Georgia
Standards of Excellence
Curriculum Frameworks

Mathematics

8th Grade

Unit 2: Exponents and Equations

Georgia Department of Education

Richard Woods, Georgia's School Superintendent

"Educating Georgia's Future"

Unit 2
Exponents and Equations

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OVERVIEW

In this unit student will:

- distinguish between rational and irrational numbers and show the relationship between the subsets of the real number system;
- recognize that every rational number has a decimal representation that either terminates or repeats;
- recognize that irrational numbers must have decimal representations that neither terminate nor repeat;
- understand that the value of a square root can be approximated between integers and that non–perfect square roots are irrational;
- locate rational and irrational numbers on a number line diagram;
- use the properties of exponents to extend the meaning beyond counting-number exponents;
- recognize perfect squares and cubes, understanding that non-perfect squares and non-perfect cubes are irrational;
- recognize that squaring a number and taking the square root of a number are inverse operations; likewise, cubing a number and taking the cube root are inverse operations;
- express numbers in scientific notation;
- compare numbers, where one is given in scientific notation and the other is given in standard notation;
- compare and interpret scientific notation quantities in the context of the situation;
- use laws of exponents to add, subtract, multiply and divide numbers written in scientific notation;
- solve one-variable equations with the variables being on both sides of the equals sign, including equations with rational numbers, the distributive property, and combining like terms; and
- analyze and represent contextual situations with equations, identify whether there is one, none, or many solutions, and then solve to prove conjectures about the solutions.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under "Evidence of Learning" be reviewed early in the planning process. A variety of resources should be utilized to supplement this unit. This unit provides much needed content information, but excellent learning activities as well. The tasks in this unit illustrate the types of learning activities that should be utilized from a variety of sources.

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

STANDARDS FOR MATHEMATICAL PRACTICE

Refer to the Comprehensive Course Overview for more detailed information about the Standards for Mathematical Practice.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

STANDARDS FOR MATHEMATICAL CONTENT

Work with radicals and integer exponents.

MGSE8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{(-5)} = 3^{(-3)} = \frac{1}{3^3} = \frac{1}{27}$.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \le 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \le 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

MGSE8.EE.3 Use numbers expressed in scientific notation to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.*

MGSE8.EE.4 Add, subtract, multiply and divide numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Understand scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g. use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology (e.g. calculators).

MGSE8.EE.7 Solve linear equations in one variable.

MGSE8.EE.7a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, *or* $a = b$ results (where a and b are different numbers).

MGSE8.EE.7b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

MGSE8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

MGSE8.NS.2 Use rational approximation of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line, and estimate the value of expressions (e.g., estimate π^2 *to the nearest tenth). For example, by truncating the decimal expansion of* $\sqrt{2}$ *(square root of 2), show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

BIG IDEAS

- Exponential Notation is a way to express repeated products of the same number. Specifically, powers of 10 express very large and very small numbers in an economical manner.
- Many numbers are not rational; the irrationals can be expressed only symbolically or approximately by using a close rational number. Examples include $\sqrt{2} \sim 1.41421\ldots$ and $\pi \sim 3.14159\ldots$
- Square roots can be rational or irrational.
- Every number has a decimal expansion. Decimal expansions of some rational numbers repeat. Decimal expansions of irrational numbers do not repeat.
- All real numbers, rational and irrational, can be plotted on a number line. Rational approximations of irrationals can be used to compare sizes, estimate locations on a number line and estimate values of expressions.
- Linear equations in one variable can have one solution, infinitely many solutions, or no solutions.

ESSENTIAL QUESTIONS

- How can I apply the properties of integer exponents to generate equivalent numerical expressions?
- How can I represent very small and large numbers using integer exponents and scientific notation?
- How can I perform operations with numbers expressed in scientific notation?
- How can I interpret scientific notation that has been generated by technology?
- What are some applications of scientific notation?
- Why is it useful for me to know the square root of a number?
- How do I simplify and evaluate numeric expressions involving integer exponents?
- How can the properties of exponents and knowledge of working with scientific notation help me interpret information?
- What is the difference between rational and irrational numbers?
- Why do we approximate irrational numbers?
- How do we locate approximate locations of irrational numbers on a number line and estimate the values of irrational numbers?
- What strategies can I use to create and solve linear equations with one solution, infinitely many solutions, or no solutions?

