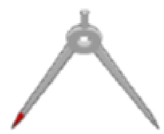
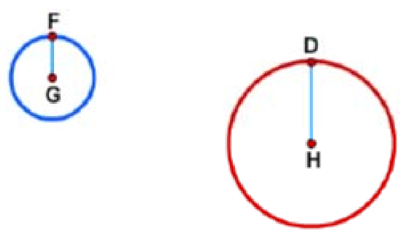
Opening Activity

1. To dilate a circle, start by drawing a circle and a point, A, that is not on the circle. Point A will be the center of the dilation. Using what you learned about constructing dilations of triangles, how would you dilate circle C by a scale factor of 3?



2. Now that you know how to dilate a circle, consider this question. Given any two circles, can you always find a dilation that maps one circle onto another?



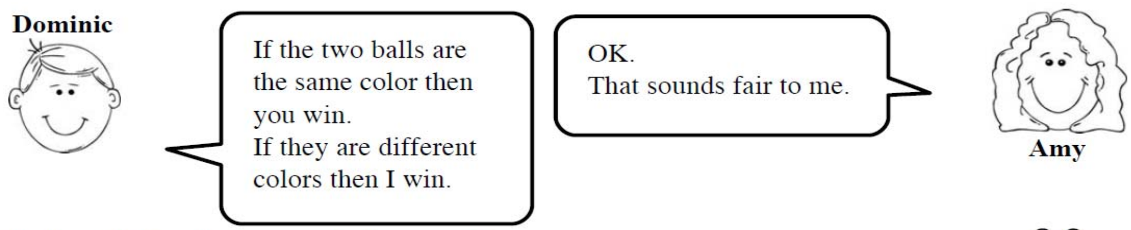
The Mathematics Common Core Toolbox
CCSSTOOLBOX.ORG

Focus Activity

A Fair Game

Dominic has made up a simple game.
Inside a bag he places nine balls. These balls are either black or white.
He then shakes the bag.

He asks Amy to take two balls from the bag without looking.



Amy is right, Dominic has made the game fair.

How many white balls (n) and how many black balls (m) has Dominic put in the bag?

Fully explain your answer.

Hints: You could think about likely combinations of black and white balls and then check to see if they would make the game fair.

Or you could use algebra to work out the number of black and white balls in the bag.

<u>Coherence Activity</u>

The Circle and the Line

1. Sketch the circle with equation $x^2 + y^2 = 1$ and the line with equation $y = 2x - 1$ on the same pair of axes.

   a) There is one solution to the pair of equations $\begin{matrix} x^2 + y^2 = 1 \\ y = 2x - 1 \end{matrix}$ that is clearly identifiable from the sketch. What is it? Verify that it is a solution.

   b) Find all solutions to this pair of equations.

The Illustrative Mathematics Project
Illustrativemathematics.org

Overview of High School Math

CCGPS Coordinate Algebra

The fundamental purpose of Coordinate Algebra is to formalize and extend the mathematics that students learned in the middle grades. The critical areas, organized into units, deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Coordinate Algebra uses algebra to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

CCGPS Analytic Geometry:

The focus of Analytic Geometry on the coordinate plane is organized into 6 critical areas. Transformations on the coordinate plane provide opportunities for the formal study of congruence and similarity. The study of similarity leads to an understanding of right triangle trigonometry and connects to quadratics through Pythagorean relationships. The study of circles uses similarity and congruence to develop basic theorems relating circles and lines. The need for extending the set of rational numbers arises and real and complex numbers are introduced so that all quadratic equations can be solved. Quadratic expressions, equations, and functions are developed; comparing their characteristics and behavior to those of linear and exponential relationships from Coordinate Algebra. Circles return with their quadratic algebraic representations on the coordinate plane. The link between probability and data is explored through conditional probability. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

