Georgia Standards of Excellence Curriculum Frameworks

Mathematics

GSE Fifth Grade
Unit 6: Volume and Measurement

Georgia Department of Education

Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

Unit 6: VOLUME AND MEASUREMENT

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IF YOU HAVE NOT READ THE 5th GRADE CURRICULUM OVERVIEW IN ITS ENTIRETY PRIOR TO USE OF THIS UNIT, PLEASE **STOP** AND CLICK HERE:
https://www.georgiastandards.org/Georgia-Standards/Frameworks/5th-Math-Grade-Level-Overview.pdf
Return to the use of this unit once you've completed reading the Curriculum Overview. Thank you!

OVERVIEW

CONVERT LIKE MEASUREMENT UNITS WITHIN A GIVEN MEASUREMENT SYSTEM

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **conversion/convert, metric and customary measurements.**
From previous grades: relative size, liquid volume, mass, weight, length, kilometer (km), meter (m), centimeter (cm), kilogram (kg), gram (g), liter (L), milliliter (mL), inch (in), foot (ft), yard (yd), mile (mi), ounce (oz), pound (lb), cup (c), pint (pt), quart (qt), gallon (gal), hour, minute, second

REPRESENT AND INTERPRET DATA

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **line plot, length, mass, liquid volume.**

GEOMETRIC MEASUREMENT: UNDERSTAND CONCEPTS OF VOLUME AND RELATE VOLUME TO MULTIPLICATION AND TO ADDITION

Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **measurement, attribute, volume, solid figure, right rectangular prism, unit, unit cube, gap, overlap, cubic units (cubic cm, cubic in. cubic ft. nonstandard cubic units), multiplication, addition, edge lengths, height, area of base.**

In this unit students will:

- change units to related units within the same measurement system by multiplying or dividing using conversion factors.
- use line plots to display a data set of measurements that includes fractions.
- use operations to solve problems based on data displayed in a line plot.
- recognize volume as an attribute of three-dimensional space.

- understand that volume can be measured by finding the total number of same size units of volume required to fill the space without gaps or overlaps.
- understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume.
- select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume.
- decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes.
- measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.
- communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language.

Students convert measurements within the same system of measurement in the context of multi-step, real world problems. Both metric and customary measurement systems are included, but the emphasis in the standards is on metric measure. Although students should be familiar with the relationships between units within either system, the conversion may be provided to them when they are solving problems. For example, when determining the number of feet there are in 28 inches, students may be provided with 12 inches = 1 foot. Students will explore how the base ten system supports conversions within the metric system. For example, 100 cm = 1 meter; 1.5 m = 150 cm. This builds on previous knowledge of placement of the decimal point when multiplying and dividing by powers of 10.

Students use measurements with fractions to collect data and graph it on a line plot. Data may include measures of length, weight, mass, liquid volume and time. Students will use data on the line plots to solve problems that may require application of operations used with fractions in this grade level. Operations with fractions may include addition and subtraction with unlike denominators, fraction multiplication, and fraction division which involve a whole number and a unit fraction.

Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

For more detailed information about unpacking the content standards, unpacking a task, math routines and rituals, maintenance activities and more, please refer to the Grade Level Overview.

STANDARDS FOR MATHEMATICAL PRACTICE

This section provides examples of learning experiences for this unit that support the development of the proficiencies described in the Standards for Mathematical Practice. These proficiencies correspond to those developed through the Literacy Standards. The statements provided offer a few examples of connections between the Standards for Mathematical Practice and the Content Standards of this unit. The list is not exhaustive and will hopefully prompt further reflection and discussion.

1. **Make sense of problems and persevere in solving them.** Students make sense that square units are used to measure 2-dimensional objects which have both length and width, and cubic units are used to measure 3-dimensional objects which have length, width, and height.

2. **Reason abstractly and quantitatively.** Students use reasoning skills to determine an average time by analyzing data and equally redistributing each data point. Students demonstrate abstract reasoning to create a display of square and cubic units in order to compare/contrast the measures of area and volume.

3. **Construct viable arguments and critique the reasoning of others.** Students construct and critique arguments regarding their knowledge of what they know about measurement, area and volume.

4. **Model with mathematics.** Students use line plots to show time measurements. Students use snap cubes to build cubes and rectangular prisms in order to generalize a formula for the volume of rectangular prisms.

5. **Use appropriate tools strategically.** Students select measurement tools to use for measuring length, weight, mass and liquid volume. Students also select and use tools such as tables, cubes, and other manipulatives to represent situations involving the relationship between volume and area.

6. **Attend to precision.** Students select appropriate scales and units to use for measuring length, weight, mass and liquid volume. Students attend to the precision when comparing and contrasting the prisms made using the same amount of cubes.

7. **Look for and make use of structure.** Students use their understanding of number lines to apply the construction of line plots. Students recognize volume as an attribute of solid figures and understand concepts of volume measurement. Students use their understanding of the mathematical structure of area and apply that knowledge to volume.

8. **Look for and express regularity in repeated reasoning.** Through experiences measuring different types of attributes, students realize that measurements in larger units always produce smaller measures and vice versa. Students relate new experiences to

experiences with similar contexts when studying a solid figure that can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

*****Mathematical Practices 1 and 6 should be evident in EVERY lesson*****

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.MD.1 Convert among different-sized standard measurement units (mass, weight, length, time, etc.) within a given measurement system (customary and metric) (e.g., convert 5cm to 0.05m), and use these conversions in solving multi-step, real word problems.

MGSE5. MD.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

MGSE5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

MGSE5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

MGSE5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

BIG IDEAS *From Teaching Student Centered Mathematics, Van de Walle & Lovin, 2006.*

- When changing from smaller units to larger related units within the same measurement system, there will be fewer larger units.
- A line plot can provide a sense of the shape of the data, including how spread out or how clustered the data points are. Each data point is displayed on the line plot along a continuous numeric scale, similar to a number line.
- Three-dimensional (3-D) figures are described by their faces (surfaces), edges, and vertices (singular is "vertex").
- Volume can be expressed in both customary and metric units.
- Volume is represented in cubic units – cubic inches, cubic centimeters, cubic feet, etc.
- Volume refers to the space taken up by an object itself.
- Measurement involves a comparison of an attribute of an item with a unit that has the same attribute. Lengths are compared to units of length, areas to units of area, time to units of time, and so on.
- Data sets can be analyzed in various ways to provide a sense of the shape of the data, including how spread out they are (range, variance).
- Volume is a term for measures of the "size" of three-dimensional regions.
- Volume typically refers to the amount of space that an object takes up.
- Volume is measured with units such as cubic inches or cubic centimeters-units that are based on linear measures.
- Two types of units can be used to measure volume: solid units and containers.

ESSENTIAL QUESTIONS

- What strategies can you use to estimate measurements?
- What happens to a measurement when you change its unit of measure to a related unit?
- How is data collected and displayed on a line plot?
- What strategies help when solving problems with line plots?
- How do we measure volume?
- How are area and volume alike and different?
- How can you find the volume of cubes and rectangular prisms?
- What is the relationship between the volumes of geometric solids?
- Why are some tools better to use than others when measuring volume?
- Why is volume represented with cubic units and area represented with square units?

CONCEPTS/SKILLS TO MAINTAIN

MGSE5.MD.1: Students progress through the underlying concepts of the measurement trajectory with the use of non-standard and standard units of measurement interchangeably. By the end of third grade, they will have learned how to use tools and appropriate units to measure metric and customary length, time, liquid volume, mass and weight. In fourth grade, students make conversions from larger units to smaller related units within the same measurement system by multiplying. All of these skills will be needed when they begin to make conversions from smaller units to larger units by dividing in fifth grade.

MGSE5.MD.2: In kindergarten, students begin working with categorical data. By the end of second grade, they will have learned how to draw line plots, picture graphs and bar graphs. In third and fourth grades, student draw scaled picture and bar graphs, graph measurement data on line plots, and solve problems using information from all three types of graphs. All of these skills will be applied in fifth grade when students use measurement data from line plots to solve problems.

MGSE5.MD.3, MGSE5.MD.4, and **MGSE5.MD.5:** These standards represent the first time that students begin exploring the concept of volume. In third grade, students begin working with area and covering spaces. The concept of volume should be extended from area with the idea that students are covering an area (the bottom of the cube) with a layer of unit cubes and then adding layers of unit cubes on top of the bottom layer. Students should have ample experiences with concrete manipulatives before moving to pictorial representations. Students' prior experiences with volume were restricted to liquid volume. As students develop their understanding volume they understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. This cube has a length of 1 unit, a width of 1 unit and a height of 1 unit and is called a cubic unit. This cubic unit is written with an exponent of 3 (e.g., in^3, m^3). Students connect this notation to their understanding of powers of 10 in our place value system. Models of cubic inches, centimeters, cubic feet, etc. are helpful in developing an image of a cubic unit. Students estimate how many cubic yards would be needed to fill the classroom or how many cubic centimeters would be needed to fill a pencil box.

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- number sense
- computation with whole numbers and decimals, including application of order of operations
- addition and subtraction of common fractions with like denominators
- angle measurement
- measuring length and finding perimeter and area of rectangles and squares
- characteristics of 2-D and 3-D shapes
- data usage and representations, including line plots, bar graphs and picture graphs
- convert from larger units to smaller metric or customary units using previous knowledge of relationships between related units

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply "how to get the answer" and instead support students' ability to access concepts from a number of perspectives. Therefore, students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students:

- flexibly use a combination of deep understanding, number sense, and memorization.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.

For more about fluency, see: http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and: https://bhi61nm2cr3mkdgk1dtaov18-wpengine.netdna-ssl.com/wp-content/uploads/nctm-timed-tests.pdf

STRATEGIES FOR TEACHING AND LEARNING

Convert like measurement units within a given measurement system.
MGSE5.MD.1

This standard calls for students to convert measurements within the same system of measurement in the context of multi-step, real-world problems. Both customary and standard measurement systems are included; students worked with both metric and customary units of length in second grade. In third grade, students work with metric units of mass and liquid volume. In fourth grade, students work with both systems and begin conversions within systems in length, mass and liquid volume.

To convert from one unit to another unit in the standard and metric system, the relationship between the units must be known. In order for students to have a better understanding of the relationships between units, they need to use measuring tools in class. The number of units must relate to the size of the unit.
Example 1: 100 cm = 1 meter
Example 2: 12 inches = 1 foot and 3 feet = 1 yard

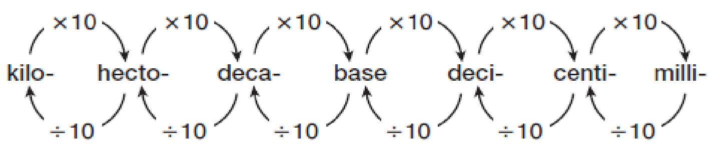
When converting in the metric system, have students extend their prior knowledge of the base-ten system as they multiply or divide by powers of ten (as referenced in Units 1 and 2). Teaching conversions should focus on the relationship of the measurements, not merely rote memorization. The questions ask the student to find out the size of each of the subsets. Students are not expected to know e.g. that there are 5280 feet in a mile. If this is to be used as an assessment task, the conversion factors should be given to the students. However, in a teaching situation it is worth having them realize that they need that information rather than giving it to them upfront; having students identify what information they need to have to solve the problem and knowing where to go to find it allows them to engage in Standard for Mathematical Practice 5, Use appropriate tools strategically.

Retrieved from Illustrative Mathematics
http://www.illustrativemathematics.org/standards/k8

The metric system of measurement is based on 10 and powers of 10. The prefixes used for length, capacity, and mass tell what part of the basic unit is being considered. The symbols for each unit of measure are given in parentheses ( ). The most commonly used units are shown in **bold** below.

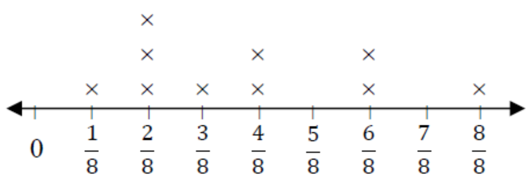
prefix	meaning	measure of length	measure of capacity	measure of mass
kilo-	1,000	**kilometer** (km)	kiloliter (kL)	**kilogram** (kg)
hecto-	100	hectometer (hm)	hectoliter (hL)	hectogram (hg)
deka-	10	dekameter (dkm)	dekaliter (dkL)	dekameter (dkg)
base unit	1	**meter** (m)	**liter** (L)	**gram** (g)
deci-	0.1	**decimeter** (dm)	deciliter (dL)	decigram (dg)
centi-	0.01	**centimeter** (cm)	centiliter (cL)	centigram (cg)
milli-	0.001	**millimeter** (mm)	**milliliter** (mL)	**milligram** (mg)

To change from a larger unit to a smaller unit, multiply by the appropriate power of 10.
To change from a smaller unit to a larger unit, divide by the appropriate power of 10.



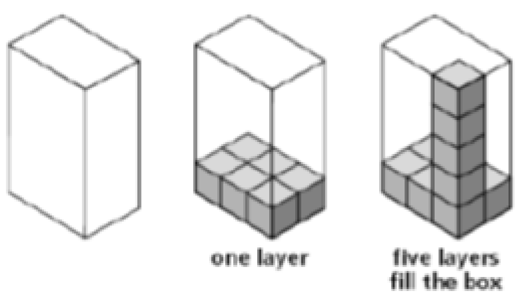
Represent and interpret data.
MGSE5.MD.2
This standard provides a context for students to work with fractions by measuring objects to one-eighth of a unit. This includes length, mass, and liquid volume. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot.

Example 1:
Students measured objects in their desk to the nearest 1/2, 1/4, or 1/8 of an inch then displayed data collected on a line plot. How many objects measured 1/4? 1/2? If you put all the objects together end to end what would be the total length of all the objects?



Geometric measurement: Understand Concepts of volume and relate volume to multiplication and to addition.
MGSE.MD.3 – MGSE.MD.4 – MGSE.MD.5

These standards involve finding the volume of right rectangular prisms and extend their understanding of finding the area of composite figures into the context of volume. Students should have experiences to describe and reason about why the formula is true. Specifically, that they are covering the bottom of a right rectangular prism (length x width) with multiple layers (height). Therefore, the formula (length × width × height) is an extension of the formula for the area of a rectangle.

	(3×2) represents the number of blocks in the first layer $(3 \times 2) \times 5$ represents the number of blocks in 5 layers 6×5 represents the number of blocks to fill the figure 30 blocks fill the figure

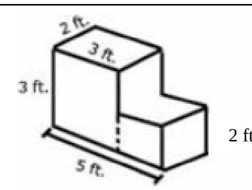
Example:

When given 24 cubes, students make as many rectangular prisms as possible with a volume of 24 cubic units. Students build the prisms and record possible dimensions.

Length	**Width**	**Height**
1	2	12
2	2	6
4	2	3
8	3	1

Example:

Students determine the volume of concrete needed to build the steps in the diagram at the right.

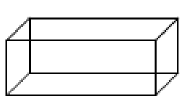


Volume refers to the amount of space that an object takes up and is measured in cubic units such as cubic inches or cubic centimeters.

Students need to experience finding the volume of rectangular prisms by counting unit cubes, in metric and standard units of measure, before the formula is presented. Provide multiple opportunities for students to develop the formula for the volume of a rectangular prism with activities similar to the one described below.

Give students one block (a 1- or 2- cubic centimeter or cubic-inch cube), a ruler with the appropriate measure based on the type of cube, and a small rectangular box. Ask students to determine the number of cubes needed to fill the box. Have students share their strategies with the class using words, drawings or numbers. Allow them to confirm the volume of the box by filling the box with cubes of the same size.

By stacking geometric solids with cubic units in layers, students can begin understanding the concept of how addition plays a part in finding volume. This will lead to an understanding of the formula for the volume of a right rectangular prism, b x h, where b is the area of the base. A right rectangular prism has three pairs of parallel faces that are all rectangles.



