Georgia Standards of Excellence Grade Level Curriculum Overview

Mathematics

GSE First Grade

Georgia Department of Education

Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

TABLE OF CONTENTS *(* indicates recent addition to the Grade Level Overview)*

***Please note that all changes made will appear in green.

Georgia Standards of Excellence
First Grade

GSE First Grade Curriculum Map						
Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7
Creating Routines Using Data	**Developing Base Ten Number Sense**	**Operations and Algebraic Thinking**	**Sorting, Comparing and Ordering**	**Understanding Place Value**	**Understanding Shapes and Fractions**	**Show What We Know**
MGSE1.NBT.1 **MGSE1.MD.4**	**MGSE1.NBT.1** **MGSE1.NBT.7** **MGSE1.MD.4**	**MGSE1.OA.1** **MGSE1.OA.2** **MGSE1.OA.3** **MGSE1.OA.4** **MGSE1.OA.5** **MGSE1.OA.6** **MGSE1.OA.7** **MGSE1.OA.8** **MGSE1.MD.4**	**MGSE1.MD.1** **MGSE1.MD.2** **MGSE1.MD.3** **MGSE1.MD.4**	**MGSE1.NBT.2** **MGSE1.NBT.3** **MGSE1.NBT.4** **MGSE1.NBT.5** **MGSE1.NBT.6** **MGSE1.NBT.7** **MGSE1.MD.4**	**MGSE1.G.1** **MGSE1.G.2** **MGSE1.G.3** **MGSE1.MD.4**	**ALL**
These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units. All units will include the Mathematical Practices and indicate skills to maintain. However, the progression of the units is at the discretion of districts.						

NOTE: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics.

Grades K-2 Key: CC = Counting and Cardinality, G= Geometry, MD=Measurement and Data, NBT= Number and Operations in Base Ten, OA = Operations and Algebraic Thinking.

STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education.

> *The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections.*
>
> *The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).*

Students are expected to:

1. Make sense of problems and persevere in solving them.
In first grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They are willing to try other approaches.

2. Reason abstractly and quantitatively.
Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities.

3. Construct viable arguments and critique the reasoning of others.
First graders construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They also practice their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" "Explain your thinking," and "Why is that true?" They not only explain their own thinking, but listen to others' explanations. They decide if the explanations make sense and ask questions.

4. Model with mathematics.
In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.

5. Use appropriate tools strategically.

In first grade, students begin to consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, first graders decide it might be best to use colored chips to model an addition problem.

6. Attend to precision.
As young children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning.

7. Look for and make use of structure.
First graders begin to discern a pattern or structure. For instance, if students recognize 12 + 3 = 15, then they also know 3 + 12 = 15. (Commutative property of addition.) To add 4 + 6 + 4, the first two numbers can be added to make a ten, so 4 + 6 + 4 = 10 + 4 = 14.

8. Look for and express regularity in repeated reasoning.
In the early grades, students notice repetitive actions in counting and computation, etc. When children have multiple opportunities to add and subtract "ten" and multiples of "ten" they notice the pattern and gain a better understanding of place value. Students continually check their work by asking themselves, "Does this make sense?"

******Mathematical Practices 1 and 6 should be evident in EVERY lesson******

CONTENT STANDARDS

OPERATIONS AND ALGEBRAIC THINKING (OA)

CLUSTER #1: REPRESENT AND SOLVE PROBLEMS INVOLVING ADDITION AND SUBTRACTION.

Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Prior to first grade students should recognize that any given group of objects (up to 10) can be separated into sub groups in multiple ways and remain equivalent in amount to the original group (Ex: A set of 6 cubes can be separated into a set of 2 cubes and a set of 4 cubes and remain 6 total cubes).

MGSE1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

> This standard builds on the work in Kindergarten by having students use a variety of mathematical representations (e.g., objects, drawings, and equations) during their work. The unknown symbols should include boxes or pictures, and not letters.

Teachers should be cognizant of the three types of problems. There are three types of addition and subtraction problems: Result Unknown, Change Unknown, and Start Unknown.

Use informal language (and, minus/subtract, the same as) to describe joining situations (putting together) and separating situations (breaking apart).

Use the addition symbol (+) to represent joining situations, the subtraction symbol (-) to represent separating situations, and the equal sign (=) to represent a relationship regarding quantity between one side of the equation and the other.

A helpful strategy is for students to recognize sets of objects in common patterned arrangements (0-6) to tell how many without counting (subitizing).

Here are some Addition Examples:

Result Unknown	Change Unknown	Start Unknown
There are 9 students on the playground. Then 8 more students showed up. How many students are there now? *(9 + 8 = ____)*	There are 9 students on the playground. Some more students show up. There are now 17 students. How many students came? *(9 + ____ = 17)*	There are some students on the playground. Then 8 more students came. There are now 17 students. How many students were on the playground at the beginning? *(____ + 8 = 17)*

<u>MGSE1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.</u>

This standard asks students to add (join) three numbers whose sum is less than or equal to 20, using a variety of mathematical representations.

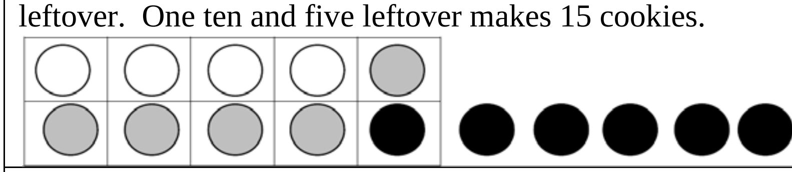
This objective does address multi-step word problems.

Example:

There are cookies on the plate. There are 4 oatmeal raisin cookies, 5 chocolate chip cookies, and 6 gingerbread cookies. How many cookies are there total?

Student 1: *Adding with a Ten Frame and Counters*

I put 4 counters on the Ten Frame for the oatmeal raisin cookies. Then I put 5 different color counters on the ten-frame for the chocolate chip cookies. Then I put another 6 color counters out for the gingerbread cookies. Only one of the gingerbread cookies fit, so I had 5 leftover. One ten and five leftover makes 15 cookies.

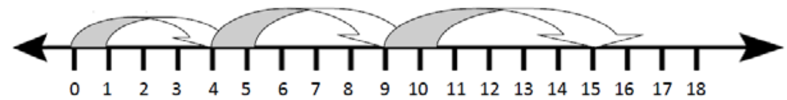


Student 2: *Look for Ways to Make 10*

I know that 4 and 6 equal 10, so the oatmeal raisin and gingerbread equals 10 cookies. Then I add the 5 chocolate chip cookies and get 15 total cookies.

Student 3: *Number Line*

I counted on the number line. First I counted 4, and then I counted 5 more and landed on 9. Then I counted 6 more and landed on 15. So there were 15 total cookies.



CLUSTER #2: UNDERSTAND AND APPLY PROPERTIES OF OPERATIONS AND THE RELATIONSHIP BETWEEN ADDITION AND SUBTRACTION.

Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

MGSE1.OA.3 Apply properties of operations as strategies to add and subtract. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)

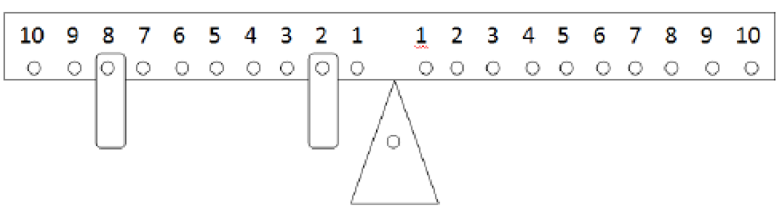
This standard calls for students to apply properties of operations as strategies to add and subtract. Students do not need to use formal terms for these properties. Students should use mathematical tools, such as cubes and counters, and representations such as the number line and a 100 chart to model these ideas.

Example:

Student can build a tower of 8 green cubes and 3 yellow cubes and another tower of 3 yellow and 8 green cubes to show that order does not change the result in the operation of addition. Students can also use cubes of 3 different colors to "prove" that (2 + 6) + 4 is equivalent to 2 + (6 + 4) and then to prove 2 + 6 + 4 = 2 + 10.

Commutative Property of Addition Order does not matter when you add numbers. For example, if 8 + 2 = 10 is known, then 2 + 8 = 10 is also known.	Associative Property of Addition When adding a string of numbers you can add any two numbers first. For example, when adding 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12

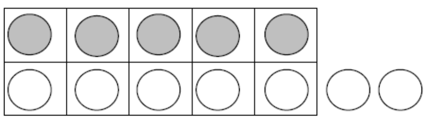
Student Example: *Using a Number Balance to Investigate the Commutative Property*
If I put a weight on 8 *first* and *then* 2, I think that will balance if I put a weight on 2 *first* this time and *then* on 8.



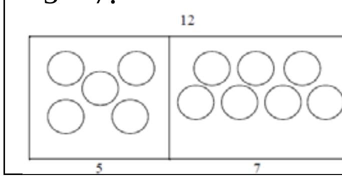
<u>MGSE1.OA.4 Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8. Add and subtract within 20.*</u>

This standard asks for students to use subtraction in the context of unknown addend problems. Example: 12 – 5 = __ could be expressed as 5 + __ = 12. Students should use cubes and counters, and representations such as the number line and the100 chart, to model and solve problems involving the inverse relationship between addition and subtraction.

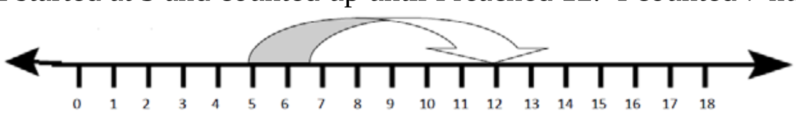
Student 1
I used a ten-frame. I started with 5 counters. I knew that I had to have 12, which is one full ten frame and two leftovers. I needed 7 counters, so 12 – 5 = 7.



Student 2
I used a part-part-whole diagram. I put 5 counters on one side. I wrote 12 above the diagram. I put counters into the other side until there were 12 in all. I know I put 7 counters on the other side, so 12 – 5 = 7.



Student 3: *Draw a Number Line*
I started at 5 and counted up until I reached 12. I counted 7 numbers, so I know that 12 – 5 = 7.



CLUSTER #3: ADD AND SUBTRACT WITHIN 20.

<u>MGSE1.OA.5 Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).</u>

This standard asks for students to make a connection between counting and adding and subtraction. Students use various counting strategies, including counting all, counting on, and counting back with numbers up to 20. This standard calls for students to move beyond counting all and become comfortable at counting on and counting back. The counting all strategy requires students to count an entire set. The counting and counting back strategies occur when students are able to hold the —start number‖ in their head and count on from that number.

