Georgia
Standards of Excellence
Middle School Support

Mathematics

GSE Grade 8

Connections/Support Materials for Remediation

Georgia Department of Education

Richard Woods, Georgia's School Superintendent

"Educating Georgia's Future"

Table of Contents

OVERVIEW

The tasks in this document are from the high school course Foundations of Algebra. Foundations of Algebra is a first year high school mathematics course option for students who have completed mathematics in grades 6 – 8 yet will need substantial support to bolster success in high school mathematics. The course is aimed at students who have reported low standardized test performance in prior grades and/or have demonstrated significant difficulties in previous mathematics classes.

In many cases, students enter middle school with the same mathematics difficulties that inhibit their success in algebra. Due to this, the Foundations of Algebra lessons have been sorted into collections aligned to 6th, 7th, and 8th grade mathematics standards and prerequisite skills. These tasks are suggested for use in middle school math connections/support classes and with other students who are in the process of mastering the identified standards.

SETTING THE ATMOSPHERE FOR SUCCESS

"There is a huge elephant standing in most math classrooms, it is the idea that only some students can do well in mathematics. Students believe it; parents believe and teachers believe it. The myth that mathematics is a gift that some students have and some do not, is one of the most damaging ideas that pervades education in the US and that stands in the way of students' mathematics achievement." (Boaler, Jo. "Unlocking Children's Mathematics Potential: 5 Research Results to Transform Mathematics Learning" youcubed at Stanford University. Web 10 May 2015.)
Some students believe that their ability to learn mathematics is a fixed trait, meaning either they are good at mathematics or not. This way of thinking is referred to as a **fixed mindset**. Other students believe that their ability to learn mathematics can develop or grow through effort and education, meaning the more they do and learn mathematics the better they will become. This way of thinking is referred to as a **growth mindset**.

In the fixed mindset, students are concerned about how they will be viewed, smart or not smart. These students do not recover well from setbacks or making mistakes and tend to "give up" or quit. In the growth mindset, students care about learning and work hard to correct and learn from their mistakes and look at these obstacles as challenges.

The manner in which students are praised greatly affects the type of mindset a student may exhibit. Praise for intelligence tends to put students in a fixed mindset, such as "You have it!" or "You are really good at mathematics". In contrast, praise for effort tends to put students in a growth mindset, such as "You must have worked hard to get that answer." or "You are developing mathematics skills because you are working hard". Developing a growth mindset produces motivation, confidence and resilience that will lead to higher achievement. (Dweck, Carol. Mindset: The New Psychology of Success. Ballantine Books: 2007.)

"Educators cannot hand students confidence on a silver platter by praising their intelligence. Instead, we can help them gain the tools they need to maintain their confidence in learning by keeping them focused on the process of achievement." (Dweck, Carol S. "The Perils and Promises of Praise." *ASCD*. Educational Leadership. October 2007. Web 10 May 2015.)

Teachers know that the business of coming to know students as learners is simply too important to leave to chance and that the peril of not undertaking this inquiry is not reaching a learner at all. Research suggests that this benefit may improve a student's academic performance. Surveying students' interests in the beginning of a year will give teachers direction in planning activities that will "get students on board". Several interest surveys are available and two examples can be located through the following websites:

https://www.scholastic.com/content/collateral_resources/pdf/student_survey.pdf
http://www.niu.edu/eteams/pdf_s/VALUE_StudentInterestInventory.pdf

CONCEPTS/SKILLS TO MAINTAIN FROM PREVIOUS GRADES

Students are expected to have prior knowledge/experience related to the concepts and skills identified below. A pre-assessment may be necessary in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

The web links may be used as needed for additional resources aimed at intervention, and even though the interventions may be needed throughout the course, these concepts correspond to the 8th grade units where the needed skills first appear as prerequisites.

Unit 1

- **plotting on the coordinate plane**
  https://www.illustrativemathematics.org/content-standards/5/G/A/1
  https://www.illustrativemathematics.org/content-standards/5/G/A/2/tasks/1516
  https://www.illustrativemathematics.org/content-standards/tasks/489
  https://www.illustrativemathematics.org/content-standards/tasks/1999
  https://www.illustrativemathematics.org/content-standards/6/G/A/3/tasks/1188
- **describing the characteristics of 2-D and 3-D shapes**
  Students may need to investigate angles, parallel lines, and polygons to determine relationships using tools such as a protractor, turn measurer, or other angle measuring device. For these students, the following mini-lessons may be used in small groups or even as a whole class prior to some of the other tasks in this unit.
  http://nzmaths.co.nz/resource/angles-parallel-lines-and-polygons

  https://www.illustrativemathematics.org/content-standards/7/G/A/3/tasks/1532
  https://www.illustrativemathematics.org/content-standards/tasks/1188
  https://www.illustrativemathematics.org/content-standards/tasks/1245
  https://www.illustrativemathematics.org/content-standards/tasks/1941
  https://www.illustrativemathematics.org/content-standards/6/G/A/3/tasks/1188
  https://www.illustrativemathematics.org/content-standards/6/G/A/3/tasks/1997
  https://www.illustrativemathematics.org/content-standards/5/G/B/4/tasks/1943

https://www.illustrativemathematics.org/content-standards/6/G/A/4/tasks/1985
http://nzmaths.co.nz/resource/angles-parallel-lines-and-polygons

- **solving equations**
  http://map.mathshell.org/download.php?fileid=1154
  https://www.illustrativemathematics.org/content-standards/7/EE/B/tasks/712
  https://www.illustrativemathematics.org/content-standards/7/EE/B/4/tasks/643
  https://www.illustrativemathematics.org/content-standards/7/EE/B/4/tasks/986
  http://robertkaplinsky.com/work/what-does-2000-calories-look-like/
  https://illuminations.nctm.org/Lesson.aspx?id=2148
- **performing operations with fractions and decimals**
  https://www.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/314
  https://www.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/317
  https://www.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/591
  https://www.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/998
  https://www.illustrativemathematics.org/content-standards/7/NS/A/2/tasks/1542
  https://www.illustrativemathematics.org/content-standards/7/NS/A/2/tasks/1541
  https://www.illustrativemathematics.org/content-standards/7/NS/A/2/tasks/604
  https://www.illustrativemathematics.org/content-standards/7/NS/A/2/tasks/593
  http://map.mathshell.org/download.php?fileid=1183

Unit 2

- **computing with whole numbers and decimals, including application of order of operations**
  https://www.illustrativemathematics.org/content-standards/5/NBT/A/3
  https://www.illustrativemathematics.org/content-standards/5/NBT/A/2
  https://www.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/1475
  https://www.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/46
  https://www.illustrativemathematics.org/content-standards/7/NS/A/3/tasks/298
  https://www.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/1987
  https://www.illustrativemathematics.org/content-standards/7/NS/A/2/tasks/1667
  https://www.illustrativemathematics.org/content-standards/5/NBT/A/2/tasks/1524
- **distinguishing between independent and dependent variables**
  https://www.illustrativemathematics.org/content-standards/tasks/940
  https://www.illustrativemathematics.org/content-standards/tasks/806
  https://www.illustrativemathematics.org/content-standards/tasks/552
- **understanding the characteristics of a proportional relationship**
  https://www.illustrativemathematics.org/content-standards/4/MD/A/1
  https://www.illustrativemathematics.org/content-standards/7/RP/A/tasks/1564
  https://www.illustrativemathematics.org/content-standards/7/RP/A/tasks/2041
  https://www.illustrativemathematics.org/content-standards/7/RP/A/1/tasks/1176
  https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/181
  https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/1526
  https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/1983
  https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/1527

Unit 3

- **knowing the properties of similarity, congruence, and right triangles**
  https://www.illustrativemathematics.org/content-standards/tasks/1998
  https://www.illustrativemathematics.org/content-standards/tasks/1231
  https://www.illustrativemathematics.org/content-standards/tasks/1936
  https://www.illustrativemathematics.org/content-standards/tasks/1230
  https://www.illustrativemathematics.org/content-standards/tasks/1232
  https://www.illustrativemathematics.org/content-standards/tasks/1935
  https://www.illustrativemathematics.org/content-standards/tasks/1228
- **representing radical expressions in radical form (irrational) or approximating these numbers as rational**
  https://www.illustrativemathematics.org/content-standards/tasks/1221
  https://www.illustrativemathematics.org/content-standards/tasks/338
  https://www.illustrativemathematics.org/content-standards/tasks/336
- **finding square roots of perfect squares**
  http://www.crctlessons.com/numbers-and-operations.html
  http://www.aplusmath.com/Flashcards/sqrt.html
- **measuring length and finding perimeter and area of quadrilaterals**
  https://www.illustrativemathematics.org/content-standards/tasks/2133
  https://www.illustrativemathematics.org/content-standards/6/EE/A/2/tasks/540
  https://www.illustrativemathematics.org/content-standards/6/EE/A/2/tasks/421
  https://www.illustrativemathematics.org/content-standards/6/G/A/3/tasks/1188
  https://www.illustrativemathematics.org/content-standards/6/G/A/3/tasks/1997
- **using properties of exponents and real numbers (commutative, associative, distributive, inverse and identity) and order of operations**
  https://www.illustrativemathematics.org/content-standards/tasks/2225
  https://www.illustrativemathematics.org/content-standards/tasks/2224
  https://www.illustrativemathematics.org/content-standards/tasks/2208
  https://www.illustrativemathematics.org/content-standards/tasks/2209
  https://www.illustrativemathematics.org/content-standards/tasks/891
  https://www.illustrativemathematics.org/content-standards/tasks/395

Unit 4

- **writing algebraic equations**
  https://www.illustrativemathematics.org/content-standards/7/NS/A/3/tasks/298
  https://www.illustrativemathematics.org/content-standards/7/EE/A/tasks/433
  https://www.illustrativemathematics.org/content-standards/7/EE/A/1/tasks/541
  https://www.illustrativemathematics.org/content-standards/7/EE/A/2/tasks/1450
  https://www.illustrativemathematics.org/content-standards/7/EE/B/3/tasks/478
  https://www.illustrativemathematics.org/content-standards/7/EE/B/3/tasks/108

Unit 5

- **determining unit rate**
  https://www.illustrativemathematics.org/content-standards/tasks/104
  https://nzmaths.co.nz/resource/rates-change
- **recognizing a function in various forms**
  https://www.illustrativemathematics.org/content-standards/tasks/2050

https://www.illustrativemathematics.org/content-standards/tasks/1928
https://www.illustrativemathematics.org/content-standards/tasks/715
https://www.illustrativemathematics.org/content-standards/tasks/1552
https://www.illustrativemathematics.org/content-standards/tasks/674
https://www.illustrativemathematics.org/content-standards/tasks/247

- **understanding of writing rules for sequences and number patterns**
  https://www.illustrativemathematics.org/content-standards/tasks/1369
  https://nzmaths.co.nz/resource/graphic-detail
  https://nzmaths.co.nz/resource/holistic-algebra
- **differentiating between the graphing of discrete and continuous data**
  https://www.illustrativemathematics.org/content-standards/tasks/1165
  https://www.illustrativemathematics.org/content-standards/tasks/713

Unit 6

- **identifying and calculating slope**
  https://www.illustrativemathematics.org/content-standards/tasks/1537
  https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/104
  https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/181
- **identifying the *y*-intercept**
  https://www.illustrativemathematics.org/content-standards/tasks/471
  http://www.crctlessons.com/algebra-lessons.html
- **creating graphs using given data**
  https://www.illustrativemathematics.org/content-standards/tasks/813
  https://www.illustrativemathematics.org/content-standards/tasks/641
  https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/180
  https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/1983
- **analyzing graphs**
  https://www.illustrativemathematics.org/content-standards/tasks/477
  https://www.illustrativemathematics.org/content-standards/tasks/628
  https://www.illustrativemathematics.org/content-standards/tasks/633
  https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/1186
  https://www.illustrativemathematics.org/content-standards/5/G/A/1/tasks/489
  https://www.illustrativemathematics.org/content-standards/5/G/A/2/tasks/1516
  https://www.illustrativemathematics.org/content-standards/6/EE/B/6/tasks/425
  https://www.illustrativemathematics.org/content-standards/6/EE/B/7/tasks/1032
  https://www.illustrativemathematics.org/content-standards/7/EE/B/4/tasks/643
  https://www.illustrativemathematics.org/content-standards/7/EE/B/4/tasks/1475
  https://www.illustrativemathematics.org/content-standards/7/EE/B/4/tasks/1602
  https://www.illustrativemathematics.org/content-standards/7/EE/B/4/tasks/884

STANDARDS FOR MATHEMATICAL CONTENT

The content standards for Foundations of Algebra are an amalgamation of mathematical standards addressed in grades 3 through high school. After each Foundations of Algebra standard there is a list of reference standards. These reference standards refer to the standards used to form those for Foundations of Algebra.

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ2. Students will conceptualize positive and negative numbers (including decimals and fractions).

a. Explain the meaning of zero. (MGSE6.NS.5)
b. Represent numbers on a number line. (MGSE6.NS.5,6)
c. Explain meanings of real numbers in a real world context. (MGSE6.NS.5)

MFANSQ3. Students will recognize that there are numbers that are not rational, and approximate them with rational numbers.

a. Find an estimated decimal expansion of an irrational number locating the approximations on a number line. For example, for $\sqrt{2}$ show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue this pattern in order to obtain better approximations. (MGSE8.NS.1,2)

MFANSQ4. **Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.**

a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)
e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MGSE7.NS.3, MGSE7.EE.3, MGSE9-12.N.Q.3)

Students will extend arithmetic operations to algebraic modeling.

MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.

a. Apply properties of operations emphasizing when the commutative property applies. (MGSE7.EE.1)
b. Use area models to represent the distributive property and develop understandings of addition and multiplication (all positive rational numbers should be included in the models). (MGSE3.MD.7)
c. Model numerical expressions (arrays) leading to the modeling of algebraic expressions. (MGSE7.EE.1,2; MGSE9-12.A.SSE.1,3)
d. Add, subtract, and multiply algebraic expressions. (MGSE6.EE.3, MGSE6.EE.4, MC7.EE.1, MGSE9-12.A.SSE.3)

e. Generate equivalent expressions using properties of operations and understand various representations within context. For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between "3 more" and "3 times". (MGSE4.0A.2; MGSE6.EE.3, MGSE7.EE.1, 2, MGSE9-12.A.SSE.3)
f. Evaluate formulas at specific values for variables. For example, use formulas such as A = l x w and find the area given the values for the length and width. (MGSE6.EE.2)

MFAAA2. Students will interpret and use the properties of exponents.

a. Substitute numeric values into formulas containing exponents, interpreting units consistently. (MGSE6.EE.2, MGSE9-12.N.Q.1, MGSE9-12.A.SSE.1, MGSE9-12.N.RN.2)
b. Use properties of integer exponents to find equivalent numerical expressions. For example, $3^2\ x\ 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$. (MGSE8.EE.1)
c. Evaluate square roots of perfect squares and cube roots of perfect cubes (MGSE8.EE.2)
d. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. (MGSE8.EE.2)
e. Use the Pythagorean Theorem to solve triangles based on real-world contexts (Limit to finding the hypotenuse given two legs). (MGSE8.G.7)

<u>Students will use ratios to solve real-world and mathematical problems.</u>

MFAPR1. Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations.* (MGSE6.RP.3)

MFAPR2. Students will recognize and represent proportional relationships between quantities.

a. Relate proportionality to fraction equivalence and division. *For example, $\frac{3}{6}$ is equal to $\frac{4}{8}$ because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3; MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

MFAPR3. Students will graph proportional relationships.

a. Interpret unit rates as slopes of graphs. (MGSE8.EE.5)
b. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. (MGSE8.EE.6)
c. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (MGSE8.EE.5)

<u>Students will solve, interpret, and create linear models using equations and inequalities.</u>

MFAEI1. Students will create and solve equations and inequalities in one variable.

a. Use variables to represent an unknown number in a specified set (conceptual understanding of a variable). (MGSE6.EE.2, 5, 6)
b. Explain each step in solving simple equations and inequalities using the equality properties of numbers. (MGSE9-12.A.REI.1)
c. Construct viable arguments to justify the solutions and methods of solving equations and inequalities. (MGSE9-12.A.REI.1)
e. Use variables to solve real-world and mathematical problems. (MGSE6.EE.7, MGSE7.EE.4)

MFAEI2. Students will use units as a way to understand problems and guide the solutions of multi-step problems.

a. Choose and interpret units in formulas. (MGSE9-12.N.Q.1)
b. Choose and interpret graphs and data displays, including scale and comparisons of data. (MGSE3.MD.3, MGSE9-12.N.Q.1)
c. Graph points in all four quadrants of the coordinate plane. (MGSE6.NS.8)

MFAEI3. Students will create algebraic models in two variables.

a. Create an algebraic model from a context using systems of two equations. (MGSE6.EE.6, MGSE8.EE.8, MGSE9-12.A.CED.2)
b. Find approximate solutions using technology to graph, construct tables of values and find successive approximations. (MGSE9-12.A.REI.10, 11)
c. Represent solutions to systems of equations graphically or by using a table of values. (MGSE6.EE5; MSGE7.EE.3;MGSE8.EE.8, MGSE9-12.A.CED.2)
d. Analyze the reasonableness of the solutions of systems of equations within a given context. (MGSE6.EE.5, 6, MGSE7.EE.4)

MFAEI4. Students will solve literal equations.

a. Solve for any variable in a multi-variable equation. (MGSE6.EE.9, MGSE9-12.A.REI.3)
b. Rearrange formulas to highlight a particular variable using the same reasoning as in solving equations. *For example, solve for the base in* $A = \frac{1}{2}bh$. (MGSE9-12.A.CED.4)

<u>Students will create function statements and analyze relationships among pairs of variables using graphs, table, and equations.</u>

MFAQR1. Students will understand characteristics of functions.

a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. (MGSE9-12.F.IF.1)
b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*. (MGSE9-12.F.IF.5)
c. Graph functions using sets of ordered pairs consisting of an input and the corresponding output. (MGSE8. F.1, 2)

MFAQR2. Students will compare and graph functions.

a. Calculate rates of change of functions, comparing when rates increase, decrease, or stay constant. *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (MGSE6.RP.2; MGSE7.RP.1, 2, 3; MGSE8.F.2,5; MGSE9-12.F.IF.6)
b. Graph by hand simple functions expressed symbolically (use all four quadrants). (MGSE9- 12.F.IF.7)
c. Interpret the equation y = mx + b as defining a linear function whose graph is a straight line. (MGSE8.F.3)
e. Analyze graphs of functions for key features (intercepts, intervals of increase/decrease,maximums/minimums, symmetries, and end behavior) based on context. (MGSE9-12.F.IF.4,7)
f. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the great rate of change.* (MGSE8.F.2)

MFAQR3. Students will construct and interpret functions.

a. Write a function that describes a relationship between two quantities. (MGSE8.F.4, MGSE9-12.F.BF.1)
b. Use variables to represent two quantities in a real-world problem that change in relationship to one another (conceptual understanding of a variable). (MGSE6.EE.9)

ELEMENTARY REFERENCE STANDARDS

These reference standards refer to the standards used to form the standards for the Foundations of Algebra course. Below, you will find the elementary reference standards with instructional strategies and common misconceptions.

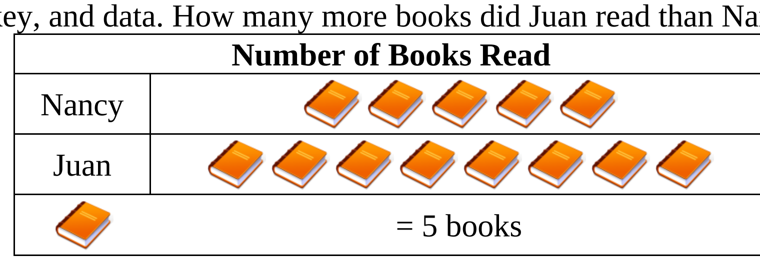
MGSE.3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. ***For example, draw a bar graph in which each square in the bar graph might represent 5 pets.***

Students should have opportunities to read and solve problems using scaled graphs before being asked to draw one. The following graphs all use five as the scale interval, but students should experience different intervals to further develop their understanding of scale graphs and number facts. While exploring data concepts, students should **P**ose a question, **C**ollect data, **A**nalyze data, and **I**nterpret data. Students should be graphing data that is relevant to their lives

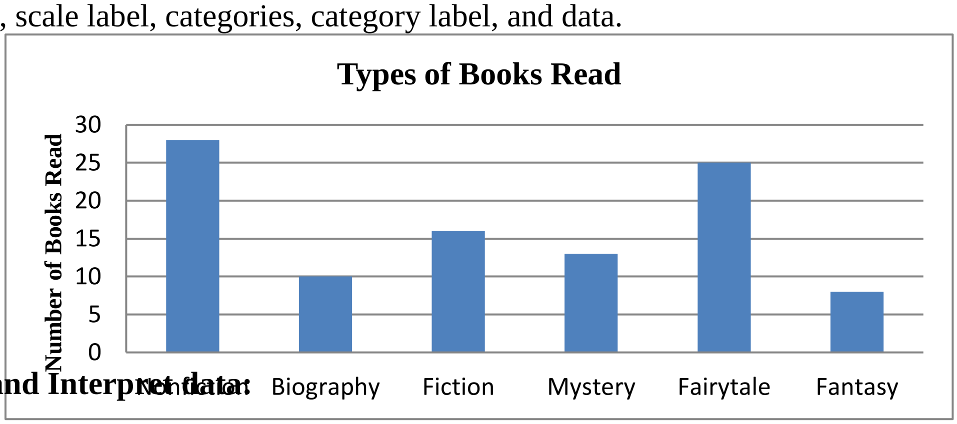
Example:

Pose a question: Student should come up with a question. What is the typical genre read in our class?

Collect and organize data: student survey
Pictographs: Scaled pictographs include symbols that represent multiple units. Below is an example of a pictograph with symbols that represent multiple units. Graphs should include a title, categories, category label, key, and data. How many more books did Juan read than Nancy?



Single Bar Graphs: Students use both horizontal and vertical bar graphs. Bar graphs include a title, scale, scale label, categories, category label, and data.



Analyze and Interpret data:

- How many more nonfiction books were read than fantasy books?
- Did more people read biography and mystery books or fiction and fantasy books?
- About how many books in all genres were read?

- Using the data from the graphs, what type of book was read more often than a mystery but less often than a fairytale?
- What interval was used for this scale?
- What can we say about the types of books read? What is a typical type of book read?
- If you were to purchase a book for the class library, which would be the best genre? Why?

MGSE.3.MD.7 Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

Students should tile rectangles, then multiply their side lengths to show it is the same.

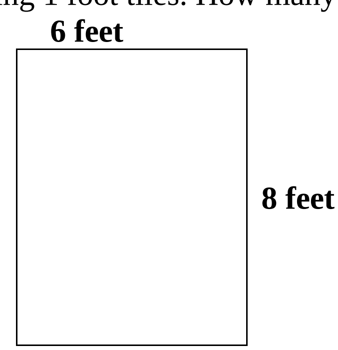
To find the area, one could count the squares or multiply $3 \times 4 = 12$.

1	2	3	4
5	6	6	8
9	10	11	12

b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

Students should solve real world and mathematical problems

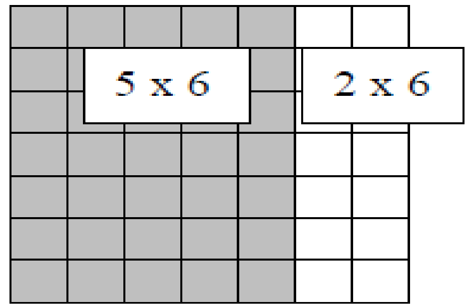
Example:
Drew wants to tile the bathroom floor using 1 foot tiles. How many square foot tiles will he need?



The area of the rectangle is 48 square feet, and since each tile is 1 square foot, 48 tiles will be needed.

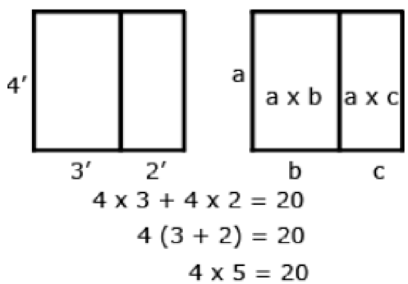
c. **Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.**

This standard extends students' work with the distributive property. For example, in the picture below the area of a 7 × 6 figure can be determined by finding the area of a 5 × 6 and 2 × 6 and adding the two sums.



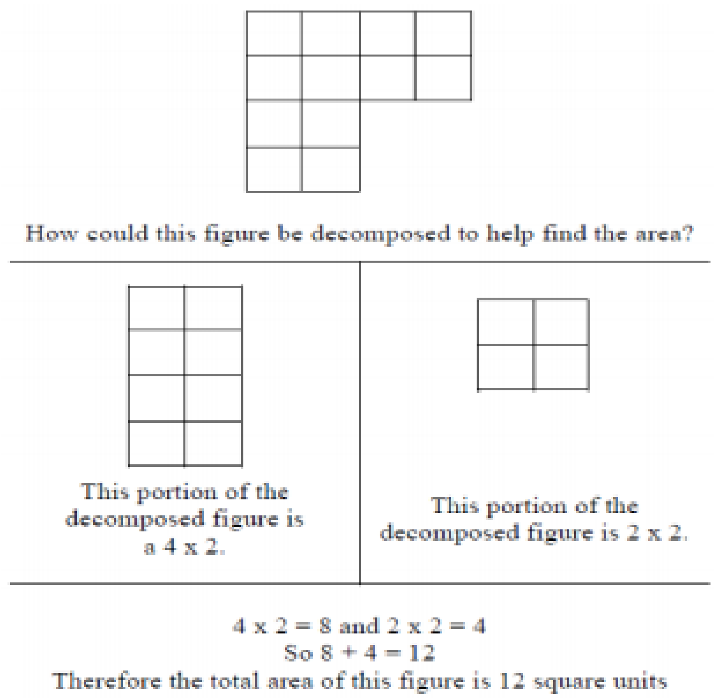
So, 7 x 6 = (5 + 2) x 6 = 5 x 6 + 2 x 6 = 30 + 12 =42

Example:



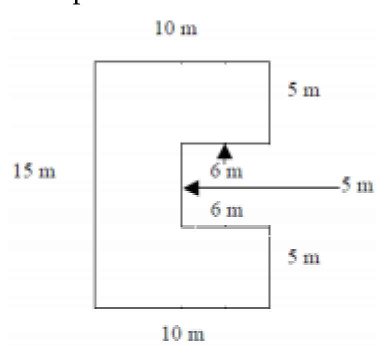
d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

This standard uses the word rectilinear. A rectilinear figure is a polygon that has all right angles.

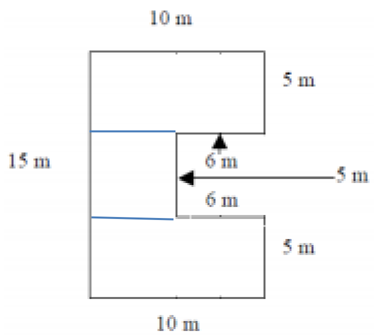


Example 1:

A storage shed is pictured below. What is the total area? How could the figure be decomposed to help find the area?

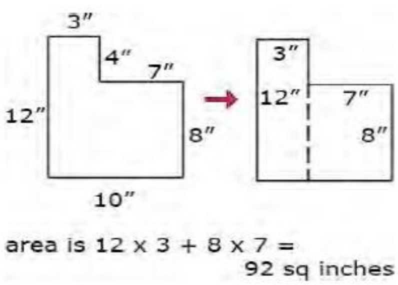


The area can be found by using 3 rectangles:



The top and bottom of the figure will be 10 m x 5 m for 50 m^2 x 2 = 100 m^2
The center rectangle will be a square with dimentions 5m x (15 – 5 – 5)m = 5m x 5 m or 25m^2
So, the area of the storage shed is 125 square meters.

Example 2:
As seen above, students can decompose a rectilinear figure into different rectangles. They find the area of the figure by adding the areas of each of the rectangles together.



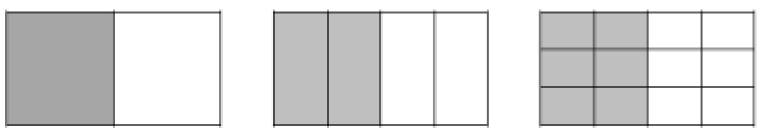
Common Misconceptions

Students may confuse perimeter and area when they measure the sides of a rectangle and then multiply. They think the attribute they find is length, which is perimeter. Pose problems situations that require students to explain whether they are to find the perimeter or area.

MGSE.4.NF.1 Explain why two or more fractions are equivalent. $\frac{a}{b} = \frac{n\,x\,a}{n\,x\,b}$. $\frac{1}{4} = \frac{3\,x\,1}{3\,x\,4}$ by using visual fraction models. Focus attention on how the number and size of the parts differ even though the fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

This standard refers to visual fraction models. This includes area models or number lines. This standard extends the work in third grade by using additional denominators (5, 10, 12, and 100). The standard addresses equivalent fractions by examining the idea that equivalent fractions can be created by multiplying both the numerator and denominator by the same number or by dividing a shaded region into various parts.

Example:



1/2 = 2/4 = 6/12

Technology Connection: http://illuminations.nctm.org/activitydetail.aspx?id=80

MGSE.4.OA.2. Multiply or divide to solve word problems involving multiplicative comparison. Use drawings and equations with a symbol or letter for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

This standard calls for students to translate comparative situations into equations with an unknown and solve. Students need many opportunities to solve contextual problems.

Examples:
***Unknown Product*:** A blue scarf costs \$3. A red scarf costs 6 times as much. How much does the red scarf cost? $(3 \times 6 = p)$
Group Size Unknown: A book costs \$18. That is 3 times more than a DVD. How much does a DVD cost? $(18 \div p = 3 \text{ or } 3 \times p = 18)$
Number of Groups Unknown: A red scarf costs \$18. A blue scarf costs \$6. How many times as much does the red scarf cost compared to the blue scarf? $(18 \div 6 = p \text{ or } 6 \times p = 18)$
When distinguishing multiplicative comparison from additive comparison, students should note the following:

- Additive comparisons focus on the difference between two quantities. For example, Deb has 3 apples and Karen has 5 apples. How many more apples does Karen have? A simple way to remember this is, “How many more?”
- Multiplicative comparisons focus on comparing two quantities by showing that one quantity is a specified number of times larger or smaller than the other. For example, Deb ran 3 miles. Karen ran 5 times as many miles as Deb. How many miles did Karen run? A simple way to remember this is “How many times as much?” or “How many times as many?”

MGSE.5.NBT.7 ADD, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

This standard also builds on work begun in 4^{th} grade when students were introduced to decimals and asked to compare them. In 5th grade, students begin adding, subtracting, multiplying and dividing decimals. This work should focus on concrete models and pictorial representations, rather than relying solely on the algorithm. The use of symbolic notations involves having students record the answers to computations ($2.25 \times 3= 6.75$), but this work should not be done without models or pictures. This standard requires that students explain their reasoning and how they use models, pictures, and strategies. Students are expected to extend their understanding of whole number models and strategies to decimal values. Before students are asked to give exact answers, they should estimate answers based on their understanding of operations and the value of the numbers.

Examples:

5.4 – 0.8

A student might estimate the answer to be a little more than 4.4 because a number less than 1 is being subtracted.

6 × 2.4

A student might estimate an answer between 12 and 18 since 6×2 is 12 and 6×3 is 18. Another student might give an estimate of a little less than 15 because s/he figures the answer to be very close, but smaller than $6 \times 2\frac{1}{2}$ and think of 2½ groups of 6 as 12 (2 groups of 6) + 3(½ of a group of 6).

When adding or subtracting decimals, students should be able to explain that tenths are added or subtracted from tenths and hundredths are added or subtracted from hundredths. So, students will need to communicate that when adding or subtracting in a vertical format (numbers beneath each other), it is important that digits with the same place value are written in the same column. This understanding can be reinforced by linking the decimal addition and subtraction process to addition and subtraction of fractions. Adding and subtracting fractions with like denominators of 10 and 100 is a standard in fourth grade.

Common Misconceptions

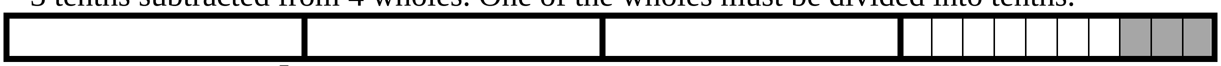
Students might compute the sum or difference of decimals by lining up the right-hand digits as they would whole number. For example, in computing the sum of 15.34 + 12.9, students will write the problem in this manner:

15.34
<u>+ 12.9</u>
16.63

To help students add and subtract decimals correctly, have them first estimate the sum or difference. Providing students with a decimal-place value chart will enable them to place the digits in the proper place.

Example 1: 4 - 0.3

3 tenths subtracted from 4 wholes. One of the wholes must be divided into tenths.

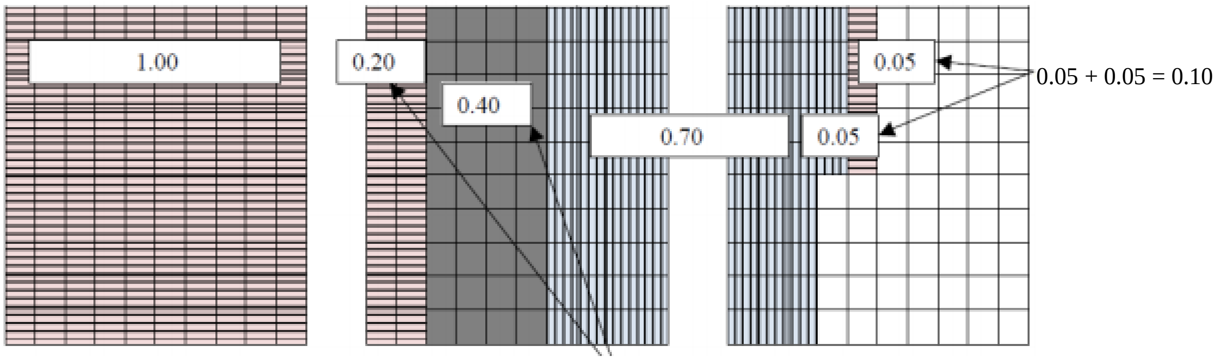


The solution is $3 + {}^{7}/_{10}$ or 3.7.

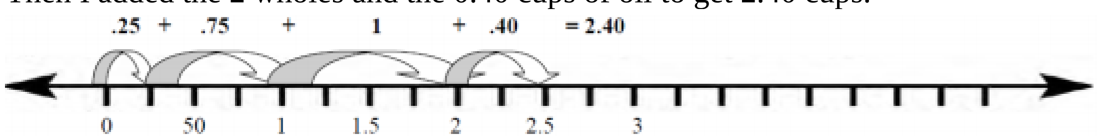
Example 2:

A recipe for a cake requires 1.25 cups of milk, 0.40 cups of oil, and 0.75 cups of water. How much liquid is in the mixing bowl?

Student 1: 1.25 + 0.40 + 0.75
First, I broke the numbers apart. I broke 1.25 into 1.00 + 0.20 + 0.05. I left 0.40 like it was. I broke 0.75 into 0.70 + 0.05.
I combined my two 0.05's to get 0.10. I combined 0.40 and 0.20 to get 0.60. I added the 1 whole from 1.25. I ended up with 1 whole, 6 tenths, 7 more tenths, and another 1 tenth, so the total is 2.4.

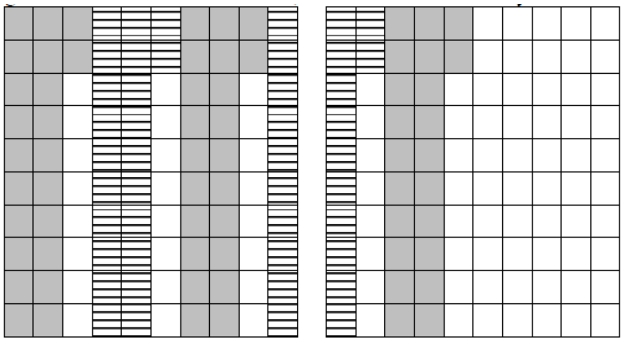


Student 2:
I saw that the 0.25 in the 1.25 cups of milk and the 0.75 cups of water would combine to equal 1 whole cup. That plus the 1 whole in the 1.25 cups of milk gives me 2 whole cups. Then I added the 2 wholes and the 0.40 cups of oil to get 2.40 cups.



Example of Multiplication 1

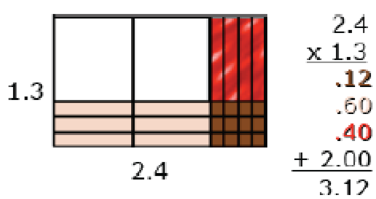
A gumball costs $0.22. How much do 5 gumballs cost? Estimate the total, and then calculate. Was your estimate close?



I estimate that the total cost will be a little more than a dollar. I know that 5 20's equal 100 and we have 5 22's. I have 10 whole columns shaded and 10 individual boxes shaded. The 10 columns equal 1 whole. The 10 individual boxes equal 10 hundredths or 1 tenth. My answer is $1.10.
My estimate was a little more than a dollar, and my answer was $1.10. I was really close.

Multiplication Example 2:

An area model can be useful for illustrating products.

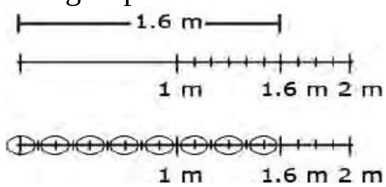


Students should be able to describe the partial products displayed by the area model.
For example, "$^{3}/_{10}$ times $^{4}/_{10}$ is $^{12}/_{100}$.
$^{3}/_{10}$ times 2 is $^{6}/_{10}$ or $^{60}/_{100}$.
1 group of $^{4}/_{10}$ is $^{4}/_{10}$ or $^{40}/_{100}$.
1 group of 2 is 2."

Division Example 1: Joe has 1.6 meters of rope. He has to cut pieces of rope that are 0.2 meters long. How many can he cut?

Finding the number of groups

Students could draw a segment to represent 1.6 meters. In doing so, s/he would count in tenths to identify the 6 tenths, and be able identify the number of 2 tenths within the 6 tenths. The student can then extend the idea of counting by tenths to divide the one meter into tenths and determine that there are 5 more groups of 2 tenths.

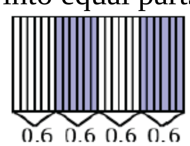


Students might count groups of 2 tenths without the use of models or diagrams. Knowing that 1 can be thought of as $^{10}/_{10}$, a student might think of 1.6 as 16 tenths. Counting 2 tenths, 4 tenths, 6 tenths, up to16 tenths, a student can count 8 groups of 2 tenths.
Use their understanding of multiplication and think, “8 groups of 2 is 16, so 8 groups of $^{2}/_{10}$ is $^{16}/_{10}$ or $1^{6}/_{10}$.”

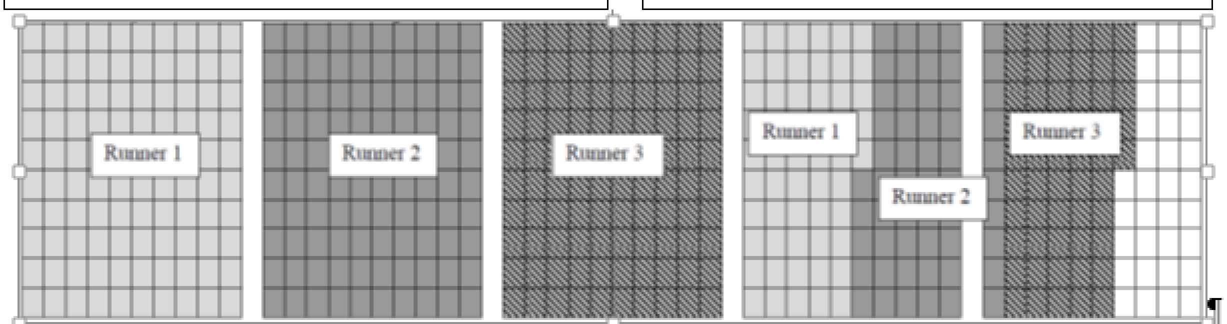
Division Example 2: 2.4 ÷ 4

Finding the number in each group or share

Students should be encouraged to apply a fair sharing model separating decimal values into equal parts such as 2.4 ÷ 4 = 0.6.



