

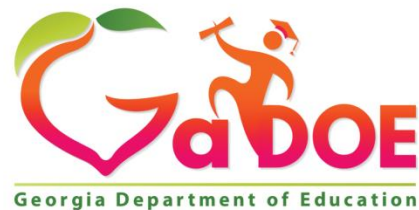


Georgia Standards of Excellence

Mathematics

Standards

Kindergarten – Fifth Grade



Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

K-12 Mathematics Introduction

The Georgia Mathematics Curriculum focuses on actively engaging the students in the development of mathematical understanding by using manipulatives and a variety of representations, working independently and cooperatively to solve problems, estimating and computing efficiently, and conducting investigations and recording findings. There is a shift towards applying mathematical concepts and skills in the context of authentic problems and for the student to understand concepts rather than merely follow a sequence of procedures. In mathematics classrooms, students will learn to think critically in a mathematical way with an understanding that there are many different ways to a solution and sometimes more than one right answer in applied mathematics. Mathematics is the economy of information. The central idea of all mathematics is to discover how knowing some things well, via reasoning, permit students to know much else—without having to commit the information to memory as a separate fact. It is the connections, the reasoned, logical connections that make mathematics manageable. As a result, implementation of Georgia Standards of Excellence places a greater emphasis on problem solving, reasoning, representation, connections, and communication.

Toward greater focus and coherence

Mathematics experiences in early childhood settings should concentrate on (1) number (which includes whole number, operations, and relations) and (2) geometry, spatial relations, and measurement, with more mathematics learning time devoted to number than to other topics. Mathematical process goals should be integrated in these content areas.

—Mathematics Learning in Early Childhood, National Research Council, 2009

The composite standards [of Hong Kong, Korea and Singapore] have a number of features that can inform an international benchmarking process for the development of K–6 mathematics standards in the U.S. First, the composite standards concentrate the early learning of mathematics on the number, measurement, and geometry strands with less emphasis on data analysis and little exposure to algebra. The Hong Kong standards for grades 1–3 devote approximately half the targeted time to numbers and almost all the time remaining to geometry and measurement.

— Ginsburg, Leinwand and Decker, 2009

Because the mathematics concepts in [U.S.] textbooks are often weak, the presentation becomes more mechanical than is ideal. We looked at both traditional and non-traditional textbooks used in the US and found this conceptual weakness in both.

— Ginsburg et al., 2005

There are many ways to organize curricula. The challenge, now rarely met, is to avoid those that distort mathematics and turn off students.

— Steen, 2007

For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is “a mile wide and an inch deep.” These Standards are a substantial answer to that challenge.

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It is important to recognize that “fewer standards” are no substitute for focused standards. Achieving “fewer standards” would be easy to do by resorting to broad, general statements. Instead, these Standards aim for clarity and specificity. Assessing the coherence of a set of standards is more difficult than assessing their focus.

William Schmidt and Richard Houang (2002) have said that content standards and curricula are coherent if they are: *articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives. That is, what and how students are taught should reflect not only the topics that fall within a certain academic discipline, but also the key ideas that determine how knowledge is organized and generated within that discipline. This implies that to be coherent, a set of content standards must evolve from particulars (e.g., the meaning and operations of whole numbers, including simple math facts and routine computational procedures associated with whole numbers and fractions) to deeper structures inherent in the discipline. These deeper structures then serve as a means for connecting the particulars (such as an understanding of the rational number system and its properties).* (emphasis added)

These Standards endeavor to follow such a design, not only by stressing conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the properties of operations to structure those ideas. In addition, the “sequence of topics and performances” that is outlined in a body of mathematics standards must also respect what is known about how students learn. As Confrey (2007) points out, developing “sequenced obstacles and challenges for students...absent the insights about meaning that derive from careful study of learning, would be unfortunate and unwise.” In recognition of this, the development of these Standards began with research-based learning progressions detailing what is known today about how students’ mathematical knowledge, skill, and understanding develop over time.

