Georgia
Standards of Excellence
Curriculum Frameworks

Mathematics

GSE Grade 6

Unit 4: One Step Equations and Inequalities

Georgia Department of Education

Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

Unit 4
One Step Equations and Inequalities

Table of Contents

OVERVIEW

In this unit students will:

- Determine if an equation or inequality is appropriate for a given situation.
- Solve mathematical and real-world problems with equations.
- Represent real-world situations as inequalities.
- Interpret the solutions to equations and inequalities.
- Represent the solutions to inequalities on a number line.
- Analyze the relationship between dependent and independent variables through the use of tables, equations and graphs.

Beginning experiences in solving equations will require students to understand the meaning of the equation as well as the question being asked. **The use of illustrations, drawings, and balance models to represent and solve equations and inequalities will help students to develop this understanding.** Solving equations will also require students to develop effective strategies such as fact families, and inverse operations. As effective strategies are developed, students will revisit rate and proportional reasoning problems and solve them using strategies developed in solving similar one-step equations.

Students will represent and model equations and inequalities that are based on mathematical and real-world problems. Presented with these situations, students must determine if a single value is required as a solution or if the situation allows for multiple solutions. This creates the need for both equations (single solution for the situation) and inequalities (multiple solutions for the situation). When working with inequalities, students will work with situations in which the solution is not limited to the set of positive whole numbers but includes positive rational numbers. As an extension to this concept, certain situations may require a solution between two numbers. Therefore, the exploration with students as to what this would look like both on a number line and symbolically will be explored.

The process of translating between mathematical phrases and symbolic notation is essential in the writing of equations and inequalities for a situation. This is a two-way process and students will be able to write a mathematical phrase for an equation.

The goal is to help students connect the pieces. This is done by having students use multiple representations for mathematical relationships. Students will translate freely among the story, words (mathematical phrases), models, tables, graphs and equations/inequalities. Given any one of these representations, students should be able to develop the others.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic representations for given contexts and can create contexts given equations or inequalities.
2. Reason abstractly and quantitatively. Students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, graphs, and tables.
4. Model with mathematics. Students model problem situations in symbolic, graphic, tabular, and contextual formats. Students form expressions, equations, and inequalities from real-world contexts and connect symbolic and visual representations.
5. Use appropriate tools strategically. Students use number lines to graph equations and inequalities. Students use tables to organize information to write equations and inequalities.
6. Attend to precision. Students precisely define variables.
7. Look for and make use of structure. Students seek patterns or structures to model and solve problems using tables and equations. Students seek patterns or structures to model problems using tables and inequalities. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 3(2 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality, $c = 6$ by division property of equality).
8. Look for and express regularity in repeated reasoning. Students generalize effective processes for representing equations and inequalities based upon experiences. Students will be solving for equations.

STANDARDS FOR MATHEMATICAL CONTENT

Reason about and solve one-variable equations and inequalities.

MGSE6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

MGSE6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

MGSE.6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

MGSE.6.EE.8 Write an inequality of the form $x < c$ or $x > c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x < c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

<u>Represent and analyze quantitative relationships between dependent and independent variables.</u>

MGSE6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another.

a. Write an equation to express one quantity, the dependent variable, in terms of the other quantity, the independent variable.
b. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*

<u>Understand ratio concepts and use ratio reasoning to solve problems.</u>

MGSE.6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.

MGSE.6.RP.3a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

MGSE.6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed.

MGSE.6.RP.3c Find a percent of a quantity as a rate per 100 (e.g. 30% of a quantity means 30/100 times the quantity); given a percent, solve problems involving finding the whole given a part and the part given the whole.

MGSE.6.RP.3d Given a conversion factor, use ratio reasoning to convert measurement units within one system of measurement and between two systems of measurements (customary and metric); manipulate and transform units appropriately when multiplying or dividing quantities. *For example, given 1 in. = 2.54 cm, how many centimeters are in 6 inches?*

BIG IDEAS

- Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules.
- Relate and compare different forms of representation for a relationship.
- Use values from specified sets to make an equation or inequality true.
- Develop an initial conceptual understanding of different uses of variables.
- Graphs can be used to represent all of the possible solutions to a given situation.
- Many problems encountered in everyday life can be solved using proportions, equations or inequalities.
- Students will solve one-step equations.

ESSENTIAL QUESTIONS

- How is an equation like a balance? How can the idea of balance help me solve an equation?
- What strategies can I use to help me understand and represent real situations using proportions, equations and inequalities?
- How can I write, interpret and manipulate proportions, equations, and inequalities?
- How can I solve a proportion and an equation?
- How can I tell the difference between an expression, equation and an inequality?
- How are the solutions of equations and inequalities different?
- What does an equal sign mean mathematically?
- How can proportions be used to solve problems?
- How can proportional relationships be described using the equation $y = kx$?
- How can proportional relationships be represented using rules, tables, and graphs?
- How can the graph of $y = kx$ be interpreted for different contexts?
- How does a change in one variable affect the other variable in a given situation?
- Which tells me more about the relationship I am investigating, a table, a graph or a formula?

It is expected that students will continue to develop and practice strategies to build their capacity to become fluent in mathematics and mathematics computation. The eventual goal is automaticity with math facts. This automaticity is built within each student through strategy development and practice. The following section is presented in order to develop a common understanding of the ideas and terminology regarding fluency and automaticity in mathematics.

CONCEPTS/SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Using parentheses, brackets, or braces in numerical expressions and evaluate expressions with these symbols.
- Write and interpret numerical expressions.

- Generating two numerical patterns using two given rules.
- Interpret a fraction as division
- Operations with whole numbers, fractions, and decimals

FLUENCY

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply "how to get the answer" and instead support students' ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students:

- flexibly use a combination of deep understanding, number sense, and memorization.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.

For more about fluency, see:
http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and
http://joboaler.com/timed-tests-and-the-development-of-math-anxiety/

STRATEGIES FOR TEACHING AND LEARNING

The skill of solving an equation must be developed *conceptually* before it is developed *procedurally*. This means that students should be thinking about what numbers could possibly be a solution to the equation before solving the equation. For example, in the equation $x + 21 = 32$ students know that $21 + 9 = 30$ therefore the solution must be 2 more than 9 or 11, so $x = 11$. Provide multiple situations in which students must determine if a single value is required as a solution, or if the situation allows for multiple solutions. This creates the need for both types of equations (single solution for the situation) and inequalities (multiple solutions for the situation). **Solutions to equations should not require using the rules for operations with negative numbers since the conceptual understanding of negatives and positives is being introduced in Grade 6.**

When working with inequalities, provide situations in which the solution is not limited to the set of positive whole numbers **but includes rational numbers.** This is a good way to practice fractional numbers and introduce negative numbers. Students need to be aware that numbers less than zero could be part of a solution set for a situation. As an extension to this concept, certain situations may require a solution between two numbers. For example, a problem situation may have a solution that requires more than 10 but not greater than 25. Therefore, the exploration with students as to what this would look like both on a number line and symbolically is a reasonable extension.

The process of translating between mathematical phrases and symbolic notation will also assist students in the writing of equations/inequalities for a situation. This process should go both ways; students should be able to write a mathematical phrase for an equation. Additionally, the writing of equations from a situation or story **does not come naturally** for many students. **A strategy for assisting with this is to give students an equation and ask them to come up with the situation/story that the equation could be referencing.**

The goal is to help students connect the pieces together. This can be done by having students use multiple representations for the mathematical relationship. Students need to be able to translate freely among the story, words (mathematical phrases), models, tables, graphs and equations. They also need to be able to start with any of the representations and develop the others.

Provide multiple situations for the student to analyze and determine what unknown is dependent on the other components. For example, how far I travel is dependent on the time and rate at which I am traveling.

Throughout the expressions and equations domain in Grade 6, students need to have an understanding of how the expressions or equations relate to situations presented, as well as the process of solving them.

The use of technology, including computer apps and other hand-held technology allows the collection of real-time data or the use of actual data to create tables and charts. It is valuable for students to realize that although real-world data often is not linear, a line sometimes can model the data.

COMMON MISCONCEPTIONS

The equal sign represents that the two sides of the equation balance, have the exact same value. Some students have developed the notion that the equal sign means "the answer is," based often on limited experiences in elementary school where the answer is always on the right side of the equal sign. In a study conducted in the late 1990s, researchers found that sixth-grade students struggled to interpret the following equation:

$$8 + 4 = \square + 5$$

All 145 students who were given this problem incorrectly believed that either 12 or 17 should go in the box above (Falkner, Levi, and Carpenter, 1999). This finding reiterates the need for teachers to constantly push their students' understanding of equality.

Students need practice translating verbal expressions into expressions and equations, and also translating expressions and equations into verbal expressions. The wording must dictate the order of the terms. For example, "three more than a number" is transcribed "x + 3", not "3 + x" even though those expressions are equivalent. In the example, "three less than a number", it is imperative that the order be "x – 3".

Students often confuse the statement "five less than a number" ("x – 5") with the inequality, "five is less than a number" (5 < x). The difference between an expression and an inequality needs to be clearly distinguished.

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for middle school students. **Note – Different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**

http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary

http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

http://www.corestandards.org/Math/Content/mathematics-glossary/glossary

- **Addition Property of Equality:** Adding the same number to each side of an equation produces an equivalent expression.

- **Constant of proportionality**: The constant value of the ratio of two proportional quantities x and y; usually written $y = kx$, where k is the constant of proportionality. In a proportional relationship, $y = kx$, k is the constant of proportionality, which is the value of the ratio between y and x.

- **Dependent variable**- A variable that depends on other factors. For example, a test score could be a dependent variable because it could change depending on several factors such as how much you studied, how much sleep you got the night before you took the test, or even how hungry you were when you took it.

- **Direct Proportion (Direct Variation)**: The relation between two quantities whose ratio remains constant. When one variable increases the other increases proportionally: When one variable doubles the other doubles, when one variable triples the other triples, and so on. When A changes by some factor, then B changes by the same factor: $A=kB$, where k is the constant of proportionality.

- **Division Property of Equality:** States that when both sides of an equation are divided by the same number, the remaining expressions are still equal

- **Equation:** A mathematical sentence that contains an equal sign

- **Independent variable:** A variable that stands alone and isn't changed by the other variables you are trying to measure. For example, someone's age might be an independent variable.

- **Inequality:** A mathematical sentence that contains the symbols $>$, $<$, $\geq$, or $\leq$.

- **Inverse Operation:** A mathematical process that combines two or more numbers such that its product or sum equals the identity.

- **Multiplication Property of Equality:** States that when both sides of an equation are multiplied by the same number, the remaining expressions are still equal.

- **Proportion:** An equation which states that two ratios are equal.

- **Solution:** the set of all values which, when substituted for unknowns, make an equation true.

- **Substitution:** the process of replacing a variable in an expression with its actual value.

- **Subtraction Property of Equality:** States that when both sides of an equation have the same number subtracted from them, the remaining expressions are still equal.

- **Term:** A number, a variable, or a product of numbers and variables.

- **Variable:** A letter or symbol used to represent a number or quantities that vary.

INSTRUCTIONAL RESOURCES/TOOLS

Use graphic organizers as tools for connecting various representations.

- Hands On Equations
- Pedal Power – NCTM illuminations lesson on translating a graph to a story
- Interactive grapher from the National Library of Virtual Manipulatives
- Algebra Balance Scales from the National Library of Virtual Manipulatives
- Manipulatives in order to solve equations

FORMATIVE ASSESSMENT LESSONS (FAL)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Guide.

SPOTLIGHT TASKS

For middle and high schools, each Georgia Standards of Excellence mathematics unit includes at least one Spotlight Task. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Georgia Standards of Excellence, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky.

3-ACT TASKS

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.
More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

TASKS

Task	Task Type / Grouping Strategy	Content Addressed	Performance Standards
Set It Up	Performance Task/ *Individual/Partner Task*	One Step Equations	MGSE.6.EE.5 MGSE.6.EE.6 MGSE.6.EE.7
Building with Toothpicks **(Spotlight Task)**	Performance Task/ *Individual/Partner Task*	Algebraic Expressions	MGSE.6.EE.5 MGSE.6.EE.6 MGSE.6.EE.7
Fruit Punch **(Spotlight Task)**	Performance Task/ *Partner/Group Task*	Proportional Relationships	MGSE.6.EE.5 MGSE.6.EE.6 MGSE.6.EE.7 MGSE.6.RP.3
Graphing Stories	Learning Task *Partner/Small Group Task*	Dependent and Independent Variables	MGSE.6.EE.9
Making Sense of Graphs	Performance Task/ *Individual/Partner Task*	Proportions, Rules, and Algebra	MGSE.6.EE.5 MGSE.6.EE.6 MGSE.6.EE.7 MGSE.6.EE.9 MGSE.6.RP.3
Analyzing Tables	Performance Task/ *Individual/Partner Task*	Equations	MGSE.6.EE.5 MGSE.6.EE.6 MGSE.6.EE.7 MGSE.6.EE.9 MGSE.6.RP.3
Who Has Faulty Thinking? **(FAL)**	Formative Assessment Lesson *Individual/Partner*	Equations	MGSE.6.EE.6
Real-life Equations (FAL)	Formative Assessment Lesson *Individual/Partner*	Equations	MGSE.6.EE.5 MGSE.6.EE.6 MGSE.6.EE.7
When is it Not Equal?	Performance Task/ *Individual/Partner Task*	Inequalities	MGSE.6.EE.5 MGSE.6.EE.8
Evaluating Statements about	Formative Assessment Lesson	Inequalities	MGSE.6.EE.5 MGSE.6.EE.6

Number Operations (FAL)	*Individual/Partner*		MGSE.6.EE.7 MGSE.6.EE.8
It's on Sale!	Performance Task/ *Individual/Partner Task*	Ratios and Percents	MGSE.6.RP.3c
The Catering Job	Performance Task/ *Individual/Partner Task*	Unit Rates	MGSE.6.RP.3b
Culminating Task: Want Ads	Culminating Task/ *Individual*	Culminating	MGSE.6.EE.5 MGSE.6.EE.6 MGSE.6.EE.7 MGSE.6.EE.8 MGSE.6.EE.9 MGSE.6.RP.3

Set It Up

Back to Task Table

Adapted from *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School (Carpenter, Franke, and Levi, 2003).*

In this task, students will use the balance method to write and solve one-step equations and is designed as a learning task. Students will also learn why conventions are an integral part of mathematics.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
MGSE6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
MGSE.6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic representations for given contexts and can create contexts given equations.
2. Reason abstractly and quantitatively. Students represent real-world contexts through the use of real numbers and variables in mathematical expressions, equations.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using verbal or written explanations accompanied by equations, models, and tables.
4. Model with mathematics. Students model problem situations in symbolic, tabular, and contextual formats. Students form expressions, equations, or from real-world contexts and connect symbolic and visual representations.
6. Attend to precision. Students precisely define variables. Students substitute solutions into equations to verify their results.
5. Use appropriate tools strategically. Students use tables to organize information to write equations.
7. Look for and make use of structure. Students seek patterns or structures to model and solve problems using tables, equations.
8. Look for and express regularity in repeated reasoning. Students generalize effective processes for representing and solving equations based upon experiences.

