CCGPS Frameworks

Mathematics

CCGPS Pre-Calculus
Unit 2: Trigonometric Functions

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"Making Education Work for All Georgians"

Unit 2
Trigonometric Functions

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OVERVIEW

In this unit students will:

- Build upon understanding of the trigonometric functions
- Use special right triangles to determine the x- and y-coordinates of angles on the unit circle.
- Investigate how the symmetry of the unit circle helps to extend knowledge to angles outside of the first quadrant
- Use the symmetry of the unit circle to define sine and cosine as even and odd functions
- Investigate inverse trigonometric function
- Use trigonometric inverses to solve equations and real-world problems.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under "Evidence of Learning" be reviewed early in the planning process. A variety of resources should be utilized to supplement this unit. This unit provides much needed content information, but excellent learning activities as well. The tasks in this unit illustrate the types of learning activities that should be utilized from a variety of sources.

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

KEY STANDARDS

Build new functions from existing functions

MCC9-12.F.BF.4 Find inverse functions.

MCC9-12.F.BF.4d (+) Produce an invertible function from a non-invertible function by restricting the domain.

Extend the domain of trigonometric functions using the unit circle

MCC9-12.F.TF.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

MCC9-12.F.TF.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

MCC9-12.F.TF.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows it's inverse to be constructed.

MCC9-12.F.TF.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

RELATED STANDARDS

MCC9-12.F.IF.7e Graph trigonometric functions showing period, midline, and amplitude

MCC9-12.F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers interpreted as radian measures of angles traversed counterclockwise around the unit circle.

MCC9-12.F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★

STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. **Make sense of problems and persevere in solving them.** High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between

equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. **Reason abstractly and quantitatively.** High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others.** High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. **Model with mathematics.** High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **Use appropriate tools strategically.** High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra

system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. **Attend to precision.** High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. **Look for and make use of structure.** By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8. **Look for and express regularity in repeated reasoning.** High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who do not have an understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a missing mathematical knowledge effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

ENDURING UNDERSTANDINGS

- Understand the relationship between right triangle trigonometry and unit circle trigonometry
- Use the unit circle to define trigonometric functions
- There are many instances of periodic data in the world around us and trigonometric functions can be used to model real world data that is periodic in nature.
- The inverses of sine, cosine and tangent functions are not functions unless the domains are limited.

ESSENTIAL QUESTIONS

- How can special right triangles help us find the coordinates of certain angles on the unit circle?
- How does symmetry help us extend our knowledge of the unit circle to an infinite number of angles?
- Why does the calculator only give one answer for an inverse trig function? Aren't there infinite answers?
- How do inverse trigonometric functions help us solve equations?

CONCEPTS/SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Knowledge of unit circle in radians and degrees
- Graphing trigonometric functions

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.
Definitions and activities for these and other terms can be found on the Intermath website. http://intermath.coe.uga.edu/dictnary/homepg.asp

- **Co-terminal Angle:** Two angles are co-terminal if they are drawn in the standard position and both have their terminal sides in the same location.

- **Even Function:** A function f is even if the graph of f is symmetric with respect to the y-axis. Algebraically, f is even if and only if f(-x) = f(x) for all x in the domain of f.

- **Odd Function:** A function f is odd if the graph of f is symmetric with respect to the origin. Algebraically, f is odd if and only if f(-x) = -f(x) for all x in the domain of f.

- **Reference Angle:** A reference angle for angle θ is the positive acute angle made by the terminal side of angle θ and the x-axis.

- **Special Right Triangles**: Refers to the 45-45-90 and 30-60-90 right triangles

- **Terminal side of angle:** The initial side of an angle lies on the x-axis. The other side, known as the terminal side, is the one that can be anywhere and defines the angle.

- **Unit Circle:** A unit circle is a circle that has a radius of one unit.

CLASSROOM ROUTINES

The importance of continuing the established classroom routines cannot be overstated even in a high school classroom. Daily routines must include such obvious activities as estimating, analyzing data, describing patterns, and answering daily questions. They should also include less obvious routines, such as how to select materials, how to use materials in a productive manner, how to put materials away, how to access classroom technology such as computers and calculators. An additional routine is to allow plenty of time for students to explore new materials before attempting any directed activity with these new materials. The regular use of routines is important to the development of students' number sense, flexibility, fluency, collaborative skills and communication. These routines contribute to a rich, hands-on standards based classroom and will support students' performances on the tasks in this unit and throughout the school year.

Throughout this unit students should work with small groups or in pairs to discover the concepts, so a culture of cooperative learning is necessary. Students should feel comfortable sharing thoughts, asking and answering questions, and making conjectures within their group. Concepts will require some individual practice to master the embedded skills. Thus the environment should be developed to make quick and smooth transitions from group work to individual or paired practice. While technology would make the learning tasks more engaging for some students, it is not necessary for this task. However classrooms should be equipped with compasses, protractors, and rulers (to use as a straight edge). Prior to planning the lessons for this unit, the teacher should read through the tasks carefully to anticipate other classroom routines that may need to be instilled.

STRATEGIES FOR TEACHING AND LEARNING

- Students should be actively engaged by developing their own understanding.
- Mathematics should be represented in as many ways as possible by using graphs, tables, pictures, symbols and words.
- Interdisciplinary and cross curricular strategies should be used to reinforce and extend the learning activities.
- Appropriate manipulatives and technology should be used to enhance student learning.
- Students should be given opportunities to revise their work based on teacher feedback, peer feedback, and metacognition which includes self-assessment and reflection.
- Students should write about the mathematical ideas and concepts they are learning.
- Consideration of all students should be made during the planning and instruction of this unit. Teachers need to consider the following:
    - What level of support do my struggling students need in order to be successful with this unit?
    - In what way can I deepen the understanding of those students who are competent in this unit?
    - What real life connections can I make that will help my students utilize the skills practiced in this unit?

EVIDENCE OF LEARNING

By the conclusion of this unit, students should be able to demonstrate the following competencies:

- Use and apply special right triangles to the unit circle
- Find inverse trigonometric functions
- Use inverse trigonometric functions to solve equations

TASKS

The following tasks represent the level of depth, rigor, and complexity expected of all Pre-Calculus students. These tasks or tasks of similar depth and rigor should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as a performance task, they may also be used for teaching and learning (learning task).

Scaffolding Task	Tasks that build up to the learning task.
Learning Task	Constructing understanding through deep/rich contextualized problem solving tasks.
Practice Task	Tasks that provide students opportunities to practice skills and concepts.
Performance Task	Tasks which may be a formative or summative assessment that checks for student understanding/misunderstanding and or progress toward the standard/learning goals at different points during a unit of instruction.
Culminating Task	Designed to require students to use several concepts learned during the unit to answer a new or unique situation. Allows students to give evidence of their own understanding toward the mastery of the standard and requires them to extend their chain of mathematical reasoning.
Formative Assessment Lesson (FAL)	Lessons that support teachers in formative assessment which both reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards.

Task Name	Task Type *Grouping Strategy*	Content Addressed
Right Triangles and the Unit Circle	Learning Task *Individual/Partner Task*	Write a rational function in different forms
Inverse Trigonometric Functions	Learning Task *Partner/Small Group Task*	Add, subtract, multiply, and divide rational expressions

<u>Right Triangles and the Unit Circle:</u>

Standards

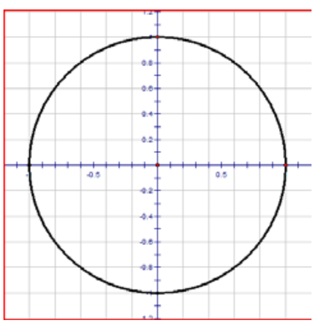
MCC9-12.F.TF.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π - x, π + x, and 2π - x in terms of their values for x, where x is any real number.

MCC9-12.F.TF.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Students may complete the work for this task on the large unit circle provided under #1. Colored pencils would be very helpful for the activity. You may want to make multiple copies of the unit circle graph so that students can use a "clean" unit circle for each reference angle group.

<u>Right Triangles and the Unit Circle:</u>

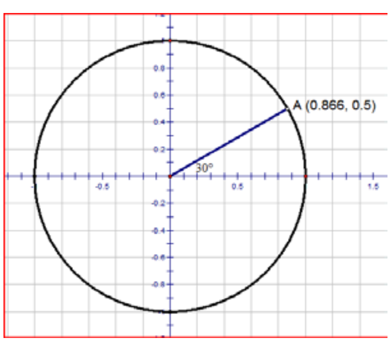
1. The circle below is referred to as a "unit circle." Why is this the circle's name?



It is called a unit circle because the radius is 1 unit.

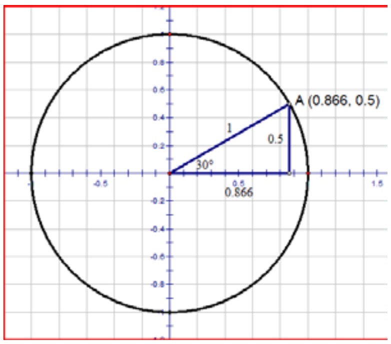
Part I

2. Using a protractor, measure a 30° angle with vertex at the origin, initial side on the positive x-axis and terminal side above the x-axis. (Any angle with vertex at the origin, initial side on the positive x-axis and measured counterclockwise from the x-axis is called an angle in standard position.) Label the point where the terminal side intersects the circle as "A". Approximate the coordinates of point A using the grid.



