Georgia Standards of Excellence Curriculum Frameworks

Mathematics

GSE Second Grade

Unit 6: Developing Multiplication

Georgia Department of Education

Richard Woods, Georgia's School Superintendent

"Educating Georgia's Future"

Unit 6: Developing Multiplication

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IF YOU HAVE NOT READ THE SECOND GRADE CURRICULUM OVERVIEW IN ITS ENTIRETY PRIOR TO USE OF THIS UNIT, PLEASE **STOP** AND CLICK HERE: https://www.georgiastandards.org/Georgia-Standards/Frameworks/2nd-Math-Grade-Level-Overview.pdf
Return to the use of this unit once you've completed reading the Curriculum Overview.
Thank you!

OVERVIEW

In this unit students will:

- Understand and model multiplication as repeated addition and as rectangular arrays.
- Determine if a number is odd or even (within twenty).
- Create and interpret picture graphs and bar graphs.

The standard M.2.OA.3 calls for students to apply their work with doubles addition facts to the concept of odd or even numbers. Van de Walle states, "All too often students are simply told that the even numbers are those that end in 0, 2, 4, 6 or 8 and odd numbers are those that end in 1, 3, 5, 7 or 9. While of course this is true, it is only an attribute of even and odd numbers rather than a definition that explains what even or not even really means" (Teaching Student Centered Mathematics, page 291).

Students should have ample experiences exploring the concept that if a number can be decomposed (broken apart) into two equal addends (e.g., 10 = 5 +5), then that number (10 in this case) is an even number. Students should explore this concept with concrete objects (e.g., counters, place value cubes, etc.) before moving towards pictorial representations such as circles or arrays.

The standard calls for students to use rectangular arrays to work with repeated addition. This is a building block for multiplication in 3rd Grade. Students should explore this concept with concrete objects (e.g., counters, bears, square tiles, etc.) as well as pictorial representations on grid paper or other drawings. Based on the commutative property of addition, students can add either the rows or the columns and still arrive at the same solution.

The standard calls for students to work with categorical data by organizing, representing and interpreting data using four categories. Students should have experiences with interpreting and gaining meaning from picture and bar graphs.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as counting, time, money, positional words, patterns, tallying, and graphing should be addressed on an ongoing basis through the use of calendar, centers, and games.

To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under "Evidence of Learning" be reviewed early in the planning process. A variety of resources can be utilized to supplement this unit. The tasks in this unit illustrate the types of learning activities that should be utilized from a variety of sources.

NUMBER TALKS

Between 5 and 15 minutes each day should be dedicated to "*Number Talks*" in order to build students' mental math capabilities and reasoning skills. Sherry Parrish's book *Number Talks* provides examples of K-5 number talks. The following video clip from Math Solutions is an excellent example of a number talk in action. https://www.teachingchannel.org/video/number-talk-math-lesson-2nd-grade

During the Number Talk, the teacher is not the definitive authority. The teacher is the facilitator and is listening for and building on the students' natural mathematical thinking. The teacher writes a problem horizontally on the board in whole group or a small setting. The students mentally solve the problem and share with the whole group **how** they derived the answer. They must justify and defend their reasoning. The teacher simply records the students' thinking and poses extended questions to draw out deeper understanding for all.

The effectiveness of Numbers Talks depends on the routines and environment that is established by the teacher. Students must be given time to think quietly without pressure from their peers. To develop this, the teacher should establish a signal, other than a raised hand, of some sort to identify that one has a strategy to share. One way to do this is to place a finger on their chest indicating that they have one strategy to share. If they have two strategies to share, they place out two fingers on their chest and so on.

Number Talk problem possible student responses:

	Possible Strategy #1	**Possible Strategy #2**
29 + 8	29 can become 30 and take 1 from 8 reducing it to 7.	9 and 8 becomes 17 17 plus 20
54 + 86	50 + 80 + 10 =	Add 6 to 54 to get 60. Then 60 + 80 = 140

Number talks often have a focus strategy such as "making tens" or "compensation." Providing students with a string of related problems, allows students to apply a strategy from a previous problem to subsequent problems. Some units lend themselves well to certain Number Talk topics. For example, the place value unit may coordinate well with the Number Talk strategy of "making ten. "For additional information on Number Talks please see the Grade Level Overview.

STANDARDS FOR MATHEMATICAL PRACTICE

This section provides examples of learning experiences for this unit that support the development of the proficiencies described in the Standards for Mathematical Practice. The statements provided offer a few examples of connections between the Standards for Mathematical Practice and the Content Standards of this unit. The list is not exhaustive and will hopefully prompt further reflection and discussion.

1. Make sense of problems and persevere in solving them. *Students develop an understanding of odd/even numbers, arrays, and repeated addition and use that knowledge to solve mathematical problems.*
2. Reason abstractly and quantitatively. *Students make connections between equal groups of objects and arrays and the concept of adding equal addends.*
3. Construct viable arguments and critique the reasoning of others. *Students develop and explain strategies for using arrays to solve a variety of mathematical problems.*
4. Model with mathematics. *Students use a growing understanding of odd/even numbers and model to determine solutions for various mathematical problems.*
5. Use appropriate tools strategically. *Students use mathematical tools such as number lines, graphs, arrays, and pictures to solve an assortment of problems.*
6. Attend to precision. *Students use precise mathematical language to communicate an understanding of odd/even numbers, rows, columns, arrays, equal addends, repeated addition, and graphs.*
7. Look for and make use of structure. *Students look for mathematical patterns using odd/ even numbers, arrays, and repeated addition to create strategies for solving problems.*
8. Look for and express regularity in repeated reasoning. *Students make connections between how odd/even numbers, arrays, and repeated addition can be used to solve math problems.*

*****<u>Mathematical Practices 1 and 6 should be evident in EVERY lesson.</u> ****

STANDARDS FOR MATHEMATICAL CONTENT

Work with equal groups of objects to gain foundations for multiplication.

MGSE2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

MGSE2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Represent and interpret data.

MGSE2.MD.10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

BIG IDEAS

By the conclusion of this unit, students should be able to demonstrate the following competencies:

- Understand the similarities between skip counting, repeated addition, and multiplication.
- Construct arrays for a given repeated addition sentence.
- Write a repeated addition equation for a given array.
- Determine how the addition sentence for a given array changes when the array is rotated ¼ turn.
- Understand that multiplication is repeated addition.
- Write an equation to express an even number.
- Identify if a number is even or odd by modeling the number in pairs.
- Draw and interpret a picture and a bar graph to represent a data set with up to four categories.
- Repeatedly adding the same quantity, using a grouping picture or forming a rectangular array are strategies for representing repeated addition equations.
- Arrays are a way of representing both repeated addition and skip counting.
- Arrays should be identified in rows and then columns.
- Explore and be able to explain even and odd numbers while using manipulatives.
- An even number can be decomposed into two equal addends.
- Double addition facts assist in recognizing even numbers.
- Tables and charts can help make solving problems easier.
- Questions can be solved by collecting and interpreting data.

ESSENTIAL QUESTIONS

- How are odd and even number lines identified on the number line?
- How do I determine if a number is odd or even?
- What strategies can I use to tell if a number is odd or even?
- What is odd? What is even?
- How are arrays and repeated addition related?
- How can rectangular arrays help us with repeated addition?
- How can we model repeated addition on the number line?
- How can we a model repeated addition equation with an array?
- How does skip counting help us solve repeated addition problems?
- What is an array?

- What is repeated addition?

CONCEPTS AND SKILLS TO MAINTAIN

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply "how to get the answer" and instead support students' ability to access concepts from a number of perspectives. Therefore, students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency and automaticity. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, and make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between varieties of strategies in context.

Fluent students:

- flexibly use a combination of deep understanding, number sense, and memorization.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.

For more about fluency, see:
http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and:
https://bhi61nm2cr3mkdgk1dtaov18-wpengine.netdna-ssl.com/wp-content/uploads/nctm-timed-tests.pdf

Skills from Grade 1:

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20;

- Developing understanding of whole number relationships and place value, including grouping in tens and ones;

Second Grade Year Long Concepts:

- Organizing and graphing data as stated in MGSE.MD.10 should be regularly incorporated in activities throughout the year. **Students should be able to draw a picture graph and a bar graph to represent a data set with up to four categories as well as solve simple put-together, take-apart, and compare problems using information presented in a bar graph.**
- Routine topics such as counting, time, money, positional words, patterns, and tallying should be addressed on an ongoing basis throughout instructional time.
- Students will be asked to use estimation and benchmark numbers throughout the year in a variety of mathematical situations.

STRATEGIES FOR TEACHING AND LEARNING

(Information adapted from Mathematics Georgia Standards of Excellence State Standards and Model Curriculum, Ohio Department of Education Teaching)

Work with equal groups of objects to gain foundations for multiplication.

MGSE2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

MGSE2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Instructional Strategies

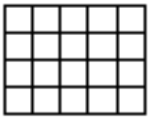
Students need to understand that a collection of objects can be one thing (a group), and a group contains a given number of objects. Investigate separating no more than 20 objects into two equal groups. Find objects (the total number of objects in collections up to 20 members) that will have some objects and no objects remaining after separating the collections into two equal groups. Odd numbers will have one object remaining, while even numbers will not. For an even number of objects in a collection, show students the total as the sum of equal addends (repeated addition). For example, 10 objects separated into two equal groups can be represented as $5 + 5 = 10$.

Another strategy is for students to think of numbers as a collection of objects. If each object can be paired, has a partner, then it is an even number. If not, the number is odd. For example, students represent the number 6 (even) as XX XX XX (every X has a partner); whereas 7 (odd) is represented as XX XX XX X (one X does not have a partner).

A rectangular array is an arrangement of objects in horizontal rows and vertical columns. Arrays can be made out of any number of objects that can be put into equal rows and columns. Making a connection to real world objects will aid students in differentiating between rows (rows in a

garden) and columns (columns holding a roof up). All rows contain the same number of items and all columns contain an equal number of items.

Have students use objects to build all the arrays possible with no more than 25 objects. Their arrays should have up to 5 rows and up to 5 columns. Ask students to draw the arrays on grid paper and write two different repeated addition equations under the arrays: one showing the total as a sum by rows (how many are in each row added by using the number of rows) and the other showing the total as a sum by columns (how many are in each column added by using the number of columns). Both equations will show the total as a sum of equal addends (repeated addition).



The equation by rows: 20 = 5 + 5 + 5 +5
The equation by columns: 20 = 4 + 4 + 4 + 4 + 4

If students are asked to identify the equation that matches the array, it would be 4 x 5 = 20 which relates to 20 = 5+5+5+5.

Or any object, such as Xs.

X X X X X
X X X X X
X X X X X
X X X X X

The equation by rows: 20 = 5 + 5 + 5 +5
The equation by columns: 20 = 4 + 4 + 4 + 4 + 4

To build understanding, teachers should ask students questions such as:

- What direction do rows go? (across)
- What direction do columns go? (up and down)
- How many rows do you see? How many ____ are in each row? What number is repeated? How many times?
- How many columns do you see? How many ____ are in each column? What number is repeated? How many times?

Build on knowledge of composing and decomposing numbers to investigate arrays with up to 5 rows and up to 5 columns in different orientations. For example, form an array with 3 rows and 4 objects in each row. Represent the total number of objects with equations showing a sum of equal addends (repeated addition) two different ways: by rows, 12 = 4 + 4 + 4; by columns, 12 = 3 + 3 + 3 + 3. Rotate the array 90° to form 4 rows with 3 objects in each row. Write two different equations to represent 12 as a sum of equal addends: by rows, 12 = 3 + 3 + 3 + 3; by columns, 12 = 4 + 4 + 4. Have students discuss this statement and explain their reasoning: The two arrays are different (the equations) and yet the same (the sum).

Ask students to think of a full ten-frame showing 10 circles as an array. One view of the ten-frame is 5 rows with 2 circles in each row. Students count by rows to 10 and write the equation 10 = 2 + 2 + 2 + 2 + 2, or students count by columns to 10 and write the equation 5 + 5 = 10

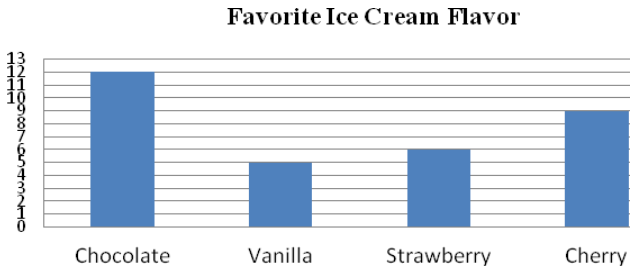
Represent and Interpret Data.

MGSE2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

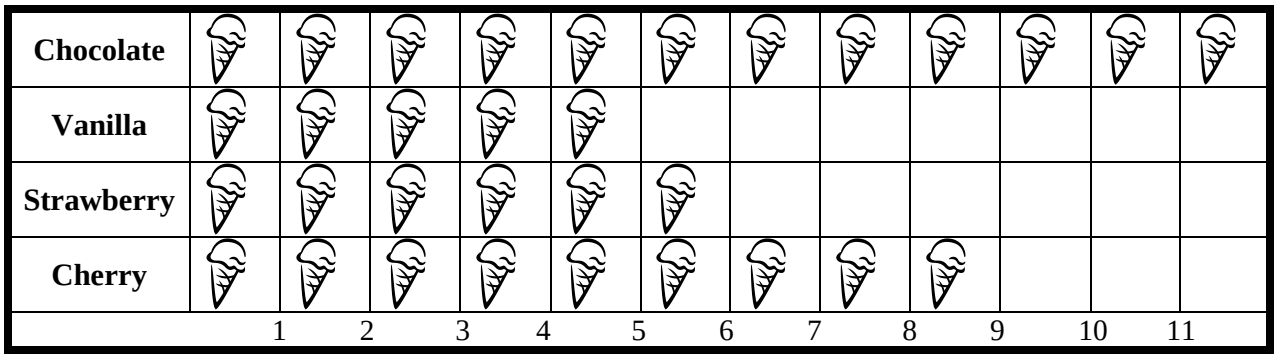
Instructional Strategies
At first students should create real object and picture graphs so each row or bar consists of countable parts. These graphs show items in a category and do not have a numerical scale. For example, a real object graph could show the students' shoes (one shoe per student) lined end to end in horizontal or vertical rows by their color. Students would simply count to find how many shoes are in each row or bar. The graphs should be limited to 2 to 4 rows or bars. Students would then move to making horizontal or vertical bar graphs with two to four categories and a single-unit scale.

