Unit 5: Understanding Plane and Solid Figures

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OVERVIEW

In this unit students will cultivate spatial awareness by:

- further developing understandings of basic geometric figures
- identifying plane figures and solid figures based on geometric properties
- describing plane figures and solid figures according to geometric properties
- expanding the ability to see geometry in the real world
- partitioning shapes into equal shares by cutting, slicing, or dividing
- represent halves, thirds, and fourths using rectangles and circles to create fraction models
- compare fractions created through partitioning same-sized rectangular or circular wholes in different ways
- understand what an array is and how it can be used as a model for repeated addition
- organize and record data using tallies, simple tables and charts, picture graphs, and bar graphs

Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as counting, time, money, positional words, patterns, and tallying should be addressed on an ongoing basis through the use of calendar, centers, and games.

To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important to study the tasks in this unit early in the planning process. The tasks in this unit illustrates the types of learning activities that should be utilized from a variety of sources in order for students to gain a solid foundation in geometry to meet or exceed grade level standards.

NUMBER TALKS

Between 5 and 15 minutes each day should be dedicated to “Number Talks” in order to build students’ mental math capabilities and reasoning skills. Sherry Parrish’s book *Number Talks* provides examples of K-5 number talks. The following video clip from Math Solutions is an excellent example of a number talk in action.

During the Number Talk, the teacher is not the definitive authority. The teacher is the facilitator and is listening for and building on the students’ natural mathematical thinking. The teacher writes a problem horizontally on the board in whole group or a small setting. The students mentally solve the problem and share with the whole group how they derived the answer. They must justify and defend their reasoning. The teacher simply records the students’ thinking and poses extended questions to draw out deeper understanding for all.

The effectiveness of Numbers Talks depends on the routines and environment that is established by the teacher. Students must be given time to think quietly without pressure from their peers. To develop this, the teacher should establish a signal, other than a raised hand, of some sort to identify that one has a strategy to share. One way to do this is to place a finger on their chest indicating that they have one strategy to share. If they have two strategies to share, they place out two fingers on their chest and so on.

Number Talk problem possible student responses:

<table>
<thead>
<tr>
<th>Possible Strategy #1</th>
<th>Possible Strategy #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>29 + 8</td>
<td>29 can become 30 and take 1 from 8 reducing it to 7.</td>
</tr>
<tr>
<td>54 + 86</td>
<td>50 + 80 + 10 = Add 6 to 54 to get 60. Then 60 + 80 = 140</td>
</tr>
</tbody>
</table>

Number talks often have a focus strategy such as “making tens” or “compensation.” Providing students with a string of related problems, allows students to apply a strategy from a previous problem to subsequent problems. Some units lend themselves well to certain Number Talk topics. For example, the place value unit may coordinate well with the Number Talk strategy of “making ten.” For additional information please refer to the Grade Level Overview.

**STANDARDS FOR MATHEMATICAL PRACTICE**

This section provides examples of learning experiences for this unit that support the development of the proficiencies described in the Standards for Mathematical Practice. The statements provided offer a few examples of connections between the Standards for Mathematical Practice and the Content Standards of this unit. The list is not exhaustive and will hopefully prompt further reflection and discussion.

1. **Make sense of problems and persevere in solving them.**
   
   *Students will use nets to create cubes and discover different attributes about them.*

2. **Reason abstractly and quantitatively.**
   
   *Students will use partitioning and equal groups to break shapes into different pieces.*
3. Construct viable arguments and critique the reasoning of others.
   Students will use known information/attributes of different shapes to construct viable arguments about them.

4. Model with mathematics.
   Students will create cubes to learn about sides, edges, vertices and angles.

5. Use appropriate tools strategically.
   Students use tangrams to help make/create different shapes.

6. Attend to precision.
   Students will create and draw shapes and will have to make sure to keep their lines straight, form the correct angles, and keep lines congruent if needed.

7. Look for and make use of structure.
   Students will use different shapes to create another object.

8. Look for and express regularity in repeated reasoning.
   Students will use knowledge of equal parts and portioning to develop strategies for sharing and grouping items.

***Mathematical Practices 1 and 6 should be evident in EVERY lesson.***

STANDARDS FOR MATHEMATICAL CONTENT

Reason with shapes and their attributes.

MGSE2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

MGSE2.G.2 Partition a rectangle into rows and columns of same-size squares to find the total number of them.

MGSE2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Represent and interpret data.

MGSE2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

BIG IDEAS

By the conclusion of this unit, students should be able to demonstrate the following competencies:

- Describe plane figures according to their characteristics (sides, corners, angles).
• Describe solid figures according to their characteristics (faces, edges, vertices).
• Describe and understand the relationships (similarities and differences) between solid figures and plane figures.
• Recognize the relationship between geometry and the environment.
• Compare geometric figures to similar objects in everyday life.
• Identify and represent the fractional parts of a whole or of a set (halves, thirds, fourths).
• Recognize and represent that differently partitioned fractional parts of same-sized rectangles or circles are equal.
• Identify the number of rows and columns in an array and count the same-size squares to find the total.
• Pose questions that will result in data that can be shown on a bar graph or picture graphs.
• Use charts, simple tables, and surveys to collect data that can be shown on a bar graph or picture graph.
• Graph data on a bar graph or picture graph and in a simple table. Interpret data shown on a bar graph or picture graph.
• Identify plane figures and solid or hollow figures according to geometric properties
• Describe plane figures and solid or hollow figures according to geometric properties
• Develop an understanding of the relationship between solid or hollow figures and plane figures
• Understand that the faces of solid or hollow figures are plane figures
• Further develop spatial awareness of geometric solids and figures
• Investigate what happens when geometric figures are combined
• Investigate what happens when geometric figures are cut apart
• Recognize plane and solid figures in the real world
• Repeatedly adding the same quantity or forming a rectangular array are strategies for repeated addition.
• Fractional parts are equal shares of a whole number, whole object, or a whole set.
• The more equal sized pieces that form a whole, the smaller the pieces (fraction) will be.
• When the numerator and denominator are the same number, the fraction equals the number one or one whole (entire object or set).
• The fraction name (half, third, fourth) indicates the number of equal parts in the whole.
• Equal shares of identical wholes may not have the same shape. For example, fourths can be represented in multiple ways (i.e. with diagonal, horizontal, vertical cuts) and although they look different they represent the same amount/size piece.

**ESSENTIAL QUESTIONS**

• How do we describe geometric figures?
• Where can we find geometric figures in the world around us?
How do we use the following terms: angle, vertex, face, side, and edge to describe geometric figures?
How do we apply the use of fractions in everyday life?
How do we know how many fractional parts make a whole?
When is it appropriate to use fractions?
How can we use a picture graph, bar graph, chart, or table to organize data and answer questions?

CONCEPTS/SKILLS TO MAINTAIN

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply “how to get the answer” and instead support students’ ability to access concepts from a number of perspectives. Therefore, students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency and automaticity. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, and make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between varieties of strategies in context.

Fluent students:

- flexibly use a combination of deep understanding, number sense, and memorization.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.


Skills from Grade 1:
It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.
Developing understanding of linear measurement and measuring lengths as iterating length units.

**Second Grade Year Long Concepts:**
Organizing and graphing data as stated in MGSE2.MD.10 should be incorporated in activities throughout the year. Students should be able to draw a picture graph and a bar graph to represent a data set with up to four categories as well as solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

Specifically, it is expected that students will have prior knowledge/experience related to concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop understanding of these ideas.

- Composition and decomposition of two-dimensional shapes
- Recognition of shapes from different perspectives and orientations
- Basic geometric figures and spatial relationships
- Sides, vertices, and other geometric attributes
- Fractions: halves, fourths
- Tally marks
- Picture graphs

**STRATEGIES FOR TEACHING AND LEARNING**
(Information adapted from the North Carolina DPI Instructional Support Tools)

**General Strategies:**

- Students should be actively engaged by developing their own understanding.
- Mathematics should be represented in as many ways as possible by using graphs, tables, pictures, symbols, and words.
- Appropriate manipulatives and technology should be used to enhance student learning.
- Students should be given opportunities to revise their work based on teacher feedback, peer feedback, and metacognition which includes self-assessment and reflection.
- Math journals are an excellent way for students to show what they are learning about a concept. These could be spiral bound notebooks that students could draw or write in to describe the day’s math lesson. Second graders love to go back and look at things they have done, so journals could also serve as a tool for a nine-week review, parent conferencing, etc.

**Reason with shapes and their attributes**

MGSE2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
MGSE2.G.2 Partition a rectangle into rows and columns of same-size squares to find the total number of them.

MGSE2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Instructional Strategies
(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades K-3, page 188)

“Not all people think about geometric ideas in the same manner. Certainly, we are all not alike, but we are all capable of growing and developing in our ability to think and reason in geometric contexts. The research of two Dutch educators, Pierre van Hiele and Dina van Hiele-Geldof, has provided insight into the differences in geometric thinking and how the differences come to be.

The most prominent feature of the model is a five-level hierarchy of ways of understanding spatial ideas. Each of the five levels describes the thinking processes used in geometric contexts. The levels describe how we think and what types of geometric ideas we think about, rather than how much knowledge we have. A significant difference from one level to the next is the objects of thought-what we are able to think about geometrically.”

- Level 0: Visualization
- Level 1: Analysis
- Level 2: Informal Deduction
- Level 3: Deduction
- Level 4: Rigor

For more information on the van Hiele Levels, refer to Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades K-3, Chapter 7.

MGSE2.G.1 Calls for students to identify (recognize) and draw shapes based on a given set of attributes. These include triangles, quadrilaterals (squares, rectangles, and trapezoids), pentagons, hexagons and cubes.

Example: Draw a closed shape that has five sides. What is the name of the shape?

Student 1
I drew a shape with 5 sides.
It is called a pentagon.

Prior to teaching the unit, you can use the plane shapes graphic organizer as a whole class assessment or give each child a copy and have them list everything they know about the given shapes.
MGSE2.G.2 calls for students to partition a rectangle into squares (or square-like regions) and then determine the total number of squares. This relates to the standard 2.OA.4 where students are arranging objects in an array of rows and columns. Modeling repeated addition with partitioned rectangles provides the foundation for student understanding of multiplication. While discussions of multiplication may arise as an offshoot of work in this standard, the emphasis should be on understanding repeated addition through the array model.

Tell students that they will be drawing a square on grid paper. The length of each side is equal to 2 units. Ask them to guess how many 1 unit by 1 unit squares will be inside this 2-unit by 2-unit square. Students now draw this square and count the 1 by 1 unit squares inside it. They compare this number to their guess. Next, students draw a 2-unit by 3-unit rectangle and count how many 1 unit by 1 unit squares are inside. Now they choose the two dimensions for a rectangle, predict the number of 1 unit by 1 unit squares inside, draw the rectangle, count the number of 1 unit by 1 unit squares inside and compare this number to their guess. Students repeat this process for different-size rectangles. Finally, ask them to share what they observed as they worked on the task.