Division Example 3:
A relay race lasts 4.65 miles. The relay team has 3 runners. If each runner goes the same distance, how far does each team member run? Make an estimate, find your actual answer, and then compare them.
A possible solution is shown on the next page.



My estimate is that each runner runs between 1 and 2 miles. If each runner went 2 miles, that would be a total of 6 miles which is too high. If each runner ran 1 mile, that would be 3 miles, which is too low. I used the 5 grids above to represent the 4.65 miles. I am going to use all of the first 4 grids and 65 of the squares in the 5th grid. I have to divide the 4 whole grids and the 65 squares into 3 equal groups. I labeled each of the first 3 grids for each runner, so I know that each team member ran at least 1 mile. I then have 1 whole grid and 65 squares to divide up. Each column represents one-tenth. If I give 5 columns to each runner, that means that each runner has run 1 whole mile and 5 tenths of a mile. Now, I have 15 squares left to divide up. Each runner gets 5 of those squares. So each runner ran 1 mile, 5 tenths and 5 hundredths of a mile. I can write that as 1.55 miles.

My answer is 1.55 and my estimate was between 1 and 2 miles. I was pretty close.

STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe varieties of expertise that educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in education. The first of these are the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning and proof, communication, representation, and connections. (*Principles and Standards for School Mathematics.* NCTM: 2000.) The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy) (National Academies Press, 2001.)

Students are expected to:

1. Make sense of problems and persevere in solving them.
Students begin in elementary school to solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. Students seek the meaning of a problem and look for efficient ways to represent and solve it. In middle school, students solve real world problems through the application of algebraic and geometric concepts.

2. Reason abstractly and quantitatively.
Earlier grade students should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.

In middle school, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

3. Construct viable arguments and critique the reasoning of others. In earlier grades, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication.

In middle school, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (e.g., box plots, dot plots, histograms). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. The students pose questions like “How did you get that?”, “Why is that true?”, and “Does that always work?” They explain their thinking to others and respond to others’ thinking.

4. Model with Mathematics.
Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Elementary students should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.

In middle school, students model problem situations with symbols, graphs, tables, and context. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students solve systems of linear equations and compare properties of functions provided in different forms. Students use scatterplots to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.

5. Use appropriate tools strategically.
Elementary students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real world data.

Students in middle school may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal.

6. Attend to precision.
Students in earlier grades continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they

record their answers in cubic units. Students in middle school use appropriate terminology when referring to the number system, functions, geometric figures, and data displays.

7. Look for and make use of structure.
In elementary grades, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply, and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation.

Students in middle school routinely seek patterns or structures to model and solve problems. Students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.

8. Look for and express regularity in repeated reasoning.
Students in elementary grades use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms, to fluently multiply multi-digit numbers, and to perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations.

Middle school students use repeated reasoning to understand algorithms and make generalizations about patterns. Students use iterative processes to determine more precise rational approximations for irrational numbers. During multiple opportunities to solve and model problems, they notice that the slope of a line and rate of change are the same value. Students flexibly make connections between covariance, rates, and representations showing the relationships between quantities.

Connecting the Standards for Mathematical Practice to the Content Standards
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who are missing the understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, an absence of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward the central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics. See Inside Mathematics for more resources.

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The terms below are for teacher reference only and are not to be memorized by the students. Teachers should present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

- Addition Property of Equality
- Algebraic Expression
- Associative Property
- Coefficient
- Commutative Property
- Coordinate plane
- Coordinates
- Cube Root
- Cubic Number
- Difference
- Distributive property
- Division Property of Equality
- Domain of a function
- Equation
- Equivalent expressions
- Equivalent ratios
- Exponent
- Formula
- Function
- Hypotenuse
- Identity Properties
- Integer
- Inverse Operations
- Irrational Number
- Multiplication Property of Equality
- Numeric expression
- Opposite of a Number
- Origin
- Proportional relationship
- Pythagorean Theorem
- Quadrant
- Range of a function
- Rate of change
- Rational Number
- Simultaneous Equations
- Slope
- Solution
- Square Number
- Square Root
- Substitution
- Subtraction Property of Equality
- Sum
- Unit rate
- Variable
- x-axis
- x-coordinate
- y-axis
- y-coordinate
- Zero

Again, discuss terminology as it naturally arises in discussion of the problems. Allow students to point out words or phrases that lead them to the model and solution of the problems. Words that imply mathematical operations vary based on context and should be delineated based on their use in the particular problem. A couple of suggested methods for students to record vocabulary are

TIP Charts (https://www.youtube.com/watch?v=Rbts9h_ruu8) and Frayer Models (https://wvde.state.wv.us/strategybank/FrayerModel.html).

The websites below are interactive and include a math glossary suitable for middle school children.

- http://www.amathsdictionaryforkids.com/
- http://intermath.coe.uga.edu/dictnary/homepg.asp

TABLE OF INTERVENTIONS

Deep Freeze	Hills and Valleys	Allow students to build fluency and automatize their use and operations with integers.	Hills and Valleys
Debits and Credits	Bucket Balance	Allow students to build fluency and automatize their use and operations with integers.	Bucket Balance
Decimal Approximation of Roots	Powerful Numbers	Solve problems involving square roots	Powerful Numbers
Distributing and Factoring Using Area	Multiplication Smorgasbord	I am learning to solve multiplication problems using a variety of mental strategies	Multiplication Smorgasbord
	Smiley Face	In this activity, students are encouraged to solve multiplication problems by deriving from known facts, looking for groupings and skip counting. Students are encouraged to explain and share their thinking.	Smiley Face
Squares, Area, Cubes, Volume, Connected?	Cube and Cube Roots	Calculate square and cube roots. Understand that squaring is the inverse of square rooting, and cubing is the inverse of cube rooting.	Cube and Cube Roots
What's the Hype about Pythagoras?	The Pythagorean Relationship	Find lengths of objects using Pythagoras' theorem.	The Pythagorean Relationship
	Pythagoras Plus	Explore Pythagoras' Theorem.	Pythagoras Plus
What is a Unit Rate?	Rates of Change	Hands on activities where students can determine unit rate.	Rates of Change
Deconstructing Word Problems	Writing Expressions	Use symbolic notation to record simple word phrases and explain what symbol notation says should be done to the numbers.	Writing Expressions
Literal Equations	What's My Line?	Form and solve simple linear equations. Use graphs, tables, and rules, to describe linear relationships found in number and spatial patterns	What's My Line?

SCAFFOLDED INSTRUCTIONAL LESSONS

Lesson Name	Lesson Type/ Grouping Strategy	Content Addressed	Standard(s)
Beads Under the Cloud	Scaffolding Lesson *Partners or small groups*	Identifying patterns in a realistic context	MFAQR1a,b MFAQR3a,b
FAL: Using Positive and Negative Numbers in Context	*Individual/Partner/ Whole Group*	Understanding and use integers in real world context.	MFANSQ2.a,b,c MFANSQ4.a,b,d
Deep Freeze	*Partner/Whole Group*	Using positive and negative numbers in real world context.	MFANSQ2.a,b,c MFANSQ4.a,b,d,e
Pattern of Multiplication and Division	*Individual/Partner*	Determining the relationship in integers when multiplied or divided.	MFANSQ2.a,b,c MFANSQ4.a,b,d
Debits and Credits	*Individual/Partner*	Using a check register to record debits and credits and calculate a running total.	MFANSQ2.a,b,c MFANSQ4.a,b,d,e
Estimating the Square Root of a Number	*Individual/Partner*	Estimating the square root of a number using a number line or colored tiles.	MFANSQ3.a, b MFANSQ4.b,d
Decimal Approximation of Roots	*Individual/Partner*	Exploring why some square roots are irrational and determine the decimal approximation of these roots.	MFANSQ3.a,b MFANSQ4.b,d
Distributing and Factoring Using Area	**Scaffolding Lesson/ *Individual/Partner***	Area models to represent and discover the distributive property	MFAAA1.b.d.e
Triangles and Quadrilaterals	Learning Lesson/ *Partner or Small Group*	Expressions and equations based on properties of triangles and quadrilaterals.	MFAAA1.a,c,d,e,f
Excursions with Exponents	Learning Lesson/ *Partner or Small Group*	Integer exponents	MFAAA2.b
Squares, Area, Cubes, Volume, Connected?	Learning Lesson/ *Partner or Small Group*	Square Roots, Area, Cube Roots, Volume	MFAAA2.c,d

Lesson Name	Lesson Type/ Grouping Strategy	Content Addressed	Standard(s)
What's the Hype about Pythagoras?	Constructing Lesson *Individual/Partner*	Square Roots, Squares, Pythagorean Theorem	MFAAA2.c,d,e
Fabulous Formulas	Learning Lesson/ *Partner or Small Group*	Formulas	MFAAA1.a,f MFAAA2.a
What is a Unit Rate?	Scaffolding Lesson *Partners or small groups*	Find rates and unit rates and interpret those rates in the context of problems	MFAPR.1 MFAPR.2
Fish in a Lake	Learning Lesson *Partners or small groups*	Use proportional reasoning to solve problems with a connection to statistics	MFAPR.2 b,c
Nate and Natalie's Walk	Performance Lesson *Partners or small groups*	Determine whether a proportional relationship exists using tables, graphs, or equations	MFAPR.2
Rectangle Families	Learning Lesson *Partners or small groups*	Compare ratios of side lengths of rectangles to sort appropriately	MFAPR.1 MFAPR.2 MFAPR.3
"Illustrative" Review	Formative Assessment *Individual*	A compilation of problem related to percentages, slopes, and rates.	
Let's Open a Business	Performance Lesson *Partner/Small Group*	Using variables, equations and expressions in real-world scenarios	MFAEI1a, e
Deconstructing Word Problems	Learning Lesson *Individual/Partner*	Representing and solving equations in context	MFAEI1a, e
Steps to Solving an Equation (FAL)	Formative Assessment Lesson *Partner/Small Group*	Using variables to represent values; using equations to solve problems in context	MFAEI1 b, c

Lesson Name	Lesson Type/ Grouping Strategy	Content Addressed	Standard(s)
Acting Out	Scaffolding Lesson *Individual/Partner*	Modeling and writing an equation in one variable; representing constraints as inequalities	MFAEI1a, e; MFAEI2a, b, c
Literal Equations	Learning Lesson *Individual/Partner*	Defining literal equations and using them to determine values	MFAEI4a, b
Function Rules	Formative Lesson *Partners or small group*	Connecting a function described by a verbal rule with corresponding values in a table	MFAQR1a-c
Equations of Attack	Formative Lesson *Individual or partners*	Writing equations given slope and intercept, and determine algebraically if a point lies on a line.	MFAQR2a,b,c,e MFAQR3a,b
Analyzing Linear Functions	Formative Assessment for Learning *Partners*	Determining slope as rate of change, finding the meaning of x and y intercepts applied to real-world situations, and explaining graphs and tables that represent realistic situation	MFAQR2c,f MFAQR3a,b
Is it Cheaper to Pay Monthly or Annually?	Lesson *Individual/Partner*	Creating and using a system of linear equations to solve a real world problem	MFAEI3 a, b, c, d

Beads Under the Cloud - FAL

This task was created by Kentucky Department of Education Mathematics Specialists and is available for downloading in its entirety at
http://education.ky.gov/educational/diff/Documents/BeadsUnderTheCloud.pdf

This problem solving lesson is intended to help you assess how well students are able to identify patterns in a realistic context: the number of beads of different colors that are hidden behind the cloud.

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class. Suggested time, 1 to 1.5 hours.

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR1. Students will understand characteristics of functions.
a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. (MCC9-12.F.IF.1)
b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. (MCC9-12.F.IF.5)

MFAQR3. Students will construct and interpret functions.
a. Write a function that describes a relationship between two quantities. (MGSE8.F.4, MGSE9-12.F.BF.1)
b. Use variables to represent two quantities in a real-world problem that change in relationship to one another (conceptual understanding of a variable). (MGSE6.EE.9)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of the problem presented in the lesson and continue to work toward a solution.
3. Construct viable arguments and critique the reasoning of others. Working with a partner, students identify functions matching equations, tables and rules.
7. Look for and make use of structure. Students use the structure of function notation in context.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to define functions.

EVIDENCE OF LEARNING/LEARNING TARGET

Students will demonstrate understanding and proficiency in:

- choosing an appropriate, systematic way to collect and organize data.
- examining the data and looking for patterns
- describing and explaining findings clearly and effectively.

MATERIALS

- Each individual student will need one copy of the *Beads under the Cloud* sheet and one copy of the *How did you work?* Sheet.
- Each small group of students will need a copy of *Sample Responses to Discuss* and samples of student work.

ESSENTIAL QUESTION

- How can patterns be identified in order to find a function rule that applies to a set of data?

EXTENSION

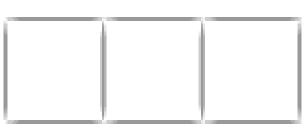
Adapted from Annenberg Learner
http://www.learner.org/courses/learningmath/algebra/session5/part_a/index.html

A function expresses a relationship between variables. For example, consider the number of toothpicks needed to make a row of squares. The number of toothpicks needed depends on the number of squares we want to make. If we call the number of toothpicks T and the number of squares S, we could say that T is a function of S. S is called the **independent variable** in this case, and T the **dependent variable** -- the value of T *depends upon* whatever we determine the value of S to be.

Predict, Explain, Observe Probe: Before the lesson, have students **PREDICT** what might happen as they build squares using toothpicks. How many will they start with? How many do they think each square will need? How many toothpicks will be needed for a row of 5 squares?

Groups: People will likely have varying degrees of comfort and familiarity with the content, so working with partners may be helpful. Work on Problems 1-3, completing the table and coming up with the function rule. You may want to share rules before moving on. If students are familiar with spreadsheets, creating the table on a computer may be helpful.

In this section, we will **EXPLORE** and **EXPLAIN** the dependence of one variable on another. Make a row of squares using toothpicks. The squares are joined at the side.



OBSERVE the pattern. Have students construct a function table as they explore the toothpick squares.

Problem 1
How many toothpicks are needed for one square? For two squares? For five squares? Make a table of these values.

Squares	**Toothpicks**
1	***4***
2	***7***
3	***10***
4	***13***
5	***16***

Problem 2
Develop a formula describing the number of toothpicks as a function of the number of squares.
The formula is $T = 3S + 1$.

[Using Positive and Negative Numbers in Context](http://map.mathshell.org/materials/lessons.php?LESSONid=453&subpage=concept)

A Formative Assessment Lesson by the Shell Center
http://map.mathshell.org/materials/lessons.php?LESSONid=453&subpage=concept

SUGGESTED TIME FOR THIS LESSON:

90 - 120 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

> **MFANSQ2.** Students will conceptualize positive and negative numbers (including decimals and fractions).
> a. Explain the meaning of zero. (MGSE6.NS.5)
> b. Represent numbers on a number line. (MGSE6.NS.5,6)
> c. Explain meanings of real numbers in a real world context. (MGSE6.NS.5)
>
> **MFANSQ4**. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
> a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
> b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
> d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.

5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to understand and use integers in real world context.

MATERIALS

- Temperature Change Assessment Lesson
- Temperature Change (Revisited) Assessment Lesson
- City Temperature Cards
- Changes in Temperature Cards

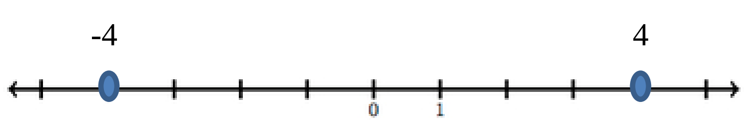
ESSENTIAL QUESTION

- How can you use real numbers in real world context?

Grouping: Individual/Partner/Whole Group

OPENER/ACTIVATOR

It was 4 degrees today in Anchorage, AK. How would you represent Anchorage's temperature on a number line? If it is – 4 degrees in Snow Hill, Antarctica, how many degrees difference is there? Justify your answer using a number line.



There is an 8 degree difference.

WORK SESSION

Teacher Notes: It is important that you read the lesson guide. It provides you with very explicit directions every step of the way. The lesson is very teacher-friendly.
Click this link to access all materials:
http://map.mathshell.org/materials/download.php?fileid=1304

CLOSING/SUMMARIZER

Temperature Change (Revisited) Assessment Lesson (See link above in Work Session)

Additional Problems:

One afternoon the temperature was -17 degrees in Lima, OH, and 62 degrees in Leesburg, GA. How many degrees warmer was it in Leesburg than in Lima on that afternoon?

There is a 79 degree difference.

The highest point on Earth is Mount Everest at 29,029 feet. The lowest point on Earth is Challenger Deep at the bottom of Mariana Trench. It is 36, 201 feet below sea level. What is the elevation difference between this two?

There is a 65,230 foot difference.

Deep Freeze

SUGGESTED TIME FOR THIS LESSON:

60-90 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ2. Students will conceptualize positive and negative numbers (including decimals and fractions).
a. Explain the meaning of zero. (MGSE6.NS.5)
b. Represent numbers on a number line. (MGSE6.NS.5,6)
c. Explain meanings of real numbers in a real world context. (MGSE6.NS.5)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)
e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MGSE7.NS.3, MGSE7.EE.3, MGSE9-12.N.Q.3)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.

5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Identify the placement of integers on the number line.
- Use positive and negative numbers in real world context.
- Use positive and negative numbers to describe opposites.

MATERIALS

- Videos for Deep Freeze – 3-Act Task
- Recording sheet (attached)

ESSENTIAL QUESTIONS

- How does a number line model addition or subtraction of rational numbers?
- How can models make sense of the real world application of rational numbers?
- How do I model addition and subtraction of integers on a vertical number line?
- What patterns are present when adding and subtracting integers?

Grouping: Whole Class

Teacher Notes
In this task, students will watch the video, then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they do not have the information they need, and ask for it, it will be given to them

Task Description
The following 3-Act Task can be found at: http://vimeo.com/94462885

OPENER/ACTIVATOR

ACT 1:

Watch the video and answer these questions on the student recording sheet:

- What's the temperature in Duluth, Minnesota? Estimate
- Write an estimate you know is too high. Write an estimate you know is too low.

WORK SESSION

ACT 2:

- It is 7 degrees in Atlanta, Georgia
- There is a 31 degree difference in the temperature between Atlanta and Duluth, Minnesota.

There's a 31 degree difference between Atlanta, Georgia and Duluth, Minnesota

.

INTERVENTIONS

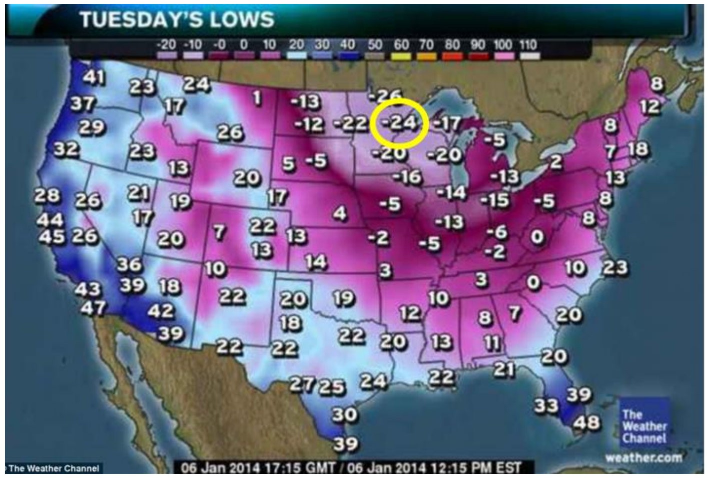
For extra help with integer operations, please open the hyperlink Intervention Table.

CLOSING/SUMMARIZER

ACT 3:

Students will compare and share solution strategies.

- Reveal the answer. Discuss the theoretical math versus the practical outcome.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.



ACT 4

- Have students identify the temperature difference between 5 different cities

Task Title: ________________________ Name: ________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question: ____

Estimate the result of the main question? Explain?
Place an estimate that is too high and too low on the number line
Low estimate *Place an "x" where your estimate belongs* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc.…)

If possible, give a better estimate using this information: _______________________________

Act 2 (cont.)

Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Pattern of Multiplication and Division

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

> **MFANSQ2.** Students will conceptualize positive and negative numbers (including decimals and fractions).
> a. Explain the meaning of zero. (MGSE6.NS.5)
> b. Represent numbers on a number line. (MGSE6.NS.5,6)
> c. Explain meanings of real numbers in a real world context. (MGSE6.NS.5)
>
> **MFANSQ4**. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
> a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
> b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
> d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

Common Misconceptions:
The section on division of positive and negative numbers using two color counters can be confusing upon first glance. Encourage students to persevere in working through the examples in order to gain understanding.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.

4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to determine the relationship of integers when multiplied or divided.

MATERIALS

- Patterns of Multiplication and Division Lesson Sheet
- Colored pencils – red and yellow
- Extra blank number lines (optional)
- Two-color counters (red/yellow)

ESSENTIAL QUESTIONS

- How are multiplication and division of integers related to one another?
- How do I model division of integers on a number line?

Grouping: Individual/Partners

OPENER/ACTIVATOR

Complete these mathematical sentences:

3 + __ = 5	**10 + 6 = __**	**-4 + 2 = __**
2 + 3 = __	**6 + __ = 16**	**2 + -4 = __**
5 – 2 = __	**16 – 6 = __**	**-2 – (- 4) = __**
5 – 3 = __	**16 – __ = 6**	**-2 – 2 = __**

What is the relationship all 3 sets have in common?

WORK SESSION

LESSON COMMENTS

This lesson uses the number line model to illustrate division of integers. When teaching this concept, it is important to revisit number line models of multiplication and addition. Revisiting patterns that are found in multiplying integers is also recommended.

LESSON DESCRIPTION

To introduce the lesson, begin with student’s knowledge of addition and subtraction facts and the relationship between the two processes. This same type of relationship occurs between multiplication and division. Point out that for any multiplication fact, we can write another multiplication fact and two different related division facts.

2 x 5 = 10 5 x 2 = 10 10 ÷ 5 = 2 10 ÷ 2 = 5

Revisit multiplication of positive and negative integers.

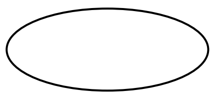
The following examples are adapted from Van de Walle, J., &Lovin, L.H. (2006). *Teaching student-centered mathematics: Grades 5-8*. Boston, MA: Pearson Education and Allyn& Bacon.

*******Show the examples with the color counters and the number line*******

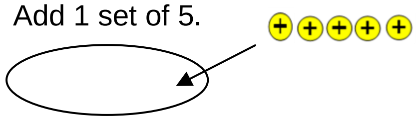
Example 1: 10 ÷ 5 = 2

How many sets of 5 will make a set of 10?

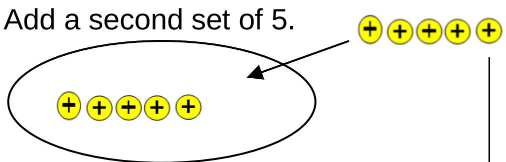
Begin with zero.



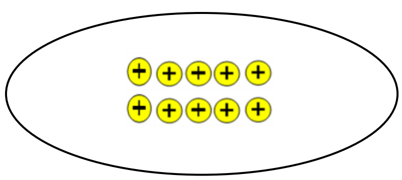
Add 1 set of 5.



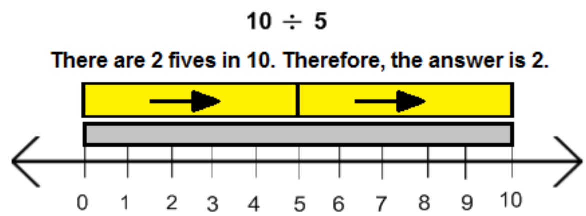
Add a second set of 5.



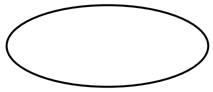
It took 2 sets of 5 to make 10.

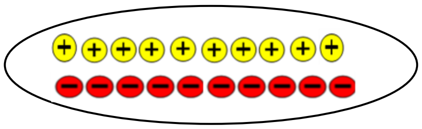


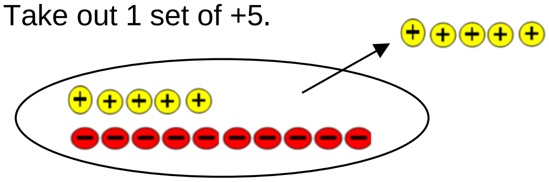
To arrive at the answer of +2, notice that on the number line we are moving forward. We move forward 2 times.

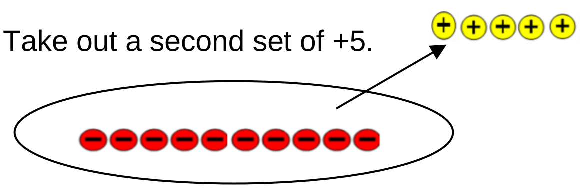


<table>
<tr><th colspan="2">Example 2: (-10) ÷ 5 = (-2)</th></tr>
<tr><td>How many sets of +5 will make -10?

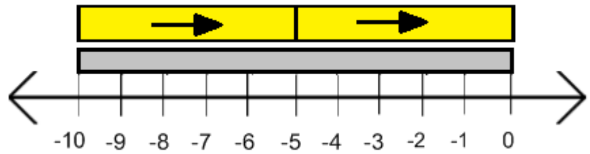
Begin with zero. 

Change the representation. Add 10 neutral pairs.


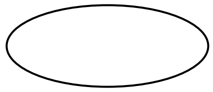
Take out 1 set of +5.


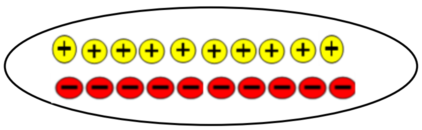
Take out a second set of +5.


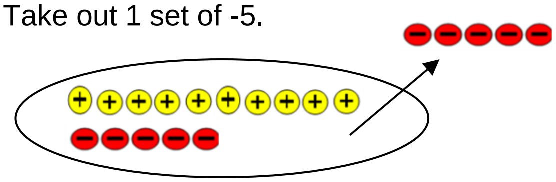
2 sets of +5 were removed to make -10 or -2 sets of 5 were used to make -10.</td>
<td>To arrive at the answer of (-2), notice that on the number line we are backing up. We back up 2 times.

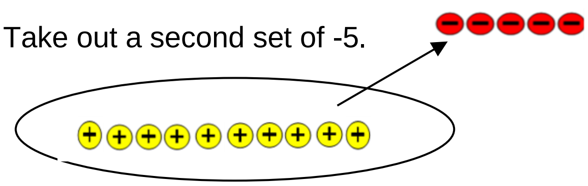
</td></tr>
</table>

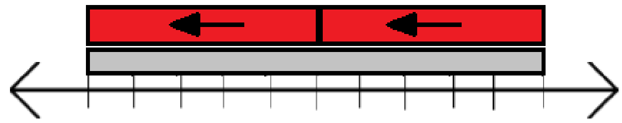
<table>
<tr><th colspan="2">Example 3: $10 \div (-5) = (-2)$</th></tr>
<tr>
<td>How many sets of -5 will make +10?

Begin with zero. 

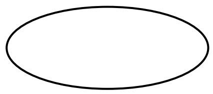
Change the representation. Add 10 neutral pairs.


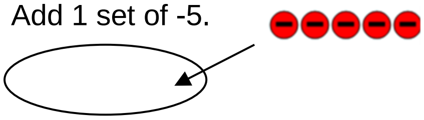
Take out 1 set of -5.


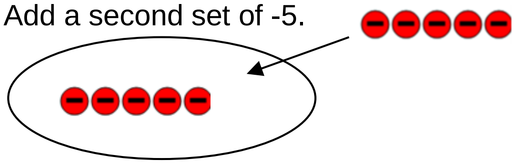
Take out a second set of -5.


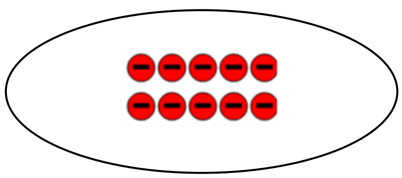
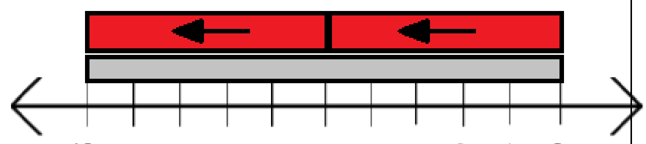
2 sets of -5 were removed to make +10 or -2 sets of -5 were used to make +10.</td>
<td>To arrive at the answer of (-2), notice that on the number line we are backing up (-5). We back up 2 times.

</td>
</tr>
</table>

<table>
<tr><th colspan="2">Example 4: (-10) ÷ (-5) = 2</th></tr>
<tr><td>How many sets of -5 will make a set of -10?

Begin with zero. 

Add 1 set of -5.


Add a second set of -5.


It took 2 sets of -5 to make -10.
</td>
<td>To arrive at the answer of +2, notice that on the number line we are moving forward. We move (-5) forward 2 times.

</td></tr>
</table>

As you teach the division model using counters, keep in mind these simple steps:

- Determine how many sets of the divisor are needed to make the dividend.
- Begin with zero or a representation of zero using neutral pairs.
- Remove or add sets of the divisor to make the dividend.
- The number of sets removed or added determines the answer.

As you teach the division model using a number line, keep in mind these simple steps:

- Identify the dividend on the number line.
- Look at the divisor, is it positive (yellow with right arrow) or negative (red with left arrow).
- Determine how many times the divisor will have to move forward (+) or backward (-) to equal the dividend.
- **The number of times it must move and the type of movement determine the answer.**

For further clarification of the division model using a number line, please watch the following video.
http://www.youtube.com/watch?v=Lh0tBKOTq8I

Have students model the problems.
LESSON DIRECTION

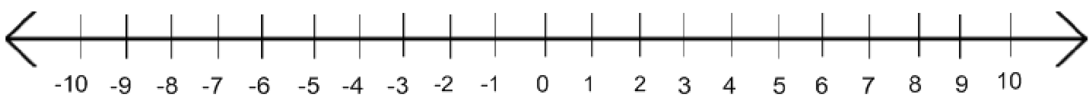
You have recently practiced multiplying positive and negative integers on a number line. It is now your turn to model how to divide. Below are "hints" to help you get started.

When you divide, keep in mind these simple steps:

- Identify the **dividend** on the number line.
- Look at the **divisor**, is it **positive (yellow with right arrow)** or **negative (red with left arrow)**.
- Determine how many times the **divisor** will have to **move forward (+)** or **backward (-) to equal the dividend**.
- The **number of times** it must move and the **type of movement determine the answer.**

<u>Model the following on the number line</u>.

8 ÷ 2

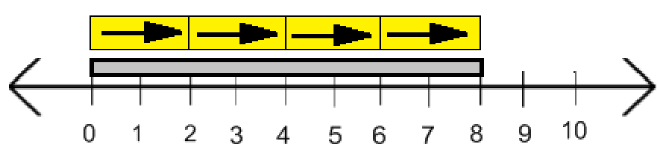


1. What is the dividend? ________

2. What is the divisor? _________

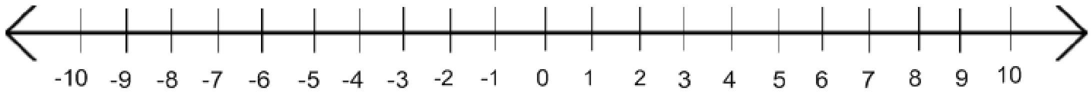
3. What is the solution and how did you find it?

<u>Solution:</u>
1. 8
2. 2
3. 4



The dividend is 8, the divisor is 2, so I counted by two's.
I counted 4 times forward.

$(-9) \div 3$

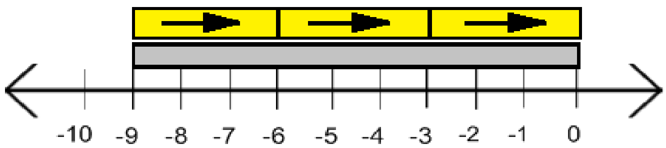


4. What is the dividend?________

5. What is the divisor?________

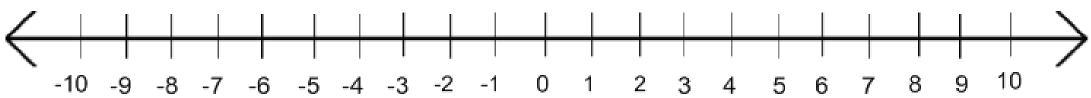
6. What is the solution and how did you find it?

Solution:
4. (-9)
5. 3
6. (-3)



The dividend is (-9), the divisor is 3, so I counted by three's.
I counted 3 times backward.

$6 \div (-2)$

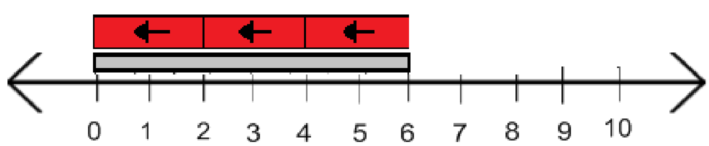


7. What is the dividend? ________

8. What is the divisor? ________

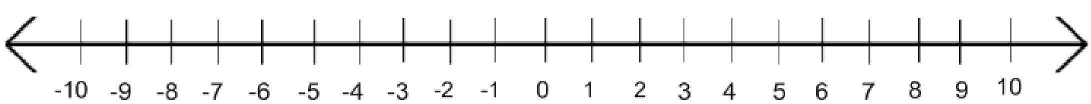
9. What is the solution and how did you find it?

Solution:
10. 6
11. (-2)
12. (-3)



The dividend is 6, the divisor is (-2), so I counted by two's.
I counted 3 times backward.

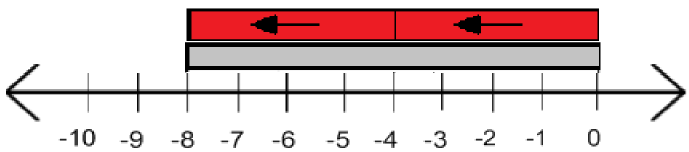
$(-8) \div (-4)$



10. What is the dividend?________
11. What is the divisor?________

12. What is the solution and how did you find it?

Solution:
16. (-8)
17. (-4)
18. 2



The dividend is (-8), the divisor is (-4), so I counted by four's.
I counted 2 times forward.

CLOSING/SUMMARIZER

Let's look to see if there are any patterns.

1. When given a **positive integer** as the **dividend**…
   a. What was the result of **dividing** by a **positive integer**?

   b. What was the result of **dividing** by a **negative integer?**

2. When given a **negative integer** as the **dividend**…
   a. What was the result of **dividing** by a **positive integer**?

   b. What was the result of **dividing** by a **negative integer?**

Student Edition Learning LESSON: Patterns of Multiplication and Division

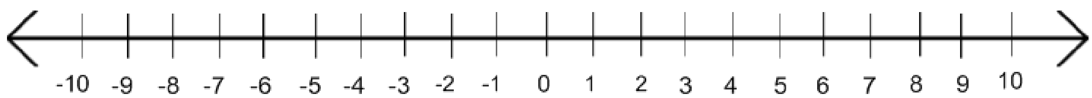
You have recently practiced dividing positive and negative integers on a number line. It is now your turn to model how to divide. Below are "hints" to help you get started.

When you divide, keep in mind these simple steps:

- Identify the **dividend** on the number line.
- Look at the **divisor**, is it **positive (yellow with right arrow)** or **negative (red with left arrow).**
- Determine how many times the **divisor** will have to **move forward (+)** or **backward (-) to equal the dividend.**
- The **number of times** it must move and the **type of movement determine the answer**.

<u>Model the following on the number line</u>.

$8 \div 2$

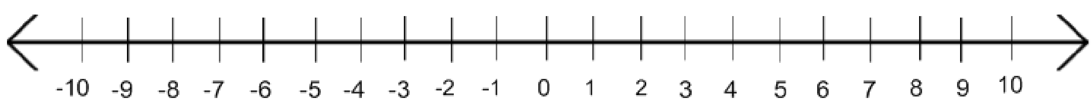


1. What is the dividend?___________

2. What is the divisor? ___________
3. What is the solution and how did you find it?

____

$(-9) \div 3$



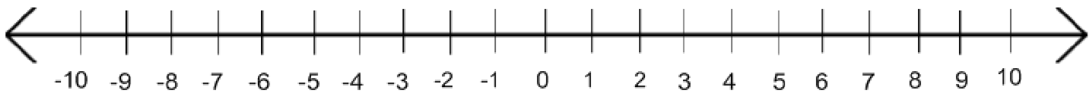
4. What is the dividend? ___________

5. What is the divisor? ___________

6. What is the solution and how did you find it?

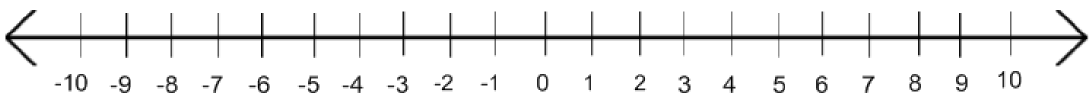
____

$6 \div (-2)$



7. What is the dividend?__________
8. What is the divisor?__________
9. What is the solution and how did you find it?

$(-8) \div (-4)$



10. What is the dividend?__________
11. What is the divisor?__________
12. What is the solution and how did you find it?

CLOSING/SUMMARIZER

Let's look to see if there are any patterns.

1. When given a **positive integer** as the **dividend**…
   a. What was the result of **dividing** by a **positive integer**?

   b. What was the result of **dividing** by a **negative integer**?

2. When given a **negative integer** as the **dividend**…
   a. What was the result of **dividing** by a **positive integer**?

   b. What was the result of **dividing** by a **negative integer**?

Debits and Credits

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ2. Students will conceptualize positive and negative numbers (including decimals and fractions).
a. Explain the meaning of zero. (MGSE6.NS.5)
b. Represent numbers on a number line. (MGSE6.NS.5,6)
c. Explain meanings of real numbers in a real world context. (MGSE6.NS.5)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)
e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MGSE7.NS.3, MGSE7.EE.3, MGSE9-12.N.Q.3)

Common Misconceptions:
Visual representations may be helpful as students begin this work. If they do not have a visual to illustrate what is happening when they are adding and subtracting integers, they will get lost in the symbols and will not know how to combine the absolute value of the integers.
Students may struggle with the vocabulary since there are many different ways to debit your account.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.

5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to use a check register to record debits and credits and calculate a running total balance.

MATERIALS

- Student Ledger Sheet

ESSENTIAL QUESTION

- When keeping a ledger, why are the rules for adding and subtracting rational numbers important?

Grouping: Partner/Individual

OPENER/ACTIVATOR

Discuss and define:

Suppose you have been given a checkbook. Your checkbook has a ledger for you to record your transactions. There are two types of transactions that may take place, (1) deposits (money placed in the account) and (2) debits/ payments (money out of the account). The difference between debits/payments and deposits tells the value of the account. If there are more credits than debits, the account is positive, or "in the black." If there are more debits than credits, the account is in debt, shows a negative cash value, or is "in the red."

Vocabulary key –
Transaction = debit or credit from an account
Debit (withdrawal) = Check or debit card usage written out of the checking account
Credit= Deposit of money put in the account

Suggestion- Bring an actual check register to show and demonstrate the described situations in the lesson.

WORK SESSION
LESSON DESCRIPTION

<u>Situation #1</u>:
Use the ledger to record the information and answer the questions.
Note: On August 12, your beginning balance is $0.00 ***(This will be the first line in the ledger.)***
1. On August 16, you receive a check from your Grandmother for $40 for your birthday.
2. On August 16, you receive a check from your Parents for $100 for your birthday.
3. On August 17, you purchase a pair of pants from Old Navy for $23.42.
4. On August 18, you find $5.19 in change during the day.
5. On August 19, you purchase socks from Wal-Mart for $12.76.

<u>Comment</u>
The ledger below shows the transactions.