Understanding mathematics

These Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student’s mathematical maturity, *why* a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with

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disabilities, reading should allow for use of Braille, screen reader technology, or other assistive devices, while writing should include the use of a scribe, computer, or speech-to-text technology. In a similar vein, speaking and listening should be interpreted broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of college and career readiness for all students.

How to read the grade level standards

Standards define what students should understand and be able to do. **Clusters** are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject. **Domains** are larger groups of related standards. Standards from different domains may sometimes be closely related.

Number and Operations in Base Ten 3.NBT (DOMAIN)

Use place value understanding and properties of operations to perform multi-digit arithmetic.

1. Use place value understanding to round whole numbers to the nearest 10 or 100. **(STANDARD)**
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction) **(STANDARD)**
3. Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9×80 , 5×60) using strategies based on place value and properties of operations. . **(STANDARD)**

(Cluster)

These Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B. What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, “Students who already know ... should next come to learn” But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. Of necessity therefore, grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. One promise of common state standards is that over time they will allow research on learning progressions to inform and improve the design of standards to a much greater extent than is possible today. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding. These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible

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arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

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7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Mathematics | Kindergarten

In Kindergarten, instructional time should focus on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

Content standards for Kindergarten are arranged within the following domains and clusters:

Counting and Cardinality

- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.

Operations and Algebraic Thinking

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten

- Work with numbers 11–19 to gain foundations for place value.

Measurement and Data

- Describe and compare measurable attributes.
- Classify objects and count the number of objects in categories.

Geometry

- Identify and describe shapes.
- Analyze, compare, create, and compose shapes.

Standards for Mathematical Practice

Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

Students are expected to:

1. Make sense of problems and persevere in solving them.

In Kindergarten, students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” or they may try another strategy.

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2. Reason abstractly and quantitatively.

Younger students begin to recognize that a number represents a specific quantity. Then, they connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities.

3. Construct viable arguments and critique the reasoning of others.

Younger students construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They also begin to develop their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking.

4. Model with mathematics.

In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.

5. Use appropriate tools strategically.

Younger students begin to consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, kindergarteners may decide that it might be advantageous to use linking cubes to represent two quantities and then compare the two representations side-by-side.

6. Attend to precision.

As kindergarteners begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning.

7. Look for and make use of structure.

Younger students begin to discern a pattern or structure. For instance, students recognize the pattern that exists in the teen numbers; every teen number is written with a 1 (representing one ten) and ends with the digit that is first stated. They also recognize that $3 + 2 = 5$ and $2 + 3 = 5$.

8. Look for and express regularity in repeated reasoning.

In the early grades, students notice repetitive actions in counting and computation, etc. For example, they may notice that the next number in a counting sequence is one more. When counting by tens, the next number in the sequence is “ten more” (or one more group of ten). In addition, students continually check their work by asking themselves, “Does this make sense?”

Counting and Cardinality

K.CC

Know number names and the count sequence.

MGSEK.CC.1 Count to 100 by ones and by tens.

MGSEK.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

MGSEK.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

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Count to tell the number of objects.

- MGSEK.CC.4** Understand the relationship between numbers and quantities; connect counting to cardinality.
- When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. (one-to-one correspondence)
 - Understand that the last number name said tells the number of objects counted (cardinality). The number of objects is the same regardless of their arrangement or the order in which they were counted.
 - Understand that each successive number name refers to a quantity that is one larger.
- MGSEK.CC.5** Count to answer “how many?” questions.
- Count to answer “how many?” questions about as many as 20 things arranged in a variety of ways (a line, a rectangular array, or a circle), or as many as 10 things in a scattered configuration.
 - Given a number from 1-20, count out that many objects.
 - Identify and be able to count pennies within 20. (Use pennies as manipulatives in multiple mathematical contexts.)