Have students build a prism in layers. Then, have students determine the number of cubes in the bottom layer and share their strategies. Students should use multiplication based on their knowledge of arrays and its use in multiplying two whole numbers.

Instructional Resources/Tools

- Cubes
- Rulers (marked in standard or metric units)
- Grid paper

http://illuminations.nctm.org/ActivityDetail.aspx?ID=6: Determining the Volume of a Box by Filling It with Cubes, Rows of Cubes, or Layers of Cubes
This cluster is connected to the third Critical Area of Focus for Grade 5, **Developing understanding of volume.**

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The terms below are for teacher reference only and are not to be memorized by the students.

- measurement
- attribute
- conversion/convert
- metric and customary systems
- metric and customary units of measure
- line plot
- length
- mass
- weight
- liquid volume
- volume
- solid figure
- right rectangular prism
- unit
- unit cube
- gap
- overlap
- cubic units (cubic cm, cubic in, cubic ft, nonstandard cubic units)

- edge lengths
- height
- area of base

MATHEMATICS GLOSSARY
http://www.corestandards.org/Math/Content/mathematics-glossary/glossary

TASKS
The following tasks represent the level of depth, rigor, and complexity expected of all fifth-grade students. These tasks or tasks of similar depth and rigor should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as a performance task, they also may be used for teaching and learning.

Scaffolding Task	Tasks that build up to the learning task.
Constructing Task	Constructing understanding through deep/rich contextualized problem-solving tasks.
Practice Task	Tasks that provide students opportunities to practice skills and concepts.
Performance Task	Tasks which may be a formative or summative assessment that checks for student understanding/misunderstanding and or progress toward the standard/learning goals at different points during a unit of instruction.
Culminating Task	Designed to require students to use several concepts learned during the unit to answer a new or unique situation. Allows students to give evidence of their own understanding toward the mastery of the standard and requires them to extend their chain of mathematical reasoning.
Intervention Table	The Intervention Table provides links to interventions specific to this unit. The interventions support students and teachers in filling foundational gaps revealed as students work through the unit. All listed interventions are from New Zealand's Numeracy Project.
Formative Assessment Lesson (FAL)	Lessons that support teachers in formative assessment which both reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards.
CTE Classroom Tasks	Designed to demonstrate how the Career and Technical Education knowledge and skills can be integrated. The tasks provide teachers with realistic applications that combine mathematics and CTE content.

3-Act Task	A Three-Act Task is a whole-group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the ***Guide to Three-Act Tasks*** on georgiastandards.org.

Task Name	Task Type *Grouping Strategy*	Content Addressed	Standard(s)	Task Description
Estimate, Measure, Estimate	Practice Task *Partner/Small Group Task*	Estimate and measure length, time, liquid volume, weight or mass and make conversions	MGSE5.MD.1	Estimating and measuring length, time, liquid, volume, weight or mass, and converting measurements
Water, Water	Practice Task *Partner/Small Group Task*	Compare and estimate quantities of water and make conversions using units of liquid volume and weight or mass	MGSE5.MD.1	Comparing amounts of water consumed with water used by astronauts
Sing a Song	Practice Task *Small Group Task*	Record times as fractions and mixed numbers on a line plot and redistribute those times equally to determine the average	MGSE5.MD.2	Recording amount of minutes used to sing a song or a poem, then plotted the times on a line plot
Survival Badge	Practice Task *Individual/Partner Task*	Use line plots to display a set of measurements	MGSE5.MD.2	Creating line plots to distribute water supply
Differentiating Area and Volume	Scaffolding Task *Small Group Task*	Investigate the relationships between area and volume	MGSE5.MD.3a-b	Creating square and cubic units to compare/contrast area and volume
Got Cubes?	Practice Task 3-Act Task *Partner/Small Group Task*	Determine the number of cubic units fill a rectangular prism	MGSE5.MD.3a-b MGSE5.MD.4 MGSE5.MD.5a-c	Wondering the volume of 3-D shapes
How Many Ways?	Constructing Task *Individual/Partner Task*	Develop a formula for determining the volume of cubes and rectangular prisms	MGSE5.MD.3a-b MGSE5.MD.4 MGSE5.MD.5a-c MGSE5.MD.1	Using snap-cubes to build rectangular prisms to generalize a formula for volume

Exploring with Boxes	Practice Task *Individual/Partner Task*	Use a chart to find volume	MGSE5.MD.3a-b MGSE5.MD.4 MGSE5.MD.5a-c	Creating boxes and discover how volume is related to length, width and height of cubes
Rolling Rectangular Prisms	Practice Task *Individual/Partner Task*	Find the volume of rectangular prisms	MGSE5.MD.3a-b MGSE5.MD.5a-c	Drawing and labeling rectangular prisms and then determining the volume
Books, Books, and More Books	Constructing Task *Individual/Partner*	Add to find the combined volume of multiple rectangular prisms	MGSE5.MD.3a-b MGSE5.MD.4 MGSE5.MD.5a-c	Determining volume as an additive
Super Solids	Practice Task *Individual/Partner Task*	Estimate and calculate the volume of rectangular prisms	MGSE5.MD.5a-c	Estimating and finding volumes of real-world objects
Toy Box Designs	Performance Task *Individual/Partner Task*	Design a toy box with a given volume	MGSE5.MD.3a-b MGSE5.MD.4 MGSE5.MD.5a-c	Designing a toy box with a given volume
Breakfast for All	Performance Task *Individual/Partner Task*	Create 3 different sized boxes for cereal	MGSE5.MD.3a-b MGSE5.MD.4 MGSE5.MD.5a-c	Designing three cereal boxes and determining the volume
Boxing Boxes	**Culminating Task** *Individual/Partner Task*	Consider volume and capacity to determine guidelines for packing boxes	MGSE5.MD.1 MGSE5.MD.3a-b MGSE5.MD.4 MGSE5.MD.5a-c	Exploring volume of various shaped boxes
The Fish Tank	**Culminating Task** **3-Act Task** *Partner/Small Group*	Use volume, time and liquid volume measurements to problem solve	MGSE5.MD.1 MGSE5.MD.4 MGSE5.MD.5a-c	Wondering the volume of a fish tank

If you need further information about this unit visit the GaDOE website and reference the unit webinars.
https://www.georgiastandards.org/Archives/Pages/default.aspx

INTERVENTION TABLE

The Intervention Table below provides links to interventions specific to this unit. The interventions support students and teachers in filling foundational gaps revealed as students work through the unit. All listed interventions are from New Zealand's Numeracy Project.

Cluster of Standards	Name of Intervention	Snapshot of summary or Student I can statement. . .	Materials Master
Convert like measurements within a given measurement system **MGSE5.MD.1**	Fill them Up	Explore capacity by ordering various containers based on estimated capacity	Problem Master
	A Thousand Seconds	Use seconds, minutes, and hours to solve a problem.	Problem Master
	Adam's Watch	Solve a rich problem involving time	Problem Master
Geometric measurement: Understand concepts of volume **MGSE5.MD.3** **MGSE5.MD.4** **MGSE5.MD.5**	Areas of Rectangles	Use multiplication to calculate area of rectangles	
	Making Benchmarks - Volume	Make reasonable estimates about the volume of given objects	
	Penny's Box	Determine dimensions of a box with a given volume and reason about the economy of the box design	Problem Master
	Cuboid Construction	Given a specific volume, build rectangular prisms	Problem Master Isometric Dot Paper
	Spaced Out	Use a formula to calculate the volume of a rectangular prism.	Copy Master

Practice Task: Estimate, Measure, Estimate

Back to Task Table

Adapted from "Teaching Student-Centered Mathematics Grades 3-5," by John A. Van de Walle and LouAnn H. Lovin
Approximately 1 day

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.MD.1 Convert among different-sized standard measurement units (mass, weight, length, time, etc.) within a given measurement system (customary and metric) (e.g., convert 5cm to 0.05m), and use these conversions in solving multi-step, real word problems.

BACKGROUND KNOWLEDGE

In order to do this task, students will need to be familiar with benchmarks and conversion factors for units of liquid volume, weight and mass. In fourth grade, students make conversions from smaller units to larger units by multiplying. In fifth grade, students extend their work with conversions to dividing larger units using appropriate factors to convert to smaller units. While it is not necessary for students to memorize the conversion factors, they need to know relative sizes of measurement units and recognize when they need to use them.

COMMON MISCONCEPTIONS

Students have difficulty remembering when they need to multiply or divide to make conversions. According to John Van de Walle, "it is fruitless to attempt explaining to students that larger units will produce a smaller measure and vice versa." Instead, students should engage in many activities in which they measure something with a specified unit, and then measure it again with a different related unit. For example, they could measure objects' lengths in inches, then in feet and then in yards and compare to see that the yard measurements are always the smallest quantities while the inches are always the largest. Students can make use of the structure seen to create generalizations about the larger unit producing a smaller measure and vice versa.

ESSENTIAL QUESTIONS

- What strategies can you use to estimate measurements?
- What happens to a measurement when you change its unit of measure to a related unit?

MATERIALS

- "Estimate, Measure, Estimate" recording sheet
- Measurement tools, including rulers, yard sticks, meter sticks, measuring cups for customary and metric units, scales and weights for customary and metric units, timers
- Items to be measured as listed on the recording sheet

GROUPING

Partner or small group task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this lesson students will estimate and make actual measurements for length, time, liquid volume, weight or mass. They will convert those measurements to a different unit within the same measurement system and use that to estimate the measurement of a similar item.

This lesson begins by having a discussion about attributes that can be measured and how they can be measured. Students will have the opportunity to make estimates, measurements and conversions. As students are working on this task, the teacher can monitor groups and provide guidance by asking questions such as whether the new unit is smaller or larger than the original unit, how the unit size will affect the converted number of units (more of them or fewer), and how close their estimate was to the actual measurement. Students may need teacher guidance with measurement precision. For example, if an actual measurement is between 2 pounds and 3 pounds, they may need help in deciding how to record that measurement. An important goal for this task is for students to understand that when the unit is larger, the measure is smaller and vice versa, so that when they make conversions to a larger or smaller unit, they will know whether the quantity should be larger or smaller.

TASK

- Show students an object such as a water bottle and ask them how much they think there is. Be intentionally vague so that this can lead to a discussion of attributes and measurement units. As students share ideas, guide them to understand that "how much" can mean several different aspects of the water bottle, including its weight, length, and capacity.
- As a class, discuss and list attributes of objects that can be measured. These attributes should include length, weight or mass, and liquid volume.
- Next, discuss and list the customary and metric units, conversion factors, and tools that are used to measure each attribute. Also discuss benchmarks or references that students can use to help them make measurement estimates. Examples could include a slice of bread for an ounce, a loaf of bread for a pound, a paper clip for a gram, and a hardcover math textbook for a kilogram.

- As a class, go through the process described on the recording sheet to estimate the measurement of one of the items, determine its actual measurement, convert that measurement to a different unit, and use your actual measurement to estimate the measure of a similar item. Discuss whether students should make measurements to the nearest whole unit or whether they should include fractions in their measurements.
- Next, students will work as partners or in small groups to complete the recording sheet.
- When students are finished, the teacher may lead a discussion for students to share their results and discuss strategies they used in step 3 to make conversions and step 4 to make estimates in the new unit.

FORMATIVE ASSESSMENT QUESTIONS

- If you converted a measurement to a smaller unit, would there be more or less of the smaller units?
- If you converted a measurement to a larger unit, would there be more or less of the larger unit?
- What strategies did you use to determine your estimates?
- What benchmarks or referents did you find helpful when making your estimates?
- How close were your estimates to your actual measurements? What would help you to make estimates that are more accurate?

DIFFERENTIATION

Extension

- Students can select other items to estimate and measure. They should measure as many attributes as possible of that item and convert to as many different units as possible within customary or metric units.
- Students can find and carry out real-world applications that require measurement conversions, such as changing recipe quantities.
- Students can investigate the history of the metric system, identify which countries don't use the metric system, and investigate the reasons that the U.S. still uses customary units of measure.
- Additional attributes of objects can be identified and measured such as perimeter, area, and solid volume.

Intervention

- Students may have difficulty making conversions when fractions are involved. They can round their measurements to the nearest whole unit before making conversions.
- Students who have difficulty deciding whether to multiply or divide by the conversion factor may need to first identify whether the conversion should result in more units because it's a smaller unit of measure or whether the conversion should result in fewer units because it's a larger unit of measure.

- Students may need more strategies for estimating units. Benchmarks or referents may need to be provided, along with suggestions such as trying to mentally subdivide the object being measured or mentally iterate the benchmark unit.
- Instead of making one estimate, students can specify a range.

Intervention Table

TECHNOLOGY CONNECTION

http://www.classzone.com/cz/books/msmath_1_ga/resources/applications/animations/chapter_5/html/g7_5_5.html Students reason about various customary units of measurement.

http://www.classzone.com/cz/books/msmath_1_ga/resources/applications/animations/chapter_5/html/g7_5_6.html Students practice converting customary measurements. This site also provides linear models as hints for reasoning about the relationship between two different units of measure.

Name ___________________________ Date ___________________________

Estimate-Measure-Estimate

You will measure similar items, as listed below. Please follow these steps:

1. Before measuring, write your estimate and using the specified unit of measure.
2. Use measurement tools, such as a ruler, yardstick, balance, or measuring cups to find the actual measurement.
3. Convert your actual measurement to a different unit as shown in the table.
4. Use your actual measurement to estimate a similar item.

1. Estimate		2. Actual	3. Convert units	4. Estimate	
Window width	_____ in.	_____ in.	_____ ft.	Door width	_____ ft.
Liquid volume of coffee mug	_____ c.	_____ c.	_____ pt.	Liquid volume of pitcher	_____ pt.
Distance between your eyes	_____ cm	_____ cm	_____ m	Width of your head	_____ m
Weight of a handful of candies	_____ oz.	_____ oz.	_____ lbs.	Weight of a bag of candies	_____ lbs.
Length of your math book	_____ cm	_____ cm	_____ m	Length of your desk	_____ m

Mass of a pencil	_____ g	_____ g	_____ kg	Mass of an eraser	_____ kg
Liquid volume of milk carton	_____ mL	_____ mL	_____ L	Liquid volume of water bottle	_____ L
Height of a chair	_____ ft.	_____ ft.	_____ yd.	Height of a door	_____ yd.
Liquid volume of a spray bottle	_____ c.	_____ c.	_____ qt.	Liquid volume of a sink	_____ qt.
Weight of your full backpack	_____ oz.	_____ oz.	_____ lbs.	Your weight	_____ lbs.
Time to sing Happy Birthday	_____ sec.	_____ sec.	_____ min.	Time to sing National Anthem	_____ min.

This task was adapted from "Teaching Student-Centered Mathematics Grades 3-5," by John A. Van de Walle and LouAnn H. Lovin.

<u>Practice Task:</u> Water, Water

Back to Task Table

Adapted from Illuminations
Lesson: http://illuminations.nctm.org/Lesson.aspx?id=1141
Approximately 1 day

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.MD.1 Convert among different-sized standard measurement units (mass, weight, length, time, etc.) within a given measurement system (customary and metric) (e.g., convert 5cm to 0.05m), and use these conversions in solving multi-step, real word problems.

BACKGROUND KNOWLEDGE

In order to do this task, students will need to be familiar with benchmarks and conversion factors for units of liquid volume, weight and mass. In fourth grade, students make conversions from smaller units to larger units by multiplying. In fifth grade, students extend their work with conversions to dividing larger units using appropriate factors to convert to smaller units. While it is not necessary for students to memorize the conversion factors, they need to know relative sizes of measurement units and recognize when they need to use them.

COMMON MISCONCEPTIONS

Students have difficulty remembering when they need to multiply or divide to make conversions. According to John Van de Walle, "it is fruitless to attempt explaining to students that larger units will produce a smaller measure and vice versa." Instead, students should engage in many activities in which they measure something with a specified unit, and then measure it again with a different related unit. For example, they could make measurements of objects' lengths in inches, then in feet, and then in yards and compare to see that the yard measurements are always the smallest quantities while the inches are always the largest.