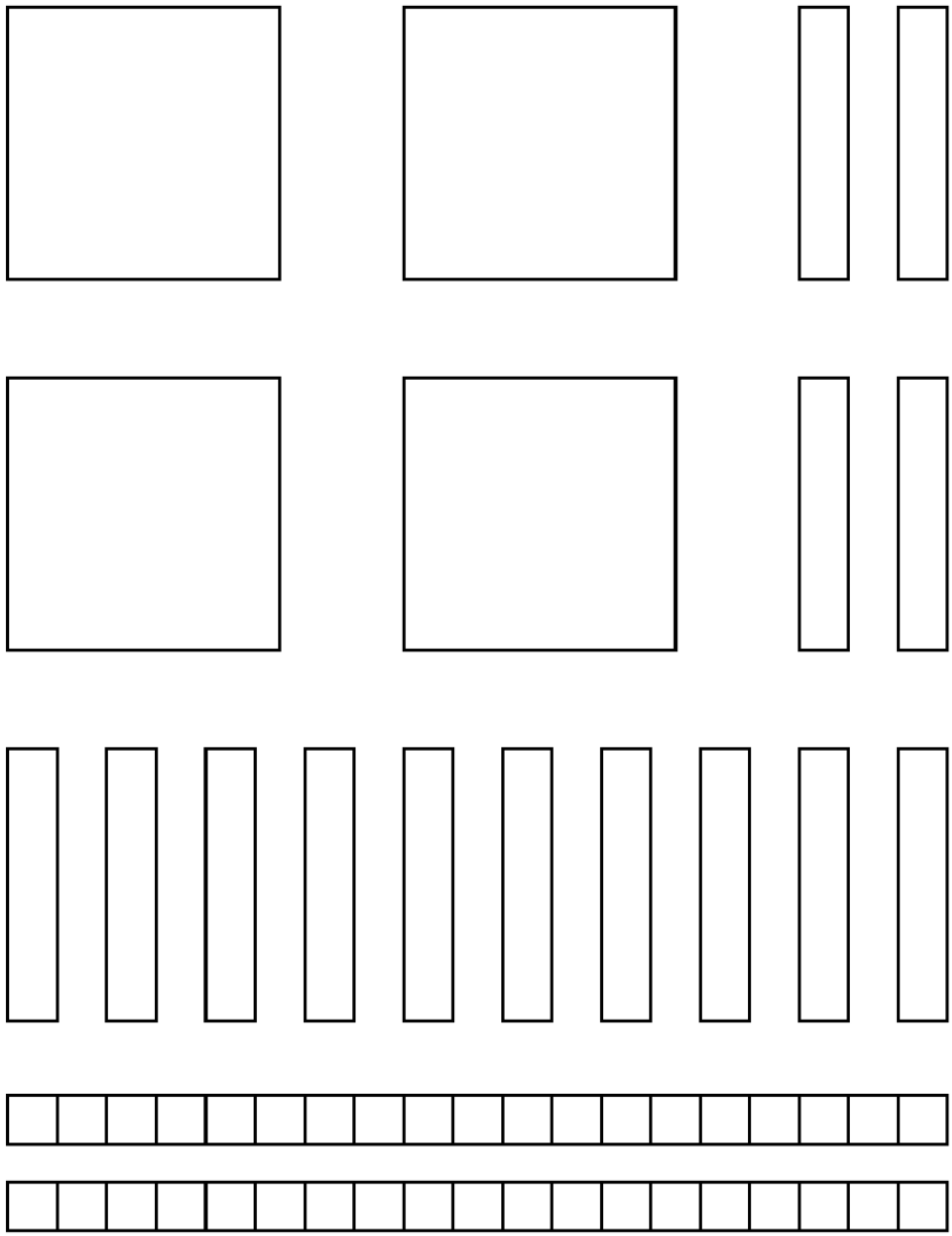
CCGPS Advanced Algebra:

It is in Advanced Algebra that students pull together and apply the accumulation of learning that they have from their previous courses, with content grouped into six critical areas, organized into units. They apply methods from probability and statistics to draw inferences and conclusions from data. Students expand their repertoire of functions to include polynomial, rational, and radical functions. They expand their study of right triangle trigonometry to model periodic phenomena. And, finally, students bring together all of their experience with functions and geometry to create models and solve contextual problems. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Deep Understanding Activity

Completing the Square

1. Convert the function $y = x^2 + 6x + 11$ from general to vertex form using algebra tiles.



a. Use the tiles to model the general form of the quadratic expression $x^2 + 6x + 11$.

b. Use the x^2 tile, x tiles, and as many unit tiles as possible from your model to form a square.

c. What is the area of the square you have built?

d. This square represents only part of the original quadratic expression $x^2 + 6x + 11$. How many unit tiles are left over?

e. Now that you have completed the square (with left over tiles), rewrite the quadratic function $y = x^2 + 6x + 11$ using the algebraic form of the completed square and the left over tiles.

Application Activity

Going Round in Circles Again

1. Complete the drawing of the graph of the equation $(x + 1)^2 + (y - 3)^2 = 9$ to show the x- axis and y-axis. Add numbers to each of these axes.

