Georgia
Standards of Excellence Curriculum Frameworks

Mathematics

GSE Grade 6

Unit 3: Expressions

Georgia Department of Education

Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

Unit 3
Expressions

Table of Contents

OVERVIEW 3
STANDARDS FOR MATHEMATICAL PRACTICE 3
STANDARDS FOR MATHEMATICAL CONTENT 4
BIG IDEAS 5
ESSENTIAL QUESTIONS 5
CONCEPTS & SKILLS TO MAINTAIN 5
STRATEGIES FOR TEACHING AND LEARNING 6
COMMON MISCONCEPTIONS 8
SELECTED TERMS AND SYMBOLS 8
INSTRUCTIONAL RESOURCES/TOOLS 9
FORMATIVE ASSESSMENT LESSONS (FAL) 9
SPOTLIGHT TASKS 10
TASKS 11
- The Best Offer (Spotlight Task) 13
- Exponents 18
- Rules for Exponents 22
- Conjectures About Properties 28
- Visual Patterns (Spotlight Task) 35
- Perimeter and Area Expressions (Spotlight Task) 41
- The Algebra of Magic Part 1 45
- The Algebra of Magic Part 2 53
- The Algebra of Magic Part 3 60
- Writing Expressions 67
- Writing and Evaluating Expressions 70
- Laws of Arithmetic - Formative Assessment Lesson 79
- Are We Equal? 80
- Culminating Task: Sweet Tooth Chocolate Shop 95
- Unit Web Links 104

OVERVIEW

In this unit students will:

- Represent repeated multiplication with exponents.
- Evaluate expressions containing exponents to solve mathematical and real world problems.
- Translate verbal phrases and situations into algebraic expressions.
- Identify the parts of a given expression.
- Use the properties to identify equivalent expressions.
- Use the properties and mathematical models to generate equivalent expressions.

Working with expressions and equations containing variables allows students for them to form generalizations. Students should think of variables as quantities that vary instead of as letters that represent set values. When students work with expressions involving variables without a focus on a specific number or numbers that the variable may represent they can better recognize the patterns that occur. It is these patterns that lead to generalizations that lay the foundation for their future work in algebra.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of expressions and formulas by connecting them to real world contexts when evaluating.
2. Reason abstractly and quantitatively. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3. Construct viable arguments and critique the reasoning of others. Students construct and critique arguments regarding the equivalence of expressions and the use of variable expressions to represent real-world situations.
4. Model with mathematics. Students form expressions from real world contexts. Students use algebra tiles to model algebraic expressions.
5. Use appropriate tools strategically. Students determine which algebraic representations are appropriate for given contexts.
6. Attend to precision. Students use the language of real-world situations to create appropriate expressions.
7. Look for and make use of structure. Students apply properties to generate equivalent expressions. They interpret the structure of an expression in terms of a context. Students identify a "term" in an expression.
8. Look for and express regularity in repeated reasoning. Students can work with expressions involving variables without the focus on a specific number or numbers that the variable may represent. Students focus on the patterns that lead to generalizations that lay the foundation for their future work in algebra. Students work with the structure of the distributive property $2(3x + 5) = 6x + 10$.

STANDARDS FOR MATHEMATICAL CONTENT

Apply and extend previous understandings of arithmetic to algebraic expressions.

MGSE6.EE.1 Write and evaluate expressions involving whole-number exponents.

MGSE6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

MGSE6.EE.2a Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5-y.*

MGSE6.EE.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.

MGSE6.EE.2c Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$.

MGSE6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

MGSE6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them.) *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

MGSE6.NS.4 Find the common multiples of two whole numbers less than or equal to 12 and the common factors of two whole numbers less than or equal to 100.

a. Find the greatest common factor of 2 whole numbers and use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factors. (GCF) *Example: 36 + 8 = 4(9 + 2)*
b. Apply the least common multiple of two whole numbers less than or equal to 12 to solve real-world problems.

BIG IDEAS

- Variables can be used as unique unknown values or as quantities that vary.
- Exponential notation is a way to express repeated products of the same number.
- Algebraic expressions may be used to represent and generalize mathematical problems and real life situations.
- Properties of numbers can be used to simplify and evaluate expressions.
- Algebraic properties can be used to create equivalent expressions.
- Two equivalent expressions form an equation.

ESSENTIAL QUESTIONS

- How are “standard form” and “exponential form” related?
- What is the purpose of an exponent?
- How are exponents used when evaluating expressions?
- How is the order of operations used to evaluate expressions?
- How are exponents useful in solving mathematical and real world problems?
- How are properties of numbers helpful in evaluating expressions?
- What strategies can I use to help me understand and represent real situations using algebraic expressions?
- How are the properties (Identify, Associative and Commutative) used to evaluate, simplify and expand expressions?
- How is the Distributive Property used to evaluate, simplify and expand expressions?
- How can I tell if two expressions are equivalent?

It is expected that students will continue to develop and practice strategies to build their capacity to become fluent in mathematics and mathematics computation. The eventual goal is automaticity with math facts. This automaticity is built within each student through strategy development and practice. The following section is presented in order to develop a common understanding of the ideas and terminology regarding fluency and automaticity in mathematics:

CONCEPTS & SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Using parentheses, brackets, or braces in numerical expressions and evaluate expressions with these symbols
- Writing and interpreting numerical expressions
- Generating two numerical patterns using two given rules
- Interpreting a fraction as division

- Operating with whole numbers, fractions, and decimals

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply "how to get the answer" and instead support students' ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students:

- flexibly use a combination of deep understanding, number sense, and memorization.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.

For more about fluency, see: http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and: http://joboaler.com/timed-tests-and-the-development-of-math-anxiety/

STRATEGIES FOR TEACHING AND LEARNING

The skills of reading, writing and evaluating expressions are essential for future work with expressions and equations, and are a Critical Area of Focus for Grade 6. In earlier grades, students added grouping symbols ( ) to reduce ambiguity when solving equations. Now the focus is on using ( ) to denote terms in an expression or equation. Students should now focus on what terms are to be solved first rather than invoking the PEMDAS rule. Likewise, the division symbol (3 ÷ 5) was used and should now be replaced with a fraction bar $\left(\frac{3}{5}\right)$. Less confusion will occur as students write algebraic expressions and equations if *x* represents only variables and not multiplication. The use of a dot $(\bullet)$ or parentheses between number terms is preferred.

Provide opportunities for students to write expressions for numerical and real-world situations. Write multiple statements that represent a given algebraic expression. For example, the expression $x - 10$ could be written as "ten less than a number," "a number minus ten," "the temperature fell ten degrees," "I scored ten fewer points than my brother," etc. Students should also read an algebraic expression and write a corresponding statement.

Through modeling, encourage students to use proper mathematical vocabulary when discussing terms, factors, coefficients, etc.

Provide opportunities for students to write equivalent expressions, both numerically and with variables. For example, given the expression $x + x + x + x + 4 \bullet 2$, students could write $2x + 2x + 8$ or some other equivalent expression. Make the connection to the equivalent form of this expression, $4x + 8$. Because this is a foundational year for building the bridge between the concrete concepts of arithmetic and the abstract thinking of algebra, using hands-on materials (such as algebra tiles, counters, cubes, "Hands on Algebra") to help students translate between concrete numerical representations and abstract symbolic representations is critical.

Provide expressions and formulas to students, along with values, for the variables so students can evaluate the expressions. Evaluate expressions using the order of operations with and without parentheses. **Include whole-number exponents, fractions, decimals, etc.** Provide a model that shows step-by-step thinking when rewriting an expression. This demonstrates how two lines of work maintain equivalent algebraic expressions and establishes the need to have a way to review and justify thinking.

Provide a variety of expressions and problem situations for students to practice and deepen their skills. Start with simple expressions to evaluate and move to more complex expressions. Likewise start with simple whole numbers and move to fractions and decimal numbers. The use of negatives and positives should mirror the level of introduction in the 6th grade standards in The Number System domain; students are developing the concept and not generalizing operation rules**. Students are not performing operations with negative numbers.**

The use of technology can assist in the exploration of the meaning of expressions. Many calculators will allow you to store a value for a variable and then use the variable in expressions. This enables the student to discover how the calculator deals with expressions like x^2, $5x$, xy, and $2(x + 5)$.

COMMON MISCONCEPTIONS

- *The mnemonic PEMDAS can mislead students into thinking that addition must come before subtraction and multiplication must come before division.*
- *Students fail to see juxtaposition (side by side) as indicating multiplication. For example, evaluating 3x as 35 when x = 5 instead of 3 times 5 = 15. Also, students may rewrite 8 – 2a as 6a.*
- *Students also miss the understood "1" in front of a lone variable like a* or *x* or *p. For example, not realizing that 4a + a is 5a.*
- *Many of the misconceptions when dealing with expressions stem from the misunderstanding/reading of the expression. For example, knowing the operations that are being referenced with notation like* x^3, *4x, 3(x + 2y) is critical. The fact that* x^3 *means (x)(x)(x) which is x times x times x, not 3x or 3 times x; 4x means 4 times x or x + x + x + x, not forty-something.*

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for middle school students. **Note – Different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**

http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary

http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

http://www.corestandards.org/Math/Content/mathematics-glossary/glossary

- **Algebraic expression**: A mathematical phrase involving at least one variable and sometimes numbers and operation symbols.

- **Associative Property of Addition**: The sum of a set of numbers is the same no matter how the numbers are grouped.
- **Associative Property of Multiplication:** The product of a set of numbers is the same no matter how the numbers are grouped.
- **Coefficient:** A number multiplied by a variable in an algebraic expression.
- **Commutative Property of Addition:** The sum of a group of numbers is the same regardless of the order in which the numbers are arranged
- **Commutative Property of Multiplication:** The product of a group of numbers is the same regardless of the order in which the numbers are arranged.
- **Constant:** A quantity that does not change its value.
- **Distributive Property:** The sum of two addends multiplied by a number is the sum of the product of each addend and the number.
- **Exponent:** The number of times a number or expression (called base) is used as a factor of repeated multiplication. Also called the power.
- **Like Terms:** Terms in an algebraic expression that have the same variable raised to the same power. Only the coefficients of like terms are different.
- **Order of Operations:** The rules to be followed when simplifying expressions.
- **Term:** A number, a variable, or a product of numbers and variables.
- **Variable:** A letter or symbol used to represent a number or quantities that vary

INSTRUCTIONAL RESOURCES/TOOLS

- Algebra Tiles or Alge-blocks (download printable Algebra Tiles here)
- From the National Library of Virtual Manipulatives: Online algebra tiles that can be used to represent expressions and equations.
- Online game **Late Delivery.** In this game, the student helps the mail carrier deliver five letters to houses with numbers such as $3(a + 2)$.

FORMATIVE ASSESSMENT LESSONS (FAL)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Guide.

SPOTLIGHT TASKS

For middle and high schools, each Georgia Standards of Excellence mathematics unit includes at least one Spotlight Task. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Georgia Standards of Excellence, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky.

3-ACT TASKS

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.
More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

TASKS

Task Name	Task Type *Grouping Strategy*	Content Addressed	Standards Addressed
The Best Offer **(Spotlight Task)**	Constructing Task *Individual/Partner Task*	Exponents	MGSE6.EE.1
Exponents	Formative Task *Individual/Partner Task*	Exponents	MGSE6.EE.1
Rules for Exponents	Formative Task *Individual/Partner Task*	Exponent Conventions	MGSE6.EE.1
Conjectures About Properties	Formative Task *Partner/Small Group*	Properties of Numbers	MGSE6.EE.3 MGSE6.NS.4
Visual Patterns **(Spotlight Task)**	Constructing Task *Individual/Partner Task*	Read, write, and evaluate expressions and formulas	MGSE6.EE.2 MGSE6.EE.2a MGSE6.EE.2b MGSE6.EE.2c
Perimeter and Area Expressions **(Spotlight Task)**	Constructing Task *Individual/Partner Task*	Read, write, and evaluate expressions and formulas	MGSE6.EE.2 MGSE6.EE.2a MGSE6.EE.2b MGSE6.EE.2c
The Algebra of Magic Part 1	Constructing Task *Individual/Partner Task*	Read, write, and evaluate expressions and formulas	MGSE6.EE.2 MGSE6.EE.2a MGSE6.EE.2b MGSE6.EE.2c
The Algebra of Magic Part 2	Constructing Task *Individual/Partner Task*	Read, write, and evaluate expressions and formulas	MGSE6.EE.2 MGSE6.EE.2a MGSE6.EE.2b MGSE6.EE.2c
The Algebra of Magic Part 3	Constructing Task *Individual/Partner Task*	Read, write, and evaluate expressions and formulas	MGSE6.EE.2 MGSE6.EE.2a MGSE6.EE.2b MGSE6.EE.2c
Writing Expressions	Formative Task *Individual/Partner Task*	Writing expressions	MGSE6.EE.2a
Writing and Evaluating Expressions	Formative Task *Group Task*	Read, write, and evaluate expressions and formulas	MGSE6.EE.2 MGSE6.EE.2a MGSE6.EE.2b MGSE6.EE.2c
Laws of Arithmetic	Formative Assessment Lesson *Individual/Partner Task*	Expressions	MGSE6.EE.3

Are We Equal?	Formative Task *Individual/Partner Task*	Identify equivalent expressions, Use properties to generate equivalent expressions	MGSE6.EE.3 MGSE6.EE.4 MGSE6.NS.4
Culminating Task: **Sweet Tooth Chocolate Shop**	Summative Performance Task *Individual/Partner Task*	Expressions	MGSE6.EE.1 MGSE6.EE.2 MGSE6.EE.2a MGSE6.EE.2b MGSE6.EE.3 MGSE6.EE.4

The Best Offer (Spotlight Task)

[Back to Task Table]

Task adapted from http://www.illustrativemathematics.org/

In this problem-based task, students will write and evaluate exponents to help determine which monetary offer is worth the greatest amount.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.1 Write and evaluate expressions involving whole-number exponents.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students must make sense of the problem in order to find an entry point to its solution.
2. Reason abstractly and quantitatively. Students will use their ability to contextualize when determining the meaning of the expressions they evaluate.
3. Construct viable arguments and critique the reasoning of others. In both small-group and whole-group discussions, students will need to defend their own approach to the problem, critique the reasoning of others' approaches, and if necessary, will provide useful questions to improve or clarify the arguments of others.
4. Model with mathematics. Students will develop a mathematical model to represent the situation and will interpret their results in the context of the problem.
5. Use appropriate tools strategically. Students should be familiar with tools available to them and make decisions on which tools are most appropriate in order to solve the problem.
7. Look for and make use of structure. Students should see a connection between the exponential and standard forms of the expression and understand the structure of the exponential form in terms of the standard form.
8. Look for and express regularity in repeated reasoning. Students will notice that Option B is doubling in value each day. They should continually compare their results to Option A keeping in mind the context of the problem at hand.

ESSENTIAL QUESTIONS

- How can expressions be used to make real-world decisions?
- How can an expression be written in exponential form?

MATERIALS REQUIRED

- 3-Act Task
- Act 1 video
- Act 2 picture (see picture in Act 2 below.)
- Act 3 video
- 3-Act Task Recording Sheet
- Manipulatives for modeling (two-color counters, centimeter cubes, etc.)

TIME NEEDED

- 1 day

TEACHER NOTES

In this task, students will watch the video, and then tell what they noticed. The objective of Act 1 is to introduce the problem and allow students to begin thinking about a route to its solution. At this point in the task, the teacher should avoid announcing the mathematical objectives. After watching Act 1, students will be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need.

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:
Watch the video of Act 1 http://real.doe.k12.ga.us/vod/gso/math/videos/Act-1-Best-Offer.mp4. Using a think-pair-share, ask students the questions: *What did you notice? What questions do you have?* Record students' questions on the board for everyone to see and, as a class, decide on the main question. Students should write down their initial guess.