DATE	TRANSACTION	PAYMENT (-)	DEPOSIT (+)	BALANCE
8.12	***Beginning Balance***			***0.00***
8.16	***Money from Grandma***		***$40.00***	***$40.00***
8.16	***Money from Parents***		***$100.00***	***$140.00***
8.17	***Old Navy***	***$23.42***		***$116.58***
8.18	***Found Change***		***$5.19***	***$121.77***
8.19	***Wal-Mart***	***$12.76***		***$109.01***

A. What is your balance after five transactions?
<u>Solution</u>
$109.01

B. How much money did you deposit (show as a positive value)?
<u>Solution</u>
$145.19

C. How much money did you pay or withdraw (show as a negative value)?
<u>Solution</u>
$-36.18

Situation #2:
Use the ledger to record the information and answer the questions.
Note: On May 5, your beginning balance is $8.00

1. On May 6, you spent $4.38 on a gallon of ice cream at Marty's Ice Cream Parlor.
2. On May 7, you spent $3.37 on crackers, a candy bar, and a coke from Circle H convenience store.
3. On May 8, you received $10 for cutting the neighbor's grass.
4. On May 8, you spent $14.80 on a downloaded book for your Kindle.

Comment
The ledger below shows the transactions.

DATE	TRANSACTION	PAYMENT (-)	DEPOSIT (+)	BALANCE
5.5	***Beginning Balance***			***$8.00***
5.6	***Marty's Ice Cream Parlor***	***$4.38***		***$3.62***
5.7	***Circle H Convenience***	***$3.37***		***$.25***
5.8	***Cutting Grass***		***$10.00***	***$10.25***
5.8	***Book for Kindle***	***$14.80***		***$-4.55***

A. What is your balance after four transactions?
Solution
There is no money left. There is a negative balance of $4.55.

B. How much money did you deposit (show as a positive value)?
Solution
$10.00

C. How much money did you pay or withdraw (show as a negative value)?
Solution
$-22.55
D. Can you really afford to spend $14.80 on a book for your Kindle? If not, how much money do you need to earn to have an account balance of $0?

Solution
No. I need to deposit another $4.55 to have an account balance of $0.

Situation #3:
Use the ledger to record the information and answer the questions.
Note: On July 4, your beginning balance is (-$40).
Show, using at least eight transactions, a way you can have an ending account balance of more than $145. You must include debit and credit amounts that have cents in at least five of your transactions. Your ledger must show both credits and debits. Be sure to fill out the ledger as you go.

Comment

Answers will vary. An example is given below.

DATE	TRANSACTION	PAYMENT (-)	DEPOSIT (+)	BALANCE
7.4	*Beginning balance*			*$-40.00*
7.5	*Allowance*		*$20.50*	*$-19.50*
7.12	*Allowance*		*$20.50*	*$1.00*
7.14	*Cutting Grass*		*$10.00*	*$11.00*
7.15	*Yard Sale: Video Game*	*$8.25*		*$2.75*
7.17	*Birthday money*		*$100.00*	*$102.75*
7.19	*Wal-Mart*	*$8.43*		*$94.32*
7.20	*Allowance*		*$20.50*	*$114.82*
7.27	*Allowance*		*$20.50*	*$135.32*
7.31	*Money for Extra Chore*		*$15.50*	*$150.82*

DIFFERENTIATION

Extensions:
Situation #3 could be used as an extension activity. Also, you could extend the assignment to include a monthly budget. Use newspapers or technology so students find an apartment they can afford, a grocery budget, entertainment, and within a given income (e.g. $1000 a month).

Intervention:
Rewrite: The standards associated with this task are addressed by completing situations 1 and 2. For extra help with integer operations, please open the hyperlink Intervention Table.

Student Edition Performance Lesson: Debits and Credits

Suppose you have been given a checkbook. Your checkbook has a ledger for you to record your transactions. There are two types of transactions that may take place, (1) deposits (money placed in the account) and (2) debits/ payments (money –which you spend and it comes out of the account). The difference between debits and the deposits tells the value of the account. If there are more credits than debits, the account is positive, or "in the black". "in the black." If there are more debits than credits, the account is in debt, shows a negative cash value, or is "in the red."

Vocabulary key –
Transaction = debit or credit from an account
Debit (withdrawal) = Check or debit card usage written out of the checking account
Credit= Deposit of money put in the account

Situation #1:
Use the ledger to record the information and answer the questions.
Note: On August 12, your beginning balance is $0.00
1. On August 16, you receive a check from your Grandmother for $40 for your birthday.
2. On August 16, you receive a check from your Parents for $100 for your birthday.
3. On August 17, you purchase a pair of pants from Old Navy for $23.42.
4. On August 18, you find $5.19 in change during the day.
5. On August 19, you purchase socks from Wal-Mart for $12.76.

DATE	TRANSACTION	PAYMENT (-)	DEPOSIT (+)	BALANCE
8/16	Beginning balance			$0.00

A. What is your balance after five transactions?

B. How much money did you deposit (show as a positive value)?

C. How much money did you pay or withdraw (show as a negative value)?

Situation #2:
Use the ledger to record the information and answer the questions.

Note: On May 5, your beginning balance is $8.00

1. On May 6, you spent $4.38 on a gallon of ice cream at Marty's Ice Cream Parlor.
2. On May 7, you spent $3.37 on crackers, a candy bar, and a coke from Circle H convenience store.
3. On May 8, you received $10 for cutting the neighbor's grass.
4. On May 8, you spent $14.80 on a downloaded book for your Kindle.

DATE	TRANSACTION	PAYMENT (-)	DEPOSIT (+)	BALANCE

A. What is your balance after four transactions?

B. How much money did you deposit (show as a positive value)?

C. How much money did you pay or withdraw (show as a negative value)?

D. Can you really afford to spend $14.80 on a book for your Kindle? If not, how much money do you need to earn to have an account balance of $0?

<u>Situation #3:</u>
Use the ledger to record the information and answer the questions.
Note: On July 4, your beginning balance is (-$40).

Requirements:

- Use at least eight transactions, four of which are debits and four are credits.
- You must have an ending balance of $145.
- You must include debit and credit amounts that have cents in at least five of your transactions.

Be sure to fill out the ledger as you go.

DATE	TRANSACTION	PAYMENT (-)	DEPOSIT (+)	BALANCE

CLOSING/SUMMARIZER

Journal: George had $230 in his checking account. His car payment is due for $250. After he makes that payment, what is the balance in his account? What might be some of the consequences?

Estimating the Square Root of a Number

SUGGESTED TIME FOR THIS LESSON:

60-90 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

> **MFANSQ3.** Students will recognize that there are numbers that are not rational, and approximate them with rational numbers.
> a. Find an estimated decimal expansion of an irrational number locating the approximations on a number line. For example, for $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue this pattern in order to obtain better approximations. (MGSE8.NS.1,2)
> b. Explain the results of adding and multiplying with rational and irrational numbers. (MGSE9-12.N.RN.3)
>
> **MFANSQ4**. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
> b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
> d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

Common Misconceptions

- *Students often mistake the square root symbol for long division.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.

5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to estimate the square root of a number using the number line or colored tiles.

MATERIALS

- Colored Tiles
- “Estimating the Square Root of a Number” recording sheet

ESSENTIAL QUESTION

- How can you estimate the square root of a number?

Grouping: Individual/Partner/Whole Group

OPENER/ACTIVATOR

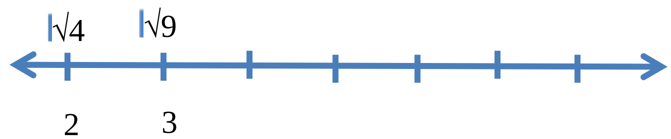
App: HOODAMATH Use the “Roots of Life” game. Great practice with perfect squares.
Teacher Notes: The teacher can project the game to the board and the class can yell out the answer or each student could load it on their own device. Fun quick game to get the brain pumping!!!

WORK SESSION

Teacher Notes: Read through these two options. Use the one you feel would best fit your students based on the manipulatives you have in your classroom.

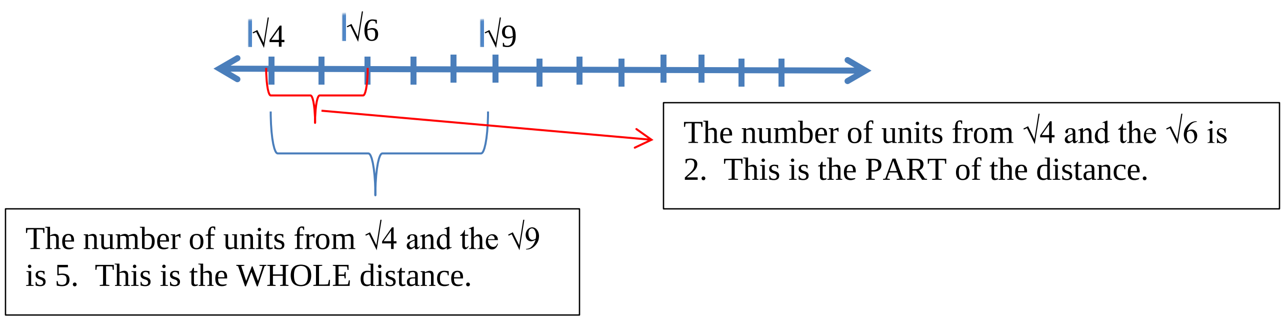
Most of us grew up narrowing down the two perfect squares that the number was between. For example, the √6 is between the √4 and the √9; therefore, the answer is 2 point something. We then used trial and error until we were satisfied. There is an easier way which helps to reinforce the part to whole concept.

NUMBER LINE APPROACH:
We begin by using a number line to estimate the square root of a number (6).



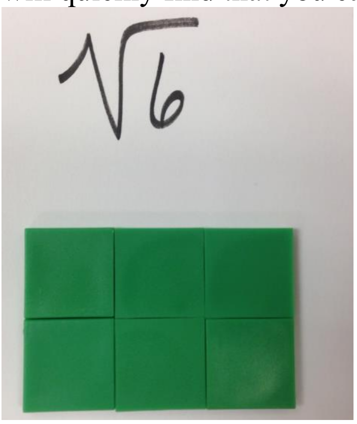
We can easily see the square root of the number 6 is located between the whole numbers of 2 and 3.

Let's take a closer look. This time zooming in and looking at the square root format.

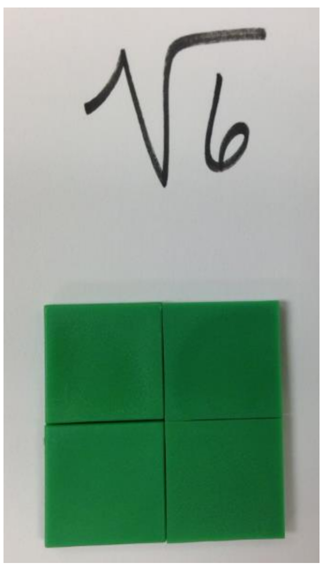


So the √6 is $\frac{2}{5}$ of the way to 3; therefore, the estimated value of √6 is $2\frac{2}{5}$ which is about 2.4.

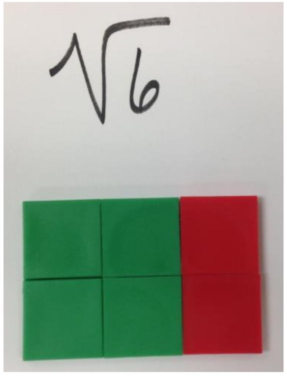
<u>COLOR TILE APPROACH:</u>
Let us take a different approach to estimating the $\sqrt{6}$, place 6 colored tiles in an array. Since we are trying to find the square root, the array should be in the form of a square; however, students will quickly find that you can't make a square out of 6 colored tiles.



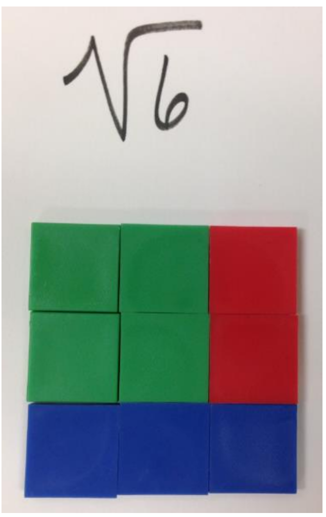
Ask the students, "What is the largest square that can be made with the 6 tiles?" The answer as you can see below is a 2 by 2 which uses 4 tiles. But….



We cannot just toss away the 2 extras after all we are looking for the $\sqrt{6}$. Again, take note that we know the $\sqrt{6}$ is 2 point something because $\sqrt{6}$ is greater than the $\sqrt{4}$. In order to remind ourselves of this, place the other 2 colored tiles down in a different color. Also remember that we are thinking in terms of part to whole. How many more color tiles do you need to add to get to the next square?



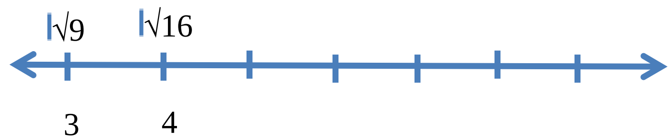
Using yet another color and by adding 3 more colored tiles, the next square has been created. Again focus on the part to whole, we had 2 tiles and it took a total of 5 tiles to get to the next size perfect square of 9. The estimated value of √6 is $2\frac{2}{5}$ which is about 2.4.



Let's look at another example, the √15 is between the √9 and the √16; therefore, the answer is 3 point something. We then used trial and error until we were satisfied. There is an easier way which helps to reinforce the part to whole concept.

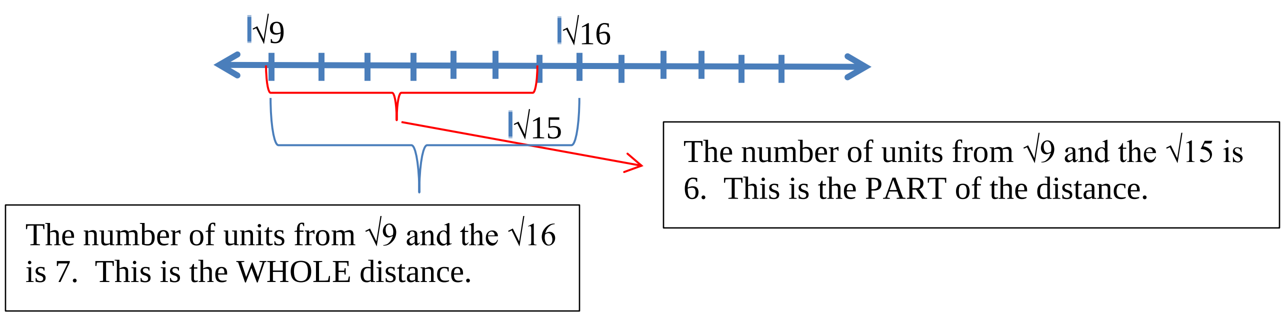
NUMBER LINE APPROACH:

We begin by using a number line to estimate the square root of a number.



We can easily see the number is located between the whole numbers of 3 and 4.

Let's take a closer look. This time zooming in and looking at the square root format.

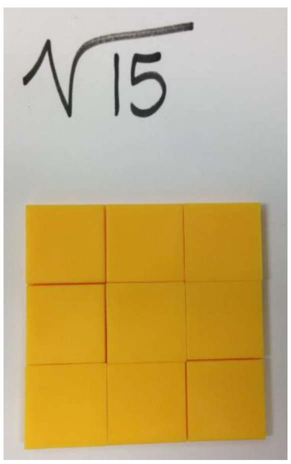


So the $\sqrt{15}$ is $\frac{6}{7}$ of the way to 4; therefore, the estimated value of $\sqrt{15}$ is $3\frac{6}{7}$ which is about 3.9.

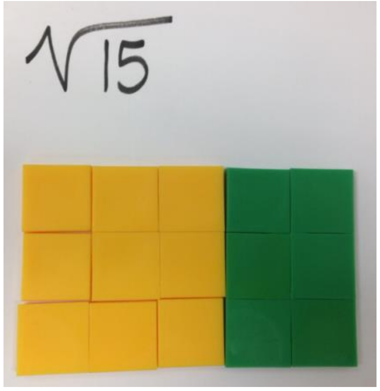
COLOR TILE APPROACH:

To estimate the $\sqrt{15}$, place 15 colored tiles in an array. Since we are trying to find the square root, the array should be in the form of a square; however, once again the students will quickly find that you cannot make a square out of 15 colored tiles

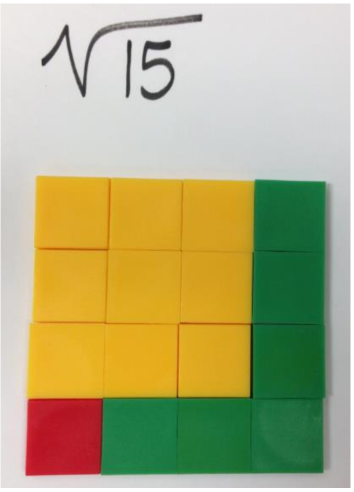
Again ask the students, "What is the largest square that can be made with the 15 tiles?" The answer as you can see below is a 3 by 3 which uses 9 tiles. But….



We cannot just toss away the 6 extras after all we are looking for the $\sqrt{15}$. Again, take note that we know the $\sqrt{15}$ is 3 point something because $\sqrt{15}$ is greater than the $\sqrt{9}$. In order to remind ourselves of this, place the other 6 colored tiles down in a different color. Also remember that we are thinking in terms of part to whole. How many more color tiles do you need to add to get to the next square?



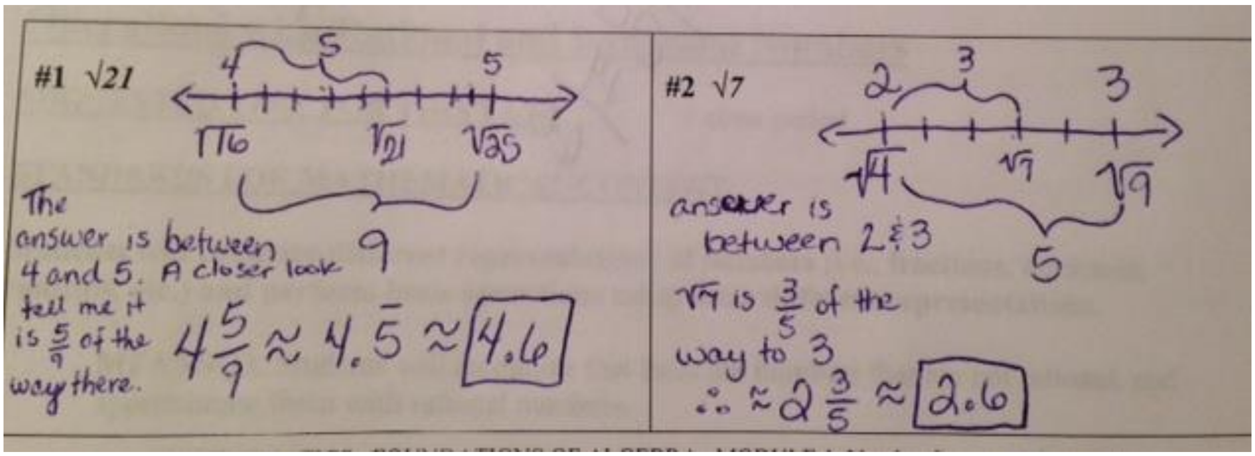
Using yet another color and by adding just 1 more colored tile, the next square has been created. Again focus on the part to whole, we had 6 tiles and it took a total of 7 tiles to get to the next size perfect square of 16. The estimated value of $\sqrt{15}$ is $3\frac{6}{7}$ which about 3.9.

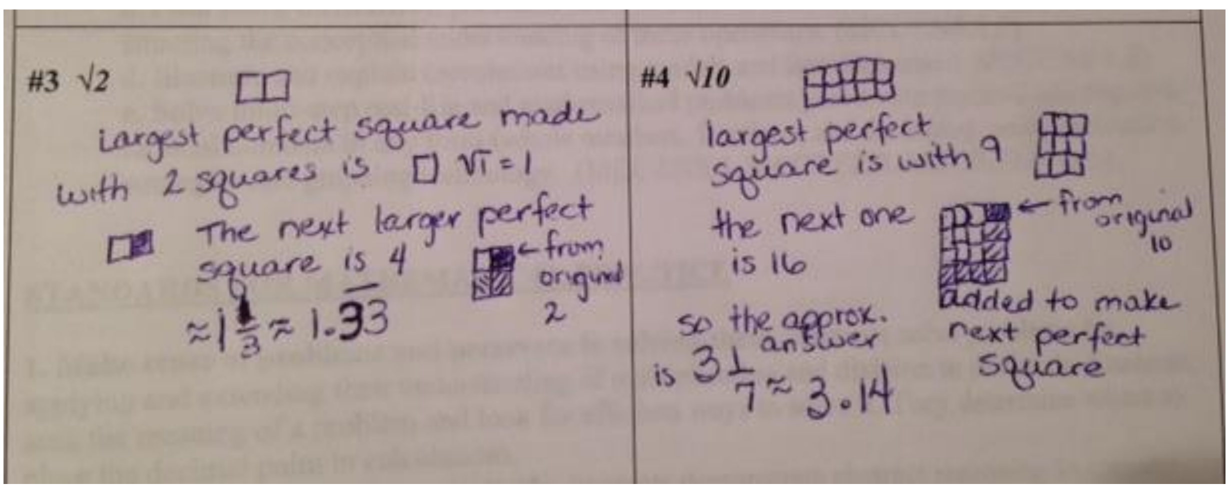


After working a few examples with the class, have the students work in pair to model estimating a few square roots of their own. Working with a partner, they both work the same problem and then share the results with each other. The students should continue this method as the teacher walks around the room checking progress.

Teacher Edition: ESTIMATING THE SQUARE ROOT OF A NUMBER

Directions: Find the estimate of the square root of a number. Draw a model to illustrate your findings.





Student Edition: ESTIMATING THE SQUARE ROOT OF A NUMBER

Directions: Find the estimate of the square root of a number. Draw a model to illustrate your findings.

#1 $\sqrt{21}$	**#2** $\sqrt{7}$
#3 $\sqrt{2}$	**#4** $\sqrt{10}$

CLOSING/SUMMARIZER

Journal – Estimate the $\sqrt{27}$. Use a model to explain your estimation.

Decimal Approximation of Roots

http://www.openmiddle.com/decimal-approximations-of-roots/#prettyPhoto
Source: Bryan Anderson

This task is included in the grade level framework; so teachers should consult with each other to decide whether to use this activity in support or in the regular classroom.

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ3. Students will recognize that there are numbers that are not rational, and approximate them with rational numbers.
a. Find an estimated decimal expansion of an irrational number locating the approximations on a number line. For example, for $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue this pattern in order to obtain better approximations. (MGSE8.NS.1,2)
b. Explain the results of adding and multiplying with rational and irrational numbers. (MGSE9-12.N.RN.3)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

Common Misconceptions:

Students may only focus on the idea that square roots are related to multiplication, so they may just divide by the number in the radical. For example, students may see $\sqrt{2}$ and think "what can I do to a number to get a 2 in the square root?" Often, students will just divide by the number, in this case 2, and get $\sqrt{2} = 1$. To address this misconception, students need to experience square numbers as actual squares, built and/or drawn, and then find the root of the square to be the length of a side of that square.
The symbol we use for square, cube, or any other root is just that, a symbol to represent how two quantities are related, not a concept in itself. In order to help students understand this relationship, give them the following pattern:

1, 4, 9, 16, . .

Ask students to find the next 3 numbers in the sequence. When they find the next three, ask them to explain how they determined the next 3 numbers. Many students will likely say that *they found the pattern 1 + 3 = 4, 4 + 5 = 9, 9 + 7 = 16, so they kept going with + 9, + 11, + 13, to get each successive number in the pattern.*

Now, ask students to build rectangles for each of the numbers in the pattern using square tiles. Students will find that all of the numbers create squares and from there, students can be guided to the idea that the root of the squares is the length of one side of the square. This also connects back to the original pattern in that the square root is the number of the term in the sequence, i.e., the square root of 1 is 1 - 1st term of the sequence. The square root of 4 is 2 – the second term of the sequence. The square root of 9 is 3 – the third term of the sequence, etc.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to explain why some square roots are irrational and determine decimal approximations for these roots.

MATERIALS

- One-inch grid paper (multi-weight grid lines-see attached)
- Scissors
- Decimal Approximations of Roots Lesson
- Open Middle Recording Sheet

ESSENTIAL QUESTIONS

- How can we determine approximations of square roots that are irrational?
- How do we know if our approximations are reasonable?

Grouping: Individual/Partner

OPENER/ACTIVATOR

Give students the following pattern:

1, 4, 9, 16,. .

Ask them to determine the next three terms in the sequence and justify how they found them (identify a pattern). Most will find the pattern mentioned in the Common Misconceptions section above.

Now, ask students to build rectangles using the number of square tiles indicated by the number in the sequence (i.e. use 4 square tiles to build a rectangle for the number 4).

Discuss the results. Ask students questions about the relationship between the side length and the square. The side length or square root, is the number of the term in the sequence. The square root of 1 is 1 – the first term in the sequence. The square root of 4 is 2 – the second term in the sequence, etc.

BACKGROUND KNOWLEDGE

When students memorize procedures for square roots without making sense of them, misconceptions develop. The opener, above, is a first step in making sure this does not happen. Students need to make sense of the conceptual idea of a square root. Once students conceptualize the idea of a square root, using the square root symbol to communicate their ideas mathematically is all that is needed.

NUMBER TALKS

Several Number Talks strategies can help student build a stronger understanding of square roots. Repeated subtraction, partial quotients, multiplying up and proportional reasoning are all valuable strategies that students can explore through number talks. For more information, refer to pgs. 286-299 in *Number Talks.* (Number Talks, 2010, Sherry Parrish)

LESSON DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Part I:

Start this lesson with a whole group discussion regarding square roots. Guide the discussion so that the conceptual understanding is the focus.

Students will work with materials to determine approximations for square roots of numbers that are not perfect squares.

Lesson Directions

Give pairs of students a sheet of the multi-weight grid paper (copy provided on following pages) and ask them to cut out two large squares (1”). Ask students to discuss how the squares are divided, then share with the group. Each square is divided into tenths (columns/rows) and hundredths (there are 100 small squares within each large square.

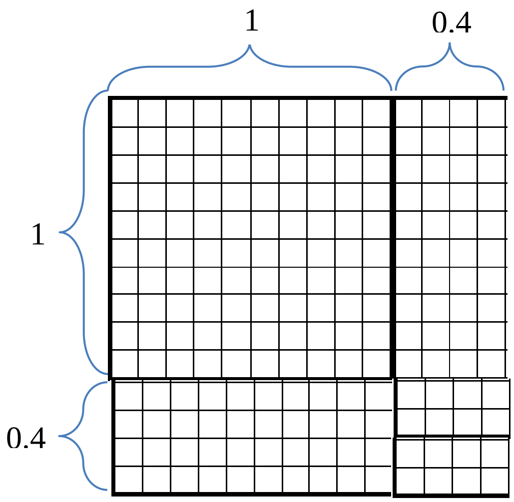
The lesson for the students is to approximate the square root of a square that has an area of two square units. Begin by asking students what this means. This may be the time for students to think about what the square root of a square with an area of 4 square units would be, then pose the lesson question again.

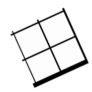
Once students have the idea that they need to find out what the a square with an area of 2 square units might look like, then determine the approximate length of the side of that square to find the square root, they can begin the lesson.

Begin by estimating.
The following questions should be discussed by students before proceeding to the lesson. Have pairs share their reasoning for each.

- Which square number is 2 close to? (1).
- Will the side of the square (the square root) of 2 be more than the square root of 1 or less?

This part may take a full class period. Many students may arrive at some different approximations. Some with different approximations. One possible student solution is shown below.





Leftover hundredths need to be shared with the whole square.

Students' approximations should be close to 1.4 and a little bit more (1.41).

Try this lesson again with another number close to a perfect square, such as 5.

Begin by estimating.

The following questions should be discussed by students before proceeding to the lesson. Have pairs share their reasoning for each.

- Which square number is 5 close to? (4).
- Will the side of the square (the square root) of 5 be more than the square root of 4 or less?

Allow students to cut their squares and make approximations based on the cuts.

Tip: As students discover their squares, have them keep a record of them in their notebook/journal as evidence of their progression of understanding square roots and the approximations.

Part II
Students will complete an Open Middle Problem from www.openmiddle.com. A copy of the **student worksheets** is included in this lesson.

Lesson Directions

Students will use their understandings of approximating square roots of decimals to complete the following problem:

Directions: Using only numbers 1-6 (and only once per inequality), fill in the boxes to create a true statement with the smallest possible interval:

$$\square.\square\square < \sqrt{2} < \square.\square\square$$

$$\square.\square\square < \sqrt{3} < \square.\square\square$$

$$\square.\square\square < \sqrt{4} < \square.\square\square$$

$$\square.\square\square < \sqrt{5} < \square.\square\square$$

$$\square.\square\square < \sqrt{6} < \square.\square\square$$

Give students time to make sense of the problem and discuss what it means with their partners, then share with the class. Making sense of the problem is a huge part of any problem solving lesson, and students need to learn how to do this by sharing their ideas during the learning process before being asked to do this independently.

FORMATIVE ASSESSMENT QUESTIONS

- What place value will impact the interval the most?
- How can you decide which numbers to place in the units (ones) place?
- For each of the problems, which approximation is closest? How can you tell?

DIFFERENTIATION

Intervention

- http://nzmaths.co.nz/resource/square-and-cube-roots
- For extra help with square roots, please open the hyperlink Intervention Table.

Decimal Approximations of Roots

Directions: Using only numbers 1-6 (and only once per inequality), fill in the boxes to create a true statement with the smallest possible interval:

$$\square.\square\square < \sqrt{2} < \square.\square\square$$

$$\square.\square\square < \sqrt{3} < \square.\square\square$$

$$\square.\square\square < \sqrt{4} < \square.\square\square$$

$$\square.\square\square < \sqrt{5} < \square.\square\square$$

$$\square.\square\square < \sqrt{6} < \square.\square\square$$

Student Worksheet

Name: ______________________________ Period: _________ Date: _________________

First attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Second attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Third attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Fourth attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Fifth attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Sixth attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Distributing and Factoring Using Area

Adapted from NCTM: Illuminations

In this lesson, students will use area models to represent and discover the distributive property. Students will be using rectangles whose sides may be variables in order to further their understanding of the distributive property.

SUGGESTED TIME FOR THIS LESSON

Approximately 60-120 minutes
The suggested time for the class will vary depending upon the needs of the students.

STANDARDS ADDRESSED IN THIS LESSON

MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.
b. Use area models to represent the distributive property and develop understandings of addition and multiplication (all positive rational numbers should be included in the models). (MGSE3.MD.7)
d. Add, subtract, and multiply algebraic expressions. (MGSE6.EE.3, MGSE6.EE.4, MC7.EE.1, MGSE9-12.A.SSE.3)
e. Generate equivalent expressions using properties of operations and understand various representations within context. For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between “3 more” and “3 times”. (MGSE4.0A.2; MGSE6.EE.3, MGSE7.EE.1,2, MGSE9-12.A.SSE.3)

COMMON MISCONCEPTIONS

As students begin to build and work with expressions containing more than two operations, students tend to set aside the order of operations. For example, having a student rewrite an expression like 8 + 4(2x - 5) + 3x can bring to light several misconceptions. Do the students immediately add the 8 and 4 before distributing the 4? Do they only multiply the 4 and the 2x and not distribute the 4 to both terms in the parenthesis? Do they collect all like terms 8 + 4 – 5, and 2x + 3x? Each of these shows gaps in students' understanding of how to rewrite numerical expressions with multiple operations.

Students have difficulties understanding equivalent forms of numbers, their various uses and relationships, and how they apply to a problem. Make sure to expose students to multiple examples and in various contexts.

Students usually have trouble remembering to distribute to both parts of the parenthesis. They also try to multiply the two terms created after distributing instead of adding them.
Students also want to add the two final terms together whether they are like terms or not since they are used to a solution being a single term.

Finally, students need to be careful to make sure negative signs are distributed properly.

STANDARDS FOR MATHEMATICAL PRACTICE

2. **Reason abstractly and quantitatively.** In this LESSON students will address the dimensions of a rectangle as they relate to its area in the application of the distributive property.

4. **Model with mathematics.** Students will explore the area model for multiplication to compare the concepts of area as a sum and area as a product.

EVIDENCE OF LEARNING/LEARNING TARGETS

By the conclusion of this lesson, students should be able to:

- Use the properties of operations to simplify linear expressions
- Apply the area representation of the distributive property.

MATERIALS

- Activity Sheet

*Optional: Colored Sheets of paper cut into rectangles. These can be used to introduce the concepts found in this lesson and to create models of the rectangles as needed.

ESSENTIAL QUESTIONS

- What are strategies for finding the area of figures with side lengths that are represented by variables?
- How can area models be used to represent the distributive property?

KEY VOCABULARY

The following terms should be reviewed/discussed as they arise in dialogue with the LESSON:

- distributive property
- commutative property

GROUPING

Individual/Partner

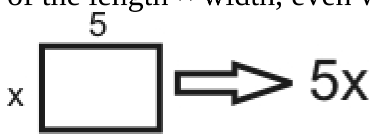
LESSON DESCRIPTION

This lesson is designed to help students understand the distributive property using area models. It is important to allow students time to develop their own solutions for how the distributive property can be used to solve problems.

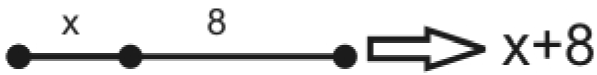
Area Representation of the Distributive Property

The first part of this activity can be used to help students recall information regarding area as a precursor for the distributive property. The following examples could be featured as anchor charts and filled in as the students work through the parts. Anchor charts could then serve as a visual reminder during work time in the lesson.

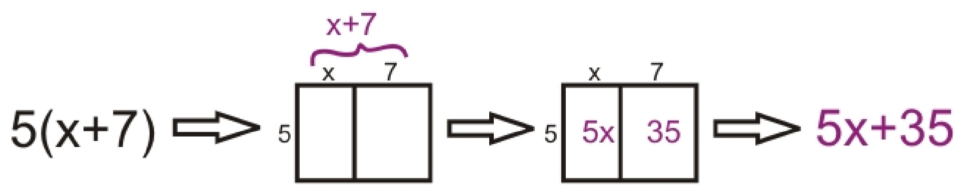
The first section introduces students to the idea of writing the area of a rectangle as an expression of the length × width, even when one or more dimensions may be represented by a variable.



The next section demonstrates how two measurements of a segment can be added together to represent the sum of the entire length of the segment.



The key section is next, having students represent the area of each rectangle *two ways* to distribute the common factor among all parts of the expression in parentheses.



<u>OPENER/ACTIVATOR</u>

To get students thinking about area in a non-traditional way, use the site Estimation180. Open the link http://www.estimation180.com/day-166.html to access a problem where students will be asked to complete the estimation activity "How much area of the paper does my *wife's* hand cover?"

This activity serves two purposes:

1. engages the students in dialogue about area
2. engages the students in estimation

Andrew Stadel provides daily estimation challenges that get kids thinking about numbers and how they are used to quantify "real world things". His activities foster number sense and problem solving accessible to students at all levels of mathematical abilities.

If you are able to access the estimation activity via the internet, students will be able to enter their estimate and compare to others all over the world. If you are unable to access the site, the activity is provided below for you to use in class on either a display board or handout for students.

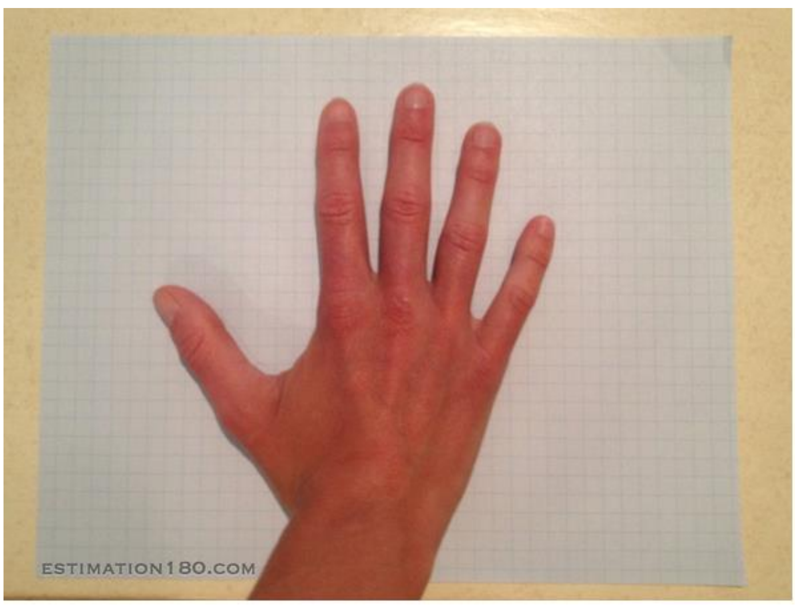
http://www.estimation180.com/uploads/1/3/8/8/13883834/estimation_180_handout_2013-08-01_font.pdf provides access to a handout students could use to keep track of their estimation activities.

By deliberately planning estimation activities into your math class, students should grow in their ability to assess the reasonableness of their responses to all types of problems. According to NCTM, "Estimation is critical to building number sense, mental math skill, and computational fluency. NCTM's *Principles and Standards for School Mathematics* assert that the development of computational fluency requires students to make a "connection between conceptual understanding and computational proficiency" (NCTM 2000, p. 35).

For more Estimation 180 activities, go to http://www.estimation180.com/

A Printable copy of the activity appears on the next page.

How much area of the paper does my *wife's* hand cover?



What's too LOW? *

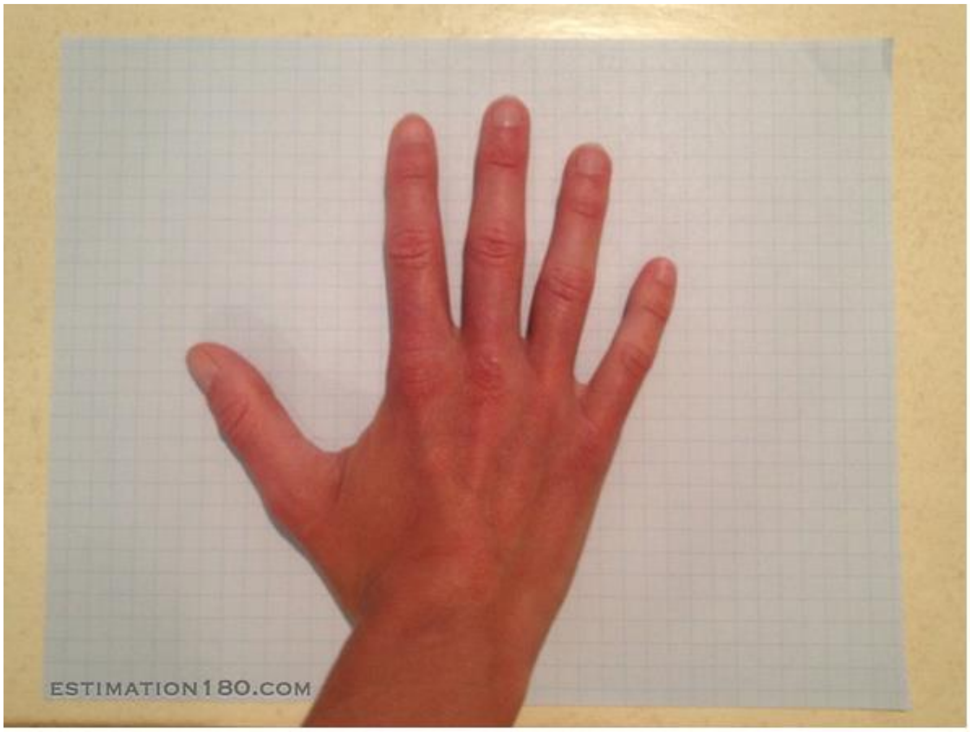
What's too HIGH? *

Your estimate. *

Your reasoning. *
Do better than "I guessed."

Your name. *

How much *area of the paper* does my *wife's* hand cover?



Use the space below to explain your strategy or make computations.

What's too LOW? *

What's too HIGH? *

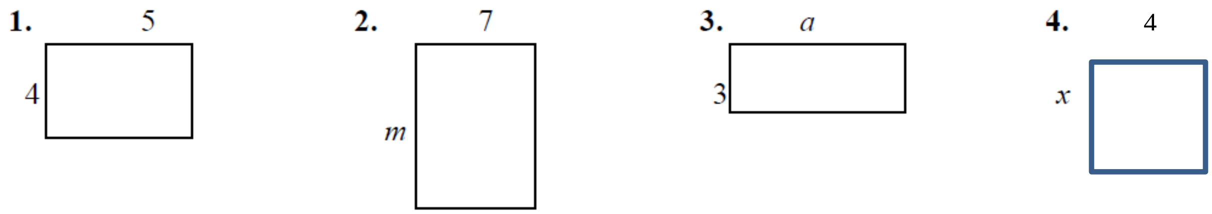
Your estimate. *

Your reasoning. *
Do better than "I guessed."