ESSENTIAL QUESTIONS

- What strategies can you use to estimate measurements?
- What happens to a measurement when you change its unit of measure to a related unit?

MATERIALS

- "Water, Water" recording sheet
- Scales and weights for customary and metric units

GROUPING

Partner or small group task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this lesson students will compare the amount of water they use daily with the amount of water allocated to astronauts living on the space station. Students will estimate their daily water use using units of liquid volume and weight or mass and make conversions.

When NASA operated the space shuttle program, each space shuttle astronaut was allotted 6 gallons of water a day. This restriction was necessary because water is heavy. Extra weight on the space shuttle required extra fuel for liftoff. Water also took up space that could be used for other payloads and experiments.

TASK

- As a class, brainstorm how students use water each day and discuss amounts of water they use. This discussion should review units of measure for customary and metric liquid volume, conversion factors and benchmarks.
- Students should work in pairs or small groups to complete the recording sheet. There are two versions of the recording sheet. One is to practice customary unit conversions and the other is to practice metric conversions. Allow some groups to estimate and measure using customary units while other groups use metric units, or all students can complete both recording sheets. It is not necessary to make conversions between metric and customary units.
- For question #5, students will need to weigh a pint of water. They may need assistance using the scale and weights. They should find that a gallon of water weighs a little more than 8 pounds and a little under 4 kg. 1 liter of water weighs 1 kg.
- When students are finished, the teacher may lead a discussion for students to share their results and justify their thinking. Their work on this activity can lead to further investigations with volume (as opposed to liquid volume) by asking students to create containers to package 6 gallons of water. The volume of the containers can be measured in cubic inches or cubic cm, depending on whether the customary or metric recording sheet was used by that group of students.

FORMATIVE ASSESSMENT QUESTIONS

- How did you determine your estimates for your daily water usage?
- If you measured water to use to make a lemonade recipe, would it require more cups or more pints? How do you know?
- Would your weight be expressed in more pounds or more ounces? How do you know?
- What would be a reasonable amount of water for one person to use each day? Explain your thinking.

DIFFERENTIATION

Extension

- Students can create a container that will hold 6 gallons of water. They can determine its volume in cubic inches or cubic cm.
- Students can investigate water consumption. For example, how much water is used to wash a load of clothes or a load of dishes? How much water do manufacturing plants use? How much water is used to make a can of soda?
- Students can track how much water they use in a day and express that amount in different units of measure.
- Students can track their water usage for an extended period of time and show their data on a line plot.
- Students can explore advances that have been made in water recycling in space and current water use in the International Space Station. Each astronaut aboard the International Space Station, also known as the ISS, uses about three gallons of water daily. The average American on Earth uses about 35 gallons of water per day.

Intervention

- Students may need more experiences measuring things using related units of liquid volume, weight and mass. Provide measurement tools such as containers that demonstrate the size of a cup, pint, quart, gallon, milliliter and liter. Allow students to measure water using these tools so that they develop a better understanding of the size of each unit and how units are related within the customary system and the metric system.

Intervention Table

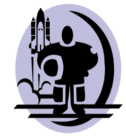
TECHNOLOGY CONNECTION

- http://illuminations.nctm.org/Lesson.aspx?id=1141
  This links to the inspiration for this task and provides additional instructions for students to measure the volume of water, graph the results, explore volume of containers, and design containers for water.

- http://www.classzone.com/cz/books/msmath_1_ga/resources/applications/animations/chapter_5/html/g7_5_5.html Students reason about various customary units of measurement.

- http://www.classzone.com/cz/books/msmath_1_ga/resources/applications/animations/chapter_5/html/g7_5_6.html Students practice converting customary measurements. This site also provides linear models as hints for reasoning about the relationship between two different units of measure.

Name ____________________________ Date ____________________________



Water, Water: Customary Units

When NASA operated the space shuttle program, each space shuttle astronaut was allotted 6 gallons of water a day. This restriction was necessary because water is heavy. Extra weight on the space shuttle required extra fuel for liftoff. Water also took up space that could be used for other payloads and experiments.

1. Estimate your daily water usage and express each amount in pints, cups and gallons.
    - List all of the ways that you use water each day.
    - Estimate how much you need for each use of water. Express your estimates in pints.
    - Convert each estimate of pints to cups. Write the operation that's needed to convert pints to cups and make the conversion.
    - Convert each estimate of pints to gallons. Write the operation that's needed to convert pints to gallons and make the conversion.

Water use	**Estimated amount of water (pints)**	**Operation required to convert to cups**	**Estimated amount of water (cups)**	**Operation required to convert to gallons**	**Estimated amount of water (gallons)**
Example: brush teeth	*$\frac{1}{4}$ pint*	*Multiply by 2*	*$\frac{1}{4}$ x 2 = 2/4 or $\frac{1}{2}$ cup*	*Divide by 8*	*$\frac{1}{4}$ ÷ 8 = 1/32 gallons*

2. Based on your estimates, about how much water do you use each day?

    ____

    ____

    ____

3. Do you think you could manage in space on 6 gallons of water a day? Why or why not?

    ____

    ____

    ____

    ____

4. How much do you think 1 gallon of water weighs?

    ____

    ____

5. Use a scale to weigh 1 pint of water and determine its weight in ounces. Use that weight to determine how many ounces one gallon of water weighs.

    Weight of 1 pint of water (ounces)

    ____

    ____

    Weight of 1 gallon of water (ounces)

    ____

    ____

6. Use your estimated daily water usage to determine the weight of the water you use each day.

    ____

    ____

    ____

7. It's recommended that people drink 8 cups of water each day.
    - How many pints does that equal?

        ____

    - How many quarts does that equal?

        ____

- How many gallons does that equal?

  ____

- How many ounces would 8 cups of water weigh?

  ____

- How many pounds would 8 cups of water weigh?

  ____

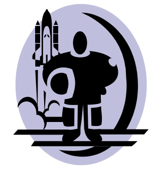
- Did you include 8 cups of water in your estimated daily use?

  ____

8. If 7 astronauts went on a trip on the space shuttle for 2 weeks and each astronaut is allocated 6 gallons of water per day, how much would their water weigh for the entire trip? Express your answer first in ounces, then in pounds.

   ____
   ____
   ____
   ____
   ____
   ____

Name ___________________________ Date ___________________________



Water, Water: Metric Units

When NASA operated the space shuttle program, each space shuttle astronaut was allotted 6 gallons of water a day. That is equivalent to about 23 liters. This restriction was necessary because water is heavy. Extra weight on the space shuttle required extra fuel for liftoff. Water also took up space that could be used for other payloads and experiments.

1. Estimate your daily water usage and express each amount in milliliters and liters.
   - List all of the ways that you use water each day.
   - Estimate how much you need for each use of water. Express your estimates in milliliters
   - Convert each estimate of milliliters to liters. Write the operation that's needed to convert milliliters to liters and make the conversion.

Water use	**Estimated amount of water (milliliters)**	**Operation required to convert to liters**	**Estimated amount of water (liters)**
Example: brush teeth	*100 ml*	*Divide by 1000*	*100 ml ÷ 1000 ml/l = 0.1 liters*

2. Based on your estimates, about how much water do you use each day?

____

____

____

3. Do you think you could manage in space on 23 liters of water a day? Why or why not?

____

____

____

____

4. How much do you think 1 liter of water weighs?

____

____

5. Use a scale to weigh 1 liter of water and determine its weight in grams. Use that weight to determine how many kg one liter of water weighs.

   Weight of 1 liter of water (grams)

   ____

   ____

   Weight of 1 liter of water (kg)

   ____

   ____

6. Use your estimated daily water usage to determine the weight of the water you use each day.

____

____

____

7. It's recommended that people drink 2 liters of water each day.
   - How many milliliters does that equal?

     ____

   - How many grams would 2 liters of water weigh?

     ____

- How many kg would 2 liters of water weigh?

  ____

- Did you include 2 liters of water in your estimated daily use?

  ____

8. If 7 astronauts went on a trip on the space shuttle for 2 weeks and each astronaut is allocated 23 liters of water per day, what would the mass of their water be for the entire trip? Express your answer first in grams, then in kg.

   ____
   ____
   ____
   ____
   ____
   ____

Practice Task: Sing a Song

Back to Task Table

Approximately 1 day

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

BACKGROUND KNOWLEDGE

In order to do this task, students will need to be able to measure and record time in minutes, including the fraction of a minute to the nearest five minutes, or twelfth. Knowledge of equivalent fractions can be applied since the times may also be recorded using denominators of two, three, or four. Students will need to know how to set up and record data on a line plot. Although students will use their data to equally redistribute the times to determine an average time, understanding and computing the mean of a set of data is a sixth-grade standard.

COMMON MISCONCEPTIONS

Students might confuse a line graph with a line plot. Review the purpose of a line plot (a graphic representation that shows the frequency of data using x's or dots along a number line) versus that of a line graph (a graphic representation that shows how data changed over time).

ESSENTIAL QUESTIONS

- How is data collected and displayed on a line plot?
- What strategies help when solving problems with line plots?

MATERIALS

- "Sing a Song" recording sheet
- Clock with second hand or other timer device

GROUPING

Small group task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this lesson students will record the number of minutes that it takes to sing a song or read a poem. They will record their times as fractions and mixed numbers on a line plot and redistribute those times equally to determine the average.

Students will work in small groups to sing a song or read a poem and measure the time it takes to sing or read to the nearest five seconds or twelfth of a minute. They will record their fractions or mixed numbers and apply their knowledge of equivalent fractions to express each fraction or mixed number in simplest form. Students will use the data from each person in the group to create a line plot. As a final step, students will analyze that data to determine an average time by equally redistributing each data point.

TASK

- As a class, decide on a song that students will sing, like Happy Birthday, or a poem to read. Estimate the amount of time that it will take to sing or read it. Discuss how to time someone while they sing or read and how to record the time in minutes and/or fraction of a minute. Time measurements will be made to the nearest 1/12 of a minute which is the nearest 5 second increment. If the time is more than 1 minute, then it should be recorded as a mixed number.
- Students should work in small groups taking turns singing/reading and timing each other. Students should record their information on the recording sheet.
- Students will determine a scale and construct a line plot to show their data.
- Students will use their data points to redistribute them fairly, so that the times for each data point are as equal as possible. If students have not done this before, they may need help determining how to do this. For example, if they move the data point that is the lowest toward the center by increasing it from 1 1/12 minutes to 1 5/12 minutes, then they will balance that by moving the data point that is highest to an amount that is 4/12 smaller. These equal moves of data points will continue until all data points are moved to the same point in the middle or as close as possible through balanced movement of data points. This is the average of the data.

FORMATIVE ASSESSMENT QUESTIONS

- How did you determine your estimate?
- How did you determine the fraction or mixed number that represented the amount of time it took you to sing the song?
- What did you need to know in order to create your line plot?
- Was there much difference in the times between the members of your group? How much difference was there?

DIFFERENTIATION

Extension

- Students can determine other data to measure and record on a line plot, such as their heights or the length of their hands or feet.
- Without being taught the procedure or algorithm, students can investigate how to write equations to record the sum of their original data, the sum of their redistributed data, and how to write equations that could be used to compute their group average.

Intervention

- Some students may find it difficult to use the beginning time and ending time to determine the elapsed time. They could either use a timer, or they could use an open number line to help determine the elapsed time.
- Students can use a number balance to build understanding of how to redistribute the data points equally.
- If students need more experience measuring length and creating a line plot from that data, they can work on the Constructing Task: Leap Frog, from Grade 3, Unit 2. That task may be modified to include length measurements to the nearest 1/8 inch.

Intervention Table

Name ___________________________ Date ___________________________

Sing a Song

Directions:

1. Decide what song each person in your group will sing. Your group may decide on a poem to read instead of singing a song, but everyone will sing or read the same thing. Write the name of your selection.

   ____

2. Estimate the time that you think it will take to sing the song. Write your estimate as a fraction or mixed number to the nearest twelfth.

   _____________________ minutes

3. Take turns timing each person in your group. Use the table below to record the time in twelfths.

Student	**Time**	**Time expressed as a fraction or mixed number**	**Time in simplest form**
Example: Sam	*1 minute, 15 seconds*	$1\frac{3}{12}$ *minutes*	$1\frac{1}{4}$ *minutes*

4. Construct a line plot using your data.

5. How did you determine the scale on your line plot?

____

____

____

6. How long did it take for <u>all</u> members of your group to sing the song?

____

____

____

7. Redistribute your times by making equal, balanced moves of your data points toward the middle of the line plot. This will be the average of your times.
8. What is the average time for your group?

____

____

____

9. How close is your group's average to your original estimate?

____

____

____

____

Practice Task - Survival Badge

Back to Task Table

Approximately 1 day

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving
2. Reason abstractly and quantitatively.
4. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots.

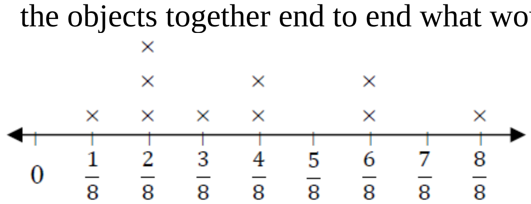
BACKGROUND KNOWLEDGE

One example of using line plots to solve real world problems might be illustrated in the following scenario. Given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were equal.

This standard provides a context for students to work with fractions by measuring objects to one-eighth of a unit. This includes length, mass, and liquid volume. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot.

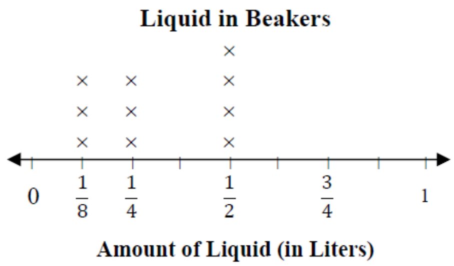
Example:

Students measured objects in their desk to the nearest $^1/_2$, $^1/_4$, or $^1/_8$ of an inch then displayed data collected on a line plot. How many objects measured $^1/_4$? $^1/_2$? If you put all the objects together end to end what would be the total length of **all** the objects?



Example:

Ten beakers, measured in liters, are filled with a liquid.



The line plot above shows the amount of liquid in liters in 10 beakers. If the liquid is redistributed equally, how much liquid would each beaker have? (This amount is the mean.)

Students apply their understanding of operations with fractions. They use either addition and/or multiplication to determine the total number of liters in the beakers. Then the sum of the liters is shared evenly among the ten beakers.

COMMON MISCONCEPTIONS

Students may not understand that in order to share the items equally, you must first find the total number of items. This portion of the standard gives them a visual model and becomes the background for finding the mean in grade 6.

ESSENTIAL QUESTIONS

- How can we use a line plot to show fractional parts of a whole?
- How can the information on the line plot be used to re-distribute the items equally?

MATERIALS

- Graph paper
- “Survival Badge” recording sheet - 2 versions are included: 1 that includes mixed numbers and 1 that only has fractions

GROUPING

Pairs/small group task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

This task provides students with the opportunity to explore using information in a table to create a line plot. They will use the line plot to re-distribute the supply of water so that the same amount is in each canteen.

TASK

In this activity, students will create line plots to evenly distribute a supply of water for a scout troop.

A Boy Scout Troop is working on a badge for survival. In order to earn the badge, they must decide how to use their available water supply equally. The water is in 12 canteens with varying amounts in each canteen. Students will use the data in the table to construct a line plot showing the various amounts of water in the canteens. Then they will re-distribute the water so that each canteen holds the same amount of water.

FORMATIVE ASSESSMENT QUESTIONS

- How can you show the various amounts of water in each canteen?
- How did you share the water equally?
- How do you know the amounts in the canteens are equal?