Main Question: ***Which offer is the best?***

Important Note: Although the MAIN QUESTION of this lesson is "*Which offer is the best?"* it is important for the teacher to not ignore student-generated questions and try to answer as many of them as possible (before, during, and after the lesson as a follow-up).

ACT 2:
Ask students to think about other information they might need to know in order to answer the main question. Although all necessary information is provided in Act 1, students may not have noticed that the 5 stacks of money in Option A each contain $10,000 for a total of $50,000.

Students may work individually for 5-10 minutes to come up with a plan, test ideas, and problem solve. After individual time, students may work in groups (3-4 students) to collaborate and solve this task together.

ACT 3:
Students should share their predictions with the class and explain how they arrived at their answer. Reveal the answer http://real.doe.k12.ga.us/vod/gso/math/videos/Act-3-Best-Offer.mp4 and allow groups with differing approaches to share out with the whole group if they haven't done so already. This is an opportunity to discuss sources of error and the reliability of our model.

Additionally, Act 3 should include time for:

- Students to revisit initial guess.
- The teacher to revisit questions from Act 1.
- Students to decide on a title for the lesson.

The overall goal of this lesson is for students to write and evaluate expressions using whole-number exponents; however, this task also gives students the opportunity to see how quickly exponential expressions grow. In 5th grade, students were introduced to exponents to denote powers of ten. Along with evaluating $2^{16} = \$65{,}536$ in this task, students should also recognize 2^{16} as a much more efficient way to express $2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2$. If no students write 2^{16} in exponential form during Act 2, make sure this is discussed thoroughly in the summary. Students can use other data from Option B in this task to practice writing exponential expressions before moving on.

Extension: For students who are ready to go farther with this task, here are some possible extension questions to ask:

- How would your answer change if Option B began with \$3 on day 1 instead of \$2?
- How would your answer change if the value of Option B (in the original problem) tripled, rather than doubled each day?
- Assume that Option B stayed the same as in the original task and Option A changed in the following way:

Day	Total Amount of Option A
1	2
2	4
3	6
4	8

  - Which amount is growing faster, Option A or Option B? How do you know?
  - Will Option A ever be greater than Option B? Why or why not?

The Best Offer ______________ Name:________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Make an initial guess and explain why.

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:_______________________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Exponents

Back to Task Table

In this task, students practice writing and evaluating exponential expressions using dice.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.1 Write and evaluate expressions involving whole-number exponents.

STANDARDS FOR MATHEMATICAL PRACTICE

3. Construct viable arguments and critique the reasoning of others. Students construct and critique arguments regarding the equivalence of exponential expressions.
6. Attend to precision. Students use the mathematical language with relation to exponents.
7. Look for and make use of structure. Students use a table to organize and compare base, exponent, exponential forms, standard form, and value.

ESSENTIAL QUESTIONS

- How are "standard form" and "exponential form" related?
- What is the purpose of an exponent?

INTRODUCTION

An exponent is simply shorthand for repeated multiplication of a number times itself (e.g., $2^5 = 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2$). This is a rule with no conceptual basis. It is simply an agreed upon convention. This lesson is intended to follow the Spotlight Task *The Best Offer* and serves as practice for students in writing and evaluating exponential expressions.

MATERIALS REQUIRED

One red and one green dice for each pair of students.

TIME NEEDED

1 day

You and a partner need a red and a green die. The red die will be the base and the green die will be the exponent. Take turns rolling the dice. Record the base and the exponent and then write the exponential expression. After you have recorded 10 numbers, find the standard form of each exponential expression and calculate the value. Race your partner and see who finishes first.

Base (red die)	**Exponent (green die)**	**Exponential Form**	**Standard Form**	**Value**

SUMMARY

Have students share selected responses and insights into the use of exponents. You are watching here for the correct use of the base and the exponent.

What was the largest number each team generated?

Did anyone have to solve 6^6?

Which makes an exponential expression grow faster: a large base or a large exponent?

How does 3^2 compare to 3 x 2?

Teacher Notes

It is important to compare 3^2 and 3×2 since the most common mistake that students make with exponential expressions is to multiply the base and exponent together.

Solutions

Solutions will vary.

Name___________________________

Exponents

You and a partner need a red and a green die. The red die will be the base and the green die will be the exponent. Take turns rolling the dice. Record the base and the exponent and then write the exponential expression. After you have recorded 10 numbers, find the standard form of each exponential expression and calculate the value. Race your partner and see who finishes first.

Base (red die)	Exponent (green die)	Exponential Form	Standard Form	Value

Rules for Exponents

Back to Task Table

In this task, students will use the order of operations to evaluate exponential expressions for equivalence. Students will also discover the necessity for the order of operations.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.1 Write and evaluate expressions involving whole-number exponents.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of expressions using grouping symbols to rewrite expressions to obtain a different answer.
3. Construct viable arguments and critique the reasoning of others. Students explain how the order of operations is useful in solving real-world problems.
5. Use appropriate tools strategically. Students determine which algebraic representations are appropriate for given contexts.
6. Attend to precision. Students use the correct order of operations to evaluate expressions.
8. Look for and express regularity in repeated reasoning. Students will notice patterns in solutions for expressions based on the order in which operations are presented.

ESSENTIAL QUESTIONS

- How are exponents used when evaluating expressions?
- How is order of operations used to evaluate expressions?

INTRODUCTION

There are two conventions of symbolism that must be learned. The first is that an exponent applies to its immediate base. For example, in the expression $3 + 4^2$ the exponent 2 applies only to the 4 so the expression is equal to $3 + 4 \times 4$. If we write $(3 + 4)^2$, the 2 is an exponent of the quantity $3 + 4$ and is evaluated $(3 + 4) \times (3 + 4)$ or 7×7. Likewise, in the quantity $3x^2$, only the x is squared ($3 \bullet x \bullet x$).

The other convention involves the order of operations: Multiplication and division are always done before addition and subtraction within the same grouping symbols. Since exponentiation is repeated multiplication, it is also done before addition and subtraction within the same grouping symbols. **NOTE - Multiplication does not always come before division. Division and multiplication are worked left to right. Also, addition does not always come before subtraction. Subtraction and addition and are worked left to right, whichever comes first.**

MATERIALS REQUIRED

- Basic (four-function) calculators
- Scientific calculators
- Student sheet

TIME NEEDED

- 1 day

BEFORE THE LESSON

Have students work in pairs to find the solutions to the problems in the table below. It is important to allow one student to use a basic four-function calculator and one student to use a scientific calculator. After completing the table, have students circle the rows that yield the same solution in both calculators and ask them to conjecture about why this happens. This is a great context for discussing why an order of operations is important.

Expression	Basic Calculator Solution	Scientific Calculator Solution
$3 + 4 \times 5$	*35*	*23*
$3 \times 4 + 5$	*17*	*17*
$3 + 5 \times 4$	*32*	*23*
$3 \times 5 + 4$	*19*	*19*
$4 + 3 \times 5$	*35*	*19*
$4 \times 3 + 5$	*17*	*17*
$4 + 5 \times 3$	*27*	*19*
$4 \times 5 + 3$	*23*	*23*
$5 + 3 \times 4$	*32*	*17*
$5 \times 3 + 4$	*19*	*19*
$5 + 4 \times 3$	*27*	*17*
$5 \times 4 + 3$	*23*	*23*

Ask: "What do you notice?"
Students should recognize that the solutions are the same when multiplication comes first in the expression and that the solutions are different when addition precedes multiplication.

Now, present the following problem:

James and Alexis used calculators to simplify the expression $5 + 4^2 - 6 \div 3$. James' calculator showed 10, while Alexis' calculator showed 19. Whose calculator is correct?

Have students work independently and then ask for each of the steps:

Teacher note: If you have some students evaluate this expression with a four-function calculator, and others with a scientific calculator, they will arrive at different solutions because the four-function calculator does not apply order of operations (=10) and the scientific calculator does (=19). James probably had a four-function calculator while Alexis likely had a scientific calculator. This leads to discovery of the need for the convention of order of operations.

Solution
4^2 and $6 \div 3$ must be evaluated first to give us the new expression 5 + 16 – 2.
Then, we do addition and subtraction in the order they appear from left to right
21 – 2 = 19.

If I rewrite the expression in this way $(5 + 4)^2 - 6 \div 3$ will we get a different answer?

EXPLORATION

With a partner determine if the following expressions are equivalent

a. $2^2 \cdot 3^2 - 2^3 - 1$
b. $2^2 \cdot (3^2 - 2^3) - 1$
c. $(2 \cdot 3)^2 - 2^3 - 1$

1. Write an expression of your own using all the operations as well as exponents.
2. Rewrite the expression using grouping symbols to give a different answer.
3. Explain the Order of Operations and how it is useful in solving mathematical and real world problems.

SUMMARY

Have students share solutions and the expressions that they have written.

Solutions
The first expression is equal to 27, the second expression is equal to 3, the third expression is equal to 27, so the second expression is not equivalent to either of the other two expressions (but the first and third expressions are equivalent.)

Solutions will vary on the second part of the exploration. Make sure that the order of operations is correct. One way to do this is to have students swap created problems with another pair and check the work.

Name____________________________________

Rules For Exponents

Expression	Basic Calculator Solution	Scientific Calculator Solution
$3 + 4 \times 5$		
$3 \times 4 + 5$		
$3 + 5 \times 4$		
$3 \times 5 + 4$		
$4 + 3 \times 5$		
$4 \times 3 + 5$		
$4 + 5 \times 3$		
$4 \times 5 + 3$		
$5 + 3 \times 4$		
$5 \times 3 + 4$		
$5 + 4 \times 3$		
$5 \times 4 + 3$		

What do you notice?

1. James and Alexis used calculators to simplify the expression, $5 + 4^2 - 6 \div 3$. James' calculator showed 10, while Alexis' calculator showed 19. Whose calculator is correct?

2. With a partner determine if the following expressions are equivalent

   a. $2^2 \cdot 3^2 - 2^3 - 1$

   b. $2^2 \cdot (3^2 - 2^3) - 1$

   c. $(2 \cdot 3)^2 - 2^3 - 1$

3. Write an expression of your own using all the operations as well as exponents.

4. Rewrite the expression using grouping symbols to give a different answer.

5. Explain the Order of Operations and how it is useful in solving mathematical and real world problems.

Conjectures About Properties

Back to Task Table

In this task, students will make generalizations about sets of equations based on properties of operations.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

MGSE6.NS.4 Find the common multiples of two whole numbers less than or equal to 12 and the common factors of two whole numbers less than or equal to 100.

a. Find the greatest common factor of 2 whole numbers and use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factors. (GCF) *Example: 36 + 8 = 4(9 + 2)*
b. Apply the least common multiple of two whole numbers less than or equal to 12 to solve real-world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of properties by identifying a rule that matches a group of equations.
3. Construct viable arguments and critique the reasoning of others. Students construct explanations about properties.
6. Attend to precision. Students use the language of the Commutative, Associative, Identity, Zero Property of Multiplication, and Distributive properties.
7. Look for and make use of structure. Students apply properties to generate equivalent expressions.
8. Look for and express regularity in repeated reasoning. Students will identify properties and reason why that group is identified with a certain property.

ESSENTIAL QUESTIONS

- How can I tell if a group of equations satisfies a property? (Commutative, Associative, Identity, Zero Property of Multiplication, and Distributive)
- How are properties of numbers helpful in computation?

INTRODUCTION

Numerical expressions are meaningful combinations of numbers and operation signs. A *variable* is a letter or other symbol that is a placeholder for an unknown number or a quantity that varies. An expression that contains at least one variable is called an *algebraic expression*.

When each variable in an algebraic expression is replaced by a number, the result is a numerical expression whose value can be calculated. This process is called *evaluating the algebraic expression*.

In this task, the idea of substituting variables to represent numbers is introduced in the context of making conjectures about number properties.

TIME NEEDED

2 days

BEFORE THE LESSON

Post the following expression on the board:

36 + 45 = 45 + 36
Ask students: Is it true or false?

123 + 24 = 24 + 123
Is it true or false?

4 + 6 = 6 + 4
Is it true or false?

Looking at these three equations, what do you notice?

Do you think this is true for all numbers?

Can you state this idea without using numbers?

By using variables, we can make an equation that shows your observation that "order does not matter when we add numbers."

$a + b = b + a$

Do you think this is also true for subtraction?

Do you think it is true for multiplication?

Do you think it is true for division?

Teacher Notes

The full class should discuss the various conjectures, asking for clarity or challenging conjectures with counterexamples. Conjectures can be added to the class word wall with the formal name for the property as well as written in words and in symbols.

Students are almost certainly not going to know or understand why division by zero is not possible. You will need to provide contexts for them to make sense of this property. Some children are simply told, "Division by zero is not allowed," often when teachers do not fully understand this concept. To avoid an arbitrary rule, pose problems to be modeled that involve zero: "Take thirty counters. How many sets of zero can be made?" or "Put twelve blocks in zero equal groups. How many are in each group?"

EXPLORATION

With a partner, look at the following sets of equations and determine if what you observe would be true for all numbers. Create statements with words about what you observe in each set of equations then write the equations using variables to represent numbers.

Property articulations by students may vary. They do not have to be exact language but mathematical ideas must be sound and variable representations precise.

$12 + 0 = 12$ $37 + 0 = 37$ $64 + 0 = 64$	$12 - 0 = 12$ $37 - 0 = 37$ $64 - 0 = 64$	$12 \cdot 1 = 12$ $37 \cdot 1 = 37$ $64 \cdot 1 = 64$
Identity Property of Addition $a + 0 = a$	***Identity Property of Subtraction*** $a - 0 = a$	***Identity Property of Multiplication*** $a \times 1 = a$
$12 \div 1 = 12$ $37 \div 1 = 37$ $64 \div 1 = 64$	$12 \cdot 0 = 0$ $37 \cdot 0 = 0$ $64 \cdot 0 = 0$	$12 \div 0 = 12$ $45 \div 0 = 45$ $64 \div 0 = 64$
Identity Property of Division $a \div 1 = a$ or $\frac{a}{1} = a$	***Zero Property of Multiplication*** $a \text{ x } 0 = 0$	***Undefined – See Summary Notes***

$12(4 + 3) = 48 + 36$

$6(7 + 2) = 42 + 12$

$4(10 + 3) = 40 + 12$

Distributive Property
a(b + c) = ab + ac
(This will need to be revisited during integer exploration.)

$12(4 - 3) = 48 - 36$

$6(7 - 2) = 42 - 12$

$4(10 - 3) = 40 - 12$

Distributive Property
a(b−c) = ab = ac

$4 \cdot 8 = (2 \cdot 8) + (2 \cdot 8)$

$8 \cdot 16 = (4 \cdot 16) + (4 \cdot 16)$

$5 \cdot 14 = (2.5 \cdot 14) + (2.5 \cdot 14)$

Distributive Property
$$\boldsymbol{a \cdot b = \left(\frac{a}{2} \cdot b\right) + \left(\frac{a}{2} \cdot b\right)}$$

$4 + 8 = (2 + 8) + (2 + 8)$

$8 + 16 = (4 + 16) + (4 + 16)$

$5 + 14 = (2.5 + 14) + (2.5 + 14)$

False

$(32 + 24) + 16 = 32 + (24 + 16)$

$(450 + 125) + 75 = 450 + (125 + 75)$

$(33 + 17) + 3 = 33 + (17 + 3)$

Associative Property
(a + b) + c = a + (b + c)

$6 \cdot (4 \cdot 3) = (6 \cdot 4) \cdot 3$

$10 \cdot (5 \cdot 2) = (10 \cdot 5) \cdot 2$

$(11 \cdot 2) \cdot 3 = 11 \cdot (2 \cdot 3)$

Associative Property
a (b × c)= (a × b) × c

The Commutative Property
Changing the order of the values you are adding or multiplying does not change the sum or product.
6 + 4 = 4 + 6 ***a • b = b • a***

The Associative Property
Changing the grouping of the values you are adding or multiplying does not change the sum or product.
(2 + 7) + 3 = 2 + (7 + 3) ***(ab)c = a(bc)***

The Identity Property
The sum of any number and zero is the original number (additive identity). The product of any number and 1 is the original number (multiplicative identity).

