Georgia Standards of Excellence Curriculum Frameworks

Mathematics

GSE Grade 6

Unit 1: Number System Fluency

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"Educating Georgia's Future"

Unit 1
Number System Fluency

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OVERVIEW

In this unit students will:

- Find the greatest common factor of two whole numbers less than or equal to 100.
- Find the least common multiple of two whole numbers less than or equal to 12.
- Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor.
- Interpret and compute quotients of fractions.
- Solve word problems involving division of fractions by fractions using visual fraction models and equations to represent the problem.
- **Fluently** divide multi-digit numbers using the standard algorithm.
- **Fluently** add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

At each grade level in the standards, one or two fluencies are expected. For sixth graders the expected fluencies are multi-digit whole number division and multi-digit decimal operations. Procedural fluency is defined by the Common Core as "skill in carrying out procedures flexibly, accurately, efficiently and appropriately". Students may not achieve fluency within the scope of one unit but it is expected the fluency will be obtained by the conclusion of the course.

In the past, fraction and decimal computation have been dominated by rules but research-based best practices have proven that **students who are taught to focus on the pencil-and-paper rules for decimal computation do not even consider the actual values of the numbers**. Therefore a good place to begin decimal computation is with **estimation**. It helps children to look at answers in terms of a **reasonable range**.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, **routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis**. Ideas related to the eight practice standards should be addressed constantly as well. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under "Evidence of Learning" be reviewed early in the planning process. A variety of resources should be utilized to supplement this unit. This unit provides much needed content information, but excellent learning activities as well. The tasks in this unit illustrate the types of learning activities that should be utilized from a variety of sources.

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics especially with respect to fluency.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of real-world fraction and decimal problem situations by representing the context in tactile and/or virtual manipulatives, visual, or algebraic models.

2. Reason abstractly and quantitatively. Students will apply the constructs of multiplication, division, addition, and subtraction of rational numbers to solve application problems.

3. Construct viable arguments and critique the reasoning of others. Students construct and critique arguments regarding the portion of a whole as represented in the context of real-world situations. Students explain why they do not always get a smaller number when dividing with fractions and decimals. Students have to reason the steps in modeling division of fractions.

4. Model with mathematics. Students will model real-world situations to show division of fractions. Students use number lines and tape diagrams to find least common multiple and greatest common factor.

5. Use appropriate tools strategically. Students will use visual or concrete tools for division of fractions with understanding.

6. Attend to precision. Students attend to the language of problems to determine appropriate representations and operations for solving real-world problems. In addition, students attend to the precision of correct decimal placement used in real-world problems.

7. Look for and make use of structure. Students examine the relationship of rational numbers (positive decimal and fraction numbers) to the number line and the place value structure as related to multi-digit operations. They also use their knowledge of problem solving structures to make sense of word problems.

8. Look for and express regularity in repeated reasoning. Students demonstrate repeated reasoning when dividing fractions by fractions and connect the inverse relationship to multiplication. Students also use repeated reasoning when solving real-world problems using rational numbers.

STANDARDS FOR MATHEMATICAL CONTENT

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

MGSE6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, including reasoning strategies such as using visual fraction models and equations to represent the problem.
For example:

- *How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally?*
- *How many 3/4-cup servings are in 2/3 of a cup of yogurt?*
- *How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*
- *Three pizzas are cut so each person at the table receives ¼ pizza. How many people are at the table?*
- *Create a story context for (2/3)÷(3/4) and use a visual fraction model to show the quotient;*
- *Use the relationship between multiplication and division to explain that (2/3)÷(3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc)*

Compute fluently with multi-digit numbers and find common factors and multiples

MGSE6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.

MGSE6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

MGSE6.NS.4 Find the common multiples of two whole numbers less than or equal to 12 and the common factors of two whole numbers less than or equal to 100.

a. Find the greatest common factor of 2 whole numbers and use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factors. (GCF) *Example: 36 + 8 = 4(9 + 2)*
b. Apply the least common multiple of two whole numbers less than or equal to 12 to solve real-world problems.

BIG IDEAS

- The meanings of each operation on fractions are consistent with the meanings of the operations on whole numbers. For example: **It is possible to divide fractions without multiplying by the inverse or reciprocal of the second fraction**.
- Least common multiple and greatest common factor are helpful when solving real-world problems.
- When dividing by a fraction, there are two ways of thinking about the operation – **partition and measurement**, which will lead to two different thought processes for division.
- When we divide one number by another, we may get a quotient that is bigger than the original number, smaller than the original number, or equal to the original number.

ESSENTIAL QUESTIONS FOR THIS UNIT

- Why would it be useful to know the greatest common factor of a set of numbers?
- Why would it be useful to know the least common multiple of a set of numbers?
- How can the distributive property help me with computation?
- Why does the process of invert and multiply work when dividing fractions?
- When I divide one number by another number, do I always get a quotient smaller than my original number?
- When I divide a fraction by a fraction what do the dividend, quotient and divisor represent?
- What kind of models can I use to show solutions to word problems involving fractions?
- Which strategies are helpful when dividing multi-digit numbers?
- Which strategies are helpful when performing operations on multi-digit decimals?

CONCEPTS & SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- number sense
- computation with multi-digit whole numbers and decimals (to hundredths), including application of order of operations
- addition, subtraction, multiplication, and division of common fractions
- familiarity with factors and multiples
- data usage and representations

FLUENCY

It is expected that students will continue to develop and practice strategies to build their capacity to become fluent in mathematics and mathematics computation. The eventual goal is automaticity with math facts. This automaticity is built within each student through strategy development and practice. The following section is presented in order to develop a common understanding of the ideas and terminology regarding fluency and automaticity in mathematics:

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply “how to get the answer” and instead support students’ ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students:

- flexibly use a combination of deep understanding, number sense, and memorization.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.

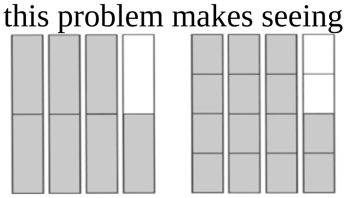
For more about fluency, see: http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and: http://joboaler.com/timed-tests-and-the-development-of-math-anxiety/

STRATEGIES FOR TEACHING AND LEARNING

Computation with fractions is best understood when it builds upon the familiar understandings of whole numbers and is paired with visual representations. **Solve a simpler problem with whole numbers, and then use the same steps to solve a fraction divided by a fraction**. Looking at the problem through the lens of "How many groups?" or "How many in each group?" helps visualize what is being sought.

For example: $12 \div 3$ means "How many groups of three would make 12?" Or, "How many in each of 3 groups would make 12?" Thus $\frac{7}{2} \div \frac{1}{4}$ can be solved the same way. "How many groups of $\frac{1}{4}$ make $\frac{7}{2}$?"

Creating the picture that represents this problem makes seeing and proving the solutions easier:



Set the problem in context and represent the problem with a concrete or pictorial model.

$\frac{5}{4} \div \frac{1}{2} = \frac{5}{4}$ cups of nuts fills $\frac{1}{2}$ of a container.

Teaching "invert and multiply" without developing an understanding of why it works first leads to confusion as to when to apply the shortcut.

Learning how to compute fraction division problems is only part of the grade level expectation; being able to relate the problems to real-world situations is important. Providing opportunities to create stories for fraction problems or writing equations for situations is needed.

As students study whole numbers in the elementary grades, a foundation is laid in the conceptual understanding of each operation. Discovering and applying multiple strategies for computing creates connections which evolve into the proficient use of standard algorithms. Fluency with an algorithm denotes an ability that is efficient, accurate, appropriate and flexible. Division was introduced in Grade 3 conceptually, as the inverse of multiplication. In Grade 4, division continues using place-value strategies, properties of operations, the relationship with multiplication, area models, and rectangular arrays to solve problems with one digit divisors. In Grade 6, fluency with the algorithms for division and all operations with decimals is developed. Fluency is something that develops over time; practice should be given over the course of the year as students solve problems related to other mathematical studies. Opportunities to determine when to use paper-pencil algorithms, mental math, or a computing tool are also necessary and should be provided in problem solving situations.

Greatest common factor and *least common multiple* are usually taught as a means of combining fractions with unlike denominators. This cluster builds upon the previous learning of the multiplicative structure of whole numbers, as well as prime and composite numbers in 4th grade. Although the process is the same, the point is to become aware of the relationships between numbers and their multiples. For example, consider answering the question: **"If two numbers are multiples of four, will the sum of the two numbers also be a multiple of four?"** Being able to see and write the relationships between numbers will be beneficial as further algebraic understandings are developed. Another focus is to be able to see how the GCF is useful in expressing the numbers using the distributive property, $(36 + 24) = 12(3 + 2)$, where 12 is the GCF of 36 and 24. **This concept will be extended in Expressions and Equations (Unit 3) as work progresses from applying the distributive property with arithmetic expressions to algebraic expressions, and then towards simplifying and solving algebraic equations in 7th grade**.

***Multiplication of fractions* is really about scaling**. If you scale something by a factor of 2, you multiply it by 2. When you scale by 1 (1 times the size), the amount is unchanged (identity property of multiplication). Similarly, multiplying by ½ means taking half of the original size, while multiplying by 1½ means the original size plus half the original size. **This scaling concept can enhance students' ability to decide whether their answers are reasonable.** The story problems that you use to pose multiplication tasks to students need not be elaborate, but it is important to think about the numbers and contexts that you use in the problems. (Van de Walle, vol. 3, Teaching Student-Centered Mathematics, p. 129).

Prime Factorization. The following response is from Bill McCallum, one of the lead writers for the mathematics standards, in regards to teaching prime factorization and using it to find greatest common factor: "Greatest common factors and least common multiplies are treated with a very light touch in the standards. They are not a major topic, and limited to numbers less than or equal to 100 (6.NS.4). For such numbers, listing the factors or multiplies is probably the most efficient method, and has the added benefit of reinforcing number facts. It also supports the meaning of the terms: you can see directly that you are finding the greatest common factor or the least common multiple. The prime factorization method can be a bit mysterious in this regard. And, as you point out, **prime factorization is not a topic in the standards**, although prime numbers are mentioned in 4.OA.4. So, the standards do indeed remove this topic from the curriculum. Achieving the focus of the standards means giving some things up, and this is one of those things. (Of course personally, as a number theorist, I love the topic!)"

<u>COMMON MISCONCEPTIONS</u>

- *Students may believe that dividing by ½ is the same as dividing in half. Dividing by half means to find how many one-halves there are in a quantity, whereas, dividing in half means to take a quantity and split it into two equal parts.* $7 \div \frac{1}{2} = 14$ *and* $7 \div \frac{1}{2} \neq 3\frac{1}{2}$
- *Students may understand that* $1\frac{1}{2} \div \frac{1}{4}$ *means, "How many fourths are in* $1\frac{1}{2}$*?" So, they set out to count how many fourths (6). But in recording their answer, they can get confused as to what the 6 refers to and think it should be a fraction, and they record* $\frac{6}{4}$ *when actually it is 6 groups of one-fourths, not 6 fourths (Cramer et al., 2010).*
- *As noted above,* ***knowing what the unit is (the divisor) is critical and must be understood in giving the remainder****. In the problem* $3\frac{3}{8} \div \frac{1}{4}$, *students are likely to count 4 fourths for each whole number (12 fourths) and one more for* $\frac{2}{8}$, *but then not know what to do with the extra eighth. It is important to be sure they understand the measurement concept of division. Ask, "How much of the next piece do you have?" Context can also help. In this case, if the problem was about pizza servings, there would be 13 full servings and* $\frac{1}{2}$ *of the next serving. (Van de Walle, vol. 3, <u>Teaching Student-Centered Mathematics</u>, p. 140).*

- *The most common error in adding fractions is to add both the numerators and the denominators. For example, one teacher asked her fifth graders if the following was correct:* $\frac{3}{8} + \frac{2}{8} = \frac{5}{16}$. *A student correctly replied, "No, because they are eighths (holds up* *one-eighth of a fraction circle). If you put them together, you still have eighths (shows this with the fraction circles). See, you didn't make them into sixteenths when you put them together. They are still eighths." (Mack, 2004, p. 229). Develop fraction number sense by routinely asking students to estimate using benchmarks before computing.*

- *Many students have trouble finding common denominators because they are not able to come up with common multiples of the denominators quickly. This skill requires having a good command of multiplication facts. Students benefit from knowing that any common denominator will work. Least common denominators are preferred because the computation is more manageable with smaller numbers, and there is less simplifying to do after adding or subtracting.* ***Do not require least common multiples, support all common denominators, and through discussion students will see that finding the smallest multiple is more efficient.*** *(Van de Walle, vol. 3, Teaching Student-Centered Mathematics, p. 128-129).*

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for middle school students. **Note – Different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**

http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary

http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

http://www.corestandards.org/Math/Content/mathematics-glossary/glossary

- **Algorithm**: a step-by-step solution to a problem.

- **Difference:** The amount left after one number is subtracted from another number.

- **Distributive Property:** The sum of two addends multiplied by a number equals the sum of the product of each addend and that number.

- **Dividend:** A number that is divided by another number.

- **Divisor:** A number by which another number is to be divided.

- **Factor:** When two or more integers are multiplied, each number is a factor of the product. "To factor" means to write the number or term as a product of its factors.

- **Greatest Common Factor:** The largest factor that two or more numbers have in common.

- **Least Common Multiple:** The smallest multiple (other than zero) that two or more numbers have in common.

- **Measurement Model of Division:** When we know the original amount and the size or measure of ONE part, we use measurement division to find the number of parts. Ex: 20 is how many groups of 4?

- **Minuend:** The number that is to be subtracted from.

- **Multiple:** The product of a given whole number and an integer.

- **Quotient:** A number that is the result of division.

- **Partitive Model of Division:** When we know the original amount and the number of parts, we use partitive division to find the size of each part. Ex: 20 is 4 groups of what unit?

- **Reciprocal:** Two numbers whose product is 1. The reciprocal of a fraction can be found by inverting that fraction (switching the denominator and numerator).

- **Sum:** The number you get by adding two or more numbers together.

- **Subtrahend:** The number that is to be subtracted.

- **Product:** A number that is the result of multiplication.

INSTRUCTIONAL RESOURCES AND TOOLS

- Base Ten Blocks and/or Unit Cubes
- Centimeter Grid Paper
- Color Tiles
- Fraction Tiles/Circles
- Number Lines

- Visual Fractions This website provides interactive practice with both area and linear models for exploring fraction multiplication and division (as well as earlier fraction skills and concepts).
- Models for Multiplying and Dividing Fractions This teacher resource shows how the area model can be used in multiplication and division of fractions. There is also a section on the relationship to decimals.
- From the National Library of Virtual Manipulatives: Fractions - Rectangle Multiplication Use this virtual manipulative to graphically demonstrate, explore, and practice multiplying fractions.

FORMATIVE ASSESSMENT LESSONS (FAL)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Guide.

SPOTLIGHT TASKS

For middle and high schools, each Georgia Standards of Excellence mathematics unit includes at least one Spotlight Task. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Georgia Standards of Excellence, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky.

3-ACT TASKS

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

TASKS

Task Name	Task Type *Grouping Strategy*	Content Addressed	Standards Addressed
Geared Up (Spotlight Task)	Constructing Task *Individual/Partner Task*	Factors and Multiples	MGSE6.NS.4
Counting and Building Rectangles (Spotlight Task)	Constructing Task *Individual/Partner Task*	Factors, Multiples, Primes & Composites	MGSE6.NS.4
Factors and Multiples Puzzles	Constructing Task *Individual/Partner Task*	Factors and Multiples	MGSE6.NS.4
Back To School	Practice Task *Individual/Partner Task*	Least Common Multiple	MGSE6.NS.4
Secret Number	Practice Task *Individual/Partner Task*	Factors and Multiples	MGSE6.NS.4
Let's Distribute	Formative Task *Individual/Partner Task*	Distributive Property	MGSE6.NS.4
Hanging By a Hair (Spotlight Task)	Scaffolding/Learning Task *Individual/Partner Task*	Dividing with the Standard Algorithm	MGSE6.NS.3
School Fund Raiser	Scaffolding/Learning Task *Individual/Partner Task*	Dividing with the Standard Algorithm	MGSE6.NS.2
Scaffolding Division Through Strip Model Diagramming (Spotlight Task)	Scaffolding/Learning Task *Individual/Partner Task*	Dividing with the Standard Algorithm	MGSE6.NS.2
Discovering an Algorithm for Dividing Fractions	Scaffolding/Learning Task *Individual/Partner Task*	Division of Fractions	MGSE6.NS.1
The Kool-Aid Kid (Spotlight Task)	Scaffolding/Learning Task *Individual/Partner Task*	Division of Fractions	MGSE6.NS.1
Dividing Fractions in Context	Scaffolding/Learning Task *Individual/Partner Task*	Division of Fractions	MGSE6.NS.1
Fractional Divisors	Scaffolding/Learning Task *Individual/Partner Task*	Division of Fractions	MGSE6.NS.1
Dividing Fractions with Modeling	Scaffolding/Learning Task *Individual/Partner Task*	Division of Fractions	MGSE6.NS.1
Understanding Algorithms	Scaffolding/Learning Task *Individual/Partner Task*	Division of Fractions	MGSE6.NS.1

Do It Yourself	Formative Task *Partner/Small Group Task*	Division of Fractions	MGSE6.NS.1
Modeling Fraction Division (FAL)	Formative Assessment Lesson *Individual/Partner*	Division of Fractions	MGSE6.NS.1
Estimating is the Root of Fluency – Addition and Subtraction	Formative Task *Individual Task*	Decimal Fluency	MGSE6.NS.3
Where Does the Decimal Go? (Multiplication)	Constructing Task *Individual/Partner Task*	Decimal Fluency	MGSE6.NS.3
Where Does the Decimal Go? (Division)	Constructing Task *Individual/Partner Task*	Decimal Fluency	MGSE6.NS.3
Culminating Task: Pick a Number, Any Number	Summative Performance Task *Individual/Partner Task*	Factors, Multiples, Distributive Property, Division of Fractions	MGSE6.NS.1 MGSE6.NS.2 MGSE6.NS.3 MGSE6.NS.4

Geared Up (Spotlight Task)

Back to Task Table

Task adapted from gfletchy.com/geared-up/

In this inquiry-based task, students will explore the concept and application of least common multiple.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS.4 Find the common multiples of two whole numbers less than or equal to 12 and the common factors of two whole numbers less than or equal to 100.

a. Find the greatest common factor of 2 whole numbers and use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factors. (GCF) *Example:* $36 + 8 = 4(9 + 2)$
b. Apply the least common multiple of two whole numbers less than or equal to 12 to solve real-world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students must make sense of the problem by identifying what information they need to solve it.
2. Reason abstractly and quantitatively. Students were asked to make an estimate (high and low).
3. Construct viable arguments and critique the reasoning of others. After writing down their own question, students discussed their question with tablemates, creating the opportunity to construct the argument of why they chose their question, as well as critiquing the questions that others came up with.
4. Model with mathematics. Once the given information was communicated, the students used that information to develop a mathematical model.
5. Use appropriate tools strategically. In the "debrief", the teacher discussed the different use of tools. By doing this, students are provided with more tools in their toolbox for future problem solving.
6. Attend to precision. Students correctly use of the terms factors, multiple, and spurs to ensure that they are communicating precisely.
7. Look for and make use of structure. The students had to develop an understanding of the physical structure in order to develop a mathematical model that had a numerical structure of its own. The student had to make the connection between the physical structure and the numerical structure of the mathematical model.

ESSENTIAL QUESTIONS

- How are multiples applicable in everyday life?
- How can I use models to represent multiples of a number?
- How is Least Common Multiple used to solve problems?

MATERIALS REQUIRED

- 3-Act Task http://gfletchy.com/geared-up/
- Act-1 Video https://vimeo.com/90593553
- Act-2 Picture http://gfletchy3act.files.wordpress.com/2014/03/how-many-spurs-on-each-wheel.docx
- Act-3 Video https://vimeo.com/90593611
- 3-Act Task Recording Sheet

TIME NEEDED

- 1 day

TEACHER NOTES

In this task, students will watch the video and then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need—and ask for it—it will be given to them.

TASK DESCRIPTION

The following 3-Act Task can be found at: http://gfletchy.com/geared-up/

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

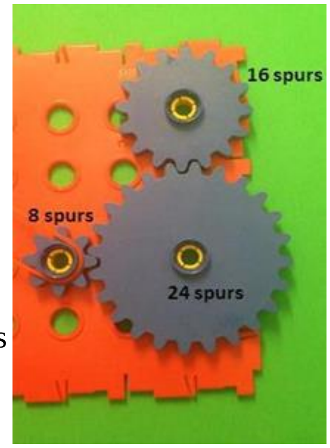
ACT 1:
Watch the video: https://vimeo.com/90593553

Students are asked what they noticed in the video. Students record what they noticed or wondered on the recording sheet. Students are asked to discuss and share what they wondered (or are curious about) as related to what they saw in the video.

Important Note: Although the MAIN QUESTION of this lesson is "*How many times must each wheel turn to make all of the dots line up again"* it is important for the teacher to not ignore student generated questions and try to answer as many of them as possible (before, during, and after the lesson as a follow up).

Main Question: *How many times must each wheel turn to make all of the dots line up again?* Write down an estimate you know is too high, and one you know is too low.

ACT 2:
Students will realize that they do not have enough information to complete the problem. Release the following information to students ONLY AFTER they have identified what information they need.



Required information:

- Small gear has 8 spurs
- Medium gear has 16 spurs
- Large gear has 24 spurs

Students may work individually for 5-10 minutes to come up with a plan, test ideas, and problem-solve. After individual time, students may work in groups (3-4 students) to collaborate and solve this task together.

ACT 3

Show students the Act-3 Video https://vimeo.com/90593611
Students will compare and share solution strategies.

- Reveal the answer. Discuss the theoretical math versus the practical outcome.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren’t answered.

When wrapping up and summarizing this lesson it will be important for students to make the connection between this task and *least common multiple* if the connection has not already been made.

FORMATIVE ASSESSMENT QUESTIONS

- How did your understanding of multiples and factors help you to make sense of this problem?
- What models did you create?
- What organizational strategies did you use?

Geared Up Name:________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate how many turns each wheel must make until the dots are lined up again.

