Georgia
Standards of Excellence
Middle School Support

Mathematics

GSE Grade 7

Connections/Support Materials for Remediation

Georgia Department of Education

Richard Woods, Georgia's School Superintendent

"Educating Georgia's Future"

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OVERVIEW

The tasks in this document are from the high school course Foundations of Algebra. Foundations of Algebra is a first year high school mathematics course option for students who have completed mathematics in grades 6 – 8 yet will need substantial support to bolster success in high school mathematics. The course is aimed at students who have reported low standardized test performance in prior grades and/or have demonstrated significant difficulties in previous mathematics classes.

In many cases, students enter middle school with the same mathematics difficulties that inhibit their success in algebra. Due to this, the Foundations of Algebra lessons have been sorted into collections aligned to 6^{th}, 7^{th}, and 8^{th} grade mathematics standards and prerequisite skills. These tasks are suggested for use in middle school math connections/support classes and with other students who are in the process of mastering the identified standards.

SETTING THE ATMOSPHERE FOR SUCCESS

"There is a huge elephant standing in most math classrooms, it is the idea that only some students can do well in mathematics. Students believe it; parents believe and teachers believe it. The myth that mathematics is a gift that some students have and some do not, is one of the most damaging ideas that pervades education in the US and that stands in the way of students' mathematics achievement." (Boaler, Jo. "Unlocking Children's Mathematics Potential: 5 Research Results to Transform Mathematics Learning" youcubed.org at Stanford University. Web 10 May 2015.)

Some students believe that their ability to learn mathematics is a fixed trait, meaning either they are good at mathematics or not. This way of thinking is referred to as a **fixed mindset**. Other students believe that their ability to learn mathematics can develop or grow through effort and education, meaning the more they do and learn mathematics the better they will become. This way of thinking is referred to as a **growth mindset**.

In the fixed mindset, students are concerned about how they will be viewed, smart or not smart. These students do not recover well from setbacks or making mistakes and tend to "give up" or quit. In the growth mindset, students care about learning and work hard to correct and learn from their mistakes and look at these obstacles as challenges.

The manner in which students are praised greatly affects the type of mindset a student may exhibit. Praise for intelligence tends to put students in a fixed mindset, such as "You have it!" or "You are really good at mathematics". In contrast, praise for effort tends to put students in a growth mindset, such as "You must have worked hard to get that answer." or "You are developing mathematics skills because you are working hard". Developing a growth mindset produces motivation, confidence and resilience that will lead to higher achievement. (Dweck, Carol. Mindset: The New Psychology of Success. Ballantine Books: 2007.)

"Educators cannot hand students confidence on a silver platter by praising their intelligence. Instead, we can help them gain the tools they need to maintain their confidence in learning by keeping them focused on the process of achievement." (Dweck, Carol S. "The Perils and Promises of Praise." *ASCD.* Educational Leadership. October 2007. Web 10 May 2015.)

Teachers know that the business of coming to know students as learners is simply too important to leave to chance and that the peril of not undertaking this inquiry is not reaching a learner at all. Research suggests that this benefit may improve a student's academic performance. Surveying students' interests in the beginning of a year will give teachers direction in planning activities that will "get students on board". Several interest surveys are available and two examples can be located through the following websites:

https://www.scholastic.com/content/collateral_resources/pdf/student_survey.pdf
http://www.niu.edu/eteams/pdf_s/VALUE_StudentInterestInventory.pdf

CONCEPTS/SKILLS TO MAINTAIN FROM PREVIOUS GRADES

Students are expected to have prior knowledge/experience related to the concepts and skills identified below. A pre-assessment may be needed in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

The web links may be used as needed for additional resources aimed at intervention, and even though the interventions may be needed throughout the course, these concepts correspond to the 7th grade units 1,2,3, and 5 where the needed skills first appear as prerequisites.

Unit 1
Intervention Note: The use of color counters or algebra tiles may help students learn patterns or rules.

- **use of number lines to order whole number integers**
https://www.illustrativemathematics.org/content-standards/6/NS/C/6/tasks/1665
https://www.illustrativemathematics.org/content-standards/6/NS/C/6/tasks/2009
https://www.illustrativemathematics.org/content-standards/6/NS/C/7/tasks/288
https://www.illustrativemathematics.org/content-standards/6/NS/C/7/tasks/286
http://map.mathshell.org/download.php?fileid=1183

- **addition, subtraction, division, and multiplication of whole numbers**
Intervention Note: In the beginning, students may need to work with smaller numbers.
https://www.illustrativemathematics.org/content-standards/6/NS/B/2/tasks/1893
https://www.illustrativemathematics.org/content-standards/6/NS/B/2/tasks/270
http://map.mathshell.org/tasks.php?unit=MA11&collection=9&redir=1
http://map.mathshell.org/download.php?fileid=1175

- **addition, subtraction, division and multiplication of fractions**
  Intervention Note: Fractions strips and number tiles may help students with fraction operations. Make an explicit connection between adding and subtracting whole numbers to adding and subtracting fractions. This relationship can be shown using a number line. Students need to see the movements occur on a number line to make the connection.
  https://www.illustrativemathematics.org/content-standards/6/NS/A/tasks/2166
  https://www.illustrativemathematics.org/content-standards/6/NS/A/tasks/2169
  https://www.illustrativemathematics.org/content-standards/6/NS/A/tasks/2170
  https://www.illustrativemathematics.org/content-standards/6/NS/A/tasks/2167
  https://www.illustrativemathematics.org/content-standards/6/NS/A/1/tasks/50
  https://www.illustrativemathematics.org/content-standards/6/NS/A/1/tasks/463
  https://www.illustrativemathematics.org/content-standards/6/NS/A/1/tasks/330
  https://www.illustrativemathematics.org/content-standards/6/NS/A/1/tasks/412
  https://www.illustrativemathematics.org/content-standards/6/NS/A/1/tasks/413
  http://map.mathshell.org/download.php?fileid=1151

Unit 2

- **number sense**
  https://www.illustrativemathematics.org/content-standards/6/NS/A/1/tasks/932
  https://www.illustrativemathematics.org/content-standards/6/NS/A/1/tasks/692
  https://www.illustrativemathematics.org/content-standards/6/NS/A/1/tasks/408
  https://www.illustrativemathematics.org/content-standards/6/NS/A/1/tasks/407
  https://www.illustrativemathematics.org/content-standards/6/NS/A/1/tasks/411
  https://www.illustrativemathematics.org/content-standards/6/NS/A/1/tasks/409
  https://www.illustrativemathematics.org/content-standards/6/NS/A/1/tasks/267
  https://www.illustrativemathematics.org/content-standards/6/NS/B/3/tasks/272
  https://www.illustrativemathematics.org/content-standards/6/NS/B/3/tasks/275
  https://www.illustrativemathematics.org/content-standards/6/NS/C/5/tasks/277
  https://www.illustrativemathematics.org/content-standards/6/NS/C/5/tasks/278

- **computation with whole numbers and decimals, including application of order of operations**
  https://www.illustrativemathematics.org/content-standards/6/NS/B/tasks/2199
  https://www.illustrativemathematics.org/content-standards/6/NS/B/tasks/2189
  https://www.illustrativemathematics.org/content-standards/6/NS/B/2/tasks/2000
  https://www.illustrativemathematics.org/content-standards/6/NS/B/3/tasks/1291
  https://www.illustrativemathematics.org/content-standards/6/NS/B/3/tasks/2190
  https://www.illustrativemathematics.org/content-standards/6/NS/B/3/tasks/2182
  https://www.illustrativemathematics.org/content-standards/6/NS/B/3/tasks/2195
  https://www.illustrativemathematics.org/content-standards/6/NS/B/3/tasks/2196
  https://www.illustrativemathematics.org/content-standards/6/NS/B/3/tasks/2197
  https://www.illustrativemathematics.org/content-standards/6/NS/B/3/tasks/274

  https://www.illustrativemathematics.org/content-standards/6/NS/B/3/tasks/2216
  https://www.illustrativemathematics.org/content-standards/6/NS/B/3/tasks/374
  https://www.illustrativemathematics.org/content-standards/6/NS/B/3/tasks/273

https://www.illustrativemathematics.org/content-standards/6/NS/B/3/tasks/1299
https://www.illustrativemathematics.org/content-standards/6/NS/B/3/tasks/1300
https://www.illustrativemathematics.org/content-standards/6/NS/B/3/tasks/2188
https://www.illustrativemathematics.org/content-standards/6/NS/B/3/tasks/2198

- **computation with all positive and negative rational numbers**
  https://www.illustrativemathematics.org/content-standards/6/NS/C/7/tasks/284
  https://www.illustrativemathematics.org/content-standards/7/NS/A/2/tasks/1667
  https://www.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/1987
  https://www.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/314
  https://www.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/317

- **data usage and representations**
  https://www.illustrativemathematics.org/content-standards/6/SP/B/4/tasks/877
  https://www.illustrativemathematics.org/content-standards/6/SP/B/4/tasks/2043
  https://www.illustrativemathematics.org/content-standards/6/SP/B/4/tasks/2047

Unit 3

- **perimeter and area of rectangles and squares**
  https://www.illustrativemathematics.org/content-standards/6/G/A/1/tasks/1523
  https://www.illustrativemathematics.org/content-standards/6/G/A/1/tasks/656
  http://robertkaplinsky.com/work/giant-pizza/
  https://illuminations.nctm.org/Lesson.aspx?id=3737

- **characteristics of 2-D and 3-D shapes**
  https://www.illustrativemathematics.org/content-standards/6/G/A/tasks/545
  https://www.illustrativemathematics.org/content-standards/6/G/A/tasks/135
  https://www.illustrativemathematics.org/content-standards/6/G/A/2/tasks/657
  https://www.illustrativemathematics.org/content-standards/6/G/A/2/tasks/657
  https://www.illustrativemathematics.org/content-standards/6/G/A/2/tasks/535
  https://illuminations.nctm.org/Lesson.aspx?id=2917
  https://illuminations.nctm.org/Lesson.aspx?id=2044

Unit 5

- **graphical representations**

https://illuminations.nctm.org/Lesson.aspx?id=2847
https://www.illustrativemathematics.org/content-standards/6/SP/B/4/tasks/1026

- **samples of populations**

https://www.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/317
https://www.illustrativemathematics.org/content-standards/6/SP/B/5/tasks/2048

STANDARDS FOR MATHEMATICAL CONTENT

The content standards for Foundations of Algebra are an amalgamation of mathematical standards addressed in grades 3 through high school.

After each Foundations of Algebra standard there is a list of reference standards. These reference standards refer to the standards used to form those for Foundations of Algebra.

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ1. Students will analyze number relationships.

a. Solve multi-step real world problems, analyzing the relationships between all four operations. For example, understand division as an unknown-factor problem in order to solve problems. Knowing that 50 x 40 = 2000 helps students determine how many boxes of cupcakes they will need in order to ship 2000 cupcakes in boxes that hold 40 cupcakes each. (MGSE3.OA.6, MGSE4.OA.3)
b. Understand a fraction a/b as a multiple of 1/b. (MGSE4.NF.4)
c. Explain patterns in the placement of decimal points when multiplying or dividing by powers of ten. (MGSE5.NBT.2)
d. Compare fractions and decimals to the thousandths place. For fractions, use strategies other than cross multiplication. For example, locating the fractions on a number line or using benchmark fractions to reason about relative size. For decimals, use place value. (MGSE4.NF.2; MGSE5.NBT.3,4)

MFANSQ2. Students will conceptualize positive and negative numbers (including decimals and fractions).

a. Explain the meaning of zero. (MGSE6.NS.5)
b. Represent numbers on a number line. (MGSE6.NS.5,6)
c. Explain meanings of real numbers in a real world context. (MGSE6.NS.5)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.

a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
c. Interpret and solve contextual problems involving division of fractions by fractions. For example, how many 3/4-cup servings are in 2/3 of a cup of yogurt? (MGSE6.NS.1)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MGSE7.NS.3, MGSE7.EE.3, MGSE9-12.N.Q.3)

Students will extend arithmetic operations to algebraic modeling.

MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.

a. Apply properties of operations emphasizing when the commutative property applies. (MGSE7.EE.1)
b. Use area models to represent the distributive property and develop understandings of addition and multiplication (all positive rational numbers should be included in the models). (MGSE3.MD.7)
c. Model numerical expressions (arrays) leading to the modeling of algebraic expressions. (MGSE7.EE.1,2; MGSE9-12.A.SSE.1,3)
d. Add and subtract algebraic equations. (MGSE6.EE.3, MGSE6.EE.4, MC7.EE.1)
e. Generate equivalent expressions using properties of operations and understand various representations within context. For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between “3 more” and “3 times”. (MGSE4.0A.2; MGSE6.EE.3, MGSE7.EE.1, 2, MGSE9-12.A.SSE.3)
f. Evaluate formulas at specific values for variables. For example, use formulas such as A = l x w and find the area given the values for the length and width. (MGSE6.EE.2)

MFAAA2. Students will interpret and use the properties of exponents.

a. Substitute numeric values into formulas containing exponents, interpreting units consistently. (MGSE6.EE.2, MGSE9-12.N.Q.1, MGSE9-12.A.SSE.1, MGSE9-12.N.RN.2)
b. Use properties of integer exponents to find equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$. (MGSE8.EE.1)

Students will use ratios to solve real-world and mathematical problems.

MFAPR1. Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations*. (MGSE6.RP.3)

MFAPR2. Students will recognize and represent proportional relationships between quantities.

a. Relate proportionality to fraction equivalence and division. For example, 3/6 is equal to 4/8 because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator. (MGSE4.NF.1)
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

MFAPR.3 Students will graph proportional relationships.

a. Interpret unit rates as slopes of graphs. (MGSE8.EE.5)

b. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. (MGSE8.EE.6)
c. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. (MGSE8.EE.5)

Students will solve, interpret, and create linear models using equations and inequalities.

MFAEI1. Students will create and solve equations and inequalities in one variable.

a. Use variables to represent an unknown number in a specified set. (MGSE.6.EE2,5,6)
b. Explain each step in solving simple equations and inequalities using the equality properties of numbers. (MGSE9-12.A.REI.1)
d. Represent and find solutions graphically.
e. Use variables to solve real-world and mathematical problems. (MGSE6.EE.7,MGSE7.EE.4)

Students will create function statements and analyze relationships among pairs of variables using graphs, tables, and equations.

MFAQR2. Students will compare and graph functions.

a. Calculate rates of change of functions, comparing when rates increase, decrease, or stay constant. *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (MGSE6.RP.2;MGSE7.RP.1,2,3, MGSE8.F.2,5, MGSE9-12.F.IF.6)

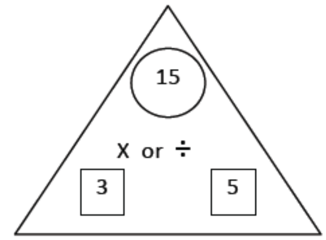
ELEMENTARY REFERENCE STANDARDS

These reference standards refer to the standards used to form the standards for the Foundations of Algebra course. Below, you will find the elementary reference standards with instructional strategies and common misconceptions.

MGSE.3.OA.6. Understand division as an unknown-factor problem. ***For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.***
Since multiplication and division are inverse operations, students are expected to solve problems and explain their processes of solving division problems that can also be represented as unknown factor multiplication problems.

Example: A student knows that 2 x 9 = 18. How can they use that fact to determine the answer to the following question: 18 people are divided into pairs in a P.E. class? How many pairs are there? Write a division equation and explain your reasoning.

Multiplication and division are inverse operations and that understanding can be used to find the unknown. Fact family triangles demonstrate the inverse operations of multiplication and division by showing the two factors and how those factors relate to the product and/or quotient. Examples:



$3 \times 5 = 15$	$5 \times 3 = 15$
$15 \div 3 = 5$	$15 \div 5 = 3$

MGSE.4.OA.3. Solve multistep word problems with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a symbol or letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding

The focus in this standard is to have students use and discuss various strategies. The reference is to estimation strategies, including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer. Students need many opportunities solving multistep story problems using all four operations.

Example 1:

On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many miles did they travel total?

Some typical estimation strategies for this problem are shown below:

Student 1:	**Student 2:**	**Student 3:**
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.	I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.	I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30, I know my answer will be about 530.

The assessment of estimation strategies should have a reasonable answer (500 or 530), or a range (between 500 and 550). Problems will be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

Example 2:

Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 containers with 6 bottles in each container. Sarah wheels in 6 containers with 6 bottles in each container. About how many bottles of water still need to be collected?

Student 1: First I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I'm trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles.	**Student 2:** First I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. 40 + 20 = 60. 300 – 60 = 240, so we need about 240 more bottles.

This standard also references interpreting remainders. Remainders should be put into context for interpretation. Ways to address remainders:

- Remained as a left over
- Partitioned into fractions or decimals
- Discarded leaving only the whole number answer
- Increased the whole number answer up one
- Rounded to the nearest whole number for an approximate result

Example 1:

Write different word problems involving **44 ÷ 6 = ?** where the answers are best represented as:

- Problem A: 7
- Problem B: 7 r 2
- Problem C: 8
- Problem D: 7 or 8
- Problem E: $7\ ^2/_6$

Possible solutions:

Problem **A: 7.**
Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches did she fill? 44 ÷ *6 = p; p = 7 r 2. Mary can fill 7 pouches completely.*

Problem **B: 7 r 2.**
Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches could she fill and how many pencils would she have left? 44 ÷ *6 = p; p = 7 r 2; Mary can fill 7 pouches and have 2 left over.*

Problem **C: 8.**

Mary had 44 pencils. Six pencils fit into each of her pencil pouches. What would the fewest number of pouches she would need in order to hold all of her pencils? 44 ÷ *6 = p; p = 7 r 2; Mary needs 8 pouches to hold all of the pencils.*

Problem **D: 7 or 8.**
Mary had 44 pencils. She divided them equally among her friends before giving one of the leftovers to each of her friends. How many pencils could her friends have received? 44 ÷ *6 = p; p = 7 r 2; some of her friends received 7 pencils. Two friends received 8 pencils.*

Problem **E: $7^2/_6$.**
Mary had 44 pencils and put six pencils in each pouch. What fraction represents the number of pouches that Mary filled? 44 ÷ *6 = p; p =* $7^2/_6$

Example 2:

There are 128 students going on a field trip. If each bus held 30 students, how many buses are needed? *(128 ÷ 30 = b; b = 4 R 8). They will need 5 buses because 4 buses would not hold all of the students).*

Students need to realize in problems, such as the examples above, that an extra bus is needed for the 8 students that are left over. Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies include, but are not limited to the following:

- **Front-end estimation with adjusting** (Using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts)
- **Clustering around an average** (When the values are close together, an average value is selected and multiplied by the number of values to determine an estimate.)
- **Rounding and adjusting** (Students round down or round up and then adjust their estimate depending on how much the rounding affected the original values.)
- **Using friendly or compatible numbers such as factors** (Students seek to fit numbers together; e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000.)
- **Using benchmark numbers that are easy to compute** (Students select close whole numbers for fractions or decimals to determine an estimate.)

MGSE.3.MD.7 Relate area to the operations of multiplication and addition.
a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

Students should tile rectangles, then multiply their side lengths to show it is the same.

To find the area, one could count the squares or multiply $3 \times 4 = 12$.

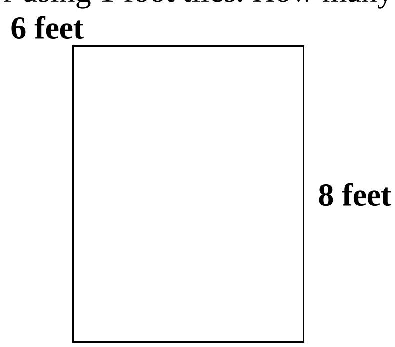
1	2	3	4
5	6	6	8
9	10	11	12

b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

Students should solve real world and mathematical problems

Example:

Drew wants to tile the bathroom floor using 1 foot tiles. How many square foot tiles will he need?

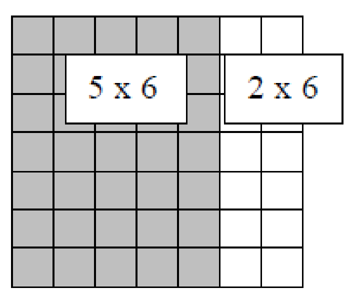


The area of the rectangle is 48 square feet, and since each tile is 1 square foot, 48 tiles will be needed.

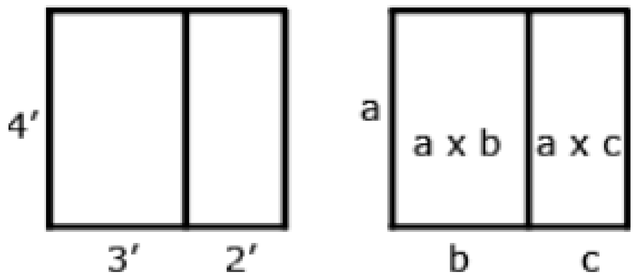
c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning.

This standard extends students' work with the distributive property. For example, in the picture below the area of a 7×6 figure can be determined by finding the area of a 5×6 and 2×6 and adding the two sums.

So, 7 x 6 = (5 + 2) x 6 = 5 x 6 + 2 x 6 = 30 + 12 =42



Example:



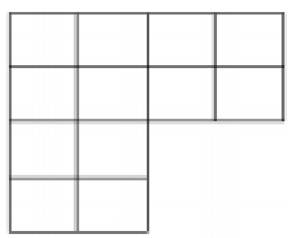
4 x 3 + 4 x 2 = 20

4 (3 + 2) = 20

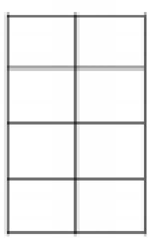
4 x 5 = 20

a. **Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.**

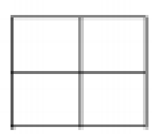
This standard uses the word rectilinear. A rectilinear figure is a polygon that has all right angles.



How could this figure be decomposed to help find the area?



This portion of the decomposed figure is a 4 x 2.

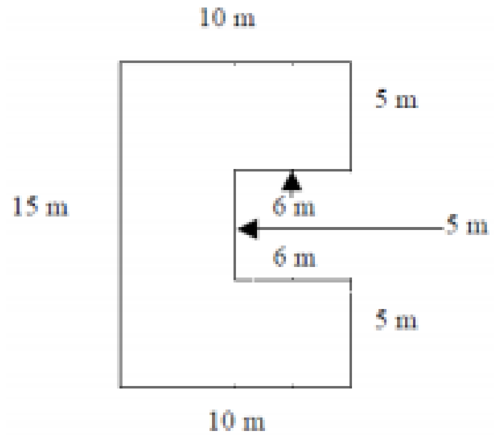


This portion of the decomposed figure is 2 x 2.

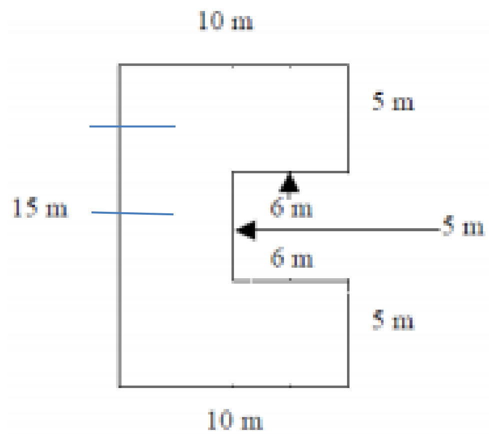
4 x 2 = 8 and 2 x 2 = 4
So 8 + 4 = 12
Therefore the total area of this figure is 12 square units

Example 1:

A storage shed is pictured below. What is the total area? How could the figure be decomposed to help find the area?



The area can be found by using 3 rectangles:

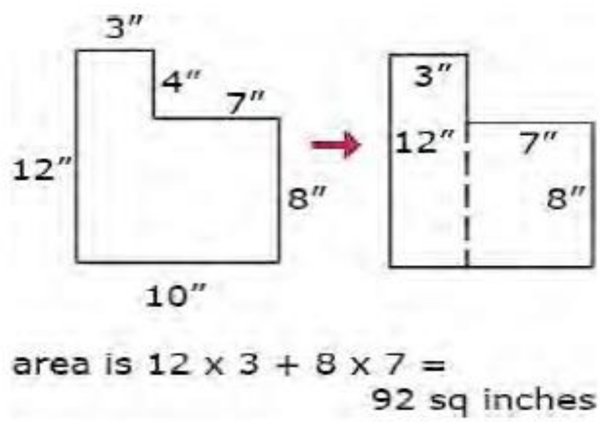


The top and bottom of the figure will be 10 m x 5 m for 50 m^2 x 2 = 100 m^2

The center rectangle will be a square with dimentions 5m x (15 – 5 – 5)m = 5m x 5 m or 25m^2
So, the area of the storage shed is 125 square meters.

Example 2:

As seen above, students can decompose a rectilinear figure into different rectangles. They find the area of the figure by adding the areas of each of the rectangles together.

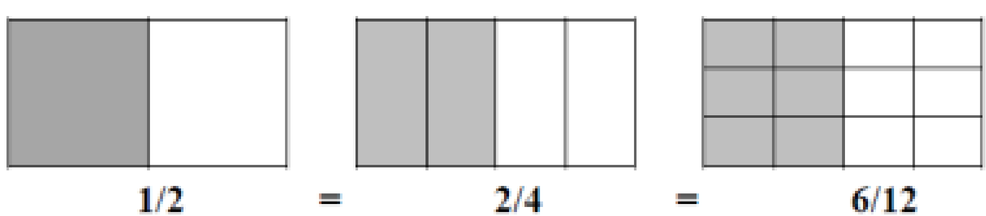


<u>Common Misconceptions</u>
Students may confuse perimeter and area when they measure the sides of a rectangle and then multiply. They think the attribute they find is length, which is perimeter. Pose problems situations that require students to explain whether they are to find the perimeter or area.

MGSE.4.NF.1 Explain why two or more fractions are equivalent. $\frac{a}{b} = \frac{n \times a}{n \times b}$. $\frac{1}{4} = \frac{3 \times 1}{3 \times 4}$ **by using visual fraction models. Focus attention on how the number and size of the parts differ even though the fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.**

This standard refers to visual fraction models. This includes area models or number lines. This standard extends the work in third grade by using additional denominators (5, 10, 12, and 100).

The standard addresses equivalent fractions by examining the idea that equivalent fractions can be created by multiplying both the numerator and denominator by the same number or by dividing a shaded region into various parts.
Example:



Technology Connection: http://illuminations.nctm.org/activitydetail.aspx?id=80

MGSE.4.OA.2. Multiply or divide to solve word problems involving multiplicative comparison. Use drawings and equations with a symbol or letter for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

This standard calls for students to translate comparative situations into equations with an unknown and solve. Students need many opportunities to solve contextual problems.

Examples:

***Unknown Product*:** A blue scarf costs \$3. A red scarf costs 6 times as much. How much does the red scarf cost? *($3 \times 6 = p$)*
Group Size Unknown: A book costs \$18. That is 3 times more than a DVD. How much does a DVD cost? *($18 \div p = 3$ or $3 \times p = 18$)*
Number of Groups Unknown: A red scarf costs \$18. A blue scarf costs \$6. How many times as much does the red scarf cost compared to the blue scarf? *($18 \div 6 = p$ or $6 \times p = 18$)*

When distinguishing multiplicative comparison from additive comparison, students should note the following:

- Additive comparisons focus on the difference between two quantities. For example, Deb has 3 apples and Karen has 5 apples. How many more apples does Karen have? A simple way to remember this is, "How many more?"
- Multiplicative comparisons focus on comparing two quantities by showing that one quantity is a specified number of times larger or smaller than the other. For example, Deb ran 3 miles. Karen ran 5 times as many miles as Deb. How many miles did Karen run? A simple way to remember this is "How many times as much?" or "How many times as many?"

MGSE.4.NF.2. Compare two fractions with different numerators and different denominators, e.g., by using visual fraction models, by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions.

This standard asks students to compare fractions by creating visual fraction models or finding common denominators or numerators. **Students' experiences should focus on visual fraction models rather than algorithms.** When tested, models may or may not be included. Students should learn to draw fraction models to help them compare. Students must also recognize that they must consider the size of the whole when comparing fractions (i.e., ½ and $\frac{1}{8}$ of two medium pizzas is very different from ½ of one medium and $\frac{1}{8}$ of one large).

Example 1:

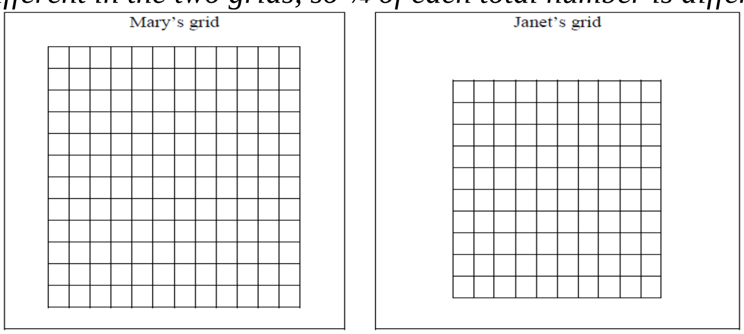
Use patterns blocks.

- If a red trapezoid is one whole, which block shows $\frac{1}{3}$?
- If the blue rhombus is $\frac{1}{3}$, which block shows one whole?
- If the red trapezoid is one whole, which block shows $\frac{2}{3}$?

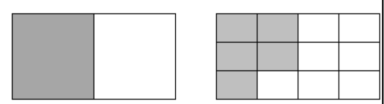
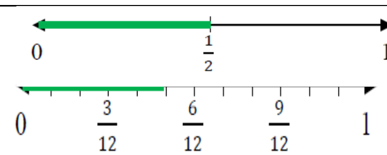
Example 2:

Mary used a 12 × 12 grid to represent 1 and Janet used a 10 × 10 grid to represent 1. Each girl shaded grid squares to show ¼. How many grid squares did Mary shade? How many grid squares did Janet shade? Why did they need to shade different numbers of grid squares?

Possible solution: Mary shaded 36 grid squares; Janet shaded 25 grid squares. The total number of little squares is different in the two grids, so ¼ of each total number is different.

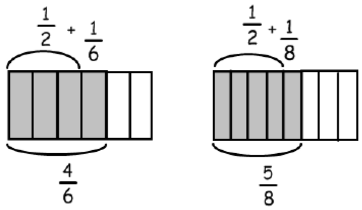


Example 3:
There are two cakes on the counter that are the same size. The first cake has ½ left. The second cake has $\frac{5}{12}$ left. Which cake has more left?

Student 1: ***Area Model*** The first cake has more left over. The second cake has $\frac{5}{12}$ left which is smaller than ½ .	
Student 2: ***Number Line Model*** The first cake has more left over: ½ is bigger than $\frac{5}{12}$	
Student 3: ***Verbal Explanation*** I know that $\frac{6}{12}$ equals ½, and $\frac{5}{12}$ is less than ½ . Therefore, the second cake has less left over than the first cake. The first cake has more left over.	

Example 4:

When using the benchmark of $\frac{1}{2}$ to compare $\frac{4}{6}$ and $\frac{5}{8}$, you could use diagrams such as these:



$\frac{4}{6}$ is $\frac{1}{6}$ larger than $\frac{1}{2}$, while $\frac{5}{8}$ is $\frac{1}{8}$ larger than $\frac{1}{2}$. Since $\frac{1}{6}$ is greater than $\frac{1}{8}$, $\frac{4}{6}$ is the greater fraction.

Common Misconceptions

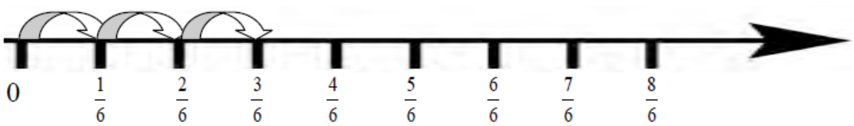
Students think that when generating equivalent fractions they need to multiply or divide either the numerator or denominator by the same number rather than the numerator **and** denominator. For example, when making equivalent fractions for $\frac{5}{6}$, a student may multiply just the numerator by 2 resulting in $\frac{10}{6}$instead of correctly multiplying by $\frac{2}{2}$ with the result $\frac{10}{12}$. This misconception comes about because students do not understand that they **need to use a fraction in the form of one**, such as $\frac{2}{2}$ to generate an equivalent fraction. Reviewing the identity property with students reemphasizing what happens when we multiply by 1 is an essential component of instruction when addressing this misconception. Conversation centered around "what one (in disguise) could we use to create an equivalent fraction?"

MGSE.4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number e.g., by using a visual such as a number line or area model.
a. Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent $\frac{5}{4}$ as the product 5 × ($\frac{1}{4}$), recording the conclusion by the equation $\frac{5}{4}$= 5 × ($\frac{1}{4}$).*

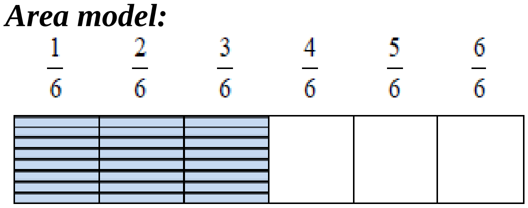
This standard builds on students' work of adding fractions and extending that work into multiplication.

Example: $\frac{3}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 3 \times \frac{1}{6}$

Number line:

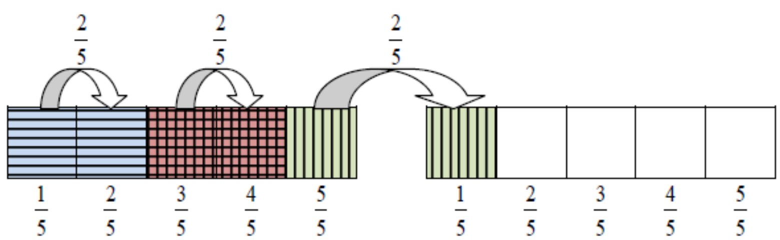


Area model:



b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. ***For example, use a visual fraction model to express 3 × ($\frac{2}{5}$) as 6 × ($\frac{1}{5}$), recognizing this product as ($\frac{6}{5}$). (In general, n × (a/b) = (n × a)/b.)***

This standard extended the idea of multiplication as repeated addition. For example, $3 \times \frac{2}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{6}{5} = 6 \times \frac{1}{5}$.
Students are expected to use and create visual fraction models to multiply a whole number by a fraction.



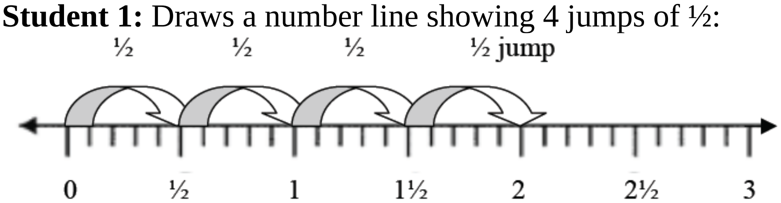
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. ***For example, if each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?***

This standard calls for students to use visual fraction models to solve word problems related to multiplying a whole number by a fraction.

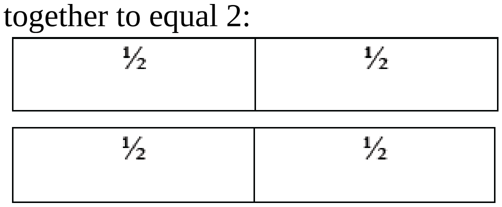
Example 1:

In a relay race, each runner runs ½ of a lap. If there are 4 team members how long is the race?

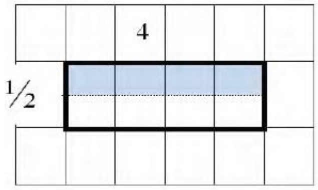
Student 1: Draws a number line showing 4 jumps of ½:



Student 2: Draws an area model showing 4 pieces of ½ joined together to equal 2:



Student 3: Draws an area model representing 4 × ½ on a grid, dividing one row into ½ to represent the multiplier:



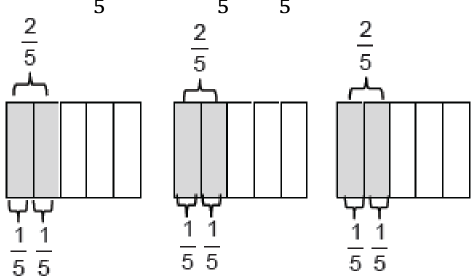
Example 2:

Heather bought 12 plums and ate $\frac{1}{3}$ of them. Paul bought 12 plums and ate $\frac{1}{4}$ of them. Which statement is true? Draw a model to explain your reasoning.

a. Heather and Paul ate the same number of plums.
b. Heather ate 4 plums and Paul ate 3 plums.
c. Heather ate 3 plums and Paul ate 4 plums.
d. Heather had 9 plums remaining.

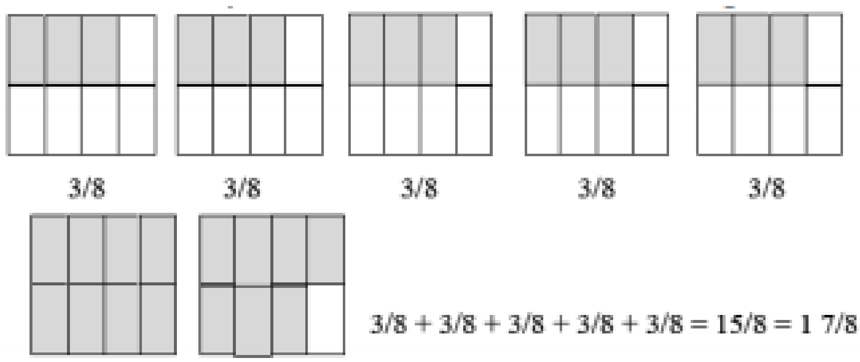
Example 3: Students need many opportunities to work with problems in context to understand the connections between models and corresponding equations. Contexts involving a whole number times a fraction lend themselves to modeling and examining patterns.

a. $3 \times \frac{2}{5} = 6 \times \frac{1}{5} = \frac{6}{5}$



b. If each person at a party eats $\frac{3}{8}$ of a pound of roast beef, and there are 5 people at the party, how many pounds of roast beef are needed? Between what two whole numbers does your answer lie?

A student may build a fraction model to represent this problem:



Common Misconception
Students think that it does not matter which model to use when finding the sum or difference of fractions. They may represent one fraction with a rectangle and the other fraction with a circle. They need to know that the models need to represent the same whole.

MGSE.5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

This standard includes multiplying by multiples of 10 and powers of 10, including 10^2 which is $10 \times 10 = 100$, and 10^3 which is $10 \times 10 \times 10 = 1{,}000$. Students should have experiences working with connecting the pattern of the number of zeros in the product when you multiply by powers of 10.

Example 1: $2.5 \times 10^3 = 2.5 \times (10 \times 10 \times 10) = 2.5 \times 1{,}000 = 2{,}500$

Students should reason that the exponent above the 10 indicates how many places the decimal point is moving (not just that the decimal point is moving but that you are multiplying or making the number 10 times greater three times) when you multiply by a power of 10. Since we are multiplying by a power of 10 the decimal point moves to the right.

Example 2: $350 \div 10^3$

$350 \div 10^3 = 350 \div 1{,}000$ (This extra representation should be added so that the decimal is just not "moving") $= 0.350 = 0.35$

Example 3: ${}^{350}/_{10}$

${}^{350}/_{10} = (350 \times {}^{1}/_{10}) = {}^{35}/_{1} = 35$ The second step emphasizes that the "0" just does not disappear. This example shows that when we divide by powers of 10, the exponent above the 10 indicates how many places the decimal point is moving (how many times we are dividing by 10 , the number becomes ten times smaller). Since we are dividing by powers of 10, the decimal point moves to the left. Students need to be provided with opportunities to explore this concept and come to this understanding; this **should not just be taught procedurally**.

MGSE.5.NBT.3. Read, write, and compare decimals to thousandths.

b. **Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 x (1/100) + 2 × (1/1000).**
c. **Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.**

This standard references the expanded form of decimals with fractions included. Students should build on their work from 4th grade, where they worked with both decimals and fractions interchangeably. Expanded form is included to build upon work in **MGSE.5.NBT.2** and to deepen students' understanding of place value.

Students will also build on the understanding they developed in fourth grade to read, write, and compare decimals to thousandths. They will connect to their prior experiences of using decimal notation for fractions in the addition of fractions with denominators of 10 and 100. When dealing with tenths and hundredths, conversation about money (cents) can help students make connections to using decimals in real world contexts.

Students will benefit by using concrete models and number lines to read, write, and compare decimals to the thousandths. Models may include base ten blocks, place value charts, grids, pictures, drawings, manipulatives, website virtual manipulatives etc. They will need to read decimals using fractional language and to write decimals in fractional form, as well as in expanded notation. This investigation leads to understanding equivalence of decimals (0.8 = 0.80 = 0.800).

Example:

Some equivalent forms of 0.72 are:

$^{72}/_{100}$	$^{70}/_{100} + {}^{2}/_{100}$
$^{7}/_{10} + {}^{2}/_{100}$	0.720
$7 \times (^{1}/_{10}) + 2 \times (^{1}/_{100})$	$7 \times (^{1}/_{10}) + 2 \times (^{1}/_{100}) + 0 \times (^{1}/_{1000})$
0.70 + 0.02	$^{720}/_{1000}$

Students need to conceptually understand the size of decimal numbers and to relate them to common benchmarks such as 0, 0.5 (0.50 and 0.500), and 1. Comparing tenths to tenths, hundredths to hundredths, and thousandths to thousandths is simplified if students use their understanding of fractions to compare decimals.

Example:

Comparing 0.25 and 0.17, a student might think, “25 hundredths is more than 17 hundredths”. They may also think that it is 8 hundredths more. They may write this comparison as 0.25 > 0.17 and recognize that 0.17 < 0.25 is another way to express this comparison. Additionally, connecting to money, “$0.25 is more than $0.17” or “$0.17 is less than $0.25”

Comparing 0.207 to 0.26, a student might think, “Both numbers have 2 tenths, so I need to compare the hundredths. The second number has 6 hundredths and the first number has no hundredths so the second number must be larger. Another student might think while writing fractions, “I know that 0.207 is 207 thousandths (and may write $^{207}/_{1000}$). 0.26 is 26 hundredths (and may write $^{26}/_{100}$) but I can also think of it as 260 thousandths ($^{260}/_{1000}$). So, 260 thousandths is more than 207 thousandths.

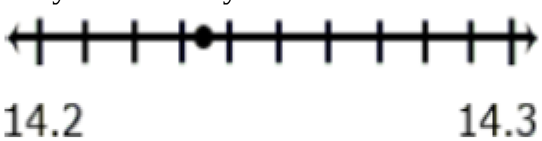
MGSE.5.NBT.4. Use place value understanding to round decimals up to the hundredths place

This standard refers to rounding. Students should go beyond simply applying an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line to support their work with rounding.

Example:

Round 14.235 to the nearest tenth.

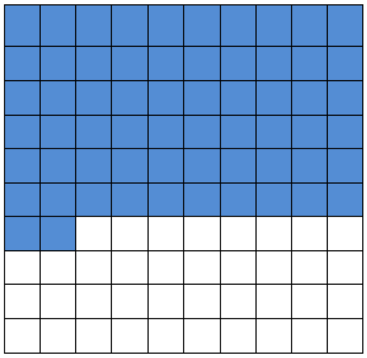
Students **must** recognize that the possible answer must be in tenths thus, it is either 14.2 or 14.3. They then identify that 14.235 is closer to 14.2 (14.20) than to 14.3 (14.30).



Students should use benchmark numbers to support this work. Benchmarks are convenient numbers for comparing and rounding numbers. 0, 0.5, 1, 1.5 are examples of benchmark numbers.

Example:

Which benchmark number is the best estimate of the shaded amount in the model below? Explain your thinking.



In this case 0.62 would be the number to be rounded to the nearest tenth. By seeing that the "extra blocks" are closest to $\frac{60}{100}$, the colored blocks show that 0.6 would be the closest value.

Common Misconceptions
A misconception that is directly related to the comparison of whole numbers and the comparison of decimals is the idea that the more digits a number contains means the greater the value of the number. With whole numbers, a 5-digit number is always greater that a 1-, 2-, 3-, or 4-digit number.

However, with decimals, a number with one decimal place may be greater than a number with two or three decimal places. For example, 0.5 is greater than 0.12, 0.009 or 0.499. One method for comparing decimals is to rewrite all numbers so that they have the same number of digits to the right of the decimals point, such as 0.500, 0.120, 0.009 and 0.499. A second method is to use a place-value chart to place the numerals for comparison. A third would be to think of the amount as money and have students' first compare the "cents" decimal positions (tenths and hundredths) to determine how much "change" they are looking at for each example.

MGSE.5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

This standard also builds on work begun in 4^{th} grade when students were introduced to decimals and asked to compare them. In 5th grade, students begin adding, subtracting, multiplying and dividing decimals. This work should focus on concrete models and pictorial representations, rather than relying solely on the algorithm. The use of symbolic notations involves having students record the answers to computations ($2.25 \times 3 = 6.75$), but this work should not be done without models or pictures.

This standard requires that students explain their reasoning and how they use models, pictures, and strategies. Students are expected to extend their understanding of whole number models and strategies to decimal values. Before students are asked to give exact answers, they should estimate answers based on their understanding of operations and the value of the numbers.
Examples:

5.4 – 0.8

A student might estimate the answer to be a little more than 4.4 because a number less than 1 is being subtracted.

6 × 2.4

A student might estimate an answer between 12 and 18 since 6 × 2 is 12 and 6 × 3 is 18. Another student might give an estimate of a little less than 15 because s/he figures the answer to be very close, but smaller than 6 × 2½ and think of 2½ groups of 6 as 12 (2 groups of 6) + 3(½ of a group of 6).

When adding or subtracting decimals, students should be able to explain that tenths are added or subtracted from tenths and hundredths are added or subtracted from hundredths. So, students will need to communicate that when adding or subtracting in a vertical format (numbers beneath each other), it is important that digits with the same place value are written in the same column. This understanding can be reinforced by linking the decimal addition and subtraction process to addition and subtraction of fractions. Adding and subtracting fractions with like denominators of 10 and 100 is a standard in fourth grade.

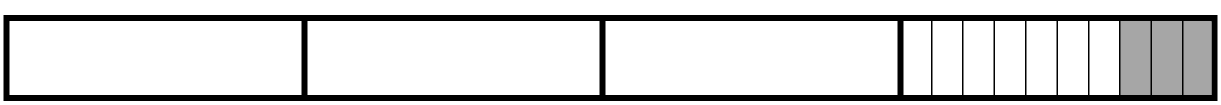
Common Misconceptions
Students might compute the sum or difference of decimals by lining up the right-hand digits as they would whole number. For example, in computing the sum of 15.34 + 12.9, students will write the problem in this manner:

15.34
<u>+ 12.9</u>
16.63

To help students add and subtract decimals correctly, have them first estimate the sum or difference. Providing students with a decimal-place value chart will enable them to place the digits in the proper place.

Example 1: 4 - 0.3

3 tenths subtracted from 4 wholes. One of the wholes must be divided into tenths.



The solution is $3 + {}^{7}/_{10}$ or 3.7.

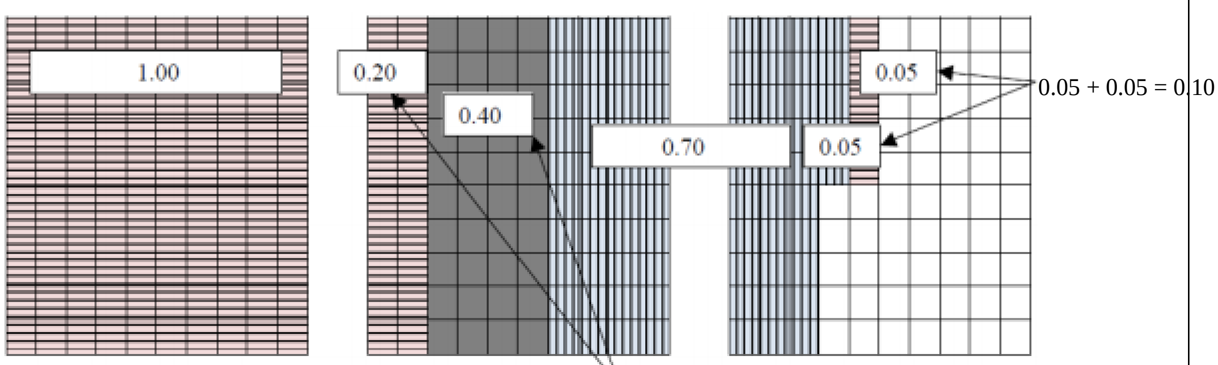
Example 2:

A recipe for a cake requires 1.25 cups of milk, 0.40 cups of oil, and 0.75 cups of water. How much liquid is in the mixing bowl?

Student 1: 1.25 + 0.40 + 0.75

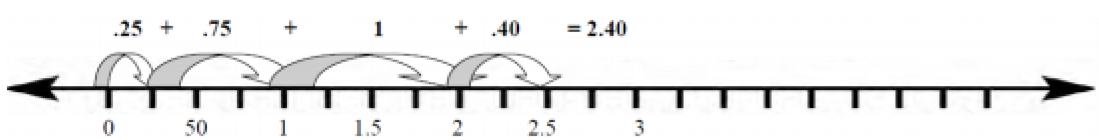
First, I broke the numbers apart. I broke 1.25 into 1.00 + 0.20 + 0.05. I left 0.40 like it was. I broke 0.75 into 0.70 + 0.05.