Flavor	Number of People
Chocolate	12
Vanilla	5
Strawberry	6
Cherry	9

Students display their data using a picture graph or bar graph using a single unit scale.



Favorite Ice Cream Flavor



As students continue to develop their use of reading and interpreting data it is highly suggested to incorporate these standards into daily routines. It is not merely the making or filling out of the graph but the connections made from the date represented that builds and strengthens mathematical reasoning.

SELECTED TERMS AND SYMBOLS

The following terms and symbols are not an inclusive list and should not be taught in isolation. Instructors should pay particular attention to them and how their students are able to explain and apply them (**i.e. students should not be told to memorize these terms).**

Teachers should present these concepts to students with models and real-life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

For specific definitions, please reference the Georgia Standards of Excellence State Standards Glossary.

- **addends**
- **addition**
- **array**
- **bar graph**
- **columns**
- **data**
- **equal sharing/forming equal sized groups**
- **equation**
- **even**
- **odd**
- **pairing**
- **picture graph**
- **product**
- **rectangular**
- **rows**
- **scale**
- **sum**
- **total**

COMMON MISCONCEPTIONS

(As stated in Teaching Student-Centered Mathematics Developmentally Appropriate Instruction for Grades K-2, Van de Walle, Lovin, Karp, Bay-Williams):
"Children must come to understand that errors provide opportunities for growth as they are uncovered and explained. Trust must be established with an understanding that it is okay to make mistakes. Without this trust, many ideas will never be shared."

Regarding odd and even numbers, students may initially have difficulty "proving" how they know numbers are odd or even. Students may struggle with communicating their growing knowledge using precise mathematical language. Teachers should provide multiple opportunities for students manipulate numbers of objects and express their observations.

With regard to an understanding of arrays, students may at first confuse rows and columns. Children should be provided with numerous chances to make real life connections to examples of rows and columns (some picture cards are provided in this unit). To further support an understanding, teachers may encourage students to use their hands to show the direction rows and columns go. As students move on to explore arrays, they may struggle with creating arrays. For example, students may not know where to begin as they attempt to create an array with 18 objects. Teachers should encourage students to look for equal groups (repeating number patterns) that can be added to make a total of 18. Multiple opportunities will increase student fluency with these tasks. When relating arrays to repeated addition equations, students who do not naturally make connections between the two may be guided towards discovery with questions such as "How many rows do you see in the array" "How many objects are in each row?" "Does each row have the same amount of objects?" "How can you add these numbers to find the total number of objects?"

TASK TYPES

Scaffolding Task	Tasks that build up to the learning task.
Constructing Task	Constructing understanding through deep/rich contextualized problem solving tasks.
Practice Task	Tasks that provide students opportunities to practice skills and concepts.
Culminating Task	Designed to require students to use several concepts learned during the unit to answer a new or unique situation. Allows students to give evidence of their own understanding toward the mastery of the standard and requires them to extend their chain of mathematical reasoning.
Formative Assessment Lesson (FAL)	Lessons that support teachers in formative assessment which both reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards.
3-Act Task	A Three-Act Task is a whole-group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the ***Guide to Three-Act Tasks*** on georgiastandards.org.

The following tasks represent the level of depth, rigor, and complexity expected of all second-grade students. These tasks or tasks of similar depth and rigor should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as a performance task, they also may be used for teaching and learning. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks be reviewed prior to instruction. The tasks in this unit illustrate the types of learning activities that should be conducted to meet the GSE. A variety of additional resources should be utilized to supplement these tasks. Below is a description of the types of tasks you will see in this unit and their purpose.

Task Name	Task Type/Grouping Strategy	Content Addressed	Standard(s)	Task Description Students will:
Drink Up	3-Act Task *Whole Group*	Arrays	MGSE2.OA.4	Be presented with a problem and phot and asked to solve the problem. They will generate questions and be given additional information only upon realizing they haven't been given enough information to solve the task.
Bumpy or Not Bumpy?	Scaffolding Task *Large Group, Small Groups*	Odd and Even Numbers	MGSE2.OA.3	Using square tiles explore the difference between odd and even numbers. Classify number into categories: odd or even.
Are We Odd or Even?	Scaffolding Task *Large Group, Partners*	Odd and Even Numbers	MGSE2.OA.3	Determine strategies to use to decide if a particular group of students are even or odd.
What's in the Bag?	Scaffolding Task *Large Group, Small Group*	Odd and Even Numbers	MGSE.2OA.3	Engage in experiences with repeated addition using real life objects and explore how even numbers can be split into two equal groups.
Two of Everything!	Practice Task, *Large Group, Individual*	Equal Addends	MGSE2.OA.3	Continue to develop and apply the concept of equal addends to the mathematical problem solving opportunities.
Add it Up!	Scaffolding Task *Large group, Small Group*	Odd and Even Numbers	MGSE2.OA.3	Begin to understand that adding 2 equal addends produces an even sum; adding 2 odd addends produces an even sum; but combining an even and odd addend results in an odd sum.
Cookie Monster	Constructing Task *Large Group, Small Group*	Odd and Even Numbers	MGSE2.OA.3 MGSE2.OA.4	Apply understanding of odd and even numbers to begin understanding arrays through exploration with manipulatives.

Cereal Arrays	Practice Task *Small Groups*	Arrays	MGSE2.OA.4	Use manipulatives to create arrays to develop an understanding of the relationship between repeated addition and arrays.
Roll an Array	Practice Task *Large group, partners*	Arrays, repeated addition	MGSE2.OA.3	Continue to develop an understanding of rows, columns, arrays, and repeated addition equations by using number cubes to create arrays.
Seating the Class	Constructing Task *Small Groups*	Repeated addition	MGSE2.OA.3 MGSE2.MD.10	Apply understanding of arrays by suggesting an array for seating the class and explaining reasoning.
Pattern Block Drop	Practice Task *Small Groups*	Arrays	MGSE2.OA.3 MGSE2.OA.4	Apply knowledge of rows, columns, arrays, and repeated addition equations to determine the total number of pattern block shapes.
The Queen's Dilemma	Constructing Task *Partners*	Arrays	MGSE2.OA.4	Use manipulatives to create a variety of arrays and explain their mathematical reasoning.
Mathemagicians	Practice Task *Partners*	Arrays	MGSE2.OA.4	Practice finding the total number of objects and model with arrays and repeated addition sentences.
No, You Can't	Constructing Task *Partners*	Arrays	MGSE2.OA.4	Justify thinking after solving a problem and create an anchor chart of supporting details.
The Candy Box	Culminating Task *Individual*	Arrays	MGSE2.OA.3 MGSE2.OA.4 MGSE2.MD.10	Apply understanding of odd/even, arrays, repeated addition equations, and graphing to design a candy box array and create a related bar graph.
Staples	3-Act Task *Whole Group*	Arrays	MGSE2.OA.4	Use knowledge of arrays and repeated addition to solve the problem.

If you would like more information about this unit, please view the Unit 6 Webinar at https://www.georgiastandards.org/Archives/ located in the Professional Learning section.

INTERVENTION TABLE

The Intervention Table below provides links to interventions specific to this unit. The interventions support students and teachers in filling foundational gaps revealed as students work through the unit. All listed interventions are from New Zealand's Numeracy Project

Cluster of Standards	Name of Intervention	Snapshot of summary or Student I can statement. . .
Operations and Algebraic Thinking Work with equal groups to gain foundations for multiplication. MGSE2.OA.3 MGSE2.OA.4	Beep	Skip count to find the answer to problems involving equal groups
	Number Path	Skip counting by 2s, 5s, and 10s
	Bead String Benchmarks	Say the forwards and backwards skip–counting sequences in the range 0–100 for twos, fives, and tens
	Animal Arrays	Create equal sized groups and count the total set either by counting all or by skip counting
	Smiley Face	Solving multiplication and division problems using skip counting by twos, fives, and tens
	Three's Company	Solve multiplication problems by using repeated addition

3-ACT TASK: Drink Up

Back to Task Table

Approximately One Class Session
In this task, students use their knowledge of arrays and repeated addition to solve the problem.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students are required to figure out a question to work through, the information they need to solve the problem, and then persevere until solving it.
2. Reason abstractly and quantitatively. Students are asked to make an estimate both high and low.
3. Construct viable arguments and critique the reasoning of others. Students will use known information about arrays to construct viable arguments about them.
4. Model with mathematics. In early grades, students experiment with representing problem situations in multiple ways including numbers, pictures, and creating equations.
6. Attend to precision. Students will use clear and precise language when discussing their strategies and sharing their solutions with others.
8. Look for and express regularity in repeated reasoning. In the early grades, students notice repetitive actions in counting and computations.

ESSENTIAL QUESTIONS

- When is using arrays helpful?

MATERIALS

- Soda Picture
- Student Handout

GROUPING

Individual/Partner Task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this task, students will view the picture and tell what they noticed. Next, they will be asked to discuss what they wonder about or are curious about. These questions will be recorded on a class chart or on the board and on the student recording sheet. Students will then use mathematics to answer their own questions. Students will be given information to solve the

problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them.

Background Knowledge:

This task follows the 3-Act Math Task format originally developed by Dan Meyer. More information on this type of task may be found at http://blog.mrmeyer.com/category/3acts/. A Three-Act Task is a whole-group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the ***Guide to Three-Act Tasks*** on georgiastandards.org.

In this task students will be shown a picture of can drinks. They will look for the array made by the drinks. Encourage students to not count by ones when calculating how many drinks are in the picture.

Common Misconceptions:

With regard to an understanding of arrays, students may at first confuse rows and columns. Children should be provided with numerous chances to make real life connections to examples of rows and columns (some picture cards are provided in this unit). To further support an understanding, teachers may encourage students to use their hands to show the direction rows and columns go. As students move on to explore arrays, they may struggle with creating arrays. For example, students may not know where to begin as they attempt to create an array with 18 objects. Teachers should encourage students to look for equal groups (repeating number patterns) that can be added to make a total of 18. Multiple opportunities will increase student fluency with these tasks. When relating arrays to repeated addition equations, students who do not naturally make connections between the two may be guided towards discovery with questions such as "How many rows do you see in the array?" "How many objects are in each row?" "Does each row have the same amount of objects?" "How can you add these numbers to find the total number of objects?"

Task Directions:

Act I – Whole Group - Pose the conflict and introduce students to the scenario by showing Act I picture. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/**)**
"Introduce the central conflict of your story/task clearly, visually, viscerally, using as few words as possible."

1. Show picture of soda boxes to students.
2. Ask students what they noticed in the picture. The teacher records this information.

3. Ask students what they wonder about and what questions they have about what they saw. Students should share with each other first, and then the teacher records these questions (think-pair-share). The teacher may need to guide students so that the questions generated are math-related.
4. Ask students to estimate answers to their questions (think-pair-share). Students will write their best estimate, then write two more estimates – one that is too low and one that is too high so that they establish a range in which the solution should occur.

Anticipated questions students may ask and wish to answer:

- How many boxes are there?*
- How many boxes would there be if the stars were not covering some?*
- Are there more red boxes or silver boxes?
- How many boxes are behind the stars?

*Main question(s) to be investigated

Act 2 – **Student Exploration** - Provide additional information as students work toward solutions to their questions. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/**)**
"The protagonist/student overcomes obstacles, looks for resources, and develops new tools."

- During Act 2, students use the main question(s) from Act 1 and decide on the facts, tools, and other information needed to answer the question(s). When students decide what they need to solve the problem, they should ask for those things. It is pivotal to the problem solving process that students decide what is needed without being given the information up front. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin.

- The teacher provides guidance as needed during this phase. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin. Questioning is an effective strategy that can be used, with questions such as:
    - What is the problem you are trying to solve?
    - What do you think affects the situation?
    - Can you explain what you've done so far?
    - What strategies are you using?
    - What assumptions are you making?
    - What tools or models may help you?
    - Why is that true?
    - Does that make sense?

Important note: Although students will only investigate the main question(s) for this task, it is important for the teacher to not ignore student generated questions. Additional questions may be answered after they've found a solution to the main question, or as homework or extra projects.

Act 3 – Whole Group – Share solutions and strategies.

- Students to present their solutions and strategies and compare them.
- Reveal the solution picture.
- Lead discussion to compare these, asking questions such as:
    - How reasonable was your estimate?
    - Which strategy was most efficient?
    - Can you think of another method that might have worked?
    - What might you do differently next time?

Act 4, The Sequel - "The goals of the sequel task are to a) challenge students who finished quickly so b) I can help students who need my help. It can't feel like punishment for good work. It can't seem like drudgery. It has to entice and activate the imagination." Dan Meyer
http://blog.mrmeyer.com/2013/teaching-with-three-act-tasks-act-three-sequel/

- Challenge students to pick a different question from the student generated list to answer.
- Challenge students to figure out how many cans there are if each box has 12 cans.
- Challenge students to calculate the cost of all the boxes if each box is $2.50

FORMATIVE ASSESSMENT QUESTIONS

- How reasonable was your estimate?
- What might you do differently next time?
- What worked well for you this time?
- What model did you use?
- What organizational strategies did you use?

DIFFERENTIATION

Extension

- Have students calculate how many boxes it would take to make an 8 x 14 array.

Intervention

- Allow students to count by ones and then look for groupings within the picture.