ESSENTIAL QUESTIONS

- Why do we need conventions in mathematics?
- How do I solve a one-step equation?

Set It Up

Part I.

Marcus has 6 pet rabbits. He keeps them in two cages that are connected so they can go back and forth between the cages. One cage is orange and the other cage is blue.

1. Show all the ways that 6 rabbits can be in two cages.

   Solution

Orange	*Blue*
6	*0*
5	*1*
4	*2*
3	*3*
2	*4*
1	*5*
0	*6*

   Comment

   Students may choose to illustrate the above situation with a pictorial representation.

2. Write an equation that represents the rabbits.

   Solution

   $r + b = 6$ ***or*** $b = 6 - r$ ***or*** $r = 6 - b$

   Comment

   Students may need to add an additional column to their table to assist them in writing an equation.

Orange	*Blue*	*Total*
6	*0*	*6*
5	*1*	*6*
4	*2*	*6*
3	*3*	*6*
2	*4*	*6*
1	*5*	*6*
0	*6*	*6*

   Students may also use their understanding of fact families to write multiple equations that represent the rabbits.

3. Write a different equation that represents the rabbits.

   Solution

   $r + b = 6$ ***or*** $b = 6 - r$ ***or*** $r = 6 - b$

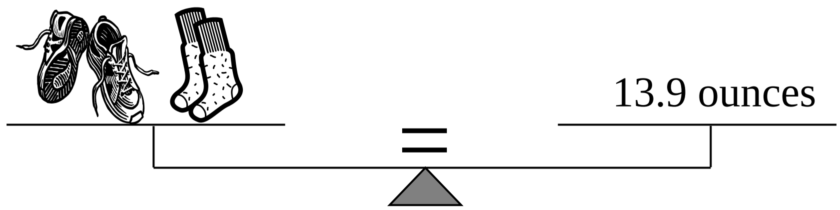
4. Write a different equation that represents the rabbits.

   Solution

   $r + b = 6$ ***or*** $b = 6 - r$ ***or*** $r = 6 - b$

Note: Students should analyze the different equations and reflect on the addition and subtraction properties of equalities. Teachers can also begin a discussion on the relationship between the properties of equality and inverse operations.

Part II.
Find the weight of the pair of shoes and pair of socks.



1. Write an equation that represents the above balance scale.
   Solution

    +  = ***13.9 ounces***

2. What does 13.9 represent in the equation?
   Solution
   Thirteen and 9 tenths (13.9) represents the combined weight of the shoes and socks.

3. What do you notice about the shoes if the pair of socks weighs 0.8 ounces? How can you find the weight of the pair of shoes if the pair of socks weighs 0.8 ounces?
   Solution
   The shoes weigh more than the socks and less than 13.9 ounces if the total weight is 13.9 ounces.

   $$13.9 - 0.8 = 13.1$$

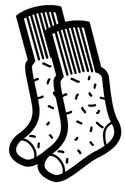
   Comment
   If students use trial and error to determine the weight of the socks, guide them to use a different method to also find the weight. Guiding questions may be necessary to help students discover and use the inverse operation to "undo" the given operation to find the weight. Students need to determine that they can use the inverse operation to solve problems. This will help students understand why conventions are put in place to solve equations and will assist them when solving more complicated equations.

4. How can you find the weight of the pair of socks if the pair of shoes weighs 13.1 ounces?
   Solution
   The socks weigh less than the shoes and less than 13.9 ounces if the total weight is 13.9 ounces.

   $$13.9 - 13.1 = 0.8$$

   Comment

If students use trial and error to determine the weight of the socks, guide them to use a different method to also find the weight. Guiding questions may be necessary to help students discover and use the inverse operation to "undo" the given operation to find the weight. Students need to determine that they can use the inverse operation to solve problems. This will help students understand why conventions are put in place to solve equations and will assist them when solving more complicated equations.

5.  +  = 13.9 ounces

a. Select a variable to represent the athletic shoes (tennis shoes).
Comment
Students may select any letter to represent the shoes. To stay consistent we will select a to represent the athletic shoes.

b. Select a variable to represent the socks.
Comment
Students may select any letter to represent the shoes. To stay consistent we will select s to represent the athletic shoes.

c. Write an equation that represents the above equations using variables instead of pictures.
Solution
$a + s = 13.9$

Comment
Students may need to first write the equation in words, and then write the equation using variables.

d. Write an equation in terms of athletic shoes.
Solution
$a = 13.9 - s$

Comment
Students may think about fact families to help them develop this equation.

e. Write an equation in terms of socks.
Solution
$s = 13.9 - a$

Set It Up

Adapted from *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School (Carpenter, Franke, and Levi, 2003).*

Part I.

Marcus has 6 pet rabbits. He keeps them in two cages that are connected so they can go back and forth between the cages. One cage is orange and the other cage is blue.

1. Show all the ways that 6 rabbits can be in two cages.

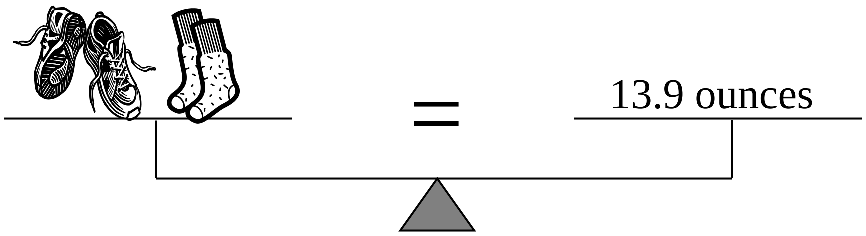
2. Write an equation that represents the rabbits.

3. Write a different equation that represents the rabbits.

4. Write a different equation that represents the rabbits.

Part II.

Find the weight of the pair of shoes and pair of socks.



6. Write an equation that represents the above balance scale.

7. What does 13.9 represent in the equation?

8. What do you notice about the shoes if the pair of socks weighs 0.8 ounces? How can you find the weight of the pair of shoes if the pair of socks weighs 0.8 ounces?

9. How can you find the weight of the pair of socks if the pair of shoes weighs 13.1 ounces?

10.  +  = 13.9 ounces

a. Select a variable to represent the athletic shoes (tennis shoes).

b. Select a variable to represent the socks.

c. Write an equation that represents the above equations using variables instead of pictures.

d. Write an equation in terms of athletic shoes.

e. Write an equation in terms of socks.

Building with Toothpicks (Spotlight Task)

Back to Task Table

In this inquiry-based task, students will generalize a formula for expressing the perimeter of a figure built with toothpicks.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
MGSE6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
MGSE.6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will reflect on the reasonableness of their solutions and conjectures and revise their thinking accordingly.
3. Construct viable arguments and critique the reasoning of others. Students will discuss their observations and questions using the language of mathematics.
4. Model with mathematics. Students will use toothpicks to represent visual and spatial patterns.
7. Look for and make use of structure. Students will recognize the structure implicit in the visual and spatial patterns and use this to derive a formula for determining the perimeter of the *n*th figure in the sequence.
8. Look for and express regularity in repeated reasoning. Through repeated reasoning in the first four figures of a sequence, students will generalize mathematical relationships into a formula for determining the perimeter of the *n*th figure in the sequence.

ESSENTIAL QUESTIONS

- How do I evaluate an algebraic expression?
- How can variables be used to describe patterns?
- How do I solve a one-step equation?

MATERIALS

- Toothpicks (optional)
- Copy of generic student work sheet

TEACHER NOTES

In this task, students will watch the video *(Act 1),* then ask what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions *(Act 2).* Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them. Once students have made their discoveries, it is time for the great reveal *(Act 3)*. Teachers should support good student dialogue and take advantage of comments and questions to help guide students into correct mathematical thinking.

TASK COMMENTS

While students will probably come up with a wide variety of questions, this task, however, is designed to encourage students to describe and generalize patterns using algebraic expressions. Students will also write and solve one-step equations.

While the original task asks students about the perimeter of the shapes, students may go in a variety of directions. BE FLEXIBLE!!!!! As long as you are working towards finding the formula for the pattern, you are meeting the goal!

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:

Watch the video: http://youtu.be/sIIzAzBSp-M

ACT 2:

Student work time to discover how to find the perimeter (area, triangular numbers, etc) of Shape *n*.

ACT 3

Students will compare and share solution strategies.

- Reveal the answer. Test your theory.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.

ACT 4

Extension:

- Does this same rule apply to area? If not, what is the new rule?
- Make a new design and find the pattern.
- Figurate Numbers: https://www.learner.org/courses/mathilluminated/interactives/prim/

Intervention:

- Provide students with toothpicks to use as manipulatives.
- Encourage students to draw out the situation on graph paper.

Building with Toothpicks

Name:__________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate the result of the main question? Explain?

Place an estimate that is too high and too low on the number line

Low estimate *Place an "x" where your estimate belongs* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:________________________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Fruit Punch (Spotlight Task)

Back to Task Table

This task was adapted from ACHIEVE

In this inquiry-based task, students use proportional reasoning to determine a solution for "fixing" a mix of fruit punch that has too much fruit punch mix.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
MGSE6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
MGSE.6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
MGSE.6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students determine the desired ratio for fruit punch mix to water and evaluate whether or not the actual mix is proportionate to the ratio from the recipe.
3. Construct viable arguments and critique the reasoning of others. Students will discuss their observations and questions using the language of mathematics.
4. Model with mathematics. Students may use concrete representations (tiles, chips, etc.) or drawings (e.g., tape diagrams, number lines, rate tables) to reason about the problem.
6. Attend to precision. Students will attend to units of measure and precise academic language.
7. Look for and make use of structure. Students will look for proportionality in various ratios of fruit punch mix to water in order to determine a solution to the problem.

ESSENTIAL QUESTIONS

- What is a proportion?
- How can proportional relationships be represented using rules, tables, and graphs?

MATERIALS

- Tiles or chips (optional)
- Copy of generic student work sheet (end of this task)

TEACHER NOTES

In this task, students will watch the video *(Act 1)*, then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions *(Act 2)*. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them. Once students have made their discoveries, it is time for the great reveal *(Act 3)*. Teachers should support good student dialogue and take advantage of comments and questions to help guide students into correct mathematical thinking.

TASK COMMENTS

While students will probably come up with a wide variety of questions, this task, however, is designed to promote a deeper understanding of proportional reasoning and connect their understanding of similarity to direct proportions.

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:

Watch the video: http://youtu.be/QgmBR68ufxU

ACT 2:

Student work time to discover how fix the situation of the extra scoop being added. What is too little water? What is too much water?

ACT 3

Students will compare and share solution strategies.

- Reveal the answer. Watch the video: http://youtu.be/NLbS39Q_Bzc. Discuss the theoretical math versus the practical outcome.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.

ACT 4

Extension:

- Instead of adding an extra scoop of crystals, what would you do if an extra cup of water was added?
- The recipe calls for 2.5 scoops of crystals for 7 cups of water. How much water is needed for each scoop of crystal?

Intervention:

- Provide students with scoops and cups or tiles to use as manipulatives.
- Encourage students to draw out the situation to develop the proportions.

Fruit Punch

Name:________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate the result of the main question? Explain?		
	Place an estimate that is too high and too low on the number line	
Low estimate	*Place an "x" where your estimate belongs*	High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:_______________________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Graphing Stories

Back to Task Table

Adapted from Van de Walle, Karp, and Bay-Williams. (2010). *Elementary and Middle School Mathematics: Teaching Developmentally*, and Lambdin, D. Lynch, R. and H. McDaniel. (2000). Algebra in the Middle Grades. *Mathematics Teaching in the Middle School, 6,* 195-198.

In this task, students will reason about the varying relationship between a variety of dependent and independent variables by representing and discussing stories on open coordinate grids.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another.

a. Write an equation to express one quantity, the dependent variable, in terms of the other quantity, the independent variable.
b. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them. Students make sense of graphs, and in particular, how one variable relates to another.
2. Reason abstractly and quantitatively. Students represent a wide variety of real-world contexts through the use stories and graphs.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments about how to represent stories with graphs and vice versa.
4. Model with mathematics. Students model problem situations with graphs.
6. Attend to precision. Students precisely identify variables and corresponding units of measure.

ESSENTIAL QUESTIONS

- How does a change in one variable affect the other variable in a given situation?

MATERIALS

- Student pages (included in task) and a clock or stopwatch

TIME NEEDED

- 1 to 2 days

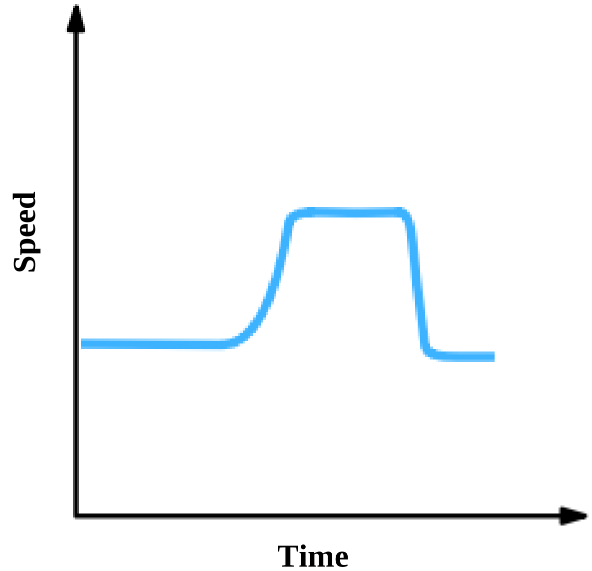
TASK COMMENTS

In this task, students will start to reason about the quantitative relationship between two variables that change in relationship to one another by graphing stories on empty coordinate grids. Students will not actually express these relationships with equations in this task – the purpose, rather, is to get students thinking about how change in one variable relates to change in another variable.