I combined my two 0.05's to get 0.10. I combined 0.40 and 0.20 to get 0.60. I added the 1 whole from 1.25. I ended up with 1 whole, 6 tenths, 7 more tenths, and another 1 tenths, so the total is 2.4.

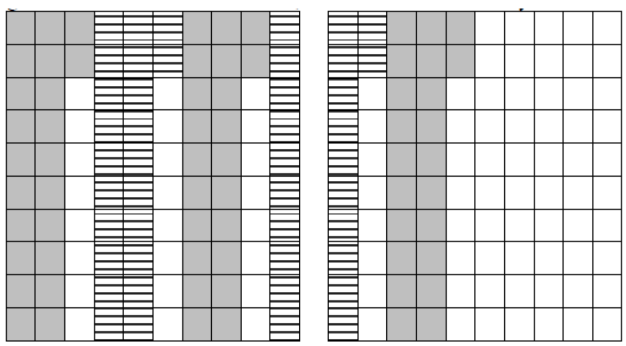


Student 2:

I saw that the 0.25 in the 1.25 cups of milk and the 0.75 cups of water would combine to equal 1 whole cup. That plus the 1 whole in the 1.25 cups of milk gives me 2 whole cups. Then I added the 2 wholes and the 0.40 cups of oil to get 2.40 cups.



Example of Multiplication 1
A gumball costs \$0.22. How much do 5 gumballs cost? Estimate the total, and then calculate. Was your estimate close?

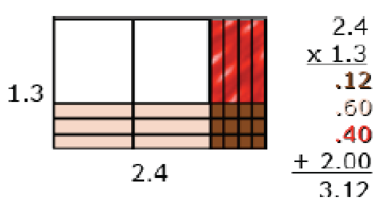


I estimate that the total cost will be a little more than a dollar. I know that 5 20's equal 100 and we have 5 22's. I have 10 whole columns shaded and 10 individual boxes shaded. The 10 columns equal 1 whole.

The 10 individual boxes equal 10 hundredths or 1 tenth. My answer is \$1.10.

My estimate was a little more than a dollar, and my answer was \$1.10. I was really close.

Multiplication Example 2:
An area model can be useful for illustrating products.



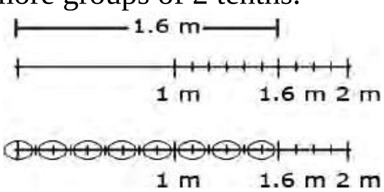
Students should be able to describe the partial products displayed by the area model.

For example, "$^{3}/_{10}$ times $^{4}/_{10}$ is $^{12}/_{100}$.
$^{3}/_{10}$ times 2 is $^{6}/_{10}$ or $^{60}/_{100}$.
1 group of $^{4}/_{10}$ is $^{4}/_{10}$ or $^{40}/_{100}$.
1 group of 2 is 2."

Division Example 1: Joe has 1.6 meters of rope. He has to cut pieces of rope that are 0.2 meters long. How many can he cut?

Finding the number of groups

Students could draw a segment to represent 1.6 meters. In doing so, s/he would count in tenths to identify the 6 tenths, and be able identify the number of 2 tenths within the 6 tenths. The student can then extend the idea of counting by tenths to divide the one meter into tenths and determine that there are 5 more groups of 2 tenths.



Students might count groups of 2 tenths without the use of models or diagrams. Knowing that 1 can be thought of as $^{10}/_{10}$, a student might think of 1.6 as 16 tenths. Counting 2 tenths, 4 tenths, 6 tenths, up to16 tenths, a student can count 8 groups of 2 tenths.

Use their understanding of multiplication and think, "8 groups of 2 is 16, so 8 groups of $^{2}/_{10}$ is $^{16}/_{10}$ or $1^{6}/_{10}$."

Division Example 2: 2.4 ÷ 4

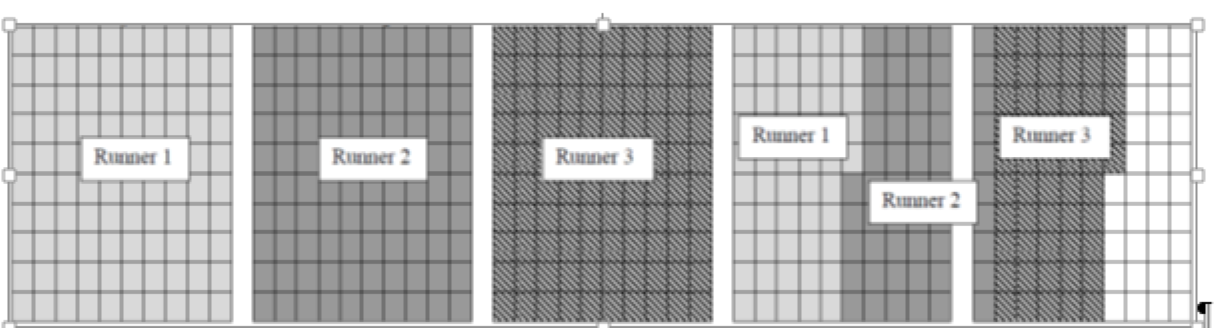
Finding the number in each group or share

Students should be encouraged to apply a fair sharing model separating decimal values into equal parts such as 2.4 ÷ 4 = 0.6.

Division Example 3:

A relay race lasts 4.65 miles. The relay team has 3 runners. If each runner goes the same distance, how far does each team member run? Make an estimate, find your actual answer, and then compare them.

A possible solution is shown on the next page.



My estimate is that each runner runs between 1 and 2 miles. If each runner went 2 miles, that would be a total of 6 miles which is too high. If each runner ran 1 mile, that would be 3 miles, which is too low. I used the 5 grids above to represent the 4.65 miles.

I am going to use all of the first 4 grids and 65 of the squares in the 5th grid. I have to divide the 4 whole grids and the 65 squares into 3 equal groups. I labeled each of the first 3 grids for each runner, so I know that each team member ran at least 1 mile. I then have 1 whole grid and 65 squares to divide up. Each column represents one-tenth. If I give 5 columns to each runner, that means that each runner has run 1 whole mile and 5 tenths of a mile. Now, I have 15 squares left to divide up. Each runner gets 5 of those squares. So each runner ran 1 mile, 5 tenths and 5 hundredths of a mile. I can write that as 1.55 miles.

My answer is 1.55 and my estimate was between 1 and 2 miles. I was pretty close.

STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe varieties of expertise that educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in education. The first of these are the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning and proof, communication, representation, and connections. (*Principles and Standards for School Mathematics.* NCTM: 2000.) The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy) (National Academies Press, 2001.)

Students are expected to:

1. Make sense of problems and persevere in solving them.
Students begin in elementary school to solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. Students seek the meaning of a problem and look for efficient ways to represent and solve it. In middle school, students solve real world problems through the application of algebraic and geometric concepts.

2. Reason abstractly and quantitatively.
Earlier grade students should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.

In middle school, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

3. Construct viable arguments and critique the reasoning of others.
In earlier grades, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication.

In middle school, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (e.g., box plots, dot plots, histograms). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. The students pose questions like “How did you get that?”, “Why is that true?”, and “Does that always work?” They explain their thinking to others and respond to others’ thinking.

4. Model with Mathematics.
Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Elementary students should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.

In middle school, students model problem situations with symbols, graphs, tables, and context. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students solve systems of linear equations and compare properties of functions provided in different forms. Students use scatterplots to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.

5. Use appropriate tools strategically.
Elementary students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real world data.

Students in middle school may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal.

6. Attend to precision.
Students in earlier grades continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units. Students in middle school use appropriate terminology when referring to the number system, functions, geometric figures, and data displays.

7. Look for and make use of structure.
In elementary grades, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply, and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation.

Students in middle school routinely seek patterns or structures to model and solve problems. Students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.

8. Look for and express regularity in repeated reasoning.
Students in elementary grades use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms, to fluently multiply multi-digit numbers, and to perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations.

Middle school students use repeated reasoning to understand algorithms and make generalizations about patterns. Students use iterative processes to determine more precise rational approximations for irrational numbers. During multiple opportunities to solve and model problems, they notice that the slope of a line and rate of change are the same value. Students flexibly make connections between covariance, rates, and representations showing the relationships between quantities.

Connecting the Standards for Mathematical Practice to the Content Standards
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who are missing the understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, an absence of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward the central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics. See Inside Mathematics for more resources.

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The terms below are for teacher reference only and are not to be memorized by the students. Teachers should present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

- **Algebraic Expression**
- **Approximation**
- **Array**
- **Associative Property**
- **Benchmark Fraction**
- **Coefficient**
- **Commutative Property**
- **Coordinate Plane**
- **Coordinates**
- **Denominator**
- **Difference**
- **Digit**
- **Distributive Property**
- **Dividend**
- **Divisor**
- **Equation**
- **Equivalent Expressions**
- **Equivalent Ratios**
- **Factors**
- **Formula**
- **Fraction**
- **Function**
- **Identity Properties**
- **Inequality**
- **Integer**
- **Inverse Operation**
- **Irrational Number**
- **Line Diagram**
- **Numerator**
- **Numeric expression**
- **Opposite of a Number**
- **Origin**
- **Place Value**
- **Power of Ten**
- **Product**
- **Proportional Relationship**
- **Quadrant**
- **Quotient**
- **Rational Number**
- **Solution**
- **Substitution**
- **Sum**
- **Unit rate**
- **Variable**
- **Variable**
- **x-axis**
- **x-coordinate**
- **y-axis**
- **y-coordinate**
- **Zero**

Again, discuss terminology as it naturally arises in discussion of the problems. Allow students to point out words or phrases that lead them to the model and solution of the problems. Words that imply mathematical operations vary based on context and should be delineated based on their use in the particular problem. A couple of suggested methods for students to record vocabulary are

TIP Charts (https://www.youtube.com/watch?v=Rbts9h_ruu8) and Frayer Models (https://wvde.state.wv.us/strategybank/FrayerModel.html).

The websites below are interactive and include a math glossary suitable for middle school children.

- http://www.amathsdictionaryforkids.com/
- http://intermath.coe.uga.edu/dictnary/homepg.asp

TABLE OF INTERVENTIONS

Lesson Name	Name Of Intervention	Snapshot of summary or student I can statement …	Book, Page Or link
Pattern-R-Us	Powers of Powers	Use these activities to help your students develop knowledge of place value and powers of 10 to support multiplicative thinking	Powers of Powers
Are These Equivalent?	Bead Strings	Find equivalents for decimals and fractions.	Bead Strings
Multiplying Rational Numbers	Multiplying Integers	Find a pattern in the multiplication facts of signed numbers.	Multiplying Integers
Olympic Cola Display	Animal Arrays	I am learning to find other ways to solve repeated addition problems.	Animal Arrays
	Array Game	The game allows students to practice their multiplication skills, and reinforces the 'array' concept of multiplication	Array Game
Conjectures About Properties	Conjectures About Properties	The purpose is to develop the students' deeper understanding of the way numbers behave, to enable them to use everyday language to make a general statement about these behaviors, and to understand the symbolic representation of these 'properties' of numbers and operations.	Conjectures About Properties

Equivalent Fractions	Equivalent Fractions	The purpose of this activity is to help your child to practice finding equivalent fractions for numbers up to 100.	Equivalent Fractions
	Addition, subtraction and equivalent fractions	The purpose of this series of lessons is to develop understanding of equivalent fractions and the operations of addition and subtraction with fractions.	Addition, subtraction and equivalent fractions
Snack Mix	Sharing in Ratios	Solve problems involving ratios.	Sharing in Ratios
Which Bed, Bath, and Beyond Coupon Should You Use?	Percentages Increases & Decreases in One Step	An activity to help develop automaticity with percentages.	Percentages Increases & Decreases in One Step
Set it Up	Set It Up	Relate tables, graphs, and equations to linear relationships found in number and spatial patterns.	Set It Up
Solving Equations using Bar Diagrams	Unknowns & Variables: Solving One Step Equations	Form and solve simple linear equations.	Unknowns & Variables: Solving One Step Equations
Reviewing Rate of Change	Rates of Change	Solve problems involving rates.	Rates of Change

SCAFFOLDED INSTRUCTIONAL LESSONS

Lesson Name	Lesson Type/ *Grouping Strategy*	Content Addressed	Standard(s)
Problem Solving Assessments	*Individual*	Sampling of assessment questions focusing on multiplication and division and building models.	MFANSQ1.a MFANSQ4.b,d
Quick Check I	Formative Assessment *Individual*	Prerequisite knowledge necessary to be successful with fractions (May be used AFTER Birthday Cake and Fraction Clues depending on your students)	MFANSQ1.b MFANSQ4.b
Chance of Surgery	*Individual*	Multiplying a whole number by a fraction using models.	MFANSQ1.b MFANSQ4.b,c,d,e
Fractional Divisors	*Individual/Partner/Small Group*	Dividing a fraction by a fraction using models.	MFANSQ1.b MFANSQ4.b,c,d,e
Dividing Fractions with Models	*Individual/Partner*	Dividing a fraction by a fraction using models.	MFANSQ4.b,c,d,e
Quick Check II	Formative Assessment *Individual*	Fractions on a number line and Fraction Operations	MFANSQ1. b MFANSQ4.b,c,d,e
Patterns-R-Us	*Partner/Small Group*	Explaining patterns when multiplying or dividing by a power of ten.	MFANSQ1.c,d MFANSQ4.a,b,d
Are These Equivalent?	*Individual/Partner*	Determining equivalency of fractions and decimals.	MFANSQ1.d MFANSQ4.a,b,d
Quick Check III	Formative Assessment *Individual*	Equivalency of fractions	MFANSQ1.b MFANSQ2 b, c MFANSQ4.b,c,e
Penny Cube	*Partner/Whole Group*	Determine the value of a cube of pennies.	MFANSQ4.a,b,d,e
Multiplying Rational Numbers	*Partner/Whole Group*	Using the number line to multiply integers and apply the patterns to divide integers.	MFANSQ2 a,b,c MFANSQ4.a,b,d,e
Arithmetic to Algebra	Review to be used throughout unit	Strategies to bridge number sense to algebra	

Lesson Name	**Lesson Type/** ***Grouping Strategy***	**Content Addressed**	**Standard(s)**
Olympic Cola Display	3 Act Task/ *Individual/Partner and/or Small Group*	Distributive Property of Multiplication	MFAAA1.b
Tiling Lesson	Learning Lesson/ *Whole group then Partner or Small Group*	Area model to find the area of a rectangle with fractional side lengths and express/simplify the perimeter of rectangles with variable side lengths	MFAAA1.a,b,c,f
Conjectures About Properties	Formative Lesson *Partner/Small Group*	Properties of Numbers	MFAAA1.a,c,e
Quick Check IV	Mini Assessment Lesson *Individual*	Equivalent expressions, arrays to model expressions	MFAAA.a,b,c,d, e
Visual Patterns	Constructing Lesson *Individual/Partner*	Expressions and formulas	MFAAA1.c,d,e,f
Exploring Expressions	Constructing Lesson *Individual/Partner*	Properties of Numbers, Applications of Formulas, Evaluating Expressions	MFAAA1.a,c,d,e,f
Equivalent Fractions	Learning Lesson *Individual or partners*	Explore relationships between equivalent fractions	MFAPR.2
Snack Mix	3-Act Task *Partners or small groups*	Find equivalent ratios to solve a problem	MFAPR.1 MFAPR.2
Proportional Relationships	Learning Lesson *Small groups*	Determine whether two quantities are in a proportional relationship	MFAPR.2
Quick Check V	Formative Assessment *Individual*	Determine whether two quantities are in a proportional relationship and comparing quantities	MFAPR.1 MFAPR.2
Which Bed, Bath, & Beyond Coupon Should You Use?	Learning Lesson *Partners or small groups*	Compare coupons to make purchasing decisions	MFAPR.2 b,c

Lesson Name	Lesson Type/ *Grouping Strategy*	Content Addressed	Standard(s)
Quick Check VI	Formative Assessment *Individual*	Use ratios and proportions to solve problems including those involving percentages	MFAPR.1 MFAPR.2 MFAPR.3
Analyzing Solutions of Equations and Inequalities	Constructing Lesson *Individual/Partner*	Solving equations and inequalities by inspection; understanding the difference between equality and inequality	MFAEI1a, e
Quick Check VII	Formative Assessment *Individual*	Find similar figures by comparing lengths and widths	MFAPR.1 MFAPR.2 MFAPR.3
Set It Up	Performance Lesson *Individual/Partner*	Setting up and solving one step equations	MFAEI1a, e
Solving Equations using Bar Diagrams	Constructing Lesson *Individual/Partner*	Using bar models to solve equations	MFAEI1a, b,e
When Is It Not Equal?	Performance Lesson *Individual/Partner*	Setting up inequalities	MFAEI1 a,d,e
Which Ticket is the Best Deal?	Three-Act Task *Partners or small groups*	Finding rate of change, graphing functions	MFAQR2a
Reviewing Rate of Change	Formative Assessment *Individual*	Reviewing rates of change from prior grades	MFAQR2a

Problem Solving Assessments

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ1. Students will analyze number relationships.
a. Solve multi-step real world problems, analyzing the relationships between all four operations. For example, understand division as an unknown-factor problem in order to solve problems. Knowing that 50 x 40 = 2000 helps students determine how many boxes of cupcakes they will need in order to ship 2000 cupcakes in boxes that hold 40 cupcakes each. (MGSE3.OA.6, MGSE4.OA.3)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.

6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to solve real-world problems involving number operations.

MATERIALS

- Assessment Questions

ESSENTIAL QUESTIONS

- How can you solve real world problems?

Grouping: Individual

OPENER/ACTIVATOR

Given the Problem:
The 9th grade class at Jones County 9th Grade Campus is going on their annual field trip to Savannah, GA. The teachers reserved two buses to take the 123 students and 14 chaperones. Each bus has 32 seats that can hold up to three people per seat. Will they need to sit in groups of 3 in each seat or can they have more room and only sit with a partner?

Analyze Dustin's Response Below:

If each bus has 32 seats, then 2 buses x 32 seats means there are 64 seats total. If there are 64 seats with 2 people on each seat, the two buses can carry 128 students comfortably. Since we only have 123 students, we will have plenty of room and no one will have to sit 3 to a seat.

How would you grade Dustin's answer?

A great rubric can be found at http://bit.ly/1eNEEBN

WORK SESSION:

Below are sample assessment questions. Pick and choose the question(s) you deem appropriate for your class. Questions not utilized in this activity could be placed on your End of Module Assessment.
Additional problems along with a rubric can be found at http://bit.ly/1eNEEBN

QUESTION #1
Kameron went to the arcade in Panama City Beach and played a game that involved throwing beanbags at clowns. She had one minute to hit as many clowns as she could, and she hit 33 clowns before time ran out. When she was done, she earned 2 points for each clown she hit.

At the prize counter, Kameron was able to buy one prize for every 7 points she earned.

Kameron bought as many prizes as she could with her points. How many prizes was Kameron able to buy? Use the space to below to show your thinking as you figure out the number of prizes Kameron bought. Then, fill in the blank to show your answer.

Answer: Kameron bought 9 prizes.

QUESTION #2
Landon and Grant were both selling candles for a school fundraiser.

Landon sold 69 candles and was awarded a prize point for every 4 candles he sold.

Grant's school only awarded a prize point for every 3 candles sold. However, Grant ended up earning the same number of points as Landon.

How many candles could Jaylen have sold? Use the space below to show your thinking, and then fill in the blank with your answer.

Answer: Grant could have sold 51, 52, or 53 tickets to earn the same number of points as Landon. Landon earned 17 points (69 ÷ 4). Since Grant's school awarded a point for every 3 tickets sold, then to earn the same amount of tickets- 17 x 3= 51 tickets. He could have also sold 52 or 53 tickets to earn the same 17 points.

QUESTION #3

Sophie worked hard to save for a senior trip. The trip costs $1500. She earned $9 an hour babysitting and $4 an hour raking leaves. In October, she worked 52 hours babysitting and 12 hours raking leaves. Then, in November, she babysat 2 times as much as she had in October. Each month she set aside $50 to spend and saved the rest. Did she make enough money to pay for her trip? If not, how much more money does she need to pay off the trip in full?

Solve using numbers, pictures and/or words.

Answer: Total of 156 babysitting hours at $9/hr = $1,404 and a total of 21 hours raking leaves at $4/hr is $84. Their sum gives you a total of $1,488. Take out the $100 ($50/month for 2 months of spending) gives you $1,388. $1,500 - $1,388 = $112. Sophie needs to earn $112 more dollars to attend her Senior Class Trip.

Student Edition: Assessment Questions Practice

QUESTION #1
Kameron went to the arcade in Panama City Beach and played a game that involved throwing beanbags at clowns. She had one minute to hit as many clowns as she could, and she hit 33 clowns before time ran out. When she was done, she earned 2 points for each clown she hit.

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Solve using numbers, pictures and/or words.

CLOSING/SUMMARIZER

Using the rubric at http://bit.ly/1eNEEBN *, have students rate themselves based on how they feel they did.*

Quick Check I

STANDARDS FOR MATHEMATICAL CONTENT

MFANSQ1. Students will analyze number relationships.
b. Understand a fraction a/b as a multiple of 1/b. (MGSE4.NF.4)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)

Instructional Tip

Quick Check I and Quick Check II are placed in this module as formative assessments pieces to assist the teacher in determining whether their students are ready for the upcoming lessons. If students struggle with these Quick Checks it is recommended to devote more instructional time on the Building Number Sense Activities before moving further into the lessons within Module I.
As you do so, listen carefully to students conjectures to adjust the instructions as needed, determining whether students are ready to work with more complex ideas.

Quick Checks III, IV, and V are placed in this module as formative assessments pieces to assist the teacher in determining whether their students have mastered the basic concepts from the lessons previously taught.

Quick Check I - Formative Assessment

1. Complete the following sentence. An improper fraction b) has a value more than 1
a) has a 1 in the numerator
b) has a value more than 1
c) has a value less than 1
d) is equal to 1

2. What is a numerator? a
a) the top part of a fraction
b) the bottom part of a fraction
c) the value of a fraction
d) a number equal to 1

3. If you have $\frac{3}{8}$, how much do you need to make a whole (Hint: a whole is equal to 1)? d

a) 5 b) $\frac{3}{8}$ c) $\frac{8}{8}$ d) $\frac{5}{8}$

4. If the numerator is larger than the denominator, then the value of the fraction is c
a) equal to 1 b) less than 1 c) more than 1 d) equal to 0

5. Fill in the blank. *A fraction is equal to 1 (one) when* numerator and denominator are the same

6. a) Add the following fractions. $\frac{3}{5}+\frac{4}{5}=?$ 7/5 = 1 2/5

   b) Draw a picture of your problem.
   Pictures will vary, but 3/5 and 4/5 added together to equal 1 and 2/5 needs to be clear

7. Fill in the blank. $\frac{5}{11}+\frac{6}{11}\quad=\frac{11}{11}=1$
   6/11 is the answer

8. Subtract the following fractions. $\frac{5}{8}-\frac{2}{8}=?$ Answer should be 3/8

9. Add the following fractions. Give the answer as an improper fraction and a mixed numeral.
$\frac{2}{3}+\frac{6}{3}=$ Answer should be 8/3 and 2 2/3

10. Given, $\frac{4}{9}+\frac{5}{9}=\frac{9}{9}=1$, what fraction would you add to your answer to make it equal to 2? 9/9

Quick Check I - Formative Assessment

1. Complete the following sentence. An improper fraction ________________________________

a) has a 1 in the numerator　　b) has a value more than 1
c) has a value less than 1　　d) is equal to 1

2. What is a numerator?
a) the top part of a fraction　　b) the bottom part of a fraction
c) the value of a fraction　　d) a number equal to 1

3. If you have $\frac{3}{8}$, how much do you need to make a whole (Hint: a whole is equal to 1)?

a) 5　　b) $\frac{3}{8}$　　c) $\frac{8}{8}$　　d) $\frac{5}{8}$

4. If the numerator is larger than the denominator, then the value of the fraction is
a) equal to 1　　b) less than 1　　c) more than 1　　d) equal to 0

5. Fill in the blank. *A fraction is equal to 1 (one) when* _________________________________.

6. a) Add the following fractions. $\frac{3}{5}+\frac{4}{5}=?$
   b) Draw a picture of your problem.

7. Fill in the blank. $\frac{5}{11}+ \qquad - \qquad =\frac{11}{11}=1$

8. Subtract the following fractions. $\frac{5}{8}-\frac{2}{8}=?$

9. Add the following fractions. Give the answer as an improper fraction and a mixed numeral.
$\frac{2}{3}+\frac{6}{3}=$

10. Given, $\frac{4}{9}+\frac{5}{9}=\frac{9}{9}=1$, what fraction would you add to your answer to make it equal to 2?

Chance of Surgery

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ1. Students will analyze number relationships.
b. Understand a fraction a/b as a multiple of 1/b. (MGSE4.NF.4)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
c. Interpret and solve contextual problems involving division of fractions by fractions. For example, how many 3/4-cup servings are in 2/3 of a cup of yogurt? (MGSE6.NS.1)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)
e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MGSE7.NS.3, MGSE7.EE.3, MGSE9-12.N.Q.3)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.

6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:
- Use models to multiply a whole number by a fraction.
- Complete a multi-step lesson.

MATERIALS

- "A Chance of Surgery" recording sheet

ESSENTIAL QUESTIONS

- How can you model the multiplication of a whole number and a fraction?
- What fractional understanding do you need to mupltiply a fraction by a whole number?
- How do you solve a multi-step problem?

Grouping: Individual/Partner

OPENER/ACTIVATOR:

Students will need to determine $\frac{2}{3}$ and $\frac{1}{3}$ of 15 in order to correctly complete the multi-step lesson.

WORK SESSION

BACKGROUND KNOWLEDGE

Students need to be able to apply their understanding of mathematical concepts with multi-step problems. In accordance with SMP 1, students need to make sense of the problems in an attempt to solve them. For this lesson, it is essential that students make sense of the information provided as students will need to apply their fractional understanding in order to multiply accurately. The operation of multiplication is applied to various kinds of numbers.

LESSON with SOLUTIONS

Dr. Clifton is a surgeon at Children's Healthcare of Atlanta Egleston. In 2012, he performed 15 surgeries to treat biliary atresia. Studies have shown that $\frac{2}{3}$of the patients treated for biliary atresia eventually need a liver transplant. Of Dr. Clifton's patients last year, how many will eventually need a liver transplant?

$\frac{2}{3}$ x 15 = 10

According to the statistic, how many patients will not need a transplant?

If $\frac{2}{3}$ need a liver transplant later, then $\frac{1}{3}$ have success and do NOT need the transplant.
$\frac{1}{3}$ x 15 = 5
Or
15 surgeries – 10 who later need the transplant = 5 find success with a transplant

If it typically takes Dr. Clifton 4 hours to complete a biliary atresia surgery, about how many hours did he perform this operation last year?

4 hour x 15 surgeries = 60 hours in surgery

If Dr. Clifton's case load doubles in 2013, how many patients can he expect to have successful surgery and possibly not need a liver transplant?

If he did 15 surgeries in 2012 and doubles the number is 2013, he will do 30 surgeries in 2013.
2 x 15 = 30

How many hours should he anticipate conducting the surgery in 2013?

At 4 hours a surgery for 30 surgeries, the result is 120 hours.
4 x 30 = 120

DIFFERENTIATION

Extension

- Determine the amount of patients who will not need a liver transplant and who will need a liver transplant if Dr. Clifton's caseload quadrupled. How many hours would he be in surgery?
- Create a function table that shows the effect of the caseload on the amount of hours in surgery. Include at least 5 data points within the table.

Intervention

- Allow students to use manipulatives such as counters, beans, bears, etc. to represent the 15 patients.

TECHNOLOGY

- http://illuminations.nctm.org/LessonDetail.aspx?ID=U123 This four part lesson can be used for additional practice or to extend understanding of the concepts.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L341 This lesson reinforces fractional parts. It can be used to extend understanding of the concept.

Name ____ Date__________________________

Chance of Surgery

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If it typically takes Dr. Clifton 4 hours to complete a biliary atresia surgery, about how many hours did he perform this operation last year?

If Dr. Clifton's case load doubles in 2013, how many patients can he expect to have successful surgery and possibly not need a liver transplant? How many hours should he anticipate conducting the surgery in 2013?

CLOSING/SUMMARIZER

Kamron answers $\frac{4}{5}$ of her Physical Science questions correctly. There were 35 questions total. How many questions did she answer correctly?

Fractional Divisors

SUGGESTED TIME FOR THIS LESSON:

90-120 minutes
Exact timings will depend on the needs of yours class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

> **MFANSQ4**. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
> b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
> c. Interpret and solve contextual problems involving division of fractions by fractions. For example, how many 3/4-cup servings are in 2/3 of a cup of yogurt? (MGSE6.NS.1)
> d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)
> e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MGSE7.NS.3, MGSE7.EE.3, MGSE9-12.N.Q.3)

Common Misconceptions:
Students often misapply the invert-and-multiply procedure for dividing by a fraction because they are missing the conceptual understanding of the procedure.

- *not inverting either fraction; for example, a student may solve the problem* $\frac{2}{3} \div \frac{4}{5}$ *by multiplying the fractions without inverting* $\frac{4}{5}$ *(e.g., writing that* $\frac{2}{3} \div \frac{4}{5} = \frac{8}{15}$*)*
- *inverting the wrong fraction (e.g.,* $\frac{2}{3} \div \frac{4}{5} = \frac{3}{2} x \frac{4}{5}$*)*
- *inverting both fractions (*$\frac{2}{3} \div \frac{4}{5} = \frac{3}{2} x \frac{5}{4}$*)*

Such errors generally reflect a missing conceptual understanding of why the invert-and-multiply procedure produces the correct quotient. The invert-and-multiply procedure translates a multi-step calculation into a more efficient procedure.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.

3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson students should be able to:
- Model division of fractions.
- Explain how to divide fractions.

MATERIALS

- Small bag (1 per group) of chocolate candies ***(any bag of candy where students can receive 17 pieces each, or students can be provided with a zip-lock of 17 small paper circles instead of candy)***
- Base Ten Blocks (If base 10 blocks are not available, you can print your own representation. See the sample https://www.illustrativemathematics.org/content-standards/5/NBT/A/2/LESSONs/1620. ***Another option would be to have students create their own base 10 blocks.***
- Unit Cubes
- Rulers

ESSENTIAL QUESTIONS

- Why does the process of invert and multiply work when dividing fractions?
- When you divide one number by another number, do you always get a quotient smaller than your original number?
- When you divide a fraction by a fraction, what do the dividend, quotient and divisor represent?
- What kind of models can you use to show solutions to word problems involving fractions?

Grouping: Individual/Partner/Groups of 3-4

OPENING/ACTIVATOR:

What does it mean to divide?
Provide each group of 3-4 students with a small bag of candy ***(any bag of candy where students can receive 17 pieces each, or students can be provided with a zip-lock of 17 small paper circles instead of candy)).*** Have the students "divide" the candies evenly among the group. Ask each group what they did to complete this process or if they encountered any obstacles. If a group did encounter an obstacle, ask what it was and how they persevered through the obstacle.

WORK SESSION:

Teacher Notes
"Multiply" by the reciprocal is probably one of the most mysterious rules in mathematics. Van de Walle urges us to avoid this mystery by allowing students to come to their own conclusions about why we multiply by the reciprocal. (Van de Walle, John. Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades 3 – 5. Pearson: 2013.)This lesson allows students to explore partitive and measurement interpretations of fractions with fractional divisors. Keeping in mind that the partitive interpretation asks the question, "How much is one?" while measurement is equal subtraction and can have remainders.

Before the lesson, review the student work from the previous task with a focus on partitive and measurement interpretations of fractions with fractional divisors.

When students finish, get answers from the class for each problem. If more than one answer is offered, simply record them and offer no evaluation.

Have students explain their strategies for thinking about the problem either on the board, with a document camera, etc. You may need to ask questions about drawings or explanations to make sure everyone in the class follows the rationale. Encourage the class to comment or ask questions about the student's representation or thinking. Ask if others used a different representation or solved the problem in a different way. If so, have the students come forward to share their solutions If there are different answers, the class should evaluate the solution strategies and decide which answer is correct and why.

It is important to have students compare and contrast the two problems and the methods for solving them.

In what ways are the two problems similar?

In what ways are they different?

What does the quotient represent in each of the problems?

What does the divisor represent in each of the problems?

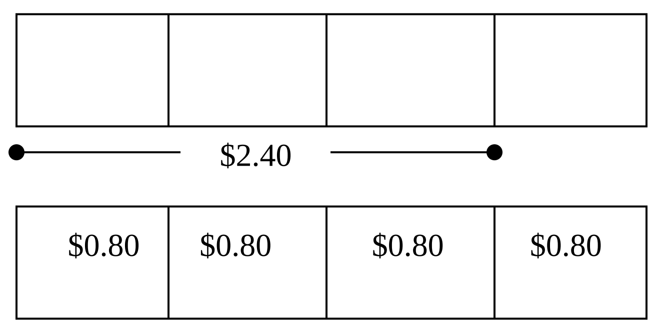
What does the dividend represent in each of the problems?

SOLUTIONS

Partitive Interpretation of Division with Fractional Divisors

Use a model (e.g., manipulative materials, pictures, number line) to find the answers to the problems below. Be sure to write a number sentence to illustrate each situation.

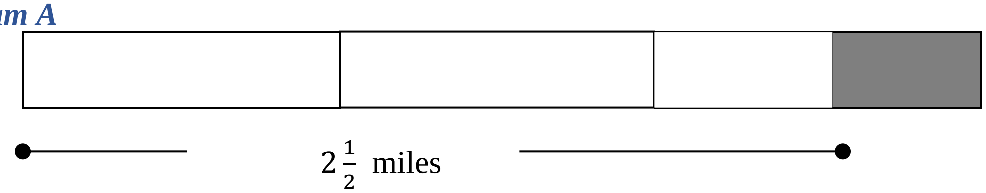
1. Michael's mom paid \$2.40 for a $\frac{3}{4}$ - pound box of cereal. How much is that per pound?



One-fourth of a pound is \$0.80(\$2.40 ÷3), so one pound will be 4 x \$0.80 = \$3.20 per pound.

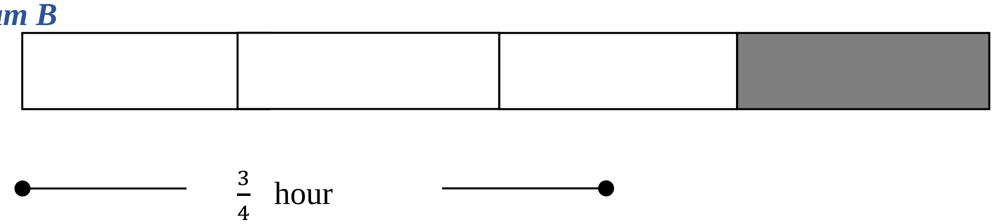
2. Melitta found out that if she walks really fast during her morning exercise, she can cover $2\frac{1}{2}$ miles in $\frac{3}{4}$ of an hour. How fast is she walking in miles per hour?

Diagram A

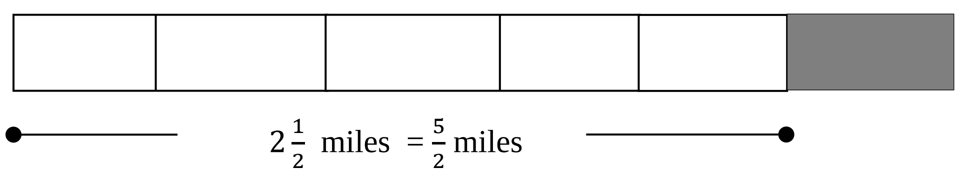


Melitta walks $2\frac{1}{2}$ miles in $\frac{3}{4}$ of an hour. In order to model this situation with tape diagrams, the student must understand that $2\frac{1}{2}$ is being divided into three quarters. Both diagrams are important because you have three fourths of an hour and need four fourths to determine miles per hour. You will have to find the distance walked in one quarter hour (one box) in order to determine the distance traveled in one whole hour.

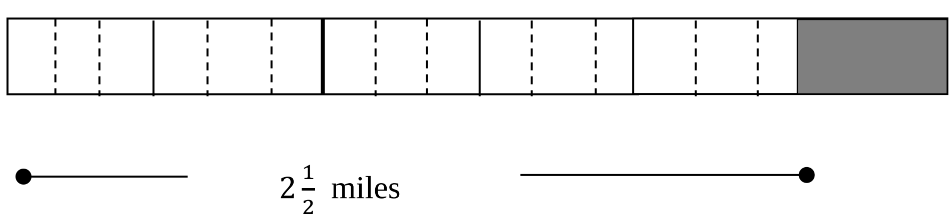
Diagram B



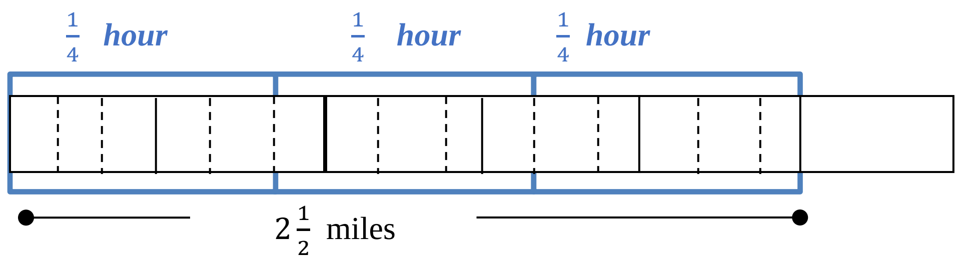
In Diagram A, each whole must be divided into 2 halves in order to have equal parts. Now there are 5 halves.



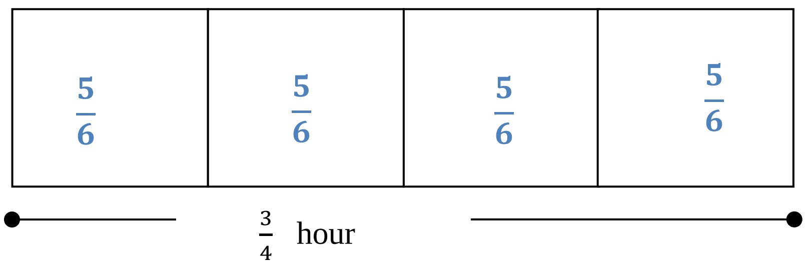
Each box of Diagram A now needs to be subdivided into three equal sections because there are five parts that cannot currently be divided into 3 equal parts ($\frac{3}{4}$ hour). Since you cannot divide 5 evenly into three boxes, divide each one-half piece into three parts. Now you have 15 even pieces.



Each piece now represents $\frac{1}{6}$ of an hour. NOW, divide the 15 one-sixth pieces into 3 groups. This represents 3quarters of an hour.



Each quarter hour has 5 pieces. Each piece is $\frac{1}{6}$ of an hour. Another way to say this is that each quarter hour she walks $\frac{5}{6}$ mile.



So, Melitta is walking $\frac{20}{6}$ or $3\frac{1}{3}$ miles per hour.

Measurement Interpretation of Division with Fractional Divisors

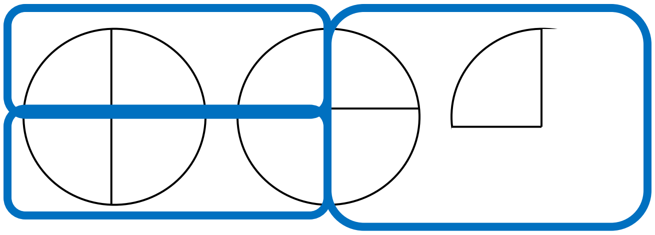
Use a model (e.g., manipulative materials, pictures, number line) to find the answers to the problems below. Be sure to write a number sentence to illustrate each situation.

3. It is your birthday and you are going to have a party. From the grocery store you get 6 pints of ice cream. If you serve $\frac{3}{4}$ of a pint of ice cream to each of your guests, how many guests can be served?

Typically students draw pictures of six items divided into fourths and count out how many servings of ¾ can be found. The difficulty is in seeing this as $6 \div \frac{3}{4}$, and that requires some guidance on the teacher's part. Try to compare the problem to one involving whole numbers (6 pints, 2 per guest).

4. Sam is a landscaper. He found that he had $2\frac{1}{4}$ gallons of liquid fertilizer concentrate. It takes $\frac{3}{4}$ gallon to make a tank of mixed fertilizer. How many tankfuls can he mix?

This question asks, "How many sets of three-fourths are in a set of 9 fourths?" Sam can mix 3 tankfuls.



Name_______________________

TASK: Fractional Divisors

Partitive Interpretation of Division with Fractional Divisors

Use a model (e.g., manipulative materials, pictures, number line) to find the answers to the problems below. Be sure to write a number sentence to illustrate each situation.

1. Michael's mom paid $2.40 for a $\frac{3}{4}$ - pound box of cereal. How much is that per pound?

2. Melitta found out that if she walks really fast during her morning exercise, she can cover $2\frac{1}{2}$ miles in $\frac{3}{4}$ of an hour. How fast is she walking in miles per hour?

Measurement Interpretation of Division with Fractional Divisor

Use a model (e.g., manipulative materials, pictures, number line) to find the answers to the problems below. Be sure to write a number sentence to illustrate each situation.

3. It is your birthday and you are going to have a party. From the grocery store you get 6 pints of ice cream. If you serve $\frac{3}{4}$ of a pint of ice cream to each of your guests, how many guests can be served?

4. Sam is a landscaper. He found that he had $2\frac{1}{4}$ gallons of liquid fertilizer concentrate. It takes $\frac{3}{4}$ gallon to make a tank of mixed fertilizer. How many tankfuls can he mix?

CLOSING:

Journal Entry: When you divide a fraction by a fraction what do the dividend, quotient and divisor represent?

Additional Practice: Dividing Fractions in Context

1. Suppose you have $2\frac{1}{2}$ apples. If a student serving consists of $\frac{3}{4}$ of an apple, how many student servings (including parts of a serving) can you make?

   $3\frac{1}{3}$ *student servings*

2. Suppose instead that you have $1\frac{1}{2}$ apples. If this is enough to make $\frac{3}{5}$ of an adult serving, how many apples (and parts of an apple) make up one adult serving?

   $2\frac{1}{2}$ *apples*

3. Emma is making posters by hand to advertise her school play, but her posters are not the same length as a standard sheet of paper (the width is the same, though). She has $3\frac{1}{2}$ sheets of paper left over, which she says is enough to make $2\frac{1}{3}$ posters. How many sheets of paper (and parts of a sheet) does each poster use?

   $1\frac{1}{2}$ *sheets of paper*

4. If Connor is also making posters, but his posters only use $\frac{2}{3}$ of a sheet of paper, how many of Connor's posters will those $3\frac{1}{2}$ sheets of paper make?

   $5\frac{1}{4}$ *posters*

5. Laura is tying ribbons in bows on boxes. She uses $2\frac{1}{4}$ feet of ribbon on each box. If she has $7\frac{1}{2}$ feet of ribbon left, how many bows (or parts of a bow) can she make?

   $3\frac{1}{3}$ *bows*

6. Audrey is also tying ribbons into bows. Audrey sees the same $7\frac{1}{2}$ feet of ribbon measured out and says, "Since my bows are bigger than Lura's, that's only enough for me to make $2\frac{1}{4}$ bows." How much ribbon does Audrey use on each bow?

   $3\frac{1}{3}$ *feet of ribbon*

7. Alex has been serving $\frac{2}{3}$ cup of lemonade to each student. If he has $1\frac{1}{2}$ cups of lemonade left, how many students can still get lemonade? How much of a serving will the last student get?

   2 students, $\frac{1}{4}$ *of a serving*

SE: Dividing Fractions in Context

1. Suppose you have 2 $\frac{1}{2}$ apples. If a student serving consists of $\frac{3}{4}$ of an apple, how many student servings (including parts of a serving) can you make?

2. Suppose instead that you have 1 $\frac{1}{2}$ apples. If this is enough to make $\frac{3}{5}$ of an adult serving, how many apples (and parts of an apple) make up one adult serving?

3. Emma is making posters by hand to advertise her school play, but her posters are not the same length as a standard sheet of paper (the width is the same, though). She has 3 $\frac{1}{2}$ sheets of paper left over, which she says is enough to make 2 $\frac{1}{3}$ posters. How many sheets of paper (and parts of a sheet) does each poster use?

4. If Connor is also making posters, but his posters only use $\frac{2}{3}$ of a sheet of paper, how many of Connor's posters will those 3 $\frac{1}{2}$ sheets of paper make?

5. Laura is tying ribbons in bows on boxes. She uses 2 $\frac{1}{4}$ feet of ribbon on each box. If she has 7 $\frac{1}{2}$ feet of ribbon left, how many bows (or parts of a bow) can she make?

6. Audrey is also tying ribbons into bows. Audrey sees the same 7 $\frac{1}{2}$ feet of ribbon measured out and says, "Since my bows are bigger than Lura's, that's only enough for me to make 2 $\frac{1}{4}$ bows." How much ribbon does Audrey use on each bow?

7. Alex has been serving $\frac{2}{3}$ cup of lemonade to each student. If he has 1 $\frac{1}{2}$ cups of lemonade left, how many students can still get lemonade? How much of a serving will the last student get?

Dividing Fractions with Models

SUGGESTED TIME FOR THIS LESSON:

60-90 minutes
Exact timings will depend on the needs of yours class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

> **MFANSQ4**. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
> b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
> c. Interpret and solve contextual problems involving division of fractions by fractions. For example, how many 3/4-cup servings are in 2/3 of a cup of yogurt? (MGSE6.NS.1)
> d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)
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Common Misconceptions:
Students often misapply the invert-and-multiply procedure for dividing by a fraction because they are missing the conceptual understanding of the procedure.

- *not inverting either fraction; for example, a student may solve the problem* $\frac{2}{3} \div \frac{4}{5}$ *by multiplying the fractions without inverting* $\frac{4}{5}$ *(e.g., writing that* $\frac{2}{3} \div \frac{4}{5} = \frac{8}{15}$*)*
- *inverting the wrong fraction (e.g.,* $\frac{2}{3} \div \frac{4}{5} = \frac{3}{2} x \frac{4}{5}$*)*
- *inverting both fractions (*$\frac{2}{3} \div \frac{4}{5} = \frac{3}{2} x \frac{5}{4}$*)*

Such errors generally reflect a missing conceptual understanding of why the invert-and-multiply procedure produces the correct quotient. The invert-and-multiply procedure translates a multi-step calculation into a more efficient procedure.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.

3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson students should be able to:
- Model division of fractions.
- Explain how to divide fractions.

MATERIALS

- Freezer Pops (paper – see template in task)
- Chocolate Bar Models (paper – see template in task)
- Color Tiles
- Grid Paper

ESSENTIAL QUESTIONS

- Why does the process of invert and multiply work when dividing fractions?
- When I divide one number by another number, do I always get a quotient smaller than my original number?
- When I divide a fraction by a fraction what do the dividend, quotient and divisor represent?
- What kind of models can I use to show solutions to word problems involving fractions?

Grouping: Individual/Partner

OPENING/ACTIVATOR:

TEACHER NOTES

In this lesson students represent division of fractions using manipulatives, such as freezer pops, candy bars, and models such as drawing squares. Students develop an algorithm from these examples and solve problems using fractions.

The concept of division of fractions has been greatly misunderstood. Developing an understanding of what happens when you divide by a fraction prior to development of the algorithm is essential in the thought process. This is accomplished by having the student visually see and understand what dividing by a fraction means with physical examples.

This task was adapted from
http://ims.ode.state.oh.us/ODE/IMS/Lessons/Content/CMA_LP_S01_BH_L06_I08_01.pdf

Activity

Ask five volunteers to come to the front of the classroom. Give each student a freezer pop (use pops with two sticks) and ask if they have ever eaten one. Then ask if they had eaten the entire freezer pop or split it in half. Because of the two sticks, one student may answer that he/she splits the freezer pop in half. Ask students to split the pops in half and have a student count the total number of halves. Use *Frozen Juice Pops*, as a visual representation for the situation.

Freezer pops could be made out of construction paper and craft sticks to illustrate this model. See template.

Ask students if they notice anything about the size of the 10 pieces compared to the original 5 freezer pops. Student should note that they are smaller. Elicit that they are half the size of the original freezer pops.

Ask a volunteer for an equation to represent the 5 freezer pops divided in half.