FLUENCY

It is expected that students will continue to develop and practice strategies to build their capacity to become fluent in mathematics and mathematics computation. The eventual goal is automaticity with math facts. This automaticity is built within each student through strategy development and practice. The following section is presented in order to develop a common understanding of the ideas and terminology regarding fluency and automaticity in mathematics:

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply “how to get the answer” and instead support students’ ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students:

- **flexibly use a combination of deep understanding, number sense, and memorization.**
- **are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.**
- **are able to articulate their reasoning.**
- **find solutions through a number of different paths.**

For more about fluency, see: http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and: http://joboaler.com/timed-tests-and-the-development-of-math-anxiety/

It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas below.

- computation with whole numbers and decimals, including application of order of operations
- solving equations
- plotting points in four quadrant coordinate plane
- understanding of independent and dependent variables
- characteristics of a proportional relationship

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for middle school students. **Note – Different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.** The definitions below are from the Common Core State Standards Mathematics Glossary and/or the Common Core GPS Mathematics Glossary when available.

Visit http://intermath.coe.uga.edu or http://mathworld.wolfram.com to see additional definitions and specific examples of many terms and symbols used in grade 8 mathematics.

- **Addition Property of Equality**
- **Additive Inverses**
- **Algebraic Expression**
- **Cube Root**
- **Decimal Expansion**
- **Equation**
- **Evaluate an Algebraic Expression**
- **Exponent**
- **Exponential Notation**
- **Inverse Operation**
- **Irrational**
- **Like Terms**
- **Linear Equation in One Variable**
- **Multiplication Property of Equality**
- **Multiplicative Inverses**
- **Perfect Square**
- **Radical**
- **Rational**
- **Scientific Notation**
- **Significant Digits**
- **Solution**
- **Square Root**
- **Variable**

EVIDENCE OF LEARNING

By the conclusion of this unit, students should be able to demonstrate the following competencies:

- use properties of integer exponents to evaluate and simplify numerical expressions containing exponents;
- apply the properties of integer exponents to generate equivalent numerical expressions;
- estimate very large and very small quantities and compare quantities expressed in the form of a single digit times an integer power of 10;
- perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used;
- use scientific notation and choose units of appropriate size for measurements of very large or very small quantities;
- interpret scientific notation that has been generated by technology;
- find square roots and cube roots of perfect squares and perfect cubes;
- explain the difference between a rational and an irrational number;

- show flexibility with the use of rational numbers in any form;
- write a decimal approximation for an irrational number to a given decimal place;
- plot rational and irrational numbers on a number line;
- solve linear equations in one variable using distributive property, combining like terms, and equations with variables on both sides; and write, solve, and explain linear equations in one variable with one solution, infinitely many solutions, or no solutions.

FORMATIVE ASSESSMENT LESSONS (FAL)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Guide.

SPOTLIGHT TASKS

A Spotlight Task has been added to each CCGPS mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Common Core Georgia Performance Standards, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky.

3-ACT TASKS

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

TASKS

Task Name	Task Type *Grouping Strategy*	Content Addressed	Standard(s)
Edges of Squares and Cubes	Learning Task *Partner/ Small Group*	Determine square roots and Distinguish between rational and irrational numbers	MGSE8.EE.2 MGSE8.NS.1 MGSE8.NS.2
Decimal Approximations of Roots	Learning Task *Partner/Small Group*	Determine square roots and Estimate irrational roots	MGSE8.EE.2 MGSE8.NS.1 MGSE8.NS.2
Rational or Irrational Reasoning?	Learning Task *Individual/Partner/ Small Group*	Distinguish between rational and irrational numbers including misconceptions.	MGSE8.NS.1 MGSE8.NS.2 MGSE8.EE.2
Repeating Decimals (FAL)	Formative Assessment Lesson *Partner/Small Group*	Translate between decimal and fraction notation (particularly when the decimals are repeating), create and solve simple linear equations to find the fractional equivalent of a repeating decimal, and understand the effect of multiplying a decimal by a power of 10.	MGSE8.NS.1 MGSE8.NS.2 MGSE8.EE.7
A Few Folds	Learning Task *Partner/Small Group*	Explore and continue patterns. Represent findings with integer outcomes.	MGSE8.EE.1
Got Cubes? Revisited (Spotlight Task)	Learning Task *Partner/Small Group*	Explore and continue patterns.	MGSE8.EE.1
Alien Attack	Learning Task *Partner/Small Group*	Discover and generalize the properties of exponents.	MGSE8.EE.1
Nesting Dolls	Performance Task *Partner/Small Group*	Use the properties of exponents to extend and generalize a pattern as applied to a real world problem.	MGSE8.EE.1 MGSE8.EE.2
Exponential Exponents	Performance Task *Individual/Partner*	Apply the properties of integer exponents to generate equivalent numerical expressions.	MGSE8.EE.1 MGSE8.EE.2