The Distributive Property of Multiplication over Addition and Subtraction helps to evaluate expressions that have a number multiplying a sum or a difference.
Example:

$9(4 + 5) = 9(4) + 9(5)$	$a(b + c) = ab + ac$
$5(8 - 2) = 5(8) - 5(2)$	$a(b - c) = ab - ac$

Name______________________

Conjectures About Properties

With a partner look at the following sets of equations and determine if what you observe would be true for all numbers. Create statements with words about what you observe in each set of equations then write the equations using variables to represent numbers.

$12 + 0 = 12$ $37 + 0 = 37$ $64 + 0 = 64$	$12 - 0 = 12$ $37 - 0 = 37$ $64 - 0 = 64$	$12 \cdot 1 = 12$ $37 \cdot 1 = 37$ $64 \cdot 1 = 64$
$12 \div 1 = 12$ $37 \div 1 = 37$ $64 \div 1 = 64$	$12 \cdot 0 = 0$ $37 \cdot 0 = 0$ $64 \cdot 0 = 0$	$12 \div 0 = 12$ $45 \div 0 = 45$ $64 \div 0 = 64$

$12(4 + 3) = 48 + 36$ $6(7 + 2) = 42 + 12$ $4(10 + 3) = 40 + 12$	$12(4 - 3) = 48 - 36$ $6(7 - 2) = 42 - 12$ $4(10 - 3) = 40 - 12$

$4 \cdot 8 = (2 \cdot 8) + (2 \cdot 8)$

$8 \cdot 16 = (4 \cdot 16) + (4 \cdot 16)$

$5 \cdot 14 = (2.5 \cdot 14) + (2.5 \cdot 14)$

$4 + 8 = (2 + 8) + (2 + 8)$

$8 + 16 = (4 + 16) + (4 + 16)$

$5 + 14 = (2.5 + 14) + (2.5 + 14)$

$(32 + 24) + 16 = 32 + (24 + 16)$

$(450 + 125) + 75 = 450 + (125 + 75)$

$(33 + 17) + 3 = 33 + (17 + 3)$

$6 \cdot (4 \cdot 3) = (6 \cdot 4) \cdot 3$

$10 \cdot (5 \cdot 2) = (10 \cdot 5) \cdot 2$

$(11 \cdot 2) \cdot 3 = 11 \cdot (2 \cdot 3)$

Visual Patterns (Spotlight Task)

Back to Task Table

Source: http://www.visualpatterns.org/

In this task, students will reason about visual and spatial patterns by modeling, extending, expressing, and explaining the patterns in various ways.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.1 Write and evaluate expressions involving whole-number exponents.

MGSE6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

MGSE6.EE.2a Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5-y.*

MGSE6.EE.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.

MGSE6.EE.2c Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$.

MGSE6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

MGSE6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them.) *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will make sense of the problems by building, extending, creating and explaining visual patterns.
2. Reason abstractly and quantitatively. Students will reason with quantities of objects in the patterns, and then create abstract generalizations to write expressions and equations that explain the patterns.
3. Construct viable arguments and critique the reasoning of others. Students will share their expressions/equations with other groups and students and discuss the validity of each. Students will likely discover equivalent expressions/equations within the class.
4. Model with mathematics. Students will model the patterns they are studying using materials, diagrams and tables as well as equations.
5. Use appropriate tools strategically. Students will choose appropriate tools to solve the visual patterns.
6. Attend to precision. Students will attend to precision through their use of the language of mathematics as well as their computations.
7. Look for and make use of structure. Students will apply properties to generate equivalent expressions. They interpret the structure of an expression in terms of a context. Students will identify a "term" in an expression.
8. Look for and express regularity in repeated reasoning. The repeated reasoning required to explain patterns is evident in this lesson. Students will express this regularity through their conversations with one another and the class.

ESSENTIAL QUESTIONS

- What strategies can I use to help me understand and represent real situations using algebraic expressions?
- How are the properties (Identify, Associative and Commutative) used to evaluate, simplify and expand expressions?
- How is the Distributive Property used to evaluate, simplify and expand expressions?
- How can I tell if two expressions are equivalent?

MATERIALS REQUIRED

- Various manipulatives such as two color counters, color tiles, connecting cubes, etc.
- Visual Patterns Handout (attached)

TIME NEEDED

Initially, 2 days. Several days should be set aside for students to investigate these types of patterns. Students should be given multiple chances to engage in these investigations with different visual patterns.

TEACHER NOTES

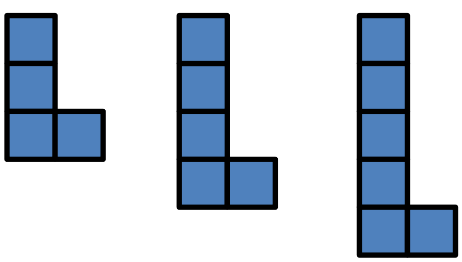
The use of visual patterns to reify algebraic relationships and teach students to think algebraically is not uncommon. Unfortunately, students are often directed to make a table to find a pattern, rather than building strong figural reasoning. This often oversimplifies the task meaning that students merely count to complete a table and ignore the visual model. **This creates iterative thinking such as (I just need to add 5 each time), guessing and checking, or the application of rote procedures, rather than an understanding of the structural relationships within the model.**

This task was created to introduce teachers and students to a resource filled with visual patterns. Teachers can assign 2 or 3 patterns per pair of students, initially. As students become more confident in their ability, visual patterns can be assigned individually.

Fawn Nguyen, a middle school math teacher in California, has put together a library of visual patterns here. In the teacher section on this site is a helpful tool for assigning random visual patterns to students as well as a form for students to use to help them organize their thinking.

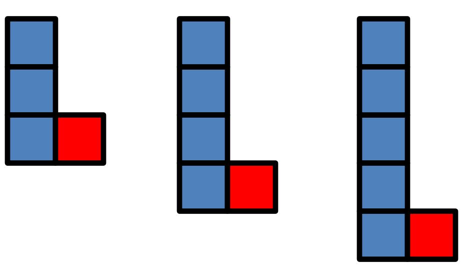
Students need multiple experiences working with and explaining patterns. Giving students a visual pattern to build concretely, allows students to experience the growth of the pattern and explain it based on that experience.

As students begin investigating visual patterns, ask them to look at how it grows from one stage (or step) to the next. When they build the pattern using manipulatives, have them use one color to begin with. Does any part of the pattern seem to be staying the same? **Whatever students "see" as staying the same have them replace those pieces with a different color.** For example:



Step 1 Step 2 Step 3

Students may be investigating the visual pattern on the left. Initially, students will build the pattern with materials such as color tiles and use all blue (for example) tiles to build it.



Step 1 Step 2 Step 3

Next, after students look for parts that stay the same as the pattern grows, they use a different color tile to show what parts stay the same. These students thought that the "part that sticks out on the right" stay the same. Another group may say that "the bottom two of each step stay the same. Either way, students are encouraged to go with it

Once students have identified the part that stays the same (the constant) and the part that changes (the variable), students can begin to organize their thinking. One way to do this is through the use of an expanded t-table. This provides a place to organize all parts and the whole of the visual pattern based on the how students see it growing.

One version of this kind of table can be seen below. It's important that students determine what information is important to keep up with in the table.

Step (Stage)	Sketch	Stays Same	Changes	Total (tiles in this case)
1	This column allows students to keep a record of their thinking. If they run out of time, they can pick up where they left off later without having to start from scratch.	1	3	4
2		1	4	5
3		1	5	6
4		1	6	7
5		1	7	8
13		1	15	16
37		1	39	40
n		1	n +2	n + 2 + 1

As students reason about the visual patterns, be sure to ask questions to check their understandings and address misconceptions.

To help students move away from iterative reasoning (I just need to add 5 each time) to explicit reasoning, it's best to ask students to build the next two steps or stages (4 and 5 in the table above), then skip some. Choose a number that is not a multiple of 1 through 5, but fairly close. Some students will continue with their iterative reasoning to fill in the table for this stage. Next, choose a larger number, again not a multiple of any of the previous stage numbers. This gives students a subtle nudge to begin thinking about finding relationships within the data that has already been collected. When students determine a rule, they should check to see that it works for all of the stages. This is important because some rules or expressions may work for 2 or three stages, but not the rest. This is an act of being precise with the mathematics (SMP#6).

Students should also represent the patterns in other ways, such as on a coordinate grid. This gives students a preview of mathematics to come later on. It's also a nice way for students to attach the equations they create to a series of points on a line that represent the growth of the pattern. **IMPORTANT NOTE:** When students plot the points on the coordinate grid, they should not connect the points. In this context, it does not make sense to connect the points, since we can't have a fraction of a stage or a fraction of a square tile.

During the closing of the lesson, students should share their expressions/equations using precise mathematical language. Pairs of students who investigate similar patterns should discuss the expressions/equations they create – especially if they look different.

For example – with the "L" shaped pattern mentioned above, depending on what students "see" as staying the same, the following expressions or equations may be derived:

- $1 + n + 2$
- $3 + n$
- $4 + n – 1$

Students should determine whether or not these expressions work for the pattern, then determine why they all work, since some are very different looking.

DIFFERENTIATION

Extension:

Students should be encouraged to create their own growth patterns. This will allow for student creativity. Students should not only create the pattern, but also find the expression/equation that explains it with several examples as proof that their equation is true. Also, some patterns are much more challenging than others. Using these patterns can provide students the challenge they need. Finally, looking at the quantity of squares in the pattern is only one possible pattern to explore. Another option would be to look at perimeter for the same pattern. Are the patterns for these two different ideas similar or very different? Why? This gets even more interesting when the visual patterns are three dimensional!

Intervention:

Students needing support can be given a graphic organizer like the one above. Using this can help students make sense of the patterns they are investigating based on how they see the pattern growing. Also, some patterns are easier to explain than others. Patterns using different shapes (such as pattern blocks) can be helpful to students needing support since the differing shapes can help them focus on what is changing and staying the same.

For more visual patterns to try with your students, go to the source: *www.visualpatterns.org*

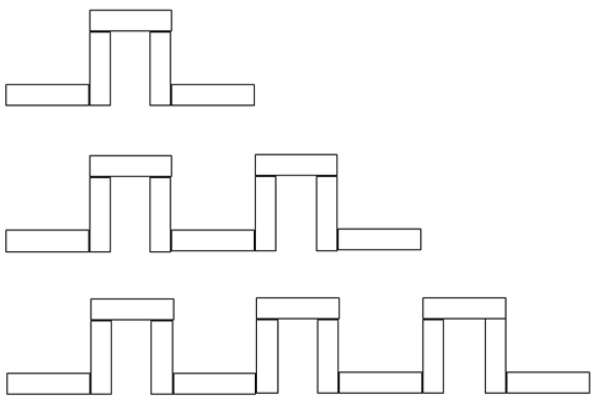
<u>Visual Patterns</u> (Spotlight Task)

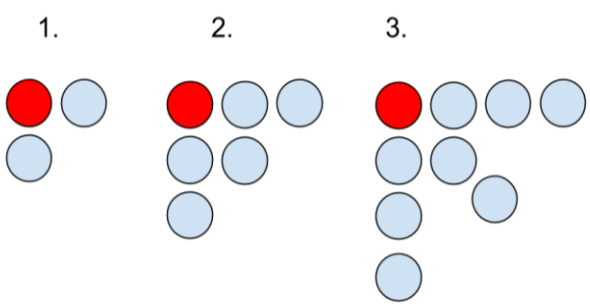
Look at the visual patterns below. Choose any two to investigate.

Write an algebraic expression to explain the pattern.

Using your expression, write an equation for the total: t = _____________

Graph your visual pattern on a coordinate grid.





Perimeter and Area Expressions (Spotlight Task)

Back to Task Table

Adapted from http://nrich.maths.org

In this task, students will use algebraic reasoning to express the perimeter and area of a rectilinear figure.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

MGSE6.EE.2a Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as* 5 – y.

MGSE6.EE.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression* 2(8 + 7) *as a product of two factors; view* (8 + 7) *as both a single entity and a sum of two terms.*

MGSE6.EE.2c Evaluate expressions at specific values for their variables. Include expressions that arise from formulas in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $\boldsymbol{V = s^3}$ *and* $\boldsymbol{A = 6s^2}$ *to find the volume and surface area of a cube with sides of length* ***s* = ½.**

MGSE6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression* ***3(2 + x)*** *to produce the equivalent expression* ***6 + 3x****; apply the distributive property to the expression* ***24x + 18y*** *to produce the equivalent expression* ***6(4x + 3y)****; apply properties of operations to* ***y + y + y*** *to produce the equivalent expression* ***3y****.*

MGSE6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions* ***y + y + y*** *and* ***3y*** *are equivalent because they name the same number regardless of which number* ***y*** *stands for.* Reason about and solve one-variable equations and inequalities.

MGSE6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will make sense of the problem using rectangles to write and evaluate expressions.

2. Reason abstractly and quantitatively. Students may be asked to reason through this task quantitatively before assigning variables to represent lengths on the smallest rectangle.

3. Construct viable arguments and critique the reasoning of others. Students will share expressions and models with partners and discuss their reasonableness.
4. Model with mathematics. Students will use rectangles to build shapes and write algebraic expressions to represent the area and perimeters for these shapes.
6. Attend to precision. Students will attend to precision through the use of the language of mathematics in their discussions as well as in the expressions they write.
7. Look for and make use of structure. Students will use their rectangles to describe how the rectangular shapes are related and write expressions showing this understanding.

ESSENTIAL QUESTIONS

- How are properties of numbers helpful in evaluating expressions?
- What strategies can I use to help me understand and represent real situations using algebraic expressions?

MATERIALS REQUIRED

- 1 sheet of paper per student
- Task directions (attached)

TIME NEEDED

- 2 to 3 days

TEACHER NOTES

This problem gives students plenty of opportunity to practice manipulating algebraic expressions within a purposeful context. Along the way, the challenges will provoke some insights that will be worth sharing. Issues relating to the dimensions in formulas for areas and lengths may emerge.

One way to introduce the lesson would be to hand out sheets of paper (any size will work) and introduce the problem, giving students time to cut out their five rectangles.

Show the image of the shape that the student from last year made and ask students to work out the perimeter in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$. If students are not confident enough in their algebra to express area in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$, the area of the smallest rectangle could be designated $\boldsymbol{R}$ and used as a unit to express other areas.

Once everyone is happy that the area is 9R or $9\boldsymbol{ab}$ and the perimeter is $10\boldsymbol{a} + 4\boldsymbol{b}$, ask them to combine the largest and smallest rectangles (edge to edge, corner to corner) to make other shapes, and work out the areas and perimeters. How many different perimeters can they find?

When students have finished investigating this, discuss. Are they surprised to only find one other possible perimeter, and that the two answers could be reached in several different ways?

The next challenge is to make different shapes using at least two of the rectangles, and again work out the area and perimeter. Students could make a shape in pairs and work out the area and perimeter to give to another pair (without letting them see the shape), to see if they can create a shape with the same area and perimeter. There is opportunity for fruitful discussion when the two shapes are compared, to see what is similar and what is different, and to see if one can be transformed into the other.

