An efficient strategy is one that the student can carry out easily, keeping track of subproblems and making use of intermediate results to solve the problem.
Accuracy depends on several aspects of the problem-solving process, among them careful recording, knowledge of number facts and other important number relationships, and double-checking results.
Flexibility requires the knowledge of more than one approach to solving a particular kind of problem, such as two-digit multiplication. Students need to be flexible in order to choose an appropriate strategy for the problem at hand, and also to use one method to solve a problem and another method to double-check the results. Fluency demands more of students than does memorization of a single procedure. *For further explanation, see:* http://bit.ly/1AcIwjV

[2] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

Fraction. A number expressible in the form *a*/*b* where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also: rational number*.

Hierarchy. An arrangement or classification of things according to relative importance or inclusiveness. *For further information on hierarchy of quadrilaterals, see:* http://bit.ly/1Fvrbc3

Identity property of 0. *See Table 3 in this Glossary*.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form *a* or –*a* for some whole number *a*.

Interpreting multiplication expressions. When interpreting multiplication expressions, the factors may be read as a groups of b, or b groups of a. For example, 3x6 means how many are in 3 groups of 6 things each: three sixes, or 3 x6 means how many are 3 things taken 6 times (6 groups of 3 things each): six threes. The context of the expression will determine which interpretation is required. *For further explanation, see:* http://bit.ly/1Kr1wkI

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also*: first quartile, third quartile.

Interval. For a ≤ b, the closed interval [a,b] is the set of elements x satisfying a ≤ x ≤ b (i.e. a ≤ x and x ≤ b). It contains at least the elements a and b. Using the corresponding strict relation "<", the open interval (a,b) is the set of elements x satisfying a < x < b (i.e. a < x and x < b). http://en.wikipedia.org/wiki/Interval_%28mathematics%29

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mathematical proficiency. Conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, productive disposition (National Research Council, 2001)

Mathematize. http://lmgtfy.com/?q=mathematize

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[4] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Memorize. http://bridges1.mathlearningcenter.org/resources/blog/201501/memorization-versus-memory

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Mindset. http://mindsetonline.com/whatisit/themindsets/index.html

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[3] Adapted from Wisconsin Department of Public Instruction, *op. cit*.

[4] To be more precise, this defines the *arithmetic mean*.

Modeling with mathematics. Moving explicitly between real-world scenarios and mathematical representations of these scenarios. http://bit.ly/1odeWYW

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

Net. A plane diagram of a polyhedron.

Number. a count of a quantity of units (example: ************, the quantity of * counted which can be represented with the numeral, 12) *See also:* digit and numeral.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Numeracy. The ability to use mathematical ideas efficiently to make sense of the world. http://www.curriculumsupport.education.nsw.gov.au/primary/mathematics/numeracy/what/index.htm; http://www.nzmaths.co.nz/numeracy-projects; http://www.numeracycontinuum.com/index.php/continuum-chart

Numeral. A symbol or name used alone or in a group to represent a number (example: 12 is a symbol used to represent a quantity or number of objects. 12 is composed of two digits: 1 and 2) *See also:* digit and number.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Perseverance. Continued effort to do or achieve something despite difficulties, failure, or opposition.

Place, value, and place value. Place- the name of the place a digit is in within a number; Value- the value a digit holds within a number based on its place; Place Value- the relationship between a digit's place and the value it represents in that place. (*Krystal Shaw*)

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Procedure- *See:* Computational algorithm.

Properties of operations. *See Table 3 in this Glossary.*

Properties of equality. *See Table 4 in this Glossary.*

Properties of inequality. *See Table 5 in this Glossary.*

Properties of operations. *See Table 3 in this Glossary.*

Proportional relationship. Proportions are the comparison of two equal ratios. Therefore, proportional relationships are relationships between two equal ratios. For example, oranges are sold in a bag of 5 for $2. The ratio of oranges to their cost is 5:2 or 5/2. I can find the cost of 20 oranges by setting up a proportion. 5/2 = 20/x. (Intermath)

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form *a*/*b* or – *a*/*b* for some fraction *a*/*b*. The rational numbers include the integers.

Reason quantitatively. http://bit.ly/1scmzhj

Rectangular array. A set of objects arranged into rows and columns. Each row must contain the same number of objects as other rows, and each column must contain the same number of objects as other columns. http://nrich.maths.org/2466; http://nrich.maths.org/2469

Rectilinear figure. A polygon all angles of which are right angles.

Repeating decimal. A number whose decimal representation eventually becomes periodic (i.e., the same sequence of digits repeats indefinitely. The repeating portion of a decimal expansion is conventionally denoted with a vinculum ($^{-}$) For example, 1/3 +0.3333333…= $0.\overline{3}$. *See also:* terminating decimal.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Rigor. Mathematical rigor is the depth of interconnecting concepts and the breadth of supporting skills students are expected to know and understand. http://rightquestion.org/teaching-strategy-elementary-math ; http://bit.ly/1bR0Bi8

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Standard algorithm. An algorithm based on place-value decomposition. (Note: The standard algorithm is not exclusive to the US Traditional algorithm.) *See also*: composition/decomposition of number and http://bit.ly/1DTZYKs; http://www.mathedleadership.org/docs/resources/journals/NCSMJournal_ST_Algorithms_Fuson_Beckmann.pdf

Subitize. Instantly see how many objects are in a group without counting. http://www.nwaea.k12.ia.us/documents/filelibrary/pdf/connections/Subitizing_B2518BBFE8FCF.pdf

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median *M*, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

Tile/tiling. Covering a space (area) by fitting individual tiles (color tiles, pattern blocks, or other plane figures) together with no gaps or overlaps. http://mathforum.org/sum95/suzanne/whattile.html

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

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[5] Adapted from Wisconsin Department of Public Instruction, *op. cit.*

Trapezoid. A quadrilateral with at least one pair of parallel sides. (*Georgia uses the inclusive definition of trapezoid.*) https://www.illustrativemathematics.org/content-standards/tasks/1505

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Unit fraction. A fraction where the numerator is one and the denominator is a positive integer.

Variability. How spread out or closely clustered a set of data is.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, ….

Table 1. Common addition and subtraction situations.[6]

	Result Unknown	**Change Unknown**	**Start Unknown**
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 – 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 – ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? – 2 = 3$
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**[7]
Put Together/ Take Apart[8]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 – 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[9]	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 – 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 – 3 = ?$, $? + 3 = 5$

[6] Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

[7] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[8] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[9] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2. Common multiplication and division situations.[10]

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	3 x 6 = ?	3 x ? = 18 , and 18 ÷ 3 = ?	? x 6 = 18 , and 18 ÷ 6 = ?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays[11], Area[12]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	a x b = ?	a x ? = p , and p ÷ a = ?	? x b = p , and p ÷ b = ?

[10] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.
[11] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[12] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3. The properties of operations. Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$.
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$.
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality. Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality. Here a, b and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.