Next example in the series: Split the rectangle into 2 rows and 4 columns. How many small squares did you make?

Note: This standard is laying the foundation for student understanding of area which will be studied in 3rd grade. They are creating an area model when they partition a rectangle into squares. It is important to help them see that for this there should be no gaps or overlaps between squares.

MGSE2.G.3 calls for students to partition (split) circles and rectangles into 2, 3 or 4 equal shares (regions). Students should be given ample experiences to explore this concept with paper strips and pictorial representations. Students should also work with the vocabulary terms halves, thirds, half of, third of, and fourth (or quarter) of. While students are working on this standard, teachers should help them to make the connection that a whole is composed of two halves, three thirds, or four fourths.

This standard also addresses the idea that equal shares of identical wholes may not have the same shape. Example: Divide each rectangle into fourths a different way.

It is vital that students understand different representations of fair shares. Provide a collection of different-size circles and rectangles cut from paper. Ask students to fold some shapes into halves, some into thirds, and some into fourths. They compare the locations of the folds in their shapes.
as a class and discuss the different representations for the fractional parts. To fold rectangles into thirds, ask students if they have ever seen how letters are folded to be placed in envelopes. Have them fold the paper very carefully to make sure the three parts are the same size. Ask them to discuss why the same process does not work to fold a circle into thirds. Use an analog clock as a model and allow children to draw a line from the center of the clock to the place where the 12, 4 and 8 are on the clock face. This will divide the circle into three equal sections. This clock connection can also be made for discussing halves and fourths and discovering to which numbers (or hours) you would draw the lines to in order to create two or four equal parts/pieces.

Represent and Interpret Data

MGSE2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

INSTRUCTIONAL STRATEGIES

At first students should create real object and picture graphs so each row or bar consists of countable parts. These graphs show items in a category and do not have a numerical scale. For example, a real object graph could show the students’ shoes (one shoe per student) lined end to end in horizontal or vertical rows by their color. Students would simply count to find how many shoes are in each row or bar. The graphs should be limited to 2 to 4 rows or bars. Students would then move to making horizontal or vertical bar graphs with two to four categories and a single-unit scale.

<table>
<thead>
<tr>
<th>Flavor</th>
<th>Number of People</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chocolate</td>
<td>12</td>
</tr>
<tr>
<td>Vanilla</td>
<td>5</td>
</tr>
<tr>
<td>Strawberry</td>
<td>6</td>
</tr>
<tr>
<td>Cherry</td>
<td>9</td>
</tr>
</tbody>
</table>

Students display their data using a picture graph or bar graph using a single unit scale.

Favorite Ice Cream Flavor
As students continue to develop their use of reading and interpreting data it is highly suggested to incorporate these standards into daily routines. It is not merely the making or filling out of the graph but the connections made from the data represented that builds and strengthens mathematical reasoning.

**SELECTED TERMS AND SYMBOLS**

The following terms and symbols are not an inclusive list and should not be taught in isolation. Instructors should pay particular attention to them and how their students are able to explain and apply them (*i.e.* students should not be told to memorize these terms).

Teachers should present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

For specific definitions, please reference the [Georgia Standards of Excellence Glossary](http://www.ncesd.org/Page/983).

**Note:** GA uses the inclusive definition of a trapezoid. This card set contains the exclusive definition.

<table>
<thead>
<tr>
<th>Favorite Ice Cream Flavor</th>
<th>Chocolate</th>
<th>Vanilla</th>
<th>Strawberry</th>
<th>Cherry</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

- angle
- attribute
- bar graph
- circle
- column
- cone
- cube
- cylinder
- data set
- edge
- face
- fourths
- fraction
- halves
- hexagon
- irregular polygon
- partition
- pentagon
- picture graph
- plane figure
- polygon
- quadrilateral
- rectangle
- regular polygon
• row
• scale
• shapes
• solid figure
• square
• thirds
• trapezoid
• triangle
• unit fraction
• vertex/vertices
• whole
Task Types
The following tasks represent the level of depth, rigor, and complexity expected of all second grade students. These tasks or tasks of similar depth and rigor should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them.

<table>
<thead>
<tr>
<th>Task Types</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scaffolding Task</td>
<td>Tasks that build up to the learning task.</td>
</tr>
<tr>
<td>Constructing Task</td>
<td>Constructing understanding through deep/rich contextualized problem solving tasks.</td>
</tr>
<tr>
<td>Practice Task</td>
<td>Tasks that provide students opportunities to practice skills and concepts.</td>
</tr>
<tr>
<td>Culminating Task</td>
<td>Designed to require students to use several concepts learned during the unit to answer a new or unique situation. Allows students to give evidence of their own understanding toward the mastery of the standard and requires them to extend their chain of mathematical reasoning.</td>
</tr>
<tr>
<td>Formative Assessment Lesson (FAL)</td>
<td>Lessons that support teachers in formative assessment which both reveal and develop students’ understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards.</td>
</tr>
<tr>
<td>3-Act Task</td>
<td>A Three-Act Task is a whole-group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the Guide to Three-Act Tasks on georgiastandards.org.</td>
</tr>
<tr>
<td>Task Name</td>
<td>Task Type/Grouping Strategy</td>
</tr>
<tr>
<td>-------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>Natural Shapes</td>
<td>3-Act Task Whole Group</td>
</tr>
<tr>
<td>Shape Robot</td>
<td>Scaffolding Task Large Group/Partners</td>
</tr>
<tr>
<td>The Shape of Things</td>
<td>Practice Task Large Group/Partners</td>
</tr>
<tr>
<td>Greedy Shapes</td>
<td>Practice Task Large Group/Partners</td>
</tr>
<tr>
<td>The Curious Case of the Cube</td>
<td>Scaffolding Task Large Group / Small Group</td>
</tr>
<tr>
<td>Net or Not?</td>
<td>Performance Task Partners</td>
</tr>
<tr>
<td>Sharing Equally</td>
<td>Constructing Task Partners</td>
</tr>
<tr>
<td>Grandma’s Quilts</td>
<td>Constructing Task Partners</td>
</tr>
<tr>
<td>Making Rectangles</td>
<td>Practice Task</td>
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<tr>
<td>Ribbon Fractions</td>
<td>Constructing Task</td>
</tr>
<tr>
<td>Making a Cake</td>
<td>Constructing Task</td>
</tr>
<tr>
<td>Fraction Cookies</td>
<td>Constructing Task</td>
</tr>
<tr>
<td>My Country’s Flag</td>
<td>Culminating Task</td>
</tr>
<tr>
<td>Teeth Graph</td>
<td>Scaffolding Task</td>
</tr>
</tbody>
</table>
The Intervention Table below provides links to interventions specific to this unit. The interventions support students and teachers in filling foundational gaps revealed as students work through the unit. All listed interventions are from New Zealand’s Numeracy Project.

<table>
<thead>
<tr>
<th>Cluster of Standards</th>
<th>Name of Intervention</th>
<th>Snapshot of summary or Student I can statement...</th>
<th>Materials Master</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometry</strong></td>
<td><strong>Shape Makers</strong></td>
<td>Describe and classify 2D and 3D shapes</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Picasso</strong></td>
<td>Explore two and three-dimensional shapes to recognize their features and develop appropriate vocabulary</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Shape Explorers</strong></td>
<td>Identify and classify shapes by name, number of sides and corners and describe similarities and differences between shapes</td>
<td></td>
</tr>
<tr>
<td>MGSE2.G.1 MGSE2.G.2</td>
<td><strong>The Folding Problem</strong></td>
<td>Create squares, triangles and rectangles by folding paper and identify and describe shapes</td>
<td></td>
</tr>
<tr>
<td>MGSE.G.3</td>
<td><strong>Mosaic Puzzles</strong></td>
<td>Join shapes together to form other shapes and identify and describe shapes</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Arty Shapes</strong></td>
<td>Name 2-dimensional shapes and describe shape attributes</td>
<td></td>
</tr>
</tbody>
</table>
Plane Shape Organizer to be used to determine students’ prior knowledge. Have students write or draw everything they know about each shape in the appropriate boxes.
3-ACT TASK: Natural Shapes
Approximate Time: One Class Session
In this task, students use their knowledge of shapes to identify shapes in nature.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students are required to figure out a question to work through, the information they need to solve the problem, and then persevere until solving it.
2. Reason abstractly and quantitatively. Students are asked to make an estimate both high and low.
3. Construct viable arguments and critique the reasoning of others. Students will use known information/attributes of different shapes to construct viable arguments about them.
4. Model with mathematics. Students will put shapes together to form different shapes to learn more about sides, vertices, and angles.
5. Attend to precision. Students will use clear and precise language when discussing their strategies and sharing their solutions with others.
6. Look for and make use of structure. Students will use different shapes to create another object.

ESSENTIAL QUESTIONS

• How do we describe geometric figures?

MATERIALS

• Nature Picture
• Student Handout

GROUPING

Individual/Partner Task
TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this task, students will view the picture and tell what they noticed. Next, they will be asked to discuss what they wonder about or are curious about. These questions will be recorded on a class chart or on the board and on the student recording sheet. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don’t have the information they need, and ask for it, it will be given to them.

Background Knowledge:

This task follows the 3-Act Math Task format originally developed by Dan Meyer. More information on this type of task may be found at http://blog.mrmeyer.com/category/3acts/. A Three-Act Task is a whole-group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the Guide to Three-Act Tasks on georgiastandards.org.

In this task students will be shown a picture of reeds in water. They will then try to find different shapes in the picture. In students’ discussions of the shapes, encourage use of mathematical vocabulary such as side, edge, vertices, and angles.

Students should have had prior experiences and/or instruction with plane figures from first grade and kindergarten. Students should be familiar with identifying sides, vertices (corners), angles, circles, triangles, quadrilaterals (squares, rectangles,) and pentagons. Students should have been exposed to these terms since as early as Kindergarten. Teachers may want to spend some time watching this video to assist in teaching the necessary vocabulary.


Common Misconceptions:

Some students may think that a shape is changed by its orientation. They may see a rectangle with the longer side as the base, but claim that the same rectangle with the shorter side as the base is a different shape. This is why it is so important to have young students handle shapes and physically feel that the shape does not change regardless of the orientation.

This task requires students to classify shapes. “As young students work at classification of shapes, be prepared for some of them to notice features that you do not consider to be “real” geometric attributes, such as “curvy” or “looks like a rocket.” Children at this level will also attribute to shapes ideas that are not part of the shape, such as “points up” or “has a side that is the same as the edge of the board.

Task Directions:
Act I – Whole Group - Pose the conflict and introduce students to the scenario by showing Act I picture. (Dan Meyer [http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/])

“Introduce the central conflict of your story/task clearly, visually, viscerally, using as few words as possible.”

1. Show picture of reeds to students.
2. Ask students what they noticed in the picture. The teacher records this information.
3. Ask students what they wonder about and what questions they have about what they saw. Students should share with each other first, and then the teacher records these questions (think-pair-share). The teacher may need to guide students so that the questions generated are math-related.
4. Ask students to estimate answers to their questions (think-pair-share). Students will write their best estimate, then write two more estimates – one that is too low and one that is too high so that they establish a range in which the solution should occur.