DIFFERENTIATION:

Extension

- Students can add additional canteens to their line plots and re-distribute the water again.

Intervention

- Students could use linking cubes to model the line plot and physically move them for re-distribution.

Intervention Table

TECHNOLOGY CONNECTIONS

http://illuminations.nctm.org/LessonDetail.aspx?ID=L520 In this lesson, one of a multi-part unit from Illuminations, students conduct a survey based on a food court theme and then create pictographs and line plots.

Name_______________________________________ Date________________________

Survival Badge

The 132nd Troop Boy Scouts were on a wilderness adventure to earn one of their survival badges. The 12 boys in the troop were only given pocket knives and water canteens. Each canteen could hold 3 cups of water but only one of them was full. As part of their survival training the boys recognized that they needed to divide the water evenly amongst the troop.

Below is a table containing the amount of water that each canteen was holding.

Canteen	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Amount of Water in Cups	2 ¼	2 ¾	2 ¼	2 ¾	2 ¼	2 ½	2 ½	2 ¾	3	2	2 ½	2 ½

1. Create a line plot to represent the data.

2. If the boys shared the water evenly amongst the 12 canteens, how much water would each boy get in their canteen? Explain your thinking.

Name____ Date_________________________

Survival Badge

The 132nd Troop Boy Scouts were on a wilderness adventure to earn one of their survival badges. The 12 boys in the troop were only given pocket knives and water canteens. Each canteen could hold water but only one of them was full. As part of their survival training the boys recognized that they needed to divide the water evenly amongst the troop.

The table below contains how full each canteen was full.

Canteen	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
How much of the canteen was full	¼	¾	¼	¾	¼	½	½	¾	full	empty	½	½

1. Create a line plot to represent the data.

2. If the boys shared the water evenly amongst the 12 canteens, how would each canteen be after sharing? Explain your thinking.

<u>Scaffolding Task:</u> Differentiating Area and Volume

[Back to Task Table]

Approximately 1 day

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

STANDARDS FOR MATHEMATICAL CONTENT

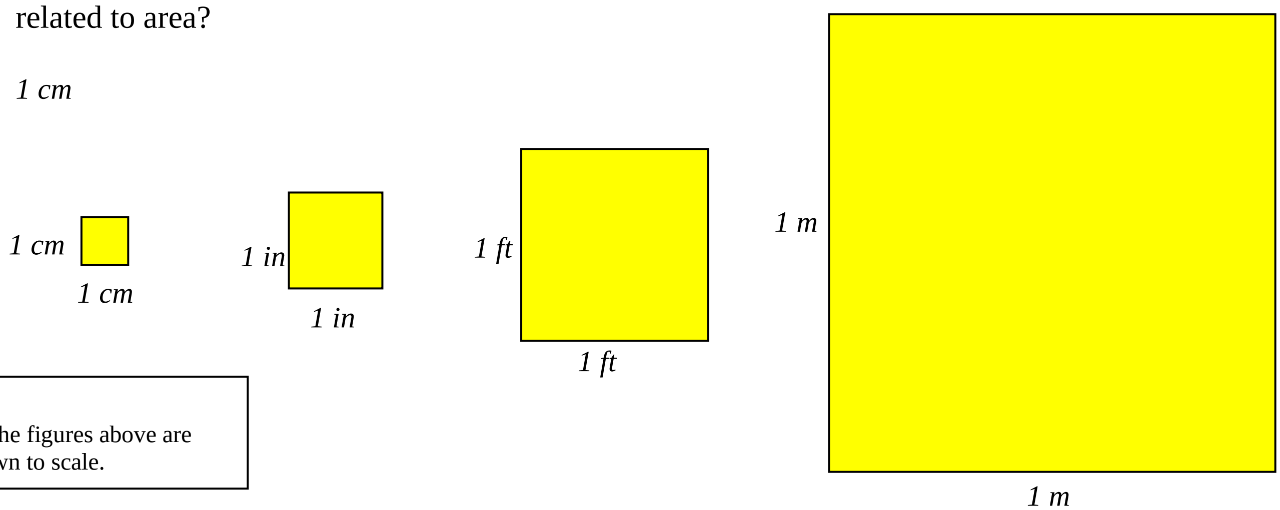
MGSE5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

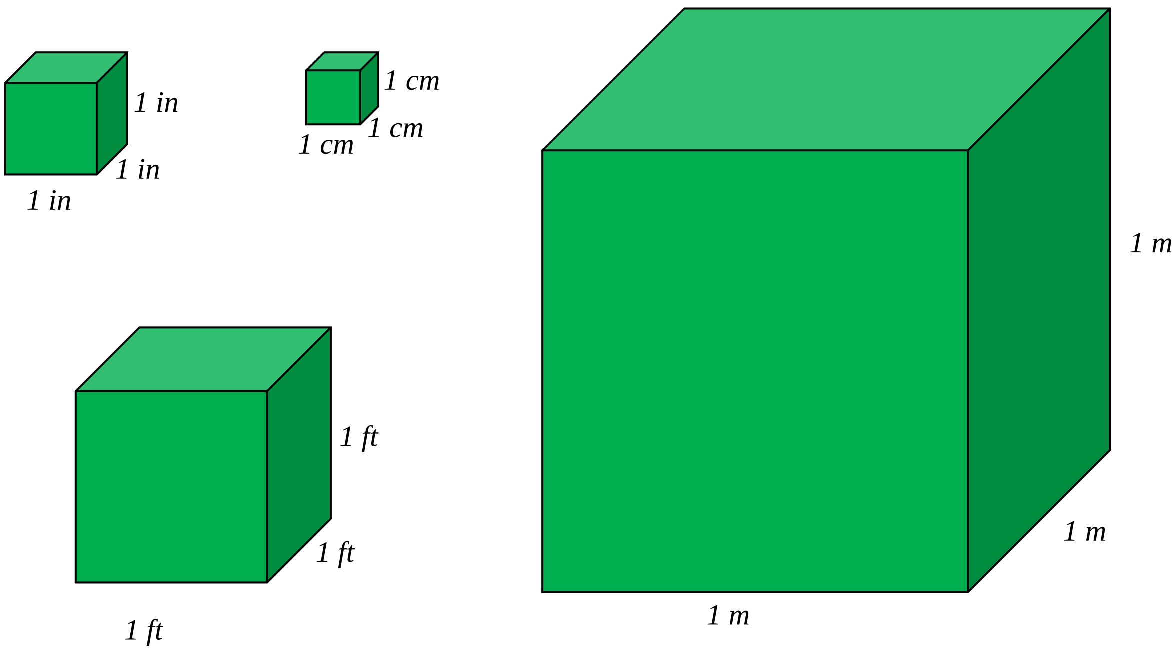
BACKGROUND KNOWLEDGE

Students should realize that the square units represent 2-dimensional objects and have both length and width. If students are having difficulty determining how to create these, have a class discussion about the word “square.” What comes to mind? How do you think this word might be related to area?



Note: The figures above are not drawn to scale.

Students should also realize that the cubic units represent 3-dimensional objects and have length, width, and height. If students are having difficulty determining how to create these, have a class discussion about the words “cube” and “cubic.” What comes to mind? How do you think these words might be related to volume?



Note: The figures above are not drawn to scale.

COMMON MISCONCEPTIONS:

Some students may think the term “square” refers only to the geometric figure with equal length sides. They will need to understand that area of any rectangle is measured in square units. The same idea may be present in “cubic units”. Students may think it only has to do with the geometric solid “cube”. They need to understand that “cubic units” are used to measure any rectangular prism.

ESSENTIAL QUESTIONS

- Why is volume represented with cubic units and area represented with square units?
- How are area and volume alike and different?

MATERIALS

- “Differentiating Area and Volume” student recording sheet
- newspaper
- construction paper

- copy paper
- grid paper (cm, in)
- scissors
- masking tape
- rulers
- meter sticks
- measuring tape
- cardstock or poster board
- markers

GROUPING

Small Group

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Students create a display of square and cubic units in order to compare/contrast the measures of area and volume.

Comments
This is a cooperative learning activity in problem solving. Students are provided with materials, but no initial instruction is given on how to build the models. This task will help give students a tangible model of square units and cubic units.

To open this task, students can discuss in their small groups what they know about area and volume. Key points of a class discussion can be recorded on chart paper.

Students will work in small groups to build models to represent units of area and units of volume. When the groups have completed their projects they will share with the class what they built, what each is called, and how each compares to some of the other models built by other groups.

Task Directions
Students will follow the directions below from the "Differentiating Area and Volume" student recording sheet.

> Create a display for area and volume by creating the following models. Use newspaper, construction paper, copy paper, grid paper, scissors, masking tape, meter sticks, markers and/or cardboard to build the models.
>
> - Area models – 1 cm^2, 4 cm^2, 1 in^2, 4 in^2, 1 ft^2
> - Volume models – 1 cm^3, 8 cm^3, 1 in^3, 8 in^3, 1 ft^3
>
> At the end of the work period, each group will share their completed models and explain what has been built, what each is called, and how your models compare with some of the other models built by the other groups.

Individually, answer the following questions:

- How are area and volume alike?
- How are area and volume different?
- Why is area labeled with square units?
- Why is volume labeled with cubic units?
- Think about your home – bedroom, kitchen, bathroom, living room.
  - What would you measure in square units? Why?
  - What would you measure in cubic units? Why?

FORMATIVE ASSESSMENT QUESTIONS

- What does cm^2 mean? cm^3? How do you know?
- What does in^2 mean? in^3? How do you know?
- What does ft^2 mean? ft^3? How do you know?
- What objects in everyday life could you use to represent cm^2? cm^3? in^2? in^3? ft^2? ft^3?
- How can you create a shape that represents 4 cm^2? What length would you use? How do you know?
- How can you create a shape that represents 8 cm^3? What length would you use? How do you know?

DIFFERENTIATION

Extension

- Ask students to describe the relationship between 4 cm^2 and 8 cm^3 as well as 9 cm^2 and 27 cm^3. Then have students generate other pairs of numbers that have the same relationship. What do they notice? (Students may use 1 cm cubes placed on a 4 cm^2 or 9 cm^2 square to determine the dimensions of a cube built on the square.)

Intervention

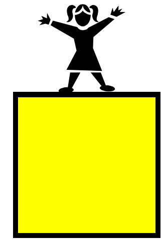
- Allow students to create at least some of the figures using a word processing or a drawing computer program. This will allow students to easily create right angles, equal side lengths, and cubes with equal edge lengths.
- Students may benefit from using 1" square tiles, 1" cubes, and similar 1 cm materials to create some of these models, especially 4 cm^2, 4 in^2, 8 cm^3, and 8 in^3.

Intervention Table

TECHNOLOGY CONNECTIONS

- http://illuminations.nctm.org/ActivityDetail.aspx?id=6 This student interactive, from Illuminations, helps students explore the volume of a box based on the amount of unit cubes that can fit inside of it.

- http://illuminations.nctm.org/LessonDetail.aspx?ID=L240 In this lesson, from Illuminations, students explore how variations in solar collectors affect the energy absorbed. They make rectangular prisms that have the same volume but different linear dimensions. Students investigate relationships among the linear dimensions, the area, and the volume of rectangular prisms.



Name________________________________ Date ____________________

Differentiating Area and Volume

Create a display for area and volume by creating the following models. Use newspaper, construction paper, copy paper, grid paper, scissors, masking tape, meter sticks, markers and/or cardboard to build the models.

- Area models – 1 cm^2, 4 cm^2, 1 in^2, 4 in^2, 1 ft^2
- Volume models – 1 cm^3, 8 cm^3, 1 in^3, 8 in^3, 1 ft^3

At the end of the work period, each group will share their completed models and explain what has been built, what each is called, and how your models compare with some of the other models built by the other groups.

Individually, answer the following questions:

1. How are area and volume alike?

____

____

____

____

2. How are area and volume different?

____

____

____

____

3. Why is area labeled with square units?

____

____

____

____

4. Why is volume labeled with cubic units?

____

____

____

____

5. Think about your home – bedroom, kitchen, bathroom, living room.

   What would you measure in square units? Why?

   ____

   ____

   ____

   ____

   What would you measure in cubic units? Why?

   ____

   ____

   ____

   ____

<u>3-Act Task:</u> Got Cubes?

Back to Task Table

Task adapted from Graham Fletcher (http://gfletchy3act.wordpress.com/got-cubes/)

Approximate time 1 day

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

MGSE5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

MGSE5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. After viewing a picture of a plastic rectangular prism and a snap cube, students will "analyze givens, constraints, relationships and goals" as they make sense of the problem presented in Act 1. Before determining a solution, students will take time to discuss the meaning of the problem with peers in order to reason through the ideas in the problem, as well as develop a plan of action for solving the problem. (www.corestandards.org/Math/Practice)

2. Reason abstractly and quantitatively. During Act 2, students will determine how many snap cubes will fit inside the large rectangular prism. As students determine the solution, they will need to discover the relationship between the snap cube and the large rectangular prism. Students will request information about the length, width and height of both the large rectangular prism and the snap cube. Once students receive that information, they will make meaning of the quantities given instead of just trying to compute using the measurements.

3. Construct viable arguments and critique the reasoning of others. As students work through Act 2, students may find that they arrive at a different solution than their peers. Students need to be able to explain how and why their answer is different. Using math vocabulary and stating specific examples during student discussion will help construct an argument to prove an individual student's thinking.

5. Use appropriate tools strategically. Students can use a variety of tools to solve this problem. During Act 1, students will provide three estimates – one estimate is too low, one estimate that is too high and one estimate that is within that range. For the work being completed in Act 2, the tools chosen will depend on how much prior knowledge students access as they work on determining a solution. Students could use tools such as snap cubes and rulers to process through the problem.

6. Attend to precision. Students will make calculations that are accurate and complete the calculations in an efficient manner. Students will precisely communicate the unit of measure being used. Students will also defend their thinking using appropriate mathematics vocabulary.

8. Look for and express regularity in repeated reasoning. In Act 4, students can see how many centimeter cubes it would take to fill the rectangular prism. Students can take the ideas learned from the three act problem and apply them to see how many centimeter cubes it takes to fill the large plastic prism.

ESSENTIAL QUESTIONS

During Act 1, students view a picture of a large plastic prism on a table with one snap cube inside it and one snap cube outside it. Students will get information about the height, length and width of each item shown in the picture and find out how many cubes it takes to fill the large rectangular prism. It is imperative that teachers allow students to ask questions of each other and participate in discussion that will lead the students to infer that information during Act 2. The essential questions below can be shared at the beginning of Act 2 to define the emphasis of the problem-solving opportunity being presented.

- How can I find the volume of a cube and a rectangular prism?
- Why is volume represented with cubic units?

MATERIALS

Act 1 picture "Got Cubes?" http://gfletchy3act.wordpress.com/got-cubes/
Student recording sheet (attached)
Act 2 "Got Cubes?" images (Use the link above to locate images.)
Act 3 "Got Cubes" reveal picture (Use the link above to locate the reveal picture.)

GROUPING

Whole group, partners or small groups

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Students will view a picture of a large plastic prism on a table with one snap cube inside it and one snap cube outside it. Next, they will be asked to discuss what they wonder about or are curious about. These questions will be recorded on a class chart or on the board and on the student recording sheet. Students will then use mathematics to answer one of the questions generated on the chart. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them.

Background Knowledge:

This task follows the 3-Act Math Task format originally developed by Dan Meyer. More information on this type of task may be found at http://blog.mrmeyer.com/category/3acts/. A Three-Act Task is a whole-group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the ***Guide to Three-Act Tasks*** on georgiastandards.org.

In third grade, students learn about perimeter, which is one dimensional in nature. Students determine the perimeter of polygons by finding the sum of the length of the sides. In addition, third graders also learn about area, which is a two dimensional measurement. Third graders start determining the area of plane figures by tiling with square units. Through the tiling experiences, students learn the number of square units it takes to cover a rectangle or square is equivalent to multiplying the length and the width. In fourth grade, students continue working with area and perimeter concepts by solving word problems using the formulas. Students work with the formulas as equations and find the unknown as they solve problems about area and perimeter. By fifth grade, students are comfortable with finding area and perimeter measurements and can apply learning of fractions and decimals to determine area and perimeter.