Answer 5÷ ½ = 10

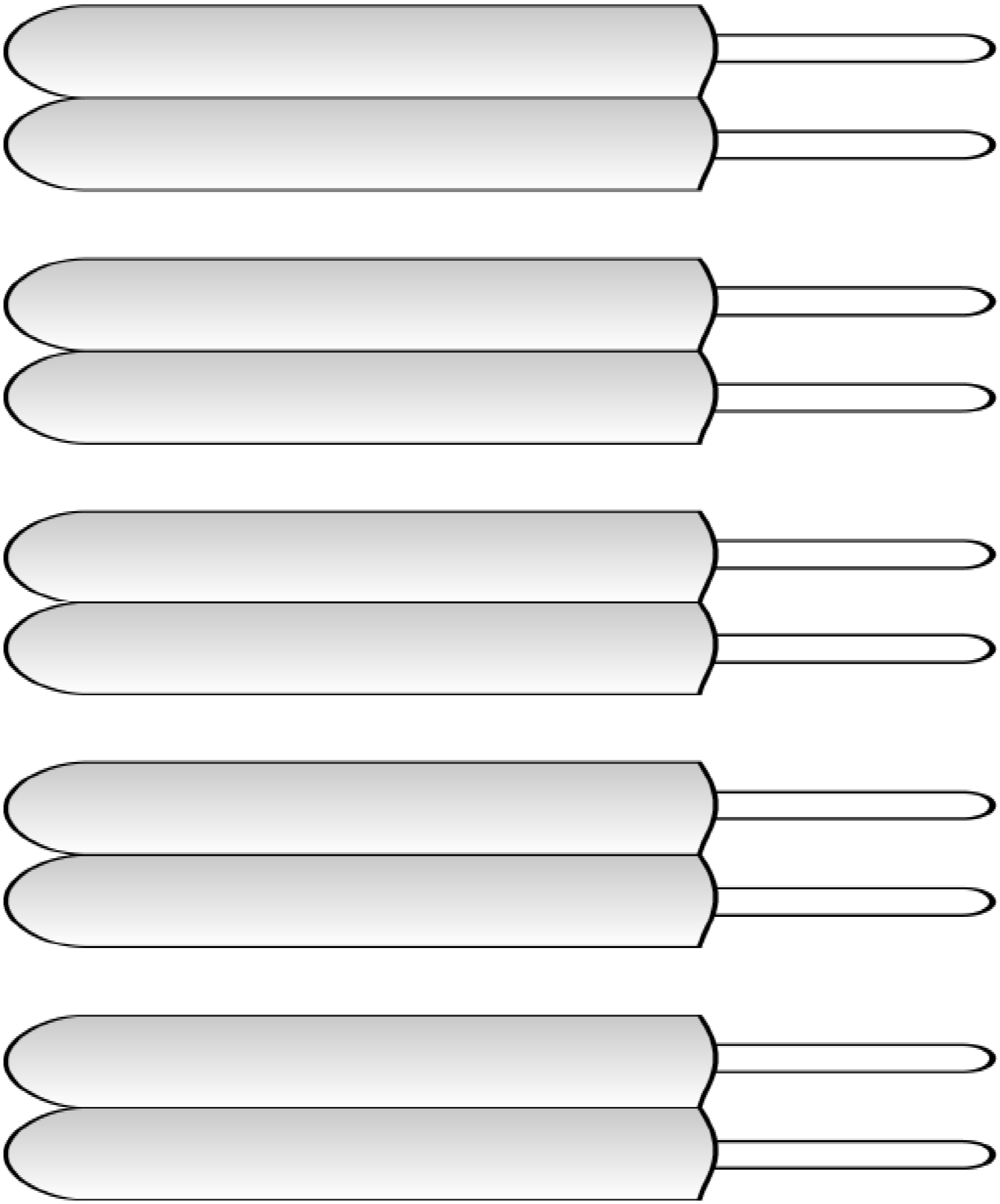
Write the equation on the board for the class to see. If students need help determining this equation, ask "How many half-size freezer pops were contained in the original 5 whole freezer pops?" Then, remind the class that when we ask how many of something there are in something else, that is a division situation (e.g., if we want to know how many 3's are in 12, we divide 12 by 3).

Distribute a variety of chocolate bars that are made with divided sections or use *Chocolate Bar Models*. Ask students to describe how the bars could be divided and give equations to represent this division. $5 \div \frac{1}{4} = 20$, $3 \div \frac{1}{12} = 36$

Again write the equations on the board. Discuss with the students that the size of the pieces desired is not the same as the original pieces. Note: Some students may come up with an equation such as $4 \div 16 = \frac{1}{4}$, which is also a correct way to model this situation. Encourage these students to find a second equation that also models the situation (How many little pieces are in the original big piece?).

Instructional Tip

Make sure students are able to recognize that even though they end with a greater number of pieces after completing the division, the size of the portion is less.

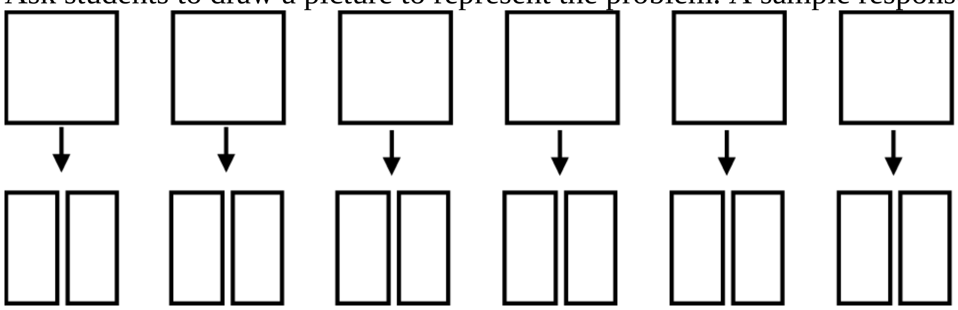


<u>WORK SESSION:</u>

<u>Model Example #1:</u>
Pose the following situation to the class.

I have six squares that I want to divide by one half. How many pieces would I have?

Ask students to draw a picture to represent the problem. A sample response should be



Step 1

Step 2

Ask the following guiding questions;

How many squares did I have? ***(6)*** What size did I want? ***(*** $\frac{1}{2}$ ***)***

How many pieces of that size do we have? ***(12)***

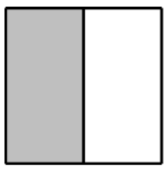
Ask students how this situation would be represented as an equation. Guide the discussion to obtain the equation $6 \div \frac{1}{2} = 12$.

<u>Model Example #2:</u>

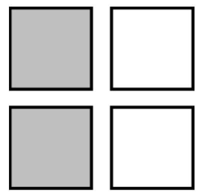
Place students in pairs and pose another situation. Ask them to model it and write an equation that represents the situation.

I have $\frac{1}{2}$ of a square and I want to divide it by $\frac{1}{4}$. How many pieces would I have?

Monitor the partners working on the task and ask the same type of guiding questions when students appear to be struggling with how to represent the situation. The solution should resemble the following example:



This represents having one half of the square.



This represents dividing the square into pieces whose size is one-fourth. The students then need to answer the question of how many pieces of size one-fourth do I have?

Ask the partners to write an equation for the problem. $\frac{1}{2} \div \frac{1}{4} = 2.$

Ask for a volunteer to provide the equation. Ask the student why he/she placed the numbers in that order.

Teacher Notes: Write the equations on the board after each situation, noting the relationships among the numbers in the equations. Have students look for any patterns or relationships they note in the equations.

Have partners make conjectures or descriptions as to what they believe is happening when they divide a number by a fraction. Ask partners to share their conjectures with the class. Record the conjectures and descriptions on the board or chart paper.

Possible conjectures include:

When you divide by a fraction you get a whole number.
When you divide by a fraction you get a larger number.
When you divide by a fraction you multiply the whole number by the denominator.

Instructional Tip

Use the conjectures to adjust the instructions as needed, determining whether students are ready to work with more complex fractions or dividing a fraction by a whole number. Students can test their conjectures and refine their descriptions. The goal is to enable students to determine the algorithm for dividing by a fraction.

<u>Model Example #3:</u>

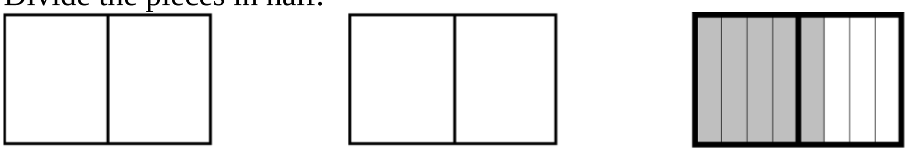
Present the following situation:

Cierra has $2\frac{5}{8}$ meters of yarn that she wants to cut into $\frac{1}{2}$ -meter lengths. How many $\frac{1}{2}$ -meter lengths of yarn will Cierra have?

Instruct students to draw a model to solve the problem.

A sample model may be

Divide the pieces in half.

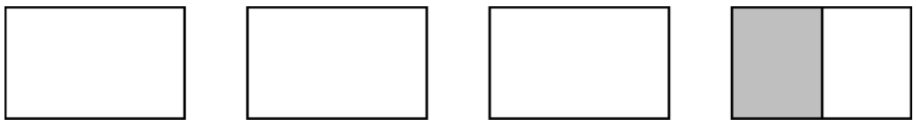


There are 5 halves in $2\frac{5}{8}$. There is also $\frac{1}{8}$ left, which is $\frac{1}{4}$ of the remaining half of the whole. Therefore, there are $5\frac{1}{4}$ halves in $2\frac{5}{8}$. ***Cierra will have 5 complete $\frac{1}{2}$ meter lengths of yarn.***

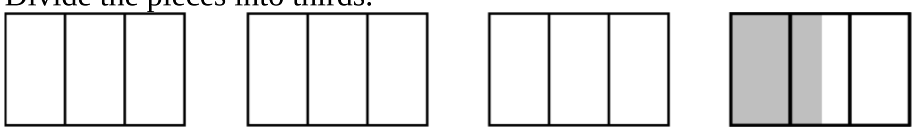
Select a student to model the problem on the board. Note: there will be a piece left over after finding 5 halves. If students are not sure what this represents, ask, "What is the relationship of $\frac{1}{8}$ to the remaining half?" ***($\frac{1}{8}$ is $\frac{1}{4}$ of the remaining half.)***

<u>Model Example #4:</u>
Present a similar situation, such as:

Jonathan has $3\frac{1}{2}$ cups of chocolate chips to make cookies. The recipe uses $\frac{1}{3}$ cup of chips in each batch. How many batches of cookies can Jonathan make?



Divide the pieces into thirds.



Jonathan can make $10\frac{1}{2}$ ***batches of cookies.***

Instructional Tip

Students should see that once they divide the whole part(s) into the desired fractional part, the remaining part, which is a fractional part of another whole piece, needs to be divided also. Students may see a fractional piece left over and should determine the relationship of this piece to the original piece.

<u>Model Example #5:</u>
Present the following problem:

I have one fourth of a square and I want to divide it by one half. How many pieces would I have?

Have students work as partners to solve the problem. Remind students to draw a picture to represent the problem.

Ask guiding questions used with other situations such as, "What are you looking for (How many sets of $\frac{1}{2}$ are in $\frac{1}{4}$)?"

A sample response could be $\frac{1}{2}$; $\frac{1}{4} \div \frac{1}{2} = \frac{1}{2}$ ***I have one half of the desired size.***

Have students compare this problem with the previous problem noting any differences and similarities. Discuss the meaning of the answer in each problem.

Have the partners test and refine the conjectures and descriptions by completing *Dividing By a Fraction.* Observe the partners working on the situations and provide intervention, reminding them of the guiding questions that were used with the other situations.

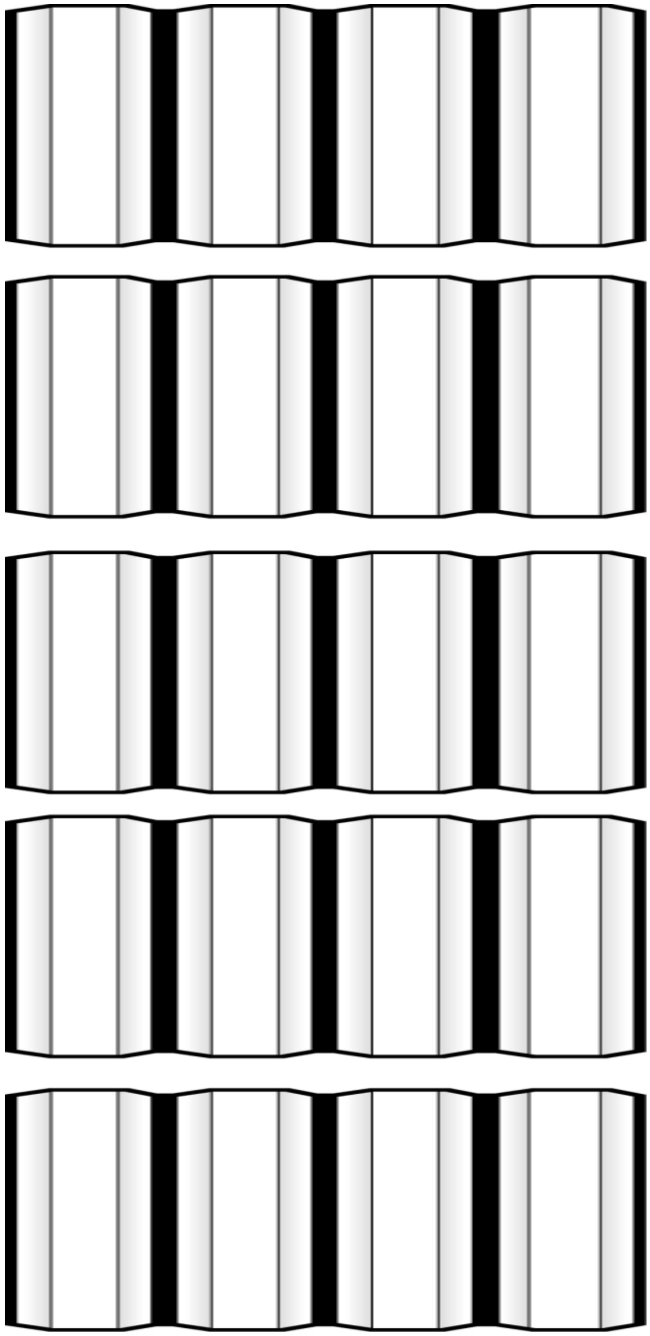
Put partners together to make groups of four. Have each group check the solutions obtained, then review answers as a class, asking each group to provide the solutions to the situations.
Use questioning to modify incorrect models.

Ask groups to review the conjectures and descriptions based on the new information. Lead students through the revision process by looking at each conjecture or description and determining if any of the situations contradicted the statement or if clarification is needed.

Have students determine if the statements refer to the process or the results of the problem.

Partner/Individual Work
Give each student a copy of *Models for Dividing Fractions*. They should complete this individually or with a partner.

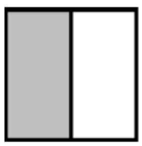
Chocolate Bar Models

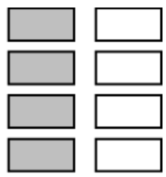


TASK PART 1: Dividing by a Fraction

1. I have one-half of a square and I want to divide it by one-eighth. How many pieces would I have?

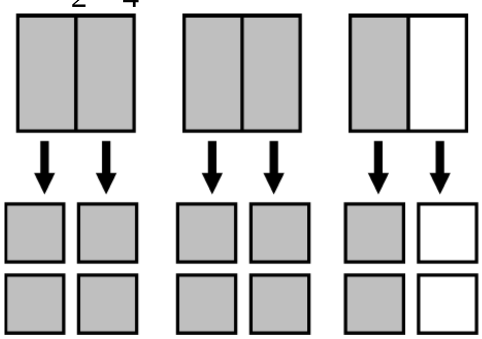
   4 ; $\frac{1}{2} \div \frac{1}{8} = 4$ ***I have 4 pieces that are one eighth of the square.***

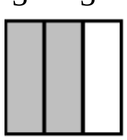
2. I have two and one-half squares and I want to divide them by one-fourth. How many pieces would I have?

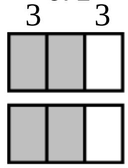
   10 ; $\frac{5}{2} \div \frac{1}{4} = 10$ ***I have 10 pieces that are one fourth of a square.***

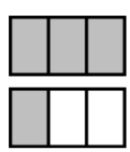
3. I have two- thirds of a square and I want to divide it by one-half. How many pieces would I have?

   $\frac{4}{3} or 1\frac{1}{3}$; $\frac{2}{3} \div \frac{1}{2} = \frac{4}{3} or 1\frac{1}{3}$ ***I have four thirds of the size one half of a square.***

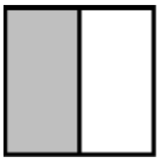
   

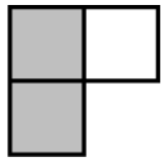
   OR

4. I have one-half of a square and I want to divide it by three-fourths. How many pieces would I have?

$\frac{2}{3}$***;*** $\frac{1}{2} \div \frac{3}{4} = \frac{2}{3}$ ***I have two thirds of the size three fourths of a square.***

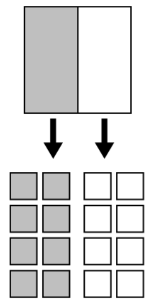




Name____________________

TASK PART 2: Models for Dividing Fractions

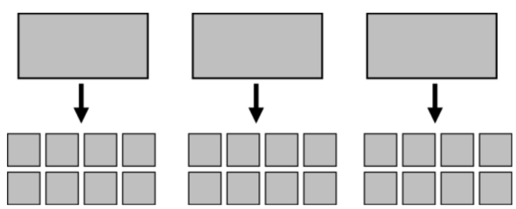
1. I have a one-half gallon container of ice cream and want to divide it into one-cup servings to share with the students in my class. A cup is one sixteenth of a gallon. How many serving dishes would I need?



Model the problem situation.

Write an equation and show how to solve the problem $\frac{1}{2} \div \frac{1}{16} = 8$

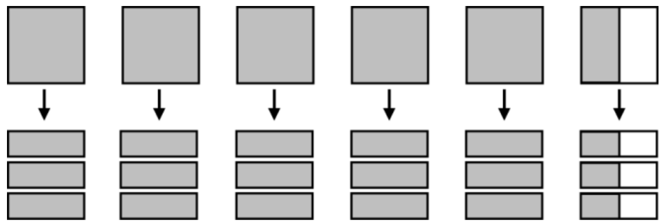
2. I also have three large chocolate candy bars that are perforated into eight sections each. If I divide the bars into these sections how many sections will I have altogether?



Model the problem situation.

Write an equation and show how to solve the problem $3 \div \frac{1}{8} = 24$

3. Becca works for the Humane Society and had to buy food for the dogs. She bought $5\frac{1}{2}$ pounds of dog food. She feeds each dog about one-third of a pound. How many dogs can she feed?



Model the problem situation.

Write an equation and show how to solve the problem. ***Answer:*** $5\frac{1}{2} \div \frac{1}{3} = \frac{33}{2} = 16\frac{1}{2}$

Name______________________________

TASK PART 1: Dividing by a Fraction

Directions: Model each situation using squares; write the equation for each.

1. I have one-half of a square and I want to divide it by one-eighth. How many pieces would I have?

2. I have two and one half squares and I want to divide them by one-fourth. How many pieces would I have?

3. I have two-thirds of a square and I want to divide it by one-half. How many pieces would I have?

4. I have one-half of a square and I want to divide it by three-fourths. How many pieces would I have?

TASK PART 2: Models for Dividing Fractions

Directions: Read the situation, draw a picture to represent the situation and then write an equation to represent the situation.

1. I have a one-half gallon container of ice cream and want to divide it into one-cup servings to share with the students in my class. A cup is one-sixteenth of a gallon. How many serving dishes would I need?

   Model the problem situation.

   Write an equation and show how to solve the problem.

2. I also have three large chocolate candy bars that are perforated into eight sections each. If I divide the bars into these sections how many sections will I have altogether?

   Model the problem situation.

   Write an equation and show how to solve the problem

3. Becca works for the Humane Society and had to buy food for the dogs. She bought $5\frac{1}{2}$ pounds of dog food. She feeds each dog about one-third of a pound. How many dogs can she feed?

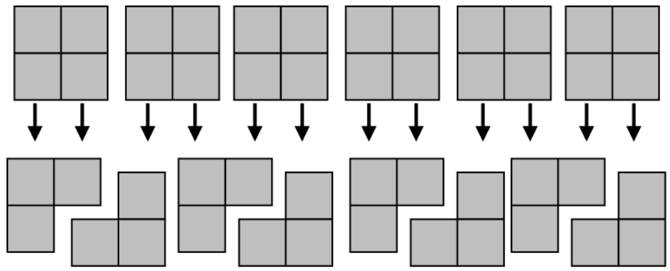
   Model the problem situation.

   Write an equation and show how to solve the problem.

CLOSING:

Ticket Out the Door:

Julie goes to the park across the street from her house several times a day and jogs a total of six miles every day. She jogs three-fourths of a mile at a time. How many times each day does she go to the park to run?



Model the problem situation.

Write an equation and show how to solve the problem. ***Solution:*** $6 \div \frac{3}{4} = 8$

Student Edition: TICKET OUT THE DOOR

Julie goes to the park across the street from her house several times a day and jogs a total of six miles every day. She jogs three-fourths of a mile at a time. How many times each day does she go to the park to run?

Model the problem situation.

Write an equation and show how to solve the problem.

Quick Check II

STANDARDS FOR MATHEMATICAL CONTENT

MFANSQ1. Students will analyze number relationships.
b. Understand a fraction a/b as a multiple of 1/b. (MGSE4.NF.4)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
c. Interpret and solve contextual problems involving division of fractions by fractions. For example, how many 3/4-cup servings are in 2/3 of a cup of yogurt? (MGSE6.NS.1)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)
e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MGSE7.NS.3, MGSE7.EE.3, MGSE9-12.N.Q.3)

Quick Check II - Formative Assessment

1. A recipe calls for $1\frac{1}{3}$ cups of flour to make cookies. If you only want to make half of the recipe, which of the following expressions would help you figure out how much flour to use?

   a) $1\frac{1}{3} \div \frac{1}{2}$ b) $1\frac{1}{3} \times \frac{1}{2}$ c) $1\frac{1}{3} \times 2$ d) $\frac{1}{2} \div 1\frac{1}{3}$

   The answer is (b).

2. Which of the following situations could be modeled by the expression $6 \div \frac{1}{4}$?

   a) You have one-fourth of a pizza and want to split it with six friends
   b) You want to know how much are six quarters is worth
   c) You share six pizzas with your friends and they each get one-fourth of a pizza
   d) You have six dollars and a quarter

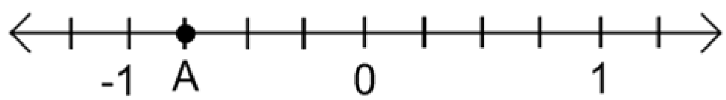
   The answwer is (c).

3. Which of the following shows the fractions in order from **least to greatest**?

   a) $\frac{1}{2}, \frac{3}{4}, \frac{1}{8}, \frac{3}{8}$ b) $\frac{1}{8}, \frac{3}{8}, \frac{1}{2}, \frac{3}{4}$ c) $\frac{1}{8}, \frac{3}{4}, \frac{1}{2}, \frac{3}{8}$ d) $\frac{3}{4}, \frac{1}{2}, \frac{3}{8}, \frac{1}{8}$

   The answer is (b).

4. What is the value of point A on the number line?



   a) -2 b) $-1\frac{1}{4}$ c) $-\frac{3}{4}$ d) $-\frac{1}{3}$

   The answer is (c).

5. Your mom ordered three cookie cakes for a party. You want each person at the party to get one-fifth (1/5) of a cookie cake. How many people can be at the party? **Write an expression that would help you solve this problem.**

3 divided by 1/5 ($3 \div \frac{1}{5}$)

Quick Check II - Formative Assessment

For problems 6-9 Simplify the expression completely (this includes changing an improper fraction to a mixed numeral when possible). **You must show all of your work to receive full credit**. Use scratch paper provided if needed. Remember, you get points for your work as well as your answer.

The work for these is shown below. Give credit for all work shown.

6. $3\frac{3}{4}\times\frac{2}{3}= \quad \frac{15}{4}\times\frac{2}{3}= \quad \frac{30}{12}= \quad \frac{5}{2}= \quad 2\frac{1}{2}$

7. $3\frac{1}{3}\times1\frac{3}{5}= \quad \frac{10}{3}\times\frac{8}{5}= \quad \frac{80}{15}= \quad \frac{16}{3}= \quad 5\frac{1}{3}$

8. $\frac{5}{6}\div\frac{4}{3}= \quad \frac{5}{6}\times\frac{3}{4}= \quad \frac{15}{24}= \quad \frac{5}{8}$

9. $7\frac{1}{2}\div1\frac{1}{4}= \quad \frac{15}{2}\div\frac{5}{4}= \quad \frac{15}{2}\times\frac{4}{5}= \quad \frac{60}{10}= \quad 6$

<u>Quick Check II - Formative Assessment</u>

1. A recipe calls for $1\frac{1}{3}$ cups of flour to make cookies. If you only want to make half of the recipe, which of the following expressions would help you figure out how much flour to use?

a) $1\frac{1}{3} \div \frac{1}{2}$ b) $1\frac{1}{3} \times \frac{1}{2}$ c) $1\frac{1}{3} \times 2$ d) $\frac{1}{2} \div 1\frac{1}{3}$

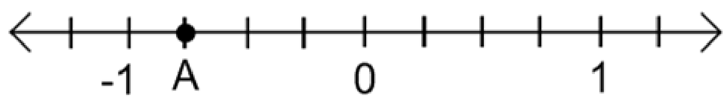
2. Which of the following situations could be modeled by the expression $6 \div \frac{1}{4}$?

a) You have one-fourth of a pizza and want to split it with six friends
b) You want to know how much are six quarters is worth
c) You share six pizzas with your friends and they each get one-fourth of a pizza
d) You have six dollars and a quarter

3. Which of the following shows the fractions in order from **least to greatest**?

a) $\frac{1}{2}, \frac{3}{4}, \frac{1}{8}, \frac{3}{8}$ b) $\frac{1}{8}, \frac{3}{8}, \frac{1}{2}, \frac{3}{4}$ c) $\frac{1}{8}, \frac{3}{4}, \frac{1}{2}, \frac{3}{8}$ d) $\frac{3}{4}, \frac{1}{2}, \frac{3}{8}, \frac{1}{8}$

4. What is the value of point A on the number line?



a) -2 b) $-1\frac{1}{4}$ c) $-\frac{3}{4}$ d) $-\frac{1}{3}$

5. Your mom ordered three cookie cakes for a party. You want each person at the party to get one-fifth (1/5) of a cookie cake. How many people can be at the party? **Write an expression that would help you solve this problem.**

Quick Check II - Formative Assessment

For problems 6-9 **you must show all of your work to receive full credit**. Use scratch paper provided if needed. Remember, you get points for your work as well as your answer.

Answer each of the following problems. Remember to use the rules for operations. SIMPLIFY completely for extra credit (this includes changing an improper fraction to a mixed numeral when possible).

6. $3\frac{3}{4} \times \frac{2}{3} =$

7. $3\frac{1}{3} \times 1\frac{3}{5} =$

8. $\frac{5}{6} \div \frac{4}{3} =$

9. $7\frac{1}{2} \div 1\frac{1}{4} =$

Patterns-R-Us

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ1. Students will analyze number relationships.
c. Explain patterns in the placement of decimal points when multiplying or dividing by powers of ten. (MGSE5.NBT.2)
d. Compare fractions and decimals to the thousandths place. For fractions, use strategies other than cross multiplication. For example, locating the fractions on a number line or using benchmark fractions to reason about relative size. For decimals, use place value. (MGSE4.NF.2;MGSE5.NBT.3,4)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1, 2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1, 2)

Common Misconception

- *Students may not think about the place value of the digits when multiplying.*
- *Multiplication can increase or decrease a number. From previous work with computing whole numbers, students understand that the product of multiplication is greater than the factors. However, multiplication can have a reducing effect when multiplying a positive number by a decimal less than one or multiplying two decimal numbers together. We need to put the term multiplying into a context with which we can identify and which will then make the situation meaningful. Also, using the terms times and groups of interchangeably can assist with the contextual understanding.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:
Identify, describe, and explain any patterns noticed when multiplying or dividing numbers by 1000, 100, 10, 0.1, and 0.01.

MATERIALS

- "Patterns-R-Us" recording sheet
- Calculators

ESSENTIAL QUESTION

- How does multiplying or dividing by a power of ten affect the product?

Grouping: Small Group

BACKGROUND KNOWLEDGE

Students should develop an understanding that when a number is multiplied by a number less than 1, the product is less than the original number, and when a number is divided by a decimal number less than 1, the quotient will be greater than the dividend. This is important, yet often difficult for students to understand because it is counterintuitive based on students' previous experiences with multiplication and division.

Students have learned how to multiply and divide with decimals, but they could benefit from solving the problems in part one without a calculator. Students **may use a calculator** for the rest of the lesson. Students may also benefit from a discussion reviewing the powers of ten and discussing what patterns appear as they increase and decrease the power (exponent) each time.

OPENER/ACTIVATOR:

An introduction for this lesson could be a round of "What's My Rule?" The rule could be x1000 which is 10^3, x100 which is 10^2, x10 which is 10^1, x0.1 which is 10^{-1}, or x0.01 which is 10^{-2}. Also, the rule could be ÷1000 which is 10^3, ÷100 which is 10^2, ÷10 which is 10^1, ÷0.1 which is 10^{-1}, or ÷0.01 which is 10^{-2}.

For example:

Input	Output
5	5000
6	6000
23	23,000

Multiply by 1,000

Input	Output
700	70
80	8
10	1

Divide by 10

WORK SESSION:

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments: This lesson is designed to serve as a discovery opportunity for the students. Students should notice that a pattern is created when a number is multiplied or divided by a power of 10. While students may notice patterns in each individual part of the lesson, encourage them to look for a pattern when considering the overall lesson. Students should be able to explain and defend their solutions through multiple representations. For example, students should try several numbers for each part to verify that each number follows the same pattern. This activity lends itself to working in pairs for reinforcement.

LESSON

Students will follow the directions below from the "Patterns-R-Us" Recording Sheet.

A statistician is interested in finding out what pattern is created, if any, under certain situations. Your mission is to help come up with concrete rules for certain mathematical situations. Record all of your work and explain your thinking in order to defend your answer. Good luck!

Part 1

1. Start with any whole number, for example 18.
2. Multiply that number by 10^3, 10^2, 10, 10^{-1}, and 10^{-2}.
   18,000; 1,800; 180; 1.8; 0.18
3. What is happening?
   Decimal moves; If multiplied by a number larger than 1 the product is larger. If multiplied by a factor smaller than 1, the product is smaller.
4. Is there a pattern?
   Yes – At this point, students may just say move it the number of zeros. If zeros are the zeros are on the right then move it to the right that number of zeros. Same idea exists for zero to the left then move to the left.
5. What do you think would happen if you multiplied your number by 10^6? 10^{-5}?
   18,000,000; 0.00018

Part 2

1. Pick any decimal as your number, for example 12.3.
2. Multiply that number by 10^3, 10^2, 10, 10^{-1}, and 10^{-2}.
   12,300; 1,230; 123; 1.23; 0.123
3. What is happening?
   Moving the decimal
4. Is there a pattern?
   Yes – The idea of the number of zeros should be evolving. Hopefully, students view the move in terms of PLACE VALUE.
5. What do you think would happen if you multiplied your number by 10^6? 10^{-5}?
   12,300,000; 0.000123

Part 3

1. Start with any whole number, for example 18.
2. Divide that number by 10^3, 10^2, 10, 10^{-1}, and 10^{-2}.
   0.018; 0.18; 1.8; 180; 1,800
3. What is happening?
   Moving the decimal
4. Is there a pattern?
   Yes – you are dividing now. So look at multiplying by the reciprocal
5. What do you think would happen if you divided your number by 10^6? 10^{-5}?

Part 4

1. Pick any decimal as your number, for example 10.8.

2. Predict what will happen when you divide that number by 10^3, 10^2, 10, 10^{-1}, and 10^{-2}.
   0.0108; 0.108; 1.08; 108; 1,080

3. After working out the problem, is your prediction correct? Why or why not?
   Answers will vary
4. Is there a similar pattern that you recognize?
   Yes – Inverse – Same principle as multiplication just moving the opposite direction

FORMATIVE ASSESSMENT QUESTIONS

- How do you know your answer is correct?
- What would happen if you started with a different number?
- What patterns are you noticing?
- Can you predict what would come next in the pattern?

DIFFERENTIATION

Extension
- Have students multiply a number by 0.1. Now ask them to divide that same number by 10. What happened? Repeat this with several numbers. Can a conjecture be made based on the results? Have students write their conjecture. Now, share their conjecture with a partner. Are the two conjectures the same? (You may also use 0.01 and 100 as another example.)

Intervention
- Pair students who may need additional time so that they will have time needed to process this lesson.
- For extra help, please open the hyperlink Intervention Table.

Name______________________________________ Date______________________________

Patterns-R-Us

A statistician is interested in finding out what pattern is created, if any, under certain situations. Your mission is to help come up with concrete rules for certain mathematical situations. Record all of your work and explain your thinking in order to defend your answer. Good luck!

PART ONE

1. Start with any whole number, for example 18.
2. Multiply that number by 10^3, 10^2, 10, 10^{-1}, and 10^{-2}.
3. What is happening?
4. Is there a pattern?
5. What do you think would happen if you multiplied your number by 10^6? 10^{-5}?

PART TWO

1. Pick any decimal as your number, for example 12.3.
2. Multiply that number by 10^3, 10^2, 10, 10^{-1}, and 10^{-2}.
3. What is happening?
4. Is there a pattern?
5. What do you think would happen if you multiplied your number by 10^6? 10^{-5}?

PART THREE

1. Start with any whole number, for example 18.
2. Divide that number by 10^3, 10^2, 10, 10^{-1}, and 10^{-2}.
3. What is happening?
4. Is there a pattern?
5. What do you think would happen if you divided your number by 10^6? 10^{-5}?

PART FOUR

1. Pick any decimal as your number, for example 10.8.
2. Predict what will happen when you divide that number by 10^3, 10^2, 10, 10^{-1}, and 10^{-2}.
3. After working out the problem, is your prediction correct? Why or why not?
4. Is there a similar pattern that you recognize?

CLOSING/SUMMARIZER

Journal Entry: Have students summarize their findings by writing a note to their partner. Have them exchange journals and edit each other's work.

Are These Equivalent?

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ1. Students will analyze number relationships.
d. Compare fractions and decimals to the thousandths place. For fractions, use strategies other than cross multiplication. For example, locating the fractions on a number line or using benchmark fractions to reason about relative size. For decimals, use place value. (MGSE4.NF.2;MGSE5.NBT.3,4)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.

5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to determine equivalency of fractions and decimals.

MATERIALS

- Grid Paper (optional)

ESSENTIAL QUESTIONS

- How can you compare decimals and fractions?

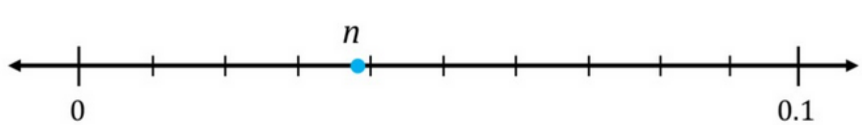
Grouping: Individual/Partner

OPENER/ACTIVATOR

TEACHER NOTE: Students will need to review rounding decimals.

Use Illustrative Math Activity: Rounding to Tenths and Hundredths at
https://www.illustrativemathematics.org/content-standards/5/NBT/A/4/tasks/1804

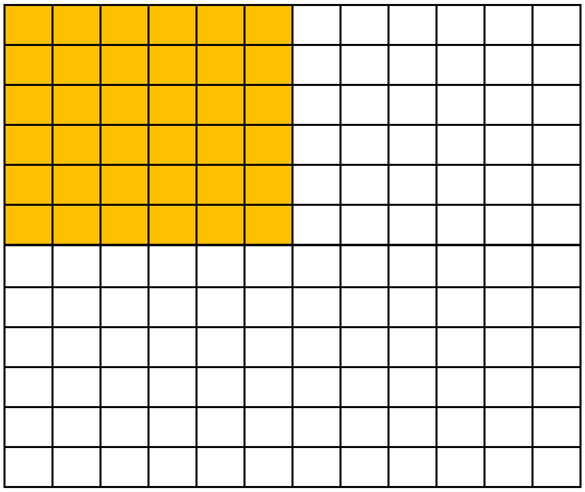
A number *n* is shown on the number line.



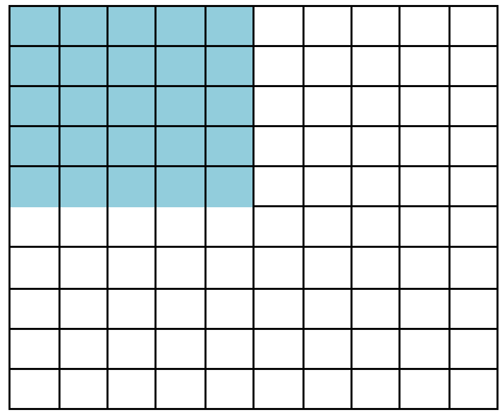
1. The tick marks are evenly spaced. Label them.
2. What is *n* rounded to the nearest hundredth?
3. What is *n* rounded to the nearest tenth?

WORK SESSION

Given Melissa's grid below, what fraction of the 12 x 12 grid did she shade? How do you know? How does Melissa's Grid compare to Joey's Grid? Why did they need to shade different numbers of grid squares?



Melissa's Grid



Joey's Grid

Melissa shaded 36 out of 144 squares which is ¼ of the grid.
We know this because ¼ of 144 is 36. Students may say they can make 4 equal groups of 36 out of the 144.

Joey shaded 25 out of his 100 block grid. This is also ¼ of his grid. So while they did not shade the same number of squares in their respective grids, they did shade the same fractional amount.

¼ of 144 is 36 while ¼ of 100 is 25. ¼ depends on the total amount in the whole.

Teacher Notes: Encourage the students to draw models. (For example – grid paper, decimal place value chart, base ten blocks) Once the models have been drawn have the students place them in order from least to greatest on a number line.

Jeremiah, Tatum, and Dustin had a disagreement. Jeremiah said that $4\frac{7}{100}$ is larger than .47 and 4.7. Tatum disagrees with him and claims that .47 is the largest number. Dustin promises they are both wrong and neither one knows what they are talking about because 4.7 is the largest number. Who is correct? Explain.

Dustin is correct in that 4.7 is larger than $4\frac{7}{100}$ and 0.47. First of all, 0.47 isn't even in the running because it is only a part of a whole unit while both 4.7 and $4\frac{7}{100}$ have 4 whole units plus part of another. If you compare the 0.7 and $\frac{7}{100}$, you will find that 0.7 is larger because 0.7 is equivalent to 0.70. There are 70 squares shaded in a 100 square grid while 7/100 has only 7 squares shaded in a 100 square grid. After looking at the models, it is easy to conclude that 4.7 is the larger number.

Teacher Notes: Encourage the students to draw models. (For example – grid paper, decimal place value chart, base ten blocks) Discuss the use of benchmark fractions – common fractions between 0 and 1 such as halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths, and hundredths.

There are two cakes on the counter that are the same size. The first cake has $\frac{1}{2}$ of it left. The second cake has $\frac{5}{12}$ left. Which cake has more left?

Again you could draw model or look at benchmark fractions as mentioned above.

In the first cake, ½ is left. The second cake has $\frac{5}{12}$ left. $\frac{5}{12}$ is less than the benchmark fraction of ½ ; therefore, the first cake has more left.

Student Edition:

QUESTION #1
Jeremiah, Tatum, and Dustin had a disagreement. Jeremiah said that $4\frac{7}{100}$ is larger than .47 and 4.7. Tatum disagrees with him and claims that .47 is the largest number. Dustin promises they are both wrong and neither one knows what they are talking about because 4.7 is the largest number. Who is correct? Explain.

QUESTION #2
There are two cakes on the counter that are the same size. The first cake has $\frac{1}{2}$ of it left. The second cake has $\frac{5}{12}$ left. Which cake has more left?

CLOSING/SUMMARIZER

This lesson is adapted from https://www.illustrativemathematics.org/content-standards/NBT/5/A/3/LESSONs/1813

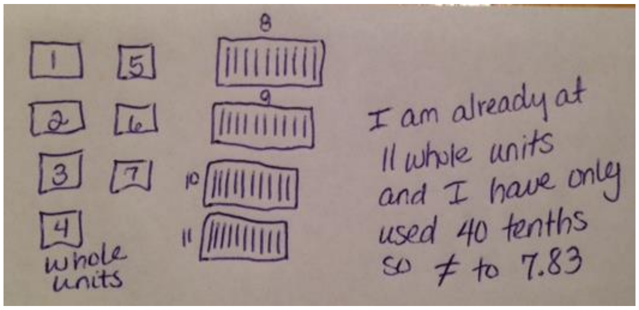
Have the students draw models to look for equivalency. The understanding after modeling all of these examples is POWERFUL!!!!

Answer: b, e, f

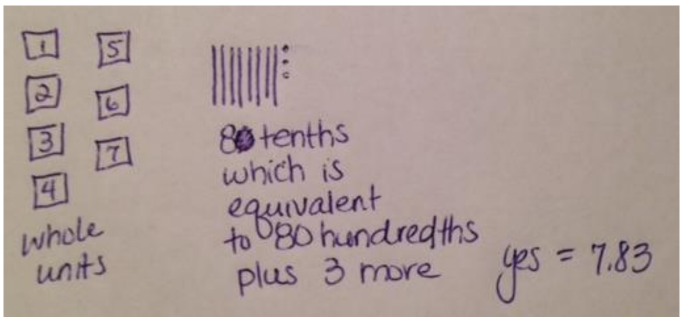
For extra help, please open the hyperlink Intervention Table.

Mr. Mears is thinking of the number 7.83 in his head. Decide whether each of these has the same value as 7.83 and discuss your reasoning.

a) Seven and eighty-two tenths



b) $7 + 0.8 + 0.03$



c) $9\frac{13}{25}$

c) calls for 9 whole units so it is already
to big

d) 783 tenths

783 tenths

10 tenths
is 1 whole

there are 78 sets of ten in 783 with
3 leftover.
78 sets of ten means 78 whole units
$\therefore \neq 7.83$

e) 783 hundredths

783 hundredths

100 hundredths
is 1 whole

There are 7 whole units in
783 hundredth w/ 83 leftover
$\therefore$ 7.83 = 783 hundredths

f) $7\frac{83}{100}$

$7\frac{83}{100}$ is just the fraction equivalent
to 7.83 so yes they are =

Student Edition:

Mr. Mears is thinking of the number 7.83 in his head. Decide whether each of these has the same value as 7.83 and discuss your reasoning.

a) Seven and eighty-two tenths

b) $7 + 0.8 + 0.03$

c) $9\frac{13}{25}$

d) 783 tenths

e) 783 hundredths

f) $7\frac{83}{100}$

Quick Check III

STANDARDS FOR MATHEMATICAL CONTENT

MFANSQ1. Students will analyze number relationships.
b. Understand a fraction a/b as a multiple of 1/b. (MGSE4.NF.4)

MFANSQ2. Students will conceptualize positive and negative numbers (including decimals and fractions).
b. Represent numbers on a number line. (MGSE6.NS.5,6)
c. Explain meanings of real numbers in a real world context. (MGSE6.NS.5)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
c. Interpret and solve contextual problems involving division of fractions by fractions. For example, how many 3/4-cup servings are in 2/3 of a cup of yogurt? (MGSE6.NS.1)
e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MGSE7.NS.3, MGSE7.EE.3, MGSE9-12.N.Q.3)

Quick Check III - Formative Assessment

1. Which of these fractions is equivalent to $\frac{2}{3}$?

a) $\frac{1}{2}$ b) $\frac{3}{4}$ c) $\frac{6}{9}$ d) $\frac{2}{6}$

The answer is (c).

2. Are the following fractions equivalent? $\frac{4}{6}$ *and* $\frac{8}{12}$

a) no, the denominators are not the same
b) no, you add 6 to the denominator and 4 to the numerator
c) yes, you multiply the numerator and denominator by the same number
d) yes, you add 4 to the numerator and 6 to the denominator

The answer is (c).

3. What denominator would you use to add the fractions $\frac{5}{6}+\frac{1}{3}$

a) 3 b) 6 c) 9 d) 15

The answer is (b).

4. Subtract. $\frac{8}{10}-\frac{1}{5}=?$ (Hint: Simplify your answer - use equivalent fractions)

a) $\frac{3}{5}$ b) $\frac{4}{10}$ c) $\frac{7}{10}$ d) $\frac{7}{5}$

The answer is (a).

For problems 5 and 6, find **two** equivalent fractions for each of the given fractions.

5. $\frac{1}{3}$ Some examples are 2/6, 3/9, 4/12, 10/30

6. $\frac{3}{4}$ Some examples are 6/8, 9/12, 12/16, 30/40

7. Add the following fractions. **Show your work.** $\frac{7}{24}+\frac{4}{16}=$ The answer is 13/24

8. Subtract the following fractions. **Show your work.** $\frac{8}{28}-\frac{6}{21}=$ The answer is zero.

Quick Check III - Formative Assessment

1. Which of these fractions is equivalent to $\frac{2}{3}$?

a) $\frac{1}{2}$ b) $\frac{3}{4}$ c) $\frac{6}{9}$ d) $\frac{2}{6}$

2. Are the following fractions equivalent? $\frac{4}{6}\ and\ \frac{8}{12}$

a) no, the denominators are not the same
b) no, you add 4 to the numerator and 6 to the denominator
c) yes, you multiply the numerator and denominator by the same number
d) yes, you add 4 to the numerator and 6 to the denominator

3. What denominator would you use to add the fractions $\frac{5}{6}+\frac{1}{3}$

a) 3 b) 6 c) 9 d) 15

4. Subtract. $\frac{8}{10}-\frac{1}{5}=?$ (Hint: Simplify your answer - use equivalent fractions)

a) $\frac{3}{5}$ b) $\frac{4}{10}$ c) $\frac{7}{10}$ d) $\frac{7}{5}$

For problems 5 and 6, find **two** equivalent fractions for each of the given fractions.

5. $\frac{1}{3}$

6. $\frac{3}{4}$

7. Add the following fractions. **Show your work.** $\frac{7}{24}+\frac{4}{16}=$

8. Subtract the following fractions. **Show your work.** $\frac{8}{28}-\frac{6}{21}=$

Penny Cube

SUGGESTED TIME FOR THIS LESSON:

60-90 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)
e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MGSE7.NS.3, MGSE7.EE.3, MGSE9-12.N.Q.3)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.

6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Multiply decimals.
- Make sense of rational numbers in the real world.

MATERIALS

- Videos for Penny Cube – 3-Act Task
- Recording sheet (attached)

ESSENTIAL QUESTIONS

- How do you calculate the amount of money in a cube of pennies?

Grouping: Learning Partners; Whole Class

Teacher Notes
In this lesson, students will watch the video, then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they do not have the information they need, and ask for it, it will be given to them

LESSON Description

The following 3-Act Task can be found at: http://mikewiernicki.com/penny-cube/

OPENER/ACTIVATOR

ACT 1:

Watch the video and answer these questions on the student recording sheet:

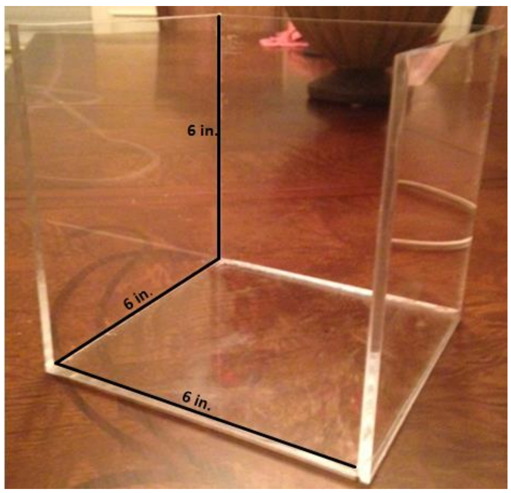
- Ask the students what they wonder after seeing the video.
  - How many pennies is that?
  - How much money is that?
- Have students record their estimates on the recording sheet. What would be an estimate that is too high? Too low?

WORK SESSION

ACT 2:

What do students need to know to answer the questions above? Below you will find some of the information they may ask for.

- Penny Cube Dimensions



- Coin Specifications

Denomination	Penny	Nickel	Dime	Quarter Dollar
Weight	About 0.09 oz	About 0.18 oz	About 0.08 oz	About 0.2 oz
Diameter	0.750 in	0.835 in	0.705 in.	0.955 in.
Thickness	About 0.06 in.	About 0.08 in.	About 0.05 in	About 0.07 in.

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CLOSING/SUMMARIZER

ACT 3:

Students will compare and share solution strategies.

- Reveal the answer. ***Video solution is located on the website above.*** Discuss the theoretical math versus the practical outcome.
- How reasonable was your estimate?
- Share student solution paths. Start with most common strategy.
- What might you do differently if you were to do this again?
- Revisit any initial student questions that weren't answered.

ACT 4 - Extensions

- How much does this penny cube weigh?
- Quarters would fit nicely in this cube as well. Which would you rather have, a cube of pennies or a cube of quarters?
- If this cube was one cubic foot, how much money would it hold?

LESSON Title:________________________
Name:________________________

Adapted from Andrew Stadel

ACT 1

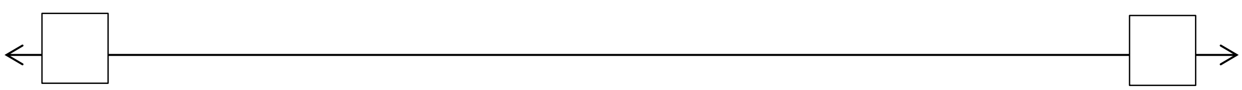
What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate the result of the main question? Explain?