Students should use the grid to approximate the x and y coordinates of A. These values should be close the answers shown, but may vary slightly.

3. Now, drop a perpendicular segment from the point you just put on the circle to the x-axis. You should notice that you have formed a right triangle. How long is the hypotenuse of your triangle? Using trigonometric ratios, specifically sine and cosine, determine the lengths of the two legs of the triangle. How do these lengths relate the coordinates of point A? How should these lengths relate to the coordinates of point A? Now use the properties of special right triangles to determine the lengths of the two legs. How do these lengths relate to the lengths found using trigonometric ratios? Which length is the exact solution and which is an estimate?



The hypotenuse is 1 unit long.

Horizontal Leg: $\sin 30° = \frac{y}{1}$ ***y = 0.5***

Vertical Leg: $\cos 30° = \frac{x}{1}$ ***x = 0.866***

The values should be the same or similar to the estimates from the graph in #2.

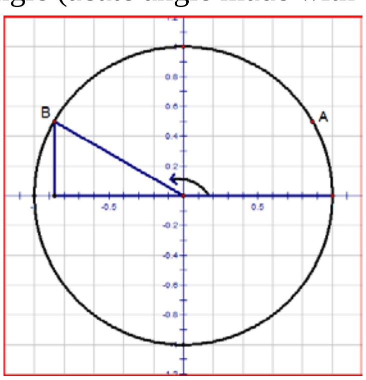
Using special right triangles:

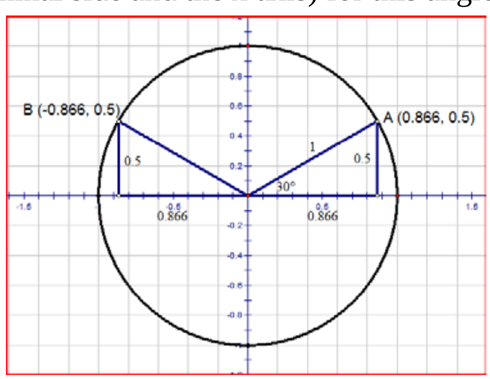
Horizontal Leg: $\frac{\sqrt{3}}{2}$

Vertical Leg: $\frac{1}{2}$

These values are the exact versions of the estimates previously determined using the trig ratios.

4. Using a Mira or paper folding, reflect this triangle across the y-axis. Label the resulting image point as point B. What are the coordinates of point B? How do these coordinates relate to the coordinates of point A? What obtuse angle (standard position angle) was formed with the positive x-axis (the initial side) as a result of this reflection? What is the reference angle (acute angle made with the terminal side and the x axis) for this angle?

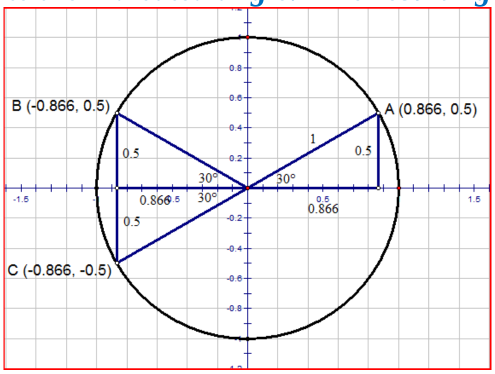




The coordinates of B are (-0.866, 0.5). The x-values of A and B are opposites while the y-values are the same. The angle of rotation formed is 150°. The reference angle is 30°.

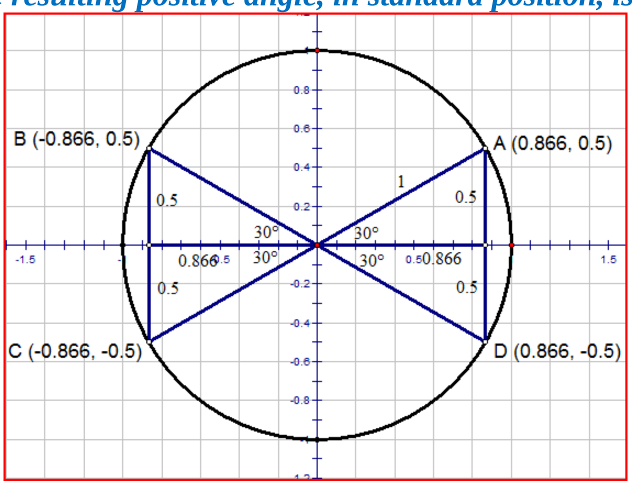
5. Which of your two triangles can be reflected to create the angle in the third quadrant with a 30^o reference angle? What is this angle measure? Complete this reflection. Mark the lengths of the three legs of the third quadrant triangle on your graph. What are the coordinates of the new point on circle? Label the point C.

The triangle from #4 (corresponding to the 150^o angle) can be reflected over the x-axis to create the indicated angle. The resulting angle measures 210^o.



6. Reflect the triangle in the first quadrant over the x-axis. What is this angle measure? Complete this reflection. Mark the lengths of the three legs of the triangle formed in quadrant four on your graph. What are the coordinates of the new point on circle? Label this point D.

The resulting positive angle, in standard position, is 330^o.



7. Let's look at what you know so far about coordinates on the unit circle. Complete the table.

θ	x-coordinate (estimate)	y-coordinate (estimate)	x-coordinate (exact)	y-coordinate (exact)
30^o	***0.866***	***0.5***	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
150^o	***-0.866***	***0.5***	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
210^o	***-0.866***	***-0.5***	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
330^o	***0.866***	***-0.5***	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$

Notice that all of your angles so far have a reference angle of 30^o.

Use a calculator to verify your conclusions for cos 30° and sin 30°. Use your calculator to find trig values of other angles.

Based on these relationships, on the unit circle, x = _____________ and y = _____________. This is a special case of the general trigonometric coefficients (rcosθ, rsinθ) where r = 1.

x = cosθ ***y=sinθ***

Part II

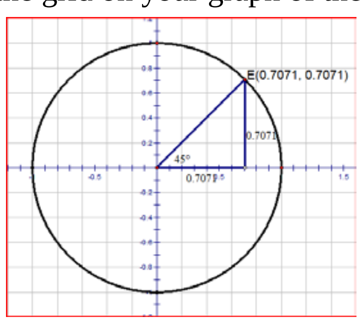
8. Now, let's look at the angles on the unit circle that have 45^o reference angles. What are these angle measures?

The angles are 45^o, 135^o, 225^o, 315^o.

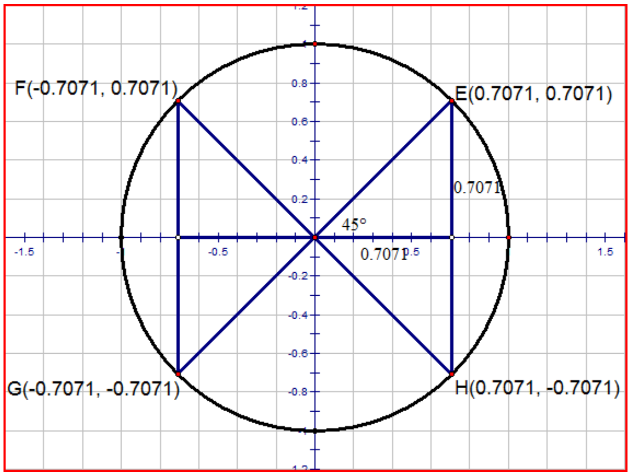
9. Mark the first quadrant angle from #8 on the unit circle. Draw the corresponding right triangle as you did in Part I. What type of triangle is this? Use the Pythagorean Theorem or the properties of special right triangles to determine the lengths of the legs of the triangle. Confirm that these lengths match the coordinates of the point where the terminal side of the 45^o angle intersects the unit circle using the grid on your graph of the unit circle.

This is an isosceles right triangle.

$$a^2 + a^2 = 1$$
$$2a^2 = 1$$
$$a^2 = \frac{1}{2}$$
$$a = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} \approx 0.7071$$



10. Using the process from Part I, draw the right triangle for each of the angles you listed in #8. Determine the lengths of each leg and match each length to the corresponding x- or y-coordinate on the unit circle. List the coordinates on the circle for each of these angles in the table.



θ	x-coordinate (estimate)	y-coordinate (estimate)	x-coordinate (exact)	y-coordinate (exact)
45^{o}	***0.7071***	***0.7071***	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
135^{o}	***-0.7071***	***0.7071***	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
225^{o}	***-0.7071***	***-0.7071***	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
315^{o}	***0.7071***	***-0.7071***	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$

Part III

11. At this point, you should notice a pattern between the length of the horizontal leg of each triangle and one of the coordinates on the unit circle. Which coordinate on the unit circle is given by the length of the horizontal leg of the right triangles?

The length of the horizontal leg provides the x-value of the coordinate on the unit circle The x-coordinate is the cosine of the angle on the unit circle.

