

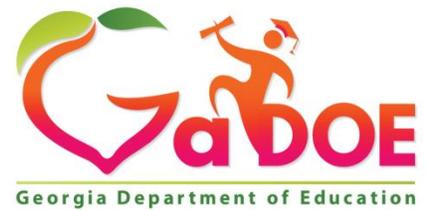


# Georgia Standards of Excellence Curriculum Frameworks

## Mathematics

GSE Grade 7

Unit 2: Expressions and  
Equations



Richard Woods, Georgia's School Superintendent  
*"Educating Georgia's Future"*

**UNIT 2**  
**Expressions and Equations**

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## **OVERVIEW**

The units in this instructional framework emphasize key standards that assist students to develop a deeper understanding of numbers. They learn how to solve multi- step equations and discuss the difference between equations and expressions. The Big Ideas that are expressed in this unit are integrated with such routine topics as estimation, mental and basic computation. All of these concepts need to be reviewed throughout the year.

Take what you need from the tasks and modify as required. These tasks are suggestions, something that you can use as a resource for your classroom.

## **STANDARDS ADDRESSED IN THIS UNIT**

### **THE STANDARDS FOR MATHEMATICAL PRACTICE**

- 1. Make sense of problems and persevere in solving them.** Students solve real world problems through the application of algebraic concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”.
- 2. Reason abstractly and quantitatively.** Students demonstrate quantitative reasoning by representing and solving real world situations using visuals, equations, inequalities and linear relationships into real world situations.
- 3. Construct viable arguments and critique the reasoning of others.** Students will discuss the differences among expressions, equations and inequalities using appropriate terminology and tools/visuals. Students will apply their knowledge of equations and inequalities to support their arguments and critique the reasoning of others while supporting their own position.
- 4. Model with mathematics.** Students will model an understanding of expressions, equations, inequalities, and graphs using tools such as algebra tiles/blocks, counters, protractors, compasses, and visuals to represent real world situations.
- 5. Use appropriate tools strategically.** Students demonstrate their ability to select and use the most appropriate tool (pencil/paper, manipulatives, calculators, protractors, etc.) while rewriting/evaluating/analyzing expressions, solving and representing and analyzing linear relationships.
- 6. Attend to precision.** Students demonstrate precision by correctly using numbers, variables and symbols to represent expressions, equations and linear relationships, and correctly label units. Students use precision in calculation by checking the reasonableness of their answers and making adjustments accordingly. Students will use appropriate algebraic language to describe the steps in rewriting expressions and solving equations.
- 7. Look for and make use of structure.** Students routinely seek patterns or structures to model and solve problems. Students apply properties to generate equivalent expressions (i.e.  $6 + 2x = 2(3 + x)$  by distributive property) and solve equations (i.e.  $2c + 3 = 15$ ,  $2c = 12$  by subtraction property of equality;  $c=6$  by division property of equality).

- 8. Look for and express regularity in repeated reasoning.** In grade 7, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that  $a/b \div c/d = ad/bc$  and construct other examples and models that confirm their generalization. They extend their thinking to include complex fractions and rational numbers.

## **CONTENT STANDARDS**

### **Use properties of operations to generate equivalent expressions.**

**MGSE7.EE.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

**MGSE7.EE.2** Understand that rewriting an expression in different forms in a problem context can clarify the problem and how the quantities in it are related. *For example  $a + 0.05a = 1.05a$  means that adding a 5% tax to a total is the same as multiplying the total by 1.05.*

### **Solve real-life and mathematical problems using numerical and algebraic expressions and equations**

**MGSE7.EE.3** Solve multistep real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals) by applying properties of operations as strategies to calculate with numbers, converting between forms as appropriate, and assessing the reasonableness of answers using mental computation and estimation strategies.

*For example:*

- *If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for a new salary of \$27.50.*
- *If you want to place a towel bar  $9\frac{3}{4}$  inches long in the center of a door that is  $27\frac{1}{2}$  inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

**MGSE7.EE.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

**MGSE7.EE.4a** Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

**MGSE7.EE.4b** Solve word problems leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example, as a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*

**MGSE7.EE.4c** Solve real-world and mathematical problems by writing and solving equations of the form  $x + p = q$  and  $px = q$  in which  $p$  and  $q$  are rational numbers.

## **BIG IDEAS**

- Variables can be used to represent numbers in any type mathematical problem.
- Understand the difference in an expression and an equation.
- Write and solve multi-step equations including all rational numbers.
- Some equations may have more than one solution
- There are differences and similarities between equations and inequalities.

## **ESSENTIAL QUESTIONS**

- How is the distributive property applied when rewriting and evaluating algebraic expressions?
- How can we represent value using variables?
- What properties are required in order to rewrite and evaluate algebraic expressions and solve equations?
- How are verbal expressions translated to algebraic expression?
- Is there more than one way to represent a linear equation?
- How can information from a word problem be translated to create an equation?
- What are the similarities and differences between equations and inequalities?
- What strategies can be used to solve and graph inequalities?
- How are the rules of order of operations used when rewriting expressions?
- How can rewriting an expression in different forms show how the quantities in it are related?

## **CONCEPTS AND SKILLS TO MAINTAIN**

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- number sense
- computation with whole numbers and decimals, including application of order of operations
- addition and subtraction of common fractions with like denominators
- computation with all positive and negative rational numbers
- data usage and representations

## **FLUENCY**

**It is expected that students will continue to develop and practice strategies to build their capacity to become fluent in mathematics and mathematics computation. The eventual goal is automaticity with math facts. This automaticity is built within each student through strategy development and practice. The following section is presented in order to develop a common understanding of the ideas and terminology regarding fluency and automaticity in mathematics:**

**Fluency:** Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

**Deep Understanding:** Teachers teach more than simply “how to get the answer” and instead support students’ ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

**Memorization:** The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

**Number Sense:** Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

### **Fluent students:**

- flexibly use a combination of deep understanding, number sense, and memorization.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.

**For more about fluency, see:** <http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf> and: <http://joboaler.com/timed-tests-and-the-development-of-math-anxiety/>

## **SELECTED TERMS AND SYMBOLS**

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for middle school students. **Note – Different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.** The definitions below are from the CCSS glossary <http://www.corestandards.org/Math/Content/mathematics-glossary/glossary>, when applicable.

Visit <http://intermath.coe.uga.edu> or <http://mathworld.wolfram.com> to see additional definitions and specific examples of many terms and symbols used in grade 7 mathematics.

- **Algebraic expression**
- **Coefficient**
- **Constant**
- **Equation**
- **Inequality**
- **Term**
- **Numerical expression**
- **Variable**

### **FORMATIVE ASSESSMENT LESSONS (FAL)**

**Formative Assessment Lessons** are intended to support teachers in formative assessment. They reveal and develop students’ understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students’ understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student’s mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Guide.

### **SPOTLIGHT TASKS**

A Spotlight Task has been added to each CCGPS mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Common Core Georgia Performance Standards, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky.

**Georgia Department of Education**  
 Georgia Standards of Excellence Framework  
*GSE Grade 7 · Unit 2*

**TASKS**

<b>Task Name</b>	<b>Task Type</b> <i>Grouping Strategy</i>	<b>Content Addressed</b>	<b>Standards Addressed</b>
<a href="#"><u>Distributing and Factoring Using Area</u></a>	Scaffolding Task <i>Individual/Partner Task</i>	Students will rewrite/evaluate algebraic expressions related to area and perimeter.	MGSE7.EE.1 MGSE7.EE.2
<a href="#"><u>Triangles and Quadrilaterals</u></a>	Learning Task <i>Partner Task</i>	Students write expressions and equations based on properties of triangles and quadrilaterals.	MGSE7.EE.1 MGSE7.EE.2
<a href="#"><u>Area and Algebra</u></a>	Learning Task <i>Individual/Partner Task</i>	Students will rewrite/evaluate algebraic expressions related to area and perimeter.	MGSE7.EE.1 MGSE7.EE.2
<a href="#"><u>Guess My Number</u></a>	Spotlight Task <i>Partner Task</i>	Students will model and write expressions and equations	MGSE7.EE.3 MGSE7.EE.4
<a href="#"><u>Algebra Magic</u></a>	Performance Task <i>Individual Task</i>	Students model and write expressions and equations.	MGSE7.EE.1 MGSE7.EE.2 MGSE7.EE.3 MGSE7.EE.4a
<a href="#"><u>Deconstructing Word Problems</u></a>	Learning Task <i>Individual / Partner Task</i>	Students will represent and solve real-world mathematical problems	MGSE7.EE.1-2 MGSE7.EE.4a
<a href="#"><u>Solving Linear Equations (FAL)</u></a>	Formative Assessment Lesson <i>Partner/Small Group</i>	Students will use variables to represent quantities in the real-world and solve word problems using equations.	MGSE7.EE.1-3 MGSE7.EE.4a
<a href="#"><u>T.V. Time and Video Games</u></a>	Learning Task <i>Partner/Small Group</i>	Students will write inequality statements to describe members of a group.	MGSE7.EE.4b
<a href="#"><u>Population Equations</u></a>	Culminating Task <i>Individual/Partner Task</i>	Students will compute using a formula and write and solve equations to problem solve.	MGSE7.EE.1-3 MGSE7.EE.4

## **Distributing And Factoring Using Area**

[Back to task table](#)

Adapted from NCTM Illuminations <http://illuminations.nctm.org/Lesson.aspx?id=2682>

In this task, students will use area models to represent and discover the distributive property as well as factor monomials. Students will be using rectangles whose sides may be variables in order to further their understanding of the distributive property.

### **STANDARDS FOR MATHEMATICAL CONTENT**

**MGSE7.EE.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

**MGSE7.EE.2** Understand that rewriting an expression in different forms in a problem context can clarify the problem and how the quantities in it are related. *For example  $a + 0.05a = 1.05a$  means that adding a 5% tax to a total is the same as multiplying the total by 1.05.*

### **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Solve problems and persevere.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.

### **BACKGROUND KNOWLEDGE**

In order for students to be successful, the following skills and concepts need to be maintained:

- positive and negative numbers are used together to describe quantities having opposite directions, use positive and negative numbers to represent quantities in real-world contexts
- apply properties of operations as strategies to multiply and divide rational numbers
- understand how to rewrite numerical and algebraic expressions

### **COMMON MISCONCEPTIONS**

- *EE1. As students begin to build and work with expressions containing more than two operations, students tend to set aside the order of operations. For example having a student rewrite an expression like  $8 + 4(2x - 5) + 3x$  can bring to light several misconceptions. Do the students immediately add the 8 and 4 before distributing the 4? Do they only multiply the 4 and the  $2x$  and not distribute the 4 to both terms in the parenthesis? Do they collect all like terms  $8 + 4 - 5$ , and  $2x + 3x$ ? Each of these shows gaps in students' understanding of how to rewrite numerical expressions with multiple operations.*
- *EE2. Students have difficulties understanding equivalent forms of numbers, their various uses and relationships, and how they apply to a problem. Make sure to expose students to multiple examples and in various contexts*
- *Students usually have trouble remembering to distribute to both parts of the parenthesis. They also try to multiply the two terms created after distributing instead of adding them.*

*Students also want to add the two final terms together whether they are like terms or not since they are used to a solution being a single term. Finally, students need to be careful to make sure negative signs are distributed properly.*

### **ESSENTIAL QUESTIONS**

- How is the distributive property applied when rewriting and evaluating algebraic expressions?

### **MATERIALS**

- Activity Sheet  
\*Optional: Colored Sheets of paper cut into rectangles. These can be used to introduce the concepts found in this lesson.

### **GROUPING**

- Individual/Partner

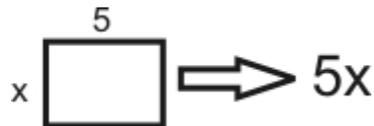
### **TASK DESCRIPTION**

This task is designed to help students understand the distributive property using area models. It is important to allow students time to develop their own solutions for how the distributive property can be used to solve problems.

#### **Area Representation of the Distributive Property**

The first part of this activity can be used to help students recall information regarding area as a precursor for the distributive property.

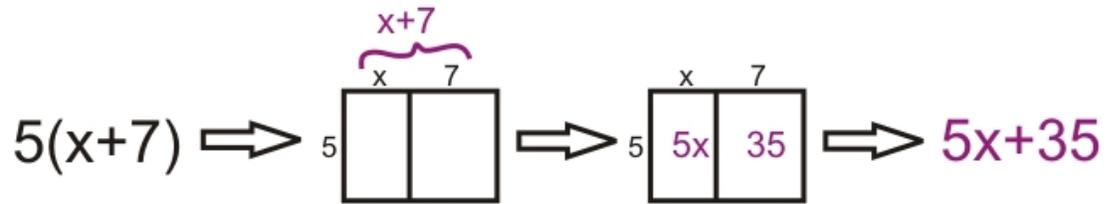
The first section introduces students to the idea of writing the area of a rectangle as an expression of the length  $\times$  width, even when one or more dimensions may be represented by a variable.