Intervention Table

Name _____ Date _____

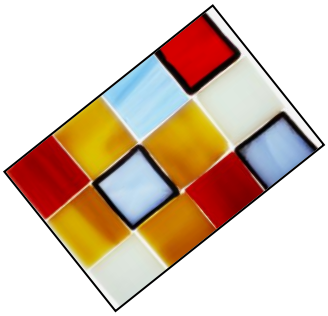
Drink Up

What problem are you trying to figure out?		
What information do you already know?	What information do you need to solve the problem?	
Make an estimate.	Write an estimate that's too low.	Write an estimate that's too high.
Show your estimates on a number line:		
Show your work.		
What is your conclusion?		

Scaffolding Task: Bumpy or Not Bumpy

Back to Task Table

Approximately 2 days (Activity originally found in Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades K-3, page 292)
In this task, students will use square tiles to begin to explore and understand the difference between odd and even numbers.



STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. **Make sense of problems and persevere in solving them.**
3. **Construct viable arguments and critique the reasoning of others.**
   Independently written story problems are shared and explained with classmates.
4. **Model with mathematics.**
   Students represent numbers in a variety of ways.
6. **Attend to precision.**

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades K-3, page 291)

"The categorization of numbers as odd or even is an important regularity in our number system. All too often students are simply told that the even numbers are those that end in 0, 2, 4, 6, or 8 and odd numbers are those that end in 1, 3, 5, 7, or 9. While of course this is true, it is only an attribute of even and odd numbers rather than a definition that explains what *even* or *not even* (i.e., *odd*) really means."

After concluding the Bumpy or Not Bumpy Task, "students should be able to classify numbers into the categories that we call odd and even. **After they have conceptualized these classes of numbers**, the appropriate labels of *odd* and *even* can be applied."

ESSENTIAL QUESTIONS

- How do I determine if a number is odd or even?
- What strategies can I use to tell if a number is odd or even?
- What is odd? What is even?

MATERIALS

- "Bumpy" or "Not Bumpy" Master
- Paper, crayons, pencils
- Envelope

GROUPING

Small Group, partners

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Special Note: This task can be repeated several times in small groups or in a center.

Part I
Duplicate the Bumpy or Not Bumpy for each student. Have students cut them and keep them in an envelope. Explain how each piece (except the single square) is made of two columns of squares. Have students work in pairs or small groups to see how many things they can find to tell about the pieces. (For example: There is a piece for each number 1 to 10. Some are like rectangles. Some have a square sticking out.) For those who might need a start, suggest that they put the pieces in order from one square to ten. Have students share with the whole group what they have discovered. For example:


1 "bumpy" odd


2 "not bumpy" even


3 "bumpy" odd


4 "not bumpy" even

Part II
Next, have students sort their pieces into two sets. It is very likely that some group will sort their pieces into "bumpy" (odd) and "not bumpy" (even). Refer to the two groups of pieces as "bumpy" numbers and "not bumpy" numbers (or whatever labels your students prefer to use). Students will then share their groupings with their classmates and have the class guess what rule they used to categorize the pieces.

Part III
Next, assign groups of students three or four numbers between 11 and 40 or 50 and have students decide whether two-column cards for these numbers would be bumpy or not bumpy. They can use words and pictures to explain their conclusions.

FORMATIVE ASSESSMENT QUESTIONS

- What did you notice about your pieces?
- How do the pieces differ from each other? What do the pieces have in common?
- What characteristics did you use to sort your pieces?

DIFFERENTIATION

Extension

- Have students create their own pieces to represent various numbers. Then trade their pieces with a classmate and they will describe the pieces a "bumpy" or "not bumpy".

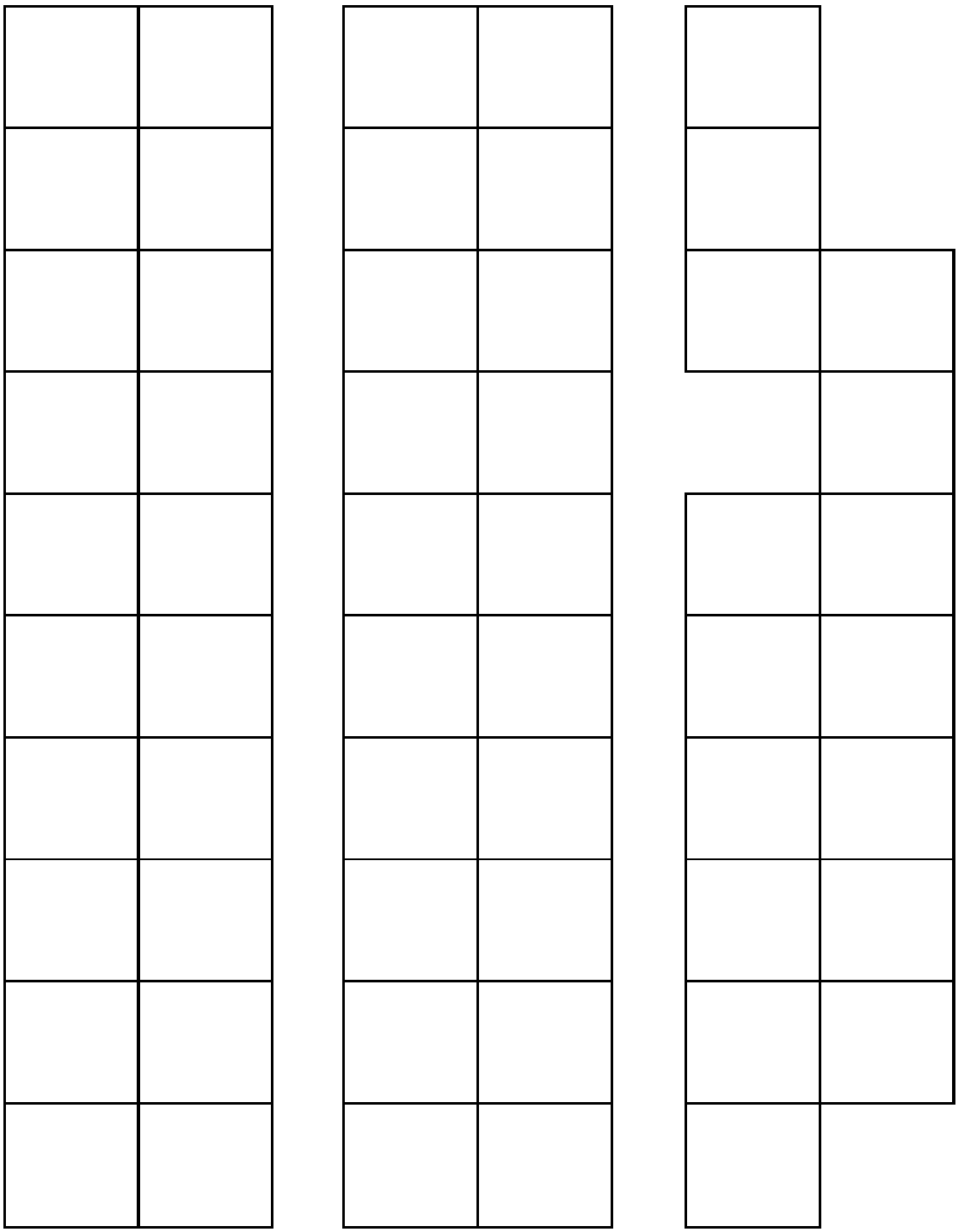
Intervention

- Have students use unifix or connecting cubes to build each piece. They can then manipulate the pieces to pair up each square. This will help then to better understand the meaning of "bumpy" and "not bumpy"
- The teacher may wish to prepare the Bumpy, Not Bumpy Master with bold lines dividing them into the groups of 1, 2, 3,…

Intervention Table

Name _____ **Date** _____

"Bumpy" or "Not Bumpy"



Scaffolding Task: Are We Odd or Even?

Back to Task Table

Approximately 1 day
In this task, students will determine strategies to use classmates to decide whether the class has an odd or even amount of students.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. **Make sense of problems and persevere in solving them.**

3. **Construct viable arguments and critique the reasoning of others.**
   Students develop and explain strategies for determining if their class has an odd or even number of students.
4. **Model with mathematics.**
   Students use classmates to determine if their class has an odd or even number.
6. **Attend to precision.**
   Students use mathematical communication skills to explain their strategies to determine
   the difference between odd and even numbers.

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades K-3, page 291)

"The categorization of numbers as odd or even is an important regularity in our number system. All too often students are simply told that the even numbers are those that end in 0, 2, 4, 6, or 8 and odd numbers are those that end in 1, 3, 5, 7, or 9. While of course this is true, it is only an attribute of even and odd numbers rather than a definition that explains what *even* or *not even* (i.e., *odd*) really means."

After concluding the Bumpy or Not Bumpy Task, "students should be able to classify numbers into the categories that we call odd and even. **After they have conceptualized these classes of numbers**, the appropriate labels of *odd* and *even* can be applied."

ESSENTIAL QUESTIONS

- How do I determine if a number is odd or even?
- What strategies can I use to tell if a number is odd or even?

- What is odd? What is even?

MATERIALS

- Unifix cubes, or similar manipulatives
- Chart paper
- Markers

GROUPING

Whole group, partners

NUMBER TALK

This task will lend itself to a number talk about these numbers:

3,5,7,9
2,4,6,8

(For more information, refer to Sherry Parrish, Number Talks, grades K-5.)

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I
Units 2 and 4 have provided students with opportunities to explore how some numbers (up to 20) can be separated into two equal groups. This prior knowledge will allow students to understand that even numbers can be separated into two equal groups (equal addends). For example, students begin to understand 12 is even because 6 + 6 = 12 (two equal groups of 6). Whereas, students begin to understand 13 is odd because it cannot be separated into two equal groups. Additionally, students may realize that when there is an even number of objects, each object will have a partner and with an odd number of objects, there will be one object left over. For example, 8 is an even number because it can be shown as XX XX XX XX, whereas 9 is an odd number because it can be shown as XX XX XX XX X.

Pass out 20 unifix cubes, or other similar manipulatives to each partner pair. Review observations students made about odd and even numbers during the "Bumpy or Not Bumpy" activity (even numbers are "not bumpy" and odd numbers are "bumpy"). Ask students how they might be able to use cubes to determine whether numbers are even or odd. Students should suggest separating the cubes into equal groups or partners to determine if the number is even or odd. If students do not make these suggestions, teachers should refer to the first part of Part I for possible discussions and explorations.

Invite students to work with partners to show how various numbers (up to 20) are even or odd. Make observations, listen to student conversations, and ask students formative assessment questions such as:

- How are you using the cubes to decide whether ____ is even or odd?

- How are the cubes helping you to decide whether ____ is even or odd? Why?
- How can you use cubes to show ____ is an even number?
- How can you use cubes to show ____ is an odd number?

Part II

Special note: Because many classrooms have more than 20 students, this activity will allow students to apply their knowledge to larger numbers.

Gather students together and pose the question "How can we use ourselves (teacher not included) to determine if our class has an odd or even number of students?" Record various student responses on chart paper. At this point, students should make suggestions such as "We can see if we can separate ourselves into two equal groups." and "We can see if we each have a partner."

As a class, decide which strategy to use first and how students should begin "sorting" themselves. Monitor student progress. Once students have sorted themselves, have students determine whether they are even or odd.

As a class, decide which strategy to use next and how students should begin "sorting" themselves. Monitor student progress. Once students have sorted themselves, have students determine whether they are even or odd (students should notice that they should have the same answer as in the previous attempt; however, if they do not have the same answer, this would provide a good opportunity for students to "check" their results). Encourage students to discuss the two strategies they used.

At this point, if the class is an even number, ask students what the two equal addends are that add up to the total number of students. For example, if your class has a sum of 22, students 11 + 11 = 22. If you class has an odd number, students should understand that odd numbers cannot be separated into two equal groups and the sum cannot be represented with two equal addends.

FORMATIVE ASSESSMENT QUESTIONS

- How can you use manipulatives to determine whether a number is even or odd?
- What strategies can you use to decide if a number is even or odd?
- How can you show (prove) ____ is even?
- How can you show (prove) ____ is odd?

DIFFERENTIATION

Extension

- Challenge students to determine if larger numbers are even or odd.

Intervention

- Provide additional opportunities for students to work with manipulatives to develop an understanding of even and odd.

Intervention Table

Scaffolding Task: What's in the Bag?

Back to Task Table

Approximately 2 days

In this task, students will use real life objects to determine whether the number of objects is odd or even. Specifically, students will explore how even numbers can be split into two equal groups.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. **Make sense of problems and persevere in solving them.**
4. **Model with mathematics.**
   Students represent even and odd numbers with manipulatives.
5. **Use appropriate tools strategically.**
   Students use number lines and bar graphs.
6. **Attend to precision.**

BACKGROUND KNOWLEDGE

(Information adapted from North Carolina DPI Instructional Support Tools)

Students should have had prior experiences and/or instruction with addition. They should begin to relate multiplication as repeated addition. Please see Units 2 and 4 for addition support.
If you have not already done tasks where students have split a group of 20 (or fewer) items into two equal groups then this needs to be done before attempting this task. Provide several experiences where students are able to investigate all the numbers 0-20 to see which ones can be split into two equal groups. This is a good opportunity to review the concepts of "not bumpy" (even) and "bumpy" (odd) numbers and now build on the understanding of how this connects to repeated addition. Having students write addition equations for the even numbers they are able to split into two equal groups is a good way to introduce the concept of repeated addition. Students should recognize that all even numbers can be expressed using two of the same addends (ex. 2+2=4, 3+3=6, again focusing on equal addends sets the stage for repeated addition, leading into multiplication.)

This task will focus on the use of strategies; however, it is important to note the focus is on conversations as students engage in experiences with repeated addition. Initially, students apply base-ten concepts and use direct modeling with physical objects or drawings to find different ways to solve problems. They move to inventing strategies that do not involve physical materials or counting by ones to solve problems. Student-invented strategies likely will be based on place-value concepts, the commutative and associative properties, and the relationship between

addition and subtraction. These strategies should be done mentally or with a written record for support. It is vital that student-invented strategies be shared, explored, recorded and tried by others. Recording the expressions and equations in the strategies horizontally encourages students to think about the numbers and the quantities they represent instead of the digits. Not every student will invent strategies, but all students can and will try strategies they have seen that make sense to them. Different students will prefer different strategies.