12. Which coordinate on the unit circle is given by the length of the vertical leg of the right triangles?

The length of the vertical leg provides the y-value of the coordinate on the unit circle. The y-coordinate is the sine of the angle on the unit circle.

13. Is it necessary to draw all four of the triangles with the same reference angle to determine the coordinates on the unit circle? What relationship(s) can you use to determine the coordinates instead?

It is helpful to draw the triangles, but it is not necessary. The quadrant determines the sign of the requested trigonometric value.

14. Use special right triangles to determine the exact (x, y) coordinates where each angle with a 60^o reference angle intersects the unit circle. Sketch each angle on the unit circle and clearly label the coordinates. Record your answers in the table.

θ	x-coordinate (estimate)	y-coordinate (estimate)	x-coordinate (exact)	y-coordinate (exact)
60^o	***0.5***	***0.866***	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
120^o	***-0.5***	***0.866***	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
240^o	***-0.5***	***-0.866***	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
300^o	***0.5***	***-0.866***	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$

15. Think about what happens as you get to angles greater than 360^o. How can you predict the value of the sine of 420°? What about the cosine of 600^o?

sin 420^o = sin 60^o – Explanations will vary. The two angles are coterminal and therefore have the same sine value. Likewise, sin 600^o = sin 240^o

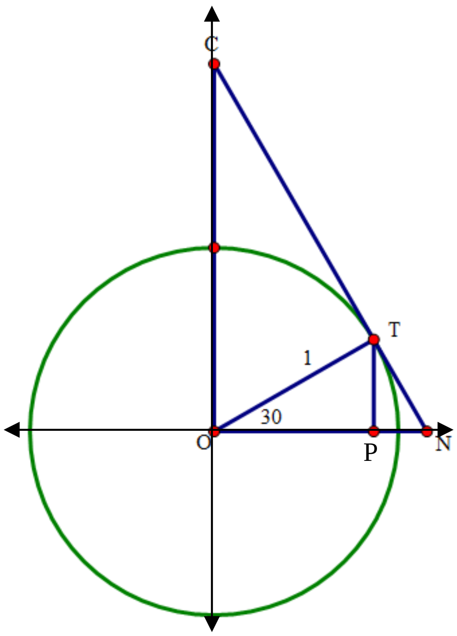
16. Does the same thing happen for negative angles? What is the largest negative angle that has the same sign and cosine as 120^o?

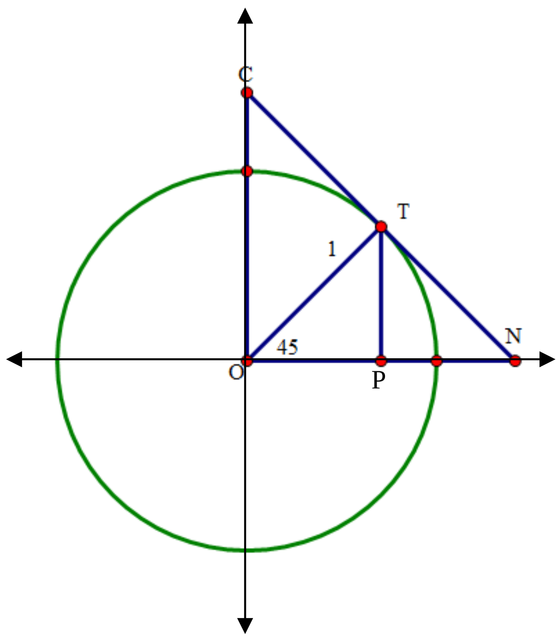
-240^o ***which you can find by subtracting 360^o from 120^o***

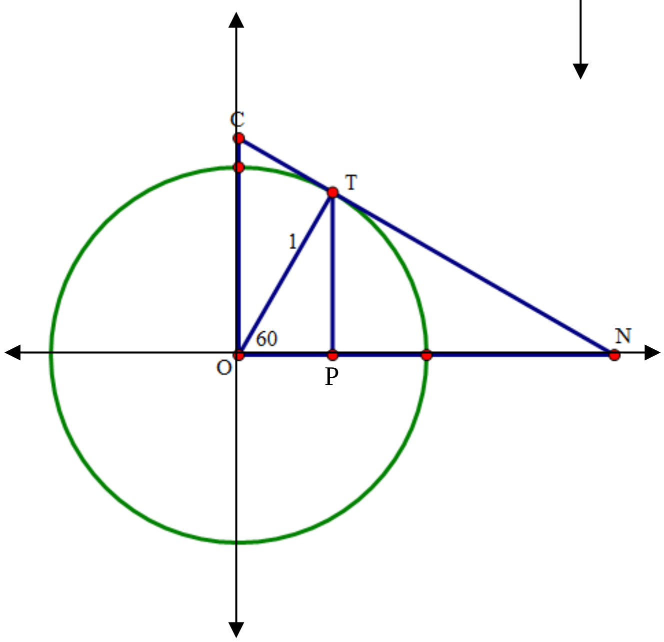
Part IV

17. In the figures below segment TP is perpendicular to segment ON. Line CN is tangent to circle O at T. N is the point where the line intersects the x-axis and C is the where the line intersects the y-axis. (Only segment CN is shown.)

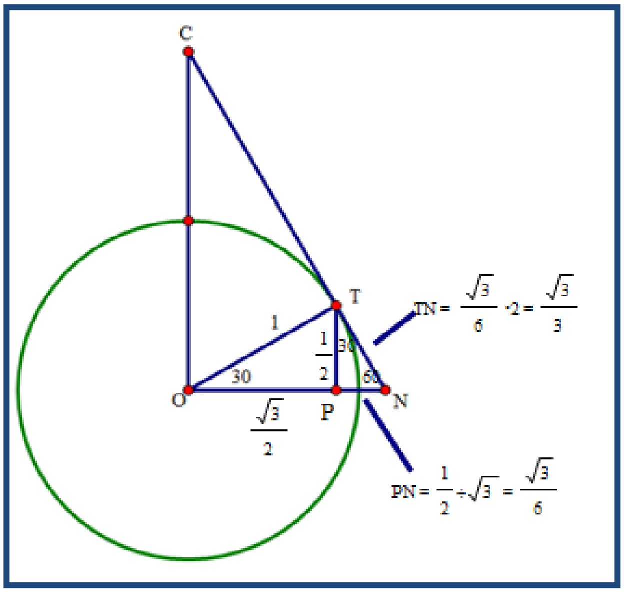
Use your knowledge of sine and cosine to determine the length of segment TN. Use exact answers, no decimal approximations.

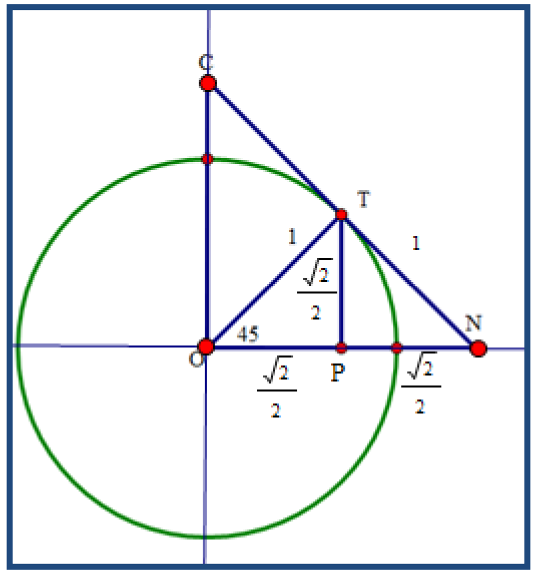


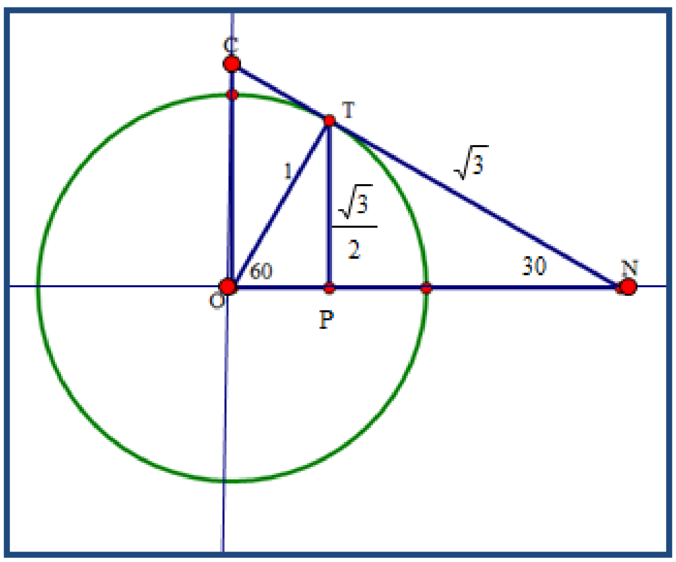




Solutions:







18. Using your understanding of the unit circle and $tangent\ \theta = \frac{sin\theta}{cos\theta}$, to complete the chart below for the indicated angles.