Exploring Powers of 10	Learning Task *Partner/Small Group*	Develop a conceptual understanding of scientific notation through exploration.	MGSE8.EE.1 MGSE8.EE.3 MGSE8.EE.4
"Ponzi" Pyramid Scheme	Short Cycle Task *Individual/Partner*	Real-world application of exponents using a chain letter.	MGSE8.EE.1
Estimating Length Using Scientific Notation (FAL)	Formative Assessment Lesson *Partner/Small Group*	Estimating lengths of everyday objects, converting between decimal and scientific notation, and making comparison of the size of numbers expressed in both decimal and scientific notation.	MGSE8.EE.3 MGSE8.EE.4
E. coli	Performance Task *Partner/Small Group*	Apply properties of exponents to problems involving numbers expressed in scientific notation.	MGSE8.EE.1 MGSE8.EE.3 MGSE8.EE.4
Giantburgers	Short Cycle Task *Partner/Small Group*	Use properties of exponents and scientific notation.	MGSE8.EE.1 MGSE8.EE.3 MGSE8.EE.4
100 People	Short Cycle Task *Individual/Partner*	Real-world application of scientific notation using population.	MGSE8.EE.1 MGSE8.EE.2 MGSE8.EE.3 MGSE8.EE.4
Geology Rocks Equations	Learning Task *Individual / Partner*	Students will model and solve one and two step equations.	MGSE8.EE.7
Solving Linear Equations in One Variable (FAL)	Formative Assessment Lesson *Partner/Small Group*	Solve linear equations and categorize as one, no, or infinite solutions.	MGSE8.EE.5 MGSE8.EE.6 MGSE8.EE.7 MGSE8.F.3
Writing for a Math Website	Performance Task *Partner/Small Group*	Write equations with various solution types and provide step-by-step solutions with justifications.	MGSE8.EE.7
Culminating Task: Integer Exponents & Scientific Notation	Culminating Task *Partner/Small Group*	Real-world application of exponents and scientific notation.	MGSE8.EE.1 MGSE8.EE.2 MGSE8.EE.3 MGSE8.EE.4

Edges Of Squares And Cubes

Source: Teaching Student Centered Mathematics, by John Van de Walle Back to Task Table

In this task, students will determine square roots and cube roots of squares and cubes that are not perfect.

STANDARDS FOR MATHEMATICAL CONTENT:

Work with radicals and integer exponents.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

Know that there are numbers that are not rational, and approximate them by rational numbers.

MGSE8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

MGSE8.NS.2 Use rational approximation of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line, and estimate the value of expressions (e.g., estimate π^2 *to the nearest tenth). For example, by truncating the decimal expansion of* $\sqrt{2}$ *(square root of 2), show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.

BACKGROUND KNOWLEDGE:

The importance of allowing students to conceptually develop understandings in mathematics is incredibly important. Some concepts can be challenging to figure out how we can make this happen for our students. When thinking about irrational numbers, it is best to allow students to use their knowledge of rational numbers to strategize and determine the best way to make sense of irrationals.