Place an estimate that is too high and too low on the number line



Low estimate Place an “x” where your estimate belongs High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:_______________________________

Act 2 (cont.)

Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?

□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Multiplying Rational Numbers

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

> **MFANSQ2.** Students will conceptualize positive and negative numbers (including decimals and fractions).
> a. Explain the meaning of zero. (MGSE6.NS.5)
> b. Represent numbers on a number line. (MGSE6.NS.5,6)
> c. Explain meanings of real numbers in a real world context. (MGSE6.NS.5)
>
> **MFANSQ4**. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
> a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
> b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
> d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

Common Misconceptions

- *Visualizing multiplication is a very difficult concept for students to comprehend. You may have some learners that find the lesson more confusing than helpful depending on their understanding of how concrete examples apply to abstract concepts.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain

calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Use a number line to multiply positive and negative integers.
- Apply the patterns found in multiplying integers to division of positive and negative integers.
- Use a number line to show multiplication should be used.

MATERIALS

- Two color counters

ESSENTIAL QUESTIONS

- How do you use a number line to multiply rational numbers?
- What patterns in multiplication can you relate to division?
- How do multiplication and division of rational numbers relate to one another?

Grouping: Partner/Whole Group

OPENER/ACTIVATOR

If everyone in class owes you $2, what is the total debt? Explain.

WORK SESSION

Students should be asked to answer these questions and prove their responses.

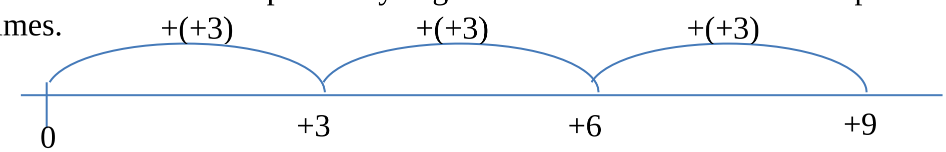
- ***Is it always true that multiplying a negative factor by a positive factor results in a negative product?***
- ***Does a positive factor times a positive factor always result in a positive product?***
- ***What is the sign of the product of two negative factors?***
- ***When three factors are multiplied, how is the sign of the product determined?***
- ***How is the numerical value of the product of any two numbers found?***

LESSON DESCRIPTION

To introduce the concept of multiplying integers, help students make the connection between this and multiplying positive whole numbers.

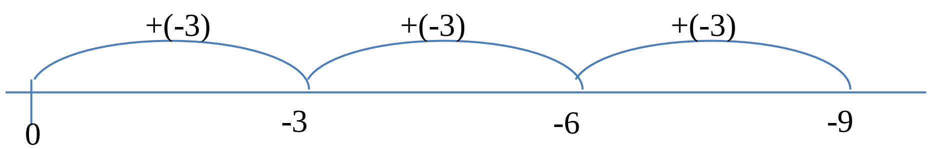
Let's begin with 3 x 3. Using counters, have students model 3 groups of positive 3. Ask a student to explain how they modeled the expression. Record their thinking on an empty number line. An empty number line is a line without designated numbers.

The student should explain they began with 0 counters and added 3 positive counters a total of 3 times.



Have students model 3 x -3 using the counters. Ask a student to explain how they modeled the expression. Record their thinking on an empty number line.

The student should explain they began with 0 counters and added 3 negative counters a total of 3 times.

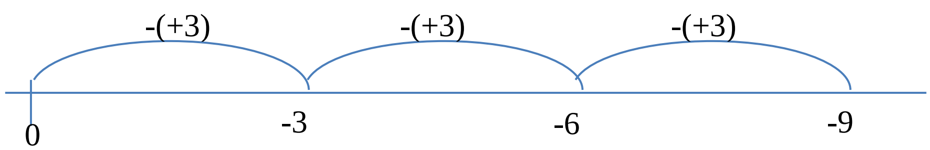


Have students model -3 x 3 using the counters. Circulate around the room to see how students are grappling with the idea of having negative 3 groups of 3. Look for students who:

- Apply the commutative property to make sense of the expression.
- Attempt to make zero pairs to make sense of the expression but become stuck.
- Attempt to make zero pairs and remove 3 groups of positive 3.

Explain to students, with the previous examples, they began with 0 and repeatedly added the quantities, so they used repeated addition when creating the positive groups. Since the groups are no longer positive, they are the opposite; students should perform the opposite operation, repeated subtraction. With this model, students should remove 3 groups of positive 3 from 0.

Allow students the opportunity to discuss with a partner how to model removing from 0.
0 can be created through the use of zero pairs, +a + (-a) = 0. Instruct students create enough zero pairs in which 3 groups of positive 3 can be removed. Once this is modeled, instruct students to remove the 3 groups of +3. Ask students what quantity is left over. With a total of 9 positive counters removed, there should be -9 counters left over.
Record this thinking on an empty number line.

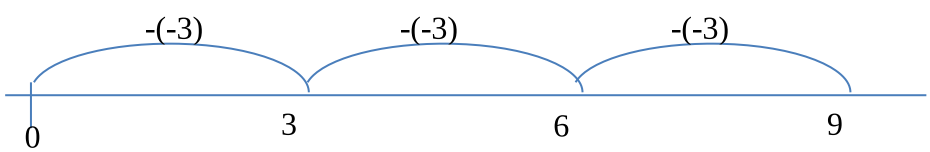


Have students model -3 x -3 using the counters. Circulate around the room to see how students are grappling with the idea of having negative 3 groups of -3. Look for students who:

- Attempt to apply a pattern determined from the previous models.
- Attempt to make zero pairs to make sense of the expression but become stuck.
- Attempt to make zero pairs and remove 3 groups of negative 3.

Allow students the opportunity to discuss with a partner how to model removing from 0.
0 can be created through the use of zero pairs, +a + (-a) = 0. Instruct students create enough zero pairs in which 3 groups of negative 3 can be removed. Once this is modeled, instruct students to remove the 3 groups of -3. Ask students what quantity is left over. With a total of 9 negative counters removed, there should be 9 counters left over.

Record this thinking on an empty number line.



Apply the conventions within a context.

1. Mr. Fletcher's bank account is assessing a $5 fee withdrawal for everyday it is under $500. It has been a total of 4 days under $500. What amount will the bank withdraw from his account?

   4 x -$5 = -$20

2. Billy is participating in a biggest Loser competition. His goal is to lose 3 pounds a week. If Billy meets his goal every week for 6 weeks, how much weight will he lose?

-3 x 6 = -18

3. Mike's son has to pay his dad $7 for every pound he loses. Mike has lost 5 pounds. How much money does his son owe him?

-$7 x -5 = $35
His son owes him $35.

Modeling the Multiplication of Integers

Try these problems on your own. Model each problem using counters or an empty number line. Record the equation and model on your paper.

1. Suppose the temperature outside is dropping 3 degrees each hour. How much will the temperature change in 8 hours?

   -3 x 8 = -24
2. A computer stock gained 2 points each hour for 6 hours. Describe the total change in the stock after 6 hours.

   2 x 6 = 12
3. A drought can cause the level of the local water supply to drop by a few inches each week. Suppose the level of the water supply drops 2 inches each week. How much will it change in 4 weeks?

   -2 x 4 = -8
4. Mike's son has to pay his dad $8 for every pound he loses. Mike has lost 10 pounds. How much money does his son owe him?

   -8 x -10 = 80

DIFFERENTIATION

Extension:

- Have students develop generalized conjectures about multiplying integers and explain them. For example, +a X –b = -c because a groups of –b added to 0 is –c.

Intervention:

- For students who struggle with the empty number line. Encourage them to continue modeling the problems using the two colored counters.
- For extra help, please open the hyperlink Intervention Table.

Student Edition Learning Lesson: Multiplying Rational Numbers

Try these problems on your own. Model each problem using counters or an empty number line. Record the equation and model on your paper.

1. Suppose the temperature outside is dropping 3 degrees each hour. How much will the temperature change in 8 hours?

2. A computer stock gained 2 points each hour for 6 hours. Describe the total change in the stock after 6 hours.

3. A drought can cause the level of the local water supply to drop by a few inches each week. Suppose the level of the water supply drops 2 inches each week. How much will it change in 4 weeks?

4. Mike's son has to pay his dad $8 for every pound he loses. Mike has lost 10 pounds. How much money does his son owe him?

CLOSING/SUMMARIZER
Write a story problem that involves multiplying integers. Solve using a number line.

Arithmetic to Algebra

The following problem sets serve as a bridge between the number sense and quantity focus of Module 1 to the connection between arithmetic and algebra in Module 2. The problem sets are grouped around central strategies and build on patterns of mathematical thought. These sets may be used as warm ups or activators, math burst sets between activities within a class, as exit tickets, or as needed based on your students. Ideas for development are featured as needed prior to a set, and all solutions are provided at the conclusion of the sets. The strategies provided are only examples and should not be considered all inclusive. Other student strategies that promote mathematical understanding should be welcomed. Many problems are provided for each set to allow multiple samples for practice. All problems do not need to be completed at the same time or even within the same lesson. Strategies may be revisited as needed.

These additional resources for consideration as Number/Strategy Talks based on Numeracy Stages from the IKAN screener are provided below. These resources along with many others may be found on the New Zealand Numeracy website at http://nzmaths.co.nz/resource.

AC
EA
AA
AM
AP

Addition Strategies
Adding in Parts: add numbers by splitting them into parts that are easier to combine

Sometimes problems can be done by splitting up one of the numbers so that the other number can be made into a “tidy number”.

e.g. $19 + 7$

Split 7 into $1 + 6$ and make 19 into a tidy number by adding 1

$$\begin{aligned} 19 + 7 &= 19 + 1 + 6 \\ &= 20 + 6 \\ &= 26 \end{aligned}$$

e.g.

$$\begin{aligned} 34 + 18 &= 32 + 2 + 18 \\ &= 32 + 20 \\ &= 52 \end{aligned}$$

Exercise 1

1) Use the strategy of splitting a number into parts to do these additions.
2) Do the problems in your head first.
3) Check you are correct by writing them down. Show them like the examples above.

1)	28 + 14	(2)	76 + 9	(3)	37 + 15
4)	46 + 17	(5)	68 + 24	(6)	48 + 37
7)	29 + 62	(8)	18 + 63	(9)	55 + 17
10)	29 + 54	(11)	38 + 17	(12)	37 + 54
13)	69 + 73	(14)	78 + 45	(15)	27 + 57
16)	19 + 64	(17)	27 + 46	(18)	38 + 75
19)	47 + 86	(20)	34 + 68	(21)	58 + 86
22)	74 + 38	(23)	95 + 29	(24)	88 + 36

Using larger numbers

$$\begin{aligned}146 + 38 &= 146 + 4 + 34\\ &= 150 + 34\\ &= 184\end{aligned} \qquad \text{or} \qquad \begin{aligned}146 + 38 &= 144 + 2 + 38\\ &= 144 + 40\\ &= 184\end{aligned}$$

Exercise 2 larger numbers:

1) Use the strategy of splitting a number into parts to do these additions.
2) Do the problems in your head first.
3) Check you are correct by writing them down. Show them like the examples above.

1)	294 + 87	(2)	392 + 118	(3)	698 + 77
4)	247 + 45	(5)	329 + 68	(6)	488 + 36
7)	539 + 83	(8)	495 + 126	(9)	597 + 363
10)	296 + 438	(11)	794 + 197	(12)	899 + 73
13)	998 + 115	(14)	724 + 89	(15)	1098 + 89
16)	3996 + 257	(17)	5997 + 325	(18)	2798 + 275

Decimals can be added in a similar way. Make one number into a whole number and adjust the other number.

e.g. 32.8 + 24.7
2 tenths or 0.2 is needed to make 32.8 into a whole number and 2 tenths or 0.2 taken from 24.7 gives 24.5 so 32.8 + 24.7= 33 + 24.5 = 57.5

Exercise 3: adding tenths

Knowledge Check – What number goes in the ☐ to make the decimal into a whole number?

1)	4.8 + ☐ = 5	(2)	7.6 + ☐ = 8	(3)	2.9 + ☐ = 3
4)	12.7 + ☐ = 13	(5)	15.9 + ☐ = 16	(6)	23.5 + ☐ = 24
7)	32.8 + ☐ = 33	(8)	74.7 + ☐ = 75	(9)	42.6 + ☐ = 43

Exercise 4: using decimals

1) Use the strategy of splitting a number into parts to do these additions.
2) Do the problems in your head first.
3) Check you are correct by writing them down. Show them like the examples above.

1)	3.9 + 6.7	(2)	4.8 + 7.3	(3)	5.9 + 8.4
4)	7.4 + 1.8	(5)	8.8 + 7.6	(6)	10.6 + 7.8
7)	2.8 + 0.9	(8)	16.7 + 22.8	(9)	34.9 + 12.6
10)	52.7 + 16.8	(11)	21.9 + 17.8	(12)	63.8 + 34.7
13)	42.6 + 16.7	(14)	14.5 + 33.9	(15)	27.8 + 31.7
16)	42.9 + 35.6	(17)	54.9 + 22.7	(18)	36.9 + 52.7
19)	54.8 + 23.4	(20)	83.7 + 12.5	(21)	86.8 + 23.2

<u>SAMPLE for Exercise 5</u>: Jane knows 34 + 18 = 32 + 20
How does she know this without working out the answer?
She knows this is true by thinking of the 34 as 32+2 and then adding the 2 onto the 18 to get 20 so 32+20 is the same.

Exercise 5:

1) Decide whether each statement is True or False.
2) Do this without working out the answer.
3) Provide an explanation of your reasoning.

1)	68 + 34 = 70 + 32	(2)	36 + 57 = 39 + 54	(3)	87 + 15 = 90 + 18
4)	74 + 18 = 72 + 20	(5)	95 + 37 = 98 + 40	(6)	65 + 17 = 68 + 14
7)	153 + 19 = 154 + 20	(8)	325 + 216 = 329 + 220		
9)	274 + 38 = 272 + 40	(10)	73.8 + 15.4 = 74 + 15.6		
11)	45.7 + 11.6 = 46 + 11.3	(12)	82.3 + 17.7 = 82 + 18		

Exercise 6: What number goes in the ☐ to make a true statement?

1)	26 + 35 = 30 + ☐	(2)	18 + 77 = 20 + ☐	(3)	37 + 18 = 40 + ☐
4)	44 + 78 = ☐ + 80	(5)	54 + 28 = ☐ + 30	(6)	93 + 79 = ☐ + 80
7)	46 + 48 = 47 + ☐	(8)	47 + 85 = ☐ + 88	(9)	54 + 28 = 58 + ☐
10)	56 + 38 = ☐ + 35	(11)	74 + 49 = 77 + ☐	(12)	73 + 45 = 70 + ☐
13)	26 + 57 = 23 + ☐	(14)	56 + 17 = ☐ + 13	(15)	93 + 79 = 90 + ☐

Exercise 7
Find two numbers that can be placed in the ☐ and the △ to make a true statement.
Do each problem in three different ways.

1) 38 + ☐ = 36 + △   38 + ☐ = 36 + △   38 + ☐ = 36 + △
Describe the relationship between the ☐ and the △.

2) 26 + ☐ = 29 + △   26 + ☐ = 29 + △   26 + ☐ = 29 + △
Describe the relationship between the ☐ and the △.

3) 51 + ☐ = 56 + △   51 + ☐ = 56 + △   51 + ☐ = 56 + △
Describe the relationship between the ☐ and the △.

4) 75 + ☐ = 72 + △   75 + ☐ = 72 + △   75 + ☐ = 72 + △
Describe the relationship between the ☐ and the △.

5) 87 + ☐ = 83 + △   87 + ☐ = 83 + △   87 + ☐ = 83 + △
Describe the relationship between the ☐ and the △.

6) 93 + ☐ = 90 + △   93 + ☐ = 90 + △   93 + ☐ = 90 + △
Describe the relationship between the ☐ and the △.

7) 86 + ☐ = 100 + △   86 + ☐ = 100 + △   86 + ☐ = 100 + △
Describe the relationship between the ☐ and the △.

8) 148 + ☐ = 150 + △   148 + ☐ = 150 + △   148 + ☐ =150 + △
Describe the relationship between the ☐ and the △.

9) 574 + ☐ = 600 + △   574 + ☐ = 600 + △   574 + ☐ = 600 + △
Describe the relationship between the ☐ and the △.

10) 423 + ☐ = 450 + △   423 + ☐ = 450 + △   423 + ☐ = 450 + △
Describe the relationship between the ☐ and the △.

Exercise 8: Equations and Inequalities

Without working out the answer how do you know that each of the following are true? Write an explanation and then discuss with other members of your group.

1) $0.7 + 0.5 > 1$

2) $54 + 28 = 52 + 30$

3) $246 + 238 > 480$

4) $49 + 51 + 52 > 3 \times 50$

5) $28 + 33 + 38 = 33 + 33 + 33$

6) $16 + {}^{-}4 = 12 + 0$

7) $\triangle + \bigcirc = (\triangle + 2) + (\bigcirc - 2)$ where $\triangle$ and $\bigcirc$ are any numbers

Exercise 9: generalizing the relationship

1) Fill in the parenthesis to make a true statement.
2) The letter stands for *any number*.

Example: 34 + n = 40 + (………) the numbers 34 and 40 are 6 units apart so to make this equation true the value in the parenthesis would have to be n-6.

1) 27 + n = 30 + (………)	(2) 68 + e = 70 + (………)
3) 89 + a = 100 + (………)	(4) 46 + (………) = 50 + y
5) 37 + (………) = 60 + b	(6) 34 + w = 30 + (………)
7) 14.6 + m = 15 + (………)	(8) 48 + g = 47.9 + (………)
9) 65 + (………) = 50 + b	(10) 36 + (………) = 50 + p
11) n + 37 = (………) + 50	(12) (………) + 113 = c + 100
13) (………) + 388 = h + 400	(14) s + 227 = (………) + 250

Answers to Numeracy Strategies

Exercise 1

1)	42	(2)	85	(3)	52
4)	63	(5)	92	(6)	85
7)	91	(8)	81	(9)	72
10)	83	(11)	55	(12)	91
13)	142	(14)	123	(15)	84
16)	83	(17)	73	(18)	113
19)	133	(20)	102	(21)	144
22)	112	(23)	124	(24)	124

Exercise 2

1)	381	(2)	510	(3)	775
4)	292	(5)	397	(6)	524
7)	622	(8)	621	(9)	960
10)	734	(11)	991	(12)	972
13)	1113	(14)	813	(15)	1187
16)	4253	(17)	6322	(18)	3073

Exercise 3

1)	0.2	(2)	0.4	(3)	0.1
4)	0.3	(5)	0.1	(6)	0.5
7)	0.2	(8)	0.3	(9)	0.4

Exercise 4

1)	10.6	(2)	12.1	(3)	14.3
4)	9.2	(5)	16.4	(6)	18.4
7)	3.7	(8)	39.5	(9)	47.5
10)	69.5	(11)	39.7	(12)	98.5
13)	59.3	(14)	48.4	(15)	59.5
16)	78.5	(17)	77.6	(18)	89.6
19)	78.2	(20)	96.2	(21)	110

Exercise 5

1)	True	(2)	True	(3)	False
4)	True	(5)	False	(6)	True
7)	False	(8)	False	(9)	True
10)	False	(11)	True	(12)	True

Exercise 6

1)	*31*	*(2)*	*75*	*(3)*	*15*
4)	*42*	*(5)*	*52*	*(6)*	*92*
7)	*47*	*(8)*	*44*	*(9)*	*24*
10)	*59*	*(11)*	*46*	*(12)*	*48*
13)	*60*	*(14)*	*60*	*(15)*	*82*

Exercise 7

Statements for each question will vary.

1) *the number in the △ is 2 more than the number in the □*
2) *the number in the △ is 3 less than the number in the □*
3) *the number in the △ is 5 less than the number in the □*
4) *the number in the △ is 3 more than the number in the □*
5) *the number in the △ is 4 more than the number in the □*
6) *the number in the △ is 3 more than the number in the □*
7) *the number in the △ is 14 less than the number in the □*
8) *the number in the △ is 2 less than the number in the □*
9) *the number in the △ is 26 less than the number in the □*
10) *the number in the △ is 27 less than the number in the □*

Exercise 8

Answers will vary.

1) *0.7 > 0.5 so total will be more than 0.5 + 0.5*
2) *Adding 2 to 28 gives 30 and subtracting 2 from 54 gives 52*
3) *2 lots of 240 gives 480 and 246 is further from 240 than 238*
4) *49 is one below 50, other two numbers are both above 50*
5) *28 is five below 33 and 38 is five above 33 so their total is the same as 33 + 33*
6) *$^{-}4$ is increased by 4 and 16 is decreased by 4 so total is unchanged*
7) *△ is increased by 2 and ○ is decreased by 2 so total is unchanged*

Exercise 9

1)	*n - 3*	*(2)*	*e - 2*	*(3)*	*a - 11*
4)	*y + 4*	*(5)*	*b + 23*	*(6)*	*w + 4*
7)	*m – 0.4*	*(8)*	*g + 0.1*	*(9)*	*b - 15*
10)	*p + 14*	*(11)*	*n - 13*	*(12)*	*c - 13*
13)	*h + 12*	*(14)*	*s - 23*		

Olympic Cola Display

Lesson adapted from: http://mikewiernicki3act.wordpress.com/olympic-display/

SUGGESTED TIME FOR THIS LESSON:

60-90 minutes
The suggested time for the lesson will vary depending upon the needs of the students.

In this lesson, students will use their understanding of area models to represent the distributive property to solve problems associated with an Olympic cola display.

STANDARDS FOR MATHEMATICAL CONTENT

> **MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.**
> b. Use area models to represent the distributive property and develop understandings of addition and multiplication (all positive rational numbers should be included in the models). (MGSE3.MD.7)

Common Misconceptions:

A common misconception is that students should learn their multiplication tables 0-12 in order, and if the student has not internalized their multiplication facts before middle grades (and especially before high school), they will never "learn" them. Van de Walle states that students need to see multiplication as patterns and use different strategies in determining the product of factors such as the base-ten model and partial products. Students who are not fluent in multiplication facts benefit from looking for patterns and relationships as opposed to memorization. Using the distributive property in relation to partial products helps students gain confidence in other methods for multiplication. For example, if a student does not quickly know the product of 6 and 7, they could think of (5x7) and (1x7). Understanding patterns and relationships between numbers translates into a stronger foundation for algebraic reasoning.

STANDARDS FOR MATHEMATICAL PRACTICE

1. **Make sense of problems and persevere in solving them.** Students must make sense of the problem by identifying what information they need to solve it.
2. **Reason abstractly and quantitatively.** Students are asked to make an estimate (high and low).
3. **Construct viable arguments and critique the reasoning of others.** After writing down their own questions, students discuss their question with partners, creating the opportunity to construct the argument of why they chose their question, as well as critiquing the questions that others came up with.
4. **Model with mathematics.** Once given the information, the students use that information to develop a mathematical model to solve their question.

5. **Use appropriate tools strategically.** Students write their best estimate and two more estimates – one that is too low and one that is too high to establish a range in which the solution would occur.
6. **Attend to precision.** Students use clear and precise language when discussing their strategies and sharing their own reasoning with others.
7. **Look for and make sense of structure.** Students use their understanding of properties of operations and area models to make sense of the distributive property.

EVIDENCE OF LEARNING/LEARNING TARGETS

By the conclusion of this lesson, students should be able to:

- Analyze the relationship between the concepts of area, multiplication, and addition.
- Solve word problems involving area of rectangular figures.
- Use models to represent the context of an area problem.

MATERIALS for Coca Cola LESSON

- Act 1 picture -Olympic Cola Display
- Pictorial representations of the display
- Student recording sheet

ESSENTIAL QUESTIONS

- Which strategies do we have that can help us understand how to multiply a two-digit number?
- How does understanding partial products (using the distributive property) help us multiply larger numbers?

In order to maintain a student-inquiry-based approach to this lesson, it may be beneficial to wait until Act 2 to share the Essential Questions (EQ’s) with your students. By doing this, students will be allowed the opportunity to be very creative with their thinking in Act 1. By sharing the EQ’s in Act 2, you will be able to narrow the focus of inquiry so that the outcome results in student learning directly related to the content standards aligned with this lesson.

KEY VOCABULARY

The following terms should be reviewed/discussed as they arise in dialogue during the lesson:

- distributive property
- commutative property
- area model

SUGGESTED GROUPING FOR THIS LESSON

Individual/Partner and or Small Group

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this lesson, students will view the picture and tell what they notice. Next, they will be asked to discuss what they wonder about or are curious about. These questions will be recorded on a class chart or on the board and on the student recording sheet. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on **need**. When they realize they do not have the information they need, and ask for it, the information will be given to them.

In this lesson, students should build upon what they already know about arrays and area models to answer their questions. Specifically, in finding the total number of 12-packs in the display, students should construct strategies for decomposing the display into smaller areas (the distributive property). Note: Students should not be expected to find the total number of 12-packs by multiplying 14 x 23. They should use the area model with partial products to find the product. The Opener/Activator focuses on the area model with partial products as a way to review that strategy.

Although many of the students will want to use a calculator, **withhold the use of a calculator** to build the need for the distributive property in the area model.

This lesson follows the 3-Act Math Task format originally developed by Dan Meyer. More information on this type of lesson may be found at http://blog.mrmeyer.com/category/3acts/.

Students need **multiple** experiences with arrays to build their understanding of multiplication. Students should also understand how arrays and multiplication are connected to the concept of area, and how their flexibility with number can help them develop strategies for solving complex problems such as the one in this lesson.

Students need to have a good understanding of basic multiplication facts. They should also understand the various ways that multiplication number sentences can be written using an x, a dot, or parentheses.

OPENER/ACTIVATOR

Post the following problem on the board and ask students to find the answer as many ways as they can. Suggest that each student comes up with at least three different ways to find the product.
7 x 23
17 x 34
14 x 43

Ask for students to share in small groups or as a class. Look for strategies to highlight during the class discussion. Make sure to demonstrate the area/array model using partial products as shown below:

7 x 23 can be modeled as shown below:

	20	3
7		

Students should see the problem as 7(20 + 3) which is easier to evaluate as 7(20) + 7(3) to get 140 +21 or 161.

This same process can be applied to larger multiplication problems such as 17x34.

	30	4
10	300	40
7	210	28

You can represent this model as 10(30 +4) + 7(30 +4) to get a final answer of 578. Use of this strategy can have long term effects on understanding and visualizing multiplication of algebraic expressions as well as factoring algebraic expressions.

Further development of this model will happen in upcoming lessons (Distributing Using Area and Tiling Lesson)

Access the Learnzillion video on area model at https://learnzillion.com/lessons/1879-use-an-area-model-for-multiplication-of-two-digit-numbers-by-two-digit-numbers
for more practice or instruction. A guided practice video is also included after the lesson video at https://learnzillion.com/resources/54500

Interactives to examine the area model for multiplication may be found on the Annenberg learning site at
http://www.learner.org/courses/learningmath/number/session4/part_b/multiplication.html

Lesson Directions:

Act 1 – Whole Group - Pose the conflict and introduce students to the scenario by showing Act I picture. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/**)**
"Introduce the central conflict of your story/lesson clearly, visually, viscerally, using as few words as possible."

Show Act 1 picture to students.

- Ask students what they noticed in the picture, what they wonder about, and what questions they have about what they saw in the picture. Do a think-pair-share so that students have an opportunity to talk with each other before sharing questions with the whole group.

- Share and record students' questions. The teacher may need to guide students so that the questions generated are math-related. Consider using the Concept Attainment strategy to place those questions and responses that are accountable to the 3-Act Task in the "Yes, Let's Consider" circle and those that are not in the "Not Now" circle. Examples of "Yes, Let's Consider" and "Not Now" are listed below. Students may come up with additional ideas, and ideas that fall in the "Not Now" category should still be validated and are not to be considered "bad ideas".

Anticipated questions students may ask and wish to answer: (*Main question(s) to be investigated)

<u>**"Yes, Let's Consider"**</u> <u>**"Not Now"**</u>

How many 12 packs of Coke are there?	How tall is it?
*How many 12 packs are there in the display?	How wide is it?
*How many cans of soda is that?	What is the area of the front of the display?
How many cans of each kind of soda are in the display?	How much time did it take to make that display?
What are the dimensions of the display?	Where is it?
OTHER	OTHER

Once students have their question, ask the students to estimate answers to their questions (think-pair-share). Students will write their best estimate, then write two more estimates – one that is too low and one that is too high so that they establish a range in which the solution should occur.

Students should plot their three estimates on an empty number line. To facilitate the discussion of possible solutions, each student could be given three post it notes to record their three estimates on. Then each student could post their estimates on the open number line to arrive at a class set of data to consider.

If you feel the students do not feel comfortable displaying their own estimates, you could have students "ball up" their three estimates in "snow balls" and throw them around the room for a minute, then end up with three "snow balls" at the end to post on the open number line.

Note: As the facilitator, you may choose to allow the students to answer their own posed questions, one question that a fellow student posed, or a related question listed above. For students to be completely engaged in the inquiry-based problem solving process, it is important for them to experience ownership of the questions posed.

***<u>Important note:</u>* Although students may only investigate the main question(s) for this lesson, it is important for the teacher to not ignore student generated questions. Additional questions may be answered after they have found a solution to the main question, or as homework or extra projects.**

Act 2 – **Student Exploration -** Provide additional information as students work toward solutions to their questions. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/ **) "The protagonist/student overcomes obstacles, looks for resources, and develops new tools."**

During Act 2, students decide on the facts, tools, and other information needed to answer the question(s) (from Act1). When students decide what they need to solve the problem, they should ask for those things. **It is pivotal to the problem solving process that students decide what is needed without being given the information up front.**

Students need the opportunity to work with manipulatives on their own or with a partner in order to develop the understanding of multiplication. From the manipulatives, students will be able to move to pictorial representations of the display (attached), then more abstract representations (such as sketches), and finally to abstract representation of multiplication using numbers. It is important to remember that this progression begins with concrete representations using manipulatives.

The goal of the lesson is to use area models to represent the distributive property and develop understandings of addition and multiplication. By sectioning the model, students can apply the distributive property to solve the problem. For example, using concrete manipulatives such as color counters to represent the different types of soda and combine like terms by grouping. Or, students could section the model into components noting parts that are equal such as the top left and top right ring. Others may see the model as a large rectangle with three partial rings above it. Listed below are some suggestions to guide students to make connections between adding all the units of soda to multiplication to simplify the process to applying the distributive property. These are only sample ideas and students should not be limited to the one provided below.

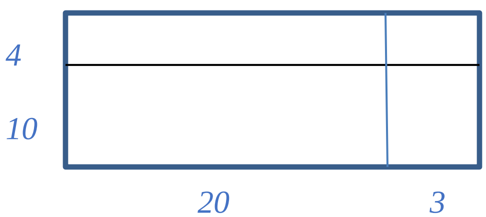
Suggested Questions (if needed) to guide the discussion toward area model and distributive property (CAUTION: Using too many direct questions can stifle the different ways students may solve the problem. These questions are only designed to give insight to the instructor.)

1. When looking at the drawing of the display, how can you divide it into sections to make computation easier? (note: a unit is defined as a 12 pack of soda)

Sample responses might include "a large rectangle with three partial circles above it"

2. How can you break apart those sections to calculate the total number of containers?

Sample responses might include "the large rectangle is 23 units long and 14 units tall so I could decompose that into (20+3)(10+4) which could be shown as



So the four sections would represent the total area of the large rectangular region as

$$\mathbf{20x10 + 3x10 + 20x4 + 3x4}$$

for a total of 322 units of soda for that portion of the display.

Next students will need to consider the top portion of the display which could be modeled as:

3 (7 + 5 +3) since there are three congruent regions containing sections of 7,5, and 3 units of soda for a total of 3(7) + 3(5) + 3(3) or 21+15+9 or 45 units of soda for the top portion of the display.

Now students must combine the two sections for a grand total of 322 + 45 or 367 units of soda.

The teacher provides guidance as needed during this phase. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin. Questioning is an effective strategy that can be used. Consider using these questions to help guide students during the problem solving phase. The questions are based on Polya's Problem Solving Approach.

Stage 1: Understand the Problem

- What are you asked to find or show?
- Can you restate the problem in your own words?
- What is the problem you are trying to solve?
- What do you think affects the situation?
- How can the picture or diagram help you understand the problem?
- What are the knowns? What are the unknowns?
- What information do you obtain from the problem?
- What information, if any, is missing or not needed?

Stage 2: Devise a Plan (Sample provided above)

- What strategy or strategies will you use to solve the problem?
- What strategies are you using?
- What assumptions are you making?
- What tools or models may help you?

Stage 3: Carry out the Plan

- Can you explain what you've done so far?
- Can you explain clearly your steps to solve the problem?
- Can you use an array to represent your mathematics?
- Can you use partial products to represent your mathematics?

Stage 4: Looking Back

- Can you check your result?
- Does the answer make sense? Is it reasonable?
- Why is that true?
- Does that make sense?

Additional Information for Act 2

It is during Act 2 that you may provide the students with the pictorial representation of the display that is attached.

Act 3 – Whole Group – Share solutions and strategies.

- Students to present their solutions and strategies and compare them.
- Teacher facilitates discussion to compare these strategies and solutions, asking questions such as:
    - How reasonable was your estimate?
    - Which strategy was most efficient?
    - Can you think of another method that might have worked?
    - What might you do differently next time?

OPTIONAL (Act 4) – Not a Required Part of the 3 Act Task

Act 4, The Sequel – The following is a quote for Dan Meyer about the purpose of the OPTIONAL 4th Act: "The goals of the sequel lesson are to a) challenge students who finished quickly so b) I can help students who need my help. It can't feel like punishment for good work. It cannot seem like drudgery. It has to entice and activate the imagination." Dan Meyer http://blog.mrmeyer.com/2013/teaching-with-three-act-LESSONs-act-three-sequel/

Act 4: Share ideas (see extensions) or reference other student-generated questions that could be used for additional classwork, projects or homework. Students may choose to revisit one of the questions posed at the beginning but not investigated during Act 2. Allow students who complete the first TWO acts of the process to move on to Act 4 if they are ready. They may join the group discussion/debriefing during Act 3 when all students are ready.

FORMATIVE ASSESSMENT QUESTIONS (These may be used during Act 3)

What partial products did you create?
What organizational strategies did you use?
What are the dimensions of your array(s)?
What product/area does your model represent?

DIFFERENTIATION

Extension
Give students a base-ten block array or a drawing of an array and have them determine the product and its factors.

Have students create their own display, build it with base 10 blocks or connecting cubes, and then trade seats with a neighbor to determine the factors and find the product.

Have students use an array to write/solve division problems.

Intervention
Begin with much smaller arrays, such as 2 x 3, 3 x 4, and 2 x 6. Have students describe the dimensions and area of each array. Then connect dimensions and area to the actual multiplication sentence.

Use grid paper and allow students to place the base-ten blocks onto the grid paper first and then to count the grid squares as part of their calculations.

If necessary, allow students to use a times table chart or other cueing device if full mastery of the basic multiplication facts has not yet been attained.

For extra help, please open the hyperlink Intervention Table.

CLOSING/SUMMARIZER
Use the "Math Mistakes" activity from http://mathmistakes.org/what-is-the-distributive-property/ to review the distributive property.

The Math Mistakes site is about compiling, analyzing and discussing the mathematical errors that students make. The site is edited by **Michael Pershan**, a middle school and high school math teacher from NYC.

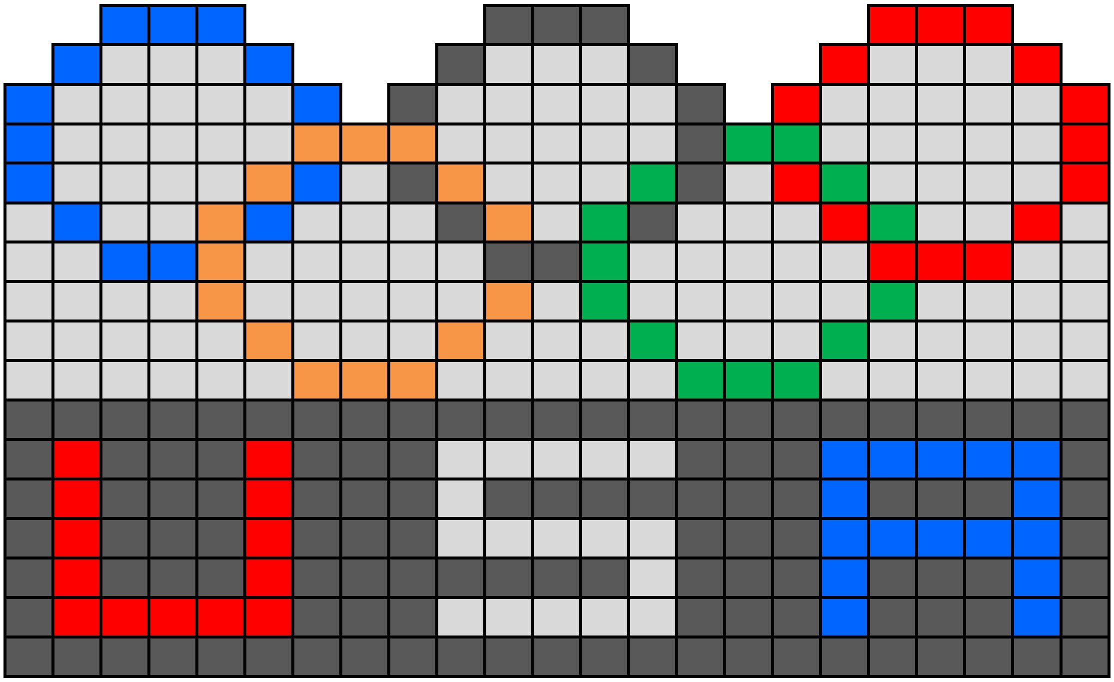
Using student work for error analysis can be an effective strategy to increase understanding of a standard. This site (Math Mistakes) provides a way to analyze mistakes without using work from your class or school. You could assign them the role of "teacher" and ask them the following question:

If this is your class, how do you respond to these student responses?

After students have time to respond to the prompt in small groups, allow several groups to share their ideas.

Act 1 Picture:

Pictorial Representation of the Display:



LESSON Title: ________________________ Name: _________________________
Adapted from Andrew Stadel

ACT 1

What did/do you notice?
What questions come to your mind?

Main Question: ____

What is your 1st estimate and why?

On an empty number line, record an estimate that is too low and an estimate that is too high.

ACT 2

What information would you like to know or need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimation with this information: ______________________________

Act 2 (cont.)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

The following problems may be used as additional practice after completing the 3 Act Task. These problems may be modified to meet the needs of your students. Additionally, you may select problems based on student understanding for a differentiated assignment.

(Teacher's Edition) **Sample Practice Problems** to consider after completion of the **Olympic Coca Cola** 3 Act LESSON and class discussion.

Use area models to represent the distributive property and develop understandings of addition and multiplication (all positive rational numbers should be included in the models). (MGSE3.MD.7)

1. The 9th grade class at Georgia High School wanted to go on a field trip to a soda factory. The trip will cost $100. The students decided to write a class newspaper and sell it to the kids at their school. Each of the 20 students will be given a 16 inch square for his/her article in the newspaper. (adapted from **Read All About It** lesson 3rd grade Unit 3)

How many pages long will the newspaper be if they used paper that was 8 ½ x 11 inches?

Suggest students draw a diagram of a page in the newspaper. Questions might arise about the layout of the page such as:

- ***Is there a margin?***
- ***How wide is the margin?***
- ***Is there a header or footer?***
- ***How much space will that take up?***

Depending on student ideas on margins and header/footer, they may decide that you can fit four of those blocks on a page. Thus, we would need 5 pages for each student to have one article.

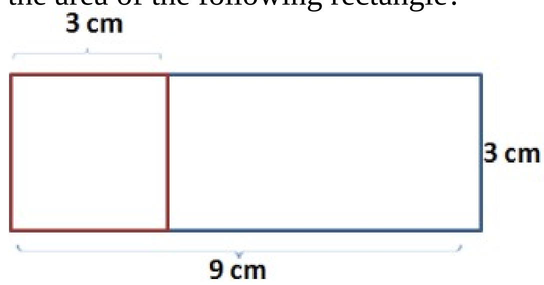
Will there be enough room for additional graphics on the pages once the articles have been written? What do you need to know in order to answer this question?
Since the total area of the page is 93.5 in^2 and we will put 4 squares at 16 in^2 each for a total amount of 64 in^2, there will be 29.5 in^2 of space left for graphics.

How did you determine your solution to part b?

I had to compare the total amount of area available on the page to the amount of area that the articles would take up on the page.
****NOTE: Discussion should address the fact that there will be 29.5 in^2 left on the page which is greater than the area of one square. Why can we not add one more to the page?***

Questions 2-4 originally found at http://www.orglib.com/home.aspx used under a creative commons license

2. Which distributive property represents the area of the following rectangle?



- ○ (3 x 3) + (3 x 4)
- ○ (3 x 3) + (3 x 9)
- ○ (9 x 3) + (6 x 9)
- ○ $\boxed{(3 \times 3) + (3 \times 6)}$

How did you make your decision?

I could tell that the missing side of the blue rectangle would be 6 cm. I also know that to find the area I would multiply the width (3cm) by the length for each rectangle separated (6cm +3cm). That means the area would be (3x3) + (3x6).

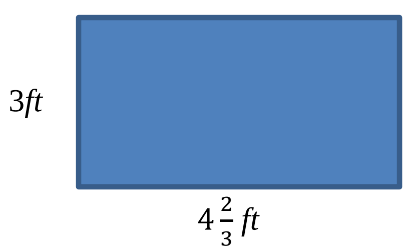
3. The diagram below (under part c) is a model of Samantha's kitchen table.

a) What is the area of Samantha's table? ***The area would be 3ft x $4\frac{2}{3}$ ft or 14 ft^2.***

b) Show a way to use the distributive property to make the problem easier to solve.

$3(4 + \frac{2}{3}) = (3x4) +(3x \frac{2}{3}) = 12 + 2 = 14$ ft^2

c) If you have a table cloth that is 2 yd^2, will it cover the table? Justify your response.



<u>*Note: part c of the problem requires understanding of unit conversions. This part could be used as an extension.</u>

One square yard covers 9 square feet (3ft x 3ft = 9ft^2) so two square feet cover 18ft^2 thus YES the table cloth will cover the table.

Practice Problems Name ____

1. The 9th grade class at Georgia High School wanted to go on a field trip to a soda factory. The trip will cost $100. The students decided to write a class newspaper and sell it to the kids at their school. Each of the 20 students will be given a 16 inch square for his/her article in the newspaper. (adapted from **Read All About It** lesson 3rd grade Unit 3)

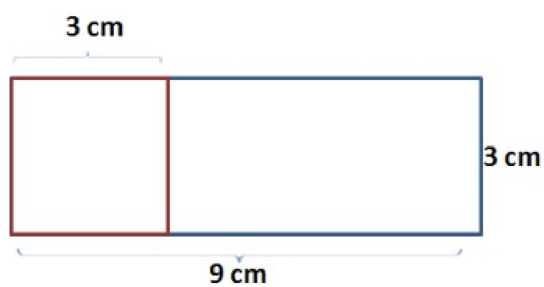
How many pages long will the newspaper be if they used paper that was 8 ½ x 11 inches?

Will there be enough room for additional graphics on the pages once the articles have been written? What do you need to know in order to answer this question?

How did you determine your solution to part b?

Questions 2-3 originally found at http://www.orglib.com/home.aspx used under a creative commons license

2. Which distributive property represents the area of the following rectangle?



- (3 x 3) + (3 x 4)
- (3 x 3) + (3 x 9)
- (9 x 3) + (6 x 9)
- (3 x 3) + (3 x 6)

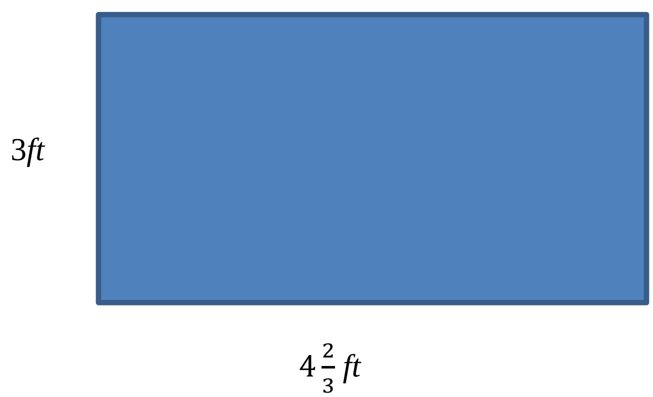
How did you make your decision?

3. The diagram below is a model of Samantha's kitchen table.

a) What is the area of Samantha's table?

b) Show a way to use the distributive property to make the problem easier to solve.

c) If you have a table cloth that is 2 yd^2, will it cover the table? Justify your response.



Tiling Lesson

Adapted from EngageNY *A Story of Units*
This lesson extends the area model to find the area of a rectangle with fractional side lengths.

SUGGESTED TIME FOR THIS LESSON

60-90 minutes
The suggested time for the class will vary depending upon the needs of the students.

**Also note that the teacher will need to create rectangles in advance for this lesson for the student activity (see notes in Materials Section).*

STANDARDS FOR MATHEMATICAL CONTENT

MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.
a**.** Apply properties of operations emphasizing when the commutative property applies. (MGSE7.EE.1)
b. Use area models to represent the distributive property and develop understandings of addition and multiplication (all positive rational numbers should be included in the models). (MGSE3.MD.7)
c. Model numerical expressions (arrays) leading to the modeling of algebraic expressions. (MGSE7.EE.1,2; MGSE9-12.A.SSE.1,3)
f. Evaluate formulas at specific values for variables. *For example, use formulas such as A = l x w and find the area given the values for the length and width*. (MGSE6.EE.2)

COMMON MISCONCEPTIONS

Students have difficulties understanding equivalent forms of numbers, their various uses and relationships, and how they apply to a problem. Make sure to expose students to multiple examples and in various contexts.
Students usually have trouble remembering to distribute to both parts of the parenthesis. They also try to multiply the two terms created after distributing instead of adding them.
Students can struggle with operations with fractions. Students should be reminded of the activities with fractions used in Module 1 as needed.

STANDARDS FOR MATHEMATICAL PRACTICE

1. **Reason abstractly and quantitatively**. Students make sense of quantities and their relationships when they analyze a geometric shape or real life scenario and identify, represent, and manipulate the relevant measurements. Students decontextualize when they represent geometric figures symbolically and apply formulas
4. **Model with mathematics**. Students model with mathematics as they make connections between addition and multiplication as applied to area. They represent the area of geometric figures with equations and models, and represent fraction products with rectangular areas.

7**. Look for and make use of structure**. Students discern patterns and structures as they apply additive and multiplicative reasoning to determine area through application of the distributive property.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Find the area of rectangles with whole-by-mixed and whole-by-fractional number side lengths by tiling, record by drawing, and relate to fraction multiplication.
- Find the area of rectangles with mixed-by-mixed and fraction-by-fraction side lengths by tiling, record by drawing, and relate to fraction multiplication.
- Measure to find the area of rectangles with fractional side lengths and variable side length.
- Multiply mixed number factors, and relate to the distributive property and the area model.
- Solve real world problems involving area of figures with fractional side lengths using visual models and the area formula.

MATERIALS

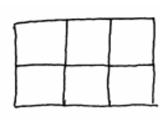
- patty paper units for tiling
- (Teacher) 3 unit × 2 unit rectangle
- (Students) 5 large mystery rectangles lettered A–E (1 of each size per group)

LESSON Note: The lesson is written such that the length of one standard patty paper (5½” by 5½”) is one unit. Hamburger patty paper (available from craft stores in boxes of 1,000) is the ideal square unit for this lesson due to its translucence and size. Measurements for the mystery rectangles are given in generic units so that any size square unit may be used to tile, as long as the tiling units can be folded. Any square paper (such as a square post it notes) may be used if patty paper is not available.

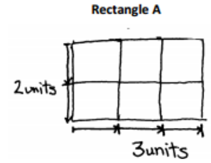
Consider color-coding Rectangles A–E for easy reference.

Preparation: Each group needs one copy of Rectangles A–E. **The most efficient way of producing these rectangles is to use the patty paper to measure and trace the outer dimensions of one rectangle. Then use that rectangle as a template to cut the number required for the class.** Rectangles should measure as follows:

Demo Rectangle A: 3 units × 2 units (See image)
Rectangle B: 3 units × 2 ½ units
Rectangle C: 1 ½ units × 5 units
Rectangle D: 2 units × 1 ¾ units
Rectangle E: ¾ unit × 5 units



Rectangle A



ESSENTIAL QUESTIONS

- What are strategies for finding the area of figures with side lengths that are represented by fractions or variables?
- How can area models be used to represent the distributive property?
- How is the commutative property of multiplication evident in an area model?
- How can the distributive property help me with computation?
- How can perimeter be used to find area in a rectangle?

SUGGESTED GROUPING THIS LESSON

This lesson begins as a whole group activity but transitions into a small group or partner activity. The first two problems will be completed as a group before students breaking into partners or small groups.

KEY VOCABULARY

The vocabulary terms listed below should be addressed (along with any others you feel students need support with) in the context of the LESSON and class discussion.