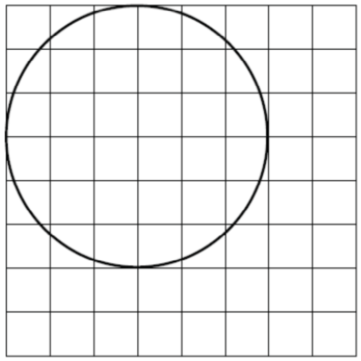


Figure out the co-ordinates of any x-intercepts and y-intercepts.

Explain your answer(s).

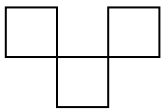
2. Write an equation of a second circle that has two x-intercepts, but just one y-intercept and a radius of 6.

Explain your answer.

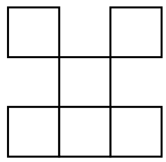
Balanced Approach Activity

Mosaics

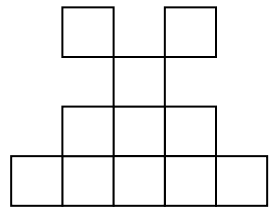
Mosaic 1



Mosaic 2



Mosaic 3



Reuben learned in art class that a mosaic is made by arranging small pieces of colored material (such as glass or tile) to create a design. Reuben created a mosaic using tiles, then decided on a growing pattern and created a second and third mosaic. Reuben continued his pattern by building additional mosaics. He counted the number of tiles in each mosaic and then represented the data in multiple ways. He thinks he sees a relationship between the mosaic number and the total number of tiles in the mosaic.

1. Represent Reuben's data from the mosaics problem in at least three ways, including a general function rule, to determine the number of tiles in any mosaic.

2. Write a description of how your rule is related to the mosaic picture. Include a description of what is constant and what is changing as tiles are added.

3. How many tiles would be in the tenth mosaic? Use two different representations to show how you determined your answer.

4. Would there be a mosaic in Reuben's set that uses exactly 57 tiles? Explain your reasoning using at least one representation.

Adapted from the Charles A. Dana Center
at the University of Texas at Austin

What's in Analytic Geometry B / Advanced Algebra

Unit 1	Unit 2	Unit 3
Extending the Number System	**Quadratic Functions**	**Modeling Geometry**
<u>Extend the properties of exponents to rational exponents.</u> **MCC9-12.N.RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. **MCC9-12.N.RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents. **<u>Use properties of rational and irrational numbers.</u>** **MCC9-12.N.RN.3** Explain why the sum or product of rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. **<u>Perform arithmetic operations with complex numbers.</u>** **MCC9-12.N.CN.1** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real. **MCC9-12.N.CN.2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **MCC9-12.N.CN.3 (+)** Find the conjugate of a complex number; use conjugates to find ~~moduli and~~ quotients of complex numbers. **<u>Perform arithmetic operations on polynomials</u>** **MCC9-12.A.APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. *(Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.)*	**<u>Use complex numbers in polynomial identities and equations.</u>** **MCC9-12.N.CN.7** Solve quadratic equations with real coefficients that have complex solutions. **<u>Interpret the structure of expressions</u>** **MCC9-12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.★ *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **MCC9-12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.★ *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **MCC9-12.A.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.★ *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **MCC9-12.A.SSE.2** Use the structure of an expression to identify ways to rewrite it. *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **<u>Write expressions in equivalent forms to solve problems</u>** **MCC9-12.A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **MCC9-12.A.SSE.3a** Factor a quadratic expression to reveal the zeros of the function it defines.★ **MCC9-12.A.SSE.3b** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.★ **<u>Create equations that describe numbers or relationships</u>** **MCC9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from ~~linear and~~ quadratic functions, ~~and simple rational and exponential functions~~.★ **MCC9-12.A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★ *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **MCC9-12.A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **<u>Solve equations and inequalities in one variable</u>** **MCC9-12.A.REI.4** Solve quadratic equations in one variable.	**<u>Translate between the geometric description and the equation for a conic section</u>** **MCC9-12.G.GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **MCC9-12.G.GPE.2** Derive the equation of a parabola given a focus and directrix. **<u>Use coordinates to prove simple geometric theorems algebraically</u>** **MCC9-12.G.GPE.4** Use coordinates to prove simple geometric theorems algebraically. *(Restrict to context of circles and parabolas)*