COMMON MISCONCEPTIONS:

- *The students may view the square roots as dividing by two.*
- *The students may infer that all rational numbers are even and all irrational numbers are odd.*
- *A few irrational numbers are given special names (pi and e), and much attention is given to sqrt(2). Because we name so few irrational numbers, students sometimes conclude that irrational numbers are unusual and rare. In fact, irrational numbers are much more plentiful than rational numbers, in the sense that they are "denser" in the real line.*

Misconceptions can be an important part of learning, unless they go undiscovered. Knowing the possible misconceptions that are common ahead of time can be a great tool to prevent these from becoming problematic. Asking questions beginning with "Why. . ." or "How. . ." and listening to/reading your students' responses is a great way to identify any misconceptions fairly easily.

ESSENTIAL QUESTIONS:

- Why is it useful for me to know the square root of a number?
- What is the difference between rational and irrational numbers?
- How do we locate approximate locations of irrational numbers on a number line and estimate the values of irrational numbers?
- What strategies can I use to create and solve linear equations with one solution, infinitely many solutions, or no solutions?

MATERIALS:

- **Copies of task for each student/pair of students/or small group**

GROUPING:

- **Partners**

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Students work with a partner to determine the side length of a square and edge length of a cube. They will need to determine the closest approximation possible for the square and the cube using a calculator (they are not allowed to use the square root key). Students should discuss strategy as they determine their "next steps" throughout this task.

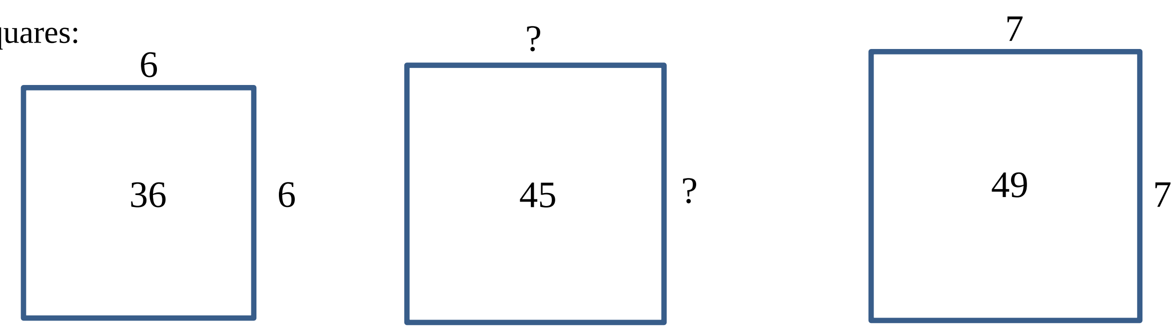
This task is quickly solved if students understand and use the square root key on their calculators. However, the estimation required of students strengthens their understanding of squares and square roots and the relative sizes of numbers.

From this introduction, students can be challenged to find solutions to equations such as ${}^2 = 6$. Students have gained some understanding of roots of a number as related to squares and cubes and are now prepared to understand a general definition of the nth root of a number, x, as a number that when raised to the nth power equals x.

THE TASK:

Students are shown pictures of 3 squares (or 3 cubes). The sides of the squares (edges of the cubes) on the left and on the right are consecutive whole numbers. The areas (or volumes) of all three figures are provided. Students are to work together, using a calculator, to find the length of the sides (square) or edges (cube) of the figure in the center. Students should continue to estimate until they find a value to the hundredths place that gets as close to 45 as possible (or 30 in the case of the cube).

Squares:



Solutions will satisfy these equations:

$$\square \times \square = 45 \qquad \text{or} \qquad \square^2 = 45$$

and

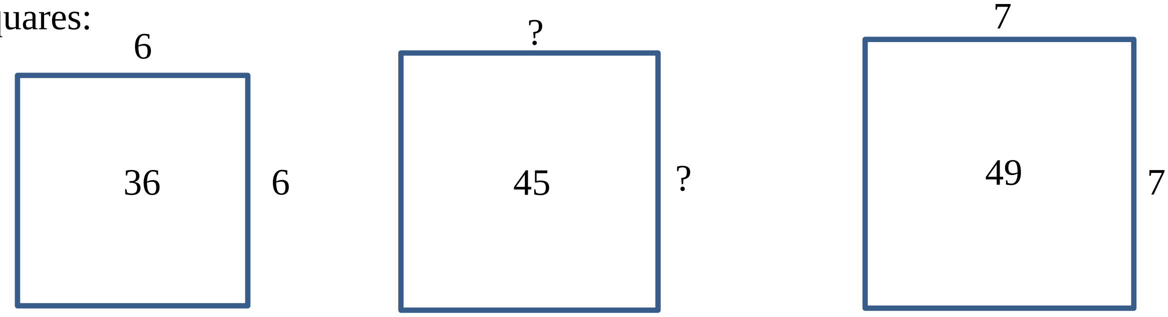
$$\square \times \square \times \square = 30 \qquad \text{or} \qquad \square^3 = 30$$

