

# *Georgia Performance Standards in Mathematics*

## *Algebra I*

### *Mathematics Georgia Performance Standards*

#### **K-12 Mathematics Introduction**

Georgia Mathematics focuses on actively engaging the student in the development of mathematical understanding by working independently and cooperatively to solve problems, estimating and computing efficiently, using appropriate tools, concrete models, and a variety of representations, and conducting investigations and recording findings. There is a shift toward applying mathematical concepts and skills in the context of authentic problems and student understanding of concepts rather than merely following a sequence of procedures. In mathematics classrooms, students will learn to think critically in a mathematical way with an understanding that there are many different solution pathways and sometimes more than one right answer in applied mathematics. Mathematics is the economy of information. The central idea of all mathematics is to discover how knowing some things leads, via reasoning, to knowing more—without having to commit the information to memory as a separate fact. It is the reasoned, logical connections that make mathematics manageable. The implementation of Georgia Performance Standards in Mathematics places the expected emphasis on sense-making, problem solving, reasoning, modeling, representation, connections, and communication.

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#### ***Algebra I***

**Algebra I** is the first course in a sequence of three required high school courses designed to ensure career and college readiness. The course represents a discrete study of algebra with correlated statistics applications.

The standards in the three-course high school sequence specify the mathematics that all students should study in order to be college and career ready. Additional mathematics content is provided in fourth credit courses and advanced courses including pre-calculus, calculus, advanced statistics, discrete mathematics, and mathematics of finance courses. High school course content standards are listed by conceptual categories including Number and Quantity, Algebra, Functions, Geometry, and Statistics and Probability. Conceptual categories portray a coherent view of high school mathematics content; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus. Standards for Mathematical Practice provide the foundation for instruction and assessment.

#### **Mathematics | Standards for Mathematical Practice**

*Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction.*

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of

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problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

### **1 Make sense of problems and persevere in solving them.**

High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

### **2 Reason abstractly and quantitatively.**

High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

### **3 Construct viable arguments and critique the reasoning of others.**

High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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#### **4 Model with mathematics.**

High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

#### **5 Use appropriate tools strategically.**

High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

**6 Attend to precision.** High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

**7 Look for and make use of structure.** By high school, students look closely to discern a pattern or structure. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ . High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

#### **8 Look for and express regularity in repeated reasoning.**

High school students notice if calculations are repeated, and look both for general methods and for shortcuts.

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Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

#### Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. **Students who do not have an understanding of a topic may rely on procedures too heavily.** Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. **In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.**

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

# Georgia Performance Standards in Mathematics

## Algebra I

### Mathematics | High School—Number and Quantity

**Numbers and Number Systems** During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3... Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers. With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings. Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that  $(5^{1/3})^3$  should be  $5^{(1/3)^3} = 5^1 = 5$  and that  $5^{1/3}$  should be the cube root of 5. Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

**Quantities** In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

### The Real Number System

N.RN

#### Use properties of rational and irrational numbers.

**MCC9-12.N.RN.3 Explain why the sum or product of rational numbers is rational; why the sum of a rational number and an irrational number is irrational; and why the product of a nonzero rational number and an irrational number is irrational.**

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## Algebra I

### Quantities

N.Q

#### Reason quantitatively and use units to solve problems.

**MCC9-12.N.Q.1** Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

- Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
- Convert units and rates using dimensional analysis (English-to-English and Metric-to-Metric without conversion factor provided and between English and Metric with conversion factor);
- Use units within multi-step problems and formulas; interpret units of input and resulting units of output.

**MCC9-12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling. Given a situation, context, or problem, students will determine, identify, and use appropriate quantities for representing the situation.