Example: 5 + 2 = ___

Student 1: *Counting All*
5 + 2 = ___. The student counts five counters. The student adds two more. The student counts 1, 2, 3, 4, 5, 6, 7 to get the answer.

Student 2: *Counting On*
5 + 2 = ___. Student counts five counters. The student adds the first counter and says 6, then adds another counter and says 7. The student knows the answer is 7, since they counted on 2.

Example: 12 – 3 = ___

Student 1: *Counting All*
12 – 3 = ___. The student counts twelve counters. The student removes 3 of them. The student counts 1, 2, 3, 4, 5, 6, 7, 8, 9 to get the answer.

Student 2: *Counting Back*
12 – 3 = ___. The student counts twelve counters. The student removes a counter and says 11, removes another counter and says 10, and removes a third counter and says 9. The student knows the answer is 9, since they counted back 3.

MGSE1.OA.6 Add and subtract within 20.

a. **Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).**

b. **Fluently add and subtract within 10.**

This standard mentions the word fluency when students are adding and subtracting numbers within 10. Fluency means accuracy (correct answer), efficiency (within 4-5 seconds), and flexibility (using strategies such as making 5 or making 10).
The standard also calls for students to use a variety of strategies when adding and subtracting numbers within 20. Students should have ample experiences modeling these operations before working on fluency. Teacher could differentiate using smaller numbers.
It is importance to move beyond the strategy of counting on, which is considered a less important skill than the ones here in 1.OA.6. Many times teachers think that counting on is all a child needs, when it is really not much better skill than counting all and can becomes a hindrance when working with larger numbers.

Example: 8 + 7 = ___

Student 1: *Making 10 and Decomposing a Number*
I know that 8 plus 2 is 10, so I decomposed (broke) the 7 up into a 2 and a 5. First I added 8 and 2 to get 10, and then added the 5 to get 15.
8 + 7 = (8 + 2) + 5 = 10 + 5 = 15

Student 2: *Creating an Easier Problem with Known Sums*
I know 8 is 7 + 1. I also know that 7 and 7 equal 14 and then I added 1 more to get 15.
8 + 7 = (7 + 7) + 1 = 15

Example: 14 – 6 = ___

Student 1: *Decomposing the Number You Subtract*
I know that 14 minus 4 is 10 so I broke the 6 up into a 4 and a 2. 14 minus 4 is 10. Then I take away 2 more to get 8.
14 – 6 = (14 – 4) – 2 = 10 – 2 = 8

Student 2: *Relationship between Addition and Subtraction*
6 + □ is 14. I know that 6 plus 8 is 14, so that means that 14 minus 6 is 8.
6 + 8 = 14 so 14 – 6 = 8

Algebraic ideas underlie what students are doing when they create equivalent expressions in order to solve a problem or when they use addition combinations they know to solve more difficult problems. Students begin to consider the relationship between the parts. For example, students notice that the whole remains the same, as one part increases the other part decreases. 5 + 2 = 4 + 3

CLUSTER #4: WORK WITH ADDITION AND SUBTRACTION EQUATIONS.

MGSE1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.

This standard calls for students to work with the concept of equality by identifying whether equations are true or false. Therefore, students need to understand that the equal sign does not mean ―answer comes next‖, but rather that the equal sign signifies a relationship between the left and right side of the equation.
The number sentence 4 + 5 = 9 can be read as, ―Four plus five is the same amount as nine.‖ In addition, Students should be exposed to various representations of equations, such as: an operation on the left side of the equal sign and the answer on the right side (5 + 8 = 13) an operation on the right side of the equal sign and the answer on the left side (13 = 5 + 8) numbers on both sides of the equal sign (6 = 6) operations on both sides of the equal sign (5 + 2 = 4 + 3). Students need many opportunities to model equations using cubes, counters, drawings, etc.

MGSE1.OA.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ – 3, 6 + 6 = _.*

This standard extends the work that students do in 1.OA.4 by relating addition and subtraction as related operations for situations with an unknown. This standard builds upon the ―think addition‖ for subtraction problems as explained by Student 2 in MGSE1.OA.6.

Student 1 5 = ___ – 3 I know that 5 plus 3 is 8. So 8 minus 3 is 5.

NUMBERS AND OPERATIONS IN BASE TEN (NBT)

CLUSTER #1: EXTEND THE COUNTING SEQUENCE.

MGSE1.NBT.1 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

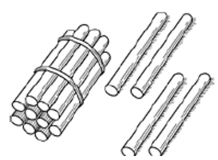
This standard calls for students to rote count forward to 120 by Counting On from any number less than 120. Students should have ample experiences with the hundreds chart to see patterns between numbers, such as all of the numbers in a column on the hundreds

chart have the same digit in the ones place, and all of the numbers in a row have the same digit in the tens place.
This standard also calls for students to read, write and represent a number of objects with a written numeral (number form or standard form). These representations can include cubes, place value (base 10) blocks, pictorial representations or other concrete materials. As students are developing accurate counting strategies they are also building an understanding of how the numbers in the counting sequence are related—each number is one more (or one less) than the number before (or after).

CLUSTER#2: UNDERSTAND PLACE VALUE.
Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

MGSE1.NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones – called a "ten."
This standard asks students to unitize a group of ten ones as a whole unit: a ten. This is the foundation of the place value system. So, rather than seeing a group of ten cubes as ten individual cubes, the student is now asked to see those ten cubes as a bundle – one bundle of ten.



b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

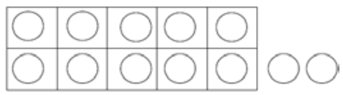
This standard asks students to extend their work from Kindergarten when they composed and decomposed numbers from 11 to 19 into ten ones and some further ones. In Kindergarten, everything was thought of as individual units: ―ones‖. In First Grade, students are asked to unitize those ten individual ones as a whole unit: ―*one* ten‖. Students in first grade explore the idea that the teen numbers (11 to 19) can be expressed as *one* ten and some leftover ones. Ample experiences with ten frames will help develop this concept.

Example:

For the number 12, do you have enough to make a ten? Would you have any leftover? If so, how many leftovers would you have?

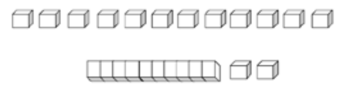
Student 1:

I filled a ten-frame to make one ten and had two counters left over. I had enough to make a ten with some left over. The number 12 has 1 ten and 2 ones.



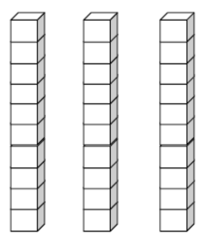
Student 2:

I counted out 12 place value cubes. I had enough to trade 10 cubes for a ten-rod (stick). I now have 1 ten-rod and 2 cubes left over. So the number 12 has 1 ten and 2 ones.



<u>c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).</u>

This standard builds on the work of MGSE.1.NBT.2b. Students should explore the idea that decade numbers (e.g., 10, 20, 30, 40) are groups of tens with no left over ones. Students can represent this with cubes or place value (base 10) rods. (Most first grade students view the ten stick (numeration rod) as ONE. It is recommended to make a ten with unfix cubes or other materials that students can group. Provide students with opportunities to count books, cubes, pennies, etc. Counting 30 or more objects supports grouping to keep track of the number of objects.)



<u>MGSE1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.</u>

This standard builds on the work of MGSE.1.NBT.1 and MGSE.1.NBT.2 by having students compare two numbers by examining the amount of tens and ones in each number. Students are introduced to the symbols greater than (>), less than (<) and equal to (=). Students should have ample experiences communicating their comparisons using words, models and in context before using only symbols in this standard.

Example: 42 ___ 45

Student 1:

42 has 4 tens and 2 ones. 45 has 4 tens and 5 ones. They have the same number of tens, but 45 has more ones than 42. So 45 is greater than 42. So, 42 < 45.

Student 2:

42 is less than 45. I know this because when I count up I say 42 before I say 45. So, 42 < 45.

CLUSTER #4: USE PLACE VALUE UNDERSTANDING AND PROPERTIES OF OPERATIONS TO ADD AND SUBTRACT.

MGSE1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of ten (e.g., 24 + 9, 13 + 10, 27 + 40), using concrete models or drawings and strategies based on place value, properties of operations, and/or relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

This standard calls for students to use concrete models, drawings and place value strategies to add and subtract within 100. Students should not be exposed to the standard algorithm of carrying or borrowing in first grade as this standard focuses on development of conceptual understanding which provides the foundation for flexible use of the standard algorithm.

The inclusion of the statement in the standard, “including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of ten” is not intended to suggest that we exclude problems such as 37+23, but rather that we make sure problems involving two-digit added to one-digit and two-digit added to a multiple of ten are not overlooked.

Example:

There are 37 children on the playground. When a class of 23 students come to the playground, how many students are on the playground altogether?

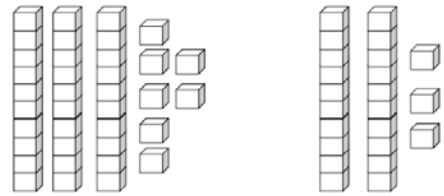
Student 1

I used a hundreds chart. I started at 37 and moved over 3 to land on 40. Then to add 20 I moved down 2 rows and landed on 60. So there are 60 people on the playground.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

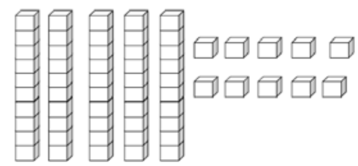
Student 2

I used place value blocks and made a pile of 37 and a pile of 23. I joined the tens and got 50. I then joined the ones and got 10. I then combined those piles and got 60. So there are 60 people on the playground. Relate models to symbolic notation.



Student 3

I broke 37 and 23 into tens and ones. I added the tens and got 50. I added the ones and got 10. I know that 50 and 10 more is 60. So, there are 60 people on the playground. Relate models to symbolic notation.

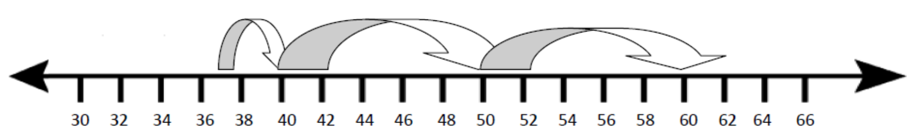


Student 4

Using mental math, I started at 37 and counted on 3 to get 40. Then I added 20 which is 2 tens, to land on 60. So, there are 60 people on the playground.

Student 5

I used the number line. I started at 37. Then I broke up 23 into 20 and 3 in my head. Next, I added 3 ones to get to 40. I then jumped 10 to get to 50 and 10 more to get to 60. So there are 60 people on the playground.



<u>MGSE1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.</u>

This standard builds on students' work with tens and ones by mentally adding ten more and ten less than any number less than 100. Ample experiences with ten frames and the hundreds chart help students use the patterns found in the tens place to solve such problems.