Your name. *

Teacher's Edition Distributing Using Area

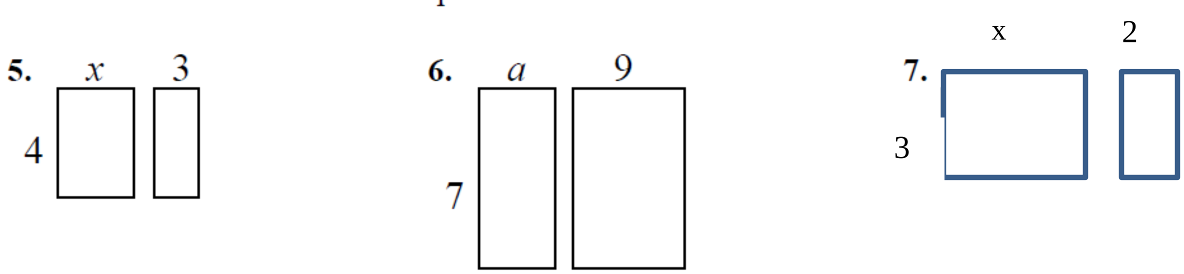
Write the expression that represents the area of each rectangle.



Area is found by multiplying the length and width of a figure together.
1) 5(4) =20 2) 7(m)=7m 3) 3(a)=3a 4) x(4) =4x

It is very important that students are encouraged to show their work on all activities of this lesson. The role of reviewing the step by step calculations will gauge the level of procedural fluency and conceptual fluency that is embedded within the work. During the formative review of the student work, prepare to intervene with strategies to address the procedural or computational error.

Find the area of each box in the pair.



Area is found by multiplying the length and width of a figure together.

This section allows students to begin to piece together some of the fundamental concepts for the distributive property.

We suggest that students write the areas of each of the figures within the corresponding boxes.
5) Area of first box: 4(x)=4x Area of second box: 4(3) =12
****Note: Students will need to recognize that the width of both figures is the same.***

6) Area of first box: 7(a)=7a Area of second box: 7(9) =63
****Note: Students will need to recognize that the width of both figures is the same.***

7) Area of first box: 3(x)=3x Area of second box: (3)(2) =6

Write the expression that represents the total length of each segment.

8. 

____

9. 

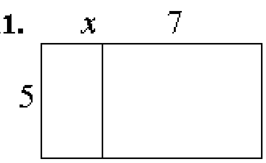
____

10. 

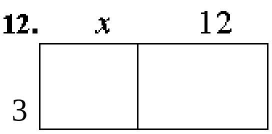
____

Solution:
8) The total length of this segment can be written as an expression: x+9 units
9) The total length of this segment can be written as an expression: x+4 units
10) The total length of this segment can be written as an expression: a+2 units
****Teachers can scaffold this section and demonstrate the ways in which to measure the total length of a segment. Other representations of these types of segments can be added in order to help students think about the same concept in multiple ways.***

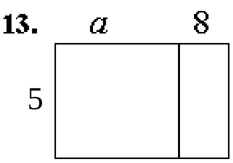
Write the area of each rectangle as the product of ***length***$\times$***width*** and also as a sum of the areas of each box.

11. 

AREA AS PRODUCT	AREA AS SUM
5(X+7)	5X+35

12. 

AREA AS PRODUCT	AREA AS SUM

13. 

AREA AS PRODUCT	AREA AS SUM

Solution:
The problems in this section demonstrate the multiple ways to represent area, which leads to the fundamental concepts of the distributive property.

- ***The area as a product section requires students to think about how to represent the area of the entire rectangle without using the area of each of the individual rectangles. Area of a rectangle can be found by multiplying the length (of the entire rectangle) and width of the rectangle.***
- ***The area as a sum section requires students to think about how to represent the area of the rectangle by using the area of each individual rectangle and taking the sum of the areas to find the area of the whole rectangle.***

11) Area as a Product: The length of the figure can be written as an expression x+7 (this has been referenced in 8-10 and teachers can use the previous questions to help students come to this realization). The width of this figure is 5 units. The area is found by multiplying $(x+7)5$ ***which is equivalent to*** $5(x+7)$ ***by the commutative property.***

***Area as a Sum:** The area of the first (left) rectangle can be found by multiplying the length, x, and the width, 5. Thus, the area of the first rectangle is x(5) or 5x by the commutative property. The area of the second rectangle can be found by multiplying the length, 7, by the width, 5. Thus the area of the first rectangle is 7(5) =35.*
In order to find the total combined area, students must add together the areas of both figures. Therefore, the total combined area is found as the expression 5x+35.

***12) Area as a Product:** The length of the figure can be written as an expression x+12 (this has been referenced in 8-10 and teachers can use the previous questions to help students come to this realization). The width of this figure is 3 units. The area is found by multiplying* $(x+12)3$ *which is equivalent to* $3(x+12)$ *by the commutative property.*

***Area as a Sum:** The area of the first (left) rectangle can be found by multiplying the length, x, and the width 3. Thus, the area of the first rectangle is* $3(x) = 3x$
The area of the second rectangle can be found by multiplying the length, 12, by the width, 3. Thus the area of the second rectangle is $12(3) = 36$.
In order to find the total combined area, students must add together the areas of both figures. Therefore, the total combined area is found as the expression $3x + 36$.

***13) Area as a Product:** The length of the figure can be written as an expression a+8. The width of this figure is 5 units. The area is found by multiplying* $(a+8)5$ *which is equivalent to* $5(a+8)$ *by the commutative property.*

***Area as a Sum:** The area of the first left rectangle can be found by multiplying the length, a, and the width, 5. Thus, the area of the first rectangle is* $(a) = 5a$.
The area of the second rectangle can be found by multiplying the length, 8, by the width, 5. Thus the area of the second rectangle is $8(5) = 40$.
In order to find the total combined area, students must add together the areas of both figures. Therefore, the total combined area is found as the expression$5a + 40$.

After finishing these questions, teachers need to help students come to the realization that the two expressions that they generated from these questions are equivalent and represent the same information in different ways.

Use the distributive property to find sums that are equivalent to the following expressions. (You may want to use a rectangle to help you)

14. $4(x+7) =$ __4x+28__	15. $7(x-3) =$ __7x-21__
16. $-2(x+4) =$ __-2x-8 *__	17. $3(x+9) =$ __3x+27__
18. $4(a-1) =$ __4a-4__	19. $3(m+2) =$ __3m+6__
20. $-4(a-4) =$ __-4a+16 *__	21. $\frac{1}{2}(a-12) =$ __½ a – 6__

For 14-21, teachers can ask students to use rectangles to solve the problems. This helps students recognize how to solve the problems while using the area model generated from questions 11-13.

****Note: This section also gives students practice with negative numbers. Students may struggle with the idea of a "negative area." Please see below for Illumination's description for how to simplify for the total area, especially taking note of their suggestion for handling the subtraction problems or the negative signs.***
Adapted from NCTM: Illuminations http://illuminations.nctm.org/LessonDetail.aspx?id=L744

FORMATIVE ASSESSMENT QUESTIONS

These questions can be used to help further develop understanding of the distributive property.

- What is the relationship between the *product* and *sum* representation of the area model?
- How does the area model help to explain the distributive property?
- Why do you think this property was named the distributive property?

DIFFERENTIATION

Extension

Have students create and explain models to demonstrate the sum of four or more positive and negative numbers.

Intervention

Have students use models other than those suggested in the lesson to add positive and negative numbers, for example, the stack or row model and hot air balloon model.

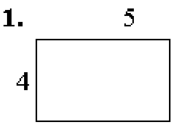
For extra help with multiplication problems, please open the hyperlink Intervention Table.

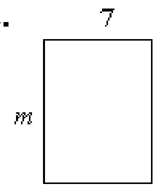
CLOSING

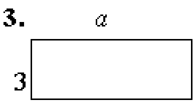
Ask students to create a multiplication problem that would be easier to solve using the distributive property. Allow students to share with their neighbor. Select a few to pose to the class.

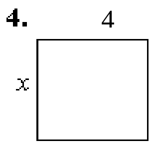
Student Edition Learning Lesson: Distributing Using Area

Write the expression that represents the area of each rectangle.

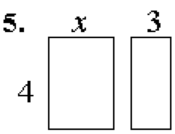
1.


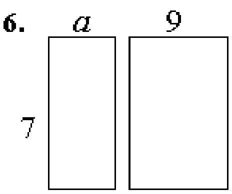
2.


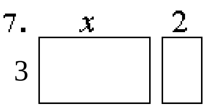
3.


4.


Find the area of each box in the pair.

5.


6.


7.


Write the expression that represents the total length of each segment.

8.


____

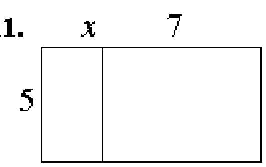
9.


____

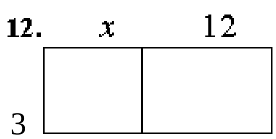
10.


____

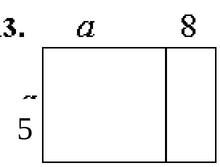
Write the area of each rectangle as the product of ***length*** $\times$ ***width*** and also as a sum of the areas of each box.

11.


AREA AS PRODUCT	AREA AS SUM
$5(x+7)$	$5x+35$

12.


AREA AS PRODUCT	AREA AS SUM

13.


AREA AS PRODUCT	AREA AS SUM

Use the distributive property to find sums that are equivalent to the following expressions. (You may want to use a rectangle to help you)

14. $4(x+7)=$______________
15. $7(x-3)=$_______________
16. $-2(x+4)=$______________
17. $3(x+9)=$_______________
18. $4(a-1)=$_______________
19. $3(m+2)=$_______________
20. $-4(a-4)=$______________
21. $\frac{1}{2}(a-12)=$_________

Triangles and Quadrilaterals

Adapted from Engage NY *A Story of Ratios*
In this lesson, students will work with variables, variable expressions, combining like terms, and evaluating variable expressions for defined values.

SUGGESTED TIME FOR THIS LESSON

60 – 90 minutes
The suggested time for the lesson will vary depending upon the needs of the students.

STANDARDS FOR MATHEMATICAL CONTENT

MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.
a. Apply properties of operations emphasizing when the commutative property applies. (MGSE7.EE.1)
c. Model numerical expressions (arrays) leading to the modeling of algebraic expressions. (MGSE7.EE.1,2; MGSE9-12.A.SSE.1,3)
d. Add, subtract, and multiply algebraic expressions. (MGSE6.EE.3, MGSE6.EE.4, MC7.EE.1, MGSE9-12.A.SSE.3)
e. Generate equivalent expressions using properties of operations and understand various representations within context. For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between "3 more" and "3 times". (MGSE4.0A.2; MGSE6.EE.3, MGSE7.EE.1,2, MGSE9-12.A.SSE.3)
f. Evaluate formulas at specific values for variables. For example, use formulas such as A = l x w and find the area given the values for the length and width. (MGSE6.EE.2)

COMMON MISCONCEPTIONS

Students tend to make the following mistakes when working with variable expressions:

- *Not recognizing that a single letter variable has a coefficient of 1 (r = 1r)*
- *Adding the coefficients of unlike terms*
- *Failing to identify all like terms in a sum*
- *Making sign errors (failing to recognize that –n = + - n)*

Additionally, students confuse application of the distributive property when the multiplier is written to the right (behind) of the parenthesis instead of to the left (in front) of the parenthesis. Clarify this misunderstanding with several examples based on student need. Additionally, students may ask if it is OK to rearrange the terms so that the multiplier is in front. Take this opportunity to explore the commutative property with the class.

Call attention to examples such as $3(2x-4)$ *being the same as* $(2x-4)3$.

STANDARDS FOR MATHEMATICAL PRACTICE

2. **Reason abstractly and quantitatively**. Students make sense of how quantities are related within a given context and formulate algebraic equations to represent this relationship.
4. **Model with mathematics**. Throughout the module, students use equations and inequalities as models to solve mathematical and real-world problems. Students test conclusions with a variety of objects to see if the results hold true, possibly improving the model if it has not served its purpose.
6. **Attend to precision**. Students are precise in defining variables. They understand that a variable represents one number. They use appropriate vocabulary and terminology when communicating about expressions and equations
7. **Look for and make use of structure**. Students recognize the repeated use of the distributive property as they write equivalent expressions. Students recognize how equations leading to the form $px + q = r$ and $(x + q) = r$ are useful in solving variety of problems.

EVIDENCE OF LEARNING/LEARNING TARGETS

By the conclusion of this lesson, students should be able to:

- Use the properties of operations to simplify linear expressions
- Combine like terms to generate equivalent expressions.

MATERIALS REQUIRED

- Envelopes containing triangles and quadrilaterals
- Match up cards for closing activity
- Template for Like Terms closing activity

ESSENTIAL QUESTIONS

- How can we represent values using variables?
- How can we determine which terms may be combined when adding or subtracting variable expressions?
- How can we evaluate variable expressions when the variable is assigned a value?

KEY VOCABULARY

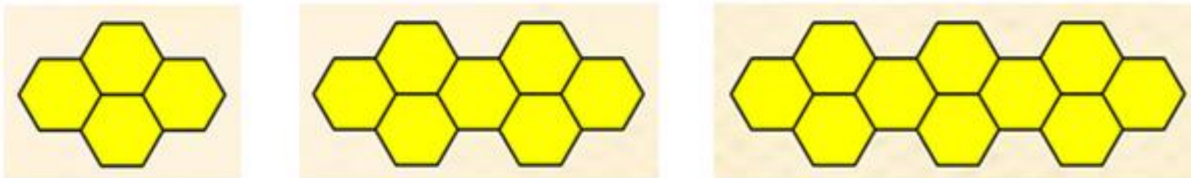
The following terms should be reviewed/discussed as they arise in dialogue within the LESSON:

- Variable expression
- commutative property

ACTIVATOR/OPENER

To get students thinking about properties of shapes (specifically the number of sides) use the following pattern from the website www.visualpatterns.org.

http://www.visualpatterns.org/41-60.html will show the following pattern: PATTERN 51 is shown below:



A **student handout** is provided on the next page to give students a place to gather their thoughts about the pattern they see. The goal is to figure out the pattern being used to create the next stage and to come up with a formula or equation to find the number of hexagons at any stage. Then, students are asked to answer the pattern (how many hexagons) for the 43 stage. In the image above, we would note that stage one has 4 hexagons, stage 2 has 7 hexagons, stage 3 has 10 hexagons.

The purpose of this introduction to the Visual Patterns Site is to get students analyzing patterns and making predictions based on the patterns and eventually to express the pattern using algebraic methods. Students will need time and repeated exposure to these activities to gain confidence and fluency. For the purpose of activating this lesson, the goal is to have dialogue about the number of sides in the shape and how many sides total are present in each stage. **It is not essential that students complete the entire visual pattern activity prior to the "Triangles and Quadrilaterals" lesson.**

URL: http://visualpatterns.org

Name KEY Pattern #: 51

Teacher's Edition of a Sample Pattern

Student Page Follows with Open Space to show work

Complete this table:

Step n	# of Hexagons
1	4
2	7
3	10
4	13
5	16
6	19
10	31
37	112

What do you notice? What do you wonder?
Number of hexagons increases by 3 each stage
Write the equation:
Total number of hexagons = 3n+1

Name: ___________________
Pattern #: _____
Draw the next step:
Complete this table:

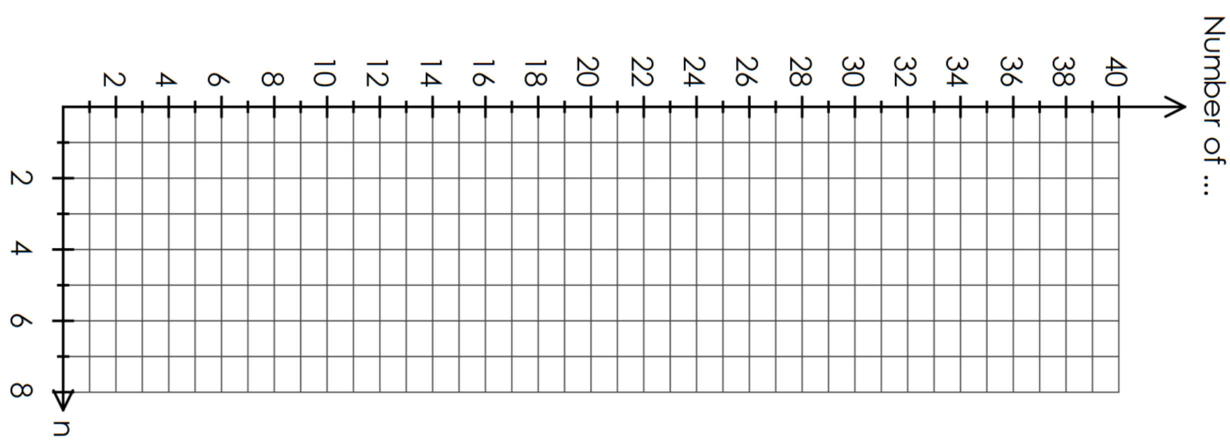
Step n	# of ________________
1	
2	
3	
4	
5	
6	
10	
37	

What do you notice?

What do you wonder?

Write the expression to get the next stage:

Graph the relationship below.



ACTIVITY for Triangles and Quadrilaterals Lesson

PART I

Each student is given an envelope containing triangles and quadrilaterals.

How might expressions be generated based on the contents of your envelope? What expressions could you write? First work individually. Next, compare with a partner. Finally, share as a whole class.

PART II

Each envelope contains a number of triangles and a number of quadrilaterals. For this exercise, let t represent the number of triangles, and let q represent the number of quadrilaterals.

a. Write an expression, using t and q, that represents the total number of sides in your envelope. Explain what the terms in your expression represent.

$3t+4q$

Triangles have 3 sides, so there will be 3 sides for each triangle in the envelope. This is represented by $3t$. Quadrilaterals have 4 sides, so there will be 4 sides for each quadrilateral in the envelope. This is represented by $4q$. The total number of sides will be the number of triangle sides and the number of quadrilateral sides together.

b. You and your partner have the same number of triangles and quadrilaterals in your envelopes. Write an expression that represents the total number of sides that you and your partner have. If possible, write more than one expression to represent this total.

$3t+4q+3t+4q$ $\quad$ $2(3t+4q)$ $\quad$ $6t+8q$

Discuss the variations of the expression in part (b) and whether those variations are equivalent. This discussion helps students understand what it means to combine like terms; some students have added their number of triangles together and number of quadrilaterals together, while others simply doubled their own number of triangles and quadrilaterals since the envelopes contain the same number. This discussion further shows how these different forms of the same expression relate to each other. Students then complete part (c).

c. Each envelope in the class contains the same number of triangles and quadrilaterals. Write an expression that represents the total number of sides in the room.

Answer depends on the seat size of the classroom. For example, if there are 12 students in the class, the expression would be $12(3t+4q)$, or an equivalent expression.
Next, discuss any variations (or possible variations) of the expression in part (c), and discuss whether those variations are equivalent. Are there as many variations in part (c), or did students use multiplication to consolidate the terms in their expressions? If the latter occurred, discuss the students' reasoning.

Choose one student to open his/her envelope and count the numbers of triangles and quadrilaterals. Record the values of t and q as reported by that student for all students to see. Next, students complete parts (d), (e), and (f).

d. Use the given values of t and q, and your expression from part (a), to determine the number of sides that should be found in your envelope.

$t=4$ and $q = 2$

$3t + 4q$
$3(4) + 4(2)$
$12 + 8$
20

There should be 20 sides contained in my envelope.

e. Use the same values for t and q, and your expression from part (b), to determine the number of sides that should be contained in your envelope and your partner's envelope combined.

Variation #1	*Variation #2*	*Variation #3*
$2(3t + 4q)$	$3t + 4q + 3t + 4q$	$6t + 8q$
$2(3(4) + 4(2))$	$3(4) + 4(2) + 3(4) + 4(2)$	$6(4) + 8(2)$
$2(12 + 8)$	$12 + 8 + 12 + 8$	$24 + 16$
$2(20)$	$20 + 12 + 8$	40
40	40	

My partner and I have a combined total of 40 sides.

f. Use the same values for t and q, and your expression from part (c), to determine the number of sides that should be contained in all of the envelopes combined

Answer will depend on the seat size of your classroom. Sample responses for a class size of 12:

Variation 1	*Variation 2*	*Variation 3*
$12(3t+4q)$	$\overbrace{3t+4q}^{1}+\overbrace{3t+4q}^{2}+\cdots+\overbrace{3t+4q}^{12}$	$36t+48q$
$12\big(3(4)+4(2)\big)$	$\overbrace{3(4)+4(2)}^{1}+\overbrace{3(4)+4(2)}^{2}+\cdots+\overbrace{3(4)+4(2)}^{12}$	$36(4)+48(2)$
$12(12+8)$	$\overbrace{3(4)+4(2)}^{1}+\overbrace{3(4)+4(2)}^{2}+\cdots+\overbrace{3(4)+4(2)}^{12}$	$144+96$
$12(20)$	$\overbrace{12+8}^{1}+\overbrace{12+8}^{2}+\cdots+\overbrace{12+8}^{12}$	240
240	$\overbrace{20}^{1}+\overbrace{20}^{2}+\cdots+\overbrace{20}^{12}$	240

For a class size of 12 students, there should be 240 sides in all of the envelopes combined.

Have all students open their envelopes and confirm that the number of triangles and quadrilaterals matches the values of t and q recorded after part (c). Then, have students count the number of sides contained on the triangles and quadrilaterals from their own envelope and confirm with their answer to part (d). Next, have partners count how many sides they have combined and confirm that number with their answer to part (e). Finally, total the number of sides reported by each student in the classroom and confirm this number with the answer to part (f).

g. What do you notice about the various expressions in parts (e) and (f)?

The expressions in part (e) are all equivalent because they evaluate to the same number: 40. The expressions in part (f) are all equivalent because they evaluate to the same number: 240. The expressions themselves all involve the expression $3t+4q$ in different ways. In part (e), $3t+3t$ is equivalent to $6t$, and $4q+4q$ is equivalent to $8q$. There appear to be several relationships among the representations involving the commutative, associative, and distributive properties.

When finished, have students return their triangles and quadrilaterals to their envelopes for use by other classes.

Extension:
Numbers of triangles and quadrilaterals can vary among students. Students can set up equivalent expressions (equations) using their available triangles and quadrilaterals.

Intervention:
Envelopes can have adjusted numbers of triangles and quadrilaterals to make computation less cumbersome for students who need support with the concept of writing expressions using variables, and combining them.

PART III:

Using shapes other than triangles and quadrilaterals, generate and model expressions and have a partner recreate your expressions given clues and instructions (from their partner).

For example, I am going to put squares and hexagons in an envelope (or get pattern blocks or draw models). Then, my partner will ask "yes or no" questions to decide what two shapes I am have chosen and how many of each I have.

CLOSING

Two options for the closing are discussed below. One would be for students who still need additional practice on combining like terms. For students who are ready to combine like terms in expressions involving distributive property, a second option is available.

Like Terms Closing Activity
Pass out index cards (one per student) or cards from template at the end of this lesson, and tell students to find two other people who have a term that can be added to theirs. They may not talk as they get up and silently look for their partners. Once the teams have been formed give them the following questions to answer:

1. What is the sum of your terms? Do you notice anything? Why do you think that happened?
2. If the value for your variable is -3, what is the value for your term? Each team member has an answer.
3. Based on the values found in #2, what is the difference between double the largest and double the smallest? What was the difference between the largest and smallest before you doubled them? What do you notice? Why do you think that happened?

NOTE: 24 cards are being provided. Modify the number of cards as needed. ***Since some of the values will get VERY Large/Small when making the substitution in for the variable, calculators can be available for student use.***

Match Up Card Closing Activity
Students will review the concept of equivalent expressions by playing the "Match Up" game in pairs. Each team will need a set of "Match Up" cards, a rule sheet, and a recording sheet. Two different versions (of equal difficulty) are provided.

Allow students to play the Match up game for 10 -15 minutes. Students who finish early may play again with the other set of cards provided.

NOTE: Printing the cards on card stock and laminating them will prolong their use. Using different colors of paper (or gluing them to different colors of index cards) for set A and set B will help keep the sets together. It is useful to have the cards cut and put in envelopes prior to playing the game.

Cards, Rule Sheet, and Recording Sheet are provided at the conclusion of this LESSON.

Other suggested lessons and activities

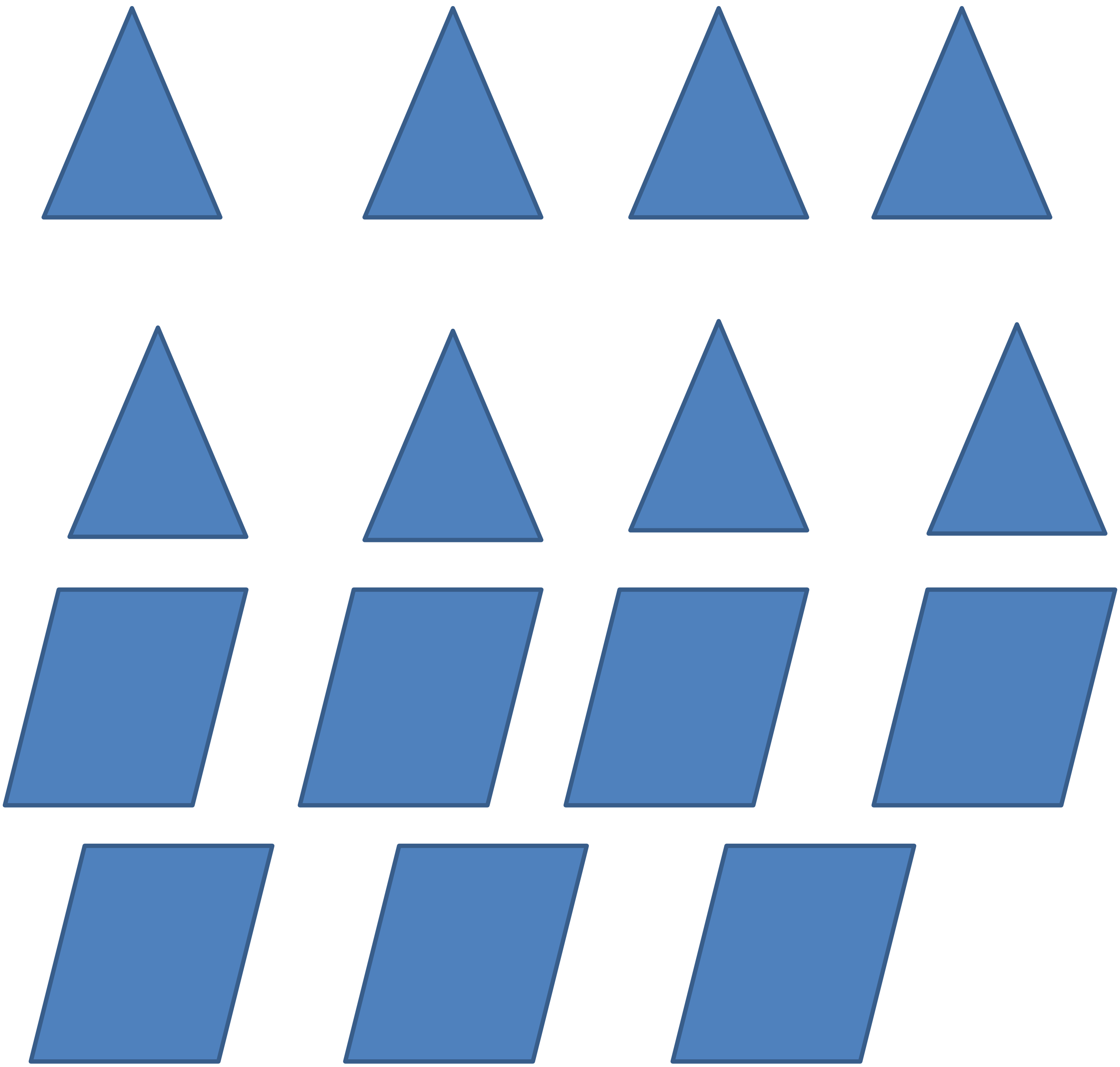
Join the Club: Identifying and Combining Like Terms from NCTM Illuminations
http://illuminations.nctm.org/Lesson.aspx?id=3642 In this lesson, students learn the definition of like terms and gain practice in identifying key features to sort and combine them.

Teaching Channel Video (for teachers) on the distributive property with variables
https://www.teachingchannel.org/videos/teaching-the-distributive-property

Like Terms Game from Math Warehouse
http://www.mathwarehouse.com/games/our-games/like-terms-games/matching-action/

TEMPLATE for Triangles and Quadrilaterals

***Use these templates to create triangles and quadrilaterals for the envelopes in the activity. Or, use pattern block as the triangles and quadrilaterals.**



TEMPLATE for Like Terms Closing Activity

Cut the cards below out for use in the closing activity. You may print on cardstock or tape to index cards for more durability.

$3x^2$	$\frac{1}{3}x^2$	$-3x^2$
$3x^3$	$\frac{1}{3}x^3$	$-3x^3$
$3x^4$	$\frac{1}{3}x^4$	$-3x^4$
$3x^5$	$\frac{1}{3}x^5$	$-3x^5$
$3x^6$	$\frac{1}{3}x^6$	$-3x^6$
$3x^7$	$\frac{1}{3}x^7$	$-3x^7$
$3x^8$	$\frac{1}{3}x^8$	$-3x^8$
$3x$	$\frac{1}{3}x$	$-3x$

LESSON: Triangles and Quadrilaterals

Adapted from Engage NY

Name ____

PART I
Each student is given an envelope containing triangles and quadrilaterals.

How might expressions be generated based on the contents of your envelope? What expressions could you write?

PART II

Each envelope contains a number of triangles and a number of quadrilaterals. For this exercise, let t represent the number of triangles, and let q represent the number of quadrilaterals.

a. Write an expression, using t and q, that represents the total number of sides in your envelope. Explain what the terms in your expression represent.

b. You and your partner have the same number of triangles and quadrilaterals in your envelopes. Write an expression that represents the total number of sides that you and your partner have. If possible, write more than one expression to represent this total.

c. Each envelope in the class contains the same number of triangles and quadrilaterals. Write an expression that represents the total number of sides in the room.

d. Use the given values of t and q, and your expression from part (a), to determine the number of sides that should be found in your envelope.

e. Use the same values for t and q, and your expression from part (b), to determine the number of sides that should be contained in your envelope and your partner's envelope combined.

f. Use the same values for t and q, and your expression from part (c), to determine the number of sides that should be contained in all of the envelopes combined

g. What do you notice about the various expressions in parts (e) and (f)?

PART III:

Using shapes other than triangles and quadrilaterals, generate and model expressions and have a partner recreate your expressions given clues and instructions (from their partner).

Excursions with Exponents

This lesson is designed to extend operations with exponents as introduced in the previous lesson. "Excursions with Exponents" is an adaptation and extension of lesson 8.2 in "A Story of Ratios" from EngageNY.

SUGGESTED TIME FOR THIS LESSON

60 minutes
The suggested time for the class will vary depending upon the needs of the students.

STANDARDS FOR MATHEMATICAL CONTENT

MFAAA2. Students will interpret and use the properties of exponents.
b. Use properties of integer exponents to find equivalent numerical expressions. For example, $3^2\ x\ 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$. (MGSE8.EE.1)

COMMON MISCONCEPTIONS

Students multiply the base numbers. For example, students multiply $3^2\ x\ 3^{-5}$ *to get* 9^{-3} *instead of the correct answer of* 3^{-3} *which would be written in final form as* $\frac{1}{3^3} = \frac{1}{27}$.
Students multiply the exponents together. For example, students multiply $3^2 \times 3^{-5}$ *to mistakenly answer* 3^{-10} *instead of correctly evaluating as* $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.

STANDARDS FOR MATHEMATICAL PRACTICE

8. **Look for and make use of structure.** Students notice patterns as calculations are repeated, and look both for generalizations about operations with integer exponents. They continually evaluate the reasonableness of their results.

EVIDENCE OF LEARNING/LEARNING TARGETS

By the conclusion of this lesson, students should be able to:

- Apply properties of integer exponents to find equivalent expressions.
- Represent the exponential integer expressions in multiple ways.

.

MATERIALS

- Student handout/note taking guide

ESSENTIAL QUESTIONS

- How can you multiply exponential expressions with a common base?
- How can you represent exponential expressions in multiple ways?
- How can you divide exponential expressions with a common base?

SUGGESTED GROUPING

Students should work independently as indicated in the lesson or with a partner as indicated in parts of the lesson.

KEY VOCABULARY (should be defined in context of the lesson)

- Base
- Exponent
- Exponential form

OPENER/ACTIVATOR

Post the following problems on the board for students to consider while working independently:

1. Can you represent the number 8 using only prime numbers? Can that answer be expressed in exponential form?

2 x 2 x 2 or 2^3

2. Can you represent the number 4 using only prime numbers? Can that answer be expressed in exponential form?

2 x 2 or 2^2

3. What is the product of 8 and 4? Can you represent their product using only prime numbers? Can that answer be expressed in exponential form?

2 x 2 x 2 x 2 x 2 or 2^5 (also point out this is 2^3 x 2^2)

4. Do you notice any relationship between these problems?

Discuss that the base number (2) is the same and the exponent is the sum of the two exponents.

After students have time to think about the problem set above, lead a class discussion about things they notice. If students do not notice the pattern of 2^3 x 2^2 = 2^5 or are unable to generalize the operation, repeat the process with another example. Or, use the following example to test their hypothesis:

Try 9 x 27 in expanded form and then in exponential form
3 x 3 x 3 x 3 x 3 = 3^5
3^2 x 3^3 = 3^5

What conclusions can you make?
Discuss that the base number (3) is the same and the exponent is the sum of the two exponents.

LESSON INSTRUCTIONS

This lesson builds from the previous lessons on exponential operations so students should see connections to previous activities.

Continue the class discussion with the following examples:

1. How about

$2^3 \times 2^5 \times 2^7 \times 2^9$ $\quad$ $2^3 \times 2^{-5} \times 2^7 \times 2^9$ $\quad$ $2^3 \times 2^5 \times 2^{-7} \times 2^{-9}$

$\mathbf{2^{24}}$ $\quad$ $\mathbf{2^{14}}$ $\quad$ $\mathbf{2^{-8}}$ **or** $\mathbf{1/2^8}$

2. Consider the following

$\mathbf{14^{23} \times 14^8}$ $\quad$ $\mathbf{(-72)^{10} \times (-72)^{13}}$ $\quad$ $a^{23} \cdot a^8$

$\mathbf{14^{31}}$ $\quad$ $\mathbf{(-72)^{23}}$ $\quad$ $\boldsymbol{a^{31}}$

Ask students to complete the following generalization:

In general, if $\boldsymbol{x}$ is any number and $\boldsymbol{m}, \boldsymbol{n}$ are positive integers, then

$$\boldsymbol{x^m \cdot x^n = x^{m+n}}$$

Because

$$\boldsymbol{x^m \times x^n = \underbrace{(x \cdots x)}_{m \text{ times}} \times \underbrace{(x \cdots x)}_{n \text{ times}} = \underbrace{(x \cdots x)}_{m+n \text{ times}} = x^{m+n}}$$

3. Consider this problem: $\frac{5^8}{5^6}$

How can you simplify the fraction? Can you use expanded form to help?

$\frac{5x5x5x5x5x5x5x5}{5x5x5x5x5x5}$ which can simplify to $\frac{5x5}{1}$ or 5^2 or 25

4. What would happen if we changed the problem a little? $\frac{5^6}{5^8}$

How can you simplify the fraction? Can you use expanded form to help? Is there another way to write the fraction?

$\mathbf{\frac{5x5x5x5x5x5}{5x5x5x5x5x5x5x5}}$ **which can simplify to** $\mathbf{\frac{1}{5x5}}$ **or** $\mathbf{\frac{1}{5^2}}$ **or** $\mathbf{\frac{1}{25}}$

5. Now, try these and formalize a "rule" for division of exponential expressions such as we did above for multiplication of exponential expressions.

$\frac{7^9}{7^6}$ $\frac{7^3}{1}$ **or** 7^3 $\frac{6^3}{6^{10}}$ $\frac{1}{6^7}$ **or** 6^{-7} $\frac{\left(\frac{8}{5}\right)^9}{\left(\frac{8}{5}\right)^2}$ $\frac{\left(\frac{8}{5}\right)^7}{1}$ **or** $\left(\frac{8}{5}\right)^7$ **or** $\frac{8^7}{5^7}$

Ask students to complete the following generalization:

In general, if x is nonzero and m, n are positive integers, then

$$\frac{x^m}{x^n} = x^{m-n}, \text{ if } m > n.$$

CLOSING/SUMMARIZER

Have students complete the exit activity in pairs to practice multiplication and division of exponential expressions with integer exponents.

EXIT TICKET (student edition provided at the end of this LESSON)

$(-19)^5 \cdot (-19)^{11} = (-19)^{16}$	$2.7^5 \times 2.7^3 = 2.7^8$
$\frac{7^{10}}{7^3} = 7^7$	$\left(\frac{1}{5}\right)^2 \cdot \left(\frac{1}{5}\right)^{15} = \left(\frac{1}{5}\right)^{17}$
$\left(-\frac{9}{7}\right)^m \cdot \left(-\frac{9}{7}\right)^n = \left(-\frac{9}{7}\right)^{m+n}$	$\frac{b^3}{b^{10}} = \frac{1}{b^7}$

INTERNET ACTIVITY

The lesson set for 8.EE.1 on Learnzillion provides several short lessons to support this standard. https://learnzillion.com/lessonsets/307-know-and-apply-the-properties-of-integer-exponents-to-generate-equivalent-numerical-expressions

FORMATIVE ASSESSMENT QUESTIONS

In this lesson, the formative assessment questions are embedded within the questions. Teachers should work with small groups as they complete the activity to formatively assess understanding and gauge review/extension needs.

EXTENSION ACTIVITY

For students who complete the exit activity quickly and demonstrate understanding, pose the following problems for them to consider:

Can the following expressions be simplified? If so, write an equivalent expression. If not, explain why not.

1. $6^5 \times 4^9 \times 4^3 \times 6^{14}$

2. $(-4)^2 \cdot 17^5 \cdot (-4)^3 \cdot 17^7$

3. $15^2 \cdot 7^2 \cdot 15 \cdot 7^4$

4. $5^4 \times 2^{11}$

5. Are the following expressions equivalent? Justify your response.

$$2^4 \times 8^2 \; and \; 2^4 \times 2^6$$

6. Let x be a number. Simplify the expression of the following number:
$(2x^3)(17x^7)$

<u>Excursions with Exponents Note Taking Guide</u>

1. Write the following products as a single term

$2^3 \times 2^5 \times 2^7 \times 2^9$ $\quad$ $2^3 \times 2^{-5} \times 2^7 \times 2^9$ $\quad$ $2^3 \times 2^5 \times 2^{-7} \times 2^{-9}$

2. Write the following products as a single term

$14^{23} \times 14^8$ $\quad$ $(-\mathbf{72})^{\mathbf{10}} \times (-\mathbf{72})^{\mathbf{13}}$ $\quad$ $a^{23} \cdot a^8$

In general, if x is any number and m, n are positive integers, then

$$\boldsymbol{x^m \cdot x^n = x^{\_\_\_\_\_\_}}$$

Because

$$\boldsymbol{x^m \times x^n = \underbrace{(x \cdots x)}_{m \text{ times}} \times \underbrace{(x \cdots x)}_{n \text{ times}} = \underbrace{(x \cdots x)}_{m+n \text{ times}} = x^{\_\_\_\_\_\_\_}}$$

3. Consider this problem: $\dfrac{5^8}{5^6}$

How can you simplify the fraction? Can you use expanded form to help?

4. What would happen if we changed the problem to $\dfrac{5^6}{5^8}$
How can you simplify the fraction?

Can you use expanded form to help?

Is there another way to write the fraction?