θ	Sin θ	Cos θ	Tan θ
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

19. How are these values of tangent related to the length you found in #4?

The tangent of the angle is equal to the length of the segment TN.

20. What would happen to the length of TN if the angle was changed to 0°?

The segment would no longer be a segment. It would be a point since T and N would be the same point. So the length would be equal to 0.

21. What would happen to the length of TN if the angle was changed to 90°?

If the angle is changed to 90°, the line perpendicular to the circle at point T would not intersect the x-axis since it would be parallel to the x-axis. Therefore, the length of the segment would be undefined.

22. Based on the chart above fill in the sine, cosine, and tangent values for all the angles with the indicated reference angle.

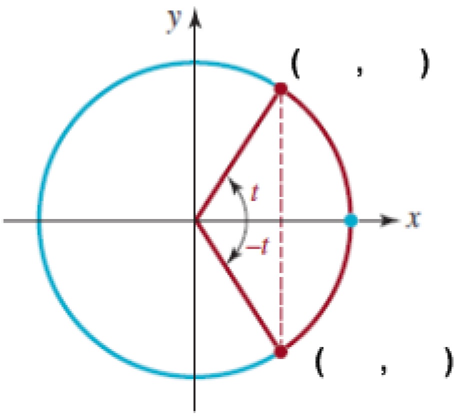
	θ	Sin θ	Cos θ	Tan θ
x	***30°***	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\pi - x$	***150°***	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
$\pi + x$	***210°***	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$2\pi - x$	***330°***	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$

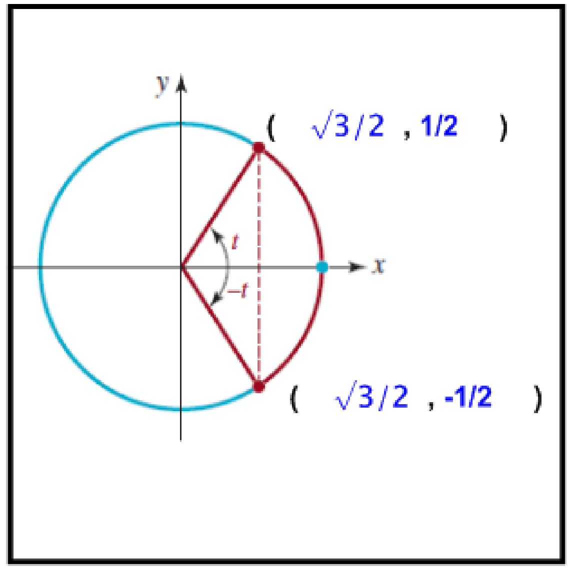
	θ	Sin θ	Cos θ	Tan θ
x	***45°***	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\pi - x$	***135°***	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1
$\pi + x$	***225°***	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1
$2\pi - x$	***315°***	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1

	θ	Sin θ	Cos θ	Tan θ
x	***60°***	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\pi - x$	***120°***	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
$\pi + x$	***240°***	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
$2\pi - x$	***300°***	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$

Part V

23. The symmetry of the unit circle is useful is generating the trigonometric values for an infinite number of angles. It is also useful in illustrating whether the sine and cosine functions are even or odd. Let's take 30^o as an example. In the picture below $t = 30^o$. Fill in the missing information.





24. Based on the graphs of sine and cosine that we studied in Advanced Algebra, we know that sine is sine is an odd function and cosine is an even function. What about the graphs help us to know this?

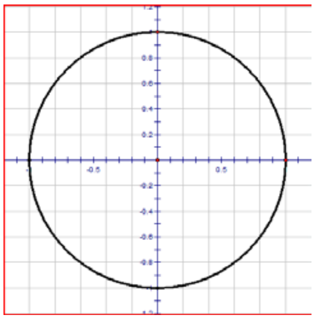
The graph of $y = \sin x$ is symmetrical about the origin and the graph of $y = \cos x$ is symmetrical about the y-axis.

25. How can we use the information in #23 to further show that sine is an odd function and cosine is an even function?

Since the x-values of t are the same that means that the cosine values for 30^o and -30^o are equal which is the definition of an even function: $\cos(-t) = \cos(t)$. The values for the sine of 30^o and -30^o are opposites which is the definition of an odd function: $\sin(-t) = -\sin(t)$

<u>Right Triangles and the Unit Circle:</u>

1. The circle below is referred to as a "unit circle." Why is this the circle's name?



Part I

2. Using a protractor, measure a 30^o angle with vertex at the origin, initial side on the positive x-axis and terminal side above the x-axis. (Any angle with vertex at the origin, initial side on the positive x-axis and measured counterclockwise from the x-axis is called an angle in standard position.) Label the point where the terminal side intersects the circle as "A". Approximate the coordinates of point A using the grid.

3. Now, drop a perpendicular segment from the point you just put on the circle to the x-axis. You should notice that you have formed a right triangle. How long is the hypotenuse of your triangle? Using trigonometric ratios, specifically sine and cosine, determine the lengths of the two legs of the triangle. How do these lengths relate the coordinates of point A? How should these lengths relate to the coordinates of point A? Now use the properties of special right triangles to determine the lengths of the two legs. How do these lengths relate to the lengths found using trigonometric ratios? Which length is the exact solution and which is an estimate?

4. Using a Mira or paper folding, reflect this triangle across the y-axis. Label the resulting image point as point B. What are the coordinates of point B? How do these coordinates relate to the coordinates of point A? What obtuse angle (standard position angle) was formed with the positive x-axis (the initial side) as a result of this reflection? What is the reference angle (acute angle made with the terminal side and the x axis) for this angle?

5. Which of your two triangles can be reflected to create the angle in the third quadrant with a 30^{o} reference angle? What is this angle measure? Complete this reflection. Mark the lengths of the three legs of the third quadrant triangle on your graph. What are the coordinates of the new point on circle? Label the point C.

6. Reflect the triangle in the first quadrant over the x-axis. What is this angle measure? Complete this reflection. Mark the lengths of the three legs of the triangle formed in quadrant four on your graph. What are the coordinates of the new point on circle? Label this point D.

7. Let's look at what you know so far about coordinates on the unit circle. Complete the table.

θ	x-coordinate (estimate)	y-coordinate (estimate)	x-coordinate (exact)	y-coordinate (exact)
30^{o}				
150^{o}				
210^{o}				
330^{o}				

Notice that all of your angles so far have a reference angle of 30^{o}.

Use a calculator to verify your conclusions for cos 30° and sin 30°. Use your calculator to find trig values of other angles.

Based on these relationships, on the unit circle, x = _____________ and y = _____________.
This is a special case of the general trigonometric coefficients ($r\cos\theta$, $r\sin\theta$) where r = 1.

Part II

8. Now, let's look at the angles on the unit circle that have 45^o reference angles. What are these angle measures?

9. Mark the first quadrant angle from #8 on the unit circle. Draw the corresponding right triangle as you did in Part I. What type of triangle is this? Use the Pythagorean Theorem or the properties of special right triangles to determine the lengths of the legs of the triangle. Confirm that these lengths match the coordinates of the point where the terminal side of the 45^o angle intersects the unit circle using the grid on your graph of the unit circle.

10. Using the process from Part I, draw the right triangle for each of the angles you listed in #8. Determine the lengths of each leg and match each length to the corresponding x- or y-coordinate on the unit circle. List the coordinates on the circle for each of these angles in the table.

θ	x-coordinate (estimate)	y-coordinate (estimate)	x-coordinate (exact)	y-coordinate (exact)
45^o				
135^o				
225^o				
315^o				

Part III

11. At this point, you should notice a pattern between the length of the horizontal leg of each triangle and one of the coordinates on the unit circle. Which coordinate on the unit circle is given by the length of the horizontal leg of the right triangles?

12. Which coordinate on the unit circle is given by the length of the vertical leg of the right triangles?

13. Is it necessary to draw all four of the triangles with the same reference angle to determine the coordinates on the unit circle? What relationship(s) can you use to determine the coordinates instead?

14. Use special right triangles to determine the exact (x, y) coordinates where each angle with a 60^o reference angle intersects the unit circle. Sketch each angle on the unit circle and clearly label the coordinates. Record your answers in the table.

θ	x-coordinate (estimate)	y-coordinate (estimate)	x-coordinate (exact)	y-coordinate (exact)
60^o				
120^o				
240^o				
300^o				

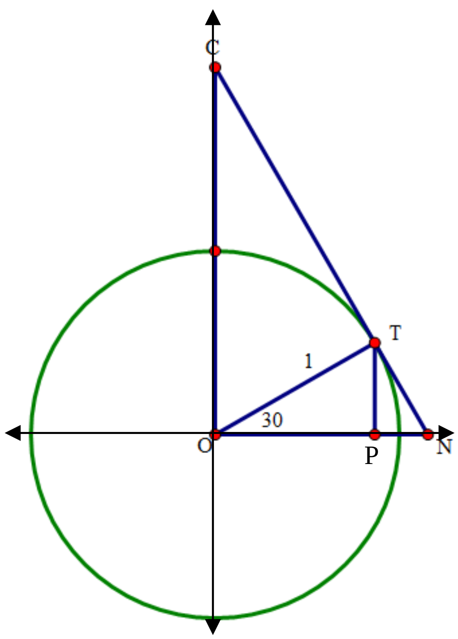
15. Think about what happens as you get to angles greater than 360^o. How can you predict the value of the sine of 420°? What about the cosine of 600^o?