- Distributive Property: The sum of two addends multiplied by a number equals the sum of the product of each addend and that number.
- Algebraic expression: An expression consisting of at least one variable and also consisting of numbers and operations.
- Numerical expression: An expression consisting of numbers and operations.

INSTRUCTIONAL STRATEGIES

Strategies for this lesson will build on the area model and distributive property from 3rd grade. Students will draw a rectangular grid to model the distributive property as it relates to fractional components to see the correlation to their work with whole number dimensions. Video support for this strategy may be found at https://learnzillion.com/lessons/1544-find-the-area-of-a-rectangle-by-multiplying-a-fraction-and-a-whole-number

Help students to gain a fundamental understanding that the ***distributive property*** works "on the right" as well as "on the left," in addition to "forwards" as well as "backwards" by sharing examples that lend to a discussion of these facets of the distributive property. Examples such as: $3(2x + 4)$, $(2x +4)3$, $6x + 12$ could be used to show equivalent expressions with the distributive property.

OPENER/ACTIVATOR

The following Number Talk activity could be used prior to the introduction to the Tiling Lesson. Using this Number Talk can open dialogue about mental strategies and number sense that will flow into the Tiling Lesson.

The purposes of a "Number Talk" include giving students opportunities to think and reason with numbers, helping students develop place value, number, and operation concepts, and helping students develop computational fluency. The activity could take anywhere from five to 15 minutes. Using a timer can help with keeping the Number Talk from extending too long. Teachers can explain the process and tell students at the onset about the time allowed for the given Number Talk.

In a Number Talk the teacher will give the class a problem to solve mentally. Students may use pencil and paper to keep track of the steps as they do the mental calculations but the goal is to avoid using traditional algorithms and encourage the use of student-led strategies. After all students have had time to think about the problem, students' strategies are shared and discussed to help all students think more flexibly as they work with numbers and operations.

Video footage from a class Number Talk may be seen at http://www.insidemathematics.org/classroom-videos/number-talks/3rd-grade-math-one-digit-by-two-digit-multiplication. Although many classrooms featured in Number Talk segments may be elementary grade level, middle and high school students can benefit from the flexible thinking patterns that number talks promote.

Directions for this Number Talk:

Write a multiplication expression **horizontally** on the board. (For example: 12 x 25)

Give students a minute to estimate the answer and record a few estimates on the board. High school students may be hesitant to do this out loud until they become more comfortable with the process. To still allow this important step in a "safe" manner, students could record their estimate on a sticky note or slip of paper WITHOUT putting their names and the teacher could quickly sort the estimates to share some with the class. This will allow the teacher to see how students are developing their number sense and operational use of strategies in solving problems.

Ask students to **mentally** find the solution using a strategy that makes sense to them.
After all students have had a chance to think about the problem, they can then share their strategy (or strategies) with a partner or small group.

As students discuss their strategies, listen to their explanations and find a few strategies you want to share with the whole class. For example: Using 10 x 25 to get 250 and 2x25 to get 50 then combining the results to get 300.

Ask a student to fully explain the steps he/she followed to solve the problem and record the steps carefully while asking for clarification as needed. This is your chance as the teacher to point out the "big ideas" of the strategy used by the student. In the example above, this student used partial product to make the problem easier to solve.

Ask other students to share different methods they used for solving the equation and ask questions about why their strategies work.

Repeat the process with other similar problems as time allows.

Other problems to consider in the same nature as the one used in the example include:
16 x 25 32 x 50 13 x 12 102 x 9 12 x 25 19 x 99

More Number Talk information and examples may be found at:
http://schoolwires.henry.k12.ga.us/Page/37071,
http://www.mathsolutions.com/documents/numbertalks_sparrish.pdf
https://sites.google.com/site/get2mathk5/home/number-talks

INTERVENTION/EXTENSION

Use this link http://www.raftbayarea.org/readpdf?isid=604 to access a card game called "Algebra Rummy". This game reviews key terminology such as coefficient, term, like terms, and expressions and also provides extension opportunities for students who need more of a challenge. Students will try to get "three of a kind" with like terms and combine them. Other options for game play are explained on the RAFT site. Teams of 3-4 students should be able to play the game in 10-15 minutes. Teachers may set a timer and declare the winner after a set number of minutes.

The NRICH article on multiplication using arrays is a supporting document found at **http://nrich.maths.org/8773**

Teacher's Edition: <u>Tiling LESSON</u> (student pages follow teacher pages)

Directions for 1-5:

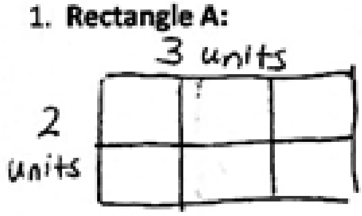
- **Sketch** the rectangles and your tiling.
- **Write** the dimensions and the units you counted in the blanks.
- **Show** the steps you take to solve the problems and justify your calculations.
- **Use multiplication** to confirm the area

Class Note: We will do Rectangles A and B together

*Snapshot solutions to 1-5 courtesy of EngageNY

Rectangle A: Sketch Below Showing Tiles

1. Rectangle A:



$3 \times 2 = 6$

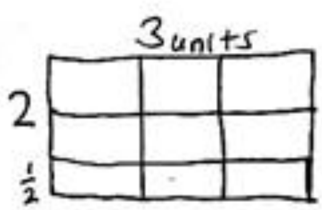
Rectangle A is

__3__ units long __2__ units wide

Area = __6__ units²

Rectangle B: Sketch Below Showing Tiles

2. Rectangle B:



3 units × 2 units = 6 units²

3 units × $\frac{1}{2}$ unit = $1\frac{1}{2}$ units²

6 units² + $1\frac{1}{2}$ units² = $7\frac{1}{2}$ units²

Rectangle B is

__3__ units long __$2\frac{1}{2}$__ units wide

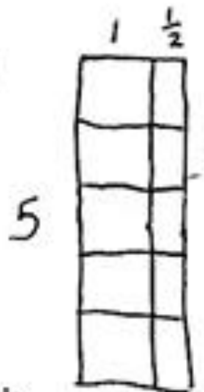
Area = __$7\frac{1}{2}$__ units²

Question students as you draw the model about partitioning into sections to make the multiplication easier and apply the distributive property to solve the model.

$$3\left(2\,\frac{1}{2}\right) = 3\left(2+\frac{1}{2}\right) = 3(2) + 3\left(\frac{1}{2}\right) = 6 + \frac{3}{2} = 6 + 1\,\frac{1}{2} = 7\frac{1}{2}$$

Rectangle C: Sketch Below Showing Tiles

3. Rectangle C:



1 unit × 5 units: 5 units^2

$\frac{1}{2}$ unit × 5 units $= 2\frac{1}{2}$ units^2

$$\begin{array}{r} 5 \text{ units}^2 \\ +\ 2\frac{1}{2} \text{ units}^2 \\ \hline 7\frac{1}{2} \text{ units}^2 \end{array}$$

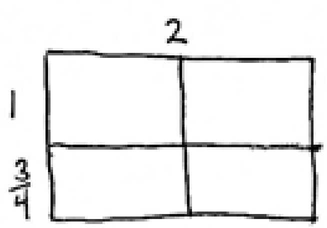
Rectangle C is

__5__ units long __$1\frac{1}{2}$__ units wide

Area = __$7\frac{1}{2}$__ units^2

Rectangle D: Sketch below Showing Tiles

4. Rectangle D:



$2 \times 1\frac{3}{4}$

$= (2\times1) + (2\times\frac{3}{4})$

$= 2 + \frac{6}{4}$

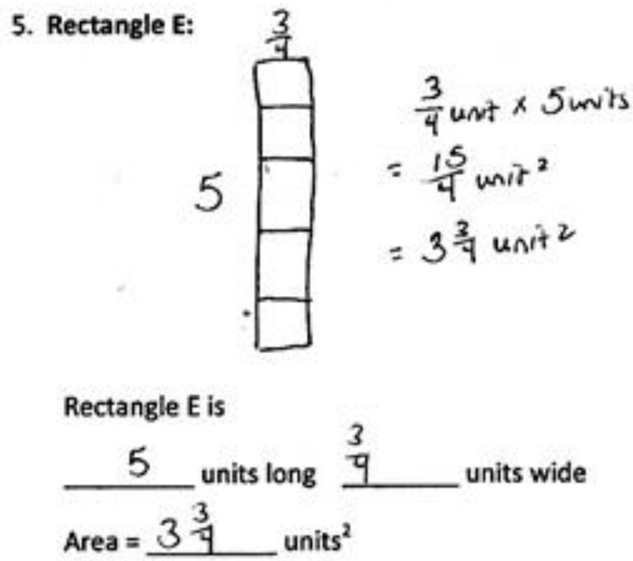
$= 3\frac{2}{4} = 3\frac{1}{2}$

Rectangle D is

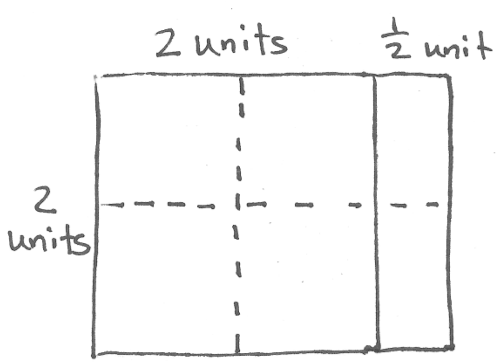
__2__ units long __$1\frac{3}{4}$__ units wide

Area = __$3\frac{1}{2}$__ units^2

Rectangle E: Sketch Below Showing Tiles



6. You found a rectangle on the floor that had been split into an array as shown below.



a) Write an expression to find the area of this rectangle.

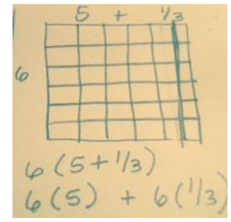
2 (2 + ½) or 2(2) + 2 $\left(\frac{1}{2}\right)$ or 2$\left(2\frac{1}{2}\right)$

b) Find the area of this rectangle and explain mathematically how you arrived at your solution.

The area of this rectangle is 5 units^2. I used the distributive property to evaluate the expression for the area represented by length x width or 2 (2 + ½) which simplifies to 2(2) + 2 $\left(\frac{1}{2}\right)$ and can be evaluated as 4 + 1 for a total of 5 units^2 .

7. Draw a rectangle whose dimensions are $5\frac{1}{3}$ units by 6 units.

a) Construct an array model within that rectangle to make computing its area easier and compute the area.



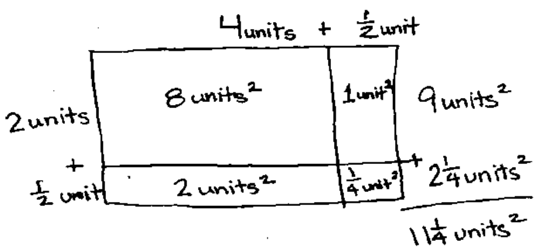
b) Justify your solution and model.

In this model I can multiply the width of 6 by the sum of the parts of the length of 5 and 1/3 to make the computation easier. I can use the distributive property to compute 6(5 + 1/3) as 6(5) + 6(1/3) to arrive at 30 + 2 or 32 units^2.

8. A rectangle has dimensions $2\frac{1}{2}$ units × $4\frac{1}{2}$ units.

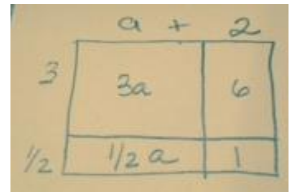
a) Draw an array showing where you would divide it to make your computation of the area of the rectangle easier.

b) Find the area and justify your answer. c) Find the perimeter of this rectangle.



$$4\tfrac{1}{2} \times 2\tfrac{1}{2} = (4+\tfrac{1}{2}) \times (2+\tfrac{1}{2})$$
$$= (2\times4)+(2\times\tfrac{1}{2})+(\tfrac{1}{2}\times4)+(\tfrac{1}{2}\times\tfrac{1}{2})$$
$$= 8 + 1 + 2 + \tfrac{1}{4}$$
$$= 11\tfrac{1}{4}$$

9. A rectangle has dimensions of a+2 by $3\frac{1}{2}$.



a) Draw an array showing where you would divide it to make your computation of the area of the rectangle easier.

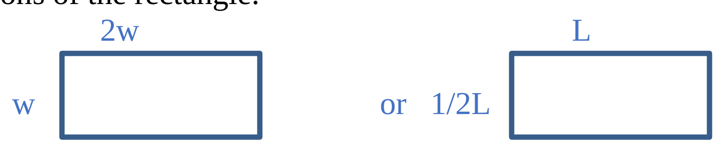
b) Find the area and justify your answer.
(a + 2) (3 + ½) = a (3 + ½) + 2 (3 + ½) = (a*3) + (a*1/2) + (2*3) + (2*1/2) =
3a + ½ a + 6 + 1= (3 ½ a + 7) units^2

I found this expression for the area using the distributive property based on my partial product area model.

c) Find the perimeter of this rectangle. ***The perimeter of this rectangle would represent the distance around the rectangle: length + length + width + width or 2*length + 2*width would be 2(a+2) + 2(3 +1/2) =2*a + 2*2 + 2*3 + 2*1/2 = 2a + 4 + 6 +1 = (2a + 11) units.***

10. You have a rectangle whose length is twice as much as its width. The perimeter of this rectangle is 36 units.

****Note: Answers may vary based on the dimension chosen to be the variable side.***
a) Draw a rectangle to model this relationship and label the sides based on the relationship between the length and the width. Use a single variable to represent this relationship but do not find the actual dimensions of the rectangle.



b) Which dimension, length or width, did you decide to make your primary variable?
Students may choose the width as w since the length is twice the width. Or, students may choose the length as L since it is twice as much as the width.

c) How can you represent the other dimension using the primary variable you have chosen and the relationship stated in the problem?
If the student chose the width as the variable then the length would be 2w. If the student chose the length as the variable then the width would be ½ L since the width is half as much as the length.

d) Write a variable expression to represent the perimeter of this rectangle.
P= 2(w+2w) which could simplify to 2(3w) or 2w + 4w which could simplify to 6w
Or P= 2(1/2L +L) which could simplify to 2(3/2 L) or 2(1/2 L) + 2L which could simplify to 3L.

e) Using any method you choose, find the dimensions of the rectangle. Show all your steps to support your solution.
Students could use a variety of methods to find the dimensions. A few options are shown below but are not to be considered a complete list. Allow students to solve and justify their work.

One option would be to solve the equation 36= 6w or 36= 3L (depending on the choice of variable) to yield w=6 units and L=12 units.

Another option could be the use of a tape diagram where the unit could be w so L would get 2 units (since it is twice as much as w) and then the other side (width) would be another unit and the final side (length) would also be 2 units. The tape would then reveal that all 6 units total 36 giving each a value of 6.

W	W	W	w	w	w

width ***___________Length__________*** ***width*** ***____________Length________***

So 6w = 36 units with each bar equal to 6 units

Students might have worked with tape diagrams (also called bar models) in elementary grades. For more information about tape diagrams (bar models) please visit one or more of the following suggested websites:
https://www.engageny.org/sites/default/files/resource/attachments/04-introduction_to_tape_diagrams.pdf
https://learnzillion.com/lessons/589-solve-ratio-problems-using-a-tape-diagram
http://www.mathplayground.com/tb_ratios/thinking_blocks_ratios.html
http://www.thesingaporemaths.com/
http://www.utahsmathfuture.com/usj.pdf

f) Write a variable expression to represent the area of this rectangle.
Students could represent area as either w (2w) or $2w^2$ or as L (½ L) or ½ L^2.

g) Based on the dimensions you found, what is the area of this rectangle?
Since students found the width to be 6 units and the length to be 12 units, the area of the rectangle would be 72 $units^2$.

EXTENSION

Challenge students to investigate different ways to break apart the arrays in this lesson. Students can summarize ways they find which make the computation easier or just different. Students could also look for area of composite figures using the distributive property as appropriate or consider problems where dimensions such as perimeter and one side length are given and they are asked to find the area.

Sample Extension Problems:

- Rachel made a mosaic from different color rectangular tiles. Three tiles measured $3\frac{1}{2}$ inches × 3 inches. Six tiles measured 4 inches × $3\frac{1}{4}$ inches. What is the area of the whole mosaic in square inches?

- A rectangle has a perimeter of $35\frac{1}{2}$ feet. If the length is 12 feet, what is the area of the rectangle?

REVIEW/INTERVENTION

For students who are struggling with the concept of area and the distributive property, provide more opportunities to count the actual units of the models to better understand the connection between the formula for area, multiplication using the distributive property, and the actual area of rectangular regions divided into arrays.

CLOSING/SUMMARIZER

To close or summarize this LESSON, please revisit the essential questions listed at the beginning. Have students discuss and explain in multiple ways the concepts listed in the essential questions.

Possible Internet Activities:

The **NRICH Project** aims to enrich the mathematical experiences of all learners. To support this aim, members of the NRICH team work in a wide range of capacities, including providing professional development for teachers wishing to embed rich mathematical LESSONs into everyday classroom practice and is part of the family of activities in the Millennium Mathematics Project from the University of Cambridge.

Perimeter Possibilities http://nrich.maths.org/9691 allows for further investigation between area and perimeter and whole number versus fractional dimensions.

Illustrative Mathematics
7.EE.1– Equivalent Expressions: The purpose of this lesson is to directly address a common misconception held by many students who are learning to solve equations. Because a frequent strategy for solving an equation with fractions is to multiply both sides by a common denominator (so all the coefficients are integers), students often forget why this is an "allowable" move in an equation and try to apply the same strategy when they see an expression. Two expressions are equivalent if they have the same value no matter what the value of the variables in them. After learning to transform expressions and equations into equivalent expressions and equations, it is easy to forget the original definition of equivalent expressions and mix up which transformations are allowed for expressions and which are allowed for equations.

Engage NY Curriculum 7th grade Module 3 – Expressions and Equations Expressions

LESSON: Tiling **Name** ________________________________

Directions for 1-5:

- **Sketch** the rectangles and your tiling.
- **Write** the dimensions and the units you counted in the blanks.
- **Show** the steps you take to solve the problems and justify your calculations.
- **Use multiplication** to confirm the area.

*We will do Rectangles A and B together

1. Rectangle A: Sketch Below Showing Tiles

Rectangle A is _________ units long _________ units wide Area = _________ $units^2$
***Remember each tile is one unit long and one unit wide**

2. Rectangle B: Sketch Below Showing Tiles

Rectangle B is _________ units long _________ units wide Area = _________ $units^2$

3. Rectangle C: Sketch Below Showing Tiles

Rectangle C is _________ units long _________ units wide Area = _________ $units^2$

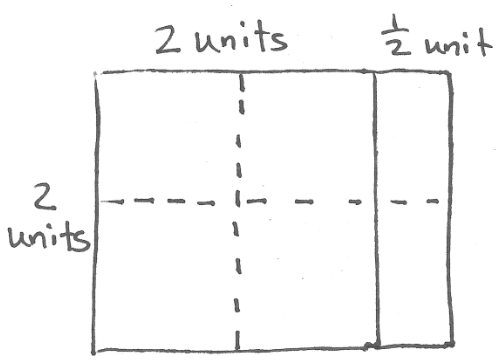
4. Rectangle D: Sketch Below Showing Tiles

Rectangle D is _________ units long _________ units wide Area = _________ units^2

5. Rectangle E: Sketch Below Showing Tiles

Rectangle E is _________ units long _________ units wide Area = _________ units^2

6. You found a rectangle on the floor that had been split into an array as shown below.



a) Write an expression to find the area of this rectangle.

b) Find the area of this rectangle and explain mathematically how you arrived at your solution.

7. Draw a rectangle whose dimensions are $5\frac{1}{3}$ units by 6 units.
a) Construct an array model within that rectangle to make computing its area easier.

b) Justify your solution and model.

8. A rectangle has dimensions $2\frac{1}{2}$ units × $4\frac{1}{2}$units.

a) Draw an array showing where you would divide it to make your computation of the area of the rectangle easier.

b. Find the area and justify your answer.

c. Find the perimeter of this rectangle.

9. A rectangle has dimensions of a+2 by $3\frac{1}{2}$.

a) Draw an array showing where you would divide it to make your computation of the area of the rectangle easier.

b) Find the area and justify your answer.

c) Find the perimeter of this rectangle.

10. You have a rectangle whose length is twice as much as its width. The perimeter of this rectangle is 36 units.

a) Draw a rectangle to model this relationship and label the sides based on the relationship between the length and the width. Use a single variable to represent the relationship but do not find the actual dimensions of the rectangle.

b) Which dimension, length or width, did you decide to make your variable?

c) How can you represent the other dimension using the variable you have chosen and the relationship stated in the problem?

d) Write a variable expression to represent the perimeter of this rectangle.

e) Using any method you choose, find the dimensions of the rectangle. Show all your steps to support your solution.

f) Write a variable expression to represent the area of this rectangle.

g) Based on the dimensions you found, what is the area of this rectangle?

Conjectures About Properties

This lesson is designed to connect properties of arithmetic to algebra. Students will examine properties in the context of variable expressions.

SUGGESTED TIME FOR THIS LESSON

90 -120 minutes
The suggested time for the lesson will vary depending upon the needs of the students.

STANDARDS FOR MATHEMATICAL CONTENT

MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.

a. Apply properties of operations emphasizing when the commutative property applies. (MGSE7.EE.1)

c. Model numerical expressions (arrays) leading to the modeling of algebraic expressions. (MGSE7.EE.1,2; MGSE9-12.A.SSE.1,3)

e. Generate equivalent expressions using properties of operations and understand various representations within context. For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between "3 more" and "3 times". (MGSE4.0A.2; MGSE6.EE.3, MGSE7.EE.1,2,MGSE9-12.A.SSE.3)

COMMON MISCONCEPTIONS

Students are almost certainly not going to know or understand why division by zero is not possible. You will need to provide contexts for them to make sense of this property. To avoid an arbitrary rule pose problems to be modeled that involve dividing by zero: "Take thirty counters. How many sets of zero can be made?" or "Put twelve blocks in zero equal groups. How many are in each group?"

STANDARDS FOR MATHEMATICAL PRACTICE

1**. Make sense of problems and persevere in solving them.** Students make sense of properties by identifying a rule that matches a group of equations.
3**. Construct viable arguments and critique the reasoning of others.** Students construct explanations about properties.
6. **Attend to precision.** Students use the language of the Commutative, Associative, Identity, Zero Property of Multiplication, and Distributive properties.
7. **Look for and make use of structure.** Students apply properties to generate equivalent expressions.
8**. Look for and express regularity in repeated reasoning.** Students will identify properties and reason why that group is identified with a certain property.

EVIDENCE OF LEARNING/LEARNING TARGETS

- Apply properties to simplify or evaluate expressions
- Apply properties to generate equivalent expressions

MATERIALS

- lesson sheet
- manipulative to show grouping (put 12 counters into groups of zero) if needed

ESSENTIAL QUESTIONS

- How can I tell if a group of equations satisfies a property? (Commutative, Associative, Identity, Zero Property of Multiplication, and Distributive)
- How are properties of numbers helpful in computation?

KEY VOCABULARY

The vocabulary terms listed below should be addressed in the context of the lesson and class discussion. The terms are defined and explained in the teacher notes section of the lesson in blue:

- Commutative Property
- Associative Property
- Identity Property
- Zero Property of Multiplication Property
- Distributive Property

ACTIVATOR/OPENING

The following problem was originally featured on Dan Meyers' Blog:
http://blog.mrmeyer.com/the-most-interesting-math-problems-to-me-right-now/

Ask students to follow the steps listed below:

- Everybody pick a number.
- Multiply it by four.
- Add two.
- Divide by two.
- Subtract one.
- Divide by two again.
- Now subtract your original number.

On the count of three, everybody say the number you have.

- What do you notice?
- Why do you think that happened?
- Does it matter what number we pick?
- Can we prove/show that this will always be true?

At this point you could lead a discussion of the problem where the chosen number is represented by a variable and make the parallel between operations with numbers and operations with variable.

*Extension Opportunity for Number Patterns from http://mathprojects.com/2012/10/05/number-tricks-student-sample/

Number Tricks is a lesson that involves writing and simplifying expressions. It demands higher order thinking skills in several ways. 1) The students are to write a mathematical model for a trick given to them. 2) They are to create their own trick and offer the algebraic expression that represents it. 3) It presses the students to understand the concept of a variable; in this case, the variable represents the number originally chosen. 4) The students are asked to compare their simplified expression to the pattern generated by the various numbers tested. The lesson offers a great opportunity for a high level of critical thinking with a rather low level piece of content.

On this site you can find the following Error Analysis opportunity:
*Here is an **erroneous** submission from my Algebra class. I want to analyze the mistake and discuss why this lesson was so very good for this student even though the "answer was wrong." This was Dewey's response to creating his own Number Trick, including 3 numbers to generate the pattern, and the algebraic expression it represents:*

Pick a number	3	10	-7	x
Add 4	7	14	-3	x + 4
Multiply by 2	14	28	-6	2x + 4
Subtract 3	11	25	-9	2x + 4 – 3
Subtract the Original Number	8	15	-2	2x + 4 – 3 – x
Simplified:				x + 1
Common Result: one more than the number picked				

Have students look for and explain the mistake made in the table above. Error analysis helps students see math in a different light and learn to be more evaluative of their own work.

Introducing variable operations and reviewing number operations sets a mental stage for the Properties Lesson.

Introduction to Lesson

Numerical expressions are meaningful combinations of numbers and operation signs. A *variable* is a letter or other symbol that is a placeholder for an unknown number or a quantity that varies. An expression that contains at least one variable is called an *algebraic expression*.

When each variable in an algebraic expression is replaced by a number, the result is a numerical expression whose value can be calculated. This process is called *evaluating the algebraic expression*.

In this lesson the idea of substituting variables to represent numbers is introduced in the context of making conjectures about number properties.

Before the Lesson: Post the following expressions on the board. Ask students to decide if they are true or false. And, ask them to explain, model, show, or prove their response.

$36 + 45 = 45 + 36$ $\qquad$ $123 + 24 = 24 + 123$ $\qquad$ $4 + 6 = 6 + 4$

- ***Looking at these three number sentences, what do you notice?***
- ***Do you think this is true for all numbers?***
- ***Can you state this idea without using numbers?***

By using variables we can make a number sentence that shows your observation that "order does not matter when we add numbers."

$a + b = b + a$

- ***Do you think this is also true for subtraction?***
- ***Do you think it is true for multiplication?***
- ***Do you think it is true for division?***

Repeat with examples to consider for other operations as needed.

Teacher Notes

The full class should discuss the various conjectures, asking for clarity or challenging conjectures with counterexamples. Conjectures can be added to the class word wall with the formal name for the property as well as written in words and in symbols. The example provided above was for the commutative property. More properties will be explored in the following exploratory lesson.

A Closer Look at Cluster 6.EE (part of MFAAA1 and 2)

In the standards for grades K–5, arithmetic is both a life skill and a thinking subject—a rehearsal for algebra. Students in grades K–5 *calculated*, but they also *operated*. For example, students used the distributive property and other properties of operations as they came to learn the standard algorithms for multi-digit multiplication in grades 3 through 5. And students learned about the meanings of operations as they solved word problems with the basic operations. (**Note that the four operations mean the same thing, model the same quantitative relationships, and solve the same kinds of word problems regardless of whether the numbers involved are whole numbers, fractions, decimals, or any combination of these—or even a variable standing for any of these**.) In grade 6, students use properties of operations and meanings of operations as a pivot from arithmetic to algebra.

6.EE.A. Apply and extend previous understandings of arithmetic to algebraic expressions.

For example, standard 6.EE.A.3 requires students to apply the properties of operations to generate equivalent expressions. This table highlights the coherence between the grade 6 expectations and the expectations of previous grades.

Property of Operations	Previous Grade Expectation	Grade 6 Expectation from 6.OA.A.3
Distributive	8 × (5 + 2) = (8 × 5) + (8 × 2) which is 56 (3.OA.B.5)	24x + 18y = 6 (4x + 3y)
Associative	3 × 5 × 2 = 15 × 2 OR 3 × 10 (3.OA.B.5)	3 × r × 5 = 15 × r OR 3r × 5 OR 3 × 5r
Commutative	4 × 6 = 24, so 6 × 4 = 24 (3.OA.B.5)	r × 6 = 24, so 6r = 24
Addition and Multiplication	Interpret 5 × 7 as 5 groups of 7 objects	Interpret y + y + y as 3y

The cluster is not only about extending these skills, but also applying them.

Additional Suggested Activities

The Teaching Channel https://www.teachingchannel.org/videos/think-pair-share-lesson-idea
Provides lesson plan and video support for a think-pair-share activity aligned to 7.EE.1 Simplifying Expressions

The Teaching Channel https://www.teachingchannel.org/videos/class-warm-up-routine
Provides a warm up activity called “My Favorite No” which uses an example problem on simplifying expressions.

EXPLORATION LESSON: CONJECTURE ABOUT PROPERTIES

With a partner look at the following sets of number sentences and determine if what you observe would be true for all numbers. Create statements with words about what you observe in each set of number sentences then write the number sentences using variables to represent numbers.

Property articulations by students may vary. They do not have to be exact language but mathematical ideas must be sound and variable representations precise.

1

$$12 + 0 = 12$$
$$37 + 0 = 37$$
$$x + 0 = x$$

Identity Property of Addition
$a + 0 = a$

2

$$12 - 0 = 12$$
$$Y - 0 = 0$$
$$64 - 0 = 64$$

Identity Property of Subtraction
$a - 0 = a$

3

$$a * 1 = a$$
$$37 * 1 = 37$$
$$64 * 1 = 64$$

Identity Property of Multiplication
$a \text{ x } 1 = a$

4

$$12 \div 1 = 12$$
$$37 \div 1 = 37$$
$$b \div 1 = b$$

Identity Property of Division
$a \div 1 = a$ or $\frac{a}{1} = a$

5

$$c * 0 = 0$$
$$37 * 0 = 0$$
$$64 * 0 = 0$$

Zero Property of Multiplication
$a \text{ x } 0 = 0$

6

$$12 \div 0 = 12$$
$$f \div 0 = f$$
$$64 \div 0 = 64$$

Undefined – See summary notes above

7

$$12(4 + 3) = 48 + 36$$
$$5(6 + c) = 30 + 5c$$
$$4(10 + 3) = 40 + 12$$

Distributive Property
$a(b + c) = ab + ac$
(This will need to be revisited during integer exploration.)

8

$$12(4 - 3) = 48$$
$$6(x - 2) = 6x - 12$$
$$4(10 - 3) = 40 - 12$$

Distributive Property
$a(b\text{-}c) = ab - ac$

9

$$4 \, x \, 8 = (2 \, x \, 8) + (2 \, x \, 8)$$
$$8 \, x \, c = (4 \, x \, c) + (4 \, x \, c)$$
$$5 \, x \, 14 = (2.5 \, x \, 14) + (2.5 \, x \, 14)$$

Distributive Property

$$a \cdot b = \left(\frac{a}{2} \cdot b\right) + \left(\frac{a}{2} \cdot b\right)$$

10

$$4 + 8 = (2 + 8) + (2 + 8)$$
$$8 + c = (4 + c) + (4 + c)$$
$$5 + 14 = (2.5 + 14) + (2.5 + 14)$$

False

11

$$(32 + 24) + 16 = 32 + (24 + 16)$$
$$(450 + 125) + 75 = 450 + (125 + 75)$$
$$(33 + v) + 3 = 33 + (v + 3)$$

Associative Property
(a + b) + c = a + (b + c)

12

$$6 \cdot (4 \cdot 3) = (6 \cdot 4) \cdot 3$$
$$10 \cdot (g \cdot 2) = (10 \cdot g) \cdot 2$$
$$(11 \cdot 2) \cdot 3 = 11 \cdot (2 \cdot 3)$$

Associative Property
a (b x c) = (a x b) x c

SUMMARY OF PROPERTIES

The Commutative Property: Changing the order of the values you are adding or multiplying does not change the sum or product.
6 + 4 = 4 + 6 ***a • b = b • a***

The Associative Property: Changing the grouping of the values you are adding or multiplying does not change the sum or product.
(2 + 7) + 3 = 2 + (7 + 3) ***(ab)c = a(bc)***

The Identity Property: The sum of any number and zero is the original number (additive identity). The product of any number and 1 is the original number (multiplicative identity).

The Distributive Property of Multiplication over Addition and Subtraction helps to evaluate expressions that have a number multiplying a sum or a difference.
Example:
9(4 + 5) = 9(4) + 9(5) ***a(b + c) = ab + ac***
5(8-2) = 5(8) – 5(2) ***a(b – c) = ab – ac***

After completion of the Properties lesson, the following closing activity is suggested.

CLOSING ACTIVITY: Number Talks Activity
Adapted from New Zealand Numeracy Project
http://nzmaths.co.nz/resource/homework-sheet-stage-6-7-revision-add-sub-and-mult-div-2
For extra help, please open the hyperlink Intervention Table.

This Number Talks activity revisits those operations and connects to the properties examined in the "Conjecture about Properties" Lesson.

The following problems open discussion about how numbers relate to each other and how number properties allow efficient methods for solving problems mentally.

- Allow students independent working time on the following problem (approximately 10-30 minutes depending on the level of your students).
- Ask students to share ideas with a partner or small group (10 minutes or more depending on the level of your students and their attention to LESSON) about methods they used to solve the problems as you walk around the room noting methods you would like to highlight during full group discussion.
- Bring the group back together to highlight uses of number properties that you heard during small group discussion. Students should share ideas and strategies for solving the problems.

NOTE: The following problems could be given to the students in written form, posted around the room for review, or written on the board.

Mental Math

Work out the answers to these problems in your head. Use the quickest method for each problem, and then record your strategy on paper so that other people can understand how you have done the problem

1)	2 + 25 +8	2)	57 – 15 + 3	3)	5 x 37 x 2
4)	257 + 199	5)	478 - 125	6)	4 × 37
7)	140 ÷ 4	8)	198 - 74	9)	125 ÷ 10
10)	195 × 4	11)	235 - 77	12)	198 - 99
13)	179 + 56	14)	128 ÷ 5	15)	345 + 60
16)	63 × 10	17)	26 × 5	18)	612 ÷ 6
19)	264 ÷ 6	20)	257 + 356	21)	374 – 189

Note: There are multiple ways to look at each problem; therefore, solutions are not listed for all problems. A representative sample of problems is listed below. The strategies listed DO NOT include all options. More information about mental strategies may be found on the New Zealand Numeracy website http://nzmaths.co.nz/ or in Number Talks resources with sample video clips at http://www.insidemathematics.org/classroom-videos/number-talks. Additional sample problems and strategies may be accessed at the end of this module. Use these strategy explorations as time permits each day in class to see strengthening of number sense in students.

1) 2 + 25 + 8 could be solved using the commutative property and associative property:
2 + 8 + 25 commutative property
(2+8) + 25 associative property
10 + 25
35

10) 195 x 4 could be solved by
(200 x 4) – (5 x 4)
800 – 20
780

Or by partial products
(100 x 4) + (90 x 4) + (5 x 4)
400 + 360 + 20
780
14) 128 ÷ 5 could be decomposed to (100 ÷ 5) + (20 ÷ 5) + (8 ÷ 5) to get 20 + 4 + 1 + 3/5 to get $25\frac{3}{5}$

Name______________________

LESSON: CONJECTURES ABOUT PROPERTIES

With a partner look at the following sets of number sentences and determine if what you observe would be true for all numbers. Create statements with words about what you observe in each set of number sentences then write the number sentences using variables to represent numbers.

1
$12 + 0 = 12$
$37 + 0 = 37$
$x + 0 = x$

2
$12 - 0 = 12$
$y - 0 = y$
$64 - 0 = 64$

3
$a \cdot 1 = a$
$37 \cdot 1 = 37$
$64 \cdot 1 = 64$

4
$12 \div 1 = 12$
$37 \div 1 = 37$
$b \div 1 = b$

5
$c \cdot 0 = 0$
$37 \cdot 0 = 0$
$64 \cdot 0 = 0$

6
$12 \div 0 = 12$
$f \div 0 = f$
$64 \div 0 = 64$

7
$12(4 + 3) = 48 + 36$
$5(6 + c) = 30 + 5c$
$4(10 + 3) = 40 + 12$

8
$12(4 - 3) = 48$
$6(x - 2) = 6x - 12$
$4(10 - 3) = 40 - 12$

9
$4\ x\ 8 = (2\ x\ 8) + (2\ x\ 8)$
$8\ x\ c = (4\ x\ c) + (4\ x\ c)$
$5\ x\ 14 = (2.5\ x\ 14) + (2.5\ x\ 14)$

10
$4 + 8 = (2 + 8) + (2 + 8)$
$8 + c = (4 + c) + (4 + c)$
$5 + 14 = (2.5 + 14) + (2.5 + 14)$

11
$(32 + 24) + 16 = 32 + (24 + 16)$
$(450 + 125) + 75 = 450 + (125 + 75)$
$(33 + v) + 3 = 33 + (v + 3)$

12
$6 \cdot (4 \cdot 3) = (6 \cdot 4) \cdot 3$
$10 \cdot (g \cdot 2) = (10 \cdot g) \cdot 2$
$(11 \cdot 2) \cdot 3 = 11 \cdot (2 \cdot 3)$

<u>Additional Practice on Simplifying Expressions</u> **TEACHER'S EDITION**

Please use the following problems as needed for additional practice on simplifying variable expressions.

Create a like term for the given term (NOTE: answers will vary as there are unlimited options)

1. 4x <u>2x</u> 2. -7y <u>3/4 y_</u> 3. 12ab <u>-3ab</u>

4. $\frac{2}{3}x^2$ <u>$5x^2$</u> 5. $-5ax^2$ <u>$4ax^2$</u> 6. 14c <u>-7c</u>

Simplify the expression if possible by combining like terms

6. $7y + 10y$ <u>17y</u> 7. $6x + 8y + 2x$ <u>8x + 8y</u>

8. $5x + 2(x + 8)$ <u>7x + 16</u> 9. $5x - 2(x - 8)$ <u>3x + 16</u>

10. $9(x + 5) + 7(x - 3)$ <u>16x + 24</u> 11. $8 - (x - 4)2$ <u>16 – 2x</u>

12. $\frac{2}{3}(12x - 6)$ <u>8x – 4</u> 13. $\frac{1}{2}(12x - 6) + \frac{1}{2}(12x - 6)$ <u>12x - 6</u>

14. $9y + 4y - 2y + y$ <u>12y</u> 15. $x + 5x - 17 + 12 + x$ <u>7x – 5</u>

16. Add your answer from #10 to your answer for #13 <u>28x + 18</u>

17. Double your answer for #8 and add it to half of your answer for #13 <u>20x + 29</u>

18. Subtract your answer for #9 from triple your answer for #12 <u>21x – 28</u>

<u>Simplifying Expressions</u> **Name ___________________________________**

Create a like term for the given term

1. 4x ___________ 2. -7y ___________ 3. 12ab ___________

4. $\frac{2}{3}x^2$ ___________ 5. -5ax^2 ___________ 6. 14c ___________

Simplify the expression if possible by combining like terms

6. $7y + 10y$ ___________ 7. $6x + 8y + 2x$ ___________

8. $5x + 2(x + 8)$ ___________ 9. $5x - 2(x - 8)$ ___________

10. $9(x + 5) + 7(x - 3)$ ___________ 11. $8 - (x - 4)2$ ___________

12. $\frac{2}{3}(12x - 6)$ ___________ 13. $\frac{1}{2}(12x - 6) + \frac{1}{2}(12x - 6)$___________

14. $9y + 4y - 2y + y$ _______________ 15. $x + 5x - 17 + 12 + x$ ____________

16. Add your answer from #10 to your answer for #13______________________________

17. Double your answer for #8 and add it to half of your answer for #13 _________________

18. Subtract your answer for #9 from triple your answer for #12 ________________________

Quick Check IV

STANDARDS FOR MATHEMATICAL CONTENT

MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.
a. Apply properties of operations emphasizing when the commutative property applies. (MGSE7.EE.1)
b. Use area models to represent the distributive property and develop understandings of addition and multiplication (all positive rational numbers should be included in the models). (MGSE3.MD.7)
c. Model numerical expressions (arrays) leading to the modeling of algebraic expressions. (MGSE7.EE.1,2; MGSE9-12.A.SSE.1,3)
d. Add, subtract, and multiply algebraic expressions. (MGSE6.EE.3, MGSE6.EE.4, MC7.EE.1, MGSE9-12.A.SSE.3)
e. Generate equivalent expressions using properties of operations and understand various representations within context. For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between "3 more" and "3 times". (MGSE4.0A.2; MGSE6.EE.3, MGSE7.EE.1, 2, MGSE9-12.A.SSE.3)

Quick Check IV

TEACHER'S EDITION

Simplify the expression

1. $2.3 + 0.5(3 + x) = \mathbf{0.5x + 3.8}$ (or equivalent values)

2. $\frac{8}{6} + \left(\frac{3}{8} + x\right)(2) = \mathbf{2x + \frac{50}{24}}$ (or equivalent values)

3. $2y + 3x + 5y + 4(6x) = \mathbf{27x + 7y}$

4. Circle all the equivalent expressions listed below:
The correct answers are circled below:

$7(b + 5) + 3$ $\quad$ $b + 38$ $\quad$ $7b + 7 \times 8$

$7b + 38$ $\quad$ $7b + (7 \times 5) + 3$

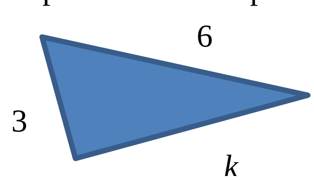
5. Justify and explain how you know the expressions circled above are equivalent.

Students may explain this in a variety of ways.
For example: "I distributed the 7, so $\mathbf{7(b + 5) + 3 = 7 \times b + (7 \times 5) + 3}$. Because $\mathbf{7 \times b}$ can be expressed as 7b, I know that $\mathbf{7(b + 5) + 3}$ is equivalent to $\mathbf{7b + (7 \times 5) + 3}$. Then, I found that the value of $\mathbf{(7 \times 5) + 3}$ is 38. So, $\mathbf{7b + (7 \times 5) + 3}$ is equivalent to $\mathbf{7b + 38}$. That means all three expressions are equivalent."

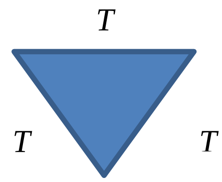
6. There is one mistake in the work shown below. Next to the first incorrect equation, write the correct result, and then, correct the rest of the problem.

P + P + 6(3P + 4) + 4P	=	2P + 6(3P + 4) + 4P	
	=	6(3P + 4) + 2P + 4P	
	=	3P + 4 + 6 + 2P + 4P	18P + 24 + 2P + 4P
	=	3P + 4 + 6 + 6P	18P + 2P + 4P + 24
	=	9P + 10	24P + 24

7. Write an expression for the perimeter of this triangle. _***k + 6 + 3 or 9 + k or equivalent***_



8. Write an expression for the perimeter of this triangle. _***T + T + T or 3T or equivalent***__



9. Draw a rectangular array in order to find the solution to $2\frac{1}{3} \times 3\frac{1}{4}$.

	2	$\frac{1}{3}$
3	6	1
$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{12}$

10. Explain your solution to #9 above.

Students may explain this in a variety of ways.

For example: $\mathbf{2\frac{1}{3} \times 3\frac{1}{4} = \left(2 + \frac{1}{3}\right) \times \left(3 + \frac{1}{4}\right)}$

$$\mathbf{= (2 \times 3) + \left(2 \times \frac{1}{4}\right) + \left(\frac{1}{3} \times 3\right) + \left(\frac{1}{3} \times \frac{1}{4}\right)}$$

$$\mathbf{= 6 + \frac{1}{2} + 1 + \frac{1}{12}}$$

$$\mathbf{= 7 + \frac{7}{12}}$$

$$\mathbf{= 7\frac{7}{12}}$$

I found the area of each rectangle and then added those areas together.

Quick Check IV Name: ____

Simplify the expression

1. $2.3 + 0.5(3 + x) =$ ____$\boldsymbol{x}$ + ____

2. $\frac{8}{6} + \left(\frac{3}{8} + x\right)(2) =$ ____$\boldsymbol{x}$ + ____

3. $2y + 3x + 5y + 4(6x) =$ ____$\boldsymbol{x}$ + ____$\boldsymbol{y}$

4. Circle all the equivalent expressions listed below:

$7(b + 5) + 3$ $b + 38$ $7b + 7 \times 8$

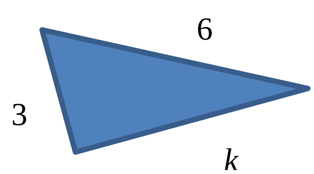
$7b + 38$ $7b + (7 \times 5) + 3$

5. Justify and explain how you know the expressions circled above are equivalent.

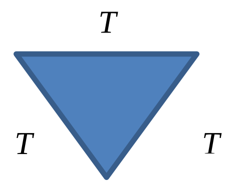
6. There is one mistake in the work shown below. Next to the first incorrect equation, write the correct result, and then, correct the rest of the problem.

P + P + 6(3P + 4) + 4P	=	2P + 6(3P + 4) + 4P	
	=	6(3P + 4) + 2P + 4P	
	=	3P + 4 + 6 + 2P + 4P	
	=	3P + 4 + 6 + 6P	
	=	9P + 10	

7. Write an expression for the perimeter of this triangle. ______________________________



8. Write an expression for the perimeter of this triangle. ______________________________



9. Draw a rectangular array in order to find the solution to $2\frac{1}{3} \times 3\frac{1}{4}$.

10. Explain your solution to #9 above.

Visual Patterns

Original Source for lesson idea: http://www.visualpatterns.org/
This lesson uses visual patterns to provide a connection from concrete or pictorial representations to abstract algebraic relationships and to help students to think algebraically. Unfortunately, students are often directed to make a table to find a pattern, rather than building strong figural reasoning.

SUGGESTED TIME FOR THIS LESSON

120 minutes
The suggested time for the lesson will vary depending upon the needs of the students.

Initially, 120 minutes (depending on your students) should be planned for the activity outlined below. Continued time (throughout this module and into future modules) should be set aside for students to investigate these types of patterns. Students should be given multiple chances to engage in these investigations with different visual patterns.

STANDARDS FOR MATHEMATICAL CONTENT

MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.
c. Model numerical expressions (arrays) leading to the modeling of algebraic expressions. (MGSE7.EE.1,2; MGSE9-12.A.SSE.1,3)
d. Add, subtract, and multiply algebraic expressions. (MGSE6.EE.3, MGSE6.EE.4, MC7.EE.1, MGSE9-12.A.SSE.3)
e. Generate equivalent expressions using properties of operations and understand various representations within context. For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between "3 more" and "3 times". (MGSE4.0A.2; MGSE6.EE.3, MGSE7.EE.1,2,MGSE9-12.A.SSE.3)
f. Evaluate formulas at specific values for variables. For example, use formulas such as A = l x w and find the area given the values for the length and width. (MGSE6.EE.2)

COMMON MISCONCEPTIONS

When working with visual patterns to predict future stages and generalize for all stages, students confuse parts of the patterns that remain and parts that change. Using color-coded tiles of models can help students arrive at the pattern more readily. Also, as students graph the stage versus pattern (i.e. number of squares or triangles), they want to connect the points on the graph. This data is not continuous data (it is discrete) so points should not be connected. There is no way to have "half a stage".

STANDARDS FOR MATHEMATICAL PRACTICE

1. **Make sense of problems and persevere in solving them.** Students will make sense of the problems by building, extending, creating and explaining visual patterns.
2. **Reason abstractly and quantitatively.** Students will reason with quantities of objects in the patterns, and then create abstract generalizations to write expressions and equations that explain the patterns.
3. **Construct viable arguments and critique the reasoning of others.** Students will share their expressions/equations with other groups and students and discuss the validity of each. Students will likely discover equivalent expressions/equations within the class.
4. **Model with mathematics.** Students will model the patterns they are studying using materials, diagrams and tables as well as equations.
5. **Use appropriate tools strategically.** Students will choose appropriate tools to solve the visual patterns.
6. **Attend to precision.** Students will attend to precision through their use of the language of mathematics as well as their computations.
7. **Look for and make use of structure.** Students will apply properties to generate equivalent expressions. They interpret the structure of an expression in terms of a context. Students will identify a "term" in an expression.
8. **Look for and express regularity in repeated reasoning.** The repeated reasoning required to explain patterns is evident in this lesson. Students will express this regularity through their conversations with one another and the class.

EVIDENCE OF LEARNING/LEARNING TARGETS

By the conclusion of this lesson, students should be able to:

- Represent the next stage in a pattern.
- Generalize a pattern to represent future stages.
- Recognize equivalent representations of algebraic expressions.

MATERIALS REQUIRED

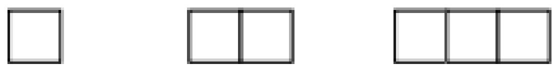
- Various manipulatives such as two color counters
- Color tiles
- Connecting cubes
- Visual Patterns Handout (proceeds teacher notes)

ESSENTIAL QUESTIONS

- What strategies can I use to help me understand and represent real situations using algebraic expressions?
- How are the properties (Identify, Associative and Commutative) used to evaluate, simplify and expand expressions?
- How can I represent a pattern using a variable expression?
- How can I tell if two expressions used to generate a pattern are equivalent?