Alternatively, collect together the areas and perimeters on the board. Then, in pairs, challenge students to combine rectangles to make each of the area/perimeter combinations listed on the board, checking each other's work as they go along.

Finally, below are some "questions to consider." These can provide some interesting points for discussion. Students could be invited to suggest questions they think mathematicians might ask themselves about this task. The list below may be used to prompt ideas if necessary.

Questions to consider:

- What's the largest perimeter you can make using ALL the pieces?
- Can you make two different shapes which have the same area **and** perimeter as each other?
- Can you make two different shapes which have the same perimeter but different areas?
- How do you combine any set of rectangles to create the largest possible perimeter?
- A student thinks he has found a shape with the perimeter $7a + 4b$. Can you find his shape?
- What can you say about the perimeters it is possible to make, if a and b are the dimensions of one of the other rectangles?

Formative Assessment Questions:

- How do we know whether an expression represents an area rather than a perimeter?
- What does the area expression tell us about the pieces used to make the shape?
- Are there multiple ways to make a given area/perimeter combination?

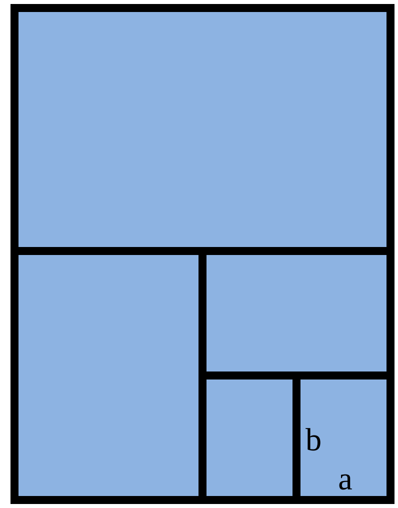
Extension:

Number Pyramids from http://nrich.maths.org builds on this activity by challenging students to create algebraic expressions to explain a relationship.

Intervention:

For students who need support, use numerical values, instead of $\boldsymbol{a}$ and $\boldsymbol{b}$, and cut the rectangles out from squared paper. Begin by looking for numerical relationships rather than algebraic ones, and perhaps introduce the algebra to explain the patterns students find.

Perimeter and Area Expressions (Spotlight Task)



Last year, when we were working with area, one of my students took a sheet of paper and cut it in half. Then she cut one of those pieces in half, and repeated until she had five pieces altogether.

She labeled the sides of the smallest rectangle, ***a*** for the shorter side and ***b*** for the longer side.

Here is a shape that she made by combining the largest and smallest rectangles:

She decided that the perimeter of her shape was $10a + 4b$. Do you agree? Show how you came to that conclusion.

Her partner combined the largest and smallest rectangles in a different way. According to him, the shape that he made had perimeter $8a + 6b$. Is this possible? How might he have put the shapes together?

These two partners made sure their rectangles always met along an edge, with vertices touching. Can you combine the largest and smallest rectangles in this way to create other perimeters?

Create some other shapes by combining two or more rectangles, making sure they meet edge to edge and corner to corner. What can you say about the areas and perimeters of the shapes you can make?

With your partner:

1. *Each person creates a shape using two or more rectangles.*
2. *Work out the area and perimeter for your shape.*
3. *Tell your partner the area and perimeter of your shape without showing them the shape.*
4. *Try to recreate your partner's secret shape using only the area and perimeter. Can this be done? Show your mathematical thinking!*

The Algebra of Magic Part 1

Back to Task Table

In this inquiry-based task, students will decode a "number trick" using algebraic reasoning.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

MGSE6.EE.2a Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as* 5 – y.

MGSE6.EE.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression* 2(8 + 7) *as a product of two factors; view* (8 + 7) *as both a single entity and a sum of two terms.*

MGSE6.EE.2c Evaluate expressions at specific values for their variables. Include expressions that arise from formulas in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $\boldsymbol{V = s^3}$ *and* $\boldsymbol{A = 6s^2}$ *to find the volume and surface area of a cube with sides of length* ***s* = ½.**

MGSE6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression **3(2 + x)** to produce the equivalent expression **6 + 3x**; apply the distributive property to the expression **24x + 18y** to produce the equivalent expression **6(4x + 3y)**; apply properties of operations to **y + y + y** to produce the equivalent expression **3y**.*

MGSE6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions **y + y + y** and **3y** are equivalent because they name the same number regardless of which number **y** stands for.* Reason about and solve one-variable equations and inequalities.

MGSE6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will be making sense of "magic "tricks"" involving algebraic expressions as well as creating their own.
2. Reason abstractly and quantitatively. Students will be reasoning through each of the ""tricks"" as they determine how each "trick" works. Students will need to reason with quantities of "stuff" initially before generalizing an abstract rule or expression that represents all of the steps involved.

3. Construct viable arguments and critique the reasoning of others. Students will create expressions and defend these expressions with their peers. Students may discover equivalent expressions and end up proving their equivalence.
4. Model with mathematics. Students will use models to represent what happens in each "trick" in order to understand the "trick", undo the "trick", invent new ""tricks"", and realize the value of representation.
6. Attend to precision. Students will attend to precision through their use of the language of mathematics as well as in their use of operations.
7. Look for and make use of structure. Students will show an understanding of how numbers and variables can be put together as parts and wholes using representations of operations and properties.

ESSENTIAL QUESTIONS

- How are the properties (Identify, Associative and Commutative) used to evaluate, simplify and expand expressions?
- How is the Distributive Property used to evaluate, simplify and expand expressions?
- How can I tell if two expressions are equivalent?

MATERIALS REQUIRED

- Computer and projector or student devices (BYOT) (optional) *Note: this computer program will not work on Apple mobile devices due to Flash player.*
- Directions for mathematical magic ""tricks"" (attached)
- Counters
- Sticky notes or blank pieces of paper-all the same size (several per student)

TIME NEEDED

- 1 or 2 days

TEACHER NOTES

In part one of this series of tasks, students will interact with a computer program that can "read minds." Then, students will tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics, specifically algebraic reasoning, to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them.

The goals of this task are for students to:

- develop an understanding of linear expressions and equations in a context;
- make simple conjectures and generalizations;
- add expressions, "collect" like terms;
- use the distributive law of multiplication over addition in simple situations;
- develop an awareness that algebra may be used to prove generalizations in number situations.

> Note: The context of magic "“tricks”" can often turn students off since it may lead students to imply that math is just a bunch of "tricks". Presenting this with a video of a middle school student performing the "trick" (see links below) for someone else helps to dispel this myth. When students see another student perform this "trick", they are more apt to ask "How did they do that?" or, better yet, "Why does that work?" This is what we should all strive for in our lessons. **Students asking how to do something or wondering why something works means we've evoked curiosity and wonder.**

While you can perform the "tricks" yourself, please be mindful of the fact that most people don't like to be fooled and middle school students can be intimidated by this especially in front of their peers. Another option would be to teach a "trick" to a student in one of your classes ahead of time. They will know the "trick" (how it works), but they will probably not know why. They can still investigate this.

After each "trick" is presented, either through the video or in another manner, students can go the link posted for each "trick" and try different numbers and look for patterns. The link is not necessary, but it does take the "trick" out of the picture and allows students to investigate without being intimidated by being "tricked".

Explain to students that, for each "trick" in this task, they should:

- investigate the "trick", trying different numbers;
- work out how the "trick" is done. This usually involves spotting a connection between a starting number and a finishing number. Algebra will be helpful here: representing the unknown number (the number that is thought of) with something that can contain a quantity may be helpful at first;
- improve the "trick" in some way.

<u>DIRECTIONS</u>

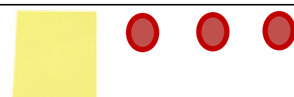
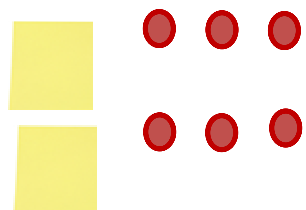
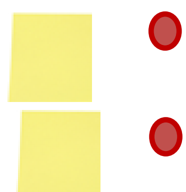
"Trick" 1: A Math-ic Prediction
This 3-act task can be found at: http://mikewiernicki3act.wordpress.com/a-math-ic-prediction/

Students choose any number and follow the steps given by the teacher or the interactive app (link is at the end of this description). The prediction of "1" is revealed after the steps are followed.
Steps to follow for this "trick":

Think of a number.
Add 3.
Double the result.
Subtract 4.
Divide the result by 2.
Subtract your original number.

The prediction for this “trick” is the same every time: 1.
Students may wish to try larger or even smaller numbers, integers, fractions or decimals to see if any numbers won’t work. This can be a nice way to check their understandings and fluency with operations using rational numbers.

An example:

Think of a number.		X
Add 3.		$x + 3$
Double the result.		$2(x + 3)$ *or* $2x + 6$
Subtract 4.		$2(x + 1)$ *or* $2x + 2$
Divide by 2.		$x + 1$
Subtract your original number.		1

The post-its work well because as students begin to visualize what is happening, they can easily write a variable on the post-it. The post-it becomes a variable and can be written in an expression as seen in the right hand column.

NOTE: Students should not be shown the table above. Students should make sense of the mathematics involved in this prediction, creating their own representations and expressions.

A Math-ic Prediction interactive app: http://scratch.mit.edu/projects/21248672/

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:
Open the link above and have the program perform for students. Alternatively, perform this "trick" for students. Ask students what they noticed and what they wonder (are curious about). Record student responses. Have students hypothesize how the "trick" works. How can it come out to be 1 for any number chosen?

ACT 2:
Students work on determining how the "trick" works based on their hypothesis. They should be guided to show what is happening in the "trick" first through the use of some model that can be represented in a diagram, then later written as an expression. Students may ask for information such as what were the steps in the "trick". When they ask, give them the steps:

Think of a number.
Add 3.
Double the result.
Subtract 4.
Divide the result by 2.
Subtract your original number.

Students may also ask for materials to use to model what is happening – these can be suggested, carefully, by the teacher.

ACT 3
Students will compare and share solution strategies.

- Share student solution paths. Start with most common strategy.
- Students should explain their thinking about the mathematics in the "trick".
- Ask students to hypothesize again about whether any number would work – like fractions or decimals. Have them work to figure it out.
- Revisit any initial student questions that weren't answered.

Students can be given practice after showing understanding, with a table similar to the following:

Words	Pictures	Diagrams	Hailee	Connor	Lura	Maury
Think of a number			7			
Double it				22		
Add 8					16	
Divide by 2						7

You have many options for stretching your students' understanding of number relationships. One option might be to have students figure out a peer's starting number after only providing an intermediate clue, such as "Connor has a value of 22 after the second step – what was his starting number?" or "Lura has a value of 16 after the third step – how might you determine her starting number?"

Another option here is to fill in a table like the one above (with the help of your students). After completing the table, simply ask "What do you notice?" Students might notice, for example, that the value after the 2nd step is an even number in each case. Encourage students to conjecture whether or not this will always be true and to explore such conjectures by trying to find more examples and possible counterexamples.

Extension:
To extend this, students could find a way to give this "trick" more of a "wow" factor or to make it more impressive. Students could also develop their own "trick" with representations and algebraic expressions that explain it. Finally, students should be encouraged to develop (code) their own computer program for a "trick" like this. The free online coding program used for the "tricks" in this task and others to come can be found at www.scratch.mit.edu.

Intervention:
Students needing support might be given simpler "tricks" at first, building up to the "trick" presented above. A simple "trick" might be:

- Start with 2.
- Think of a number and add it to the 2.
- Add 4.
- Subtract your original number. *(the result is always 6 for this "trick")*

Work with students on making sense of this and build the "tricks" up to the "trick" presented above.

The Algebra of Magic Part 1 Name:________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Make a hypothesis. How do you think this works?

ACT 2

What information do you have, would like to know, or do you need to help you answer your MAIN question?

Record the given information (measurements, materials, etc…)

Act 2 (con't)
Use this area for your work, tables, calculations, sketches or other representations, and final solution.

ACT 3

What was the result? How do you know this is correct?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

The Sequel: How would you give this Algebra Magic "trick" more of a "Wow" factor?

The Algebra of Magic Part 2

Back to Task Table

In this inquiry-based task, students will decode a "number trick" using algebraic reasoning.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

MGSE6.EE.2a Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as* 5 – y.

MGSE6.EE.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression* 2(8 + 7) *as a product of two factors; view* (8 + 7) *as both a single entity and a sum of two terms.*

MGSE6.EE.2c Evaluate expressions at specific values for their variables. Include expressions that arise
from formulas in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $\boldsymbol{V = s^3}$ *and* $\boldsymbol{A = 6s^2}$ *to find the volume and surface area of a cube with sides of length* ***s* = ½.**

MGSE6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression* ***3(2 + x)*** *to produce the equivalent expression* ***6 + 3x****; apply the distributive property to the expression* ***24x + 18y*** *to produce the equivalent expression* ***6(4x + 3y)****; apply properties of operations to* ***y + y + y*** *to produce the equivalent expression* ***3y****.*

MGSE6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions* ***y + y + y*** *and* ***3y*** *are equivalent because they name the same number regardless of which number* ***y*** *stands for.* Reason about and solve one-variable equations and inequalities.

MGSE6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will be making sense of magic ""tricks"" involving algebraic expressions as well as creating their own.
2. Reason abstractly and quantitatively. Students will be reasoning through each of the ""tricks"" as they determine how each "trick" works. Students will need to reason with quantities of "stuff" initially before generalizing an abstract rule or expression that represents all of the steps involved.

3. Construct viable arguments and critique the reasoning of others. Students will create expressions and defend these expressions with their peers. Students may discover equivalent expressions and end up proving their equivalence.
4. Model with mathematics. Students will use models to represent what happens in each "trick" in order to understand the "trick", undo the "trick", invent new ""tricks"", and realize the value of representation.
6. Attend to precision. Students will attend to precision through their use of the language of mathematics as well as in their use of operations.
7. Look for and make use of structure. Students will show an understanding of how numbers and variables can be put together as parts and wholes using representations of operations and properties.

ESSENTIAL QUESTIONS

- How are the properties (Identify, Associative and Commutative) used to evaluate, simplify and expand expressions?
- How is the Distributive Property used to evaluate, simplify and expand expressions?
- How can I tell if two expressions are equivalent?

MATERIALS REQUIRED

- Computer and projector or student devices (BYOT) (optional) *Note: this computer program will not work on Apple mobile devices due to Flash player.*
- Directions for mathematical magic "tricks" (attached)
- Counters
- Sticky notes or blank pieces of paper-all the same size (several per student)

TIME NEEDED

- 1 or 2 days

TEACHER NOTES

In part 2 of this series of tasks, students will watch a screencast of a student performing a super quick calculation. Students will then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics, specifically algebraic reasoning, to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them.

The goals of this task are for students to:

- develop an understanding of linear expressions and equations in a context;
- make simple conjectures and generalizations;
- add expressions, "collect" like terms;
- use the distributive law of multiplication over addition in simple situations;
- develop an awareness that algebra may be used to prove generalizations in number situations.

Note: The context of magic ““tricks”” can often turn students off since it may lead students to imply that math is just a bunch of ““tricks””. Presenting this with a video of a middle school student performing the “trick” (see links below) for someone else helps to dispel this myth. When students see another student perform this “trick”, they are more apt to ask “How did they do that?” or, better yet, “Why does that work?” This is what we should all strive for in our lessons. Students asking how to do something or wondering why something works means we've evoked curiosity and wonder.

> While you can perform the ““tricks”” yourself, please be mindful of the fact that most people don't like to be fooled and middle school students can be intimidated by this especially in front of their peers. Another option would be to teach a “trick” to a student in one of your classes ahead of time. They will know the “trick” (how it works), but they will probably not know why. They can still investigate this.