	Your estimate	Estimate (too low)	Estimate (too high)
Small wheel			
Medium wheel			
Large wheel			

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:_______________________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Counting and Building Rectangles (Spotlight Task)

Back to Task Table

In this hands-on task, students will explore the relationship between a number and its factors.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE.6.NS.4 Find the common multiples of two whole numbers less than or equal to 12 and the common factors of two whole numbers less than or equal to 100.

a. Find the greatest common factor of 2 whole numbers and use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factors. (GCF) *Example: 36 + 8 = 4(9 +2)*
b. Apply the least common multiple of two whole numbers less than or equal to 12 to solve real-world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will make sense of factors as they build rectangles for each of the numbers. This can help students build connections to primes and composites investigated in 4th grade.
2. Reason abstractly and quantitatively. As students investigate the numbers and shapes of rectangles made from different numbers of tiles, they will be asked to look for and explain the patterns they discover.
4. Model with mathematics. Students will model numbers by creating different rectangles for specific numbers of square tiles.
6. Attend to precision. Students will use math vocabulary to communicate ideas clearly and compute accurately.
7. Look for and make use of structure. For each number students investigate, they will discover that the number of ways to make rectangles is directly related to the number of factors.
8. Look for and express regularity in repeated reasoning. Students will likely make sense of the commutative property of multiplication as they build rectangles, realizing that a 4×6 rectangle has the same factors as a 6×4 rectangle.

ESSENTIAL QUESTIONS

- What are common factors? Common multiples?
- What can factors tell me about a number?
- What can multiples tell me about a number?
- What are some other characteristics numbers can have?

MATERIALS REQUIRED

- *Two Ways to Count to Ten, by Susan Meddaugh* (optional)
- Introduction to Building Rectangles (attached)
- Chart Paper with a table as shown below or other recording media (excel spreadsheet, Word Document, etc.)
- Color tiles (or other square tiles)

TIME NEEDED

- 2 days (at least)

TEACHER NOTES

Students need to build an understanding of multiples and factors through tasks which help them build connections between the two as well as among other number characteristics. Teachers should allow students to struggle, ready with a question, to support students building their own ideas about these concepts.

Students can, and often do, create their own misconceptions in all areas of mathematics. Common factors and common multiples are no exceptions. In order to minimize the creation of misconceptions, teachers should ask students to clarify their understandings throughout this task as well as others in this unit. Questioning students and asking students to comment on what other students have stated help to build a learning community in which the expectation is that we all can learn and make sense of the mathematics. The students, *not the teacher(s)*, need to own the math.

One way to open this task, is to read the book *Two Ways to Count to Ten, by Susan Meddaugh.* Students may be familiar with this book from elementary school, but it is worth a re-read! The concept of multiples is apparent and as it is read aloud, math teachers can ask questions like, "What animal do you think will become king?" "How will this animal win the contest?" These questions are often reserved for Language Arts, History, and Science. Students need to become comfortable making predictions in math class, too. Using a familiar form of media (the book) helps initiate students into this.

After you have read the book, have students make a table in their journals/notebooks similar to the one below:

Number	Ways to Count	Number of Ways
1		
2		
3		
. .	. .	. .
25		

Students should complete the table by looking at each number and talking with their partner about all of the ways to count to that number (similar to what happened in the story). So when students get to the number six, they should write down in the "ways to count" column 1, 2, 3, 6 since we can count to six by ones, twos, threes, and sixes. There are 4 ways to count to six so a 4 would be written in the "number of ways" column.

Build a class chart of the same table on chart paper (or using other media). Students should fill in the chart using the tables in their journals/notebooks.

Ask students to look for patterns and to explain their thinking behind the patterns they notice. For example, students may notice that in the "ways to count" column, every second number can be counted to by twos. Students should be asked to explain all patterns they find. One way to do this is to have students first identify several patterns as a class. Assign students to find explanations for these patterns as homework to report back the following day.

As you open class the next day, begin a discussion of the explanations for the patterns. Pay attention to the reasoning of students and address any misconceptions. It is likely that you will have students develop their own rules for divisibility through this discussion and the next part of this lesson.

Introduce students to the Introduction to Building Rectangles page below. Show students the examples of rectangles and non-rectangles and have them define how rectangles should be built with tiles:

- Sides match (not offset)
- No stacking tiles
- No gaps or holes in the rectangle

Ask students to build rectangles for each of the following three situations:

1) Build a rectangle that is four tiles long and two tiles wide.

2) Build as many different rectangles as you can with 12 tiles in each.

3) Using 25 tiles, build a rectangle with a length of 5 tiles.

Discuss the results. Common misconceptions here:
Number 2: A 4×3 rectangle is different from a 3×4 rectangle. It doesn't matter, really, for this context whether they think it is or it isn't, but a quick student explanation of how you can rotate a 4×3 rectangle to make a 3×4 rectangle is usually enough to convince most students that they are the same. The bonus is that this is a great student created visual to show the commutative property of multiplication!

Number 3: It doesn't make a rectangle, it makes a square. This is really a geometry misconception, but it can be resolved fairly quickly and then brought back for a more in-depth discussion later. Ask students for the definition of a rectangle. You will likely get multiple definitions, many of which are incomplete or incorrect. Since you have multiple definitions, look in a math glossary to compare their definitions to (you can find one here). When students read that a rectangle is defined as a quadrilateral with four 90 degree angles, ask them if the square fits that description. It does. As students complete the next part of this task, they will be looking at rectangles (and numbers) that make squares.

Have students build a table in their journals/notebooks similar to the one below:

Number	Rectangles	Number of Rectangles	Factors
1	*1 × 1*	*1*	*1*
2	*1 × 2* *2 × 1*	*2*	*1, 2*
. . .	. . .	. . .	. . .
25			

Using the color tiles, students will build all of the different rectangles they can by using each number of tiles. For example for the number 14, students will use 14 tiles to build a 1 x 14 (or 14 × 1) rectangle and a 2 × 7 (or 7 × 2) rectangle. They write these dimensions in the "Rectangles" column. In the "Number of Rectangles" column, students will write "4" since there are 4 different rectangles that can be made with 14 tiles (students should be pressed to realize that this number always corresponds with the number of factors). In the "Factors" column students will write 1, 2, 7, 14, since those are all of the factors (lengths of each of the sides) of the rectangles made with 14 tiles.

Make a class chart of this using chart paper or other media. Discuss patterns students see in the table and again, assign homework. Students may request to see the chart from the previous "Ways to Count" portion of this task. Give them the opportunity to compare the data from both tables.

Again, ask for explanations of any patterns. If it doesn't come up, ask about numbers that made special rectangles (squares). What were they? This is another characteristic that some numbers have. Someone may ask about triangular numbers, give them some triangles from a set of pattern blocks and ask them to build some triangular numbers. Investigate & label these on the table as well.

The last column has no heading. As students make observations about patterns, ask if they notice anything about the factors of some numbers (from either table). (Some numbers have an odd number of factors, some have two, some have an even number of factors, one number has only one factor).

When they identify this attribute of numbers, tell them that there is a name for numbers with exactly two factors – **Prime.** Numbers with more than two factors also have a name – **Composite**. Students should fill in the last column of their tables with the appropriate labels for each number. *Note: prime and composite numbers were taught in 4th grade.*

There is a lot of discussion over these two days. Be sure to vary the discussion each time in order to get keep students engaged. For example, before the first discussion about the patterns in the "Ways to Count" task, have students write in their journals, **all of the patterns they notice, then have them share their patterns with a partner, then share as a group (Think-Pair-Share). For the next discussion, the next day, when students share their explanations, have students Give One – Get One. Students get up from their seats, and go to one person in the room, give them one of their explanations, and get one from the other person. Give a time limit of 2-3 minutes to give and get as many as they can.** This is a great way to build confidence and engage more students in discussion since everyone will have 4-5 new explanations that they might not have had before.

There are many reasons for engaging students in this task. Though the teacher does a lot of facilitating in this task, it is expected that students make sense of the mathematics. The teacher asks questions and poses new ideas to think about throughout the investigations in this task in order to help students make sense, internalize number relationships and build understandings of multiples and factors. Students do this while revisiting previous concepts such as prime and composite numbers, factors, multiples, and arrays.

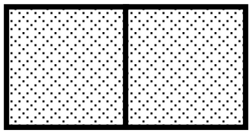
DIFFERENTIATION

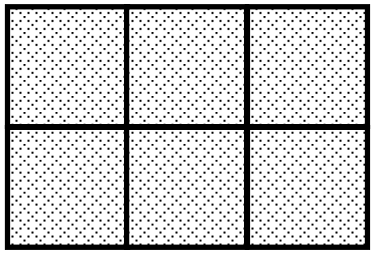
Extension:
This can be an ongoing investigation, where each week groups look at a new range of numbers to build with tiles. The class chart can be revised as students' investigations are completed. New discoveries and patterns can be discussed and connected to current standards being taught. Other, related, number characteristics can be investigated as well, such as rectangular numbers, triangular numbers, hexagonal numbers, etc.

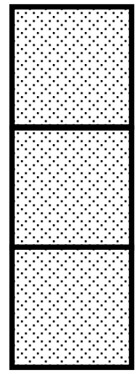
Intervention:
Students should be given support regarding the numbers they are investigating. Giving students a smaller range of numbers or a particular list of numbers with a specific relationship students are struggling with might be helpful here. For example, students struggling with finding all of the factors of a number might be given a small set of numbers with many factors, such as {24, 32, 48}. Using the tiles to build every possible rectangle with each of these numbers and recording them will give struggling students the practice they need, while building their confidence and competence with mathematics.

Introduction To Building Rectangles

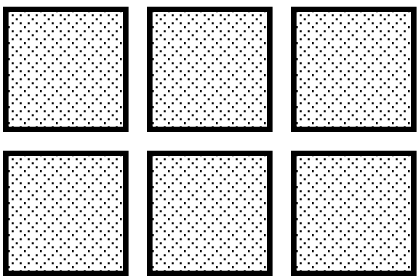
These are rectangles:

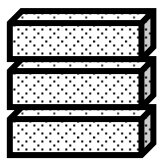
a) 

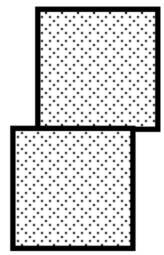
b) 

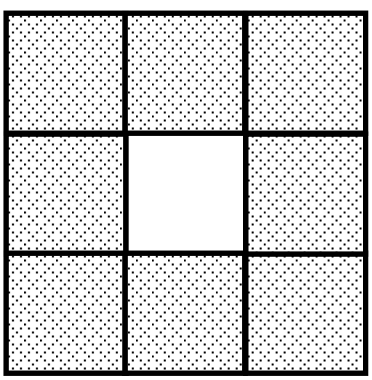
c) 

These are not rectangles:

a) 

b) 

c) 

d) 

Use what you know from the above examples to complete the following:

1) Build a rectangle that is four tiles long and two tiles wide.

2) Build as many different rectangles as you can with 12 tiles in each.

3) Using 25 tiles, build a rectangle with a length of 5 tiles.

Factors And Multiples Puzzle

Back to Task Table

Adapted from NRICH Maths. http://nrich.maths.org

In this student-centered task, students will grapple with the relationship between various properties and characteristics of number, such as common factors and multiples.

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b. Apply the least common multiple of two whole numbers less than or equal to 12 to solve real-world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

1. **Make sense of problems and persevere in solving them.**
Students will persevere and make sense as they wrestle with the productive struggle of this task.
2. **Reason abstractly and quantitatively.**
Students will reason quantitatively as they make decisions about where to place the number and heading cards.
3. **Construct viable arguments and critique the reasoning of others**
As students work with their partner/group, they should be questioning each other and proving their understandings.
4. **Model with mathematics.**
The models students use to prove their understandings to their partner(s) should be based on their own understandings.
6. **Attend to precision.**
Students will attend to precision as they discuss their ideas with their partner(s) through their correct use of vocabulary and by performing calculations accurately.
7. **Look for and make use of structure.**
Students will look for and make use of structure by making sense of how numbers are organized (by factors and multiples), taken apart and put together.

ESSENTIAL QUESTIONS

- Why might it be useful to find the greatest common factor of two numbers?
- Why might it be useful to find the least common multiple of two numbers?
- What kinds of mathematical models can I use to show understanding of least common multiple and greatest common factor?

MATERIALS REQUIRED

- Copies of number cards, heading cards and playing board, attached.

TIME NEEDED

- 2 days (use more time to investigate curious questions generated by students)

TEACHER NOTES

The ideas of highest common factor and lowest common multiple are often taught separately, but dealing with them together can help learners to appreciate the connections between them.

This puzzle provides an interesting context which challenges students to apply their knowledge of the properties of numbers, specifically factors and multiples. This task allows students to make sense of factors and multiples by placing them in the same puzzle (context). Students need to work with various types of numbers at the same time and consider their relationships to each other (e.g. primes, squares and specific sets of multiples), make sense of these relationships, and build new understandings from them.

Misconceptions learners have about LCM and GCF indicates an incorrect interpretation of a mathematical idea as a result of a student's personal experience. To help students address these misconceptions, we need to give them time to investigate tasks in which they make sense of these ideas through their own experiences.

One possible approach to introducing this problem is to show a 3 × 3 grid with six headings on an overhead, doc cam, or the board. Ask pupils to suggest numbers that could fit into each of the nine segments (in some cases, there will not be a number that satisfies both conditions – it is important to include such cases, so that students do not develop misconceptions that a "solution" *always* exists).

	Multiple of 4	Prime	Factors of 80
Multiple of 7			
Square number			
Factors of 24			

The students, working with a partner or small group, can then begin the challenge of filling in the 5 × 5 grid. There is no one solution, so students could display their different arrangements and do a gallery walk to look at and give commentary on other students' work. When pairs/groups finish filling in their grid, they should record the grid headings and how many numbers they placed.

Empty heading cards may be filled with replacement number characteristics to support students through interventions (see interventions below) or to extend the problem further (see extensions below).

Possible questions to guide students' thinking:

- Which numbers are hard to place?
- Which heading cards should not intersect? How do you know?
- Encourage students to pay attention to the order in which they place numbers in the grid – identifying key cells to fill, and key numbers to place.

A concluding question to end the task might be to ask students to share (using a journal, tweet, or other media) any insights and strategies that helped them succeed at this task.

Watch and listen for students making connections between factors and multiples. Have students share observations/discoveries about factors and multiples (as well as other connections) and which headings should not intersect.

DIFFERENTIATION

Extension:
Teachers can adapt the task by changing the heading cards or by asking students to create a new set of heading cards and a set of numbers that make it possible to fill the board. Students could then swap their new puzzles.

Is it possible to create a puzzle that can be filled with 25 consecutive numbers?

Intervention:
Students needing support could be given a larger range of numbers to choose from, or offered a smaller grid and appropriately restricted numbers. This could work by having students choose from the full set of 10 categories, or with an adapted set.

Factors And Multiples Puzzle

1. Cut out the 10 heading cards and put one in each of the 10 spaces on the top and left of the playing board.
2. Cut out the 25 number cards and place each one in a different square on the playing board so that the number satisfies the condition given by the heading card for that row *and* the condition given by the heading card for that column.
3. By rearranging the heading cards and the number cards, try to fill as many squares on the playing board as possible.

1	2	3	4	5
6	7	9	10	11
12	15	16	18	20
21	23	24	25	30
35	36	45	55	60

PRIME NUMBERS	TRIANGULAR NUMBERS	ODD NUMBERS
SQUARE NUMBERS	FACTORS OF 60	EVEN NUMBERS
NUMBERS LESS THAN 20	MULTIPLES OF 3	
NUMBERS GREATER THAN 20	MULTIPLES OF 5	

Back To School!

Back to Task Table

In this task, students will apply the concept of least common multiples to various situations. The use of multiple number lines may be very helpful to all students in order to visualize the problems and provide justification of their work.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS.4 Find the common multiples of two whole numbers less than or equal to 12 and the common factors of two whole numbers less than or equal to 100.

a. Find the greatest common factor of 2 whole numbers and use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factors. (GCF) *Example: 36 + 8 = 4(9 + 2)*
b. Apply the least common multiple of two whole numbers less than or equal to 12 to solve real-world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

1. **Make sense of problems and persevere in solving them.** Students make sense of real-world problems using least common multiples.
3. **Construct viable arguments and critique the reasoning of others.** Students construct and explanations for how they found least common multiple when solving problems.
4. **Model with mathematics.** Students model least common multiple on number lines to solve real-world problems.
7. **Look for and make use of structure.** Students examine the relationship of whole numbers and their multiples to make sense of real-world problems.

ESSENTIAL QUESTIONS

- How are multiples applicable in everyday life?
- How can I use models to represent multiples of a number?
- How is Least Common Multiple used to solve problems?

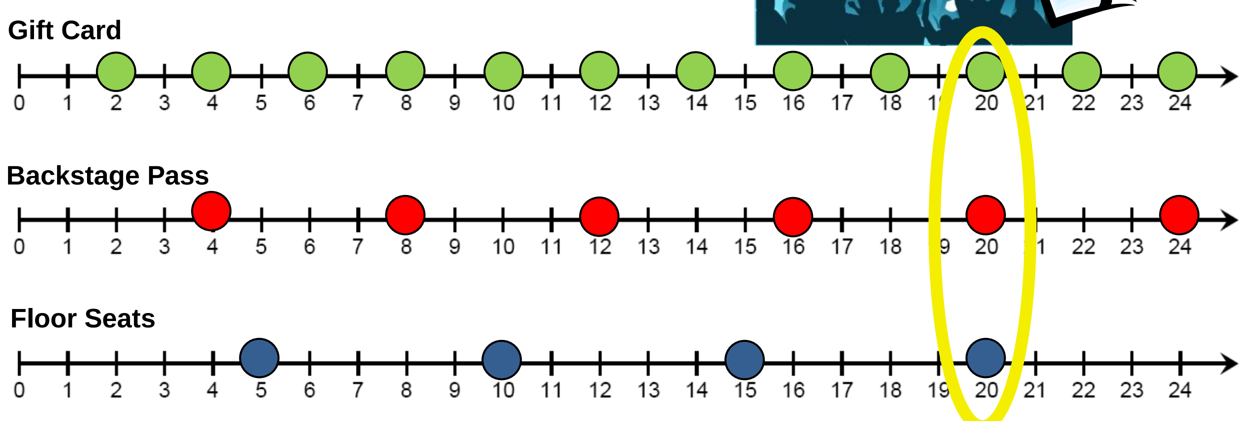
<u>Back To School!</u>

Part 1: Music

You and your friends have tickets to attend a music concert. While standing in line, the promoter states he will give a gift card for a free album download to each person that is a multiple of 2. He will also give a backstage pass to each fourth person and floor seats to each fifth person.

Which person will receive the free album download, backstage pass, and floor seats? Explain the process you used to determine your answer.

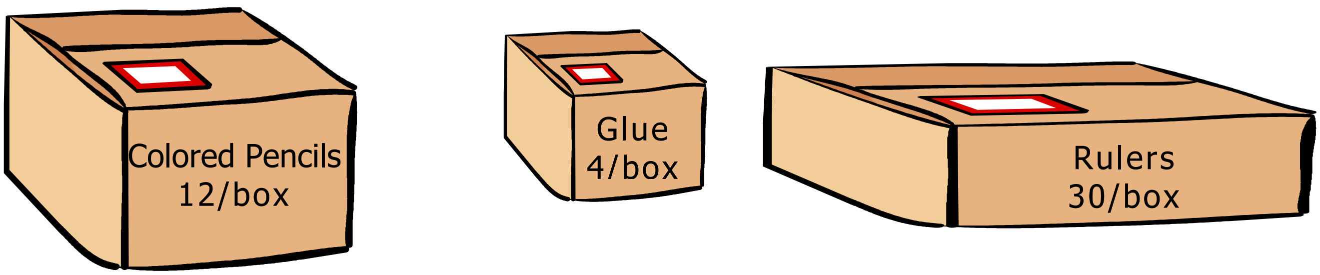
<u>Solution</u>:



The gift card number line illustrates multiples of 2. The backstage pass number line illustrates multiples of 4. The floor seats number line illustrates multiples of 5. The number 20 is a multiple of 2, 4, and 5 which is illustrated on each of the above number lines. For that reason, the number 20 is a common multiple of 2, 4, and 5. Since the number 20 is the smallest common multiple, it is the least common multiple of 2, 4, and 5. This concept is illustrated as the first common multiple on the above number lines. In view of that, the number 20 represents the 20th person in line who will receive a free album download gift card, a backstage pass, and floor seats to the concert.

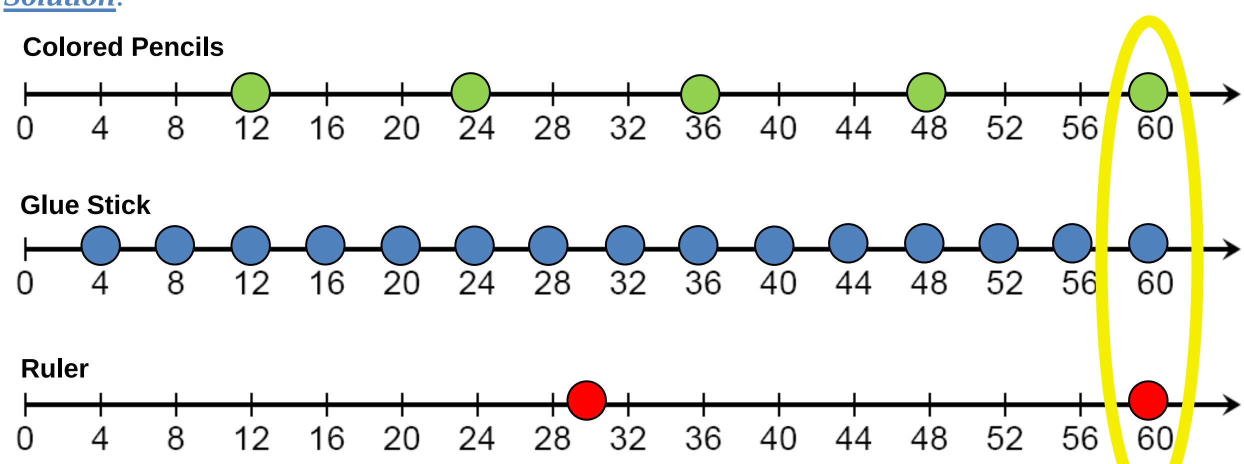
Part 2: School Supplies

The Parents Teachers Association (PTA) at your school donated school supplies to help increase student creativity and student success in the classroom. Your teacher would like you to create kits that include one package of colored pencils, one glue stick, and one ruler. When you receive the supplies, you notice the colored pencils are packaged 12 boxes to a case, the rulers are packaged 30 to a box, and glue sticks are packaged 4 to a box.



