Unit 6
Coordinate Geometry
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Introduction:
This unit investigates the properties of geometric figures on the coordinate plane. Students develop and use the formulas for the distance between two points, the distance between a point and a line, and the midpoint of segments. In addition, many topics that were addressed in previous units will be revisited relative to the coordinate plane. Focusing students’ attention on a coordinate grid as a reference for locations and descriptions of geometric figures strengthens their recognitions of algebraic and geometric connections.

Enduring Understandings:
• Algebraic formulas can be used to find measures of distance on the coordinate plane.
• The coordinate plane allows precise communication about graphical representations.
• The coordinate plane permits use of algebraic methods to obtain geometric results.

Key Standards Addressed:
MM1G1: Students will investigate properties of geometric figures in the coordinate plane.
   a. Determine the distance between two points.
   b. Determine the distance between a point and a line.
   c. Determine the midpoint of a segment.
   d. Understand the distance formula as an application of the Pythagorean Theorem.
   e. Use the coordinate plane to investigate properties of and verify conjectures related to triangles and quadrilaterals.

Related Standards Addressed:
MM1G2: Students will understand and use the language of mathematical argument and justification.
   a. Use conjecture, inductive reasoning, deductive reasoning, counterexample, and indirect proof as appropriate.
   b. Understand and use the relationships among a statement and its converse, inverse, and contrapositive.

MM1G3: Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
   d. Understand, use, and prove properties of and relationships among special quadrilaterals: parallelogram, rectangle, rhombus, square, trapezoid, and kite.
   e. Find and use points of concurrency in triangles: incenter, orthocenter, circumcenter, and centroid.
MM1P1. **Students will solve problems (using appropriate technology).**
   a. Build new mathematical knowledge through problem solving.
   b. Solve problems that arise in mathematics and in other contexts.
   c. Apply and adapt a variety of appropriate strategies to solve problems.
   d. Monitor and reflect on the process of mathematical problem solving.

MM1P2. **Students will reason and evaluate mathematical arguments.**
   a. Recognize reasoning and proof as fundamental aspects of mathematics.
   b. Make and investigate mathematical conjectures.
   c. Develop and evaluate mathematical arguments and proofs.
   d. Select and use various types of reasoning and methods of proof.

MM1P3. **Students will communicate mathematically.**
   a. Organize and consolidate their mathematical thinking through communication.
   b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
   c. Analyze and evaluate the mathematical thinking and strategies of others.
   d. Use the language of mathematics to express mathematical ideas precisely.

MM1P4. **Students will make connections among mathematical ideas and to other disciplines.**
   a. Recognize and use connections among mathematical ideas.
   b. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
   c. Recognize and apply mathematics in contexts outside of mathematics.

MM1P5. **Students will represent mathematics in multiple ways.**
   a. Create and use representations to organize, record, and communicate mathematical ideas.
   b. Select, apply, and translate among mathematical representations to solve problems.
   c. Use representations to model and interpret physical, social, and mathematical phenomena.

**Unit Overview:**

This unit continues to develop concepts, skills, and problem solving utilizing the coordinate plane. In fifth grade, students began plotting points in the first quadrant. Throughout sixth, seventh, and eighth grade they continued to progress from working in the first quadrant to using all four quadrants. Students have made scatter plots and have worked with both lines and systems of lines, including finding equations of lines, finding slopes of lines, and finding the slope of a line perpendicular to a given line. This unit offers the opportunity to
use calculators, especially when computing distances. The explorations in this unit lend
themselves to computing with a calculator allowing students to focus on the emerging
patterns and not the arithmetic process.

In eighth grade, students discovered and used the Pythagorean Theorem. This unit allows
students to extend this theorem to the coordinate plane while developing the distance
formula. It includes work with distance between two points and distance between a point
and a line. Students are expected to discover and use the midpoint formula. They will
revisit the properties of special quadrilaterals while using slope and distance on the
coordinate plane.

**Formulas and Definitions**

Distance Formula: \[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

Midpoint Formula: \[ \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]

**Tasks:** The following are tasks that develop the concepts, skills, and problem solving
necessary for mastery of the standards in this unit:
Video Game Learning Task

John and Mary are fond of playing retro style video games on hand held game machines. They are currently playing a game on a device that has a screen that is 2 inches high and four inches wide. At the start, John's token starts $\frac{1}{2}$ inch from the left edge and halfway between the top and bottom of the screen. Mary's token starts out at the extreme top of the screen and exactly at the midpoint of the top edge.