After each “trick” is presented, either through the video or in another manner, students can go the link posted for each “trick” and try different numbers and look for patterns. The link is not necessary, but it does take the “trick” out of the picture and allows students to investigate without being intimidated by being “tricked”.

Explain to students that, for each “trick” in this series of tasks, they should:

- investigate the “trick”, trying different numbers;
- work out how the “trick” is done. This usually involves spotting a connection between a starting number and a finishing number. Algebra will be helpful here: representing the unknown number (the number that is thought of) with something that can contain a quantity may be helpful at first;
- improve the “trick” in some way.

“trick” 2: Consecutive Number Sum

This 3-act task can be found at: https://mikewiernicki3act.wordpress.com/consecutive-number-sums/

Students choose any start number. The next four numbers are the four consecutive numbers that follow the chosen number. The “trick” is to tell the sum of the series of numbers knowing only the start number.

In the investigation of this “trick”, students may vary the starting numbers and make conjectures about the sums produced.

This table should not be shown to students.

Start Number	n
	$n + 1$
	$n + 2$
	$n + 3$
+	$n + 4$
	$5n + 10$

For example:
Students may decide that the sum is always a multiple of 5.
Some may use the following reasoning:

You start with a number, and then add a number that's one more, and then you add a number that's two more, then three more, then four more. That makes ten more altogether. So you add ten to five times the number.

Encourage students to show this more formally, by using a post-it or a blank card for the first number (n), a blank card and a counter ($n + 1$) for the second, a blank card and 2 counters for the third ($n + 2$) and so on. The final total obtained is 5 cards and 10 counters ($5n + 10$) or $5(n + 2)$. A quick way to predict the total from any starting number is to multiply by 5 and then add 10, or add 2 and then multiply by 5. Students may be encouraged to develop this situation into a more complex number "trick". It could be made more impressive by having more addends, or changing consecutive numbers to consecutive even (or odd) numbers, for example.

An interactive app for this "trick" can be found here: http://scratch.mit.edu/projects/20832805/

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:
Watch the video for Act 1. Alternatively, perform this "trick" for students. Ask students what they noticed and what they wonder (are curious about). Record student responses.

Have students hypothesize how the "trick" works. How can the performer know the sum so quickly for any number chosen?

ACT 2:
Students work on determining how the "trick" works based on their hypothesis. They should be guided to show what is happening in the "trick" first through the use of some model that can be represented in a diagram, and then later written as an expression. Students may ask for information such as: "How were the other numbers generated after the start number was chosen?" When they ask, point them to the link below or copy the numbers generated in the video on the board so students can use them.

Students may ask if they can do the "trick" using technology:
http://scratch.mit.edu/projects/20832805/

Students may also ask for materials to use to model what is happening. (they may even ask to use similar materials from the previous task) – materials can also be suggested, carefully, by the teacher.

ACT 3
Students will compare and share solution strategies.

- Share student solution paths. Start with most common strategy.
- Students should explain their thinking about the mathematics in the "trick".
- Ask students to hypothesize again about whether any number would work – like fractions or decimals. Have them work to figure it out.
- Be sure to help students make connections between equivalent expressions (i.e. the rules $5n + 10$ and $5(n + 2)$.
- Revisit any initial student questions that weren't answered.

Extension:
To extend this, students could find a way to give this "trick" more of a "wow" factor or to make it more impressive. Students could also develop their own "trick" with representations and algebraic expressions that explain it. Finally, students should be encouraged to develop (code) their own computer program for a "trick" like this. The free online coding program used for the "tricks" in this task and others to come can be found at www.scratch.mit.edu.

Intervention:
Students needing support might be given simpler "tricks" at first, building up to the "trick" presented above. A simple "trick" might be to only use 3 numbers rather than 5 to determine the sum. Use materials as well as variables to build the algebraic understanding through the use of the quantities being represented.

The Algebra of Magic Part 2

Name:________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Make a hypothesis. How do you think this works?

ACT 2

What information do you have, would like to know, or do you need to help you answer your MAIN question?

Record the given information (measurements, materials, etc…)

Act 2 (con't)
Use this area for your work, tables, calculations, sketches or other representations, and final solution.

ACT 3

What was the result? How do you know this is correct?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

The Sequel: How would you give this Algebra Magic "trick" more of a "Wow" factor?

The Algebra of Magic Part 3

Back to Task Table

In this inquiry-based task, students will decode a "number trick" using algebraic reasoning.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

MGSE6.EE.2a Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as* 5 – y.

MGSE6.EE.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression* 2(8 + 7) *as a product of two factors; view* (8 + 7) *as both a single entity and a sum of two terms.*

MGSE6.EE.2c Evaluate expressions at specific values for their variables. Include expressions that arise from formulas in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $\boldsymbol{V = s^3}$ *and* $\boldsymbol{A = 6s^2}$ *to find the volume and surface area of a cube with sides of length* ***s* = ½.**

MGSE6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression* ***3(2 + x)*** *to produce the equivalent expression* ***6 + 3x****; apply the distributive property to the expression* ***24x + 18y*** *to produce the equivalent expression* ***6(4x + 3y)****; apply properties of operations to* ***y + y + y*** *to produce the equivalent expression* ***3y****.*

MGSE6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions* ***y + y + y*** *and* ***3y*** *are equivalent because they name the same number regardless of which number* ***y*** *stands for.* Reason about and solve one-variable equations and inequalities.

MGSE6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will be making sense of magic "tricks" involving algebraic expressions as well as creating their own.
2. Reason abstractly and quantitatively. Students will be reasoning through each of the "tricks" as they determine how each "trick" works. Students will need to reason with quantities of "stuff" initially before generalizing an abstract rule or expression that represents all of the steps involved.

3. Construct viable arguments and critique the reasoning of others. Students will create expressions and defend these expressions with their peers. Students may discover equivalent expressions and end up proving their equivalence.
4. Model with mathematics. Students will use models to represent what happens in each "trick" in order to understand the "trick", undo the "trick", invent new "tricks", and realize the value of representation.
6. Attend to precision. Students will attend to precision through their use of the language of mathematics as well as in their use of operations.
7. Look for and make use of structure. Students will show an understanding of how numbers and variables can be put together as parts and wholes using representations of operations and properties.

ESSENTIAL QUESTIONS

- How are the properties (Identify, Associative and Commutative) used to evaluate, simplify and expand expressions?
- How is the Distributive Property used to evaluate, simplify and expand expressions?
- How can I tell if two expressions are equivalent?

MATERIALS REQUIRED

- Computer and projector or Students with personal technology (BYOT) (optional) Note: this computer program will not work on apple mobile devices due to Flash player.
- Directions for mathematical magic "tricks" (attached)
- Counters
- Sticky notes or blank pieces of paper-all the same size (several per student)

TIME NEEDED

- 1 or 2 days

TEACHER NOTES

In each part of this task, students will watch a screencast video, then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics, specifically algebraic reasoning, to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them.

The goals of this task are for students to:

- develop an understanding of linear expressions and equations in a context;
- make simple conjectures and generalizations;
- add expressions, "collect" like terms;
- use the distributive law of multiplication over addition in simple situations;

- develop an awareness that algebra may be used to prove generalizations in number situations.

Note: The context of magic "tricks" can often turn students off since it may lead students to imply that math is just a bunch of "tricks". Presenting this with a video of a middle school student performing the "trick" (see links below) for someone else helps to dispel this myth. When students see another student perform this "trick", they are more apt to ask "How did they do that?" or, better yet, "Why does that work?" This is what we should all strive for in our lessons. Students asking how to do something or wondering why something works means we've evoked curiosity and wonder.

While you can perform the "tricks" yourself, please be mindful of the fact that most people don't like to be fooled and middle school students can be intimidated by this especially in front of their peers. Another option would be to teach a "trick" to a student in one of your classes ahead of time. They will know the "trick" (how it works), but they will probably not know why. They can still investigate this.

After each "trick" is presented, either through the video or in another manner, students can go the link posted for each "trick" and try different numbers and look for patterns. The link is not necessary, but it does take the "trick" out of the picture and allows students to investigate without being intimidated by being "tricked".

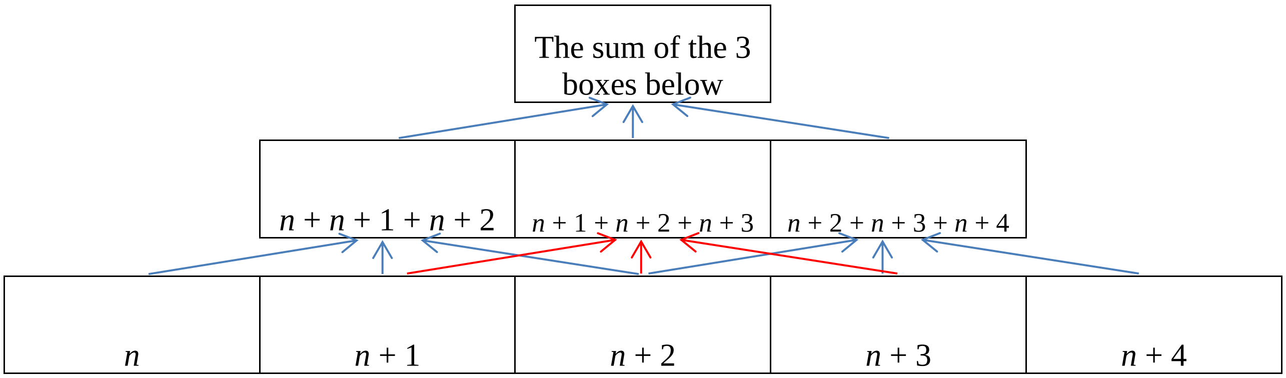
Explain to students that, for each "trick" in this series of tasks, they should:

- investigate the "trick", trying different numbers;
- work out how the "trick" is done. This usually involves spotting a connection between a starting number and a finishing number. Algebra will be helpful here: representing the unknown number (the number that is thought of) with something that can contain a quantity may be helpful at first;
- improve the "trick" in some way.

"Trick" 3: Triangle Mystery . . .
The following 3-act task can be found at: http://mikewiernicki3act.wordpress.com/triangle-mystery/

This "trick" extends the previous "trick". There are more patterns to look for and many ways to determine the rules for the patterns (it's more open!). Students should be told how the triangle mystery works: in the bottom row are consecutive numbers beginning with the number chosen. Each box in the second row is the sum of the 3 boxes below *(see the diagram on the next page)*. The top box is the sum of the three boxes in the middle row. The "trick" is to determine the top number of the triangle, given only the start number (in the lower left hand box).

This "trick" is again, presented in such a way that the "magic" doesn't overtake the mathematics. If you perform this for your students rather than use the video, please be mindful of the fact that most people don't like to be fooled and middle school students can be intimidated by this especially in front of their peers.



This table (with the expressions) should not be shown to students.

Tell students, prior to the "trick", that:

- The bottom row of this triangle contains consecutive numbers.
- Each other number is found by adding the three numbers beneath it.

Presentation:
Give students the opportunity change the start number in the bottom left hand rectangle and tell what it is.
You immediately say the top number.
How is the "trick" done?
Try to make the "trick" more impressive.

Here is what the students' productive struggle will lead to:
In Triangle Mystery, if the bottom left hand number is called n, then:

- the bottom row is n, $n + 1$, $n + 2$, $n + 3$, $n + 4$;
- the second row is $3n + 3$, $3n + 6$, $3n + 9$;
- the top row is $9n + 18 = 9(n + 2)$.

So, the short cut is simply to add 2 to the bottom left hand number and then multiply by 9 (or multiply the first number by 9 and add 18). Students may like to try creating larger Pyramids that follow different rules.

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:
Watch the video for Act 1. Alternatively, perform this "trick" for students. Ask students what they noticed and what they wonder (are curious about). Record student responses.

Have students hypothesize how the "trick" works. How can the performer know the number at the top of the triangle so quickly for any number chosen?

ACT 2:
Students work on determining how the "trick" works based on their hypothesis. They should be guided to show what is happening in the "trick" first through the use of some model that can be represented in a diagram, and then later written as an expression. Students may ask for information such as: "How were the other numbers generated after the start number was chosen?" When they ask, you can tell them the numbers in the bottom row are consecutive numbers after the start number. Each number in the middle row is the sum of the 3 numbers below. The top number is the sum of the numbers in the middle row. OR you can give them the technology link below for further investigation.

Students may ask if they can investigate the "trick" using technology:
http://scratch.mit.edu/projects/20831707/

Students may also ask for materials to use (they may even ask to use similar materials from the previous task) – these can also be suggested, carefully, by the teacher.

ACT 3
Students will compare and share solution strategies.

- Share student solution paths. Start with most common strategy.
- Students should explain their thinking about the mathematics in the "trick".
- Ask students to hypothesize again about whether any number would work – like fractions or decimals. Have them work to figure it out.
- Be sure to help students make connections between equivalent expressions (i.e. the expressions $9n + 18$ and $9(n + 2)$.
- Revisit any initial student questions that weren't answered.

Extension:
To extend this, students could find a way to give this "trick" more of a "wow" factor or to make it more impressive. Students could also develop their own "trick" with representations and algebraic expressions that explain it. Finally, students should be encouraged to develop (code) their own computer program for a "trick" like this. The free online coding program used for the "tricks" in this task and others to come can be found at www.scratch.mit.edu.

Intervention:
Students needing support might be given simpler "tricks" at first, building up to the "trick" presented above. A simple "trick" might use 3 rows, but the second row may be determined by adding only the two numbers below it. Use materials as well as variables to build the algebraic understanding through the use of the quantities being represented.

The Algebra of Magic Part 3

Name:________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Make a hypothesis. How do you think this works?

ACT 2

What information do you have, would like to know, or do you need to help you answer your MAIN question?

Record the given information (measurements, materials, etc…)

Act 2 (con't)
Use this area for your work, tables, calculations, sketches or other representations, and final solution.

ACT 3

What was the result? How do you know this is correct?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

The Sequel: How would you give this Algebra Magic "trick" more of a "Wow" factor?

Writing Expressions

Back to Task Table

In this task, students will translate between verbal and symbolic algebraic expressions and equations.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.2a Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5-y.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of real world situations by writing expressions to represent the situation.
4. Model with mathematics. Students form expressions from real world contexts.
6. Attend to precision. Students use the language of real-world situations to write expressions.

ESSENTIAL QUESTIONS

- How do I translate between word phrases and expressions?

(Answers)

1) *Answers vary.*
2) 6 less than $3t$ $3t - 6$ | the product of w and 8 $8w$

r divided by 15 $\frac{r}{15}$ or $r \div 15$ | 9 more than twice x $2x + 9$

the quotient of 12 and x $\frac{12}{x}$ or $12 \div x$ | the product of x and 6 $6x$

the sum of three times a and 35 $3a + 35$ | 6 times the sum of x and 8 $6(x + 8)$

a number, x, decreased by 9 $x - 9$ | 5 increased by the quotient of x and 7 $5 + \frac{x}{7}$ or $5 + (x \div 7)$

15 less than 4 times w $4w - 15$ | 12 decreased by the difference of x and 7 $12 - (x - 7)$

3) **Part A:**
Katie's age = k;
Hannah's age = k – 3;
Joey's age = 2(k – 3) = 2k – 6
Part B: *Answers vary depending on substituted value for Katie's age.*
Part C: *It depends. If Katie is over 6 years old, then Joey is the oldest. If Katie is 6, then she and Joey are the same age. If Katie is under 6 years old, she is the oldest. Allow students to debate their conjectures rather than just telling them the answer.*

Name_______________________________

Writing Expressions

1. Within your classroom, have the students find situations where they can role play to compare known and unknown quantities (e.g., Student A (Dory) and Student B (Colleen). For example Dory says, "I have two sisters." Colleen says, "I have Dory – 1 sister." Dory says, "You have $d - 1$ sister. You have one sister.") Make sure all operations are included. Write expressions here.