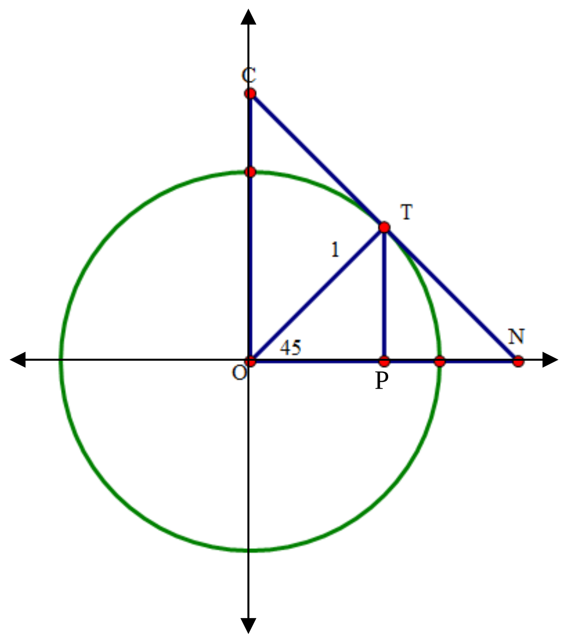
16. Does the same thing happen for negative angles? What is the largest negative angle that has the same sign and cosine as 120^o?

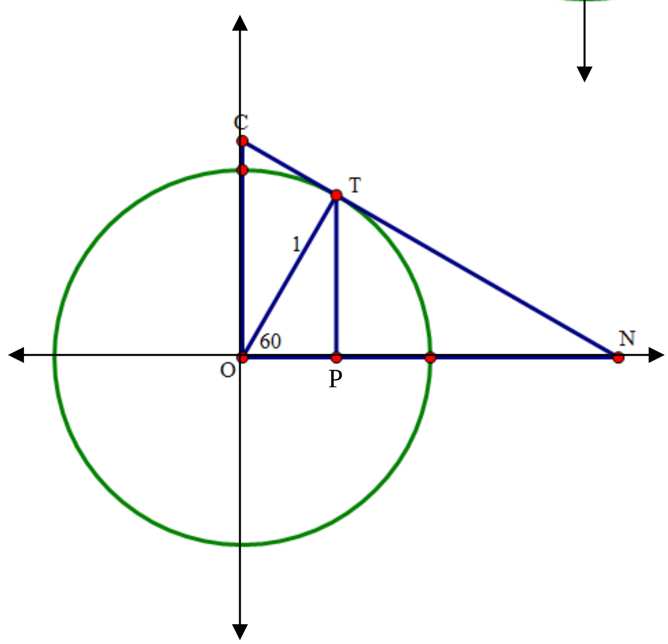
Part IV

17. In the figures below segment TP is perpendicular to segment ON. Line CN is tangent to circle O at T. N is the point where the line intersects the x-axis and C is the where the line intersects the y-axis. (Only segment CN is shown.)

Use your knowledge of sine and cosine to determine the length of segment TN. Use exact answers, no decimal approximations.







18. Using your understanding of the unit circle and $tangent\ \theta = \frac{sin\theta}{cos\theta}$, to complete the chart below for the indicated angles.

θ	Sin θ	Cos θ	Tan θ
30°			
45°			
60°			

19. How are these values of tangent related to the length you found in #4?

20. What would happen to the length of TN if the angle was changed to 0°?

21. What would happen to the length of TN if the angle was changed to 90°?

22. Based on the chart above fill in the sine, cosine, and tangent values for all the angles with the indicated reference angle.

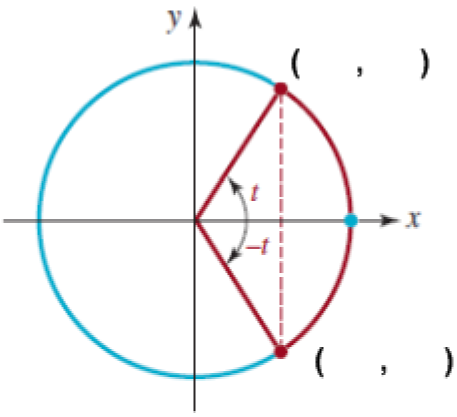
	θ	Sin θ	Cos θ	Tan θ
x	***30º***			
$\pi - x$				
$\pi + x$				
$2\pi - x$				

	θ	Sin θ	Cos θ	Tan θ
x	***45º***			
$\pi - x$				
$\pi + x$				
$2\pi - x$				

	θ	Sin θ	Cos θ	Tan θ
x	***60º***			
$\pi - x$				
$\pi + x$				
$2\pi - x$				

Part V

23. The symmetry of the unit circle is useful is generating the trigonometric values for an infinite number of angles. It is also useful in illustrating whether the sine and cosine functions are even or odd. Let's take 30^{o} as an example. In the picture below $t = 30^{o}$. Fill in the missing information.



24. Based on the graphs of sine and cosine that we studied in Advanced Algebra, we know that sine is sine is an odd function and cosine is an even function. What about the graphs help us to know this?

25. How can we use the information in #23 to further show that sine is an odd function and cosine is an even function?

Inverse Trigonometric Functions

Standards:
MCC9-12.F.BF.4 Find inverse functions.

MCC9-12.F.BF.4d (+) Produce an invertible function from a non-invertible function by restricting the domain.

MCC9-12.F.TF.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows it's inverse to be constructed.

MCC9-12.F.TF.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

This task is designed to give students a chance to explore the meaning of inverse function both numerically and algebraically. Students will solve some basic trigonometric equations using inverse functions. It is important for students to see that the notation sin^{-1}(x) refers to the inverse sine function and not $\frac{1}{sinx}$. It is also interesting to note that the term arcsin refers to the inverse sine relation and the term Arcsin refers to the inverse sine function. A graphing calculator or graphing utility on a computer will be helpful for students to use while working through the task.

Inverse Trigonometric Functions:

Trigonometric functions can be useful models for many real life phenomena. Average monthly temperatures are periodic in nature and can be modeled by sine and/or cosine functions. The function below models the average monthly temperatures for Asheville, NC. (The average monthly temperature is an average of the daily highs and daily lows.)

$f(t) = 18.5\sin\left(\frac{\pi}{6}t - 4\right) + 54.5$ where t=1 represents January.

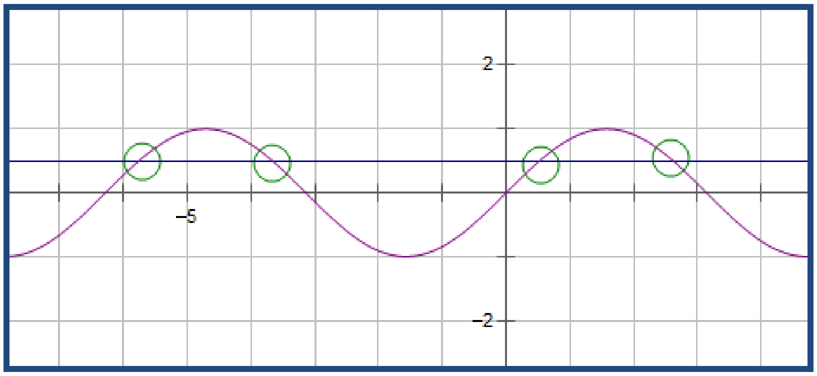
How can you use this model to find the month that has a specific average temperature?

Recall your work with inverse relationships. The inverse of a function can be found by interchanging the coordinates of the ordered pairs of the function. In this case, the ordered pair (month, temperature) would become (temperature, month).

This task will allow you to explore the inverses the trigonometric functions from a geometric and algebraic perspective.

Part I

1. Graph $f(\theta) = \sin\theta$ and the line y = ½ .



a. How many times do these functions intersect between -2π and 2π?

The functions intersect 4 times.

b. How is this graph related to finding the solution to ½ = sin θ?

The function y = ½ is the left side of the equation.
The function y = sinθ is the right side of the equation.
The intersections of the two functions represent the solutions to the equation.

c. If the domain is not limited, how many solutions exist to the equation ½ = sin θ?

There are an infinite number of solutions.

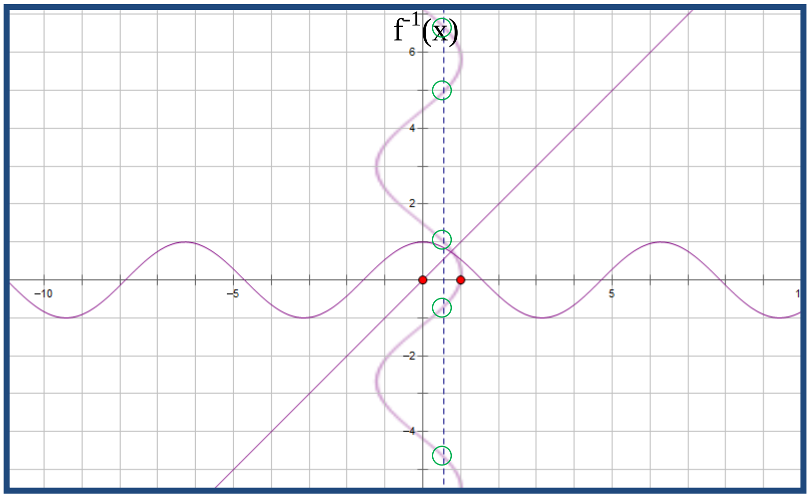
d. Would this be true for the other trigonometric functions? Explain.