OPENING/ACTIVATOR

Display the following pattern on the board and ask students to generate statements about what they notice and what they wonder. Encourage them to list as many things as they can for each part (notice and wonder).



After students have time to think and list a few items, make a list of things the class noticed and what they are curious about. Use their ideas as a launch into the visual patterns below.

TEACHER NOTES

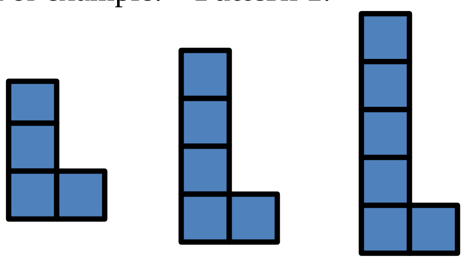
The use of visual patterns to concretize algebraic relationships and teach students to think algebraically is not uncommon. Unfortunately, students are directed to make a table to find a pattern, rather than building strong figural reasoning. This often oversimplifies the lesson meaning that students merely count to complete a table and ignore the visual model. This creates iterative thinking such as (I just need to add 5 each time), guessing and checking, or the application of rote procedures, rather than an understanding of the structural relationships within the model.

This lesson was created to introduce teachers and students to a resource filled with visual patterns. Teachers can assign 2 or 3 patterns per pair of students, initially. As students become more confident in their ability, visual patterns can be assigned individually.

Fawn Nguyen, a middle school math teacher in California, has put together a library of visual patterns here. In the teacher section on this site is a helpful tool for assigning random visual patterns to students as well as a form for students to use to help them organize their thinking.

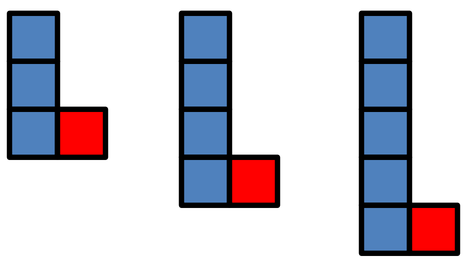
Students need multiple experiences working with and explaining patterns. Giving students a visual pattern to build concretely, allows students to experience the growth of the pattern and explain it based on that experience.

As students begin investigating visual patterns, ask them to look at how it grows from one stage (or step) to the next. When they build the pattern using manipulatives, have them use one color to begin with. Does any part of the pattern seem to be staying the same? Whatever students “see” as staying the same have them replace those pieces with a different color.
For example: **Pattern 1**:



Step 1 Step 2 Step 3

Students may be investigating the visual pattern on the left. Initially, students will build the pattern with materials such as color tiles and use all blue (for example) tiles to build it.



Step 1 Step 2 Step 3

Next, after students look for parts that stay the same as the pattern grows, they use a different color tile to show what parts stay the same. These students thought that the “part that sticks out on the right” stay the same. Another group may say that “the bottom two of each step stay the same. Either way, students are encouraged to go with it.

Once students have identified the part that stays the same (the constant) and the part that changes (the variable), students can begin organize their thinking. One way to do this is through the use of an expanded t-table. This provides a place to organize all parts and the whole of the visual pattern based on the how students see it growing.

One version of this kind of table can be seen below. It’s important that students determine what information is important to keep up with in the table.

Step (Stage)	**Sketch**	**Stays Same**	**Changes**	**Total (tiles in this case)**
1	This column allows students to keep a record of their thinking. If they run out of time, they can pick up where they left off later without having to start from scratch.	**1**	**3**	**4**
2		**1**	**4**	**5**
3		**1**	**5**	**6**
4		**1**	**6**	**7**
5		**1**	**7**	**8**
13		**1**	**15**	**16**
37		**1**	**39**	**40**
N		**1**	**n +2**	**n + 2 + 1**

As students reason about the visual patterns, be sure to ask questions to check their understandings and address misconceptions.

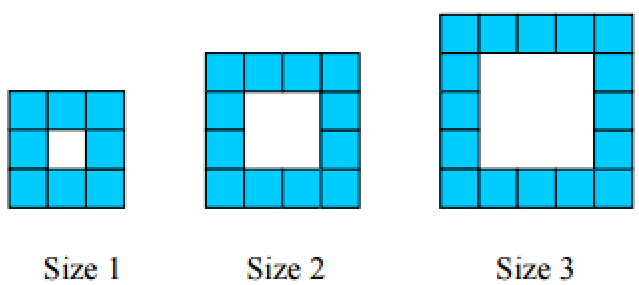
To help students move away from iterative reasoning (I just need to add 5 each time) to explicit reasoning, it is best to ask students to build the next two steps or stages (4 and 5 in the table above), then skip some. Choose a number that is not a multiple of 1 through 5, but fairly close. Some students will continue with their iterative reasoning to fill in the table for this stage. Next, choose a larger number, again not a multiple of any of the previous stage numbers. This gives students a subtle nudge to begin thinking about finding relationships within the data that has already been collected. When students determine a rule, they should check to see that it works for all of the stages. This is important because some rules or expressions may work for 2 or three stages, but not the rest. This is an act of being precise with the mathematics.

Students should also represent the patterns in other ways, such as on a coordinate grid. This gives students a preview of mathematics to come later on. It's also a nice way for students to attach the equations they create to a series of points on a line that represent the growth of the pattern. **IMPORTANT NOTE:** When students plot the points on the coordinate grid, they should not connect the points. In this context, it does not make sense to connect the points, since we can't have a fraction of a stage or a fraction of a square tile.

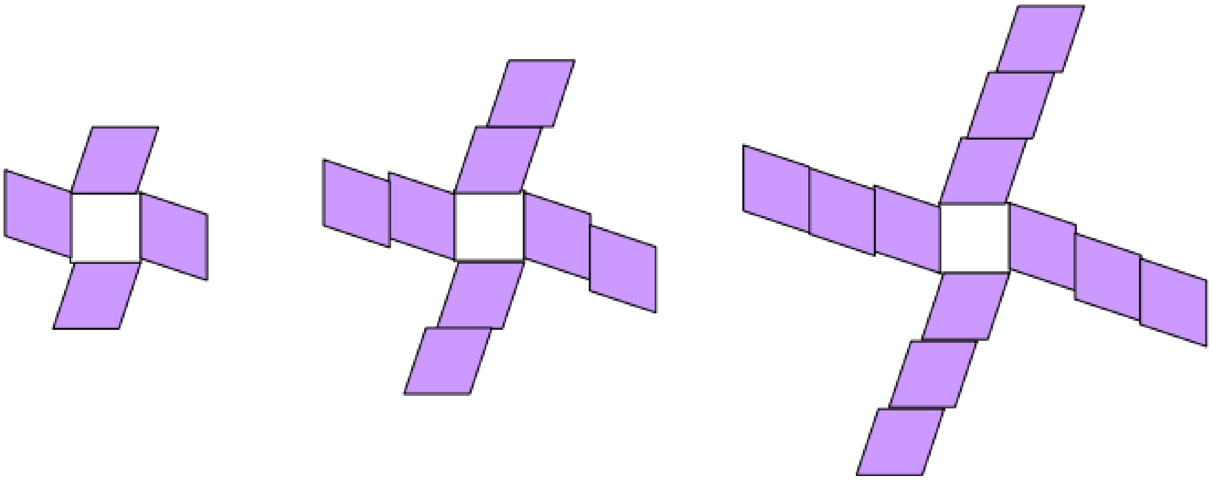
Continue to practice developing expressions to generate specific stages of visual patterns using samples listed below or developed on your own. A sheet of the patterns given as examples is provided as a handout after the activity.

Sample Patterns adapted from http://www2.edc.org/mathpartners

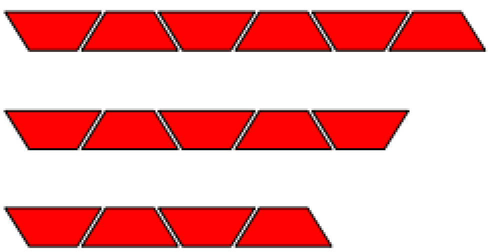
Pattern 2:



Pattern 3:



Pattern 4:



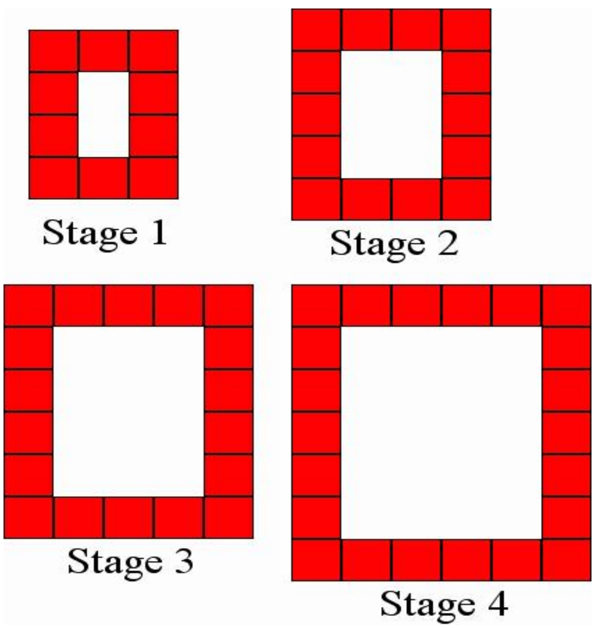
Number Challenge Extension Opportunity

After students have had experience with visual patterns, offer the extension opportunities listed below. Students are asked to build patterns numerically. Have students generate a rule for the pattern, draw a model (or use manipulatives), and ask others to solve their pattern challenge. You could have each student put one pattern on their desk and then have the class move around the room to solve as many as they can in a set period of time.

1. Build growing or shrinking patterns that have the number 20 as the fourth term.

2. Start at 100. Build as many shrinking patterns as possible that end exactly at zero.

FORMATIVE ASSESSMENT QUESTION



1. How many tiles are needed for a model at design 5?

2. How many tiles are needed for a model at design 11?

3. Explain how you determined the number needed for design 11.

4. Determine an expression for the number of tiles in a model of any design, *n*.

CLOSING

During the closing of the lesson, students should share their expressions/equations using precise mathematical language. Pairs of students who investigate similar patterns should discuss the expressions/equations they create – especially if they look different. For example – with the "L" shaped pattern mentioned above (pattern 1), depending on what students "see" as staying the same, the following expressions or equations may be derived:

$1 + n + 2$
$3 + n$
$4 + n – 1$

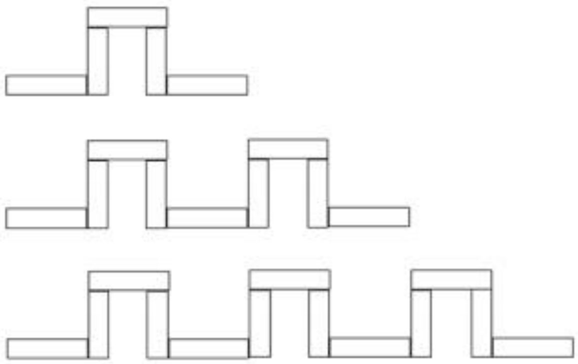
Students should determine whether or not these expressions work for the pattern, then determine why they all work, since some are very different looking.

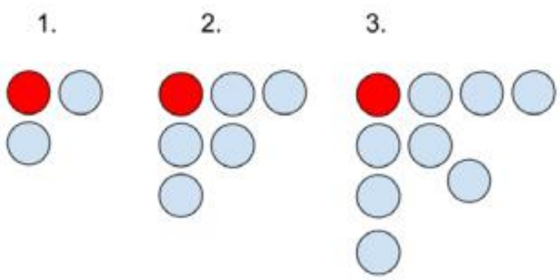
Extension:
Students should be encouraged to create their own growth patterns. This will allow for student creativity. Students should not only create the pattern, but also find the expression/equation that explains it with several examples as proof that their equation is true. Also, some patterns are much more challenging than others. Using these patterns can provide students the challenge they need. Finally, looking at the quantity of squares in the pattern is only one possible pattern to explore. Another option would be to look at perimeter for the same pattern. Are the patterns for these two different ideas similar or very different? Why? This gets even more interesting when the visual patterns are three-dimensional!

Intervention:
Students needing support can be given a graphic organizer like the one above. Using this can help students make sense of the patterns they are investigating based on how they see the pattern growing. Also, some patterns are easier to explain than others. Patterns using different shapes (such as pattern blocks) can be helpful to students needing support since the differing shapes can help them focus on what is changing and staying the same.

VISUAL PATTERNS PRACTICE

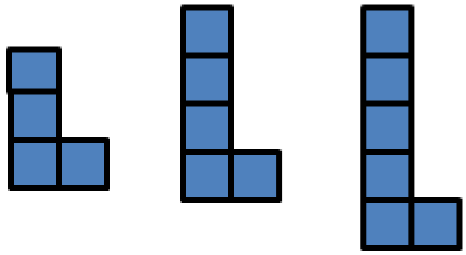
Look at the visual patterns below. Choose any two to investigate.
Write an algebraic expression to explain the pattern.
Using your expression, write an equation for the total: t = ____________
Graph your visual pattern on a coordinate grid.



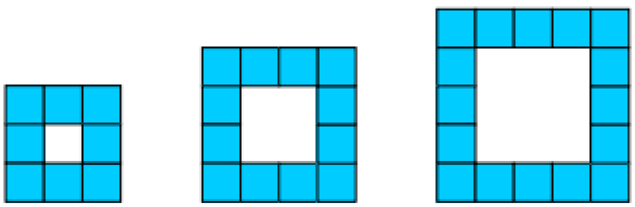


HANDOUT OF PATTERNS FEATURED IN THE LESSON

PATTERN 1

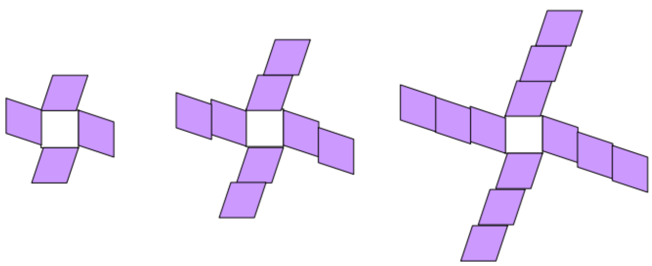


PATTERN 2

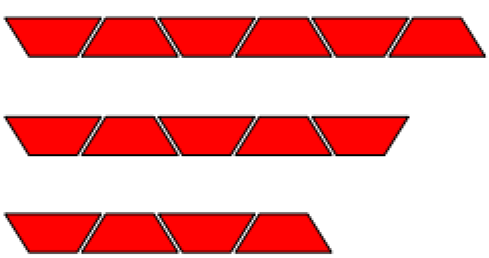


Size 1 Size 2 Size 3

PATTERN 3



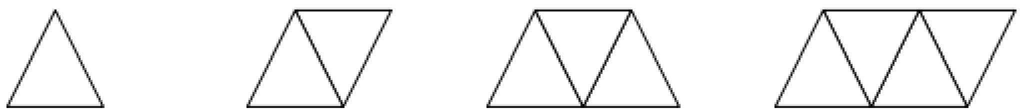
PATTERN 4



ADDITIONAL PRACTICE PROBLEMS

Please select problems from the set below or generate your own problems based on the performance of your students in the lesson above. Some students will not need as much practice as others. You may scaffold the assignment to increase the rigor or provide additional cues to open the problems to learners who might need more direction.

1. Consider the following pattern constructed by connecting toothpicks: (**adapted from NCTM Illuminations**)



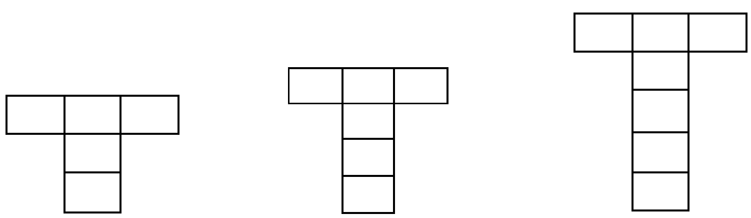
Stage 1 Stage 2 Stage 3 Stage 5

a) How can you represent the number of triangles in each stage of the pattern?

b) How can you represent the number of toothpicks needed to construct each stage?

c) Based on you representation of the toothpicks, how many toothpicks would you need for stage 8?

2. Consider the following pattern:



Stage 1 Stage 2 Stage 3

a) How can you represent the number of squares in each stage of the pattern?

b) How can you represent the number of toothpicks needed to construct each stage?

c) Based on you representation of the toothpicks, how many toothpicks would you need for stage 8?

For more visual patterns to try with your students, go to the source: *www.visualpatterns.org*

Additional Internet Resources

Patterns and Functions, Grades 6–8
http://www2.edc.org/MathPartners/pdfs/6-8%20Patterns%20and%20Functions.pdf

Investigating Growing Patterns, Mathwire
http://mathwire.com/algebra/growingpatterns.html

Inside the Math TOY TRAINS
http://bit.ly/1ETe2UN
This lesson challenges a student to use algebra to represent, analyze, and generalize a variety of functions including linear relationships. A student must be able to:

- relate and compare different forms of representation for a relationship including words, tables, graphs, and writing an equation to describe a functional pattern
- use rules of operations to extend a pattern and use its inverse

The lesson was developed by the Mathematics Assessment Resource Service (MARS) and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the lesson, the number of core points, and the percent of students that scored at standard on the lesson. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the LESSON, are included in the LESSON packet.

Beads Under the Clouds Lesson
http://education.ky.gov/educational/diff/Documents/BeadsUnderTheCloud.pdf
This problem solving lesson is intended to help you assess how well students are able to identify patterns in a realistic context: the number of beads of different colors that are hidden behind the cloud. In particular, this problem solving lesson aims to identify and help students who have difficulties with:

- Choosing an appropriate, systematic way to collect and organize data.
- Examining the data and looking for patterns.
- Describing and explaining findings clearly and effectively.

Exploring Expressions

Adapted from New York City Department of Education.

SUGGESTED TIME FOR THIS LESSON:

60-120 minutes
The suggested time for the class will vary depending upon the needs of the students.

STANDARDS FOR MATHEMATICAL CONTENT

MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.
a. Apply properties of operations emphasizing when the commutative property applies. (MGSE7.EE.1)
c. Model numerical expressions (arrays) leading to the modeling of algebraic expressions. (MGSE7.EE.1, 2; MGSE9-12.A.SSE.1, 3)
d. Add, subtract, and multiply algebraic expressions. (MGSE6.EE.3, MGSE6.EE.4, MC7.EE.1, MGSE9-12.A.SSE.3)
e. Generate equivalent expressions using properties of operations and understand various representations within context. For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between "3 more" and "3 times". (MGSE4.0A.2; MGSE6.EE.3, MGSE7.EE.1, 2, MGSE 9-12.A.SSE.3)
f. Evaluate formulas at specific values for variables. For example, use formulas such as A = l x w and find the area given the values for the length and width. (MGSE6.EE.2)

COMMON MISCONCEPTIONS

Students may see 3^2 *and think it means 3 x 2, and multiply the base by the exponent. Also, students may mistake* 3^2 *for 3 + 3 and vice versa, or p +p +p for* p^3 *(Here they may realize how many of the base they need in expanded form, but they add or count by the base, rather than multiplying.)*

STANDARDS FOR MATHEMATICAL PRACTICE

2. **Reason abstractly and quantitatively**. Students will work in verbal, expanded, and exponential forms of number representations.
3. **Construct a viable argument and critique the reasoning of others**. Students will be asked to evaluate responses of others to determine the accuracy along with providing an explanation in support of their choice.
7. **Look for and make use of structure**. Students will follow and apply properties of algebra to represent equivalent expressions.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Represent numerical relationships as expressions.
- Use algebraic expressions solve contextual based problems.

MATERIALS

- Personal white boards (or sheet protectors) if you have decided to use for Activator
- Student lesson pages
- Marbles and a bag (if using the differentiation activity)

ESSENTIAL QUESTIONS

- How can mathematical relationships be expressed with symbols?
- What does it mean to evaluate an expression?
- How can numerical and algebraic expressions be used to represent real life situations?
- How do number properties effect the creation of equivalent expressions?

KEY VOCABULARY

The following vocabulary terms should be discussed in the context of the lesson:
Exponent, base, expression, power, factor, evaluate, expanded form (of multiplication), exponential form, variable, coefficient, constant, equation, notation for multiplication, grouping symbol such as ( ), twice as much, more than, less than, the product of, the quotient of, difference, sum, total, increasing, decreasing, fewer than, equal to, associative property, commutative property, distributive property, additive inverse property, identity element, multiplicative inverse, equivalent expressions, substitution principle.

SUGGESTED GROUPING FOR THIS LESSON

Independent thought time for the activator sections, then followed by partner or small group for the short lessons followed by whole class discussion. This lesson will repeat these grouping cycles as new short lessons are introduced. Activators may be worked on personal white boards (or heavy duty sheet protectors with card stock inserts) for quick discussion and engagement.

<u>OPENER/ACTIVATOR</u> (A) NOTE: Student pages of lessons follow the Closing Activity

Pose the following problems as an activator for LESSON A:

1. Write 3^4 in repeated multiplication form. ***3 x 3 x 3 x 3***
2. Evaluate10^3. ***1000***
3. Find the area of a square computer desk with side of 18 inches. ***18x18 = 324 inches²*** $\boldsymbol{18 \times 18 = 324\ inches^2}$
4. Find the volume of a cube with sides of 3 inches. $\boldsymbol{3 \times 3 \times 3 = 27\ inches^3}$
5. Explain the difference between 4^3 and 3^4. **4^3 means 4x4x4 whereas 3^4 means 3x3x3x3**

After a short discussion on the activating problems, have students work in teams to answer the following:

Short LESSON Section A

LESSON 1: Mrs. Alexander's Offer Mrs. Alexander is offered a job with a computer company that will pay her $100,000. Represent the salary using exponential form.
Answer: ____10^5___

LESSON 2: Mr. Baily, a scientist who liked to express numbers with exponents, used a table to represent the number of mold spores in his bread experiment

Month	1	2	3	4	5
Number of mold spores	3	3x3	3x3x3	3x3x3x3	3x3x3x3x3

a) Mr. Baily asked his students to write an exponential expression to represent the total number of mold spores on the fourth day. Marcus wrote the expression 3^4 and John wrote 4^3. Who was correct? ***Answer: Marcus***

b) Justify your answer with a detailed explanation.
Marcus was correct because the number of 3's in the multiplication is 4 indicating a base of 3 and an exponent of 4.

c) If the pattern continued, how many mold spores would Mr. Baily have on the fifth day? Write your answer in exponential form. ***Answer:*** __3^5___

After students have time to work the short lesson problems, initiate a discussion based on their work. Then, post/ask activator (B).

ACTIVATOR (B)
Pose the following problems as an activator for lessons 3 and 4

Translate each verbal expression into an algebraic expression:
a) 4 less than the product of a number and 7 ***7n - 4***
b) The quotient of y and 6 $\frac{y}{6}$ ***or*** $y \div 6$
c) The sum of a number and 225 ***n + 22***
d) Twice a number ***2n***

After a short discussion on the activating problems, have students work in teams to answer the following:

<u>LESSON 3</u>: Algebra Homework
Let h represent the number of hours Jordan spent on homework last week.

Part A: Jack spent 1/2 as much time on his homework as Jordan, plus an additional 3 hours. Write an expression for the number of hours he spent on his homework.

Expression: $\frac{h}{2} + 3$

Part B: Identify the number of terms, coefficient and constant in your expression.

Number of terms: **<u>2</u>** Coefficient: $\frac{1}{2}$ Constant: **<u>3</u>**

Part C: Jordan spent 12 hours doing her homework. How many hours did it take Jack to complete his homework?
Show your work. $\frac{12}{2} + 3$ ***Simplifies to 6 + 3 which is 9 hours*** Jack: ***9 hours***

<u>LESSON 4</u>: Algebra for Dogs

Caesar, a pit bull, weighs p pounds.

For Part A, write an expression for the weight in pounds of each of the dogs.
Each expression should include the variable p.

Part A
• Bruno, a pug, weighs 49 pounds less than Caesar: p - 49

• Mia, a Chihuahua, weighs $\frac{1}{17}$ as much as Caesar: $\frac{1}{17}$ p or $\frac{p}{17}$

• Lucy, a Great Dane, weighs twice as much Caesar, minus 15 pounds: 2p - 15

Part B
Set up a chart or organizer to represent the weights of the four dogs.

Answers may vary based on student preferences

Caesar	Bruno	Mia	Lucy
P	p - 49	$\frac{1}{17}$ p or $\frac{p}{17}$	2p - 15

Part C
Caesar weighs 68 pounds. How much do Bruno, Mia, and Lucy weigh?

Show or explain how you found this.

Bruno***:*** ***68-49 or 19 pounds***; Mia: $\frac{68}{17}$ ***or 4 pounds***; Lucy: ***2(68) – 15 or 121 pounds***

After students have time to work the short LESSON problems, initiate a discussion based on their work. Then, post/ask activator (C).

ACTIVATOR (C)
Pose the following problems as an activator for LESSON 5 and 6

Find an equivalent expression for each of the following:
1) 3(x – 2) ***3x - 6***
2) 8x + 12y ***4(2x + 3y) using factoring (NOT REQUIRED) or 12y + 8x using commutative property***
NOTE: Problem (2) as factoring is included as an extension. Factoring is not required in this module. Commutative property is part of this module
3) p +p +p +p +p ***5p***
4) 7y + 2p ***2p + 7y***
5) 3x + (5m + 5p) ***(3x + 5m) + 5p using Associative Property; 3x + (5p + 5m) using Commutative Property; 3x + 5(m + p) using Factoring (see explanation in #2)***

After a short discussion on the activating problems, have students work in teams to answer the following:

LESSON 5: Distributive Property of Colored Pencils

Miss Nix opens a box of pencils that contains 6 green, 8 blue, and 10 red pencils.
Write two equivalent expressions to show the total number of pencils in 5 boxes.
Expressions: 5 (6g + 8b + 10 r) or 5(6g) + 5(8b) + 5(10r) or 30g + 40b + 50r

Explain how you know that these two expressions are equivalent
Answers may vary based on student choice

<u>LESSON 6</u>: Equivalent Expressions

Are the following expressions equivalent? Explain your reasoning.

1) a + a + 2 + 2	2a + 4	***Yes. If you combine like terms on the left set of terms you match the right set of terms***
2) 6(p + 3)	6p + 6	***No. If you distribute on the left group of terms you get 6p + 18 instead of 6p + 6***
3) 10(y – x)	10y – 10x	***Yes. If you distribute on the left group you get 10y – 10x which matches the right set of terms.***

4) Tony says the three expressions below are equivalent to 4 + 24y. Is she correct?

Explain your reasoning for each.

1) (1 + 3) + (4y + 20y)

2) 4 + 10y + 14y

3) (20 – 16) + (44y – 20y)

1. Yes. If you combine like terms in each parenthesis you get 4 + 24y.

2. Yes, If you combine the 10y and the 14y you get 24y and keep the 4. The result is 4 + 24y.

3. Yes. If you combine like terms in each parenthesis you get 4 + 24y.

<u>CLOSING/SUMMARIZER</u>

Have students choose three (or more) of the key terms used during the lesson to connect in a sentence. You may choose to have students share their sentences or turn them in to provide feedback for an opening discussion for the next class.

For Example: A student may choose the words exponent, base, and expanded form. They could generate a sentence such as: “When writing an expression in expanded form, the base is multiplied by itself the number of times equivalent to the exponent. 5^3 would be 5x5x5 in expanded form.

<u>Differentiation Idea</u>: The following lesson is designed to help students develop connections between concrete experiences, pictorial representations of mathematical ideas, and abstract mathematical symbols. Based on the needs of your students, you may choose to use the following activities to introduce students to variables, expressions, and equations. You could have a bag of marbles (or other item) to model the situation.

Marbles Expression Mrs. Wright has a bag and puts some marbles inside the bag. There are 6 marbles on the floor. How can we represent the total number of marbles there are all together?

Ideas to consider as you discuss the unknown number of marbles in the bags:

1. The number of marbles in the bag is unknown, so choose a variable to represent it. ***m***.

2. The number of marbles on the floor stays the same (is a constant). What is that number? ***6*****.**

3. To find the number of marbles all together, we could add the number of marbles in the bag and the 6 marbles on the floor. What would that expression be? ***m + 6 would then represent the total number of marbles.***

4. Suppose you have 80 in the bag. Therefore, m = 80. Can you find the total number of marbles in the bag? ***By substituting 80 in the expression m + 6 we get 80 + 6. There are 86 marbles in all together.***

<u>Sample Formative Assessment Questions/Practice Problems</u>
The following problems/lessons could be used as formative assessments or as practice problems for students.

1. Mark has 12 comic books, Jeremy has 14 comic books, and Sam has half as many comic books as Jeremy.

a) Write a numerical expression to represent how many comic books the three boys have together.

Mark + Jeremy + Sam
12 + 14 + 7

b) If each comic book cost $3.50, write two equivalent expressions to represent how much the boys have spent total.

3.50 (12 + 14 + 7) or 3.50(12) + 3.50(14) + 3.50(7)

c) How do you know that these two expressions are equivalent? Justify your response.

Using the distributive property you can show that they are equal.
3.50 (12 + 14 + 7) = 3.50(12) + 3.50(14) + 3.50(7)

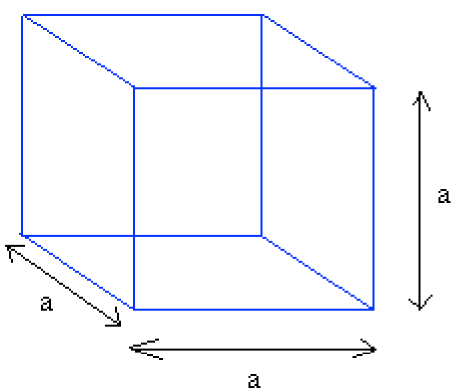
d) If Mark lost a few of his cards, write a new expression to represent how many cards Mark has now.

Since Mark had 12 cards and lost some unknown amount, we will represent the number of cards he lost with a variable such as x. His new amount would be 12 – x.

e) Explain the differences between a numeric expression and an algebraic expression.

A numeric expression contains numbers and operations where as an algebraic expression also contains unknown values represented by variables.

2. Samantha wishes to save $1,024 to buy a laptop for her birthday. She is saving her money in a cube shaped bank (shown below).



a) Write a variable expression to represent the volume of the bank.

$\boldsymbol{V = a \times a \times a \text{ or } a^3}$

b) If the length of the side of the bank is 8 inches, write two numerical expressions to find the volume of the bank. One expression should be in exponential form and the other should be in expanded form.

$\mathbf{V = 8 \times 8 \times 8 \text{ or } 8^3 \text{ in}^3}$

3) Mrs. Owens is making a quilt in the shape of a square for her younger sister. The border of the quilt is made of 1-foot by 1-foot patches. She asks one of her students, Kayla, to use the picture of the quilt below to write an expression to illustrate the number of patches needed to border the square quilt with side length ***s feet.***

a) Kayla writes: $(s + s + s + s) + (1 + 1 + 1 + 1)$ Is Kayla's expression correct? Explain your reasoning.

Yes, since the sides of the quilt are "s" feet long, it will take 4s or s + s + s + s to border each side and you will also need four additional squares for the corners. So, Kayla is correct.

Quilt WITH the border is shown.
The **Inner Square** is the quilt and the **Outer Area** shows the **Border**.
b) Mrs. Owens asks four other students (Sarah, Jeff, Nathan, and Bill) to generate expressions that are equivalent to Kayla's expression.
1. Sarah's expression: $4s + 4$
2. Jeff's expression: $4(s + 1)$
3. Nathan says the correct expression is $4s + 1$
4. Bill writes: $2s + 2(s + 2)$

How many of the students wrote a correct or an incorrect expression? Justify your answer with a detailed explanation.

- ***Sarah is correct because her expression represents Kayla's with like terms combined.***
- ***Jeff is also correct. Using the distributive property would generate an equivalent expression.***
- ***Bill is also correct using distributive property and like terms being combined.***
- ***Nathan is incorrect. He did not account for all four corners of the quilt.***

Numeric and Algebraic Expressions LESSON Name ______________________________

Short lesson Section A

LESSON 1: Mrs. Alexander's Offer Mrs. Alexander is offered a job with a computer company that will pay her $100,000. Represent the salary using exponential form.

LESSON 2: Mr. Baily, a scientist who liked to express numbers with exponents, used a table to represent the number of mold spores in his bread experiment

Month	1	2	3	4	5
Number of mold spores	3	3 x 3	3 x 3 x 3	3 x 3 x 3 x 3	

a) Mr. Baily asked his students to write an exponential expression to represent the total number of mold spores on the fourth day. Marcus wrote the expression 3^4 and John wrote 4^3. Who was correct?

b) Justify your answer with a detailed explanation.

c) If the pattern continued, how many mold spores would Mr. Baily have on the fifth day? Write your answer in exponential form.

<u>LESSON 3</u>: Algebra Homework
Let h represent the number of hours Jordan spent on homework last week.

Part A: Jack spent ½ as much time on his homework as Jordan, plus an additional 3 hours. Write an expression for the number of hours he spent on his homework.

Expression: ____

Part B: Identify the number of terms, coefficient and constant in the expression above.

Number of terms: ________ Coefficient: ________ Constant: ________

Part C: Jordan spent 12 hours doing her homework. How many hours did it take Jack to complete his homework?

Show your work. Jack: _______________ hours

<u>LESSON 4</u>: Algebra for Dogs

Caesar, a pit bull, weighs p pounds.

For Part A, write an expression for the weight in pounds of each of the dogs.
Each expression should include the variable p.

Part A
- Bruno, a pug, weighs 49 pounds less than Caesar: ______________________________
- Mia, a Chihuahua, weighs 1/17 as much as Caesar: ___________________________
- Lucy, a Great Dane, weighs twice as much Caesar, minus 15 pounds:________________

Part B Set up a chart or organizer to represent the weights of the four dogs.

Part C
Caesar weighs 68 pounds. How much do Bruno, Mia, and Lucy weigh?

Show or explain how you found this.

Bruno: _____________ pounds; Mia: ______________ pounds; Lucy: ______________ pounds

LESSON 5: Distributive Property of Color Pencils

Miss Nix opens a box of pencils that contains 6 green, 8 blue, and 10 red pencils.
Write two equivalent expressions to show the total number of pencils in 5 boxes.

Expressions: ____________________________ and __________________________________

Explain how you know that these two expressions are equivalent

LESSON 6: Equivalent Expressions

Are the following expressions equivalent? Explain your reasoning.

1) a + a + 2 + 2 = 2a + 4

2) 6(p + 3) = 6p + 6

3) 10(y – x) = 10y – 10x

4) Tony says the three expressions below are equivalent to 4 + 24y. Is she correct?

Explain your reasoning for each.

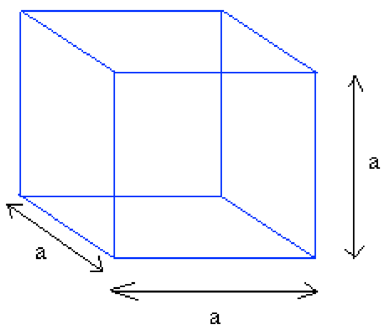
1) (1 + 3) + (4y + 20y)

2) 4 + 10y + 14y

3) (20 – 16) + (44y – 20y)

Formative Assessment Questions/Practice Problems **Name ______________________**

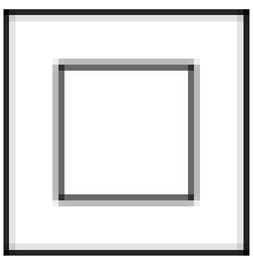
1. Samantha wishes to save $1,024 to buy a laptop for her birthday. She is saving her money in a cube shaped bank (shown below).



a) Write a variable expression to represent the volume of the bank.

b) If the length of the side of the bank is 8 inches, write two numerical expressions to find the volume of the bank. One expression should be in exponential form and the other should be in expanded form.

2. Mrs. Owens is making a quilt in the shape of a square for her younger sister. The border of the quilt is made of 1-foot by 1-foot patches. She asks one of her students, Kayla, to use the picture of the quilt below to write an expression to illustrate the number of patches needed to border the square quilt with side length ***s feet***.




Patch

a) Kayla writes: $(s + s + s + s) + (1 + 1 + 1 + 1)$
Is Kayla's expression correct? Explain your reasoning.

Quilt WITH the border is shown.
The **Inner Square** is the quilt and the **Outer Area** shows the **Border**.

b) Mrs. Owens asks four other students (Sarah, Jeff, Nathan, and Bill) to generate expressions that are equivalent to Kayla's expression.

1. Sarah's expression: $4s + 4$
2. Jeff's expression: $4(s + 1)$
3. Nathan says the correct expression is $4s + 1$
4. Bill writes: $2s + 2(s + 2)$

How many of the students wrote a correct or an incorrect expression? Justify your answer with a detailed explanation.

Equivalent Fractions

Adapted from Van De Walle, J. (2007). *Elementary and middle school mathematics: Teaching developmentally* (6th ed.) Boston: Pearson Education, Inc.

This lesson allows students to explore the relationship between equivalent fractions and to write equations for equivalent fractions using the product of a fraction equivalent to one.

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class.
Recommended time: 45-60 minutes. Recommended arrangement: individual or partners.

STANDARDS FOR MATHEMATICAL CONTENT

> **MFAPR2. Students will recognize and represent proportional relationships between quantities.**
> a. Relate proportionality to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students must understand what is happening to the part and the whole when horizontal lines are drawn and be able to make connections between the area models and their equations.
6. Attend to precision. Students should use precise mathematical language when explaining both in writing and in whole-group discussions what patterns they notice in the area models and equations.
7. Look for and make use of structure. Students should understand how equivalent fractions are formed by multiplication or division. They should also be able to connect the area model with its corresponding equation.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Create equivalent fractions using multiplication and division.
- Relate an area model used to create equivalent fractions with an equation that explains the model.

MATERIALS

- Student lesson sheet

ESSENTIAL QUESTIONS

- What happens to the value of a fraction when the numerator and denominator are multiplied or divided by the same number?
- How are equivalent fractions related?

The first essential question should not be shared with students until after students have completed the lesson.

OPENER/ACTIVATOR

Number Talks
Students can use friendly number strategies to begin discussing multiples and specifically, combining multiples to arrive at a larger product. For example, students could solve the following number string:

$1 \times 12 = 12$
$2 \times 12 = 24$

by combining the products 12 and 24 as well as combining the factors of 1 and 2, the students will produce another multiple of 12, namely $3 \times 12 = 36$.
Doubling and halving is also a helpful strategy in building equivalent fractions. (*Number Talks*, 2010, Sherry Parrish).

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

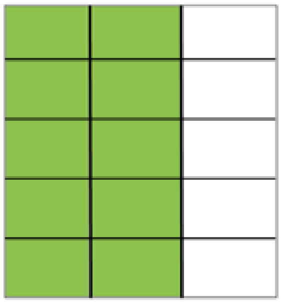
Background Knowledge:

This lesson was designed for students to learn an algorithm for finding equivalent fractions by multiplying a fraction by a fraction equivalent to one. Although some students in the Foundations of Algebra course may remember this algorithm, some may not. Therefore, it is important not to immediately remind students of this algorithm before they have had a chance to do the lesson and discover for themselves.

Lesson Directions:

This lesson will give students the opportunity to explore equivalent fractions. Students should follow the directions given in part 1 of the lesson by drawing horizontal line segments to create different but equivalent fractions. Before allowing students to continue on to part 2, initiate a whole-group discussion about any patterns students may have noticed in part 1.

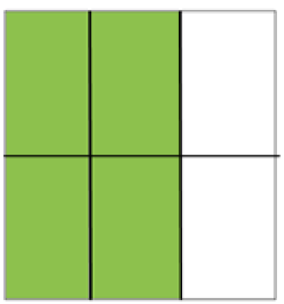
It is very important that students are making a connection between their equations and the corresponding area models. For example,



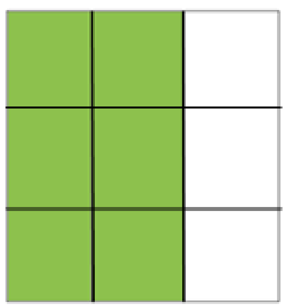
rather than drawing 5 horizontal line segments and then using that 5 to multiply $\frac{2}{3}$ by $\frac{5}{5}$, students should notice that 2×5 comes from the 2 original shaded sections being sliced into 2×5 smaller sections. Likewise, the whole which was originally comprised of 3 sections was sliced to create 3×5 smaller sections. Therefore, $\frac{2}{3}\times\frac{5}{5}=\frac{10}{15}$.

Ask several students to share equations that they wrote in part 1 and write these on the board. Engaging students in the above explanation may help students who are having trouble making connections between the area models and the equations. Ask students to record what they notice at the bottom of the page before continuing on to part 2. If necessary, have a similar discussion at the end of part 2.

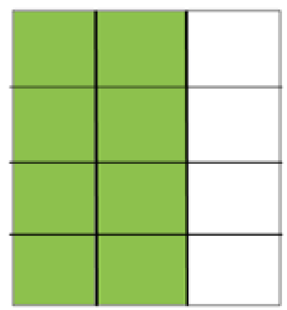
Possible Solutions to Part 1: Equivalent Fractions — $\frac{2}{3}$



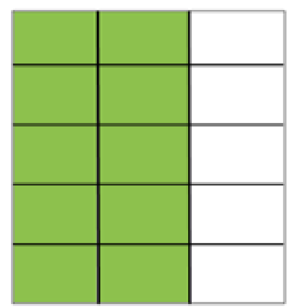
$\frac{2}{3}\times\frac{2}{2}=\frac{4}{6}$



$\frac{2}{3}\times\frac{3}{3}=\frac{6}{9}$

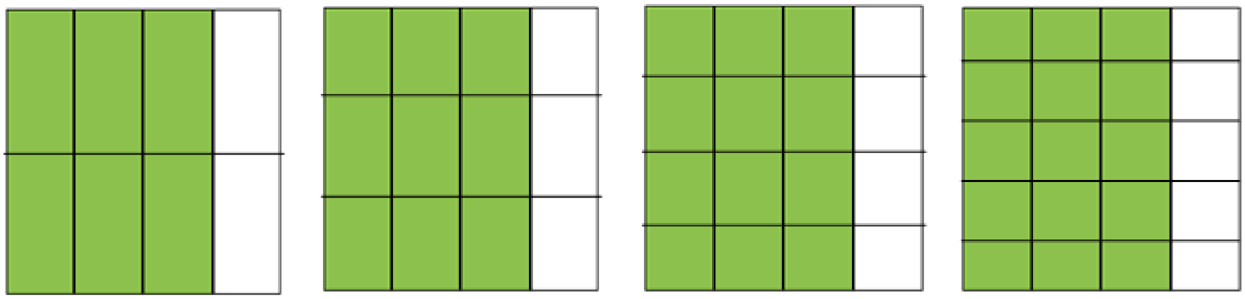


$\frac{2}{3}\times\frac{4}{4}=\frac{8}{12}$



$\frac{2}{3}\times\frac{5}{5}=\frac{10}{15}$

Possible Solutions to Part 2: Equivalent Fractions — $\frac{3}{4}$



$\frac{3}{4}\times\frac{2}{2}=\frac{6}{8}$ $\frac{3}{4}\times\frac{3}{3}=\frac{9}{12}$ $\frac{3}{4}\times\frac{4}{4}=\frac{12}{16}$ $\frac{3}{4}\times\frac{5}{5}=\frac{15}{20}$

Before moving on to part 3, ask students the following question:

What are some fractions equivalent to $\frac{8}{12}$?

Listen carefully to students' ideas to see if anyone mentions the possibility of creating equivalent fractions by *dividing* both the numerator and denominator by the same number. If not, a probing question such as this might help:

Are there any fractions equivalent to $\frac{8}{12}$ with numerators less than 8 and denominators less that 12?

Ultimately, we want students to see that equivalent fractions can be obtained both by multiplying and dividing. In this case,

$$\frac{8}{12}\times\frac{2}{2}=\frac{16}{24} \textit{ AND } \frac{8}{12}\div\frac{4}{4}=\frac{2}{3}.$$

For part 3, students should show evidence of both multiplying and dividing for the fractions $\frac{6}{8}$, $\frac{12}{15}$, and $\frac{20}{24}$.

For more information on this lesson, see *Slicing Squares* on p. 311 of Van De Walle, J. (2007). *Elementary and middle school mathematics: Teaching developmentally* (6th ed.) Boston: Pearson Education, Inc.

FORMATIVE ASSESSMENT QUESTIONS

- What product tells how many parts are shaded?
- What product tells how many parts are in the whole?

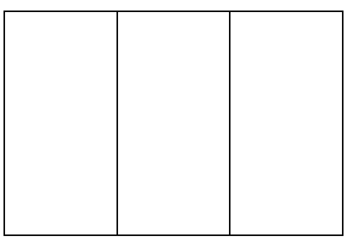
DIFFERENTIATION

Extension

This game will allow students the opportunity to use their knowledge of equivalent fractions.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=18

Intervention

For students struggling to make connections between the area model and the equations, provide them several unshaded "wholes" divided vertically into three equal parts (like the one below) and ask them to shade in $\frac{2}{3}$, and then draw the horizontal line segments to help them see where the fraction equivalent to one can be "seen" in the area model. This may need to be repeated for various numbers of horizontal lines.

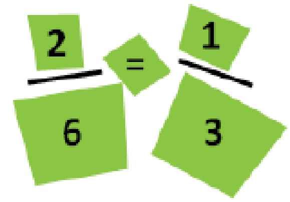


Additional practice can be found at http://illuminations.nctm.org/activity.aspx?id=3510. This activity allows students to create equivalent fractions and connect them to their location on the number line.
For extra help, please open the hyperlink Intervention Table.

CLOSING/SUMMARIZER

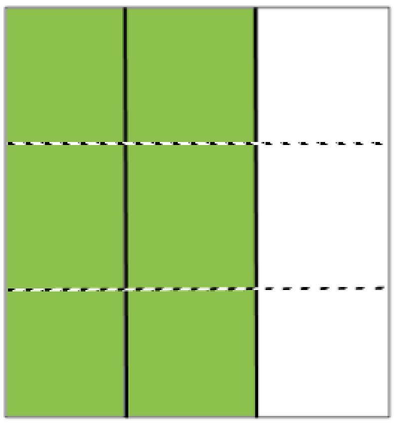
https://www.illustrativemathematics.org/content-standards/lessons/743
This lesson offers students an opportunity to explain why two fractions are equivalent. Students may complete it individually or can be given time to discuss with a partner before writing their own individual response.

Equivalent Fractions

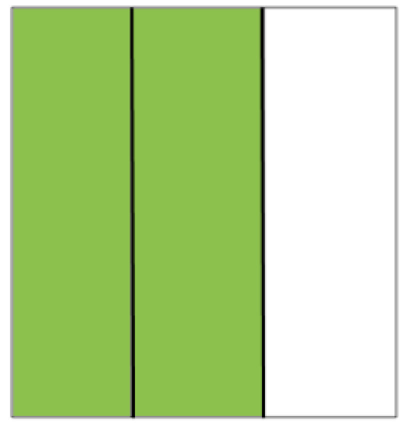


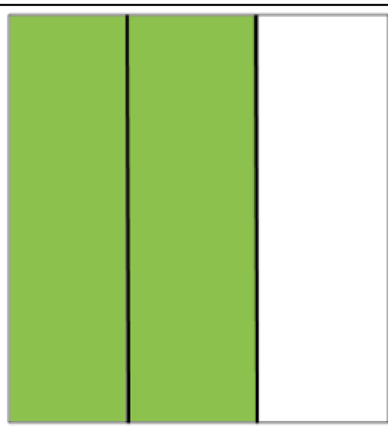
Part 1: Equivalent Fractions — $\frac{2}{3}$

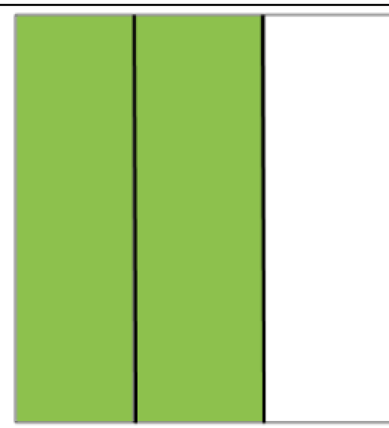
Find fractions that are equivalent to the fraction shown in each square below. Slice the squares by drawing horizontal line segments in each square to create a different but equivalent fraction. Then write an equation for each square. See the example below.



$$\frac{2}{3} = \frac{2}{3} \times \frac{3}{3} = \frac{6}{9}$$

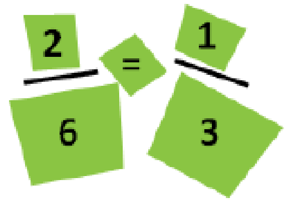




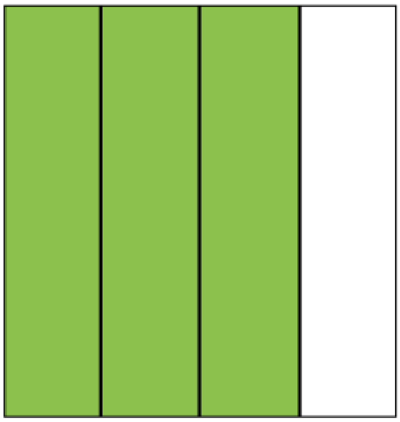


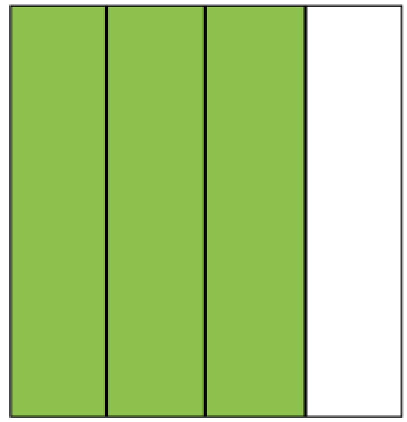
What patterns do you notice?