While some students may have prior knowledge of volume, it is formally introduced for the first time in fifth grade. Learning about volume should be accessed through their prior learning about perimeter and area. Volume is a three-dimensional measurement that is similar to area measurement. However, instead of tiling with square units, students are packing with cubic units because now the solid figures being measured have a third dimension called height. Students need to spend time finding volume measurements using snap cubes, centimeter cubes/base ten units and base ten cubes. These experiences are crucial to understanding how the length, width and height are determined in a three-dimensional figure.

COMMON MISCONCEPTIONS:

Since this is the first formal experience students have with volume, students may have trouble comparing volumes of three dimensional cubes or rectangular prisms. Students might only focus on one of the three dimensions necessary to find volume. For example, "They will decide that a tall object has lots of volume because they only focus on the height and fail to take into account the other two dimensions." (http://homepages.math.uic.edu/~dmiltner/download7.pdf, pg.2) "Children should encounter activity oriented measurement situations by doing and experimenting rather than passively observing. The activities should encourage discussion and stimulate the refinement and testing of ideas and concepts." (Reys, Lindquist, Lambdin, et.al, *Helping Children Learn Mathematics*; pg. 394)

Students need first-hand experiences comparing volumes of multiple rectangular prisms and cubes so they can see that rectangular prisms and cubes may have different appearances because of varying heights, lengths or widths, but that it is possible that the volumes could be the same. Students can also see that a rectangular prism that has a shorter height, but a longer length and width could possibly have a larger volume than a rectangular prism that has a taller height, but a shorter length and width.

Task Directions:

Act 1 – Whole Group - Pose the conflict and introduce students to the scenario by showing Act I video or picture. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/**)**
"Introduce the central conflict of your story/task clearly, visually, viscerally, using as few words as possible."

- Show the Act 1 picture "Got Cubes?" to students. http://gfletchy3act.wordpress.com/got-cubes/
- Ask students what they noticed mathematically in the picture, what they wonder about, and what questions they have about what they saw in the picture.
- Give each student a copy of the Student Recording Sheet. Have students record their questions and curiosities in the Act 1 section that asks "What questions come to your mind?" Consider doing a think-pair-share so that students have an opportunity to talk with each other before sharing questions with the whole group. The picture should be posted for students to view as they develop and record their questions.
- Share and record students' questions. The teacher may need to guide students so that the questions generated are math-related.

- Share the main question that will be investigated during today's lesson. In the list below it is denoted with an asterisk. (*) Students will record the main question on their recording sheet.
- Ask students to estimate how many snap cubes will fit inside the large plastic rectangular prism. Students will write their best estimate, then write two more estimates – one that is too low and one that is too high so that they establish a range in which the solution should occur. Students should plot their three estimates on a number line. Space is available on the recording sheet for students to record open number line with all three estimates.

Anticipated questions students may ask and wish to answer:

- What are the dimensions of the large, plastic rectangular prism?
- What are the dimensions of the small snap cube?
- *How many snap cubes will fit in the large, plastic rectangular prism?
- What is the difference in the volume of the large, rectangular prism and the small snap cube?

**Main question(s) to be investigated*

Act 2 – **Student Exploration** - Provide additional information as students work toward solutions to their questions. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/**)**
"The protagonist/student overcomes obstacles, looks for resources, and develops new tools."

- During Act 2, students review the main question from Act 1 and decide on the facts, tools, and other information needed to answer the question. The main question for this task is "How many snap cubes will fit inside the large, plastic rectangular prism?" When students decide what they need to solve the problem, they should ask for those things. The Act 2 picture "Got Cubes?" contains images of both the large rectangular prism and the snap cubes with their length, height and width measurements. Copies of the images can be given to the students at their request. The document can be found under "Act 2" using the link to the picture. It is pivotal to the problem-solving process that students decide what is needed without being given the information up front.
- Students can record information that they need to solve the problem, given information, estimates and work on the student recording sheet under Act 2.
- The teacher provides guidance as needed during this phase. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin. Questioning is an effective strategy that can be used, with questions such as:
    - What is the problem you are trying to solve?
    - What do you think affects the situation?
    - Can you explain what you've done so far?
    - What strategies are you using?

- What assumptions are you making?
- What tools or models may help you?
- Why is that true?
- Does that make sense?

Additional Information for Act 2

"Got Cubes?" images and measurements http://gfletchy3act.wordpress.com/got-cubes/

Important note: Although students will only investigate the main question for this task, it is important for the teacher to not ignore student generated questions. Additional questions may be answered after they've found a solution to the main question, or as homework or extra projects.

Act 3 – Whole Group – Share solutions and strategies.

- Students present their solutions and strategies and compare them. Have students share the solutions they arrived at when determining how many snap cubes fit inside the large, plastic rectangular prism. Record student solutions on the board, as you would in a Number Talk.
- Reveal the solution by projecting or displaying the Act 3 reveal pictures. The images can be found under "Act 3 – The Reveal" using the link to the picture. Work as a whole group to determine how many snap cubes fit inside the large, plastic rectangular prism using the strategies students shared as they presented their solutions. Some students may calculate the area of the base first and then multiply the area of the base by the number of layers, or the height of the rectangular prism to determine to solution in the reveal photo. Some students may visually determine how many snap cubes it takes to measure the length, width and height of the rectangular prism and then determine how many snap cubes it takes to fill the rectangular prism by multiplying the dimensions.
- After comparing the different solutions, have students discuss why their solutions were different from what was revealed in the Act 3 photos.
- As the discussion progresses, students can discuss why different solutions were reached as they solved the problem. If students don't pick up on this idea through the course of the discussion, it is appropriate for the teacher to ask students to think about why various solutions were given.
- Have students record their result in the Act 3 section of the student recording sheet. Students can also plot their solution on the open number line on the student recording sheet.
- Lead discussion to compare these, asking questions such as:
    - How reasonable was your estimate?
    - Which strategy was most efficient?
    - Can you think of another method that might have worked?
    - What might you do differently next time?

Act 4, The Sequel - "The goals of the sequel task are to a) challenge students who finished quickly so b) I can help students who need my help. It can't feel like punishment for good work. It can't seem like drudgery. It has to entice and activate the imagination." Dan Meyer http://blog.mrmeyer.com/2013/teaching-with-three-act-tasks-act-three-sequel/

Give students a centimeter cube. Repeat the three-act task, this time having students determine how many centimeter cubes would fit in the large, plastic rectangular prism. Students can make estimates and discuss with peers whether more or less centimeter cubes are needed to fill the prism. Have students determine the dimensions of the centimeter cube in inches. Then, have students determine how many centimeter cubes would be needed to fill the large, plastic rectangular prism.

FORMATIVE ASSESSMENT QUESTIONS

- What models did you create?
- What organizational strategies did you use?
- How was your result different than other results in the class? Give examples that prove or disprove your results.
- How is volume determined in a rectangular prism or a cube?

DIFFERENTIATION

Extension
Students can complete the investigation in Act 4, The Sequel to extend their learning of volume.

Intervention
Students can use rulers to mark off the dimensions of the large, plastic rectangular prism on a piece of paper. Students can then take snap cubes and see how many snap cubes it takes to equal the length, the width and the height of the prism.

Intervention Table

TECHNOLOGY CONNECTIONS

- Building Rectangular Prisms with a Given Volume http://www.k-5mathteachingresources.com/5th-grade-measurement-and-data.html
  This activity uses centimeter cubes to find various ways to construct a rectangular prism that has a volume of 24 cubic centimeters. Students can record a drawing of each prism constructed along with its dimensions. Students can compare and contrast the rectangular prisms in writing by discussing the similarities and differences among them.

Three Act Task Student Recording Sheet Name ______________________

ACT 1

What questions come to your mind?

Main Question: ____

What is your first estimate and why?
Record an estimate that is too low and an estimate that is too high.
On an empty number line, record all three estimates made above.

ACT 2

What information would you like to know or need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimation with this information: _______________________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

<u>Constructing Task:</u> How Many Ways?

Back to Task Table

Approximately 1 day

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

MGSE5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

MGSE5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

BACKGROUND KNOWLEDGE

Students should have had experiences with the attributes of rectangular prisms, such as faces, edges, and vertices, in fourth grade. This task will build upon this understanding.

The "How Many Ways?" student recording sheet asks students to determine the area of the base of each prism using the measurements of base and height of the solid's BASE. The general formula for the area of a parallelogram is A = bh. Knowing the general formula for the area of a parallelogram enables students to memorize ONE formula for the area of rectangles, squares, and parallelograms since each of these shapes is a parallelogram.

The general formula for the volume of a prism is V = Bh, where B is the area of the BASE of the prism and h is the height of the prism. Knowing the general formula for the volume of a prism prevents students from having to memorize different formulas for each of the types of prisms they encounter.

There are six possible rectangular prisms that can be made from 24 snap cubes.

$1 \times 1 \times 24$
$1 \times 2 \times 12$
$1 \times 3 \times 8$
$1 \times 4 \times 6$
$2 \times 2 \times 6$
$2 \times 3 \times 4$

Students may identify rectangular prisms with the same dimensions in a different order, for example, $1 \times 4 \times 6$, $1 \times 6 \times 4$, $6 \times 1 \times 4$, $6 \times 4 \times 1$, $4 \times 1 \times 6$, $4 \times 6 \times 1$. All of these are the same rectangular prism, just oriented differently. It is okay for students to include these different orientations on their recording sheet. However, some students may need to be encouraged to find different rectangular prisms.

COMMON MISCONCEPTIONS:

Students may have difficulty with the concept of the formula V=Bh representing 3 factors. (length, width, height). They may leave out one of the components because of that misconception.

ESSENTIAL QUESTIONS

- Why is volume represented with cubic units?
- How do we measure volume?
- How can you find the volume of cubes and rectangular prisms?
- In the formula *V=Bh or, V=bh,* what does the *B/b* represent?

MATERIALS

- "How Many Ways?" student recording sheet

- Snap cubes

GROUPING

Partner/Small Group Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will use 24 snap cubes to build cubes and rectangular prisms in order to generalize a formula for the volume of rectangular prisms.

Comments

To introduce this task, ask students to make a cube and a rectangular prism using snap cubes. Discuss the attributes of cubes and rectangular prisms – faces, edges, and vertices. Initiate a conversation about the figures:

- What is the shape of the cube's base?
- What is the shape of the rectangular prism's base? The base of each is a rectangle (remember a square is a rectangle!).

Students should notice that the cube and rectangular prism are made up of repeated layers of the base. Describe the base of the figure as the first floor of a rectangular-prism-shaped building. Ask students, "What is the area of the base? Next, discuss the height of the figure. Ask students, "How many layers high is the cube?" or "How many layers high is the prism?" The number of layers will represent the height. DO NOT LEAD THE DISCUSSION TO THE VOLUME FORMULA. Students will use the results of this task to determine the volume formula for rectangular prisms on their own.

While working on the task, students do not need to fill in all ten rows of the "How Many Ways?" student recording sheet. Some students may recognize that there are only six different ways to create a rectangular prism using 24 snap cubes. For students who have found four or five ways to build a rectangular prism, tell them they have not found all of the possible ways **without telling them exactly how many ways are possible**. It is important for students to recognize when they have found all possible ways and to prove that they have found all of the possible rectangular prisms.

Once students have completed the task, lead a class discussion about the similarities and differences between the rectangular prisms they created using 24 snap cubes. Allow students to explain what they think about finding the volume of each prism they created. Also, allow students to share their conjectures about an efficient method to find the volume of any rectangular prism. Finally, as a class, come to a consensus regarding an efficient method for finding the volume of a rectangular prism.

Task Directions

Students will follow the directions below from the "How Many Ways?" student recording sheet.

1. Count out 24 cubes.

2. Build all the rectangular prisms that can be made with the 24 cubes. For each rectangular prism, record the dimensions and volume in the table below.
3. What do you notice about the rectangular prisms you created?
4. How can you find the volume without building and counting the cubes?

Shape #	Area of the BASE of the Solid A = *bh*		Number of Layers of the Base (Height of Solid)	Volume in cubic centimeters
	base	height		
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

FORMATIVE ASSESSMENT QUESTIONS

- What is the shape of the rectangular prism's base? Explain how to calculate the base of 3-dimensional objects.
- How did you determine the height of the rectangular prism? How do you know? (How many layers or "floors" does it have?)
- What is the volume of the rectangular prism? How do you know? (How many snap cubes did you use to make the rectangular prism? How do you know?)

DIFFERENTIATION

Extension

- Ask students to suggest possible dimensions for a rectangular prism that has a volume of 42 cm^3 without using snap cubes.
- Ask students to explore the similarities and differences of a rectangular prism with dimensions 3 cm x 4 cm x 5 cm and a rectangular prism with dimensions 5 cm x 3 cm x 4 cm. Students can consider the attributes and volumes of each of the prisms.
- Students can calculate the area of each surface of the solid and determine the total surface area.

Intervention

- Some students may need organizational support from a peer or by working in a small group with an adult. This person may help students recognize duplications in their table as well as help them recognize patterns that become evident in the table.
- Some students may benefit from using the “Cubes” applet on the Illuminations web site (see link in “Technology Connection” below). It allows students to easily manipulate the size of the rectangular prism and then build the rectangular prism using unit cubes.

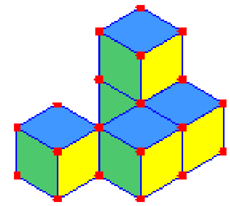
Intervention Table

TECHNOLOGY CONNECTIONS

- http://illuminations.nctm.org/ActivityDetail.aspx?id=6 This student interactive, from Illuminations, helps students explore the volume of a box based on the amount of unit cubes that can fit inside of it.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L240 In this lesson, from Illuminations, students explore how variations in solar collectors affect the energy absorbed. They make rectangular prisms that have the same volume but different linear dimensions. Students investigate relationships among the linear dimensions, the area, and the volume of rectangular prisms.

Name____ Date _______________

How Many Ways?



1. Count out 24 cubes.
2. Build all the rectangular prisms that can be made with the 24 cubes. For each rectangular prism, record the dimensions and volume in the table below.
3. What do you notice about the rectangular prisms you created?
4. How can you find the volume without building and counting the cubes?

Shape #	Area of the BASE of the Solid A = *bh*		Number of Layers of the Base (Height of Solid)	Volume in cubic centimeters
	base	height		
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Practice Task: Exploring with Boxes

Back to Task Table

Adapted from K-5 Math Teaching Resources
Approximately 1 day

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

MGSE5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

MGSE5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

BACKGROUND KNOWLEDGE

Students should have experience with drawing boxes on grid paper. They also need to understand how to cut and fold the patterns to make boxes. Teachers may need to model and let students practice before the task.

COMMON MISCONCEPTIONS:

When filling a solid figure, there can be no gaps or overlaps with the cubes filling the object.

ESSENTIAL QUESTIONS

- What is the relationship between the size of the box and the number of cubes it will hold?
- How does the volume change as the dimensions of the box change?

MATERIALS

- cube patterns
- scissors
- tape
- cm cubes
- ruler
- recording sheet

GROUPING

Individual/Partners

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will create boxes and discover how volume is related to the length, width, and height of cubes.

Comments: To introduce this task, show the cube pattern and ask this question? What could be done to this pattern so that the top of the cube will be open? Students should be able to tell that the top square could be cut off. Tell students that they will be building open cubes of different sizes and filling them with cubes. Explain that they will need to measure the dimensions of each cube to complete the chart.

Once students have completed the task, lead a class discussion about the patterns they noticed. Allow students to explain their findings and any relationships they noticed. Also, allow students to share their conclusions about the relationships between volume and the dimensions of cubes. Finally, allow students to write about their findings in their math journals.