When students complete the task and have had time to reflect on their work, make sure to have students share their reflections and strategies. Allow students to try to estimate to another place and see if it will work exactly. Ask them to explain why. At the end of the discussion, be sure to guide the conversation to a review of what rational numbers are and which numbers from the task they consider rational. Also, have them determine which numbers in this task do not fit the rational number category. Ask students what they think the name for these numbers that don't fit the rational number definition should be called (Someone will probably name it correctly). Let them have some fun with this. Finally, introduce the term Irrational number.

Edges Of Squares And Cubes

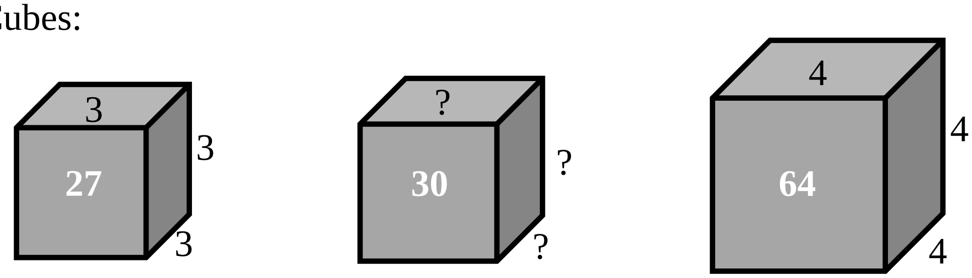
The area of each of the squares below is shown in the center of each square. The lengths of the sides of the first and last square are also given. Your task is to work with your partner, using a calculator, to find the length of the sides of the square in the center. You are not to use the square root key, but to estimate what you think the length of the side would be and test it by squaring it. Continue to estimate until you have found a value to the hundredths place that gets as close to 45 as possible.

Squares:



The volume of each of the cubes below is shown in the center of each cube. The lengths of the edges of the first and last cubes are also given. Your task is to work with your partner, using a calculator, to find the length of the edges of the cube in the center. You are not to use the cube root key, but to estimate what you think the length of the edge would be and test it by cubing it. Continue to estimate until you have found a value to the hundredths place that gets as close to 30 as possible.

Cubes:



Solutions will satisfy these equations:

$$\square \times \square = 45 \quad \text{or} \quad \square^2 = 45$$

and

$$\square \times \square \times \square = 30 \quad \text{or} \quad \square^3 = 30$$

Reflection:

1. How did you decide what the whole number part of your estimate should be for the square?

What do you think you could do to get a side length of the square that would produce a more accurate area?

Is it possible to find a side length that would be perfect for a square with an area of 45 square units? Explain your reasoning.

2. How did you decide what the whole number part of your estimate should be for the cube?

What do you think you could do to get an edge length for the cube that would produce a more accurate volume?

Is it possible to find an edge length that would be perfect for a cube with a volume of 30 cubic units? Explain your reasoning.

Decimal Approximation Of Roots

Back to Task Table

http://www.openmiddle.com/decimal-approximations-of-roots/#prettyPhoto

Source: Bryan Anderson

STANDARDS FOR MATHEMATICAL CONTENT:

MGSE8.NS.1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

MGSE8.NS.2 Use rational approximation of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line, and estimate the value of expressions (e.g., estimate π^2*to the nearest tenth). For example, by truncating the decimal expansion of* $\sqrt{2}$ *(square root of 2), show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it, based on their own errors.

2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to square root values, and to compare relative values of decimal numbers.

3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and decimal values, based upon estimation their understanding of square roots. They explain their thinking to others and respond to others' thinking.