* Graphing Stories

<u>Introduction: Creating and Comparing Graphs of Two Stories</u>

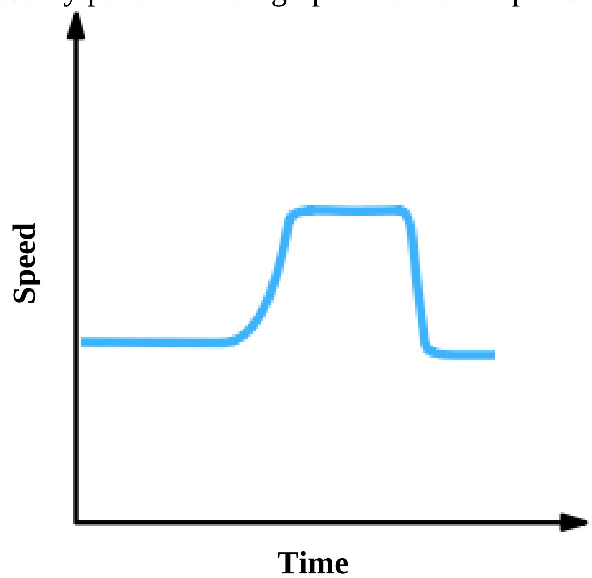
A girl walks down a set of stairs at a steady pace. When she gets to the bottom, she runs back up the stairs. Once she gets to the top, she walks around in a circle at a steady pace. Draw a graph that could represent this story.



Answers will vary slightly. Here is one possible interpretation.

Look for misconceptions. Some students may incorrectly represent walking "down" with a downward line, or running "up" with an upward line. Point out the units of measure on each axis to ensure students are attending to precision.

A boy walks up a hill at a steady pace. When he gets to the top, he runs down the hill. He then continues walking at a steady pace. Draw a graph that could represent this story.



Answers will vary slightly. Here is one possible interpretation.

Look for misconceptions. Some students may incorrectly represent walking "up" with an upward line, or running "down" with an downward line. Point out the units of measure on each axis to ensure students are attending to precision.

Should the shapes of both graphs be similar? Why or why not?
Yes, they should be mostly similar in shape because in both stories, the child is traveling at a slow but steady pace, then moving faster at a steady pace, before finally returning to a slower but steady speed.

Part II: Matching scenarios to graphs.

Match each story below to a graph on the following page. Be sure to justify your choices with mathematical evidence.

a. The temperature of a frozen dinner from 30 minutes before it is removed from the freezer until it is removed from the microwave and placed on the table. (Consider time 0 to be the moment the dinner is removed from the freezer.) ***Graph 5***

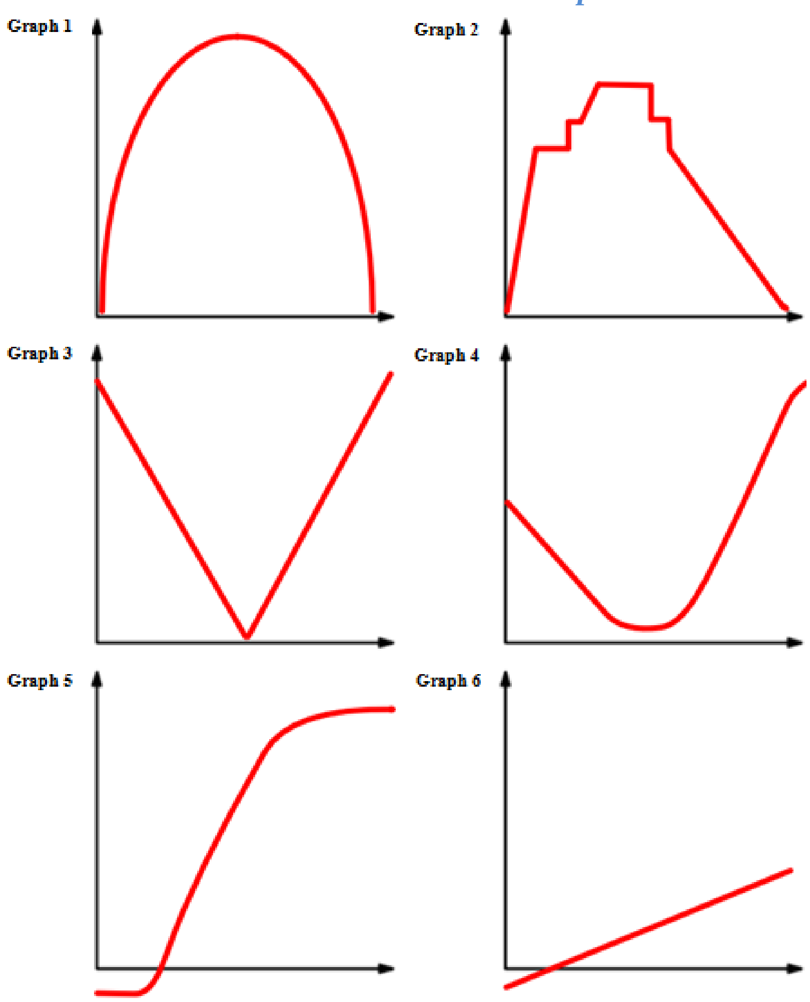
b. The value of a 1970 Volkswagen Beetle from the time it was purchased to the present. (It was kept in great condition). ***Graph 4***

c. The level of water in the bathtub from the time you begin to fill it to to the time it is completely empty after your bath. ***Graph 2***

d. Profit in terms of the number of items sold. ***Graph 6***

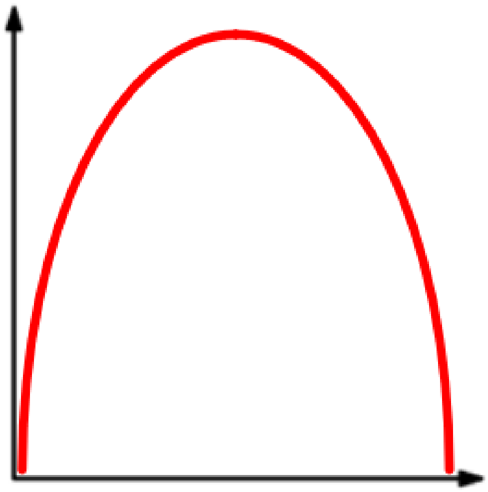
e. The height of a baseball thrown straight up into the air, from when it is thrown to the time it hits the ground. ***Graph 1***

f. The speed of the baseball in the situation in item e. ***Graph 3***

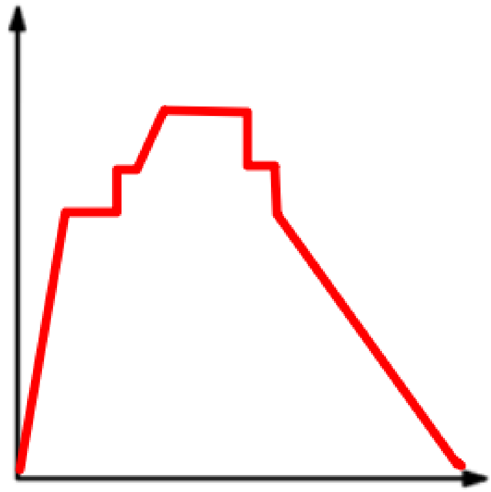


You may wish to project this image on the board.

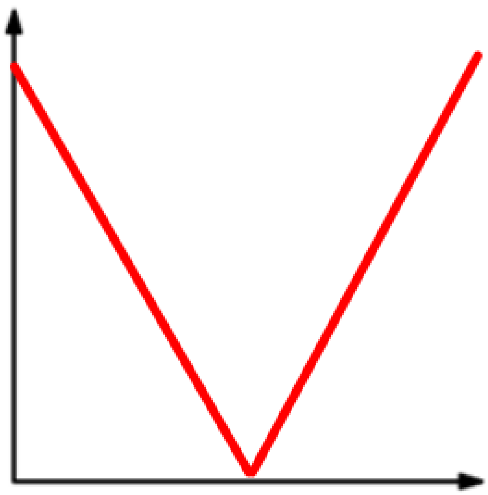
Graph 1



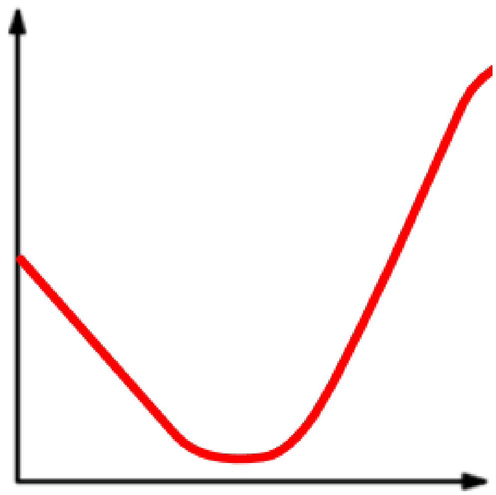
Graph 2



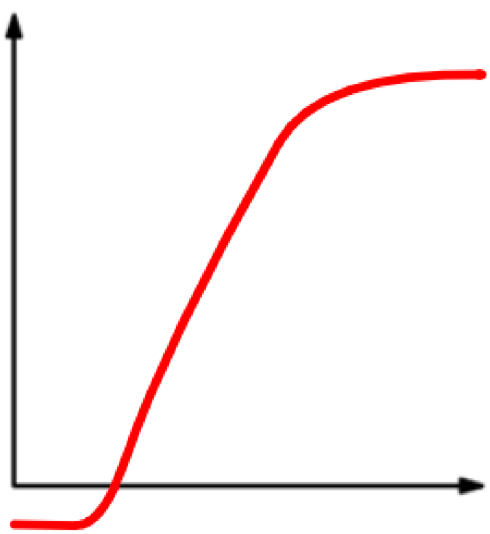
Graph 3



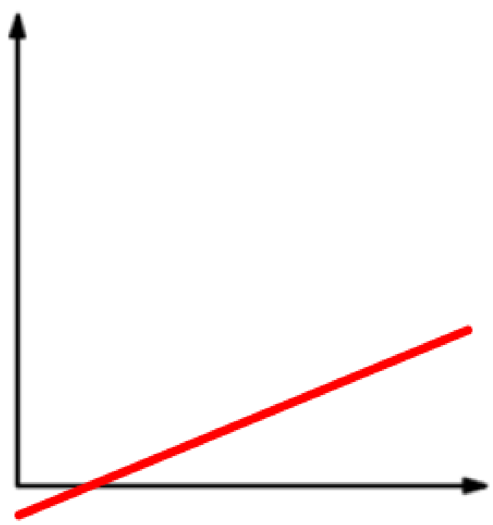
Graph 4



Graph 5



Graph 6

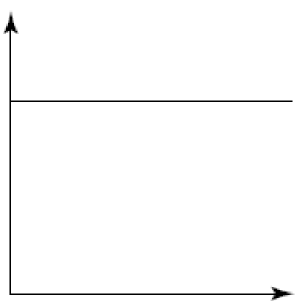


Part III Jumping Jack Activity

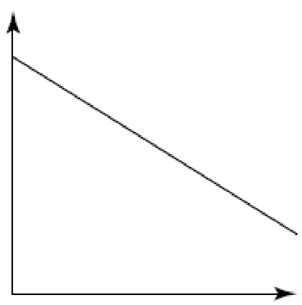
In groups of 4, you will conduct the following activity. Each student will need to take on one of the following roles: 1) jumper, 2) counter, 3) timer, and 4) recorder.

- The jumper will do jumping jacks for 2 minutes straight.
- The counter will count the number of jumping jacks performed aloud.
- The timer will monitor a stopwatch/clock and call out, "*Time!*" every 10 seconds.
- The recorder will keep a written record of the total number of jumping jacks performed at every 10-second interval.

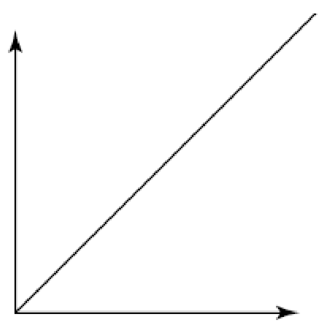
Before conducting this activity, look at the graphs below. If time is represented on the x-axis, and the number of jumping jacks performed is shown on the y-axis, make a prediction: which shape do you think will most likely match your data? Explain your reasoning in the space below.
Answers will vary. Be sure to allow students to debate which ones make sense and which ones do not make sense (based on this particular context). You may want to ask students to describe and/or translate each of these representations as a sense-making strategy.