Yes. If sine was replaced by cosine, tangent, or cotangent there would be an infinite number of solutions.
If the value was greater than 1or less than -1 this would also be true for the cosecant and secant functions.

2. What happens to the axes and coordinates of a function when you reflect it over the line y = x?

The coordinates and axes of a function switch when you reflect over the line y=x. For examples, (x,y) coordinates become (y,x).

a. Sketch a graph of $f(\theta) = \cos\theta$ and its reflection over the line y = x.

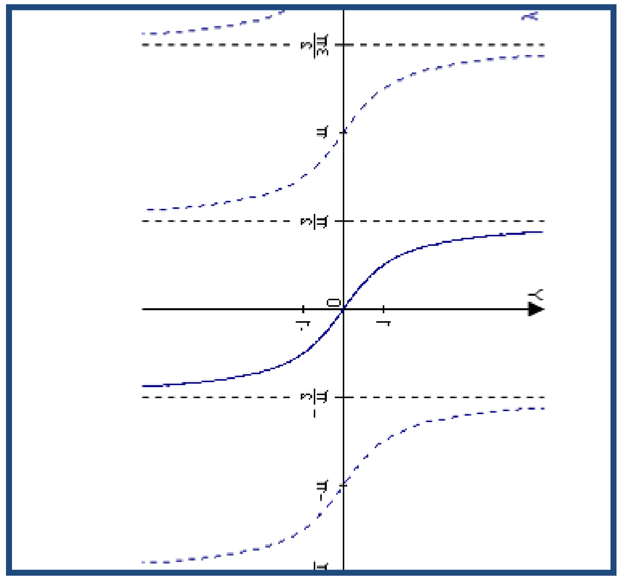


b. How many times does the line x= ½ intersect the reflection of cos x?

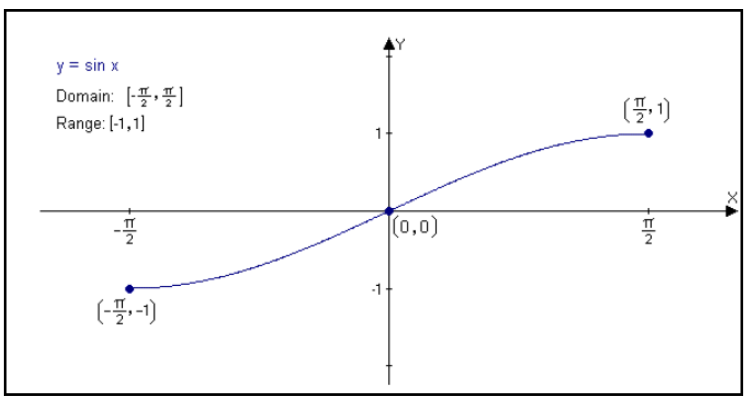
Solutions are shown on the graph above. There are an infinite number of solutions.

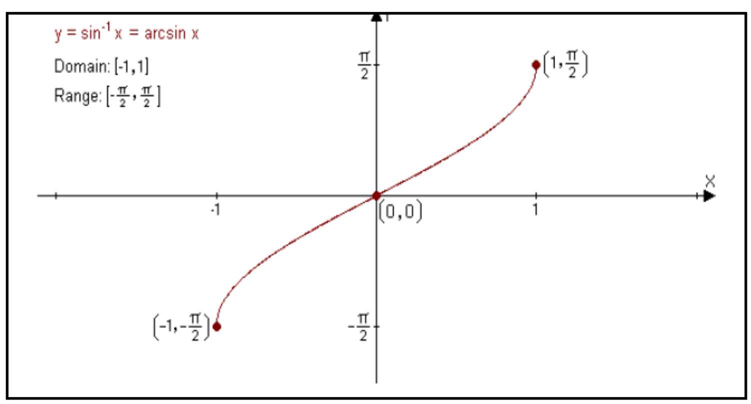
3. Sketch the inverse of $f(\theta) = \tan\theta$ and determine if it is a function.

The inverse of the tangent function is not a function.



4. Look at this graph of $f(\theta) = \sin\theta$ with domain $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ and its inverse.



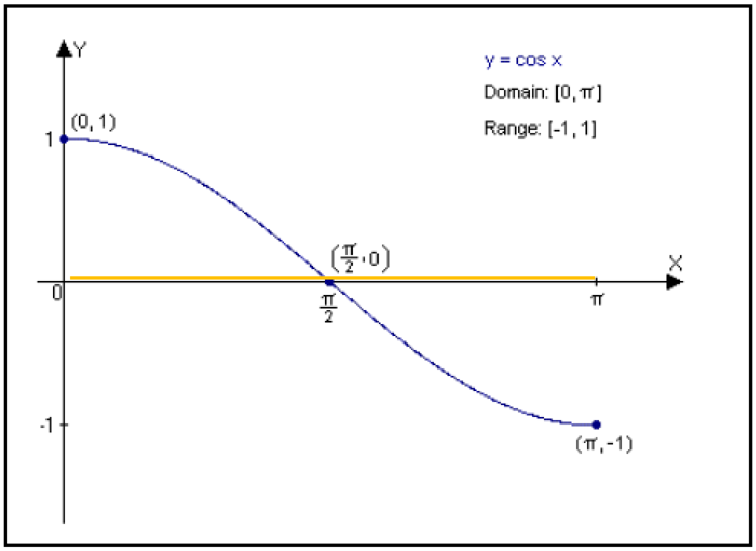


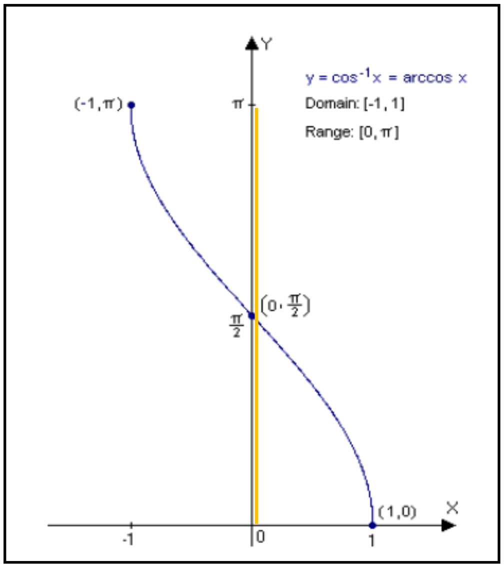
Is the inverse a function now?

Yes. The inverse is now a function.

5. Use the following graphs to determine the limited domains on the cosine function used to insure the inverse is a function.

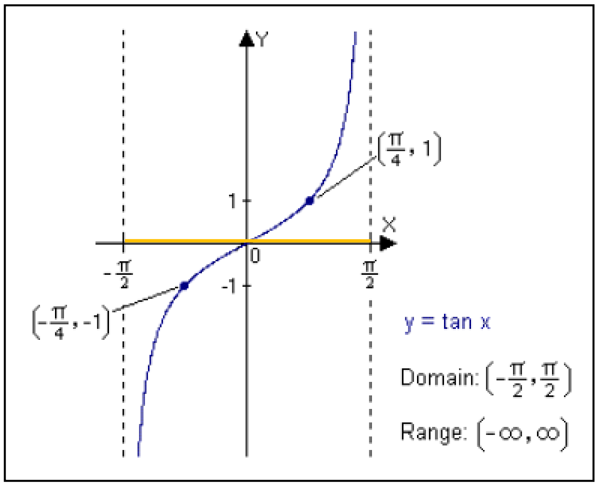
Highlight the axes that represent the angle measure. (on both graphs)

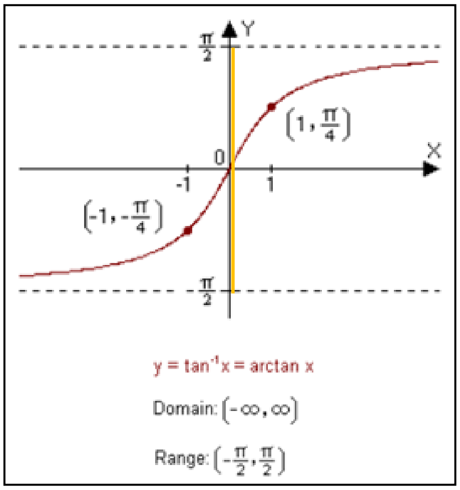




For cos x, the domain is limited to $0 \le x \le \pi$.

6. Use the following graphs to determine the limited domains on the tangent function used to insure the inverse is a function. Mark the axes that represent the angle measure.