ESSENTIAL QUESTIONS

- How do I determine if a number is odd or even?
- What strategies can I use to tell if a number is odd or even?
- What is odd? What is even?

MATERIALS

- Various manipulatives (counters, base-ten blocks, unifix cubes, beans in bags labeled A – J, 1 set per partner)
- Paper, crayons, pencils

GROUPING

Whole Group, Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Special Note: This task can be repeated several times in small groups or in a center.

Part I
Gather students together in class meeting area. Display the questions, "What is even? What is odd?" The teacher will need to guide discussion into mathematical talk and not story sharing. Be prepared to guide students' thinking into conversations about something such as sharing carrot snacks between two friends.

Part II
Have two students come up and practice sharing the cubes the teacher has placed in front of them. For example, the teacher would place 6 cubes in front of 2 students and ask them if they can share the total evenly (fairly). As students are sharing, record each shared quantity on a chart labeled "We can share equally between 2 groups/ We cannot share equally between 2 groups." After several student pairs share the cubes (different quantities each time), lead class in discussion about information on the chart. The conversation should be directed to build the understanding that groups shared evenly are called even numbers and ones which do not share evenly are called odd numbers. The chart can be relabeled as EVEN and ODD.

Part III
Students work in partners with 10 different bags of items. These should be made in advance and could be shared between various partners. Each bag should be labeled A –J. Once students have determined which groups are odd and which are even, they will work together and create a bar graph of the number of odd and even draws they had with their partner. Students should be prepared to share their graph with others.

Part IV
Students individually will create their own number line from 0-20. The teacher calls out numbers and students first label the numbers as they teacher calls them out and then students labels as odd or even using red and blue crayons. Students will share with a table partner to check their labeling.

FORMATIVE ASSESSMENT QUESTIONS

- What strategies did you use to decide if a number of odd or even?
- Can you show that answer in a different way?
- How can you demonstrate this with a picture?
- How could you write this in a number sentence?

DIFFERENTIATION

Extension
- Encourage students to determine whether or not they can come up with a rule for any number that would tell whether or not the number is odd or even. Have students record their rule on an anchor chart and present their even/odd rule to the class.

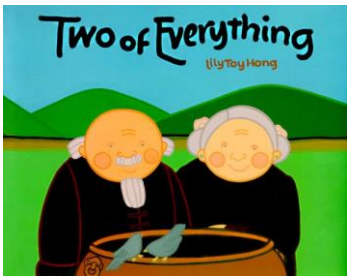
Intervention
- Some students will need to use manipulatives to help to determine or represent the number of objects in each group.
- Give the student a 0-20 chart to help them skip count to determine the number of objects in each group.

Intervention Table

Practice Task: Two of Everything!

Back to Task Table

Approximately 1 day
In this task, students will continue to develop and apply the concept of equal addends to mathematical problem solving opportunities.



STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
   Students represent even numbers using equations with equal addends.
4. **Model with mathematics.**
   Students use pictures to represent equations with equal addends.
6. **Attend to precision.**
   Students use precise mathematical language to explain their pictures and solutions.

BACKGROUND KNOWLEDGE

Students should have had multiple opportunities to split groups of 20 (or fewer) items into two equal groups and discuss/write addition equations for them. Students should understand that even numbers can be separated into two equal groups and equation can be expressed at the sum of two equal addends (for example, 4 + 4 = 8, 10 + 10 =20). An understanding of equal addends serves as a foundation for the concepts of repeated addition and eventually multiplication.

ESSENTIAL QUESTIONS

- How can even numbers be represented as a sum of two equal addends?
- What is odd? What is even?

MATERIALS

- Crayons, pencils
- Chart paper
- Two of Everything practice activity
- Two of Everything by Lily Toy Hong

GROUPING
Whole Group, individual

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I
Gather students together and read Two of Everything by Lily Toy Hong, or a similar book about counting two groups of objects. Before reading, ask the students "What are some things you would like to have two of?" List student responses on chart paper for later discussion. Explain that this book will have two of many things and challenge students to keep track of the objects mentioned.

After reading, ask students what some of the objects were and list them on the chart paper. Ask students to look for similarities and differences between the items the children wanted two of and the objects Mr. and Mrs. Haktak had two of.

Ask students questions like the following (or similar questions if you are not using Two of Everything**):**

- How many purses did Mrs. Haktak pull out of the pot? (2)
- How many coins were in each purse? (5)
- How many coins were there in all (the sum)? (10)
- How can you use two equal addends to show the sum is an even number? (5 + 5 = 10)

Part II
Have students return to their seats. Pass out the Two of Everything practice activity for each student to work on independently. Read the story problem aloud while students follow along. Explain to the students that they are to solve the problem using a picture.

Once students have completed the activity, invite students to share their pictures and solutions. Encourage students to use precise mathematical language as they explain their pictures and solutions.

FORMATIVE ASSESSMENT QUESTIONS

- How does your picture represent the information in the story problem?
- How does your picture show (prove) the sum is an even number?
- How does your picture help you write an equation (number sentence)?

DIFFERENTIATION
Extension

- Encourage students to create their own story problems.

Intervention

- Students may need to use manipulatives to help to determine solutions.

Intervention Table

Name _____ Date _____

Two of Everything!

Mia and Max are twins. They are always kind and helpful. Mia and Max only have one toy box with 6 toys inside. As a reward for their kindness and helpfulness, a nice fairy waved her magic wand over Mia and Max's toy box and suddenly there were two toy boxes, each with 6 toys inside. How many toys do Mia and Max have now?

Draw a picture to represent the information in the story problem and find the answer.

Write an equation with two equal addends and the sum to represent your answer.

_____ + _____ = _____

Is your answer and even number or an odd number? _____

Constructing Task: Add it Up!

Back to Task Table

Approximately 2 days

In this task, students will use a growing understanding of odd and even numbers to form problems solving strategies for adding numbers. Students will begin to understand that adding two equal numbers will produce an even number, adding two odd numbers will produce an even number, however, adding an even number to an odd number will produce an odd number.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. **Make sense of problems and persevere in solving them.**
3. **Construct viable arguments and critique the reasoning of others.**
   Independently written story problems are shared and explained with classmates.
4. **Model with mathematics.**
   Students represent numbers in a variety of ways.
6. **Attend to precision.**

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle, Karp, and Bay-Williams, Elementary and Middle School Mathematics: Teaching Developmentally, page 266-267)

"An interesting category of number structures is that of odd and even numbers. Students will often observe that the sum of two even numbers is even, that the sum of two odd numbers is even, or that the sum of an even and odd number is always odd. Similar statements can be made about multiplication.

Students will provide a variety of interesting proofs of odd/even conjectures. As with other conjectures, they typically begin by trying lots of numbers. But here it is a bit easier to imagine that there just might be two numbers 'out there' that don't work. Then students turn to the definition or a model that illustrates the definition. For example, if a number is odd and you split in two, there will be a leftover. If you do this with the second odd number, it will have a leftover also. So if you put these two together, the two leftovers will go together so there won't be a leftover in the sum. Students frequently use models such as bars of snap cubes to strengthen their arguments."

ESSENTIAL QUESTIONS

- How do I determine if a number is odd or even?
- What strategies can I use to tell if a number is odd or even?
- What is odd? What is even?
- What is repeated addition?
- How do I use what I know about odd and even to help me with repeated addition?

MATERIALS

- Various manipulatives (counters, base-ten blocks, unifix cubes, beans) in bags. Need multiple bags with even number of items and multiple bags with an odd number of items
- Group Recording Sheet

GROUPING

Whole Group, Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Special Note: This task will take a few days as students repeat the work several times. Each opportunity will provide students with more time to describe their thinking and deepen their conceptual understanding of how these two concepts connect.

Part I
The teacher will gather students together for large group work. In advance, the teacher should create multiple bags of even materials and multiple bags of odd materials. Guide students in making connections about what happens when we combine two bags with even amounts, what happens when we combine bags with odd amounts, and what about when we combine bags with one of each. What is the result?

Students will need to record their findings as they combine bags such as: Bag A had 7 and Bag B had 4, therefore adding an odd and an even number which resulted in an odd number; however when I combined Bag A with 7 and Bag C with 3, I had a total of 10 which I know is an even number. This is all with manipulatives, students are not writing the number sentences yet. The teacher should allow time for students to predict whether the total will be odd or even and why they think the number will be odd or even.

Part II
After students have had extensive work describing all these combinations, go back to the bag combinations with writing addition sentences talking about addition sentences. This time students will connect the combinations with writing the number sentences, connecting repeating addition to even and odd. Use rich math language as you question students and repeat the questions about combinations. What are our addends? What do we know about those numbers? How will knowing if it's even or odd help us determine the answer?

This task is one that can be moved to a center once students have had ample time in class discussion. This task is designed to be repeated several times.

FORMATIVE ASSESSMENT QUESTIONS

- What strategies are you using to determine how many _____ are in your group?
- Can you show that answer in a different way?
- How can you demonstrate this with a picture?
- How could you write this in a number sentence?
- Do you have the same number of any of your objects? Why do you think this is the case?
- What makes a number even? What makes a number odd?
- How can knowing if a number is even or odd help you with addition sentences?
- What is an addend?

DIFFERENTIATION

Extension

- Students work independently with grab bags of items and justify their answers.

Intervention

- Students use manipulatives to show their work.

Intervention Table

CONSTRUCTING TASK: Cookie Monster

Back to Task Table

Approximately 1-2 Days
In this task, students will apply a growing understanding of odd and even numbers to a beginning understanding of arrays. Students will use manipulatives to begin this exploration.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

MGSE2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. **Make sense of problems and persevere in solving them.**
3. **Construct viable arguments and critique the reasoning of others.**
   Students share observations of patterns in even and odd numbers.
6. **Attend to precision.**
7. **Look for and make use of structure.**
   Students look for patterns in even and odd numbers. (equal addends)

BACKGROUND KNOWLEDGE

(Information adapted from North Carolina DPI Instructional Support Tools)
The standard addressed in this task calls for students to apply their work with doubles addition facts to the concept of odd or even numbers. Students should have ample experiences exploring the concept that if a number can be decomposed (broken apart) into two equal addends (e.g., 10 = 5 +5), and then that number (10 in this case) is an even number. Students should explore this concept with concrete objects (e.g., counters, place value cubes, etc.) before moving towards pictorial representations such as circles or arrays. This task will also introduce students to the basic concept of arrays.

Example: Is 8 an even number? Prove your answer.

Student 1

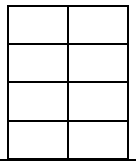
I grabbed 8 counters. I paired counters up into groups of 2. Since I didn't have any counters left over, I know that 8 is an even number.

Student 2

I grabbed 8 counters. I put them into 2 equal groups. There were 4 counters in each group, so 8 is an even number.

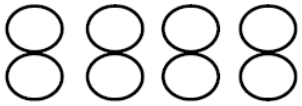
Student 3

I drew 8 boxes in a rectangle that had two columns. Since every box on the left matches a box on the right, I know 8 is even.



Student 4

I drew 8 circles. I matched one on the left with one on the right. Since they all match up, I know that 8 is an even number.



Student 5 I know that 4 plus 4 equals 8. So 8 is an even number.

Example: Is 7 an even number? Prove your answer.

Student 1

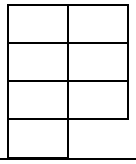
I grabbed 7 counters. I paired counters up into groups of 2. Since I had 1 counter left over, I know that 7 is NOT an even number. It is an odd number

Student 2

I grabbed 7 counters. I tried to put them into 2 equal groups. There were 4 counters in one group and 3 in the other group. I know that 7 is NOT an even number. It is an odd number.

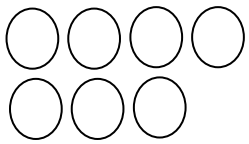
Student 3

I tried to draw a rectangle with 7 boxes. Every box on the left did not match a box on the right, so I know7 is NOT an even number. It is an odd number.



Student 4

I drew 7 circles. I matched one on the left with one on the right. Since they all do NOT match up, I know that 7 is NOT an even number. It is an odd number



Student 5 I know that 4 plus 3 equals 7. It is not a double fact, so 7 is NOT an even number. It is an odd number.

ESSENTIAL QUESTIONS

- What is an array?
- What is repeated addition?
- How are arrays and repeated addition related?
- How does skip counting help us solve repeated addition problems?
- How can we model repeated addition equation with an array?
- How are arrays used in our daily lives?

MATERIALS

- Counters, square tiles, other manipulatives (or use real cookies – mini or snack size – to engage students)
- Cookie Monster Student Recording Form
- *Stacks of Trouble* by Martha F. Brenner

GROUPING

Large Group, Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I
Review the concepts of arrays by reading *Stacks of Trouble* by Martha F. Brenner, or a similar text, aloud to the class. With each situation have students discuss how the dishes are stacked to create an array.

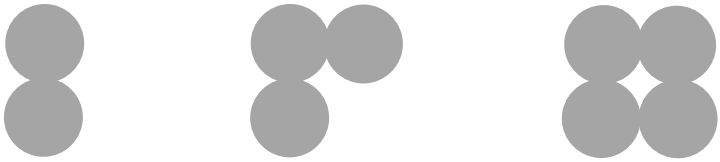
Part II
In small groups or partners, present the student with this task:

The cookie monster loved cookies. Every day he would take out one more cookie than he did the day before. Each time before eating the cookies, he would line them up on the table and try to make an array with only two rows.

On the first day, the cookie monster had 2 cookies and he was able to build his array with two equal rows. On day 2, he had 3 cookies but he couldn't make an array with two equal rows. If the pattern continued 9 more times, on what days would the cookie monster be able to make an array with his cookies using only two rows? Show and explain your mathematical thinking.