Starting Position

As the game starts, John's token moves directly to the right at a speed of 1 inch per second. For example, John’s token moves 0.1 inches in 0.1 seconds, 2 inches in 2 seconds, etc. Mary's token moves directly downward at a speed of 0.8 inches per second.

After One Second
Let time be denoted in this manner: \( t = 1 \) means the positions of the tiles after one second

1. Draw a picture on graph paper showing the positions of both tokens at times \( t = \frac{1}{4}, t = \frac{1}{2}, t = 1 \), and other times of your choice.

2. Discuss the movements possible for John’s token.

3. Discuss the movements possible for Mary’s token.

4. Discuss the movements of both tokens relative to each other.

5. Find the distance between John and Mary’s tokens at times \( t = 0, t = \frac{1}{4}, t = \frac{1}{2}, t = 1 \).

If Mary’s token gets closer than \( \frac{1}{4} \) inch to John’s token, then Mary’s token will destroy John’s, and Mary will get 10,000 points. However, if John presses button A when the tokens are less than 1/2 inch apart and more than \( \frac{1}{4} \) inch apart, then John’s token destroys Mary’s, and John gets 10,000 points.

6. Find a time at which John can press the button and earn 10,000 points. Draw the configuration at this time.

7. Compare your answers with your group. What did you discover?

8. Estimate the longest amount of time John could wait before pressing the button.

9. Drawing pictures gives an estimate of the critical time, but inside the video game, everything is done with numbers. Describe in words the mathematical concepts needed in order for this video game to work.
Inside the computer game the distance between John and Mary’s token are computed using a mathematical formula based on the coordinates of the tokens. Our goal now is to develop this formula.

To help us think about the distance between the tokens in our video game, it may help us to look first at a one-dimensional situation. Let’s look at how you determine distance between two locations on a number line:

10. What is the distance between 5 and 7? 7 and 5? -1 and 6? 5 and -3?

11. Can you find a formula for the distance between two points, a and b, on a number line?

Now that you can find the distance on a number line, let’s look at finding distance on the coordinate plane:

12. Plot the points A= (0, 0), B = (3, 0) and C = (3, 4) on centimeter graph paper.

13. Find the distance from the point (0, 0) to the point (3, 4) using a ruler.

14. Consider the triangle ABC, what kind of triangle is formed? Find the lengths of the two shorter sides. Use these lengths to calculate the length of the hypotenuse. Is this consistent with your prior measurement? Why or why not?

15. Using the same graph paper, find the distance between:

   a. (1, 0) and (4, 4)
   b. (-1, 1) and (11, 6)
   c. (-1, 2) and (2, -6)
16. Find the distance between points (a, b) and (c, d) shown below.

17. Using your solutions from 16, find the distance between the point \((x_1, y_1)\) and the point \((x_2, y_2)\). Solutions written in this generic form are often called formulas.

18. Do you think your formula would work for any pair of points? Why or why not?

Let’s revisit the video game. Draw a diagram of the game on a coordinate grid placing the bottom left corner at the origin.

19. Place John and Mary’s tokens at the starting positions.

20. Write an ordered pair for John’s token and an ordered pair for Mary’s token when \(t = 0\), when \(t = \frac{1}{2}\), when \(t = 1 \frac{1}{2}\), and when \(t = 2\).

21. Find the distance between their tokens when \(t = 0\), when \(t = \frac{1}{2}\), when \(t = 1 \frac{1}{2}\), and when \(t = 2\).

22. Write an ordered pair for John and Mary’s tokens at any time \(t\).

23. Write an equation for the distance between John and Mary’s tokens at any time \(t\).

24. Using a graphing utility, graph the equation you derived for the distance between the two tokens.
25. What does the graph look like? What are the characteristics of this graph?

26. What do the variables represent?

27. Recall that when John and Mary are between ¼ and ½ inches apart, John may press the button to earn 10,000 points. What interval of time represents John’s window of opportunity to score points?
New York Learning Task

Emily works at a building located on the corner of 9th Avenue and 61st Street in New York City. Her brother, Gregory, is in town on business. He is staying at a hotel at the corner of 9th Avenue and 43rd Street.

The streets of New York City were laid out in a rectangular pattern. In this part of town, Avenues run in a North-South direction and they are numbered from east to west, in other words the further east you go, the lower the number. That means the Avenues east of 9th Ave are 8th Ave, 7th Ave, etc. Streets run in an east-west direction. They increase in number as you proceed north. So, north of 41st Street is 42nd Street, then 43rd Street, etc. The distance between the avenues is the same as the distance between the streets. All the blocks are approximately the same size.