Compare numbers.

MGSEK.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.¹

MGSEK.CC.7 Compare two numbers between 1 and 10 presented as written numerals.

Operations and Algebraic Thinking

K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

MGSEK.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings², sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

MGSEK.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

MGSEK.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation. (drawings need not include an equation).

MGSEK.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

¹ Include groups with up to ten objects.

² Drawings need not show details, but should show the mathematics in the problem.

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MGSEK.OA.5 Fluently add and subtract within 5.

Number and Operations in Base Ten

K.NBT

Work with numbers 11–19 to gain foundations for place value.

MGSEK.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones to understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., $18 = 10 + 8$).

Measurement and Data

K.MD

Describe and compare measurable attributes.

MGSEK.MD.1 Describe several measurable attributes of an object, such as length or weight. *For example, a student may describe a shoe as, “This shoe is heavy! It is also really long!”*

MGSEK.MD.2 Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

Classify objects and count the number of objects in each category.

MGSEK.MD.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.³

Geometry

K.G

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

MGSEK.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.

MGSEK.G.2 Correctly name shapes regardless of their orientations or overall size.

MGSEK.G.3 Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).

³ Limit category counts to be less than or equal to 10.

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Analyze, compare, create, and compose shapes.

MGSEK.G.4 Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).

MGSEK.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.

MGSEK.G.6 Compose simple shapes to form larger shapes. *For example, “Can you join these two triangles with full sides touching to make a rectangle?”*

Mathematics | Grade 1

In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

Content standards for Grade 1 are arranged within the following domains and clusters:

Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Understand and apply properties of operations and the relationship between addition and subtraction.
- Add and subtract within 20.
- Work with addition and subtraction equations.

Number and Operations in Base Ten

- Extend the counting sequence.
- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data

- Measure lengths indirectly and by iterating length units.
- Tell and write time.
- Represent and interpret data.

Geometry

- Reason with shapes and their attributes.

Standards for Mathematical Practice

Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

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Students are expected to:

1. Make sense of problems and persevere in solving them.

In first grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They are willing to try other approaches.

2. Reason abstractly and quantitatively.

Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities.

3. Construct viable arguments and critique the reasoning of others.

First graders construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They also practice their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?” “Explain your thinking,” and “Why is that true?” They not only explain their own thinking, but listen to others’ explanations. They decide if the explanations make sense and ask questions.

4. Model with mathematics.

In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.

5. Use appropriate tools strategically.

In first grade, students begin to consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, first graders decide it might be best to use colored chips to model an addition problem.

6. Attend to precision.

As young children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning.

7. Look for and make use of structure.

First graders begin to discern a pattern or structure. For instance, if students recognize $12 + 3 = 15$, then they also know $3 + 12 = 15$. (*Commutative property of addition.*) To add $4 + 6 + 4$, *the first two numbers can be added to make a ten, so $4 + 6 + 4 = 10 + 4 = 14$.*

8. Look for and express regularity in repeated reasoning.

In the early grades, students notice repetitive actions in counting and computation, etc. When children have multiple opportunities to add and subtract “ten” and multiples of “ten” they notice the pattern and gain a better understanding of place value. Students continually check their work by asking themselves, “Does this make sense?”

Represent and solve problems involving addition and subtraction.

MGSE1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.⁴

MGSE1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Understand and apply properties of operations and the relationship between addition and subtraction.

MGSE1.OA.3 Apply properties of operations as strategies to add and subtract.⁵

Examples: If $8 + 3 = 11$ is known, then $3 + 8 = 11$ is also known. (Commutative property of addition.) To add $2 + 6 + 4$, the second two numbers can be added to make a ten, so $2 + 6 + 4 = 2 + 10 = 12$. (Associative property of addition.)

MGSE1.OA.4 Understand subtraction as an unknown-addend problem. For example, subtract $10 - 8$ by finding the number that makes 10 when added to 8.