2. Write each word phrase as an algebraic expression.

6 less than 3*t*

the product of *w* and 8

***r* divided by 15**

9 more than twice *x*

the quotient of 12 and *x*

the product of *x* and 6

the sum of three times *a* and 35

6 times the sum of *x* and 8

a number, *x*, decreased by 9

5 increased by the quotient of *x* and 7

15 less than 4 times *w*

12 decreased by the difference of *x* and 7

3. Part A:
   Hannah is 3 years younger than Katie.
   Joey is twice as old as Hannah.
   Let *k* stand for Katie's age.

Write an expression below to represent Hannah's age.

Using *k*, write an expression below for Joey's age.

Part B:
Now, test your expressions out to see if they are accurate. Pick an age for Katie and then substitute that value in for the variable *k* to see if your expressions make sense.

Part C: Who is the oldest? Justify your thinking by providing an example/explanation.

Writing and Evaluating Expressions

Back to Task Table

In this task, students will evaluate expressions for given values, translate between verbal and symbolic algebraic expressions and equations, and generalize an algebraic expression for various problems.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

MGSE6.EE.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.

MGSE6.EE.2c Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of expressions and formulas by connecting them to real world contexts when evaluating.
2. Reason abstractly and quantitatively. Students contextualize to understand the meaning of the number or variable as related to the problem.
3. Construct viable arguments and critique the reasoning of others. Students construct and explanations for their use of the operations they chose and compare their reasoning with others.
4. Model with mathematics. Students model expressions algebra tiles.
6. Attend to precision. Students attend to precision when evaluating formulas with fractions and decimals.

ESSENTIAL QUESTIONS

- How is the order of operations used to evaluate expressions and given formulas?
- How will can I read, write, and evaluate expressions and equations in which letters stand for numbers?

INTRODUCTION

Mathematics is a language. Complete sentences in mathematics are called equations. They are called equations because of the equal sign. When working with equations, operations are like verbs and numbers are adjectives that describe the units, which are like nouns.

In mathematics, incomplete sentences are called expressions. They are just part of a number sentence since they do not have an equal sign.

At times we need the freedom to change a value or a value is unknown in an expression or an equation so we use a symbol to stand for the number. In these cases, we use letters to represent those values and we call those letters variables.

When we substitute numerical values for variables in expressions we say that we are "evaluating the expression." When we find an answer for the variable in an equation we call it the solution. Evaluating is also useful with formulas like volume, area or perimeter.

An extension of this lesson is to use Algebra Tiles or Algeblocks to model each of the expressions.

TIME NEEDED
2 days

BEFORE THE LESSON
Have students generate a list of words that help them identify operations. The list below is a guide. Do not give them to students; instead, present an operation and ask them to list words they associate with that operation

Key Words	**Operation**
Add	+
Plus	+
Sum	+
Total	+
Increased by	+
More Than	+
Product	×
Times	×
Multiply	×
Of	×
Minus	–
Difference	–
Subtract	–
Less than	–
Decreased by	–
Quotient	÷
Divide	÷

Teacher Notes: ***Discuss with students that this is not an exhaustive list and that they will often have to interpret the situation presented in the problem in order to determine which mathematical operations are appropriate. Be careful not to use Key Words as a strategy in working with expressions. This should be used to show there are many ways to express the operations.***

Teacher Notes: ***It is recommended that students work in groups to complete this exploration. Have each group post their solutions around the room and then provide time for groups to look at other students work before the summary portion of the lesson.***

Part I

Mr. Green's Math class is planning a trip to the IMAX Theater. It will cost $10 for the school bus and the price of a ticket is $13 dollars per student. What will determine the amount of money the class will have to make? ***The number of students who are able to go***

How will the number of students affect the price? ***The more students the higher the price***

How will they know how much money they need to make? ***When they know how many students will attend***

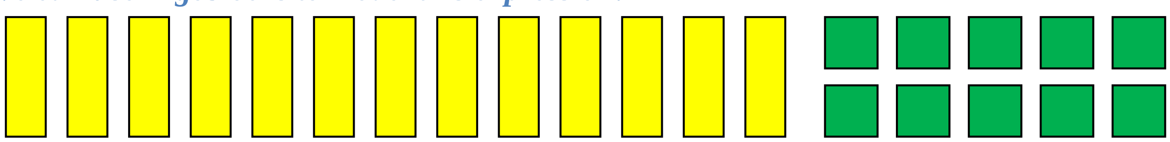
What value varies in this example? ***The number of students***

Write an expression to show the amount of money the class needs to make. ***$10 + $13n where n is the number of students who will attend.***

How much will it cost if 10 students attend? ***$140***

How much will it cost if 17 students attend? ***$231***

We can use Algeblocks to model this expression:



It is recommended that students work in groups to complete this exploration. Have each group post their solutions around the room and then provide time for groups to look at other students work before the summary portion of the lesson.

Part II

1. Mr. White drives 55 km a day for work. How many km will he drive in:
   a. 5 days? ***275 km***
   b. 8 days? ***440 km***
   c. 15 days? ***825 km***
   d. Write an expression to represent the number of km he will drive in *d* days ***55d, where d = number of work days***

2. Sean's father is working on a crew that will build a skyscraper. He found out that each story is 13 ft tall. How tall, in feet, would the skyscraper be if it were:
   a. 55 floors? ***715 feet***
   b. 65 floors? ***845 feet***
   c. 75 floors? ***975 feet***
   d. Write an expression to represent the height of a skyscraper with f stories ***13f, where f = number of floors***

2. 55 figurines of a miniature porcelain doll can be safely shipped in a case. A distributor is investigating to find which size box is the safest to hold the largest number of cases. How many figurines could be shipped in a box that could hold:
   a. 750 cases? ***41,250 figurines***
   b. 1000 cases? ***55,000 figurines***
   c. 1250 cases? ***68,750 figurines***
   d. Write an expression to represent the number of figurines that can be shipped in a box that holds c cases ***55c, where c = number of cases***

3. The rental fee for a mo-ped is $10 plus $3 for each hour the bike is used. How much will it cost if you rent the mo-ped for:
   a. 1 hour? ***$13***
   b. 8 hours? ***$34***
   c. 1 day? ***$82***
   d. Write an expression that represents the cost for h hours ***$10 + $3h, where h = number of hours the bike is used***

4. A wireless service provider charges $29.99 per month for service plus $0.10 for each text message. How much will it cost if:
   a. 35 text messages are sent? ***$33.49***
   b. 105 text messages are sent? ***$40.49***
   c. 217 text messages are sent? ***$51.69***
   d. Write an expression to represent the cost if t text messages are sent ***$29.99 +$0.10t, where t = number of text messages***

6. The formula for finding the Volume of a rectangular prism can be stated as $V = l \text{ x } w \text{ x } h$, where l = length of the prism, w = width of the prism and h = height of the prism. What is the Volume of a prism with:
   a. l = 33 ft, w = 47 ft, and h = 15 ft? ***23,265 ft^3***
   b. l = 22.5 cm, w = 33.7 cm, and h = 12.5 cm? ***9,478.125 cm^3***
   c. l = 122.25 inches, w = 50.75 inches, and h = 16.5 inches? ***102,369.09375 in^3***

7. The formula for finding the volume of a prism is $V=Bh$. What is the volume of the prism with:
   a. Area of the base is 16 cm and height is 2.4 cm? $V=38.4 \text{ cm}^3$
   b. Area of the base is 12½ cm and height is 7cm? $V=87.5 \text{ cm}^3$

c. Area of the base is $3\frac{3}{4}$ cm and height is $3\frac{1}{5}$ cm? V=12 cm^3

8. The formula for finding the Volume of a cube is $V = s^3$, where s is equal to the length of one side of the cube. What is the volume of the cube with side length of:
   a. $\frac{1}{2}$ $inch$? $\left(\frac{1}{2}\right)^3 = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}\ in^3$
   b. 2.4 $meters$? ***13.824 m^3***
   c. 100 *feet?* ***1,000,000 ft^3***

9. The formula for finding the Surface Area of a cube is $A = 6s^2$, where s is the length of one side of the cube. What is the surface area of a cube with side length of:
   a. $\frac{1}{2}$ $inches$? $6\left(\frac{1^2}{2}\right) = 6\left(\frac{1}{4}\right) = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}\ in^2$
   b. 2.4 $meters$? ***34.56 m^2***
   c. 100 *feet?* ***60,000 ft^2***

DURING THE LESSON

Allow students to struggle with this process. Direct their attention to the list created at the beginning of the class for guidance. Ask questions to guide student thinking. Suggest manipulatives or drawing pictures for modeling.

Fluency with multi-digit computations is reinforced through this task along with the focus of being able to translate words into expressions.

SUMMARY

When students are sharing their work make sure they explain why they chose the operation that they chose.

Name____________________________________

Writing and Evaluating Expressions

Part I
Mr. Green's Math class is planning a trip to the IMAX Theater. It will cost $10 for the school bus and the price of a ticket is $13 dollars per student. What will determine the amount of money the class will have to make?

How will the number of students affect the price?

How will they know how much money they need to make?

What value varies in this example?

Write an expression to show the amount of money the class needs to make.

How much will it cost if 10 students attend?

How much will it cost if 17 students attend? Draw a model to represent this situation

Part II: For the first five problems read each carefully and write an expression that includes numbers and variables. Then, evaluate the expression using the numbers indicated. For the last five problems evaluate the expression for the numbers provided.

1. Mr. White drives 55 km a day for work. How many km will he drive in:

    a. 5 days?

    b. 8 days?

    c. 15 days?

d. Write an expression to represent the number of km he will drive in d days

2. Sean's father is working on a crew that will build a skyscraper. He found out that each story is 13 ft tall. How tall, in feet, would the skyscraper be if it were:

    a. 55 stories?

    b. 65 stories?

    c. 75 stories?

    d. Write an expression to represent the height of a skyscraper with f stories

3. 55 figurines of a porcelain doll can be safely shipped in a case. A distributor is investigating to find which size box is the safest to hold the largest number of cases. How many figurines could be shipped in a box that could hold:

    a. 750 cases?

    b. 1000 cases?

    c. 1250 cases?

    d. Write an expression to represent the number of figurines that can be shipped in a box that holds *c* cases.

4. The rental fee for a bike is $10 plus $3 for each hour the bike is used. How much will it cost if you rent the bike for:

    a. 1 hour?

    b. 8 hours?

    c. 1 day?

    d. Write an expression that represents the cost for h hours.

5. A wireless service provider charges $29.99 per month for service plus $0.10 for each text message. How much will it cost if:

    a. 35 text messages are sent?

    b. 105 text messages are sent?

    c. 217 text messages are sent?

    d. Write an expression to represent the cost if t text messages are sent

6. The formula for finding the Volume of a rectangular prism can be stated as $V = l \text{ x } w \text{ x } h$, where l = length of the prism, w = width of the prism and h = height of the prism. What is the Volume of a prism with:

    a. $l = 33$, $w = 47$, and $h = 15$?

    b. $l = 22.5$, $w = 33.7$, and $h = 12.5$?

c. $l = 122.25$, $w = 50.75$, and $h = 16.5$?

7. The formula for finding the volume of a prism is V=Bh. What is the volume of the prism with:

   a. Area of the base is 16 cm and height is 2.4 cm?

   b. Area of the base is 12½ cm and height is 7cm?

   c. Area of the base is $3\frac{3}{4}$ cm and height is $3\frac{1}{5}$ cm?

8. The formula for finding the volume of a cube is $V = s^3$, where s is equal to the length of one side of the cube. What is the volume of the cube with side length of:

   a. $\frac{1}{2}\ inches$?

   b. $26.4\ meters$?

   c. 100 *feet?*

9. The formula for finding the surface area of a cube is $A = 6s^2$, where s is the length of one side of the cube. What is the surface area of a cube with side length of:

   a. $\frac{1}{2}$ inches?

   b. 26.4 meters?

   c. 100 feet?

Laws of Arithmetic - Formative Assessment Lesson

Back to Task Table

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1358

In this task, students will write and evaluate arithmetic expressions to find the area of compound rectangles. Students will also identify equivalent expressions.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

STANDARDS FOR MATHEMATICAL PRACTICE

This lesson uses all of the practices with emphasis on:

1. **Make sense of problems and persevere in solving them.**
5. **Use appropriate tools strategically.**
7. **Look for and make use of structure.**

TASK COMMENTS:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Laws of Arithmetic*, is a Formative Assessment Lesson (FAL) that can be found at the website: http://map.mathshell.org/materials/lessons.php?taskid=484&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1358

Are We Equal?

Back to Task Table

In this hands-on task, students will model algebraic expressions in order to explore the concepts of combining like terms and equivalent expressions.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

MGSE6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them.) *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

MGSE6.NS.4 Find the common multiples of two whole numbers less than or equal to 12 and the common factors of two whole numbers less than or equal to 100.

a. Find the greatest common factor of 2 whole numbers and use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factors. (GCF) *Example: 36 + 8 = 4(9 + 2)*
b. Apply the least common multiple of two whole numbers less than or equal to 12 to solve real-world problems.

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them. Students make sense of expressions by connecting them to real world contexts when evaluating.
4. Model with mathematics. Students use algebra tiles to model equivalent expressions.
5. Use appropriate tools strategically. Students determine which algebraic representations are appropriate for given contexts.
7. Look for and make use of structure. Students apply properties to generate equivalent expressions. They interpret the structure of an expression in terms of a context. Students identify a “term” in an expression.
8. Look for and express regularity in repeated reasoning. Students can work with expressions involving variables without the focus on a specific number or numbers that the variable may represent. Students focus on the patterns that lead to generalizations that lay the foundation for their future work in algebra. Students work with the structure of the distributive property $2(3x + 5) = 6x + 10$.

ESSENTIAL QUESTIONS

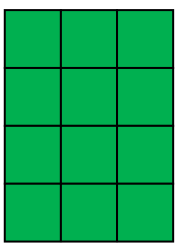
- How can I identify when two expressions are equal?
- How can I generate equivalent expressions by applying the properties of numbers?

INTRODUCTION

Manipulatives are a great way to allow students to explore the idea of equivalency with expressions and simplifying expressions. If you have Algeblocks or Algebra Tiles, both have lessons ready-made for this purpose. You can also access the use of Algebra Tiles on the National Library of Virtual Manipulatives. If you do not have access to any of these things, do not worry, this lesson will help you accomplish the same task by having students draw representations or cut models with paper.

We want students to be able to represent the idea of a variable as an unknown quantity or quantities using letters or symbols. Then we would like them to be able to model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.

This may make more sense if students understand how to represent multiplication and division of numbers as rectangular arrays. For example, 3 x 4 can be represented by:



Using this model students can answer the question, "What is four groups of three?" or "What is three groups of four?"

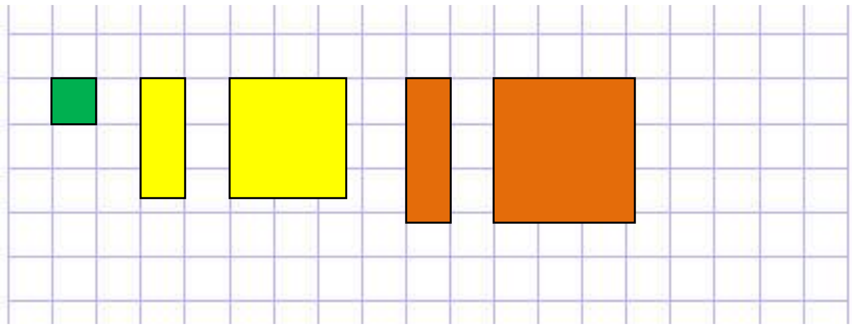
Students should also be able to recognize that the same question is being asked in a problem involving one or more variables. For example, the problem $3(x+ 1) = ?$ can be stated, "How many is 3 groups of $(x + 1)$?" Students should recognize that multiplication is the operation best suited for answering questions of the form, "how many is ___ groups of ___ things?" The first step in representing multiplication or division using algebra tiles is making sure you know what question is being asked and what operation is best suited for answering that question.