1. What is the smallest number of each supply you will need in order to make the kits and not have supplies left over? Explain your thought process.

Solution:

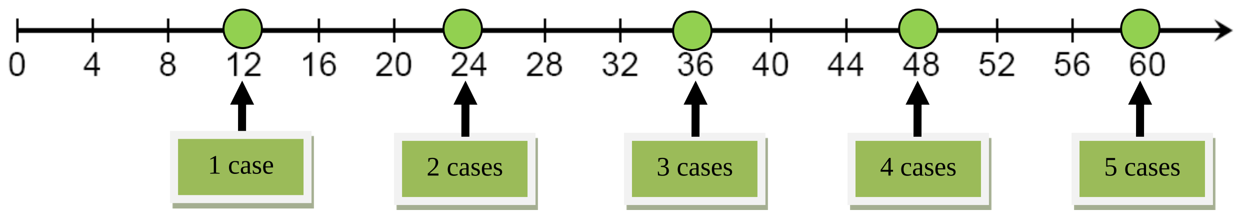


The colored pencils number line illustrates multiples of 12. The glue stick number line illustrates multiples of 4. The ruler number line illustrates multiples of 30. The number 60 is the smallest multiple of 12, 4, and 30, which is illustrated on each of the above number lines. For that reason, the number 60 is a common multiple of 12, 4, and 30. Since the number 60 is the smallest common multiple, it is the least common multiple of 12, 4, and 30. This concept is illustrated as the first common multiple on the above number lines. In view of that, 60 represents the smallest number of each supply needed to create kits consisting of one box of colored pencils, one ruler, and one glue stick and not have supplies left over. Therefore, the students must create 60 kits.

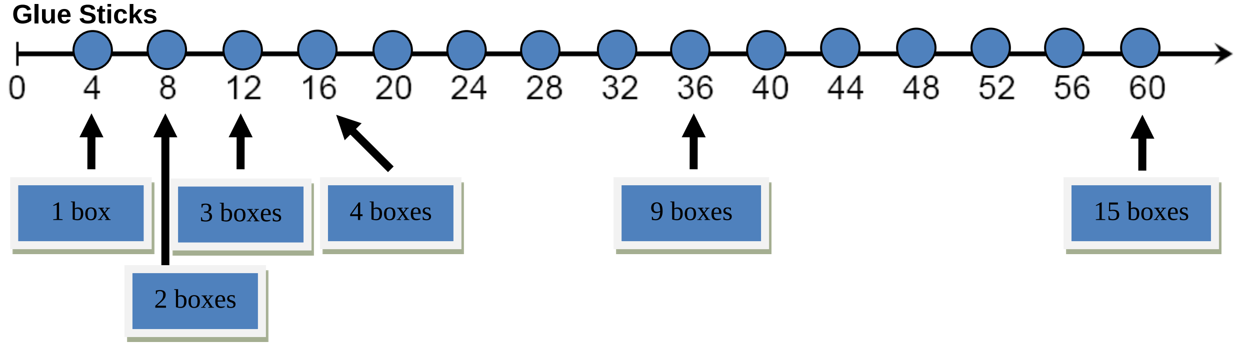
2. How many packaged rulers, colored pencils, and glue sticks will you need in order to make the kits? Explain the process you used to determine how many packages are needed for each supply.

Solution:

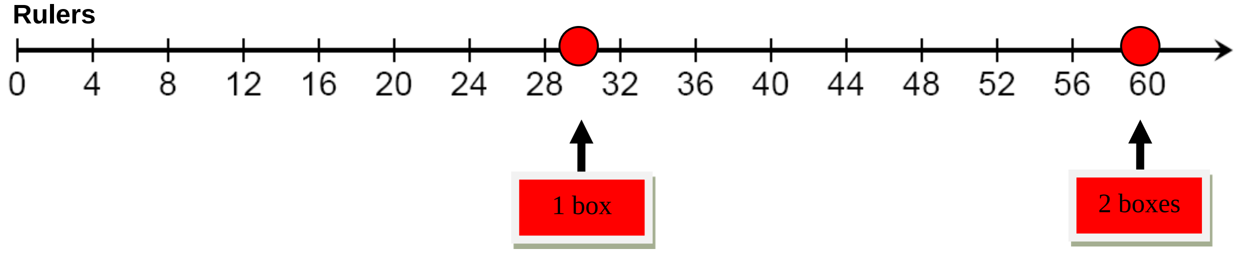
Colored Pencils



Glue Sticks



Rulers



The students need 5 cases of colored pencils, 15 boxes of glue sticks, and 2 boxes of rulers.
Colored pencils: students should recognize that each multiple of 12 is divisible by 12 and the quotient represents the number of cases needed.
Glue sticks: students should recognize that each multiple of 4 is divisible by 4 and the quotient represents the number of boxes needed.
Rulers: students should recognize that each multiple of 30 is divisible by 30 and the quotient represents the number of boxes needed.

EXTENSION

School Lunch

The Yearbook club at your school is sponsoring a fall festival to kick off the annual yearbook drive. The club sponsor has asked for your help in determining what food items to package together and sell and how much of each item he needs to buy. Write a budget report supporting your decision.

The budget report should include a dialogue about common multiples and least common multiples and how this concept is related to the amount of items the sponsor needs to purchase. Students can also include the cost of each item and total costs.

Name___________________________

Back to School!

Part 1: Music

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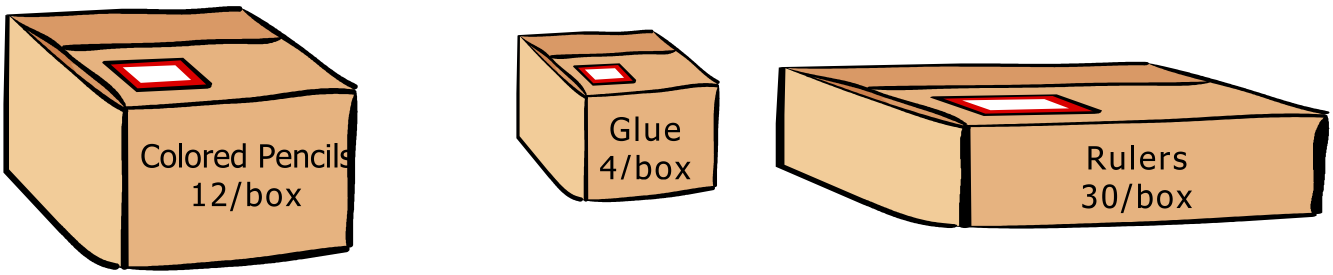

1. What is the smallest number of each supply you will need in order to make the kits and not have supplies left over? Explain your thought process.

2. How many packaged rulers, colored pencils, and glue sticks will you need in order to make the kits? Explain the process you used to determine how many packages are needed for each supply.

Secret Number

Back to Task Table

This task is from *Balanced Assessment for the Mathematics Curriculum, Middle Grades Assessment, Package 2.* Dayle Seymour Publications, Copyright 2000, pages 189-200.

In this task, students will demonstrate understanding of factors and multiples. Students should be directed to conduct a peer review by conversing with a different group or multiple groups. During this conversation, students need to justify and defend their work and explanations.

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b. Apply the least common multiple of two whole numbers less than or equal to 12 to solve real-world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students use least common multiple, greatest common factor and other characteristics of numbers to persevere in finding an unknown number.
3. Construct viable arguments and critique the reasoning of others. Students construct (rules) to describe a secret number using factors, multiples and other characteristics of numbers.
5. Use appropriate tools strategically. Students will use least common multiple and greatest common factor to find a specific number given the "clues".
8. Look for and express regularity in repeated reasoning. Students demonstrate reasoning when they apply given clues to find a number and create rules for their number.

ESSENTIAL QUESTIONS

- Why is it and useful to know the factors and multiples of a number?
- What features does a number have if the number is prime?
- How are multiples and factors applicable in everyday life?
- How can I use models to represent multiples of a number?

Secret Number

Juanita has a secret number. Read her clues and then answer the questions that follow:
Juanita says, "Clue 1: My secret number is a factor of 60."

1. Can you tell what Juanita's secret number is? Explain your reasoning.
   No. Possible numbers could be 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, or 60.

2. Daren said that Juanita's number must also be a factor of 120. Do you agree or disagree with Daren? Explain your reasoning.
   Yes. Since 60 is a factor of 120 then all the factors of 60 will be factors of 120.
   (60)(2)=120

3. Malcolm says that Juanita's number must also be a factor of 15. Do you agree or disagree with Malcolm? Explain your reasoning.
   No. These numbers are factors of 60, but not factors of 15.
   2, 4, 6, 10, 12, 20, 30, 60.

4. What is the smallest Juanita's number could be? Explain.
   1 1 is the smallest number that is a factor of 60.

5. What is the largest Juanita's number could be. Explain.
   60 1 is the largest number that is a factor of 60.

Suppose for Juanita's second clue she says, "Clue 2: My number is prime."

6. Can the class guess her number and be certain? Explain your answer.
   No. 2, 3, 5 are all prime factors of 60.

Suppose for Juanita's third clue she says, "Clue 3: 15 is a multiple of my secret number."

7. Now can you tell what her number is? Explain your reasoning.
   No. 3 and 5 have multiples of 15 and are all prime factors of 60.

Suppose for Juanita's fourth clue she says, "Clue 4: My secret number is a factor of 20."

8. What is Juanita's secret number? **5.**

9. Your secret number is 36. Write a series of interesting clues using factors, multiples, and other number properties needed for somebody else to identify your number.

Examples:

Clue 1 - My secret number is a factor of 180.

Clue 2 - My secret number is a multiple of 9.

Clue 3 - Six is a factor of my secret number.

Clue 4 - My secret number does not end in zero.

Rule 5 - Four is a factor of my secret number.

Name______________________

Secret Number

Juanita has a secret number. Read her clues and then answer the questions that follow:
Juanita says, "Clue 1: My secret number is a factor of 60."

1. Can you tell what Juanita's secret number is? Explain your reasoning.

2. Daren said that Juanita's number must also be a factor of 120. Do you agree or disagree with Daren? Explain your reasoning.

3. Malcolm says that Juanita's number must also be a factor of 15. Do you agree or disagree with Malcolm? Explain your reasoning.

4. What is the smallest Juanita's number could be? Explain.

5. What is the largest Juanita's number could be. Explain.

Suppose for Juanita's second clue she says, "Clue 2: My number is prime."

6. Can the class guess her number and be certain? Explain your answer.

Suppose for Juanita's third clue she says, "Clue 3: 15 is a multiple of my secret number."

7. Now can you tell what her number is? Explain your reasoning.

Suppose for Juanita's fourth clue she says, "Clue 4: My secret number is a factor of 20."

8. What is Juanita's secret number?

9. Your secret number is 36. Write a series of interesting clues using factors, multiples, and other number properties needed for somebody else to identify your number.

Let's Distribute

Back to Task Table

In this task, students will use the distributive property to express the sum of two numbers using a variety of common factors.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS.4 Find the common multiples of two whole numbers less than or equal to 12 and the common factors of two whole numbers less than or equal to 100.

a. Find the greatest common factor of 2 whole numbers and use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factors. (GCF) *Example: 36 + 8 = 4(9 + 2)*
b. Apply the least common multiple of two whole numbers less than or equal to 12 to solve real-world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of using the distributive property to express a sum of two whole numbers.
2. Reason abstractly and quantitatively. Students will apply the constructs of multiplication in using distributive property.
3. Construct viable arguments and critique the reasoning of others. Students explain their reasoning for determining if the distributive property was used correctly.
4. Model with mathematics. Students will model the distributive property using an array model.
5. Use appropriate tools strategically. Students will use color tiles or draw an array model to show the distributive property.
6. Attend to precision. Students attend to the language of using the distributive property.
7. Look for and make use of structure. Students examine the relationship of the sum of two whole numbers using the distributive property.

ESSENTIAL QUESTIONS

- How can the distributive property help me with computation?
- What is the distributive property?

INTRODUCTION

The distributive property of multiplication over addition allows us to "distribute" a factor to two different addends (or "over" two different addends). One of two factors in a product can be split into two or more parts and each part multiplied separately and then added. The result is the same as when the original factors are multiplied.

Making use of the distributive property helps students develop strategies for computing and because applying the distributive property depends on the specific numbers involved, as well as develop number sense.

The distributive property is also the basis of why standard algorithms work, producing partial products for multiplication and the individual digits in quotients of long division problems. The focus of lessons should be on finding ways to make complicated computations simpler and more manageable, not on learning what the distributive property is. Keep the emphasis on the specifics of numerical calculations. It is fine to identify that what you are doing is applying the distributive property, but do not make the distributive property the ultimate goal of instruction.

[NCTM Illuminations](https://illuminations.nctm.org/Activity.aspx?id=3511) (https://illuminations.nctm.org/Activity.aspx?id=3511) provides an applet for creating and representing array models of factors of numbers.

MATERIALS REQUIRED

- Centimeter grid paper or color tiles

TIME NEEDED

- 1 day

BEFORE THE LESSON

Post this problem on the board: $42 + 12 = 6(7 + 2)$
Ask students: Is it true or false? Have them explain their thinking.

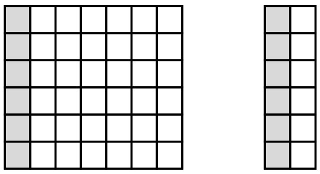
Then post: $6(7 + 2) = (6 \times 7) + (6 \times 2)$
Ask students: Is it true or false? Have them explain their thinking.

Then post: $6(7 + 2) = (6 \times 7) + (6 \times 2) = 42 + 12$
Ask students: Is it true or false? Have them explain their thinking.

Looking at the expression on the left side of the equation, can you explain the use of the number 6?

How is 6 related to 42 and 12? ***Six is a common factor of 42 and 12***

Use an array model to illustrate the expression 42 + 12 and show the common factor.



Supply students with several sheets of centimeter grid paper or color tiles.

For each numerical expression:

A. Find all of the common factors for both numbers in each expression.
B. For each common factor, write an expression.
C. Choose one common factor for each expression and draw a model.
D. Decide which expression would help you do mental computation more easily and accurately.

Here are some examples to get you started:

64 + 32

72 + 12

45 + 18

51 + 21

SUMMARY

Conduct a whole class discussion about the solutions to each of the problems. Make sure that students share their thinking process when finding the common factors. The last part of the question is the most important. Which of the new number sentences using common factors makes mental computation easiest and most accurate?

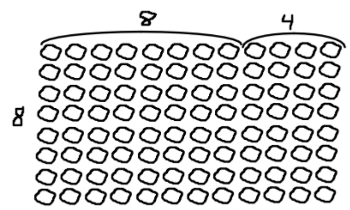
Looking at the list of expressions that you wrote, determine which of the sentences uses the GCF and how?

Now that you have isolated the expression that uses the GCF of the two addends, compare that expression to the others you have written.

Solutions

64 + 32
64 ⟶ 1, 2, 4, 8, 16, 32, 64
32 ⟶ 1, 2, 4, 8, 16, 32

64 and 32 have the common factors of 2, 4, 8, 16, and 32. So, we can write the following number sentences:



Sample model for 8(8 + 4)

64 + 32 = 2(32 + 16)
64 + 32 = 4(16 + 8)
64 + 32 = 8(8 + 4)
64 + 32 = 16(4 + 2)
64 + 32 = 32(2 + 1)
Student answers will vary about which makes it easier to compute. Their reasoning and justification should be mathematically sound.

72 + 12
72 ⟶ 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
12 ⟶ 1, 2, 3, 4, 6, 12

72 + 12 = 12(6 + 1)
72 + 12 = 6(12 + 2)
72 + 12 = 4(18 + 3)
72 + 12 = 3(24 + 4)
72 + 12 = 2(36 + 6)
Student answers will vary about which makes it easier to compute. Their reasoning and justification should be mathematically sound.

45 + 18
45 → 1, 3, 5, 9, 15, 45
18 → 1, 2, 3, 6, 9, 18
45 + 18 = 3(15 +6)
45 + 18 = 9(5 +2)
Student answers will vary about which makes it easier to compute. Their reasoning and justification should be mathematically sound.

51 + 21
51 → 1, 3, 17, 51
21 → 1, 3, 7, 21
51 + 21 = 3(17 + 7)

INTERVENTION
https://illuminations.nctm.org/Activity.aspx?id=3511 provides an applet for creating and representing array models of factors of numbers. Students who struggle with identifying lists of factors may use this tool as a scaffold.

Name______________________

Let's Distribute

$42 + 12 = 6(7 + 2)$ Is it true or false? Explain your thinking.

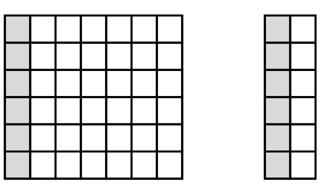
$6(7 + 2) = (6 \times 7) + (6 \times 2)$ Is it true or false? Explain your thinking.

$6(7 + 2) = (6 \times 7) + (6 \times 2) = 42 + 12$ Is it true or false? Explain your thinking.

Looking at the expression on the left side of the equation, can you explain the use of the number 6?

How is 6 related to 42 and 12?

Look at the array model that illustrates the expression $42 + 12$ and shows the common factor is 6.



Use centimeter grid paper, color tiles or array models for each numerical expression.

A. Find all of the common factors for both numbers in each expression.

B. For each common factor write an expression.

C. Choose one common factor for each expression and draw a model.

D. Decide which expression would help you do mental computation more easily and accurately.

1. 64 + 32

2. 72 + 12

3. 45 + 18

4. 51 + 21

Hanging by a Hair (Spotlight Task)

Back to Task Table

Task adapted from http://gfletchy.com/hanging-by-a-hair/

In this inquiry-based task, students will use a variety of arithmetic operations with whole numbers and decimals to solve an engaging problem.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will persevere and make sense as they wrestle with the productive struggle of this task.
2. Reason abstractly and quantitatively. Students will reason quantitatively as they make decisions about where to place the number and heading cards.
3. Construct viable arguments and critique the reasoning of others. As students work with their partner/group, they should be questioning each other and proving their understandings.
4. Model with mathematics. The models students use to prove their understandings to their partner(s) should be based on their own understandings.
6. Attend to precision. Students will attend to precision as they discuss their ideas with their partner(s) through their correct use of vocabulary and by performing calculations accurately.

ESSENTIAL QUESTIONS

- How can computation of whole numbers and estimation help us accurately solve questions involving multiplication and division of decimals?

MATERIALS REQUIRED

- 3-Act Task: http://gfletchy.com/hanging-by-a-hair/
- Act-1 video: http://www.youtube.com/watch?v=eHk_sNTNxe8
- stack of books
- pencil
- tape
- 35 pennies for each group of 3-4 students

TIME NEEDED

- 1 day

TEACHER NOTES

While it is not obvious when handling a single hair, you only need to try to break a small lock to be convinced that hair is extremely strong. The organization of keratin within its cortex allows a single strand to support approximately 100 grams. A lock of 100 hairs can thus withstand a weight of 10 kilograms. As to the average head of hair, it could withstand 12 tons – if the scalp were strong enough!

In this exploration students will engage in patient problem solving to test the strength of their hair. Students will develop a simple expression that will determine the amount of hairs required to support any person their weight.

TASK DESCRIPTION

The following 3-Act Task can be found at: http://gfletchy.com/hanging-by-a-hair/

In this task, students will watch the video and then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need—and ask for it—it will be given to them.

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:
Watch the video: http://www.youtube.com/watch?v=eHk_sNTNxe8

Students are asked what they noticed in the video. Students record what they noticed or wondered on the recording sheet. Students are asked to discuss and share what they wondered (or are curious about) as related to what they saw in the video.

Important Note: Although the MAIN QUESTION of this lesson is ***"How many pieces of hair would you need to support your body weight?"*** it is important for the teacher to not ignore student generated questions and try to answer as many of them as possible (before, during, and after the lesson as a follow up).

Main Question: ***How many pieces of hair would you need to support your body weight?***
Write down an estimate you know is too high, and one you know is too low.

After students have estimated, have them explore how many pennies a single strand of hair can hold using the following steps:

- Securely tape one end of the strand of hair onto a pencil.
- Wedge the pencil into the stack of books so that the pencil is sticking out and the hair is hanging down.

- Securely tape one penny onto the strand of hair and see if the hair can hold it.

- Keep taping on pennies to the hair until the strand breaks.

After the hair breaks, have students compare the actual amount of pennies it took to break the single strand of hair to their estimates. Revisit the questions list of "*what do you wonder?"* questions and see if any of them can be answered with the information students just discovered. Engage students in a brainstorming session that requires them to identify what they could find out based from the information they just found. In the case that students don't inquire about how many hairs they would need to support their weight, guide them in that direction.

ACT 2:
Students will realize that they do not have enough information to complete the problem. Release the following information to students ONLY AFTER they have identified what information they need.

Required information:

- a single penny weighs 2.5 grams
- A target weight? *(to differentiate students could solve to identify the weight of a member in their group or the students could be given a target weight offered by an individual in the class)*
- approximately 28 grams = 1 ounce

Students may work individually for 5-10 minutes to come up with a plan, test ideas, and problem-solve. After individual time, students may work in groups (3-4 students) to collaborate and solve this task together.

Using the released information in Act 2, students will solve and identify the amount of hairs required to support their weight. Have students write a single equation that identifies how many hairs they would need.

ACT 3
Students will compare and share solution strategies.

- Reveal the answer. Discuss the theoretical math versus the practical outcome.

- How appropriate was your initial estimate?

- Share student solution paths. Start with the most common strategy.

- Revisit any initial student questions that weren't answered.

- **ACT 4**

After students have found an equation, have them develop an expression that could be used to find the amount of hairs required to support a person at any given weight.

- Can you find the rule that would determine how many hairs would be needed for a person of any weight?
- Can you write it as an expression?

FORMATIVE ASSESSMENT QUESTIONS

- What models did you create?
- What organizational strategies did you use?
- Does your rule for finding the amount of hair for any weight work? How do you know?

EXTENSION:

After students have identified the expression, have them graph and develop generalizations on hair types and their strengths in regards to color and race.