The next section demonstrates how two measurements of a segment can be added together to represent the sum of the entire length of the segment.

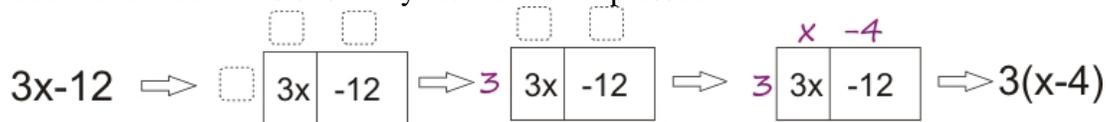


The key section is next, having students represent the area of each rectangle *two ways* to distribute the common factor among all parts of the expression in parentheses.



### Area Representation to Find a Common Factor

When students are presented with a figure such as the one below, they can be asked to determine the dimensions that yield the area expressed.



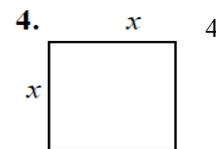
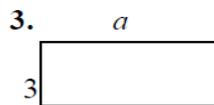
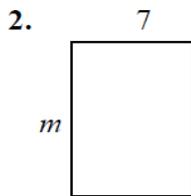
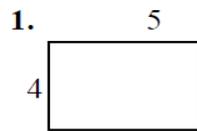
- The second page helps students recognize and factor out an *integer* common factor.

**Note:** Help students who pause when the common factor is a negative number. Be sure that they *change the sign* of the second term. (Example:  $-2a + 10 = -2(a - 5)$  )

Students should recognize that when they find a common factor, it may not be the *greatest* common factor. If students do not reach this conclusion, teachers need to ensure that students do come to this conclusion.

## Distributing And Factoring Area

Write the expression that represents the area of each rectangle.

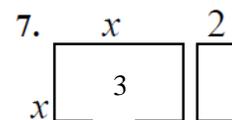
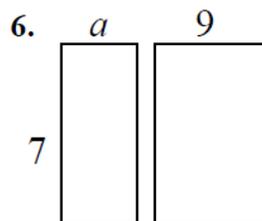
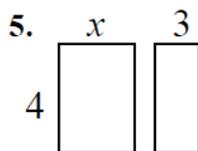


### Solutions

*Area is found by multiplying the length and width of a figure together.*

1.  $5(4)=20\text{units}$
2.  $7(m)=7m\text{ units}$
3.  $3(a)=3a\text{ units}$
4.  $x(4)=4x\text{ units}$

Find the area of each box in the pair.



### Solutions

*Area is found by multiplying the length and width of a figure together.*

*This section allows students to begin to piece together some of the fundamental concepts for the distributive property.*

*We suggest that students write the areas of each of the figures within the corresponding boxes.*

5. *Area of first box:  $4(x)=4x\text{ units}$   
Area of second box:  $4(3)=12\text{ units}$*

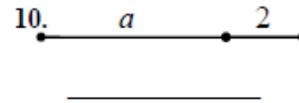
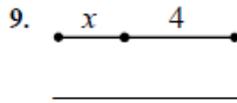
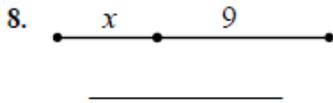
*\*Note: Students will need to recognize that the width of both figures is the same.*

6. *Area of first box:  $7(a)=7a\text{ units}$   
Area of second box:  $7(9)=63\text{ units}$*

*\*Note: Students will need to recognize that the width of both figures is the same.*

7. *Area of first box:  $3(x)=3x\text{ units}$   
Area of second box:  $(3)(2)=6\text{ units}$*

Write the expression that represents the total length of each segment.



*Solution:*

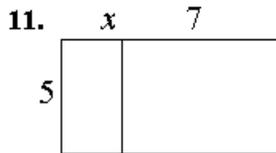
8. The total length of this segment can be written as an expression:  $x+9$  units

9. The total length of this segment can be written as an expression:  $x+4$  units

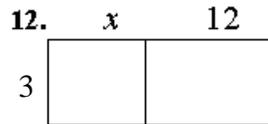
10. The total length of this segment can be written as an expression:  $a+2$  units

*\*Teachers can scaffold this section and demonstrate the ways in which to measure the total length of a segment. Other representations of these types of segments can be added in order to help students think about the same concept in multiple ways*

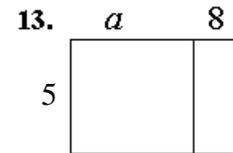
Write the area of each rectangle as the product of **length**  $\times$  **width** and also as a sum of the areas of each box.



AREA AS PRODUCT	AREA AS SUM
$5(x+7)$	$5x+35$



AREA AS PRODUCT	AREA AS SUM



AREA AS PRODUCT	AREA AS SUM

*rectangle while using the area of each individual rectangle and taking the sum of the areas to find the area of the whole rectangle.*

**11. Area as a Product**

*The length of the figure can be written as an expression  $x+7$  (this has been referenced in 8-10 and teachers can use the previous questions to help students come to this realization).*

*The width of this figure is 5 units.*

*The area is found by multiplying  $(x + 7)5 = 5(x + 7)$  by the commutative property*

**Area as a Sum**

*The area of the first left rectangle can be found by multiplying the length,  $x$ , and the width, 5, together. Thus, the area of the first rectangle is  $x(5)=5x$  by the commutative property*

*The area of the second rectangle can be found by multiplying the length, 7, by the width, 5. Thus the area of the first rectangle is  $7(5)=35$ .*

*In order to find the total combined area, students must add together the areas of both figures. Therefore, the total combined area is found as the expression  $5x+35$*

**12. Area as a Product**

*The length of the figure can be written as an expression  $x+12$  (this has been referenced in 8-10 and teachers can use the previous questions to help students come to this realization).*

*The width of this figure is 3 units.*

*The area is found by multiplying  $(x + 12)3 = 3(x + 12)$  by the commutative property*

### Area as a Sum

*The area of the first left rectangle can be found by multiplying the length,  $x$ , and the width,  $x$ , together. Thus, the area of the first rectangle is  $3(x) = 3x$*

*The area of the second rectangle can be found by multiplying the length,  $12$ , by the width,  $x$ . Thus the area of the first rectangle is  $12(x) = 36$ .*

*In order to find the total combined area, students must add together the areas of both figures. Therefore, the total combined area is found as the expression  $3x + 36$*

### 13. Area as a Product

*The length of the figure can be written as an expression  $a+8$  (this has been referenced in 8-10 and teachers can use the previous questions to help students come to this realization).*

*The width of this figure is  $a$  units.*

*The area is found by multiplying  $(a + 8)a = a(a + 8)$  by the commutative property*

### Area as a Sum

*The area of the first left rectangle can be found by multiplying the length,  $a$ , and the width,  $a$ , together. Thus, the area of the first rectangle is  $a(a) = a^2$*

*The area of the second rectangle can be found by multiplying the length,  $8$ , by the width,  $a$ . Thus the area of the first rectangle is  $8(a) = 8a$*

*In order to find the total combined area, students must add together the areas of both figures. Therefore, the total combined area is found as the expression  $a^2 + 8a$*

*After finishing these questions, teachers need to help students come to the realization that the two expressions that they generated from these questions are equivalent and represent the same information in different ways.*

*Use the distributive property to find sums that are equivalent to the following expressions. (You may want to use a rectangle to help you)*

14.  $4(x + 7) =$  \_\_\_\_\_

15.  $7(x - 3) =$  \_\_\_\_\_

16.  $-2(x + 4) =$  \_\_\_\_\_

17.  $3(x + 9) =$  \_\_\_\_\_

18.  $4(a - 1) =$  \_\_\_\_\_

19.  $3(m + 2) =$  \_\_\_\_\_

20.  $-4(a - 4) =$  \_\_\_\_\_

21.  $\frac{1}{2}(a - 12) =$  \_\_\_\_\_

*For 14-21, teachers can ask students to use rectangles to solve the problems. This helps students recognize how to solve the problems while using the area model generated from questions 11-13*

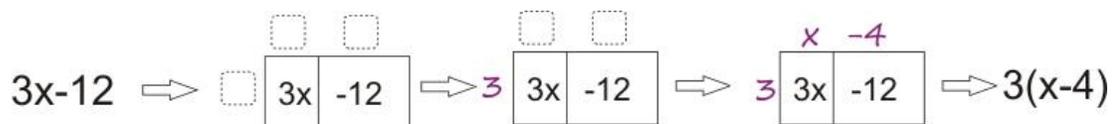
*Solutions*

- 14.  $4x + 28$
- 15.  $7x - 21$
- 16.  $-2x - 8$  \*
- 17.  $3x + 27$
- 18.  $4a - 4$
- 19.  $3m + 6$
- 20.  $-4a + 16$ \*
- 21.  $\frac{1}{2}a - 6$

*\*\*Note: This section also give students negative numbers. Students may struggle with the idea of a “negative area.” Please see below for Illumination’s description for how to simplify for the total area, especially taking note of their suggestion for handling the subtraction problems or the negative signs.*

*Taken from NCTM: Illuminations* <http://illuminations.nctm.org/LessonDetail.aspx?id=L744>

When students are presented with a figure such as the one below, they can be asked to determine the dimensions that yield the area expressed.



Help students who pause when the common factor is a negative number. Be sure that they *change the sign* of the second term. (Example:  $-2a + 10 = -2(a - 5)$  )

### Factoring Using Area Models

*This section requires students to think about how distributive property works in reverse, which is a key for true understanding of the distributive property.*

Fill in the missing information for each: dimensions, area as product, and area as sum

<p>1.</p> <table border="1" style="margin-left: 20px; border-collapse: collapse;"> <tr> <td style="padding: 5px;"><math>x</math></td> <td style="padding: 5px;"><math>6</math></td> </tr> <tr> <td style="padding: 5px;"><math>2</math></td> <td style="padding: 5px;"><math>\square</math></td> </tr> <tr> <td style="padding: 5px;"><math>\square</math></td> <td style="padding: 5px;"><math>\square</math></td> </tr> </table> <p style="margin-left: 20px;"><u>          </u> <u>          </u></p>	$x$	$6$	$2$	$\square$	$\square$	$\square$	<p>2.</p> <table border="1" style="margin-left: 20px; border-collapse: collapse;"> <tr> <td style="padding: 5px;"><math>\square</math></td> <td style="padding: 5px;"><math>\square</math></td> </tr> <tr> <td style="padding: 5px;"><math>5</math></td> <td style="padding: 5px;"><math>5x</math></td> </tr> <tr> <td style="padding: 5px;"><math>\square</math></td> <td style="padding: 5px;"><math>20</math></td> </tr> </table> <p style="margin-left: 20px;"><u>          </u> <u>          </u></p>	$\square$	$\square$	$5$	$5x$	$\square$	$20$	<p>3.</p> <table border="1" style="margin-left: 20px; border-collapse: collapse;"> <tr> <td style="padding: 5px;"><math>\square</math></td> <td style="padding: 5px;"><math>8</math></td> </tr> <tr> <td style="padding: 5px;"><math>\square</math></td> <td style="padding: 5px;"><math>6x</math></td> </tr> <tr> <td style="padding: 5px;"><math>\square</math></td> <td style="padding: 5px;"><math>48</math></td> </tr> </table> <p style="margin-left: 20px;"><u>          </u> <u>          </u></p>	$\square$	$8$	$\square$	$6x$	$\square$	$48$	<p>4.</p> <table border="1" style="margin-left: 20px; border-collapse: collapse;"> <tr> <td style="padding: 5px;"><math>x</math></td> <td style="padding: 5px;"><math>\square</math></td> </tr> <tr> <td style="padding: 5px;"><math>\square</math></td> <td style="padding: 5px;"><math>10x</math></td> </tr> <tr> <td style="padding: 5px;"><math>\square</math></td> <td style="padding: 5px;"><math>30</math></td> </tr> </table> <p style="margin-left: 20px;"><u>          </u> <u>          </u></p>	$x$	$\square$	$\square$	$10x$	$\square$	$30$
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$\square$	$10x$																										
$\square$	$30$																										

*This section is very important to help students recall information about area models learned in the previous section.*

**Solutions:**

*Students must fill in the empty spaces to explain the area for the first and second rectangles.*