Example:

There are 74 birds in the park. 10 birds fly away. How many are left?

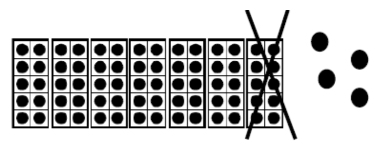
Student 1

I used a 100s board. I started at 74. Then, because 10 birds flew away, I moved back one row. I landed on 64. So, there are 64 birds left in the park.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Student 2

I pictured 7 ten-frames and 4 left over in my head. Since 10 birds flew away, I took one of the ten-frames away. That left 6 ten-frames and 4 left over. So, there are 64 birds left in the park.



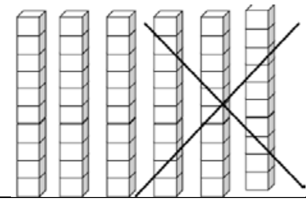
MGSE1.NBT.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range of 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (e.g.,70 – 30, 30 – 10, 60 – 60)

This standard calls for students to use concrete models, drawings and place value strategies to subtract multiples of 10 from decade numbers (e.g., 30, 40, 50).
Example:
There are 60 students in the gym. 30 students leave. How many students are still in the gym?

Student 1
I used a 100s chart and started at 60. I moved up 3 rows to land on 30. There are 30 students left.

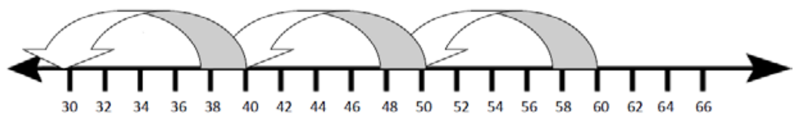
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Student 2
I used place value blocks or unifix cubes to build towers of 10. I started with 6 towers of 10 and removed 3 towers. I had 3 towers left. 3 towers have a value of 30. So there are 30 students left.



Student 3
Using mental math, I solved this subtraction problem. I know that 30 plus 30 is 60, so 60 minus 30 equals 30. There are 30 students left..

Student 4
I used a number line. I started with 60 and moved back 3 jumps of 10 and landed on 30. There are 30 students left.



<u>MGSE1.NBT.7 Identify dimes, and understand ten pennies can be thought of as a dime. (Use dimes as manipulatives in multiple mathematical contexts.)</u>

Information quoted from Van de Walle and Lovin, Teaching Student-Centered mathematics: Grades K-3, page 150)

“The recognition of coins is not a mathematical skill at all. The names of our coins are conventions of our social system. Students learn these names the same way that they learn the names of any physical object in their daily environment-through exposure and repetition.” The value of each coin, a dime is worth 10¢, and so on- is also a convention that students must simply be told. However, a student can say, “A dime is worth 10 cents” and have not really understood what that means. For these values to make sense, students have to have an understanding of 5, 10, 25. More than that, they need to be able to think of these quantities without seeing countable objects. Nowhere else do we say, “this is five,” while pointing to a single item. A child whose number concepts remain tied to counts of objects is not going to be able to understand the values of coins. The social concept of having an equivalent worth or value is nontrivial for the young child. If your students seem to have good concepts of small numbers but still have difficulties with the values of single coins, then your lessons should focus on purchase power-a dime can *buy the same thing* that 10 pennies can buy.”

Give students multiple opportunities to work with dimes and pennies daily. Ideas include (but are not limited to):

- Coin rubbings to help identify pennies and dimes.

- Using the date as a sum of coins by amount and asking what combination of coins could equal today's date. Example: September 14, "What combinations of pennies and dimes could equal 14 cents?"
- Using dimes and pennies as manipulatives.

Developing an understanding of equivalency using resources such as: Coins for Unitary Thinkers- downloadable visual/mats http://ccgpsmathematicsk-5.wikispaces.com/1st+Grade .

Teachers are strongly encouraged to refrain from using strategies such as "hairy money" as this does not develop the connection with place value intended by the standard.

MEASUREMENT AND DATA (MD)

MGSE CLUSTER #1: MEASURE LENGTHS INDIRECTLY AND BY ITERATING LENGTH UNITS.
Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.1
1Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.

MGSE1.MD.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object.

This standard calls for students to indirectly measure objects by comparing the length of two objects by using a third object as a measuring tool. This concept is referred to as transitivity.

Example: Which is longer: the height of the bookshelf or the height of a desk?

Student 1:	**Student 2:**
I used inch cubes to measure the height of the bookshelf and it was 36 cubes long. I used the same pencil to measure the height of the desk and the desk was 24 inch cubes long. Therefore, the bookshelf is taller than the desk.	I used a 1 foot piece of string to measure the bookshelf and it was 3 strings long. I used the same string to measure the height of the desk and it was 2 strings long. Therefore, the bookshelf is taller than the desk.

It is beneficial to use informal units for beginning measurement activities at all grade levels because they allow students to focus on the attributes being measured. The units need to correspond to standard units of measurement and this relationship should always be expressed by the teacher.

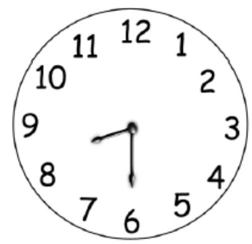
MGSE1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length

<u>measurement of an object is the number of same-size length units that span it with no gaps or overlaps. (Iteration)</u>

This standard asks students to use multiple copies of one object to measure a larger object. This concept is referred to as iteration. Through numerous experiences and careful questioning by the teacher, students will recognize the importance of making sure that there are not any gaps or overlaps in order to get an accurate measurement. This concept is a foundational building block for the concept of area in 3rd Grade.
Example:
How long is the paper in terms of 1 inch paper clips?

MGSE CLUSTER #2: TELL AND WRITE TIME.

<u>MGSE1.MD.3 Tell and write time in hours and half-hours using analog and digital clocks.</u>



This standard calls for students to read both analog and digital clocks and then orally tell and write the time. Times should be limited to the hour and the half-hour. Students need experiences exploring the idea that when the time is at the half-hour the hour hand is between numbers and not on a number. Further, the hour is the number before where the hour hand is. For example, in the clock at the right, the time is 8:30. The hour hand is between the 8 and 9, but the hour is 8 since it is not yet on the 9.

MGSE CLUSTER #3: REPRESENT AND INTERPRET DATA.

<u>MGSE1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.</u>

This standard calls for students to work with categorical data by organizing, representing and interpreting data. Students should have experiences posing a question with 3 possible responses and then work with the data that they collect. For example:
Students pose a question and the 3 possible responses: *Which is your favorite flavor of ice cream? Chocolate, vanilla or strawberry?* Students collect their data by using tallies or another way of keeping track. Students organize their data by totaling each category in a chart or table. Picture and bar graphs are introduced in 2nd Grade.

What is your favorite flavor of ice cream?	
Chocolate	12
Vanilla	5
Strawberry	6

Students interpret the data by comparing categories.
Examples of comparisons:

- What does the data tell us? Does it answer our question?

- More people like chocolate than the other two flavors.
- Only 5 people liked vanilla.
- Six people liked Strawberry.
- 7 more people liked Chocolate than Vanilla.
- The number of people that liked Vanilla was 1 less than the number of people who liked Strawberry.
- The number of people who liked either Vanilla or Strawberry was 1 less than the number of people who liked chocolate.
- 23 people answered this question.

GEOMETRY (G)

MGSE CLUSTER #1: REASON WITH SHAPES AND THEIR ATTRIBUTES.

Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

MGSE1.G.1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size) ; build and draw shapes to possess defining attributes.

This standard calls for students to determine which attributes of shapes are defining compared to those that are non-defining. Defining attributes are attributes that must always be present. Non-defining attributes are attributes that do not always have to be present. The shapes can include triangles, squares, rectangles, and trapezoids.

Asks students to determine which attributes of shapes are defining compared to those that are non-defining. Defining attributes are attributes that help to define a particular shape (#angles, # sides, length of sides, etc.). Non-defining attributes are attributes that do not define a particular shape (color, position, location, etc.). The shapes can include triangles, squares, rectangles, and trapezoids. MGSE1.G.2 includes half-circles and quarter-circles.

Example:

All triangles must be closed figures and have 3 sides. These are defining attributes. Triangles can be different colors, sizes and be turned in different directions, so these are non-defining.

Which figure is a triangle? How do you know this is a tri angle?

Student 1

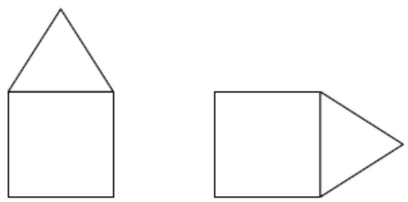
The figure on the left is a triangle. It has three sides. It is also closed.

<u>MGSE1.G.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[1] *This is important for the future development of spatial relations which later connects to developing understanding of area, volume, and fractions.*</u>

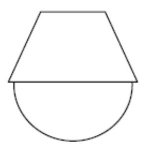
This standard calls for students to compose (build) a two-dimensional or three-dimensional shape from two shapes. This standard includes shape puzzles in which students use objects (e.g., pattern blocks) to fill a larger region. Students do not need to use the formal names such as ―right rectangular prism.‖

Example:

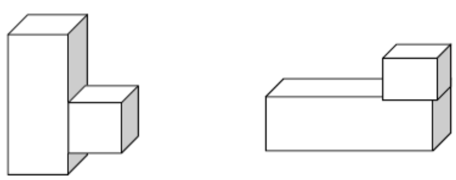
Show the different shapes that you can make by joining a triangle with a square.



Show the different shapes that you can make by joining trapezoid with a half-circle.



Show the different shapes that you can make with a cube and a rectangular prism.



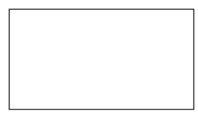
[1] Students do not need to learn formal names such as "right rectangular prism."

<u>MGSE1.G.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.</u>

This standard is the first time students begin partitioning regions into equal shares using a context such as cookies, pies, pizza, etc. This is a foundational building block of fractions, which will be extended in future grades. Students should have ample experiences using the words, *halves, fourths,* and *quarters,* and the phrases *half of, fourth of,* and *quarter of.* Students should also work with the idea of the whole, which is composed of two halves, or four fourths or four quarters.

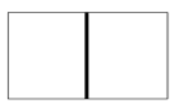
Example:

How can you and a friend share equally (partition) this piece of paper so that you both have the same amount of paper to paint a picture?



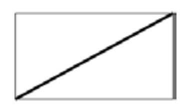
Student 1:

I would split the paper right down the middle. That gives us 2 halves. I have half of the paper and my friend has the other half of the paper.