Anticipated questions students may ask and wish to answer:
- What is in the picture?
- How many shapes are in the picture?
- How many different shapes are in the picture? *
- How many quadrilaterals are in the picture? *
- What shapes can I find?
*Main question(s) to be investigated

Act 2 – Student Exploration - Provide additional information as students work toward solutions to their questions. (Dan Meyer [http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/])

“The protagonist/student overcomes obstacles, looks for resources, and develops new tools.”

- During Act 2, students determine the main question(s) from Act 1 and decide on the facts, tools, and other information needed to answer the question(s). When students decide what they need to solve the problem, they should ask for those things. It is pivotal to the problem solving process that students decide what is needed without being given the information up front. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin.

- The teacher provides guidance as needed during this phase. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin. Questioning is an effective strategy that can be used, with questions such as:
  - What is the problem you are trying to solve?
  - What do you think affects the situation?
  - Can you explain what you’ve done so far?
  - What strategies are you using?
What assumptions are you making?
What tools or models may help you?
Why is that true?
Does that make sense?

Important note: Although students will only investigate the main question(s) for this task, it is important for the teacher to not ignore student generated questions. Additional questions may be answered after they’ve found a solution to the main question, or as homework or extra projects.


- Students to present their solutions and strategies and compare them.
- Discuss the solutions.
- Lead discussion to compare these, asking questions such as:
  - How reasonable was your estimate?
  - Which strategy was most efficient?
  - Can you think of another method that might have worked?
  - What might you do differently next time?

Act 4, The Sequel - “The goals of the sequel task are to a) challenge students who finished quickly so b) I can help students who need my help. It can’t feel like punishment for good work. It can’t seem like drudgery. It has to entice and activate the imagination.” Dan Meyer


- Challenge students to pick a different question from the student generated list to answer.

FORMATIVE ASSESSMENT QUESTIONS

- How reasonable was your estimate?
- What might you do differently next time?
- What worked well for you this time?
- What model did you use?
- What organizational strategies did you use?

DIFFERENTIATION

Extension
- Students have to draw a different type of each shape (different orientation, size, etc.)

Intervention
- Give students a sheet of pictures of each type of shape for reference when looking for shapes.

Intervention Table
Scaffolding Task: Shape Robot
Approximately 1 Day
Adapted from www.k-5mathteachingresources.com
In this task, students identify different shapes and create a robot from these shapes.
Technology link: http://nces.ed.gov/nceskids/createAGraph/

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

MGSE2.MD.10 Draw a picture graph and a bar graph (with single-unit scales) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

STANDARDS FOR MATHEMATICAL PRACTICE
Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. Make sense of problems and persevere in solving them.
5. Use appropriate tools strategically.
6. Attend to precision.

BACKGROUND KNOWLEDGE
Students should have had prior experiences and/or instruction with plane figures from first grade and kindergarten. Students should be familiar with identifying sides, vertices (corners), angles, circles, triangles, quadrilaterals (squares, rectangles,) and pentagons. Students should have been exposed to these terms since as early as Kindergarten. Teachers may want to spend some time watching this video to assist in teaching the necessary vocabulary.

ESSENTIAL QUESTIONS

- How do we use the terms: angle, vertices, faces, sides, and edges to describe geometric figures?
- How do we describe geometric figures?

MATERIALS

- The Shape of Things by Dayle Ann Dodds or similar text
- Tangrams
- Crayons
GROUPING

Whole group, Individual, Partners

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Part I
Gather students together to read *The Shape of Things* by Dayle Ann Dodds, or a similar book about shapes. During the story, ask students questions about the shapes and their observations from the book. Create an anchor chart of vocabulary generated during this discussion for future reference. It is imperative that students understand the definition of a rectangle as a quadrilateral with opposite sides that are parallel and equal in length. (This definition lays the foundation for area and perimeter in 3rd grade and is crucial for 3rd grade success.)

Part II
After completing the story, explain to the students that they will be building a robot from shapes and drawing the food that a robot will eat for lunch.

Allow the students to build their robot using tangrams or pattern blocks and then draw their robot, either tracing the shapes used or free hand, in the space provided. Have students take a photo of their robot. Instruct each child to create a tally chart of the shapes used in his/her robot. Students are to create a bar graph or picture graph based on the data collected. The following link is useful for integrating math and technology: [http://nces.ed.gov/nceskids/createAgraph/](http://nces.ed.gov/nceskids/createAgraph/)

Ask the students to label each piece of their robot using the shape vocabulary that they know.

This will give you an idea of the shapes that the students can identify. This is a great opportunity to integrate writing by having the students create a story about their robot.

*This writing could include a story about a day in the life of a robot (narrative writing), an advertisement advising the reader of all the uses of your robot (persuasive), or a descriptive paragraph of all the shapes that make up the robot (informational).*

FORMATIVE ASSESSMENT QUESTIONS

- What do you notice about the shapes?
- How do we categorize shapes?
- Can you identify the number of vertices?
- Can you identify the number of sides?
- How many of each shape did you use to create your robot?
- Did you use any shapes that you do not know the name of?
- Was it easier to use one shape more than another? Why or why not?
What did you notice about the (adjacent) sides when you drew two shapes?

DIFFERENTIATION

Extension

• Students who demonstrate an understanding of the shapes presented in this lesson may combine shapes to form a number of different polygons, and name them, based on their understanding of shapes, or by researching the shapes.

Intervention

• Students who have difficulty drawing the shape can choose from pre-cut shapes or build the shapes with tangram pieces and trace the shape.
• Use dot paper to assist students in drawing various shapes such as triangles, quadrilaterals, pentagons, hexagons, etc.
• Use a geoboard to assist students in creating various shapes.
SHAPE ROBOT is READY FOR LUNCH

Draw/create your Shape Robot here.
Practice Task: The Shape of Things
Approximately 1 Day
In this task students identify different shapes that they come across in their daily routine.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

STANDARDS FOR MATHEMATICAL PRACTICE
Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
6. Attend to precision.

BACKGROUND KNOWLEDGE
(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades K-2, page 193)

“Children need experiences with a rich variety of both two- and three-dimensional shapes. It is useful for students to be able to identify common shapes, notice likenesses and difference among shapes, become aware of the properties that different shapes have, and eventually use these properties to further define and understand their geometric world. As students find out more about shapes over time, they can begin to appreciate how definitions of special shapes come to be.”

ESSENTIAL QUESTIONS

- How do we use the terms: angle, vertices, faces, sides, and edges to describe geometric figures?
- How do we describe geometric figures?

MATERIALS

- When a Line Bends, a Shape Begins by Rhonda G. Greene
- “The Shape of Things” Student recording form.

GROUPING
Individual, partners
TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Part I
Gather students together to read *When a Line Bends, a Shape Begins* by Rhonda G. Greene. During the story, question the students about the attributes of the shapes mentioned in the book. Students should have demonstrated prior knowledge of shapes in the previous task. This questioning will provide you with knowledge of the students’ understanding. After reading the story, ask students to identify the shapes around the classroom. Reinforce the correct terminology of words such as: triangles, quadrilaterals, pentagons, hexagons, and cubes. Don’t use this conversation to “teach” these terms, but rather to have students explain what they are noticing around them. If the students produce misconceptions, use that opportunity to better clarify.

Part II
After the class discussion of shapes around the room, present the students with “The Shape of Things” Student Recording Form. Create a connection between the conversation that you had about the classroom and the playground shown on the Student Recording Form. Take this opportunity to go over the expectations of a clear explanation and the illustration of the shapes.

Student Recording Form Directions:
Examine the scene of the playground. There are many different shapes that make up the toys and play-equipment. Name, draw, and describe clearly all shapes you can see in the playground picture.

Part III
Take class outside to investigate and record all the shapes the see on the playground(s) at your school. This could easily be extended into a homework assignment by having students describe the shapes of any play equipment near their home.

FORMATIVE ASSESSMENT QUESTIONS

- What shapes do you see around the room?
- How do you know it is that shape?
- What attributes or characteristics does that shape have?
- How are these shapes similar?
- How are these shapes different?
- Are there any shapes on your playground that you can see around the room?
DIFFERENTIATION

Extension
- Students who demonstrate an understanding of the shapes presented in this lesson may draw a picture using the shapes in this lesson and include the heptagon (7-sided polygon), octagon (8-sided polygon), nonagon (9-sided polygon), and decagon (10-sided polygon).

Intervention
- Assist students in creating a chart labeling the polygons in various orientations. This could be created by gluing on pre-cut shapes, or by having students draw and cut out shapes. This student-created chart becomes an anchor chart for whole class reference after being created as an intervention strategy.

Intervention Table
Examine the scene of the playground above. There are a lot of different shapes that make up the toys and play-equipment. Name, draw, and clearly describe all the shapes you can see in the playground picture.

(Information adapted from the Noyce Foundation, Problems of the Month)
Practice Task: Greedy Shapes

Approximately 2 Days
In this task, students explore attributes of a triangle and start to reason with other shapes and learn more about polygons.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

STANDARDS FOR MATHEMATICAL PRACTICE

Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
6. Attend to precision.

BACKGROUND KNOWLEDGE

Students should have had prior experiences and/or instruction with plane figures from first grade and kindergarten. Students should be familiar with identifying sides, vertices (corners), angles, circles, triangles, quadrilaterals (squares and rectangles), and pentagons. Students should have been exposed to these terms since as early as Kindergarten.

Teachers may want to spend some time watching this video to assist in teaching the necessary vocabulary.

http://gadoe.georgiastandards.org/mathframework.aspx?pageReq=MathName

Some students may think that a shape is changed by its orientation. They may see a rectangle with the longer side as the base, but claim that the same rectangle with the shorter side as the base is a different shape. This is why it is so important to have young students handle shapes and physically feel that the shape does not change regardless of the orientation, as illustrated below.

This task requires students to classify shapes. “As young students work at classification of shapes, be prepared for some of them to notice features that you do not consider to be “real” geometric attributes, such as “curvy” or “looks like a rocket.” Children at this level will also attribute to shapes ideas that are not part of the shape, such as “points up” or “has a side that is the same as the edge of the board. In any sorting activity, the students should decide how to sort, not the teacher. This allows the students to do the activity using ideas they own and understand. By listening to the kinds of attributes that they use in their sorting, you will be able to tell what properties they know and use and how they think about their shapes.”
ESSENTIAL QUESTIONS

- How do we use the terms: angle, vertices, faces, sides, and edges to describe geometric figures?
- How do we describe geometric figures?

MATERIALS

- *The Greedy Triangle* by Marilyn Burns or similar book
- Geoboards
- Rubber bands
- Describing Plane Shapes Class Chart
- Describing Plane Shapes Student Chart
- “What’s my rule” task sheet

GROUPING

Large Group, Partners

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Part I
Read students the book, *The Greedy Triangle* by Marilyn Burns. Before reading, discuss the terms side, angle, and vertex. Draw a triangle on the board and have students determine the number of sides and vertices. As you read the book, have the students predict the shape which the greedy triangle will become next.