Students should be allowed to use the manipulative until they decide they do not need it anymore.

MATERIALS REQUIRED

Algebra Tiles or Algeblocks
If these are not available, have students cut their own set from construction paper or from this template. Make sure that the sizes you use correspond to one another, that is the x and y tiles are

the same width as the unit and different lengths from each other and the x^2 the y^2 are the same height and width as the x and y respectively. You can use grid paper to create them as shown below, making sure that the length of x and y do not correspond to an equal measure of the unit.



Provide students with twenty of each tile.

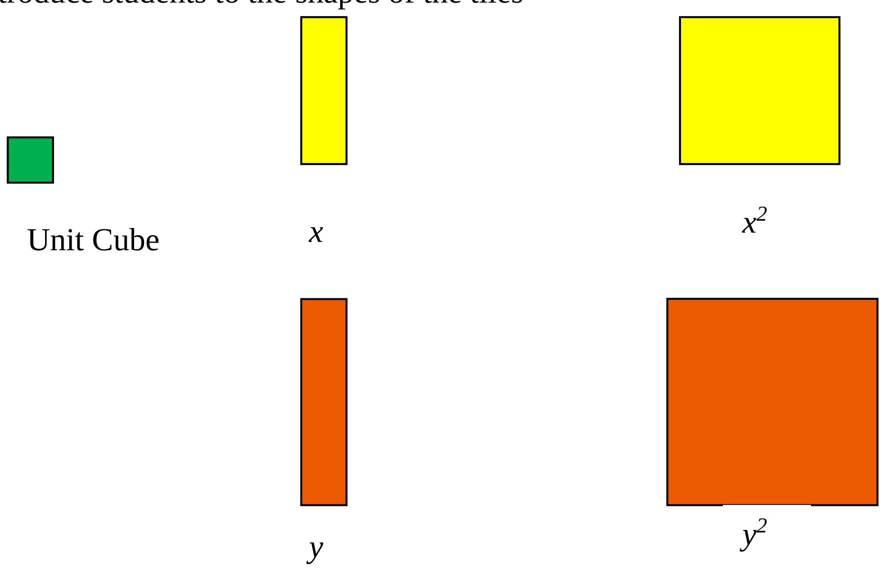
When using a manipulative it is important for students to be able to use and manipulate it themselves and not just watch the teacher demonstrate. There are on-line applets for Algebra Tiles. If you use one of these with a SmartBoard make sure that students are able to do it themselves.

Time Needed

2 days

Before the Lesson

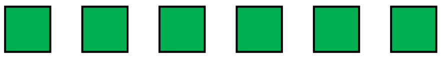
Introduce students to the shapes of the tiles

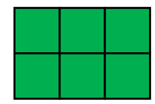


Are We Equal?

Numerical Expressions

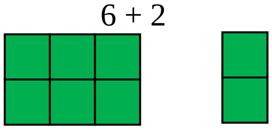
To model whole numbers we use unit cubes. For example:

6  or we can arrange them as an array to show the factors

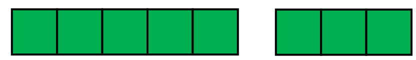


To model a numeric expression we would use groups of unit cubes

6 + 2



5 + 3

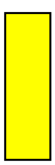
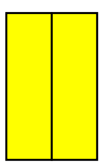


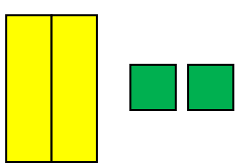
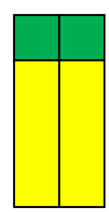
1. Are these two numeric expressions equivalent?

Yes, because both drawings contain 8 unit cubes

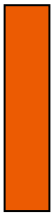
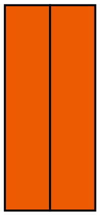
To model variable expressions, we use the *x* and *y* tiles. They don't have to be called *x* and *y*. They could be *a* and *b* or *s* and *t* or *b* and *w*. The point is that they represent one of an unknown.

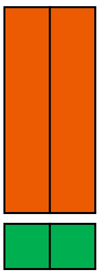
Algebraic Expressions

x  2*x*  We have a rectangle that has dimensions of 2 and *x* or 2*x*

2*x* + 2  We can also arrange them in this way 

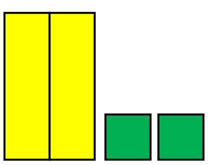
We can model these same examples with our y

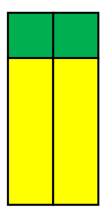
y  $2y$ 

$2y + 2$ 

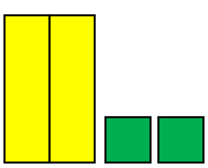
Right now, we are not concerned with the value of x and y. We just want to know how many x's or how many y's.

2. Draw two different models of $2(x + 1)$?

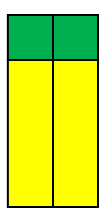




3. Draw a model to represent $2x + 2$



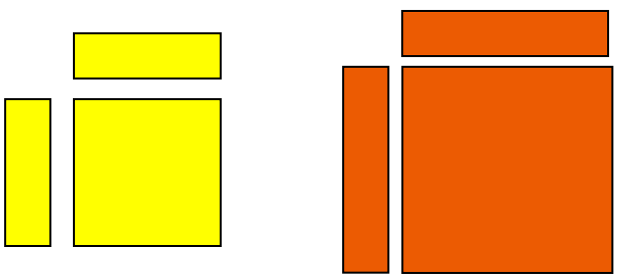
or



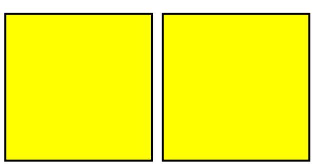
4. Based on your two models, what conclusion can you draw about $2(x + 1)$ and $2x + 2$. Be sure to justify your conclusion. ***They are equal. Justifications should include a comparison of the models and some students may mention the distributive property***

Now let's look at two more tiles

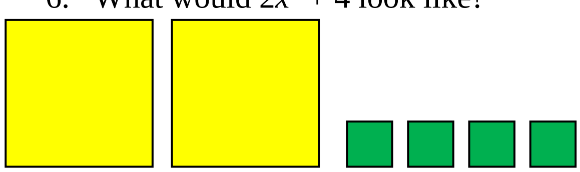
They are called x^2 and y^2 because the length of the sides of each is equal to our x and y tiles



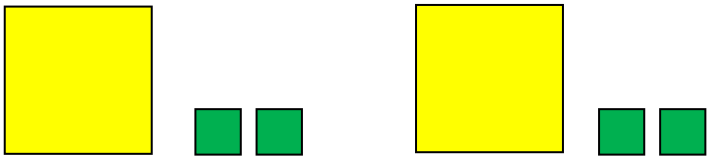
5. What would $2x^2$ look like?



6. What would $2x^2 + 4$ look like?



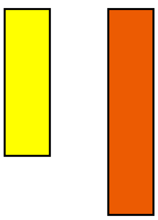
7. Can you divide these into equal groups? Draw a model to represent this and write an algebraic expression to represent your model



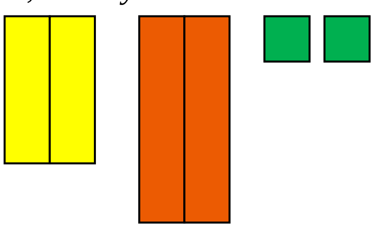
8. Based on the models what conclusion can you draw about $2x^2 + 4$ and $2(x^2 + 2)$? Be sure to justify your conclusion. ***They are equal. Justifications should include a comparison of the models and some students may mention the distributive property***

Let's try combining variables.

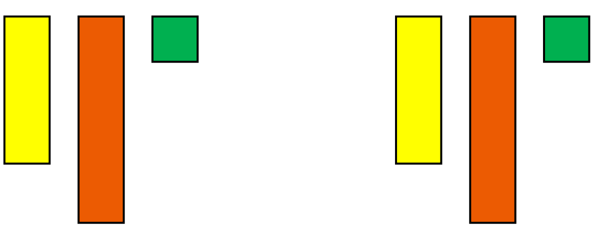
What would $x + y$ look like?



So, $2x + 2y + 2$ could be modeled

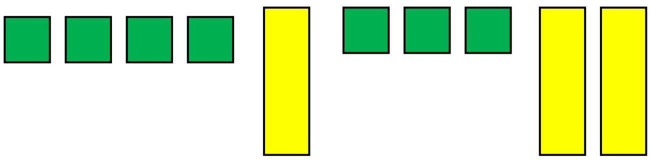


9. Is it possible to divide these into equal groups? Draw a model to represent this and write an algebraic expression to represent your model ***2(x + y + 1)***

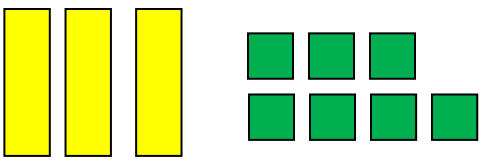


10. Based on the models what conclusion can you draw about $2x + 2y + 2$ and $2(x + y + 1)$? Be sure to justify your conclusion. ***They are equal. Justifications should include a comparison of the models and some students may mention the distributive property***

Let's look at this example: $4 + x + 3 + 2x$



11. Is there a way to rearrange things so it is a little neater? Draw a model to justify your answer



12. Can we write a new expression to represent this? ***3x + 7***

13. What conclusion can you draw about $4 + x + 3 + 2x$ and $3x + 7$? ***They are equal. Justifications should include a comparison of the models***

With a partner, determine whether each of the following pairs of expressions are equivalent. Some of them may not be equivalent. Be sure to justify your conclusions

14. $6y + 12$ and $6(y + 2)$ ***Equivalent, student justifications could include models and/or the application of the properties***

15. $3x + y$ and $y + 3x$ ***Equivalent, student justifications could include models and/or the application of the properties***

16. $3x + 2$ and $3(x + 2)$ ***Not Equivalent, student justifications could include models and/or the application of the properties***

17. $5x^2 + 15$ and $5(x^2 + 3)$ ***Equivalent, student justifications could include models and/or the application of the properties***

18. $3y^2 + 6x^2$ and $3(y^2 + 2x^2)$ ***Equivalent, student justifications could include models and/or the application of the properties***

Now, find an equivalent expression or expressions for each of the following. Draw your representations. Write an expression to show the expressions are equal. Explain why you know that they are equivalent (properties of numbers or operations).

19. $3y^2 + 2 + 1$ ***$= 3y^2 + 3$, combining like terms***

20. $2y + y + 4 + 2$ ***$= 3y + 6$ or $3(y + 2)$, combining like terms or the distributive property***

21. $2y^2 + 4x^2$ ***$= 2(y^2 + 2x^2)$, distributive property***

22. $2 + 3x^2 + x + 2x + 1$ ***$= 3x^2 + 3x + 3$ or $3(x^2 + x + 1)$) combining like terms or the distributive property***

23. y + y + y + y ***= 4y combining like terms***

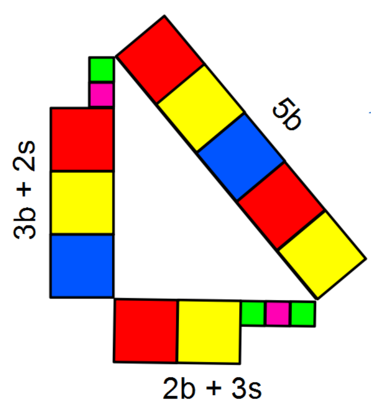
DURING THE LESSON

Make sure students are actually modeling the expressions. Ask questions to guide.

SUMMARY

Have students share solutions. Make sure that the mathematical reasoning is sound and they demonstrate their thought processes.

Support: Using square tiles and centimeter cubes, have students construct polygons (such as the triangle below) by laying the tiles/cubes on the exterior of the shape. Then have students express the length of each side algebraically by using *b* to represent the bigger tiles and *s* to represent the smaller cubes. Then have students find the perimeter of the polygon by combining the like



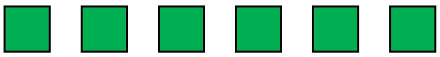
Perimeter = (3b + 2s) + 5b + (2b + 3s)
or
10b + 5s

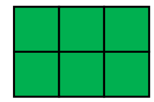
Name________________________

<u>Are We Equal</u>?

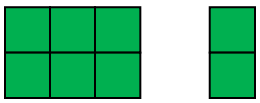
Numerical Expressions

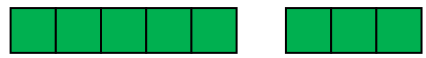
To model whole numbers we use unit cubes. For example:

6  or we can arrange them as an array to show the factors



To model a numeric expression we would use groups of unit cubes

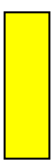
6 + 2 

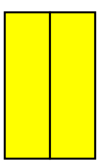
5 + 3 

1. Are these two numeric expressions equivalent?

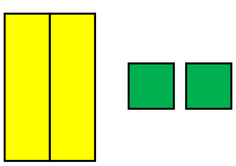
To model variable expressions we use the *x* and *y* tiles. They don't have to be called *x* and *y*. They could be *a* and *b* or *s* and *t* or *b* and *w*. The point is that they represent one of an unknown.

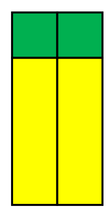
Algebraic Expressions

x 

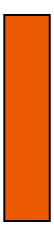
2*x* 

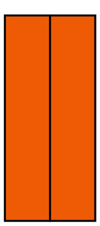
We have a rectangle that has dimensions of 2 and *x* or 2*x*

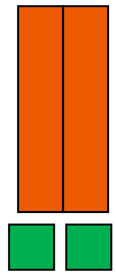
2*x* + 2 

We can also arrange them in this way 

We can model these same examples with our *y*

y 

2*y* 

$2y + 2$ 

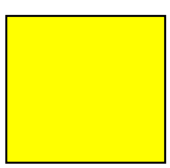
Right now we are not concerned with the value of x and y. We just want to know how many x's or how many y's.

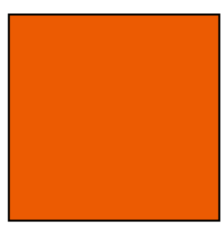
2. Draw two different models of $2(x + 1)$?

3. Draw a model to represent $2x + 2$

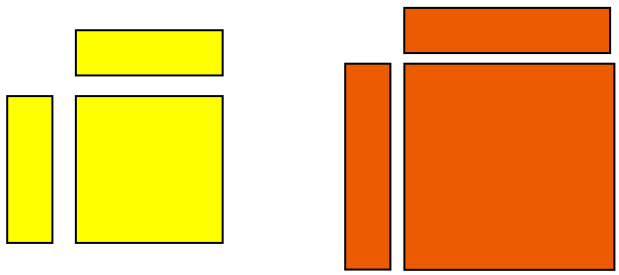
4. Based on your two models what conclusion can you draw about $2(x + 1)$ and $2x + 2$. Be sure to justify your conclusion.

Now let's look at two more tiles





They are called x^2 and y^2 because the length of the sides of each is equal to our x and y tiles



5. What would $2x^2$ look like?

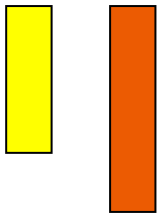
6. What would $2x^2 + 4$ look like?