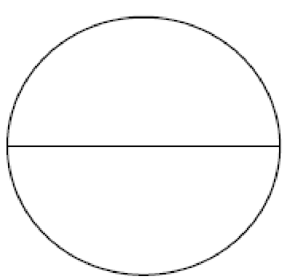
Student 2:

I would split it from corner to corner (diagonally). She gets half the paper. See, if we cut here (along the line), the parts are the same size.



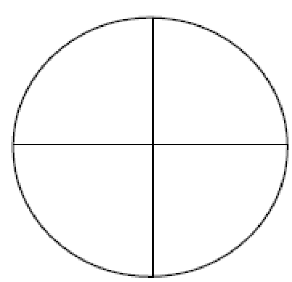
Example:

Teacher: There is pizza for dinner. What do you notice about the slices on the pizza?



Student: There are two slices on the pizza. Each slice is the same size. Those

Teacher: If we cut the same pizza into four slices (fourths), do you think the slices would be the same size, larger, or smaller as the slices on this pizza?



are big slices!	**Student:** When you cut the pizza into fourths, the slices are smaller than the other pizza. More slices mean that the slices get smaller and smaller. I want a slice of that first pizza!

MINDSET and MATHEMATICS

Growth mindset was pioneered by Carol Dweck, Lewis and Virginia Eaton Professor of Psychology at Stanford University. She and her colleagues were the first to identify a link between growth mindset and achievement. They found that students who believed that their ability and intelligence could grow and change, otherwise known as growth mindset, outperformed those who thought that their ability and intelligence were fixed. Additionally, students who were taught that they could grow their intelligence actually did better over time. Dweck's research showed that an increased focus on the process of learning, rather than the outcome, helped increase a student's growth mindset and ability.
(from WITH+MATH=I CAN)

Jo Boaler, Professor of Mathematics Education at the Stanford Graduate School of Education and author of *Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages, and Innovative Teaching*, was one of the first to apply growth mindset to math achievement.
You can learn how to use the power of growth mindset for yourself and your students here:

https://www.amazon.com/gp/withmathican

https://www.mindsetkit.org/topics/about-growth-mindset

https://www.youcubed.org/

Growth and Fixed Mindset images courtesy of Katherine Lynas (katherinelynas.com). Thank you, Katherine!

VERTICAL UNDERSTANDING of the MATHEMATICS LEARNING TRAJECTORY

Why does it matter if you know what happens in mathematics in the grades before and after the one you teach? Isn't it enough just to know and understand the expectations for your grade?

There are many reasons to devote a bit of your time to the progression of standards.

You'll:

- Deepen your understanding of how development of algebraic thinking has proven to be a critical element of student mathematics success as they transition from elementary to middle school. Elementary and middle school teachers must understand how algebraic thinking develops prior to their grade, in their grade, and beyond their grade in order to support student algebraic thinking

- Know what to expect when students show up in your grade because you know what they should understand from the years before
- Understand how conceptual understanding develops, making it easier to help students who have missing bits and pieces
- Be able to help students to see the connections between ideas in mathematics in your grade and beyond, helping them to connect to what they already know and what is to come
- Assess understanding more completely, and develop better assessments
- Know what the teachers in the grades to come expect your students to know and understand
- Plan more effectively with same-grade and other-grade colleagues
- Deepen your understanding of the mathematics of your grade

We aren't asking you to take a month off to study up, just asking that you reference the following resources when you want to deepen your understanding of where students are in their mathematics learning, understand why they are learning what they are learning in your grade, and understand the mathematical ideas and connections within your grade and beyond.

Resources:

The Coherence Map:

http://achievethecore.org/page/1118/coherence-map This resource diagrams the connections between standards, provides explanations of standards, provides example tasks for many standards, and links to the progressions document when further detail is required.

A visual learning trajectory of:

Addition and Subtraction - http://gfletchy.com/2016/03/04/the-progression-of-addition-and-subtraction/
Multiplication - http://gfletchy.com/2015/12/18/the-progression-of-multiplication/
Division - http://gfletchy.com/2016/01/31/the-progression-of-division/
(Many thanks to Graham Fletcher, the genius behind these videos)

The Mathematics Progression Documents:

http://math.arizona.edu/~ime/progressions/

Learning Trajectories in Mathematics-

http://www.cpre.org/images/stories/cpre_pdfs/learning%20trajectories%20in%20math_ccii%20report.pdf

RESEARCH of INTEREST to MATHEMATICS TEACHERS

Social Emotional Learning and Math- http://curry.virginia.edu/uploads/resourceLibrary/Teachers_support_for_SEL_contributes_to_improved_math_teaching_and_learning,_et_al.pdf

Why how you teach math is important- https://www.youcubed.org/

GloSS/IKAN

GloSS and IKAN information can be found here: http://ccgpsmathematics6-8.wikispaces.com/GLoSS+%26+IKAN While this page 'lives' on the 6-8 math wiki, the information provided is useful to teachers of grades K-8.

The GloSS and IKAN professional learning video found here: https://www.georgiastandards.org/Georgia-Standards/Pages/FOA/Foundations-of-Algebra-Day-1.aspx provides an in-depth look at the GloSS and IKAN. While it was created for teachers of Foundations of Algebra, the information is important for teachers of grades K- 12.

The GloSS and IKAN prezi found on georgiastandards.org, here: https://www.georgiastandards.org/Georgia-Standards/Pages/Global-Strategy-Stage-GloSS-and-Individual-Knowledge-Assessment-of-Number-IKAN.aspx

FLUENCY

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply "how to get the answer" and instead support students' ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students:

- flexibly use a combination of deep understanding, number sense, and memorization.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.

For more about fluency, see:

http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf

and:

https://bhi61nm2cr3mkdgk1dtaov18-wpengine.netdna-ssl.com/wp-content/uploads/nctm-timed-tests.pdf

ARC OF LESSON (OPENING, WORK SESSION, CLOSING)

"When classrooms are workshops-when learners are inquiring, investigating, and constructing-there is already a feeling of community. In workshops learners talk to one another, ask one another questions, collaborate, prove, and communicate their thinking to one another. The heart of math workshop is this: investigations and inquiries are ongoing, and teachers try to find situations and structure contexts that will enable children to mathematize their lives- that will move the community toward the horizon. Children have the opportunity to explore, to pursue inquiries, and to model and solve problems on their own in creative ways. Searching for patterns, raising questions, and constructing one's own models, ideas, and strategies are the primary activities of math workshop. The classroom becomes a community of learners engaged in activity, discourse, and reflection." *Young Mathematicians at Work- Constructing Addition and Subtraction* by Catherine Twomey Fosnot and Maarten Dolk.

"Students must believe that the teacher does not have a predetermined method for solving the problem. If they suspect otherwise, there is no reason for them to take risks with their own ideas and methods." *Teaching Student-Centered Mathematics, K-3* by John Van de Walle and Lou Ann Lovin.

Opening: Set the stage
Get students mentally ready to work on the task
Clarify expectations for products/behavior
How?

- Begin with a simpler version of the task to be presented
- Solve problem strings related to the mathematical idea/s being investigated
- Leap headlong into the task and begin by brainstorming strategies for approaching the task

- Estimate the size of the solution and reason about the estimate

Make sure everyone understands the task before beginning. Have students restate the task in their own words. Every task should require more of the students than just the answer.

Work session: Give 'em a chance
Students- grapple with the mathematics through sense-making, discussion, concretizing their mathematical ideas and the situation, record thinking in journals
Teacher- Let go. Listen. Respect student thinking. Encourage testing of ideas. Ask questions to clarify or provoke thinking. Provide gentle hints. Observe and assess.

Closing: Best Learning Happens Here
Students- share answers, justify thinking, clarify understanding, explain thinking, question each other
Teacher- Listen attentively to all ideas, ask for explanations, offer comments such as, "Please tell me how you figured that out." "I wonder what would happen if you tried…"
Anchor charts
Read Van de Walle K-3, Chapter 1

BREAKDOWN OF A TASK (UNPACKING TASKS)

How do I go about tackling a task or a unit?

1. Read the unit in its entirety. Discuss it with your grade level colleagues. Which parts do you feel comfortable with? Which make you wonder? Brainstorm ways to implement the tasks. Collaboratively complete the culminating task with your grade level colleagues. As students work through the tasks, you will be able to facilitate their learning with this end in mind. The structure of the units/tasks is similar task to task and grade to grade. This structure allows you to converse in a vertical manner with your colleagues, school-wide. There is a great deal of mathematical knowledge and teaching support within each grade level guide, unit, and task.

2. Read the first task your students will be engaged in. Discuss it with your grade level colleagues. Which parts do you feel comfortable with? Which make you wonder? Brainstorm ways to implement the tasks.

3. If not already established, use the first few weeks of school to establish routines and rituals, and to assess student mathematical understanding. You might use some of the tasks found in the unit, or in some of the following resources as beginning tasks/centers/math tubs which serve the dual purpose of allowing you to observe and assess.

   Additional Resources:
   Math Their Way: http://www.center.edu/MathTheirWay.shtml
   NZMaths- http://www.nzmaths.co.nz/numeracy-development-projects-books?parent_node=

K-5 Math Teaching Resources- http://www.k-5mathteachingresources.com/index.html
(this is a for-profit site with several free resources)
Winnepeg resources- http://www.wsd1.org/iwb/math.htm
Math Solutions- http://www.mathsolutions.com/index.cfm?page=wp9&crid=56

4. **Points to remember:**
   - **Each task begins with a list of the standards specifically addressed in that task, however, *that does not mean that these are the only standards addressed in the task*. Remember, standards build on one another, and mathematical ideas are connected.**
   - **Tasks are made to be modified to match your learner's needs. If the names need changing, change them. If the specified materials are not available, use what is available. If a task doesn't go where the students need to go, modify the task or use a different resource.**
   - **The units are not intended to be all encompassing. Each teacher and team will make the units their own, and add to them to meet the needs of the learners.**

ROUTINES AND RITUALS

Teaching Math in Context and Through Problems

"By the time they begin school, most children have already developed a sophisticated, informal understanding of basic mathematical concepts and problem solving strategies. Too often, however, the mathematics instruction we impose upon them in the classroom fails to connect with this informal knowledge" (Carpenter et al., 1999). The 8 Standards of Mathematical Practices (SMP) should be at the forefront of every mathematics lessons and be the driving factor of HOW students learn.

One way to help ensure that students are engaged in the 8 SMPs is to construct lessons built on context or through story problems. "Fosnot and Dolk (2001) point out that in story problems children tend to focus on getting the answer, probably in a way that the teacher wants. "Context problems, on the other hand, are connected as closely as possible to children's lives, rather than to 'school mathematics'. They are designed to anticipate and to develop children's mathematical modeling of the real world."