Task Directions: Using the open cube pattern, have students construct cubes of different dimensions and fill them with cm cubes. Have them measure the dimensions and record them in the appropriate boxes on the recording sheet. Then they will count the number of cubes it took to fill the cube and record the volume of each cube. Have students discuss their findings to generalize statements about the relationship between the dimensions of the cubes and their volume.

FORMATIVE ASSESSMENT QUESTIONS

- What do you notice about the size of the open cubes and the number of cm cubes they can hold? Explain your thinking.
- Could you predict how many cm cubes a container can hold, based on its measurements? Justify your answer.

DIFFERENTIATION

Extension:
- Students may create their own open cubes with grid paper.
- Students may present a demonstration on drawing cubes to the class.

Intervention:
- Students may work with partners.
- Students may need support to measure dimensions accurately.
- Students may need support with differentiating between the length, width, and height on an open cube.

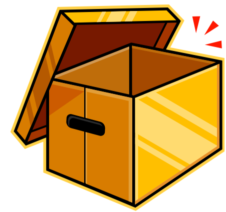
Intervention Table

TECHNOLOGY CONNECTIONS

- http://illuminations.nctm.org/ActivityDetail.aspx?id=6 This student interactive, from Illuminations, helps students explore the volume of a box based on the amount of unit cubes that can fit inside of it.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L240 In this lesson, from Illuminations, students explore how variations in solar collectors affect the energy absorbed. They make rectangular prisms that have the same volume but different linear dimensions. Students investigate relationships among the linear dimensions, the area, and the volume of rectangular prisms.

Name____Date_________________

Exploring With Boxes



Materials: open cube patterns, scissors, tape, cm ruler, cm cubes, recording sheet

Directions:

1. Work with a partner. Cut out the patterns for the open cubes, fold up the sides, and tape them together.
2. Measure each open cube and record your findings in the chart below.
3. Fill each box (open cube) with cm cubes and count them to find the volume.
4. Record your findings in the chart below.
5. Write in your math journal and describe how the size of the box is related to its volume.

Box (Open Cube)	**Length of Base**	**Width of Base**	**Height of Cube**	**Volume**
A				
B				
C				

Findings____

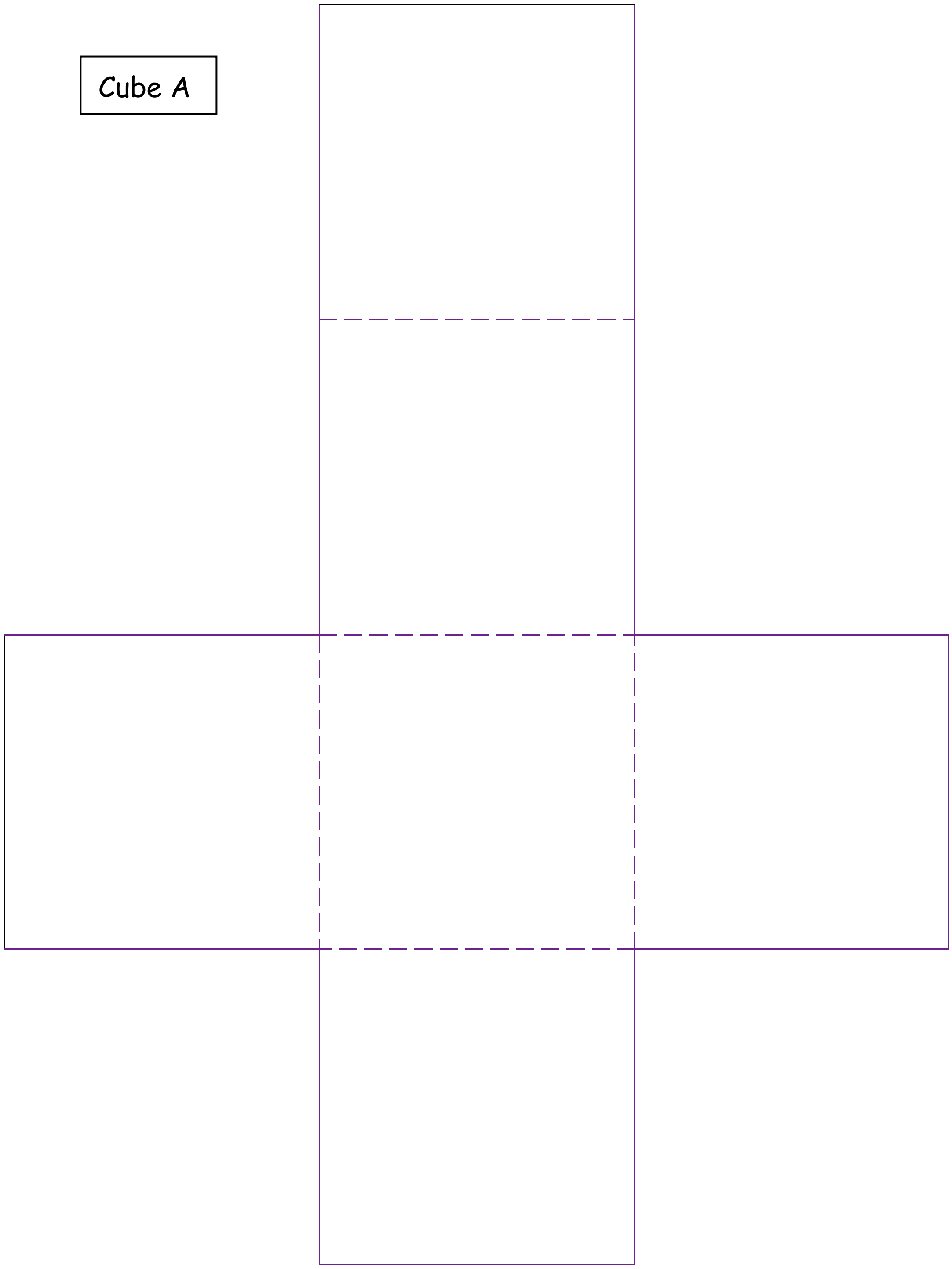
____

____

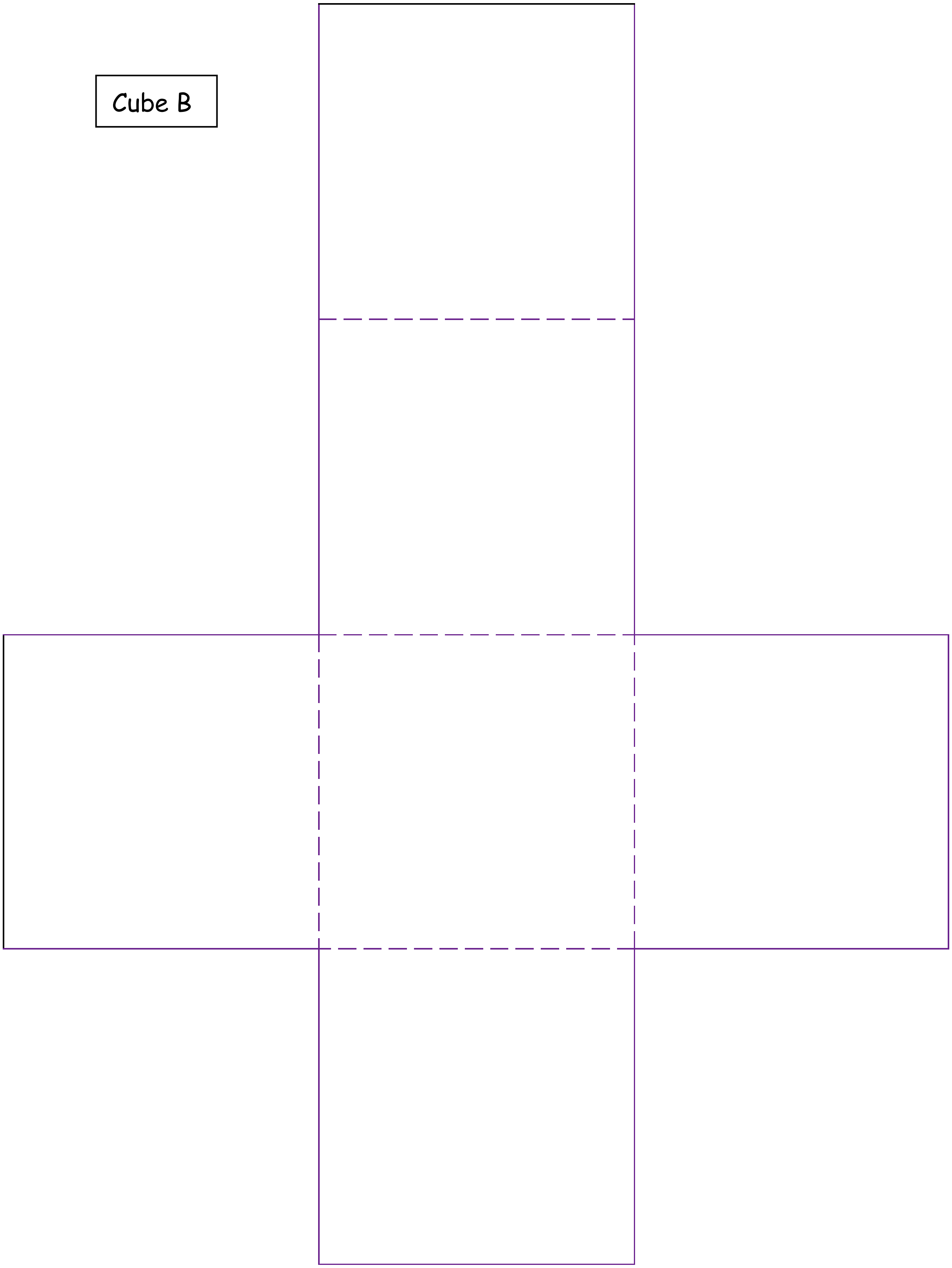
____

____

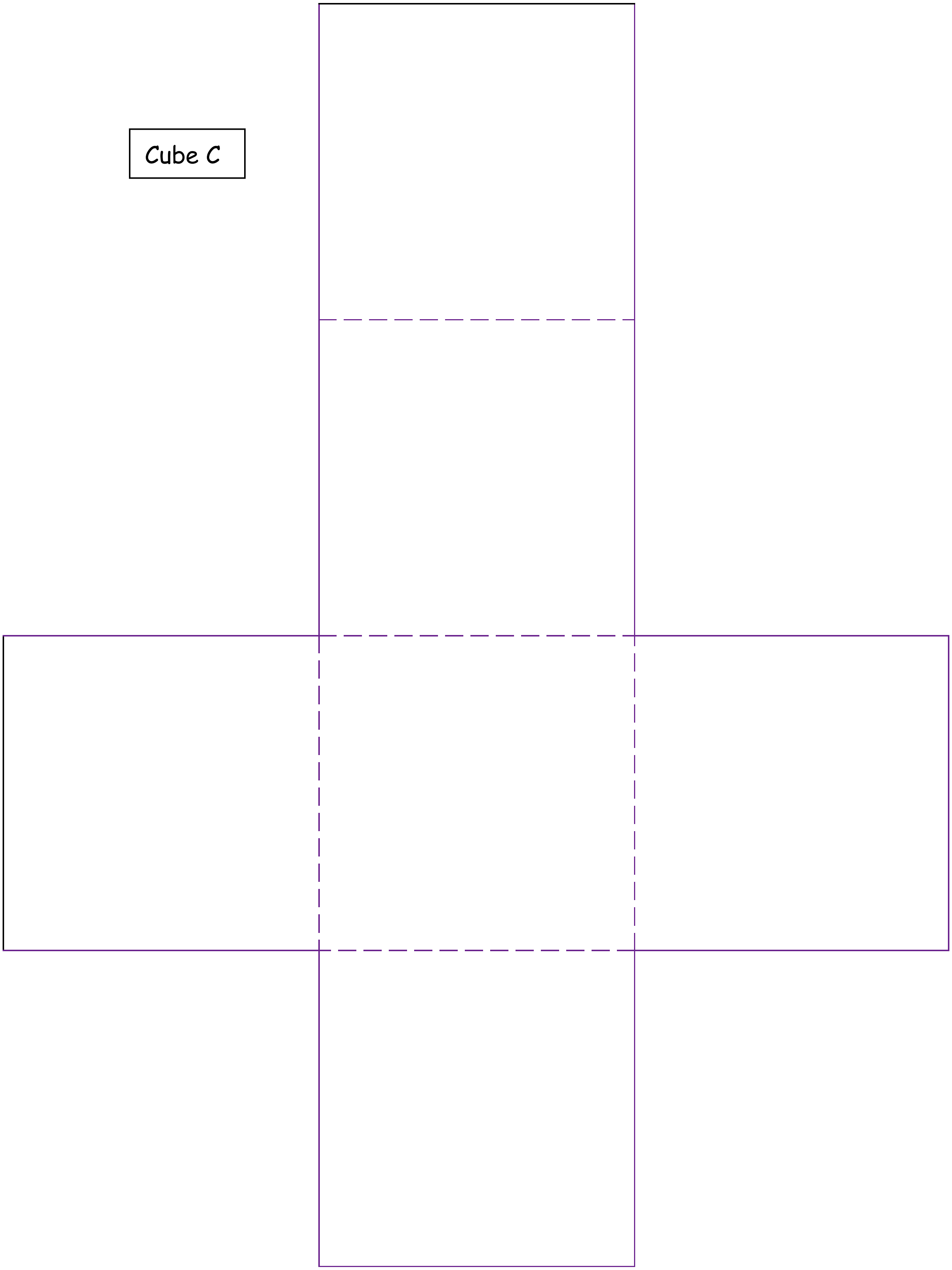
Cube A



Cube B



Cube C



PRACTICE TASK: Rolling Rectangular Prisms

Back to Task Table

Adapted from K-5 Math Teaching Resources
Approximately 1 day

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

MGSE5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

BACKGROUND KNOWLEDGE

Students will need to know the names of the dimensions of rectangular prisms (length, width, height) and have some experience with the formulas $V = l \times w \times h$ and $V = b \times h$. Additionally, students will need to understand multiplication with 3 factors.

COMMON MISCONCEPTIONS:

Students may believe that converting customary units is like converting metric units; using the base ten system. They will need to be reminded of equivalent measures in customary units if they are confused.

ESSENTIAL QUESTIONS

- Do all the dimensions have to be the same in a rectangular prism? Justify your answer.
- How are cubes and rectangular prisms the same? How are they different?

MATERIALS

- Dice
- Recording sheet

GROUPING

- Individual/Partner Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will draw rectangular prisms and determine the dimensions of the prism by rolling a die. Students will label the prism drawn with the dimensions rolled and calculate its volume.

Comments: To introduce this task, remind them of the formula for volume and that precision is very important in calculating volume.

Task Directions: Model drawing a rectangular prism and have someone roll the die to determine its measurements (length, width, and height) in centimeters. Label the drawing and model multiplying the three measurements to determine the volume. Have the students follow the directions on the task sheet to complete the task.

FORMATIVE ASSESSMENT QUESTIONS

- What do you notice about the measurements and the volume of the rectangular prisms?
- What is the greatest possible volume for a rectangular prism in this game?

DIFFERENTIATION

Extension:

- Students may use both dice to increase the size of their rectangular prisms.

- Students may convert the dimensions of each rectangular prism from centimeters to millimeters. Next, students find the volume of each rectangular prism in cubic millimeters. Students can compare the volumes in cubic centimeters and cubic millimeters and use repeated reasoning to determine what happens to the volume when the dimensions are converted from centimeters to millimeters. Students can also explain why this pattern occurs.

Intervention:

- Students may work with partners.
- Students may use calculators to determine volume.