Hanging by a Hair Name:________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate the result of the main question? Explain?
Place an estimate that is too high and too low on the number line
Low estimate *Place an "x" where your estimate belongs* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:______________________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

School Fund-Raiser (Understanding the Long Division Standard Algorithm)

Back to Task Table

In this task, students will explore the conceptual underpinnings of the standard algorithm for division by modeling a problem with base ten blocks and semi-concrete representations.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS.2 Fluently divide multi-digit numbers using the standard algorithm

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will need to solve this problem specifically using the most efficient strategy possible.
2. Reason abstractly and quantitatively. Students will work through this task and reason abstractly by opening and breaking down the fewest number of pallets, cartons and boxes as possible.
3. Construct viable arguments and critique the reasoning of others. As students work throughout the task they should be questioning each other on an ongoing basis and not just in the summary portion.
4. Model with mathematics. The modeling of the math in this task is extremely important because it will allow students to conceptually understand how the standard algorithm for division works.
6. Attend to precision. Students should be referring to numbers based on place value understanding and not as digits.
7. Look for and make use of structure. Students make the connection between evenly distributing the donuts and relate their modeling to the standard division algorithm.

ESSENTIAL QUESTIONS

- How is place value important in long division?
- Which strategies are helpful when dividing multi-digit numbers?
- How can I use models to explain the process and meaning of long division?

MATERIALS REQUIRED

- Base-ten blocks
- Base-ten grid paper (click to download)

TIME NEEDED

- 1 day

TEACHER NOTES

Language plays an enormous role in thinking conceptually about the standard division algorithm. Most teachers and students are accustomed to saying *"goes into"* which is hard to let go. Traditionally if we were to do a problem such as 583 ÷ 4, we might say "4 goes into 5 one time." Initially, this is mysterious to students. How can you just ignore the "83" and keep changing the problem? Preferably, you want students to think of 583 as 5 hundreds, 8 tens, and 3 ones **NOT AS INDEPENDENT DIGITS 5, 8, AND 3!**

This task asks that students disseminate the donuts the most efficient way possible. The most efficient way to share the donuts is to keep as many pallets, cartons and boxes unopened as possible. The underlying mathematics of this task mimics the standard algorithm and it will be important for students to make that connection.

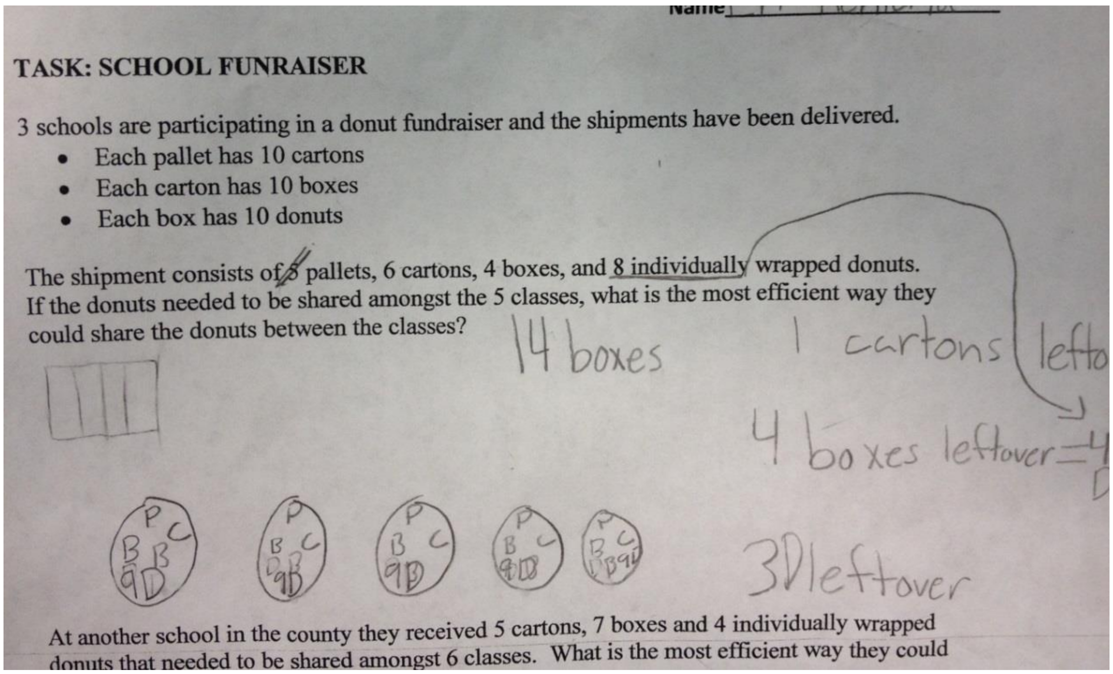
School Fund-Raiser

3 schools are participating in a donut fund-raiser and the shipments have been delivered.

- Each pallet has 10 cartons
- Each carton has 10 boxes
- Each box has 10 donuts

The shipment consists of 5 pallets, 6 cartons, 4 boxes, and 8 individually wrapped donuts. If the donuts needed to be shared amongst the 5 classes, what is the most efficient way they could share the donuts between the classes?

Example of possible student work.



TASK: SCHOOL FUNRAISER

3 schools are participating in a donut fundraiser and the shipments have been delivered.

- Each pallet has 10 cartons
- Each carton has 10 boxes
- Each box has 10 donuts

The shipment consists of 5 pallets, 6 cartons, 4 boxes, and 8 individually wrapped donuts. If the donuts needed to be shared amongst the 5 classes, what is the most efficient way they could share the donuts between the classes?

14 boxes

1 cartons lefto

4 boxes leftover = 4

3D leftover

At another school in the county they received 5 cartons, 7 boxes and 4 individually wrapped donuts that needed to be shared amongst 6 classes. What is the most efficient way they could

1 pallet, 1 carton, 2 boxes, and 9 individually wrapped donuts for each class with three leftover donuts.

At another school in the county, they received 5 cartons, 7 boxes and 4 individually wrapped donuts that needed to be shared amongst 6 classes. What is the most efficient way they could share the donuts?

9 boxes and 5 individually wrapped donuts for each class with 4 leftover donuts.

The 3rd school participating in the fund-raiser has 12 classes to share their shipment with. They have received 7 pallets, 8 cartons, 2 boxes, and 5 individually wrapped donuts. What is the most efficient way they could share the donuts?

6 cartons, 5 boxes, and 2 individually wrapped donuts for each class with 1 leftover donut.

School Fund-Raiser

Name________________________

3 schools are participating in a donut fund-raiser and the shipments have been delivered.

- Each pallet has 10 cartons
- Each carton has 10 boxes
- Each box has 10 donuts

The shipment consists of 5 pallets, 6 cartons, 4 boxes, and 8 individually wrapped donuts. If the donuts needed to be shared amongst the 5 classes, what is the most efficient way they could share the donuts between the classes?

At another school in the county they received 5 cartons, 7 boxes and 4 individually wrapped donuts that needed to be shared amongst 6 classes. What is the most efficient way they could share the donuts?

The 3rd school participating in the fund-raiser has 12 classes to share their shipment with. They have received 7 pallets, 8 cartons, 2 boxes, and 5 individually wrapped donuts. What is the most efficient way they could share the donuts?

Scaffolding Division through Strip Model Diagramming -Understanding of the Standard Algorithm (Spotlight Task) ……

Back to Task Table

Adapted from Joe Schwartz http://exit10a.blogspot.com/

In this task, students will model fair sharing of a dividend on a tape diagram. Connections between the model and the symbolic representation of the standard algorithm for division will be made.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS.2 Fluently divide multi-digit numbers using the standard algorithm

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will need to solve this problem specifically using the most efficient strategy possible.
2. Reason abstractly and quantitatively. Students will work through this task and reason abstractly by connecting the semi-concrete tape diagram to the symbolic division algorithm.
3. Construct viable arguments and critique the reasoning of others. As students work throughout the task they should be questioning each other on an ongoing basis and not just in the summary portion.
4. Model with mathematics. The modeling of the math in this task is extremely important because it will allow students to conceptually understand how the standard algorithm for division works.
6. Attend to precision. Students should be referring to numbers based on place value understanding and not as digits.
7. Look for and make use of structure. Students make the connection between evenly distributing the donuts and relate their modeling to the standard division algorithm.

ESSENTIAL QUESTIONS

- How is place value important in long division?
- Which strategies are helpful when dividing multi-digit numbers?
- How can I use models to explain the process and meaning of long division?

MATERIALS REQUIRED

- Construction paper cut into fourths (length ways)
- Math journal or blank paper

TIME NEEDED

- Minimum of 2 days

TEACHER NOTES

Language plays an enormous role in thinking conceptually about standard division algorithm. Most teachers and students are accustomed to saying *"goes into"* which is hard to let go. Traditionally if we were to do a problem such as 583÷4, we might say "4 goes into 5, one time." Initially, this is mysterious to students. How can you just ignore the "83" and keep changing the problem? Preferably you want students to think of 583 as 5 hundreds, 8 tens, and 3 ones **NOT AS INDEPENDENT DIGITS 5, 8, AND 3!**

Although the following task is scripted, the teacher should lead the lesson through guided questioning. **This lesson suggests a minimum of 2 days but students must be moved through this task at their own pace. DO NOT FORCE or scaffold this lesson too quickly!!!** It is also extremely important for the teacher to model this task <u>using precise language such as quotient instead of answer</u> (SMP #6).

TASK DIRECTIONS

Part 1

- Hand out a piece of construction paper (cut in ¼ length ways)

$$145 \div 4$$

- Place the following equation on the board and tell the student that the value of the whole strip is 145.
- Pose the question, **"*if the value of the whole strip is 145, how many parts do we need to partition the strip into?*** Students might need to see that the strip needs to be partitioned into 4 parts. Have students fold the strip into fourths. On the board, the teacher should model the students' strip:

- ***If the whole strip is 145, how much do we want to start partitioning into each fourth?*** It will be important for students to understand that each fourth has an equal amount. What students are doing is using partial quotients. The following example is how it could be done but it is not the only way. LET THE STUDENTS DICTATE WHAT QUOTIENTS TO USE!
  - Student 1 says to put 20 in each box (it will be important to record the remaining dividend on the board next to the strip diagram)
  - Student 2 says to put 10 more in each box
  - Student 3 says that we 25 left so we'll put 6 more in each fourth
  - Student 4 says that we have 1 left over so that can be our remainder or we can partition it into ¼ or 0.25

20 10 6	20 10 6	20 10 6	20 10 6

$$\begin{array}{r} 145 \\ \underline{-80} \\ 65 \\ \underline{-40} \\ 25 \\ \underline{-24} \\ 1 \end{array}$$

- Using the strip model students just created, ask them what the ***total quotient*** is for the equation 145 ÷ 4
  - ***Total Quotient for 145 ÷ 4 is equal to 36 remainder 1 or 36¼ or 36.25***
- Hand out more strips to students and have them model the following expressions individually or in small groups for 10-15 minutes:

  201 ÷ 3   467 ÷ 5   694 ÷ 4

Note: it is important to use simple divisors, like 2, 3, 4, and 5 because the modeling can be cumbersome with larger divisors like 13 or 18. What is important in this phase of learning is to develop conceptual understanding of the algorithm before transitioning to the abstract representation of long division.

- Once students have completed modeling partial quotients using their strip diagram, have them share their solutions. This is an extremely important part of the lesson because students need to see that there are multiple solution paths. This also gives the teacher great insight into students' number sense. (*Example: a student that is able to partition the dividend by multiples of 10 has a stronger sense of number than the student that only removes 10 at a time).* There is no right or wrong way to solve, however the emphasis should be on the more efficient solution paths.

Part 2

- Tell the students you are going to try the same thing but this time you are not going to use the strips. Students will draw the strips instead.

423÷5

423				
40 40 4	40 40 4	40 40 4	40 40 4	40 40 4

$$\begin{array}{r} 423 \\ \underline{-200} \\ 223 \\ \underline{-200} \\ 23 \\ \underline{-20} \\ 3 \end{array}$$

Remainder 3 or $\frac{3}{5}$ or *0.6*

- Place some more equations on the board and have the students solve them using the horizontal strip model.

Part 3

- (Day 2) Tell students that today that you are going to do the exact same thing as yesterday but this time you want them to draw their strip diagram horizontally and instead of showing the subtraction/remaining divisor outside of the diagram you'd like them to include it as part of the diagram.

874÷4

	218 r2
200 15 3	874 -800 74
200 15 3	-60 14 -12
200 15 3	2
200 15 3	

- This time, have students record the quotient total on top of the vertical strip diagram

- Place some more equations on the board and have the students solve them using the vertical strip model.

Part 4

- After students have had had ample time and practice using the vertical strip model, pose the following question: ***"We are writing a lot of partial quotients each time. Is there anything we can eliminate or get rid of?"*** Guide the students to see that they don't need to repeatedly draw the boxes and write the exact same numbers in each one. After the students have arrived to this point, have them write the number of boxes they need next to the dividend box/strip. Partial quotients are recorded at the top of the vertical strip.

691÷8

	50+30+5+1 remainder 3
8	691 -400 291 -240 51 -40 11 -8 3

- Place some more equations on the board and have the students solve them using the vertical strip model without the partial quotient boxes. As students become more flexible in their thinking and using the largest possible product with the remaining dividend, they will be modeling the standard algorithm for division (but with place value understanding). Here is an example using the previous equation 691÷8

$$
\begin{array}{r|r|}
\multicolumn{2}{r}{80+6 \text{ remainder } 3} \\
\cline{2-2}
8 & 691 \\
 & \underline{-640} \\
 & 51 \\
 & \underline{-48} \\
 & 3 \\
\cline{2-2}
\end{array}
$$

PART 5

- Pose a problem on the board using the traditional long division symbol and tell the students that the box is not finished and if they prefer, they can either draw it or leave it open.

$$
\begin{array}{r|r|}
\multicolumn{2}{r}{70+5 \text{ r4}} \\
\cline{2-2}
7 & 529 \\
 & \underline{-490} \\
 & 39 \\
 & \underline{-35} \\
 & 4 \\
\cline{2-2}
\end{array}
$$

<u>Discovering an Algorithm for Dividing Fractions</u>

Back to Task Table

Adapted from: Sharp, J. (1998). A constructed algorithm for the division of fractions. In L. Morrow & M. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics, NCTM 1998 Yearbook* (pp. 204-207). Reston, VA: NCTM.

In this task, students will discover the common denominator algorithm for dividing fractions by modeling various expressions and looking for patterns in repeated reasoning.

<u>STANDARDS FOR MATHEMATICAL CONTENT</u>

MGSE6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, including reasoning strategies such as using visual fraction models and equations to represent the problem.
For example:

- *How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally?*
- *How many 3/4-cup servings are in 2/3 of a cup of yogurt?*
- *How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*
- *Three pizzas are cut so each person at the table receives ¼ pizza. How many people are at the table?*
- *Create a story context for (2/3)÷(3/4) and use a visual fraction model to show the quotient;*
- *Use the relationship between multiplication and division to explain that (2/3)÷(3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc)*

<u>STANDARDS FOR MATHEMATICAL PRACTICE</u>

1. Make sense of problems and persevere in solving them. Students will be given contexts for division with fractions that require them to make sense of the problem, find a solution pathway, and execute a strategy. **<u>Students should not be told to invert and multiply</u>**, rather make sense of each fraction in the context and solve based on their understanding of the problem.
2. Reason abstractly and quantitatively. Students will need to make sense of the fractional quantities in each problem and how they are related to the whole as well as each other in order to solve each practice problem.
4. Model with mathematics. Students will model their solutions with visual fraction models and equations – not just one or the other.
5. Use appropriate tools strategically. Students should be allowed to choose appropriate tools to solve the problems.
6. Attend to precision. Students will attend to specific units, particularly in interpreting remainders as it relates to the divisor.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning as they solve a variety of problems involving division of fractions. They look for patterns to help them discover the common denominator algorithm for dividing fractions.

<u>ESSENTIAL QUESTIONS</u>

- How can I apply what I know about division of whole numbers to divide fractions by fractions?
- What are some models and strategies I can create to divide fractions?

MATERIALS REQUIRED

- Fraction circles or fraction tiles

TIME NEEDED

- 1 day

TEACHER NOTES

Students need experience with concrete and semi-concrete representations of fraction division before they are simply "taught" abstract algorithms. Research has proven that students who explore concrete and semi-concrete representations tend to have stronger conceptual understanding—which reduces the need for students to rely on memorization of rules and procedures that they are likely to forget—and are more successful at applying skills to problem solving. In this task, students will "discover" the common denominator algorithm for fraction division. It is very important to allow students to actively construct meaning themselves in this task, so that they "own" the math.

The problems presented in this task will represent "division as repeated subtraction" as opposed to "fair sharing." According to Sharp (1998):

> Division as fair sharing involves the allocation of the given amount across a set of number groups, whereas division through repeated subtraction requires successive removals of a set amount from a given amount. When using the fair sharing approach, students divide objects across a certain number of groups. This approach is most intuitive when the number of groups is a whole number. Imagining the division of an amount across two and a half groups is perplexing to most learners. Repeated subtraction affords no such pitfalls, so this method will be applied to the discovery of the algorithm. (p. 198)

NOTE: <u>This task is divided into 6 parts or phases. Each phase builds on the previous phase. Although only 1 division problem is presented in each phase, use professional discretion to determine whether or not students can benefit from additional problems within each phase</u>.

Discovering an Algorithm for Dividing Fractions

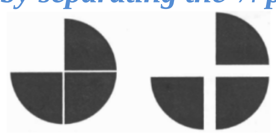
Part I: Like Denominator Problem(s) with Whole Number Quotient

Ask students to write a division expression to match the following question:
"How many groups of 2 are in 10?" ***Answer: 10 ÷ 2. Discuss and model this. For example, students can take 10 square tiles and "pull out" 5 groups of 2. Relate this process to the idea of repeated subtraction.***

Now, provide students with a model of ¾. You may use area models (e.g., circles) or linear models (e.g., fraction strips/tiles) or both. *If you have access to a document camera or projector, allow students to model the following problems for the whole class to see and evaluate.*

Say: "Think back to how you wrote the expression for the question, '*How many groups of 2 are in 10?' which was (10 ÷ 2).* Now write the expression that matches this question:"

"How many groups of ¼ are in ¾?" ***Answer: ¾ ÷ ¼. Discuss and model this with manipulatives. Now, ask students to answer the question by saying, "So, how many groups of ¼ are in ¾?" Answer: 3. Show this by separating the ¼ pieces likewise:***



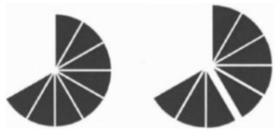
Part II: Like Denominator Problem(s) with a Non-Integer Quotient Greater than 1

Ask students to write a division expression to match the following question:
"How many groups of $\frac{2}{6}$ are in $\frac{5}{6}$?" ***Answer: $\frac{5}{6} \div \frac{2}{6}$. Discuss and model this with manipulatives. Now, ask students to answer the question by saying, "So, how many groups of $\frac{2}{6}$ are in $\frac{5}{6}$?" Answer: 2½ . Show this by separating the groups of $\frac{2}{6}$ likewise:***

Many students may think the quotient is $2\frac{1}{6}$. Do not try to "fix" this misconception – rather, let the class debate this conjecture. A good way to prod students to attend to the unit might be to ask, "What part of what I want ($\frac{2}{6}$) is left over?" This might help students to realize that ½ of the divisor is remaining, so the quotient is actually 2½ (groups of $\frac{2}{6}$).

Ask students to write a division expression to match the following question:
"How many groups of $\frac{5}{12}$ are in $\frac{8}{12}$?" ***Answer: $\frac{8}{12} \div \frac{5}{12}$. Discuss and model this with***

manipulatives. Now, ask students to answer the question by saying, "So, how many groups of $\frac{5}{12}$ are in $\frac{8}{12}$?" Answer: 1⅗ . Show this by separating the groups of $\frac{5}{12}$ likewise:

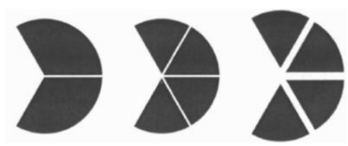


It may be helpful to draw dotted "pieces" to represent the additional 2 twelfths that we would "like to have to make a full group of 5/12". This will help students see that the leftover part is NOT 3/12 but rather 3/5.

Part III: Unlike Denominator Problem(s) with a Whole Number Quotient

Ask students to write a division expression to match the following question:

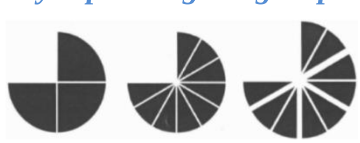
"How many groups of $\frac{1}{6}$ are in $\frac{2}{3}$?" *Answer: $\frac{2}{3} \div \frac{1}{6}$. Discuss and model this with manipulatives. You may need to ask, "What would it look like to remove groups of 1/6 from 2/3?" Students who have had experiences with fraction manipulatives and equivalence will know to exchange 2/3 for 4/6. Now, ask students to answer the question by saying, "So, how many groups of $\frac{1}{6}$ are in $\frac{4}{6}$?" Answer: 4. Show this by separating the groups of $\frac{1}{6}$ likewise:*



Part IV: Unike Denominator Problem(s) with a Non-Integer Quotient Greater than 1

Ask students to write a division expression to match the following question:

"How many groups of $\frac{1}{6}$ are in $\frac{3}{4}$?" *Answer: $\frac{3}{4} \div \frac{1}{6}$. Discuss and model this with manipulatives. You may need to ask, "What would it look like to remove groups of 1/6 from 3/4?" Students who have had experiences with fraction manipulatives and equivalence will know to exchange ¾ for 9/12. Now, ask students to answer the question by saying, "So, how many groups of $\frac{2}{12}$ are in $\frac{3}{4}$?" Answer: 4½. Show this by separating the groups of $\frac{2}{12}$ likewise:*

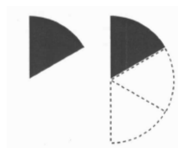


Part V: Quotient Less than 1

Ask students to write a division expression to match the following question:

"How many groups of $\frac{3}{6}$ are in $\frac{1}{6}$?" *Answer: $\frac{1}{6} \div \frac{3}{6}$. Discuss and model this with manipulatives.*

You may need to ask, "Will the quotient be greater than 1 or less than 1 and how do you know?" Allow students to think-pair-share then discuss as a whole class. Now, ask students to <u>answer</u> the question by saying, "So, how many groups of $\frac{3}{6}$ are in $\frac{1}{6}$?" Allow students to struggle/wrestle with this problem. Resist the temptation to teach by telling. You may need to remind students that we "want" a group of 3/6 but we don't have a full group. You might ask, "How might we represent what we want and what we actually have?" Show this by representing likewise:



Answer: ⅓

Part VI: Discovering the Algorithm

Display all of the exercises done up to this point on a poster/chart paper/white board. Ask students to observe closely, look for patterns, and write down what they notice. Discuss.