Add and subtract within 20.

MGSE1.OA.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

MGSE1.OA.6 Add and subtract within 20.

- a. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).
- b. Fluently add and subtract within 10.

Work with addition and subtraction equations.

MGSE1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? $6 = 6$, $7 = 8 - 1$, $5 + 2 = 2 + 5$, $4 + 1 = 5 + 2$.*

⁴ See Glossary, Table 1

⁵ Students need not use formal terms for these properties. Problems should be within 20.

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MGSE1.OA.8 Determine the unknown whole number in an addition or subtraction equation relating to three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 + ? = 11$, $5 = \square - 3$, $6 + 6 = \Delta$.

Number and Operations in Base Ten

1.NBT

Extend the counting sequence.

MGSE1.NBT.1 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Understand place value.

MGSE1.NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

- a. 10 can be thought of as a bundle of ten ones — called a “ten.”
- b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
- c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

MGSE1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols $>$, $=$, and $<$.

Use place value understanding and properties of operations to add and subtract.

MGSE1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of ten (e.g., $24 + 9$, $13 + 10$, $27 + 40$), using concrete models or drawings and strategies based on place value, properties of operations, and/or relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

MGSE1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

MGSE1.NBT.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range of 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (e.g., $70 - 30$, $30 - 10$, $60 - 60$)

MGSE1.NBT.7 Identify dimes, and understand ten pennies can be thought of as a dime. (Use dimes as manipulatives in multiple mathematical contexts.)

Measurement and Data

1.MD

Measure lengths indirectly and by iterating length units.

MGSE1.MD.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object.

MGSE1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. (Iteration)

Tell and write time.

MGSE1.MD.3 Tell and write time in hours and half-hours using analog and digital clocks.

Represent and interpret data.

MGSE1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Geometry

1.G

Reason with shapes and their attributes.

MGSE1.G.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.

MGSE1.G.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.⁶ *This is important for the future development of spatial relations which later connects to developing understanding of area, volume, and fractions.*

MGSE1.G.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

⁶ Students do not need to learn formal names such as “right rectangular prism.”

Mathematics | Grade 2

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

Content standards for Grade 2 are arranged within the following domains and clusters:

Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten

- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data

- Measure and estimate lengths in standard units.
- Relate addition and subtraction to length.
- Work with time and money.
- Represent and interpret data.

Geometry

- Reason with shapes and their attributes.

Standards for Mathematical Practice

Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

Students are expected to:

1. Make sense of problems and persevere in solving them.

In second grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. They may use

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concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They make conjectures about the solution and plan out a problem-solving approach.

2. Reason abstractly and quantitatively.

Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. Second graders begin to know and use different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Second graders may construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They practice their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?”, “Explain your thinking,” and “Why is that true?” They not only explain their own thinking, but listen to others’ explanations. They decide if the explanations make sense and ask appropriate questions.

4. Model with mathematics.

In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.

5. Use appropriate tools strategically.

In second grade, students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be better suited. For instance, second graders may decide to solve a problem by drawing a picture rather than writing an equation.

6. Attend to precision.

As children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning.

7. Look for and make use of structure.

Second graders look for patterns. For instance, they adopt mental math strategies based on patterns (making ten, fact families, doubles).

8. Look for and express regularity in repeated reasoning.

Students notice repetitive actions in counting and computation, etc. When children have multiple opportunities to add and subtract, they look for shortcuts, such as rounding up and then adjusting the answer to compensate for the rounding. Students continually check their work by asking themselves, does this make sense?

Operations and Algebraic Thinking

2.OA

Represent and solve problems involving addition and subtraction.

MGSE2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems by using drawings and equations with a symbol for the unknown number to represent the problem. Problems include contexts that involve adding to, taking from, putting together/taking apart (part/part/whole) and comparing with unknowns in all positions.⁷

⁷ See Glossary, Table 1.