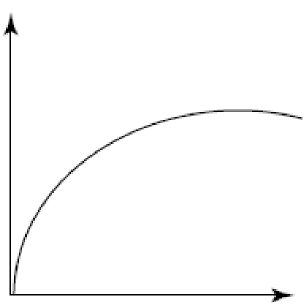
Graph A



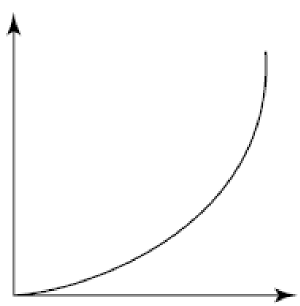
Graph B



Graph C



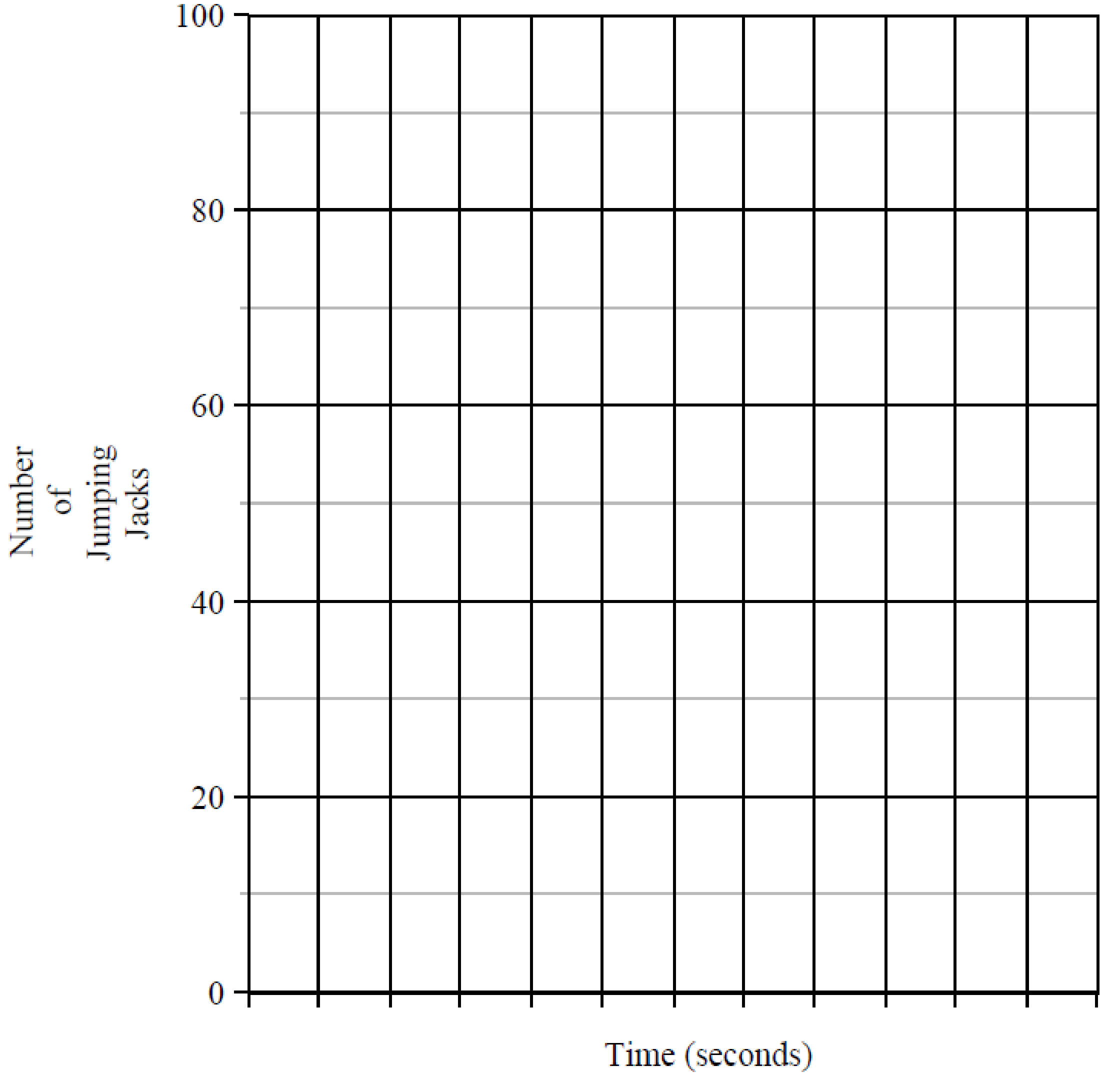
Graph D



Graph E

Now, graph your group's data on the grid below. Connect the dots with a ruler or straight edge.

Data will vary.



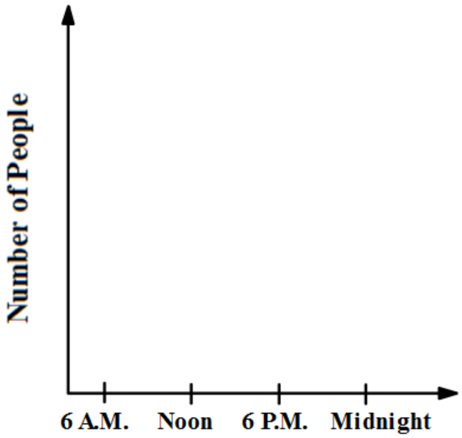
1. Explain the shape of your graph. Why is it steeper or flatter in certain parts?
   Students should explain that the flatter parts represent either resting time or periods in which fewer jumping jacks were performed, while steeper parts represent periods in which many jumping jacks were done within a shorter period of time (thus, the rate of jumping jacks was higher).
2. How would the shape of your graph change if the sizes of the intervals were changed? Give an example.
   Answers will vary. For example, if the time intervals were smaller, the graph would not be as wide. If the intervals for jumping jacks were larger, the graph would be "shorter".

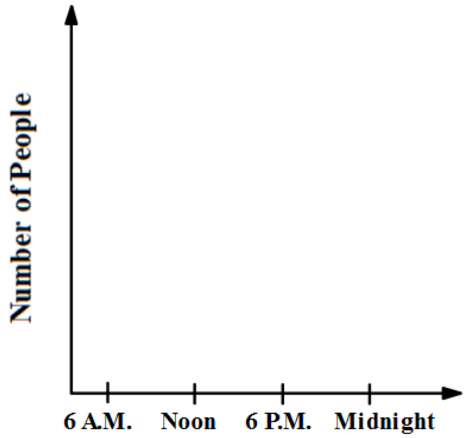
Part IV: Create a "Secret Location" Graph

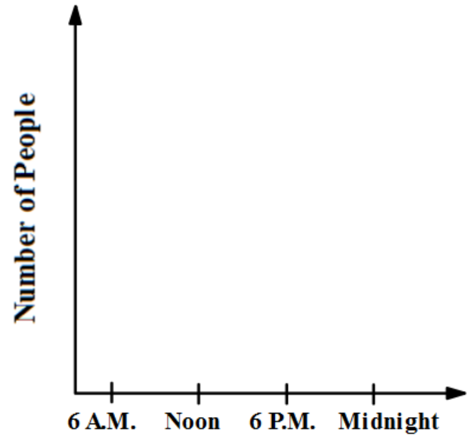
Using the grids below, create a graph that shows the varying number of people who might be present in the following "secret locations" at various times throughout a weekday.

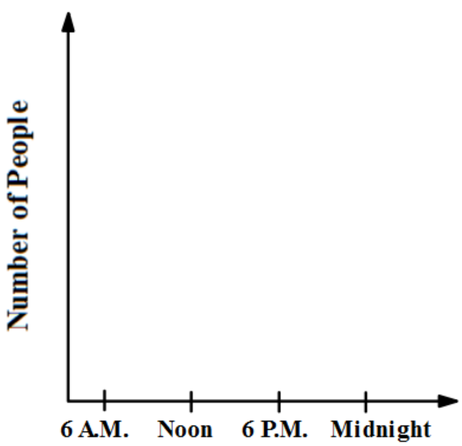
1. The number of people at the grocery store on a weekday. ***Graphs will vary.***
2. The number of people at your house on a school day. ***Graphs will vary.***
3. The number people driving a car on the road on a weekday. ***Graphs will vary.***
4. The number of people at McDonald's on a weekday. ***Graphs will vary.***
5. The number of people at school on a weekday. ***Graphs will vary.***
6. The number of people in a hospital on a weekday. ***Graphs will vary.***

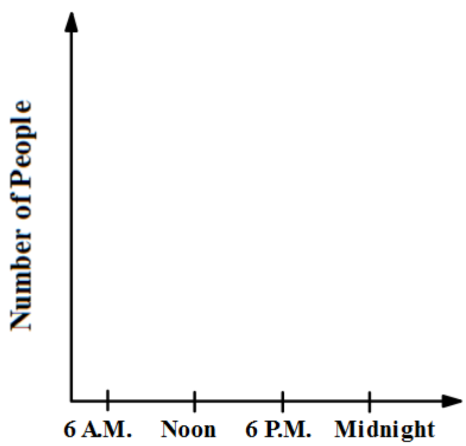
Students should be able to justify the shapes of their graphs.

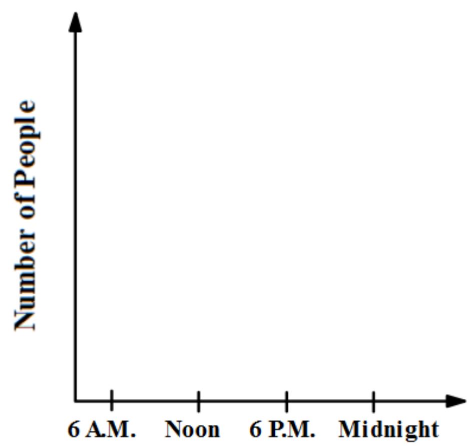












For one of the graphs above, explain the shape you drew. Be sure to relate the shape of the graph to the context of the scenario.

Responses will vary. Example response: The number of people at McDonald's spikes around 8 A.M. for breakfast, then tapers off around 9 A.M. when most people are in school or at work. It peaks again around noon because of lunch time, and then slows down before finally peaking again around 6 P.M. for dinner. After 8 P.M., the number of people at McDonald's drops until it closes at 11 P.M.

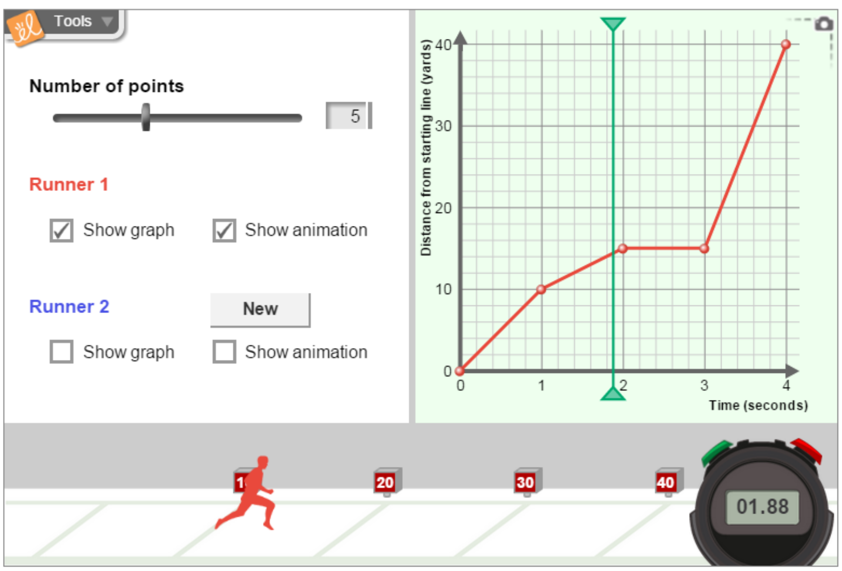
Extension

- Have students create their own stories to trade with a partner to graph. Or have students create graphs to trade with a partner to draft a corresponding story.
- In Part IV of this task, you may want to assign each scenario to small groups of students. After the groups create their graph, you can allow students to do a gallery walk and try to determine which scenarios match the graphs.

Intervention

Allow students to explore interactive graphs, such as:

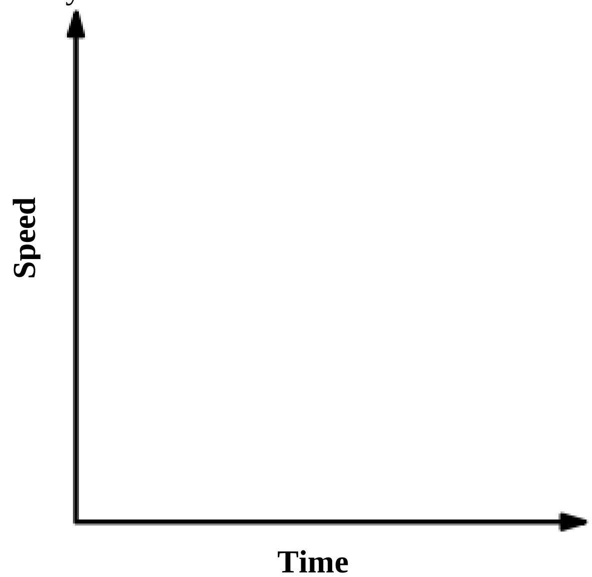
- http://www.colmanweb.co.uk/Assets/SWF/Skate_boarders.swf
- http://www.absorblearning.com/media/attachment.action?quick=wl&att=2335
- http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=260 This applet allows you to explore interactive distance-time and velocity-time graphs. You can also take screen shots of various graphical scenarios and allow students to practice interpreting the graphs (see below).



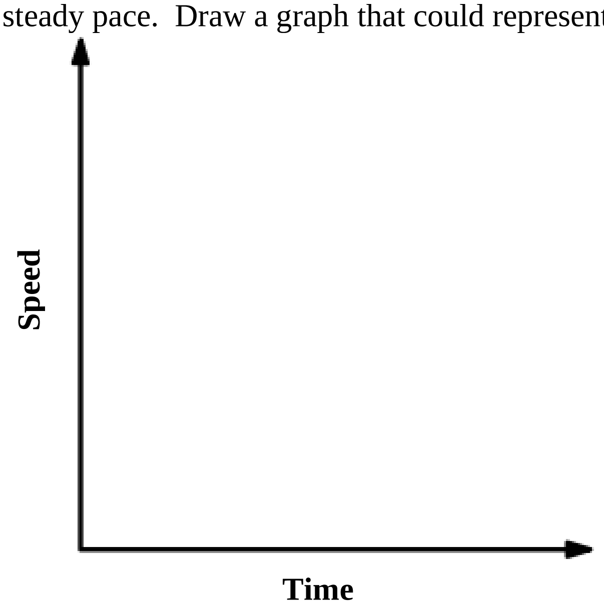
Graphing Stories Name:___________________________________

Introduction: Creating and Comparing Graphs of Two Stories

A girl walks down a set of stairs at a steady pace. When she gets to the bottom, she runs back up the stairs. Once she gets to the top, she walks around in a circle at a steady pace. Draw a graph that could represent this story.