5. Now, try these and formalize a "rule" for division of exponential expressions such as we did above for multiplication of exponential expressions.

$\frac{7^9}{7^6}$ $\frac{6^3}{6^{10}}$ $\frac{\left(\frac{8}{5}\right)^9}{\left(\frac{8}{5}\right)^2}$

In general, if x is nonzero and m, n are positive integers, then

$$\frac{x^m}{x^n} = x^{\_\_\_\_\_\_}, \text{ if } m > n.$$

Excursions with Exponents EXIT TICKET Name ________________________________

$(-19)^5 \cdot (-19)^{11} =$	$2.7^5 \times 2.7^3 =$
$\frac{7^{10}}{7^3} =$	$\left(\frac{1}{5}\right)^2 \cdot \left(\frac{1}{5}\right)^{15} =$
$\left(-\frac{9}{7}\right)^m \cdot \left(-\frac{9}{7}\right)^n =$	$\frac{b^3}{b^{10}} =$

Squares, Area, Cubes, Volume, Roots…. Connected?

This lesson is designed to explore the concepts of square numbers, square roots, cube numbers, and cube roots from a concrete application. The lesson is an adaptation and extension of "Cheez-It Activity" found on "I Speak Math".
http://ispeakmath.org/2012/05/03/square-roots-with-cheez-its-and-a-graphic-organizer/

SUGGESTED TIME FOR THIS LESSON:

60 to 120 minutes
The suggested time for the class will vary depending upon the needs of the students.

STANDARDS FOR MATHEMATICAL CONTENT

MFAAA2. Students will interpret and use the properties of exponents.
c. Evaluate square roots of perfect squares and cube roots of perfect cubes (MGSE8.EE.2) d. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. (MGSE8.EE.2)

COMMON MISCONCEPTIONS

- *Students misinterpret finding the square of a number as doubling the number.*
- *Students misinterpret finding the square root of a number as taking half of the number (or dividing by two).*
- *Students misinterpret finding the cube of a number as tripling the number.*
- *Students misinterpret finding the cube root of a number as taking one third of the number (or dividing by three).*

Using a concrete introduction followed by a pictorial representation will provide a link to the abstraction of squaring and cubing as well as provide a context for understanding using area and volume.

STANDARDS FOR MATHEMATICAL PRACTICE

1. **Make sense of problems and persevere in solving them.** Students start to examine the relationship between the number of squares and the area of the larger square by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals.
2. **Reason abstractly and quantitatively.** Students seek to make sense of quantities (smaller squares) and their relationships in problem situations (larger square). They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students make connections between the concepts of area and dimension in relation to the use of a symbol (radical sign).
8. **Look for and express regularity in repeated reasoning.** Students notice if calculations are repeated, and look both for general methods and for shortcuts. They will consider if they can arrive at the total area (or root) by considering the other component without actually having to

build the model. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

EVIDENCE OF LEARNING/LEARNING TARGETS

By the conclusion of this lesson, students should be able to:

- Evaluate the square roots of small perfect squares and cube roots of small perfect cubes.
- Represent the solutions to equations using square root and cube root symbols.
- Understand that all non-perfect square roots and cube roots are irrational.

.

MATERIALS

- One box of Cheez-Its per team (algebra tiles or other squares may be substituted)
- One box of sugar cubes per team (average 200 cubes per one-pound box) (algebra cubes, linking cubes, or other cubes may be substituted)
- Graphic Organizer for Squares
- Graphic Organizer for Cubes
- 2 Large number lines (create using bulletin board paper) to display in the class; one for square roots and one for cube roots

ESSENTIAL QUESTIONS

- How can you use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number?
- How can you evaluate square roots of small perfect squares?
- How can you evaluate cube roots of small perfect cubes?
- How can you determine if the square root of a number (such as 2) is irrational?

OPENER/ACTIVATOR

Show the Video Opener: Rubik's Cube and Juggling to get the students' attention and open dialogue about cubes, dimensions of cubes, and characteristics of cubes. To preview the lesson, teachers could ask about the number of cubes it takes to make a Rubik's cube.
https://www.youtube.com/watch?v=lhkzgjOKeLs

KEY VOCABULARY (should be defined in context of the lesson)

- square/square root
- cube
- cube root
- radical sign
- rational number
- irrational number

SUGGESTED GROUPING

Students should would in teams of three to four. Each student should have their own note taking graphic organizer.

LESSON INSTRUCTIONS

Start the lesson by having the students analyze the dimensions of one Cheez-It square (1×1), which has an area of 1 unit squared. Then, repeated this discussion with a four Cheez-it square. Next, ask students to make a square with 6 Cheez-Its. Then discuss why they can't make that model *(As an extension, ask students to consider if it might EVER be possible to construct a square with area of 6 units. Ask them HOW CLOSE they could get to that area. These extension problems might best fit after the activity but are mentioned here to alert teachers of the possibility of students' questions)* Now students can work in small teams to use their Cheez-Its to find more squares and record the dimensions and area on the chart. Often students will figure out the pattern very quickly and are easily able to complete the chart to 144 without needing 144 Cheez-Its.

Repeat the same activity using the sugar cubes or substitute cubes. Allowing students to actually build the cubes, investigate the connection between dimensions and volume, and formalize the discussion of cube roots provides a tangible connection that will support future abstraction and application.

FORMATIVE ASSESSMENT QUESTIONS

- How can you decide if a number is a perfect square?
- How can you decide is a number is a perfect cube?
- How is finding the square root of a number related to the concept of area?
- How is finding the cube root of a number related to the concept of volume?

INTERVENTION

For extra help square and cube roots, please open the hyperlink Intervention Table.

CLOSING/SUMMARIZER

Have students move to the large number line displays and locate the perfect square roots on the line. Pay close attention to the scale used on the number line. Next, have students approximate the square roots of non-perfect squares by estimating and using a calculator to check. Then, have the students record the location of the irrational numbers on the number line.

Repeat the activity for the perfect cubes number line.

Keep the number lines posted as anchor charts for approximating roots (cubes and squares)

Sample student work for this lesson may be found at http://ispeakmath.org/2012/05/03/square-roots-with-cheez-its-and-a-graphic-organizer/

Video Lesson Support from Learnzillion:
Understand and evaluate square roots and cube roots (5 Videos)
https://learnzillion.com/lessonsets/351-understanding-and-evaluating-square-roots-and-cube-roots
Understand Perfect Squares and Square Roots (6 Videos)
https://learnzillion.com/lessonsets/44-understand-perfect-squares-and-square-roots
Understanding Perfect Cubes and Cube Roots (2 Videos)
https://learnzillion.com/lessonsets/45-understanding-perfect-cubes-and-cube-roots

Learnzillion Quiz Opportunities (2 quizzes)
Cube roots https://learnzillion.com/quizzes/2706
Understanding and Evaluating Cube roots and Square roots https://learnzillion.com/quizzes/2705

Extension Activity: Approximating square roots
The following PBS video leads students to a method for approximating square roots when numbers are not perfect squares.
http://www.pbslearningmedia.org/resource/mgbh.math.ns.approxsqroot/approximating-square-roots-of-nonperfect-squares/

A process mentioned (approximating square roots) in the video can be summarized as:

1. Find the closest perfect square (smaller than your number of interest). For example, if I am trying to approximate $\sqrt{20}$, I would use 16.

2. Find the difference between your focus number (in our case 20) and that perfect square (in our case 16). For our example, the difference is 4.

3. Build (or think of) the next perfect square. For our example, the next perfect square would be 25.

4. Find the difference between the two perfect squares. In our example we would find the difference between 25 and 16 to be 9.

5. Estimate the quotient of these two numbers (focus number – perfect square) / (difference between the two perfect squares). For our problem, 4/9 would be $0.\overline{4}$. So, the final approximation of $\sqrt{20}$ would be $4.\overline{4}$. When checked against a calculator we find the actual approximation to be 4.47 which makes our calculation 0.03 off.

Sample problems to consider:

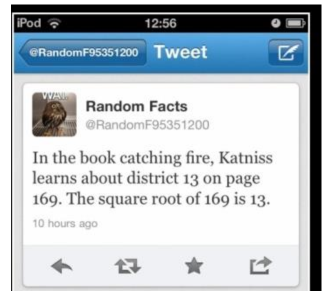
Estimate the following square roots

1. $\sqrt{41}$ 2. $\sqrt{15}$ 3. $\sqrt{147}$

Problems such as these help develop estimation skills and a strong sense of number relationships.

Video closing: http://meangreenmath.com/2014/02/11/engaging-students-square-roots/ is the host page and the Elvis Video is toward the bottom of the page https://youtu.be/AfBQGLowyKU. This video is a catchy tune to recall the basic square roots. It also mentions the inverse relationship of squaring and taking square roots.

Random Fact Tweet (taken from RandomFacts and shown in the image to the right): In the book Catching Fire, Katniss learns about district 13 on page 169.



Finish the **CLOSING** of this lesson by revisiting the essential questions of the lesson.

- How can you use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number?
- How can you evaluate square roots of small perfect squares?
- How can you evaluate cube roots of small perfect cubes?
- How can you determine if the square root of a number (such as 2) is irrational?

Squares and Cubes Activity Name Teacher's Edition

Squares Table: Complete the table as you build your squares. Fill in the number line with the radical notation on one side and the root (dimension) on the other side.

Root	Number of Squares	Root	Number of Squares
1	*1*	*11*	*121*
2	*4*	*12*	*144*
3	*9*	*13*	*169*
4	*16*	*14*	*196*
5	*25*	*15*	*225*
6	*36*	*16*	*256*
7	*49*	*17*	*289*
8	*64*	*18*	*324*
9	*81*	*19*	*361*
10	*100*	*20*	*400*

Key Vocabulary

Square Root: ***a number that produces a specified quantity when multiplied by itself***

Area = ***length x width***

Radical: ***An expression that uses a root, such as square root, cube root.***

How is area "special" for a square? ***Length and width are the same so area is the square of one side.***

How does this characteristic relate to "square roots"? ***The length of a side is the square root of the area.***

Between what two integers do the following values fall?

4 $\sqrt{20}$ *5* *8* $\sqrt{68}$ *9* *11* $\sqrt{120}$ *12* *7* $\sqrt{58}$ *8*

Squares and Cubes Activity Name________________________________

Squares Table: Complete the table as you build your squares. Fill in the number line with the radical notation on one side and the root (dimension) on the other side.

Root	Number of Squares	Root	Number of Squares

Key Vocabulary

Square Root:

Area = _____ x _____

Radical:

How is area "special" for a square?

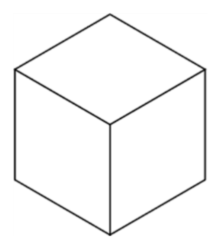
How does this characteristic relate to "square roots"?

Between what two integers do the following values fall?

____ $\sqrt{20}$ _____ _____ $\sqrt{68}$ _____ _____ $\sqrt{120}$ _____ _____ $\sqrt{58}$ _____

Cubes Table: Complete the table as you build your cubes. Fill in the number line with the radical notation on one side and the root (dimension) on the other side. Note: You may fill in the table without building the model once you establish the pattern.

Root	**Number of Cubes**	**Root**	**Number of Cubes**
1	1	11	1331
2	8	12	1728
3	27	13	2197
4	64	14	2744
5	125	15	3375
6	216	16	4096
7	343	17	4913
8	512	18	5832
9	729	19	6859
10	1000	20	8000



Key Vocabulary

Cube Root: ***a number that produces a specified quantity when multiplied by itself a total of three times.***

Radical: ***An expression that uses a root, such as square root, cube root.***

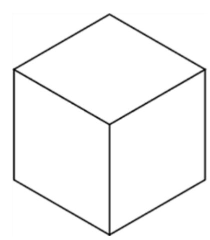
Volume = ***length*** x ***width*** x ***height***

How is volume "special" for a cube? ***Length, width, and height are all equal.***

How does this characteristic relate to "cube roots"? ***Any one dimension represents the cube root of the cube***

Cubes Table: Complete the table as you build your cubes. Fill in the number line with the radical notation on one side and the root (dimension) on the other side.

Root	Number of Cubes	Root	Number of Cubes



Key Vocabulary

Cube Root:

Radical:

Volume = _______ x ________ x ________

How is volume “special” for a cube?

How does this characteristic relate to “cube roots”?

Practice Problems for 8.EE.2 Name ***Teacher's Edition***

1. Complete the Table:

Area (square units)	Length of Side (units)
1	***1***
9	***3***
25	5
2	***4***
5	$\sqrt{5}$
13	$\sqrt{13}$
5	$\sqrt{5}$
100	***10***
400	20

Find the missing measure.

2. Side = 11cm

A = ***121 cm²***

3. Side = ***4m***

A = 16 m^2

Directions: Complete the following sentences.
Provide examples to support your statements.

4. A perfect square is created when…***you square a whole number. For example, 25 is a perfect square because you square 5.***

5. To find the area of a square given the side length of the square…***you square the length. For example, if the side of a square is 4 units, the area is 16 square units.***

6. To find the side length of a square given the area of the square…***you take the square root of the area. For example, if the area of a square is 144 square units the length of a side would be 12 units.***

7. Simplify the following.

$\sqrt{36}$ ***6***	$\sqrt{121}$ ***11***	$\sqrt{16}$ ***4***
$\sqrt{1}$ ***1***	$\sqrt{100}$ ***10***	$\sqrt{49}$ ***7***
$\sqrt{625}$ ***25***	$\sqrt{2500}$ ***50***	$\sqrt{225}$ ***15***

8. (Extension Problem) Approximate the value of the following square roots. Show the steps used in words, pictures, or diagrams. Check your approximation on a calculator.

(focus number – perfect square) / (difference between the two perfect squares)

a) $\sqrt{18}$

(18-16)/ (25-16) = 2/9

So the approximation is 4.22

Actual answer is 4.24

b) $\sqrt{45}$

(45-36)/(49-36) = 6/13 or approx. 0.46

So the approximation is 6.46

Actual answer is 6.71

Practice Problems for 8.EE.2 Name ________________________________

1. Complete the Table:

Area (square units)	Length of Side (units)
1	
9	
	5
2	
5	
	$\sqrt{13}$
	$\sqrt{5}$
100	
	20

Find the missing measure.

2. Side = 11cm

A = _____

3. Side = ________

A = 16 m^2

Directions: Complete the following sentences.
Provide examples to support your statements.

4. A perfect square is created when…

5. To find the area of a square given the side length of the square…

6. To find the side length of a square given the area of the square…

7. Simplify the following.

$\sqrt{36}$	$\sqrt{121}$	$\sqrt{16}$
$\sqrt{1}$	$\sqrt{100}$	$\sqrt{49}$
$\sqrt{625}$	$\sqrt{2500}$	$\sqrt{225}$

8. (Extension Problem) Approximate the value of the following square roots. Show the steps used in words, pictures, or diagrams. Check your approximation on a calculator.

a) $\sqrt{18}$

b) $\sqrt{45}$

What's the "Hype" About Pythagoras?

In this lesson, students work with the Pythagorean Theorem in the context of application problems. For Foundations of Algebra, students will be required to find the hypotenuse of a right triangle given the other two legs. Extension opportunities are provided for students who are ready to extend that requirement to finding a missing leg given the hypotenuse and the other leg. An additional extension activity has been provided for students to informally "discover" the Pythagorean Theorem.

SUGGESTED TIME FOR THIS LESSON:

60-90 minutes
The suggested time for the class will vary depending upon the needs of the students.

STANDARDS FOR MATHEMATICAL CONTENT

MFAAA2. Students will interpret and use the properties of exponents.
c. Evaluate square roots of perfect squares and cube roots of perfect cubes (MGSE8.EE.2)
d. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. (MGSE8.EE.2)
e. Apply the Pythagorean Theorem to solve triangles based on real-world contexts (Limit to finding the hypotenuse given the two legs). (MGSE.8.G.7)

COMMON MISCONCEPTIONS

Students will confuse parts of right triangles and inappropriately apply the Pythagorean Theorem. Students have difficulty distinguishing between the legs of the triangle and the hypotenuse of the triangle. Additionally, students want to make a distinction between "a" and "b" in terms of the legs of the triangle. Attention should be directed toward the commutative property of addition as it relates to the Pythagorean Theorem.

Another common mistake students make when applying the Pythagorean Theorem is to double the sides of the triangle as opposed to squaring the sides. Call attention to the distinction between these mathematical actions.

STANDARDS FOR MATHEMATICAL PRACTICE

1**. Make sense of problems and persevere in solving them.** Students will make sense of this lesson through their questioning and understanding of the Pythagorean Theorem.
2. **Reason abstractly and quantitatively.** Students will reason quantitatively based on the information given for right triangle problems.
3. **Construct viable arguments and critique the reasoning of others.** Students will discuss their solutions and strategies and decide whether they agree or disagree with and debate these mathematical arguments.
4. **Model with mathematics.** Students will model their thinking through diagrams and equations.
5. **Use appropriate tools strategically.** Students will use appropriate tools such as calculators

EVIDENCE OF LEARNING/LEARNING TARGETS

By the conclusion of this lesson, students should be able to:

- Apply the Pythagorean Theorem in real world context.
- Interpret the parts of a right triangle used appropriately in the Pythagorean Theorem.

MATERIALS

- Student handout
- Calculators
- Sticky notes
- Link or download version of Robert Kaplinksy's "How Can We Correct the Scarecrow?" video ***http://robertkaplinsky.com/work/wizard-of-oz***

ESSENTIAL QUESTIONS

- How is the Pythagorean Theorem useful when real world solving problems?
- When is it useful to use the Pythagorean Theorem?
- Does the Pythagorean Theorem always work?

OPENER/ACTIVATOR

To introduce the lesson, show the video clip about the Pythagorean Theorem from Robert Kaplinksy's "How Can We Correct the Scarecrow?" lesson. The clip is designed to get the students thinking about their past experience (if any) with the Pythagorean Theorem. Save the remaining parts of the activity (correcting the Scarecrow's statement) until after you have completed the lesson.

After showing the video, students will complete the Pythagorean Investigations Sheet to open dialogue about their understanding of the Pythagorean Theorem. The Pythagorean Inventory will be completed as the opener with the remaining parts developed in the LESSON.

KEY VOCABULARY

The key vocabulary for this lesson is listed in the Pythagorean Inventory and should be discussed after the Gallery Walk activity:

square numbers, square roots, right triangle, right angle, Pythagorean Theorem, legs, hypotenuse, vertex, rational number, irrational number, radical, squaring a number

LESSON DESCRIPTION

After completing the inventory part (part 1) of the Pythagorean Investigations Sheet, students will work with a partner to complete the sticky note brainstorming activity. (Allow 10-15 minutes based on student understanding.)

During this phase, students will then be asked to work with a partner to record ideas about the Pythagorean Theorem on sticky notes to be added to the class brainstorming sheet. Students should work with a partner to come up with details, definitions, diagrams, drawings to describe as many of the terms in the inventory as they can. Teams should record ideas on the sticky notes provided then post them on the charts around the room.

Teams will then go on a Gallery Walk to see peer responses on the anchor charts. Students should walk with their partner through the posted terms and review the responses from each team. Instruct students to make notes on paper about follow up questions based on what they read on the charts. Students should circle the terms on the list from the inventory that need more clarification during class discussion. Students should draw a box around the terms that they feel are completely clear after the gallery walk. (Allow 10-15 minutes based on student understanding.)

After students complete the gallery walk, provide time to discuss and clarify each of the terms from the inventory. Understanding of the terminology will be key to the application of the Pythagorean Theorem.

Next, give students the graphic organizer on the Pythagorean Theorem and have them label the parts of the triangle independently. Students will use the organizer as they discuss the application problems provided on their Application practice guide.

NOTE: When evaluating irrational roots, have students work with both radical notation and decimal notation. For the purposes of this course, students do not need to simplify radical expressions.

EXTENSION ALERT

The standard for this course limits the application of the Pythagorean Theorem to finding the hypotenuse of a right triangle when given the legs. Many students are curious about the origin of the Pythagorean Theorem and "Why" it works. Extension Lessons will be provided at the conclusion of the student pages for students who might want to investigate the proof of the Pythagorean Theorem and who might want to find the leg of a right triangle when given the hypotenuse and the other leg.

Internet Activities

- Learnzillion Video https://learnzillion.com/lessons/1262-find-the-length-of-the-hypotenuse-of-a-right-triangle-using-the-pythagorean-theorem
- Pythagorean Theorem http://mathforum.org/isaac/problems/pythagthm.html
- Biography of Pythagoras http://www-groups.dcs.st-and.ac.uk/~history/Printonly/Pythagoras.html
- Pythagorean Theorem http://www.mathsisfun.com/pythagoras.html
- Using Pythagorean Theorem http://www.pbs.org/wgbh/nova/proof/puzzle/use.html
- The Pythagorean Theorem: Square Areas Formative Assessment Lesson featured on the Math Assessment Project website at http://map.mathshell.org/materials/lessons.php?LESSONid=408 provides complete lesson plans and all student materials needed to extend the content of the Pythagorean Theorem as addressed in Foundations of Algebra.

Mathematical goals of the Formative Assessment Lesson (from Math Assessment Project):
This lesson is intended to help you assess how well students are able to:

- Use the area of right triangles to deduce the areas of other shapes.
- Use dissection methods for finding areas.
- Organize an investigation systematically and collect data.
- Deduce a generalizable method for finding lengths and areas (The Pythagorean Theorem.)

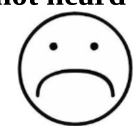
Name ________________________ Pythagorean Investigations

Part 1: <u>Pythagorean Theorem Self Inventory</u>

Put a smiley face
If you know the term

Put a blank face
if you have heard the term

Put a frown face if you
have not heard the term



square numbers _____	square roots _____	right triangle _____
right angle _____	Pythagorean Theorem _____	legs _____
hypotenuse _____	vertex_____	rational number _____
irrational number _____	radical _____	squaring a number _____

<u>Part 2: Partner Discussion</u>

- Work with your partner to come up with details, definitions, diagrams, drawings to describe as many of the terms above as you can.
- Record your ideas on the sticky notes provided, then post them on the charts around the room.

<u>Part 3: Gallery Walk</u>

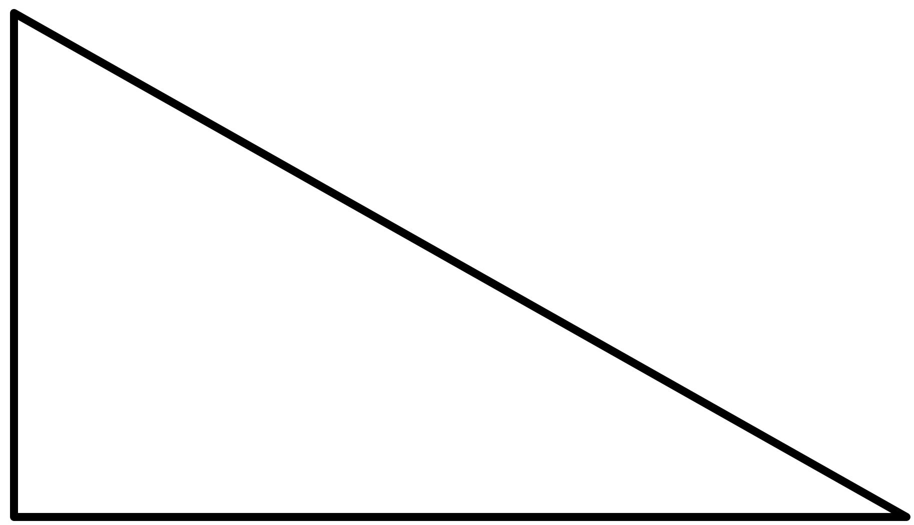
- Walk with your partner through the posted terms and review the responses from each team.
- Make notes on paper about follow up questions based on what you read on the charts.
- Circle the terms on the list above that need more clarification during class discussion.
- Draw a box around the terms that you feel are completely clear to you after the gallery walk.

Parts of a Right Triangle **Name** ______________________________

Label the right triangle shown below.

A word list is included to help you. Some words will be used more than once.

WORD LIST
leg
hypotenuse
right angle
side a
side b
side c
vertex
$a^2 + b^2 = c^2$



The Pythagorean Theorem ____

And can only be used for ____

Sample Problem:

If the sides of a right triangle are 6 inches and 8 inches, how long is the hypotenuse?

Applications of the Pythagorean Theorem **Name** ***Teacher's Edition***

Solve each of the following problems. Please include your steps in solving the problems. For answers that are irrational, include both the radical and decimal approximation (to the nearest hundredth using a calculator).

1. A baseball diamond is a square with sides of 90 feet. What is the shortest distance, to the *nearest tenth* of a foot, between first base and third base?

$90^2 + 90^2 = X^2$ $16200 = X^2$ $\sqrt{16200} = x$ $X = 127.28$ feet

2. A suitcase measures 24 inches long and 18 inches high. What is the diagonal length of the suitcase to the *nearest tenth* of an inch?

$24^2 + 18^2 = X^2$ $900 = X^2$ $\sqrt{900} = x$ $X = 30$ inches

3. The older floppy diskettes measured 5 and 1/4 inches on each side. What was the diagonal length of the diskette to the *nearest tenth* of an inch?

$5.25^2 + 5.25^2 = X^2$ $55.125 = X^2$ $\sqrt{55.125} = X$ $X = 7.4$ inches

4. Find, Fix, and Justify: Raymond was asked to solve for the length of the hypotenuse in a right triangle with legs that have side lengths of 4 and 5. His work is shown below. He made a mistake when solving. Explain the mistake and then solve the problem correctly.

Raphael's Solution:

$\mathbf{a^2 + b^2 = c^2}$
$\mathbf{4^2 + 5^2 = c^2}$
$\mathbf{16 + 25 = c^2}$
$\mathbf{41 = c}$

Correct Solution:

Raymond forgot to take the square root of 41. The correct answer should be 6.4.

<u>Applications of the Pythagorean Theorem</u> **Name ________________________**

Solve each of the following problems. Please include your steps in solving the problems. For answers that are irrational, include both the radical and decimal approximation (to the nearest hundredth using a calculator).

1. A baseball diamond is a square with sides of 90 feet. What is the shortest distance, to the *nearest tenth* of a foot, between first base and third base?

2. A suitcase measures 24 inches long and 18 inches high. What is the diagonal length of the suitcase to the *nearest tenth* of an inch?

3. The older floppy diskettes measured 5 and 1/4 inches on each side. What was the diagonal length of the diskette to the *nearest tenth* of an inch?

4. Find, Fix, and Justify: Raymond was asked to solve for the length of the hypotenuse in a right triangle with legs that have side lengths of 4 and 5. His work is shown below. He made a mistake when solving. Explain the mistake and then solve the problem correctly.

Raphael's Solution: Correct Solution:

$\mathbf{a^2 + b^2 = c^2}$
$\mathbf{4^2 + 5^2 = c^2}$
$\mathbf{16 + 25 = c^2}$
$\mathbf{41 = c}$

REVIEW ACTIVITIES

The following activities may be found on the New Zealand Numeracy Site in Project Book 8 page 30 http://nzmaths.co.nz/sites/default/files/Numeracy/2008numPDFs/NumBk8.pdf. These activities review the concepts developed during the learning lesson. All parts provided below may not be necessary for your particular classroom. Evaluate student understanding from the practice problems provided after the learning lesson and through classroom dialogue/student discussion.

Square Roots Activity (Concrete, Representational, Abstract)

Learning Target: I am learning that using the square root finds the length of the side given the area.

Equipment Needed: Square cardboard pieces or square tiles and calculators.

Using Manipulative (square tiles)

Problem: "Zoë builds a large square from 16 small square tiles. How big is Zoë's square?"

Get the students to build the large square out of 16 tiles. Then discuss its dimensions. (Answer: 4 rows of 4 tiles.)

Provide other problems to build:
Build large squares built with these numbers of small squares and give their size: choose small numbers of squares such as 4, 25, and 9

Using Imaging or a diagram (drawing without the manipulative)

Problem: "Zoë builds a large square from 36 small square tiles. **Imagine** (think about) how big the square is." Go back to building the square if needed. (Answer: 6 by 6.)

Provide other examples: Imagine large squares with these numbers of small squares and find their sizes: 49, 100, 81 ...

Using Number Properties

Problem: Zoë builds a large square from 961 small square tiles. Describe the square Zoë builds with the aid of a calculator. For example, how long are the sides? How can you estimate the length of the sides?

Discuss how the $\sqrt{x}$ button helps. (Answer: $\sqrt{961}$ = 31.)
Provide other examples: Large squares are made with these numbers of small squares. Describe the side length of the large squares: 207,936 2,025 622,521 2, 217,121 ...

INTERVENTION
For extra help with the Pythagorean Theorem, please open the hyperlink Intervention Table.

EXTENSION ACTIVITY
Locating Square Roots
Most square roots do not come out nicely. For example, $\sqrt{3}$ is an infinite non-recurring decimal starting 1.73205080 ....

A method of finding square roots involves locating the answer between two numbers. In principal, this method will locate a square root to any desired accuracy.

Learning Target: I am learning how to find the square root of numbers without using a square root button on a calculator.

*Note the resource sheets listed below are available on the New Zealand Numeracy Site (links are provided)

Equipment: Tables of squares (Material Master 8–22)
http://nzmaths.co.nz/sites/default/files/Numeracy/2007matmas/Bk8/MM%208_22.pdf

Worksheet (Material Master 8–23)
http://nzmaths.co.nz/sites/default/files/Numeracy/2007matmas/Bk8/MM%208_23.pdf

Using Number Properties
Problem: "Meriana designs a square swimming pool, which she wants to have an area of 75 m^2. Unfortunately, the square root button on her calculator is damaged, so she will have to find another way of finding square roots."

Display these tables on the board from Material Master 8–22 (linked above)

0	1	2	3	4	5	6	7	8	9	10
0	1	4	9	16	25	36	49	64	81	100

8	8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9
64	65.61	67.24	68.89	70.56	72.25	73.96	75.69	77.44	79.21	81

"How does the first table show the side of the pool is between 8 and 9 meters? How does the second table show the side of the pool is between 8.6 and 8.7 meters?"

8.6	8.61	8.62	8.63	8.64	8.65	8.66	8.67	8.68	8.69	8.7

Explore filling in the squares in this table to locate 75. (Answer: It is between 8.66 and 8.67.)

Examples: Worksheet (Material Master 8–23).

Understanding Number Properties: How could you find the square root to any desired accuracy of any number using only the square button on a calculator?

An additional activity is available in "Teaching Number Sense and Algebraic Thinking" (Book 8 linked above) that remediates the concept of cubes and cube roots.

Fabulous Formulas

SUGGESTED TIME FOR THIS LESSON

60-120 minutes
The suggested time for the class will vary depending upon the needs of the students.

STANDARDS FOR MATHEMATICAL CONTENT

MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.
a. Apply properties of operations emphasizing when the commutative property applies. (MGSE7.EE.1)
f. Evaluate formulas at specific values for variables. For example, use formulas such as A = l x w and find the area given the values for the length and width. (MGSE6.EE.2)
MFAAA2. Students will interpret and use the properties of exponents.
a. Substitute numeric values into formulas containing exponents, interpreting units consistently. (MGSE6.EE.2, MGSE9-12.N.Q.1, MGSE9-12.A.SSE.1, MGSE9-12.N.RN.2)

COMMON MISCONCEPTIONS

In evaluating formulas for specific values students make errors in the order of operations and perform ALL operations from left to right without attention to the type of operation that should be performed first. Some students also use incorrect notation when making substitutions into the given formula. For example, a student might incorrectly evaluate the formula for the volume of a cylinder, $V = \pi r^2 h$ *where* $r = 3$ *and* $h = 4$, *as* $V = \pi * 3 * 2 * 4$ *to get 24 π instead of* $V = \pi * 3^2 * 4$ *to get 36 π.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. **Make sense of problems and persevere in solving them.** Students will make sense of the problems posed in this LESSON to select the appropriate formula to use in solving the problems.
2. **Reason abstractly and quantitatively.** Students will reason quantitatively based on the constraints of the problem.
3. **Construct viable arguments and critique the reasoning of others.** Students will discuss their solutions and strategies and decide whether they agree or disagree with and debate these mathematical arguments.
5. **Use appropriate tools strategically.** Students will use appropriate tools such as calculators.
6. **Attend to precision.** Students will show precision through their use of mathematical language and vocabulary in their questioning and discussions. They will also show precision in their mathematical computations and procedures.

EVIDENCE OF LEARNING/LEARNING TARGETS

By the conclusion of this lesson, students should be able to:

- Select the appropriate formula for a given situation.
- Apply the order of operations to evaluate a formula given specific parameters.
- Assess the reasonableness of a solution based on the situation described in the problem.

MATERIALS

- Formula sheet
- Application problems
- Calculators

ESSENTIAL QUESTIONS

- How do you decide which formula applies in a given situation?
- What role does the order of operations play in evaluating a formula?

GROUPING

Pairs or small groups

ACTIVATOR/OPENING

Post the formulas from the formula sheet on the board and have students identify as many of them as they know. Have students write descriptions of what the variables represent in the formulas. After students have time to ponder the formulas, hand out the formula table and let them self-evaluate their performance. Allow time for students to share ideas or things they notice about the formulas. End the activator with a discussion of the role of "formulas" in everyday life. Point out that the empty rows at the end of the formula sheet are provided for them to enter any other formulas that they might already know and be able to apply.

LESSON DESCRIPTION

In Foundations of Algebra, students will be expected to evaluate expressions at specific values of their variables. These expressions should also be extended to include expressions that arise from formulas used in real-world problems. For example, students should be able to use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of lengths $s = \frac{1}{2}$.

Students will be given a formula sheet as reference, along with a set of situational problems. Students will work in pairs or small groups to decide which formula to use to solve the problems. Attention must be directed toward proper usage of the order of operations.

The last problem on the activity sheet asks students to create a unique problem using one of the formulas from the table. They are to record their problem on a separate sheet of paper. You may post the problems around the room for a "around the room" problem solving activity, use the problems as activators or closing activities, or use them as formative items in a future class.

FORMATIVE ASSESSMENT QUESTIONS

As you walk around during small group work discuss the essential questions of the lesson with teams to evaluate their understanding of the lesson focus.

- How do you decide which formula applies in a given situation?
- What role does the order of operations play in evaluating a formula?

CLOSING

The last problem on the activity sheet asks students to create a unique problem using one of the formulas from the table. Use a few problems to highlight student applications of the formulas.

Formula Sheet

Convert temperatures from Celsius to Fahrenheit where F is temperature in Fahrenheit and C is temperature in Celsius	$F = \frac{9}{5}C + 32$
Interest compounded once per year where P is the principal amount, A is the final amount, r is the rate per year in decimal form, and t is time in years	$A = P(1 + r)^t$
Surface area of a sphere where A is surface area and r is radius of the sphere	$A = 4\pi r^2$
Volume of a sphere where V is volume and r is radius of the sphere	$V = \frac{4}{3}\pi r^3$
Simple Interest (not compound) where A is total amount after t years at r rate (as a decimal) with the initial value of P	$A = P + Prt$
Volume of a cylinder where V is volume, r is radius and h is height of the cylinder	$V = \pi r^2 h$
Surface area of a cylinder where A is surface area, r is radius, and h is height of0 the cylinder	$A = 2\pi r^2 + 2\pi rh$ (2 circle bases) + (rolled up rectangular side)
Convert temperatures from Fahrenheit to Celsius where F is temperature in Fahrenheit and C is temperature in Celsius	$C = \frac{5}{9}(F - 32)$
Area of a square with side length *s*	$A = s^2$

<u>Fabulous Formulas</u> **Name** **Teacher's Edition**

Work with your partner or team to decide which formula to apply in each problem listed below. Show your work for each problem (calculations may be done on the calculator if needed) and decide if your answer is reasonable or not.
After partner work time, you will be asked to compare results with another team. Resolve any problems for which both teams do not have the same answer.

1. Elmer borrowed $500.00 to go on his senior trip. The interest on his loan is simple interest at a rate of 7% and he takes three years to pay off his loan.
a) Which formula will Elmer need to find the total amount he must repay for his loan?
$\boldsymbol{A = P \quad + Prt}$

b) What is the total amount of money he must repay at the end of the three years?
$A = \mathbf{500} \quad + (\mathbf{500}).\mathbf{07})(\mathbf{3})$ **= $605.00**

c) How much interest will Elmer have to pay on his loan?
Interest will be $105.00
d) If he makes equal monthly payments over the course of his loan, how many payments will Elmer have to make?

3 years x 12 months per year = 36 payments
e) How much will each monthly payment be?

$605.00 / 36 means each payment would be approximately $16.81 for each payment.

2. Hope loves to make papier-mache models. She is building a basketball model with a diameter of 10 inches.

a) What is the capacity of her basketball model?
$V = \frac{4}{3}\pi r^3$ ***with radius of 5 inches gives*** $\frac{500}{3} * \pi$ ***or*** $66\frac{2}{3}$ ***π or 523.598 in³***

b) How much material will she need to cover her model with fabric if no parts overlap?
$A = 4\pi r^2$ ***with radius of 5 inches gives 100 π in² or 314.159 in²***

3. The temperature outside is 35° Celsius. Should you wear shorts or a coat? Justify your response for someone who is not familiar with the Celsius temperature scale.
$F \ = \frac{9}{5}\, C \ + \ 32$ ***yields 95°f when it is 35°c You should wear shorts.***

4. Howard is recycling his Pringle's potato crisp can as a pencil holder. He wants to decorate the tall part (cylinder side) with fabric. If the can has a 3-inch diameter and a 10.5-inch height, how much fabric will he need?

He will use $2\pi rh$ to find the surface area of the rolled up rectangle to be approximately 98.91 in^2

5. Howard decided to cover the top with aluminum foil (no over lapping or extra). How much foil will he need to cover the top?

He will find the area of the circle on top using πr^2 to be 7.065 in^2

6. Marcus told his mother that the area of the square office at his school is 144 ft^2. His mom asked him for the length of one side of the office. How can he figure it out? What is the length of the side of the office?

He should use $x^2 = 144$ to solve using square roots to find the side of the office to be 12 feet.

7. Irina is expecting guests from Europe. They asked what kind of temperatures there will be during their visit, and Irina told them high temperatures in the 80's and low temperatures in the 60's. Her guests were very confused by this information! Irina realized they use Celsius but she gave them Fahrenheit temperatures. What temperature range should she tell them in Celsius?

The high should be 26.6 degrees Celsius and low should be 15.6 degrees Celsius

8. Select one of the formulas from the formula table and create your own application problem. Record your problem on a separate sheet of paper. Work your created problem in the space provided below making sure to show all steps.

<u>Fabulous Formulas</u> **Name** ______________________________

Work with your partner or team to decide which formula to apply in each problem listed below. Show your work for each problem (calculations may be done on the calculator if needed) and decide if your answer is reasonable or not.
After partner work time, you will be asked to compare results with another team. Resolve any problems for which both teams do not have the same answer.

1. Elmer borrowed $500.00 to go on his senior trip. The interest on his loan is simple interest at a rate of 7% and he takes three years to pay off his loan.

a) Which formula will Elmer need to find the total amount he must repay for his loan?

b) What is the total amount of money he must repay at the end of the three years?

c) How much interest will Elmer have to pay on his loan?

d) If he makes equal monthly payments over the course of his loan, how many payments will Elmer have to make?

e) How much will each monthly payment be?

2. Hope loves to make papier mache models. She is building a basketball model with a diameter of 10 inches.

a) What is the capacity of her basketball model?

b) How much material will she need to cover her model with fabric if no parts overlap?

3. The temperature outside is 35° Celsius. Should you wear shorts or a coat? Justify your response for someone who is not familiar with the Celsius temperature scale.

4. Howard is recycling his Pringle's potato crisp can as a pencil holder.

a) He wants to decorate the tall part (cylinder side) with fabric. If the can has a 3-inch diameter and a 10.5-inch height, how much fabric will he need?

b) Howard decided to cover the top with aluminum foil (no over lapping or extra). How much foil will he need to cover the top?

6. Marcus told his mother that the area of the square office at his school is 144 ft^2. His mom asked him for the length of one side of the office. How can he figure it out? What is the length of the side of the office?

7. Irina is expecting guests from Europe. They asked what kind of temperatures there will be during their visit and Irina told them high temperatures in the 80's and low temperatures in the 60's. Her guests were very confused by this information! Irina realized they use Celsius but she gave them Fahrenheit temperatures. What temperature range should she tell them in Celsius?