Row 1

Row 2



Day 1 *Day 2* *Day 3*

Be sure to describe and explain any patterns you recognize. Can you make a rule? On what days do you see equal addends? (Day 3 2 + 2 = 4)

Part III

When students have completed the task, have them explain their rule on chart part and share with the class. Students will vary in their explanations, which should give the teacher a better understanding of their misconceptions. Students may confuse rows (go across) and columns (up and down).

FORMATIVE ASSESSMENT QUESTIONS

- What arrays do you see?
- How can we create an equation for this?
- What are you noticing about the pattern?
- How are the days similar? How are they different?

DIFFERENTIATION

Extension

- Have students create their own cookie monster pattern that alternates between even and odd, but this time the cookie monster takes more than one cookie. How many cookies will he take?

Intervention

- Allow students to use circular counters to manipulate to better understand that variety of arrays.
- Allow students to use grid paper to ensure that their representation of the pattern is correctly aligned.

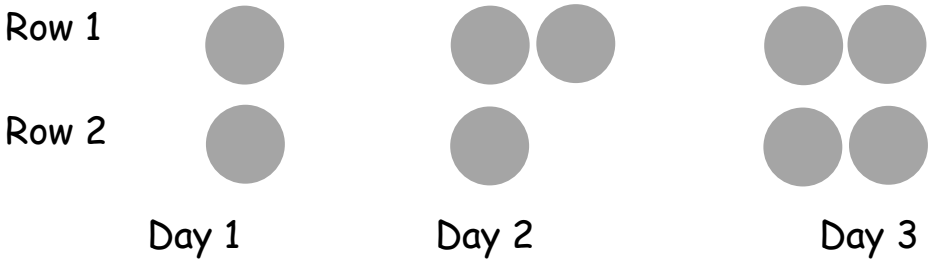
Intervention Table

Name _____ Date _____

Cookies

The cookie monster loved cookies. Every day he would take out one more cookie than he did the previous time. Each time before eating the cookies, he would line them up on the table and make an array with only two rows.

On the first day, the cookie monster had 2 cookies and he was able to build his array with two equal rows. On the second day, he had 3 cookies but he couldn't make an array with two equal rows. If the pattern continued 9 more days, on what days would the cookie monster be able to make an array with his cookies using only two rows? Show and explain your mathematical thinking.



Be sure to describe and explain any patterns you recognize. Can you make a rule?

Practice Task: Cereal Arrays

Back to Task Table

Approximately 3-4 days

In this task, students will increase their understanding of rows and columns as they use manipulatives (cereal) to create arrays. Students will begin to understand the relationship between arrays and their related repeated addition equations.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. **Make sense of problems and persevere in solving them.**
4. **Model with mathematics.**
   Students model arrays with cereal.
5. **Use appropriate tools strategically.**
   Students determine the total number of objects by using cereal arrays and pictures.
6. **Attend to precision.**

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades K-3, page 80)

"When students solve simple multiplication story problems before learning about multiplication symbolism, they will most likely write repeated-addition equations to represent what they did as an equation."

Although the focus in second grade is not learning the multiplication symbolism or facts, introducing the multiplication sign is up to the discretion of the teacher.

"The usual convention is that 4 x 8 refers to four sets of eight, not eight sets of four. There is absolutely no reason to be rigid about this convention. The important thing is that the students can tell you what each factor in their equations represents."

ESSENTIAL QUESTIONS

- What is an array?
- What is repeated addition?
- How can rectangular arrays help us with repeated addition?
- How are arrays and repeated addition related?

- How does skip counting help us solve repeated addition problems?
- How can we use model repeated addition equation with an array?

MATERIALS

- Cereal
- Glue

GROUPING

Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Each part of this task is designed to take one class period. Once the students have completed the task a version of it can be placed in a center for repeated practice throughout the year.

Part I
Explain to students that sometimes people use arrays as a model for repeated addition and that today we are going to make sure they understand what columns and rows look like in arrays. Tell students that arrays are made up of rows and columns. Ask students to think of some places that they have gone to where they might have had to sit in rows. Allow them to share and record their experiences. Show students pictures of rows in a movie theatre and at a football stadium (some examples are provided). Explain to students that rows are horizontal or they go in direction from left to right or right to left. Next ask the students if anyone knows what a column is or if they can point to an example of a column in the room or the school. If they need a hint tell them that columns travel in the opposite direction as rows. Once students have figured out that columns run "up and down." tell them that we use the word vertical to describe this direction (up and down). Ask students to think of places where they may have seen columns, and again record their ideas. Show students pictures of columns on various buildings such as a local home or the White House. (Some examples have been provided)

Allow students time to demonstrate their understanding of these new words (rows/horizontal; columns/vertical) by playing a short game of *Simon Says*. Have students get into a horizontal position by lying on the floor when you say "Simon says show me a row" or have them point to or locate things in the room that run in a horizontal direction (i.e. tray of the chalk board, bottom of the doorframe, edge of a rug, etc.). For a vertical position they can just stand up when you say, "Simon says show me a column", or point to places in the room that have lines running in a vertical direction (i.e. the flag pole, a music stand, leg of a table, etc.).

Once it is clear that students understand what columns and rows are, and the difference between them you can introduce what an array is to them using a picture that is a combination of rows and columns. Demonstrate for students an example of objects

organized in an array. You may have tubs on a shelf or desks organized in an array or using a block of tiles on the hallway floor. Ask students how these things organized as an array helps you. One possible reason would be to help you find things faster. Create a chart where you can list additional arrays they may notice in the classroom or mention having seen elsewhere. Draw several arrays on chart paper or white board large enough for students to see and be able to discuss in small groups.

Give the students some items (buttons, cubes, counters, etc.) and have them organize them into a rectangle array. You can partner students for this or have them work independently. Once they have their items in an array ask the students figure out what shape they have created (rectangle). Ask them to describe their array to the class by telling how many columns and how many rows they have made. As they are describing what they have made, have a student draw it on the chart paper or white board. Be sure to ask, "Does the picture I have drawn match your array?"

Part II

Assign each student a certain number of cereal pieces and have them decide the number of rows and columns that the array would have and then create the repeated addition. They could also create more than one based on their number. Once students come together and share their work, discuss that the number of rows is the addend that will be repeated and the number of columns is how many times you will repeat the addend. Have a student share their addition sentence and explain their reasoning for writing it that way. For example, 5+5+5 creates an array that is 5 rows and three columns. Once they show emergent understanding, allow students to explain or describe the array with an addition sentence. You can challenge students to describe the array with different kind of clues. For example, "I have an array that is 3+3+3+3." Or, "Who has an array that is 2+2+2?" You will need to have several examples of different arrays on the board for them to use as examples.

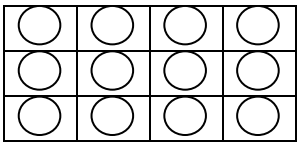
Part III

Tell students that they will now have the opportunity to create their own arrays using cereal. Have each student draw a repeated addition sentence. Encourage each student to create an array using the cereal to illustrate the card. As students create their cereal arrays, move about the room and ask questions from the formative assessment list.

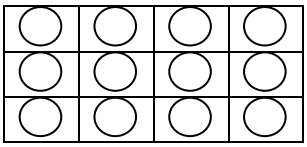
Have students glue down their cereal arrays so they can hold them up to share with the class. After students have completed the task, allow them to share their arrays. Invite discussion about how two students might view or describe the same array differently. For example:

Student 1

I see 3 pieces of cereal in each column and there are 4 columns. So I added: 3 + 3 + 3+ 3. That equals 12.



Student 2
I see 4 pieces of cereal in each row and there are 3 rows. So I added 4 + 4 + 4. That equals 12.



Encourage students to ask questions of their peers and make comments about the work and strategy used to figure out how to make their arrays.

Part IV
Start off with the question, "What happens if you rotate/turn the cereal array that you made? Does it change the number sentence you write? How and why?"
Invite further discussion about what happens when you rotate or turn someone's array a ¼ turn. Allow students to come up and act it out by actually rotating their paper. Have the class generate the new repeated addition equation that now goes with the array. Record these new repeated addition sentences on the board. Have students return to their seats, give them enough cereal to create the "new" array (the rotated one) and have them record the repeated addition equation that goes with this recreated array. This supports understanding of the commutative property, and allows students to remain flexible in their thinking.

FORMATIVE ASSESSMENT QUESTIONS

- Describe how you know how to rotate/turn your array? How does the sentence change when it turns? Does the answer change?
- What is your repeated addition number sentence?
- How many rows should you include?
- How many columns should you include?
- Why are arrays important?

DIFFERENTIATION

Extension
- Supply cards with larger addends. These students may also be encouraged to create as many arrays as possible with the same number of items.

Intervention
- Use 1 inch graph paper to line up the cereal correctly in arrays.

Intervention Table

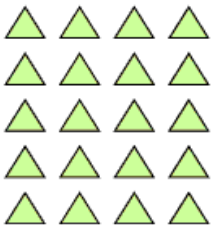
Sample Pictures for Repeated Addition

Practice Task: Roll an Array

Back to Task Table

Approximately 1 day

In this task, students will build on their understanding of rows, columns, arrays, and repeated addition equations by using dice/number cubes to create arrays.



STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. **Make sense of problems and persevere in solving them.**
4. **Model with mathematics.**
   Students use number cubes to create arrays.
5. **Use appropriate tools strategically.**
   Students use arrays determine repeated addition equations and sums.
6. **Attend to precision.**

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades K-3, page 83)

"A model not generally used for addition but extremely important and widely used for multiplication and division is the array. An *array* is any arrangement of things in rows and columns, such as a rectangle of square tiles or blocks.

To make the clear connection to addition, early multiplication activities should also include writing an addition sentence for the same model."

ESSENTIAL QUESTIONS

- What is an array?
- What is repeated addition?
- How can rectangular arrays help us with repeated addition?
- How are arrays and repeated addition related?
- How can we use model repeated addition equation with an array?

MATERIALS

- Dice or number cubes with number 1 - 9
- Roll an Array activity sheet

GROUPING

Large Group, Partners

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Special note: Students should have participated in the "Cereal Arrays" task before beginning this activity.

This task should begin with a brief discussion about what students know about rows and columns. An understanding of how the number of rows and columns can be used to create repeated addition equations and how they relate to arrays will create a readiness for students begin to understand the relationship between arrays, repeated addition, and multiplication.

Part I

Explain to students that they will be using dice/number cubes to create arrays and write repeated addition equations. Teachers should use this as an opportunity to model the expectations for this task. For example, the teacher rolls one dice/number cube to determine how many rows the array will have. Let's say the number is 3. The teacher records the number on the activity sheet. Next, the teacher rolls the same dice/number cube to determine how many columns the array will have. Let's say this number is 5. The teacher records the number on the activity sheet. At this point, teachers can engage students in a conversation about how to draw the array.
Ask:

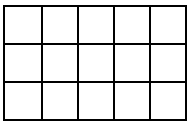
- I need to draw an array with 3 rows, which way do rows go? (across/left to right/side to side/horizontal)
- My array needs to have 5 columns, which way to columns go? (up and down/vertical)

Next, teachers are ready to model the array. Invite students to tell how to make the array using various objects (Xs, Os, etc.).

For example, students may say to draw 3 rows of Xs with 5 Xs in each row:

XXXXX
XXXXX
XXXXX

Or, students may say to draw a rectangle and divide it into 3 rows and 5 columns (this understanding will come from partitioning tasks found in Unit 5):



Explain that an array with 3 rows and 5 columns looks different than an array with 3 columns and 5 rows. Be sure to spend time reviewing rows and columns if students confuse the two.

Now teachers can ask students to determine the two possible repeated addition number models that relate to the array. You may get students thinking by saying "How many rows are there? (3) How many are in each row? (5)" "So, we can add the 5 three times."

5 + 5 + 5 =

Or, "How many columns are there? (5) How many are in each column? (3)" So, we can add the 3 five times."

3 + 3 + 3 + 3 + 3 =

Now teachers can ask students how many objects there are in all (the sum). Encourage students to explain their strategies for finding the sum.

Part II
Pass out a Roll an Array activity sheet to each student. Students will work with partners to create four arrays and write the related repeated addition equations. Explain that partners will take turns rolling the dice/number cubes, drawing arrays, and writing related repeated addition equations. Partners should work together to determine if the arrays are drawn correctly and the repeated addition equations are written correctly.

Teachers should monitor student progress and ask the following formative assessment questions.

FORMATIVE ASSESSMENT QUESTIONS

- Which way do rows go?
- Which way do columns go?
- How can arrays help you write related repeated addition equations?
- How can arrays help you find the sum of the objects?
- How can repeated addition equations help you find the sum?

DIFFERENTIATION

Extension

- Invite students to write story problems about their arrays.

Intervention

- Allow students to use square tiles, or other objects, to manipulate to better understand the various arrays.

Intervention Table

Name _____ Date _____

Roll an Array Activity Sheet

Work with your partner to create four different arrays.

- Roll one dice to determine how many rows your array will have. Record the number.
- Roll one dice to determine how many columns your array will have. Record the number.
- Draw the array.
- Write the two repeated addition equations that relate to your array.

Roll 1	Roll 2
Roll 3	Roll 4

Constructing Task: Seating the Class

Back to Task Table

Approximately 2 days
In this task, students will apply a growing understanding of arrays as they suggest an array for seating a class and explain their reasoning.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

MGSE2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

MGSE2.MD.10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)
Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. **Make sense of problems and persevere in solving them.**
1. **Construct viable arguments and critique the reasoning of others.**
   Students defend their strategies when making seating arrangement arrays.
2. **Model with mathematics.**
   Students model with arrays to determine seating arrangements.
6. **Attend to precision.**

BACKGROUND KNOWLEDGE
(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades K-3, page 83)

"In the beginning, children will be able to use the same models- sets and number lines- for all four operations. A model not generally used for addition but extremely important and widely used for multiplication and division is the array. An *array* is any arrangement of things in rows and column, such as a rectangle of square tiles or blocks.