$\frac{3}{4} \div \frac{1}{4} = 3$	$\frac{2}{3} \div \frac{1}{6} = 4$
$\frac{5}{6} \div \frac{2}{6} = 2\frac{1}{2}$	$\frac{3}{4} \div \frac{1}{6} = 4\frac{1}{2}$
$\frac{8}{12} \div \frac{5}{12} = 1\frac{3}{5}$	$\frac{1}{6} \div \frac{3}{6} = \frac{1}{3}$

Although the intent is for students to discover that the quotient is ALWAYS $\frac{a}{c}$ when $\frac{a}{b} \div \frac{c}{b}$, that may not be apparent without rewriting the problems with common denominators. Regardless, if students make other observations, like the fact that only 1 problem has a quotient that is less than 1, praise and encourage them – or put the responsibility of evaluation back on the class by asking, "Is that true?"

Students are more likely to discover the common denominator algorithm ($\frac{a}{b} \div \frac{c}{b} = \frac{a}{c}$) when the problems are represented with common denominators. So, now show the problems as such:

$$\frac{3}{4} \div \frac{1}{4} = 3 \qquad \frac{4}{6} \div \frac{1}{6} = 4$$

$$\frac{5}{6} \div \frac{2}{6} = \frac{5}{2} \qquad \frac{9}{12} \div \frac{2}{12} = \frac{9}{2}$$

$$\frac{8}{12} \div \frac{5}{12} = \frac{8}{5} \qquad \frac{1}{6} \div \frac{3}{6} = \frac{1}{3}$$

If students are still not able to make the discovery of the common denominator algorithm, show the problems like this:

$$\frac{\mathbf{3}}{4} \div \frac{\mathbf{1}}{4} = \mathbf{3} \qquad \frac{\mathbf{4}}{6} \div \frac{\mathbf{1}}{6} = \mathbf{4}$$

$$\frac{\mathbf{5}}{6} \div \frac{\mathbf{2}}{6} = \frac{\mathbf{5}}{\mathbf{2}} \qquad \frac{\mathbf{9}}{12} \div \frac{\mathbf{2}}{12} = \frac{\mathbf{9}}{\mathbf{2}}$$

$$\frac{\mathbf{8}}{12} \div \frac{\mathbf{5}}{12} = \frac{\mathbf{8}}{\mathbf{5}} \qquad \frac{\mathbf{1}}{6} \div \frac{\mathbf{3}}{6} = \frac{\mathbf{1}}{\mathbf{3}}$$

Answer: When dividing two fractions with a like denominator, the quotient can be found by simply dividing the numerator of the first term by the numerator of the second term. NOTE: Some students will likely ask, "Will that always work???" This is a GREAT sign of engagement and intellectual curiosity. Rather than saying, "Only when you have a common denominator" and explaining why, empower the students to answer the question by exploring the conjecture. The following tasks in this unit will certainly allow your students to explore this very question.

The Kool-Aid Kid (Spotlight Task)

Back to Task Table

Taken from http://www.illustrativemathematics.org/
Adapted by http://gfletchy.com/the-kool-aid-kid/

In this inquiry-based task, students will reason about mathematical relationships between parts and whole, while keeping the unit in mind. It is highly likely that students will discover the relevance of dividing fractions as it applies to the problem presented in this task.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, including reasoning strategies such as using visual fraction models and equations to represent the problem.
For example:

- *How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally?*
- *How many 3/4-cup servings are in 2/3 of a cup of yogurt?*
- *How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*
- *Three pizzas are cut so each person at the table receives ¼ pizza. How many people are at the table?*
- *Create a story context for (2/3)÷(3/4) and use a visual fraction model to show the quotient;*
- *Use the relationship between multiplication and division to explain that (2/3)÷(3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc)*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students must make sense of the problem by identifying what information they need to solve it.
3. Construct viable arguments and critique the reasoning of others. After writing down their own question, students discuss their question with tablemates, creating the opportunity to construct the argument of why they chose their question, as well as critiquing the questions that others generate.
4. Model with mathematics. Once the given information is communicated, the students use that information to develop a mathematical model.
5. Use appropriate tools strategically. In the "debrief", the teacher discusses the different uses of tools. By doing this, students are provided with more tools in their toolbox for future problem solving. Also, students are asked to use estimation as a tool (by making high and low estimates).
6. Attend to precision. Although students make an estimate based on Act 1, it is be important for them to identify exactly how much juice from the glass was drunk.
7. Look for and make use of structure. The students must develop an understanding of the physical structure in order to develop a mathematical model that has a numerical structure of its own. Students make the connection between the physical structure and the numerical structure of the mathematical model.
8. Look for and express regularity in repeated reasoning. Students may contextualize the process of dividing fractions by relating it to the process of dividing whole numbers.

ESSENTIAL QUESTIONS

- How can I apply what I know about division of whole numbers to divide fractions by fractions?
- What are some models and strategies I can create to divide fractions?

MATERIALS REQUIRED

- 3-Act Task taken from: http://gfletchy.com/the-kool-aid-kid/
- Act 1 Video link: https://vimeo.com/90479455
- Act 2 Video link: https://vimeo.com/90482305
- Act 2 picture: "*How Much was Consumed?*" http://gfletchy3act.files.wordpress.com/2014/03/act-2-how-much-was-consumed.docx
- Act 2 picture: "*How Much was in the Glass?*" http://gfletchy3act.files.wordpress.com/2014/03/act-2-how-much-was-in-the-glass.docx
- Act 3 video link: https://vimeo.com/90479475
- 3-Act Task Recording Sheet

TIME NEEDED

- 1 day

TEACHER NOTES

In this task, students will watch the video and then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need—and ask for it—it will be given to them. If students experience struggle, make it *productive* struggle by relating back to the context of whole numbers. For example, if students reason that the expression "$\frac{3}{8}$ is what fraction of $\frac{1}{2}$?" is relevant to the scenario presented in this task, reasoning about this relationship can be tied back to whole numbers as a sense-making strategy *(for example, an analogy might be 6 is what fraction 10? In such a case, one can reason:* $\frac{6}{10}$ *or 6 ÷ 10).* Similar reasoning applies to this case, because the verbal expression "$\frac{3}{8}$ is what fraction of $\frac{1}{2}$?" is the same as the symbolic expression, $\frac{3}{8} \div \frac{1}{2} = ?$ (SMP#8)

TASK DESCRIPTION

The following 3-Act Task can be found at: http://gfletchy.com/the-kool-aid-kid/

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:
Watch the video: https://vimeo.com/90479455

Students are asked what they noticed in the video. Students record what they noticed or wondered on the recording sheet. Students are asked to discuss and share what they wondered (or are curious about) as related to what they saw in the video.

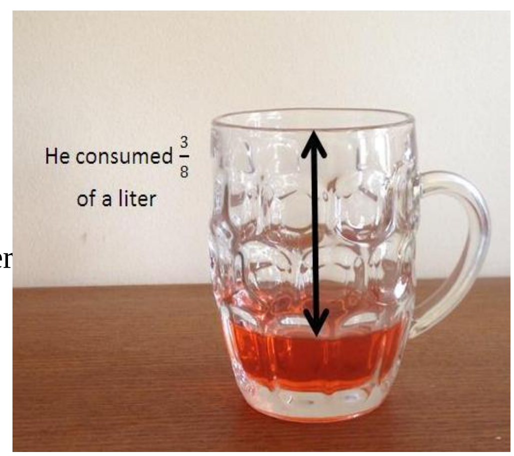
Important Note: Although the MAIN QUESTION of this lesson is "*What fraction of the juice in the glass did the Kool-Aid Kid drink?*" it is important for the teacher to not ignore student generated questions and try to answer as many of them as possible (before, during, and after the lesson as a follow up).

Main Question: *What fraction of the juice in the glass did the Kool-Aid Kid drink?*
Write down an estimate you know is too high, and one you know is too low.

ACT 2:
Students will realize that they do not have enough information to complete the problem. Release the following information to students ONLY AFTER they have identified what information they need.

Required information:

- There was $\frac{1}{2}$ a liter of Kool-Aid in the glass to begin
- The Kool-Aid Kid drank $\frac{3}{8}$ of a liter



Students may work individually for 5-10 minutes to come up with a plan, test ideas, and problem-solve. After individual time, students may work in groups (3-4 students) to collaborate and solve this task together. Note: Some students may think that the answer to the question has been revealed in Act 2 (i.e., $\frac{3}{8}$ of a liter). Still, some students may simply translate the scenario to

$\frac{3}{8}$ of $\frac{1}{2}$ to get $\frac{3}{16}$ of the glass. If so, ask how they know this makes sense (e.g., would that be more than half or less than half of the glass?) Regardless of the misconception, it is important to position students to debate these claims. You will want them to pay very close attention to the units (SMP#6). Guide students to realize that while the Kool-Aid Kid drank $\frac{3}{8}$ of a liter, the question at hand is "What fraction of what's in the glass (not a liter) has been consumed? *The whole, in this case, is not 1 whole liter, but ½ of a liter.*

ACT 3
Show students the Act-3 Video https://vimeo.com/90479475
Students will compare and share solution strategies.

- Reveal the answer. Discuss the theoretical math versus the practical outcome.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.

FORMATIVE ASSESSMENT QUESTIONS

- Does the information revealed in Act 2, (i.e., The Kool-Aid Kid drank $\frac{3}{8}$ of a liter) answer the question, "*What fraction of the juice in the glass did the Kool-Aid Kid drink?"* Why or why not?
- What models did you create?
- What organizational strategies did you use?

The Kool-Aid Kid

Name:_________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate the result of the main question? Explain?

Low estimate

Place an estimate that is too high and too low in the boxes
Place an "x" where your estimate belongs

High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:________________________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

<u>Dividing Fractions in Context</u>

Back to Task Table

Adapted from: Kribs-Zaleta, C. M. (2008). Oranges, Posters, Ribbons, and Lemonade: Concrete Computational Strategies for Dividing Fractions. *Mathematics Teaching in the Middle School*, vol. 13, no. 8, p. 454.

In this task, students will model and solve fraction division tasks in a problem-based context.

<u>STANDARDS FOR MATHEMATICAL CONTENT</u>

MGSE6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, including reasoning strategies such as using visual fraction models and equations to represent the problem.
For example:

- *How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally?*
- *How many 3/4-cup servings are in 2/3 of a cup of yogurt?*
- *How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*
- *Three pizzas are cut so each person at the table receives ¼ pizza. How many people are at the table?*
- *Create a story context for (2/3)÷(3/4) and use a visual fraction model to show the quotient;*
- *Use the relationship between multiplication and division to explain that (2/3)÷(3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc)*

<u>STANDARDS FOR MATHEMATICAL PRACTICE</u>

1. Make sense of problems and persevere in solving them. Students will be given contexts for division with fractions that require them to make sense of the problem, find a solution pathway, and execute a strategy. **<u>Students should not be told to invert and multiply</u>**, rather make sense of each fraction in the context and solve based on their understanding of the problem.
2. Reason abstractly and quantitatively. Students will need to make sense of the fractional quantities in each problem and how they are related to the whole as well as each other in order to solve each practice problem.
4. Model with mathematics. Students will model their solutions with visual fraction models and equations – not just one or the other.
5. Use appropriate tools strategically. Students should be allowed to choose appropriate tools to solve the problems.
6. Attend to precision. Students will use the language of mathematics in their discussions with each other throughout the problem solving process.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning as they solve a variety of problems involving division of fractions. As they get comfortable with their own understanding of division as it is now applied to fractions, they will find more efficient methods. Some of these methods may not always work; others will be enlightening!

<u>ESSENTIAL QUESTIONS</u>

- How can I apply what I know about division of whole numbers to divide fractions by fractions?
- What are some models and strategies I can create to divide fractions?

MATERIALS REQUIRED

- A variety of manipulatives – preferably with no fractions written on the pieces
- Handout: Dividing Fractions in Context (attached)

TIME NEEDED

- At least 2 days

TEACHER NOTES

Students begin their work with division of fractions in 5^{th} grade by investigating dividing a whole number by a unit fraction and dividing a unit fraction by a whole number. In 6^{th} grade, students extend this to include other fractions. The concept is still the same, and students should not enter (or leave) 6^{th} grade with any rules about inverting and multiplying, unless they develop these through discussions in class. **Telling students this rule only robs students of the chance to make sense of division with fractions.**

Some common misconceptions that students have are based on the very algorithms we used to teach by telling. One is the **invert and multiply rule** and the other is the **butterfly method**. Both of these are tricks that have no place in today's mathematics classroom.

> Division and multiplication are different (albeit related) operations, one cannot magically switch the operation in an expression. Plus, students confuse "cross" (diagonal) with "across" (horizontal). Not to mention, where does the answer go? Why does one product end up in the numerator and the other in the denominator? (from *Nix the Tricks by Tina Cardone and the online math community known as MTBoS)*

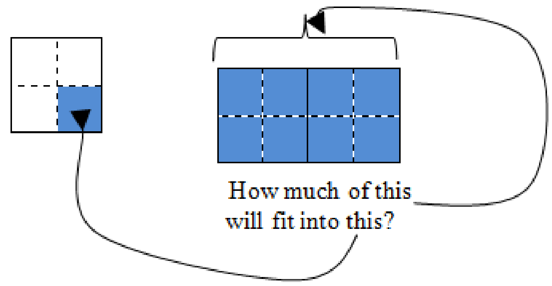
Students should be given two problems to solve in class each day, with a discussion of solutions and strategies to follow facilitated by the teacher. The problems are written in pairs. One of each pair involves partitive division **(P)** or fair sharing. The second involves measurement division **(M)**, which involves a different kind of thinking. When given the problems on the handout, the expectation should be that students make sense of the problem through some sort of model, with manipulatives or without, but definitely with a drawing at least. An equation to attach to the model(s) is also an expectation.

Some possible examples for students to try prior to the problems in this task:

A serving size of chips is ¼ of a bag. There is ¼ of a bag of chips left. There are 2 people sharing the bag. How much of a serving will each person get? **(P)**

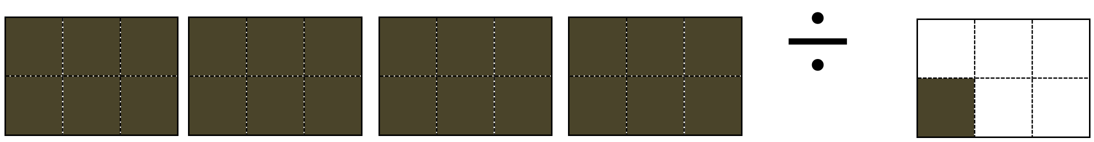
$\mathbf{\frac{1}{4} \div 2 =}$

How many 2's are there in ¼? Each person gets $\frac{1}{8}$ of a bag because ¼ of a bag shared (divided) by 2 people means each person gets half of the ¼. ½ of ¼ is $\frac{1}{8}$.



$\mathbf{4 \div \frac{1}{6} =}$

I made 4 pans of brownies. I want to give $\frac{1}{6}$ of a pan to as many friends as I can. To how many friends can I give brownies? **(M)**



As students work on their problem(s), look at students' choices for materials. An inappropriate choice of mathematical tool can often be an eye opener about the depth of their understanding of fractions. Also, allow students to share their ideas with one another, particularly if several students have difficulty getting started. Set parameters for students to do this so that one group is not just "telling" the other students what to do. One parameter to consider might be that the helpers can only ask questions, not answer them.

The idea of sharing solutions and strategies should not necessarily involve every student or group. However the discussion should involve all students. The teacher should choose students or groups based on the efficiency of strategies used. Inefficient strategies should be shared first, with more efficient strategies shared later. Novel strategies should be addressed as well – these often may lead to deeper understanding or misconceptions that need to be addressed.

DIFFERENTIATION

Extension:

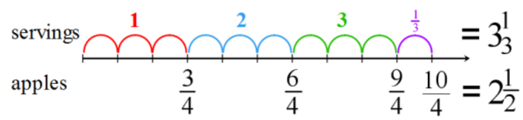
- Students can create a blog post or journal entry about their understanding of the division of fractions. Pictures and or video (if available) might make this an excellent assessment piece.

Intervention:

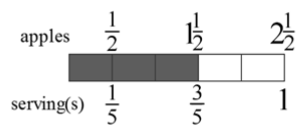
- Support struggling students by pulling them aside for a quick math check on strategies they have seen used in the classroom prior to this task. Guide students to use a strategy they feel comfortable using.

- **<u>Dividing Fractions in Context</u>**
  Note: The first two problems show possible models. Encourage multiple representations.

1. Suppose you have 2½ apples. If a student serving consists of ¾ of an apple, how many student servings (including parts of a serving) can you make? (M)



2. Suppose instead that you have 1½ apples. If this is enough to make 3/5 of an adult serving, how many apples (and parts of an apple) make up one adult serving? (P)



3. Emma is making posters by hand to advertise her school play, but her posters are not the same length as a standard sheet of paper (the width is the same, though). She has 3 ½ sheets of paper left over, which she says is enough to make 2 1/3 posters. How many sheets of paper (and parts of a sheet) does each poster use? (P)

   $1\frac{1}{2}$ *sheets of paper*

4. If Connor is also making posters, but his posters only use 2/3 of a sheet of paper, how many of Connor's posters will those 3 ½ sheets of paper make? (M)

   $5\frac{1}{4}$ *posters*

5. Lura is tying ribbons in bows on boxes. She uses 2 ¼ feet of ribbon on each box. If she has 7 ½ feet of ribbon left, how many bows (or parts of a bow) can she make? (M)

   $3\frac{1}{3}$ *bows*

6. Audrey is also tying ribbons into bows. Audrey sees the same 7 ½ feet of ribbon measured out and says, "Since my bows are bigger than Lura's, that's only enough for me to make 2 ¼ bows." How much ribbon does Audrey use on each bow? (P)

   $3\frac{1}{3}$ *feet of ribbon*

7. Alex has been serving 2/3 cup of lemonade to each student. If he has 1 ½ cups of lemonade left, how many students can still get lemonade? How much of a serving will the last student get? (M)

   2 *students*, $\frac{1}{4}$ *of a serving*

8. 3 ½ cups of lemonade will fill 2 1/3 glasses. How many cups of lemonade does each glass hold? (P) $1\frac{1}{2}$ *cups of lemonade*

Dividing Fractions in Context

1. Suppose you have 2 ½ apples. If a student serving consists of ¾ of an apple, how many student servings (including parts of a serving) can you make? (M)

2. Suppose instead that you have 1 ½ apples. If this is enough to make 3/5 of an adult serving, how many apples (and parts of an apple) make up one adult serving? (P)

3. Emma is making posters by hand to advertise her school play, but her posters are not the same length as a standard sheet of paper (the width is the same, though). She has 3 ½ sheets of paper left over, which she says is enough to make 2 1/3 posters. How many sheets of paper (and parts of a sheet) does each poster use? (P)

4. If Connor is also making posters, but his posters only use 2/3 of a sheet of paper, how many of Connor's posters will those 3 ½ sheets of paper make? (M)

5. Lura is tying ribbons in bows on boxes. She uses 2 ¼ feet of ribbon on each box. If she has 7 ½ feet of ribbon left, how many bows (or parts of a bow) can she make? (M)

6. Audrey is also tying ribbons into bows. Audrey sees the same 7 ½ feet of ribbon measured out and says, "Since my bows are bigger than Lura's, that's only enough for me to make 2 ¼ bows." How much ribbon does Audrey use on each bow? (P)

7. Alex has been serving 2/3 cup of lemonade to each student. If he has 1 ½ cups of lemonade left, how many students can still get lemonade? How much of a serving will the last student get? (M)

8. 3 ½ cups of lemonade will fill 2 1/3 glasses. How many cups of lemonade does each glass hold? (P)

Fractional Divisors

Back to Task Table

In this task, students will model and solve both partitive and measurement fraction division problems.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, including reasoning strategies such as using visual fraction models and equations to represent the problem.
For example:

- *How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally?*
- *How many 3/4-cup servings are in 2/3 of a cup of yogurt?*
- *How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*
- *Three pizzas are cut so each person at the table receives ¼ pizza. How many people are at the table?*
- *Create a story context for (2/3)÷(3/4) and use a visual fraction model to show the quotient;*
- *Use the relationship between multiplication and division to explain that (2/3)÷(3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc)*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of real-world situations that involve division of fractions.
2. Reason abstractly and quantitatively. Students will reason why the process of inverting and multiplying works when dividing fractions.
3. Construct viable arguments and critique the reasoning of others. Students explain how to divide a fraction by a whole number and a whole number by a fraction.
4. Model with mathematics. Students use tape diagrams to model the division of fractions.
6. Attend to precision. Students use appropriate terminology when referring their explanations of dividing fractions.
7. Look for and make use of structure. Students examine the structure of dividing fractions by fractions.

ESSENTIAL QUESTIONS

- Why does the process of invert and multiply work when dividing fractions?
- When I divide one number by another number, do I always get a quotient smaller than my original number?
- When I divide a fraction by a fraction, what do the dividend, quotient and divisor represent?
- What kind of models can I use to show solutions to word problems involving fractions?

MATERIALS REQUIRED

- Paper and writing utensil
- Base ten blocks
- Unit cubes
- Rulers

TIME NEEDED

- 2 days

TEACHER NOTES

Multiply by the reciprocal is probably one of the most mysterious rules in mathematics. Van de Walle urges us to avoid this mystery by allowing students to experience both interpretations of fractions and come to their own reasoning about why we multiply by the reciprocal. This lesson allows students to explore partitive and measurement interpretations of fractions with fractional divisors. Keeping in mind that the partitive interpretation asks the question, "How much is one?" while measurement is equal subtraction and can have remainders.

Before the lesson, review the student work with the two interpretations of division from the previous task.

When students finish, get answers from the class for each problem. If more than one answer is offered, simply record them and offer no evaluation.

Have students explain their strategies for thinking about the problem either on the board or with a document camera, etc. You may need to ask questions about drawings or explanations to make sure everyone in the class follows the rationale. Encourage the class to comment or ask questions about the student's representation or thinking. Ask if others used a different representation or solved the problem in a different way. If so, have the students come forward to share their solutions. If there are different answers, the class should evaluate the solution strategies and decide which answer is correct and why.

It is important to have students compare and contrast the two problems and the methods for solving them.

In what ways are the two problems similar?

In what ways are they different?

What does the quotient represent in each of the problems?

What does the divisor represent in each of the problems?