A boy walks up a hill at a steady pace. When he gets to the top, he runs down the hill. He then continues walking at a steady pace. Draw a graph that could represent this story.



Should the shapes of both graphs be similar? Why or why not?

Part II: Matching scenarios to graphs.

Match each story below to a graph on the following page. Be sure to justify your choices with mathematical evidence.

g. The temperature of a frozen dinner from 30 minutes before it is removed from the freezer until it is removed from the microwave and placed on the table. (Consider time 0 to be the moment the dinner is removed from the freezer.)

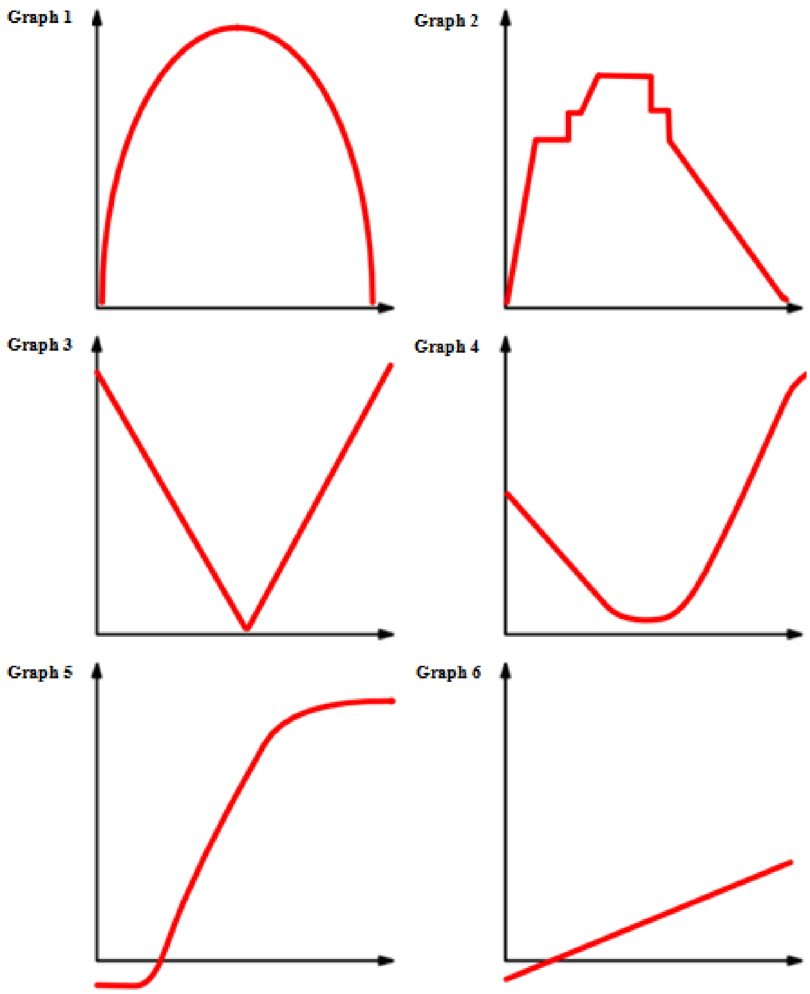
h. The value of a 1970 Volkswagen Beetle from the time it was purchased to the present. (It was kept in great condition).

i. The level of water in the bathtub from the time you begin to fill it to to the time it is completely empty after your bath.

j. Profit in terms of the number of items sold.

k. The height of a baseball thrown straight up into the air, from when it is thrown to the time it hits the ground.

l. The speed of the baseball in the situation in item e.

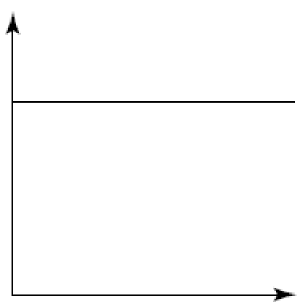


<u>Part III Jumping Jack Activity</u>

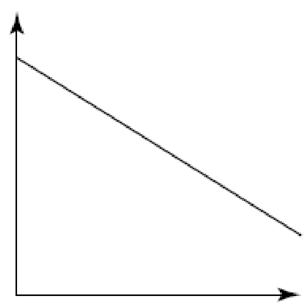
In groups of 4, you will conduct the following activity. Each student will need to take on one of the following roles: 1) jumper, 2) counter, 3) timer, and 4) recorder.

- The jumper will do jumping jacks for 2 minutes straight.
- The counter will count the number of jumping jacks performed aloud.
- The timer will monitor a stopwatch/clock and call out, "*Time!*" every 10 seconds.
- The recorder will keep a written record of the total number of jumping jacks performed at every 10-second interval.

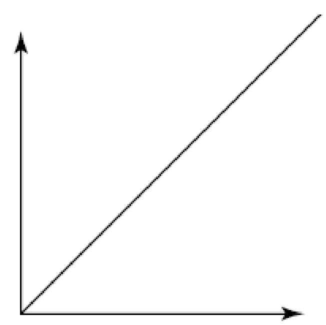
<u>Before conducting this activity</u>, look at the graphs below. If time is represented on the x-axis, and the number of jumping jacks performed is shown on the y-axis, make a prediction: which shape do you think will most likely match your data? Explain your reasoning in the space below.



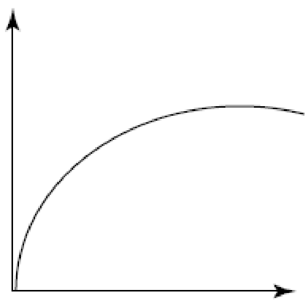
Graph A



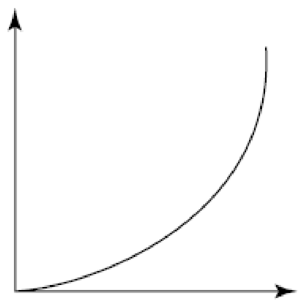
Graph B



Graph C

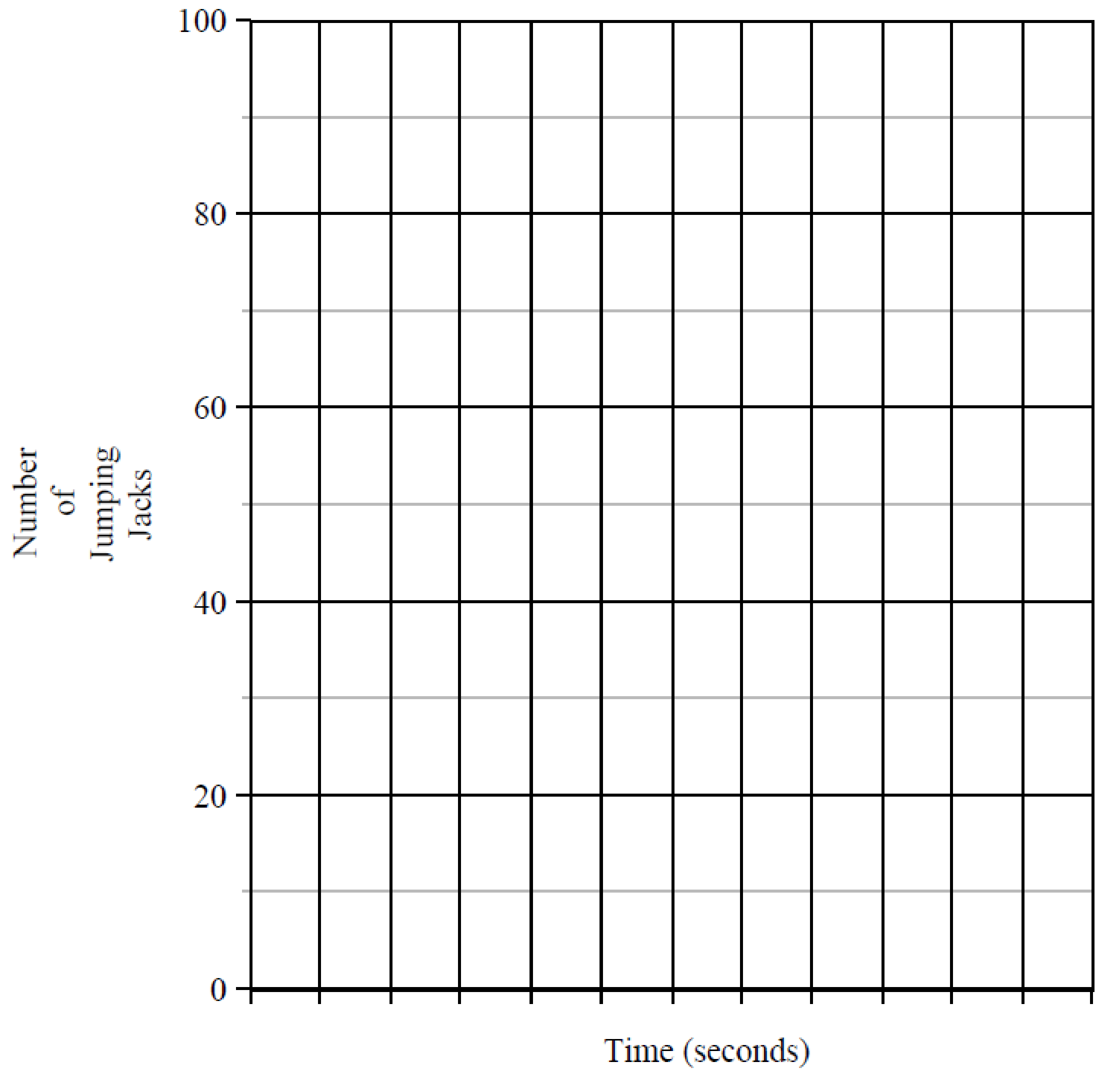


Graph D



Graph E

Now, graph your group's data on the grid below. Connect the dots with a ruler or straight edge.



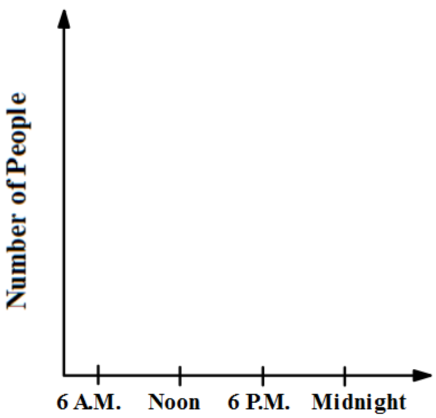
3. Explain the shape of your graph. Why is it steeper or flatter in certain parts?

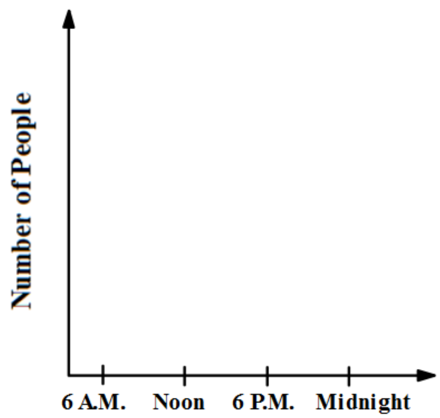
4. How would the shape of your graph change if the sizes of the intervals were changed? Give an example.

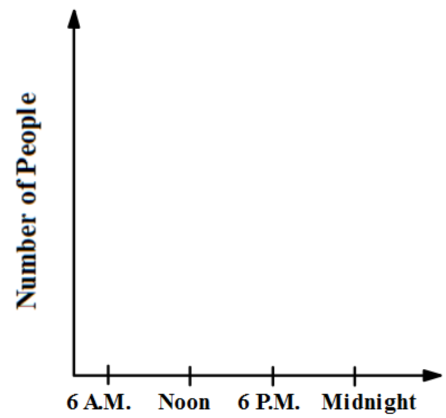
<u>Part IV: Create a "Secret Location" Graph</u>

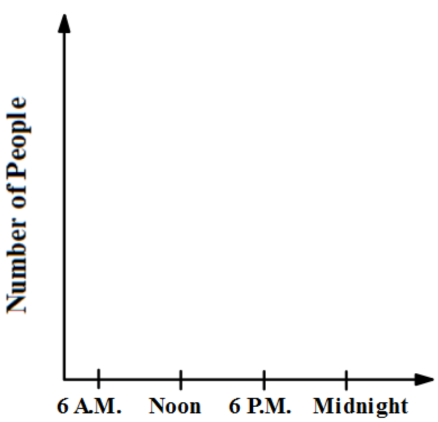
Using the grids below, create a graph that shows the varying number of people who might be present in the following "secret locations" at various times throughout a weekday.

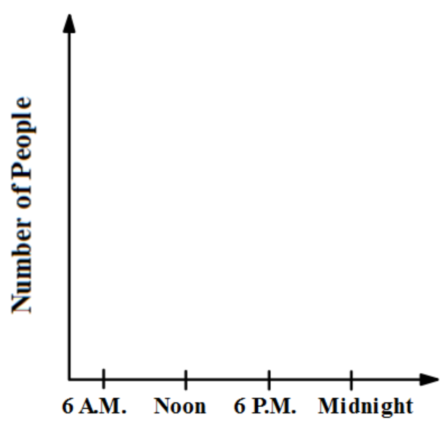
1. The number of people at the grocery store on a weekday.
2. The number of people at your house on a school day.
3. The number people driving a car on the road on a weekday.
4. The number of people at McDonald's on a weekday.
5. The number of people at school on a weekday.
6. The number of people in a hospital on a weekday.

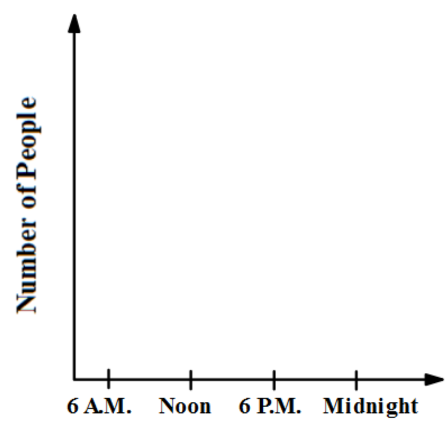












For one of the graphs above, explain the shape you drew. Be sure to relate the shape of the graph to the context of the scenario.