1. *Missing Blanks: 2x, 12*
2. *Missing Blanks: x, 4*  
*5(x+4) is the area represented as a product*  
*5x+20 is the area represented as a sum*
3. *Missing Blanks: top-x, side-6*  
*6(x+8)*  
*6x+48*
4. *Missing Blanks, top-10, side-3*  
*10(x+3)*  
*10x+30*

**Georgia Department of Education**  
 Georgia Standards of Excellence Framework  
 GSE Grade 7 • Unit 2

5. $5x + 35 = 5(\quad)$ <div style="display: flex; justify-content: center; gap: 20px;"> <div style="text-align: center;"><input type="text"/> <input type="text"/></div> <div style="text-align: center;"><input type="text"/> <input type="text"/></div> </div> <div style="display: flex; align-items: center; margin-top: 5px;"> <input style="width: 20px; height: 20px; margin-right: 5px;" type="text"/> <div style="border: 1px solid black; padding: 5px; display: flex; gap: 10px;"> <div style="border: 1px solid black; padding: 5px; width: 40px; height: 40px; display: flex; align-items: center; justify-content: center;"><input style="width: 20px; height: 20px;" type="text"/></div> <div style="border: 1px solid black; padding: 5px; width: 40px; height: 40px; display: flex; align-items: center; justify-content: center;"><input style="width: 20px; height: 20px;" type="text"/></div> </div> </div>	6. $2x + 12 = 2(\quad)$ <div style="display: flex; justify-content: center; gap: 20px;"> <div style="text-align: center;"><input type="text"/> <input type="text"/></div> <div style="text-align: center;"><input type="text"/> <input type="text"/></div> </div> <div style="display: flex; align-items: center; margin-top: 5px;"> <input style="width: 20px; height: 20px; margin-right: 5px;" type="text"/> <div style="border: 1px solid black; padding: 5px; display: flex; gap: 10px;"> <div style="border: 1px solid black; padding: 5px; width: 40px; height: 40px; display: flex; align-items: center; justify-content: center;"><input style="width: 20px; height: 20px;" type="text"/></div> <div style="border: 1px solid black; padding: 5px; width: 40px; height: 40px; display: flex; align-items: center; justify-content: center;"><input style="width: 20px; height: 20px;" type="text"/></div> </div> </div>	7. $3x - 21 = \quad(\quad)$ <div style="display: flex; justify-content: center; gap: 20px;"> <div style="text-align: center;"><input type="text"/> <input type="text"/></div> <div style="text-align: center;"><input type="text"/> <input type="text"/></div> </div> <div style="display: flex; align-items: center; margin-top: 5px;"> <input style="width: 20px; height: 20px; margin-right: 5px;" type="text"/> <div style="border: 1px solid black; padding: 5px; display: flex; gap: 10px;"> <div style="border: 1px solid black; padding: 5px; width: 40px; height: 40px; display: flex; align-items: center; justify-content: center;"><input style="width: 20px; height: 20px;" type="text"/></div> <div style="border: 1px solid black; padding: 5px; width: 40px; height: 40px; display: flex; align-items: center; justify-content: center;"><input style="width: 20px; height: 20px;" type="text"/></div> </div> </div>
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*Solutions:*

*Students will work to decompose the distributive property into components.*

*Students can use rectangles to decompose the following problems:*

5.  $(x+7)L-R(top): x;7$   $L-R(bottom): 5, 5x, 35$

6.  $(x+6)L-R(top): x;6$   $L-R(bottom): 2, 2x, 12$

7.  $3(x-7)L-R(top): x;-7$   $L-R(bottom): 3, 3x, -21$

8.  $7(x-7)L-R(top): x;-3$   $L-R(bottom): 7, 7x, -21$

9.  $(x+5)L-R(top): x;5$   $L-R(bottom): -3, -3x, -15$

10.  $-5(x-9)L-R(top): x;-9$   $L-R(bottom): -5, -5x, 45$

Factor these:

11.  $4x - 16 = \underline{\hspace{2cm}}$

12.  $-7x - 35 = \underline{\hspace{2cm}}$

13.  $9x - 81 = \underline{\hspace{2cm}}$

14.  $4x + 18 = \underline{\hspace{2cm}}$

*Solutions:*

11.  $4(x-4)$

12.  $-7(x+5)$

13.  $9(x-9)$

14.  $2(2x+9)$  or  $4(x+9/2)$  *\*this solution can be useful for discussing factoring out the greatest common factor versus a common factor. It will be important to emphasize that these expressions are equivalent*

## **FORMATIVE ASSESSMENT QUESTIONS**

*These questions can be used to help further develop understanding of the distributive property.*

- What relationship between the *product* and *sum* representation of the area model?
- How does the area model help to explain the distributive property?
- Why do you think this property was named the distributive property?

## **DIFFERENTIATION**

### **Extension**

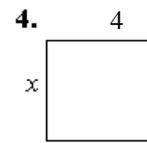
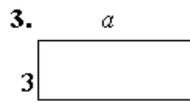
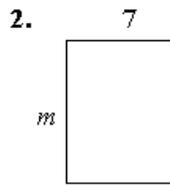
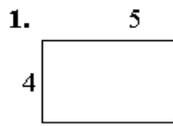
- Have students create and explain models to demonstrate the sum of four or more positive and negative numbers.

### **Intervention**

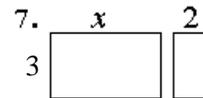
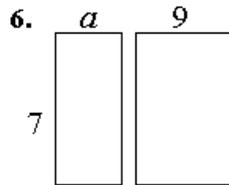
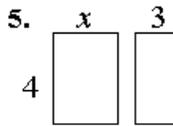
- Have students use models other than those suggested in lesson to add positive and negative numbers, for example, the stack or row model and hot air balloon model.

## Distributing And Factoring Area

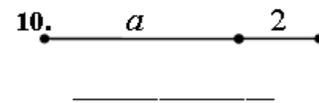
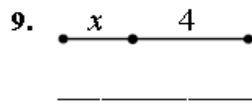
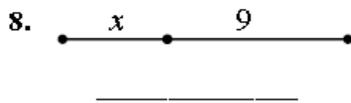
Write the expression that represents the area of each rectangle.



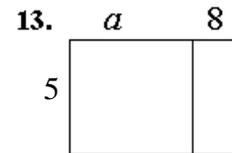
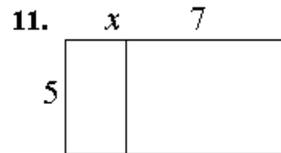
Find the area of each box in the pair.



Write the expression that represents the total length of each segment.



Write the area of each rectangle as the product of **length**  $\times$  **width** and also as a sum of the areas of each box.



AREA AS PRODUCT	AREA AS SUM
$5(x+7)$	$5x+35$

AREA AS PRODUCT	AREA AS SUM

AREA AS PRODUCT	AREA AS SUM

Use the distributive property to find sums that are equivalent to the following expressions. (You may want to use a rectangle to help you)

14.  $4(x + 7) =$  \_\_\_\_\_

15.  $7(x - 3) =$  \_\_\_\_\_

16.  $-2(x + 4) =$  \_\_\_\_\_

17.  $3(x + 9) =$  \_\_\_\_\_

18.  $4(a - 1) =$  \_\_\_\_\_

19.  $3(m + 2) =$  \_\_\_\_\_

20.  $-4(a - 4) =$  \_\_\_\_\_

21.  $\frac{1}{2}(a - 12) =$  \_\_\_\_\_

### Factoring Using Area Models

Fill in the missing information for each: dimensions, area as product, and area as sum

<p>1.</p> <table style="border: 1px solid black; margin: 0 auto; text-align: center;"> <tr> <td style="padding: 5px;"><math>x</math></td> <td style="padding: 5px;"><math>6</math></td> </tr> <tr> <td style="padding: 5px;"><math>2</math></td> <td style="padding: 5px;"><math>2</math></td> </tr> <tr> <td style="border: 1px solid black; width: 30px; height: 30px;"></td> <td style="border: 1px solid black; width: 30px; height: 30px;"></td> </tr> </table> <p>_____</p> <p>_____</p>	$x$	$6$	$2$	$2$			<p>2.</p> <table style="border: 1px solid black; margin: 0 auto; text-align: center;"> <tr> <td style="padding: 5px;"><math>5x</math></td> <td style="padding: 5px;"><math>20</math></td> </tr> <tr> <td style="padding: 5px;"><math>5</math></td> <td style="padding: 5px;"><math>2</math></td> </tr> <tr> <td style="border: 1px solid black; width: 30px; height: 30px;"></td> <td style="border: 1px solid black; width: 30px; height: 30px;"></td> </tr> </table> <p>_____</p> <p>_____</p>	$5x$	$20$	$5$	$2$			<p>3.</p> <table style="border: 1px solid black; margin: 0 auto; text-align: center;"> <tr> <td style="padding: 5px;"><math>6x</math></td> <td style="padding: 5px;"><math>48</math></td> </tr> <tr> <td style="padding: 5px;"><math>8</math></td> <td style="padding: 5px;"><math>2</math></td> </tr> <tr> <td style="border: 1px solid black; width: 30px; height: 30px;"></td> <td style="border: 1px solid black; width: 30px; height: 30px;"></td> </tr> </table> <p>_____</p> <p>_____</p>	$6x$	$48$	$8$	$2$			<p>4.</p> <table style="border: 1px solid black; margin: 0 auto; text-align: center;"> <tr> <td style="padding: 5px;"><math>10x</math></td> <td style="padding: 5px;"><math>30</math></td> </tr> <tr> <td style="padding: 5px;"><math>x</math></td> <td style="padding: 5px;"><math>3</math></td> </tr> <tr> <td style="border: 1px solid black; width: 30px; height: 30px;"></td> <td style="border: 1px solid black; width: 30px; height: 30px;"></td> </tr> </table> <p>_____</p> <p>_____</p>	$10x$	$30$	$x$	$3$		
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Fill in the missing dimensions from the expression given.

<p>5. <math>5x + 35 = 5(\quad)</math></p> <table style="border: 1px solid black; margin: 0 auto; text-align: center;"> <tr> <td style="padding: 5px;"><math>5</math></td> <td style="padding: 5px;"><math>5</math></td> </tr> <tr> <td style="padding: 5px;"><math>5</math></td> <td style="padding: 5px;"><math>5</math></td> </tr> <tr> <td style="border: 1px solid black; width: 30px; height: 30px;"></td> <td style="border: 1px solid black; width: 30px; height: 30px;"></td> </tr> </table>	$5$	$5$	$5$	$5$			<p>6. <math>2x + 12 = 2(\quad)</math></p> <table style="border: 1px solid black; margin: 0 auto; text-align: center;"> <tr> <td style="padding: 5px;"><math>2</math></td> <td style="padding: 5px;"><math>2</math></td> </tr> <tr> <td style="padding: 5px;"><math>2</math></td> <td style="padding: 5px;"><math>2</math></td> </tr> <tr> <td style="border: 1px solid black; width: 30px; height: 30px;"></td> <td style="border: 1px solid black; width: 30px; height: 30px;"></td> </tr> </table>	$2$	$2$	$2$	$2$			<p>7. <math>3x - 21 = \quad(\quad)</math></p> <table style="border: 1px solid black; margin: 0 auto; text-align: center;"> <tr> <td style="padding: 5px;"><math>3</math></td> <td style="padding: 5px;"><math>3</math></td> </tr> <tr> <td style="padding: 5px;"><math>3</math></td> <td style="padding: 5px;"><math>3</math></td> </tr> <tr> <td style="border: 1px solid black; width: 30px; height: 30px;"></td> <td style="border: 1px solid black; width: 30px; height: 30px;"></td> </tr> </table>	$3$	$3$	$3$	$3$		
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Use rectangles to factor the following problems:

Factor these:

11.  $4x - 16 = \underline{\hspace{2cm}}$

12.  $-7x - 35 = \underline{\hspace{2cm}}$

13.  $9x - 81 = \underline{\hspace{2cm}}$

14.  $4x + 18 = \underline{\hspace{2cm}}$

## **Triangles And Quadrilaterals**

[Back to task table](#)

Adapted from EngageNY

### **STANDARDS FOR MATHEMATICAL CONTENT**

**MGSE7.EE.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

**MGSE7.EE.2** Understand that rewriting an expression in different forms in a problem context can clarify the problem and how the quantities in it are related. *For example  $a + 0.05a = 1.05a$  means that adding a 5% tax to a total is the same as multiplying the total by 1.05.*

### **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Solve problems and persevere.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.

### **ESSENTIAL QUESTIONS**

- How can we represent value using variables?

### **MATERIALS REQUIRED**

- Envelopes containing triangles and quadrilaterals.

