Common Core Georgia Performance Standards
Advanced Algebra
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Secondary Mathematics Specialists
Thank you for being here today.

You will need the following materials during today’s broadcast:

- Advanced Algebra handouts
- Note-taking materials

Activate your brain

Apply geometric concepts in modeling situations.

Propane Tanks

People who live in isolated or rural areas have their own tanks of natural gas to run appliances like stoves, washers, and water heaters.

These tanks are made in the shape of a cylinder with hemispheres on the ends.

The Insenm Propane Tank Company makes tanks with this shape, in different sizes.

The cylinder part of every tank is exactly 10 feet long, but the radius of the hemispheres, r, will be different depending on the size of the tank.

The company wants to double the capacity of their standard tank, which is 6 feet in diameter.
Why Common Core Standards?

• Preparation: The standards are college- and career-ready. They will help prepare students with the knowledge and skills they need to succeed in education and training after high school.

• Competition: The standards are internationally benchmarked. Common standards will help ensure our students are globally competitive.

• Equity: Expectations are consistent for all – and not dependent on a student’s zip code.

Why Common Core Standards?

• Clarity: The standards are focused, coherent, and clear. Clearer standards help students (and parents and teachers) understand what is expected of them.

• Collaboration: The standards create a foundation to work collaboratively across states and districts, pooling resources and expertise, to create curricular tools, professional development, common assessments and other materials.
Common Core State Standards

Building on the strength of current state standards, the CCSS are designed to be:

• Focused, coherent, clear and rigorous
• Internationally benchmarked
• Anchored in college and career readiness
• Evidence and research based

Common Core State Standards in Mathematics

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Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Reasoning and explaining
Modeling and using tools
Seeing structure and generalizing

(McCallum, 2011)

Algebra

Creating Equations

★ A.CED

Create equations that describe numbers or relationships.

MCC9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

MCC9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
While the standards focus on what is most essential, they do not describe all that can or should be taught. A great deal is left to the discretion of teachers and curriculum developers. The aim of the standards is to articulate the fundamentals, not to set out an exhaustive list or a set of restrictions that limits what can be taught beyond what is specified.

corestandards.org

What’s a Advanced Algebra Teacher to do?

- Read your grade level standards
- Use the CCGPS Teaching Guide found on Georgia Standards.org and Learning Village
- Discuss the standards with your colleagues
Advanced Algebra is the culminating course in the sequence. It is designed to prepare students for fourth course options relevant to their career pursuits.

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Advanced Algebra Overview

Unit 1: Inferences and Conclusions from Data

Statistics and Probability – Interpreting Categorical and Quantitative Data
- Summarize, represent, and interpret data on a single count or measurement variable.

Statistics and Probability – Making Inferences and Justifying Conclusions
- Understand and evaluate random processes underlying statistical experiments.
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
Advanced Algebra Overview

Unit 2: Polynomial Functions

Number and Quantity – The Complex Number System
- Use complex numbers in polynomial identities and equations.

Algebra – Seeing Structure in Expressions
- Interpret the structure of expressions
- Write expressions in equivalent forms to solve problems

Algebra – Arithmetic with Polynomials and Rational Expressions
- Represent and solve equations and inequalities graphically.
- Understand the relationship between zeros and factors of polynomials
- Use polynomial identities to solve problems

Advanced Algebra Overview

Unit 2: Polynomial Functions (continued)

Algebra – Reasoning with Equations and Inequalities
- Solve systems of equations
- Represent and solve equations and inequalities graphically

Function - Interpreting Functions
- Analyze functions using different representations
Advanced Algebra Overview

Unit 3: Rational and Radical Relationships
- Algebra – Arithmetic with Polynomials and Rational Expressions
  ➢ Rewrite rational expressions.
- Algebra – Creating Equations
  ➢ Create equations that describe numbers or relationships.
- Algebra – Reasoning with Equations and Inequalities
  ➢ Understand solving equations as a process of reasoning and explain the reasoning.
  ➢ Represent and solve equations and inequalities graphically.
- Functions – Interpreting Functions
  ➢ Interpret functions that arise in applications in terms of the context.
  ➢ Analyze functions using different representations.

Unit 4: Exponential and Logarithms
- Algebra – Seeing Structure in Expressions
  ✓ Write expressions in equivalent forms to solve problems.
- Functions – Interpreting Functions
  ✓ Analyze functions using different representations.
- Functions – Building Functions
  ✓ Build new functions from existing functions.
- Functions – Linear, Quadratic, and Exponential Models
  ✓ Construct and compare linear, quadratic, and exponential models and solve problems.
Advanced Algebra Overview

Unit 5: Trigonometric Functions
Functions – Interpreting Functions
- Analyze functions using different representations.

Functions – Trigonometric Functions
- Extend the domain of trigonometric functions using the unit circle
- Model periodic phenomena with trigonometric functions
- Prove and apply trigonometric identities

Advanced Algebra Overview

Unit 6: Mathematical Modeling
Algebra – Creating Equations
- Create equations that describe numbers or relationships.