8. Select one of the formulas from the formula table and create your own application problem. Record your problem on a separate sheet of paper.

What is a Unit Rate?

Source : **http://www.cehd.umn.edu/ci/rationalnumberproject/89_4.html**

In this lesson, students will explore different ways to express rates and unit rates, and students will interpret unit rates in the context of a problem.

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class.
Recommended time: 60-90 minutes. Recommended arrangement: partners or small groups.

STANDARDS FOR MATHEMATICAL CONTENT

Students will use ratios to solve real-world and mathematical problems.

MFAPR1. Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations.* (MGSE6.RP.3)

MFAPR2. Students will recognize and represent proportional relationships between quantities.
a. Relate proportionality to fraction equivalence and division. *For example, $\frac{3}{6}$ is equal to $\frac{4}{8}$ because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator*. (MGSE4.NF.1)
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

STANDARDS FOR MATHEMATICAL PRACTICE

1. **Make sense of problems and persevere in solving them**. Students must be able to make sense of unit rates in the context of a problem.
2. **Reason abstractly and quantitatively**. Students must reason through each problem to determine which unit rate is most appropriate for the question being asked.
6. **Attend to precision**. Students must be precise in their calculations.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Find a unit rate and explain what the unit rate means in the context of a problem.
- Determine the most appropriate unit rate for solving a problem and use that unit rate to solve for missing values.

MATERIALS

- Student lesson sheet
- Colored tiles (optional)
- Cuisenaire rods (optional)

ESSENTIAL QUESTIONS

- How can a unit rate be used to answer questions in a problem?
- How can I determine which unit rate will be most useful in solving a problem?
- How can tables and equations be useful when answering questions about proprotional relationships?

NUMBER TALKS

For this Number Talk, begin with the following problem, "6 melons cost $12. How much does one melon cost?" Record the problem on the far left side of the board. Provide students with wait time as they work to mentally solve this problem. When the majority of the students have given the "thumbs up" signal, call on several students to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you. It is important to allow students ownership of their thinking.

Record, "3 watermelons cost $15. How much does one watermelon cost?" on the board next to the previous problem. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the "thumbs up" signal, call on several students to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you.

Record, "3 apples cost $.99. How much does 1 apple cost?" on the board towards the far right. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the "thumbs up" signal, call on several students to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you.

At the end of the Number Talk, discuss the strategies used to find the answers. Some of the strategies students may use are: relationship between multiplication and division, division, guess and check and skip counting. Talk with the students about which strategy was most efficient (quick, easy and accurate).

Allow for a maximum of 15 minutes to conduct the Number Talk before moving into the lesson.

Explain to students that the strategies were used to identify a unit rate.
Revisit the first problem from the Number Talk. Have students identify the rate $\frac{6\ melons}{\$12}$***, then identify the unit rate*** $\frac{1}{2}$ ***melon per dollar. Have students define a unit rate.***

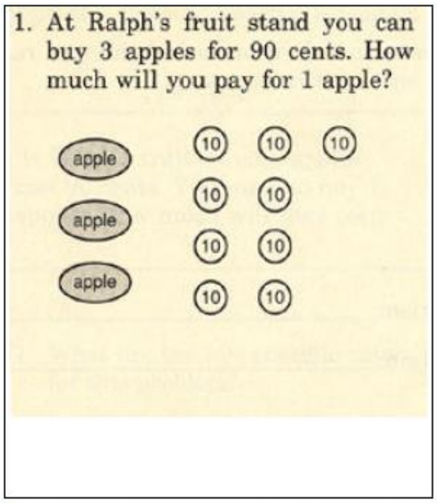
***On the recording sheet, students analyze a real life situation to create two versions of a unit rate. Then, students need to analyze the two possible rates and determine which is the most appropriate to the given problem and use this rate to find possible cost for other values within the given problem*.**

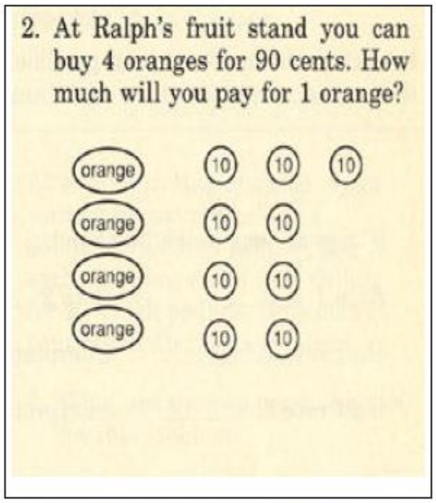
OPENER/ACTIVATOR

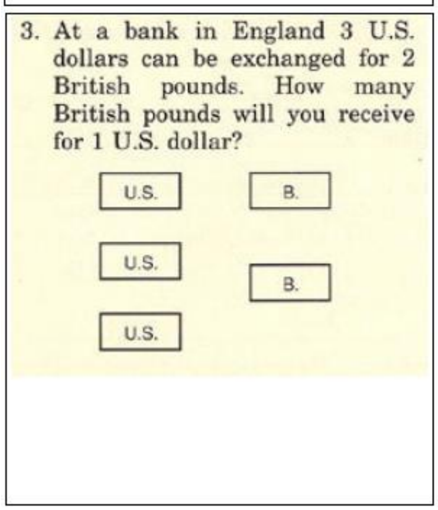
http://www.cehd.umn.edu/ci/rationalnumberproject/images/89_4/sheet_1.pdf

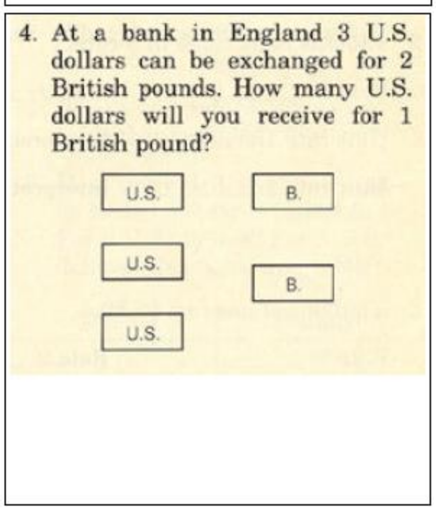
Pose the following question to students:

How could you use each picture to answer the question?


1. At Ralph's fruit stand you can buy 3 apples for 90 cents. How much will you pay for 1 apple?


2. At Ralph's fruit stand you can buy 4 oranges for 90 cents. How much will you pay for 1 orange?


3. At a bank in England 3 U.S. dollars can be exchanged for 2 British pounds. How many British pounds will you receive for 1 U.S. dollar?


4. At a bank in England 3 U.S. dollars can be exchanged for 2 British pounds. How many U.S. dollars will you receive for 1 British pound?

Possible solution for #1.

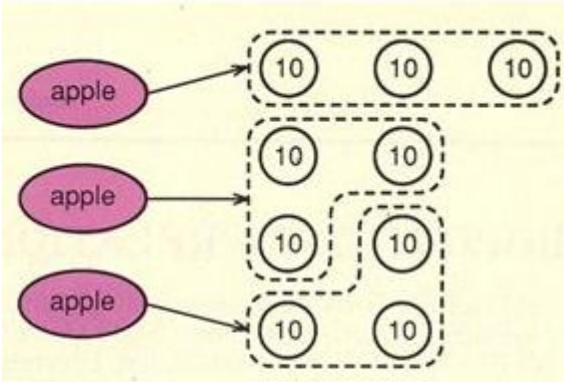


Fig. 1

After students have had time to work through all four problems, ask some students to share their work. Look specifically for students whose work looks different. Any problems that seemed to be particularly difficult for students should also be discussed at this time.

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Lesson Directions:

Each of the three parts of this lesson focuses on a different aspect of unit rates. Part 1 allows students to see that a unit rate can always be written two different ways. Most importantly, students must interpret each unit rate in the context of the problem. When interpreting the unit rate, make sure that students are not simply "reading" the unit rate as it is written, for instance on *a*, they should write something similar to this, *there are 5 pounds of flour in one bag,* as opposed to this, *5 pounds per bag.*

The problems in part 1 can be used to help prepare students for the remainder of the lesson by asking some addition questions. After students have completed part 1, some of the following prompts could be given as a think-pair-share:

> *Which unit rate makes more sense in the context of the problem? Explain your thinking.*
> *Which unit rate would be most helpful if I want to know the cost of 6 gallons of gas?*
> *If I need 20 tennis balls, which unit rate will help me determine the number of cans to buy?*

In part 2, students will select an appropriate unit rate and will use that unit rate to solve the problem. However, an important mathematical skill arises from asking students to use the other unit rate. After students have completed part 2, pose this question: *If you have $5 to spend at Ralph's fruit stand on apples, how many apples can you buy?* Begin by asking students to decide which unit rate would help them to solve this problem and then use that unit rate to solve the problem. The important skill here is to see if students correctly round their solution. Some students may think that if a decimal is greater than .5, they should always round up, but in this case, $5 will buy 16.65 apples so there is only enough money to buy 16 apples, not 17.

Lastly, part 3 requires students to use this same process on three new scenarios. Give students the opportunity to share their solutions in a whole-group discussion. Pay particular attention to their solutions in *c* where there may be misconceptions about converting time to minutes and seconds.

What is a Unit Rate?

Part 1: Finding and Interpreting the Unit Rate

In each problem, record both possible rates, use an appropriate strategy to find the unit rates, and then write a short sentence explaining each unit rate.

a. 6 bags of flour weigh 30 pounds.

Rate 1:	***Unit rate 1:***
$\frac{6\ bags}{30\ lbs}$	$\frac{.2\ bags}{lb}$
Interpretation: *There are $\frac{2}{10}$ of a bag for each pound of flour.*	

Rate 2:	***Unit rate 2:***
$\frac{30\ lbs}{6\ bags}$	$\frac{5\ lbs}{bag}$
Interpretation: *There are 5 pounds of flour per bag.*	

b. 9 tennis balls come in 3 cans.

Rate 1:	***Unit rate 1:***
$\frac{9\ tennis\ balls}{3\ cans}$	$\frac{3\ tennis\ balls}{can}$
Interpretation: *Each can contains 3 tennis balls.*	

Rate 2:	***Unit rate 2:***
$\frac{3\ cans}{9\ tennis\ balls}$	$\frac{.33\ cans}{tennis\ ball}$
Interpretation: *1 tennis ball makes up $\frac{1}{3}$ of a can.*	

c. 5 gallons of gas cost $6.50.

Rate 1:	***Unit rate 1:***
$\frac{5\ gallons\ of\ gas}{\$6.50}$	$\frac{.77\ gallons\ of\ gas}{\$1}$
Interpretation: *One dollar will buy .77 gallons of gas.*	

Rate 2:	***Unit rate 2:***
$\frac{\$6.50}{5\ gallons\ of\ gas}$	$\frac{\$1.30}{gallon\ of\ gas}$
Interpretation: *$1.30 will buy 1 gallon of gas.*	

Part 2: Selecting the Appropriate Unit Rate

At Ralph's fruit stand 3 apples cost $.90. You want to buy 7 apples. How much will they cost?

a. What are the two possible rates for this problem?

$\frac{3\ apples}{\$.90}$ *or* $\frac{\$.90}{3\ apples}$.

b. Show each rate as a unit rate.

$\frac{3.33\ apples}{\$1}$ *or* $\frac{\$.30}{apple}$.

c. What does each unit rate tell you?

3.33 apples can be purchased for $1 or 1 apple can be purchased for $.30.

d. Which unit rate will help you solve the problem?

$\frac{\$.30}{apple}$

e. Complete the table in order to determine the cost of seven apples. Then, describe the pattern you see.

As the number of apples increases by one, the cost increases by $.30.

Number of apples, n	*Cost, C (in dollars)*
1	*.30*
2	*.60*
3	*.90*
4	*1.20*
5	*1.50*
6	*1.80*
7	*2.10*

f. Since you know the unit price, write a number sentence for the cost of seven apples. Write an equation for the cost of any number of apples using the variables in the table above.

$7\ apples \times \frac{\$.30}{apple} = \$2.10$

$C = .30n$

Part 3: Applying the Unit Rate

In each problem, record the rate appropriate for the question asked, find the corresponding unit rate, write a short sentence interpreting the unit rate, and use this rate to find the solution to the problem.

a. Anne is painting her house light blue. To make the color she wants, she must add 3 cans of white paint to every 2 cans of blue paint. How many cans of white paint will she need to mix with 6 cans of blue?

Rate needed: $\frac{3\ cans\ of\ white}{2\ cans\ of\ blue}$ **Unit rate:** $\frac{1.5\ cans\ of\ white}{1\ can\ of\ blue}$

Interpretation of unit rate: *Anne needs 1.5 cans of white paint for every can of blue paint.*

Solution : $1.5 \times 6 = 9\ cans\ of\ blue\ paint$

b. Ryan is making a fruit drink. The directions say to mix 5 cups of water with 2 scoops of powdered fruit mix. How many cups of water should he use with 9 scoops of fruit mix?

Rate needed: $\frac{5\ cups\ of\ water}{2\ scoops\ of\ fruit\ mix}$ **Unit rate:** $\frac{2.5\ cups\ of\ water}{1\ scoop\ of\ fruit\ mix}$

Interpretation of unit rate: *Ryan needs 2.5 cups of water for every scoop of fruit mix*

Solution : $2.5\ \times 9 = 22.5\ \times scoops\ of\ fruit\ mix$

c. Donna is running around a track. It takes her 10 minutes to run 6 laps. If she keeps running at the same speed, how long will it take her to run 5 laps?

Rate needed: $\frac{10\ minutes}{6\ laps}$ **Unit rate:** $\frac{1.67\ minutes}{1\ lap}$

Interpretation of unit rate: *Donna can run 1 lap in 1.67 minutes.*

Solution: $1.67\ \times 5 = 8.35\ minutes$ *(Approximately 8 minutes and 20 seconds)*

d. Carla is cleaning her classroom but decides to first help out her friends, Liz and Melissa,

by cleaning both of their classrooms. It takes Carla $3\frac{1}{3}$ hours to clean both Liz and Melissa's classrooms. How long will she be working to clean all three classrooms?

Rate needed: $\frac{3\frac{1}{3}\ hours}{2\ classrooms}$ **Unit rate:** $\frac{1\frac{2}{3}\ \ hours}{1\ classroom}$

Interpretation of unit rate: *Carla can clean 1 classroom in* $1\frac{2}{3}$ ***hours.***

Solution: $1\frac{2}{3}\ x\ 3\ classrooms =\ 5\ hours$

<u>FORMATIVE ASSESSMENT QUESTIONS</u>

- What does the unit rate mean for this particular problem?
- What information in the problem can help you decide on the appropriate unit rate?
- What is the relationship between the two variables?

<u>DIFFERENTIATION</u>

Extension
This lesson gives students only one opportunity to make a table of values and write an equation, but some students may be ready to look at multiple representations of the problem. For students who are ready to move beyond the lesson, assign them one of the problems in part 3 and ask them to make a table, plot the points on a coordinate grid, and write an equation for the situation. Ask them to explain how the solution to the problem can be found in each representation.

<u>Intervention</u>
Some students may need to use manipulatives to visualize the unit rates. Provide colored tiles, Cuisenaire rods, or encourage the use of appropriate tools such as bar models and double-number lines to make sense of the problems. Additionally, students can find more practice at http://www.mathplayground.com/thinkingblocks.html.

For extra help with unit rates, please open the hyperlink Intervention Table.

CLOSING/SUMMARIZER

End the lesson by asking students to discuss the differences and similarities between using a table and an equation to find a solution (may need to refer back to part 2). We want students to realize that an equation can be more efficient than extending values in a table. This question may help to initiate that discussion: *How much would 63 apples cost?*

What is a Unit Rate?

Part 1: Finding and Interpreting the Unit Rate

In each problem, record both possible rates, use an appropriate strategy to find the unit rates, and then write a short sentence explaining each unit rate.

a. 6 bags of flour weigh 30 pounds.

Rate 1:	*Unit rate 1:*
Interpretation:	

Rate 2:	*Unit rate 2:*
Interpretation:	

b. 9 tennis balls come in 3 cans.

Rate 1:	*Unit rate 1:*
Interpretation:	

Rate 2:	*Unit rate 2:*
Interpretation:	

c. 5 gallons of gas cost $6.50.

Rate 1:	*Unit rate 1:*
Interpretation:	

Rate 2:	*Unit rate 2:*
Interpretation:	

Part 2: Selecting the Appropriate Unit Rate

At Ralph's fruit stand 3 apples cost $.90. You want to buy 7 apples. How much will they cost?

a. What are the two possible rates for this problem?

b. Show each rate as a unit rate.

c. What does each unit rate tell you?

d. Which unit rate will help you solve the problem?

e. Complete the table in order to determine the cost of seven apples. Then, describe the pattern you see.

Number of apples, n	***Cost, C (in dollars)***
1	
2	
3	
4	
5	
6	
7	

f. Since you know the unit price, write a number sentence for the cost of seven apples. Write an equation for the cost of **any number** of apples using the variables in the table above.

Part 3: Applying the Unit Rate

In each problem, record the rate appropriate for the question asked, find the corresponding unit rate, write a short sentence interpreting the unit rate, and use this rate to find the solution to the problem.

a. Anne is painting her house light blue. To make the color she wants, she must add 3 cans of white paint to every 2 cans of blue paint. How many cans of white paint will she need to mix with 6 cans of blue?

Rate needed: ___________________ Unit rate: ____________________

Interpretation of unit rate: ____

Solution: ___________________

b. Ryan is making a fruit drink. The directions say to mix 5 cups of water with 2 scoops of powdered fruit mix. How many cups of water should he use with 9 scoops of fruit mix?

Rate needed: ___________________ Unit rate: ____________________

Interpretation of unit rate: ____

Solution: ___________________

c. Donna is running around a track. It takes her 10 minutes to run 6 laps. If she keeps running at the same speed, how long will it take her to run 5 laps?

Rate needed: ___________________ Unit rate: ____________________

Interpretation of unit rate: ____

Solution: ___________________

d. Carla is cleaning her classroom but decides to first help out her friends, Liz and Melissa, by cleaning both of their classrooms. It takes Carla $3\frac{1}{3}$ hours to clean both Liz and Melissa's classrooms. How long will she be working to clean all three classrooms?

Rate needed: ___________________ Unit rate: ____________________

Interpretation of unit rate: ____

Solution: ___________________

Fish in a Lake

Source: Teaching Student Centered Mathematics, vol. 3 Grades 5-8, by John A. Van de Walle

In this lesson, students will simulate the "tagging" of fish in order to estimate the total number of fish in a lake by collecting data, examining ratios, and writing and solving proportions.

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class.
Recommended time: 60-90 minutes. Recommended arrangement: partners or small groups.

STANDARDS FOR MATHEMATICAL CONTENT

MFAPR2. Students will recognize and represent proportional relationships between quantities.
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will use a model of the situation to make sense of the problem.
2. Reason abstractly and quantitatively. Students will use their reasoning skills to understand how the ratio can help us to estimate the total population.
3. Construct viable arguments and critique the reasoning of others. Students will share results and discuss strategies with other students.
4. Model with mathematics. Students will model the problem using available materials.
6. Attend to precision. Students will attend to precision when using the language of mathematics, being precise in calculations and when assessing the reasonable of their estimations.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Compare two quantities using a ratio and understand what that ratio means in the context of the problem.
- Use proportional reasoning to solve for an unknown quantity.

MATERIALS

- Paper bags (one per group)
- White beans and red beans (alternatives to the beans include: two different colored counters, two different colored cubes, or plain Goldfish crackers and pretzel Goldfish crackers (tagged fish))

ESSENTIAL QUESTIONS

- How can I use information from a sample to help make a prediction about a population?
- How can I use a proportion to solve for an unknown quantity?

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Background Knowledge:

Proportional relationships can be confusing to students. In order for students to make sense of proportional relationships and develop strategies, students need experience with contextual problems. The context gives students a starting point to make sense of the problems and develop strategies for solving them. It is also important that students solve problems in a context other than those related to money, which is so common. This lesson embeds data collection and estimation into ratio and proportional reasoning.

Lesson Directions:

In order to save some instructional time, the teacher may want to have the bags ready for students at the beginning of class. Each bag should contain several handfuls of beans. If necessary, students can create their own lake but they should not count the beans prior to dumping them in the bag.

While this lesson can be done individually, the benefits of using partners or small groups to promote discussion of mathematical ideas and strategies may outweigh any need for an individual grade. As a teacher, listening to students' discussions of mathematical ideas and strategies can be extremely informative and valuable. This data can be used to inform instruction and to decide next steps for individuals or groups of students.

It is very important early on in the lesson that students understand the role of the red beans. In real-life, fish would not be taken out of a lake and replaced with other fish in order to tag them, but in this case, the red beans are very easily recognizable as the tagged fish. Stickers can be used to literally "tag" the fish if the teacher chooses, but if they fall off during the experiment, this can cause major inaccuracies in the data.

In order for the students to estimate the total number of fish in the lake, they must first choose a ratio of tagged fish in the sample to total number of fish in the sample. Students should be given the opportunity to decide for themselves an effective way of selecting this ratio. Amongst the many different ways of selecting a representative ratio, students could take an average of the ones in their table or they may notice one they consider to be different from the others and exclude that from the possibilities. Some will simply pick one they think looks best from their table. These different strategies should be shared in the whole-group discussion in the closing.

Lastly, students will need to be reminded of how to properly set up a proportion.

$$\frac{\text{tagged sample}}{\text{total sample}} = \frac{\text{tagged population}}{\text{total population}} \qquad \frac{\text{tagged sample}}{\text{tagged population}} = \frac{\text{total sample}}{\text{total population}}$$

Of course, writing the reciprocals of each ratio above would provide a 3rd and 4th possible way of writing a proportion for this problem.

<u>FORMATIVE ASSESSMENT QUESTIONS</u>

- What ratio of tagged to total fish would best represent the situation?
- What quantities are being compared in your ratio?
- How might you use your ratio to determine the total number of fish in the lake?
- What is the relationship between the ratio of tagged to total fish in a sample and the ratio of tagged to total fish in the population?
- How many quantities in a proportion must we know in order to be able to solve for an unknown quantity?

<u>DIFFERENTIATION</u>

Extension

Some students may be ready to answer questions about percents related to the problem. Provide these students a series of questions about the same context or different context involving percentages rather than just the ratio.

Intervention

Students may struggle with setting up the proportion correctly. Provide those students with a possible equation such as:

$$\frac{\text{tagged sample}}{\text{total sample}} = \frac{\text{tagged population}}{\text{total population}}$$

Begin by just providing the left side of the equation only and ask them how the right side could be written. Struggling students will likely need to be reminded that the proportion can be written four different ways.

CLOSING/SUMMARIZER

Close the lesson with a whole-group discussion focused on the strategies used for selecting a representative ratio and for estimating the total number of fish in the lake. Students should leave knowing that there are four different ways to write a proportion. Some students, however, may not have set up a proportion to arrive at their estimate. Ask students to share their strategies, making sure to focus on proportional reasoning, while still valuing all legitimate strategies. End by asking students to think-pair-share about the two essential questions:

How can I use information from a sample to help make a prediction about a population?
How can I use a proportion to solve for an unknown quantity?

The teacher should pay close attention as students share their answers, listening closely for any misconceptions students still may have.

Fish in a Lake

Wildlife biologists often need to estimate the number of deer in a national park or the population size of fish in a lake. Because they cannot possibly count each deer or each fish, the biologists must use a "capture-recapture" method to help them estimate the population.

In order to estimate the number of fish in a lake, biologists first capture a sample of the fish, tag them, and then release them back into the lake with all of the untagged fish. Once the biologists have given the tagged fish enough time to thoroughly mix with the untagged fish, they capture another sample and count the number of each.

In today's lesson, we are going to simulate the capture-recapture method. The bag will represent the lake, the white beans will represent the untagged fish, and the red beans will represent the tagged fish. Your objective is to estimate the total number of fish in the lake. ***The teacher only puts white beans in the bag at the beginning of the lesson.***

1. Reach into the lake and capture a handful of "fish" to tag. Count and record the number of fish removed. Tag all of the fish (white beans) removed by replacing them with the same number of tagged fish (red beans). Set the removed fish to the side and return the tagged fish to the lake.

Total number of tagged fish _________________

2. Shake the bag so that the tagged and untagged fish can mix thoroughly. Remove a handful of fish, count them all, and then count the number of tagged fish in the sample. Record the counts in the table below and then find the ratio of tagged fish to total fish in the sample.

Sample Number	*Number of tagged fish*	*Total number of fish*	*Ratio of tagged fish to total fish*
1			
2			
3			
4			
5			

s

3. Return the sample of fish back to the lake. In order to get a better idea of the actual ratio of tagged to total number of fish in the lake, we will take a few more samples.

4. Repeat the sampling process four more times, each time recording the number of tagged fish and total number of fish in the sample. Remember to mix your fish thoroughly after returning the sample to the lake.

5. Now, your job is to use the information in your table to estimate the total number of fish in the lake. Create an equation that would allow you to solve for the total number of fish in the lake.

6. Check your estimate by counting the total number of fish in the lake. How close was your estimate to the actual population? What factors might cause your estimate to be incorrect?

Nate & Natalie's Walk

Students will use proportional reasoning to compare the distance that a brother and sister walk. In the performance lesson, students will use tables and graphs to represent a proportional relationship.

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class.
Recommended time: 60-90 minutes. Recommended arrangement: partners or small groups.

STANDARD FOR MATHEMATICAL CONTENT

MFAPR2. Students will recognize and represent proportional relationships between quantities.
a. Relate proportionality to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1, 2, 3; MGSE7.RP.1, 2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2, 3)

Common Misconceptions:
Several misconceptions are associated with graphing, in general. For example, students may:
- *Confuse the x and y axes.*
- *Not remember which to plot first, x or y.*
- *Mislabel axes in a particular context.*
- *Additionally, students may struggle with setting up and/or solving the proportion.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will need to decide the differences in the scenarios and solve the problems accordingly.
2. Reason abstractly and quantitatively. Students will go from analyzing diagrams to using tables and graphs.
4. Model with mathematics. Students will set-up models for comparisons.
8. Look for and express regularity in repeated reasoning. Students will find similarities and differences in the data.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Set up tables.
- Create a graph from a table of values.
- Interpret relationships between data using rates.

MATERIALS

- *Walk to the Movies* activity sheet
- Student lesson sheet

ESSENTIAL QUESTIONS

- How can we make predictions from graphs?
- What makes a relationship "proportional"? How can I tell if a proportional relationship exists?
- How can I represent a proportional relationship?

OPENER/ACTIVATOR

A video of the Disney marathon can be found at http://www.rundisney.com/results/

Students are asked if they know their walking/running rates in marathons. If so, these rates can be discussed. If not, runners' rates in the 2012 Olympics can be found at http://www.olympic.org/olympic-results/london-2012/athletics/marathon-m

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Prior to doing this performance lesson, students should understand that graphing is a way to visually represent ratios and proportional relationships. This visual is a tool that can be used to determine the reasonableness of an equation and to draw conclusions about proportional relationships.

Lesson Directions:

Hand out the lesson and allow students to work individually for 3-5 minutes without intervening. Circulate around the classroom to get an idea about what strategies are being used to solve the problem.

After 3-5 minutes, students may work with their partner or in their small groups. Support students' problem-solving by:

- Asking questions related to the mathematical ideas, problem-solving strategies, and connections between representations.
- asking students to explain their thinking and reasoning.

CLOSING/SUMMARIZER

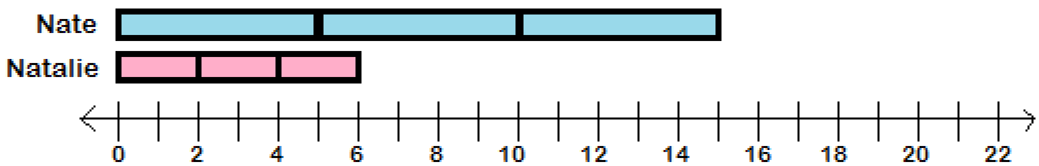
Students can calculate their own walking rates in the hallway. Activities can be found at http://illuminations.nctm.org/Lesson.aspx?id=4133

Nate and Natalie's Walk

Nate and his sister Natalie are walking around the track at school at a steady rate. Nate walks 5 feet in 2 seconds while Natalie walks 2 feet in the same amount of time.

1. Draw a diagram or picture that represents Nate and Natalie's walk around the track.

Students may use a variety of representations. For example, bar models or a number line as shown below could be used to represent the walk around the track.

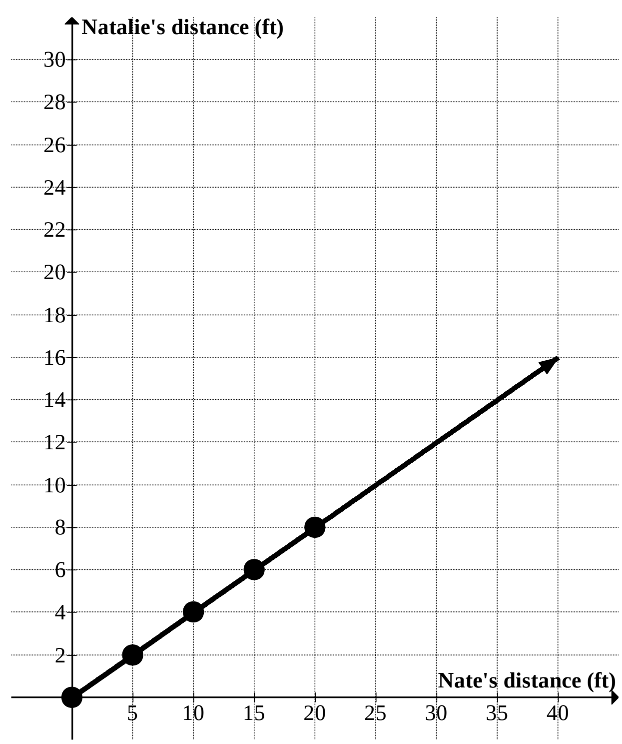


<u>Questions to encourage thinking</u>:

- ***Can you explain what your diagram shows about Nate and Natalie's walk?***
- ***Can you think of a different way to model the situation?***

2. Set up a table and create a graph to represent this situation. Let the *x*-axis represent the distance Nate walks and the *y*-axis represent the distance Natalie walks.

Nate's Distance (x)	*Natalie's Distance (y)*
0	*0*
5	*2*
10	*4*
15	*6*
20	*8*



3. What patterns do you see in the table? Explain the pattern. Express this as an equation.

Students should be able to explain that for every 5 feet Nate walks, Natalie walks 2 feet. So the ratio of Nate's distance to Natalie's distance in feet is 5:2. The equation that represents the situation is $y = \frac{2}{5}x$.

4. How do you read the graph? Explain what the coordinate (20, 8) means in the context of Nate and Natalie's walk?

For any point on the graph, the y-value represents the distance that Natalie can walk in the same amount of time that Nate can walk the distance represented in the x-value. The coordinate means that when Nate has walked 20 feet, Natalie has walked 8 feet.

5. When Nate walks 45 feet, how far will Natalie walk? Explain in writing or show how you found your answer.

Students may use the equation, table, or graph as a way to answer the problem. They may also set up a proportion to solve the problem. For example, $\frac{5}{2} = \frac{45}{x}$***. When solved the answer is 18. Natalie will have walked 18 feet when Nate has walked 45 feet.***

Comment:
For further discussion, ask, "Can you predict how far Natalie will walk if Nate walks 1000 feet?" This discussion should focus on the most efficient methods for solving the problem (equation or proportion) versus using a table or a graph.

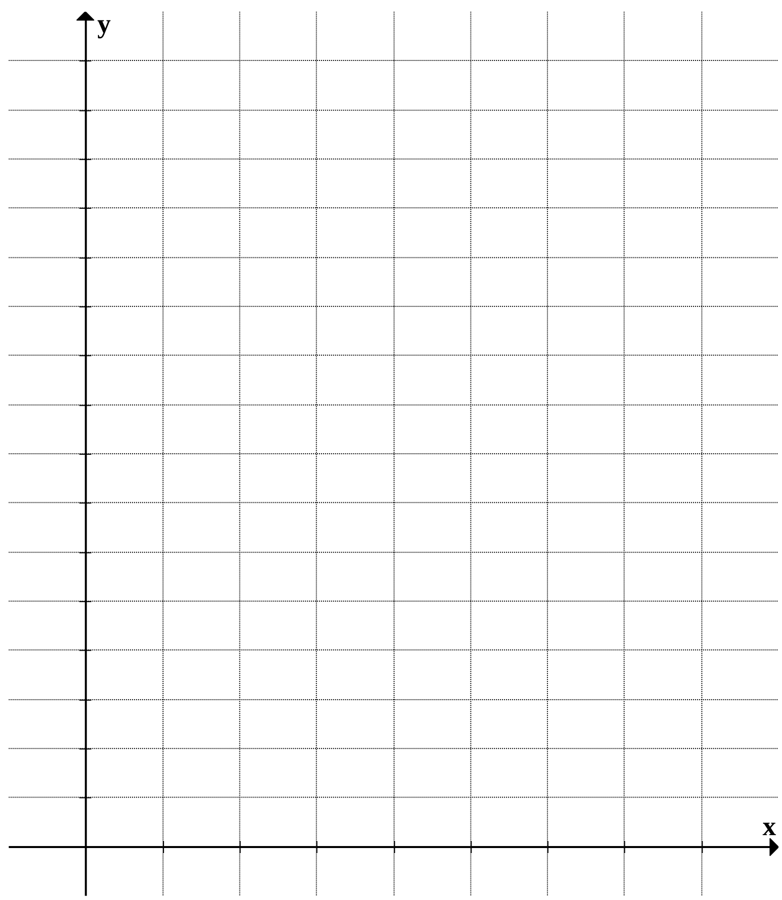
Another question to ask may be, "If the line that represents the relationship between the distance Nate and Natalie walk has a slope of 1, what does this mean?". Likewise, you could then ask students to explain what it would mean for a line to have a slope greater than 1 and a line to have a slope less than 1.

Nate & Natalie's Walk

Nate and his sister Natalie are walking around the track at school at a steady rate. Nate walks 5 feet in 2 seconds while Natalie walks 2 feet in the same amount of time.
.

1. Draw a diagram or picture that represents Nate and Natalie's walk around the track.

2. Set up a table and create a graph to represent this situation. Let the *x*-axis represent the distance Nate walks and the *y*-axis represent the distance Natalie walks.



3. What patterns do you see in the table? Explain the pattern. Express this as an equation.

4. How do you read the graph? Explain what the coordinate (20, 8) means in the context of Nate and Natalie's walk?

5. When Nate walks 45 feet, how far will Natalie walk? Explain in writing or show how you found your answer.

Rectangle Families

Adapted from Van De Walle, J. (2006). *Teaching student-centered mathematics: Grades 5-8* (Vol.3) Boston: Pearson Education, Inc.

In this lesson, students will sort rectangles into "families" by similarity and represent the relationship of the length to width on a graph, in a table, and in an equation.

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class.
Recommended time: 90-120 minutes. Recommended arrangement: partners or small groups.

STANDARDS FOR MATHEMATICAL CONTENT

MFAPR1. Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations.* (MGSE6.RP.3)

MFAPR2. Students will recognize and represent proportional relationships between quantities.
a. Relate proportionality to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator*. (MGSE4.NF.1)
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

MFAPR3. Students will graph proportional relationships.
a. Interpret unit rates as slopes of graphs. (MGSE8.EE.5)
b. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. (MGSE8.EE.6)
c. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (MGSE8.EE.5)

Common Misconceptions:
Students may believe that the width and length of the rectangles cannot be interchanged.
Students may continue to see an additive relationship rather than a multiplicative one. In other words, for the skinny rectangle, they may only notice that the width (or length) increases by 4 as the length (or width) increases by 1 rather than seeing that the width is the length multiplied by four.
Students may struggle with writing an equation to model each family of rectangles. Ask them to refer back to their mathematical sentences and replace the length and width with variables.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students must make sense of the problem in order to properly arrange the rectangles.
2. Reason abstractly and quantitatively. Students must use quantitative reasoning to see the relationship between width and length of the rectangle families.
6. Attend to precision. Students must use correct mathematical language as they communicate their thinking to their peers.
7. Look for and make use of structure. Students must notice a pattern in the lengths and widths of rectangles to understand the proportional relationship.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Recognize a proportional relationship in a table and in a graph.
- Identify similar figures by determining if corresponding sides are proportional.
- Find additional similar figures using the constant of proportionality.
- Write an equation to represent a proportional relationship.

MATERIALS

- Student lesson sheet
- Two pages of rectangles (copied single side)
- Scissors
- Colored pencils
- Rulers

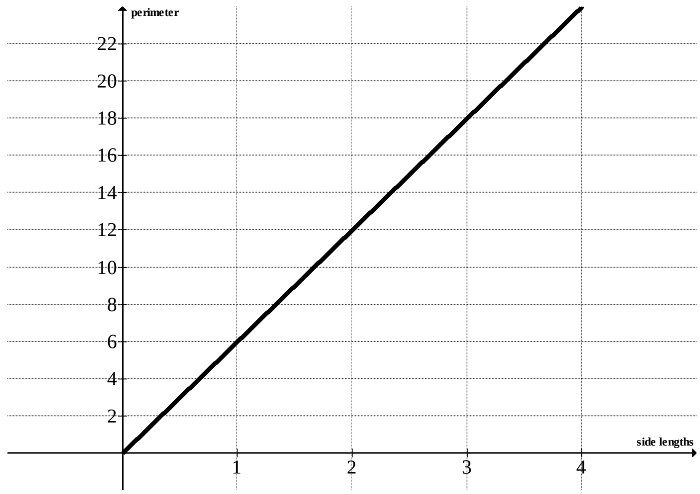
ESSENTIAL QUESTIONS

- What are some ways to prove that 2 rectangles are similar?
- How can graphs, tables, and equations help me to see proprotional relationships?

OPENER/ACTIVATOR

Determine whether the variables in each representation are in a proportional relationship or not. Justify your mathematical thinking.

1.



The side lengths of a regular hexagon and its perimeter are displayed in the graph.

2.

# of students	Total cost of fieldtrip ($)
5	50
10	100
15	150
20	200
25	250
30	300

The table shows the relationship between the number of students in a class and the total cost of a fieldtrip for the class.

3.

$$S = 15n + 45$$

The equation represents the total amount of money Max has in his savings account (S) after mowing n number of lawns.

This activator will give students the chance to review the characteristics of proportional relationships. ***Students should recognize that both #1 and #2 have 0 for the input and output and the relationships have a constant rate of change. (Students should be expected to explain this mathematical thinking in the context of the problem.) The 45 in number 3, however, indicates that Max must have already had $45 in his savings account prior to mowing lawns.***

If students are still struggling with these problems, show all representations for one of the three problems so that students can see multiple representations of the same problem.

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Lesson Directions:

Each student should receive a copy of the lesson, and each small group should receive two rectangles sheets and scissors. The lesson leads students through the steps but you may need to point out that the rectangles are separated by bold lines. They may be unsure of rectangle B which is a 1 x 1 rectangle and C which is a 2 x 1.

This is a somewhat lengthy lesson so it will be helpful to break it up into smaller chunks. Ask students to complete 1-7 first. (While walking around to all the groups, you may notice that some are not including the new rectangles in #6.) At this point, have a quick whole-group discussion about #7. Ask students to briefly share the numeric patterns that they see from the tables. This may also be a good time to point out that the rectangles can be arranged two different ways; the lengths and widths could be switched. As students work in their groups, check to make sure that students are being consistent. If the skinny rectangles are arranged with the long side horizontally, the same should be true for the other rectangles that are not square.

For the next chunk, ask the students to complete #8-11. Watch out for students on #10 who may try to graph all the data. Questions 10 and 11 are only in reference to the square data—all other data is plotted in #12. Before moving on, check for students' understanding of making another rectangle to fit in the family. This will be an important topic to discuss in the closing.

Lastly, students should complete the remainder of the lesson. Pay close attention to their answers on #12 and also to whether or not they are able to write equations for each rectangle family.

FORMATIVE ASSESSMENT QUESTIONS

- What relationship do you notice between the length and the width of each rectangle in a family?
- What method did you use to create the new rectangles in each family?
- How can you see the relationship between the lengths and widths of the rectangles in the table and in the graph?