### **TIME NEEDED**

- 1 class period

### **TASK DESCRIPTION**

#### **PART I**

Each student is given an envelope containing triangles and quadrilaterals.

**How might expressions be generated based on the contents of your envelope? What expressions could you write?**

Individually. Compare with a partner. Share as a whole class.

## PART II

Each envelope contains a number of triangles and a number of quadrilaterals. For this exercise, let  $t$  represent the number of triangles, and let  $q$  represent the number of quadrilaterals.

a. Write an expression, using  $t$  and  $q$ , that represents the total number of sides in your envelope. Explain what the terms in your expression represent.

$$3t+4q.$$

*Triangles have 3 sides, so there will be 3 sides for each triangle in the envelope. This is represented by  $3t$ . Quadrilaterals have 4 sides, so there will be 4 sides for each quadrilateral in the envelope. This is represented by  $4q$ . The total number of sides will be the number of triangle sides and the number of quadrilateral sides together.*

b. You and your partner have the same number of triangles and quadrilaterals in your envelopes. Write an expression that represents the total number of sides that you and your partner have. If possible, write more than one expression to represent this total.

$$3t+4q+3t+4q; \quad 2(3t+4q); \quad 6t+8q$$

*Discuss the variations of the expression in part (b) and whether those variations are equivalent. This discussion helps students understand what it means to combine like terms; some students have added their number of triangles together and number of quadrilaterals together, while others simply doubled their own number of triangles and quadrilaterals since the envelopes contain the same number. This discussion further shows how these different forms of the same expression relate to each other. Students then complete part (c).*

c. Each envelope in the class contains the same number of triangles and quadrilaterals. Write an expression that represents the total number of sides in the room.

*Answer depends on the seat size of the classroom. For example, if there are 12 students in the class, the expression would be  $12(3t+4q)$ , or an equivalent expression.*

*Next, discuss any variations (or possible variations) of the expression in part (c), and discuss whether those variations are equivalent. Are there as many variations in part (c), or did students use multiplication to consolidate the terms in their expressions? If the latter occurred, discuss the students' reasoning.*

*Choose one student to open his/her envelope and count the numbers of triangles and quadrilaterals. Record the values of  $t$  and  $q$  as reported by that student for all students to see. Next, students complete parts (d), (e), and (f).*

d. Use the given values of  $t$  and  $q$ , and your expression from part (a), to determine the number of sides that should be found in your envelope.

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$$3t + 4q$$

$$3(4) + 4(2)$$

$$12 + 8$$

$$20$$

*There should be 20 sides contained in my envelope.*

e. Use the same values for  $t$  and  $q$ , and your expression from part (b), to determine the number of sides that should be contained in your envelope and your partner's envelope combined.

*Variation #1*

$$2(3t + 4q)$$

$$2(3(4) + 4(2))$$

$$2(12 + 8)$$

$$2(20)$$

$$40$$

*Variation #2*

$$3t + 4q + 3t + 4q$$

$$3(4) + 4(2) + 3(4) + 4(2)$$

$$12 + 8 + 12 + 8$$

$$20 + 12 + 8$$

$$40$$

*Variation #3*

$$6t + 8q$$

$$6(4) + 8(2)$$

$$24 + 16$$

$$40$$

*My partner and I have a combined total of 40 sides.*

f. Use the same values for  $t$  and  $q$ , and your expression from part (c), to determine the number of sides that should be contained in all of the envelopes combined

*Answer will depend on the seat size of your classroom. Sample responses for a class size of 12:*

<i>Variation 1</i>	<i>Variation 2</i>	<i>Variation 3</i>
$12(3t + 4q)$	$\overbrace{3t + 4q}^1 + \overbrace{3t + 4q}^2 + \dots + \overbrace{3t + 4q}^{12}$	$36t + 48q$
$12(3(4) + 4(2))$	$\overbrace{3(4) + 4(2)}^1 + \overbrace{3(4) + 4(2)}^2 + \dots + \overbrace{3(4) + 4(2)}^{12}$	$36(4) + 48(2)$
$12(12 + 8)$	$\overbrace{3(4) + 4(2)}^1 + \overbrace{3(4) + 4(2)}^2 + \dots + \overbrace{3(4) + 4(2)}^{12}$	$144 + 96$
$12(20)$	$\overbrace{12 + 8}^1 + \overbrace{12 + 8}^2 + \dots + \overbrace{12 + 8}^{12}$	$240$
$240$	$\overbrace{20}^1 + \overbrace{20}^2 + \dots + \overbrace{20}^{12}$	$240$

*For a class size of 12 students, there should be 240 sides in all of the envelopes combined.*

Have all students open their envelopes and confirm that the number of triangles and quadrilaterals matches the values of  $t$  and  $q$  recorded after part (c). Then, have students count the number of sides contained on the triangles and quadrilaterals from their own envelope and confirm with their answer to part (d). Next, have partners count how many sides they have combined and confirm that number with their answer to part (e). Finally, total the number of sides reported by each student in the classroom and confirm this number with the answer to part (f).

**g. What do you notice about the various expressions in parts (e) and (f)?**

The expressions in part (e) are all equivalent because they evaluate to the same number: **40**. The expressions in part (f) are all equivalent because they evaluate to the same number: **240**. The expressions themselves all involve the expression  **$3t+4q$**  in different ways. In part (e),  **$3t+3t$**  is equivalent to  **$6t$** , and  **$4q+4q$**  is equivalent to  **$8q$** . There appear to be several relationships among the representations involving the commutative, associative, and distributive properties.

When finished, have students return their triangles and quadrilaterals to their envelopes for use by other classes.

**Extension:**

- Numbers of triangles and quadrilaterals can vary among students. Students can set up equivalent expressions (equations) using their available triangles and quadrilaterals.

**Intervention:**

- Envelopes can have adjusted numbers of triangles and quadrilaterals to make computation less cumbersome for students who need support with the concept of writing expressions using variables, and combining them.

**PART III:**

**Using shapes other than triangles and quadrilaterals, generate and model expressions and have a partner recreate your expressions given clues and instructions (from their partner).**

**Triangles And Quadrilaterals**

Adapted from EngageNY

Name \_\_\_\_\_

**PART I**

Each student is given an envelope containing triangles and quadrilaterals.

**How might expressions be generated based on the contents of your envelope? What expressions could you write?**

**PART II**

Each envelope contains a number of triangles and a number of quadrilaterals. For this exercise, let  $t$  represent the number of triangles, and let  $q$  represent the number of quadrilaterals.

**a. Write an expression, using  $t$  and  $q$ , that represents the total number of sides in your envelope. Explain what the terms in your expression represent.**

**b. You and your partner have the same number of triangles and quadrilaterals in your envelopes. Write an expression that represents the total number of sides that you and your partner have. If possible, write more than one expression to represent this total.**

**c. Each envelope in the class contains the same number of triangles and quadrilaterals. Write an expression that represents the total number of sides in the room.**

**d. Use the given values of  $t$  and  $q$ , and your expression from part (a), to determine the number of sides that should be found in your envelope.**

**e. Use the same values for  $t$  and  $q$ , and your expression from part (b), to determine the number of sides that should be contained in your envelope and your partner's envelope combined.**

**f. Use the same values for  $t$  and  $q$ , and your expression from part (c), to determine the number of sides that should be contained in all of the envelopes combined**

**g. What do you notice about the various expressions in parts (e) and (f)?**

**PART III:**

**Using shapes other than triangles and quadrilaterals, generate and model expressions and have a partner recreate your expressions given clues and instructions (from their partner).**

## **Area And Algebra**

[Back to task table](#)

Adapted from:

[http://mdk12.org/instruction/clg/public\\_release/algebra\\_data\\_analysis/G1\\_E1\\_I3.html](http://mdk12.org/instruction/clg/public_release/algebra_data_analysis/G1_E1_I3.html)

<http://www.regentsprep.org/Regents/math/ALGEBRA/MultipleChoiceReview/shapetrap.jpg>

To complete the task, students will revisit area and perimeter of composite figures. In conjunction with this knowledge, students will solve the task by rewriting and evaluating algebraic expressions, applying the properties of real numbers (particularly the distributive property), and reasoning and problem solving.

### **STANDARDS FOR MATHEMATICAL CONTENT**

**MGSE7.EE.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

**MGSE7.EE.2** Understand that rewriting an expression in different forms in a problem context can clarify the problem and how the quantities in it are related. *For example  $a + 0.05a = 1.05a$  means that adding a 5% tax to a total is the same as multiplying the total by 1.05.*

### **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

### **BACKGROUND KNOWLEDGE**

In order for students to be successful, the following skills and concepts need to be maintained:

- Knowledge that perimeter is used to measure the outside length of a figure
- Knowledge that area is used to measure the space inside of a figure
- Adding and subtracting positive and negative rational numbers in a real-world context

### **COMMON MISCONCEPTIONS**

- Students have difficulties understanding equivalent forms of numbers, their various uses and relationships, and how they apply to a problem. Make sure to expose students to multiple examples and in various contexts.

### **ESSENTIAL QUESTIONS**

- What properties are required in order to rewrite and evaluate algebraic expressions and solve equations?

### **MATERIALS**

- Area and Algebra Student Task sheet
- Graph paper (optional)

• **GROUPING**

Individual/Partner

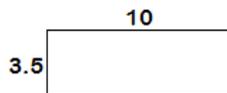
**TASK DESCRIPTION**

As an introduction to the task, problems involving area and perimeter with plane and composite figures may need to be reviewed. Problems will need to include plane and composite figures with missing side lengths so that students may reacquaint themselves with the area and perimeter concepts learned prior to 7<sup>th</sup> grade.

**I. Perimeter and Area of Figures**

Find the perimeter and area of the following figures. **Explain in words how you found the perimeter and area of each figure.** (unit: inches)

1.



Perimeter:  $10+3.5+10+3.5=27$  inches

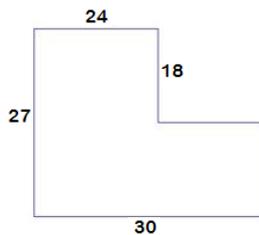
Area:  $10(3.5)=35.0$  inches

Explanation:

*Example: The perimeter of a figure is found by adding all sides. You must split this figure into two rectangles in order to find the total area.*

*Students should be able to demonstrate the difference between perimeter and area as well as write their computations out step-by-step.*

2.



Perimeter:  $27+24+18+6+9+30=114$  inches

Area:  $27(24)+9(6)=648+54=702$  inches

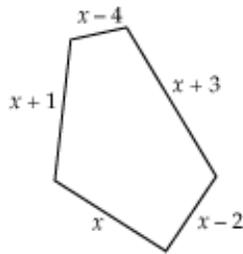
Explanation:

*Example: The perimeter of a figure is found by adding all sides. You must split this figure into two rectangles in order to find the total area.*

*Students should be able to demonstrate the difference between perimeter and area as well as write their computations out step-by-step. It is crucial that students be able to solve for the missing side lengths and their measurements*

## II. Perimeter of Algebraic Figures

Find the perimeter of each of the following figures.



1. What is the perimeter of this figure?

$$\begin{aligned} &(x + 1) + (x - 1) + (x + 3) + (x - 2) + x \\ &x + x + x + x + 1 - 1 + 3 - 2 \quad \text{by commutative property} \\ &5x - 2 \quad \text{by associative property and combining like terms} \end{aligned}$$

2. What is the perimeter of the figure if  $x=3$ in.? Show your calculations step-by-step.

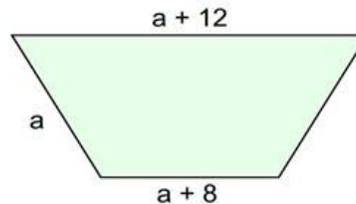
$$5x - 2 = 5(3) - 2 = 15 - 2 = 13$$

*Students must use the expression generated in Part 2 #1 in order to solve this for this equation.*

3. What is the perimeter of this figure?

$$a + a + 12 + a + 8 + a = 4a + 20$$

*\*See #1 for similar explanation*



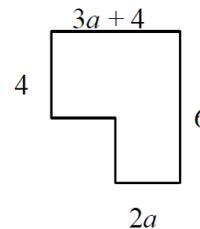
4. What is the perimeter of the figure if  $a=1/2$  in.? Show your calculations step-by-step.

$$4a + 20 = 4\left(\frac{1}{2}\right) + 20 = 2 + 20 = 22 \text{ inches}$$

[http://mdk12.org/instruction/clg/public\\_release/algebra\\_data\\_analysis/G1\\_E1\\_I3.html](http://mdk12.org/instruction/clg/public_release/algebra_data_analysis/G1_E1_I3.html)  
<http://www.regentsprep.org/Regents/math/ALGEBRA/MultipleChoiceReview/shapetrap.jpg>

## III. Perimeter and Area of Algebraic Figures

**Task Directions:** A corner has been removed from this rectangle.



1. Find an expression for the perimeter of the rectangle.

**Solution:**

*The perimeter of the rectangle is found by adding all of the sides.*

*The sum of the sides is  $4 + 3a + 4 + 6 + 2a + 2 + (3a + 4 - 2a)$ .*

$$\begin{aligned} &3a + 2a + a + 4 + 4 + 6 + 2 + 4 \quad \text{subtraction and commutative property of addition} \\ &(3a + 2a + a) + (4 + 4 + 6 + 2 + 4) \quad \text{associative property of addition} \\ &6a + 20 \end{aligned}$$

2. What is the perimeter of the rectangle if  $a = 3/4$  inch? Show your calculations step-by-step.

**Solution:**

If  $a = \frac{3}{4}$ , then  $6(\frac{3}{4}) + 20 = \frac{18}{4} + 20 = \frac{18}{4} + \frac{80}{4} = \frac{98}{4}$  or 24.5 inches.