For tan x, the domain is limited to $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

7. We use the names $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ or Arcsin, Arccos, and Arctan to represent the inverse of these functions on the limited domains you explored above. The values in the limited domains of sine, cosine and tangent are called **principal values**. (Similar to the principal values of the square root function.) Calculators give principal values when reporting $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$.

Complete the chart below indicating the domain and range of the given functions.

Function	Domain	Range
$f(\theta) = \sin^{-1}\theta$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$f(\theta) = \cos^{-1}\theta$	$-1 \le x \le 1$	$0 \le y \le \pi$
$f(\theta) = \tan^{-1}\theta$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

The inverse functions do not have ranges that include all 4 domains. Add a column to your chart that indicates the quadrants included in the range of the function. This will be important to remember when you are determining values of the inverse functions.

Function	Domain	Range	Quadrant for Range
$f(\theta) = \sin^{-1}\theta$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$	*QI and QIV*
$f(\theta) = \cos^{-1}\theta$	$-1 \le x \le 1$	$0 \le y \le \pi$	*QI and QII*
$f(\theta) = \tan^{-1}\theta$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$	*QI and QIV*

Part II

8. Alton felt like he understood inverse trig functions and he quickly evaluated the expressions below. Check Alton's answers to the problems. Mark the problems as correct or incorrect. Correct any problems she missed. (His answers are circled.)

1. $\cos(\tan^{-1} 1) = \frac{3\sqrt{2}}{2}$ ***Incorrect.*** $\tan^{-1} = 45^o$ ***so*** $\cos(45) = \frac{\sqrt{2}}{2}$	2. Arctan (-1) $= \frac{\pi}{4}$ ***Incorrect. It should be*** $-\frac{\pi}{4}$
3. $\sin(\sin^{-1} \frac{\sqrt{3}}{2}) = \frac{\sqrt{3}}{2}$ ***Correct.***	4. $\sin(\arctan\sqrt{3} + \arcsin ½) = 2$ ***Incorrect: Since neither arctan nor arcsin are functions, there are infinitely many values for both these angles. One counterexample is: sin(60 + 30) = sin(90) = 1***

9. Use what you know about trigonometric functions and their inverses to solve these simple equations. Two examples are included for you. (Unit circles can also be useful.)

Example 1:	**Example 2:**
$\text{ArcCos}\left(\frac{\sqrt{3}}{2}\right) = x$ *The answers will be an angle. Use θ to remind yourself* Let $\text{Arccos}\left(\frac{\sqrt{3}}{2}\right) = \theta$ *Ask yourself, what angle has a cos value of* $\frac{\sqrt{3}}{2}$. $\left(\frac{\sqrt{3}}{2}\right) = \text{Cos}\,\theta$ *Using the definition of Arccos.* $\left(\frac{\pi}{6}\right) = \theta$ *Why isn't* $\left(\frac{5\pi}{6}\right)$ *included?* So, $\text{ArcCos}\left(\frac{\sqrt{3}}{2}\right) = \left(\frac{\pi}{6}\right)$	$\sin(\cos^{-1} 1 + \tan^{-1} 1) = x$ *The answer will be a number, not an angle. Simplify parentheses first.* $\theta_1 = \cos^{-1} 1$ $\quad\theta_2 = \tan^{-1} 1$ $\theta_1 = 0$ $\quad\theta_2 = 45°$ $\sin(0 + 45) = x$ *Substitution* $\sin(45) = x$ $\frac{\sqrt{2}}{2} = x$ So, $\sin(\cos^{-1} 1 + \tan^{-1} 1) = \frac{\sqrt{2}}{2}$

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a. θ = Cos-1 $\frac{1}{2}$

Cos $\theta = \left(\frac{1}{2}\right)$

Using the definition of Arccos.

$\theta = \left(\frac{\pi}{3}\right)$

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b. θ = Arcsin 1

Sin θ ***=1***

Using the definition of Arcsin.

$\theta = \left(\frac{\pi}{2}\right)$

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c. $\sin^{-1} 2 = x$

$\theta = \sin^{-1}(2)$

$\text{Sin}^{-1}\,\theta = (2)$

Using the definition of Arcsin.

This is impossible. The sine function has a maximum value of 1.

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d. $\cos(\tan^{-1}\sqrt{3} - \sin^{-1}½) = x$

The answer will be a number, not an angle.
Simplify parentheses first.

$\theta_1 = \tan^{-1}\sqrt{3}$ $\quad\theta_2 = \sin^{-1}½$

$\theta_1 = 60°$ $\quad\theta_2 = 30°$

$\cos(60° - 30°) = x$

Substitution

$\cos(30°) = x$ $\quad\frac{\sqrt{3}}{2} = x$

So, $\cos(\tan^{-1}\sqrt{3} - \sin^{-1}½) = \frac{\sqrt{3}}{2}$

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e. $\cos(\tan^{-1}\frac{\sqrt{3}}{3}) = x$

The answer will be a number, not an angle.
Simplify parentheses first.

$\theta = \tan^{-1}\frac{\sqrt{3}}{3}$

$\theta = (30°)$

$\cos(\tan^{-1}\frac{\sqrt{3}}{3}) = x$

$\cos(30°) = x$

$\frac{\sqrt{3}}{2} = x$

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f. $\sin(\sin^{-1}\frac{\sqrt{3}}{2}) = x$

sin and sin-1 'cancel each other out'

$\sin(\sin^{-1}\frac{\sqrt{3}}{2}) = x$

$\frac{\sqrt{3}}{2} = x$

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Part III

10. Amy's family went to an amusement park while they are at the beach. She decides to ride the Ferris wheel so she can look out at the ocean. She was disappointed to find out that a 100 foot building blocked her view for part of the ride. Amy's height from the ground as she travels around the Ferris wheel can be found using the following equation where t = time in seconds from the beginning of Amy's ride.

$$h = 60\sin\left(\frac{2\pi}{3}t\right) + 70$$

a. Using what you know about inverse trig functions, find out how long will it take until Amy can see over the building?

$\frac{1}{4}$ of a minute or 15 seconds.

b. How long will it take Amy to reach the top of the ride?

$\frac{3}{4}$ of a minute or 45 seconds.

11. The area of an isosceles triangle can be found using the formula $A = \frac{1}{2}x^2 sin\theta$ where x is the length of the legs and θ is the vertex angle. If an isosceles triangle has a leg length of 4, then what value of θ give you an area of 4?

θ must equal 30^o.

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Trigonometric functions can be useful models for many real life phenomena. Average monthly temperatures are periodic in nature and can be modeled by sine and/or cosine functions. The function below models the average monthly temperatures for Asheville, NC. (The average monthly temperature is an average of the daily highs and daily lows.)

$f(t) = 18.5\sin\left(\frac{\pi}{6}t - 4\right) + 54.5$ where t=1 represents January.

How can you use this model to find the month that has a specific average temperature?

Recall your work with inverse relationships. The inverse of a function can be found by interchanging the coordinates of the ordered pairs of the function. In this case, the ordered pair (month, temperature) would become (temperature, month).

This task will allow you to explore the inverses the trigonometric functions from a geometric and algebraic perspective.

Part I

1. Graph $f(\theta) = \sin\theta$ and the line y = ½ .

   a. How many times do these functions intersect between -2π and 2π?

   b. How is this graph related to finding the solution to ½ = sin θ?

   c. If the domain is not limited, how many solutions exist to the equation ½ = sin θ?

   d. Would this be true for the other trigonometric functions? Explain.

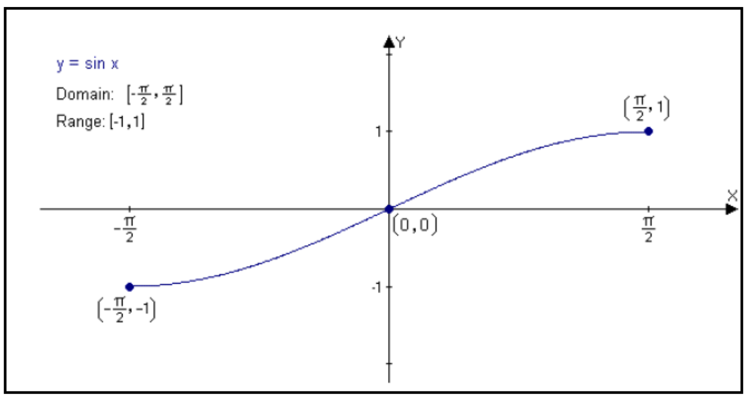
2. What happens to the axes and coordinates of a function when you reflect it over the line y = x?

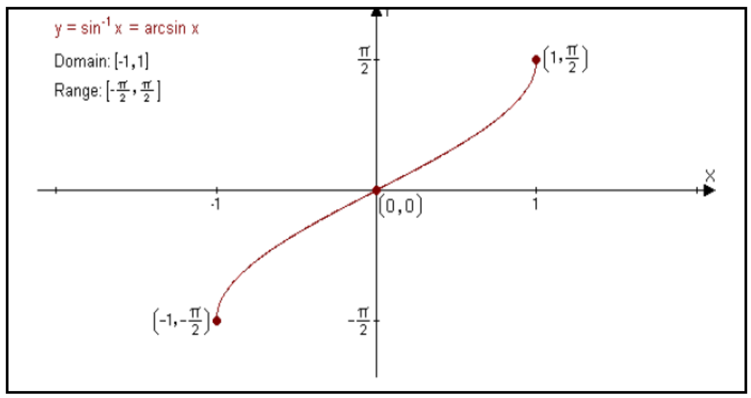
   a. Sketch a graph of $f(\theta) = \cos\theta$ and its reflection over the line y = x.

   b. How many times does the line x= ½ intersect the reflection of cos x?