4. Model with mathematics. Students may build models using different manipulatives, drawings, number lines, and equations to represent decimal values.

5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately determine decimal approximations of roots.

6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal approximations and square roots.

7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals.

8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand square roots and make generalizations about patterns. Students connect decimal approximations and values to fluently multiply and divide decimals.

BACKGROUND KNOWLEDGE:

Before students can work with square roots and cube roots that are irrational, they need to be able to visualize what these numbers might look like. The following task is hands-on and allows students to build squares with areas that are not square (2, 3, 5, etc.). By squares from squares, students build a model of an approximation for the square roots of non-perfect squares.

When students memorize procedures for square roots without making sense of them, misconceptions develop. Students need to make sense of the conceptual idea of a square root. The opener, above, is a first step in making sure this doesn't happen. Once students conceptualize the idea of a square root, using the square root symbol to communicate their ideas mathematically is all that is needed.

COMMON MISCONCEPTIONS:

Students may only focus on the idea that square roots are related to multiplication, so they may just divide by the number in the radical. For example, students may see $\sqrt{2}$ and think "what can I do to a number to get a 2 in the square root?" Often, students will just divide by the number, in this case 2, and get $\sqrt{2} = 1$. To address this misconception, students need to experience square numbers as actual squares, built and/or drawn, and then find the root of the square to be the length of a side of that square.

The symbol we use for square, cube, or any other root is just that, a symbol to represent how two quantities are related, not a concept in itself. In order to help students understand this relationship, give them the following pattern:

1, 4, 9, 16, . .

Ask students to find the next 3 numbers in the sequence. When they find the next three, ask them to explain how they determined the next 3 numbers. Many students will likely say that they found the pattern 1 + 3 = 4, 4 + 5 = 9, 9 + 7 = 16, *so they kept going with* + 9, + 11, + 13, *to get each successive number in the pattern.*

Now, ask students to build rectangles for each of the numbers in the pattern using square tiles. Students will find that all of the numbers create squares and from there, students can be guided to the idea that the root of the squares is the length of one side of the square. This also connects back to the original pattern in that the square root is the number of the term in the sequence, i.e., the square root of 1 is 1 - 1st term of the sequence. The square root of 4 is 2 – the second term of the sequence. The square root of 9 is 3 – the third term of the sequence, etc.

ESSENTIAL QUESTIONS

- How can we determine approximations of square roots that are irrational?
- How do we know if our approximations are reasonable?

MATERIALS:

- One inch grid paper (multi-weight grid lines-see attached)
- Scissors
- Decimal Approximations of Roots Task
- Open Middle Recording Sheet

GROUPING:

- Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

To begin, give students the following pattern:

1, 4, 9, 16, . .

Ask them to determine the next three terms in the sequence and justify how they found them (identify a pattern). Most will find the pattern mentioned in the Common Misconceptions section above.

Now, asks students to build rectangles using the number of square tiles indicated by the number in the sequence (i.e. use 4 square tiles to build a rectangle for the number 4).

Discuss the results. Ask students questions about the relationship between the side length and the square. The side length or square root, is the number of the term in the sequence. The square root of 1 is 1 – the first term in the sequence. The square root of 4 is 2 – the second term in the sequence, etc.

Part I:

Start this task with a whole group discussion regarding square roots. Guide the discussion so that the opener is the focus and the conceptual understanding is the focus.

Students will work with materials to determine approximations for square roots of numbers that are not perfect squares.

TASK DIRECTIONS

Give pairs of students a sheet of the multi-weight grid paper and ask them to cut out two large squares (1”). Ask students to discuss how the squares are divided, then share with the group. Each square is divided into tenths (columns/rows) and hundredths (there are 100 small squares within each large square.

The task for the students is to approximate the square root of a square that has an area of two square units. Begin by asking students what this means. This may be the time for students to think about what the square root of a square with an area of 4 square units would be, then pose the task question again.