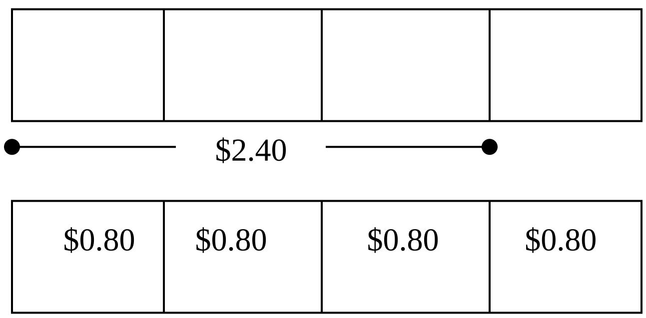
What does the dividend represent in each of the problems?

SOLUTIONS

Partitive Interpretation of Division with Fractional Divisors

Use a model (e.g., manipulative materials, pictures, number line) to find the answers to the problems below. Be sure to write an equation to illustrate each situation.

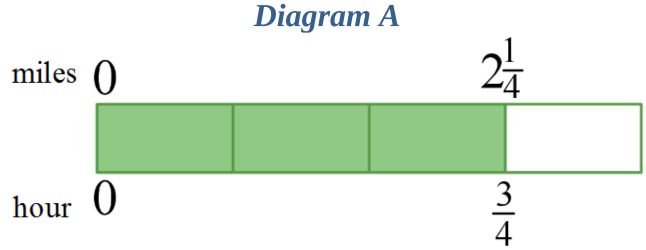
1. Michael's mom paid $2.40 for a $\frac{3}{4}$ - pound box of cereal. How much is that per pound?



One-fourth of a pound is $0.80($2.40 ÷3), so one pound will be 4 x $0.80 = $3.20 per pound.

2. Melitta found out that when she walks during her morning exercise, she can cover 2 $\frac{1}{4}$ miles in $\frac{3}{4}$ of an hour. How fast is she walking in miles per hour?

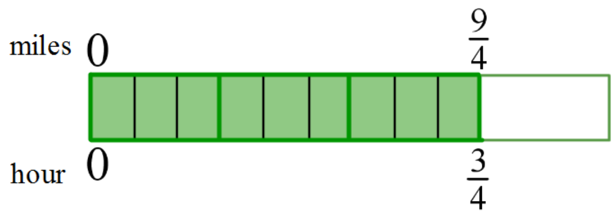
Diagram A



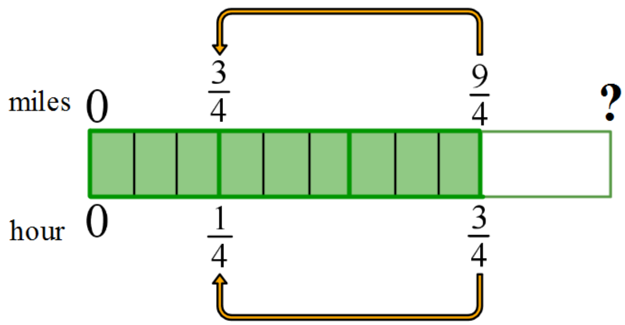
Melitta walks 2¼ miles in ¾ of an hour. In order to model this situation with tape diagrams, the student must understand that 2½ is being divided into three parts (quarters). A tape diagram is a powerful model because it can be used like a double number line to compare two quantities that are measured in different units (as in this case, miles and hours).

In order to find how many miles are walked in 1 hour, it is helpful to first consider the distance walked in ¼ of a hour – this is where the unit fraction plays an important role in the development of proportional reasoning.

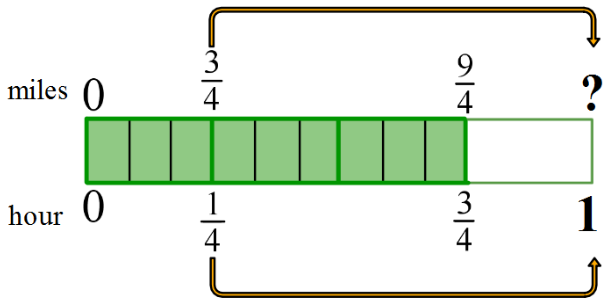
Renaming 2 ¼ to $\frac{9}{4}$ is l … hich happens to be the number of parts that names the distance between 0 hour and ¾ hour.



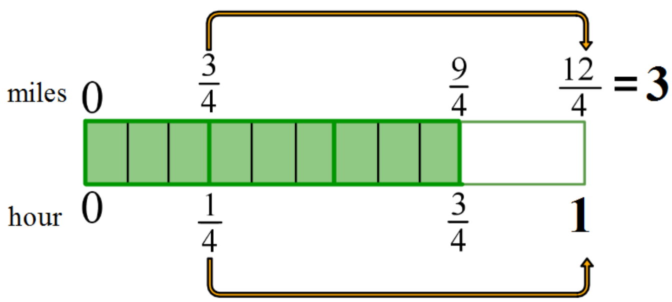
Now, it is fairly easy to reason how many miles are walked in ¼ of an hour (3/4 miles in ¼ hr).



But the question that must be answered is "How many miles does she walk in 1 hour?"



Because 1 hour is simply 4 times as much as ¼ of an hour, multiplying the distance by 4 will keep the miles proportional to the unit being compared (hours).



So, Melitta is walking 12/4 miles per hour, or 3 miles per hour.

Measurement Interpretation of Division with Fractional Divisors

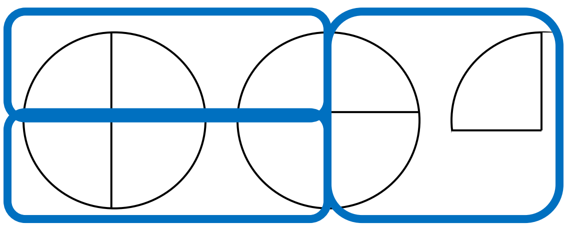
Use a model (e.g., manipulative materials, pictures, number line) to find the answers to the problems below. Be sure to write a number sentence to illustrate each situation.

3. It's your birthday and you are going to have a party. From the grocery store you get 6 pints of ice cream. If you serve $\frac{3}{4}$ of a pint of ice cream to each of your guests, how many guests can be served?

Typically students draw pictures of six items divided into fourths and count out how many servings of ¾ can be found. The difficulty is in seeing this as $6 \div \frac{3}{4}$, and that requires some guidance on the teacher's part. Try to compare the problem to one involving whole numbers (6 pints, 2 per guest).

4. Sam is a landscaper. He found that he had $2\frac{1}{4}$ gallons of liquid fertilizer concentrate. It takes $\frac{3}{4}$ gallon to make a tank of mixed fertilizer. How many tankfuls can he mix?

This question asks, "How many sets of three-fourths are in a set of 9 fourths?" Sam can mix 3 tankfuls.



Name________________________

Fractional Divisors

Partitive Interpretation of Division with Fractional Divisors

Use a model (e.g., manipulative materials, pictures, number line) to find the answers to the problems below. Be sure to write a number sentence to illustrate each situation.

1. Michael's mom paid \$2.40 for a $\frac{3}{4}$ - pound box of cereal. How much is that per pound?

2. Melitta found out that if she walks really fast during her morning exercise, she can cover $2\frac{1}{4}$ miles in $\frac{3}{4}$ of an hour. How fast is she walking in miles per hour?

Measurement Interpretation of Division with Fractional Divisor

Use a model (e.g., manipulative materials, pictures, number line) to find the answers to the problems below. Be sure to write a number sentence to illustrate each situation.

3. It's your birthday and you are going to have a party. From the grocery store you get 6 pints of ice cream. If you serve $\frac{3}{4}$ of a pint of ice cream to each of your guests, how many guests can be served?

4. Sam is a landscaper. He found that he had $2\frac{1}{4}$ gallons of liquid fertilizer concentrate. It takes $\frac{3}{4}$ gallon to make a tank of mixed fertilizer. How many tankfuls can he mix?

Dividing Fractions with Models

Back to Task Table

In this task, students will relate fraction division expressions and problems to models.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, including reasoning strategies such as using visual fraction models and equations to represent the problem.
For example:
- *How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally?*
- *How many 3/4-cup servings are in 2/3 of a cup of yogurt?*
- *How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*
- *Three pizzas are cut so each person at the table receives ¼ pizza. How many people are at the table?*
- *Create a story context for (2/3)÷(3/4) and use a visual fraction model to show the quotient;*
- *Use the relationship between multiplication and division to explain that (2/3)÷(3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc)*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively. Students will apply the constructs of division and fractions.
3. Construct viable arguments and critique the reasoning of others. Students construct and critique arguments regarding division of fractions.
4. Model with mathematics. Students will model real-world situations to show division of fractions.
5. Use appropriate tools strategically. Students will use visual or concrete tools for division of fractions with understanding.
8. Look for and express regularity in repeated reasoning. Students demonstrate repeated reasoning when dividing fractions by fractions and connect the inverse relationship to multiplication.

ESSENTIAL QUESTIONS

- Why does the process of invert and multiply work when dividing fractions?
- When I divide one number by another number, do I always get a quotient smaller than my original number?
- When I divide a fraction by a fraction, what do the dividend, quotient and divisor represent?
- What kind of models can I use to show solutions to word problems involving fractions?

TEACHER NOTES

In this lesson students represent division of fractions using manipulatives, such as freezer pops, candy bars, and models such as drawing squares. Students develop an algorithm from these examples and solve problems using fractions.

The concept of division of fractions has been greatly misunderstood. **Developing an understanding of what happens when you divide by a fraction prior to development of the algorithm is essential in the thought process.** This is accomplished by having the student visually see and understand what dividing by a fraction means with physical examples.

This task was adapted from:
http://ims.ode.state.oh.us/ODE/IMS/Lessons/Content/CMA_LP_S01_BH_L06_I08_01.pdf

Part 1
Ask five volunteers to come to the front of the classroom. Give each student a freezer pop (use pops with two sticks) and ask if they have ever eaten one. Then ask if they had eaten the entire freezer pop or split it in half. Because of the two sticks, one student may answer that he/she splits the freezer pop in half. Ask students to split the pops in half and have a student count the total number of halves. Use *Frozen Juice Pops,* as a visual representation for the situation.

Freezer pops could be made out of construction paper and craft sticks to illustrate this model. See template.

Ask students if they notice anything about the size of the 10 pieces compared to the original 5 freezer pops. Student should note that they are smaller. Elicit that they are half the size of the original freezer pops.

Ask a volunteer for an equation to represent the 5 freezer pops divided in half.

Answer 5 ÷ ½ = 10

Write the equation on the board for the class to see. If students need help determining this equation, ask "How many half-size freezer pops were contained in the original 5 whole freezer pops?" Then, remind the class that when we ask how many of something there are in something else, that is a division situation (e.g., if we want to know how many 3s are in 12, we divide 12 by 3).

Distribute a variety of chocolate bars that are made with divided sections or use *Chocolate Bar Models*. Ask students to describe how the bars could be divided and give equations to represent this division. $5 \div \frac{1}{4} = 20$, $3 \div \frac{1}{12} = 36$

Again write the equations on the board. Discuss with the students that the size of the pieces desired is not the same as the original pieces. Note: Some students may come up with an equation such as $4 \div 16 = \frac{1}{4}$, which is also a correct way to model this situation. Encourage these students to find a second equation that also models the situation (How many little pieces are in the original big piece?).

Instructional Tip

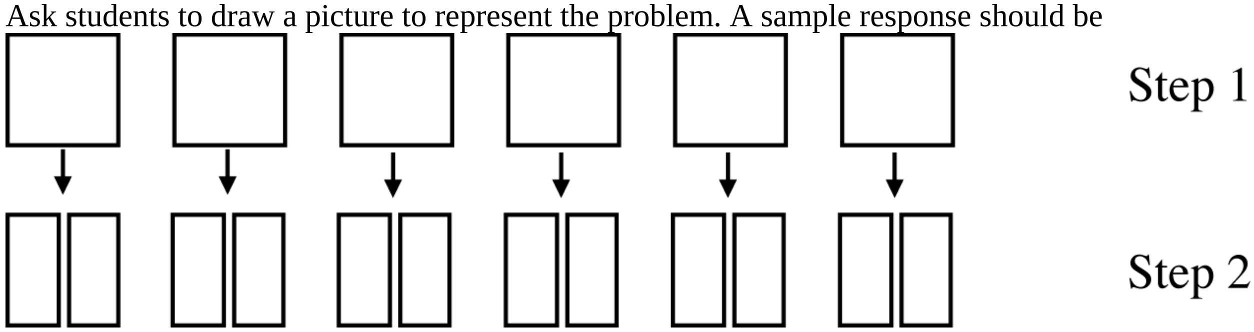
Make sure students are able to recognize that even though they end with a greater number of pieces after completing the division, the size of the portion is less.

Part 2

Pose the following situation to the class.

I have six squares that I want to divide by one half. How many pieces would I have?

Ask students to draw a picture to represent the problem. A sample response should be



Ask the following guiding questions;

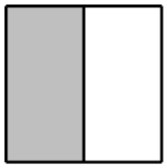
How many squares did I have? ***(6)*** What size did I want? ***($\frac{1}{2}$)***

How many pieces of that size do we have? ***(12)***

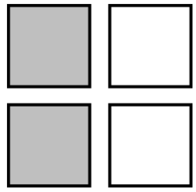
Ask students how this situation would be represented as an equation. Guide the discussion to obtain the equation $6 \div \frac{1}{2} = 12$.

Place students in pairs and pose another situation. Ask them to model it and write an equation that represents the situation.
I have ½ of a square and I want to divide it by ¼. How many pieces would I have?

Monitor the partners working on the task and ask the same type of guiding questions when students appear to be struggling with how to represent the situation. The solution should resemble the following example:



This represents having one half of the square.



This represents dividing the square into pieces whose size is one-fourth. The students then need to answer the question of how many pieces of size one-fourth do I have?

Ask the partners to write an equation for the problem. ***$\frac{1}{2} \div \frac{1}{4} = 2$.***

Ask for a volunteer to provide the equation. Ask the student why he/she placed the numbers in that order.

Write the equations on the board after each situation, noting the relationships among the numbers in the equations. Have students look for any patterns or relationships they note in the equations.

Have partners make conjectures or descriptions as to what they believe is happening when they divide a number by a fraction. Ask partners to share their conjectures with the class. Record the conjectures and descriptions on the board or chart paper.

Possible conjectures include:

> ***When you divide by a fraction you get a whole number.***
> ***When you divide by a fraction you get a larger number.***
> ***When you divide by a fraction you multiply the whole number by the denominator.***

Instructional Tip

Use the conjectures to adjust the instructions as needed, determining whether students are ready to work with more complex fractions or dividing a fraction by a whole number. Students can test their conjectures and refine their descriptions. **The goal is to enable students to determine the algorithm for dividing by a fraction.**

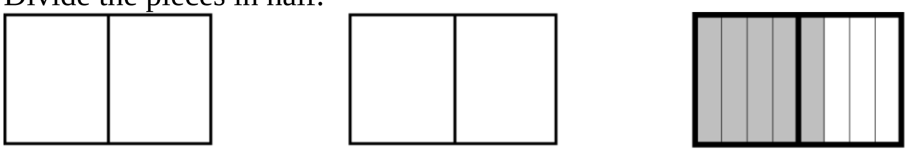
Part Three

Present the following situation:

Cierra has $2\frac{5}{8}$ meters of yarn that she wants to cut into $\frac{1}{2}$-meter lengths. How many $\frac{1}{2}$-meter lengths of yarn will Cierra have?

Instruct students to draw a model to solve the problem.
A sample model may be

Divide the pieces in half.

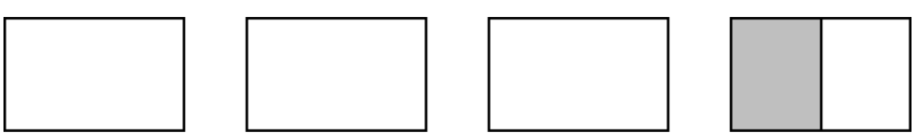


There are 5 halves in $2\frac{5}{8}$. There is also $\frac{1}{8}$ left, which is $\frac{1}{4}$ of the remaining half of the whole. Therefore, there are $5\frac{1}{4}$ halves in $2\frac{5}{8}$. ***Cierra will have 5 complete $\frac{1}{2}$ meter lengths of yarn.***

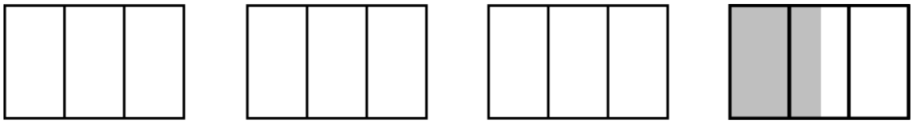
Select a student to model the problem on the board. Note: there will be a piece left over after finding 5 halves. If students are not sure what this represents, ask, "What is the relationship of $\frac{1}{8}$ to the remaining half?" ***($\frac{1}{8}$ is $\frac{1}{4}$ of the remaining half.)***

Present a similar situation, such as:

Jonathan has $3\frac{1}{2}$ cups of chocolate chips to make cookies. The recipe uses $\frac{1}{3}$ cup of chips in each batch. How many batches of cookies can Jonathan make?



Divide the pieces into thirds.



Jonathan can make $10\frac{1}{2}$ ***batches of cookies.***

Instructional Tip

Students should see that once they divide the whole part(s) into the desired fractional part, the remaining part, which is a fractional part of another whole piece, needs to be divided also. Students may see a fractional piece left over and should determine the relationship of this piece to the original piece.

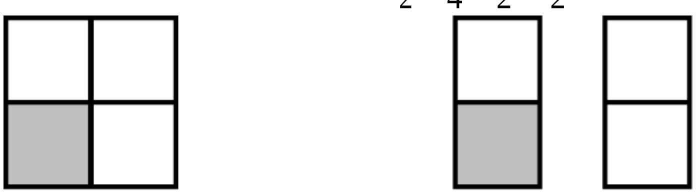
Present the following problem:

I have one fourth of a square and I want to divide it by one half. How many pieces would I have?

Have students work as partners to solve the problem. Remind students to draw a picture to represent the problem.

Ask guiding questions used with other situations such as, "What are you looking for (How many sets of $\frac{1}{2}$ are in $\frac{1}{4}$)?"

A sample response should be $\frac{1}{2}$***;*** $\frac{1}{4} \div \frac{1}{2} = \frac{1}{2}$ ***I have one half of the desired size.***



Have students compare this problem with the previous problem noting any differences and similarities. Discuss the meaning of the answer in each problem.

Have the partners test and refine the conjectures and descriptions by completing *Dividing By a Fraction.* Observe the partners working on the situations and provide intervention, reminding them of the guiding questions that were used with the other situations.

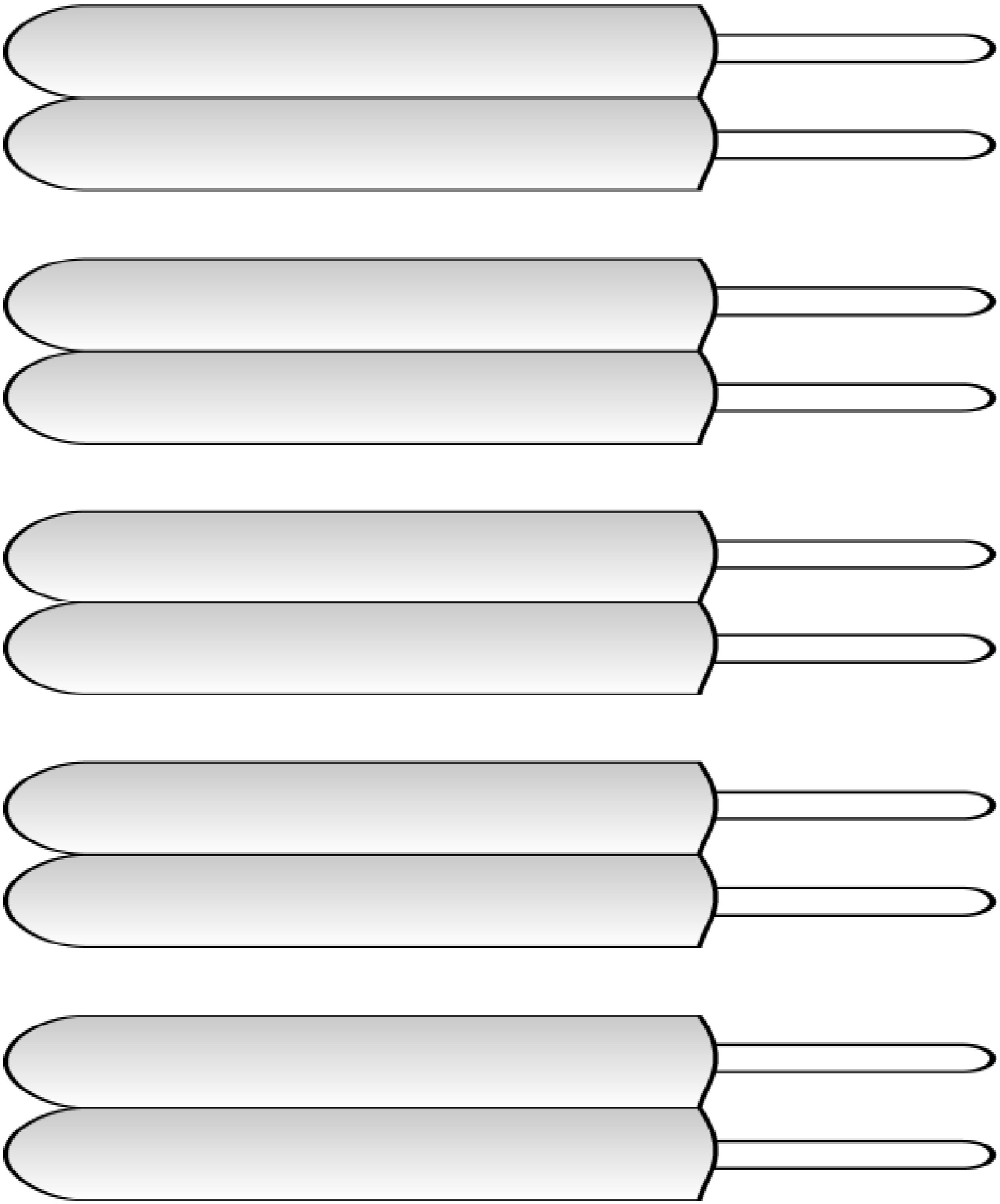
Put partners together to make groups of four. Have each group check the solutions obtained, then review answers as a class, asking each group to provide the solutions to the situations. Use questioning to modify incorrect models.

Ask groups to review the conjectures and descriptions based on the new information. Lead students through the revision process by looking at each conjecture or description and determining if any of the situations contradicted the statement or if clarification is needed.