Functions – Interpreting Functions
- Interpret functions that arise in applications in terms of the context.
- Analyze functions using different representations

Functions – Building Functions
- Build a function that models a relationship between two quantities
- Build new functions from existing functions

Geometry – Geometric Measurement and Dimensions
- Visualize relationships between two-dimensional and three-dimensional objects

Geometry – Modeling with Geometry
- Apply geometric concepts in modeling situations
Focus
Coherence
Fluency
Deep Understanding
Applications
Balanced Approach
Focus

The student…

• spends more time thinking and working on priority concepts.
• is able to understand concepts and their connections to processes (algorithms).

Focus

The teacher…

• builds knowledge, fluency and understanding of why and how certain mathematics concepts are done.
• thinks about how the concepts connect to one another.
• pays more attention to priority content and invests the appropriate time for all students to learn before moving onto the next topic.
Focus

The mile-wide inch-deep problem looks different in high school. In earlier grades it's a matter of having too many topics. In high school it's a matter of having too many separately memorized techniques, with no overall understanding of the structure to tie them altogether. So narrowing and deepening the curriculum is not so much a matter of eliminating topics, as seeing the structure that ties them together. For example, if students see that the distance formula and the trig identity \( \sin^2(t) + \cos^2(t) = 1 \) are both manifestations of the Pythagorean theorem, they have an understanding that helps them reconstruct these formulas rather than memorize them...

Bill McCallum – CCSS author

<table>
<thead>
<tr>
<th>Grade</th>
<th>Priorities in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding</th>
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<tbody>
<tr>
<td>K–2</td>
<td>Addition and subtraction, measurement using whole number quantities</td>
</tr>
<tr>
<td>3-5</td>
<td>Multiplication and division of whole numbers and fractions</td>
</tr>
<tr>
<td>6</td>
<td>Ratios and proportional reasoning; early expressions and equations</td>
</tr>
<tr>
<td>7</td>
<td>Ratios and proportional reasoning; arithmetic of rational numbers</td>
</tr>
<tr>
<td>8</td>
<td>Linear algebra</td>
</tr>
<tr>
<td>9-12</td>
<td>Modeling</td>
</tr>
</tbody>
</table>
Modeling in Advanced Algebra

What distinguishes modeling from other forms of applications of mathematics are (1) explicit attention at the beginning to the process of getting from the problem outside of mathematics to its mathematical formulation and (2) an explicit reconciliation between the mathematics and the real-world situation at the end. Throughout the modeling process, consideration is given to both the external world and the mathematics, and the results have to be both mathematically correct and reasonable in the real-world context.

“The Definition of Modeling” Henry O. Pollak

Focus task

Model periodic phenomena with trigonometric functions.
Coherence

The student...

- builds on knowledge from year to year, in a coherent learning progression.
Coherence

The teacher...

- connects mathematical ideas across grade levels.
- thinks deeply about what is being focused on.
- thinks how those ideas connect to how it was taught the years before and the years after.

What do Advanced Algebra students bring?
What are they connecting to later?

Analyze functions using different representations.
Analyze Functions Overview

8th Grade
• Define functions and linear functions

Coordinate Algebra
• Compare characteristics of linear and exponential functions

Analytic Geometry
• Compare quadratic function characteristics to linear functions
The student...

- spends time practicing skills with intensity and frequency.
Fluency

The teacher...

- pushes students to know skills at a greater level of fluency based on understanding.
- focuses on the listed fluencies by grade level.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Required Fluency</th>
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<tbody>
<tr>
<td>K</td>
<td>Add/subtract within 5</td>
</tr>
<tr>
<td>1</td>
<td>Add/subtract within 10</td>
</tr>
<tr>
<td>2</td>
<td>Add/subtract within 20 &amp; Add/subtract within 100 (pencil and paper)</td>
</tr>
<tr>
<td>3</td>
<td>Multiply/divide within 100 &amp; Add/subtract within 1000</td>
</tr>
<tr>
<td>4</td>
<td>Add/subtract within 1,000,000</td>
</tr>
<tr>
<td>5</td>
<td>Multi-digit multiplication</td>
</tr>
<tr>
<td>6</td>
<td>Multi-digit division &amp; Multi-digit decimal operations</td>
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<tr>
<td>7</td>
<td>Solve $px + q = r$, $p(x + q) = r$</td>
</tr>
<tr>
<td>8</td>
<td>Solve simple $2 \times 2$ systems by inspection</td>
</tr>
<tr>
<td>9-12</td>
<td>Algebraic manipulation in which to understand structure. Writing a rule to represent a relationship between two quantities. Seeing mathematics as a tool to model real-world situations. Understanding quantities and their relationships.</td>
</tr>
</tbody>
</table>
What does Fluency Look Like in Advanced Algebra?

- FLEXIBLY
- ACCURATELY
- EFFICIENTLY
- APPROPRIATELY
Deep Understanding

The student...

• shows mastery of material at a deep level in numerous ways.
• uses mathematical practices to demonstrate understanding of different material and concepts.

Deep Understanding

The teacher...