DIFFERENTIATION

Extension
Give students grid paper and ask them to create 3 similar, but not congruent, right triangles. They must then explain how they know they are similar.

Intervention

For students who may be overwhelmed by beginning with 14 rectangles, start them out with only two from each family. Then, as they become more comfortable with the process, introduce more rectangles for them to sort.
If students are having trouble writing an equation, ask “How many times longer is one rectangle than another?” Ask them to show where they see this value on the graph to connect the slope of the line to the constant of proportionality identified in the table of values.

CLOSING/SUMMARIZER

The focus of the closing should depend largely on how well students were able to write equations for their rectangle families. If most students were able to write correct equations in #15 and solve for the widths in 16, then assign this closing activity:

Journal Entry
Thoroughly answer the following essential question using precise mathematical language:

How can graphs, tables, and equations help me to see proprotional relationships?

If most students were not able to produce correct equations in #15, assign this alternate closing:

Alternate Closing

Below is a table and graph for a new family of rectangles.

Length	**Width**
1	5
2	10
3	15
4	20
5	25

Determine which of the equations model the relationship between the length and width of the rectangle family. Then, explain how you can tell from the graph, table, and the equation(s) that the length and width are in a proportional relationship.

$W = L + 1$	$L = 5W$	$W = L + 5$
$W = 5L$	$L = \frac{W}{5}$	$L = W + 5$

Rectangle Families

1. Cut out the set of rectangles.
2. Sort the 14 rectangles into three "families" where all the members have the same shape, but differ in size.
3. In each family, arrange the rectangles from smallest to largest. What patterns do you see within each family?

   Answers will vary. Some students may notice that one family consists of squares, one family consists of "skinny" rectangles, which the other falls somewhere in the middle. Other students might already notice patterns in the dimensions and answer from a numerical perspective.

4. Stack each family of rectangles in order of size with the largest on the bottom. Arrange the rectangles so that each one in every family shares the bottom and the left edge. What new observations can you make about each of the families?

   Answers will vary. Students may notice that for each rectangle in the family (CGIKN), the width is twice the length (or vice versa), in family (BDFHM), the length and width are equal, and in family (AEJL), the width is four times the length.

5. Describe a method for finding the dimensions of another rectangle that would fit in a family.
   Answers will vary. Possible solutions include:
   For rectangles in family (CGIKN), multiply the length by 2.
   For rectangles in family (BDFHM), the length and width are equal.
   For rectangles in family (AEJL), multiply the length by 4.

6. Record, by family, the width and length of each rectangle. Then write in the dimensions for **a new rectangle** that fits each family.

Rectangle	Length	Width
C	***1***	***2***
G	***2***	***4***
I	***3***	***6***
K	***4***	***8***
N	***5***	***10***
	6	***12***

Rectangle	Length	Width
B	***1***	***1***
D	***3***	***3***
F	***5***	***5***
H	***7***	***7***
M	***10***	***10***
	8	***8***

Rectangle	Length	Width
A	***1***	***4***
E	***2***	***8***
J	***3***	***12***
L	***4***	***16***
	5	***20***

Note to teacher: Students may interchange length and width so that in the skinny rectangle, for instance, the length is 4 and the width is 1. This too is correct but notice that it will affect the answers to the graph and the constant of proportionality.

7. List any numeric patterns you see in your chart in step 6.

Answers will vary. Possible responses include:
Family (CGIKN) – as length increases by 1, width increases by 2
Family (BDFHM) – as length increase by 2, width increases by 2
Family (AEJL) – as length increases by 1, width increases by 4

8. Make the chart into a series of ratios by placing a fraction bar between the width and length pairs. Write the ratios in simplest form.

Rectangle	Ratio	Simplified
C	***1/2***	***1/2***
G	***2/4***	***1/2***
I	***3/6***	***1/2***
K	***4/8***	***1/2***
N	***5/10***	***1/2***
	6/12	***1/2***

Rectangle	Ratio	Simplified
B	***1/1***	***1***
D	***3/3***	***1***
F	***5/5***	***1***
H	***7/7***	***1***
M	***10/10***	***1***
	8/8	***1***

Rectangle	Ratio	Simplified
A	***1/4***	***1/4***
E	***2/8***	***1/4***
J	***3/12***	***1/4***
L	***4/16***	***1/4***
	5/20	***1/4***

9. What does the pattern in the ratio tell you about the rectangles in that family?

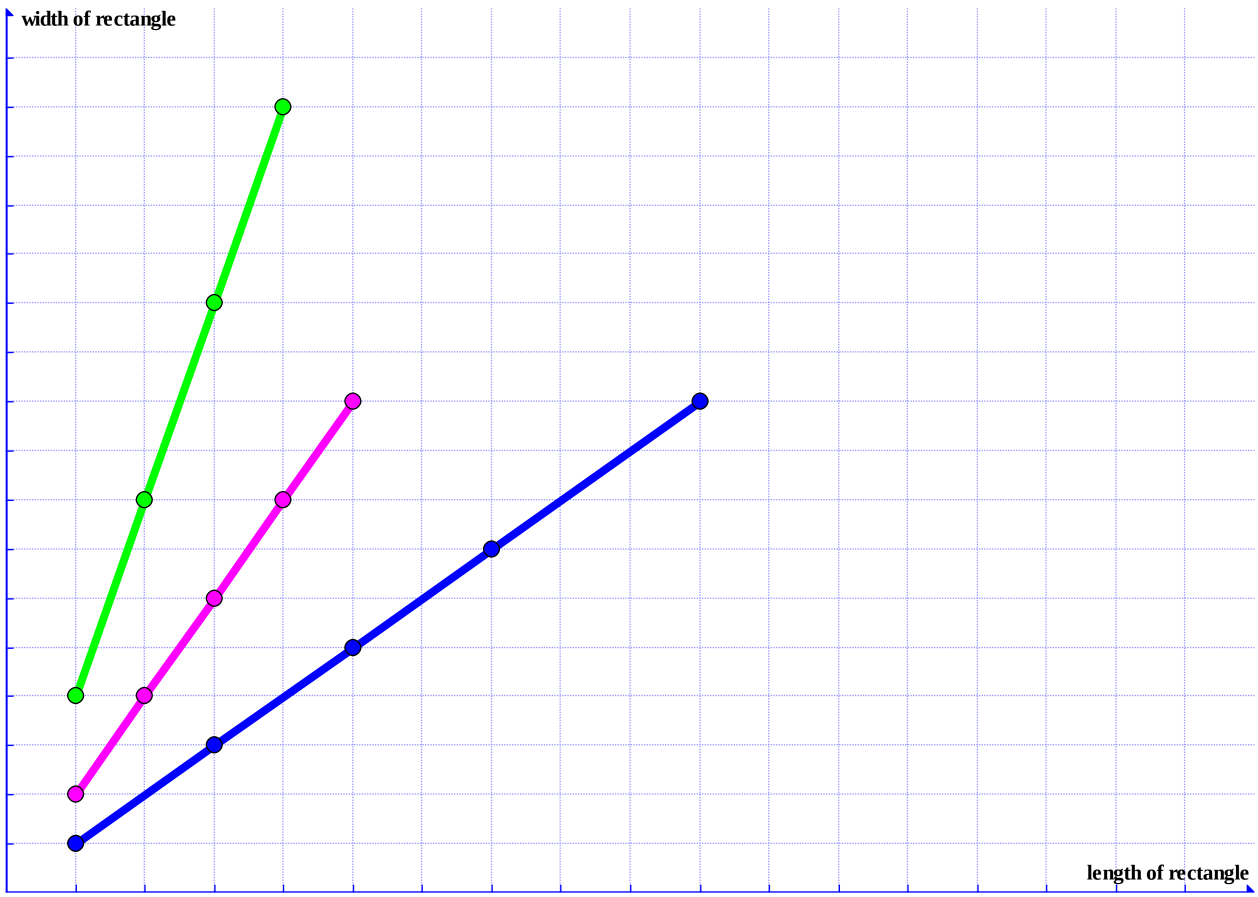
Square rectangle: __***Length and width are equal***____

Skinny rectangle: __***Width is four times the length***______________________________________

Other rectangle: ___***Width is twice the length***____

10. Using the data from the "square" rectangles, plot a graph of length vs. width. What do you notice about the plotted points?

    Answers will vary. Possible responses include:
    The points form a line.
    As the length increases by 1, the width increases by 1.



11. Use a ruler or other straight edge and a colored pencil to connect the points. How could you use what you drew to help you find the dimensions for another member of this rectangle family? Why does your method work?

    Answers will vary. Students should notice that they can find the dimensions for additional rectangles by extending the line with their straightedge. They may also notice that they can add additional points by continuing to move up 1 and to the right 1. These methods work because for this family of rectangles, the length and width are always equal. Continuing the pattern, either by extending the line or adding additional points, creates new dimensions that are equal in length and width.

12. Using different colored pencils, plot the data for each of the other families of rectangles. Do the new rectangles you created in step 6 fit the family patterns? How do you know?

    Answers will vary. If students choose dimensions in step 6 that follow the pattern in each family, then they should see that these points lie on the line (assuming their dimensions fit on the graph provided). They can explain their thinking using either the graph or the numerical pattern of the dimensions.

13. What is the constant of proportionality for each family of rectangles? How can you *see* each constant of proportionality in the table and the graph?

    Square: 1 ***Skinny:*** $\frac{1}{4}$ ***"Other":*** $\frac{1}{2}$

    Answers will vary:
    On the graph, the constant of proportionality can be seen by using $\frac{rise}{run}$ ***from one point to any other point of the line. In the table, the constant of proportionality is the ratio of length to width for each family.***

14. How could you use slope triangles to prove that your new rectangles fit the family patterns?

    Answers will vary. At this point, we want students to understand that a constant rate of change means that the ratio of the height:base of any right triangle formed from two points on a line is constant. For students struggling with this concept, ask them to compare the triangles formed by connecting two consecutive points and two non-consecutive points.

15. How would you determine how wide to make a rectangle to fit a specific family if you know how long the rectangle is supposed to be? Develop a method for each family.

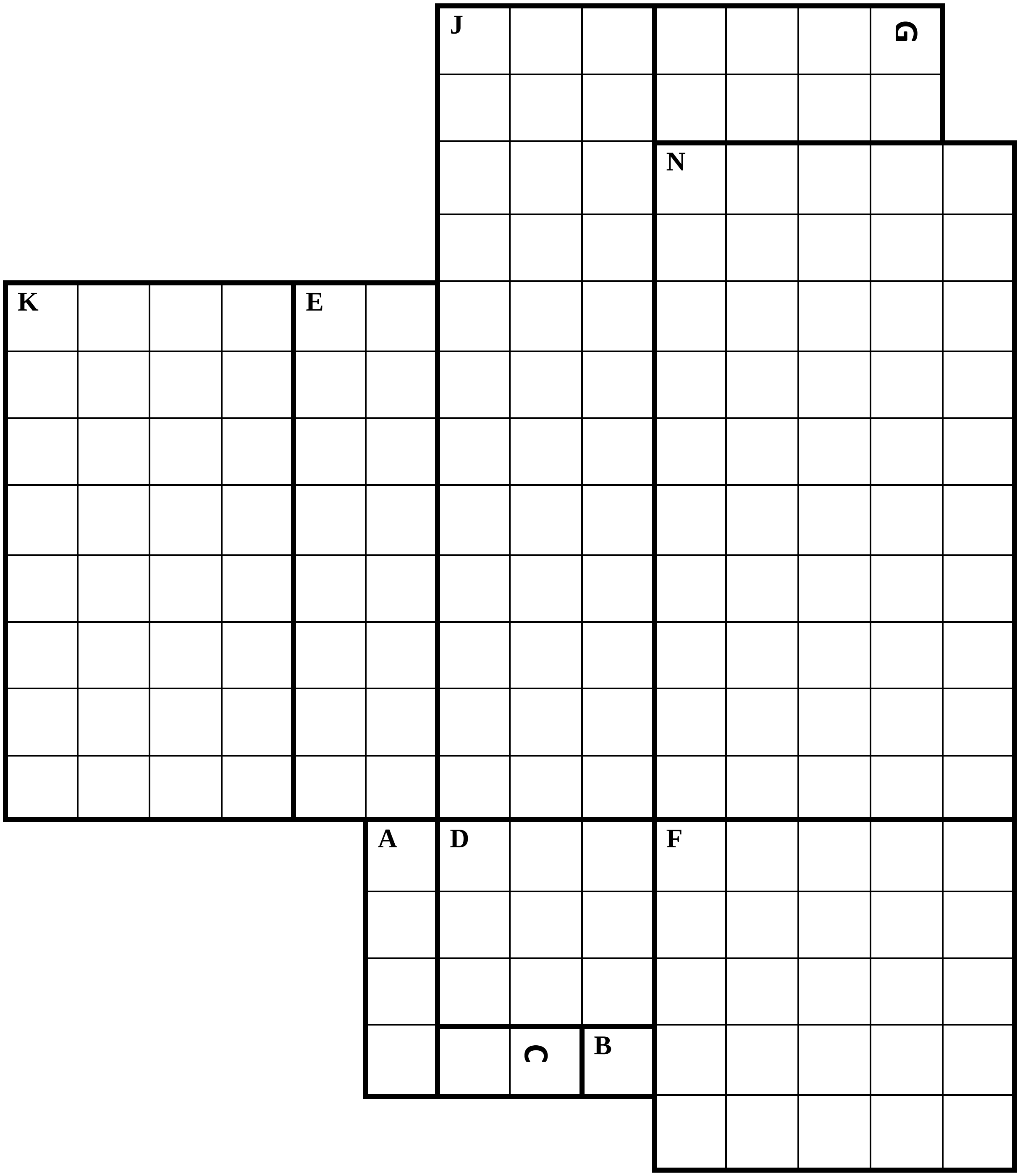
    Answers will vary. Possible responses include:
    For the family of square rectangles, the length and width are equal.
    For the family of skinny rectangles, take the given length and multiply it by four to get the width.
    For the other family of rectangles, take the given length and multiply it by two to get the width.

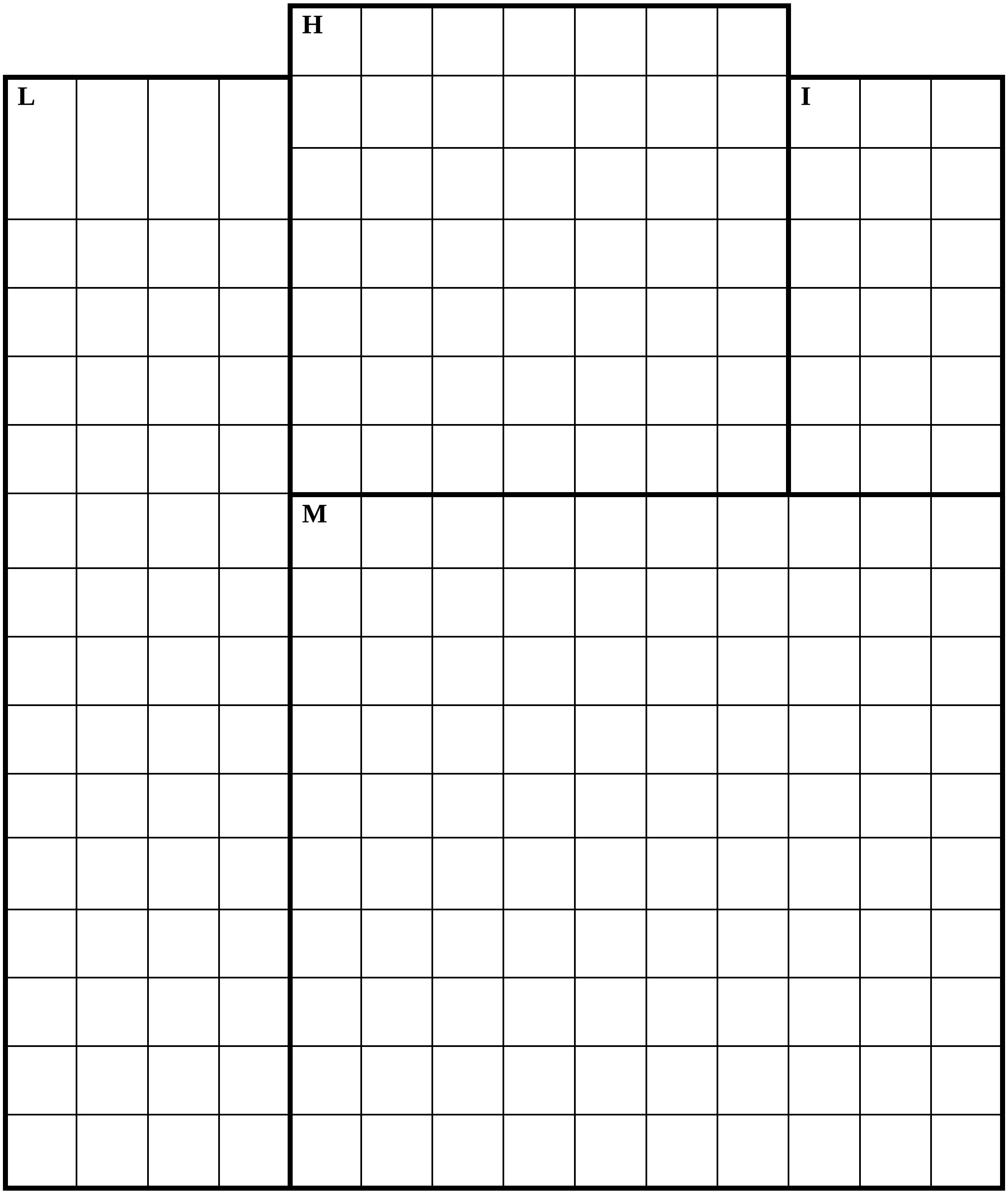
16. Write mathematical sentences to record the methods you developed for finding the width when you know the length. (Example: W = L / 8.)

Square	***Skinny***	***Other***
W = L	***W = 4L***	***W = 2L***

17. For each family, determine the widths of rectangles with lengths of 9 units. Use your graph to check your solutions.

Square	***Skinny***	***Other***
W = L ***W = 9***	***W = 4L*** ***W = 4(9)*** ***W = 36***	***W = 2L*** ***W = 2(9)*** ***W = 18***


J
G
N
K
E
A
D
F
C
B


H
L
I
M

Rectangle Families

1. Cut out the set of rectangles.
2. Sort the 14 rectangles into three "families" where all the members have the same shape, but differ in size.
3. In each family, arrange the rectangles from smallest too largest. What patterns do you see within each family?

4. Stack each family of rectangles in order of size with the largest on the bottom. Arrange the rectangles so that each one in every family shares the bottom and the left edge. What new observations can you make about each of the families?

5. Describe a method for finding the dimensions of another rectangle that would fit in a family.

6. Record, by family, the width and length of each rectangle. Then write in the dimensions for **a new rectangle** that fits each family.

Rectangle	Length	Width

Rectangle	Length	Width

Rectangle	Length	Width

7. List any numeric patterns you see in your chart in step 6.

8. Make the chart into a series of ratios by placing a fraction bar between the width and length pairs. Write the ratios in simplest form.

Rectangle	Ratio	Simplified

Rectangle	Ratio	Simplified

Rectangle	Ratio	Simplified

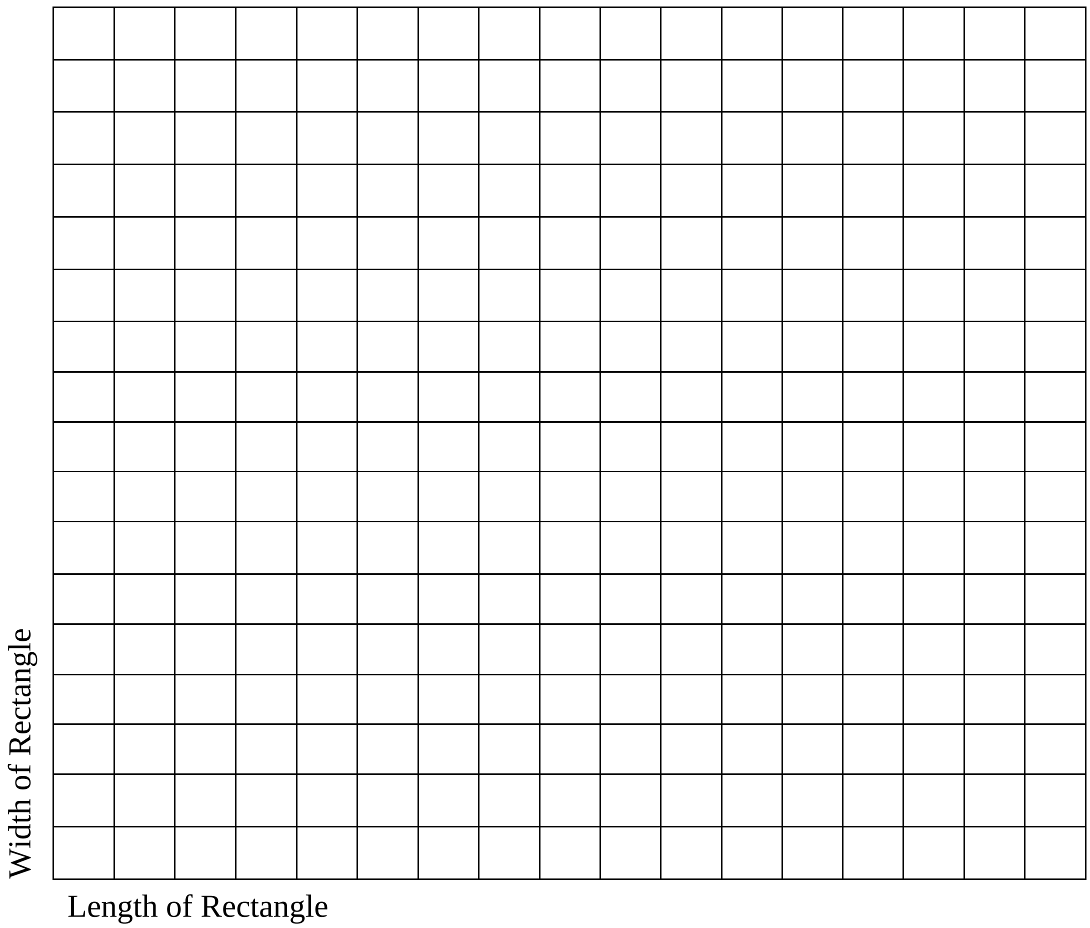
9. What does the pattern in the ratio tell you about the rectangles in that family?

Square rectangle: ____

Skinny rectangle: ____

Other rectangle: ____

10. Using the data from the "square" rectangles, plot a graph of length vs. width. What do you notice about the plotted points?



11. Use a ruler or other straight edge and a colored pencil to connect the points. How could you use what you drew to help you find the dimensions for another member of this rectangle family? Why does your method work?

12. Using different colored pencils, plot the data for each of the other families of rectangles. Do the new rectangles you created in step 6 fit the family patterns? How do you know?

13. What is the constant of proportionality for each family of rectangles? How can you *see* each constant of proportionality in the table and the graph?

14. How could you use slope triangles to prove that your new rectangles fit the family patterns?

15. How would you determine how wide to make a rectangle to fit a specific family if you know how long the rectangle is supposed to be? Develop a method for each family.

16. Write mathematical sentences to record the methods you developed for finding the width when you know the length. (Example: W = L / 8.)

17. For each family, determine the widths of rectangles with lengths of 9 units. Use your graph to check your solutions.

"Illustrative" Review

To review the concepts of percentages, slopes, and rates, Illustrative Mathematics has many problem solving sets. Descriptions and links are provided below.

Teacher Note: These problems can be used each day of the module as opening problems or tickets-out-the-door.

Rates: https://www.illustrativemathematics.org/content-standards/7/RP/A/1/tasks/82

Rates and Ratios: https://www.illustrativemathematics.org/content-standards/7/RP/A/1/tasks/470

Rates with Fractions: https://www.illustrativemathematics.org/content-standards/7/RP/A/1/tasks/828

Rates and Graphs: https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/104

Comparing Rates: https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/180

Rates and Graphs: https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/181

Rates and Graphs: https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/1178

Comparing Rates with Graphs: https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/1186

Proportions and Graphs: https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/1983

Interest https://www.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/1550

Percent Change; https://www.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/130

Percentages with Fractions: https://www.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/121

Double Percentages : https://www.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/2040

Percentages : https://www.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/105

Percentages and Rates: https://www.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/1330

Comparing Rates of Graphs: https://www.illustrativemathematics.org/content-standards/8/EE/B/5/tasks/55

Graphs to Equations: https://www.illustrativemathematics.org/content-standards/8/EE/B/5/tasks/57

Tables to Graphs: https://www.illustrativemathematics.org/content-standards/8/EE/B/5/tasks/184

Slopes as Rates of Change: https://www.illustrativemathematics.org/content-standards/8/EE/B/6/tasks/1537

Let's Open a Business

Adapted from *Teaching Student-Centered Mathematics Grades 5-8*, Van de Walle, John (2006)

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class. Recommended timing is 120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI1. Students will create and solve equations and inequalities in one variable.

a. Use variables to represent an unknown number in a specified set (conceptual understanding of a variable). (MGSE6.EE.2, 5, 6)

e. Use variables to solve real-world and mathematical problems. (MGSE6.EE.7, MGSE.7.EE.4)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic representations for real-world problems and can create real-world problems given an algebraic representation.
2. Reason abstractly and quantitatively. Students represent real-world contexts through the use of variables in expressions and equations.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using verbal or written explanations with peers.
4. Model with mathematics. Students model problem situations by forming expressions and equations that connect real-world problems to symbolic representations.
6. Attend to precision. Students precisely define variables.
7. Look for and make use of structure. Students analyze the structure of the statements and the words used to help them determine the correct expression for each scenario.

EVIDENCE OF LEARNING/LEARNING TARGET
By the conclusion of this lesson, students should be able to:

- Identify a variable in a real-world problem.
- Model real-world problems with an expression or an equation.

MATERIALS
Business Lesson Cards
Copies of additional practice

ESSENTIAL QUESTIONS

- How can you translate a word problem into an algebraic expression?
- How do you know you need an expression and not an equation?

OPENER/ACTIVATOR

Pose the following question: If you wanted to open a business, what would be some costs you would have to consider? ***Have students categorize their costs as being either variable or constant.***

LESSON

*****This lesson will be a huge jump for some students. You may want to use ability pairing/grouping as a strategy to spark rich discussion and manageable productive struggle.***

Explain to the students that they have been approved to receive a loan from the bank to become a small business owner. Assign each group of 3 (or pairs) one of the following businesses: bakery, coffee shop or clothing store. Lesson cards for each one involve identifying variables and writing expressions and equations.

Lesson card #1: Barney's Bakery will specialize in personalized cakes, cupcakes, cake pops and cookies. As the owner, you are trying to decide if you will be able to afford a space where customers can host a party during the first year open. To make this decision, you need to explore several buildings and spaces to find one that fits your needs. The cost information for one of the possibilities is listed below:

Rent (includes utilities) $1250 per month plus $750 for security deposit

1. ***What do you think is the variable in this situation? How do you know? (The number of months is the variable.)***
2. ***Write an expression to represent the total amount of money needed to cover building rent and security deposit, and use it to calculate:***
   a. ***Total amount needed for the first 3 months (1250m + 750; $4500)***
   b. ***Total amount needed for the first 6 months ($8250)***
   c. ***Total amount needed for the first year ($15,750)***
3. ***If your loan was approved in the amount of $75,000, do you think you could afford to rent this particular building? Explain your reasoning. (Answers may vary here; students may use an equation to help explain their reasoning.)***

Lesson card #2: Miko's Café is a high end coffee shop that offers gourmet coffee, specialty drinks and an in-house bakery. As the owner, you need to determine a fair wage to pay your full-time employees. After discussing this with several other business owners and employees, you narrowed it down to the following options:

Option #1: Full-time employees make $10.10 per hour; Managers make $15.00 per hour plus a $50 bonus per paycheck. (All employees are paid once every 2 weeks.)

Option #2: Full-time employees make $10.10 per hour; Managers make $17.25 per hour. (All employees are paid once every 2 weeks.)

1. ***In option #1, what would a full-time employee make if he worked 80 hours? What about a manager? ($808; $1250)***
2. ***What would the total cost be in general to pay both employees in option #1? (10.10h + (15h + 50) = 25.10h + 50)***
3. ***In option #2, what would a full-time employee make if he worked 80 hours? What about a manager? ($808; $1380)***
4. ***What would the total cost be in general to pay both employees in option #2? (10.10h + 17.25h = 27.35h)***
5. ***Which option would you choose? Why? (Answers will vary.)***

Lesson card #3: Trend City is a one-stop shop for all of your fashion needs. You realize that you will need at least one manager for the store, and you are trying to determine an estimated cost of having one. Your estimated cost will hopefully help you decide how many managers you can actually hire. They will get paid once every 2 weeks and make $15.00 per hour with a $50 bonus in each paycheck.

1. ***Write an expression that represents the relationship between the number of hours worked and earned pay. (15h + 50)***
2. ***How much will the manager make if he works 55 hours in a pay period? 75 hours? ($875; $1175)***
3. ***How many hours would you expect your manager to work within a pay period? Explain. (Answers will vary.)***
4. ***If you allotted $8000 of your payroll budget per pay period for manager's wage, how many managers could you afford? (Answers will vary.)***
5. ***Which option would you choose? Why? (Answers will vary.)***

Once all groups are finished, have each group share their small business lesson, questions and conclusions.

CLOSING/SUMMARIZER

What did the variable in each situation represent?
What are some different strategies you used to determine the expression or equation?
Then explore the following problem with students:
Zury has saved $20 more than 5 times the amount that her sister has saved. If we want to represent the amount that Zury has saved, do we need an expression or an equation? ***expression***
Suppose Zury has saved a total of $250. If we want to know the amount that her sister has saved, do we need an expression or an equation? Justify your response. ***An equation because a total number is given.***

ADDITIONAL PRACTICE

Suggested Additional Practice is attached.

1. Brian hiked 4 miles less than 3 times the amount his friend hiked. Write an expression that represents the amount of miles Brian has hiked. ***3h-4 with h representing his friend's mileage***

2. Describe a situation that can be solved by the following algebraic expression:

$$1.5s + 50$$

Responses will vary.

3. What information would we need in question #1 in order to write an equation? ***You would need to know how far Brian hiked for a one-variable equation (or you could write a two-variable equation b = 3h-4 where b is the distance Brian hiked and h is the number of miles his friend hiked.)***

Business #1: Barney’s Bakery

Barney’s Bakery will specialize in personalized cakes, cupcakes, cake pops, and cookies. As the owner, you are trying to decide if you will be able to afford a space where customers can host a party during the first year open. To make this decision, you need to explore several buildings and spaces to find on that fits your needs. The cost information for one of the possibilities is listed below:

Rent (includes utilities) $1250 per month plus $750 for security deposit

1. What do you think is the variable in this situation? How do you know?
2. Write an expression to represent the total amount of money needed to cover building rent and security deposit, and use it to calculate:
   a. Total amount needed for the first 3 months
   b. Total amount needed for the first 6 months
   c. Total amount needed for the first year
3. If your loan was approved in the amount of $75,000, do you think you could afford to rent this particular building? Explain your reasoning.

Business #2: Miko’s Café

Miko’s Café is a high end coffee shop that offers gourmet coffee, specialty drinks, and in-house bakery. As the owner, you need to determine a fair wage to pay your full-time employees. After discussing this with several other business owners and employees, you narrowed it down to the following options:

Option #1
Full-time employees make $10.10 per hour
Manager make $15.00 per hour plus a $50 bonus per paycheck. (All employees are paid once every 2 weeks.)

Option #2
Full-time employees make $10.10 per hours.
Managers make $17.25 per hour. (All employees are paid once every 2 weeks.)

1. In option #1, what would a full-time employee make if he worked 80 hours? What about a manger?
2. What would the total cost be in general to pay both employees in option #1?
3. In option #2, what would a full-time employee make if he worked 80 hours? What about a manager?
4. What would the total cost be in general to pay both employees in option #2?
5. Which option would you choose? Why?

Business #3: Trend City

Trend City is a one-stop shop for all your fashion needs. You realize that you will need at least one manager for the store, and you are trying to determine an estimated cost of having one. Your estimated cost will hopefully help you decide how many managers you can actually hire.

They will get paid once every 2 weeks and make $15.00 per hour with a $50 bonus in each paycheck.

1. Write an expression that represents the relationship between the number of hours worked and earned pay.
2. How much will the manager make if he works 55 hours in a pay period? 75 hours?
3. How many hours would you expect your manager to work within a pay period? Explain.
4. If you allotted $8000 of your payroll budget per pay period for manager's wage, how many managers could you afford?
5. Which option would you choose? Why?

Additional practice: Let's Open a Business

1. Brian hiked 4 miles less than 3 times the amount his friend hiked. Write an expression that represents the amount of miles Brian has hiked.

2. Describe a situation that can be solved by the following algebraic expression:

$$1.5s + 50$$

3. What information would we need in question #1 in order to write an equation?

Deconstructing Word Problems

(Adapted from "Deciphering Word Problems" which can be found at http://www.nsa.gov/academia/_files/collected_learning/middle_school/algebra/deciphering_world_problems.pdf)

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class. Recommended timing is 120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI1. Students will create and solve equations and inequalities in one variable.

a. Use variables to represent an unknown number in a specified set (conceptual understanding of a variable). (MGSE6.EE.2, 5, 6)
e. Use variables to solve real-world and mathematical problems. (MGSE6.EE.7, MGSE.7.EE.4)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic representations for real-world problems and can create real-world problems given an algebraic representation.
2. Reason abstractly and quantitatively. Students represent real-world contexts through the use of variables in expressions and equations.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using verbal or written explanations with peers.
4. Model with mathematics. Students model problem situations by forming expressions and equations that connect real world problems to symbolic representations.
6. Attend to precision. Students precisely define variables.
7. Look for and make use of structure. Students analyze the structure of the statements and the words used to help them determine the correct expression for each scenario.

EVIDENCE OF LEARNING/LEARNING TARGET
By the conclusion of this lesson, students should be able to build expressions and equations when presented a real-world problem.

MATERIALS
Pencil and paper
Student Handout

ESSENTIAL QUESTIONS

How can information from a word problem be translated to create an equation?
How can building and solving equations from word problems lead to a conclusion and help answer the problem being presented?

OPENER/ACTIVATOR

Have the students work through the translating verbal expression warm-up questions in order to address misconceptions about what words represent each operation. ***Keep an eye out for "less than" and how it differs in its placement in an expression. Anything that says "less than" or "from" changes the order of how the expression is written.***

Choose 3 of the 7 situations below and translate the verbal expression to an algebraic equation. ***Allowing students this choice will help you pinpoint more quickly who is having success and who is not. For those who struggle, pull them aside individually while others work on part II and have the struggling students discuss some of the "leftover" equations.***

1) Ann has the 5 newest music CD's which is 3 less than twice the amount that Bob has.
2x- 3 = 5 where x is Bob's amount.
2) Mike, who has 6 video games, has half as many games as Paul. $\frac{x}{2}$ ***= 6 where x is Paul's amount***
3) Nan rode the roller coaster 8 times, which was twice as many times as she rode the Ferris wheel. ***2x = 8 where x is the times on the Ferris wheel***
4) Janine, who bought $15 worth of make-up, spent $6 less than Leah spent.
x - 6 = 15 where x is what Leah spent
5) Rob, who has all 13 girls' phone numbers that are in his homeroom, has 3 more than half the number of girls' phone numbers that Jay has.
$\frac{x}{2}$ ***+ 3 = 13 where x stands for Jay's number***
6) Kate's 85 on her English test was 37 points less than twice the grade on her Science test.
2x – 37 = 85 where x is the Science test
7) At the Middle School Graduation Dance, the DJ played 12 slow dances, which was equal to the quotient of the number of fast dances and 2. $\frac{x}{2}$ ***= 12 where x is the fast dances***

LESSON

Create expressions for the situation described in the word problem. Your teacher may guide you through using a chart to help you do this. Then, use these expressions and the word problem to create and solve an equation. Make sure you not only solve for the variable, but also answer the question being presented. ***Students sometimes struggle with deciding which person or item is the variable. One suggestion is to tell them it is the person or item on which everything else is based.***

1. Sean sold 4 more boxes of candy for the school fundraiser than Marta. The sum of the boxes they sold was 22. How many boxes did each sell?

WHO	***NUMBER OF BOXES***
Sean	***4 + M***
Marta	***M***

$$4+M+M=22$$
$$2M+4=22$$
$$2M=18$$
$$M=9$$

Marta sold 9 boxes of candy and Sean sold 13.

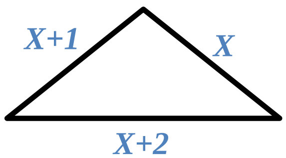
2. Ned weigh 1 ½ times as much as Jill, and Tom weighs 15 kilograms more than Jill. If their combined weight is 190 kilograms, how much does each person weigh?

WHO	***WEIGHT***
Ned	***1 ½ J***
Jill	***J***
Tom	***15 + J***

$$1\frac{1}{2}(J)+J+15+J=190$$
$$3\frac{1}{2}(J)+15=190$$
$$3\frac{1}{2}(J)=175$$
$$J=50$$

Jill weights 50 kilograms, Tom weighs 65 kg. Ned weights 75 kg.

3. The side lengths of a triangular birdcage are consecutive integers. If the perimeter is 114 centimeters, what is the length of each side? Label each side with an expression that represents its length.



$$X + X + 1 + X + 2 = 114$$
$$3X + 3 = 114$$
$$3X = 111$$
$$X = 37$$

The lengths of the sides are 37, 38, and 39.

4. Caitlyn did $\frac{6}{7}$ of the problems on her math quiz correctly and four incorrectly. She did all the problems. How many were there?

TYPE OF PROBLEM	***FRACTIONAL PART OF WHOLE***	***NUMBER***
Correct	$\frac{6}{7}$	***x - 4***
Incorrect	$\frac{1}{7}$	***4***
Total on Quiz	$\frac{7}{7}$	***x***

$$\frac{1}{7}x = 4$$
$$x = 28$$

There were 28 problems on the quiz.

5. Geri spent Friday, Saturday, and Sunday selling a total of 24 magazine orders for her school fundraiser. The amounts she sold respectively, on the three days were consecutive even integers. How many did she sell on each day?

DAY OF WEEK	***AMOUNT SOLD***
Friday	***n***
Saturday	***n + 2***
Sunday	***n + 4***

$$n + n + 2 + n + 4 = 24$$
$$3n + 6 = 24$$
$$3n = 18$$
$$n = 6$$

Geri sold 6 orders on Friday, 8 orders on Saturday, and 10 orders on Sunday.

INTERVENTION

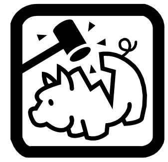
For extra help with topics in this lesson, please refer to the Intervention Table. If students struggle with the handout, suggest some active reading strategies to help them locate important information within the problem (highlighting or underlining key words, etc.).

CLOSING/SUMMARIZER

Have students look through the various scenarios they addressed in this lesson. Ask the following questions, "How did we use the information in each scenario to create an equation? What common threads can we identify across the different situations?" Have students write a letter to an "absent" student explaining the most important ideas to consider in translating scenarios into equations.

ADDITIONAL PRACTICE:

There are many more scenarios on the referenced website as well as a quiz for the end of the section if needed. Assign one or two to challenge the students.
http://www.nsa.gov/academia/_files/collected_learning/middle_school/algebra/deciphering_world_problems.pdf



Student Handout: Deconstructing Word Problems

Part I: Warm-Up
Choose 3 of the 7 situations below and translate the verbal expression to an algebraic equation.

1) Ann has the 5 newest music CD's which is 3 less than twice the amount that Bob has.
   ____

2) Mike, who has 6 video games, has half as many games as Paul.
   ____

3) Nan rode the roller coaster 8 times, which was twice as many times as she rode the Ferris wheel. _________________________________

4) Janine, who bought $15 worth of make-up, spent $6 less than Leah spent.
   ____

5) Rob, who has all 13 girls' phone numbers that are in his homeroom, has 3 more than half the number of girls' phone numbers that Jay has.
   ____

6) Kate's 85 on her English test was 37 points less than twice the grade on her Science test.
   ____

7) At the Middle School Graduation Dance, the DJ played 12 slow dances, which was equal to the quotient of the number of fast dances and 2.
   ____

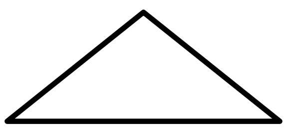
Part II: Lesson on Creating Equations from Word Problems

Create expressions for the situation described in the word problem. Your teacher may guide you through using a chart to help you do this. Then, use these expressions and the word problem in order to create and solve an equation. Make sure you not only solve for the variable, but also answer the question being presented. Show all your work when solving the equations.