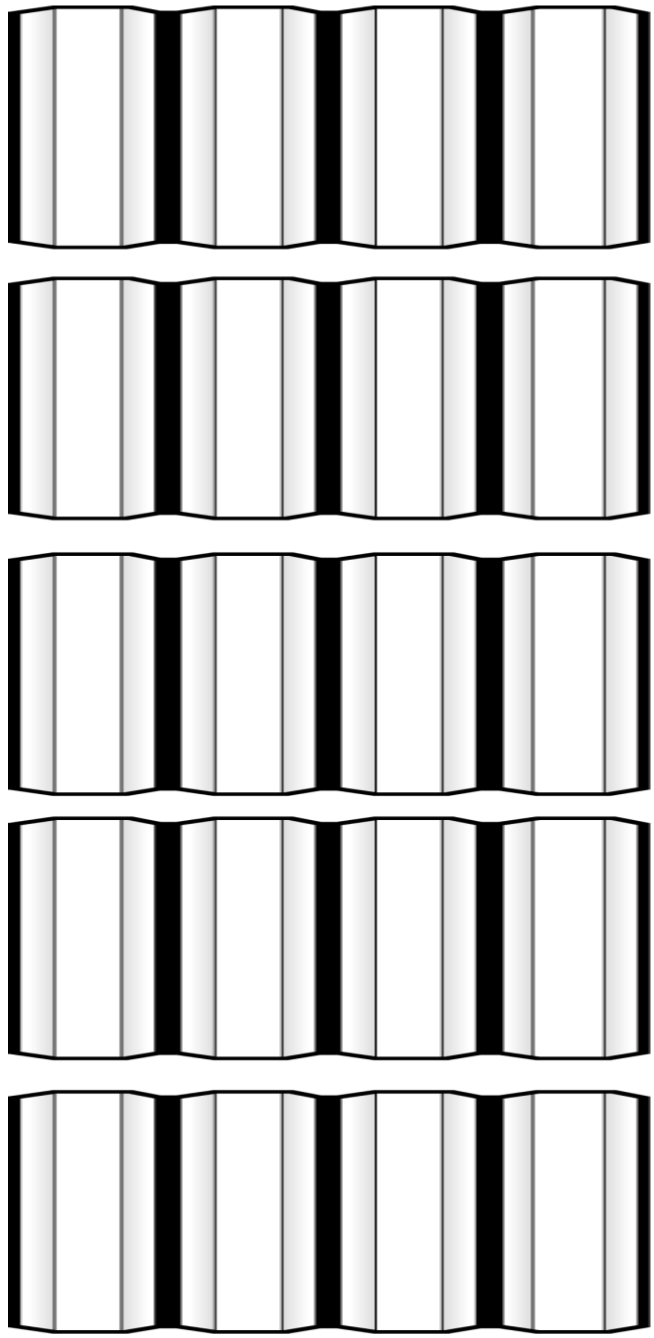
Have students determine if the statements refer to the process or the results of the problem.

Part 4

Give each student a copy of *Models for Dividing Fractions*. They should complete this individually or with a partner.



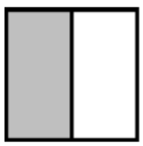
Chocolate Bar Models

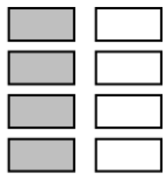


Task Part 1: Dividing By A Fraction

1. I have one-half of a square and I want to divide it by one-eighth. How many pieces would I have?

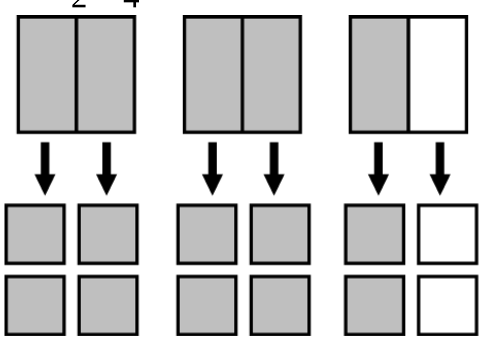
4 ; $\frac{1}{2} \div \frac{1}{8} = 4$ *I have 4 pieces that are one eighth of the square.*





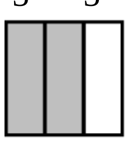
2. I have two and one-half squares and I want to divide them by one-fourth. How many pieces would I have?

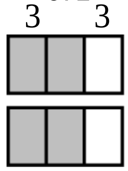
10 ; $\frac{5}{2} \div \frac{1}{4} = 10$ *I have 10 pieces that are one fourth of a square.*



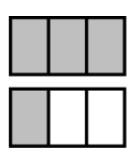
3. I have two- thirds of a square and I want to divide it by one-half. How many pieces would I have?

$\frac{4}{3}\, or\, 1\frac{1}{3}$*;* $\frac{2}{3} \div \frac{1}{2} = \frac{4}{3}\, or\, 1\frac{1}{3}$ *I have four thirds of the size one half of a square.*



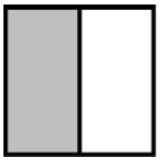


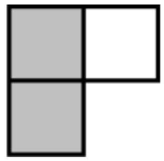
OR



4. I have one-half of a square and I want to divide it by three-fourths. How many pieces would I have?

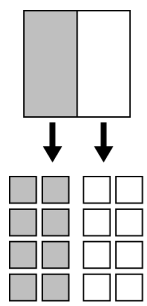
$\frac{2}{3}$; $\frac{1}{2} \div \frac{3}{4} = \frac{2}{3}$ *I have two thirds of the size three fourths of a square.*





Part 2: Models For Dividing Fractions

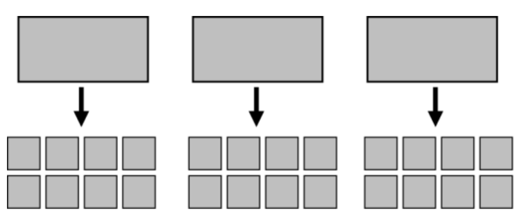
1. I have a one-half gallon container of ice cream and want to divide it into one-cup servings to share with the students in my class. A cup is one sixteenth of a gallon. How many serving dishes would I need?



Model the problem situation.

Write an equation and show how to solve the problem ***½ ÷ $^{1}/_{16}$ = 8***

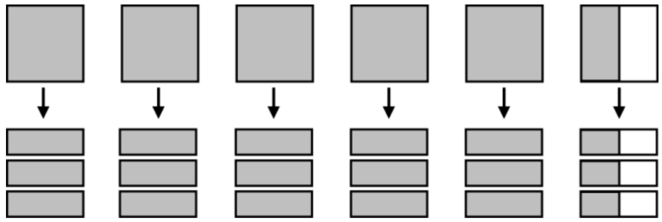
2. I also have three large chocolate candy bars that are perforated into eight sections each. If I divide the bars into these sections how many sections will I have altogether?



Model the problem situation.

Write an equation and show how to solve the problem ***3 ÷ 1/8 = 24***

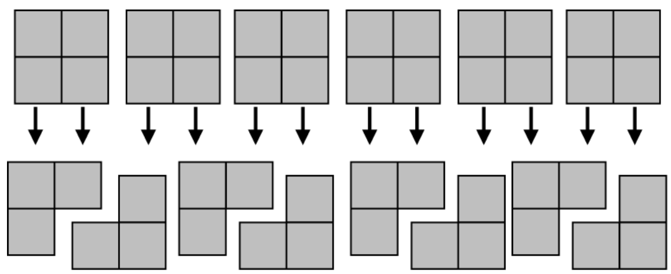
3. Becca works for the Humane Society and had to buy food for the dogs. She bought $5\frac{1}{2}$ lbs. of dog food. She feeds each dog about one-third of a pound. How many dogs can she feed?



Model the problem situation.

Write an equation and show how to solve the problem. ***Answer:*** $5\frac{1}{2} \div \frac{1}{3} = \frac{33}{2} = 16\frac{1}{2}$

4. Julie goes to the park across the street from her house several times a day and jogs a total of six miles every day. She jogs three-fourths of a mile at a time. How many times each day does she go to the park to run?

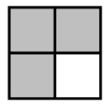


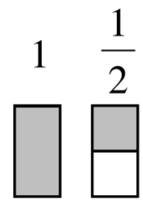
Model the problem situation.

Write an equation and show how to solve the problem. ***Solution:*** $6 \div \frac{3}{4} = 8$

For 5, 6, and 7 Model and solve each of the following equations.

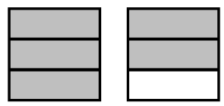
5. $\frac{3}{4} \div \frac{1}{2}$

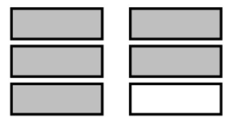




$$\frac{3}{4} \div \frac{1}{2} = \frac{3}{2} \; or \; 1\frac{1}{2}$$

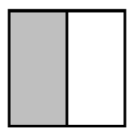
6. $\frac{5}{3} \div \frac{1}{3}$

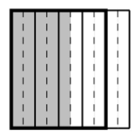




$$\frac{5}{3} \div \frac{1}{3} = 5$$

7. $\frac{1}{2} \div \frac{4}{5}$





$$\frac{1}{2} \div \frac{4}{5} = \frac{5}{8}$$

Name_______________________________

Part 1: Dividing By A Fraction

Directions: Model each situation using squares; write the equation for each.

1. I have one-half of a square and I want to divide it by one-eighth. How many pieces would I have?

2. I have two and one half squares and I want to divide them by one-fourth. How many pieces would I have?

3. I have two-thirds of a square and I want to divide it by one-half. How many pieces would I have?

4. I have one-half of a square and I want to divide it by three-fourths. How many pieces would I have?

Part 2: Models For Dividing Fractions

Directions: Read the situation, draw a picture to represent the situation and then write an equation to represent the situation.

1. I have a one-half gallon container of ice cream and want to divide it into one-cup servings to share with the students in my class. A cup is one-sixteenth of a gallon. How many serving dishes would I need?

   Model the problem situation.

   Write an equation and show how to solve the problem.

2. I also have three large chocolate candy bars that are perforated into eight sections each. If I divide the bars into these sections how many sections will I have altogether?

   Model the problem situation.

   Write an equation and show how to solve the problem

3. Becca works for the Humane Society and had to buy food for the dogs. She bought $5\frac{1}{2}$ lbs. of dog food. She feeds each dog about one-third of a pound. How many dogs can she feed?

   Model the problem situation.

   Write an equation and show how to solve the problem.

4. Julie goes to the park across the street from her house several times a day and jogs a total of six miles every day. She jogs three-fourths of a mile at a time. How many times each day does she go to the park to run?

Model the problem situation.

Write an equation and show how to solve the problem.

For 5, 6, and 7: Model and solve each of the following equations.

5. $\frac{3}{4} \div \frac{1}{2}$

6. $\frac{5}{3} \div \frac{1}{3}$

7. $\frac{1}{2} \div \frac{4}{5}$

Understanding Algorithms

Back to Task Table

In this problem-based task, students will investigate the meaning of the two most common algorithms for dividing fractions: common denominator and invert-and-multiply.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, including reasoning strategies such as using visual fraction models and equations to represent the problem.
For example:

- *How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally?*
- *How many 3/4-cup servings are in 2/3 of a cup of yogurt?*
- *How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*
- *Three pizzas are cut so each person at the table receives ¼ pizza. How many people are at the table?*
- *Create a story context for (2/3)÷(3/4) and use a visual fraction model to show the quotient;*
- *Use the relationship between multiplication and division to explain that (2/3)÷(3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc)*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of real-world situations that involve division of fractions.
2. Reason abstractly and quantitatively. Students will reason why the process of inverting and multiplying works when dividing fractions.
3. Construct viable arguments and critique the reasoning of others. Students explain how to divide a fraction by a whole number and a whole number by a fraction.
4. Model with mathematics. Students use tape diagrams to model the division of fractions.
6. Attend to precision. Students use appropriate terminology when referring their explanations of dividing fractions.
7. Look for and make use of structure. Students examine the structure of dividing fractions by fractions.

ESSENTIAL QUESTIONS

- Why would it be helpful to know the least common multiple of a set of numbers when dividing fractions?
- Why does the process of invert and multiply work when dividing fractions and mixed numbers?
- When I divide one number by another number, do I always get a quotient smaller than my original number?
- When I divide a fraction by a fraction what do the dividend, quotient and divisor represent?
- What kind of models can I use to show solutions to word problems involving fractions?

MATERIALS REQUIRED

- Paper and writing utensil
- Base ten blocks
- Unit cubes
- Rulers

TIME NEEDED

- 2 days

TEACHER NOTES

There are two different algorithms for division of fractions: **Common-Denominator Algorithm** and **Invert-and-Multiply Algorithm**. This task introduces both to students.

Before the Lesson, review the student work with the two interpretations of division from the previous task (Partitive and Measurement). This can be done in small groups or as a whole class.

For each problem, first get answers from the class. If more than one answer is offered, simply record them and offer no evaluation.

Have students explain their strategies for thinking about the problem either on the board, with a document camera, etc. You may need to ask questions about drawings or explanations to make sure everyone in the class follows the rationale. Encourage the class to comment or ask questions about the student's representation or thinking. Ask if others used a different representation or solved the problem in a different way. If so, have the students come forward to share their solutions. If there are different answers, the class should evaluate the solution strategies and decide which answer is correct and why.

What does the quotient represent in each of the problems?

What does the divisor represent in each of the problems?

What does the dividend represent in each of the problems?

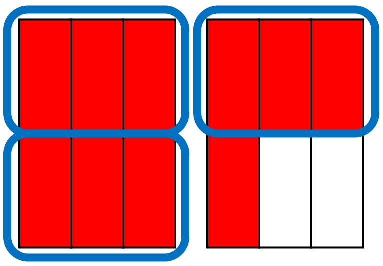
1. Consider the problem $\frac{5}{3} \div \frac{1}{2}$. Using words explain what this problem means. Restate your explanation with common denominators then solve the problem. Draw pictures first and then write number sentences.

The Common-Denominator algorithm relies on the measurement or repeated subtraction concept of division.

$\frac{5}{3} \div \frac{1}{2}$ ***means "How many sets of*** $\frac{1}{2}$ ***are in*** $\frac{5}{3}$ ***?"***

Restated with common denominators: "How many sets of $\frac{3}{6}$ ***are in*** $\frac{10}{6}$ ***?"***

$\frac{5}{3} \div \frac{1}{2} = \frac{10}{6} \div \frac{3}{6} = 10 \div 3 \text{ or } 3\frac{1}{3}$



2. Now try $1\frac{2}{3} \div \frac{3}{4}$ using the common-denominator approach.

$1\frac{2}{3} \div \frac{3}{4}$

$\frac{5}{3} \div \frac{3}{4}$

$\frac{20}{12} \div \frac{9}{12} = \frac{2\frac{2}{9}}{1}$

3. Complete the following set of problems using the methods we have been using for the last several days. Make a table of your answers to each and look for a pattern.

$3 \div \frac{1}{2} =$ How many servings of $\frac{1}{2}$ in 3 containers?
$5 \div \frac{1}{4} =$ How many servings of $\frac{1}{4}$ in 5 containers?
$3\frac{3}{4} \div \frac{1}{2} =$ How many servings of $\frac{1}{2}$ in $3\frac{3}{4}$ containers?
$6 \div \frac{1}{3} =$ How many servings of $\frac{1}{3}$ in 6 containers?
$8 \div \frac{1}{5} =$ How many servings of $\frac{1}{5}$ in 8 containers?

Serving Size	*# of Containers*	*# of servings*
$\frac{1}{2}$	*3*	*6*
$\frac{1}{4}$	*5*	*20*
$\frac{1}{2}$	$3\frac{3}{4}$	$7\frac{1}{2}$
$\frac{1}{3}$	*6*	*18*
$\frac{1}{5}$	*8*	*40*

Students should notice that they are multiplying by the denominator of the second fraction. For example, in the first example, a student might say, "You get two for every whole container, so 2 x 3 is 6".

4. Now try this set of problems. Use the results from your first table to help you.

$5 \div \frac{3}{4}$

$6 \div \frac{2}{3}$

$8 \div \frac{2}{5}$

Serving Size	*# of Containers*	*# of servings*
$\frac{3}{4}$	*5*	$6\frac{2}{3}$
$\frac{2}{3}$	*6*	*9*
$\frac{2}{5}$	*8*	*20*

5. Compare your responses from the second set of problems to the corresponding problems in the first set. What do you see?

Notice that if there are 40 one-fifths in 8, then when you group the fifths in pairs (two-fifths), you will have half as many – 20. Stated in servings, if the serving is twice as big, you will have half the number of servings. Similarly, if the fraction is $\frac{3}{4}$, after finding how many fourths, you will group in threes, which means you will get $\frac{1}{3}$ the number of servings. This means you must divide by 3.

6. Finally, try this partitioning problem:

You have $1\frac{1}{2}$ oranges, which is $\frac{3}{5}$ of an adult serving. How many oranges (and parts of oranges) make up 1 adult serving?

One-third of the oranges or one-half of an orange (notice you are dividing by the numerator) is equal to one-fifth of an adult serving. To get the whole serving you multiply one-half by 5 (the denominator) to get $2\frac{1}{2}$ oranges in 1 adult serving.

In the measurement or partition interpretation, the denominator leads you to find out how many parts you have (fifths, eighths, or sixths), and the numerator tells you the size of the serving, so you group according to how many are in the serving. From the information in the table students should be able to see the process means to multiply by the denominator and divide by the numerator.

Name________________________

Understanding Algorithms

1. Consider the expression $\frac{5}{3} \div \frac{1}{2}$. Using words explain what this problem means. Restate the expression with common denominators then solve the problem. Draw pictures first and then write number sentences.

2. Now try $1\frac{2}{3} \div \frac{3}{4}$ using the common-denominator approach.

3. Complete the following set of problems using the methods we have been using for the last several days. Make a table of your answers to each and look for a pattern.

$3 \div \frac{1}{2} =$ How many servings of $\frac{1}{2}$ in 3 containers?
$5 \div \frac{1}{4} =$ How many servings of $\frac{1}{4}$ in 5 containers?
$3\frac{3}{4} \div \frac{1}{2} =$ How many servings of $\frac{1}{2}$ in $3\frac{3}{4}$ containers?
$6 \div \frac{1}{3} =$ How many servings of $\frac{1}{3}$ in 6 containers?
$8 \div \frac{1}{5} =$ How many servings of $\frac{1}{5}$ in 8 containers?

4. Now try this set of problems:

$$5 \div \frac{3}{4}$$
$$6 \div \frac{2}{3}$$
$$8 \div \frac{2}{5}$$

5. Compare your responses from the second set of problems to the corresponding problems in the first set. What do you see?

6. Finally, try this partitioning problem:

You have $1\frac{1}{2}$ oranges, which is $\frac{3}{5}$ of an adult serving. How many oranges (and parts of oranges) make up 1 adult serving?

Do It Yourself

Back to Task Table

In this task, students will write story problems to represent given numerical expressions involving fraction division.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, including reasoning strategies such as using visual fraction models and equations to represent the problem.
For example:

- *How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally?*
- *How many 3/4-cup servings are in 2/3 of a cup of yogurt?*
- *How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*
- *Three pizzas are cut so each person at the table receives ¼ pizza. How many people are at the table?*
- *Create a story context for (2/3)÷(3/4) and use a visual fraction model to show the quotient;*
- *Use the relationship between multiplication and division to explain that (2/3)÷(3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc)*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of a given division problem and real-world word problem.
3. Construct viable arguments and critique the reasoning of others. Students construct a real-world problem from a given division problem

ESSENTIAL QUESTIONS

- When I divide one number by another number, do I always get a quotient smaller than my original number?
- When I divide a fraction by a fraction what do the dividend, quotient and divisor represent?

MATERIALS REQUIRED

- Paper and writing utensil
- Base ten blocks
- Unit cubes
- Rulers

TIME NEEDED

- 1 day

TEACHER NOTES

Students will write story problems to represent given numerical expressions

Before the lesson, briefly highlight work from previous tasks in the dividing fractions portion of the unit highlighting different ways of modeling and the meanings of the quotient, divisor, and dividend

With your group write a story problem for each of the expressions shown below.

$1\frac{3}{4} \div \frac{1}{2}$ $\quad$ $2\frac{3}{5} \div \frac{1}{3}$ $\quad$ $\frac{11}{12} \div \frac{1}{4}$ $\quad$ $\frac{2}{3} \div \frac{3}{4}$

Students will work in groups on each of the problems. Then, form new groups by pulling one person from each of the first groups where they will meet with the other students to compare their solutions and plan their presentation to the class. They will need to provide an explanation, any discrepancies they had with group members, and share their strategies.

During the lesson, be sure that students are drawing pictures or using manipulatives to help them think about how to do the problems and explain their thinking.
Look for students who use different representations to think about the problems. Highlight those different ways in the Summary portion of the lesson.

Let each group present their solution to the problem.

What does the quotient represent in each of the problems?

What does the divisor represent in each of the problems?

What does the dividend represent in each of the problems?

INTERVENTION

If students struggle to generate word problems from scratch, provide the context for them and have them provide the question that can be asked to match the context. For example, with the first set of numbers, "*Johnny has 1 ¾ cups of cupcake batter. Each cupcake needs ½ of a cup of batter*." What could be the question?

Name___________________________

Do It Yourself

With your group write a story problem for each of the expressions shown below.

$1\frac{3}{4} \div \frac{1}{2}$

$2\frac{3}{5} \div \frac{1}{3}$

$\frac{11}{12} \div \frac{1}{4}$

$\frac{2}{3} \div \frac{3}{4}$

The next three tasks are similar and involve estimation and fluency with addition, subtraction, multiplication and division.

At each grade level in the standards, one or two fluencies are expected. For sixth graders it is multi-digit division and multi-digit decimal operations. Fluent in the standards means "fast and accurate". It might also help to think of fluency as meaning more or less the same as when someone is said to be fluent in a foreign language. To be fluent is to flow; fluent isn't halting, stumbling, or reversing oneself. Fluency may not happen within the scope of one unit. Nor can fluency be taught. It is experience.

The tasks included here are meant to provide an opportunity for the teacher to gain some understanding of students' sense of number and computational expertise. To make sure that this standard is met by year end students should have the opportunity to compute using multi-digit whole numbers and decimals throughout the entire year.

This is not meant to be a task for teaching formal methods of estimation but instead to explore the methods students are bringing with them from elementary school. You may see examples of front-end methods, rounding, the use of compatible numbers, or clustering.