Ask questions as you read, such as:
- How many sides did the shape have to begin with? (3) How did you figure that out? (Counted them, or recognized the shape as a triangle and I know triangles have three sides)
- How many vertices did the shape have to begin with? How are these different from sides?
- How many sides did the shape have when it became a (quadrilateral, a pentagon, etc.)?
Part II
Introduce the term quadrilateral to the students. This word is an important term for the students to use and understand. Use a word web to deconstruct the meaning of the word polygon. Break apart the word using prior knowledge of shapes.

Tell students that they will now use geo-boards to recreate the story of *The Greedy Triangle*. Each student should have his or her own geoboard. However, students can sit in partner groups as they create shapes. This will encourage dialogue about the geometry and allow students to comment on each other’s work.

**Variation**
*Toothpicks or pretzels sticks could be used to create the shape instead of using the geoboard.*

Begin reading the book again, however, this time stop at each shape and allow the students to create that shape on their geo-boards using rubber bands. (Students who can create the shapes quickly may explore creating various sizes and irregular examples of the shapes)

While students are working, ask questions like:
- How many sides does your shape have now?
- What shape have you made? How do you know it is that shape?
- What characteristics of a shape help us figure out/determine the name of the shape?
- How did your shape change?
- What are differences between a (triangle) and a (quadrilateral)?
- Can you make that shape smaller? Larger?
- What would happen if you made that side longer? Would it still be a (triangle?)

Once all students have created the shape, allow a partner group to come to the board. Have one student demonstrate how to make the shape using the overhead or possibly a virtual geoboard: [http://nlvm.usu.edu/en/nav/vlibrary.html](http://nlvm.usu.edu/en/nav/vlibrary.html). Discuss with students the meaning of the word “regular polygon”; that the shape is a regular polygon if all the sides are equal. However, show the students that you can make the figure an irregular polygon by grasping one or more of the vertices and extending the sides. Ask the students, “What do we call this three-sided shape that does not have all sides the same length? *(a triangle)* Why do we still call it a triangle? *(because it still has three sides).*” Make sure that students understand and can explain that making sides longer or shorter does not change the name of the shape *(triangle, quadrilateral, pentagon, etc.)* because the number of sides and vertices is still the same. It is very important for students to understand and articulate that knowing the number of sides and vertices of a shape is how we determine/decide the name of the shape. Allow the other student in the partner group to record the number of sides and vertices by the shape name on the class chart. Allow all the students in the class to record the number of sides, vertices, and several of their favorite examples of each shape on their student chart. Continue with the book, stopping at each shape and repeating the process as above.
Note
When students are creating quadrilaterals, encourage them to create various kinds of quadrilaterals (parallelograms, squares, rectangles, and trapezoids).

Part III
Organize students into small groups. Distribute the sets of cut-out figures, one set per group and the recording sheet. “What’s My Rule?” task sheet cards should be distributed. Then review the rules of the game. One participant in each group is the sorter. The sorter writes down a "secret rule" to classify the set of figures into two groups and uses that rule to slowly sort the pieces as the other players observe. At any point in the game, the players can call "stop" and guess the rule. After the correct rule identification, the player who figured out the rule becomes the sorter. The correct identification from the sorter is worth five points. A correct answer, but not the written one, is worth one point. As a variation, each incorrect guess results in a two-point penalty. The winner is the first one to accumulate ten points.
You may also use this set of shapes, which provides more variety, thus expanding the possible conversations about shapes and their attributes.
BLM 20-26: http://www.ablongman.com/vandewalleseries/volume_1.html
As students are sorting the cards and making decisions about the sort, the teacher should be listening for student’s descriptions of the shapes.

FORMATIVE ASSESSMENT QUESTIONS

• What are differences between a (triangle) and a (quadrilateral)?
• How do we determine, or decide, the name of a shape?
• Can you make that shape smaller? Larger?
• What would happen if you made that side longer? Would it still be the same shape?

DIFFERENTIATION

Extension
• Students who demonstrate an understanding of the shapes presented in this lesson may be introduced to heptagons (7 sides), octagons (8 sides), nonagons (9 sides), decagons (10 sides), and dodecagons (12 sides). Students can create these shapes using their geo-boards and add information regarding these shapes to their charts.
• Choose a polygon and create a picture using the shape. Describe it with mathematical words and then create a story about your picture.
Intervention

- Some students may have difficulty using the geoboards with rubber bands. These students can use dot paper instead. They can draw lines between the dots to create the various shapes.
- Use dot paper to model various shapes such as triangles, quadrilaterals, pentagons, hexagons, etc.

Intervention Table
## Describing Plane Shapes

<table>
<thead>
<tr>
<th>Shape Name</th>
<th>Model of the Shape</th>
<th>Number of Sides</th>
<th>Number of Vertices</th>
<th>Examples</th>
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<tr>
<td>Triangle</td>
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A “What’s My Rule?” Cards for Greedy Shapes

Richard Woods, State School Superintendent
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Georgia Department of Education
Georgia Standards of Excellence Framework
GSE Understanding Plane and Solid Figures • Unit 5

Mathematics • GSE Second Grade • Unit 5: Understanding Plane and Solid Figures
Richard Woods, State School Superintendent
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### WHAT’S MY RULE?

**Part III Recording Sheet**

<table>
<thead>
<tr>
<th>GROUP MEMBERS</th>
<th>SCORE</th>
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Scaffolding Task: The Curious Case of the Cube

Approximately 1 to 2 days
In this task, students visualize and discuss the attributes of a cube. They then construct a cube using simple materials.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

STANDARDS FOR MATHEMATICAL PRACTICE

Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
6. Attend to precision.

BACKGROUND KNOWLEDGE

(from Teaching Student-Centered Mathematics by Van de Walle, Lovin, Karp, and Bay-Williams, pages 285-286)

“Visualization might be called “geometry done with the mind’s eye.” It involves being able to create mental images of shapes and then turn them around mentally, thinking about how they look from different viewpoints – predicting the results of various transformations.”

A cube always has 6 congruent square faces, 8 vertices, and 12 edges. A cube is a special type of platonic solid. It is “completely regular” meaning that all of its faces are regular polygon, in this case squares, and that every vertex has exactly the same number of faces joining at that point.

COMMON MISCONCEPTIONS

- It is often difficult for students to visualize as it requires a coordination of both two and three-dimensional shapes. Activities which require students to think about, manipulate, or transform or a shape mentally will contribute to students’ overall visualization skills.
- Students often use incorrect vocabulary to describe three-dimensional figures, calling a cube a “square.”

ESSENTIAL QUESTIONS

- How can I use visualization to determine the attributes of a cube?
- How can I use the attributes of a cube to create a cube?
MATERIALS

- large cube to use as a model
- small cubes for students to hold
- marshmallows or gumdrops
- toothpicks

(Note: marshmallows and toothpicks are just suggested materials to create the cubes. Other materials such as straws, tape, pipe cleaners, modeling clay, etc. would also work.)

GROUPING

Whole group and small groups

TASK

Part I – Visualizing
The teacher should direct the class through a visualization activity.

Teacher holds up one large cube “A cube is a very interesting object. So, we are going to examine it. What does examine mean? Who does examinations? What do you think are the parts of the cube we can examine?”

Without holding a cube, try to picture it in your mind. How many faces does a cube have? How many corners (vertices) does a cube have? How many edges does a cube have? What can we say about the size of the faces and the edges?

Teacher asks questions to have the children think about parts, especially faces, vertices, and edges.

Note: Students may not arrive with the correct vocabulary to have a mathematical discussion about cubes. As the discussion progresses, use the correct mathematical terms and expect that the students begin to use the correct terms as well.

Once students have thoroughly discussed their visualizations, they can hold a cube. How did our visualization compare with the cube in our hand? Why might visualization be important?

Part II – Build your Own Cube
Students will use materials of your choosing to create their own cube.
A “materials request form” is attached (highly recommended because this is a quick formative assessment of who can or is almost able to visualize the cube). You may want to have students predict how many gumdrops/marshmallows and toothpicks they will need to build 1 cube. Allow students who ask for too many or too few to figure out their error on their own as they work.

Here are some examples:
FORMATIVE ASSESSMENT QUESTIONS

- What might help us visualize a cube?
- What experiences do we have with cubes in everyday life?
- When you say sides, do you mean faces? (Use clarifying questions to help students gain mathematical vocabulary.)

DIFFERENTIATION

Extension

- Students may attempt to represent their three-dimensional shapes with a drawing.
- There are also commercially available in which students visualize with multiple cubes.

Intervention

- Students may have difficulty visualizing a cube. They may need to count the faces, edges, and vertices with a teacher. As you count each face, mark it with a sticker so that students do double count the faces. Do the same with edges and vertices.

Intervention Table
Materials Request Form

How many ______________ will you need to make 1 cube? ______

How many toothpicks will you need to make 1 cube? ______
Performance Task: Net or Not?
In this task, students will explore geometric patterns (nets) and determine whether or not the patterns will fold up into cubes. Then they will look at the nets that worked and visualize how the net folds into a cube and where each face ends up. Finally, they will construct the cubes to verify their ideas.

STANDARDS FOR MATHEMATICAL CONTENT
Reason with shapes and their attributes.

MGSE2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.

BACKGROUND KNOWLEDGE

Visualization with three dimensional shapes has been linked to math achievement, but it’s hard to find opportunities to include it in the regular math curriculum. This task offers students the opportunity to mentally manipulate a cube while focusing on the attributes of a cube.

COMMON MISCONCEPTIONS

- Students may be a bit confused about the change in vocabulary for three dimensional shapes. The term vertex refers to point where two sides of a polygon meet or where three edges of a solid meet, both of which could be called a corner. A side is used only with two dimensional shapes. Edge is used to refer to the straight line where two faces meet in three dimensional shapes.
- Students may think that any net with six faces can fold into a cube. Allow students to work through this idea.

ESSENTIAL QUESTIONS

- How does knowing the attributes of a cube help me solve problems about the nets?
- How can I use a model to verify my ideas?
MATERIALS

- Cube nets
  - Card stock cut to 8.5” x 8.5” (if using all nets, 72 pieces will be required)
  - Masking tape
- Stickers – one per net (12 as written)
- Student pages

GROUPING

Pairs

TASK

Before the lesson – Assemble nets. Directions follow.

Technology link to interactive Cube nets game.
http://illuminations.nctm.org/Activity.aspx?id=3544

PART I: Whole Group

- Collect information about what students know about cubes. From previous activities and/or experiences in 1st grade and kindergarten, students will likely mention that a cube has six faces.

- On an overhead or interactive white board, show students a picture of a cube net (attached at end). Tell students that 3D shapes can be represented by two dimensional patterns called nets. The net shown can be folded on the edges and taped together to form a cube. Ask students to visualize how the paper might be folded and taped to form a cube. After students have shared their ideas, demonstrate with a large-scale net. Ask the students to verify their ideas and adjust if necessary.