Making Sense of Graphs

Back to Task Table

In this task, students will create and analyze graphs to demonstrate their understanding of direct proportions.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
MGSE6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
MGSE.6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x+p=q$ and $px=q$ for cases in which p, q and x are all nonnegative rational numbers.
MGSE6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another.

a. Write an equation to express one quantity, the dependent variable, in terms of the other quantity, the independent variable.
b. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*

MGSE.6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic representations for given contexts and can create contexts given equations.
2. Reason abstractly and quantitatively. Students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical equations and graphs
3. Construct viable arguments and critique the reasoning of others. Students construct arguments to compare the equation and graph.
4. Model with mathematics. Students model problem situations in graphs from equations and tables.
5. Use appropriate tools strategically. Coordinate graph and tables.
6. Attend to precision. Students precisely place ordered pairs on the coordinate plane.
7. Look for and make use of structure. Students seek patterns or structures to model and solve problems using tables and equations.
8. Look for and express regularity in repeated reasoning. Students generalize that as “k” gets larger the line gets steeper.

ESSENTIAL QUESTIONS

- How can proportional relationships be represented using rules, tables, and graphs?
- How can algebraic expressions be used to model real-world situations?
- How can we solve simple algebraic equations, and how do we interpret the meaning of the solutions?

MATERIALS

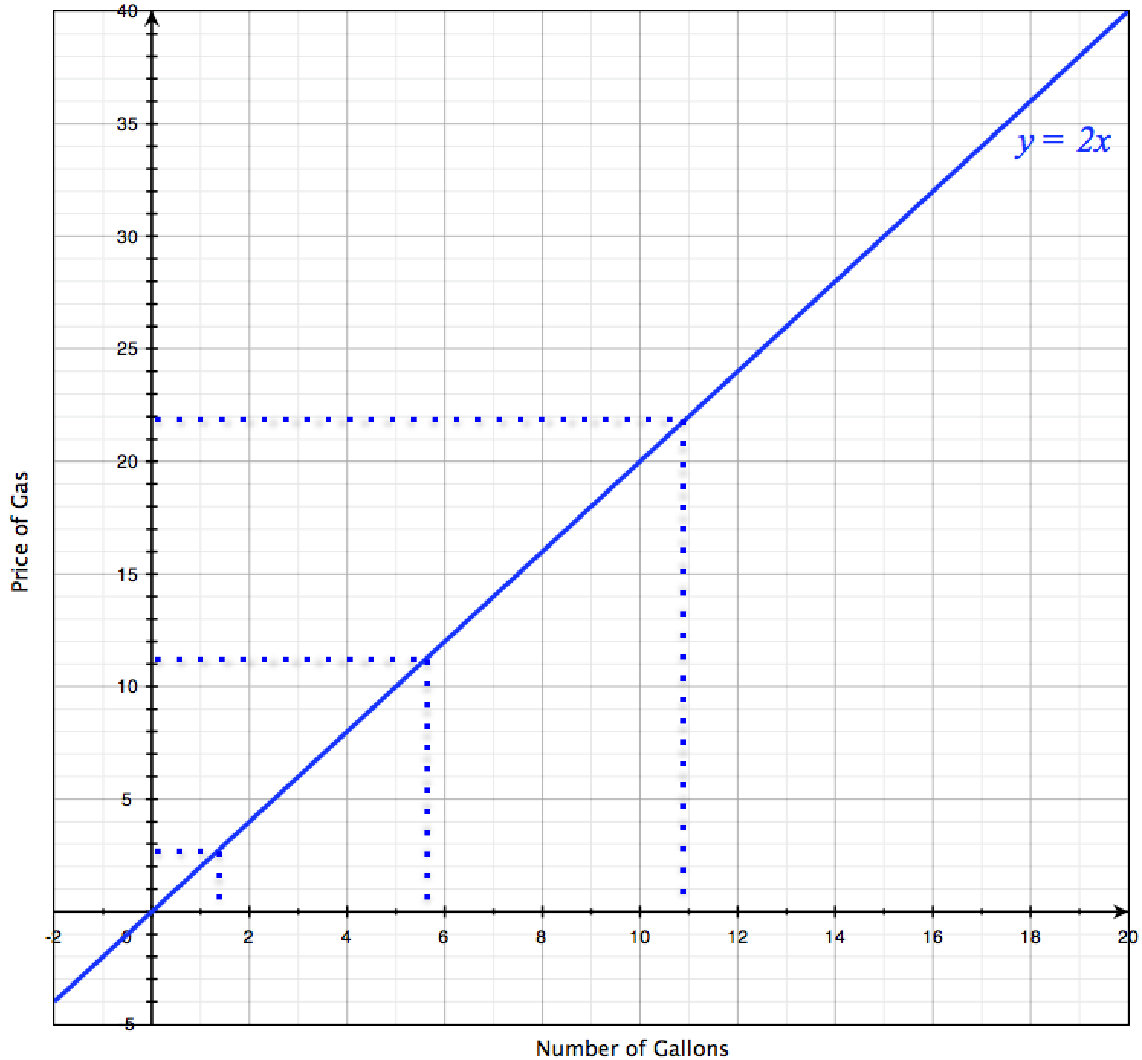
- graph paper
- colored pencils

TASK COMMENTS

It is important for students to compare multiple graphs and describe the variation that occurs between the graphs and their corresponding equations. Teachers should support good student dialogue and take advantage of comments and questions to help guide students into correct mathematical thinking.

Making Sense of Graphs

The graph below shows the amount of money required to buy gasoline if the cost per gallon is \$2.00.



a. What two quantities vary proportionally in this situation?
Solution
The price of gas varies proportionally with the number of gallons of gas purchased.

b. What is the value of the constant of proportionality? What does this value represent in the context of the problem? How is the constant of proportionality represented on the graph?
Solution
The constant of proportionality is 2. This value represents the price per gallon of gas.

c. Write an equation to represent this situation.

Solution

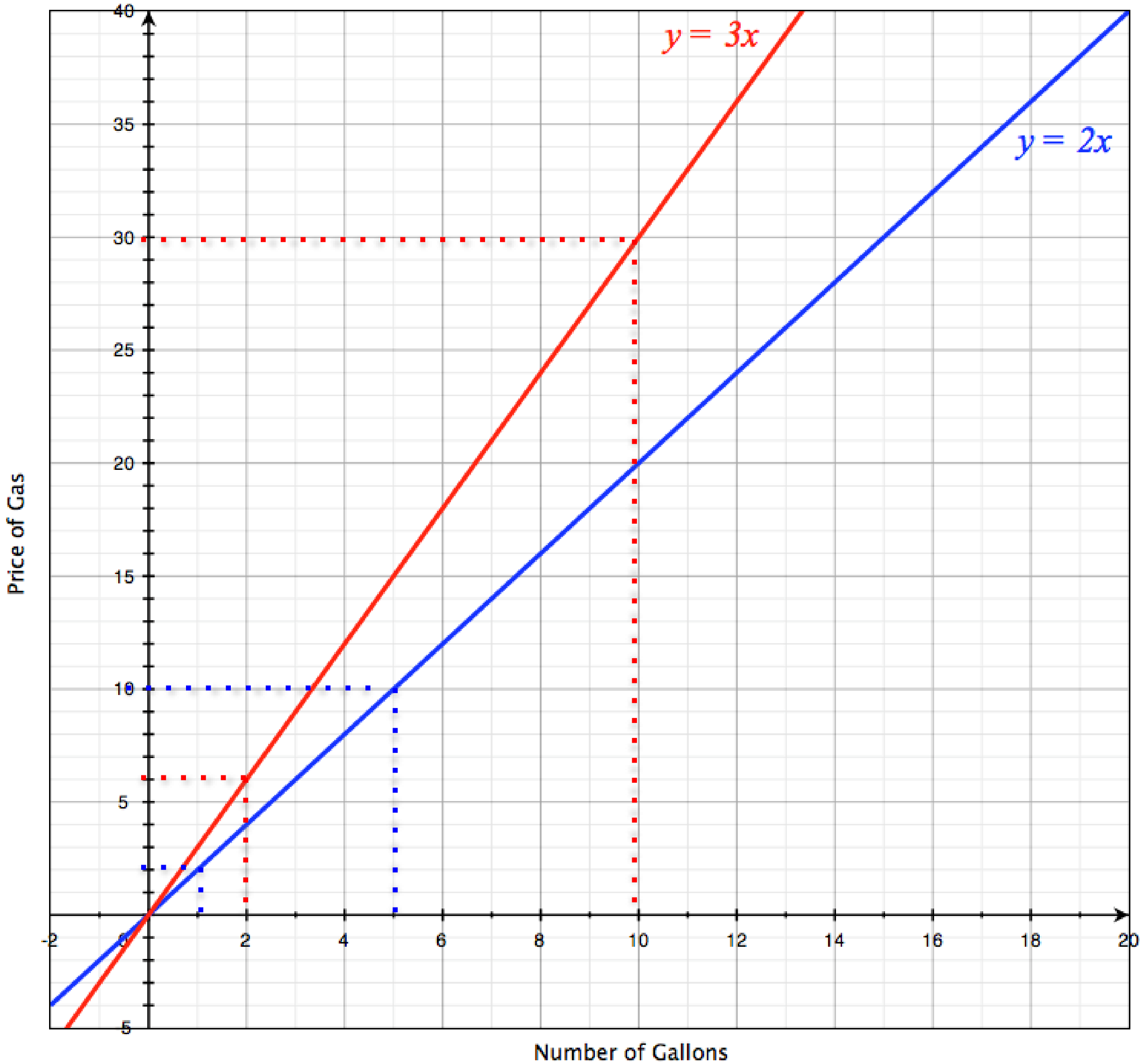
The constant of proportionality is 2 which means $\frac{y}{x} = 2$.

Therefore, $y = 2x$.

d. Suppose gas prices rose to $3.00 per gallon. How would the graph change? Explain your reasoning.

Solution

The graph would get steeper, because for every one increase in the number of gallons purchased, there is an increase of $3.00 in the cost instead of $2.00. Over the same amount of horizontal increase, there is a larger vertical increase, which would make the graph steeper.



e. Write an equation to represent the situation in part d.

Solution

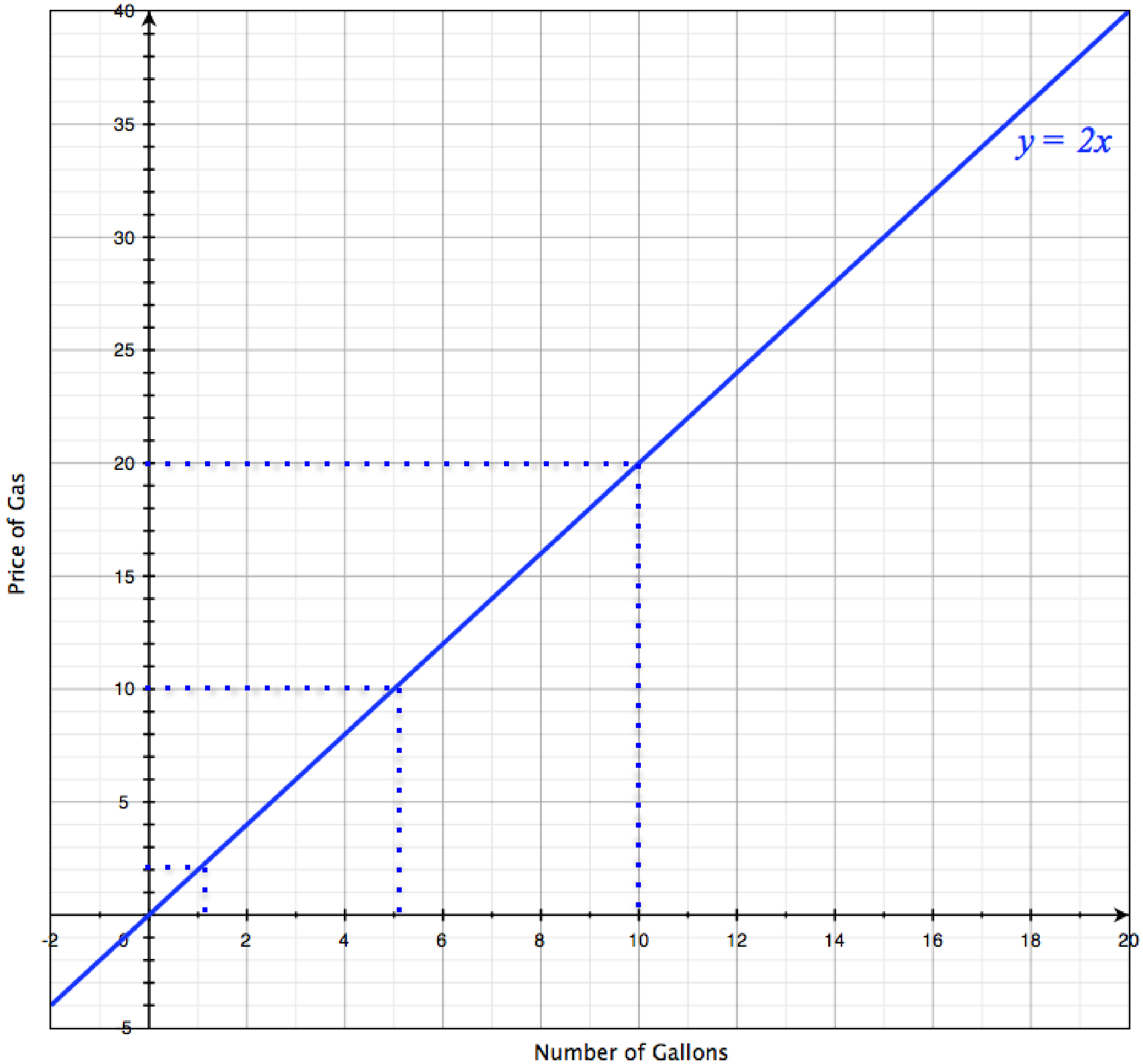
The constant of proportionality is 3 which means $\frac{y}{x} = 3$.

Therefore, $y = 3x$.