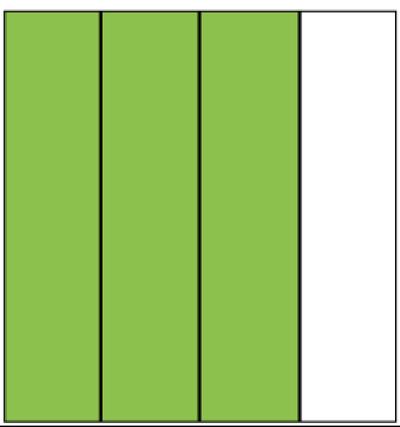
Part 2: Equivalent Fractions — $\frac{3}{4}$

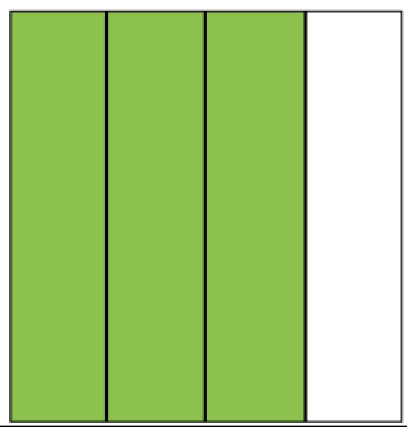


Find fractions that are equivalent to the fraction shown in each square below. Slice the squares by drawing horizontal line segments in each square to create a different but equivalent fraction. Then write an equation for each square. See the example below.



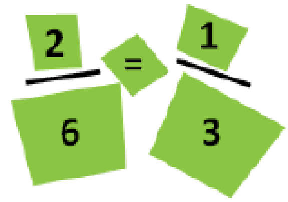






What patterns do you notice?

Part 3: Equivalent Fractions



Find fractions equivalent to the fractions in the table below. Use both multiplication and division, if possible. Record the equivalent fractions in the white boxes.

$\frac{1}{2}$						$\frac{2}{3}$		
$\frac{1}{3}$						$\frac{6}{8}$		
$\frac{1}{4}$						$\frac{2}{5}$		
$\frac{1}{5}$						$\frac{3}{5}$		
$\frac{1}{6}$						$\frac{12}{15}$		
						$\frac{20}{24}$		

[Snack Mix]

Lesson adapted from: http://mikewiernicki.com/snack-mix/

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class.
Recommended time: 45-60 minutes. Recommended arrangement: partners or small groups.

STANDARDS FOR MATHEMATICAL CONTENT

Students will use ratios to solve real-world and mathematical problems.

MFAPR1. Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations.* (MGSE6.RP.3)

MFAPR2. Students will recognize and represent proportional relationships between quantities.

a.Relate proportionality to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)

b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)

c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

Common Misconceptions

Students may see the relationship between the amount of granola and amount of candies as additive instead of multiplicative. In other words, they may think that the amount of granola will always be 2 parts more than the amount of candies, rather than seeing the amount of granola as $\frac{5}{3}$ *the amount of candies or seeing the amount of candies as* $\frac{3}{5}$ *the amount of granola. This misconception could lead students to believe that a 4-cup mixture would consist of 10 parts granola and 8 parts candies.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students must make sense of the problem in order to devise a plan for solving it.

2. Reason abstractly and quantitatively. Students must use quantitative reasoning to create a representation of the problem and to understand how the quantities relate.

3. Construct viable arguments and critique the reasoning of others. Throughout the problem, students will need to communicate their mathematical thinking to their peers as they evaluate their own and their peers' understanding of the problem, the model they create, and the reasonableness of their answer.
4. Model with mathematics. Based on Act 2, students will make a model to represent the situation.
5. Use appropriate tools strategically. Students will need to decide on an appropriate tool for their model, e.g., tables of values, tape diagrams, bar models, double number line diagrams.
6. Attend to precision. Students must use correct mathematical language as they communicate their thinking to their peers.
7. Look for and make use of structure. In order to create an accurate model, students must notice patterns in order to make equivalent ratios.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:
- Identify a multiplicative relationship.
- Use a model to find equivalent ratios.

MATERIALS

- Video and other information provided: http://mikewiernicki.com/snack-mix/
- 3-Act Recording Sheet (attached)
- Color tiles and/or Cuisenaire rods (optional)

ESSENTIAL QUESTIONS

In order to maintain a student-inquiry-based approach to this lesson, it may be beneficial to wait until Act 2 to share the Essential Questions with your students. By doing this, students will be allowed the opportunity to be very creative with their thinking in Act 1. By sharing the essential questions in Act 2, you will be able to narrow the focus of inquiry so that the outcome results in student learning directly related to the content standards aligned with this lesson.

- How can we find equivalent ratios?
- How can a model be used to find and organize equivalent ratios?

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Background Knowledge:

This lesson follows the Three-Act Math Task format originally developed by Dan Meyer. More information on this type of lesson may be found at http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/. A Three-Act Task is a whole-group mathematics lesson consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information, along with guidelines for Three-Act Tasks, may be found in each Comprehensive Course Overview.

Lesson Directions:

Act 1 – Whole Group - Pose the conflict and introduce students to the scenario by showing Act 1 picture. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/)
"Introduce the central conflict of your story/lesson clearly, visually, viscerally, using as few words as possible."

Show the Act 1 video to students: http://mikewiernicki.com/snack-mix/
Give students a copy of the 3-Act Recording Sheet.
Ask students what they noticed in the video, what they wonder about, and what questions they have about what they saw in the video. Facilitate a think-pair-share so that students have an opportunity to talk with each other before sharing questions with the whole group.
Share and record students' questions. The teacher may need to guide students so that the questions generated are math-related.

Anticipated questions students may ask and wish to answer:

- *How much of each ingredient is needed to fill all of the cups?
- How much of each ingredient does he have at the beginning of the video?
- How much of the mixture will he add to each red cup?
- How many red cups are there?

**Main question(s) to be investigated*

Once the class has decided on the main question to investigate, students should record the question on the recording sheet. Then, ask the students to estimate answers to their questions (think-pair-share). Students will write their best estimate, then write two more estimates – one that is too low and one that is too high so that they establish a range in which the solution should occur. Students should plot their three estimates on an empty number line. In this three-act lesson, students may want to plot their estimates on a double-number line to include both ingredients.

***Important note:** Although students will only investigate the main question(s) for this lesson, it is important for the teacher to not ignore student-generated questions. Additional questions may be answered after they have found a solution to the main question, or as homework or extra projects.*

Act 2 – **Student Exploration** - Provide additional information as students work toward solutions to their questions. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/**)**
"The protagonist/student overcomes obstacles, looks for resources, and develops new tools."

During Act 2, students decide on the facts, tools, and other information needed to answer the question(s) (from Act1). When students decide what they need to solve the problem, they should ask for those things. It is pivotal to the problem-solving process that students decide what is needed without being given the information up front.

Students may wish to use manipulatives to model the ratio of ingredients in the lesson. Colored tiles and/or Cuisenaire rods could be made available to students who wish to begin with the concrete representation. Other students may feel comfortable beginning with a table of values, double-number line, or a tape diagram.

The teacher provides guidance as needed during this phase. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin. Questioning is an effective strategy that can be used, with questions such as:

- What is the problem you are trying to solve?
- What do you think affects the situation?
- Can you explain what you've done so far?
- What strategies are you using?
- What assumptions are you making?
- What tools or models may help you?
- Why is that true?
- Does that make sense?
- Do the quantities have a multiplicative or additive relationship?

Additional Information for Act 2

Number of cups: 24
The ratio of ingredients video: http://mikewiernicki.com/snack-mix/
The ratio of servings per recipe video: http://mikewiernicki.com/snack-mix/

Act 3 – Whole Group – Share solutions and strategies.

Students present their solutions and strategies and compare them.
Lead discussion to compare these, asking questions such as:

- How reasonable was your estimate?
- Which strategy was most efficient?
- Can you think of another method that might have worked?
- What might you do differently next time?

Revisit initial student questions that weren't answered.

FORMATIVE ASSESSMENT QUESTIONS

- How can you tell that this relationship is multiplicative and not additive?
- How might you use the ratio of the two ingredients to determine the amount of each ingredient in 4 cups?
- What is the relationship between the amounts of each ingredient?
- What organizational strategies did you use?

DIFFERENTIATION

Extension

Ask students to create a similar problem but with a different context and different ratio. Two students working on the extension could exchange problems and provide feedback to each other.

Intervention

Encourage students to begin with the concrete representation using colored tiles or Cuisenaire rods. Ask them to model the relationship between the 2 ingredients for 2 cups. Then, ask them to show how the model would change for 4 cups.
Use grid paper to sketch a bar model of the ratio of the 2 ingredients for two cups. Ask students to extend their bars to represent the ratio of the ingredients for 4 cups.

For extra help, please open the hyperlink Intervention Table.

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question: ____

Estimate the result of the main question. Explain.

Place an estimate that is too high and too low on the number line

Low estimate	*Place an "x" where your estimate belongs.*	*High estimate*

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:_______________________________

Act 2 (cont.)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ *Make sense of problems & persevere in solving them*	□ *Use appropriate tools strategically.*
□ *Reason abstractly & quantitatively*	□ *Attend to precision.*
□ *Construct viable arguments & critique the reasoning of others.*	□ *Look for and make use of structure.*
□ *Model with mathematics.*	□ *Look for and express regularity in repeated reasoning.*

Proportional Relationships

Adapted from https://www.engageny.org/

In this lesson, students examine both proportional and non-proportional relationships, paying particular attention to the characteristics that make two variables proportional to one another.

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class.
Recommended time: 90-120 minutes. Recommended arrangement: small groups of 3-4.

STANDARDS FOR MATHEMATICAL CONTENT

Students will use ratios to solve real-world and mathematical problems.

MFAPR1. Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations.* (MGSE6.RP.3)

MFAPR2. Students will recognize and represent proportional relationships between quantities.

a. Relate proportionality to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)

b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)

c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

MFAPR3. Students will graph proportional relationships.

a. Interpret unit rates as slopes of graphs. (MGSE8.EE.5)

c. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (MGSE8.EE.5)

Common Misconceptions:

Students often forget proportional relationships always have a multiplicative relationship. When looking at a graph, some students may think every linear function is also proportional because of the constant rate of change, forgetting that the graph must also pass through the origin. Students may need to examine corresponding tables and graphs for a proportional and non-proportional relationship so they see that even though a relation may have a constant slope that does not necessarily mean the variables have a multiplicative relationship.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students must make sense of problems to be able to distinguish between variables that are in a proportional relationship and those that are not.
3. Construct viable arguments and critique the reasoning of others. Students will need to clearly communicate reasons why to variables are, or are not, proportional to one another.
4. Model with mathematics. Students will examine a variety of models to draw conclusions about relationships between variables.
6. Attend to precision. Students must exercise precision both in their mathematical explanations and in their calculations.
7. Look for and make use of structure. Students should begin to notice that when two variables are proportional, the dependent variable is always attained by multiplying the independent variable by a constant multiplier.
8. Look for and express regularity in repeated reasoning. Students will use repeated reasoning to identify a multiplicative relationship between variables.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Distinguish between variables that are proportional to one another and those that are not.
- Clearly explain how I know two variables are in a proportional relationship or how I know they are not.
- Identify a proportional relationship given a situation, table, graph, or equation.

MATERIALS

- Student lesson sheet
- Chart paper (with grid, if possible)
- Graph paper (if no grid paper is available)
- Glue sticks (if no grid paper is available)
- Envelopes
- Markers
- Sticky notes

ESSENTIAL QUESTIONS

What are the characteristics of a proportional relationship?
How can I identify a proportional relationship in a situation, table, graph, or equation?

OPENER/ACTIVATOR

Students should examine the proportional and non-proportional relationships below.

What characteristics do you see in the proportional relationships that make them different from the non-proportional relationships?

Proportional

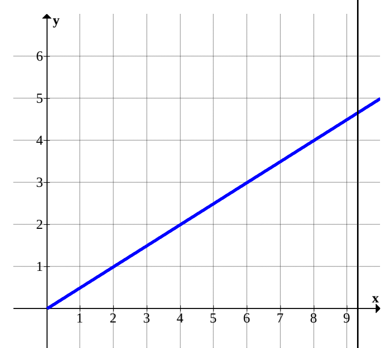
$C = 1.49n$

The number of candy bars, n, C is the total cost of the candy bars

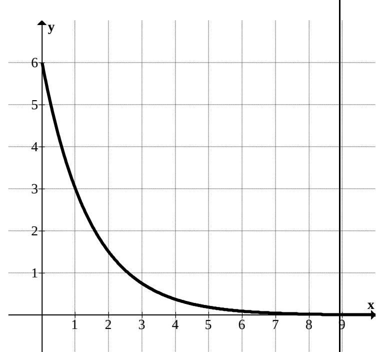
Proportional

Number of weeks	*Number of books read*
0	*0*
1	*7*
3	*21*
6	*42*

Proportional



Non-proportional



Non-proportional

$T = 35 + 10n$

Candice earned $10 a week in addition to the $35 she already had saved

Non-proportional

Input	*Output*
1	*2*
2	*4*
3	*8*
4	*16*

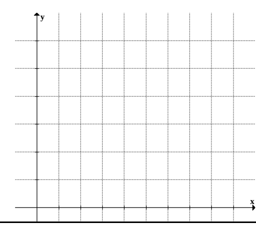
LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Due to the length of the lesson, it may be helpful to chunk this lesson. Begin by asking students to work through #6. It is important that students hear multiple ways to explain why two variables are, or are not, proportional to one another. Ask students to share their thinking and record various explanations on the board before moving on to the last problem in part 1.

Part 2 will allow students to compare a proportional relationship to a non-proportional relationship in the same scenario. Students should feel comfortable answering the questions in this section by using either the graph or the tables.

In Part 3, students will examine proportional and non-proportional relationships in multiple representations. Before class, cut and place the situations and corresponding 5 ratios in envelopes and label. Note, that even though there are cards for 8 different groups, there are only 4 different scenarios. It is likely that there may be different responses for the same scenario that can provide a rich discussion to summarize.

Each group of 3-4 students should be given one piece of chart paper, marker, and graph paper and glue stick, if chart paper does not include a grid. One member of the group should fold the chart paper into quarters like the layout to the right. Each group should read and record the problem, create a table and graph, and decide whether or not the variables are proportional. Students should be encouraged to provide a clear and concise explanation supporting their claim.

Problem:	*Table:*
Graph: 	*Proportional or Not? Explanation:*

After all groups have completed their poster, instruct students to post their work around the classroom. Students should rotate with their group around the room in a gallery walk to observe each poster leaving questions and feedback on sticky notes. Students should answer the following questions on their worksheets:

Are there any differences in groups that had the same ratios?
Do you notice any common mistakes? How might they be fixed?
Are there any groups that stood out by representing their problem and findings exceptionally well?

Group 1	*Group 2*	*Group 3*	*Group 4*
A local frozen yogurt shop is known for its monster sundaes to be shared by a group. The ratios represent the number of toppings to the total cost of the toppings. Create a table and graph and then explain if the quantities are proportional to each other.	*The school library receives money for every book sold at the school's book fair. The ratios represent the number of books sold to the amount of money the library receives. Create a table and graph and then explain if the quantities are proportional to each other.*	*Your uncle just bought a hybrid car and wants to take you and your sibling camping. The ratios represent the number of gallons of gas remaining to the number of hours of driving. Create a table and graph and then explain if the quantities are proportional to each other.*	*For a science project, Eli decided to study colonies of mold. He observed a piece of bread that was molding. The ratios represent the number of days passed to the number of colonies of mold on the bread. Create a table and graph and then explain if the quantities are proportional to each other.*
4 to 0	*1 to 5*	*8 to 0*	*1 to 1*
6 : 3	*2 to 10*	*After 1 hour of driving, there are 6 gallons of gas left in the tank.*	*2 to 4*
8 : 6	*The library received $15 for selling 3 books.*	*4 : 4*	*3 : 9*

The total cost of a 10-topping sundae is $9.	*4 : 20*	*2 to 7*	*4 : 16*
12 to 12	*5 : 25*	*0 : 8*	*Twenty-five colonies were found on the 5th day.*

Group 5	*Group 6*	*Group 7*	*Group 8*
For a science project, Eli decided to study colonies of mold. He observed a piece of bread that was molding. The ratios represent the number of days passed to the number of colonies of mold on the bread. Create a table and graph and then explain if the quantities are proportional to each other.	*Your uncle just bought a hybrid car and wants to take you and your sibling camping. The ratios represent the number of gallons of gas remaining to the number of hours of driving. Create a table and graph and then explain if the quantities are proportional to each other.*	*The school library receives money for every book sold at the school's book fair. The ratios represent the number of books sold to the amount of money the library receives. Create a table and graph and then explain if the quantities are proportional to each other.*	*A local frozen yogurt shop is known for its monster sundaes to be shared by a group. The ratios represent the number of toppings to the total cost of the toppings. Create a table and graph and then explain if the quantities are proportional to each other.*
1 to 1	*8 to 0*	*1 to 5*	*4 to 0*
2 to 4	*After 1 hour of driving, there are 6 gallons of gas left in the tank.*	*2 to 10*	*6 : 3*

3 : 9	*4 : 4*	*The library received $15 for selling 3 books.*	*8 : 6*
4 : 16	*2 to 7*	*4 : 20*	*The total cost of a 10-topping sundae is $9.*
Twenty-five colonies were found on the 5th day.	*0 : 8*	*5 : 25*	*12 to 12*

FORMATIVE ASSESSMENT QUESTIONS

- What are the characteristics of a proportional relationship?
- Does the relationship have a constant multiplier?

DIFFERENTIATION

Extension

For students who are ready for an extra challenge, ask them to create their own proportional relationship and display it in a table and on a coordinate grid. Additionally, they may write an equation to model the scenario and create a set of questions to be answered using the graph, table, and equation.

Intervention

Some students may need a graphic organizer to help them distinguish variables that are proportional to one another to those that are not. It may be helpful to provide students a copy of the 6 relationships in the opener to glue into their interactive notebook. In the notebook, they should also record explanations for why each problem has been characterized as either proportional or non-proportional.

CLOSING/SUMMARIZER

Journal entry: Suppose we invite students from another school to walk through our gallery. What would they be able to learn about proportional reasoning from our posters?

<u>Proportional Relationships</u>

Part 1: Proportional or Not?

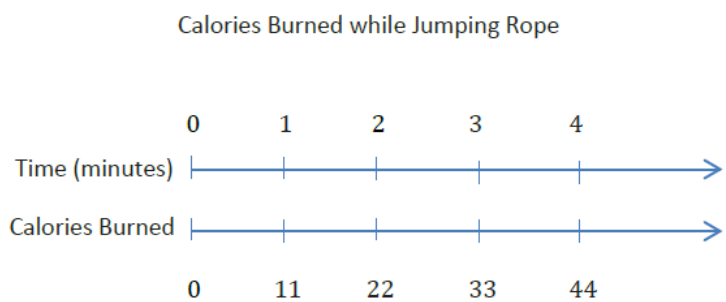
Determine whether or not the following situations represent two quantities that are proportional to each other. Explain your reasoning.

1. A new self-serve frozen yogurt store opened that sells its yogurt at a price based upon the total weight of the yogurt and its toppings in a dish. Each member of Isabelle's family weighed their dish and this is what they found.

Weight (ounces)	*12.5*	*10*	*5*	*8*
Cost ($)	*5*	*4*	*2*	*3.20*

Answers may vary. A possible solution:
Yes. Weight and cost have a multiplicative relationship.

2. During Jose's physical education class today, students visited activity stations. Next to each station was a chart depicting the number of calories (on average) that would be burned by completing the activity.



Answers may vary. A possible solution:
Yes. The number of calories burned is the time in minutes multiplied by 11. These variables are proportional because they have a multiplicative relationship.

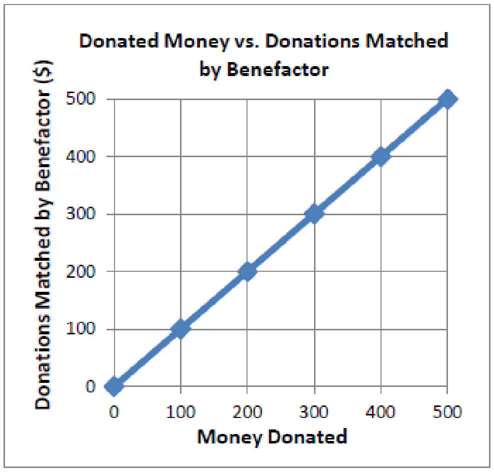
3. The table represents the relationship of the amount of snowfall (in inches) in 5 counties to the amount of time (in hours) of a recent winter storm.

x Time (h)	*2*	*6*	*8*	*2.5*	*7*

y *Snowfall (in.)*	*10*	*12*	*16*	*5*	*14*

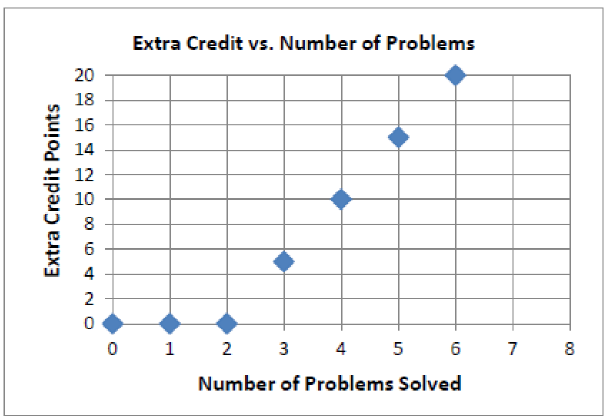
Answers may vary. A possible solution:
No. Snowfall and time do not have a constant rate of change.

4.



Answers may vary. A possible solution:
Yes. The graph passes through the origin and has a constant rate of change.

5.



Answers may vary. A possible solution:
No. Although the graph passes through the origin, it does not have a constant rate of change.

6. Jayden's favorite cake recipe calls for 5 cups of flour and 2 cups of sugar. The equation represents the relationships between the amount of flour, *F*¸ and the amount of sugar, *s*, in Jayden's recipe: $F = \frac{5}{2}s$.

Answers may vary. A possible solution: Yes. When $s = 0, F = 0$ and the independent variable is multiplied by a constant.

Alex spent the summer helping out at his family's business. He was hoping to earn enough money to buy a new $220 gaming system by the end of the summer. Halfway through the summer, after working for 4 weeks, he had earned $112. Alex wonders, "If I continue to work and earn money at this rate, will I have enough money to buy the gaming system by the end of the summer?"

To determine if he will earn enough money, he decided to make a table. He entered his total money earned at the end of Week 1 and his total money earned at the end of Week 4.

Week	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Total Earnings	***0***	*$28*	***56***	***84***	*$112*	***140***	***168***	***196***	***224***

1. Work with a partner to answer Alex's question.
Answers may vary.
Yes. By the end of the summer, Alex would have earned more than the $220 he needed.

2. Are Alex's total earnings proportional to the number of weeks he worked? How do you know?
Answers may vary.
Yes. The ratio of total earnings to the week number is always the same.

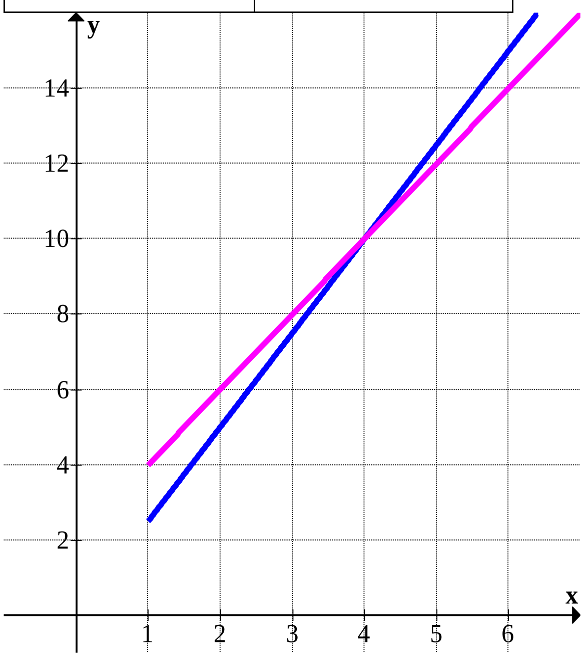
Part 2: Which Team Will Win the Race?

You have decided to walk in a long-distance race. There are two teams that you can join. Team A walks at a constant rate of 2.5 miles per hour. Team B walks 4 miles the first hour and then 2 miles per hour after that.

1. Create a table for each team showing the distances that would be walked for times of 1, 2, 3, 4, 5, and 6 hours. Then, plot the data for both teams on the same coordinate plane. Use the tables and the graphs to answer the questions that follow.

Team A	
Time (h)	*Distance (miles)*
1	***2.5***
2	***5***
3	***7.5***
4	***10***
5	***12.5***
6	***15***

Team B	
Time (h)	*Distance (miles)*
1	***4***
2	***6***
3	***8***
4	***10***
5	***12***
6	***14***



2. For which team is distance proportional to time? Explain your reasoning.

Answers may vary.
Team A. Distance and time for Team A have a multiplicative relationship.

3. Explain how you know distance for the other team is not proportional to time.

Answers may vary.
There is not a constant multiplier between distance and time.

4. At what distance in the race would it be better to be on Team B than Team A? Explain.

Answers may vary.
Team B is better up until 4 hours. The graph of Team B is higher than the graph of Team A.

5. If the members on each team walked for 10 hours, how far would each member walk on each team?
Answers may vary.
Members of Team A would walk for 25 miles in 10 hours.
Members of Team B would walk for 22 miles in 10 hours.

6. Will there always be a winning team, no matter what the length of the course? Why or why not?
Answers may vary.
No. At 4 hours, both teams have walked 10 miles.
7. If the race is 12 miles long, which team should you choose to be on if you wish to win? How do you know that team will win?

Answers may vary.
Team A will win. On the graph, Team A arrives at the 12 mile mark before Team B.

8. How much sooner would you finish on that team compared to the other team?

Answers may vary.
Some students may look at the graph or table and estimate. Although unlikely, some could calculate the hour in which Team A reaches the 12-mile mark by writing an equation and solving for the hour when the distance is 12.

Proportional Relationships

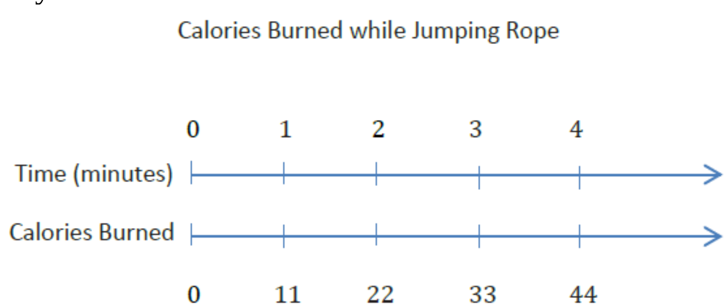
Part 1: Proportional or Not?

Determine whether or not the following situations represent two quantities that are proportional to each other. Explain your reasoning.

1. A new self-serve frozen yogurt store opened that sells its yogurt at a price based upon the total weight of the yogurt and its toppings in a dish. Each member of Isabelle's family weighed their dish and this is what they found.

Weight (ounces)	*12.5*	*10*	*5*	*8*
Cost ($)	*5*	*4*	*2*	*3.20*

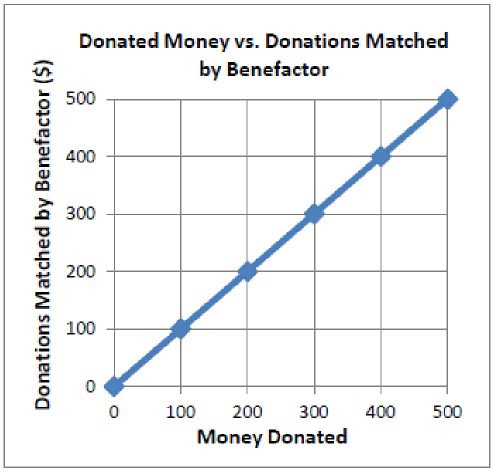
2. During Jose's physical education class today, students visited activity stations. Next to each station was a chart depicting the number of calories (on average) that would be burned by completing the activity.



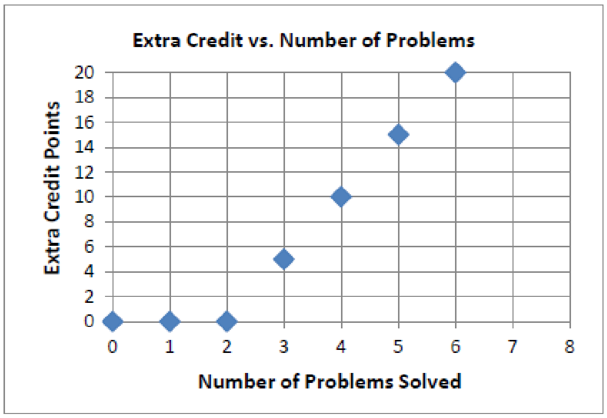
3. The table represents the relationship of the amount of snowfall (in inches) in 5 counties to the amount of time (in hours) of a recent winter storm.

x Time (h)	*2*	*6*	*8*	*2.5*	*7*
y Snowfall (in.)	*10*	*12*	*16*	*5*	*14*

4.



5.



6. Jayden's favorite cake recipe calls for 5 cups of flour and 2 cups of sugar. The equation represents the relationships between the amount of flour, F¸ and the amount of sugar, s, in Jayden's recipe: $F = \frac{5}{2}s$.

Alex spent the summer helping out at his family's business. He was hoping to earn enough money to buy a new $220 gaming system by the end of the summer. Halfway through the summer, after working for 4 weeks, he had earned $112. Alex wonders, "If I continue to work and earn money at this rate, will I have enough money to buy the gaming system by the end of the summer?"

To determine if he will earn enough money, he decided to make a table. He entered his total money earned at the end of Week 1 and his total money earned at the end of Week 4.

Week	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Total Earnings		*$28*			*$112*				

1. Work with a partner to answer Alex's question.
2. Are Alex's total earnings proportional to the number of weeks he worked? How do you know?

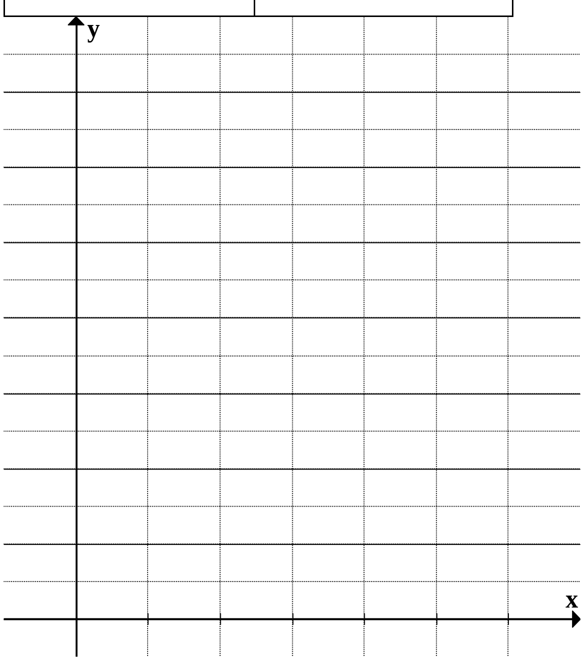
Part 2: Which Team Will Win the Race?

You have decided to walk in a long distance race. There are two teams that you can join. Team A walks at a constant rate of 2.5 miles per hour. Team B walks 4 miles the first hour and then 2 miles per hour after that.

1. Create a table for each team showing the distances that would be walked for times of 1, 2, 3, 4, 5, and 6 hours. Then, plot the data for both teams on the same coordinate plane. Use the tables and the graphs to answer the questions that follow.

Team A	
Time (h)	*Distance (miles)*

Team B	
Time (h)	*Distance (miles)*



2. For which team is distance proportional to time? Explain your reasoning.

3. Explain how you know distance for the other team is not proportional to time.

4. At what distance in the race would it be better to be on Team B than Team A? Explain.

5. If the members on each team walked for 10 hours, how far would each member walk on each team?

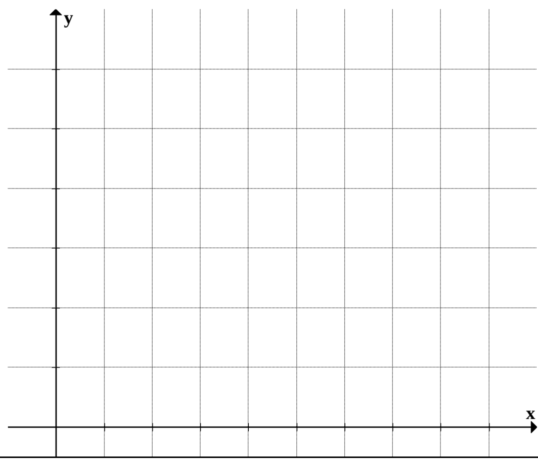
6. Will there always be a winning team, no matter what the length of the course? Why or why not?

7. If the race is 12 miles long, which team should you choose to be on if you wish to win? How do you know that team will win?

8. How much sooner would you finish on that team compared to the other team?

Part 3: Proportional and Non-Proportional Relationships in Multiple Representations

Use this layout as your group discusses your assigned problem.

Problem:	*Table:*
Graph: 	*Proportional or Not?* *Explanation:*

Gallery Walk

Use the table to record your notes on each of the posters. Think about the following questions:

Are there any differences in groups that had the same ratios?
Do you notice any common mistakes? How might they be fixed?
Are there any groups that stood out by representing their problem and findings exceptionally well?

Group 1	*Group 2*
Group 3	*Group 4*
Group 5	*Group 6*
Group 7	*Group 8*

Quick Check V

STANDARDS FOR MATHEMATICAL CONTENT

MFAPR1. Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations.* (MGSE6.RP.3)
MFAPR2. Students will recognize and represent proportional relationships between Relate proportionality **quantities.**
a. to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

Quick Check V

For each situation, determine if the identified variables are in a proportional relationship. If the variables are proportional, solve the problem.

1. If gasoline sells for $3.35 per gallon, how much will 8 gallons cost? (Amount of gasoline vs. Cost)

Amount of gasoline and cost are proportional.
$26.80

2. A plumber charges $60 to make a house call and $25 per hour for the time he works to fix the problem. If he works 3 hours, how much does he charge? (Hours worked vs. Charge)

Hours worked and charge are NOT proportional.

3. Granola sells for $2.75 per pound. How much will 7 pounds cost? (Number of pounds vs. Cost)
Number of pounds and cost are proportional.
$19.25

4. A school bus runs its route in 50 minutes going at 25 mph, on average. If it doubles its speed, how long will the run take? (Speed vs. Time)

Speed and time are NOT proportional.

5. A recipe that makes 16 cupcakes calls for $3\frac{1}{2}$ cup flour. How much flour is needed if you only wish to make 8 cupcakes? (Number of cupcakes vs. amount of flour)

Number of cupcakes and amount of flour are proportional.

$1\frac{3}{4}$ ***cup flour***

Quick Check V

For each situation, determine if the identified variables are in a proportional relationship. If the variables are proportional, solve the problem.

1. If gasoline sells for $3.35 per gallon, how much will 8 gallons cost? (Amount of gasoline vs. Cost)

2. A plumber charges $60 to make a house call and $25 per hour for the time he works to fix the problem. If he works 3 hours, how much does he charge? (Hours worked vs. Charge)

3. Granola sells for $2.75 per pound. How much will 7 pounds cost? (Number of pounds vs. Cost)

4. A school bus runs its route in 50 minutes going at 25 mph, on average. If it doubles its speed, how long will the run take? (Speed vs. Time)

5. A recipe that makes 16 cupcakes calls for $3\frac{1}{2}$ cup flour. How much flour is needed if you only wish to make 8 cupcakes? (Number of cupcakes vs. amount of flour)

Which Bed Bath & Beyond Coupon Should You Use?

Source: http://robertkaplinsky.com/work/bed-bath-beyond/

In this lesson, students will be presented with two coupons and will compare their values to determine the circumstances in which one coupon is better than (or equal) to the other.

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class.
Recommended time: 60-90 minutes. Recommended arrangement: partners or small groups.

STANDARDS FOR MATHEMATICAL CONTENT

MFAPR2. Students will recognize and represent proportional relationships between quantities.
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

Common Misconceptions:
Students who do not have a solid understanding of percents may not realize that the amount saved from the 20% off coupon varies depending on the cost of the item. They may wrongly assume because "20" is more than "5", that the 20% off coupon will always be better. When students begin comparing the coupons on specific items, they may not notice that the $5 coupon can only be applied when one has spent $15 or more. Thus, the $5 coupon cannot be applied toward the Melissa & Doug Floor Puzzle.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Making sense of this problem is crucial in being able to compare the values of coupons.
2. Reason abstractly and quantitatively. Students will need to understand the relationship between the discounted amount and the purchase price for each item.
3. Construct viable arguments and critique the reasoning of others. Students will need to analyze their mathematical work in order to construct an argument for their final decision.
6. Attend to precision. Students will need to attend to precise when calculating discounts. Additionally, their conclusions should include precise mathematical language.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Calculate purchase price given a percent discount and a flat amount discount.
- Compare the value of coupons for a variety of items and determine the circumstances in which one coupon is better than (or equal to) another.

MATERIALS

All images available to download at http://robertkaplinsky.com/work/bed-bath-beyond/ in JPEG and PowerPoint formats

- Images of both Bed Bath & Beyond coupons: $5 off any purchase of $15 or more and 20% off one single item
- Images of four items from Bed Bath & Beyond website
- Problem solving framework (provided in this document)

ESSENTIAL QUESTIONS

- Which coupon will save you the most money?
- What strategies can be useful when comparing coupons?

OPENER/ACTIVATOR

https://www.illustrativemathematics.org/content-standards/lessons/105

The sales team at an electronics store sold 48 computers last month. The manager at the store wants to encourage the sales team to sell more computers and is going to give all sales team members a bonus if the number of computers sold increases by 30% in the next month. How many computers must the sales team sell to receive the bonus? Explain your reasoning.

If the sales team is going to sell 30% more computer next month, they will have to sell

$$\mathbf{0.3 \times 48 = 14.4}$$

more computers. Of course, you cannot sell four-tenths of a computer, so that means they will have to sell 15 more computers. Since $\mathbf{48 + 15 = 63}$, they will need to sell 63 computers next month to receive a bonus.

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Lesson Directions:

Begin by showing students a picture of both Bed Bath & Beyond coupons: $5 off any purchase of $15 or more and 20% off one single item. Ask students to do a think-pair-share around the question "What problem are you trying to figure out?". This strategy will give students an opportunity to think individually about the question first, then collaborate with members of their group before finally sharing out in a whole-group discussion. Ask students to record their thoughts in the problem-solving framework provided. During the whole-group discussion when students are sharing their ideas, make a list of all questions raised by students. Together as a class, decide on the main question for this lesson. While it is important to honor *all* students' questions, the main question intended for this lesson is: *Which coupon will save you the most money?* Next, students should make a guess and record on their problem solving framework.

Now that the question has been identified, students in their small groups should brainstorm what they already know from the problem and what they need to know in order to answer the main question. Listen carefully for students who may begin realizing that it is necessary to know the amount of item(s) being purchased before being able to solve this problem. It is ok at this point if no student has come to this realization. The next steps will lead them to this discovery.

For students who do already realize the purhcase price is required, allow them time to think about the prices for which each coupon would be best and the item cost that would allow each coupon to be of equal value. Provide the sample items only when needed. It is likely, however, that many students will need to look at some sample items before making a decision. The four items provided were intentionally selected so that one is less that $15, another is between $15 and $25, one is exactly $25, and another is more than $25.

For students who need to work with the sample items, one option is to ask them to pretend they are buying each item. They will then need to compare the cost of the item after the discount has been applied to determine which coupon is the better deal. If time does not allow for them to compare coupons on all four items, give each group a different item and then ask the groups to share their findings in a whole-group discussion. After this discussion, the small groups can then construct their conclusions. During the work session, watch carefully for different strategies that would be beneficial to highlight in the closing.

FORMATIVE ASSESSMENT QUESTIONS

- Will both coupons always save you the same amount?
- How can we tell which coupon will save us more money?
- What restrictions apply to each coupon?
- Are you comparing the amount of the discount or the final amount of the product?

DIFFERENTIATION

Extension

Bed Bath & Beyond will accept multiple coupons in one purchase. Students who are ready to move beyond the initial problem may investigate how this policy affects the purchase of multiple items.

Intervention

Some students may be overwhelmed by comparing coupons on all four items. For these students, provide only one item at a time for them to compare. It may also be helpful to provide a chart to help them organize their work in order to make accurate comparisons.

For extra help with addition and subtraction, please open the hyperlink Intervention Table.

CLOSING/SUMMARIZER

Allow chosen groups to share their conclusions and strategies for solving the problem. After listening to the groups share, encourage all students to write a strong concluding statement that is rich in mathematical language and clearly communicates the solution.

Melissa & Doug® LAX Check! 48-Piece Floor Puzzle

The lacrosse sticks cross to block the shot as the blue team dives toward the goal! The intensity and beauty of this popular sport are captured in this thrilling moment--rendered extra-large and in vivid color on this 48-piece floor puzzle. The durable jigsaw pieces are made of extra-thick cardboard and coated with an easy-clean finish that keeps them looking like new. Imported.

Description	Available color	Price	Qty
Melissa & Doug® LAX Check! 48-Piece Floor Puzzle		$12.99 ea.	1

Bravo Sports Kryptonics 28-Inch Cruiser Board - Legend

Step up and cruise to your favorite haunts with confidence and style with the Kryptonics Legend longboard from Bravo Sports. Boasting eye-catching graphics and solid 9-ply maple construction, it features 61mm x 37mm PU injected wheels, heavy duty trucks with 12mm Risers, and ABEC 5 bearings. Measures 28" x 7.5". Imported. 30-day warranty.

Description	Available color	Price	Qty
Bravo Sports Kryptonics 28-Inch Cruiser Board - Legend		$29.99 ea.	1

Steering Wheel for iPhone and iPod Touch

Fulfill your need for speed with the Basic Steering Wheel for iPhone and iPod Touch. Whether car racing, motorcycle riding, or jet skiing, you can now maneuver your way through those interactive iPhone/iPod touch Apps with precision. This wheel gives you the feel of driving a vehicle instead of making sharp, erratic turns as you twist your iPhone or iPod Touch from side to side. Simply by connecting the iPhone or iPod Touch to its corresponding adapter then placing it in the middle of the wheel, you are ready to put the pedal to the metal and leave the competition in the dust. Steering column adds stability and balance, and an adjustable suction cup adheres to any flat surface. Includes adapters for iPhone, iPhone 3G, iPod Touch, iPod Touch 2G. Model #IP-SWS.

Description	Available color	Price	Qty
Steering Wheel for iPhone and iPod Touch	Black	$19.99 ea.	1

Sky Riders 36" Patriot Foam Glider

You'll have hours of outdoor fun soaring the skies with this Sky Riders Patriot Foam Glider. It features a huge 3-foot wingspan and can fly over 75 feet. Its aerodynamic design gives you extended air time, and is constructed from durable light-weight foam for easy use. Assembly is easy, too, and disassembles for convenient storage. Recommended for ages 4 and up. Constructed in the USA.

Description	Available color	Price	Qty
Sky Riders 36" Patriot Foam Glider		$25.00 ea.	1

Name ______________________________ Period: ______________ Date: ________________

What problem are you trying to figure out?	*What guesses do you have?*
What do you already know from the problem?	*What do you need to know to solve the problem?*

What should we title this lesson?

What is your conclusion? How did you reach that conclusion?

Adapted from http://robertkaplinsky.com/ Problem Solving Framework v7.1

Your work

Adapted from http://robertkaplinsky.com/ Problem Solving Framework v7.1

Quick Check VI

STANDARDS FOR MATHEMATICAL CONTENT

MFAPR1. Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations.* (MGSE6.RP.3)

MFAPR2. Students will recognize and represent proportional relationships between quantities.

a. Relate proportionality to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)

b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1, 2, 3; MGSE7.RP.1, 2)

c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2, 3)

MFAPR3. Students will graph proportional relationships.

a. Interpret unit rates as slopes of graphs. (MGSE8.EE.5)

b. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. (MGSE8.EE.6)

c. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (MGSE8.EE.5)

Quick Check VI

1. A biologist caught and tagged 250 fish. When she collected a sample of 75 fish, 5 of them were tagged. How many fish would she estimate are in the lake?

Approximately 3750 are in the lake.

2. A wildlife ranger caught and tagged 34 deer in his area of the national forest. He estimates that there are approximately 350 deer living in that area. If he is correct, what percent of the population did he tag?

The ranger tagged approximately 9.7% of the population.

3. A lake contains approximately 325 tagged fish. Results from several samples taken show that about 12% of the fish are tagged. Estimate the number of total fish in the lake.

There are approximately 2708 fish in the lake.

4. Maria is shopping for cheddar cheese:

Cubed cheese → 10 oz. for $4.50	***$.45 per ounce***
Block of cheese → 12 oz. for $3.84	***$.32 per ounce***
Shredded cheese → 16 oz. for $6.40	***$.40 per ounce***

Which package is the best deal? Explain your mathematical reasoning.

The block of cheese has a lower unit rate at $.32 per ounce, therefore the block of cheese is the best deal.

Question #5 from: https://www.illustrativemathematics.org/content-standards/lessons/2040

5. Emily has a coupon for 20 percent off of her purchase at the store. She finds a backpack that she likes on the discount rack. Its original price is $60 but everything on the rack comes with a 30 percent discount. Emily says:

Thirty percent and twenty percent make fifty percent so it will cost $30.

a. Is Emily correct? Explain.
It is true that 20% and 30% make 50%. But in the context of sale prices it is essential to keep track of the wholes to which these percents apply. For the backpack, the 30% discount applies to the original $60 price: 30% of $60 is 0.3×60=18 making the discount

on the backback$18. So after using the coupon, the backpack price becomes $42. Emily's additional 20% coupon applies not to the original backpack price but to the discounted price of $42: 20% of $42 is $8.40. Emily would need to save an additional $12 off the $42 price in order to buy the backpack for $30 so her calculations are not correct.

b. What price will Emily pay for the backpack?

As seen in part (a), Emily's coupon lowers the discount rack price by$8.40 so she will pay $42-8.40=33.60$ or $33.60.

Quick Check VI

1. A biologist caught and tagged 250 fish. When she collected a sample of 75 fish, 5 of them were tagged. How many fish would she estimate are in the lake?

2. A wildlife ranger caught and tagged 34 deer in his area of the national forest. He estimates that there are approximately 350 deer living in that area. If he is correct, what percent of the population did he tag?

3. A lake contains approximately 325 tagged fish. Results from several samples taken show that about 12% of the fish are tagged. Estimate the number of total fish in the lake.

4. Maria is shopping for cheddar cheese:

Cubed cheese → 10 oz. for $4.50
Block of cheese → 12 oz. for $3.84
Shredded cheese → 16 oz. for $6.40

Which package is the best deal? Explain your mathematical reasoning.

5. Emily has a coupon for 20 percent off of her purchase at the store. She finds a backpack that she likes on the discount rack. Its original price is $60 but everything on the rack comes with a 30 percent discount. Emily says:

Thirty percent and twenty percent make fifty percent so it will cost $30.

a. Is Emily correct? Explain.

b. What price will Emily pay for the backpack?

Analyzing Solutions of Equations and Inequalities

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class. Recommended timing is 90-120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI1. Students will create and solve equations and inequalities in one variable.

a. Use variables to represent an unknown number in a specified set (conceptual understanding of a variable). (MGSE6.EE.2, 5, 6)

e. Use variables to solve real-world and mathematical problems. (MGSE6.EE.7, MGSE.7.EE.4)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic representations for given contexts.
2. Reason abstractly and quantitatively. Students represent real world contexts through the use of variables in equations and inequalities, and reason about the possible solutions.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using verbal and written explanations accompanied by equations and inequalities.
4. Model with mathematics. Students form expressions, equations and inequalities from real world contexts.
6. Attend to precision. Students precisely define variables. Students substitute values into the equations and inequalities to verify results.
7. Look for and make use of structure. Students use patterns and structure to solve problems and reason using equations and inequalities.

EVIDENCE OF LEARNING/LEARNING TARGET
By the conclusion of this lesson, students should be able to:

- Distinguish between equality and inequality.
- Discuss how many values might make an equation or inequality true.

MATERIALS
Student Handout: Partner Problems
Mini-whiteboards (optional)

ESSENTIAL QUESTIONS
- How many values of a variable make an equation true?
- How many values of a variable make an inequality true?