- Pass out Cube Patterns sheets. Ask students to work in small groups to discuss and identify which of the cube patterns they think will fold up into a cube.

- Explain that students will work in groups to look at a series of nets. They will decide which ones they think will fold into cubes and which will not. If they decide a net won’t fold into a cube, they can write down a quick explanation of their reasoning.

PART II – Visualizing the Cube

After each group has completed the sheet of nets, they should get actual nets and masking tape to verify.

- Tell students that the face with the sticker on will be the bottom of the cube. Have them predict which face will be on the top. (This gives students the opportunity to mentally manipulate an object as referenced in the background knowledge.)
• After students have made their predictions, allow them to construct the cube by folding and taping the net.
• Individually assess each student’s knowledge of the cube’s attributes by having him/her identify an edge, vertex, and face on the cube.

**FORMATIVE ASSESSMENT QUESTIONS**

• Can you point to an edge? How many edges does your cube have?
• Can you point to a vertex? How many vertices does your cube have?
• Can you point to a face? How many faces does your cube have?
• Is that face (the one marked with a sticker) the only one that could be the bottom of the cube? If you made this face the bottom, what would be the top?

**DIFFERENTIATION**

**Extension**

• Include nets that won’t fold into a cube (e.g., a net with 7 faces, or six faces assembled in a 2x3 array). Have students describe how the net should be altered in order to make it work.
• Have students find all eleven nets of a cube. Grid paper or square tiles may be useful.

**Intervention**

• Focus on the construction aspect of the task more so than the prediction. Students who are struggling with visualizing need more experience constructing and discussing observations.
• If students are having trouble counting the edges, faces, or vertices, you can use stickers to help them keep track of the ones they have already counted.

[Intervention Table]
Teacher Directions for assembling nets:
These directions will create one net per pair of students for a class size of 24. You may want to create a few extra for early finishers.

- Cut 72 pieces of cardstock into 8.5” squares
- Tape the squares together to form two sets of each of the following nets. (These are the nets from the student worksheet that will successfully fold into a cube.)

- Label each face with a number 1 – 6. Place a sticker on one of the faces to signify that this face will be the bottom. (This step could also be done by the students to reinforce the idea that a cube has 6 faces.)
- You choose to include the nets that will not fold into a cube. This would be ideal, but students may be upset if everyone else walks away with a cube and they don’t. Allow for extra nets for those students to create a cube that works.
One Net for a Cube
Constructing Task: Sharing Equally
Approximately 1 Day
In this task, students explore equal parts and start to learn about halves and fourth.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

STANDARDS FOR MATHEMATICAL PRACTICE

Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades K-3, partial excerpts, pages 252-258)

While this task is supporting a geometry standard, it also builds fraction understanding through manipulation of shapes. It is important for teachers to understand the connections inherent in the task.

“The first goal in the development of fractions should be to help children construct the idea of fractional parts of the whole - the parts that result when the whole or unit has been partitioned into equal-sized portions or fair shares. Children seem to understand the idea of separating quantity into two or more parts to be shared fairly among friends. They eventually make connections between the idea of fair shares and fractional parts. Sharing tasks are, therefore, good places to begin the development of fractions.

Students initially perform sharing tasks (division) by distributing items one at a time. When this process leaves leftover pieces, it is much easier to think of sharing them fairly if the items can be subdivided. Typical “regions” to share are brownies (rectangles), sandwiches, pizzas, crackers, cake, candy bars and so on.

Problem difficulty is determined by the relationship between the number of things to be shared and the number of sharers. Because children’s initial strategies for sharing involving halving, a good place to begin is with two, four, or even eight sharers, many children will deal out two to each child and then halve each of the remaining brownies.”
“It is a mistake to think that fractional parts such as sixths or eighths are conceptually more difficult than halves or thirds. Note that in the discussion of sharing, halves, fourths, and eighths were explored prior to thirds, sixths, and fifths. This is done because successive halving of parts is a natural process for young children. The number of parts does not correlate with conceptual difficulty. Most state curricula would lead you to believe differently. In fact, if we want children to generalize the concept of fractional parts and connect to this generalization the numeric names of thirds, fourths, fifths, and so on, they must be exposed to more than just halves and thirds in the earliest stages of exploring fractions.”

**ESSENTIAL QUESTIONS**

- How many different ways can we divide a rectangle into two equal parts?
- How are square and rectangles related?

**MATERIALS**

- Math Journal
- “Sharing Equally” Student recording form
- *Give Me Half*, by Stewart Murphy, or similar text

**GROUPING**

Partners

**TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION**

**Part I**

Begin the lesson by asking students this question, “Have you ever had to share something before?”

Hold up an even set number of items, and ask the students for suggestions on how you could share these items with one other person. Have students record their answer in their math journals including pictures and words. This will allow you to begin to see how students see “sharing”. This will produce a variety of results, possibly including students may describe handing objects out one at a time, some students will match all the items up into pairs and then take half of each pair, and some students may even describe cutting each item in half and then giving each person a series of halves. Have students share their explanations and illustrations with the class.
Part II
Read Give Me Half by Stewart Murphy or some other similar book that describes sharing things equally between two people. As you read, stop to discuss the ways the children are sharing the items in the story.

Present students with the situation: Jessica and Katie wanted to share a cake equally. The cake was in the shape of a rectangle. Katie said, “There are two ways to cut the cake to make equal size pieces. Either cut from one vertex to the opposite vertex or cut from the midpoint of one side to midpoint of the opposite side. Those are the only two ways to make one straight cut and have two equal size pieces.” Jessica said, “No, there are other ways to make one cut and share the cake equally besides the ways that you named.” Who is right?

Students should explain their thinking on their student recording form using pictures and words.

Part III
Present students with a similar situation as above but this time they need to share the cake between 4 friends. Ask questions such as, “What will happen to the size of the pieces? Would you rather share with 4 friends or 2 friends? Why? How many different ways could they cut the cake and still have four equal pieces? What do you think we should call each of these four pieces? Why wouldn’t the name thirds or fifths or sixths make sense for these pieces? What would those pieces look like? Would they be bigger or smaller than fourths? Bigger or smaller than halves? How do you know? How many pieces (halves or fourths) do we need in order to talk about the whole cake?

FORMATIVE ASSESSMENT QUESTIONS

- When sharing, how do you make sure all parts are equal?
- Do equal parts have to be the same shape? Why?
- Is there more than one way to cut something in half?
- Is there more than one way to cut something into fourths?

DIFFERENTIATION

Extension
- Students can attempt to divide other polygons into equal halves, thirds, and fourths.

Intervention
- Students who are having difficulty can use pattern blocks, fraction strips, index cards, or construction paper shapes which they can compare and/or cut into equal parts.

Intervention Table
Katie and Jessica wanted to share a cake equally. The cake was in the shape of a rectangle.

Katie said, “There are only two ways to cut the cake to make 2 equal size pieces.”

Either cut from one vertex to the opposite vertex... or Cut from the midpoint of one side to midpoint of the opposite side.

Jessica said, “No, there are other ways to cut the cake into 2 equal pieces.”

Who is right? Use pictures and words to explain your thinking.
Constructing Task: Making Rectangles
Approximately 1 Day
Adapted from www.k-5mathteachingresources.com
In this task, students explore and create different rectangles.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.G.2 Partition a rectangle into rows and columns of same-size squares to find the total number of them.

STANDARDS FOR MATHEMATICAL PRACTICE

Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
6. Attend to precision.

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades K-3, read pages 234-238, and 252-8. Partial excerpt below.)

“It is important to understand that filling regions with units and counting does little to help students develop multiplicative formulas. Even when rectangles are filled with a grid of squares, students are more likely to count the squares than to relate the number of squares to the dimension of the rectangles.”

“IThe goal is not necessarily to develop an area formula but to apply students’ developing concepts of multiplication to the area of rectangles. Not all students will use a multiplicative approach. Many will draw copies of rectangle s and attempt to draw in all the squares. However, it is likely that some will use their rulers to determine the number of squares that will fit along each side and, from that, use multiplication to determine the total area. By having students share their strategies, more students can be exposed to the use of multiplication.”

ESSENTIAL QUESTIONS

• What strategies can I use to count the total number of squares in a rectangle?
• How are arrays and repeated addition related?
**MATERIALS**

- Square tiles
- “Making Rectangles” Student recording form

**GROUPING**

Partners

**TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION**

Working alongside a partner, students will build as many rectangles as possible using 12 square tiles. They should record their answers to the questions on the student recording form. Students create a drawing of each rectangle as well as record the number of columns and rows. Encourage discussion of rotation and dialogue about the number of rows and columns.

Student Directions for the Task:
1. Collect 12 square pattern blocks or colored tiles.
2. How many different rectangles can you make using your 12 squares?
3. Record each rectangle that you make.
4. How many rows did you make in each rectangle? How many squares were in each row?

After they have completed the task, bring students together for a class discussion. Ask the students to think about how they could use numbers to represent or describe their pictures/rectangles. Encourage them to talk about how many rows of a certain amount and how many columns of a certain amount they are seeing. Lead a discussion with a line of questions such as, “How many squares are used to create each rectangle? (12) How do you know? (by counting, or that’s how many we used for each one). If we used the same number of tiles for each rectangle why are there so many different looking rectangles? Why don’t they all look the same? (we arranged the tiles in different numbers of rows or columns). If we wanted to write an addition sentence for each rectangle what would that look like? For instance, would 3+2+4 make sense for the 3 by 4 rectangle? Why not? (that is not what is in each row or column and those numbers don’t add up/total 12). What do you notice about these addition sentences?

**FORMATIVE ASSESSMENT QUESTIONS**

- What strategies did you use to discover the rectangles?
- What is the difference between a row and a column?
- Did you notice any patterns?
- How did you organize your thinking?
- How do you think the number of square tiles affects the number of different rectangles you can create? What would happen if we used a different number of tiles? How about an odd number?
DIFFERENTIATION

Extension

• Students can receive 24 or 36 squares, or an amount of their choice.

Intervention

• Start with a small number of tiles (4), and allow the student to create arrays for the smaller numbers, building understanding to a larger number of tiles. Allow for student exploration, and question along the way to determine whether student truly has an understanding of the concept of an array and what it represents (repeated addition).

Intervention Table
Making Rectangles

1. Collect 12 square pattern blocks or colored tiles.
2. How many different rectangles can you make using your 12 squares?
3. Record each rectangle that you make.
4. How many rows did you make in each rectangle? How many squares were in each row?

<table>
<thead>
<tr>
<th>RECTANGLE</th>
<th>How many rows does your rectangle have?</th>
<th>How many columns does your rectangle have?</th>
</tr>
</thead>
<tbody>
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</tr>
</tbody>
</table>

Name: _______________________  Date ___________
Practice Task: Grandmas’ Quilts
Approximately 3 Days  (Information adapted from the Noyce Foundation, Problems of the Month)
In this task, students create a quilt using their knowledge of partitioning and arrays. (This standard and task scaffold area, which will be studied in 3rd grade.)