Extension – change the gas price to $4.50 per gallon. Graph and explain the changes to the graph. Do this with several different prices for practice. Students could also come up with their own gas prices and explain how their graph looks. Questions to ponder – Are the graphs getting steeper? Where would a line be on the graph given a certain gas price?

Making Sense of Graphs

The graph below shows the amount of money required to buy gasoline if the cost per gallon is $2.00.



a. What two quantities vary proportionally in this situation?

b. What is the value of the constant of proportionality? What does this value represent in the context of the problem? How is the constant of proportionality represented on the graph?

c. Write an equation to represent this situation.

d. Suppose gas prices rose to $3.00 per gallon. How would the graph change? Explain your reasoning.

e. Write an equation to represent the situation in part d.

Analyzing Tables

Back to Task Table

In this task, students will demonstrate a deeper understanding of direct proportions through tables, graphs, and rules (equations).

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
MGSE6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
MGSE.6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x+p=q$ and $px=q$ for cases in which p, q and x are all nonnegative rational numbers.
MGSE6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another.

a. Write an equation to express one quantity, the dependent variable, in terms of the other quantity, the independent variable.
b. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*

MGSE.6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic representations for given contexts and can create contexts given equations.
2. Reason abstractly and quantitatively. Students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical, equations, and graphs
3. Construct viable arguments and critique the reasoning of others. Students construct arguments to compare the equation and graph.
4. Model with mathematics. Students model problem situations in graphs from equations and tables.
5. Use appropriate tools strategically. Coordinate graph and tables.
6. Attend to precision. Students precisely place ordered pairs on the coordinate plane.
7. Look for and make use of structure. Students seek patterns or structures to model and solve problems using tables and equations.
8. Look for and express regularity in repeated reasoning. Students generalize that as “k” gets larger the line gets steeper.

ESSENTIAL QUESTIONS

- How can proportional relationships be represented using rules, tables, and graphs?
- How can algebraic expressions be used to model real-world situations?
- How can we solve simple algebraic equations, and how do we interpret the meaning of the solutions?

MATERIALS

- graph paper (optional)

TASK COMMENTS

It is important for students to communicate their understanding of direct proportions using multiple representations. Students should also explain how each representation demonstrates proportional reasoning. Teachers should support good student dialogue and take advantage of comments and questions to help guide students into correct mathematical thinking.

Extension

Ask students to graph each data set. Compare graphs to proportional data sets and sets that are not proportional.

Analyzing Tables

Consider the tables below where the x- and y-values represent two quantities. For each table, do the following:

a. Do the quantities vary proportionally? Explain how you know.
b. Write a rule for each table in words.
c. Write the rule as an equation.

Table 1

x	5	4	3	2	1
y	10	8	6	4	2

Solution

a. The quantities x and y vary proportionally because $\frac{y}{x} = 2$ ***for each pair of values.***
b. In other words, the ratio between the two quantities y and x is constant (2).
c. For each entry in the table, y is two times x. Rule: y = 2x

Table 2

x	50	40	30	20	10
y	5	4	3	2	1

Solution

a. The quantities x and y vary proportionally because $\frac{y}{x} = \frac{1}{10}$ ***for each pair of values.***
b. In other words, the ratio between the two quantities y and x is constant $\left(\frac{1}{10}\right)$***.***
c. For each entry in the table, y is one-tenth times x. Rule: $y = \frac{1}{10}x$ ***or*** $y = \frac{x}{10}$

Table 3

x	1	2	3	4	5
y	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$

$\frac{y}{x}$	$\frac{1}{1} = 1$	$\frac{\frac{1}{2}}{2} = \frac{1}{4}$	$\frac{\frac{1}{3}}{3} = \frac{1}{9}$	$\frac{\frac{1}{4}}{4} = \frac{1}{16}$	$\frac{\frac{1}{5}}{5} = \frac{1}{25}$

xy	$1 \cdot 1 = 1$	$\frac{1}{2} \cdot 2 = 1$	$\frac{1}{3} \cdot 3 = 1$	$\frac{1}{4} \cdot 4 = 1$	$\frac{1}{5} \cdot 5 = 1$

Solution

a. ***The quantities x and y <u>do not</u> vary proportionally because $\frac{y}{x}$ is not constant for each pair of values.***
b. ***In other words, the <u>product</u> between the two quantities y and x is constant (1).***
c. ***For each entry in the table, y times x is equal to 1 or y is 1 divided by x.***
Rule:
$xy = 1$

$y = \frac{1}{x}$

Analyzing Tables

Consider the tables below where the x- and y-values represent two quantities. For each table, do the following:

a. Do the quantities vary proportionally? Explain how you know.
b. Write a rule for each table in words.
c. Write the rule as an equation.

Table 1

x	5	4	3	2	1
y	10	8	6	4	2

Table 2

x	50	40	30	20	10
y	5	4	3	2	1

Table 3

x	1	2	3	4	5
y	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$

Who Has Faulty Thinking? (Formative Assessment Lesson)

Source: Georgia Mathematics Design Collaborative, 2013 **Back to Task Table**
Click here to download this FAL from the Georgia Mathematics Teacher Wiki Forum

In this task, students will translate between word sentences and equations and produce appropriate equations in more than one way.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

STANDARDS FOR MATHEMATICAL PRACTICE

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**

TASK COMMENTS

This Formative Assessment Lesson will help you to identify and support students who have difficulty in:

- Representing an unknown number for a given problem
- Writing an equation to represent a mathematical problem
- Read a problem situation
- Determine and identify the unknown of the problem situation
- Assign a variable to represent the unknown
- Write down what the variable represents
- Write an equation that will find the value of the unknown for the problem situation

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Georgia Mathematics Design Collaborative are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding.

The task, *Who Has Faulty Thinking?*, is a Formative Assessment Lesson (FAL) that can be found on the Georgia Mathematics online professional learning community, edWeb.net. Once logged in, the task can be found directly at: https://www.edweb.net/?14@@.5abfa3bd.

Real-life Equations (Formative Assessment Lesson)

Back to Task Table

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1529

In this task, students will make sense of equations that express the relationship between two real-world variables, as well as explore the meaning of variables in contextualized equations.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
MGSE6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
MGSE.6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x+p=q$ and $px=q$ for cases in which p, q and x are all nonnegative rational numbers.

STANDARDS FOR MATHEMATICAL PRACTICE

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Model with mathematics.**
7. **Look for and make use of structure.**

TASK COMMENTS

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Real-life Equations*, is a Formative Assessment Lesson (FAL) that can be found at the website: http://map.mathshell.org/materials/lessons.php?taskid=580&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson. **NOTE: This FAL is also called *Interpreting Equations*.**

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1529

When Is It Not Equal?

Back to Task Table

In this task, students will write and solve one-step inequalities and graph them on the number line.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
MGSE.6.EE.8 Write an inequality of the form $x < c$ or $x > c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x < c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic representations for given contexts and can create contexts given inequalities.
2. Reason abstractly and quantitatively. Students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using verbal or written explanations accompanied by inequalities and number lines.
4. Model with mathematics. Students model inequality situations on a number line.
5. Use appropriate tools strategically. Students use number lines to graph inequalities. Students use tables to organize information to write inequalities.
6. Attend to precision. Students precisely define variables.

ESSENTIAL QUESTIONS

- What strategies can I use to help me understand and represent real situations using inequalities?
- How can I write, interpret and manipulate inequalities?
- How can I solve an inequality?
- How are the solutions of equations and inequalities different?

When Is It Not Equal?

Comment:
Particular care should be used when working with inequalities not to utilize negative numbers. Students will be exposed to negative numbers in unit 6 and will also deal with graphing them at that time. This task will restrict to positive rational numbers.

Write an inequality for the following statements.

1. You need to earn at least $50.

   $x \geq 50$

2. You can spend no more than $5.60

   $x \leq 5.60$

3. The trip will take at least 4 hours.

   $x \geq 4$

4. The car ride will be less than 8 hours.

   $x < 8$

5. Four boxes of candy contained at least 48 pieces total.

   $4x \geq 48$

6. With John's 7 marbles and mine, we had less than 20 marbles together.

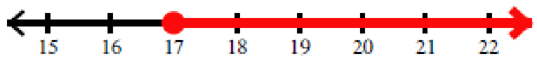
   $x + 7 < 20$

7. Seven buses can hold no more than 560 students.

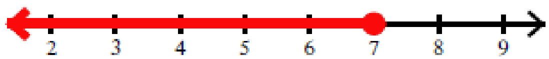
   $7x \leq 560$

Graph the following inequalities:

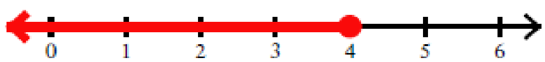
8. $p \geq 17$



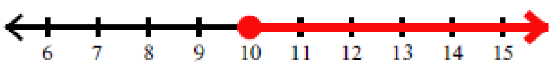
9. $b \leq 7$



10. $t < 4$

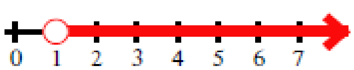


11. $r > 10$



12. $k \leq 18$

13. $m > 1$



14. $d > 2$

Circle the numbers that are part of the solution for the inequalities below.

15. $x + 2 > 5$ (0 3 4 10)

16. $v - 4 < 10$ (4 9 14 15)

17. $4b \leq 15$ (0 3 5 6)

18 $\frac{1}{3}r \geq 3\frac{1}{2}$ (6 9 15 30)

19. $0.5w > 2.3$ (2 4 5 10)

20. $t + 1.5 < 3.6$ (0.6 1.7 2.1 3.2)

Write an inequality for each situation.
Students are translating the verbal expressions into statements of inequality, not solving inequalities.

21. What is the minimum number of 80-passenger buses needed to transport 375 students? Choose and justify a solution (4, $4\frac{11}{16}$, 5)

$80b \geq 375$

$4\frac{11}{16}$ and 5 are solutions to the inequality but 5 is the answer to the question.

22. What is the minimum speed needed to travel at least 440 miles in 8 hours? Choose and justify a solution (54 mph, 55 mph, 56 mph)

$8r \geq 440$

55 and 56 are solutions the inequality but 55 is the answer to the question

23. What is the least number of boxes needed to package 300 candies if each box will hold 16 candies? Choose and justify a solution (18, $18\frac{3}{4}$, 19)

$16b \geq 300$

$18\frac{3}{4}$ and 19 are solutions to the inequality but 19 is the answer to the question

When Is It Not Equal?

Write an inequality for the following statements.

1. You need to earn at least $50.
2. You can spend no more than $5.60
3. The trip will take at least 4 hours.
4. The car ride will be no more than 8 hours.
5. Four boxes of candy contained at least 48 pieces total.
6. With John's 7 marbles and mine, we had less than 20 marbles together.
7. Seven buses can hold no more than 560 students.

Graph the following inequalities:

8. $p \geq 17$
9. $b \leq 7$
10. $t < 4$
11. $r > 10$
12. $k \leq 18$
13. $m > 1$
14. $d > 2$

Circle the numbers that are part of the solution for the inequalities below.

15. $x + 2 > 5$ (0 3 4 10)

16. $v - 4 < 10$ (4 9 14 15)

17. $4b \leq 15$ (0 3 5 6)

18 $1/3r \geq 3\frac{1}{2}$ (6 9 15 30)

19. $0.5w > 2.3$ (2 4 5 10)

20. $t + 1.5 < 3.6$ (0.6 1.7 2.1 3.2)

Write an inequality for each situation.

21. What is the minimum number of 80-passenger buses needed to transport 375 students? Choose and justify a solution (4, $4^{11}/_{16}$, 5)

22. What is the minimum speed needed to travel at least 440 miles in 8 hours? Choose and justify a solution (54 mph, 55 mph, 56 mph)

23. What is the least number of boxes are needed to package 300 candies if each box will hold 16 candies? Choose and justify a solution (18, $18\frac{3}{4}$, 19)

Evaluating Statements about Number Operations (Formative Assessment Lesson)

Back to Task Table

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1580
In this task, students will represent inequalities algebraically and in words, while substituting values into inequality statements in order to test their validity.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
MGSE6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
MGSE.6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x+p=q$ and $px=q$ for cases in which p, q and x are all nonnegative rational numbers.
MGSE.6.EE.8 Write an inequality of the form $x<c$ or $x>c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x<c$ or $x<c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
6. Attend to precision.

TASK COMMENTS

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Evaluating Statements about Number Operations*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=602&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.
The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1580

It's On Sale!

Back to Task Table

In this task, students will solve retail problems using proportional reasoning.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE.6.RP.3c Find a percent of a quantity as a rate per 100 (e.g. 30% of a quantity means 30/100 times the quantity); given a percent, solve problems involving finding the whole given a part and the part given the whole.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose the appropriate proportion to represent the given situation and solve to find the answer.
3. Construct viable arguments and critique the reasoning of others. Students construct written explanations for their work with percent.
6. Attend to precision.

ESSENTIAL QUESTIONS

- How can proportions be used to solve problems?
- How is percent used to solve problems?

It's on Sale!

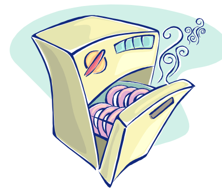
Answer Key

1. A) $170.50 B) $6,125 C) 70% D) $4,335.50	*2. 735 total cars*	*3. Lucy is 120 cm Rachel is 160 cm*	*4. The total bill was $86*	*5. A) 25% B) 3 C) 75%, 9*

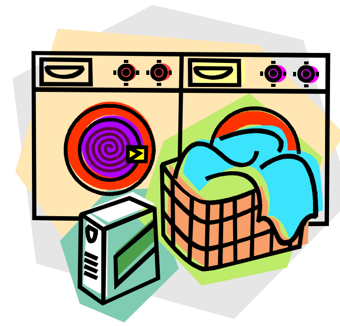
Name_____________________

It's On Sale!

Part 1



Dishwashers on sale for 45% off. Original price is $310.



Washer and Dryer combo for $2450. This is 40% off the original price.

Refrigerator was $2450 and is now $1,715.

1. Logan went to the local appliance store and bought a washer/dryer set, a refrigerator, and a dishwasher. She was thrilled to discover that all the items were on sale.