OPENER/ACTIVATOR

Before beginning the lesson, check for students' prior understanding of the terms included in this lesson, in particular: value, variable, equation, inequality, and expression.

Prompt students with the following questions: Which of the following are equations?

a) $2n - 3 = 11$ b) $3x > 7$ c) $b - 3 = 12$ d) $15 + m$

What is the difference between an equation and an expression? What is the difference between an equation and an inequality?

LESSON

Explain that solving an equation or inequality is finding one or more values that make the statement true.

Present students with a statement like $10 - \_\_\_\_ = 7$. Have students determine a value that would make the statement false, and explain why it is false. Then have students determine which number(s) would make the statement true. (Students hopefully understand that 3 is the only number that will make the statement true.)

Students should work in pairs to answer the following questions (the expectation is that students replace the variable with the value to determine equality):

- ***Consider: $4 + b = 11$. Which value(s) will make the equation true? 6, 7, 8, 9***
- ***Consider $p - 8 > 13$ Which value(s) will make the inequality true? 28, 25, 21, 18***

Ask students if they are able to find more than one value that makes the equation true. (The expectation is that they will not.)

Show the statement $p - 8 > 13$, and have them compare this to $p - 8 = 13$.
Guiding questions: How are the solutions different for each statement? Are there any values in the previous list that will make the new statement true? Are there more values of p that make the statement true?

Keep the students in pairs and distribute Student Handout. Display, and have the students write an equation for the following: "The cash prize at BINGO is $24, and there were three winners in round one. If each winner receives the same amount of money, what amount could each winner receive?" Students should discuss possible solutions to their equation. (Solution: 3x = 24; each winner receives $8.) After students determine an answer, ask what strategies they used to determine the correct solution.

Display the equation $\frac{x}{4} = 3$. In their pairs, students should determine a solution and at least three non-solutions. Then, each pair can share their solutions and the strategies they used to determine the solutions with another pair. Each pair should write a scenario that could represent this equation. (x = 12)

Have the pairs complete questions 3 and 4. Have different pairs share their equations/inequalities, and their strategies for their work.

3: $27 + g = 50$, $g = \$23$

4: $3.50s + 27 \leq 50$, $s \leq 6.57$, therefore he can buy no more than 6 Sno-cones.

CLOSING/SUMMARIZER

Revisit the essential questions. Then prompt students with the following situation: Andrew bought a tie and dress shirt for $45. If the shirt cost $30, how much was the tie? Write an equation to represent this situation. What could the cost of the tie be? Is there more than one possible value? Why or why not?

ADDITIONAL PRACTICE

Suggested Additional Practice is attached.

1. Create a scenario similar to one of the scenarios presented in the partner problems. Your scenario will be traded with a partner during the next class. ***Responses will vary.***

2. Erica went in the store to buy a loaf of bread and gallon of milk. The milk costs $2.99 a gallon. Her mom only gave her $5 to spend. Write an inequality to represent the situation. What is a possible cost of the loaf of bread? Is there more than one possible value? Why or why not? ***2.99+b≤5.00, where b is the cost of bread. The loaf of bread must be $2.01 or less. There are many possible answers, as long as the price of the bread is less than or equal to $2.01.***

3. Which of the following is a solution(s) to $x + 6 = 13$? ***C. Other equations will vary, as long as the solution is 7.***

4. Which of the following is a solution(s) to $9 - n < 7$? ***b, c, d. Other inequalities will vary.***

Student Handout: Analyzing Solutions to Equations and Inequalities
Partner Problems

1. The cash prize at BINGO is $24, and there were three winners in round one. If each winner receives the same amount of money, what amount could each winner receive? Write an equation to represent the situation and determine a solution.

2. What is a solution to $\frac{x}{4} = 3$? Provide at least three values that do not make the statement true. Write a situation that could be represented by this equation.

3. Esteban has $50.00 to spend at Six Flags. He wants to ride a few of the roller coasters and play some of the games. The student pass into the park grants unlimited roller coaster rides, and costs $27. How much money can Esteban spend on games?

4. Esteban has decided that he just wants to ride the roller coasters and eat Sno-cones. How many Sno-cones could Esteban buy if each Sno-cone costs $3.50?

5. Could Esteban buy 6 Sno-cones? Why or why not?

6. Could he buy 10 Sno-cones? Why or why not?

Additional Practice: Analyzing Solutions to Equations and Inequalities

1. Create a scenario similar to one of the scenarios presented in the partner problems. Your scenario will be traded with a partner during the next class.

2. Erica went in the store to buy a loaf of bread and gallon of milk. The milk costs $2.99 a gallon. Her mom only gave her $5 to spend. Write an inequality to represent the situation. What is a possible cost of the loaf of bread? Is there more than one possible value? Why or why not?

3. Which of the following is a solution(s) to $x + 6 = 13$?

   $a.\ x = 9$
   $b.\ x = 8$
   $c.\ x = 7$
   $d.\ x = 6$

   Write another equation that has the same solution(s).

4. Which of the following is a solution(s) to $9 - n < 7$?

   $a.\ n = 1$
   $b.\ n = 3$
   $c.\ n = 4$
   $d.\ n = 5$

   Write another inequality that has the same solution(s).

Quick Check VII

STANDARDS FOR MATHEMATICAL CONTENT

MFAPR1. Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations.* (MGSE6.RP.3)

MFAPR2. Students will recognize and represent proportional relationships between quantities.

a. . Relate proportionality to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)

b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)

c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

MFAPR3. Students will graph proportional relationships.

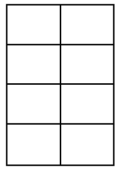
a. Interpret unit rates as slopes of graphs. (MGSE8.EE.5)

b. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. (MGSE8.EE.6)

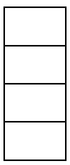
c. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (MGSE8.EE.5)

Quick Check VII

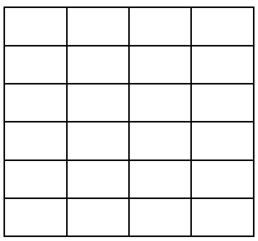
1. Given rectangle:



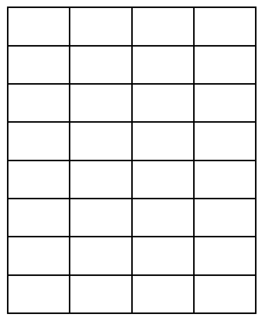
Which of the following rectangles is similar to the given rectangle?



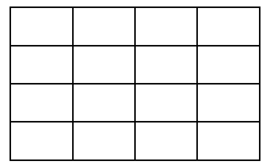
A



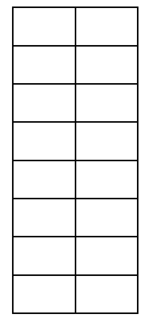
B



C



D



E

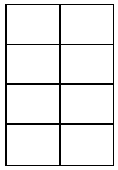
2. The ratio of the sides of a rectangle is $\frac{2}{3}$.

Which of the following could be the ratio of the sides of a similar rectangle?

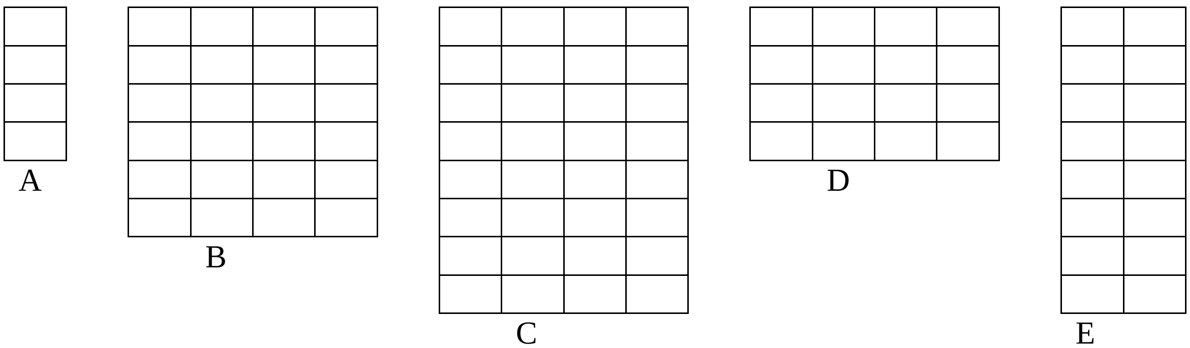
A. $\frac{4}{9}$ B. $\frac{4}{3}$ C. $\frac{2}{6}$ D. $\frac{4}{5}$ ***E.*** $\frac{6}{9}$

Quick Check VII

1. Given rectangle:



Which of the following rectangles is similar to the given rectangle?



2. The ratio of the sides of a rectangle is $\frac{2}{3}$.

Which of the following could be the ratio of the sides of a similar rectangle?

A. $\frac{4}{9}$ B. $\frac{4}{3}$ C. $\frac{2}{6}$ D. $\frac{4}{5}$ E. $\frac{6}{9}$

Set It Up

Adapted from *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*, Carpenter, Thomas, Megan Loef Franke, Linda Levi, 2003

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class. Recommended timing is 120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI1. Students will create and solve equations and inequalities in one variable.

a. Use variables to represent an unknown number in a specified set (conceptual understanding of a variable). (MGSE6.EE.2, 5, 6)

e. Use variables to solve real-world and mathematical problems. (MGSE6.EE.7, MGSE.7.EE.4)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic representations for given contexts and create contexts given equations.
2. Reason abstractly and quantitatively. Students represent real-world contexts through the use of real numbers and variables in mathematical expressions, equations.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using verbal or written explanations accompanied by equations, models, and tables.
4. Model with mathematics. Students model problem situations in symbolic, tabular, and contextual formats. Students form expressions and equations from real-world contexts and connect symbolic and visual representations.
6. Attend to precision. Students precisely define variables. Students substitute solutions into equations to verify their results.
5. Use appropriate tools strategically. Students use tables to organize information to write equations.
7. Look for and make use of structure. Students seek patterns or structures to model and solve problems using tables and equations.
8. Look for and express regularity in repeated reasoning. Students generalize effective processes for representing and solving equations based upon experiences.

EVIDENCE OF LEARNING/LEARNING TARGET
By the conclusion of this lesson, students should be able to:

- Discuss why conventions are necessary in mathematics.
- Use the balance method to write and solve one-step equations.

MATERIALS

Student Handout

ESSENTIAL QUESTIONS

- Why do we need conventions in mathematics? ***(The word "conventions" may need to be defined.)***
- How do I set up and solve a one-step equation?

OPENER/ACTIVATOR

Part I.
Marcus has 6 pet rabbits. He keeps them in two cages that are connected so they can go back and forth between the cages. One cage is red and the other cage is blue.

1. Show all the ways that 6 rabbits can be in two cages.

Solution

Red	*Blue*
6	*0*
5	*1*
4	*2*
3	*3*
2	*4*
1	*5*
0	*6*

Comment
Students may choose to illustrate the above situation with a pictorial representation.

2. Write an equation that represents the total number of rabbits.

Solution
$r + b = 6$ ***or*** $b = 6 - r$ ***or*** $r = 6 - b$

Comment
Students may need to add an additional column to their table to assist them in writing an equation.

Red	*Blue*	*Total*
6	*0*	*6*
5	*1*	*6*
4	*2*	*6*
3	*3*	*6*
2	*4*	*6*
1	*5*	*6*
0	*6*	*6*

Students may also use their understanding of fact families to write multiple equations that represent the rabbits.

3. Write a different equation that represents the rabbits.

   Solution
   $r + b = 6$ or $b = 6 - r$ or $r = 6 - b$

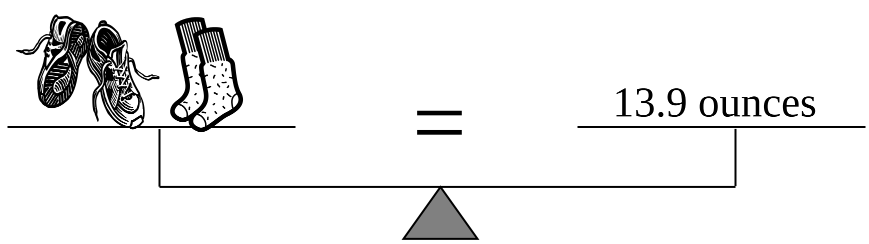
4. Write a different equation that represents the rabbits.

   Solution
   $r + b = 6$ or $b = 6 - r$ or $r = 6 - b$

STOP ***Note: Students should analyze the different equations and reflect on the addition and subtraction properties of equalities. Teachers can also begin a discussion on the relationship between the properties of equality and inverse operations.***

LESSON
Part II.

The diagram below represents a balance scale showing the combined weight of a pair of shoes and a pair of socks. This part of the lesson will lead you through the process of finding the weight of the pair of shoes and pair of socks.



1. Write an equation that represents the above balance scale.

   Solution

    +  = ***13.9 ounces***

2. What does 13.9 represent in the equation?

   Solution
   Thirteen and 9 tenths (13.9) represents the combined weight of the shoes and socks.

3. How can you find the weight of the pair of shoes if the pair of socks weighs 0.8 ounces? What do you notice about the shoes if the pair of socks weighs 0.8 ounces?

   Solution
   13.9 – 0.8 = 13.1

   The shoes weigh more than the socks and less than 13.9 ounces if the total weight is 13.9 ounces.

   Comment
   If students use trial and error to determine the weight of the shoes, guide them to use a different method to also find the weight. Guiding questions may be necessary to help students discover and use the inverse operation to "undo" the given operation to find the weight. Students need to determine that they can use the inverse operation to solve problems. This will help students understand why conventions are put in place to solve equations and will assist them when solving more complicated equations.

4. How can you find the weight of the pair of socks if the pair of shoes weighs 13.1 ounces?

   Solution
   The socks weigh less than the shoes and less than 13.9 ounces if the total weight is 13.9 ounces.

   13.9 – 13.1 = 0.8

   Comment
   If students use trial and error to determine the weight of the socks, guide them to use a different method to also find the weight. Guiding questions may be necessary to help students discover and use the inverse operation to "undo" the given operation to find the weight. Students need to determine that they can use the inverse operation to solve problems. This will help students understand why conventions are put in place to solve equations and will assist them when solving more complicated equations.

5.  +  = 13.9 ounces

   a. Using the diagram above, select a variable to represent the athletic shoes (tennis shoes).

***Comment:* Students may select any letter to represent the athletic shoes. To stay consistent we will select "a" to represent the athletic shoes.**

b. Select a variable to represent the socks.

***Comment:* Students may select any letter to represent the socks. To stay consistent we will select "s" to represent the socks.**

c. Write an equation that represents the above equations using variables instead of pictures.

Solution
$a + s = 13.9$
***Comment:* Students may need to first write the equation in words then write the equation using variables.**

d. Write an equation in terms of athletic shoes.

Solution
$a = 13.9 - s$
***Comment:* Students may think about fact families to help them develop this equation.**

e. Write an equation in terms of socks.

Solution
$s = 13.9 - a$

CLOSING/SUMMARIZER
Discuss the following questions: What conventions helped us in today's lesson scenario? Was it possible for different students to use different conventions? Why or why not? What was common about the set-up and solution methods for the two scenarios (rabbits vs. shoes/socks)?

For extra help with addition and subtraction, please open the hyperlink Intervention Table.

ADDITIONAL PRACTICE
Students should identify two scenarios that can be modeled with two variables and one constant. They should describe the scenario in their own words, define the variables and constants, and create one equation that models the situation. Scenarios can be collected, shuffled, and passed out randomly to the class for discussion. ***Responses will vary.***

Student Handout: Set It Up

Adapted from *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School.*

Part I.

Marcus has 6 pet rabbits. He keeps them in two cages that are connected so they can go back and forth between the cages. One cage is red and the other cage is blue.

1. Show all the ways that 6 rabbits can be in two cages.

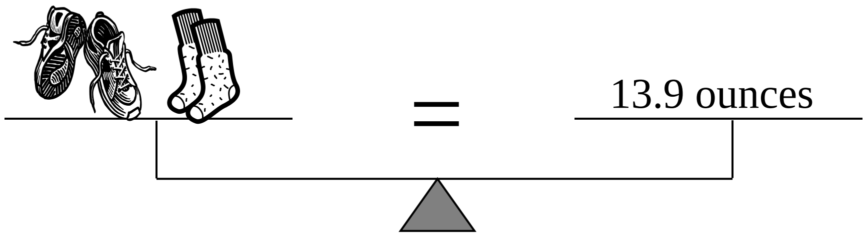
2. Write an equation that represents the rabbits.

3. Write a different equation that represents the rabbits.

4. Write a different equation that represents the rabbits.

Part II.

The diagram below represents a balance scale showing the combined weight of a pair of shoes and a pair of socks. This part of the lesson will lead you through the process of finding the weight of the pair of shoes and pair of socks.



1. Write an equation that represents the above balance scale.

2. What does 13.9 represent in the equation?

3. How can you find the weight of the pair of shoes if the pair of socks weighs 0.8 ounces? What do you notice about the shoes if the pair of socks weighs 0.8 ounces?

4. How can you find the weight of the pair of socks if the pair of shoes weighs 13.1 ounces?

5.  +  = 13.9 ounces

a. Using the diagram above, select a variable to represent the athletic shoes (tennis shoes).

b. Select a variable to represent the socks.

c. Write an equation that represents the above equations using variables instead of pictures.

d. Write an equation in terms of athletic shoes.

e. Write an equation in terms of socks.

Solving Equations Using Bar Diagrams

Adapted from *Putting the Practices into Action*, O'Connell, Susan and John San Giovanni, 2013

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class. Recommended timing is 120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI1. Students will create and solve equations and inequalities in one variable.

a. Explain each step in solving simple equations and inequalities using the equality properties of numbers. (MGSE9-12.A.REI.1)
b. Construct viable arguments to justify the solutions and methods of solving equations and inequalities. (MGSE9-12.A.REI.1)
e. Use variables to solve real-world and mathematical problems. (MGSE6.EE.7, MGSE.7.EE.4)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose an appropriate algebraic representation for given context.
4. Model with mathematics. Students model problems with equations and visual representations that connect to them.
6. Attend to precision. Students precisely define variables.
7. Look for and make use of structure. Students use structure to model and solve problems using equations and diagrams.

EVIDENCE OF LEARNING/LEARNING TARGET
By the conclusion of this lesson, students should be able to:

- Model equations using a bar diagram.
- Solve equations using bar diagram models.

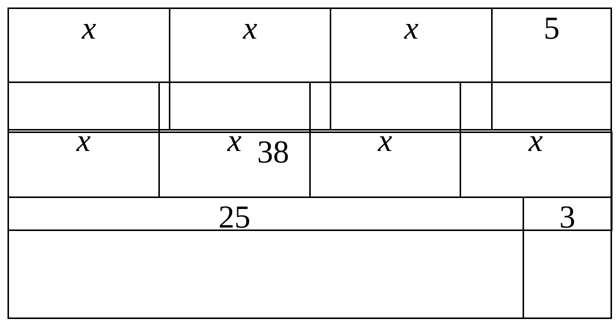
MATERIALS
Pencil
Paper
Tape
Learnzillion videos (core lesson and guided practice)

ESSENTIAL QUESTION

- How can a bar diagram help you solve an equation?

OPENER/ACTIVATOR

We can show $3x + 5 = 38$ with the following bar diagram:



Consider the following questions to guide the model:

- Is 3*x* more or less than 38?
- How much less is it? How do you know?
- How could you explain that using the bar diagram?

LESSON

Begin the lesson with the Learnzillion Core Lesson* video *entitled "Use a bar model to write and solve equations." Be sure to pause and discuss key points in the video and allow students to ask questions.

Use the Guided Practice* video *as another example. Again, it is important to pause and discuss key points in the video and allow students to ask questions.
*****There is also a downloadable PowerPoint presentation at http://www.learnzillion.com for this lesson that includes the core lesson, the guided practice problem, and extension activities.***

CLOSING/SUMMARIZER

Pose the following question to students, "How do bar diagrams help us model algebraic equations?"

Then explore the following:

We can show $4x - 3 = 25$ with the following bar diagram:

Consider the following questions to guide the model:

- Is 4*x* more or less than 25?
- How much more is it? How do you know?
- How does the bar diagram show this?

<u>ADDITIONAL PRACTICE (or could be used as Ticket Out of the Door)</u>
David went to Lenox Mall and purchased 3 shirts, all the same price, as well as a hat for $15. If he spent $47.50 at the mall, set up an equation and a bar diagram model that could be used to determine the cost of each shirt.

3x+15=47.50

For extra help, please open the hyperlink Intervention Table.

x	***x***	***x***	***15***
47.50			

When Is It Not Equal?

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class. Recommended timing is 90-120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI1. Students will create and solve equations and inequalities in one variable.

a. Use variables to represent an unknown number in a specified set (conceptual understanding of a variable). (MGSE6.EE.2, 5, 6)
d. Represent and find solutions graphically.
e. Use variables to solve real-world and mathematical problems. (MGSE6.EE.7, MGSE.7.EE.4)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic representations for given contexts and can create contexts given inequalities.
2. Reason abstractly and quantitatively. Students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using verbal or written explanations accompanied by inequalities and number lines.
4. Model with mathematics. Students model inequality situations on a number line.
5. Use appropriate tools strategically. Students use number lines to graph inequalities. Students use tables to organize information to write inequalities.
6. Attend to precision. Students precisely define variables.

EVIDENCE OF LEARNING/LEARNING TARGET
By the conclusion of this lesson, students should be able to:

- Write and solve one-step inequalities and graph them on the number line.

MATERIALS
Student Handout

ESSENTIAL QUESTIONS

- What strategies can I use to help me understand and represent real situations using inequalities?
- How are the solutions of equations and inequalities different?
- How can I write, interpret, manipulate and find solutions for inequalities?

OPENER/ACTIVATOR

Part one of this lesson is to help students recognize words in a word problem that indicate inequality. Students choose 3 statements to complete, and discuss their circled selections. As with previous lessons, pull struggling students aside individually while others work on the rest of the lesson. Have the struggling students discuss some of the "leftover" statements.

Write an inequality for three of the following seven statements. Then circle the hardest situation in numbers 1-7 and be prepared to share with the class.

1. You need to earn at least $50.
   $x \geq 50$
2. You can spend no more than $5.60
   $x \leq 5.60$
3. The trip will take at least 4 hours.
   $x \geq 4$
4. The car ride will be less than 8 hours.
   $x < 8$
5. Four boxes of candy contained at least 48 pieces total.
   $4x \geq 48$
6. With John's 7 marbles and mine, we had less than 20 marbles together.
   $x + 7 < 20$
7. Seven buses can hold no more than 560 students.
   $7x \leq 560$

LESSON

Graph the following inequalities. Then write a scenario that could be modeled by each inequality. ***Answers will vary. Check graphs for open/closed circles and direction of solution. Have students defend their responses.***

8. $p \geq 17$

9. $b \leq 7$

10. $t < 4$

11. $r > 10$

12. $k \leq 18$

13. $m > 1$

14. $d > 2$

Have students work with a partner on the following and discuss as a class:

Circle the numbers that are part of the solution for the inequalities below.

15. x + 2 > 5	(0 3 4 10)	***4 and 10 are solutions***
16. v – 4 < 10	(4 9 14 15)	***4 and 9 are solutions***
17. 4b ≤ 15	(0 3 5 6)	***0 and 3 are solutions***
18 $\frac{1}{3}r \leq 3\frac{1}{2}$	(6 9 15 30)	***6 and 9 are solutions***
19. 0.5w > 2.3	(2 4 5 10)	***5 and 10 are solutions***
20. t + 1.5 < 3.6	(0.6 1.7 2.1 3.2)	***0.6 and 1.5 are solutions***

CLOSING/SUMMARIZER

This lesson focused on inequalities. Ask students to compare and contrast the solutions they found in this lesson with how they find solutions to equations. You may decide to create a Venn diagram for similarities and differences in solution methods and conditions. You may also aid this discussion by having students rewrite their responses to the seven items of the opener of this lesson to make them equations.

ADDITIONAL PRACTICE

Suggested Additional Practice is attached. The solutions are provided below.

Write an inequality for each situation. Then use that inequality to choose and justify solutions listed for each situation. ***Students are translating the verbal expressions into statements of inequality, not solving inequalities.***

1. What is the minimum number of 80-passenger buses needed to transport 375 students? Choose and justify a solution (4, $4\ ^{11}/_{16}$, 5)
   80b ≥ 375
   $4\ ^{11}/_{16}$ and 5 are solutions to the inequality but 5 is the answer to the question.

2. What is the minimum speed needed to travel at least 440 miles in 8 hours? Choose and justify a solution (54 mph, 55 mph, 56 mph)
   8r ≥ 440
   55 and 56 are solutions the inequality but 55 is the answer to the question

3. What is the least number of boxes needed to package 300 candies if each box will hold 16 candies? Choose and justify a solution (18, 18 ¾ , 19)
   16b ≥ 300
   18 ¾ and 19 are solutions to the inequality but 19 is the answer to the question

Student Handout: When Is It Not Equal?

Write an inequality for **three** of the following seven statements. Then circle the hardest situation in numbers 1-7 and be prepared to share with the class.

1. You need to earn at least $50.
2. You can spend no more than $5.60
3. The trip will take at least 4 hours.
4. The car ride will be no more than 8 hours.
5. Four boxes of candy contained at least 48 pieces total.
6. With John's 7 marbles and mine, we had less than 20 marbles together.
7. Seven buses can hold no more than 560 students.

Graph the following inequalities. Then write a scenario that could be modeled by each inequality.

8. $p \geq 17$
9. $b \leq 7$
10. $t < 4$
11. $r > 10$
12. $k \leq 18$
13. $m > 1$
14. $d > 2$

With a partner, circle the numbers that are part of the solution for the inequalities below. Explain your reasoning.

15. $x + 2 > 5$ (0 3 4 10)

16. $v - 4 < 10$ (4 9 14 15)

17. $4b \leq 15$ (0 3 5 6)

18 $1/3r \geq 3\frac{1}{2}$ (6 9 15 30)

19. $0.5w > 2.3$ (2 4 5 10)

20. $t + 1.5 < 3.6$ (0.6 1.7 2.1 3.2)

Additional Practice: When Is It Not Equal?

Write an inequality for each situation. Then use that inequality to choose and justify solutions listed for each situation.

1. What is the minimum number of 80-passenger buses needed to transport 375 students? Choose and justify a solution (4, 4 $^{11}/_{16}$, 5)

2. What is the minimum speed needed to travel at least 440 miles in 8 hours? Choose and justify a solution (54 mph, 55 mph, 56 mph)

3. What is the least number of boxes are needed to package 300 candies if each box will hold 16 candies? Choose and justify a solution (18, 18 ¾, 19)

Which Ticket Is the Best Deal?

Adapted from Robert Kaplinsky http://robertkaplinsky.com/work/ticket-option/

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Suggested time, 1.5 hours.

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR2. Students will compare and graph functions.
a. Calculate rates of change of functions, comparing when rates increase, decrease, or stay constant. *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (MGSE6.RP.2;MGSE7.RP.1,2,3;MGSE8.F.2,5; MGSE9-12.F.IF.6)

STANDARDS FOR MATHEMATICAL PRACTICE

1**. Make sense of problems and persevere in solving them.** Students must make sense of the problem presented in the lesson and continue to work toward a solution.
2. Reason abstractly and quantitatively. Students can reason abstractly regarding options with proportions and prices.
3. Construct viable arguments and critique the reasoning of others. Students construct and defend their arguments as they determine what constitutes the "best deal" in different situations.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to calculate rates of change of functions and determine when rates change or remain the same.

MATERIALS

Copy of ticket prices

Which Ticket Is the Best Deal?

THE SITUATION

You are at a high school carnival's ticket booth and see the ticket prices below:

The Challenge(s)
•Which ticket option is the best deal?
•Which ticket option is the worst deal?
•Which ticket options are the same deal?
•How would you suggest they change their prices?

Question(s) To Ask:
•How did you reach that conclusion?
•Does anyone else have the same answer but a different explanation?

TEACHER NOTES: Consider This
It is important to note that the challenge question is not "Which ticket option should you buy?"   Asking about the "best", "worst", and "same" deals gets closer to the heart of comparing ratios and unit rates. Specifically, by "best deal" you are asking which ticket option has the lowest cost per ticket.

Looking at the painted ticket booth chart, it is clear that you are getting a better deal if you buy 12 tickets instead of 1 as well as 25 tickets instead of 12 tickets. However something strange happens when you consider purchasing 50 tickets instead of 25 tickets. You can buy the 25-ticket option twice for $10 each time which gives you 50 tickets for $20. Alternatively, you could buy the 50-ticket option for $25 and spend $5 more for the same number of tickets.

There are other interesting pricing options such as:

•12 tickets for $5 and 120 tickets for $50 (same price per ticket)
•1 ticket for $0.50 and 50 tickets for $25 (same price per ticket)
•5 sets of 25 tickets for $50 or 120 tickets for $50 (still cheaper to buy sets of 25 tickets)

The chart below lists the cost per ticket for each ticket option:

Total Cost	Tickets	Cost per Ticket
$0.50	1	$0.50
$5.00	12	$0.42
$10.00	25	$0.40
$25.00	50	$0.50
$50.00	120	$0.42

Looking at the chart, it is easy to see that the 25 tickets for $10 is the best deal. Tied for the worst deals are $0.50 for one ticket and $25 for 50 tickets. However, you could make the case

that the $25 for 50 tickets is worse in that you could have bought the tickets one at a time to have just the right amount. The same deals are the $5 for 12 tickets and the $50 for 120 tickets as well as the two worst options.

Encourage students to defend their reasoning using multiple explanations and discuss how they are connected. For the extension challenge, "How would you suggest they change their prices?", students can change the pricing structure so that the price per ticket decreases with the more tickets you buy. They should be able to explain their reasoning and show that the cost per ticket decreases when you buy larger quantities of tickets.

CLOSING/SUMMARIZER

Create a ticket poster for the carnival with at least 4 options. Determine if the unit rates for the tickets increase, decrease, or stay the same.

ADDITIONAL PRACTICE

The students' carnival posters may be used to review and practice finding rates of change.

Reviewing Rate of Change

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class. This lesson is intended for practice, perhaps for homework, small group, or paired practice. Timing will vary depending on grouping.

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR2. Students will compare and graph functions.
a. Calculate rates of change of functions, comparing when rates increase, decrease, or stay constant. *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (MGSE6.RP.2; MGSE7.RP.1,2,3;MGSE8.F.2,5; MGSE9-12.F.IF.6)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students interpret the situation and persevere in demonstrating their understanding, and use trial and error to find solutions.
4. Model with mathematics. Students use words, numbers, and pictures to solve problems.
5. Use appropriate tools strategically. Students select and use tools and strategies to interpret and solve word problems.
6. Attend to precision. Students use precise language in their discussions and explanations of proportions rate of change.

EVIDENCE OF LEARNING/LEARNING TARGET
By the conclusion of this lesson, students should be able to:
- demonstrate understanding of rate of change
- explain the concept of a unit rate as part – to – one.
- explain that the concept of a unit rate is related to a ratio.
- apply unit rates in real world situations.

MATERIALS
Student practice page

ESSENTIAL QUESTIONS

- How do you calculate a rate of change?
- What does a rate of change mean?

OPENER/ACTIVATOR

Adapted from Inside Mathematics. http://bit.ly/1BIEOPO

Sarah is organizing a party at the Vine House Hotel.

Vine House Hotel
Your fab party place!

Charges
$750 for up to 30 people
plus
$20 per person for each extra person

1 . Sarah thinks there will be 60 people at the party. Show that the cost will be $1350.

$750 + 30 X $20 = 750 + 600

2. What is the cost of a party for 100 people at the Vine House Hotel? $ ______________
Explain your reasoning.

$750 + 20 X 70 = $2150

3. What rate is being used in both the questions above? ***The rate is $20/person***

Sarah is organizing a party at the Vine House Hotel.

Vine House Hotel
Your fab party place!

Charges
$750 for up to 30 people
plus
$20 per person for each extra person

1 . Sarah thinks there will be 60 people at the party. Show that the cost will be $1350.

2. What is the cost of a party for 100 people at the Vine House Hotel? $ _____________
Explain your reasoning.

3. What rate is being used in both the questions above?

Reviewing Rate of Change

Complete the problems. Use words, numbers, and pictures to help solve the problems. Explain your process and answers.

1. Megan types $\frac{1}{6}$ of a page in $\frac{1}{12}$ of a minute. How much time does it take her to write a whole page?

½ minute or 30 seconds

2. Louis fills $\frac{1}{3}$ of a bottle in $\frac{1}{6}$ of a minute. How much time will it take him to fill the bottle?

½ minute or 30 seconds

3. Ben plays $\frac{1}{5}$ of a song on his guitar in $\frac{1}{15}$ of a minute. How much time will it take him to play the entire song?

⅓ minute or 20 seconds

4. Katie used $\frac{1}{3}$ of a gallon of water to make $\frac{1}{9}$ of a jug of tea. How much water is needed to fill the entire jug?

3 gallons

5. Lucy used ¼ of an ounce of nuts to make $\frac{1}{12}$ of a recipe of cookies. How many ounces of nuts will she need to make one full recipe of cookies?

3 ounces

Reviewing Rate of Change

Complete the problems. Use words, numbers, and pictures to help solve the problems. Explain your process and answers.

1. Megan types $\frac{1}{6}$ of a page in $\frac{1}{12}$ of a minute. How much time does it take her to write a whole page?

2. Louis fills $\frac{1}{3}$ of a bottle in $\frac{1}{6}$ of a minute. How much time will it take him to fill the bottle?

3. Ben plays $\frac{1}{5}$ of a song on his guitar in $\frac{1}{15}$ of a minute. How much time will it take him to play the entire song?

4. Katie used $\frac{1}{3}$ of a gallon of water to make $\frac{1}{9}$ of a jug of tea. How much water is needed to fill the entire jug?

5. Lucy used ¼ of an ounce of nuts to make $\frac{1}{12}$ of a recipe of cookies. How many ounces of nuts will she need to make one full recipe of cookies?

INTERVENTION

For extra help, please open the hyperlink Intervention Table.

CLOSING/SUMMARIZER

When driving an automobile, what rates of change can be calculated? ***Possible Solutions***: ***Miles per gallon for gasoline consumption, or miles per hour for speed***

ADDITIONAL PRACTICE

More practice involving rates of change can be found at https://www.ixl.com/math/grade-6/unit-rates-word-problems

WEB LINKS

The following websites are correlated to the designated 7th grade standards.

Unit 1

MGSE7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

https://www.illustrativemathematics.org/7.NS.A
http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132

MGSE7.NS.1a Show that a number and its opposite have a sum of 0 (are additive inverses). Describe situations in which opposite quantities combine to make 0. *For example, your bank account balance is -$25.00. You deposit $25.00 into your account. The net balance is $0.00.*

http://www.nzmaths.co.nz/search/node/integers
https://www.illustrativemathematics.org/7.NS.A
http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132

MGSE7.NS.1b Understand p + q as the number located a distance $|q||q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Interpret sums of rational numbers by describing real world contexts.

http://www.nzmaths.co.nz/search/node/integers
https://www.illustrativemathematics.org/7.NS.A
http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132

MGSE7.NS.1c Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

http://www.nzmaths.co.nz/search/node/integers
https://www.illustrativemathematics.org/7.NS.A
http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132

MGSE7.NS.1d Apply properties of operations as strategies to add and subtract rational numbers.

http://www.openmiddle.com/category/grade-7/the-number-system-grade-7/
http://mathmistakes.org/category/grade-7/the-number-system-grade-7/
https://www.illustrativemathematics.org/7.NS.A
http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132
http://fivetriangles.blogspot.com/2012/04/fractions.html

MGSE7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

http://www.openmiddle.com/category/grade-7/the-number-system-grade-7/
https://www.illustrativemathematics.org/7.NS.A.2
http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132

MGSE7.NS.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (- 1)(– 1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

http://www.nzmaths.co.nz/search/node/integers
https://www.illustrativemathematics.org/7.NS.A.2
http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132

MGSE7.NS.2b Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

http://www.nzmaths.co.nz/search/node/integers
https://www.illustrativemathematics.org/7.NS.A.2
http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132

MGSE7.NS.2c Apply properties of operations as strategies to multiply and divide rational numbers.

http://mathmistakes.org/category/grade-7/the-number-system-grade-7/
https://www.illustrativemathematics.org/7.NS.A.2
http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132

MGSE7.NS.2d Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
https://www.illustrativemathematics.org/7.NS.A
http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132

MGSE7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.
https://www.illustrativemathematics.org/7.NS.A
http://illuminations.nctm.org/Search.aspx?view=search&kw=integers&cc=2116_2132

Unit 2

MGSE7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

https://www.nsa.gov/academia/_files/2013_CDU_summaries/Primary-Patterns-AlgebraicThinking.pdf
https://www.illustrativemathematics.org/7.EE
http://illuminations.nctm.org/Lesson.aspx?id=3642

MGSE7.EE.2 Understand that rewriting an expression in different forms in a problem context can clarify the problem and how the quantities in it are related. *For example $a + 0.05a = 1.05a$ means that adding a 5% tax to a total is the same as multiplying the total by 1.05.*

https://www.illustrativemathematics.org/7.EE
http://illuminations.nctm.org/Lesson.aspx?id=3642

MGSE7.EE.3 Solve multistep real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals) by applying properties of operations as strategies to calculate with numbers, converting between forms as appropriate, and assessing the reasonableness of answers using mental computation and estimation strategies.
For example:

- *If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50.*
- *If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

https://www.nsa.gov/academia/_files/2013_CDU_summaries/Primary-Patterns-AlgebraicThinking.pdf

https://www.illustrativemathematics.org/7.EE
http://www.estimation180.com/lessons.html
http://illuminations.nctm.org/Lesson.aspx?id=3642

MGSE7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

https://www.nsa.gov/academia/_files/2013_CDU_summaries/Primary-Patterns-AlgebraicThinking.pdf

http://nzmaths.co.nz/resource/solving-multi-step-equations
https://www.illustrativemathematics.org/7.EE
http://www.estimation180.com/lessons.html

http://illuminations.nctm.org/Activity.aspx?id=3482

MGSE7.EE.4a Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

https://www.nsa.gov/academia/_files/2013_CDU_summaries/Primary-Patterns-AlgebraicThinking.pdf

http://nzmaths.co.nz/resource/solving-multi-step-equations
https://www.illustrativemathematics.org/7.EE
http://illuminations.nctm.org/Activity.aspx?id=3482

MGSE7.EE.4b Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example, as a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

https://www.illustrativemathematics.org/7.EE

MGSE7.EE.4c Solve real-world and mathematical problems by writing and solving equations of the form x+p = q and px = q in which p and q are rational numbers.
https://www.nsa.gov/academia/_files/2013_CDU_summaries/Primary-Patterns-AlgebraicThinking.pdf

http://nzmaths.co.nz/resource/solving-multi-step-equations
https://www.illustrativemathematics.org/7.EE
http://www.estimation180.com/lessons.html

Unit 3

MGSE7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction (1/2)/(1/4) miles per hour, equivalently 2 miles per hour.

http://nzmaths.co.nz/resource/comparing-finding-rates
http://nzmaths.co.nz/resource/ratios-and-rates
http://nzmaths.co.nz/resource/playing-money-other-point-view
http://mathmistakes.org/category/grade-7/ratios-proportional-relationships-grade-7/
http://www.estimation180.com/lessons.html

MGSE7.RP.2 Recognize and represent proportional relationships between quantities.

http://nzmaths.co.nz/resource/comparing-finding-rates
http://nzmaths.co.nz/resource/ratios-and-rates
http://nzmaths.co.nz/resource/ratios
http://nzmaths.co.nz/resource/playing-money-other-point-view
http://mathmistakes.org/category/grade-7/ratios-proportional-relationships-grade-7/

MGSE7.RP.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

http://nzmaths.co.nz/resource/comparing-finding-rates
http://www.estimation180.com/lessons.html

MGSE7.RP.2c Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.

http://nzmaths.co.nz/resource/ratios-and-rates

MGSE7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, and fees.

http://nzmaths.co.nz/resource/comparing-finding-rates
http://nzmaths.co.nz/resource/body-ratios
http://illuminations.nctm.org/Lesson.aspx?id=1049
http://nzmaths.co.nz/resource/playing-money-other-point-view
http://nzmaths.co.nz/resource/percentages
http://nzmaths.co.nz/resource/percentages-problems-two-steps
http://nzmaths.co.nz/resource/estimating-percentages
http://nzmaths.co.nz/resource/percentage-bar
http://nzmaths.co.nz/resource/getting-percentible
https://sites.google.com/site/sensiblemathematics/activities-by-concepts/percentages

MGSE7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

http://nzmaths.co.nz/resource/scale-factors-areas-and-volumes
http://nzmaths.co.nz/resource/russian-boxes
https://www.illustrativemathematics.org/content-standards/7/G/A/1/tasks
http://illuminations.nctm.org/Lesson.aspx?id=1049

Unit 4

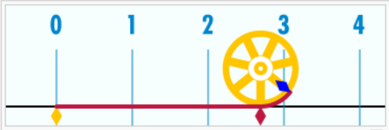
MGSE7.G.2 Explore various geometric shapes with given conditions. Focus on creating triangles from three measures of angles and/or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

http://illuminations.nctm.org/Lesson.aspx?id=2028
http://illuminations.nctm.org/Activity.aspx?id=3546
http://nzmaths.co.nz/resource/how-high-and-other-problems
http://nzmaths.co.nz/resource/inside-irregular-polygons
http://nzmaths.co.nz/resource/red-october

MGSE7.G.3 Describe the two-dimensional figures (cross sections) that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms, right rectangular pyramids, cones, cylinders, and spheres.
https://www.illustrativemathematics.org/content-standards/7/G/A/3/tasks

MGSE7.G.4 Given the formulas for the area and circumference of a circle, use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

http://www.uen.org/Lessonplan/preview.cgi?LPid=15436 A link to the activity on which this task was based.
http://en.wikipedia.org/wiki/Pi The Pi entry provides a graphic for pi, a circle's circumference is measured on a ruler created using increments equal to the diameter of the circle.



When a circle's diameter is 1, its circumference is pi.

http://www.mathgoodies.com/lessons/vol2/circumference.html
This site provides background information on circles and allows students to practice finding the circumference of circles.
http://www.kathimitchell.com/pi.html A comprehensive list of sites about pi.
http://www.joyofpi.com/pi.html The First 10,000 digits of pi.
http://curvebank.calstatela.edu/circle/circle.htm Uses animation to derive the formula for the area of a circle based on the area of a parallelogram.
http://www.rkm.com.au/ANIMATIONS/animation-Circle-Area-Derivation.html Uses animation to derive the formula for the area of a circle based on the area of a parallelogram.
http://curvebank.calstatela.edu/circle2/circle2.htm Uses animation to derive the formula for the area of a circle based on the area of a triangle.
http://www.worsleyschool.net/science/files/circle/area.html Uses graphics to derive the formula for the area of a circle.
http://illuminations.nctm.org/Activity.aspx?id=3547

MGSE7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

https://www.illustrativemathematics.org/7.G.B
http://illuminations.nctm.org/Lesson.aspx?id=1992

MGSE7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
https://www.illustrativemathematics.org/7.G.B
http://illuminations.nctm.org/Lesson.aspx?id=2009
http://illuminations.nctm.org/Lesson.aspx?id=2232
http://illuminations.nctm.org/Lesson.aspx?id=2771

Unit 5

Use random sampling to draw inferences about a population.

MGSE7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

http://nzmaths.co.nz/resource/planning-statistics-investigation-l3
http://nzmaths.co.nz/resource/channel-surfing
http://nzmaths.co.nz/resource/stork-delivery
http://illuminations.nctm.org/Activity.aspx?id=3580

MGSE7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

http://nzmaths.co.nz/resource/planning-statistics-investigation-l3
http://nzmaths.co.nz/resource/channel-surfing
http://nzmaths.co.nz/resource/stork-delivery
http://illuminations.nctm.org/Activity.aspx?id=3580

Draw informal comparative inferences about two populations.

MGSE7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the medians by expressing it as a multiple of the interquartile range.

https://www.illustrativemathematics.org/content-standards/7/SP/B/3/tasks
https://www.illustrativemathematics.org/content-standards/7/SP/B/4/tasks
http://nzmaths.co.nz/resource/number-statistics
http://nzmaths.co.nz/resource/sports-statistics
http://nzmaths.co.nz/illustrating-mathematics-standards-60
http://illuminations.nctm.org/Activity.aspx?id=3576
http://illuminations.nctm.org/Activity.aspx?id=3580
http://illuminations.nctm.org/Lesson.aspx?id=2643
http://illuminations.nctm.org/Activity.aspx?id=3580

MGSE7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

https://www.illustrativemathematics.org/content-standards/7/SP/B/3/tasks
https://www.illustrativemathematics.org/content-standards/7/SP/B/4/tasks
http://nzmaths.co.nz/resource/number-statistics
http://nzmaths.co.nz/resource/sports-statistics
http://nzmaths.co.nz/illustrating-mathematics-standards-60
http://illuminations.nctm.org/Activity.aspx?id=3576
http://illuminations.nctm.org/Lesson.aspx?id=2643
http://illuminations.nctm.org/Activity.aspx?id=3580

Unit 6

MGSE7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

http://nzmaths.co.nz/sites/default/files/curriculum/Level3StatisticsProbability.pdf
http://nzmaths.co.nz/resource/counting-probability
http://nzmaths.co.nz/probability-units-work
http://nzmaths.co.nz/resource/statements-about-probability
http://nzmaths.co.nz/resource/probability-distributions
http://nzmaths.co.nz/resource/fair-games-0

MGSE7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency. Predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

https://www.illustrativemathematics.org/content-standards/7/SP/C/6/tasks
http://nzmaths.co.nz/resource/counting-probability
http://nzmaths.co.nz/probability-units-work

http://nzmaths.co.nz/resource/statements-about-probability
http://nzmaths.co.nz/resource/probability-distributions
http://nzmaths.co.nz/resource/fair-games-0

MGSE7.SP.7 Develop a probability model and use it to find probabilities of events. Compare experimental and theoretical probabilities of events. If the probabilities are not close, explain possible sources of the discrepancy.

https://www.illustrativemathematics.org/7.SP.C.7
http://nzmaths.co.nz/sites/default/files/curriculum/Level3StatisticsProbability.pdf
http://nzmaths.co.nz/probability-units-work
http://nzmaths.co.nz/resource/probability-distributions
http://nzmaths.co.nz/resource/number-probability-1
http://nzmaths.co.nz/resource/fair-games-0

MGSE7.SP.7a Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

https://www.illustrativemathematics.org/7.SP.C.7
http://nzmaths.co.nz/sites/default/files/curriculum/Level3StatisticsProbability.pdf
http://nzmaths.co.nz/probability-units-work
http://nzmaths.co.nz/resource/probability-distributions
http://nzmaths.co.nz/resource/fair-games-0

MGSE7.SP.7b Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

https://www.illustrativemathematics.org/7.SP.C.7
http://nzmaths.co.nz/sites/default/files/curriculum/Level3StatisticsProbability.pdf
http://nzmaths.co.nz/probability-units-work
http://nzmaths.co.nz/resource/probability-distributions
http://nzmaths.co.nz/resource/fair-games-0

MGSE7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

https://www.illustrativemathematics.org/content-standards/7/SP/C/8/tasks
http://nzmaths.co.nz/probability-units-work
http://nzmaths.co.nz/resource/probability-distributions

MGSE7.SP.8a Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

https://www.illustrativemathematics.org/content-standards/7/SP/C/8/tasks
http://nzmaths.co.nz/resource/counting-probability
http://nzmaths.co.nz/resource/probability-distributions
http://nzmaths.co.nz/resource/fair-games-0

MGSE7.SP.8b Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

https://www.illustrativemathematics.org/content-standards/7/SP/C/8/tasks
http://nzmaths.co.nz/probability-units-work

MGSE7.SP.8c Explain ways to set up a simulation and use the simulation to generate frequencies for compound events. *For example, if 40% of donors have type A blood, create a simulation to predict the probability that it will take at least 4 donors to find one with type A blood?*

http://nzmaths.co.nz/probability-units-work

ADDITIONAL RESOURCES

- Georgiastandards.org provides a gateway to a wealth of instruction links and information. Open the GSE Mathematics to access specific GSE resources for this course.

- Georgia Virtual School content available on the Shared Resources Website is available for anyone to view. Courses are divided into modules and are aligned with the Georgia Standards of Excellence.

- Course/Grade Level WIKI spaces are available to post questions about a unit, a standard, the course, or any other GSE mathematics related concern. Shared resources and information are also available at the site.

- From the National Council of Teachers of Mathematics, Illuminations: Height of Students in our Class. This lesson has students creating box-and-whisker plots with an extension of finding measures of center and creating a stem-and-leaf plot.

- National Library of Virtual Manipulatives. Students can use the appropriate applet from this page of virtual manipulatives to create graphical displays of the data set. This provides an important visual display of the data without the tediousness of the student hand drawing the display.

- Bright, G., Brewer, W., McClain, K., and Mooney, E. (2001). ***Navigating Through Data Analysis in Grades 6-8***. Reston, VA: NCTM.

- ***Statistics and Probability (Grades 6-9)***. Activities that Integrate Math and Science (AIMS Foundation).

- Number Talks for Middle School: **http://mathematicssolutions.com/wp-content/uploads/14_nctm_kcohen_numbertalks.pdf**