**MCC9-12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers' precision is limited to the precision of the data given.*

### Mathematics | High School—Algebra

#### Expressions

An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example,  $p + 0.05p$  can be interpreted as the addition of a 5% tax to a price  $p$ . Rewriting  $p + 0.05p$  as  $1.05p$  shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example,  $p + 0.05p$  is the sum of the simpler expressions  $p$  and  $0.05p$ . Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

#### Equations and Inequalities

An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

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The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of  $x + 1 = 0$  is an integer, not a whole number; the solution of  $2x + 1 = 0$  is a rational number, not an integer; the solutions of  $x^2 - 2 = 0$  are real numbers, not rational numbers; and the solutions of  $x^2 + 2 = 0$  are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid,  $A = ((b_1 + b_2)/2)h$ , can be solved for  $h$  using the same deductive process. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

#### Connections to Functions and Modeling

Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

#### Seeing Structure in Expressions

A.SSE

##### Interpret the structure of expressions

**MCC9-12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.**

**MCC9-12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients, in context.**

**MCC9-12.A.SSE.1b Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.**

**MCC9-12.A.SSE.2 Use the structure of an expression to rewrite it in different equivalent forms.**

**For example, see  $x^4 - y^4$  as  $(x^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .**

##### Write expressions in equivalent forms to solve problems

**MCC9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.**

**MCC9-12.A.SSE.3a Factor any quadratic expression to reveal the zeros of the function defined by the expression.**

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**MCC9-12.A.SSE.3b** Complete the square in a quadratic expression to reveal the maximum and minimum value of the function defined by the expression.

### Arithmetic with Polynomials and Rational Expressions

A.APR

#### Perform arithmetic operations on polynomials

**MCC9-12.A.APR.1** Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations.

### Creating Equations

A.CED

#### Create equations that describe numbers or relationships

**MCC9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, and exponential functions (integer inputs only).

**MCC9-12.A.CED.2** Create linear and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which  $A = P(1 + r/n)^t$  has multiple variables.)

**MCC9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

**MCC9-12.A.CED.4** Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. *Examples: Rearrange Ohm’s law  $V = IR$  to highlight resistance  $R$ ; Rearrange area of a circle formula  $A = \pi r^2$  to highlight the radius  $r$ .*

### Reasoning with Equations and Inequalities

A.REI

#### Understand solving equations as a process of reasoning and explain the reasoning

**MCC9-12.A.REI.1** Using algebraic properties and the properties of real numbers, justify the steps of a simple, one-solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties.

#### Solve equations and inequalities in one variable

**MCC9-12.A.REI.3** Solve linear equations and inequalities in one variable including equations with coefficients represented by letters. *For example, given  $ax + 3 = 7$ , solve for  $x$ .*

**MCC9-12.A.REI.4** Solve quadratic equations in one variable.

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**MCC9-12.A.REI.4a** Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from  $ax^2 + bx + c = 0$ .

**MCC9-12.A.REI.4b** Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation.

### Solve systems of equations

**MCC9-12.A.REI.5** Show and explain why the elimination method works to solve a system of two-variable equations.

**MCC9-12.A.REI.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

### Represent and solve equations and inequalities graphically

**MCC9-12.A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane

**MCC9-12.A.REI.11** Using graphs, tables, or successive approximations, show that the solution to the equation  $f(x) = g(x)$  is the  $x$ -value where the  $y$ -values of  $f(x)$  and  $g(x)$  are the same.

**MCC9-12.A.REI.12** Graph the solution set to a linear inequality in two variables.

## Mathematics | High School—Functions

Functions describe situations where one quantity determines another. For example, the return on \$10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour,  $v$ ; the rule  $T(v) = 100/v$  expresses this relationship algebraically and defines a function whose name is  $T$ .

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like  $f(x) = a + bx$ ; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties. Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

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A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

### Connections to Expressions, Equations, Modeling, and Coordinates

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

### Interpreting Functions

F.IF

#### Understand the concept of a function and use function notation

**MCC9-12.F.IF.1** Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If  $f$  is a function,  $x$  is the input (an element of the domain), and  $f(x)$  is the output (an element of the range). Graphically, the graph is  $y = f(x)$ .