1. Gregory called Emily at work, and they agree to meet for lunch. They agree to meet at a corner half way between Emily’s work and Gregory’s hotel. Where should they meet? Justify your answer using a diagram.

2. After lunch, Emily has the afternoon off and wants to show her brother her apartment. Her apartment is three blocks north and four blocks west of the hotel. At what intersection is her apartment building located?

3. Gregory walks back to his hotel for an afternoon business meeting. He and Emily are going to meet for dinner. They decide to be fair and will meet half way.

   How far is it from Emily’s apartment to Gregory’s hotel?

   Where should they meet for dinner?

   How far are they going to walk to meet?

   Is their walking distance ½ the distance from Emily’s apartment and Gregory’s hotel? Why or why not?
On appropriate graph paper, plot the points $A = (0, 0)$, $B = (6, 0)$, and $C = (4, 12)$.

4. Find the point midway between $A$ and $B$. This point is called the midpoint of the segment $AB$. Find the distance from $A$ to midpoint and the distance from midpoint to $B$. What do you notice?

5. Find the midpoint of segment $BC$. Check using distances.

6. Find the midpoint of a segment whose endpoints are $(x_1, y_1)$ and $(x_2, y_2)$. Would this formula work for any endpoints? Why or why not?
Quadrilaterals Revisited Learning Task

Plot points $A = (-3, -1)$, $B = (-1, 2)$, $C = (4, 2)$, and $D = (2, -1)$.

1. What specialized geometric figure is quadrilateral $ABCD$? Support your answer mathematically.

2. Draw the diagonals of $ABCD$. Find the coordinates of the midpoint of each diagonal. What do you notice?

3. Find the slopes of the diagonals of $ABCD$. What do you notice?

4. The diagonals of $ABCD$ create four small triangles. Are any of these triangles congruent to any of the others? Why or why not?

Plot points $E = (1, 2)$, $F = (2, 5)$, $G = (4, 3)$ and $H = (5, 6)$.

5. What specialized geometric figure is quadrilateral $EFHG$? Support your answer mathematically using two different methods.

6. Draw the diagonals of $EFHG$. Find the coordinates of the midpoint of each diagonal. What do you notice?
7. Find the slopes of the diagonals of EFHG. What do you notice?

8. The diagonals of EFHG create four small triangles. Are any of these triangles congruent to any of the others? Why or why not?

Plot points P = (4, 1), W = (-2, 3), M = (2, -5), and K = (-6, -4).

9. What specialized geometric figure is quadrilateral PWKM? Support your answer mathematically.

10. Draw the diagonals of PWKM. Find the coordinates of the midpoint of each diagonal. What do you notice?

11. Find the lengths of the diagonals of PWKM. What do you notice?

12. Find the slopes of the diagonals of PWKM. What do you notice?

13. The diagonals of ABCD create four small triangles. Are any of these triangles congruent to any of the others? Why or why not?

Plot points A = (1, 0), B = (-1, 2), and C = (2, 5).

14. Find the coordinates of a fourth point D that would make ABCD a rectangle. Justify that ABCD is a rectangle.

15. Find the coordinates of a fourth point D that would make ABCD a parallelogram that is not also a rectangle. Justify that ABCD is a parallelogram but is not a rectangle.
Euler’s Village Learning Task

You would like to build a house close to the village of Euler. There is a beautiful town square in the village, and the road you would like to build your house on begins right at the town square.

The road follows an approximately north east direction as you leave town and continues for 3,000 feet. It passes right by a large shade tree located approximately 200 yards east and 300 yards north of the town square. There is a stretch of the road, between 300 and 1200 yards to the east of town, which currently has no houses. This stretch of road is where you would like to locate your house. Building restrictions require all houses sit parallel to the road. All water supplies are linked to town wells and the closest well to this part of the road is 500 yards east and 1200 yards north of the town square.

1. How far from the well would it be if the house was located on the road 300 yards east of town? 500 yards east of town? 1,000 yards east of town? 1,200 yards east of town? (For the sake of calculations, do not consider how far the house is from the road, just use the road to make calculations)

2. The cost of the piping leading from the well to the house is a major concern. Where should you locate your house in order to have the shortest distance to the well? Justify your answer mathematically.

3. If the cost of laying pipes is $22.5 per linear yard, how much will it cost to connect your house to the well?

4. The builder of your house is impressed by your calculations and wants to use the same method for placing other houses. Describe the method you used. Would you want him to place the other houses in the same manner?

5. Write a formula that the builder could use to find the cost of laying pipes to any house along this road. How would you have to change your formula for another road?