To make the clear connection to addition, early multiplication activities should also include writing an addition sentence for the same model."

ESSENTIAL QUESTIONS

- What is an array?
- What is repeated addition?
- How can rectangular arrays help us with repeated addition?
- How are arrays and repeated addition related?
- How does skip counting help us solve repeated addition problems?
- How can we model repeated addition equation with an array?
- How can we determine if a number is odd or even?

MATERIALS

- Counters, square tiles, other manipulatives
- Half-Sheet of Chart Paper

GROUPING

Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I
Review the concepts of repeated addition or equal groups by demonstrating examples with students on the whiteboard using the following task:

Mrs. Evans wants to rearrange the desks in her classroom. She wants to organize the students to sit in an array, with each row having the same number of desks. If her class has twenty students, how many different arrays could Mrs. Evans make her desks? Explain your thinking using pictures, equations and words.

Put the students into small groups to allow them to decide on ways that Mrs. Evans could organize her class into equal rows. You may want to incorporate the word "group" so students can begin to focus on the grouping as it associates with rows and repeated addition. **You can expose them to the word divide (separate), but it should not be a focus as you are talking to the students about what they are doing.** Encourage students to come up with as many ways possible. Have students use the chart paper to demonstrate the various ways they divided up the 20 students, and help them explain the strategy they used. Make sure to have students explain the arrays they created using pictures, words, and repeated addition. Encourage conversations on how to use their understanding of odd and even to assist in this task.

Teacher note: Understanding the connection between repeated addition and equal groups is the goal; this will develop a foundation for multiplication. As students are working, look for students who have found various ways to organize the desks.

Part II
After students have completed the task, choose students who used a variety of strategies to share with the class. Act out some of their strategies. Keep a class chart to document the different ways the class could arrange the 20 students into rows. Ask students what they notice about the number 20. Is there only one way to separate the 20 students? What addend combinations created a total of 20? In other words, how many different repeated addition equations were we able to write?

Part III
Gather students to create a bar graph displaying the number of students that made each of the different arrays for 20.
Possible student created arrays: 1 x 20, 20 x 1, 2 x 10, 10 x 2, 4 x 5, 5 x 4.
Questions to ask:

- How many students created a _____ array?
- Which array did the most students create? The fewest students?
- What is the difference between the most and fewest students?

FORMATIVE ASSESSMENT QUESTIONS

- How many total students does Mrs. Evans have?
- What is your plan to arrange up the students?
- Will your rows be equal? Should they be? Why?
- How many students will be in each row? How did you figure that out?
- How can you show this through repeated addition?
- Is there another way you could group/divide up the students?
- What strategies did you use to group the 20 students?
- Is there only one way to separate the 20 students?
- What addend combinations created a total of 20?
- How many different repeated addition equations were we able to write?

DIFFERENTIATION

Extension

- If students complete the task, tell the students the class just received a new student. How will they rearrange the desks to fit 21 students?
- What arrays do you think would work best in a typical classroom? Why?

Intervention

- Allow students to use square tiles to manipulate to better understand that variety of arrays.

Intervention Table

Name: _____ Date _____

Desk Arrays in Many Ways

Mrs. Evans wants to rearrange the desks in her classroom. She wants to organize the students to sit in an array, with each row having the same number of desks. If her class has twenty students, how many different arrays could Mrs. Evans make her desks? Explain your thinking using pictures, equations, and words.

Practice Task: Pattern Block Drop

Back to Task Table

Approximately 2 days
Adapted from Even/Odd Pattern Block Grab from K5MathTeachingResources.com
In this task, students will use their knowledge of rows, columns, arrays, and repeated addition equations to determine the total number of sides of pattern block shapes.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

MGSE2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. **Make sense of problems and persevere in solving them.**
   Students use an understanding of rows and columns to solve mathematical problems.
4. **Model with mathematics.**
   Students use pattern blocks to model arrays and repeated addition.
6. **Attend to precision.**

BACKGROUND KNOWLEDGE

(Information adapted from Mathematics Georgia Standards of Excellence State Standards and Model Curriculum, Ohio Department of Education Teaching)

Build on knowledge of composing and decomposing numbers to investigate arrays with up to 5 rows and up to 5 columns in different orientations. For example, form an array with 3 rows and 4 objects in each row. Represent the total number of objects with equations showing a sum of equal addends two different ways: by rows, 12 = 4 + 4 + 4; by columns, 12 = 3 + 3 + 3 + 3. Rotate the array 90° to form 4 rows with 3 objects in each row. Write two different equations to represent 12 as a sum of equal addends: by rows, 12 = 3 + 3 + 3 + 3; by columns, 12 = 4 + 4 + 4. Have students discuss this statement and explain their reasoning: The two arrays are different and yet the same.

Ask students to think of a full ten-frame showing 10 circles as an array. One view of the ten-frame is 5 rows with 2 circles in each row. Students count by rows to 10 and write the equation 10 = 2 + 2 + 2 + 2 + 2. Then students put two full ten-frames together end-to-end so they form 10 rows of 2 circles or 10 columns of 2 circles. They use this larger array to count by 2s up to 20 and write an equation that shows 20 equal to the sum of ten 2s.

ESSENTIAL QUESTIONS

- What is an array?
- What is repeated addition?
- How can rectangular arrays help us with repeated addition?
- How are arrays and repeated addition related?
- How does skip counting help us solve repeated addition problems?
- How can we use model repeated addition equation with an array?

MATERIALS

- 1 inch Graph paper
- Glue

GROUPING

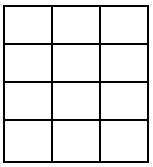
Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I
Review with the students what arrays are and why we use arrays to organize things. Ask students to describe what they know about rows and columns. Use the cereal arrays task to review how to write repeated addition equations to represent arrays. Review how to use arrays to represent objects. You may wish to model creating an array to represent the number of sides on 4 triangles. Say something like, "I know a triangle has 3 sides. I have 4 triangles. My columns are the number of sides and the rows are the number of triangles. How many columns will I have in my array? How many rows in each column? Draw the 4 x 3 and write repeated addition sentences for the array.

Student arrays may look like:

* * *
* * *
* * *
* * *



Part II
Each student will reach into a bag of pattern blocks and grab a handful. They will drop the handful of blocks onto their workspace and sort them by shape. They will then take each group of shapes (make sure to have all the same shapes in each group) and create an array to represent the number of sides included in the group. For example, I might grab 3 triangles, 7 squares and 5 hexagons. After sorting them, I will choose the first group to represent with an array. I choose the 5 hexagons. I have 5 sets of 6 sides in the hexagon group. I will create an array of 5 columns with 6 in each row. I will then describe the array with a repeated addition

equation. As students create their arrays, circulate the room and ask questions from the formative assessment list.

Part III
After students have completed part II of the task, allow them time to share their arrays. Invite discussion about the different strategies they might use to count the total number of sides represented by the array. Encourage students to ask questions of their peers and make comments about the work and strategy used to figure out how to make their arrays. **Lead a class discussion about how if you rotated these arrays ¼ of a turn that they would not represent the same group of pattern blocks. In this activity, allow students to describe their thinking and then lead them to an understanding of the columns represent the number of blocks and the rows represent the number of sides. If you rotate the array, it changes what is represented.**

FORMATIVE ASSESSMENT QUESTIONS

- Is the same information represented when you rotate the array? Why or why not?
- What is your repeated addition number sentence?
- How many rows should you include?
- How many columns should you include?
- Why are arrays important?
- How can arrays help us to multiply?

DIFFERENTIATION

Extension
- Have students create a graph to represent the shapes they drew from the bag.

Intervention
- Some students may need to use other strategies like counting by 1s, skip counting or repeated addition to determine the number of sides in the group instead of creating arrays.

Intervention Table

Constructing Task: The Queen's Dilemma

Back to Task Table

Approximately 3 days
In this, students will determine possible arrays and explain their mathematical thinking.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addend

MGSE2.MD.10. Draw a picture graph and bar graph (with single single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

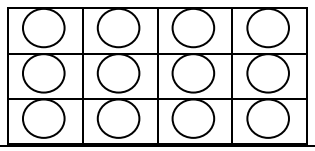
1. **Make sense of problems and persevere in solving them.**
3. **Construct viable arguments and critique the reasoning of others.**
   Students are defending strategies chosen to create arrays of ants.
6. **Attend to precision.**
8. **Look for and express regularity in repeated reasoning.**
   Students begin to make connections between how repeated addition can be used to solve math problems.

BACKGROUND KNOWLEDGE

This standard calls for students to use rectangular arrays to work with repeated addition. This is a building block for multiplication in 3rd Grade. Students should explore this concept with concrete objects (e.g., counters, bears, square tiles, etc.) as well as pictorial representations on grid paper or other drawings. Based on the commutative property of addition, students can add either the rows or the columns and still arrive at the same solution.
Example below:

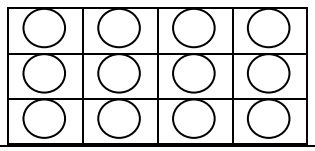
Student 1

I see 3 counters in each column and there are 4 columns. So I added: 3 + 3 + 3+ 3. That equals 12.



Student 2

I see 4 counters in each row and there are 3 rows. So I added 4 + 4 + 4. That equals 12.



ESSENTIAL QUESTIONS

- What is an array?
- What is repeated addition?
- How can rectangular arrays help us with repeated addition?
- How are arrays and repeated addition related?
- How does skip counting help us solve repeated addition problems?
- How can we use model repeated addition equation with an array?

MATERIALS

- *A Remainder of One* by Elinor J. Pinczes (Houghton Mifflin Co., 1995) or similar book such as *One Hundred Hungry Ants* by Elinor J. Pinczes
- Array recording sheet (per group)
- Half sheet of chart paper (per group)
- Various manipulatives (pop cubes, counters, tiles, etc.)

GROUPING

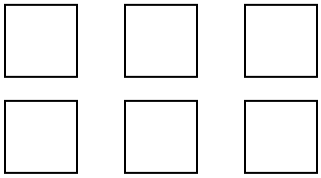
Partners

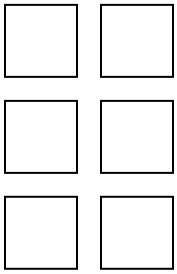
TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Part I

Begin the lesson with a review of arrays. Share pictures of arrays that can be seen in the real world and discuss the difference between a row and a column. Read aloud the book, *A Remainder of One*. Throughout the book, discuss the arrays with the students, focusing on the number of rows and columns and the product of each array. After reading the book, explain and demonstrate how the ants were not able to make equal rows with 2, 3 or 4 in a row but they could make equal rows with 5. Model the language 5 by 5. Indicate that the repeated addition sentence for this is 5+5+5+5+5. Remind the children this is called an array because you can make a rectangle with the number. Tell the students that now they will get to act as the Queen and will be given a certain number of ants to divide into an equal number of rows and an equal number of columns with nothing left over. For example, if she had only 6 soldiers in the group they could march in 2 rows with 3 in each row, a 2 by 3 array (2x3). Or they could march in 3 rows with 2 in each row, a 3 by 2 array (3x2). Keep in mind writing a multiplication equation is not something second grade students are required to master at this point. The focus is on the concept of multiplication and how we can model it with both repeated addition sentences and arrays.

(They can also march in 1 row of 6 or 6 rows of 1, in other words, single file as when going to lunch.)





So a group of six makes a rectangle (or an array).

Share the task with the students:

The queen of the ___(insert school name)____ Ant Colony has 16 ants in her army. The queen is attempting to organize her ants into arrays. She wants to know how many arrays she can create using her 16 ants. How many arrays can she make with 16 ants? Using manipulatives, drawings and words, explain your work. What arrays could she make with numbers fewer than 16?

Allow the students to work with a partner to experiment with different arrays that the queen could create when arranging the ants. Have the students use manipulatives to create their arrays. Then, encourage the students to draw their arrays on the chart paper and record the repeated addition equation on the recording sheet. While students are working, circulate the room and ask questions from the formative assessment list.

Part II
After students have completed Part I of the task, choose several students to share their discoveries and observations with the class.

If students used numbers up to 16, use this to lead discussion: The teacher or students can record the arrays the students have found on a larger version of the "The Queen's Dilemma" sheet. Ask questions which require them to look for patterns. Some things that they may notice are:

- 5, 7, 11 and 13 can only march in single file
- 4, 9 and 16 can be made into squares with equal sides (With only 1 possible repeated equation)
- 2,4,6,8,10,12,14 and 16 can all be divided into 2 equal rows

As the students "discover" this information, use this data to create a graph that shows how many of the numbers could only create single file lines, made squares, 2 equal rows or others. Graphing this data will reveal patterns about numbers, allowing for discussion of odd and even, among other things.

FORMATIVE ASSESSMENT QUESTIONS

- What manipulatives are you using to help solve this problem?
- Why are you arranging the tiles in that way?
- What are some ways the queen could arrange 6 ants? 10? 16? etc.
- What are some ways that she cannot arrange them?
- How many rows does this array have?
- How many columns does this array have?
- How can you tell the difference between rows and columns?
- What strategies are you using to help figure out ways the queen could arrange the ants?
- Are you noticing anything about the numbers that she is or is not able to use?
- How could you use repeated addition to help you solve this problem?
- Could this number be arranged in a different way?
- How would the equation be different if this array were rotated a ¼ turn?
- How are you communicating the results you have found?
- What patterns are you noticing from your chart?
- Why are we able to make different arrays for some numbers but not others?
- Do any of the arrays you have made have the same product?

DIFFERENTIATION

Extension

- If students complete the assigned task, ask students what additional arrays the ants could march in if the queen allowed 14 more ants to join the army.

Intervention

- Students who are having difficulty may need additional questioning with the use of manipulatives.