3. Find an expression for the area of the rectangle.

**Solution:**

*There are two ways to find this area. One is to treat the shape as the composition of two rectangles.*

*The area of the smaller rectangle is  $2(2a)$  or  $4a$ .*

*The area of the larger rectangle is  $4(3a + 4)$ .*

*Thus the area is:  $4a + 4(3a + 4)$*

*$4a + 12a + 16$  distributive property*

*$16a + 16$*

*A second method is to compose one rectangle and subtract away the area of the corner that has been removed.*

*The area of the composed rectangle is  $6(3a + 4)$  or  $18a + 24$ .*

*The area of the corner rectangle that has been removed is  $2(3a + 4 - 2a)$  or  $2a + 8$ .*

*Thus the area is:  $(18a + 24) - (2a + 8)$*

*$16a + 16$*

4. What is the area of the rectangle if  $a = 1.8$  feet? Show your calculations step-by-step.

**Solution:**

*If  $a = 1.8$  feet, then the area is  $16 \times 1.8 + 16 = 28.8 + 16 = 44.8$  square feet.*

**DIFFERENTIATION**

**Extension**

- Create different problems similar to the one in this task involving irregular shapes.

**Intervention**

- Have students complete problems that do not involve fractional units (fractions or decimals). Make sure that all polygons used for this lesson are regular; do not use irregular un-closed shapes.

**Area And Algebra**

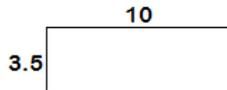
**I. Perimeter and Area of Figures**

*Find the perimeter and area of the following figures. Explain in words how you found the perimeter and area of each figure. (unit: inches)*

1.

Perimeter: \_\_\_\_\_

Area: \_\_\_\_\_

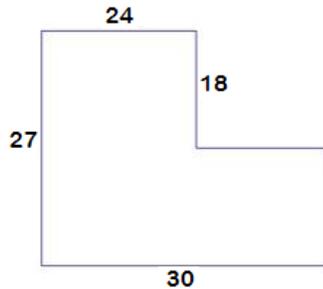


Explanation:

2.

Perimeter: \_\_\_\_\_

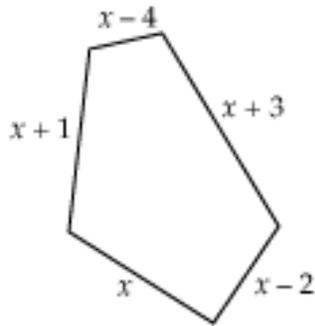
Area: \_\_\_\_\_



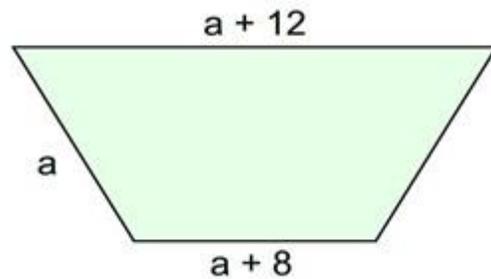
Explanation:

**II. Perimeter of Algebraic Figures**

Find the perimeter of each of the following figures.



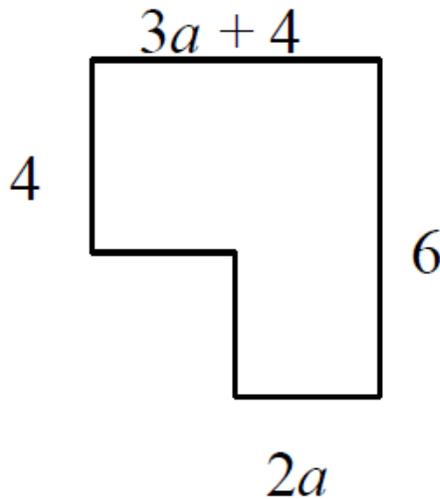
1. What is the perimeter of this figure?
2. What is the perimeter of the figure if  $x=3$ in.? Show your calculations step-by-step.



3. What is the perimeter of this figure?
4. What is the perimeter of the figure if  $a=1/2$  in.? Show your calculations step-by-step.

**III. Perimeter and Area of Algebraic Figures**

*A corner has been removed from this rectangle. Answer the following questions related to figure below.*



1. Find an expression for the perimeter of the rectangle.
2. What is the perimeter of the rectangle if  $a = \frac{3}{4}$  inch? Show your calculations step-by-step.
3. Find an expression for the area of the rectangle.
4. What is the area of the rectangle if  $a = 1.8$  feet? Show your calculations step-by-step.

## **Guess My Number! (Spotlight Task)**

[Back to task table](#)

Adapted from Illustrative Mathematics <https://www.illustrativemathematics.org/illustrations/712>

### **STANDARDS FOR MATHEMATICAL CONTENT**

**MGSE7.EE.2** Understand that rewriting an expression in different forms in a problem context can clarify the problem and how the quantities in it are related. *For example  $a + 0.05a = 1.05a$  means that adding a 5% tax to a total is the same as multiplying the total by 1.05.*

### **Solve real-life and mathematical problems using numerical and algebraic expressions and equations**

**MGSE7.EE.3** Solve multistep real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals) by applying properties of operations as strategies to calculate with numbers, converting between forms as appropriate, and assessing the reasonableness of answers using mental computation and estimation strategies.

*For example:*

- *If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for a new salary of \$27.50.*
- *If you want to place a towel bar  $9\frac{3}{4}$  inches long in the center of a door that is  $27\frac{1}{2}$  inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

### **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

### **BACKGROUND KNOWLEDGE**

This problem asks the students to represent a sequence of operations using an expression and then to write and solve simple equations. The problem is posed as a game and allows the students to visualize mathematical operations. It would make sense to actually play a similar game in pairs first and then ask the students to record the operations to figure out each other's numbers.

Some students will write  $(x+4) \cdot 5 - 7$

Many students translate word problems literally and place numbers in the problem as they appear in the sentences/paragraph. This provides a good opportunity to talk about the properties of operations and how they can be used to write the same expression in different ways.

The last part of the task in particular is meant to generate classroom discussion, and isn't meant for e.g. high-stakes assessment situations.

### **ESSENTIAL QUESTIONS**

- What strategies can be used for understanding and representing real situations using algebraic expressions and equations?

### **MATERIALS**

- Area and Algebra Student Task sheet
- Graph paper (optional)

### **GROUPING**

Individual/Partner

### **TASK DESCRIPTION**

Laila tells Julius to pick a number between one and ten. “Add three to your number and multiply the sum by five,” she tells him. Next she says, “Now take that number and subtract seven from it and tell me the new number.” “Twenty-three,” Julius exclaims.

Can you figure out what Julius’ number was?

Write down a guess that is too low.

Write down a guess that is too high.

How did you know?

Working with a partner, record your operations that Julius used. What was his number?

In the next round, Leila is supposed to pick a number between 1 and 10 and follow the same instructions. She gives her final result as 108. Julius immediately replies: “Hey, you cheated!” What might he mean?

*This problem asks the students to represent a sequence of operations using an expression and then to write and solve simple equations. The problem is posed as a game and allows the students to visualize mathematical operations. It would make sense to actually play a similar game in pairs first and then ask the students to record the operations to figure out each other's numbers.*

### ***Solutions***

*The unknown variable here is Julius’ original number. Let  $n$  be Julius’ original number. We know from Laila’s directions that first, Julius added three to his number:*

$$n+3$$

*She then told him to multiply this sum by five:*

$$5(n+3)$$

*Finally, she told him to subtract seven from this number:*

$$5(n+3)-7$$

*This is the expression that records the operations that Julius used.*

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1. *Julius exclaims that his new number after performing Laila's instructions is twenty-three. So we set our expression from part (a) equal to 23 and solve for n, Julius' original number.*

$$5(n+3)-7=23$$

$$5n+15=30$$

$$5n=15$$

$$n=3$$

*Thus, Julius' original number was three.*

2. *To find Laila's number we solve*

$$5(n+3)-7=108.$$

*We find  $n=20$ . So Laila did not follow the instructions of using a number between 1 and 10.*

**Guess My Number! (Spotlight Task)**

Name \_\_\_\_\_ Date \_\_\_\_\_



Laila tells Julius to pick a number between one and ten. “Add three to your number and multiply the sum by five,” she tells him. Next she says, “Now take that number and subtract seven from it and tell me the new number.” “Twenty-three,” Julius exclaims.

Can you figure out what Julius’ number was?

Write down a guess that is too low.

Write down a guess that is too high.

How did you know?

Working with a partner, record your operations that Julius used. What was his number?

In the next round, Leila is supposed to pick a number between 1 and 10 and follow the same instructions. She gives her final result as 108. Julius immediately replies: “Hey, you cheated!” What might he mean?

## Algebra Magic

[Back to task table](#)

Is this magic trick based on telepathy, or algebra? To find out, students model a sequence of algebraic steps. (*Adapted from The Future's Channel website*)

**\* This task is similar to the FAL “Building and Solving Equations 1” that comes after it.**

The outline provided is useful and suggested to be used for this task. It can be accessed through the link below. The activity includes a mini-lesson that introduces students to writing expressions. The lesson will guide students through the process of modeling the expression.

**The original task can be found at:**

[http://www.thefutureschannel.com/pdf/math/algebra\\_magic.pdf](http://www.thefutureschannel.com/pdf/math/algebra_magic.pdf)

## STANDARDS FOR MATHEMATICAL CONTENT

**MGSE7.EE.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

**MGSE7.EE.2** Understand that rewriting an expression in different forms in a problem context can clarify the problem and how the quantities in it are related. *For example  $a + 0.05a = 1.05a$  means that adding a 5% tax to a total is the same as multiplying the total by 1.05.*

**MGSE7.EE.3** Solve multistep real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals) by applying properties of operations as strategies to calculate with numbers, converting between forms as appropriate, and assessing the reasonableness of answers using mental computation and estimation strategies.

*For example:*

- *If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for a new salary of \$27.50.*
- *If you want to place a towel bar  $9\frac{3}{4}$  inches long in the center of a door that is  $27\frac{1}{2}$  inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

**MGSE7.EE.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

## STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

### **COMMON MISCONCEPTIONS**

**7.EE.3** Provide concrete examples for students to use and understand the properties of operations. These include: the commutative, associative, identity, and inverse properties of addition and of multiplication, and the zero property of multiplication.

Another method students can use to become convinced that expressions are equivalent is to justify each step of rewriting of an expression with an operation property.

**7.EE.4** Provide multiple opportunities for students to work with multi-step problem situations that have multiple solutions and therefore can be represented by an inequality. Students need to be aware that values can satisfy an inequality but not be appropriate for the situation, therefore limiting the solutions for that particular problem.

Write an equation or inequality to model the situation. Explain how you determined whether to write an equation or inequality and the properties of the real number system that you used to find a solution

### **ESSENTIAL QUESTIONS**

- How can mathematical relationships be represented as expressions, equations, or inequalities?

### **MATERIALS**

- 1 plastic re-sealable bag containing
  - 10 small discs (roughly 1” in diameter)
  - 2 other objects which are identical to each other but different from and somewhat larger than the discs (these objects could be wooden blocks, plastic construction bricks, or geometric shapes cut from construction paper or cardboard)
- 1 calculator (optional but recommended)

### **GROUPING**

Individual/partner

### **TASK DESCRIPTION**

This task is designed to help students develop their understanding of how to translate words into algebraic expressions. Students will also need to implement properties (commutative, associative, distributive, etc.) in order to rewrite the expressions and prove how the trick works.