• asks what mastery/proficiency really looks like and means.
• plans for progression of levels of understanding.
• spends the time to gain the depth of the understanding.
• becomes flexible and comfortable in own depth of content knowledge.
FAL Structure

- Pre-Assessment / opening
- Collaborative activity
- Whole-class discussion
- Return to the pre-assessment / opening

What does depth mean in Advanced Algebra?

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Muddying The Waters

The manager of the Riverside Center is concerned about visitor numbers. As a result, the Center’s popularity has been adversely affected by an increase in lake pollution. He feels the local Environmental Agency should do something about it.

To support his argument he measured the chemical concentration in the river each month. He also counted the number of people visiting the Center over several months. He used the results to draw this chart.

Dr. John D. Barge, State School Superintendent
“Making Education Work for All Georgians”
The student...

- applies mathematics in other content areas and situations.
- chooses the right mathematics concept to solve a problem when not necessarily prompted to do so.
Application

The teacher...
• contextualizes mathematics.
• creates real world experiences in which students use what they know, and in which they are not necessarily prompted to use mathematics.

Mathematizing Advanced Algebra

Rewrite rational expressions.

You have been hired for a summer internship at a marine life aquarium. Your job requires that you dilute brine for the saltwater fish tanks. The brine is water and 15.8% sea salt (by weight). Thus, the salt concentration of the brine is 15.8%.

You are given the function

\[ S(x) = \frac{15.8}{100} \cdot \frac{1}{x} \]

where \( x \) is the amount of brine in liters and \( S(x) \) is the amount of salt in grams in a liter of brine.

a) Describe how the graph of \( S(x) \) is related to the graph of \( y = \frac{1}{x} \).

b) Sketch the graph of \( S(x) \).

c) How much fresh water should you add to get a mixture which is 4% sea salt, approximately the salt concentration of the ocean?
Focus
Coherence
Fluency
Deep Understanding
Applications
Balanced Approach

Balance Approach

The student...
• practices mathematics skills to achieve fluency.
• practices mathematics concepts to ensure application in novel situations.
Balanced Approach

The teacher...

• finds the balance between understanding and practice.
• normalizes the productive struggle.
• ritualizes skills practice.

Balanced Approach

Build a function that models a relationship between two quantities.

Mosaics

Mosaic 1  Mosaic 2  Mosaic 3
Focus
Coherence
Fluency
Deep Understanding
Applications
Balanced Approach

What’s in Analytic Geometry B
/ Advanced Algebra

- **Extending the Number System**
  - Extend the properties of exponents to rational exponents.
  - Use properties of rational and irrational numbers.
  - Perform arithmetic operations with complex numbers.
  - Perform arithmetic operations on polynomials

- **Quadratic Functions**
  - Use complex numbers in polynomial identities and equations.
  - Interpret the structure of expressions
  - Write expressions in equivalent forms to solve problems
  - Create equations that describe numbers or relationships
  - Solve equations and inequalities in one variable
What’s in Analytic Geometry B / Advanced Algebra

• **Quadratic Functions (continued)**
  ◦ Solve systems of equations
  ◦ Interpret functions that arise in applications in terms of the context
  ◦ Analyze functions using different representations
  ◦ Build a function that models a relationship between two quantities
  ◦ Build new functions from existing functions
  ◦ Construct and compare linear, quadratic, and exponential models and solve problems
  ◦ Summarize, represent, and interpret data on two categorical and quantitative variables

• **Modeling Geometry**
  ◦ Translate between the geometric description and the equation for a conic section
  ◦ Use coordinates to prove simple geometric theorems algebraically

• **Applications of Probability**
  ◦ Understand independence and conditional probability and use them to interpret data
  ◦ Use the rules of probability to compute probabilities of compound events in a uniform probability model
CCGPS Suggestions:

1. Review the CCGPS. The teaching guide, curriculum map, and standards can all be found in Learning Village, on the Mathematics Program Page and on Georgia Standards.org

2. View the Fall 2011 Grade Level Webinar if you haven’t already seen it.

3. Review this broadcast with your team to identify key areas of focus.

4. Participate in the unit-by-unit webinars beginning in May.

5. Structure time for grade level/content areas to use framework units as a guide for planning.

6. Plan to get together with your colleagues at the end of each CCGPS unit to analyze student work samples and compare how student learning and performance look.
Advanced Algebra Support:

Now-
• Fall 2011 Grade Level Webinar
• Standards Document
• Teaching Guide
• Curriculum Map

Coming soon-
• Framework Units
• Unit-by-unit webinars

Takeaways

3 Things-
1. What’s new?
2. What’s different?
3. What resources and support are available for CCGPS mathematics?
“The resources we need in order to grow as teachers are abundant within the community of colleagues. Good talk about good teaching is what we need…”

Parker Palmer
Courage to Teach
Thank you for participating in this CCGPS Professional Learning Session. We value your feedback. Please go to the following website, take the anonymous feedback survey, and complete the participation log to receive a certificate of participation:

http://survey.sedl.org/efm/wsb.dll/s/1g10a

If you have questions, feel free to contact any of the English/Language Arts or Mathematics staff at the following email addresses:

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