**MCC9-12.F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

**MCC9-12.F.IF.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. (Generally, the scope of high school math defines this subset as the set of natural numbers 1,2,3,4...) By graphing or calculating terms, students should be able to show how the recursive sequence  $a_1=7$ ,  $a_n=a_{n-1}+2$ ; the sequence  $s_n = 2(n-1) + 7$ ; and the function  $f(x) = 2x + 5$  (when  $x$  is a natural number) all define the same sequence.

#### Interpret functions that arise in applications in terms of the context

**MCC9-12.F.IF.4** Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.

**MCC9-12.F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function.*

**MCC9-12.F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

#### Analyze functions using different representations

**MCC9-12.F.IF.7** Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

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MCC9-12.F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima (as determined by the function or by context).

MCC9-12.F.IF.7e Graph exponential functions, showing intercepts and end behavior.

MCC9-12.F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

MCC9-12.F.IF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. *For example, compare and contrast quadratic functions in standard, vertex, and intercept forms.*

MCC9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.*

### Building Functions

F-BF

#### Build a function that models a relationship between two quantities

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities.

MCC9-12.F.BF.1a Determine an explicit expression and the recursive process (steps for calculation) from context. *For example, if Jimmy starts out with \$15 and earns \$2 a day, the explicit expression “ $2x+15$ ” can be described recursively (either in writing or verbally) as “to find out how much money Jimmy will have tomorrow, you add \$2 to his total today.”  $J_n = J_{n-1} + 2, J_0 = 15$*

#### Build new functions from existing functions

MCC9-12.F.BF.3 Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

### Linear, Quadratic, and Exponential Models

F.LE

#### Construct and compare linear, quadratic, and exponential models and solve problems

MCC9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

MCC9-12.F.LE.1a Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. (This can be shown by algebraic proof, with a table showing differences, or by calculating average rates of change over equal intervals).

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**MCC9-12.F.LE.1b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.**

**MCC9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.**

**MCC9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).**

**MCC9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.**

#### **Interpret expressions for functions in terms of the situation they model**

**MCC9-12.F.LE.5 Interpret the parameters in a linear ( $f(x) = mx + b$ ) and exponential ( $f(x) = a \cdot d^x$ ) function in terms of context. (In the functions above, “m” and “b” are the parameters of the linear function, and “a” and “d” are the parameters of the exponential function.) In context, students should describe what these parameters mean in terms of change and starting value.**

### **Mathematics | High School—Statistics and Probability**

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account. Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities

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relies on an understanding of independence and conditional probability, which can be approached through the analysis of two way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

### Connections to Functions and Modeling

Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

### Interpreting Categorical and Quantitative Data

S.ID

#### Summarize, represent, and interpret data on a single count or measurement variable

**MCC9-12.S.ID.1** Represent data with plots on the real number line (dot plots, histograms, and box plots).

**MCC9-12.S.ID.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, mean absolute deviation) of two or more different data sets.

**MCC9-12.S.ID.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

#### Summarize, represent, and interpret data on two categorical and quantitative variables

**MCC9-12.S.ID.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

**MCC9-12.S.ID.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

**MCC9-12.S.ID.6a** Decide which type of function is most appropriate by observing graphed data, charted data, or by analysis of context to generate a viable (rough) function of best fit. Use this function to solve problems in context. Emphasize linear, quadratic and exponential models.

**MCC9-12.S.ID.6c** Using given or collected bivariate data, fit a linear function for a scatter plot that suggests a linear association.

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**Interpret linear models**

**MCC9-12.S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.**

**MCC9-12.S.ID.8 Compute (using technology) and interpret the correlation coefficient “ $r$ ” of a linear fit. (For instance, by looking at a scatterplot, students should be able to tell if the correlation coefficient is positive or negative and give a reasonable estimate of the “ $r$ ” value.) After calculating the line of best fit using technology, students should be able to describe how strong the goodness of fit of the regression is, using “ $r$ ”.**

**MCC9-12.S.ID.9 Distinguish between correlation and causation.**

DRAFT