### **Teacher Notes: Introduction to Algebra Magic Tricks**

Ask students to pick a number and write it on their paper. Then read through the following steps and have students follow along performing the given operations on their paper. Make sure to pause while reading to allow students time to complete the operation.

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- Start with a number.
- Add seven.
- Double the result.
- Subtract four.
- Half the result.
- Subtract five

**Solution:** *Example:  $2 + 7 = 9(2) = 18 - 4 = 14(1/2) = 7 - 5 = 2$*   
*All students should get the same number.*

Upon completion, discuss what result each student got as a result. Why do you think this occurred?

- Think about the date of your birthday.
- Multiply the number of the month of your birthday by 5.
- Add 7.
- Multiply by 4.
- Add 13.
- Multiply by 5.
- Add the day of your birthday.
- Subtract 205.
- Write your answer.
- Your answer should show the month and day of your birth.**

**USING MATH TO PROVE ALGEBRA MAGIC TRICKS**

For each magic trick below, work through the trick with numbers. Then, write an expression and rewrite it to demonstrate the operation being performed in the trick.

**Trick #1**

Steps to Trick	Numerical	Expression	Equivalent Expression
Start with an even number	<i>i.e. 4</i>	<i>x</i>	<i>X</i>
Double the number	<i>4(2) = 8</i>	<i>2x</i>	<i>2x</i>
Add 4	<i>8 + 4 = 12</i>	<i>2x + 4</i>	<i>2x + 4</i>
Subtract 6	<i>12 - 6 = 6</i>	<i>(2x + 4) - 6</i> <i>*opportunity to review associative property</i>	<i>2x - 2</i>
Half the number	<i>6/2 = 3</i>	<i>(2x - 2) / 2</i>	<i>X - 1</i>

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**Trick #2**

Steps to Trick	Numerical	Expression	Equivalent Expression
Start with a number	<i>i.e.</i> 2	$X$	$X$
Add six	$2 + 6 = 8$	$X + 6$	$X + 6$
Double the Result	$8(2) = 16$	$2(X + 6)$	$2x + 12$
Subtract 20	$16 - 20 = -4$	$(2x + 12) - 20$ <i>*associative property</i>	$2x + -8$
Subtract your original number	$-4 - 2 = -6$	$(2x + -8) - x$ <i>*associative and commutative property</i>	$X + -8$
Add 8	$-6 + 8 = 2$	$(X + -8) + 8$ <i>*associative property</i>	$X$

**Trick #3**

Steps to Trick	Numerical	Expression	Equivalent Expression
Start with a number. It must be a fraction.	<i>i.e.</i> $\frac{1}{2}$	$x$	$X$
Triple the number	$\frac{1}{2}(3) = \frac{3}{2}$	$3x$	$3x$
Add $\frac{-1}{4}$	$\frac{3}{2} - \frac{1}{4} = \frac{5}{4}$	$3x - \frac{1}{4}$	$3x + 8$ $3x - \frac{1}{4}$
Subtract twice the number	$\frac{5}{4} - (2)\left(\frac{1}{2}\right) = \frac{1}{4}$	$3x - \frac{1}{4} - 2x$	$x - \frac{1}{4}$
Add $\frac{3}{5}$	$\frac{1}{4} + \frac{3}{5} = \frac{17}{20}$	$(x - \frac{1}{4}) + \frac{3}{5} = x + \frac{7}{20}$	$x + \frac{7}{20}$
Subtract the original number	$\frac{17}{20} - \frac{1}{2} = \frac{7}{20}$	$X + \frac{7}{20} - x$	$\frac{7}{20}$

- What properties did you have to apply in order to rewrite the algebraic expressions?  
Solution: *distributive property, commutative property, combining like terms*
- Create a trick with four steps that will have a result of 5. Write the algebraic expression(s) that prove it works.  
Solution: *Answers May vary*
- Create a new trick with the distributive property that will result in starting and ending with the same value. Write the algebraic expression(s) that prove that it works.  
Solution: *Answers May vary*

4. Create a trick where you start with the day of your birth and end with the month of your birth. Write a numerical expression to show each of your steps.

*Solution: Answers may vary*

## **DIFFERENTIATION**

### **Extension:**

- Have students create more complex number tricks using decimals or fractions within their steps. Can students create number tricks that begin or end with a fraction or decimal?

### **Intervention:**

- Tricks can be simplified by having fewer steps. Questions can be changed to create tricks that are less complex or have fewer steps. Students may also find more success by working with a partner and talking through what happens in each trick. Modeling with manipulatives may be a way to provide a concrete explanation of what is taking place in a problem.



## Algebra Magic - Using Math To Prove Algebra Magic Tricks

In the space provided, follow your teacher’s instructions for the first math trick. What did you get as a result? How does this compare to your classmates answers?

For each magic trick below, work through the trick with numbers. Then, write an expression and rewrite it to demonstrate the operation being performed in the trick.

### Trick #1

Steps to Trick	Numerical	Expression	Equivalent Expression
Start with a number			
Double the number			
Add 4			
Subtract 5			
Half the number			

### Trick #2

Steps to Trick	Numerical	Expression	Equivalent Expression
Start with a number			
Add six			
Double the Result			
Subtract 20			
Subtract your original number			
Add 8			

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**Trick #3**

<b>Steps to Trick</b>	<b>Numerical</b>	<b>Expression</b>	<b>Equivalent Expression</b>
Start with a number. It must be a fraction.			
Triple the number			
Add $\frac{-1}{4}$			
Subtract twice the number			
Add $\frac{3}{5}$			
Subtract the original number			

1. What properties did you have to apply in order to rewrite the algebraic expressions?
2. Create a trick with four steps that will have a result of 5. Write the algebraic expression(s) that prove it works.
3. Create a new trick with the distributive property that will result in starting and ending with the same value. Write the algebraic expression(s) to prove it works.
4. Create a trick where you start with the day of your birth and end with the month of your birth. Write a numerical expression to show each of your steps.

## **Deconstructing Word Problems**

[Back to task table](#)

(Adapted from “Deciphering Word Problems” which can be found at [http://www.nsa.gov/academia/files/collected\\_learning/middle\\_school/algebra/deciphering\\_word\\_problems.pdf](http://www.nsa.gov/academia/files/collected_learning/middle_school/algebra/deciphering_word_problems.pdf) )

The goal of this lesson is for students to build expressions and equations when presented with a real life problem.

### **STANDARDS FOR MATHEMATICAL CONTENT:**

**MGSE7.EE.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

**MGSE7.EE.2** Understand that rewriting an expression in different forms in a problem context can clarify the problem and how the quantities in it are related. *For example  $a + 0.05a = 1.05a$  means that adding a 5% tax to a total is the same as multiplying the total by 1.05.*

**MGSE7.EE.4a** Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

### **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

### **BACKGROUND KNOWLEDGE**

In sixth grade, students were taught to solve and show work for one step equations involving whole numbers.

### **COMMON MISCONCEPTIONS**

*Students might struggle with setting up the charts and will need whole group instruction on how to set them up.*

There may also be confusion about what part of the word problem leads to the expressions and which part is for the writing of the equation.

### **ESSENTIAL QUESTIONS**

- How can information from a word problem be translated to create an equation?

### **MATERIALS:**

- Pencil and Paper
- Student Sheet with Charts

### **GROUPING**

Individual/partner

### **TASK DESCRIPTION**

In this task, student will work through creating tables in order to build expressions and equations in order to solve a word problem.

### **Introduction:**

Have the students work through the translating verbal expression warm-up questions in order to address misconceptions about what words represent each operation. (Keep an eye out for “less than” and how it differs in its placement in an expression. Anything that says than or from changes the order of how the expression is written.)

### **Warm-Up Expressions**

For each expression below, translate the verbal expression to an algebraic expression.

- 1) Ann has the 5 newest music CD’s which is 3 less than twice the amount that Bob has.  $2x - 3 = 5$
- 2) Mike, who has 6 video games, has half as many games as Paul.  $x/2 = 6$
- 3) Nan rode the roller coaster 8 times, which was twice as many times as she rode the Ferris wheel.  $2x = 8$
- 4) Janine, who bought \$15 worth of make-up, spent \$6 less than Leah spent.  
 $x - 6 = 15$
- 5) Rob, who has all 13 girls’ phone numbers that are in his homeroom, has 3 more than half the number of girls’ phone numbers that Jay has.  
 $x/2 + 3 = 13$
- 6) Kate’s 85 on her English test was 37 points less than twice the grade on her Science test.  
 $2x - 37 = 85$
- 7) At the Middle School Graduation Dance, the DJ played 12 slow dances, which was equal to the quotient of the number of fast dances and 2.  $x/2 = 12$

### **Creating Equations from Word Problems**

Use the chart provided to create expressions for the situation described in the word problem. Then, use these expressions and the word problem to create and solve an equation. Make sure you not only solve for the variable, but also answer the question being presented.

- Students sometimes struggle with deciding which person or item is the variable. One suggestion is to tell them it is the person or item on which everything else is based.
1. Sean sold 4 more boxes of candy for the school fundraiser than Marta. The sum of the boxes they sold was 22. How many boxes did each sell?

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<i>WHO</i>	<i>NUMBER OF BOXES</i>
<i>Sean</i>	$4 + M$
<i>Marta</i>	$M$

$$4 + M + M = 22$$

$$2M + 4 = 22$$

$$2M = 18$$

$$M = 9$$

*Marta sold 9 boxes of candy and Sean sold 13.*

2. Ned weighs  $1 \frac{1}{2}$  times as much as Jill and Tom weighs 15 kilograms more than Jill. If their combined weight is 190 kilograms, how much does each person weigh?

<i>WHO</i>	<i>WEIGHT</i>
<i>Ned</i>	$1 \frac{1}{2} J$
<i>Jill</i>	$J$
<i>Tom</i>	$15 + J$

$$1 \frac{1}{2} (J) + J + 15 + J = 190$$

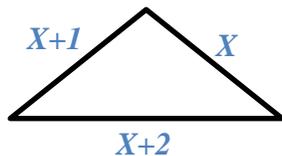
$$3 \frac{1}{2} (J) + 15 = 190$$

$$3 \frac{1}{2} (J) = 175$$

$$J = 50$$

*Jill weighs 50 kilograms, Tom weighs 65 kg. Ned weighs 75 kg.*

3. The side lengths of a triangular birdcage are consecutive integers. If the perimeter is 114 centimeters, what is the length of each side? Label each side with an expression that represents its length.



$$X + X + 1 + x + 2 = 114$$

$$3X + 3 = 114$$

$$3X = 111$$

$$X = 37$$

*The lengths of the sides are 37, 38, and 39.*

4. Caitlyn did  $\frac{6}{7}$  of the problems on her math quiz correctly and four incorrectly. She did all the problems. How many were there?

<i>TYPE OF PROBLEM</i>	<i>FRACTIONAL PART OF WHOLE</i>	<i>NUMBER</i>
<i>Correct</i>	$\frac{6}{7}$	$x - 4$
<i>Incorrect</i>	$\frac{1}{7}$	$4$
<i>Total on Quiz</i>	$\frac{7}{7}$	$x$

$$\frac{1}{7} x = 4$$

$$x = 28$$

*There were 28 problems on the quiz.*

5. Geri spent Friday, Saturday, and Sunday selling a total of 24 magazine orders for her school fundraiser. The amounts she sold respectively, on the three days were consecutive even integers. How many did she sell on each day?

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<i>DAY OF WEEK</i>	<i>AMOUNT SOLD</i>
<i>Friday</i>	<i>n</i>
<i>Saturday</i>	<i>n + 2</i>
<i>Sunday</i>	<i>n + 4</i>

$$n + n + 2 + n + 4 = 24$$

$$3n + 6 = 24$$

$$3n = 18$$

$$n = 6$$

*Geri sold 6 orders on Friday, 8 orders on Saturday, and 10 orders on Sunday.*

\*\*There are many more equations on the referenced website as well as a quiz for the end of the section if needed.

([http://www.nsa.gov/academia/files/collected\\_learning/middle\\_school/algebra/deciphering\\_world\\_problems.pdf](http://www.nsa.gov/academia/files/collected_learning/middle_school/algebra/deciphering_world_problems.pdf))

### **DIFFERENTIATION**

#### **Extension:**

- Some of the problems from the website involve more complex equations including multi-step equations.
- Allowing students the opportunity to work in their groups or with a partner to work through the equations rather than working individually with guided teacher instruction would be more challenging.