Traditionally, mathematics instruction has been centered around a lot of problems in a single math lesson, focusing on rote procedures and algorithms which do not promote conceptual understanding. Teaching through word problems and in context is difficult however, "kindergarten students should be expected to solve word problems" (Van de Walle, K-3).

A *problem* is defined as any task or activity for which the students have no prescribed or memorized rules or methods, nor is there a perception by students that there is a specific correct solution method. A problem for learning mathematics also has these features:

- *The problem must begin where the students are which makes it accessible to all learners.*
- *The problematic or engaging aspect of the problem must be due to the mathematics that the students are to learn.*
- *The problem must require justifications and explanations for answers and methods.*

It is important to understand that mathematics is to be taught *through* problem solving. That is, problem-based tasks or activities are the vehicle through which the standards are taught. Student learning is an outcome of the problem-solving process and the result of teaching within context and through the Standards for Mathematical Practice. (Van de Walle and Lovin, Teaching Student-Centered Mathematics: K-3, page 11).

Use of Manipulatives

"It would be difficult for you to have become a teacher and not at least heard that the use of manipulatives, or a "hands-on approach," is the recommended way to teach mathematics. There is no doubt that these materials can and should play a significant role in your classroom. Used correctly they can be a positive factor in children's learning. But they are not a cure-all that some educators seem to believe them to be. It is important that you have a good perspective on how manipulatives can help or fail to help children construct ideas. We can't just give students a ten-frame or bars of Unifix cubes and expect them to develop the mathematical ideas that these manipulatives can potentially represent. When a new model or new use of a familiar model is introduced into the classroom, it is generally a good idea to explain how the model is used and perhaps conduct a simple activity that illustrates this use. "
(Van de Walle and Lovin, Teaching Student-Centered Mathematics: K-3, page 6).

Once you are comfortable that the models have been explained, you should not force their use on students. Rather, students should feel free to select and use models that make sense to them. In most instances, not using a model at all should also be an option. The choice a student makes can provide you with valuable information about the level of sophistication of the student's reasoning.

Whereas the free choice of models should generally be the norm in the classroom, you can often ask students to model to show their thinking. This will help you find out about a child's understanding of the idea and also his or her understanding of the models that have been used in the classroom.

The following are simple rules of thumb for using models:

- Introduce new models by showing how they can represent the ideas for which they are intended.
- Allow students (in most instances) to select freely from available models to use in solving problems.
- Encourage the use of a model when you believe it would be helpful to a student having difficulty." (Van de Walle and Lovin, Teaching Student-Centered Mathematics: K-3, page 8-9)

- Modeling also includes the use of mathematical symbols to represent/model the concrete mathematical idea/thought process/situation. This is a very important, yet often neglected step along the way. Modeling can be concrete, representational, and abstract. Each type of model is important to student understanding. Modeling also means to "mathematize" a situation or problem, to take a situation which might at first glance not seem mathematical, and view it through the lens of mathematics. For example, students notice that the cafeteria is always out of their favorite flavor of ice cream on ice cream days. They decide to survey their schoolmates to determine which flavors are most popular, and share their data with the cafeteria manager so that ice cream orders reflect their findings. The problem: Running out of ice cream flavors. The solution: Use math to change the flavor amounts ordered.

Use of Strategies and Effective Questioning

Teachers ask questions all the time. They serve a wide variety of purposes: to keep learners engaged during an explanation; to assess their understanding; to deepen their thinking or focus their attention on something. This process is often semi-automatic. Unfortunately, there are many common pitfalls. These include:

- asking questions with no apparent purpose;
- asking too many closed questions;
- asking several questions all at once;
- poor sequencing of questions;
- asking rhetorical questions;
- asking 'Guess what is in my head' questions;
- focusing on just a small number of learners;
- ignoring incorrect answers;
- not taking answers seriously.

In contrast, the research shows that effective questioning has the following characteristics:

- Questions are planned, well ramped in difficulty.
- Open questions predominate.
- A climate is created where learners feel safe.
- A 'no hands' approach is used, for example when all learners answer at once using mini-whiteboards, or when the teacher chooses who answers.
- Probing follow-up questions are prepared.
- There is a sufficient 'wait time' between asking and answering a question.
- Learners are encouraged to collaborate before answering.
- Learners are encouraged to ask their own questions.

0-99 Chart or 1-100 Chart

(Adapted information from About Teaching Mathematics A K–8 RESOURCE MARILYN BURNS 3rd edition and Van de Walle)

Both the 0-99 Chart and the 1-100 Chart are valuable tools in the understanding of mathematics. Most often these charts are used to reinforce counting skills. Counting involves two separate skills: (1) ability to produce the standard list of counting words (i.e. one, two, three) and (2) the ability to connect the number sequence in a one-to-one manner with objects (Van de Walle, 2007). The counting sequence is a rote procedure. The ability to attach meaning to counting is “the key conceptual idea on which all other number concepts are developed” (Van de Walle, p. 122). Children have greater difficulty attaching meaning to counting than rote memorization of the number sequence. Although both charts can be useful, the focus of the 0-99 chart should be at the forefront of number sense development in early elementary.

A 0-99 Chart should be used in place of a 1-100 Chart when possible in early elementary mathematics for many reasons, but the overarching argument for the 0-99 is that it helps to develop a deeper understanding of place value. Listed below are some of the benefits of using the 0-99 Chart in your classroom:

- A 0-99 Chart begins with zero where as a hundred’s chart begins with 1. It is important to include zero because it is a digit and just as important as 1-9.
- A 1-100 chart puts the decade numerals (10, 20, 30, etc.) on rows without the remaining members of the same decade. For instance, on a hundred’s chart 20 appears at the end of the teens’ row. This causes a separation between the number 20 and the numbers 21-29. The number 20 is the beginning of the 20’s family; therefore it should be in the beginning of the 20’s row like in a 99’s chart to encourage students to associate the quantities together.
- A 0-99 chart ends with the last two digit number, 99, this allows the students to concentrate their understanding using numbers only within the ones’ and tens’ place values. A hundred’s chart ends in 100, introducing a new place value which may change the focus of the places.
- The understanding that 9 units fit in each place value position is crucial to the development of good number sense. It is also very important that students recognize that zero is a number, not merely a placeholder. This concept is poorly modeled by a typical 1-100 chart, base ten manipulatives, and even finger counting. We have no "zero" finger, "zero" block, or "zero" space on typical 1-100 number charts. Whereas having a zero on the chart helps to give it status and reinforces that zero holds a quantity, a quantity of none. Zero is the answer to a question such as, “How many elephants are in the room?”.
- Including zero presents the opportunity to establish zero correctly as an even number, when discussing even and odd. Children see that it fits the same pattern as all of the other even numbers on the chart.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

While there are differences between the 0-99 Chart and the 1-100 Chart, both number charts are valuable resources for your students and should be readily available in several places around the classroom. Both charts can be used to recognize number patterns, such as the increase or decrease by multiples of ten. Provide students the opportunity to explore the charts and communicate the patterns they discover.

The number charts should be placed in locations that are easily accessible to students and promote conversation. Having one back at your math calendar/bulletin board area provides you the opportunity to use the chart to engage students in the following kinds of discussions. Ask students to find the numeral that represents:

- the day of the month
- the month of the year
- the number of students in the class
- the number of students absent or any other amount relevant to the moment.

Using the number is 21, give directions and/or ask questions similar to those below.

- Name a number greater than 21.
- Name a number less than 21.
- What number is 3 more than/less than 21?
- What number is 5 more than/less than 21?
- What number is 10 more than/less than 21?
- Is 21 even or odd?
- What numbers live right next door to 21?

Ask students to pick an even number and explain how they know the number is even. Ask students to pick an odd number and explain how they know it is odd. Ask students to count by 2's, 5's or 10's. Tell them to describe any patterns that they see. *(Accept any patterns that students are able to justify. There are many right answers!)*

Number Lines

The use of number lines in elementary mathematics is crucial in students' development of number and mathematical proficiency. While the CCGPS explicitly state use number lines in grades 2-5, number lines should be used in all grade levels and in multiple settings.

According to John Van de Walle,

> A number line is also a worthwhile model, but can initially present conceptual difficulties for children below second grade and students with disabilities (National Research Council Committee, 2009) This is partially due to their difficulty in seeing the unit, which is a challenge when it appears in a continuous line. A number line is also a shift from counting a number of individual objects in a collection to continuous length units. There are, however, ways to introduce and model number lines that support young

learners as they learn this representation. Familiarity with a number line is essential because third grade students will use number lines to locate fractions and add and subtract time intervals, fourth graders will locate decimals and use them for measurement, and fifth graders will use perpendicular number lines in coordinate grids (CCSSO,2010).

A number line measures distance from zero the same way a ruler does. If you don't actually teach the use of the number line through emphasis on the unit (length), students may focus on the hash marks or numerals instead of the spaces (a misunderstanding that becomes apparent when their answers are consistently off by one). At first students can build a number path by using a given length, such as a set of Cuisenaire rods of the same color to make a straight line of multiple single units (Van de Walle and Lovin, Teaching Student-Centered Mathematics: 3-5 pg. 106-107)

Open number lines are particularly useful for building students' number sense. They can also form the basis for discussions that require the precise use of vocabulary and quantities, and are therefore a good way to engage students in the Standards for Mathematical Practice.

While the possibilities for integrating number lines into the mathematics classroom are endless, the following are some suggestions/ideas:

- On a bulletin board, attach a string which will function as an open number line. Each morning (or dedicated time for math routines) put a new number on each student's desk. Using the some type of adhesive (thumb tack, tape, etc.), students will place the number in the appropriate location on the string. In the beginning of the year, provide students with numbers that are more familiar to them. As the year progresses, move through more complex problems such as skip counting, fractions, decimals or other appropriate grade level problems. Through daily integration, the number line becomes part of the routine. Following the number placement, have a brief discussion/debriefing of the reasoning used by students to place the numbers.
- In the 3-Act tasks placed throughout the units, students will be provided opportunities to use an open number line to place estimates that are too low, too high and just right as related to the posed problem. Similar opportunities can also be used as part of a daily routine.

Math Maintenance Activities

In addition to instruction centered on the current unit of study, the math instructional block should include time devoted to reviewing mathematics that have already been taught, previewing upcoming mathematics, and developing mental math and estimation skills. There is a saying that if you don't use it, you'll lose it. If students don't have opportunities to continuously apply and refine the math skills they've learned previously, then they may forget how to apply what they've learned. Unlike vocabulary words for literacy, math vocabulary words are not used much outside math class, so it becomes more important to use those words in discussions regularly. Math maintenance activities incorporate review and preview of math concepts and vocabulary and help students make connections across domains. It's recommended that 15 to 30 minutes of the math instructional block be used for these math maintenance activities each day. It's not necessary nor is it recommended that teachers do every activity every day. Teachers should strive for a balance of math maintenance activities so that over the course of a week, students are exposed to a variety of these activities. Math maintenance time may occur before or after instruction related to the current math unit, or it can occur at a different time during the day.