	MCC9-12.A.REI.4a Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. **MCC9-12.A.REI.4b** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. **<u>Solve systems of equations</u>** **MCC9-12.A.REI.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. **<u>Interpret functions that arise in applications in terms of the context</u>** **MCC9-12.F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity~~.★ **MCC9-12.F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.★ *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **MCC9-12.F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **<u>Analyze functions using different representations</u>** **MCC9-12.F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **MCC9-12.F.IF.7a** Graph ~~linear and~~ quadratic functions and show intercepts, maxima, and minima.★ **MCC9-12.F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **MCC9-12.F.IF.8a** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. **MCC9-12.F.IF.9** Compare properties of two functions each represented in a different way	

	(algebraically, graphically, numerically in tables, or by verbal descriptions). *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **<u>Build a function that models a relationship between two quantities</u>** **MCC9-12.F.BF.1** Write a function that describes a relationship between two quantities.★ *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **MCC9-12.F.BF.1a** Determine an explicit expression, a recursive process, or steps for calculation from a context. *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **MCC9-12.F.BF.1b** Combine standard function types using arithmetic operations. *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **<u>Build new functions from existing functions</u>** **MCC9-12.F.BF.3** Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. *(Focus on quadratic functions; compare with linear and exponential functions studied in Coordinate Algebra.)* **<u>Construct and compare linear, quadratic, and exponential models and solve problems</u>** **MCC9-12.F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.★ **<u>Summarize, represent, and interpret data on two categorical and quantitative variables</u>** **MCC9-12.S.ID.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★ **MCC9-12.S.ID.6a** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize ~~linear~~, quadratic, ~~and exponential~~ models.★	

Analytic Geometry

What's in Analytic Geometry B / Advanced Algebra

Unit 4	Unit 5	Unit 6
Applications of Probability	**Inferences and Conclusions from Data**	**Polynomial Functions**
Understand independence and conditional probability and use them to interpret data **MCC9-12.S.CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").★ **MCC9-12.S.CP.2** Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★ **MCC9-12.S.CP.3** Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★ **MCC9-12.S.CP.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.★ **MCC9-12.S.CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.★ **Use the rules of probability to compute probabilities of compound events in a uniform probability model** **MCC9-12.S.CP.6** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.★ **MCC9-12.S.CP.7** Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.★	**Summarize, represent, and interpret data on a single count or measurement variable** **MCC9-12.S.ID.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.★ **MCC9-12.S.ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.★ **Understand and evaluate random processes underlying statistical experiments** **MCC9-12.S.IC.1** Understand statistics as a process for making inferences about population parameters based on a random sample from that population.★ **MCC9-12.S.IC.2** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. **Make inferences and justify conclusions from sample surveys, experiments, and observational studies** **MCC9-12.S.IC.3** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.★ **MCC9-12.S.IC.4** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.★ **MCC9-12.S.IC.5** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.★ **MCC9-12.S.IC.6** Evaluate reports based on data.★	**Use complex numbers in polynomial identities and equations.** **MCC9-12.N.CN.8** Extend polynomial identities to the complex numbers. **MCC9-12.N.CN.9 (+)** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **Interpret the structure of expressions** **MCC9-12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.★ **MCC9-12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.★ **MCC9-12.A.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.★ **MCC9-12.A.SSE.2** Use the structure of an expression to identify ways to rewrite it. **Write expressions in equivalent forms to solve problems** **MCC9-12.A.SSE.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems★ **Perform arithmetic operations on polynomials** **MCC9-12.A.APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **Understand the relationship between zeros and factors of polynomials** **MCC9-12.A.APR.2** Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). **MCC9-12.A.APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **Use polynomial identities to solve problems** **MCC9-12.A.APR.4** Prove polynomial identities and use them to describe numerical relationships. **MCC9-12.A.APR.5 (+)** Know and apply that the Binomial Theorem gives the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction ~~or by a combinatorial argument.~~) **Solve systems of equations** **MCC9-12.A.REI.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

		<u>Represent and solve equations and inequalities graphically</u> **MCC9-12.A.REI.11** Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are ~~linear~~, polynomial, ~~rational, absolute value, exponential, and logarithmic functions~~.★ **<u>Analyze functions using different representations</u>** **MCC9-12.F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ **MCC9-12.F.IF.7c** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.★