3. Sketch the inverse of $f(\theta) = \tan\theta$ and determine if it is a function.

4. Look at this graph of $f(\theta) = \sin\theta$ with domain $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ and its inverse.

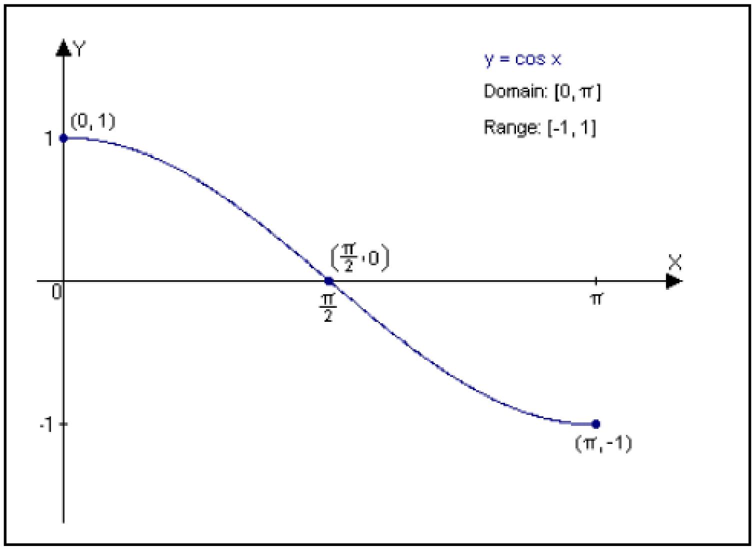


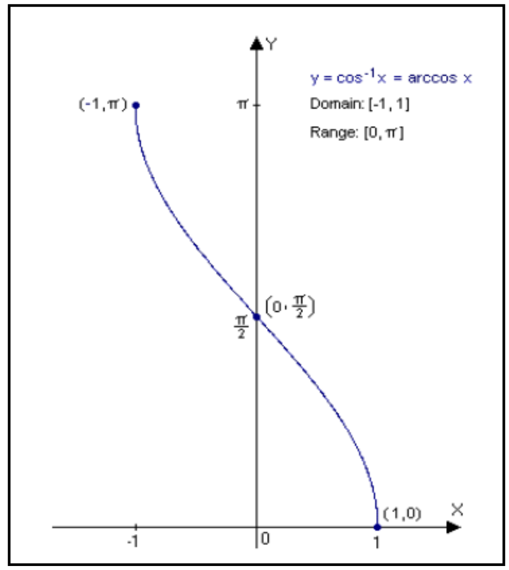


Is the inverse a function now?

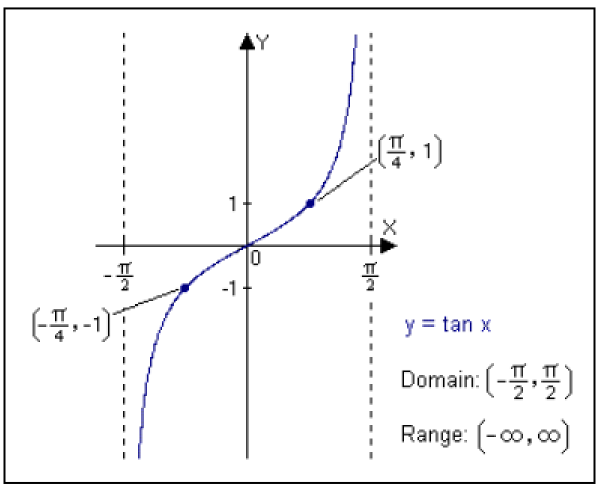
5. Use the following graphs to determine the limited domains on the cosine function used to insure the inverse is a function.

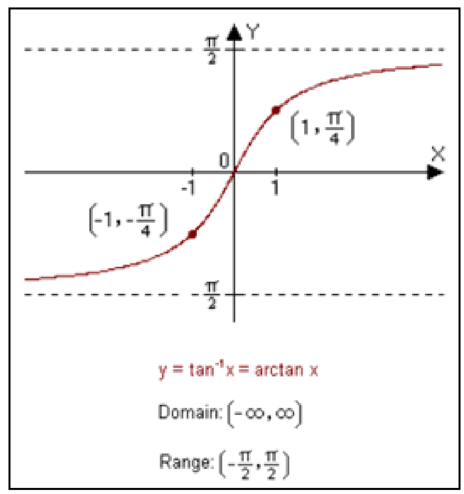
Highlight the axes that represent the angle measure. (on both graphs)





6. Use the following graphs to determine the limited domains on the tangent function used to insure the inverse is a function. Mark the axes that represent the angle measure.





7. We use the names $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ or Arcsin, Arccos, and Arctan to represent the inverse of these functions on the limited domains you explored above. The values in the limited domains of sine, cosine and tangent are called **principal values**. (Similar to the principal values of the square root function.) Calculators give principal values when reporting $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$.

Complete the chart below indicating the domain and range of the given functions.

Function	Domain	Range
$f(\theta) = \sin^{-1}\theta$		
$f(\theta) = \cos^{-1}\theta$		
$f(\theta) = \tan^{-1}\theta$		

The inverse functions do not have ranges that include all 4 domains. Add a column to your chart that indicates the quadrants included in the range of the function. This will be important to remember when you are determining values of the inverse functions.

Function	Domain	Range	Quadrant for Range
$f(\theta) = \sin^{-1}\theta$			
$f(\theta) = \cos^{-1}\theta$			
$f(\theta) = \tan^{-1}\theta$			

Part II

8. Alton felt like he understood inverse trig functions and he quickly evaluated the expressions below. Check Alton's answers to the problems. Mark the problems as correct or incorrect. Correct any problems she missed. (His answers are circled.)

1. $\cos(\tan^{-1} 1) = \frac{3\sqrt{2}}{2}$	2. Arctan $(-1) = \frac{\pi}{4}$
3. $\sin(\sin^{-1}\frac{\sqrt{3}}{2}) = \frac{\sqrt{3}}{2}$	4. sin(arctan $\sqrt{3}$ + arcsin ½) = 2

5. Use what you know about trigonometric functions and their inverses to solve these simple equations. Two examples are included for you. (Unit circles can also be useful.)

Example 1:	Example 2:
ArcCos $\left(\frac{\sqrt{3}}{2}\right) = x$ *The answers will be an angle. Use θ to remind yourself* Let Arccos $\left(\frac{\sqrt{3}}{2}\right) = \theta$ *Ask yourself, what angle has a cos value of* $\frac{\sqrt{3}}{2}$. $\left(\frac{\sqrt{3}}{2}\right) = \text{Cos } \theta$ *Using the definition of Arccos.* $\left(\frac{\pi}{6}\right) = \theta$ *Why isn't* $\left(\frac{5\pi}{6}\right)$ *included?* So, ArcCos $\left(\frac{\sqrt{3}}{2}\right) = \left(\frac{\pi}{6}\right)$	sin ($\cos^{-1}$ 1 + $\tan^{-1}$ 1) = x *The answer will be a number, not an angle. Simplify parentheses first.* $\theta_1 = \cos^{-1} 1$ $\theta_2 = \tan^{-1} 1$ $\theta_1 = 0$ $\theta_2 = 45°$ sin (0 + 45) = x *Substitution* sin(45) = x $\frac{\sqrt{2}}{2} = x$ So, sin ($\cos^{-1}$ 1 + $\tan^{-1}$ 1) = $\frac{\sqrt{2}}{2}$

a. θ = Cos-1 $\frac{1}{2}$	b. θ = Arcsin 1
c. sin-1 2 = x	d. cos (tan-1 $\sqrt{3}$ – sin-1 ½) = x
e. cos (tan-1 $\frac{\sqrt{3}}{3}$) = x	f. sin (sin-1 $\frac{\sqrt{3}}{2}$) = x

Part III

6. Amy's family went to an amusement park while they are at the beach. She decides to ride the Ferris wheel so she can look out at the ocean. She was disappointed to find out that a 100 foot building blocked her view for part of the ride. Amy's height from the ground as she travels around the Ferris wheel can be found using the following equation where t = time in seconds from the beginning of Amy's ride.

$$h = 60\sin\left(\frac{2\pi}{3}t\right) + 70$$

   a. Using what you know about inverse trig functions, find out how long will it take until Amy can see over the building?

   b. How long will it take Amy to reach the top of the ride?

7. The area of an isosceles triangle can be found using the formula $A = \frac{1}{2}x^2 sin\theta$ where x is the length of the legs and θ is the vertex angle. If an isosceles triangle has a leg length of 4, then what value of θ give you an area of 4?