The goals of this maintenance time should include:

- Deepening number sense, including subitizing, flexible grouping of quantities, counting forward and backward using whole numbers, fractions, decimals and skip counting starting at random numbers or fractional amounts
- Developing mental math skills by practicing flexible and efficient numerical thinking through the use of operations and the properties of operations
- Practicing estimation skills with quantities and measurements such as length, mass, and liquid volume, depending on grade level
- Practicing previously-taught skills so that students deepen and refine their understanding
- Reviewing previously-taught concepts that students struggled with as indicated on their assessments, including gaps in math concepts taught in previous grade levels
- Using a variety of math vocabulary terms, especially those that are used infrequently
- Practicing basic facts using strategies learned in previous grade levels or in previous units to develop or maintain fluency
- Previewing prerequisite skills for upcoming math units of study
- Participating in mathematical discussions with others that require students to construct viable arguments and critique the reasoning of others

To accomplish these goals, math maintenance activities can take many different forms. Some activities include:

- Number Corner or Calendar Time
- Number Talks
- Estimation Activities/Estimation 180
- Problem of the Day or Spiraled Review Problems

In addition, math discussions, math journals and math games are appropriate not only for the current unit of study, but also for maintaining math skills that were previously taught.

Although there are commercially-available materials to use for math maintenance activities, there are also many excellent websites and internet resources that are free for classroom use.

Here is a partial list of some recommended resources. A more detailed explanation of some of these components follows below.

Math Maintenance Activity	**Possible Resources**
Number Corner or Calendar Time	• http://teachelemmath.weebly.com/calendar.html • Every Day Counts Calendar Math from Houghton Mifflin Harcourt • Number Corner from The Math Learning Center
Number Talks	• *Number Talks* by Sherry Parrish • *http://kentuckymathematics.org/number_talk_resources.php*
Estimation Activities/Estimation 180	• http://www.estimation180.com/
Problem of the Day/Spiraled Review Problems	• www.insidemathematics.org • http://nzmaths.co.nz/teaching-material • http://www.k-5mathteachingresources.com/ • http://mathlearnnc.sharpschool.com/cms/One.aspx?portalId=4507283&pageId=5856325 • *Children's Mathematics: Cognitively Guided Instruction* by Thomas Carpentar, et. Al. • *Extending Children's Mathematics: Fractions and Decimals* by Epson and Levi

Number Corner/Calendar Time

Number Corner is a time set aside to go over mathematics skills (Standards for calendar can be found in Social Studies) during the primary classroom day. This should be an interesting and motivating time for students. A calendar board or corner can be set up and there should be several elements that are put in place. The following elements should be set in place for students to succeed during Number Corner:

1. a safe environment
2. concrete models or math tools
3. opportunities to think first and then discuss
4. student interaction

Number Corner should relate several mathematics concepts/skills to real life experiences. This time can be as simple as reviewing the months, days of the week, temperature outside, and the schedule for the day, but some teachers choose to add other components that integrate more standards. Number Corner should be used as a time to engage students in a discussion about events which can be mathematized, or as a time to engage in Number Talks.

- Find the number ___ .
- If I have a nickel and a dime, how much money do I have? (any money combination)

- What is ___ more than ___?
- What is ___ less than ___?
- Mystery number: Give clues and they have to guess what number you have.
- This number has ___tens and ___ ones. What number am I?
- What is the difference between ___ and ____?
- What number comes after ___? before ___?
- Tell me everything you know about the number ____. (Anchor Chart)

Number Corner is also a chance to familiarize your students with Data Analysis. This creates an open conversation to compare quantities, which is a vital process that must be explored before students are introduced to addition and subtraction.

- At first, choose questions that have only two mutually exclusive answers, such as yes or no (e.g., Are you a girl or a boy?), rather than questions that can be answered yes, no, or maybe (or sometimes). This sets up the part-whole relationship between the number of responses in each category and the total number of students present and it provides the easiest comparison situation (between two numbers; e.g., Which is more? How much more is it?). Keep in mind that the concept of less than (or fewer) is more difficult than the concept of greater than (or more). Be sure to frequently include the concept of less in your questions and discussions about comparisons.
  - Later, you can expand the questions so they have more than two responses. Expected responses may include maybe, I'm not sure, I don't know or a short, predictable list of categorical responses (e.g., In which season were you born?).
  - Once the question is determined, decide how to collect and represent the data. Use a variety of approaches, including asking students to add their response to a list of names or tally marks, using Unifix cubes of two colors to accumulate response sticks, or posting 3 x 5 cards on the board in columns to form a bar chart.
  - The question should be posted for students to answer. For example, "Do you have an older sister?" Ask students to contribute their responses in a way that creates a simple visual representation of the data, such as a physical model, table of responses, bar graph, etc.
  - Each day, ask students to describe, compare, and interpret the data by asking questions such as these: "What do you notice about the data? Which group has the most? Which group has the least? How many more answered [this] compared to [that]? Why do you suppose more answered [this]?" Sometimes ask data gathering questions: "Do you think we would get similar data on a different day? Would we get similar data if we asked the same question in another class? Do you think these answers are typical for first graders? Why or why not?"
  - Ask students to share their thinking strategies that justify their answers to the questions. Encourage and reward attention to specific details. Focus on relational thinking and problem solving strategies for making comparisons. Also pay attention to identifying part-whole relationships; and reasoning that leads to interpretations.

- Ask students questions about the ideas communicated by the representation used. What does this graph represent? How does this representation communicate this information clearly? Would a different representation communicate this idea better?
- The representation, analysis, and discussion of the data are the most important parts of the routine (as opposed to the data gathering process or the particular question being asked). These mathematical processes are supported by the computational aspects of using operations on the category totals to solve part-whole or "compare" problems.

Number Talks

In order to be mathematically proficient, students must be able to compute accurately, efficiently, and flexibly. Daily classroom number talks provide a powerful avenue for developing "efficient, flexible, and accurate computation strategies that build upon the key foundational ideas of mathematics." (Parrish, 2010) Number talks involve classroom conversations and discussions centered upon purposefully planned computation problems.

In Sherry Parrish's book, Number Talks: Helping Children Build Mental Math and Computation Strategies, teachers will find a wealth of information about Number Talks, including:

- Key components of Number Talks
- Establishing procedures
- Setting expectations
- Designing purposeful Number Talks
- Developing specific strategies through Number Talks

There are four overarching goals upon which K-2 teachers should focus during Number Talks. These goals are:

1. Developing number sense
2. Developing fluency with small numbers
3. Subitizing
4. Making Tens

Number talks are a great way for students to use mental math to solve and explain a variety of math problems. A Number Talk is a short, ongoing daily routine that provides students with meaningful ongoing practice with computation. Number Talks should be structured as short sessions alongside (but not necessarily directly related to) the ongoing math curriculum. A great place to introduce a Number Talk is during Number Corner/Calendar Time. It is important to keep Number Talks short, as they **are not intended to replace current curriculum or take up the majority of the time spent on mathematics**. In fact, teachers only need to spend 5 to 15 minutes on Number Talks. Number Talks are most effective when done every day. The primary goal of Number Talks is computational fluency. Children develop computational fluency while thinking and reasoning like mathematicians. When they share their strategies with others, they learn to clarify and express their thinking, thereby developing mathematical language. This in turn serves them well when they are asked to express their mathematical processes in writing. In

order for children to become computationally fluent, they need to know particular mathematical concepts that go beyond what is required to memorize basic facts or procedures.
Students will begin to understand major characteristics of numbers, such as:

- Numbers are composed of smaller numbers.
- Numbers can be taken apart and combined with other numbers to make new numbers.
- What we know about one number can help us figure out other numbers.
- What we know about parts of smaller numbers can help us with parts of larger numbers.
- Numbers are organized into groups of tens and ones (and hundreds, tens and ones and so forth).
- What we know about numbers to 10 helps us with numbers to 100 and beyond.

All Number Talks follow a basic six-step format. The format is always the same, but the problems and models used will differ for each number talk.

1. **Teacher presents the problem.** Problems are presented in many different ways: as dot cards, ten frames, sticks of cubes, models shown on the overhead, a word problem or a written algorithm. Strategies are *not explicitly taught* to students, instead the problems presented lead to various strategies.
2. **Students figure out the answer.** Students are given time to figure out the answer. To make sure students have the time they need, the teacher asks them to give a “thumbs-up” when they have determined their answer. The thumbs up signal is unobtrusive- a message to the teacher, not the other students.
3. **Students share their answers**. Four or five students volunteer to share their answers and the teacher records them on the board.
4. **Students share their thinking.** Three or four students volunteer to share how they got their answers. (Occasionally, students are asked to share with the person(s) sitting next to them.) The teacher records the student's thinking.
5. **The class agrees on the "real" answer for the problem.** The answer that together the class determines is the right answer is presented as one would the results of an experiment. The answer a student comes up with initially is considered a conjecture. Models and/or the logic of the explanation may help a student see where their thinking went wrong, may help them identify a step they left out, or clarify a point of confusion. There should be a sense of confirmation or clarity rather than a feeling that each problem is a test to see who is right and who is wrong. A student who is still unconvinced of an answer should be encouraged to keep thinking and to keep trying to understand. For some students, it may take one more experience for them to understand what is happening with the numbers and for others it may be out of reach for some time. The mantra should be, "If you are not sure or it doesn't make sense yet, keep thinking."
6. **The steps are repeated for additional problems**.

Similar to other procedures in your classroom, there are several elements that must be in place to ensure students get the most from their Number Talk experiences. These elements are:

1. A safe environment
2. Problems of various levels of difficulty that can be solved in a variety of ways
3. Concrete models
4. Opportunities to think first and then check
5. Interaction
6. Self-correction

Estimation 180

Estimation is a skill that has many applications, such as checking computation answers quickly. Engaging in regular estimation activities will develop students' reasoning skills, number sense, and increase their repertoire of flexible and efficient strategies. As students gain more experiences with estimation, their accuracy will improve.

According to John Van de Walle, there are three types of estimation that students should practice:

- Measurement estimation – determining an approximate measurement, such as weight, length, or capacity
- Quantity estimation – approximating the number of items in a collection
- Computational estimation – determining a number that is an approximation of a computation

One resource which provides contexts for all three types of estimation is Andrew Stadel's website, http://www.estimation180.com/. In his website, Mr. Stadel has posted daily estimation contexts. Here are his directions for using his website:

1. Click on a picture.
2. Read the question.
3. Look for context clues.
4. Make an estimate.
5. Tell us how confident you are.
6. Share your reasoning (what context clues did you use?).
7. See the answer.
8. See the estimates of others.