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Add and subtract within 20.

MGSE2.OA.2 Fluently add and subtract within 20 using mental strategies.⁸ By end of Grade 2, know from memory all sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.

MGSE2.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

MGSE2.OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Number and Operations in Base Ten

2.NBT

Understand place value.

MGSE2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

- a. 100 can be thought of as a bundle of ten tens — called a “hundred.”
- b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

MGSE2.NBT.2 Count within 1000; skip-count by 5s, 10s, and 100s.

MGSE2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

MGSE2.NBT.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using $>$, $=$, and $<$ symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

MGSE2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

MGSE2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.

MGSE2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method.

⁸ See standard 1.OA.6 for a list of mental strategies.

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MGSE2.NBT.8 Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

MGSE2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations.⁹

Measurement and Data

2.MD

Measure and estimate lengths in standard units.

MGSE2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

MGSE2.MD.2 Measure the length of an object twice, using length units of different measurements; describe how the two measurements relate to the size of the unit chosen. Understand the relative size of units in different systems of measurement. *For example, an inch is longer than a centimeter.* (Students are not expected to convert between systems of measurement.)

MGSE2.MD.3 Estimate lengths using units of inches, feet, centimeters, and meters.

MGSE2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Relate addition and subtraction to length.

MGSE2.MD.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

MGSE2.MD.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Work with time and money.

MGSE2.MD.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

MGSE2.MD.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

⁹ Explanations may be supported by drawings or objects.

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Represent and interpret data.

MGSE2.MD.9 Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

MGSE2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems¹⁰ using information presented in a bar graph.

Geometry

2.G

Reason with shapes and their attributes.

MGSE2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.¹¹ Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

MGSE2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

MGSE2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

¹⁰ See Glossary, Table 1.

¹¹ Sizes are compared directly or visually, not compared by measuring.

Mathematics | Grade 3

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

Content standards for Grade 3 are arranged within the following domains and clusters:

Operations and Algebraic Thinking

- Represent and solve problems involving multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations in Base Ten

- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions

- Develop understanding of fractions as numbers.

Measurement and Data

- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
- Represent and interpret data.
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Geometry

- Reason with shapes and their attributes.

Standards for Mathematical Practice

Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

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Students are expected to:

1. Make sense of problems and persevere in solving them.

In third grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.

2. Reason abstractly and quantitatively.

Third graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.

3. Construct viable arguments and critique the reasoning of others.

In third grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking.

4. Model with mathematics.

Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Third graders should evaluate their results in the context of the situation and reflect on whether the results make sense.

5. Use appropriate tools strategically.

Third graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles

6. Attend to precision.

As third graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.

7. Look for and make use of structure.

In third grade, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to multiply and divide (commutative and distributive properties).

8. Look for and express regularity in repeated reasoning.

Students in third grade should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don’t know. For example, if students are asked to find the product of 7×8 , they might decompose 7 into 5 and 2 and then multiply 5×8 and 2×8 to arrive at $40 + 16$ or 56. In addition, third graders continually evaluate their work by asking themselves, “Does this make sense?”

Represent and solve problems involving multiplication and division.

MGSE3.OA.1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5×7 .*

MGSE3.OA.2 Interpret whole number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares (How many in each group?), or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each (How many groups can you make?). *For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.*

MGSE3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities,[†] e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.¹² *See Glossary: *Multiplication and Division Within 100*.

MGSE3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers using the inverse relationship of multiplication and division. *For example, determine the unknown number that makes the equation true in each of the equations, $8 \times ? = 48$, $5 = \square \div 3$, $6 \times 6 = ?$.*

Understand properties of multiplication and the relationship between multiplication and division.

MGSE3.OA.5 Apply properties of operations as strategies to multiply and divide.¹³ *Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)*

MGSE3.OA.6 Understand division as an unknown-factor problem. *For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.*

Multiply and divide within 100.