Intervention Table

Group Members: _____ Date: _____

The Queen's Dilemma Recording Sheet

Number of ants	Arrays	Repeated Addition Equations
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		

Practice Task: Mathemagicians

Back to Task Table

Approximately 3 days

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addend

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

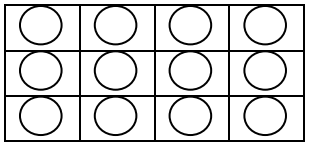
1. **Make sense of problems and persevere in solving them.**
   Students persist in finding all the arrays for each number.
6. **Attend to precision.**
7. **Look for and make use of structure.**
   Students understand that a 4 x 5 array is the same as 5 x 4.

BACKGROUND KNOWLEDGE

This standard calls for students to use rectangular arrays to work with repeated addition. This is a building block for multiplication in 3rd Grade. Students should explore this concept with concrete objects (e.g., counters, bears, square tiles, etc.) as well as pictorial representations on grid paper or other drawings. Based on the commutative property of addition, students can add either the rows or the columns and still arrive at the same solution.
Example below:

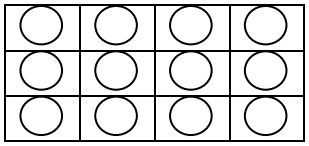
Student 1

I see 3 counters in each column and there are 4 columns. So I added: 3 + 3 + 3+ 3. That equals 12.



Student 2

I see 4 counters in each row and there are 3 rows. So I added 4 + 4 + 4. That equals 12.



ESSENTIAL QUESTIONS

- What is an array?
- What is repeated addition?
- How can rectangular arrays help us with repeated addition?

ESSENTIAL QUESTIONS

- How are arrays and repeated addition related?
- How does skip counting help us solve repeated addition problems?
- How can we use model repeated addition equation with an array?

MATERIALS

- Construction paper
- Various manipulatives (snap cubes, counters, tiles, coins etc.)

GROUPING

Partners

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I
Begin the lesson with a review of arrays. Share the pictures of arrays created in the task "The Queen's Dilemma" and discuss the difference between a row and a column. Make sure you model the language 3 by 3. Indicate that the equation for this is 3+3+3. Ask the children this is called an array because you can make a rectangle with the number. Tell the students that now they will be the Mathemagician and will magically build all of the arrays possible for each number 1-25. Remind them to divide the numbers into an equal number of rows and an equal number of columns with nothing left over. For example, with the number 10 one can create 1 row with 10 in it, 2 rows with 5 in each row, 5 rows with 2 in each row, and 10 rows with 1 in each row. Encourage the children to look for the real magic in the numbers and discover ALL of the possibilities for their given numbers and to describe the array in all of the ways possible (repeated addition, a 5 by 2 array, etc.) **Keep in mind writing a multiplication equation is not something second grade students are required to master at this point**. The focus is on finding the total number of objects and how we can model it with both repeated addition sentences and arrays.

Part II
Allow the students to work with a partner to experiment with different arrays that the Mathemagician could create when describing numbers 1-25. Have the students use manipulatives to create their arrays. Then, encourage the students to draw their arrays on the construction paper and record the repeated addition equation next to the array. While students are working, circulate the room and ask questions from the formative assessment list. Each group of students will have one index card for each number 1-25 showing the

arrays possible for that given number. You may want to consider having them create books with these index cards so that they may continue to share them after the lesson is complete.
Variations:

- The teacher may wish to assign a number or numbers to the partners, have them construct their arrays on construction paper, and make a class book to share.
- A bulletin board may be created by using graph paper cut out by the students representing all the arrays for their number or numbers.

Part III
After students have completed Part II of the task, choose several students to share their discoveries and observations with the class.

Lead a discussion about what the discovered about each number. Compare the information/data to the discoveries from the task "The Queen's Dilemma", is there any new data/information to add to the graph? Ask questions which require them to look for patterns. Some things that they may notice are:

- 1, 2, 3, 5, 7, 11, 13, 17, 19 and 23 can only have in single file rows/columns
- 4, 9, 16 and 25 can be made into squares with equal sides
- 2,4,6,8,10,12,14, 16, 18, 20, 22, and 24 can all be divided into 2 equal rows
- 15, and 21 are odd numbers but they have more arrays than just single file rows/columns

FORMATIVE ASSESSMENT QUESTIONS

- What manipulatives are you using to help solve this problem?
- Why are you arranging the tiles in that way?
- What are some ways the Mathemagician could arrange the number 6? 10? 16? etc.
- What are some ways that she/he cannot arrange them?
- How many rows does this array have?
- How many columns does this array have?
- How can you tell the difference between rows and columns?
- What strategies are you using to help figure out ways the Mathemagician could arrange the numbers?
- Are you noticing anything about the numbers that she is or is not able to use?
- How could you use repeated addition to help you solve this problem?
- Could this number be arranged in a different way?
- How would the equation be different if this array were rotated a ¼ turn?
- How are you communicating the results you have found?
- What patterns are you noticing from your chart?
- Why are we able to make different arrays for some numbers but not others?
- Do any of the arrays you have made have the same product?

DIFFERENTIATION

Extension

- Have the students identify the arrays that are similar or that have been rotated ¼ of a turn. They are different, yet the same. Ask them to tell why.

Intervention

- Students who are having difficulty may need additional questioning with the use of manipulatives.

Intervention Table

CONSTRUCTING TASK: No, You Can't

Back to Task Table

Approximately 1-2 Days

In this task, students will use an understanding of arrays to determine which brother is correct in his thinking and justify their reasoning.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. **Make sense of problems and persevere in solving them.**
3. **Construct viable arguments and critique the reasoning of others.**
   Students defend their decisions about which brother is right, as well as listen to the reasoning of others.
6. **Attend to precision.**
7. **Look for and make use of structure.**
   Students look for patterns in arrays.

*****Mathematical Practices 1 and 6 should be evident in EVERY lesson. *****

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle, Karp, and Bay-Williams, Elementary and Middle School Mathematics: Teaching Developmentally, page 266-267)

This task requires the students to build conjectures of their own and then participate in meaningful discussion. This is a direct reflection of the Standards of Mathematical Practices which should be incorporated within every task.

"It is important that all students initiate conjectures. It is important that all students actively consider the validity of all conjectures made by classmates. When deciding if a conjecture is always true, have students write their ideas before sharing with the class. If you begin with a class discussion, only a few students are likely to participate, with others content to listen whether or not they are following the arguments. You can then use both what the students write as well as their input in discussions to assess what level of reasoning they are at: authority, use of examples, or an appeal to logic."

ESSENTIAL QUESTIONS

- What is an array?
- What is repeated addition?
- How can rectangular arrays help us with repeated addition?
- How are arrays and repeated addition related?
- How does skip counting help us solve repeated addition problems?
- How can we use model repeated addition equation with an array?

MATERIALS

- Recording Sheet with table

GROUPING

Small Group, Partners

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I
Within a small group, provide students with the problem:

Lou, Stu, and Moe are triplets who love to argue. They always argue until one would prove the other two wrong. This time they're stuck and need your help.

- *Lou says every number (0 to 25) can be represented in an array that has two or more rows.*
- *Stu says you can make an array with 2 or more rows for fewer than 16 of the numbers (0 to 25).*
- *Moe says you can make an array with 2 or more rows for more than 16 of the numbers (0 to 25).*

Who is right? If you know a brother is wrong, you must prove it to them using numbers, pictures and words to show your thinking!

Students may wish to organize their thinking using the Recording Sheet with table.

Part II
Have groups create an anchor chart stating which brother they feel is right and what lead them to this understanding. After all the students have created their anchor charts, allow each group to present their discoveries. Once all the groups have shared their thoughts, open the class to a group discussion in which they can carry on a constructive, respectful, debate.

FORMATIVE ASSESSMENT QUESTIONS

- What information is important?
- Which brother do you think will be correct? Why?
- Can you see any mistakes that the brothers made?

DIFFERENTIATION

Extension

- What do all the numbers that have an array with 2 or more rows have in common? Create a rule and test it with larger numbers.

Intervention

- Provide students with a limited list of numbers 1-10. This will allow the students to still see the patterning.

Intervention Table

Name: _____ Date _____

No, You Can't!

Lou, Stu, and Moe are triplets who love to argue. They always argue until one would prove the other two wrong. This time they're stuck and need your help.

- Lou says every number (0 to 25) can be represented in an array that has two or more rows.

- Stu says you can make an array with 2 or more rows for less than 16 of the numbers (0 to 25).

- Moe says you can make an array with 2 or more rows for more than 16 of the numbers (0 to 25).

Who is right? If you know a brother is wrong, you must prove it to them using numbers, pictures and words to share your thinking! Otherwise, they will keep arguing!!!!!

Name: _____

Write the repeated addition sentence for the numbers that can be modeled in an array with 2 or more rows

	Equation
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

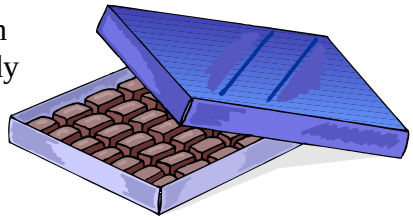
	Equation
13	
14	
15	
16	
17	
18	
19	
20	
21	
22	
23	
24	
25	

Culminating Task: The Candy Box

Back to Task Table

Approximately 2 – 3 days

In this culminating task, students will apply their understanding of odd/even numbers, arrays, repeated addition equations, and graphing to design a candy box array and create a related bar graph. This task will provide an opportunity for students to demonstrate an understanding of the concepts addressed throughout this unit.



STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

MGSE2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

MGSE2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. **Make sense of problems and persevere in solving them.** *Students use knowledge of rows and columns to determine possible arrays*
2. **Reason abstractly and quantitatively.** *Students make connections between arrays and repeated addition equations.*
3. **Construct viable arguments and critique the reasoning of others.** *Students choose arrays and explain their mathematical thinking.*
4. **Model with mathematics.** *Students use candy pictures to create arrays and use the information (data) to create bar graphs.*
3. **Use appropriate tools strategically.** *Students use arrays and bar graphs to solve mathematical problems.*
4. **Attend to precision.** *Students use precise mathematical language as they communicate their thinking.*

7. Look for and make use of structure. *Students develop mental math strategies as they make connections to number patterns (arrays/repeated addition equations).*
8. Look for and express regularity in repeated reasoning. *Students make connections between using repeated addition equations to solve mathematical problems.*

BACKGROUND KNOWLEDGE

Students should have had multiple experiences with repeated addition, arrays, and graphing. This task serves as the final opportunity for students to express their understanding of the connection between arrays and repeated addition.

ESSENTIAL QUESTIONS

- What is an array?
- What is repeated addition?
- How can rectangular arrays help us with repeated addition?
- How are arrays and repeated addition related?
- How does skip counting help us solve repeated addition problems?
- How can we model repeated addition on the number line?
- How can we use model repeated addition equation with an array?

MATERIALS

- Selection of egg cartons, candy boxes, etc.
- Half a sheet of chart paper per student.
- The Candy Box activity sheet
- The Candy Box candies master

GROUPING

Individual

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I
Bring in various containers from home that are examples of arrays, such as candy boxes, egg cartons, etc. and show them containers to students. Encourage students to discuss why they think the boxes are organized in this way. Ask students what repeated addition equations they can make and determine what to total number of objects would be for each example.

Give students this task:
The owner of a candy store wants to package candies in different sized rectangular boxes. The candy store has four kinds of candies mixed in each box. Explain that the owner is thinking about

the different rectangular boxes he could use to package the candies. The owner wants to have (no more than 24) candies in the candy box. What kind of rectangular array do you think the candy store owner will like? Be prepared to explain your thinking.

Part II
Begin by challenging students use the chart paper to create as many arrays they can for amount of candies the owner wants in the box. Each student will then choose one array to create their candy box.

Each student will need one The Candy Box activity sheet to create their candy box. It is suggested that students first organize their candy pieces into their desired array (do not glue them yet). Students will then draw a rectangle to form the shape of the candy box. Inside the candy box, the students will arrange any combination of chocolate, peppermint, caramel filled chocolate, and taffy candies in rows and columns and glue them to create their arrays. Students should use all four types of candies. At this point, students will answer the related questions. In doing so, students should demonstrate an understanding of rows, columns, arrays, and repeated addition equations.

Part III
Using information from their candy boxes, each student will create a bar graph to organize the types of candies in his/her candy box and determine how many of each there are. Students should exhibit and understanding of how the data (information) on their graphs can help them solve a variety of questions.

Part IV
After students have completed the task, gather the class together to discuss the various candy box arrays. Invite students to explain why they chose their arrays. This activity can be extended by creating a chart of similarities and differences students notice about the different candy box arrays.
This activity can be extended further with a discussion of the data collected in the graphs. Possible questions:

- If we combined all of the candy boxes, how many (chocolate/caramel filled chocolate/peppermint/taffy) candies would there be altogether?
- If we combined all of the candy boxes, what candy has the largest amount?
- If we combined all of the candy boxes, what candy has the smallest amount?
- If we combined all of the candy boxes, what is the difference between the amounts of chocolate and caramel candies?
- If we combined all of the candy boxes, how many peanut cluster or coconut candies are there?

FORMATIVE ASSESSMENT QUESTIONS

- How many total candies does the candy store owner want in each box?
- Why did you choose your candy box array?
- How many rows do you have?
- How many columns do you have?
- How can you show this through repeated addition?
- Is there another way you could arrange the candy box?
- Is there only one way to arrange the candies?
- How does your array help you determine how many candies there are in all?
- How does repeated addition help you determine how many candies there are in all?
- How does your candy box array help you organize your data for your bar graph?
- What strategies can you use to tell if a number is odd or even?

DIFFERENTIATION

Extension

- Allow the students to create a 3-D model of one of their candy box designs. Then, allow students to imagine that they are trying to sell the box model to the owner. Have them write a proposal explaining their model and why it is the best choice for the candy store.