STANDARDS FOR MATHEMATICAL CONTENT
MGSE2.G.2 Partition a rectangle into rows and columns of same-size squares to find the total number of them.

STANDARDS FOR MATHEMATICAL PRACTICE
Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
6. Attend to precision.

BACKGROUND KNOWLEDGE
(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades K-3, read pages 234-238, 252. Partial excerpt below.)

“One of the purposes of early comparison activities with areas is to help students distinguish between size (or area) and shape, length, and other dimensions. A long, skinny rectangle may have less area than a triangle with shorter sides. Piagetian experiments indicate that many 8-9 year olds do not understand that rearranging areas into different shapes does not affect the amount of area. Direct comparison of two areas is nearly always impossible except when the shapes involved have some common dimension or property. For example, two rectangles with the same width can be compared directly, as can any two circles. Comparison of these special shapes, however, fails to deal with the attribute of area. Instead, activities in which one area is rearranged are suggested. Cutting a shape into two parts and reassembling it in a different shape can show that the before and after shapes have the same areas, even though they are different shapes. This idea is not at all obvious to children in the K-2 grade range.”


ESSENTIAL QUESTIONS

- What strategies can I use to count the total number of squares in a rectangle?
- How are arrays and repeated addition related?
MATERIALS

- *A Remainder of One* by Elinor J. Pinczes (Houghton Mifflin Co., 1995) or similar book
- *Each Orange has 8 slices: A Counting Book* by P. Giganti
- Array recording sheet (per group)
- Half sheet of chart paper (per group)
- Various manipulatives (connecting cubes, counters, tiles, etc.)
- Construction paper
- Colored pencils, crayons, or markers (optional)

GROUPING

Partners

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I

Begin the lesson explaining to the students they will be creating a quilt. This task will involve the students in partitioning rectangles, which will allow that students to develop a conceptual understanding of arrays. You need not focus the lesson around explicit use of this term or the repeated addition to which it leads, as discussion of these ideas will arise naturally through completion of the task and the student discussion at the close of the task.

Share pictures of arrays that can be seen in the real world and discuss the difference between a row and a column, and of the role orientation plays in those names. Provide some real-world examples- seats in the movie theatre, ice cube tray, egg carton, candy boxes, floor tiles, etc. Read aloud the book, *Each Orange has 8 slices: A Counting Book* by P. Giganti or a similar book. Throughout the book, discuss the number groupings and model for students the rectangles that can be created throughout the story using the illustrations. Before counting the total, have students make predictions about the total number of wheels or balloons.

Ask students if they have ever seen a quilt, or know what one is. Show a photograph of a quilt with an array structure.

Explain to students that quilts are made of fabric with squares that are stitched together. Today they will help a special Grandma decide the number of fabric pieces she needs for the quilts she will make for each of her grandchildren. Each of the squares tells a story or has a message that Grandma’s wants to share. Her grandchildren are Zoe, Zachery, Cierra, and Quinton. Grandma wants to begin by creating a quilt for herself by giving each one of her grandchildren a section. Using a sheet of construction paper, have students use folding to partition the rectangular paper to leave a portion for each of her grandchildren. At this point, you may allow students to decorate each portion. Allow students to share their quilt.
Part II
Students will create a quilt for each of the grandchildren. Zoe’s quilt will have 4 rows and 5 columns. Zachery’s quilt will have 3 rows and 4 columns. Cierra’s quilt will have 2 rows and 4 columns. Quinton’s quilt will have 4 rows and 4 columns. Allow the students to work with a partner to fold a piece of construction that resembles the quilt Grandma would create when she has sewn together the pieces. Allow students to use manipulatives or paper folding to create their quilts. Then, encourage the students to draw their quilts for each grandchild on chart paper and record the strategies they used to find the total number of squares on the recording sheet. While students are working, circulate the room and ask questions like:

- What manipulatives are you using to help solve this problem?
- How can you tell the difference between rows and columns?
- What strategies are you using to help figure out the total number of squares?
- How are you communicating the results you have found?
- What patterns are you noticing from your strategies?
- How does addition help you decide how many pieces in all, or how many rows or columns to make?
- Do any of the grandchildren have the same number of pieces?
- How did working with a partner help you?

Part III
After students have completed the task, choose several students to share their discoveries and observations with the class. The teacher or students can record the arrays and strategies the students have found on a larger version of the array recording sheet, along with observations about arrays, creating an anchor chart about arrays and repeated addition.

FORMATIVE ASSESSMENT QUESTIONS

- How can you tell the difference between rows and columns? Why or when does it matter?
- How did working with a partner help you?
- What do you notice about arrays? How is this arrangement helpful?
- Where else can you find arrays? (packaging, tiling) Why might they appear in these places?

DIFFERENTIATION

Extension
If students complete the assigned task, ask students to identify the different dimensions of the quilt (the rows and columns) if Grandma wanted to create a quilt with 36 squares.

Intervention
Students may build a quilt using color tiles, or construction paper squares in order to visualize the arrays.

Intervention Table
**Grandma's Quilts Recording Sheet**

<table>
<thead>
<tr>
<th></th>
<th>Dimensions</th>
<th>Array (record with dots or X's)</th>
<th>Strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zoe</td>
<td>4 rows and 5 columns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zachary</td>
<td>3 rows and 4 columns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cierra</td>
<td>2 rows and 4 columns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quinton</td>
<td>4 rows and 4 columns</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Constructing Task: Ribbon Fractions

Approximately 2 Days
This lesson is adapted from State of Victoria, Australia: Teaching Resources. Activity 6
In this task, students use ribbon to make equal shares and relate it to creating a number line.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.G.2 Partition a rectangle into rows and columns of same-size squares to find the total number of them.

MGSE2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

STANDARDS FOR MATHEMATICAL PRACTICE
Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE
(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades K-3, page 253-254)

“During the discussions of students’ solutions (and discussions are essential!) is a good time to introduce the vocabulary of fractional parts. This can be quite casual and, at least for younger children, should not involve fraction symbolism. When a brownie or other region has been broken into equal shares, simply say, “We call these fourths. The whole is cut into four parts. All of the parts are the same size- fourths.

Children need to be aware of two aspects or components of fractional parts: (1) the number of parts and (2) the equality of the parts (in size, not necessarily in shape). Emphasize that the number of equal parts or fair shares that make up a whole determines the name of the fractional parts or shares. They will be familiar with halves but should quickly learn to describe thirds, fourths, fifths, and so on.”

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ESSENTIAL QUESTIONS

- Why is it important to identify, label, and compare fractions (halves, thirds, fourths) as representations of equal parts of a whole or of a set?
- If you have two fractions, how do you know which is greater or has more value?

MATERIALS

- For the teacher
  - Several pieces of ribbon cut into 1 yard pieces (these pieces will be used for student demonstration, 3-5 pieces may be necessary)
- For each group
  - 1 yard of ribbon (string or adding machine tape would also work)
  - String
  - paper clips
  - ruler
  - scissors
  - Unifix cubes or snap cubes
- For Differentiation groups
  - Adding machine tape
  - Unifix cubes

GROUPING

Large group, Small groups of 3 or 4

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Part I
Show the class a length of ribbon that is one yard long and ask how to divide it into two equal pieces. You may wish to have volunteers come up and demonstrate their strategies. Some might suggest folding the ribbon in half so the two parts match and then cutting on the fold. Ask, “How do you know this will work?” Make sure to have them explain their strategies, even if they are wrong! Make sure to demonstrate ALL strategies so those that are incorrect can see why what they are thinking does not work, (this is why the teacher will need so many pieces). Allow them an opportunity to change their thinking and have them explain WHY they changed their mind!

Once the class has come to an agreement about how to cut the ribbon then cut it. Make sure to tape an original uncut ribbon (the unit) to the board for comparison. Label it as 1. Discuss how each section is one half of the original ribbon. Have someone hold up the cut pieces (separately) and compare them to the uncut piece (the unit). Tape them to the board and label them with half written above each section. Ask students “Why does it take both parts to make one whole piece?” The goal of this task is to have students understand the terms “halves, thirds, and fourths”.

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**Part II**

Divide the class into groups, with some groups of three and some groups of three or four children. Give each group a piece of ribbon that is one yard long and present them with this problem: How could you divide this ribbon so that each person in your group gets an equal piece? You might use string or adding machine tape depending on what you have available. Make it clear that all of the students in the group should get pieces that are the same length. Suggest that students do not cut the ribbon until the whole group agrees that they can make equal parts. If a group cuts their ribbon and then sees that the pieces are not of equal size, provide them with another ribbon and have them try again. **Make sure they can explain to you WHY they need another ribbon and what their new strategy will be for cutting it this time!**

After ample work time is given, allow groups to share the strategies they used. Some might fold the ribbon into 3 or 4 pieces and then cut it. Others might use something to measure the ribbon and make equal parts using a ruler. The measuring could also be done with other non-standard measures – such as Unifix cubes, snapping cubes etc. During the discussion ask the following questions:

- Does each person in the group have a ribbon that is the same size?
- What fraction could be used to explain how much of the whole each person in the group received?

*Some might fold the ribbon in half and then in half again to create 4 equal parts. If this happens be sure to have those students explain why this strategy works for creating fourths. If the students demonstrate an understanding of this then you pose the following question: “Now that this ribbon is folded into fourths could I use the same strategy to create another fraction? (I.e. eighths). If I use this strategy on a ribbon folded in thirds what would I create? (I.e. sixths). Can you think of other examples of this same kind of thinking?”

After the group discussion, allow all students to carry their ribbon pieces back to their desk (or floor) and ask the following questions:

- Does each person in the class now have a piece of ribbon the same size? (no)
- Why are some ribbon pieces longer than others?
- What would you expect the pieces to be like for the members of a group of 6? 10? (Pieces would be smaller)

**Part III**

Write the fraction words halves, thirds, quarters, and fourths on the board and have a student come up and tape their piece of ribbon underneath the correct spot. Have the students explain which fractional piece they think is the largest and smallest and how they decided. Listen for them to say that if the ribbon is only shared two ways the piece is larger. If it is shared four ways, it is smaller. Make sure that the students can explain that this is because more people need a piece of the ribbon.

Draw a number line that is the length of one uncut piece of ribbon on the board with zero at one end and 1 and the other. Tape an uncut piece of ribbon underneath the one. Ask, “Why did I
place this uncut piece of ribbon here? *(because it is one whole piece of ribbon)* RIGHT! So where should we tape the half piece of ribbon? How about the third piece and the fourth piece? Listen and look for student recognition and understanding that there are numbers that live in-between other (whole) numbers. Check for transfer of understanding by asking if they can show the placement of halves, thirds, quarters, and fourths on a number line (the line goes from zero to one, so halfway would be 1/2.)