#### **Intervention:**

- Use highlighters and active reading strategies in order to help students locate important information within the word problem

## Deconstructing Word Problems

### Part I: Warm-Up Expressions

For each expression below, translate the verbal expression to an algebraic expression

- 1) Ann has the 5 newest music CD's which is 3 less than twice the amount that Bob has.

\_\_\_\_\_

- 2) Mike, who has 6 video games, has half as many games as Paul.

\_\_\_\_\_

- 3) Nan rode the roller coaster 8 times, which was twice as many times as she rode the Ferris wheel. \_\_\_\_\_

- 4) Janine, who bought \$15 worth of make-up, spent \$6 less than Leah spent.

\_\_\_\_\_

- 5) Rob, who has all 13 girls' phone numbers that are in his homeroom, has 3 more than half the number of girls' phone numbers that Jay has.

\_\_\_\_\_

- 6) Kate's 85 on her English test was 37 points less than twice the grade on her Science test.

\_\_\_\_\_

- 7) At the Middle School Graduation Dance, the DJ played 12 slow dances, which was equal to the quotient of the number of fast dances and 2.

\_\_\_\_\_

### Part II: Creating Equations from Word Problems

Use the chart provided in order to create expressions for the situation described in the word problem. Then, use these expressions and the word problem in order to create and solve an equation. Make sure you not only solve for the variable, but also answer the question being presented. Show all your work when solving the equations.

1. Sean sold 4 more boxes of candy for the school fundraiser than Marta. The sum of the boxes they sold was 22. How many boxes did each sell?

WHO	NUMBER OF BOXES

2. Ned weighs  $1\frac{1}{2}$  times as much as Jill and Tom weighs 15 kilograms more than Jill. If their combined weight is 190 kilograms, how much does each person weigh?

WHO	WEIGHT

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3. The sides of a triangular birdcage are consecutive integers. If the perimeter is 114 centimeters, what is the length of each side? Label each side with an expression that represents its length.



4. Caitlyn did  $\frac{6}{7}$  of the problems on her math quiz correctly and four incorrectly. She did all the problems. How many were there?

<b>TYPE OF PROBLEM</b>	<b>FRACTIONAL PART OF WHOLE</b>	<b>NUMBER</b>

5. Geri spent Friday, Saturday and Sunday selling a total of 24 magazine orders for her school fundraiser. The amounts she sold, respectively, on the three days were consecutive even integers. How many did she sell on each day?

<b>DAY OF WEEK</b>	<b>AMOUNT SOLD</b>

## **Solving Linear Equations (FAL)**

[Back to task table](#)

(Concept Development)

This lesson is intended to assess how well students are able to:

- form and solve linear equations using factoring and the distributive property
- use variables to represent equations in real-world problems
- represent word problems in equivalent equations.

Source: *Formative Assessment Lesson Materials from Mathematics Assessment Project*  
<http://map.mathshell.org/lessons.php?unit=7220&collection=8>

### **STANDARDS FOR MATHEMATICAL CONTENT**

**MGSE7.EE.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

**MGSE7.EE.2** Understand that rewriting an expression in different forms in a problem context can clarify the problem and how the quantities in it are related. *For example  $a + 0.05a = 1.05a$  means that adding a 5% tax to a total is the same as multiplying the total by 1.05.*

### **Solve real-life and mathematical problems using numerical and algebraic expressions and equations**

**MGSE7.EE.3** Solve multistep real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals) by applying properties of operations as strategies to calculate with numbers, converting between forms as appropriate, and assessing the reasonableness of answers using mental computation and estimation strategies.

*For example:*

- *If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for a new salary of \$27.50.*
- *If you want to place a towel bar  $9\frac{3}{4}$  inches long in the center of a door that is  $27\frac{1}{2}$  inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

**MGSE7.EE.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

**MGSE7.EE.4a** Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

**MGSE7.EE.4b** Solve word problems leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example, as a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*

**MGSE7.EE.4c** Solve real-world and mathematical problems by writing and solving equations of the form  $x + p = q$  and  $px = q$  in which  $p$  and  $q$  are rational numbers.

### **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

### **ESSENTIAL QUESTIONS**

- What are some strategies for solving real life mathematical problems involving numerical and algebraic equations and expressions?

### **TASK COMMENTS**

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:

<http://www.map.mathshell.org/materials/background.php?subpage=formative>

The PDF version of the task can be found at the link below:

<http://map.mathshell.org/lessons.php?unit=7220&collection=8>

## **T.V. Time And Video Games**

[Back to task table](#)

In this task, students will create inequalities to represent a situation. They will solve these inequalities and graph in order to provide a visual representation of the real-life situation.

*(Adapted from Connected Mathematics*

[http://connectedmath.msu.edu/pdf\\_news\\_links/cmp2\\_gr7\\_transition\\_kit1.pdf](http://connectedmath.msu.edu/pdf_news_links/cmp2_gr7_transition_kit1.pdf))

### **STANDARDS FOR MATHEMATICAL CONTENT**

**MGSE7.EE.4b** Solve word problems leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example, as a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*

### **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.

### **BACKGROUND KNOWLEDGE**

In order for students to be successful, the following skills and concepts need to be maintained:

- Creating inequality statements in order to compare two values
- How to solve one and two step equations in order to transition the same process to inequalities
- An inequality shows a range of possible values not just a single value

### **COMMON MISCONCEPTIONS**

*When working with inequalities, students struggle with the idea that the solution is a range of numbers not just a single value. The first few problems may require a discussion about what the graph is truly telling you and how you can choose possible values from the given solution.*

### **ESSENTIAL QUESTIONS**

- How can inequalities be used in order to demonstrate all possible values that are solutions to a given real life situation?
- How inequalities be displayed on a number line to provide a visual representation of a given situation?

**MATERIALS**

- T.V. Time and Video Game Student Sheet

**TASK DESCRIPTION**

Inequalities are used to show a range of possible values that meet a given criteria. In the following task, students will be creating a visual and algebraic solution to a given situation.

An inequality is a math sentence that compares two quantities. Often one of the quantities represented is a variable. Use the following symbols and descriptions to represent each type of inequality.

- |                                 |   |
|---------------------------------|---|
| $<$ means “is less than.”       | $\leq$ means “is less than or equal to.”    |
| $>$ means “is greater than.”    | $\geq$ means “is greater than or equal to.” |
| $\neq$ means “is not equal to.” |   |

How could you represent each inequality below using a variable and a constant?

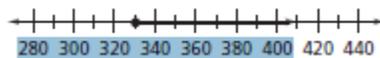
1. Nima will spend less than \$25       $L < 25$
2. Derrick ran at least 30 miles last week       $R \geq 30$
3. Emily needs at least \$200 to buy the TV she wants       $A \geq 200$

Kia volunteers with some friends at a community center. While shopping online for a new television she decides she wants one with at least a 26 in. screen. Using the chart below, write an inequality to show how much money the center will have to spend.

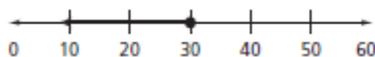
Television Prices	
Screen Size	Price
22 in.	\$300
26 in.	\$330
32 in.	\$370
40 in.	\$420

Inequality  $x \geq 330$

4. Graph the inequality on the number line.



5. Kia wants to have money left over. How can the graph be changed to show they need to have more than \$330? *Don't color in the point at \$330. The inequality would change to  $x > 330$ .*
6. The center has a stand for the television that will hold up to 30 lb of weight. Draw a graph to show how much the television she buys can weigh.



Kia plans to use money from the community center's savings account to buy a gaming system. There must be \$129 left in the savings account after she withdraws what she needs.

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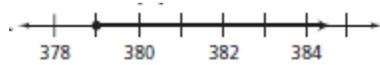
Video Game System Sale This Month Only  
 \$250.00  
 Selected Extra Games on Sale  
 \$35.00 each

7. Write and solve an inequality to represent the situation, where  $x$  represents the amount of money the center has in its savings account. What does your solution mean in terms of the problem?

$$A - 129 \geq 250 ; a \geq 379$$

*The center needs at least \$379 in the account to start in order to take out \$250 for the gaming system and still leave \$129 in the account. If they just have \$379 in savings, they will not be able to afford any games.*

8. Graph the possible values from the solution found in number seven.



The community center rents rooms for an hourly rate, plus a set-up fee.

Room Rentals		
Room	Rental Rate per Hour	Set-up Fee
Main Hall	\$15	\$40
Dining Room	\$12	\$80

9. A school group has \$140 to spend. Write and solve an inequality that represents the cost to rent the main hall, where  $h$  represents the number of hours the group can rent the room.

$$15h + 40 \leq 140; x \leq 6.666... \text{ (6 hrs and 40 min.)}$$

*The group can rent the room for a total of 6 full hours.*

10. The same group is also considering renting the dining room. Write and solve an inequality to represent this situation.

$$12x + 80 \leq 140; x \leq 5$$

*They can rent the dining room for a total of 5 hours at most.*

11. Use your solutions from 9 and 10 to justify your selection of which room the group should rent.

*They should rent the main hall since they get 6 total hours rather than the five they would get from the dining room.*

The community center has \$175 to spend on video games for its new gaming system. Games are on sale for \$35 each.

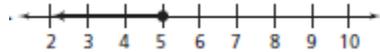
12. Write and solve an inequality to represent the number of games the center could buy. Explain your solution in reference to the problem.

$$35x \leq 175; x \leq 5 \text{ or } 175 - 35x \geq 0 ; x \leq 5$$

*In order to stay in budget the center has to buy 5 or less video games.*

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13. Graph the solution on a number line.



The center is considering signing up for an online game-rental service rather than buying the games. The table shows equipment cost and monthly fees for two services.

Game Rental Services		
Service	Equipment Cost	Monthly Fee
NetGames	\$99	\$8
Anytime Games	\$19	\$19

14. Write and solve an inequality that represents the number of months the center could rent games from NetGames with its \$175. Explain the solution in terms of the problem.

$$8m + 99 \leq 175; x \leq 9.5$$

*The center would be able to rent movies for nine and a half months. They could rent for a total of 9 full months.*

15. Write and solve an inequality to represent the number of months the center could rent games from Anytime Games. Explain the solution in terms of the problem.

$$19m + 19 \leq 175; m \leq 8.2$$

*The center would be able to rent for 8.2 months or 8 full months and still stay within budget.*

16. Use your answers from 14 and 15 to justify which service the community center should purchase.

*NetGames is the more affordable choice. The community center would get an extra month of service by choosing this service.*

### **DIFFERENTIATION**

#### **Extension:**

- Give the students a budget and have them research t.v. sets and gaming systems that they would be able to purchase and still remain within their budget. What would be the cheapest t.v./gaming combination? What would be the most expensive?

#### **Intervention:**

- Incorporate more examples of how to solve inequalities before beginning the task.

**T.V. TIME AND VIDEO Games**

An inequality is a math sentence that compares two quantities. Often one of the quantities is a variable. Use the following symbols and descriptions to represent each type of inequality.

- |                                 |   |
|---------------------------------|---|
| $<$ means “is less than.”       | $\leq$ means “is less than or equal to.”    |
| $>$ means “is greater than.”    | $\geq$ means “is greater than or equal to.” |
| $\neq$ means “is not equal to.” |   |

How could you represent each inequality below?

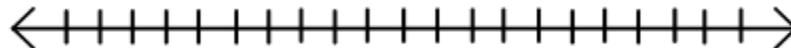
1. Nima will spend less than \$25 \_\_\_\_\_
2. Derrick ran at least 30 miles last week \_\_\_\_\_
3. Kia needs at least \$200 to buy the TV she wants \_\_\_\_\_

Kia volunteers with some friends at a community center. While shopping online for a new television she decides she wants one with at least a 26 in. screen. Using the chart below, write an inequality to show how much money the center will have to spend.

Television Prices	
Screen Size	Price
22 in.	\$300
26 in.	\$330
32 in.	\$370
40 in.	\$420

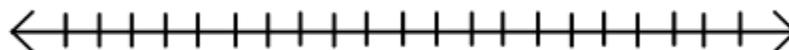
Inequality \_\_\_\_\_

4. Graph the inequality on the number line.



5. Kia wants to have money left over. How can the graph be changed to show they need to have more than \$330?

6. The center has a stand for the television that will hold up to 30 lb of weight. Draw a graph to show how much the television she buys can weigh.

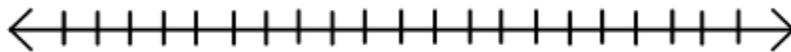


Kia plans to use money from the community center’s savings account to buy a gaming system. There must be \$129 left in the savings account after she withdraws what she needs.