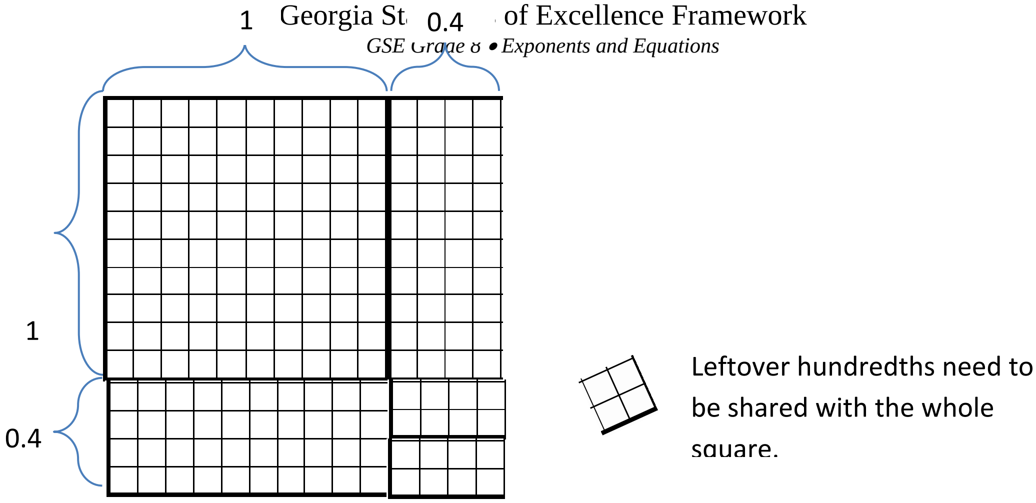
Once students have the idea that they need to find out what the a square with an area of 2 square units might look like, then determine the approximate length of the side of that square to find the square root, they can begin the task.

Begin by estimating.

The following questions should be discussed by students before proceeding to the task. Have pairs share their reasoning for each.

- Which square number is 2 close to? (1).
- Will the side of the square (the square root) of 2 be more than the square root of 1 or less?

This part may take a full class period. Many students may arrive at a different approximations. Some with different approximations. One possible student solution is shown below.



Students' approximations should be close to 1.4 and a little bit more (1.41).

Try this task again with another number close to a perfect square, such as 5.

Begin by estimating.

The following questions should be discussed by students before proceeding to the task. Have pairs share their reasoning for each.

- Which square number is 5 close to? (4).
- Will the side of the square (the square root) of 5 be more than the square root of 4 or less?

Allow students to cut their squares and make approximations based on the cuts.

Tip: As students discover their squares, have them keep a record of them in their notebook/journal as evidence of their progression of understanding square roots and the approximations.

Part II

Students will complete an Open Middle Problem from www.openmiddle.com.

<u>TASK DIRECTIONS</u>

Students will use their understandings of approximating square roots of decimals to complete the following problem:

Directions: Using only numbers 1-6 (and only once per inequality), fill in the boxes to create a true statement with the smallest possible interval:

$$\square.\square\square < \sqrt{2} < \square.\square\square$$

$$\square.\square\square < \sqrt{3} < \square.\square\square$$

$$\square.\square\square < \sqrt{4} < \square.\square\square$$

$$\square.\square\square < \sqrt{5} < \square.\square\square$$

$$\square.\square\square < \sqrt{6} < \square.\square\square$$

Give students time to make sense of the problem and discuss what it means with their partners, then share with the class. Making sense of the problem is a huge part of any problem solving task, and students need to learn how to do this by sharing their ideas during the learning process before being asked to do this independently.

FORMATIVE ASSESSMENT QUESTIONS

- What place value will impact the interval the most?
- How can you decide which numbers to place in the units (ones) place?
- For each of the tasks, which approximation is closest? How can you tell?

DIFFERENTIATION

Extension

- Teachers may wish to try more Open Middle problems with students, such as this one http://www.openmiddle.com/rational-and-irrational-numbers-2/ which involves students in making sense of differences between rational and irrational numbers.

Intervention

- http://nzmaths.co.nz/resource/square-and-cube-roots

Decimal Approximations of Roots

Directions: Using only numbers 1-6 (and only once per inequality), fill in the boxes to create a true statement with the smallest possible interval:

$$\square.\square\square < \sqrt{2} < \square.\square\square$$

$$\square.\square\square < \sqrt{3} < \square.\square\square$$

$$\square.\square\square < \sqrt{4} < \square.\square\square$$

$$\square.\square\square < \sqrt{5} < \square.\square\square$$

$$\square.\square\square < \sqrt{6} < \square.\square\square$$

Name: ____ Period: ________ Date: _______________

First attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Second attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Third attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Fourth attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Fifth attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Open Middle Worksheet - version 1.1

Sixth attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Rational Or Irrational Reasoning?