The most important part is step #6. After you make an estimate, feel free to give a brief description. It's so valuable to a classroom when students share their logic or use of context clues when formulating an estimate.

Andrew Stadel has collaborated with Michael Fenton to create a recording sheet for students to use with the estimation contexts on the website. The recording sheet can also be found at http://www.estimation180.com/. Here are his directions for the recording sheet:

Day #	Description	⬇ Too Low	Too High ⬆	My Estimate	My Reasoning	Answer	Error	|Error| as %
Ex. A	Tyler's age (months)	24	36	30	He looks a little older than my cousin (who is 2)	26	+ – 4	4/26 ≈ 15%
Ex. B	*Bohemian Rhapsody*	4:00	5:00	4:30	10% of song = 30 sec 300 sec total = 5 min	5:56	+ – 86	86/356 ≈ 24%
							+ –	

Column use descriptions from Andrew Stadel:

Day #
In Estimation 180's first year, I was just trying to keep up with creating these estimation challenges in time for use in my own classroom. There really wasn't a scope and sequence involved. That said, now that there are over 160 estimation challenges available, teachers and students can use them at any time throughout the school year and without completing them in sequential order. Therefore, use the *Day #* column simply to number your daily challenges according to the site. Tell your students or write it up on the board that you're doing the challenge from Day 135 even though you might be on the fifth day of school.

Description
In my opinion, this column is more important than the *Day #* column. Don't go crazy here. Keep it short and sweet, but as specific as possible. For example, there's a lot of scattered height estimates on the site. Don't write down "How tall?" for Day 110. Instead write "Bus height" because when you get to Day 111, I'd write in "Parking structure height". I believe the teacher has the ultimate say here, but it can be fun to poll your students for a short description in which you all can agree. Give students some ownership, right? If unit measurement is involved, try and sneak it in here. Take Day 125 for instance. I'd suggest entering "Net Wt. (oz.) of lg Hershey's bar." Keep in mind that Day 126 asks the same question, but I'd suggest you encourage your class to use pounds if they don't think of it.
*By the way, sometimes unit measurement(s) are already included in the question. Use discretion.

Too Low
Think of an estimate that is too low.
Don't accept one (1), that's just rubbish, unless one (1) is actually applicable to the

context of the challenge. Stretch your students. Think of it more as an answer that's too low, but reasonably close. After all, this is a site of estimation challenges, not gimmes.

Too High
Refer to my notes in *Too Low*. Just don't accept 1 billion unless it's actually applicable. Discuss with students the importance of the *Too Low* and *Too High* sections: we are trying to eliminate wrong answers while creating a range of possible answers.

My Estimate
This is the place for students to fill in their answer. If the answer requires a unit of measurement, we better see one. Not every estimation challenge is "How many..." marshmallows? or christmas lights? or cheese balls?
Even if a unit of measurement has already been established (see the *Description* notes), I'd still encourage your students to accompany their numerical estimate with a unit of measurement.

For example, on Day 41, "What's the height of the Giant [Ferris] Wheel?" use what makes sense to you, your students and your country's customary unit of measurement. Discuss the importance of unit measurements with students. Don't accept 108. What does that 108 represent? Pancakes? Oil spills? Bird droppings? NO! It represents 108 feet.

My Reasoning
The *My Reasoning* section is the most recent addition to the handout and I'm extremely thrilled about it. This is a student's chance to shine! Encourage their reasoning to be short and sweet. When a student writes something down, they'll be more inclined to share it or remember it. Accept bullet points or phrases due to the limited space. We don't need students to write paragraphs. However, we are looking for students to identify any context clues they used, personal experiences, and/or prior knowledge. Hold students accountable for their reasoning behind the estimate of the day.

Don't let student reasoning go untapped! If you're doing a sequence of themed estimation challenges, don't accept, "I just guessed" after the first day in the sequence. For example, if you're doing the flight distance themed estimate challenges starting on Day 136, you will establish the distance across the USA on the first day. Sure, go ahead and guess on Day 136, but make sure you hold students accountable for their reasoning every day thereafter.
Have students share their reasoning before and after revealing the answer. Utilize Think-Pair-Share. This will help create some fun conversations *before* revealing the answer.

After revealing the answer, get those who were extremely close (or correct) to share their reasoning. I bet you'll have some great mathematical discussions. I'm also curious to hear from those that are way off and how their reasoning could possibly be improved.

I'd say the *My Reasoning* section was born for Mathematical Practice 3: *Construct viable arguments and critique the reasoning of others.* Keep some of these thoughts in mind regarding Mathematical Practice 3:

- Explain and defend your estimate.
- Construct a detailed explanation referencing context clues, prior knowledge, or previous experiences.
- Invest some confidence in it.
- Try to initiate a playful and respectful argument in class.
- Ask "Was anyone convinced by this explanation? Why? Why not?" or"Are you guys going to let [student name] off the hook with that explanation?"

There's reasoning behind every estimate (not guess).

- Find out what that reasoning is!
- DON'T let student reasoning go untapped!

Answer
Jot down the revealed answer. I'd also encourage students to write down the unit of measurement used in the answer. The answer might use a different unit of measurement than what you and your class agreed upon. Take the necessary time to discuss the most relative unit of measurement. I might be subjectively wrong on some of the answers posted. As for more thoughts on unit of measurement, refer to the *My Estimate* notes above. Continue having mathematical discussion after revealing the answer. Refer to my notes regarding the use of Mathematical Practice 3 in the *My Reasoning* section.

Error
Find the difference between *My Estimate* and *Answer*. Have students circle either the "+" or the "-" if they didn't get it exactly correct.
+ Your estimate was **greater** than (above) the actual answer.
- Your estimate was **less** than (below) the actual answer.

Mathematize the World through Daily Routines
The importance of continuing the established classroom routines cannot be overstated. Daily routines must include such obvious activities such as taking attendance, doing a lunch count, determining how many items are needed for snack, lining up in a variety of ways (by height, age, type of shoe, hair color, eye color, etc.), daily questions, 99 chart questions, and calendar activities. They should also include less obvious routines, such as how to select materials, how to use materials in a productive manner, how to put materials away, how to open and close a door, how to do just about everything! An additional routine is to allow plenty of time for children to explore new materials before attempting any directed activity with these new materials. The

regular use of the routines are important to the development of students' number sense, flexibility, and fluency, which will support students' performances on the tasks in this unit.

Workstations and Learning Centers

It is recommended that workstations be implemented to create a safe and supportive environment for problem solving in a standards based classroom. These workstations typically occur during the "exploring" part of the lesson, which follows the mini-lesson. Your role is to introduce the concept and allow students to identify the problem. Once students understand what to do and you see that groups are working towards a solution, offer assistance to the next group.

Groups should consist of 2-5 students and each student should have the opportunity to work with all of their classmates throughout the year. Avoid grouping students by ability. Students in the lower group will not experience the thinking and language of the top group, and top students will not hear the often unconventional but interesting approaches to tasks in the lower group (28, Van de Walle and Lovin 2006).

In order for students to work efficiently and to maximize participation, several guidelines must be in place (Burns 2007):

1. You are responsible for your own work and behavior.
2. You must be willing to help any group member who asks.
3. You may ask the teacher for help only when everyone in your group has the same question.

These rules should be explained and discussed with the class so that each student is aware of the expectations you have for them as a group member. Once these guidelines are established, you should be able to successfully lead small groups, which will allow you the opportunity to engage with students on a more personal level while providing students the chance to gain confidence as they share their ideas with others.

The types of activities students engage in within the small groups will not always be the same. Facilitate a variety of tasks that will lead students to develop proficiency with numerous concepts and skills. Possible activities include: math games, related previous Framework tasks, problems, and computer-based activities. With all tasks, regardless if they are problems, games, etc. include a recording sheet for accountability. This recording sheet will serve as a means of providing you information of how a child arrived at a solution or the level at which they can explain their thinking (Van de Walle 2006).

Games

"A game or other repeatable activity may not look like a problem, but it can nonetheless be problem based. The determining factor is this: Does the activity cause students to be reflective about new or developing relationships? If the activity merely has students repeating procedure without wrestling with an emerging idea, then it is not a problem-based experience. However the few examples just mentioned and many others do have children thinking through ideas that are not easily developed in one or two lessons. In this sense, they fit the definition of a problem-based task.

Just as with any task, some form of recording or writing should be included with stations whenever possible. Students solving a problem on a computer can write up what they did and explain what they learned. Students playing a game can keep records and then tell about how they played the game- what thinking or strategies they used.” (Van de Walle and Lovin, Teaching Student-Centered Mathematics: K-3, page 26)

Journaling

"Students should be writing and talking about math topics every day. Putting thoughts into words helps to clarify and solidify thinking. By sharing their mathematical understandings in written and oral form with their classmates, teachers, and parents, students develop confidence in themselves as mathematical learners; this practice also enables teachers to better monitor student progress." NJ DOE

"Language, whether used to express ideas or to receive them, is a very powerful tool and should be used to foster the learning of mathematics. Communicating about mathematical ideas is a way for students to articulate, clarify, organize, and consolidate their thinking. Students, like adults, exchange thoughts and ideas in many ways—orally; with gestures; and with pictures, objects, and symbols. By listening carefully to others, students can become aware of alternative perspectives and strategies. By writing and talking with others, they learn to use more-precise mathematical language and, gradually, conventional symbols to express their mathematical ideas. Communication makes mathematical thinking observable and therefore facilitates further development of that thought. It encourages students to reflect on their own knowledge and their own ways of solving problems. Throughout the early years, students should have daily opportunities to talk and write about mathematics." NCTM

When beginning math journals, the teacher should model the process initially, showing students how to find the front of the journal, the top and bottom of the composition book, how to open to the next page in sequence (special bookmarks or ribbons), and how to date the page. Discuss the usefulness of the book, and the way in which it will help students retrieve their math thinking whenever they need it.
When beginning a task, you can ask, "What do we need to find out?" and then, "How do we figure it out?" Then figure it out, usually by drawing representations, and eventually adding words, numbers, and symbols. During the closing of a task, have students show their journals with a document camera or overhead when they share their thinking. This is an excellent opportunity to discuss different ways to organize thinking and clarity of explanations.

Use a composition notebook (the ones with graph paper are terrific for math) for recording or drawing answers to problems. The journal entries can be from Frameworks tasks, but should also include all mathematical thinking. Journal entries should be simple to begin with and become more detailed as the children's problem-solving skills improve. Children should always be allowed to discuss their representations with classmates if they desire feedback. The children's journal entries demonstrate their thinking processes. Each entry could first be shared with a "buddy" to encourage discussion and explanation; then one or two children could share their entries with the entire class. Don't forget to praise children for their thinking skills and their journal entries! These journals are perfect for assessment and for parent conferencing. The student's thinking is made visible!