MGSE3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

¹² See glossary, Table 2.

¹³ Students need not use formal terms for these properties.

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Solve problems involving the four operations, and identify and explain patterns in arithmetic.

MGSE3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.^{†14}

This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order where there are no parentheses to specify a particular order (Order of Operations).

[†] See Glossary, Table 2

MGSE3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.[‡] *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

[‡] See Glossary, Table 3

Number and Operations in Base Ten

3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.¹⁵

MGSE3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

MGSE3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

MGSE3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80 , 5×60) using strategies based on place value and properties of operations.

Number and Operations – Fractions¹⁶

3.NF

Develop understanding of fractions as numbers.

MGSE3.NF.1 Understand a fraction

$\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts (unit fraction);

understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$. For example, $\frac{3}{4}$ means there are three $\frac{1}{4}$ parts, so $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$.

MGSE3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

- Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$. Recognize that a unit fraction

¹⁴ This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order where there are no parentheses to specify a particular order (Order of Operations).

¹⁵ A range of algorithms will be used.

¹⁶ Grade 3 expectations in this domain are limited to fractions with denominators of 2, 3, 4, 6, and 8.

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- $\frac{1}{b}$ is located $\frac{1}{b}$ whole unit from 0 on the number line.
- b. Represent a non-unit fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths of $\frac{1}{b}$ (unit fractions) from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the non-unit fraction $\frac{a}{b}$ on the number line.

MGSE3.NF.3 Explain equivalence of fractions through reasoning with visual fraction models. Compare fractions by reasoning about their size.

- a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
- b. Recognize and generate simple equivalent fractions with denominators of 2, 3, 4, 6, and 8, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
- c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form $3 = \frac{6}{2}$ (3 wholes is equal to six halves); recognize that $\frac{3}{1} = 3$; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram.*
- d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

Measurement and Data

3.MD

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

MGSE3.MD.1 Tell and write time to the nearest minute and measure elapsed time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram, drawing a pictorial representation on a clock face, etc.

MGSE3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).¹⁷ Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.¹⁸

Represent and interpret data.

MGSE3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

¹⁷ Excludes compound units such as cm^3 and finding the geometric volume of a container.

¹⁸ Excludes multiplicative comparison problems (problems involving notions of “times as much”; see Glossary, Table 2).

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MGSE3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.

Geometric Measurement: understand concepts of area and relate area to multiplication and to addition.

MGSE3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

- A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
- A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

MGSE3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

MGSE3.MD.7 Relate area to the operations of multiplication and addition.

- Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
- Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
- Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

MGSE3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Geometry

3.G

Reason with shapes and their attributes.

MGSE3.G.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

MGSE3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as $1/4$ of the area of the shape.*

In Georgia resources and assessments, trapezoids are defined using the inclusive definition: At least one pair of parallel sides.

Mathematics | Grade 4

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry

Content standards for Grade 4 are arranged within the following domains and clusters:

Operations and Algebraic Thinking

- Use the four operations with whole numbers to solve problems.
- Gain familiarity with factors and multiples.
- Generate and analyze patterns.

Number and Operations in Base Ten

- Generalize place value understanding for multi-digit whole numbers.
- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions

- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- Understand decimal notation for fractions, and compare decimal fractions.

Measurement and Data

- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
- Represent and interpret data.
- Geometric measurement: understand concepts of angle and measure angles.

Geometry

- Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Standards for Mathematical Practice

Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately),

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and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

Students are expected to:

1. Make sense of problems and persevere in solving them.

In fourth grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.

2. Reason abstractly and quantitatively.

Fourth graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.

3. Construct viable arguments and critique the reasoning of others.

In fourth grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.

4. Model with mathematics.

Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense.

5. Use appropriate tools strategically.

Fourth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.

6. Attend to precision.

As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.

7. Look for and make use of structure.

In fourth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principle of counting. They generate number or shape patterns that follow a given rule.