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Video Game System Sale This Month Only \$250.00 Selected Extra Games on Sale \$35.00 each
--

7. Write and solve an inequality to represent the situation, where  $x$  represents the amount of money the center has in its savings account.
8. Graph the possible values from the solution found in number seven.



The community center rents rooms for an hourly rate, plus a set-up fee.

Room Rentals		
Room	Rental Rate per Hour	Set-up Fee
Main Hall	\$15	\$40
Dining Room	\$12	\$80

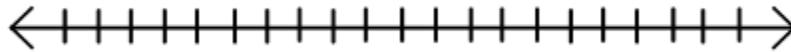
9. A school group has \$140 to spend. Write and solve an inequality that represents the cost to rent the main hall, where  $h$  represents the number of hours the group can rent the room.
10. The same group is also considering renting the dining room. Write and solve an inequality to represent this situation.
11. Use your solutions from 9 and 10 to justify your selection of which room the group should rent.

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The community center has \$175 to spend on video games for its new gaming system. Games are on sale for \$35 each.

12. Write and solve an inequality to represent the number of games the center could buy. Explain your solution in reference to the problem.

13. Graph the solution on a number line.



The center is considering signing up for an online game-rental service rather than buying the games. The table shows equipment cost and monthly fees for two services.

Game Rental Services		
Service	Equipment Cost	Monthly Fee
NetGames	\$99	\$8
Anytime Games	\$19	\$19

14. Write and solve an inequality that represents the number of months the center could rent games from NetGames with its \$175. Explain the solution in terms of the problem.
15. Write and solve an inequality to represent the number of months the center could rent games from Anytime Games. Explain the solution in terms of the problem.
16. Use your answers from 14 and 15 to justify which service the community center should purchase.

## **Culminating Task: Population Equations**

[Back to task table](#)

Students will use principals of algebra to rearrange and solve one-variable equations.

### **STANDARDS FOR MATHEMATICAL CONTENT**

**MGSE7.EE.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

**MGSE7.EE.2** Understand that rewriting an expression in different forms in a problem context can clarify the problem and how the quantities in it are related. *For example  $a + 0.05a = 1.05a$  means that adding a 5% tax to a total is the same as multiplying the total by 1.05.*

### **Solve real-life and mathematical problems using numerical and algebraic expressions and equations**

**MGSE7.EE.3** Solve multistep real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals) by applying properties of operations as strategies to calculate with numbers, converting between forms as appropriate, and assessing the reasonableness of answers using mental computation and estimation strategies.

*For example:*

- *If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for a new salary of \$27.50.*
- *If you want to place a towel bar  $9 \frac{3}{4}$  inches long in the center of a door that is  $27 \frac{1}{2}$  inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

**MGSE7.EE.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

**MGSE7.EE.4a** Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

**MGSE7.EE.4b** Solve word problems leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example, as a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*

**MGSE7.EE.4c** Solve real-world and mathematical problems by writing and solving equations of the form  $x + p = q$  and  $px = q$  in which  $p$  and  $q$  are rational numbers.

### **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Make sense of problems and persevere in solving them.
7. Look for and make use of structure.

### **COMMON MISCONCEPTIONS**

- *As students begin to build and work with expressions containing more than two operations, students tend to set aside the order of operations. For example having a student rewrite an expression like  $8 + 4(2x - 5) + 3x$  can bring to light several misconceptions. Do the students immediately add the 8 and 4 before distributing the 4? Do they only multiply the 4 and the  $2x$  and not distribute the 4 to both terms in the parenthesis? Do they collect all like terms  $8 + 4 - 5$ , and  $2x + 3x$ ? Each of these show gaps in students' understanding of how to rewrite numerical expressions with multiple operations.*
- *Most students will not be familiar with the term "attrition". It might take some extra emphasis on what this term means through using synonyms like "decline", or "death" (in this context, but not all contexts).*

### **ESSENTIAL QUESTIONS**

- How are the rules of order of operations used when rewriting expressions?
- How can rewriting an expression in different forms show how the quantities in it are related?

### **MATERIALS**

- Population Equations Student Task sheet

### **GROUPING**

- Individually/partners

### **TASK DESCRIPTION**

Distribute the handout and ensure that students understand the task.

- This task can also be found at:  
[http://www.thefutureschannel.com/pdf/algebra/population\\_equations.pdf](http://www.thefutureschannel.com/pdf/algebra/population_equations.pdf)
- Download Student Activity:  
[http://www.thefutureschannel.com/pdf/algebra/population\\_equations\\_s.pdf](http://www.thefutureschannel.com/pdf/algebra/population_equations_s.pdf)
- For any further information, go to the following site:  
[http://www.thefutureschannel.com/algebra/population\\_equations.php](http://www.thefutureschannel.com/algebra/population_equations.php)

If you are managing a wildlife population, three variables that you want to watch closely are the population (P), the rate of reproduction (R), and the rate of attrition, (A).

*Rate of reproduction* simply means how many new animals are born as a percentage of the total population. For example, if the population starts out at 60, and the rate of reproduction is 20%, then in one year there will be 20% of  $60=12$  young animals born.

*Rate of attrition* is the number of animals that die each year, as a percentage of the total population. For example, if the population starts out at 60, and the rate of attrition is 10%, then in that year there would be 10% of  $60=6$  animal deaths.

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1. *Make up 3 more examples that show the meaning of “rate of reproduction”, and 3 examples that show the meaning of “rate of attrition”.*

If no animals arrive from the outside of the region being studied, and no animals leave to the outside, then the population at the beginning of one year (say, 1996) is related to the population at the beginning of the next year (say, 1997), but this equation.

**$P_{1997} = P_{1996} + P_{1996} (R - A)$**  *Teachers should show this equation broken down into two equivalent expressions because students will be utilizing the right side of the equation when solving for the value  $P_{1997}$ .*

*Solutions will vary-Population Examples: If the population of ducks starts out at 50, and the rate of reproduction is 10%, then in one year there will be 10% of 50= 5 ducklings born. If the population of rabbits starts out at 80, and the rate of reproduction is 30%, then in one year there will be 30% of 80=24 baby rabbits born. If the population of trout starts out at 100, and the rate of reproduction is 40%, then in one year there will be 40% of 100=40 baby trout. Attrition examples will be similar to population examples, but students will acknowledge that animals are dying instead of being born.*

2. *Compute the population for 1997 for each set of values given below:  
 Possible misconception: Using row a) as an example, students might substitute(25-22) for (R-A) and leave out the % signs (25%-22%). The % signs are crucial because without them, students would be multiple 500x3, not 500x(.03). You may want to lead students through row a) to ensure that they do not make this error repeatedly in the table.*

	<b><math>P_{1996}</math></b>	<b>R</b>	<b>A</b>	<b><math>P_{1997}</math></b>
<b>a)</b>	500	25%	22%	<b>515</b>
<b>b)</b>	200	50%	40%	<b>220</b>
<b>c)</b>	1000	15%	20%	<b>950</b>
<b>d)</b>	15	30%	25%	<b>16</b>
<b>e)</b>	1500	40%	40%	<b>1500</b>

3. *Look at your answers to (c), (d), and (e) above, and, for each one, explain why it’s a reasonable answer.  
 Solutions:c)population decreases because  $A > R$   
 d) answer rounded to the nearest whole since you cannot have a part of an animal  
 e)population doesn’t change when  $A=R$*
4. *Solve the equation given above to find the missing values in each case:*

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	$P_{1996}$	R	A	$P_{1997}$
a)	700	20%	18%	714
b)	2500	30%	20%	2750
c)	8000	20%	30%	7200
d)	500	50%	60%	450
e)	20	60%	20%	28
f)	12000	10%	15%	11400

### **DIFFERENTIATION**

#### **Extension**

##### **Collaborative project incorporating life science:**

- The teacher could allow students to create their own representation of the situation with animal populations. This could be in the form of an actual 3-d model of a nature reserve, state park, zoo, aquarium, or biome using plastic, paper, or clay animals with a key showing how many animals each model animal represents.
  - Students could research specific animals of their choosing to use in this project and use actual quantitative data to estimate real rates of reproduction and attrition.

#### **Intervention**

- If students cannot take the population equation and substitute the data from the chart accurately, lead the class through the process for the first one or two letters:
- White boards could be used with struggling students to write the population equation, erase each term individually and replace it with the specific data from each data piece found in the given table. This process could be repeated for all of the missing data in the table for #2 and #4.



## Culminating Task: Population Equations

If you are managing a wildlife population, three variables that you want to watch closely are the population ( $P$ ), the rate of reproduction ( $R$ ), and the rate of attrition, ( $A$ ).

*Rate of reproduction* simply means how many new animals are born as a percentage of the total population. For example, if the population starts out at 60, and the rate of reproduction is 20%, then in one year there will be 20% of  $60=12$  young animals born.

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1. *Make up 3 more examples that show the meaning of “rate of reproduction”, and 3 examples that show the meaning of “rate of attrition”.*

If no animals arrive from the outside of the region being studied, and no animals leave to the outside, then the population at the beginning of one year (say, 1996) is related to the population at the beginning of the next year (say, 1997), but this equation.

$$P_{1997} = P_{1996} + P_{1996} (R - A)$$

2. *Compute the population for 1997 for each set of values given below:*

	$P_{1996}$	$R$	$A$	$P_{1997}$
a)	500	25%	22%	
b)	200	50%	40%	
c)	1000	15%	20%	
d)	15	30%	25%	
e)	1500	40%	40%	

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3. Look at your answers to (c), (d), and (e) above, and, for each one, explain why it's a reasonable answer.

c)

d)

e)

4. Solve the equation given above to find the missing values in each case:

	<b><math>P_{1996}</math></b>	<b>R</b>	<b>A</b>	<b><math>P_{1997}</math></b>
<b>a)</b>		20%	18%	714
<b>b)</b>	2500	30%		2750
<b>c)</b>	8000	20%		7200
<b>d)</b>		50%	60%	450
<b>e)</b>	20		20%	28
<b>f)</b>	12000		15%	11400

## **Technology Resources**

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**MGSE7.EE.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

[https://www.nsa.gov/academia/files/2013\\_CDU\\_summaries/Primary-Patterns-AlgebraicThinking.pdf](https://www.nsa.gov/academia/files/2013_CDU_summaries/Primary-Patterns-AlgebraicThinking.pdf)

<https://www.illustrativemathematics.org/7.EE>

<http://illuminations.nctm.org/Lesson.aspx?id=3642>

**MGSE7.EE.2** Understand that rewriting an expression in different forms in a problem context can clarify the problem and how the quantities in it are related. *For example  $a + 0.05a = 1.05a$  means that adding a 5% tax to a total is the same as multiplying the total by 1.05.*

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*For example:*

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[https://www.nsa.gov/academia/files/2013\\_CDU\\_summaries/Primary-Patterns-AlgebraicThinking.pdf](https://www.nsa.gov/academia/files/2013_CDU_summaries/Primary-Patterns-AlgebraicThinking.pdf)

<https://www.illustrativemathematics.org/7.EE>

<http://www.estimated180.com/lessons.html>

<http://illuminations.nctm.org/Lesson.aspx?id=3642>

**MGSE7.EE.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

[https://www.nsa.gov/academia/files/2013\\_CDU\\_summaries/Primary-Patterns-AlgebraicThinking.pdf](https://www.nsa.gov/academia/files/2013_CDU_summaries/Primary-Patterns-AlgebraicThinking.pdf)

<http://nzmaths.co.nz/resource/solving-multi-step-equations>

<https://www.illustrativemathematics.org/7.EE>

<http://www.estimated180.com/lessons.html>

<http://illuminations.nctm.org/Activity.aspx?id=3482>

**MGSE7.EE.4a** Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently.

Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

[https://www.nsa.gov/academia/files/2013\\_CDU\\_summaries/Primary-Patterns-AlgebraicThinking.pdf](https://www.nsa.gov/academia/files/2013_CDU_summaries/Primary-Patterns-AlgebraicThinking.pdf)

<http://nzmaths.co.nz/resource/solving-multi-step-equations>

<https://www.illustrativemathematics.org/7.EE>

<http://illuminations.nctm.org/Activity.aspx?id=3482>

**MGSE7.EE.4b** Solve word problems leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example, as a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*

<https://www.illustrativemathematics.org/7.EE>

**MGSE7.EE.4c** Solve real-world and mathematical problems by writing and solving equations of the form  $x + p = q$  and  $px = q$  in which  $p$  and  $q$  are rational numbers.

[https://www.nsa.gov/academia/files/2013\\_CDU\\_summaries/Primary-Patterns-AlgebraicThinking.pdf](https://www.nsa.gov/academia/files/2013_CDU_summaries/Primary-Patterns-AlgebraicThinking.pdf)

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