Intervention Table

TECHNOLOGY CONNECTIONS

- http://illuminations.nctm.org/ActivityDetail.aspx?id=6 This student interactive, from Illuminations, helps students explore the volume of a box based on the amount of unit cubes that can fit inside of it.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L240 In this lesson, from Illuminations, students explore how variations in solar collectors affect the energy absorbed. They make rectangular prisms that have the same volume but different linear dimensions. Students investigate relationships among the linear dimensions, the area, and the volume of rectangular prisms.

ROLLING A RECTANGULAR PRISM

Materials: dice, recording sheet

Directions:

1. **Draw a rectangular prism.**
2. **Roll a die three times to find the dimensions of the rectangular prism.**
3. **Label the dimensions.**
4. **Calculate the volume of the rectangular prism. Show your work.**
5. **Repeat steps 1-4 three times.**

Picture	**Length**	**Width**	**Height**	**Volume in cubic centimeters**

<u>Constructing Task:</u> Books, Books, and More Books!

Back to Task Table

Approximately 1 day

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

MGSE5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

MGSE5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

BACKGROUND KNOWLEDGE

Students will need to have had practice finding the volume of a rectangular prism. They will also need to recognize that addition can be used to combine rectangular prisms, just like they combine quantities by adding. Also, they will need to understand that real world problems require a variety of problem solving strategies.

ESSENTIAL QUESTIONS

- How can you find the combined volume of two or more rectangular prisms?
- How can you determine if your solution is correct?

MATERIALS

- Pencils
- Recording sheet

GROUPING

Individual/Partners

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will determine the combined volume of three boxes of books. They will conclude that adding the volume of each box will give the combined volume.

Comments: To introduce this task, tell them that you need to take three boxes of books home with you, but you are not sure they will fit in your truck. Tell them that they can help you figure out if they will fit, by figuring their volume. You may need to remind them of the formula for volume.

Task Directions: Determine the volume of each box of books and decide if they will all fit in the teacher's truck. Use pictures, words, and numbers to show your work.

FORMATIVE ASSESSMENT QUESTIONS

- What information do you need to be able to solve this problem?
- What is the largest size box you could fit, if all three boxes were the same size?

DIFFERENTIATION

Extension:

- Ask students if 4 boxes would fit.
- If your boxes were half the size of the originals, how many could you fit?

Intervention:

- Students may work with partners.
- Students may use calculators to determine volume.

Intervention Table

TECHNOLOGY

- http://illuminations.nctm.org/ActivityDetail.aspx?id=6 This student interactive, from Illuminations, helps students explore the volume of a box based on the amount of unit cubes that can fit inside of it.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L240 In this lesson, from Illuminations, students explore how variations in solar collectors affect the energy absorbed. They make rectangular prisms that have the same volume but different linear dimensions. Students investigate relationships among the linear dimensions, the area, and the volume of rectangular prisms.

Name____________________________________**Date**______________________________

Books, Books, and More Books

Directions: Your teacher wants to take three boxes of books home from school. She needs to know if they will all fit in her truck, or if she needs to make two trips to get all the boxes home. Here is some information you will need:

- Two of the boxes are the same size. (2 ft. long, 3ft. wide, and 2 ft. high)
- One box is larger than the others. (3 ft. long, 3 ft. wide, and 3 ft. high)
- Your teacher's truck has 60 cu. ft of space.

Can your teacher take all three boxes in one load? Show how you know with pictures, words, and numbers.

<u>Practice Task</u>: Super Solids

Back to Task Table

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

BACKGROUND KNOWLEDGE

Students should realize that square units represent 2-dimensional objects and have both length and width, while cubic units represent 3-dimensional objects and have length, width, and height.

Students should have had experiences with the attributes of rectangular prisms, such as faces, edges, and vertices, in fourth grade. This task will build upon this understanding.

The general formula for the area of a parallelogram is A = *bh*. Knowing the general formula for the area of a parallelogram enables students to memorize ONE formula for the area of rectangles, squares, and parallelograms since each of these shapes is a parallelogram.

The general formula for the volume of a prism is V = *Bh*, where *B* is the area of the BASE of the prism and *h* is the height of the prism. Knowing the general formula for the volume of a prism prevents students from having to memorize different formulas for each of the types of prisms they encounter.

COMMON MISCONCEPTIONS:

Students need to be encouraged to estimate the volume based on the information they have, but not actually calculating the answer. Estimating is not the same as guessing and students need to know that there are strategies involved in estimating. They need to be encouraged to share their strategies with each other.

ESSENTIAL QUESTIONS

- Explain the process of finding the volume of cubes and rectangular prisms?
- Why is volume represented with cubic units?
- What is the relationship between the volumes of geometric solids?
- How do we measure volume?

MATERIALS

- Empty boxes (such as shoe, cereal, cracker, etc.)
- Centimeter cubes
- Rulers or measuring tapes
- “Super Solids” task sheet

GROUPING

Partner/Small Group Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will estimate and find the volume of real-world objects.

Comments
For each object, students will estimate the number of centimeter cubes that will be needed completely fill the box. (They should NOT fill the box with centimeter cubes to estimate.) After all estimates have been recorded, students will use their measurement tools to determine the volume of each box. All measurements should be to the nearest tenth of a centimeter.
After students have found the volume of each box, compare results. Discuss any discrepancies. Allow pairs of students to share their strategies for making their estimate and determining the volume.

Task Directions

Students will follow the directions below from the "Super Solids" student recording sheet. Objects to measure could include tissue box, storage tubs, lunch box, waste basket, storage area of desk, etc.

> For each object you choose, estimate the number of centimeter cubes that will be needed to completely fill the box. Once you have recorded your estimate, measure the object to determine the volume of each box.
>
> ∗All measurements should be recorded to the nearest tenth of a centimeter.

Object	Estimate in cm^3	Area of Base $A = b \times h$	Height of Prism	Volume of Prism in cm^3 $A = B \times h$

<u>FORMATIVE ASSESSMENT QUESTIONS</u>

- What information did you use to help you estimate the volume of each rectangular prism?
- How did you find the area of the base of your prism?
- How did you find the volume of your prism?
- What is $\frac{1}{10} \times \frac{1}{10}$? What is 0.1×0.1? Where should you place your decimal in your answer? How do you know? (Students should recognize that $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$ and that $\frac{1}{100} \times \frac{1}{10} = \frac{1}{1{,}000}$. Therefore, $0.1 \times 0.1 = 0.01$ and $0.01 \times 0.1 = 0.001$.

<u>DIFFERENTIATION</u>

Extension

- Students can calculate the area of each surface of the solid and determine the total surface area.

Intervention

- Encourage students to fill their boxes with centimeter cubes. This allows students to use models when determining volume.

<u>Intervention Table</u>

TECHNOLOGY CONNECTIONS

- http://illuminations.nctm.org/ActivityDetail.aspx?id=6 This student interactive, from Illuminations, helps students explore the volume of a box based on the amount of unit cubes that can fit inside of it.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L240 In this lesson, from Illuminations, students explore how variations in solar collectors affect the energy absorbed. They make rectangular prisms that have the same volume but different linear dimensions. Students investigate relationships among the linear dimensions, the area, and the volume of rectangular prisms.

Name_______________________________________ Date______________________________

Super Solids

For each object you choose, estimate the number of centimeter cubes that will be needed to completely fill the box. Once you have recorded your estimate, measure the object to determine the volume of each box. ∗All measurements should be recorded to the nearest tenth of a centimeter.

Object	**Estimate in cm^3**	**Area of Base** $A = b \times h$	**Height of Prism**	**Volume of Prism in cm^3** $A = B \times h$

Practice Task: Toy Box Designs

Back to Task Table

Adapted from K-5 Math Teaching Resources

Approximately 1 day

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

MGSE5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

MGSE5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

BACKGROUND KNOWLEDGE

Students should be familiar with using meter sticks, rulers and other measurement tools to measure and draw rectangular prisms. The knowledge of area and area of a base will be extended as students apply that understanding to volume of solids. Students should be familiar with using unit cubes to fill an object and find the volume. Students should also be able to use their knowledge of factors to determine the measurements for the box.

COMMON MISCONCEPTIONS:

Some students may think that the box must be a cube. They need to understand that rectangular prisms (boxes) can have different measures of length, width, and height. They will need to consider which design would work best for a child. For example, they could decide to use a height of 50 cm, width of 50 cm, and a length of 60 cm. However, a child could not practically use a toy box that is 100 cm tall.

ESSENTIAL QUESTIONS

- Explain the relationship between 3D objects that have different measurements, but the same volume.
- How did you determine the most appropriate dimensions for the box based on its use?

MATERIALS

- Ruler
- Paper (grid paper works very nicely)
- Centimeter cubes (optional)

GROUPING

Individual/pairs

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will be designing a toy box for a child's bedroom. The box needs to hold 150,000 cubic cm of toys. They must design two boxes with appropriate dimensions and tell which box would be most suitable for use in a child's bedroom.

Comments: You might begin this task by asking them if they have ever seen a toy box (a box designed to hold toys) and let them describe what they know. Ask them why they think the height of toy boxes is usually less than their width. Lead a general discussion of how the size of the toy box needs to be appropriate for use by a child.

Task Directions: Draw and label two designs for a toy box. Decide which design is most appropriate for a child's bedroom. Explain your answer.

FORMATIVE ASSESSMENT QUESTIONS

- How could you determine which 3 numbers could be multiplied together to get 150,000?
- Is your answer reasonable? How do you know?
- What expression might you use to find volume?

DIFFERENTIATION

Extension:
- Have students design another toy box with a capacity of 12,000 cubic inches.

Intervention:
- Students may work with partners.
- Students may use calculators.
- Students may use centimeter cubes to create a model.

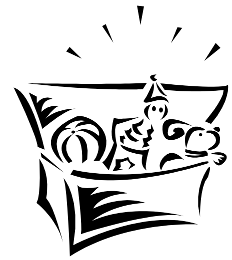
Intervention Table

TECHNOLOGY CONNECTIONS

- http://illuminations.nctm.org/ActivityDetail.aspx?id=6 This student interactive, from Illuminations, helps students explore the volume of a box based on the amount of unit cubes that can fit inside of it.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L240 In this lesson, from Illuminations, students explore how variations in solar collectors affect the energy absorbed. They make rectangular prisms that have the same volume but different linear dimensions. Students investigate relationships among the linear dimensions, the area, and the volume of rectangular prisms.

Name________________________ Date_________________________

Toy Box Designs



You are designing a toy box for child's bedroom. The toy box needs to be able to hold 150,000 cubic centimeters of toys. What might the dimensions be?

1. Draw and label two possible designs for the toy box.

2. Explain which design would work best in a child's bedroom and give reasons to support your choice.

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Practice Task: Breakfast for All

Back to Task Table

Adapted from K-5 Math Teaching Resources
Approximately 1 day

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

MGSE5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

MGSE5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

BACKGROUND KNOWLEDGE

Students should have had practice determining the volume of rectangular prisms. In addition, they should be familiar with the terminology "half the size of" and "three times the size of" and be able to determine relative dimensions. They should also be able to determine the correct unit of measure for given item (centimeters/inches or meters/feet/yards)

COMMON MISCONCEPTIONS:

Students may believe that in order to make the boxes "half the size" or "three times the size" they need to adjust each dimension (length, width, height) by half or three times. They need to investigate how the total volume is affected by changing the dimensions and determine "half" and "three time" by calculating total volume.

ESSENTIAL QUESTIONS

- How can I determine appropriate units of measure for an object?
- How did you determine the sizes for the mini-sized box and the super-sized box?

MATERIALS

- Ruler
- Grid paper

GROUPING

Individual/Pairs

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will be designing three different sizes of cereal boxes. They will need to determine the dimensions for the original box and then use the appropriate operations to enlarge or reduce the size of the original box to meet the specifications of the manufacturer.

Comments: You could begin this task by showing several cereal boxes and asking them to estimate the dimensions of the box. They could even measure a cereal box to find out what the appropriate dimensions could be.

Task Directions: Design the packaging for a new breakfast cereal in three different sized boxes. Draw a design for each box. Label the dimensions and calculate the volume of each one.

FORMATIVE ASSESSMENT QUESTIONS

- Justify why you chose which unit of measure to use?
- Is your answer reasonable? How do you know?

DIFFERENTIATION

Extension:
- Have students solve and answer the following question: *How big would a box have to be to hold enough cereal for your entire school?*

Intervention:
- Students may work with partners.
- Students may use calculators.

Intervention Table

TECHNOLOGY CONNECTIONS

- http://illuminations.nctm.org/ActivityDetail.aspx?id=6 This student interactive, from Illuminations, helps students explore the volume of a box based on the amount of unit cubes that can fit inside of it.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L240 In this lesson, from Illuminations, students explore how variations in solar collectors affect the energy absorbed. They make rectangular prisms that have the same volume but different linear dimensions. Students investigate relationships among the linear dimensions, the area, and the volume of rectangular prisms.

Name ____ Date _________________________

Breakfast for All

You have been asked to create the packaging for a new kind of cereal. The manufacturer wants three different sized boxes:

1. A standard sized cereal box
2. A mini-sized box that is half as tall, half as wide, and half as deep as the standard size
3. A super-sized box that is three times as tall, three times as wide and three times as deep as the standard size.

Using grid paper, draw a possible design for each box. Label the dimensions and calculate the volume.

Which box do you think would be the best seller? Write your answer on the lines below and tell why you think so.

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Culminating Task: Boxing Boxes

Back to Task Table

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.MD.1 Convert among different-sized standard measurement units (mass, weight, length, time, etc.) within a given measurement system (customary and metric) (e.g., convert 5cm to 0.05m), and use these conversions in solving multi-step, real word problems.

MGSE5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

MGSE5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

MGSE5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

BACKGROUND KNOWLEDGE

"*Volume* typically refers to the amount of space that an object takes up" whereas "*capacity* is generally used to refer to the amount that container will hold," Van de Walle (2006) (p. 265). To distinguish further between the two terms, consider how the two are typically measured. Volume is measured using linear measures (ft, cm, in, m, etc) while capacity is measured using liquid measures (L, mL, qt, pt, g, etc). However, Van de Walle reminds educators, "having made these distinctions [between volume and capacity], they are not ones to worry about. The term *volume* can also be used to refer to the *capacity* of a container" (p. 266).

Van de Walle, J. A. & Lovin, L. H. (2006). *Teaching students-centered mathematics: Grades 3-5*. Boston: Pearson Education, Inc.

COMMON MISCONCEPTIONS

When solving problems that require renaming units, students use their knowledge of renaming the numbers as with whole numbers. Students need to pay attention to the unit of measurement which dictates the renaming and the number to use. The same procedures used in renaming whole numbers should not be taught when solving problems involving measurement conversions.

ESSENTIAL QUESTIONS

- Can different size containers have the same volume?
- Why does the formula *V=l x w x h* work to find volume?

MATERIALS

- "Boxing Boxes" student recording sheet
- Snap cubes and/or 1" grid paper (several sheets per student), scissors, and clear tape
- "Boxing Boxes, Part II" student recording sheet (optional)

GROUPING

Individual/Partner Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students explore volume while packing shipping boxes with various-sized merchandise boxes.

Comments

This task can be introduced by asking small groups of students to create the different sized merchandise boxes using grid paper or snap cubes. If using grid paper, students will need to sketch the nets for the boxes described on 1" grid paper and then cut the nets out and fold them to create the rectangular prisms. If using snap cubes, students can create the required rectangular prisms with snap cubes using the dimensions required. Students can then use these models while working on the task.

Allow students to create their own chart for the "Boxing Boxes" task that makes sense to them. Then allow students to share their chart with students in their small group and choose two or three students who created different charts to share their work with the class.

Notice that the capacity of the standard shipping box is 12 ft^3. Therefore, the sum of the volumes of the merchandise boxes packed must equal 12 ft^3 for each packing plan (see table below).