8. Look for and express regularity in repeated reasoning.

Students in fourth grade should notice repetitive actions in computation to make generalizations. Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

Use the four operations with whole numbers to solve problems.

MGSE4.OA.1 Understand that a multiplicative comparison is a situation in which one quantity is multiplied by a specified number to get another quantity.

- Interpret a multiplication equation as a comparison e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.
- Represent verbal statements of multiplicative comparisons as multiplication equations.

MGSE4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison. Use drawings and equations with a symbol or letter for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.¹⁹

MGSE4.OA.3 Solve multistep word problems with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a symbol or letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

MGSE4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Generate and analyze patterns.

MGSE4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. Explain informally why the pattern will continue to develop in this way. *For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers.*

Generalize place value understanding for multi-digit whole numbers.

MGSE4.NBT.1 Recognize that in a multi-digit whole number, a digit in any one place represents ten times what it represents in the place to its right. *For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.*

¹⁹ See Glossary, Table 2.

²⁰ Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

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MGSE4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.

MGSE4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

MGSE4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

MGSE4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

MGSE4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Number and Operations – Fractions²¹

4.NF

Extend understanding of fraction equivalence and ordering.

MGSE4.NF.1 Explain why two or more fractions are equivalent $\frac{a}{b} = \frac{n \times a}{n \times b}$ ex: $\frac{1}{4} = \frac{3 \times 1}{3 \times 4}$ by using visual fraction models. Focus attention on how the number and size of the parts differ even though the fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

MGSE4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by using visual fraction models, by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

MGSE4.NF.3 Understand a fraction $\frac{a}{b}$ with a numerator >1 as a sum of unit fractions $\frac{1}{b}$.

- Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2 \frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$.

²¹ Grade 4 expectations in this domain are limited to fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100.

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- c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
- d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

MGSE4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number e.g., by using a visual such as a number line or area model.

- a. Understand a fraction a/b as a multiple of $1/b$. For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.
- b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.)
- c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat $3/8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Understand decimal notation for fractions, and compare decimal fractions.

MGSE4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.²² For example, express $3/10$ as $30/100$, and add $3/10 + 4/100 = 34/100$.

MGSE4.NF.6 Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as $62/100$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

MGSE4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model.

Measurement and Data

4.MD

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

MGSE4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec.

- a. Understand the relationship between gallons, cups, quarts, and pints.
- b. Express larger units in terms of smaller units within the same measurement system.
- c. Record measurement equivalents in a two column table.

²² Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But, addition and subtraction with unlike denominators in general is not a requirement at this grade.

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MGSE4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

MGSE4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

Represent and interpret data.

MGSE4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$). Solve problems involving addition and subtraction of fractions with common denominators by using information presented in line plots. *For example, from a line plot, find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Geometric Measurement: understand concepts of angle and measure angles.

MGSE4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

- a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a “one-degree angle,” and can be used to measure angles.
- b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

MGSE4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

MGSE4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol or letter for the unknown angle measure.

MGSE4.MD.8 Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Geometry

4.G

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

MGSE4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

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MGSE4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

MGSE4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

In Georgia resources and assessments, trapezoids are defined using the inclusive definition: At least one pair of parallel sides.

Mathematics | Grade 5

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

Content standards for Grade 5 are arranged within the following domains and clusters:

Operations and Algebraic Thinking

- Write and interpret numerical expressions.
- Analyze patterns and relationships.

Number and Operations in Base Ten

- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations—Fractions

- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data

- Convert like measurement units within a given measurement system.
- Represent and interpret data.
- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Geometry

- Graph points on the coordinate plane to solve real-world and mathematical problems.
- Classify two-dimensional figures into categories based on their properties.

Standards for Mathematical Practice

Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

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Students are expected to:

1. Make sense of problems and persevere in solving them.

Students solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”.

2. Reason abstractly and quantitatively.

Fifth graders should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.