**FORMATIVE ASSESSMENT QUESTIONS**

- Does each person in the group have a ribbon that is the same size? How do you know for sure?
- What fraction could be used to explain how much of the whole each person in the group received?
- Does each person in the class now have a piece of ribbon the same size? (no)
- Why are some ribbon pieces longer or shorter than others?
- What would you expect the pieces to be like for the members of a group of 6? 10? 100? (Pieces would be smaller)

**DIFFERENTIATION**

**Extension**

- Give students a new length of ribbon, and tell them it is 1/2 of a length. Allow them to decide what the whole length would be.
- Make a connection to candy bars. Ask students whether they would rather have a half, third, or a fourth of a candy bar. Again, have them explain their thinking. This would be a great topic for journal writing!

**Intervention**

- If students are having difficulty, provide them with pre-cut fractional parts of circles and rectangles to manipulate to create “the whole” from halves, thirds, and fourths

[Intervention Table]
Constructing Task: Making a Cake

Approximately 2 Days
In this task, students use their partitioning strategies for equal parts to solve different problems.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.G.2 Partition a rectangle into rows and columns of same-size squares to find the total number of them.

MGSE2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

STANDARDS FOR MATHEMATICAL PRACTICE

Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
6. Attend to precision.

BACKGROUND KNOWLEDGE

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades 3-5, page 138)

“Students should come to think of counting fractional parts in much the same way as they might count apples or other objects. Students who understand fractional parts should not need to arrange pie pieces into a circle to know that four-fourths make a whole. For each collection, tell students what type of piece is being shown and simply count them together: “one-fourth, two-fourths, three-fourths, and four-fourths. As students count each collection of parts, discuss the relationship to one whole.”

ESSENTIAL QUESTIONS

- Why is it important to identify, label, and compare fractions (halves, thirds, fourths) as representations of equal parts of a whole or of a set?
- How do you know how many fractional parts make a whole?
MATERIALS

- *My Half Day* by Doris Fisher or similar book
- Play-dough
- Circular Sheet
- Rectangular Sheet
- Chart paper
- Markers
- Craft stick
- Job descriptions (Students can refer to these to help them remember their role)
- Task Cards (copies for each group to have one task card)

GROUPING

Large group, Small group

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Part I

Begin by reading a book similar to *My Half Day* written by Doris Fisher, or *Apple Fractions* by Jerry Pallotta to the class. Use fraction circles to model sharing a pie with a small group of students in the class. You can also make copies of the fractions found within the book – enlarge them on the copy machine. Display each as you read about it in the book.

Part II

After a discussion, have students get into groups. Assign students jobs:

- The Recorder is the person who is in charge of managing the writing that goes on the chart.
- The Materials Collector is the person who is in charge of getting the materials needed and putting the materials up after the task is complete.
- The Reporter is the person who is in charge of sharing the strategies their group used and any other information they would like to share about discoveries their group made.

Encourage students that although they have certain “jobs” everyone participates and works cooperatively helping one another.

Have group’s select one task card. They are to read the situation and determine a solution to the task. Students may use cake pans, play dough, and craft sticks to act out the scenario. Students should also share their thought processes through numbers, pictures, and words.
While students are working on the task using the materials provided, circulate the room and ask questions like:

- How many people must share your cake?
- How will you separate your cake into equal pieces?
- What if everyone wanted two pieces, how would this change the fraction of each piece?
- Would that change the amount each person received? Why or Why not?
- Can you explain this process through words?

**Part III**

Allow students to share their task and solution. Have students show their work and explain the strategies they used. Discuss the differences in the sizes of the pieces based on the shape of the cake and the number of ways the cake had to be split. Ask students which cake they would rather be sharing. Allow students in the audience to ask questions, make comments, and give suggestions.

**FORMATIVE ASSESSMENT QUESTIONS**

- How many people must share your cake?
- How will you separate your cake into equal pieces?
- What if everyone wanted two pieces, how would this change the fraction of each piece?
- Would that change the amount each person received? Why or Why not?
- Can you explain this process through words?

**DIFFERENTIATION**

**Extension**

- Present students with this problem and the extension handout: I have a rectangle cake that I want to share with 4 friends. How many different ways can you cut the cake into fourths?
- How would I cut the cake if each of my friends wanted more than two pieces (e.g. 3, 4, or more)?

**Intervention**

Allow students to review the identification of fractions through the website: pbskids.org/cyber chase/games/fractions/index.html to play “13 Ways to a Half”

[Intervention Table]
Making a Cake: Task Cards

Task Card #1
Make a rectangle cake and imagine you want to serve the whole thing to three people.
How could you cut your cake to assure that each person had the same amount of cake?
What fraction would each person get to eat? Use pictures, words, and numbers to explain your answers.

Task Card #2
Make a rectangle cake and imagine you want to serve the whole thing to four people.
How could you cut your cake to assure that each person had the same amount of cake?
What fraction would each person get to eat? Use pictures, words, and numbers to explain your answers.

Task Card #3
Make a circle cake and imagine you want to serve the whole thing to three people.
How could you cut your cake to assure that each person had the same amount of cake?
What fraction would each person get to eat? Use pictures, words, and numbers to explain your answers.

Task Card #4
Make a circle cake and imagine you want to serve the whole thing to two people.
How could you cut your cake to assure that each person had the same amount of cake?
What fraction would each person get to eat? Use pictures, words, and numbers to explain your answers.
Making a Cake: Job Descriptions

Job Descriptions: These could be copied and given to each group so they remember their “job” within the group.

The Recorder is the person who is in charge of managing the writing that goes on the chart.

The Materials Collector is the person who is in charge of getting the materials needed and putting the materials up after the task is complete.

The Reporter is the person who is in charge of sharing the strategies their group used and any other information they would like to share about discoveries in their group.
Constructing Task: Fraction Cookies

Approximately 2 Days
In this task, students model different strategies for partitioning into halves, thirds, and fourths.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

MGSE2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

STANDARDS FOR MATHEMATICAL PRACTICE

Although all standards for mathematical practice should be applied regularly, this task lends itself to the standards below:

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
6. Attend to precision.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

(Information adapted from North Carolina DPI Instructional Support Tools)

Within this task, students will be partitioning circles (cookies) and discussing these circles in reference to their fractional parts. Students will use this information to create graphs. At first students should create real object and picture graphs so each row or bar consists of countable parts. These graphs show items in a category and do not have a numerical scale. For example, a real object graph could show the students’ shoes (one shoe per student) lined end to end in horizontal or vertical rows by their color. Students would simply count to find how many shoes are in each row or bar. The graphs should be limited to 2 to 4 rows or bars. Students would then move to making horizontal or vertical bar graphs with two to four categories and a single-unit scale.

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades K-3, page 254)

“All of the sharing tasks involved sharing something that could be cut into smaller parts. In these situations, the fractions are based on parts of an area or region. This is a good place to begin and is almost essential when doing sharing tasks. There are many good region models, as shown in Figure 9.3 on page 254.
Circular “pie” piece models are by far the most commonly used area model. The main advantage of the circular region is that it emphasizes the amount that is remaining to make up a whole. The strong emphasis on the circle as a whole also has disadvantages. To use the semicircle or any other piece other than the circle to represent the whole would be very confusing. So, there is no challenge for students to construct a whole given one of the pieces as a fractional part. Another disadvantage lies in the fact that each piece is a unit fraction. Drawings of circle models can mislead and be overused. Even adults have difficulty partitioning a circle in a reasonably accurate manner.” In order to overcome this concern, the flag task which follows this one involves the use of rectangular wholes.

**ESSENTIAL QUESTIONS**

- What is a fraction and how can it be represented?
- How do we add fractions?
- How do we apply the use of fractions in everyday life?
- When is it appropriate to use fractions?
- How can we use a picture graph, bar graph, chart, or table to organize data and answer questions?
- What is a survey?

**MATERIALS**

- “Fraction Cookie Order” worksheet, 2 pages (one per student)
- Crayons, colored pencils, or markers
- Survey
- Paper for constructing graphs

**GROUPING**

Individual/Partner Task

**TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION**

In this task, students will create a cookie order in which cookies are divided into fractional parts.

**Part I**

- Tell the students the following scenario:
  - You own your own bakery. Your specialty is fraction cookies. People place orders from all over the country for your cookies. You have recently received the following orders.
  - Before filling the order by making the cookies, you like to confirm your order with a drawing for the customer. (If the toppings ordered do not cover an entire cookie, customers want the remaining portion of the cookie to be left plain.)
  - Using the circle templates below, show how you would create each cookie with the correct fractional amounts of toppings.
  - Share your illustrations with your teacher.
• As students are working, be sure they are dividing the circles into equal-sized pieces and filling the sections as described in the order form.

Example:
This made to order cookie is split equally half raspberries and half plain.

![Cookie diagram]

Part II
Ask students to survey ten family members and/or classmates about their favorite cookie topping, each person can choose up to 4 toppings. The students will write a question that will generate the information needed to determine what their families or classmates like on their cookies. Brainstorm a list of possible topping and have the students make the decision for their survey choices based on this discussion. Remind students that most surveys will use information that is common to all people. This could lead to a discussion about how survey items are chosen and their purpose. Responses to the survey will be recorded on the survey form with a check mark. Point out to the students that a survey form is different from a graph.

Discuss possible ways their answers could be recorded. Students will record their answers on their “Cookies Please” survey form. Each student needs to be sure to label the top row of the table with their four topping options.

Once the items have been decided, have students determine what kind of graph will be needed to record the information. Have students create a blank graph they will use for their survey data of the choice of toppings. Allow students to take this survey home if necessary.

When the surveys come back the next day, discuss the results with the class. If students do not have ten responses, they may also ask their classmates and add these results to their survey.
FORMATIVE ASSESSMENT QUESTIONS

- How do you partition a shape?
- How do you know what name to give a fraction?
- When you were creating your survey, what difficulties did you have?

DIFFERENTIATION

Extension
- Challenge student to become the creator of the triangular or square cookie. Have them convert their illustrations showing the fractional cookies to a triangle template, or square template.
- Have students develop a list of survey questions on a topic of their choice. Allow them to survey additional classrooms or grade levels to compare results.

Intervention
- Provide manipulatives such as connecting cubes for students to model the toppings for the cookies.
- Provide the student with the questions and the survey so that their task is limited to collecting the information and recording the results.
- Provide students with shapes already divided up into the fractional shares halves, thirds, and fourths. These students should still select which they need to use, but they will not be physically partitioning them.

Intervention Table
Fraction Cookies
You own your own bakery. Your specialty is fraction cookies. People place orders from all over the country for your cookies. You recently received the following orders. Before filling the order by making the cookies, you like to confirm your order with a drawing for the customer. (If the toppings ordered do not cover an entire cookie, customers want the remaining portion of the cookie to be left plain.) Using the circle templates below, show how you would create each cookie with the correct fractional amounts of toppings. Share your illustrations with your teacher.