1. Sean sold 4 more boxes of candy for the school fundraiser than Marta. The sum of the boxes they sold was 22. How many boxes did each sell?

2. Ned weighs $1\frac{1}{2}$ times as much as Jill and Tom weighs 15 kilograms more than Jill. If their combined weight is 190 kilograms, how much does each person weigh?

3. The sides of a triangular birdcage are consecutive integers. If the perimeter is 114 centimeters, what is the length of each side? Label each side with an expression that represents its length.



4. Caitlyn did $\frac{6}{7}$ of the problems on her math quiz correctly and four incorrectly. She did all the problems. How many were there?

5. Geri spent Friday, Saturday and Sunday selling a total of 24 magazine orders for her school fundraiser. The amounts she sold, respectively, on the three days were consecutive even integers. How many did she sell on each day?

Steps to Solving an Equation (FAL)

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/lessons.php?unit=7220&collection=8&redir=1

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Recommended timing is 120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI1. Students will create and solve equations and inequalities in one variable.

b. Explain each step in solving simple equations and inequalities using the equality properties of numbers. (MGSE9-12.A.REI.1)
c. Construct viable arguments to justify the solutions and methods of solving equations and inequalities. (MGSE9-12.A.REI.1)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic representations for real-world problems and can create real-world problems given an algebraic representation.
2. Reason abstractly and quantitatively. Students represent real-world contexts through the use of variables in expressions and equations.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using verbal or written explanations with peers.
4. Model with mathematics. Students model problem situations by forming expressions and equations that connect real world problems to symbolic representations.
6. Attend to precision. Students precisely define variables.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Form and solve linear equations using factoring and the distributive property.
- Use variables to represent equations in real-world problems.
- Represent word problems in equivalent equations.

MATERIALS

- Copies of formative assessment lesson pre-and post-assessment for each student
- Card sets for each pair (or small group)
- Mini whiteboards
- Chart paper or poster board
- Glue
- Scissors (if lesson cards are not pre-cut)

ESSENTIAL QUESTION

- What are some strategies for solving real life mathematical problems involving numerical and algebraic equations and expressions?

Lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information, access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The PDF version of this lesson can be found at the link below:
http://map.mathshell.org/download.php?fileid=1635

Acting Out

Adapted from Shell Center Leaky Faucet Short Cycle Lesson
http://www.map.mathshell.org/materials/tasks.php?taskid=267&subpage=apprentice

This task is included in the grade level framework; so, teachers should consult with each other to decide whether to use this activity in support or in the regular classroom.

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Recommended timing is 120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI1. Students will create and solve equations and inequalities in one variable.

a. Use variables to represent an unknown number in a specified set (conceptual understanding of a variable). (MGSE6.EE.2, 5, 6)
e. Use variables to solve real-world and mathematical problems. (MGSE6.EE.7, MGSE.7.EE.4)

MFAEI2. Students will use units as a way to understand problems and guide the solutions of multi-step problems.

a. Choose and interpret units in formulas. (MGSE9-12.N.Q.1)
b. Choose and interpret graphs and data displays, including the scale and comparisons of data. (MGSE3.MD.3, MGSE9-12.N.Q.1)
c. Graph points in all four quadrants. (MGSE6.NS.8)

STANDARDS FOR MATHEMATICAL PRACTICE

1. **Make sense of problems and persevere in solving them.** Students determine different ways of measuring a fixed distance from a given point and strategize the most accurate way to convert measurements.
5. **Use appropriate tools strategically.** Students are given multiple tools to choose from to model the scenario in Part I.
6. **Attend to precision.** Students determine when to round their answers based on the units of measurement.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Model and write an equation in one variable and solve a problem in context.
- Create one-variable linear equations and inequalities from contextual situations.
- Solve word problems where quantities are given in different units that must be converted to understand the problem.

MATERIALS

- Student Handout: Acting Out

- Colored pencils
- Compass
- String
- Graph paper

ESSENTIAL QUESTION

- How do I choose and interpret units consistently in formulas?

GROUPING

- Part I: Small group / whole group
- Part II: Partner / Individual

OPENER/ACTIVATOR

Part I:

Erik and Kim are actors at a theater. Erik lives 5 miles from the theater and Kim lives 3 miles from the theater. Their boss, the director, wonders how far apart the actors live.

Comments

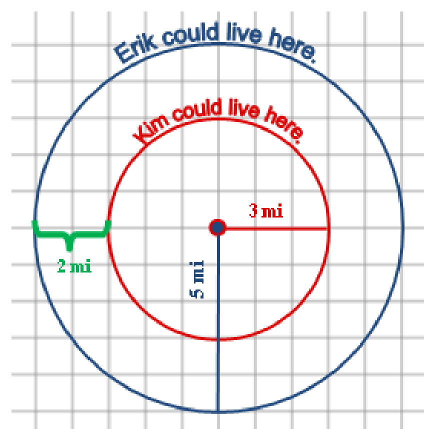
Students should understand that Erik and Kim could live anywhere on the circle with the theater as the center and the radius as the distance that they live from the theater.

1. On the given grid:

   a. Pick a point to represent the location of the theater.

   b. Illustrate all of the possible places that Erik could live on the grid paper.

   c. Using a different color, illustrate all of the possible places that Kim could live on the grid paper.



2. What is the smallest distance, *d*, that could separate their homes? How did you know?
Solution ***5 – 3 = 2 miles***

3. What is the largest distance, *d*, that could separate their homes? How did you know?
Solution ***5 + 3 = 8 miles***

4. Write and graph an inequality in terms of *d* to show their boss all of the possible distances that could separate the homes of the 2 actors.
Solution
An inequality that could represent this distance could be $2 \le d \le 8$ miles.
Graphing this inequality should look like the graph shown below.

Students should understand that the solid dots on the graph represent the fact that Erik and Kim could live exactly 2 miles or exactly 8 miles apart. Should the situation have been different and they lived more than 2 miles or less than 8 miles apart, those dots would have been left open, or not filled in. The space shaded on the number line between the 2 and 8 means that they could live any of those distances apart.

LESSON
Part II: Extension Problem

The actors are good friends since they live close to each other. Kim has a leaky faucet in her kitchen and asks Erik to come over and take a look at it.

1. Kim estimates that the faucet in her kitchen drips at a rate of 1 drop every 2 seconds. Erik wants to know how many times the faucet drips in a week. Help Erik by showing your calculations below.

Solution:

60 sec = 1 min
60 min = 1 hour
24 hours = 1 day
7 days = 1 week

(60) (60) (24) (7) = 604,800
604800 ÷ 2 = 302,400 drops per week

365 days = 1 year

$\frac{(60)(60)(24)(365)}{2}$ ***= 15,768,000 drops per year [this information will be used in item 2.]***

2. Kim estimates that approximately 575 drops fill a 100 milliliter bottle. Estimate how much water her leaky faucet wastes in a year.

Solution:

15,768,000 ÷ 575 = 27,422.608

About 27,423 of 100 millimeter bottles would be filled.
Encourage students to write this answer in the most user-friendly measurement (i.e., liters)

27,423(100) = 2,742,300 milliliters or 2,742.3 Liters

CLOSING/SUMMARIZER

Discuss the following questions with students, "What units were important in this lesson? Were there any units that were not important? What other lessons have we completed in this module in which units were important? Are there any commonalities among this lesson and other lessons we have completed?"

ADDITIONAL PRACTICE

Have students investigate modifications of the leaky faucet situation:

1. Kim's faucet drips at a rate of 2 drops every second. Complete items 1 and 2 from Part II of the lesson with this new rate of dripping. ***Solutions will be four times as large. Encourage students to estimate and analyze what is happening rather than automatically re-doing the mathematics.***

2. Kim's faucet drips at a rate of *d* drops every second. Complete items 1 and 2 from Part II of this lesson by creating algebraic expressions.
   Part I: (60) (60) (24) (7) = 604,800
   604,800d drops per week

   Part II: (604800d)/575 bottles
3. Kim's faucet drips at a rate of 1 drop every *s* seconds. Complete items 1 and 2 from Part II of this lesson by creating algebraic expressions.
   Part I: 604800/s drops per week

   Part II: (604800/s)/575 bottles

Student Handout: Acting Out

Name__________________________________ **Date**___________________

Adapted from Shell Center Leaky Faucet Short Cycle Lesson
http://www.map.mathshell.org/materials/tasks.php?taskid=267&subpage=apprentice

Part I:

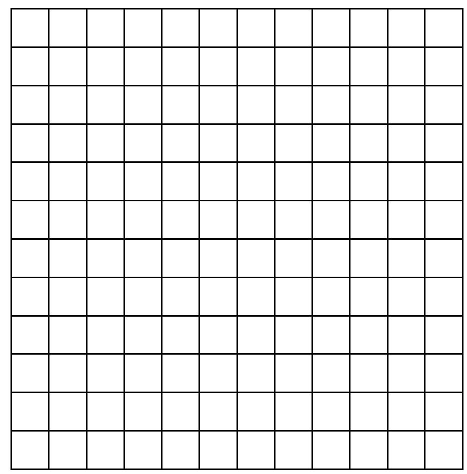
Erik and Kim are actors at a theater. Erik lives 5 miles from the theater and Kim lives 3 miles from the theater. Their boss, the director, wonders how far apart the actors live.

1. On the given grid:

   a. Pick a point to represent the location of the theater.

   b. Illustrate all of the possible places that Erik could live on the grid paper.

   c. Using a different color, illustrate all of the possible places that Kim could live on the grid paper.



2. What is the smallest distance, *d*, that could separate their homes? How did you know?

3. What is the largest distance, *d*, that could separate their homes? How did you know?

4. Write and graph an inequality in terms of *d* to show their boss all of the possible distances that could separate the homes of the 2 actors.

Part II: Extension

The actors are good friends since they live close to each other. Kim has a leaky faucet in her kitchen and asks Erik to come over and take a look at it.

1. Kim estimates that the faucet in her kitchen drips at a rate of 1 drop every 2 seconds. Erik wants to know how many times the faucet drips in a week. Help Erik by showing your calculations below.

2. Kim estimates that approximately 575 drops fill a 100 milliliter bottle. Estimate how much water her leaky faucet wastes in a year.

Literal Equations

This task is included in the grade level framework; so, teachers should consult with each other to decide whether to use this activity in support or in the regular classroom.

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class. Recommended timing is 120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI4. Students will solve literal equations.

a. Solve for any variable in a multi-variable equation. (MGSE6.EE.9, MGSE9-12.A.REI.3)
b. Rearrange formulas to highlight a particular variable using the same reasoning as in solving equations. *For example, solve for the base, b, in* $A = \frac{1}{2}bh$. (MGSE9-12.A.CED.4)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic operations in order to highlight a specific quantity in each equation.
2. Reason abstractly and quantitatively. Students interpret parts of an expression and equation in context.
6. Attend to precision. Students use the properties of equality precisely.
7. Look for and make use of structure. Students seek patterns or structures to solve equations for a specific variable.
8. Look for and express regularity in repeated reasoning. Students generalize effective processes for solving equations based upon experiences.

EVIDENCE OF LEARNING/LEARNING TARGET
By the conclusion of this lesson, students should be able to extend the properties of equality used in solving numerical equations to rearrange formulas to highlight a quantity of interest.

MATERIALS

- Projector and laptop
- Literal Equations power point
- 35 index cards with letters and operations on them

ESSENTIAL QUESTIONS

- How do I interpret parts of an expression in terms of context?
- How can I rearrange formulas to highlight a quantity of interest?
- What arithmetic and algebraic properties do I have to consider when rearranging formulas?

OPENER/ACTIVATOR

(Using slide 1 of the power point) Pose the following question to students: What does the word ‘literal’ mean? Complete the practice in the slides, waiting until the end of the lesson for the final slide question (see closing below).

LESSON

After students have completed the practice in the power point, have them do the following sorting activity. A visual and variations on this activity can be found at http://handsonmathinhighschool.blogspot.com/2012/07/made4math-3-literal-equations.html. Group the students in pairs. Give each pair a set of 35 notecards (laminated if possible) with the following letters and operations written on them: X, Y, D, A, B, S, T, Z, A, R, F, S, T, X, Z, A, +, =, +, ‘division bar,’ =, -, ‘division bar,’ =, (,), -, =, +, and 6 blank cards. They are to use the cards to set up the following literal equations, and solve for the indicated variable:

X+Y = D; solve for X $(X = D - Y)$
$\frac{A+B}{S} = T$***; solve for A*** $(A = TS - B)$
$Z - \frac{A}{R} = F$***; solve for R*** $(R = \frac{-A}{F-Z})$
$S(T - X) = Z + A$***; solve for T*** $(T = \frac{Z+A}{S} + X)$

As students work to reorganize the cards to solve for the indicated variable, they should write the inverse operation or reciprocal of the fraction on the back of the card. The blank cards are there to assist with “extra” operations (for instance, in equation #3, neither Z nor F has an explicit sign, so a student may want to create a “- “card to use when solving.

INTERVENTION
For extra help with topics in this lesson, please refer to the Intervention Table.

CLOSING/SUMMARIZER
(Using the last slide of the presentation) Pose the following question: When is solving a literal equation helpful in real life? Generate a list of situations as a class and discuss the value of being able to isolate variables for each situation. Review arithmetic and algebraic properties as needed.

ADDITIONAL PRACTICE
Assign one of the two tasks found on the Illustrative Mathematics website using the following link: https://www.illustrativemathematics.org/HSA-CED.A.4 (Read through the commentary for each task before deciding which one to assign.) A list of equations can be found at: https://www.illustrativemathematics.org/content-standards/HSA/CED/A/4/tasks/393

Function Rules

Adapted from Illustrative Mathematics, http://s3.amazonaws.com/illustrativemathematics/illustration_pdfs/000/000/715/original/illustrative_mathematics_715.pdf?1423236545

This lesson can be used to introduce/review the idea that a function assigns a unique output to every input. It also encourages students to look for less obvious patterns and encourages them to verbalize functions rules using precise language.

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. If played with a partner as a game, this could take 0.5 hours.

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR1. Students will understand characteristics of functions.
a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. (MGSE9-12.F.IF.1)
b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*. (MGSE9-12.F.IF.5)
c. Graph functions using sets of ordered pairs consisting of an input and the corresponding output. (MGSE8.F.1, 2)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students interpret the situation and persevere in demonstrating their understanding.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning in completing tables
3. Construct viable arguments and critique the reasoning of others. Students work collaboratively to explain their understanding of functions in real life situations.
4. Model with mathematics. Students model functions with graphs and tables.
6. Attend to precision. Students use clear and precise language in discussing strategies and carefully work to identify patterns.
7. Look for and make use of structure. Students use their understanding of functions to make sense of information given and extend their thinking in real-life situations.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to

- complete tables
- describe a rule in words that satisfies the pairs (given tables of input-output pairs in different forms)

MATERIALS

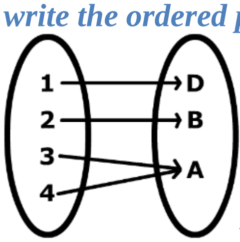
Copy of the lesson.

ESSENTIAL QUESTION

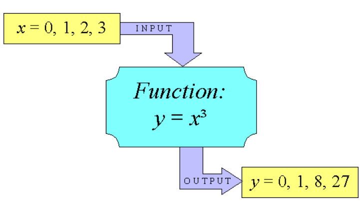
How can I find a rule that works for a given domain and range, then extend the pattern?

OPENER/ACTIVATOR

Make sure students are comfortable with the concept of identifying functions as ordered pairs using (x,y). The input and output of a function can be expressed as an ordered pair presented in any form: graphically, verbally, tabular form, or later in equation form. Students should write the ordered pairs represented by each scenario below.



{(1, D), (2, B), (3, A), (4, A)}

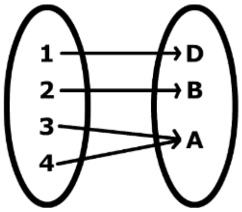


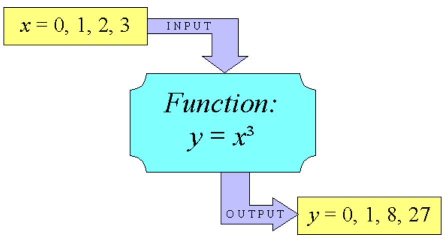
{(0, 0), (1, 1), (2, 8), (3, 27)}

x	-3	-2	-1	0	1	2	3
y	1	-2	2	4	-3	-2	-1

{(-3, 1), (-2, -2), (-1, 2), (0, 4), (1, -3), (2, -2), (3, -1)}

Students should write the ordered pairs represented by each scenario below.





x	-3	-2	-1	0	1	2	3
y	1	-2	2	4	-3	-2	-1

<u>TEACHER COMMENTARY</u>

This lesson can be played as a game where students have to guess the rule as the instructor gives more and more input-output pairs. Giving only three input-output pairs might not be enough to clarify the rule. Instructors might consider varying the inputs, e.g., the second table, to provide non-integer entries. A nice variation on the game is to have students who think they found the rule supply input-output pairs, which the teacher then confirms or refutes.

Verbalizing the rule requires precision of language. For the first part, only vowels a, e, i, o, u is counted. In the third example, we are looking at a non-leap year.

a.
Seeing that the input values can be any English word, we find the rule to be "the number of vowels" in the input word, and we designate that a vowel here is defined as a, e, i, o, u, but not y, as our input of "you" has an output of 2, not 3. Below is one possible way to complete the table.

Input	Cat	House	You	Table	Fireplace	Sky	Picture
Output	1	3	2	2	4	0	3

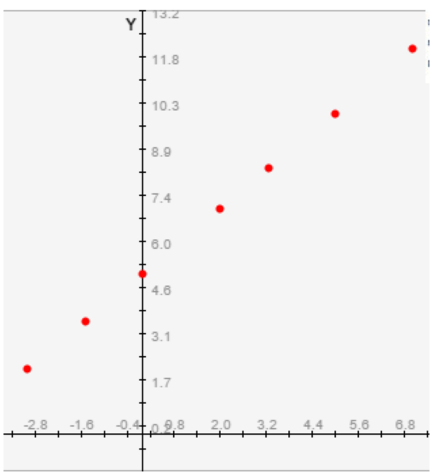
b.
To find the rule, we find the common math operation between the pairs of numbers. We can conclude that our rule here is to "add 5" to the input value. Below is one possible way to complete the table.

Input	2	5	-1.5	7	-3	3.285	0
Output	7	10	3.5	12	2	8.285	5

With only three input-output pairs, we can probably come up with many other functions, but the rule of the game is that the teacher has a function rule in mind and gives more and more input-output pairs until the students guess the teacher's function.
c.
Since our input values here are numbers, but our output values are months of the year, we find that the rule here is "the month corresponding to that day out of the year", defining January 1st to be day one. We can also assume we are using a non-leap year to determine the output values. Below is one possible way to complete the table.

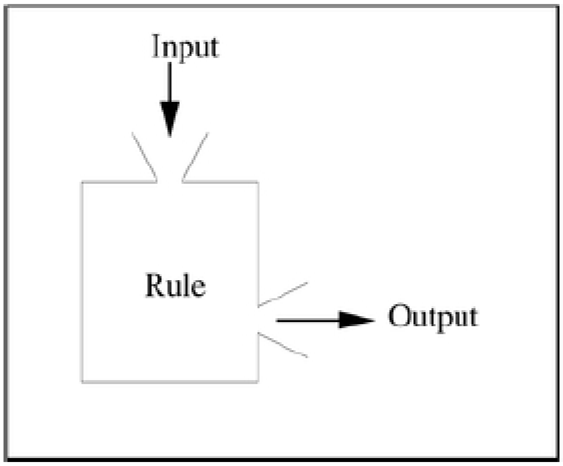
Input	25	365	35	95	330	123	200
Output	January	December	February	April	November	May	July

Can any of the functions above be easily graphed on a coordinate plane? If not, why not? If yes, graph one to demonstrate. ***b***



Function Rules

The function machine takes an input, does something to this input according to some rule, and returns a unique output.



Given below are tables of input-output pairs for different function machines. Fill in the remaining table entries and describe each function rule in words.

a. Input values can be any English word.

Input	Cat	House	You				
Output	1	3	2				

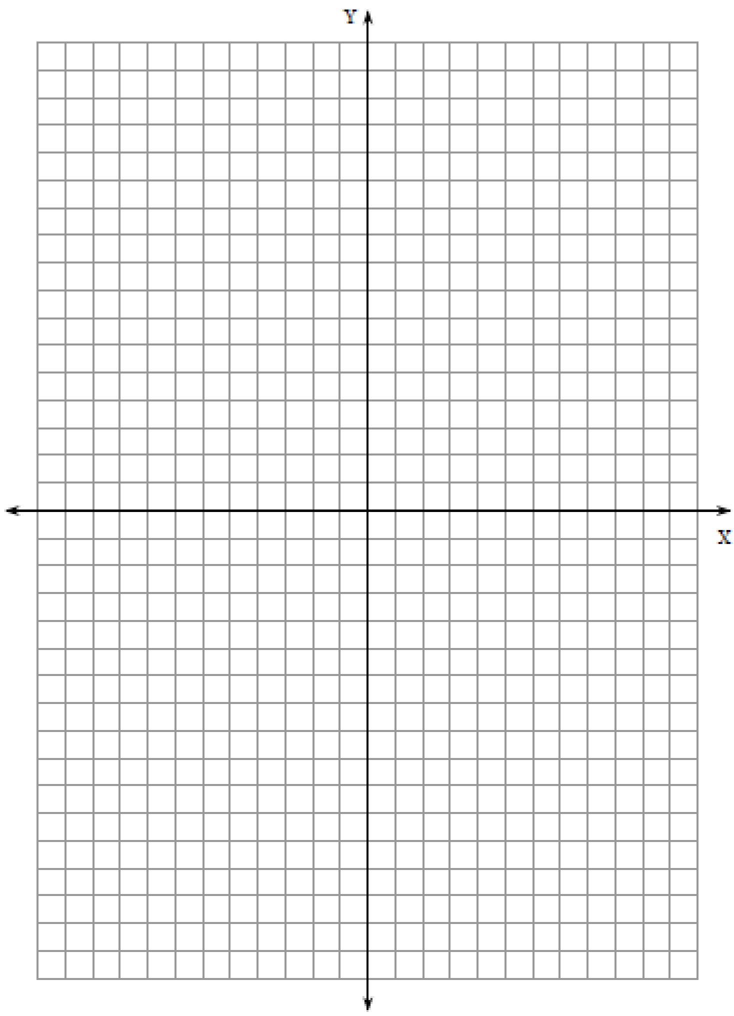
b. Input values can be any real number.

Input	2	5	-1.5				
Output	7	10	3.5				

c. Input values can be any whole number between 1 and 365.

Input	25	365	35	95	330		
Output	January	December	February	April	November		

Can any of the functions above be easily graphed on a coordinate plane? If not, why not? If yes, graph one to demonstrate.



CLOSING/SUMMARIZER

Each student should write a function with a specific pattern, and write the domain for the function.

Then, students will exchange papers with a study partner. The study partner should then find the range using the given domain. Students should provide feedback to each other by writing one thing that would make their responses more precise and accurate.

ADDITIONAL PRACTICE

For more practice with domain and range, http://www.ixl.com/math/algebra-2/domain-and-range

Equations of Attack

From NCTM Illuminations, http://illuminations.nctm.org/Lesson.aspx?id=2858

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class. Suggested time, 1 hour.

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR2. Students will compare and graph functions.

a. Calculate rates of change of functions, comparing when rates increase, decrease, or stay constant. *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (MGSE6.RP.2;MGSE7.RP.1,2,3; MGSE8.F.2,5; MGSE9-12.F.IF.6)

b. Graph by hand simple functions expressed symbolically (use all four quadrants). (MGSE9-12.F.IF.7)

c. Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line. (MGSE8.F.3)

e. Analyze graphs of functions for key features (intercepts, intervals of increase/decrease, maximums/minimums, symmetries, and end behavior) based on context. (MGSE9-12.F.IF.4,7)

MFAQR3. Students will construct and interpret functions.

a. Write a function that describes a relationship between two quantities. (MGSE8.F.4, MGSE9- 12.F.BF.1)

b. Use variables to represent two quantities in a real-world problem that change in relationship to one another (conceptual understanding of a variable). (MGSE6.EE.9)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students show perseverance in their efforts to apply their learning on a formative lesson.

3. Construct viable arguments and critique the reasoning of others. Students work collaboratively to explain their understanding of functions and defend their reasoning.

4. Model with mathematics. Students use the graph to model the rates of change, and tables to model mathematical thinking.

6. Attend to precision. Students use clear and precise language in their discussions and utilize patterns in identifying functions.

8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to define functions.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- plot and name points on a coordinate grid using correct coordinate pairs
- graph lines, given the slope and y-intercept
- write equations, given the slope and y-intercept
- determine algebraically if a point lies on a line

MATERIALS

- Copy of student page
  http://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/6-8/EquationsOfAttack-AS.pdf
- Slope cards
- http://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/6-8/EquationsOfAttack-AS-Cards.pdf
- Colored pencils or markers
- Coins or counters
- Scissors

ESSENTIAL QUESTIONS

- What are the advantages of finding equations graphically rather than algebraically?
- What are the advantages of finding equations algebraically rather than graphically?

TEACHER COMMENTARY

Students will plot points on a coordinate grid to represent ships before playing the graphing equations game with a partner. Points along the y-axis represent cannons and the slopes are chosen randomly to determine the line and equation of attacks. Students will use their math skills and strategy to sink their opponent's ships and win the game. After the game, an algebraic approach to the game is investigated.

Tell students that they will be playing a strategy game in which they must sink their opponent's ships. To win the game students will need to use their knowledge of graphing and linear equations.

Break the class up into pairs. Depending on the ability levels of your students, you may choose to allow them to pick their own partners or separate them into pre-determined pairs that are matched for mathematical ability. Distribute the Equations of Attack Activity Sheet, Slope Cards Activity Sheet, 2 different-colored pencils, a coin, and scissors to each pair of students. Note: If your class is just beginning to explore linear equations, you may wish to create your own set of slope cards with only integers (e.g., 2 and –3) and unit fractions (e.g., ¼, but not ¾). To challenge more advanced students, consider including decimal slopes (e.g., 1.5).

Analyzing Linear Functions (FAL)

Source: Georgia Mathematics Design Collaborative Formative Assessment Lessons
http://ccgpsmathematics6-8.wikispaces.com/Georgia+Mathematics+Design+Collaborative+Formative+Assessment+Lessons

SUGGESTED TIME FOR THIS LESSON

Exact timings will depend on the needs of your class. Recommended timing is 120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR2. Students will compare and graph functions.
c. Interpret the equation y = mx + b as defining a linear function whose graph is a straight line. (MGSE8.F.3)
f. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the great rate of change.* (MGSE8.F.2)

MFAQR3. Students will construct and interpret functions.
a. Write a function that describes a relationship between two quantities. (MGSE8.F.4, MGSE9-12.F.BF.1)
b. Use variables to represent two quantities in a real-world problem that change in relationship to one another (conceptual understanding of a variable). (MGSE6.EE.9)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students show perseverance in their efforts to apply their learning on a formative lesson.
2. Reason abstractly and quantitatively. Students will explain the meaning of slope and y-intercept.
3. Construct viable arguments and critique the reasoning of others. Students will work collaboratively with a partner.
4. Model with mathematics. Students will construct tables and graphs to match a real-world problem.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of the lesson, students will be able to understand

- slope as rate of change
- the meaning of x and y intercepts applied to real-world situations
- graphs and tables that represent realistic situation

MATERIALS

- Copies of formative assessment lesson pre-and post-assessment for each student

- Mini whiteboards and markers (optional)

ESSENTIAL QUESTIONS

- What can ordered pairs mean?
- What does slope mean in a problem situation?
- How can equations represent real-world scenarios?

TEACHER COMMENTARY

This formative assessment lesson will show if students truly understand the various aspects of linear equations, including slope, x-intercepts, y-intercepts, a chart of values, and graphing. It is very important they understand concepts about slope being a rate of change, how fast it changes, etc. They also need to understand and be able to communicate that the x-intercept is where the y value is zero, the y-intercept has an x that is zero, and relate linear descriptors to the context of the problem. These discussions will be critical and pivotal to their understanding of future graphs.

INTERVENTION

Prior to this lesson, students have been practicing rates of change and identifying characteristics of equations. If needed, more specific lessons on graphing equations can be found at http://crctlessons.com/algebra-lessons.html This site includes lessons, practice in the form of games, and mini-assessments.

Is It Cheaper to Pay Monthly or Annually?

Source: Robert Kaplinsky, Glenrock Consulting
http://robertkaplinsky.com/work/monthly-or-annually/

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Recommended timing is 120-150 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI3. Students will create algebraic models in two variables.

a. Create an algebraic model from a context using systems of two equations. (MGSE6.EE.6, MGSE8.EE.8, MGSE9-12.A.CED.2)
b. Find approximate solutions using technology to graph, construct tables of values and find successive approximations. (MGSE9-12.A.REI.10, 11)
c. Represent solutions to systems of equations graphically or by using a table of values. (MGSE6.EE.5;MGSE7.EE.3;MGSE8.EE.8, MGSE9-12.A.CED.2)
d. Analyze the reasonableness of the solutions of systems of equations within a given context. (MGSE6.EE.5, 6, MGSE7.EE.4)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students analyze the given information and make conjectures about the meaning of the solution and plan a means of determining a solution.
2. Reason abstractly and quantitatively. Students make sense of quantities and their relationships in context.
3. Construct viable arguments and critique the reasoning of others. Students justify their conclusions.
4. Model with mathematics. Students model the real-world problem with simultaneous equations.
5. Use appropriate tools strategically. Students use graphs and tables to help organize and provide a solution to the problem.
6. Attend to precision. Students communicate precisely and show their mathematical thinking.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Create a table of values to represent the relationship between two variables.
- Use a graph to determine a solution and discuss reasonableness of their answers.

MATERIALS

- Price charts and graphs from website
- Pictures of Disney or Universal characters

ESSENTIAL QUESTIONS

What does the solution to a system tell me about the answer to a problem situation?

Note: Tasks and lessons from Robert Kaplinsky's blog provide excellent resources that incorporate mathematical content, mathematical practices and real-world problem-solving. For more information on his strategies and philosophy, go to http://robertkaplinsky.com/

OPENER/ACTIVATOR

Present students with the following situation: Many theme parks like Disneyland and Universal Studios offer annual passes where customers can pay for a whole year in advance or pay a little in the beginning and make smaller payments each month for the rest of the year. Which payment options saves you the most money?

LESSON

Present students with "Questions to Ask" from website. Read through "Consider This" in preparation for responses. Information on Ticketing in the form of charts and graphs will then be presented to the class. The students may use the problem solving hand-out to make sense of the problem.

CLOSING/SUMMARIZER

Have students present their solutions to each other. Then, present unlabeled samples of student work from website and have students critique the sample student responses using the class-developed rubric.

ADDITIONAL PRACTICE

Assign one of the tasks found on the Illustrative Mathematics website using the following link: https://www.illustrativemathematics.org/HSA-REI (Scroll down to the tasks for standard HSA-REI.C.6 and read through the commentary for each task before deciding which one to assign.)

Student Handout: Understanding the Problem

What problem are you trying to figure out?
What do you already know about the problem?
What do you need to know to solve the problem?

Solution (include rationale, strategies, diagrams, supporting evidence)
What is your conclusion?

WEB LINKS

The following websites are correlated to the designated 8th grade standards.

Unit 1

MGSE8.G.1 Verify experimentally the properties of rotations, reflections, and translations: lines are taken to lines and line segments to line segments of the same length; angles are taken to angles of the same measure; parallel lines are taken to parallel lines.
https://www.illustrativemathematics.org/content-standards/8/G/A/1
http://nzmaths.co.nz/transformation-units-work

MGSE8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
https://www.illustrativemathematics.org/content-standards/8/G/A/2
http://nzmaths.co.nz/transformation-units-work

MGSE8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
https://www.illustrativemathematics.org/content-standards/8/G/A/3
http://nzmaths.co.nz/transformation-units-work

MGSE8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
https://www.illustrativemathematics.org/content-standards/8/G/A/4
http://nzmaths.co.nz/transformation-units-work

MGSE8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the three angles appear to form a line, and give an argument in terms of transversals why this is so.*
https://www.illustrativemathematics.org/content-standards/8/G/A/5
http://nzmaths.co.nz/resource/angles-parallel-lines-and-polygons

Unit 2

Work with radicals and integer exponents.

MGSE8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{(-5)} = 3^{(-3)} = \frac{1}{3^3} = \frac{1}{27}$.

https://www.illustrativemathematics.org/content-standards/8/EE/A/1/tasks/823
https://www.illustrativemathematics.org/content-standards/8/EE/A/1/tasks/395
https://www.illustrativemathematics.org/content-standards/8/EE/A/1/tasks/1438

MGSE8.EE.3 Use numbers expressed in scientific notation to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*

https://www.illustrativemathematics.org/content-standards/8/EE/A/3/tasks/476
https://www.illustrativemathematics.org/content-standards/8/EE/A/3/tasks/1593

MGSE8.EE.4 Add, subtract, multiply and divide numbers expressed in scientific notation, including problems where both decimal and scientific notations are used. Understand scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g. use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology (e.g. calculators).

https://www.illustrativemathematics.org/content-standards/8/EE/A/4/tasks/1981
https://www.illustrativemathematics.org/content-standards/8/EE/A/4/tasks/113

MGSE8.EE.7 Solve linear equations in one variable.

https://www.illustrativemathematics.org/content-standards/8/EE/C/7/tasks/583
https://www.illustrativemathematics.org/content-standards/8/EE/C/7/tasks/392
https://www.illustrativemathematics.org/content-standards/8/EE/C/7/tasks/999
https://www.illustrativemathematics.org/content-standards/8/EE/C/7/tasks/550

MGSE8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
https://www.illustrativemathematics.org/content-standards/8/NS/A/tasks/2066
https://www.illustrativemathematics.org/content-standards/8/NS/A/tasks/766
https://www.illustrativemathematics.org/content-standards/8/NS/A/tasks/338
https://www.illustrativemathematics.org/content-standards/8/NS/A/tasks/764
https://www.illustrativemathematics.org/content-standards/8/NS/A/1/tasks/1541
https://www.illustrativemathematics.org/content-standards/8/NS/A/1/tasks/335
https://www.illustrativemathematics.org/content-standards/8/NS/A/1/tasks/334

MGSE8.NS.2 Use rational approximation of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line, and estimate the value of expressions (e.g., estimate π^2*to the nearest tenth). For example, by truncating the decimal expansion of* $\sqrt{2}$ *(square root of 2), show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

https://www.illustrativemathematics.org/content-standards/8/NS/A/2/tasks/336
https://www.illustrativemathematics.org/content-standards/8/NS/A/2/tasks/337
https://www.illustrativemathematics.org/content-standards/8/NS/A/2/tasks/1221

Unit 3

MGSE8.G.6 Explain a proof of the Pythagorean Theorem and its converse.
http://illuminations.nctm.org/Search.aspx?view=search&kw=Pythagorean%20theorem&gr=6-8
http://nrich.maths.org/982

MGSE8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

https://www.illustrativemathematics.org/content-standards/8/G/B/7
http://illuminations.nctm.org/Search.aspx?view=search&kw=Pythagorean%20theorem&gr=6-8

MGSE8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

https://www.illustrativemathematics.org/content-standards/8/G/B/8
http://illuminations.nctm.org/Search.aspx?view=search&kw=Pythagorean%20theorem&gr=6-8

MGSE8.G.9 Apply the formulas for the volume of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

https://www.illustrativemathematics.org/content-standards/8/G/C/9
http://nrich.maths.org/1408

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

http://nrich.maths.org/2034

Unit 4

MGSE8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
https://www.illustrativemathematics.org/content-standards/8/F/A/1
http://www.visualpatterns.org/
http://illuminations.nctm.org/Unit.aspx?id=6526

MGSE8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

https://www.illustrativemathematics.org/content-standards/8/F/A/2
http://illuminations.nctm.org/Unit.aspx?id=6526
http://www.visualpatterns.org/

Unit 5
Understand the connections between proportional relationships, lines, and linear equations.

MGSE8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

https://www.illustrativemathematics.org/content-standards/8/EE/B/5
https://www.desmos.com/calculator

MGSE8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

https://www.illustrativemathematics.org/content-standards/8/EE/B/6/tasks/1537
https://www.desmos.com/calculator

Define, evaluate, and compare functions.

MGSE8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

https://www.illustrativemathematics.org/content-standards/8/F/A/3/tasks/813
http://reasonandwonder.com/match-my-graph-linear-functions/ - for practice graphing linear functions

Unit 6
Use functions to model relationships between quantities.

MGSE8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

http://nzmaths.co.nz/resource/balancing-acts
https://teacher.desmos.com/activities

https://www.illustrativemathematics.org/content-standards/8/F/B/4
http://graphingstories.com/

MGSE8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

https://www.illustrativemathematics.org/content-standards/8/F/B/5
http://graphingstories.com/

Investigate patterns of association in bivariate data.

MGSE8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

https://www.illustrativemathematics.org/content-standards/8/SP/A/1
http://nzmaths.co.nz/resource/stork-delivery
http://nzmaths.co.nz/resource/suspect-foot
http://nzmaths.co.nz/resource/data-squares-level-3

MGSE8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

https://www.illustrativemathematics.org/content-standards/8/SP/A/2
http://nzmaths.co.nz/resource/stork-delivery
http://nzmaths.co.nz/resource/suspect-foot

MGSE8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr. as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

https://www.illustrativemathematics.org/content-standards/8/SP/A/3
http://nzmaths.co.nz/resource/rates-change

MGSE8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

a. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.
b. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

https://www.illustrativemathematics.org/content-standards/8/SP/A/4
http://nzmaths.co.nz/resource/data-squares-level-3

Unit 7
MGSE8.EE.8 Analyze and solve pairs of simultaneous linear equations (systems of linear equations).

- **a.** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

https://www.desmos.com/calculator
https://www.illustrativemathematics.org/content-standards/8/EE/C/8

- **b.** Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*

https://www.illustrativemathematics.org/content-standards/8/EE/C/8
http://nzmaths.co.nz/resource/weighing-time
http://nzmaths.co.nz/resource/pigs-goats-and-sheep

- **c.** Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

https://www.desmos.com/calculator
https://www.illustrativemathematics.org/content-standards/8/EE/C/8
http://nrich.maths.org/5438
http://nzmaths.co.nz/resource/pigs-goats-and-sheep

ADDITIONAL RESOURCES

- Georgiastandards.org provides a gateway to a wealth of instruction links and information. Open the GSE Mathematics to access specific GSE resources for this course.

- Georgia Virtual School content available on the Shared Resources Website is available for anyone to view. Courses are divided into modules and are aligned with the Georgia Standards of Excellence.

- Course/Grade Level WIKI spaces are available to post questions about a unit, a standard, the course, or any other GSE mathematics related concern. Shared resources and information are also available at the site.

- From the National Council of Teachers of Mathematics, Illuminations: Height of Students in our Class. This lesson has students creating box-and-whisker plots with an extension of finding measures of center and creating a stem-and-leaf plot.

- National Library of Virtual Manipulatives. Students can use the appropriate applet from this page of virtual manipulatives to create graphical displays of the data set. This provides an important visual display of the data without the tediousness of the student hand drawing the display.

- Bright, G., Brewer, W., McClain, K., and Mooney, E. (2001). ***Navigating Through Data Analysis in Grades 6-8*****.** Reston, VA: NCTM.

- ***Statistics and Probability (Grades 6-9)***. Activities that Integrate Math and Science (AIMS Foundation).

- Number Talks for Middle School: **http://mathematicssolutions.com/wp-content/uploads/14_nctm_kcohen_numbertalks.pdf**

- For more practice matching graphs with scenarios, go to http://www.nationalstemcentre.org.uk/elibrary/resource/6658/matching-graphs-and-scenarios