A. How much did the dishwasher cost at the sale price?

B. What was the original price of the washer/dryer set?

C. The refrigerator is on sale. Logan wants to know what percent of the original price she paid for this item.

D. What is the total amount she spent on all the items she purchased? Explain how you found her total cost.

Part 2 Solve the given problems and show your work.

2. In the annual car show, 40% of the cars in the main display area are used cars. Given that 294 cars are used cars, what is the total number of cars in the show?

3. Rachel is 40 cm taller than Lucy. Rachel is 25% taller than Lucy. How tall is Lucy?

4. Mike and Chris went to dinner. Chris insisted that they leave a 20% tip for the waitress. Mike left a $17.20 tip. How much was their bill before the tip?

5. Jenny and Tim run a bed and breakfast in Crawford, Georgia. Jenny painted ¼ of her rocking chairs red to match her new door.

   A. What percent of her chairs did she paint red?

   B. Jenny has 12 chairs on her front porch. How many chairs are painted red?

   C. If she wants to paint all her chairs red, what percent still need to be painted? How many chairs need to be painted?

The Catering Job

Back to Task Table

In this task, students will reason about unit rates to determine best deals.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE.6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose he appropriate way to solve problems involving unit rates. Students will choose the best deals given several options.
3. Construct viable arguments and critique the reasoning of others. Students will construct written arguments to proof which is a deal is best using unit rate.
6. Attend to precision. Students precisely divide numbers to the hundredths place.

ESSENTIAL QUESTIONS

- How can proportions be used to solve problems?
- How is unit rate helpful when solving problems?

The Catering Job

Mrs. Smith works for a catering company. She is in charge of buying the food for a special event. It is very important to her boss that she buy the supplies at the best unit rate.

1. Below are the prices Mrs. Smith found on shrimp:

- Kroger sells a package of 15 shrimp for $8.99. ***(unit rate $0.59)***
- Publix sells a package of 12 shrimp for $8.34. ***(unit rate $0.69)***
- The Bluffton Oyster Company sells a package of 20 shrimp for $10.02. ***(unit rate $0.50)***

Where should she buy the shrimp? Show your work and WRITE an explanation for Mrs. Smith to give to her boss to show him that she chose the best deal on shrimp. ***The Bluffton Oyster Company is the best deal. Students have a written explanation that includes the unit rate for each of the three stores.***

2. Below are the prices she found on potatoes:

- Kroger sells 10 pound bags of potatoes for 3.69 ***(unit rate $0.37)***
- Publix sells 5 pound bags of potatoes for $2.45 ***(unit rate $0.49)***
- Wal-Mart sells 20 pound bags of potatoes for $7 ***(unit rate $0.35)***

Where should she buy potatoes? Show your work and WRITE an explanation for Mrs. Smith to give to her boss to show him that she chose the best deal on potatoes. ***Wal-Mart is the best deal. Students have a written explanation that includes the unit rate for each of the three stores.***

3. Below are the prices Mrs. Smith found on corn on the cobb:

- Snook's sells 25 pieces of corn for 12.50 ***(unit rate $0.50)***
- Publix sells 3 pieces of corn for $1 ***(unit rate $0.33)***
- The Farmer's Market sells 12 pieces of corn for $4.80 ***(unit rate $0.40)***

Where should she buy corn on the cob? Show your work and WRITE an explanation for Mrs. Smith to give to her boss to show him that she chose the best deal on corn on the cob. ***The Farmer's Market is the best deal. Students have a written explanation that includes the unit rate for each of the three stores.***

Name____________________

The Catering Job

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Where should she buy the shrimp? Show your work and WRITE an explanation for Mrs. Smith to give to her boss to show him that she chose the best deal on shrimp.

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Where should she buy potatoes? Show your work and WRITE an explanation for Mrs. Smith to give to her boss to show him that she chose the best deal on potatoes.

3. Below are the prices Mrs. Smith found on corn on the cobb:

- Snook's sells 25 pieces of corn for 12.50
- Publix sells 3 pieces of corn for $1
- The Farmer's Market sells 12 pieces of corn for $4.80

Where should she buy corn on the cob? Show your work and WRITE an explanation for Mrs. Smith to give to her boss to show him that she chose the best deal on corn on the cob.

Culminating Task: Want Ads

Back to Task Table

In this task, students will demonstrate their understanding of direct proportions as applied to the context of newspaper "want ads".

STANDARDS FOR MATHEMATICAL CONTENT

MGSE.6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

MGSE6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another.

a. Write an equation to express one quantity, the dependent variable, in terms of the other quantity, the independent variable.
b. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*

MGSE.6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.

MGSE.6.RP.3a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

MGSE.6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed.

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic representations for given contexts and can create contexts given equations or inequalities.

2. Reason abstractly and quantitatively. Students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities

3. Construct viable arguments and critique the reasoning of others. Students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, graphs, and tables.

4. Model with mathematics. Students model problem situations in symbolic, graphic, tabular, and contextual formats. Students form expressions, equations, and inequalities from real-world contexts and connect symbolic and visual representations.

5. Use appropriate tools strategically. Students use number lines to graph equations and inequalities. Students use tables to organize information to write equations and inequalities.

6. Attend to precision. Students precisely define variables.

7. Look for and make use of structure. Students seek patterns or structures to model and solve problems using tables, and equations. Students seek patterns or structures to model problems

using tables and inequalities. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 3(2 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality, $c = 6$ by division property of equality).
8. Look for and express regularity in repeated reasoning. Students generalize effective processes for representing equations and inequalities based upon experiences. Students will be solving for equations.

ESSENTIAL QUESTIONS
- What is a proportion?
- How can proportions be used to solve problems?
- How can proportional relationships be described using the equation $y = kx$?
- How can proportional relationships be represented using rules, tables, and graphs?
- How can the graph of $y = kx$ be interpreted for different contexts?
- How can algebraic expressions be used to model real-world situations?
- How can we solve simple algebraic equations, and how do we interpret the meaning of the solutions?

MATERIALS
- calculator

TASK COMMENTS

In this task, students will demonstrate their understanding of direct proportions. Students will demonstrate algebraic thinking using current newspaper "Want Ads". The investigation will have several parts. It is important for students to communicate their understanding of direct proportions using multiple representations. Students should also explain how each representation demonstrates proportional reasoning. Teachers should support good student dialogue and take advantage of comments and questions to help guide students into correct mathematical thinking.

While this task may serve as a summative assessment, it also may be used for teaching and learning. The task is designed assess students at the end of the unit, yet aspects may be interspersed appropriately throughout the unit so that students are aware of what is expected of them. It is important that all elements of the task be addressed throughout the learning process.

Want Ads

1. Select an advertised position with an hourly rate from the want ads.

Possible Solution (A page of want ads is attached for your use)
Answers will vary. The rest of the solutions will assume a $10.00 per hour job.

2. How much money would you make at this job if you worked one hour? Two hours? Three hours? Four hours? Five hours? Zero hours?

Possible Solution

Hours worked	***0***	***1***	***2***	***3***	***4***	***5***
Pay	***$0***	***$10***	***$20***	***$30***	***$40***	***$50***

3. What is the greatest amount of money you could earn in one week at your job? Explain your reasoning.

Possible Solution

If I ate while I worked, worked 7 days a week, and only slept 7 hours per night, I could work 17 hours a day for 7 days which would be 119 hours, and I would make $1,190.

4. Write a variable expression that represents how much you will be paid if you work x hours at your selected rate.

Possible Solution

10x

5. Evaluate your expression when $x = 35$. Explain what your answer means in this situation.

Possible Solution

350. If I worked 35 hours, I would make $350.

6. If y equals the total amount you are paid, write an equation that represents how much you will be paid if you work x hours at your selected rate.

Possible Solution

y = 10x

7. Make a table of values containing five coordinate pairs (x, y), each of which satisfies your equation.

Possible Solution

x	***20***	***25***	***30***	***35***	***40***	***45***
y	***$200***	***$250***	***$300***	***$350***	***$400***	***$450***

8. Plot the points you found in question 7. Describe what your graph looks like.

Possible Solution

A line through the origin that goes up and to the right.

9. Suppose you got a raise and made $2.00 more per hour. How do you think your graph would change?

Possible Solution

It would be steeper because for the same amount of horizontal change, there is more vertical change (for every one hour, the pay increases by $2)

10. Suppose your hourly rate is k dollars. Write the equation that represents how much you will be paid, y, if you work x hours.

Possible Solution
$y = kx$

11. If you wanted to earn $600, how many hours would you need to work at your job?

    Answers will vary based upon hourly rate.

12. If you wanted to earn $1500, how many hours would you need to work at your job?
    Answers will vary based upon hourly rate.

13. If you wanted to earn at least $1000, what is the minimum number of hours would you have to work at your job?

    Answers will vary based upon hourly rate, students should realize this question involves an inequality. Students should solve an equation (expression = 1000) and should be able to write a related inequality (greater than or equal to 1000)

Culminating Task: <u>Want Ads</u>

Entry Salaries for Professional Occupations

Career: Actuaries
Description: Actuaries analyze the financial costs of risk and uncertainty. They use mathematics, statistics and financial theory to assess the risk that the risk that an event will occur and to help businesses and clients develop policies that minimize the cost of that risk. Hourly rate: $40.00 per hour

Career: Accountants and Auditors
Description: Accountants and auditors prepare and examine financial records. They ensure that financial records are accurate and that taxes are paid properly and on time. Accountants and auditors assess financial operations and work to help ensure that organizations run efficiently. Hourly rate: $30.00

Career: Financial Analysts
Description: Financial analysts provide guidance to businesses and individuals making investment decisions. They assess the performance of stocks, bond, and other types of investments. Hourly rate: $35.00

Career: Aerospace Engineers
Description: Aerospace engineers design aircraft, spacecraft, satellites, and missiles. In addition, they test prototypes to make sure that they function according to design. Hourly rate: $45.00

Career: Computer Programmers
Description: Computer programmers write code to create software programs. They turn the program designs created by software developers and engineers into instructions that a computer can follow. Hourly rate: $35.00

Career: Cost Estimators
Description: Cost estimators collect and analyze data to estimate the time, money, resources, and labor required for product manufacturing, construction projects, or services. Some specialize in a particular industry or product type. Hourly rate: $25.00

Culminating Task: Want Ads

1. Select an advertised position with an hourly rate from the want ads.

2. How much money would you make at this job if you worked one hour? Two hours? Three hours? Four hours? Five hours? Zero hours?

3. What is the most amount of money you could earn in one week at your job? Explain your reasoning.

4. Write a variable expression that represents how much you will be paid if you work x hours at your selected rate.

5. Evaluate your expression when $x = 35$. Explain what your answer means in this situation.

6. If y equals the total amount you are paid, write an equation that represents how much you will be paid if you work x hours at your selected rate.

7. Make a table of values containing five coordinate pairs (x, y), each of which satisfies your equation.

8. Plot the points you found in question7. Describe what your graph looks like.

9. Suppose you got a raise and made $2.00 more per hour. How do you think your graph would change?

10. Suppose your hourly rate is k dollars. Write the equation that represents how much you will be paid, y, if you work x hours.

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Unit Web Links

Back to Task Table

https://illuminations.nctm.org/Search.aspx?view=search&cc=2059_2090
http://www.yummymath.com/?s=%226.EE%22

Reason about and solve one-variable equations and inequalities

MGSE6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
https://www.illustrativemathematics.org/content-standards/6/EE/B/5/tasks/673
http://www.101qs.com/2327
http://www.visualpatterns.org/

MGSE6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
https://www.illustrativemathematics.org/content-standards/6/EE/B/6/tasks/425
https://www.illustrativemathematics.org/content-standards/6/EE/B/6/tasks/1291
http://www.visualpatterns.org/

MGSE.6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
https://www.illustrativemathematics.org/content-standards/6/EE/B/7/tasks/425
https://www.illustrativemathematics.org/content-standards/6/EE/B/7/tasks/997
https://www.illustrativemathematics.org/content-standards/6/EE/B/7/tasks/1032
https://www.illustrativemathematics.org/content-standards/6/EE/B/7/tasks/1107

MGSE.6.EE.8 Write an inequality of the form $x < c$ or $x > c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x < c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
https://www.illustrativemathematics.org/content-standards/6/EE/B/8/tasks/642
https://www.illustrativemathematics.org/content-standards/6/EE/B/8/tasks/2010
http://www.estimation180.com/inequalities.html
http://www.openmiddle.com/inequality-with-no-solution/
PARCC Prototype Item:
http://www.parcconline.org/sites/parcc/files/PARCC_SampleItems_Mathematics_G6Kelvin_081513_Final.pdf

Represent and analyze quantitative relationships between dependent and independent variables.

MGSE6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another.

a. Write an equation to express one quantity, the dependent variable, in terms of the other quantity, the independent variable.
b. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation* $d = 65t$ *to represent the relationship between distance and time.*

https://www.illustrativemathematics.org/content-standards/6/EE/C/9/tasks/806
http://ccsstoolbox.agilemind.com/parcc/middle_3788_1.html
http://ccsstoolbox.agilemind.com/parcc/middle_3789_1.html
http://www.colmanweb.co.uk/Assets/SWF/Skate_boarders.swf
http://www.absorblearning.com/media/attachment.action?quick=wl&att=2335
http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=260

Understand ratio concepts and use ratio reasoning to solve problems.

MGSE.6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.
See links listed in Unit 2

MGSE.6.RP.3a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
See links listed in Unit 2

MGSE.6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed.
See links listed in Unit 2

MGSE.6.RP.3c Find a percent of a quantity as a rate per 100 (e.g. 30% of a quantity means 30/100 times the quantity); given a percent, solve problems involving finding the whole given a part and the part given the whole.
See links listed in Unit 2

MGSE.6.RP.3d Given a conversion factor, use ratio reasoning to convert measurement units within one system of measurement and between two systems of measurements (customary and metric); manipulate and transform units appropriately when multiplying or dividing quantities. *For example, given 1 in. = 2.54 cm, how many centimeters are in 6 inches?*
See links listed in Unit 2