Volume of Merchandise Boxes
(Number of Boxes × Volume of each Box in ft $^{3)}$

Packing Plans	Merchandise Box W	Merchandise Box X	Merchandise Box Y	Merchandise Box Z	Total Volume
1				12 × 1 ft^3	12 ft^3
2	2 × 6 ft^3				12 ft^3
3	1 × 6 ft^3	1 × 4 ft^3		2 × 1 ft^3	12 ft^3
4	1 × 6 ft^3			6 × 1 ft^3	12 ft^3
5		3 × 4 ft^3			12 ft^3
6		2 × 4 ft^3		4 × 1 ft^3	12 ft^3
7			1 × 8 ft^3	4 × 1 ft^3	12 ft^3
8		1 × 4 ft^3		8 × 1 ft^3	12 ft^3

Merchandise Packing Guide

Number of Merchandise Boxes in Each Standard Packing Box				
Packing Plans	**Merchandise Box W**	**Merchandise Box X**	**Merchandise Box Y**	**Merchandise Box Z**
1				12
2	2			
3	1	1		2
4	1			6
5		3		
6		2		4
7			1	4
8		1		1

The volume of the merchandise boxes are as follows:
Merchandise Box W: 1 ft x 3 ft x 2 ft = 6 ft^3
Merchandise Box X: 1 ft x 2 ft x 2 ft = 4 ft^3
Merchandise Box Y: 2 ft x 2 ft x 2 ft = 8 ft^3
Merchandise Box Z: 1 ft x 1 ft x 1 ft = 1 ft^3

The capacity of the standard shipping box is 2 ft × 3 ft × 2 ft = 12 ft^3

Additionally, students will need to write a letter to their boss explaining how to use the chart they created.

Task Directions
Students will follow the directions below from the "Boxing Boxes" student recording sheet.

You have been hired by Boxes Unlimited to help determine the best way to package merchandise for shipping.
Boxes Unlimited has a standard shipping box which will hold merchandise measuring 2 ft by 3 ft by 2 ft.
Boxes Unlimited needs to pack merchandise they receive into the standard shipping box. The merchandise arrives in four different box sizes.

Merchandise Box W is 1 ft. x 3 ft. x 2 ft.
Merchandise Box X is 1 ft. x 2 ft. x 2 ft.
Merchandise Box Y is 2 ft. x 2 ft. x 2 ft.
Merchandise Box Z is 1 ft. x 1 ft. x 1 ft.

Your task is to create a chart for employees to use as a reference when they are packing boxes for shipment. Be sure to include the volume of each merchandise box and the

capacity of the standard shipping box on your chart. Convert the capacity of the standard shipping box from cubic feet to cubic yards.
Write a report to your boss explaining how to read your chart.

FORMATIVE ASSESSMENT QUESTIONS

- Have you found all of the possible ways to fill the standard shipping box? How do you know?
- What is the total capacity of the standard shipping box? Will the merchandise completely fill the standard shipping box? How do you know?
- How are you organizing your packing chart? Why did you choose this type of organizational chart?
- Explain how your chart could be used by the employees who pack boxes?
- Will the formula *l x w x h* work to find the volume of any 3D shape? Explain your reasoning.

DIFFERENTIATION

Extension

- Ask students to consider a large shipping box with dimensions of 3 ft × 3 ft × 3 ft. What are the ways that this packing box could be filled with the given merchandise boxes? Students could work the task with this large shipping box rather than the regular shipping box. Next, students who worked with the large shipping box could be paired with students who worked on the standard shipping box. Partners could then be asked to determine which size box would be a better choice and justify their thinking.

Intervention

- Encourage students to use snap-cubes to create models of the merchandise boxes.
- Students who would benefit from a chart in which to record their work should be provided one. A sample is given below. See “Boxing Boxes, Part II” student recording sheet.

[Intervention Table]

TECHNOLOGY CONNECTIONS

- http://illuminations.nctm.org/ActivityDetail.aspx?id=6 This student interactive, from Illuminations, helps students explore the volume of a box based on the amount of unit cubes that can fit inside of it.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L240 In this lesson, from Illuminations, students explore how variations in solar collectors affect the energy absorbed. They make rectangular prisms that have the same volume but different linear dimensions. Students investigate relationships among the linear dimensions, the area, and the volume of rectangular prisms.

Name____ Date _______________________________

Boxing Boxes

You have been hired by Boxes Unlimited to help determine the best way to package merchandise for shipping.

Boxes Unlimited has a standard shipping box which will hold merchandise measuring 2 ft by 3 ft by 2 ft.

Boxes Unlimited needs to pack merchandise they receive into the standard shipping box. The merchandise arrives in four different box sizes.
Merchandise Box W is 1 ft. x 3 ft. x 2 ft.
Merchandise Box X is 1 ft. x 2 ft. x 2 ft.
Merchandise Box Y is 2 ft. x 2 ft. x 2 ft.
Merchandise Box Z is 1 ft. x 1 ft. x 1 ft.

Your task is to create a chart for employees to use as a reference when they are packing boxes for shipment. Be sure to include the volume of each merchandise box and the capacity of the standard shipping box on your chart. Convert the capacity of a standard shipping box from cubic feet to cubic yards.

Write a report to your boss explaining how to read your chart.

Name_______________________________________ Date ______________________________

Boxing Boxes
Part II

The volume of the merchandise boxes are as follows:

Merchandise Box W: _______________________________

Merchandise Box X: _______________________________

Merchandise Box Y: _______________________________

Merchandise Box Z: _______________________________

The capacity of the standard shipping box is _______________________________.

<u>3-Act Task:</u> The Fish Tank

Back to Task Table

Task adapted from Graham Fletcher (http://gfletchy3act.wordpress.com/the-fish-tank/)

Approximate time 1-2 days

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will view the Act 1 video and discuss with their peers what they are mathematically curious about. After recording the main question to be investigated, students decide the best way in which to find a solution to the problem. If students find that the means they are using is not producing a reasonable result, they will make necessary changes to their method in order to get a more reasonable solution.

2. Reason abstractly and quantitatively. Students make sense of the quantities being used in the problem by determining the relationship between the volume of the fish tank, the number of cups it takes to fill the number of cubic inches in the fish tank and the time it takes to fill the fish tank. Students will need to consider the units being used as they work through Act 2. Students will use cubic inches, cups, seconds and minutes as they determine a solution to the problem.

4. Model with mathematics. Students will apply previously learned mathematical knowledge to solve the question of how long it will take to fill the fish tank shown in the video. As they solve the problem, students will multiply and divide whole numbers and decimal numbers. They will also create mixed numbers as they find solutions throughout the problem. It will be helpful to rewrite the fractions as equivalent mixed numbers, but this is not an expectation of the standard.

6. Attend to precision. Since students are working with a variety of units in the problem, it is necessary to attend to the meaning of the quantities being used as they calculate answers. For example, students may start by finding the volume of the fish tank. After finding the volume, the quantity needs to be precisely communicated as a quantity in cubic inches. Then, students will need to request the conversion for cups to cubic inches. Once they learn the conversion, they can divide the volume of the fish tank by the number of cubic inches in one cup. Once the quantity is determined, students should report it in cups to precisely communicate the meaning of the quantity.

8. Look for and express regularity in repeated reasoning. In Act 4, students can solve a similar problem by changing the volume of the fish tank. Students can explain how the strategies used to solve the original problem would work to help solve the problems in Act 4.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE5.MD.1 Convert among different-sized standard measurement units (mass, weight, length, time, etc.) within a given measurement system (customary and metric) (e.g., convert 5cm to 0.05m), and use these conversions in solving multi-step, real word problems.

MGSE5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

MGSE5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole- number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

ESSENTIAL QUESTIONS

During Act 1, students view a video of a fish tank being filled with water for about 25 seconds. Students request necessary information to help find how long it will take to fill the fish tank with water. It is imperative that teachers allow students to ask questions of each other and participate in discussion that will lead the students to infer that information during Act 2. The essential questions below can be shared at the beginning of Act 2 to define the emphasis of the problem-solving opportunity being presented.

- How does the object being measured determine what unit of measure is needed?
- How is measurement used to solve everyday problems?

MATERIALS

Act 1 video “The Fish Tank” http://gfletchy3act.wordpress.com/the-fish-tank/
Student recording sheet (attached)
Act 2 “The Fish Tank” video and images (Use the link above to locate images.)
Act 3 “The Fish Tank” reveal video (Use the link above to locate the reveal picture.)

GROUPING

Whole group, partners or small groups

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Students will view a video of a fish tank being filled for about 25 seconds. Next, they will be asked to discuss what they wonder about or are curious about. These questions will be recorded on a class chart or on the board and on the student recording sheet. Students will then use mathematics to answer one of the questions generated on the chart. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them.

Background Knowledge:

This task follows the 3-Act Math Task format originally developed by Dan Meyer. More information on this type of task may be found at http://blog.mrmeyer.com/category/3acts/. A Three-Act Task is a whole-group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the ***Guide to Three-Act Tasks*** on georgiastandards.org.

Prior to fifth grade, students have experienced measuring liquid volume in liters and milliliters in third grade, including that milliliters are smaller than liters and that it takes 1,000 milliliters to make a liter. As fourth graders, students have experienced measuring liquid volume in both the customary and metric systems. They can express larger measurements as an equivalent amount of a smaller measurement within the same system. In both third and fourth grades, students have solved word problems involving liquid volume using all four operations. Students are only required to convert units within the same system of measurement. In this 3Act Task, students will need to know how many cubic inches are in one cup, which is beyond what students are required to know. However, students are naturally curious about conversions between systems of measurement and various types of measurement.

COMMON MISCONCEPTIONS;

Students may have trouble knowing which operation to use when converting measurements within the same system of measurement. Oftentimes this occurs because students have been told the measurement conversions rather than directly experiencing the conversions. Students better understand measurement and the units of measure when they have experienced measuring objects using a variety of units. For example, students can take a bucket and fill it using cups, pints, quarts and gallons. After recording how many of each unit it takes to fill the bucket, students can generalize that some units of measure are larger than others and that when using a larger unit of measure, it takes less of the unit to measure the item. They can also understand that when a unit of measure is smaller, it will take more of that unit to measure the item.

Task Directions:

Act 1 – Whole Group - Pose the conflict and introduce students to the scenario by showing Act I video or picture. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/**)**
"Introduce the central conflict of your story/task clearly, visually, viscerally, using as few words as possible."

- Show the Act 1 video "The Fish Tank" to students. http://gfletchy3act.wordpress.com/the-fish-tank/
- Ask students what they noticed mathematically in the video, what they wonder about, and what questions they have about what they saw in the video.
- Give each student a copy of the Student Recording Sheet. Have students record their questions and curiosities in the Act 1 section that asks "What mathematical questions come to your mind?" Consider doing a think-pair-share so that students have an opportunity to talk with each other before sharing questions with the whole group. Students may need to view the video multiple times as they develop questions.
- Share and record students' questions. The teacher may need to guide students so that the questions generated are math-related.
- Share the main question that will be investigated during today's lesson. In the list below it is denoted with an asterisk. (*) Students will record the main question on their recording sheet.
- Ask students to estimate how long it will take to fill the fish tank with water. Students will write their best estimate, then write two more estimates – one that is too low and one that is too high so that they establish a range in which the solution should occur. Students should plot their three estimates on a number line. Space is available on the recording sheet for students to record open number line with all three estimates.

Anticipated questions students may ask and wish to answer:

- What is the volume of the fish tank?
- *How long will it take to fill the fish tank with water?
- How fast is water coming from the hose?
- How much water is in the tank now?

**Main question(s) to be investigated*

Act 2 – **Student Exploration -** Provide additional information as students work toward solutions to their questions. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/**)**
"The protagonist/student overcomes obstacles, looks for resources, and develops new tools."

- During Act 2, students review the main question from Act 1 and decide on the facts, tools, and other information needed to answer the question. The main question for this task is "How long will it take to fill the fish tank with water?" When students decide what they need to solve the problem, they should ask for those things. The Act 2 video "The Fish Tank" shows that it takes 10 seconds to pour one cup. There are also two images with information that students may request. The first image is a picture of the fish tank with the measurements of each dimension. The second image shows how many cubic inches are in one cup. Copies of the images can be given to the students at their request. The video and images can be found under "Act 2" using the link to the Act 1 video. It is pivotal to the problem-solving process that students decide what is needed without being given the information up front.

- Students can record information that they need to solve the problem, given information, estimates and work on the student recording sheet under Act 2.
- The teacher provides guidance as needed during this phase. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin. Questioning is an effective strategy that can be used, with questions such as:
    - What is the problem you are trying to solve?
    - What do you think affects the situation?
    - Can you explain what you've done so far?
    - What strategies are you using?
    - What assumptions are you making?
    - What tools or models may help you?
    - Why is that true?
    - Does that make sense?

Additional Information for Act 2

"The Fish Tank?" video and images http://gfletchy3act.wordpress.com/the-fish-tank/

Important note: Although students will only investigate the main question for this task, it is important for the teacher to not ignore student generated questions. Additional questions may be answered after they've found a solution to the main question, or as homework or extra projects.

Act 3 – Whole Group – Share solutions and strategies.

- Students present their solutions and strategies and compare them. Have students share the solutions they arrived at when determining how long it will take to fill the fish tank with water. Record student solutions on the board, as you would in a Number Talk.
- Reveal the solution by showing the Act 3 Reveal video. The video can be found under "Act 3" using the link to the Act 1 video.
- After comparing the different solutions, have students discuss why their solutions were different from what was revealed in the Act 3 video.
- As the discussion progresses, students can discuss why different solutions were reached as they solved the problem. If students don't pick up on this idea through the course of

the discussion, it is appropriate for the teacher to ask students to think about why various solutions were given.

- Have students record their results in the Act 3 section of the student recording sheet. Students can also plot their solution on the open number line on the student recording sheet.
- Lead discussion to compare these, asking questions such as:
  - How reasonable was your estimate?
  - Which strategy was most efficient?
  - Can you think of another method that might have worked?
  - What might you do differently next time?

Act 4, The Sequel - "The goals of the sequel task are to a) challenge students who finished quickly so b) I can help students who need my help. It can't feel like punishment for good work. It can't seem like drudgery. It has to entice and activate the imagination." Dan Meyer
http://blog.mrmeyer.com/2013/teaching-with-three-act-tasks-act-three-sequel/

Students can think about where fish tanks that are smaller and larger than the fish tank shown in the video could be seen. There are desktop fish tanks and fish tanks that could be found in places like the Georgia Aquarium. Students can design a fish tank that is larger than the fish tank shown in the video and a fish tank that is smaller than the fish tank shown in the video. After labeling the dimensions of the fish tanks, students can calculate how long it would take to fill the fish tanks they designed with water as shown in the video.

FORMATIVE ASSESSMENT QUESTIONS

- What models did you create?
- What organizational strategies did you use?
- How was your result different than other results in the class? Give examples that prove or disprove your results.
- What tools and resources were helpful in solving this problem?
- As you solved this problem, what connections did you make to other mathematical concepts you have learned?

DIFFERENTIATION

Extension

Students can use the information to determine how long it would take to fill the fish tank with water using pints, quarts and gallons.

Intervention

Students may need help with attending to the unit being found when solving various parts of the problem. Questioning students about what the numbers being used in the problem represent and how the quantities are related will help them attach meaning to the numbers that are being used in the problem.

Intervention Table

TECHNOLOGY CONNECTIONS

- Solve Real World Liquid Volume Problems with Unit Conversions http://learnzillion.com/lessons/3502-solve-real-world-liquid-volume-problems-with-unit-conversions This LearnZillion video reviews common customary measurement conversions and shows strategies for making conversions within the same system of measurement to solve word problems.

Three Act Task Student Recording Sheet

Name ______________________

ACT 1

What questions come to your mind?

Main Question: ____

What is your first estimate and why?
Record an estimate that is too low and an estimate that is too high.
On an empty number line, record all three estimates made above.

ACT 2

What information would you like to know or need to solve the MAIN question?

Record the given information (measurements, materials, etc…)
If possible, give a better estimation with this information: ______________________________

Act 2 (con't)

Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?