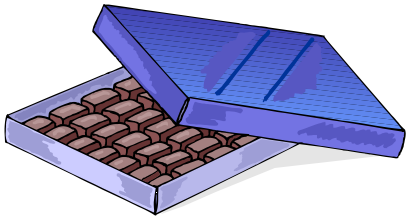
Intervention

- Some students may need to use the example boxes that you brought in from home to gain ideas about the arrangement of the candy box.
- Allow students to use manipulatives to create their arrays before they begin The Candy Box activity sheet.

Intervention Table

Name _____ Date_____

The Candy Box



The owner of a candy store wants to package candies in different sized rectangular boxes. The candy store has four kinds of candies mixed in each box. The owner is thinking about the different rectangular boxes he could use to package the candies. The owner wants to have _____ candies in the candy box. What kind of rectangular array do you think the candy store owner will like?

Create your candy box array by drawing a rectangle with rows and columns. Choose candy pieces and glue them onto your candy box array.

Answer the following questions about your array.

How many rows does your array have? ___ How many columns? ___

What is the difference between rows and columns?

_____

_____

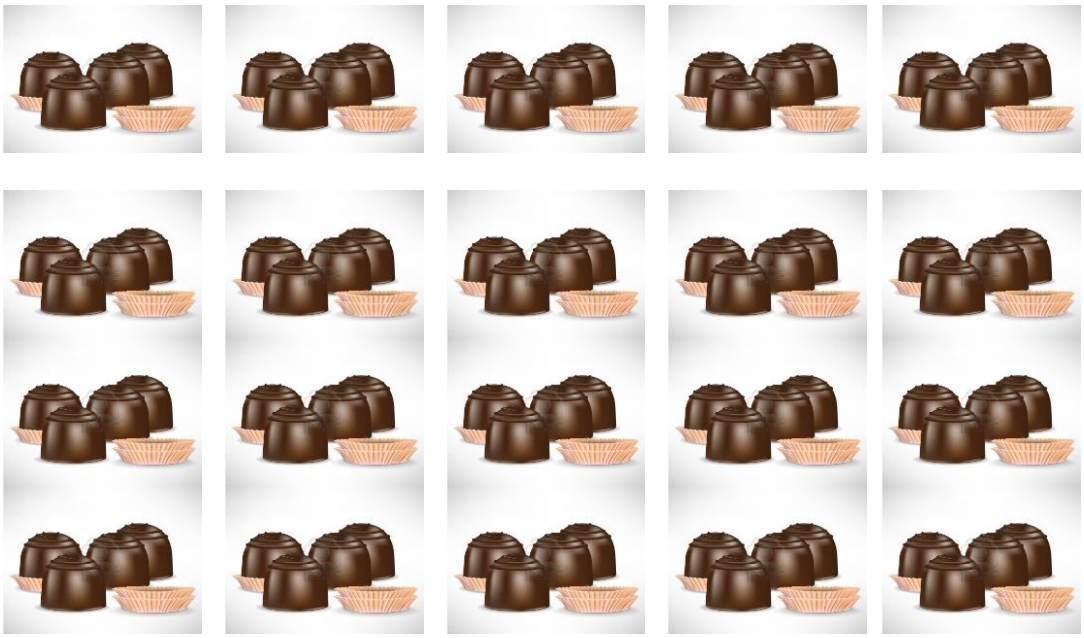
What two repeated addition equations relate to your array?

What is the total amount of candy pieces? ___ Is the number odd or even? ____
Explain how you know.

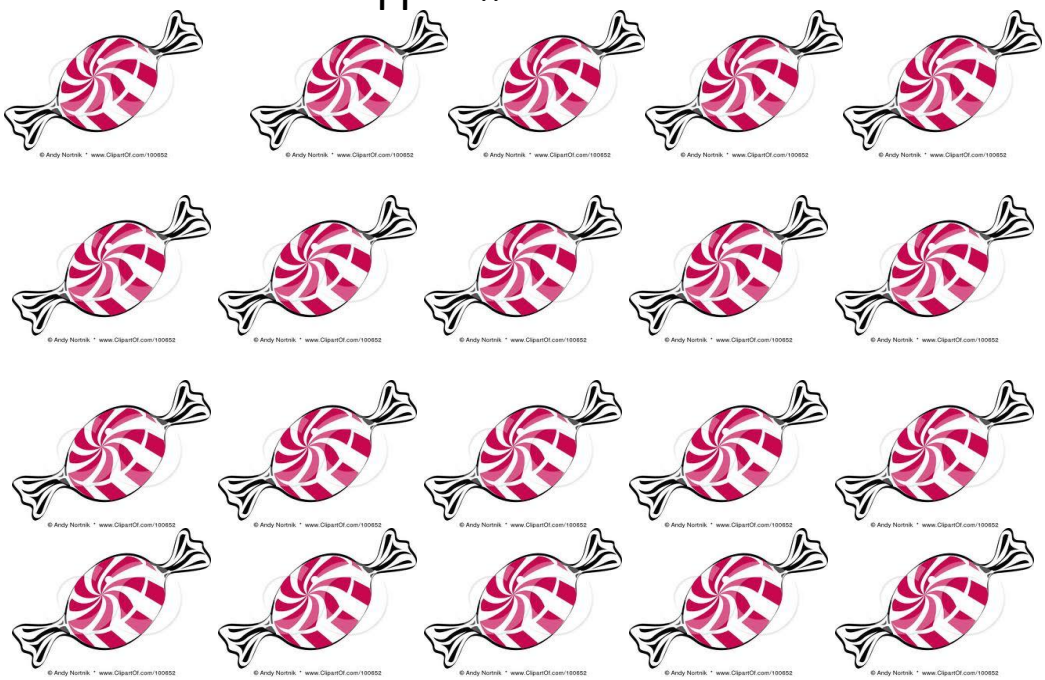
_____

_____

Caramel filled candies

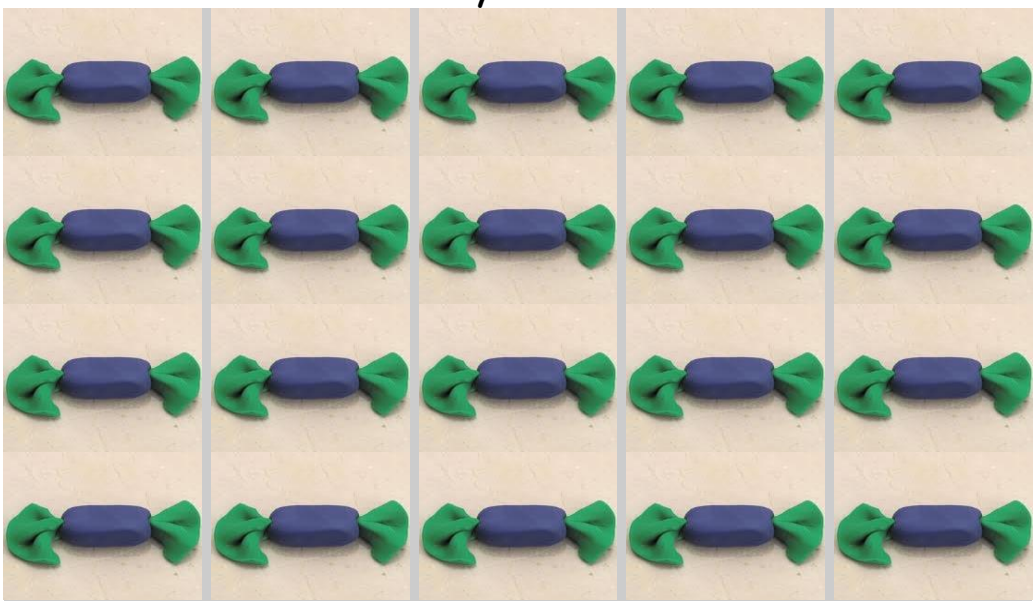


Peppermint candies



Chocolate candies

Taffy candies



Name _____ Date _____

Create a bar graph using the information from your candy box array. Be sure to label your graph. The title is already completed for you.

The Candy Box Bar Graph

Answer the following questions about your bar graph.

How many chocolate candies does your graph show? ____

How many caramel filled chocolate candies and peppermint candies does your graph show? ___

Which type of candy has the largest amount? ___

What is the difference between the largest amount and the smallest amount? ___

How can you use the bar graph to get information?

_____

3-ACT TASK: Staples

Back to Task Table

Adapted from: Estimation 180
Approximately One Class Session
In this task, students use their knowledge of arrays and repeated addition to solve the problem.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students are required to figure out a question to work through, the information they need to solve the problem, and then persevere until solving it.
2. Reason abstractly and quantitatively. Students are asked to make an estimate both high and low.
3. Construct viable arguments and critique the reasoning of others. Students will use known information about arrays to construct viable arguments about them.
4. Model with mathematics. In early grades, students experiment with representing problem situations in multiple ways including numbers, pictures, and creating equations.
6. Attend to precision. Students will use clear and precise language when discussing their strategies and sharing their solutions with others.
8. Look for and express regularity in repeated reasoning. In the early grades, students notice repetitive actions in counting and computations.

ESSENTIAL QUESTIONS

- When is using arrays helpful?

MATERIALS

- Staples Picture
- Student Handout

GROUPING

Individual/Partner Task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this task, students will view the picture and tell what they noticed. Next, they will be asked to discuss what they wonder about or are curious about. These questions will be recorded on a class chart or on the board and on the student recording sheet. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them.

Background Knowledge:

This task follows the 3-Act Math Task format originally developed by Dan Meyer. More information on this type of task may be found at http://blog.mrmeyer.com/category/3acts/. A Three-Act Task is a whole-group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the ***Guide to Three-Act Tasks*** on georgiastandards.org.

In this task students will be shown a picture of a box of staples. They will look for the array made by the ends of the staples and use that knowledge to figure out how many staples are in the box. Students should have a good understanding of addition before completing this task since repeated addition will be required to solve it.

Common Misconceptions:

With regard to an understanding of arrays, students may at first confuse rows and columns. Children should be provided with numerous chances to make real life connections to examples of rows and columns (some picture cards are provided in this unit). To further support an understanding, teachers may encourage students to use their hands to show the direction rows and columns go. As students move on to explore arrays, they may struggle with creating arrays. For example, students may not know where to begin as they attempt to create an array with 18 objects. Teachers should encourage students to look for equal groups (repeating number patterns) that can be added to make a total of 18. Multiple opportunities will increase student fluency with these tasks. When relating arrays to repeated addition equations, students who do not naturally make connections between the two may be guided towards discovery with questions such as "How many rows do you see in the array?" "How many objects are in each row?" "Does each row have the same amount of objects?" "How can you add these numbers to find the total number of objects?"

Task Directions:

Act I – Whole Group - Pose the conflict and introduce students to the scenario by showing Act I picture. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/**)**
"Introduce the central conflict of your story/task clearly, visually, viscerally, using as few words as possible."

5. Show picture of staples to students.
6. Ask students what they noticed in the picture. The teacher records this information.
7. Ask students what they wonder about and what questions they have about what they saw. Students should share with each other first, and then the teacher records these questions (think-pair-share). The teacher may need to guide students so that the questions generated are math-related.
8. Ask students to estimate answers to their questions (think-pair-share). Students will write their best estimate, then write two more estimates – one that is too low and one that is too high so that they establish a range in which the solution should occur.

Anticipated questions students may ask and wish to answer:

- How many staples are in the box? *
- How many staples are in the stapler?
- How many staples are outside the box?
- How many staples are in the box and outside the box? *
- How many more staples are in the box than outside the box? *

*Main question(s) to be investigated

Act 2 – **Student Exploration** - Provide additional information as students work toward solutions to their questions. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/**)**
"The protagonist/student overcomes obstacles, looks for resources, and develops new tools."

- During Act 2, students determine the main question(s) from Act 1 and decide on the facts, tools, and other information needed to answer the question(s). When students decide what they need to solve the problem, they should ask for those things. It is pivotal to the problem solving process that students decide what is needed without being given the information up front. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin.

- The teacher provides guidance as needed during this phase. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin. Questioning is an effective strategy that can be used, with questions such as:
  - What is the problem you are trying to solve?
  - What do you think affects the situation?
  - Can you explain what you've done so far?
  - What strategies are you using?
  - What assumptions are you making?
  - What tools or models may help you?
  - Why is that true?
  - Does that make sense?

Additional Information for Act 2

- The array is a 3 x 4 array
- There are 210 staples in each row

Important note: Although students will only investigate the main question(s) for this task, it is important for the teacher to not ignore student generated questions. Additional questions may be answered after they've found a solution to the main question, or as homework or extra projects.

Act 3 – Whole Group – Share solutions and strategies.

- Students to present their solutions and strategies and compare them.
- Reveal the solution. There are 2,520 staples in the box.
- Lead discussion to compare these, asking questions such as:
    - How reasonable was your estimate?
    - Which strategy was most efficient?
    - Can you think of another method that might have worked?
    - What might you do differently next time?

Act 4, The Sequel - "The goals of the sequel task are to a) challenge students who finished quickly so b) I can help students who need my help. It can't feel like punishment for good work. It can't seem like drudgery. It has to entice and activate the imagination." Dan Meyer
http://blog.mrmeyer.com/2013/teaching-with-three-act-tasks-act-three-sequel/

- Challenge students to pick a different question from the student generated list to answer.
- Challenge students to figure out how many boxes are needed to have 10,000 staples.
- Challenge students to solve how many staples there would be if I used two rows.

FORMATIVE ASSESSMENT QUESTIONS

- How reasonable was your estimate?
- What might you do differently next time?
- What worked well for you this time?
- What model did you use?
- What organizational strategies did you use?

DIFFERENTIATION

Extension

- Have students use a picture of a larger box of staples so that the numbers are more challenging.

Intervention

- Give students a smaller box of staples so that the numbers are easier to work with.

Intervention Table

Name _____ Date _____

Staples

What problem are you trying to figure out?		
What information do you already know?	What information do you need to solve the problem?	
Make an estimate.	Write an estimate that's too low.	Write an estimate that's too high.
Show your estimates on a number line:		
Show your work.		
What is your conclusion?		