<u>What's in Analytic Geometry B / Advanced Algebra</u>

Unit 7	Unit 8	Unit 9
Rational and Radical Relationships	**Exponential and Logarithms**	**Trigonometric Functions**
<u>Rewrite rational expressions</u> **MCC9-12.A.APR.6** Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. **MCC9-12.A.APR.7 (+)** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. **<u>Create equations that describe numbers or relationships</u>** **MCC9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from ~~linear and quadratic functions~~, and simple rational ~~and exponential~~ functions.★ **MCC9-12.A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★ *(Limit to radical and rational functions.)* **<u>Understand solving equations as a process of reasoning and explain the reasoning</u>** **MCC9-12.A.REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. **<u>Represent and solve equations and</u>**	**<u>Write expressions in equivalent forms to solve problems</u>** **MCC9-12.A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ *(Limit to exponential and logarithmic functions.)* **MCC9-12.A.SSE.3c** Use the properties of exponents to transform expressions for exponential functions. **<u>Analyze functions using different representations</u>** **MCC9-12.F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ *(Limit to exponential and logarithmic functions.)* **MCC9-12.F.IF.7e** Graph exponential and logarithmic functions, showing intercepts and end behavior, ~~and trigonometric functions, showing period, midline, and amplitude.~~★ **MCC9-12.F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. *(Limit to exponential and logarithmic functions.)* **MCC9-12.F.IF.8b** Use the properties of exponents to interpret expressions for exponential functions. *(Limit to exponential and logarithmic functions.)* **<u>Build new functions from existing functions</u>** **MCC9-12.F.BF.5 (+)** Understand the inverse relationship between exponents and logarithms and	**<u>Analyze functions using different representations</u>** **MCC9-12.F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ *(Limit to trigonometric functions.)* **MCC9-12.F.IF.7e** Graph ~~exponential and logarithmic functions, showing intercepts and end behavior~~, and trigonometric functions, showing period, midline, and amplitude.★ **<u>Extend the domain of trigonometric functions using the unit circle</u>** **MCC9-12.F.TF.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **MCC9-12.F.TF.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **<u>Model periodic phenomena with trigonometric functions</u>** **MCC9-12.F.TF.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★ **<u>Prove and apply trigonometric identities</u>** **MCC9-12.F.TF.8** Prove the Pythagorean identity $(\sin A)^2 + (\cos A)^2 = 1$ and use it to find sin A, cos A, or tan A, given sin A, cos A, or tan A, and the quadrant of the angle.

<u>inequalities graphically</u> **MCC9-12.A.REI.11** Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are ~~linear, polynomial~~, rational, ~~absolute value, exponential, and logarithmic functions~~.★ **<u>Interpret functions that arise in applications in terms of the context</u>** **MCC9-12.F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and ~~periodicity~~.★ *(Limit to radical and rational functions.)* **MCC9-12.F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *(Limit to radical and rational functions.)* **<u>Analyze functions using different representations</u>** **MCC9-12.F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ *(Limit to radical and rational functions.)* **MCC9-12.F.IF.7b** Graph square root, cube root, ~~and piecewise-defined functions, including step functions and absolute value functions.~~★ **MCC9-12.F.IF.7d (+)** Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.★ **MCC9-12.F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *(Limit to radical and rational functions.)*	use this relationship to solve problems involving logarithms and exponents. **<u>Construct and compare linear, quadratic, and exponential models and solve problems</u>** **MCC9-12.F.LE.4** For exponential models, express as a logarithm the solution to $ab^{(ct)} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.★	

What's in Analytic Geometry B / Advanced Algebra

Unit 10

Mathematical Modeling

Create equations that describe numbers or relationships

MCC9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.★

MCC9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

MCC9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.★

MCC9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Interpret functions that arise in applications in terms of the context

MCC9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★

MCC9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.★

MCC9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Analyze functions using different representations

MCC9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

MCC9-12.F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.★

MCC9-12.F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.★

MCC9-12.F.IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.★

MCC9-12.F.IF.7d (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.★

MCC9-12.F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.★

MCC9-12.F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

MCC9-12.F.IF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

MCC9-12.F.IF.8b Use the properties of exponents to interpret expressions for exponential functions.

MCC9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Build a function that models a relationship between two quantities

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities.★

MCC9-12.F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.

MCC9-12.F.BF.1b Combine standard function types using arithmetic operations.

MCC9-12.F.BF.1c (+) Compose functions.

Build new functions from existing functions

MCC9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

MCC9-12.F.BF.4 Find inverse functions.

MCC9-12.F.BF.4a Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.

MCC9-12.F.BF.4b (+) Verify by composition that one function is the inverse of another.

MCC9-12.F.BF.4c (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

Visualize relationships between two-dimensional and three-dimensional objects

MCC9-12.G.GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Apply geometric concepts in modeling situations

MCC9-12.G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★

MCC9-12.G.MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★

MCC9-12.G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★