7. Can you divide these into equal groups? Draw a model to represent this and write an algebraic expression to represent your model

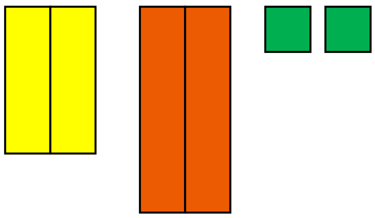
8. Based on the models what conclusion can you draw about $2x^2 + 4$ and $2(x^2 + 2)$? Be sure to justify your conclusion.

Let's try combining variables.

What would $x + y$ look like?



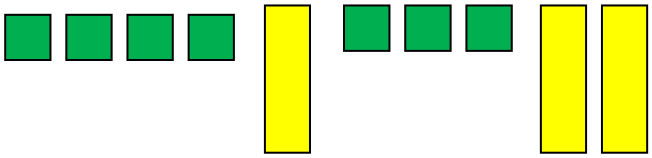
So, $2x + 2y + 2$ could be modeled



9. Is it possible to divide these into equal groups? Draw a model to represent this and write an algebraic expression to represent your model

10. Based on the models what conclusion can you draw about $2x + 2y + 2$ and $2(x + y +1)$. Be sure to justify your conclusion.

Let's look at this example: $4 + x + 3 + 2x$



11. Is there a way to rearrange things so it is a little neater? Draw a model to justify your answer

12. Can we write a new expression to represent this?

13. What conclusion can you draw about $4 + x + 3 + 2x$ and $3x + 7$?

With a partner, determine whether each of the following pairs of expressions are equivalent. Some of them may not be equivalent. Be sure to justify your conclusions

14. $6y + 12$ and $6(y + 2)$

15. $3x + y$ and $y + 3x$

16. $3x + 2$ and $3(x + 2)$

17. $5x^2 + 15$ and $5(x^2 + 3)$

18. $3y^2 + 6x^2$ and $3(y^2 + 2x^2)$

Now, find an equivalent expression or expressions for each of the following. Draw your representations. Write an equation to show the expressions are equal.

19. $3y^2 + 2 + 1$

20. $2y + y + 4 + 2$

21. $2y^2 + 4x^2$

22. $2 + 3x^2 + x + 2x + 1$

23. y + y + y + y

Culminating Task: Sweet Tooth Chocolate Shop

[Back to Task Table]

Atwood, A. & Nagy, R. adapted this activity from "Helping Students Make Sense of Algebraic Expressions: The Candy Shop", by Diana Underwood Gregg and Erna Yackel, *Teaching Mathematics in the Middle School*, v.7, no. 9, NCTM, May 2002.

In this rich, problem-based task, students will synthesize and apply multiple skills and concepts addressed in this unit to complete an inventory log for a candy company.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.1 Write and evaluate expressions involving whole-number exponents.

MGSE6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

MGSE6.EE.2a Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5-y.*

MGSE6.EE.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.

MGSE6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

MGSE6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them.) *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

MGSE6.NS.4 Find the common multiples of two whole numbers less than or equal to 12 and the common factors of two whole numbers less than or equal to 100.

a. Find the greatest common factor of 2 whole numbers and use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factors. (GCF) *Example: 36 + 8 = 4(9 + 2)*
b. Apply the least common multiple of two whole numbers less than or equal to 12 to solve real-world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of expressions and formulas by connecting them to real world contexts when evaluating expressions.

2. Reason abstractly and quantitatively. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3. Construct viable arguments and critique the reasoning of others. Students construct and critique arguments regarding the equivalence of expressions and the use of variable expressions to represent real-world situations.
4. Model with mathematics. Students form expressions from real world contexts. Students use algebra tiles to model algebraic expressions.
5. Use appropriate tools strategically. Students determine which algebraic representations are appropriate for given contexts.
6. Attend to precision. Students use the language of real-world situations to create appropriate expressions.
7. Look for and make use of structure. Students apply properties to generate equivalent expressions. They interpret the structure of an expression in terms of a context. Students identify a "term" in an expression.
8. Look for and express regularity in repeated reasoning. Students can work with expressions involving variables without the focus on a specific number or numbers that the variable may represent. Students focus on the patterns that lead to generalizations that lay the foundation for their future work in algebra. Students work with the structure of the distributive property $2(3x + 5) = 6x + 10$.

ESSENTIAL QUESTIONS

- How can I translate written information into algebraic expressions?
- How can I generate equivalent expressions by applying the properties of numbers?

DESCRIPTION OF THE TASK

As co-owner and shop keeper of Sweet Tooth Chocolates, a small candy company, you are responsible for keeping clear, accurate records of inventory and sales in the candy shop and sharing this information with your business partner, who runs the candy making end of things.

Your chocolates are sold in boxes as well as individually. Opening inventory, sales, new stock, and closing inventory data are recorded algebraically on a weekly activity sheet. (Because your operation is so small, only one type of chocolate candy is sold at a time. For instance, if there are 6 boxes and 27 individual pieces of chocolate in the store when it opens, you would write $6b + 27$ in the "opening inventory" column. If someone buys two and a half boxes of candy and another person buys a box and two pieces of chocolate, you would record it as $2.5b + (b + 2)$, or simplified as $3.5b + 2$.

You hired Rita and Rex, your niece and nephew, to run the candy shop every Monday while you were taking a chocolate making class during the month of September. Though you explained the system for recording inventory, sales, and new stock to the pair, they did not follow your directions and did not keep complete records of the store's activity for the four Mondays you were gone.

Your task is to translate Rita and Rex's written information into the language of algebra, determine the missing information, and then complete an inventory report for the days you were gone. You also decide to write a set of directions explaining your record-keeping system so that whoever works for you in the future can keep track of things properly.

Steps for completing the tasks:

1. Using the information recorded on Rita and Rex's Chocolate Shop Log A, translate their entries from written words to algebraic expressions and record these on Chocolate Shop Log B. If an expression can be rewritten, do so below your original one. Show your work in the space provided.

2. Where entries are missing on Rita and Rex's sheet, determine the missing information (boxes and pieces bought, sold, or remaining at the end of the day), and record as a simplified algebraic expression on your log. Show your work in the space provided. **On some days both the sales and new stock data are missing. In these cases, determine reasonable quantities for both that will result in the recorded inventory at the end of the day and express these sales and new stock quantities algebraically on your log.**

3. Write a set of directions explaining the proper way to record opening inventories, daily sales, and new stock information as well as how to determine the closing inventory algebraically. These directions should include examples and be clear and organized so that anyone you hire to run the shop in the future can understand the system.

4. Submit your work to your business partner (your teacher).

Specific Grading Criteria:

1. Translate written inventory, sales, and new stock information into algebraic expressions.
2. Simplify algebraic expressions.
3. Add and subtract algebraic expressions to determine missing entries.
4. Explain how to write and simplify algebraic expressions to represent candy quantities (sales and new stock), and how to add and subtract these expressions to determine ending inventories.

Sweet Tooth Chocolate Shop

Shop Log A Completed by: Rita and Rex

DATE	Opening Inventory	Sales Total	New Stock	Closing Inventory
Monday, September 3	We counted 7 boxes and 60 chocolates, but then we found another box and 4 more chocolates.	Four different people came in today, and each bought one third of a box and 13 candies.	Can't remember.	15 full boxes and thirty-two chocolates.
Monday, September 10	18 and 1/3 boxes and 20 chocolates	Three kids bought 4 chocolates each, then a bunch of people came in and bought 14 full boxes and 6 chocolates.	In the morning boxes were brought in. Some were missing pieces of chocolate. We didn't count any of it.	26 and two-thirds boxes and 11 chocolates
Monday, September 17	24.25 boxes and one dozen chocolates.	Don't remember.	Don't remember, but less than 9 boxes.	12 boxes, 4 chocolates.
Monday, September 24	34 boxes and 17 chocolates.	Too busy to keep track.	They brought a little bit in, but we didn't have time to count it.	Only two full boxes and a box that was 75% full.

Sweet Tooth Chocolate Shop

Shop Log B Completed by: ____________________________

DATE	Opening Inventory	Sales Total	New Stock	Closing Inventory
Sept. 3	Expression: ***8b + 64***	Expression: ***4(1/3b+13)= 4/3b + 52***	Show all work: ***8 1/3b + 20***	Expression: ***15b + 32***
Sept. 10	Expression: ***18 1/3b+20***	Expression: ***3(4) + 14b + 6= 14b + 18***	Show all work: ***22 1/3b + 9***	Expression: ***26 2/3b + 11***
Sept. 17	Expression: ***24.25b + 12***	Show all work for determining reasonable sales and new stock: ***Answers will vary Possible 14.25b +12 2b + 4***		Expression: ***12b+4***
Sept. 24	Expression: ***34b + 17***	Show all work for determining reasonable sales and new stock: ***Answers will vary*** ***33.25b + 17***		Show all work: ***2b + 0.75b= 2.75b***

Name____________________________

Culminating Task: <u>Sweet Tooth Chocolate Shop</u>

Description of the Task:

As co-owner and shop keeper of Sweet Tooth Chocolates, a small candy company, you are responsible for keeping clear, accurate records of inventory and sales in the candy shop and sharing this information with your business partner, who runs the candy making end of things.

Your chocolates are sold in boxes as well as individually. Opening inventory, sales, new stock, and closing inventory information is recorded algebraically on a weekly activity sheet. (Because your operation is so small, only one type of chocolate candy is sold at a time. For instance, if there are 6 boxes and 27 individual pieces of chocolate in the store when it opens, you would write 6b + 27 in the "opening inventory" column. If someone buys two and a half boxes of candy and another person buys a box and two pieces of chocolate, you would record it as 2.5b + (b + 2), or simplified as 3.5b + 2.

You hired Rita and Rex, your niece and nephew, to run the candy shop every Monday while you were taking a chocolate making class during the month of September. Though you explained the system for recording inventory, sales, and new stock to the pair, they did not follow your directions and did not keep complete records of the store's activity for the four Mondays you were gone.

Your task is to translate Rita and Rex's written information into the language of algebra, determine the missing information, then complete an inventory report for the days you were gone. You also decide to write a set of directions explaining your record-keeping system so that whoever works for you in the future can keep track of things properly.

Steps for completing the tasks:

5. Using the information recorded on Rita and Rex's Chocolate Shop Log A, translate their entries from written words to algebraic expressions and record these on Chocolate Shop Log B. If an expression can be rewritten, do so below your original one. Show your work in the space provided.

6. Where entries are missing on Rita and Rex's sheet, determine the missing information (boxes and pieces bought, sold, or remaining at the end of the day), and record as a simplified algebraic expression on your log. Show your work in the space provided. **On some days both the sales and new stock data are missing. In these cases, determine reasonable quantities for both that will result in the recorded inventory at the end of the day and express these sales and new stock quantities algebraically on your log.**

7. Write a set of directions explaining the proper way to record opening inventories, daily sales, and new stock information as well as how to determine the closing inventory algebraically. These directions should include examples and be clear and organized so that anyone you hire to run the shop in the future can understand the system.

8. Submit your work to your business partner (your teacher).

Specific Grading Criteria:

5. Translate written inventory, sales, and new stock information into algebraic expressions.
6. Simplify algebraic expressions.
7. Add and subtract algebraic expressions to determine missing entries.
8. Explain how to write and simplify algebraic expressions to represent candy quantities (sales and new stock), and how to add and subtract these expressions to determine ending inventories.

Sweet Tooth Chocolate Shop

Shop Log A Completed by: Rita and Rex

DATE	Opening Inventory	Sales Total	New Stock	Closing Inventory
Monday, September 3	We counted 7 boxes and 60 chocolates, but then we found another box and 4 more chocolates.	Four different people came in today, and each bought one third of a box and 13 candies.	Can't remember.	15 full boxes and thirty-two chocolates.
Monday, September 10	18 and 1/3 boxes and 20 chocolates	Three kids bought 4 chocolates each, then a bunch of people came in and bought 14 full boxes and 6 chocolates.	In the morning boxes were brought in. Some were missing pieces of chocolate. We didn't count any of it.	26 and two-thirds boxes and 11 chocolates
Monday, September 17	24.25 boxes and one dozen chocolates.	Don't remember.	Don't remember, but less than 9 boxes.	12 boxes, 4 chocolates.
Monday, September 24	34 boxes and 17 chocolates.	Too busy to keep track.	They brought a little bit in, but we didn't have time to count it.	Only two full boxes and a box that was 75% full.

Sweet Tooth Chocolates
Shop Log B

<table>
<tr><th>DATE</th><th>Opening Inventory</th><th>Sales Total</th><th>New Stock</th><th>Closing Inventory</th></tr>
<tr><td>Sept. 3</td><td>Expression:</td><td>Expression:</td><td>Show all work:</td><td>Expression:</td></tr>
<tr><td>Sept. 10</td><td>Expression:</td><td>Show all work:</td><td>Expression:</td><td>Expression:</td></tr>
<tr><td>Sept. 17</td><td>Expression:</td><td colspan="2">Show all work for determining reasonable sales and new stock:</td><td>Expression:</td></tr>
<tr><td>Sept. 24</td><td>Expression:</td><td colspan="2">Show all work for determining reasonable sales and new stock:</td><td>Show all work:</td></tr>
</table>

Unit Web Links

Back to Task Table

http://www.yummymath.com/?s=6.EE

MGSE6.EE.1 Write and evaluate expressions involving whole-number exponents.
https://www.illustrativemathematics.org/content-standards/6/EE/A/1/tasks/532
https://www.illustrativemathematics.org/content-standards/6/EE/A/1/tasks/891
https://www.illustrativemathematics.org/content-standards/6/EE/A/1/tasks/1523
http://www.openmiddle.com/order-of-operations-3/
http://www.openmiddle.com/order-of-operations-2/
http://www.openmiddle.com/order-of-operations/
http://nzmaths.co.nz/resource/four-fours-challenge

MGSE6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.
https://www.illustrativemathematics.org/content-standards/6/EE/A/2/tasks/421
https://www.illustrativemathematics.org/content-standards/6/EE/A/2/tasks/540

MGSE6.EE.2a Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5-y.*

MGSE6.EE.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.
http://nzmaths.co.nz/resource/cup-capers

MGSE6.EE.2c Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$.

MGSE6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*
https://www.illustrativemathematics.org/content-standards/6/EE/A/3/tasks/997
http://www.openmiddle.com/product-of-distributive-property/
http://www.pbslearningmedia.org/resource/mgbh.math.nf.commprop/commutative-and-associative-equations/
http://media.mivu.org/mvu_pd/a4a/homework/applets_multiplication.html
http://illuminations.nctm.org/Lesson.aspx?id=2682

https://www.teachervision.com/tv/printables/0134356500_M1MUTM27.pdf

MGSE6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them.) *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*
https://www.illustrativemathematics.org/content-standards/6/EE/A/4/tasks/461
https://www.illustrativemathematics.org/content-standards/6/EE/A/4/tasks/542
http://www.learner.org/courses/learningmath/algebra/session6/part_c/index.html
http://illuminations.nctm.org/activity.aspx?id=3529

MGSE6.NS.4 Find the common multiples of two whole numbers less than or equal to 12 and the common factors of two whole numbers less than or equal to 100.

a. Find the greatest common factor of 2 whole numbers and use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factors. (GCF) *Example: 36 + 8 = 4(9 + 2)*
b. Apply the least common multiple of two whole numbers less than or equal to 12 to solve real-world problems.

https://illuminations.nctm.org/Activity.aspx?id=3530
http://www.rda.aps.edu/mathtaskbank/pdfs/tasks/6-8/t68gears.pdf
http://www.learner.org/courses/learningmath/number/session6/part_a/area.html