3. Construct viable arguments and critique the reasoning of others.

In fifth grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking.

4. Model with mathematics.

Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fifth graders should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.

5. Use appropriate tools strategically.

Fifth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real world data.

6. Attend to precision.

Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units.

7. Look for and make use of structure.

In fifth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation.

8. Look for and express regularity in repeated reasoning.

Fifth graders use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit

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numbers and perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations.

Operations and Algebraic Thinking

5.OA

Write and interpret numerical expressions.

MGSE5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

MGSE5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.

Analyze patterns and relationships.

MGSE5.OA.3 Generate two numerical patterns using a given rule. Identify apparent relationships between corresponding terms by completing a function table or input/output table. Using the terms created, form and graph ordered pairs on a coordinate plane.

Number and Operations in Base Ten

5.NBT

Understand the place value system.

MGSE5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $1/10$ of what it represents in the place to its left.

MGSE5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

MGSE5.NBT.3 Read, write, and compare decimals to thousandths.

- Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.
- Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.

MGSE5.NBT.4 Use place value understanding to round decimals up to the hundredths place.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

MGSE5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm (or other strategies demonstrating understanding of multiplication) up to a 3 digit by 2 digit factor.

MGSE5.NBT.6 Fluently divide up to 4-digit dividends and 2-digit divisors by using at least one of the following methods: strategies based on place value, the properties of operations, and/or the

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relationship between multiplication and division. Illustrate and explain the calculation by using equations or concrete models. (e.g., rectangular arrays, area models)

MGSE5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Number and Operations – Fractions

5.NF

Use equivalent fractions as a strategy to add and subtract fractions.

MGSE5.NF.1 Add and subtract fractions and mixed numbers with unlike denominators by finding a common denominator and equivalent fractions to produce like denominators.

MGSE5.NF.2 Solve word problems involving addition and subtraction of fractions, including cases of unlike denominators (e.g., by using visual fraction models or equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

MGSE5.NF.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *Example: $\frac{3}{5}$ can be interpreted as “3 divided by 5 and as 3 shared by 5”.*

MGSE5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

- a. Apply and use understanding of multiplication to multiply a fraction or whole number by a fraction.
Examples: $\frac{a}{b} \times q$ as $\frac{a}{b} \times \frac{q}{1}$ and $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$
- b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths.

MGSE5.NF.5 Interpret multiplication as scaling (resizing), by:

- a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. *Example: 4×10 is twice as large as 2×10 .*
- b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

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MGSE5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

MGSE5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.²³

- Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for $(1/3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1/3) \div 4 = 1/12$ because $(1/12) \times 4 = 1/3$.
- Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.
- Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual *fraction* models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share $1/2$ lb of chocolate equally? How many $1/3$ -cup servings are in 2 cups of raisins?

Measurement and Data

5.MD

Convert like measurement units within a given measurement system.

MGSE5.MD.1 Convert among different-sized standard measurement units (mass, weight, length, time, etc.) within a given measurement system (customary and metric) (e.g., convert 5cm to 0.05m), and use these conversions in solving multi-step, real world problems.

Represent and interpret data.

MGSE5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit ($1/2$, $1/4$, $1/8$). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

Geometric Measurement: understand concepts of volume and relate volume to multiplication and division.

MGSE5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

- A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.
- A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

MGSE5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

²³ Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

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MGSE5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

- a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
- b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems.
- c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Geometry

5.G

Graph points on the coordinate plane to solve real-world and mathematical problems.

MGSE5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x -axis and x -coordinate, y -axis and y -coordinate).

MGSE5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Classify two-dimensional figures into categories based on their properties.

MGSE5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*

MGSE5.G.4 Classify two-dimensional figures in a hierarchy based on properties (*polygons, triangles, and quadrilaterals*).

In Georgia resources and assessments, trapezoids are defined using the inclusive definition: At least one pair of parallel sides.