GENERAL QUESTIONS FOR TEACHER USE

Adapted from *Growing Success* and materials from Math GAINS and *TIPS4RM*

Reasoning and Proving

- How can we show that this is true for all cases?
- In what cases might our conclusion not hold true?
- How can we verify this answer?
- Explain the reasoning behind your prediction.
- Why does this work?
- What do you think will happen if this pattern continues?
- Show how you know that this statement is true.
- Give an example of when this statement is false.
- Explain why you do not accept the argument as proof.
- How could we check that solution?
- What other situations need to be considered?

Reflecting

- Have you thought about…?
- What do you notice about…?
- What patterns do you see?
- Does this problem/answer make sense to you?
- How does this compare to…?
- What could you start with to help you explore the possibilities?
- How can you verify this answer?
- What evidence of your thinking can you share?
- Is this a reasonable answer, given that…?

Selecting Tools and Computational Strategies

- How did the learning tool you chose contribute to your understanding/solving of the problem? Assist in your communication?
- In what ways would [name a tool] assist in your investigation/solving of this problem?
- What other tools did you consider using? Explain why you chose not to use them.
- Think of a different way to do the calculation that may be more efficient.
- What estimation strategy did you use?

Connections

- What other math have you studied that has some of the same principles, properties, or procedures as this?
- How do these different representations connect to one another?
- When could this mathematical concept or procedure be used in daily life?

- What connection do you see between a problem you did previously and today's problem?

Representing
- What would other representations of this problem demonstrate?
- Explain why you chose this representation.
- How could you represent this idea algebraically? Graphically?
- Does this graphical representation of the data bias the viewer? Explain.
- What properties would you have to use to construct a dynamic representation of this situation?
- In what way would a scale model help you solve this problem?

QUESTIONS FOR TEACHER REFLECTION

- How did I assess for student understanding?
- How did my students engage in the 8 mathematical practices today?
- How effective was I in creating an environment where meaningful learning could take place?
- How effective was my questioning today? Did I question too little or say too much?
- Were manipulatives made accessible for students to work through the task?
- Name at least one positive thing about today's lesson and one thing you will change.
- How will today's learning impact tomorrow's instruction?

MATHEMATICS DEPTH-OF-KNOWLEDGE LEVELS

Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include "identify," "recall," "recognize," "use," and "measure." Verbs such as "describe" and "explain" could be classified at different levels depending on what is to be described and explained.

Level 2 (Skill/Concept) includes the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include "classify," "organize," "estimate," "make observations," "collect and display data," and "compare data." These actions imply more than one step. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Some action verbs, such as "explain," "describe," or "interpret" could be classified at different levels depending on the object of the action. For example, if an item required students to explain how light affects mass by indicating there is a relationship between light and heat, this is considered a Level 2. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a Level 3. Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly, as primarily numerical skills, and such interpretation excludes from this level other skills such as visualization skills and probability skills, which may be more complex simply because they are less common. Other Level 2 activities include explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

DOK cont'd…

Level 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking most likely over an extended period of time. The extended time period is not a distinguishing factor if the required work is only repetitive and does not require applying significant conceptual understanding and higher-order thinking. For example, if a student has to take the water temperature from a river each day for a month and then construct a graph, this would be classified as a Level 2. However, if the student is to conduct a river study that requires taking into consideration a number of variables, this would be a Level 4. At Level 4, the cognitive demands of the task should be high and the work should be very complex. Students should be required to make several connections—relate ideas *within* the content area or *among* content areas—and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level. Level 4 activities include designing and conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.

DEPTH AND RIGOR STATEMENT

By changing the way we teach, we are not asking children to learn less, we are asking them to learn more. We are asking them to mathematize, to think like mathematicians, to look at numbers before they calculate, to think rather than to perform rote procedures. Children can and do construct their own strategies, and when they are allowed to make sense of calculations in their own ways, they understand better. In the words of Blaise Pascal, "We are usually convinced more easily by reasons we have found ourselves than by those which have occurred to others."

By changing the way we teach, we are asking teachers to think mathematically, too. We are asking them to develop their own mental math strategies in order to develop them in their students. Catherine Twomey Fosnot and Maarten Dolk, *Young Mathematicians at Work.*

While you may be tempted to explain and show students how to do a task, much of the learning comes as a result of making sense of the task at hand. Allow for the productive struggle, the grappling with the unfamiliar, the contentious discourse, for on the other side of frustration lies understanding and the confidence that comes from "doing it myself!"

Problem Solving Rubric (K-2)

SMP	1-Emergent	2-Progressing	3- Meets/Proficient	4-Exceeds
Make sense of problems and persevere in solving them.	The student was unable to explain the problem and showed minimal perseverance when identifying the purpose of the problem.	The student explained the problem and showed some perseverance in identifying the purpose of the problem, and selected and applied an appropriate problem solving strategy that lead to a partially complete and/or partially accurate solution.	The student explained the problem and showed perseverance when identifying the purpose of the problem, and selected an applied and appropriate problem solving strategy that lead to a generally complete and accurate solution.	The student explained the problem and showed perseverance by identifying the purpose of the problem and selected and applied an appropriate problem solving strategy that lead to a thorough and accurate solution.
Attends to precision	The student was unclear in their thinking and was unable to communicate mathematically.	The student was precise by clearly describing their actions and strategies, while showing understanding and using appropriate vocabulary in their process of finding solutions.	The student was precise by clearly describing their actions and strategies, while showing understanding and using grade-level appropriate vocabulary in their process of finding solutions.	The student was precise by clearly describing their actions and strategies, while showing understanding and using above-grade-level appropriate vocabulary in their process of finding solutions.
Reasoning and explaining	The student was unable to express or justify their opinion quantitatively or abstractly using numbers, pictures, charts or words.	The student expressed or justified their opinion either quantitatively OR abstractly using numbers, pictures, charts OR words.	The student expressed and justified their opinion both quantitatively and abstractly using numbers, pictures, charts and/or words.	The student expressed and justified their opinion both quantitatively and abstractly using a variety of numbers, pictures, charts and words.
Models and use of tools	The student was unable to select an appropriate tool, draw a representation to reason or justify their thinking.	The student selected an appropriate tools or drew a correct representation of the tools used to reason and justify their response.	The student selected an efficient tool and/or drew a correct representation of the efficient tool used to reason and justify their response.	The student selected multiple efficient tools and correctly represented the tools to reason and justify their response. In addition this students was able to explain why their tool/ model was efficient
Seeing structure and generalizing	The student was unable to identify patterns, structures or connect to other areas of mathematics and/or real-life.	The student identified a pattern or structure in the number system and noticed connections to other areas of mathematics or real-life.	The student identified patterns or structures in the number system and noticed connections to other areas of mathematics and real-life.	The student identified various patterns and structures in the number system and noticed connections to multiple areas of mathematics and real-life.

Many thanks to Richmond County Schools for sharing this rubric!

SUGGESTED LITERATURE

- *Just Enough Carrots* by Stuart Murphy
- *Tally O'Malley* by Stuart J. Murphy
- *Two Ways to Count to Ten* by Ruby Dee
- *Best Vacation Ever* by Stuart Murphy
- *Corduroy* by Don Freeman
- *More or Less* by Stuart J Murphy
- *Centipede's One Hundred Shoe*s by Tony Ross
- *1, 2, 3 Sassafras* by Stuart J. Murphy
- *The Greedy Triangle* by Marilyn Burns
- *Shapes, Shapes, Shapes* by Tanya Hoban
- *Captain Invincible and the Space Shapes* by Stuart J. Murphy
- *Shapes That Roll* by Karen Berman Nagel
- *A Fair Bear Share* by Stuart J Murphy
- *Give Me Half!* by Stuart J Murphy
- *Eating Fractions* by Bruce McMillan
- *How Tall, How Short, How Far Away* by David Adler
- *How Big Is a Foot?* by Rolf Myller
- *Measuring Penny* by Loreen Leedy
- *Length* by Henry Pluckrose
- *A Second is a Hiccup* by Hazel Hutchins and Kady MacDonald Denton
- *It's About Time!* By Stuart J. Murphy
- *The Grouchy Lady Bug* by Eric Carle
- *It's About Time, Max!* by Kitty Richards
- *The Clock Struck One: A Time-Telling Tale* by Trudy Harris
- *Clocks and More Clocks* by Pat Hutchins
- *What Time is it Mr. Crocodile?* by Judy Sierra
- *The Blast Off Kid* by Laura Driscoll
- *The King's Commissioners* by Aileen Friedman
- *Animals on Board* by Stuart J. Murphy
- *Chrysanthemum* by Kevin Henkes

MGSE WIKI RESOURCES

You are strongly encouraged to view the unit webinars on the First Grade page of the K-5 wiki to get an idea of how the Georgia Standards of Excellence should look in your classroom.

https://www.georgiastandards.org/Archives/Pages/default.aspx

http://ccgpsmathematicsk-5.wikispaces.com/

TECHNOLOGY LINKS

http://catalog.mathlearningcenter.org/apps

http://nzmaths.co.nz/digital-learning-objects

http://www.fi.uu.nl/toepassingen/00203/toepassing_rekenweb.xml?style=rekenweb&language=en&use=game

https://www.georgiastandards.org/resources/Pages/Tools/LearningVillage.aspx

https://www.georgiastandards.org/Common-Core/Pages/Math.aspx

https://www.georgiastandards.org/Common-Core/Pages/Math-PL-Sessions.aspx

http://kentuckymathematics.org/pimser_printables.php

RESOURCES CONSULTED

Content:
Mathematics Progressions Documents
http://ime.math.arizona.edu/progressions/
Ohio DOE
http://www.ode.state.oh.us/GD/Templates/Pages/ODE/ODEPrimary.aspx?page=2&TopicRelationID=1704
Arizona DOE
http://www.azed.gov/standards-practices/mathematics-standards/
Nzmaths
http://nzmaths.co.nz/
Illustrative Mathematics
https://www.illustrativemathematics.org/

Teacher/Student Sense-making:
http://www.youtube.com/user/mitcccnyorg?feature=watch
http://www.insidemathematics.org/index.php/video-tours-of-inside-mathematics/classroom-teachers/157-teachers-reflect-mathematics-teaching-practices
https://www.georgiastandards.org/Common-Core/Pages/Math.aspx or
http://secc.sedl.org/common_core_videos/

Journaling:
http://www.kindergartenkindergarten.com/2010/07/setting-up-math-problemsolving-notebooks.html

Community of Learners:
http://www.edutopia.org/math-social-activity-cooperative-learning-video

http://www.edutopia.org/math-social-activity-sel
http://www.youtube.com/user/responsiveclassroom/videos
http://www.responsiveclassroom.org/category/category/first-weeks-school