Cookie Orders

<table>
<thead>
<tr>
<th>Order Number</th>
<th>M &amp; Ms</th>
<th>Walnuts</th>
<th>Chocolate Chips</th>
<th>Vanilla Icing</th>
<th>Sprinkles</th>
<th>Chocolate Icing</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>Half</td>
<td>Half</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#2</td>
<td></td>
<td></td>
<td>Two fourth</td>
<td>One fourth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#3</td>
<td>One fourth</td>
<td></td>
<td>One fourth</td>
<td>One fourth</td>
<td>Two fourths</td>
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<tr>
<td>#4</td>
<td>One fourth</td>
<td></td>
<td>One fourth</td>
<td></td>
<td></td>
<td>Two fourths</td>
</tr>
<tr>
<td>#5</td>
<td>One third</td>
<td></td>
<td>One third</td>
<td>One third</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#6</td>
<td></td>
<td></td>
<td>Two fourths</td>
<td>Two fourths</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#7</td>
<td>One fourth</td>
<td></td>
<td>One fourth</td>
<td>One fourth</td>
<td>One fourth</td>
<td></td>
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<tr>
<td>#8</td>
<td>One third</td>
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<td></td>
<td></td>
<td>Two thirds</td>
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</tbody>
</table>
COOKIES PLEASE!

Survey Question: 

<table>
<thead>
<tr>
<th>Topping #1</th>
<th>Topping #2</th>
<th>Topping #3</th>
<th>Topping #4</th>
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<td>Total</td>
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<td>#1</td>
<td>Half</td>
<td>Half</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#2</td>
<td></td>
<td></td>
<td>Two fourth</td>
<td>One fourth</td>
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<tr>
<td>#3</td>
<td>One fourth</td>
<td></td>
<td></td>
<td>One fourth</td>
<td>Two fourths</td>
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<tr>
<td>#4</td>
<td>One fourth</td>
<td>One fourth</td>
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<td>#5</td>
<td>One third</td>
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<td>#6</td>
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<td>Two fourths</td>
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<tr>
<td>#7</td>
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<tr>
<td>#8</td>
<td>One third</td>
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<td></td>
<td>Two thirds</td>
</tr>
</tbody>
</table>

Mathematics • GSE Second Grade • Unit 5: Understanding Plane and Solid Figures
Richard Woods, State School Superintendent
July 2018 • Page 82 of 94
All Rights Reserved
Fraction Cookie Orders
**COOKIES PLEASE!**

Survey Question:

_________________________________________________________

_________________________________________________________

<table>
<thead>
<tr>
<th>Topping #1</th>
<th>Topping #2</th>
<th>Topping #3</th>
<th>Topping #4</th>
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</thead>
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<td>Total</td>
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</tbody>
</table>
Performance Task: My Country’s Flag
In this task, students will create a flag using their knowledge of shapes and their attributes. They also explore partitioning and creating a flag that has more than one part.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

MGSE2.G.2 Partition a rectangle into rows and columns of same-size squares to find the total number of them.

MGSE2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

This task should help to conclude Unit 5. This unit serves to address fourth Critical Area of Focus. “Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.”
ESSENTIAL QUESTIONS

- Why is it important to identify, label, and compare fractions (thirds, sixths, eighths, tenths) as representations of equal parts of a whole or of a set?
- What is a fraction and how can it be represented?
- What do the parts of a fraction tell us?
- If we have two fractions, how do we know which is greater or has more value?
- How do we know how many fractional parts make a whole?
- How are arrays and repeated addition related?
- How do we describe geometric figures?

MATERIALS

- Flags from Various Countries
- 1 inch grid paper
- Colored tiles
- Pencils
- Centimeter or dot paper
- Tape or glue stick
- Colored Pencils, markers, or crayons

GROUPING

Individual

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Part I
Show students flags from various countries. Discuss the fractions found within each of the flags. Discuss what happens when you add up each of the fractions. Tell students that today they will get to create their own flag for a make-believe country. They will have to:

- Use 24 square tiles to create the rectangular array for the measurements. The array does not have to result in the shape for a typical flag. Allow students to create a flag from anything 1 row of 24 to 24 rows of 1, as long as the students can explain the number of rows and columns and how many tiles they are using in all. Then trace the array to draw the boundary of the flag.
Students will use up to 4 colors to decorate their flag. Each color must be used for an equal share, allowing certain colors to have more than one share. (Example, you may have your flag colored one fourth red, one fourth black and two fourths white.)

Draw figures with centimeter paper, dot paper, or using a ruler. Create at least one of the following plane shapes to be included on the flag: triangle, quadrilateral, pentagon, and hexagon.

Part II
Give each student 2 pieces of 1-inch grid paper, centimeter or dot paper, and glue, paper colored tiles, and colored pencils. Have students brainstorm ideas for flags on one sheet of grid paper to begin with. Then, allow students to create their fraction flags. Students could cut out their shapes and affix them to the flag. While students are working, ask questions such as:

- Can you tell me about your design?
- How did you decide the number of rows and columns?
- How can you use fractions to describe the colors in this flag?
- What will happen when you add all the fractions within your flag together?
- Which fractions within your flag are the largest? Smallest?
- What are the shapes of your fractions?
- Are any of your fractions equal?
- What are the properties of your geometric shapes?

Part III
Students will create a written sample that responds to the prompts below.

- Identify the number of rows and columns in your flag.
- How did you decide the number of rows and columns?
- Explain the fractions that describe the colors in this flag?
- What geometric shapes did you use in your flag?
- What are the attributes of your geometric shapes?

Part IV
After students have created their flags, have them share the flags and the writing sample with the class. Allow other students to ask questions and make comments about the flags their peers have created. Display the flags, their descriptions, and feedback for others to see.

Variation
Students could make/use quilt squares instead of flags.

FORMATIVE ASSESSMENT QUESTIONS

- Identify the number of rows and columns in your flag.
- How did you decide the number of rows and columns?
- Explain the fractions that describe the colors in this flag?
- What geometric shapes did you use in your flag?
- What are the attributes of your geometric shapes?
Can you tell me about your design?
How did you decide the number of rows and columns?
How can you use fractions to describe the colors in this flag?
What will happen when you add all the fractions within your flag together?
Which fractions within your flag are the largest? Smallest?
What are the shapes of your fractions?
Are any of your fractions equal?
What are the properties of your geometric shapes?

DIFFERENTIATION

Extension
- Students could be given 36 tiles to create array and asked to identify the number of tiles that make-up each fraction.

Intervention
- Students could have a choice of rectangles that have been divided into halves, thirds, or fourths.

Intervention Table
My Country’s Flag

Identify the number of rows and columns in your flag.

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

How did you decide the number of rows and columns?

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

What other arrays did you consider?

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

Explain the fractions that describe the colors in this flag?

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

What geometric shapes did you use in your flag?

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

What are the attributes of your geometric shapes?

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________
SCAFFOLDING TASK: Teeth Graph

Approximately 1 day
(Adapted from http://www.scoe.org/files/mars-grade2.pdf)

Note: The graphing standard MD.10 is stretched throughout the year. By the end of the year, students will be creating their own picture and bar graphs. This scaffolding task is designed to prepare them for future construction tasks.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put together, take-apart, and compare problems using information presented in a bar graph.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
5. Use appropriate tools strategically.
6. Attend to precision.

BACKGROUND INFORMATION

(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades K-3, pages 319 – 321)

“Bar graphs and tally charts are among the first ways to group and present data and are especially useful in grades K-3. At this early level, bar graphs should be made so that each bar consists of countable parts such as squares, objects, tallies, or pictures of objects period. A “real graph” uses the actual objects being graphed. Examples include types of shoes, sea shells, and books. Each item can be placed in a square so that comparisons and counts are easily made. Picture graphs use a drawing of some sort that represents what is being graphed. Students can make their own drawings, or you can duplicate drawings to be colored or cut out to suit particular needs.”

ESSENTIAL QUESTION

How do you use a bar graph to gain information?

MATERIALS

Graphing Sheet: Teeth Lost in 2nd Grade
GROUPING

Grouping should be determined based on students’ prior experiences with graphing. For students unfamiliar with graphing, completing the task as a whole group would be appropriate. While students already familiar may benefit from working in pairs or individually.

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Part I – Create a “Real Graph”
(Information quoted from Van de Walle and Lovin, Teaching Student-Centered Mathematics: Grades K-3, pages 321)
“A “real graph” uses the actual objects being graphed. Examples include types of shoes, sea shells, and books. Each item can be placed in a square so that comparisons and counts are easily made.”

Creating a “real graph” will help students connect representing information (data) with the graphical representation of a graph. Use shoes or another easily accessible resource. Put the items in a pile and have students identify categories that they could be divided into (i.e. sneakers, sandals, crocs, jellies, etc.)

Use the tiles on the floor in the classroom or in the hallway as squares to keep your graph accurate. Make sure to use one square for each shoe. Simply lining the shoes up may create a student misconception as some shoes, such as sneakers, may be bulkier than sandals. There may be more sandals, but a line of sneakers tip to tip may look longer.

Types of questions can include:
- How many sneakers?
- How many sneaker and jellies?
- How many more sneakers than jellies?
- How many fewer sandals than sneakers?
- How many shoes in all?

Part II – Teeth Lost by 2nd Graders
Give students a copy of the “Teeth Lost by 2nd Graders” graphing task. Are the students able to connect the experience with “real graphs” with the teeth graph?

After students have completed the task, look over their work and consider which students have a solid understanding of how graphs are created and how to use the graph to solve questions?

FORMATIVE ASSESSMENT QUESTIONS

What does this graph represent?
What questions can we ask and answer using this graph?
DIFFERENTIATION

Extension

- Students can create their own graph either on teeth lost or another topic. Students can either use a data set provided by the teacher or collect their own data with a spinner or using a survey.
- Students can create a picture graph based on the same or different data set.
- The 2\textsuperscript{nd} grade standard only requires students to graph with a single-unit scale (i.e. intervals of 1). Have students graph the same or different data set using an interval other than 1.

Intervention

- Students may find “how many more” or “how many fewer” type questions particularly challenging. These questions are asking for the difference between two groups. Provide a variety of experiences for students to count up from the smaller quantity to the larger quantity. When students are ready, show how and why a subtraction equation can be used to find the difference between the quantities.
- Students may also have difficulty with a question that has more than 1 possible answer such as question #7. Help students determine if they answered the question being asked.

[Intervention Table](#)
Georgia Department of Education
Georgia Standards of Excellence Framework
GSE Understanding Plane and Solid Figures • Unit 5

Name ________________________________________    Date___________

Teeth Lost by 2nd Graders

10 9 8 7 6 5 4 3 2 1

Ping Samir Angel Laura Franco Imani Tory

1. Tory lost 4 teeth. Show this on the graph.
2. How many teeth did Franco lose? ______
3. How many teeth did Samir and Angel lose together? ______
4. Which student lost the greatest number of teeth? ______
5. How many more teeth did Ping lose than Imani? ______
6. How many teeth did the students lose in all? ______
   Show your calculations. Explain how you figured it out.

7. Sydney lost more teeth than Franco and fewer teeth than Laura. How many teeth could Sydney have lost? ______
   Explain how you figured it out.