In the past, decimal computation was dominated by following the rules: Line up the decimal points (addition & subtraction), and shift the decimal point in the divisor and dividend so that the divisor is a whole number (division). However, Van de Walle and other standards-based curricula take the position that specific rules are not necessary if computation is built on a firm understanding of place value and a connection between decimals and fractions.

Fluency is something that develops over time; practice should be given over the course of the year as students solve problems related to other mathematical studies. Opportunities to determine when to use paper pencil algorithms, mental math or a computing tool is also a necessary skill and should be provided in problem solving situations.

In order to achieve fluency, students must have a firm understanding of place value and a connection between decimals and fractions. Students who are taught only to focus on the pencil-and-paper rules for decimal computation do not even consider the actual values of the numbers. Therefore a good place to begin decimal computation is with estimation. It helps children to look at answers in terms of a reasonable range and can form a check on calculator computation. This emphasis on estimation is very important, even for students in the middle grades who have been exposed to and have used rules. There is evidence that understanding is the basis for developing procedural fluency. It is never too late to begin.

TEACHER NOTES: FLUENCY

About MGSE6.NS.2 Fluently divide multi-digit numbers using the standard algorithm: Procedural fluency is defined as "skill in carrying out procedures flexibly, accurately, efficiently and appropriately". In the elementary grades, students were introduced to division through concrete models and various strategies to develop an understanding of this mathematical operation (limited to 4-digit numbers divided by 2-digit numbers). In 6th grade, students become fluent in the use of the standard division algorithm. This understanding is foundational for work with fractions and decimals in the 6th & 7th grade.

About MGSE6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation: Procedural fluency is defined as "skill in carrying out procedures flexibly, accurately, efficiently and appropriately". In 4th and 5th grades, students added and subtracted decimals. Multiplication and division of decimals was introduced in 5th grade (decimals to the hundredth place). At the elementary level, these operations were based on concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. In 6th grade, students become fluent in the use of the standard algorithms of each of these operations.

Modeling Fraction Division (Formative Assessment Lesson)

Back to Task Table

Source: Georgia Mathematics Design Collaborative

In this task, students will use real-world situations to represent the meaning of fraction division.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, including reasoning strategies such as using visual fraction models and equations to represent the problem.
For example:

- *How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally?*
- *How many 3/4-cup servings are in 2/3 of a cup of yogurt?*
- *How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*
- *Three pizzas are cut so each person at the table receives ¼ pizza. How many people are at the table?*
- *Create a story context for (2/3)÷(3/4) and use a visual fraction model to show the quotient;*
- *Use the relationship between multiplication and division to explain that (2/3)÷(3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc)*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.

TASK COMMENTS

This Formative Assessment Lesson will help you to identify and support students who have difficulty in:

- Representing fraction division.
- Writing an equation to represent a fraction division problem.
- Interpreting the remainder of a fraction division problem.

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Georgia Mathematics Design Collaborative are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding.

The task, *Modeling Fraction Division*, is a Formative Assessment Lesson (FAL) that can be found on the Georgia Mathematics online professional learning community, edWeb.net. Once logged in, the task can be found directly at: https://www.edweb.net/?14@@.5abfa3bd.

Estimating is the Root of Fluency – Addition and Subtraction

Back to Task Table

In this task, students will use number sense to reason about adding and subtracting decimals (rather than relying exclusively on paper-pencil algorithms).

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.

MGSE6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

STANDARDS FOR MATHEMATICAL PRACTICE

6. Attend to precision. Students attend to precision by correctly adding and subtracting numbers with decimals.
7. Look for and make use of structure. Students examine the relationship of decimal places and the sums and differences.
8. Look for and express regularity in repeated reasoning. Students demonstrate repeated reasoning when adding and subtracting with decimals.

ESSENTIAL QUESTIONS

- When adding a subtracting numbers with decimals, what must be done before adding or subtracting?
- How is estimation helpful when adding and subtracting numbers with decimals?
- Which strategies are helpful when performing operations on multi-digit decimals?

BEFORE THE LESSON

Consider this problem as a class:

Jessica and MacKenna each timed her own quarter-mile run with a stopwatch. Jessica says that she ran the quarter in 74.5 seconds. MacKenna was more precise. She reported her run as 81.34 seconds. How many seconds faster did Jessica run than McKenna?

Ask for student's estimate (no paper or pencil please) of the difference and an explanation of their estimation strategy.

Students who understand decimal numeration should be able to tell that the difference is approximately 7 seconds.

Now, take out your pencil and paper and find the exact answer.

EXPLORATION

Give students a sum involving different numbers of decimal places. For example: 67.23 + 9.3 + 0.758. The first task is to make an estimate and explain how the estimate was made. The second task is to compute the exact answer and explain how that was done (no calculators, please). In the third and final task students devise a method for adding decimal numbers that they can use with any two numbers.

Repeat the series of tasks for subtraction. 67.23 - 9.3 - 0.758

DURING

If students have difficulty with this series of small tasks, it is an indication that they have a weak understanding of decimal concepts and the role of the decimal point. Be wary of student who can get a correct sum or difference by using a rule they learned but who have difficulty with their explanations. The teacher will have to plan some remediation for these students. It is recommended that the remediation begin with student invented strategies which can be found in Van de Walle's Elementary and Middle School Mathematics: Teaching Developmentally.

SUMMARY

Allow students to share solutions making sure that explanations are mathematically sound.

Solutions

Estimates and explanations will vary.

67.23 + 9.3 + 0.758 = 77.288 ***67.23 - 9.3 - 0.758 = 57.172***

Extra Practice

The following online games encourage mental math/estimation with simple decimal addition and subtraction problems:

http://www.decimalsquares.com/dsGames/games/laserbeam.html

http://www.decimalsquares.com/dsGames/games/blackjack.html

http://www.decimalsquares.com/dsGames/games/tugowar.html

Where Does the Decimal Go? - Using Estimation When Multiplying Decimals

(Modification of *Estimation is the Root of Fluency – Multiplication with Decimals)*

Back to Task Table

In this task, students will use number sense to reason about multiplying decimals (rather than relying exclusively on paper-pencil algorithms).

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.
MGSE6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

STANDARDS FOR MATHEMATICAL PRACTICE

6. Attend to precision. Students attend to precision by placing decimals in the correct place in a division problem through the use of estimation and sense making.
7. Look for and make use of structure. Students examine the relationship of decimal places and the answer of a multiplication problem with decimals.
8. Look for and express regularity in repeated reasoning. Students demonstrate repeated reasoning when multiplying with decimals.

ESSENTIAL QUESTIONS

- What strategies can I use to determine whether my product or quotient is reasonable?
- Which strategies are efficient when performing operations on multi-digit decimals?

MATERIALS REQUIRED

Post the exploration problem or provide copies to students. Sense making and reasoning should allow students to determine if their answers are reasonable.

TIME NEEDED

- 1 day

TEACHER NOTES

This task is meant to give students a chance to use estimation as a tool to determine the placement of a decimal point in multiplication problems involving decimals. The procedures students use involve estimation and strategies for computation with whole numbers.

Division and multiplication of decimals can be approached in similar manners. Estimation can produce reasonable results but you still may require an exact answer. Van de Walle (2013) suggests "ignoring the decimal points, and doing the computation as if all the numbers were whole first. When finished, place the decimal back in but reason through estimation." Employing this technique will increase students' number sense and eliminate the need for teachers and students to use procedural rules such as counting "spots" relating to place value with no understanding.

BEFORE THE LESSON

As a class, consider this problem:
The water machine at the local grocery store fills a jug with 3.7 liters of water. If you fill 4 jugs, how many liters of water is that?

Begin with an estimate. Have students estimate about how many liters of water that is. Have them give 3 total estimates – one they think is close (share these either on post-it notes or by writing estimates on the board), one that is too low and one that is too high.

Ask students what computation they could use to determine the correct digits in the multiplication problem (37×4). If this gives the digits, based on the estimates, where should the decimal be placed for 3.7×4?

Now, compare two more multiplication problems:

43.4×4.3 and 434×43

Again, make an estimate for the first. Then, compute the exact answer to the second problem.

Based on your estimate, where should the decimal be placed in the product of the first problem? What similarities do you see between the two products? What difference do you see between the two products?

EXPLORATION

Students should work on this task individually. After they complete the computations and rationale they can share their work with a partner.

Give students the following instructions:
Compute the following product: 35×37. Using only the result of this computation and estimation, give the exact answer to each of the following:
0.35×3.7 35×0.37 3.5×37 0.35×0.37

For each computation write a rationale for how you made the placement of the decimal point in each answer. When you have finished, you make check your results with a calculator. Acknowledge any errors you may have made and adjust your rationale to correct the error.

DURING

Make sure that students are not doing actual computations and that they are placing decimals by using estimation strategies. Reassure them that they will get the chance to correct their work and their thinking.

SUMMARY

Have students share rationales for placing the decimal point. Make sure that these rationales are sound. Encourage those students who made errors to share their error and their correction. You may make decisions about who is to share during the exploration portion of the lesson.

DIFFERENTIATION

Extension:
Assign students the task of identifying benchmark decimals to use for estimation other than 0.25, 0.5 and 0.75. Students should choose a benchmark decimal and explain why it can be helpful to estimate using these benchmarks. Example: 0.125 is the same as 1/8 or half of a quarter (extending previously mastered benchmarks).

Intervention:
For students in need of support, discuss and use benchmark decimals such as 0.25, 0.5 and 0.75 initially. Allow students to make sense of using these decimals which have familiar connections to benchmark fractions of ¼, ½, and ¾.

SOLUTIONS
$0.35 \times 3.7 = 1.295$ $35 \times 0.37 = 12.95$ $3.5 \times 37 = 129.5$ $0.35 \times 0.37 = 0.1295$

The method of placing the decimal point in a product by way of estimation is more difficult as the product gets smaller.

Even if students have already learned the traditional algorithm, they need to know the conceptual rationale centered on place value and the powers of ten for "counting" and shifting the decimal places. By focusing on rote applications of rules, students lose out on approaches that emphasize opportunities to understand the meaning and effects of operations and are more prone to misapply procedures.

Name________________________________

<u>Where Does the Decimal Go? – Using Estimation When Multiplying Decimals</u>

The water machine at the local grocery store fills a jug with 3.7 liters of water. If you fill 4 jugs, how many liters of water is that?

Begin with an estimate. Estimate about how many liters of water that would be. Write down two more estimates – one that you know is too low and one that you know is too high.

Ask students what computation they could use to determine the correct digits in the multiplication problem. Find the product of these numbers.

Based on the estimates you made, where should the decimal be placed for 3.7×4?

Now, compare two more multiplication problems:

43.4×4.3 and 434×43

Again, make an estimate for the first. Make two more estimates – one that is too low, and one that is too high.

Compute the exact answer to the second problem.

Based on your estimate, where should the decimal be placed in the product of the first problem? Explain your reasoning.

What similarities do you see between the two products? What difference do you see between the two products?

EXPLORATION

Compute the following product: 35×37.

Using only the result of this computation and estimation, give the exact answer to each of the following: For each computation write a rationale for how you made the placement of the decimal point in each answer. When you have finished, you make check your results with a partner. Acknowledge any errors you may have made and adjust your rationale to correct the error.

Problem	Solution	Reasoning
0.35×3.7		
35×0.37		
3.5×37		
0.35×0.37		

Where Does the Decimal Go? – Using Estimation When Dividing Decimals

(Modified version of *Estimation is the Root of Fluency – Division with Decimals*)

Back to Task Table

In this task, students will use number sense to reason about dividing decimals (rather than relying exclusively on paper-pencil algorithms).

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.
MGSE6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

STANDARDS FOR MATHEMATICAL PRACTICE

6. Attend to precision. Students attend to precision by placing decimals in the correct place in a division problem through the use of estimation and sense making.
7. Look for and make use of structure. Students examine the relationship of decimal places and the quotient of a division problem with decimals.
8. Look for and express regularity in repeated reasoning. Students demonstrate repeated reasoning when dividing with decimals.

ESSENTIAL QUESTIONS

- What strategies can I use to determine whether my product or quotient is reasonable?
- Which strategies are efficient when performing operations on multi-digit decimals?

MATERIALS REQUIRED

- Post the exploration problem or provide copies to students. Calculators may be used for checking work after students have complete computations by hand.

TIME NEEDED

- 2 days

TEACHER NOTES

This task is meant to give students a chance to use estimation as a tool to determine the placement of a decimal point in division problems involving decimals. The procedures students use involve estimation and strategies for computation with whole numbers.

Division and multiplication of decimals can be approached in similar manners. Estimation can produce reasonable results but you still may require an exact answer. Van de Walle (2013) suggests "ignoring the decimal points, and doing the computation as if all the numbers were whole first. When finished, place the decimal back in but reason through estimation." Employing this technique will increase students' number sense and eliminate the need for teachers and students to use procedural rules such as counting "spots" relating to place value with no understanding.

BEFORE THE LESSON

Consider the expression:

$45.7 \div 1.83$

Have students make an estimate of this quotient. Have them write the estimate on a post-it or write estimates on the board.

Students should work on this task individually. After they complete the computations and rationale they can share their work with a partner.

Give students the following problem:
Provide this quotient $146 \div 7 = 20857$ or one that is similar meaning correct to five digits but without a decimal point. The task is to use only this information and estimation to give a precise answer to each of the following:

$146 \div 0.7$ $1.46 \div 7$ $14.6 \div 0.7$ $1460 \div 70$

For each computation students should write a rationale for their answers and then check their results with a calculator. Any errors should be acknowledged, and the rationale that produced the error adjusted.

DURING

Make sure that students are not doing actual computations and that they are placing decimals by using estimation strategies. Reassure them that they will get the chance to correct their work and their thinking.

SUMMARY

Have students share rationales for placing the decimal point. Make sure that these rationales are sound. Encourage those students who made errors to share their error and their correction. You may make decisions about who is to share during the exploration portion of the lesson.

DIFFERENTIATION

Extension:
Assign students the task of identifying benchmark decimals to use for estimation other than 0.25, 0.5, and 0.75. Students should choose a benchmark decimal and explain why it can be helpful to estimate using these benchmarks. Example: 0.125 is the same as 1/8 or half of a quarter (extending previously mastered benchmarks).

Intervention:
For students in need of support, discuss and use benchmark decimals such as 0.25, 0.5, and 0.75 initially. Allow students to make sense of using these decimals which have familiar connections to benchmark fractions of ¼, ½, and ¾.

SOLUTIONS

146 ÷ 0.7 = 208.57 ***1.46 ÷ 7 = 0.20857*** ***14.6 ÷ 0.7 = 20.857*** ***1460 ÷ 70 = 20.857***

Name________________________________

Where Does the Decimal Go? – Using Estimation When Dividing Decimals

Consider the number sentence 146 ÷ 7 = 20857, is it true? If not use what you know about estimation to determine the correct placement of the decimal point. Justify your solution.
The task is to use only this information and estimation to give a fairly precise answer to each of the following: Be sure to justify each of your solutions.

Problem	Solution	Reasoning
146 ÷ 0.7		
1.46 ÷ 7		
14.6 ÷ 0.7		
1460 ÷ 70		

Culminating Task: Pick A Number, Any Number

Back to Task Table

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, including reasoning strategies such as using visual fraction models and equations to represent the problem.
For example:

- *How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally?*
- *How many 3/4-cup servings are in 2/3 of a cup of yogurt?*
- *How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*
- *Three pizzas are cut so each person at the table receives ¼ pizza. How many people are at the table?*
- *Create a story context for (2/3)÷(3/4) and use a visual fraction model to show the quotient;*
- *Use the relationship between multiplication and division to explain that (2/3)÷(3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc)*

MGSE6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.

MGSE6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

MGSE6.NS.4 Find the common multiples of two whole numbers less than or equal to 12 and the common factors of two whole numbers less than or equal to 100.

a. Find the greatest common factor of 2 whole numbers and use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factors. (GCF) *Example: 36 + 8 = 4(9 + 2)*
b. Apply the least common multiple of two whole numbers less than or equal to 12 to solve real-world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of real-world fraction and decimal problem situations by representing the context in tactile and/or virtual manipulatives, visual, or algebraic models.
2. Reason abstractly and quantitatively. Students will apply the constructs of multiplication and division of rational numbers to solve application problems, including finding the area of nets.
4. Model with mathematics. Students will model real-world situations to show multiplication and division of fractions and decimals.
6. Attend to precision. Students attend to the language of problems to determine appropriate representations and operations for solving real-world problems. In addition, students attend to the units of measure used in real-world problems.

7. Look for and make use of structure. Students examine the place value structure as related to multi-digit operations. They also use their knowledge of problem solving structures to make sense of word problems.
8. Look for and express regularity in repeated reasoning. Students demonstrate repeated reasoning when finding LCM, GCF, and the operations with decimals.

ESSENTIAL QUESTIONS

- Why would it be useful to know the greatest common factor of a set of numbers?
- Why would it be helpful to know the least common multiple of a set of numbers?
- How can the distributive property help me with computation?
- What do the dividend, quotient and divisor represent when dividing a fraction by a fraction?
- Which strategies are helpful when dividing multi-digit numbers?
- Which strategies are helpful when performing operations on multi-digit decimals?

Culminating Task – Pick A Number, Any Number

1. Choose a number between 2 and 12. Write **YOUR NUMBER** here ______ ***example: 5***

2. Write a word problem (story problem) to find the greatest common factor of two numbers. Let **YOUR NUMBER** be the solution to the problem. You can use pictures to model the solution.

Answers will vary – one example: Kathy is making fruit baskets. She has 15 oranges and 20 bananas. She wants to make baskets with the most amount of fruit but wants each basket to be the same. How many baskets can she make?

3. Write a word problem (story problem) to find the least common multiple of two numbers. Let **YOUR NUMBER** be ONE of the numbers. The solution will be a multiple of **YOUR NUMBER**. You can use pictures to model the solution.

Answers will vary – one example: Jesse and his mom are making treat bags for students at his school. Pencils come in packs of 12 and erasers come in packs of 5. What is the minimum numbers of items he will need to give his friends one eraser and one pencil and have nothing left over? 60 is the solution to my problem.

4. Write a word problem (story problem) involving division of a fraction and a fraction. Use **YOUR NUMBER** as the denominator of one of the fractions. The other fraction can be anything you want. Use mixed numbers for extra points.

Answers will vary – one example: Mrs. Waltz bought 6 pizzas, each cut into 10 slices. After Marcus ate some, there were $4\,{}^3/_5$ pizzas left. How many people could the rest of the pizza serve if each one eats ${}^2/_5$ of a pizza (4 slices)? Answer: 11 servings with a ½ serving (2 slices) left over.

5. Write a number sentence to show an example of the distributive property using **YOUR NUMBER** as a common factor of two numbers.

Answers will vary – one example: $5(12 + 7) = 60 + 35$

6. Make-up a four digit number that has **YOUR NUMBER** in the tenths place. Second, choose a four digit number that has the last digit of **YOUR NUMBER** in the hundredths place. Create an addition and subtraction problem with these two numbers. Compute the sum and difference.
Answers will vary – one example:

216.5_____ (first-digit four number) ***87.25***________ (second four-digit number)

Sum ***303.75*** Difference ***129.25***

7. Use the "first four digit number" above and **YOUR NUMBER** to create a multiplication

problem. Set up the problem and compute the product.

Answers will vary – one example: **$216.5 \times 5 = 1082.5$**

8. Use the "first four digit number" above to create a division problem. Choose a two-digit divisor. Set up the problem and compute the quotient.

Answers will vary – one example: **$216.5 \div 15 = 14.4\overline{3}$**

Name_________________

Culminating Task – Pick A Number, Any Number

1. Choose a number between 2 and 12. Write **YOUR NUMBER** here ______

2. Write a word problem (story problem) to find the greatest common factor of two numbers. Let **YOUR NUMBER** be the solution to the problem. You can use pictures to model the solution.

3. Write a word problem (story problem) to find the least common multiple of two numbers. Let **YOUR NUMBER** be ONE of the numbers. The solution will be a multiple of **YOUR NUMBER**. You can use pictures to model the solution.

4. Write a word problem (story problem) involving division of a fraction and a fraction. Use **YOUR NUMBER** as the denominator of one of the fractions. The other fraction can be anything you want. Use mixed numbers if you want.

5. Write a number sentence to show an example of the distributive property using **YOUR NUMBER** as a common factor of two numbers.

6. Make-up a four digit number that has **YOUR NUMBER** in the tenths place. Second, choose a four digit number that has the last digit of **YOUR NUMBER** in the hundredths place. Create an addition and subtraction problem with these two numbers. Compute the sum and difference.

_________________ (first-digit four number) _______________ (second four-digit number)

Sum Difference

7. Use the "first four digit number" above and **YOUR NUMBER** to create a multiplication problem. Set up the problem and compute the product.

8. Use the "first four digit number" above to create a division problem. Choose a two-digit divisor. Set up the problem and compute the quotient.

Unit Web Links

Back to Task Table

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

MGSE6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, including reasoning strategies such as using visual fraction models and equations to represent the problem.
For example:

- *How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally?*
- *How many 3/4-cup servings are in 2/3 of a cup of yogurt?*
- *How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*
- *Three pizzas are cut so each person at the table receives ¼ pizza. How many people are at the table?*
- *Create a story context for (2/3)÷(3/4) and use a visual fraction model to show the quotient;*
- *Use the relationship between multiplication and division to explain that (2/3)÷(3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc)*

https://www.illustrativemathematics.org/content-standards/6/NS/A/1/tasks
http://www.visualfractions.com/divide.htm
http://www.yummymath.com/wp-content/uploads/fathers-day-muffins.pdf
http://www.openmiddle.com/dividing-fractions/
http://www.openmiddle.com/dividing-mixed-numbers/
http://www.openmiddle.com/undefined-products-with-fraction-division/
http://www.101qs.com/3043
http://nzmaths.co.nz/resource/dividing-fractions
http://nzmaths.co.nz/resource/dividing-fractions-0
http://www.pbslearningmedia.org/resource/mgbh.math.nf.lindadiv/division-of-fractions/

Compute fluently with multi-digit numbers and find common factors and multiples

MGSE6.NS.4 Find the common multiples of two whole numbers less than or equal to 12 and the common factors of two whole numbers less than or equal to 100.

a. Find the greatest common factor of 2 whole numbers and use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factors. (GCF) *Example: 36 + 8 = 4(9 + 2)*
b. Apply the least common multiple of two whole numbers less than or equal to 12 to solve real-world problems.

http://www.learner.org/courses/learningmath/number/session6/part_a/area.html