Back to Task Table

In this task, students will distinguish between rational and irrational numbers, including misconceptions.

STANDARDS FOR MATHEMATICAL CONTENT:

Work with radicals and integer exponents.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

Know that there are numbers that are not rational, and approximate them by rational numbers.

MGSE8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

MGSE8.NS.2 Use rational approximation of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line, and estimate the value of expressions (e.g., estimate π^2 *to the nearest tenth). For example, by truncating the decimal expansion of* $\sqrt{2}$ *(square root of 2), show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

2. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.

BACKGROUND KNOWLEDGE:

The importance of allowing students to conceptually develop understandings in mathematics is incredibly important. Some concepts can be challenging to figure out how we can make this happen for our students. When thinking about irrational numbers, it is best to allow students to use their knowledge of rational numbers to strategize and determine the best way to make sense of irrationals.

In order for students to be successful, the following skills and concepts need to be maintained:

MGSE6.NS.6

COMMON MISCONCEPTIONS:

- *The students may view the square roots as dividing by two.*
- *The students may infer that all rational numbers are even and all irrational numbers are odd.*
- *A few irrational numbers are given special names (pi and e), and much attention is given to sqrt(2). Because we name so few irrational numbers, students sometimes conclude that irrational numbers are unusual and rare. In fact, irrational numbers are much more plentiful than rational numbers, in the sense that they are "denser" in the real line.*

Misconceptions can be an important part of learning, unless they go undiscovered. Knowing the possible misconceptions that are common ahead of time can be a great tool to prevent these from becoming problematic. Asking questions beginning with "Why. . ." or "How. . ." and listening to/reading your students' responses is a great way to identify any misconceptions fairly easily.

ESSENTIAL QUESTIONS:

- Why is it useful for me to know the square root of a number?
- What is the difference between rational and irrational numbers?
- How do we locate approximate locations of irrational numbers on a number line and estimate the values of irrational numbers?
- What strategies can I use to create and solve linear equations with one solution, infinitely many solutions, or no solutions?

MATERIALS:

- Copies of task for each student/pair of students/or small group

GROUPING:

- Individual/Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Students work individually, with a partner, or in a small group to analyze the response to a test question. They must determine if the student had a deep understanding of the topic or misconceptions. They should complete a study guide over these standards for this student to help clear up the misconceptions. The study guide should contain written explanation, examples, graphics, etc.

DIFFERENTIATION:

Extension:

- We begin our study of Algebra by looking at numbers. Numbers play a role in our life in many ways. From counting the money we make in a year to describing how many megs of ram we have in our computer. In this section we will learn how to group numbers that are like together into sets.

  A **set** is a collection of objects. The objects in the set are called the **elements** of the set.

  Roster Notation: When braces are used to enclose the elements in the set.

  The set of ages of people in a class = $\{18, 19, 20, 21, 23, 27, 32, 38, 43\}$

  The Roster Method is used when the set is finite.

  Set Builder Notation: Used when the sets are infinite. Can be used when they are finite.

  $\{x \mid x$ is an age of someone in the class$\} = \{18, 19, 20, 21, 23, 27, 32, 38, 43\}$

  Read "The set of x such that x is an age of someone in the class"

Give a description of the following using help from:

http://www.uiowa.edu/~examserv/mathmatters/tutorial_quiz/arithmetic/realnumbersystem.html

The set of **Real Numbers**:

The set of **Natural Numbers**:

The set of **Whole Numbers**:

The set of **Integers**:

The set of **Rational Numbers**:

Rational Numbers have a decimal expansion that a.) terminates or b.) doesn't terminate

a.) $\frac{3}{4} = .75$ b.) $\frac{2}{3} = .66\overline{6}$

EXAMPLE: True or False

a.) Every integer is a rational number
b.) Every rational number is a whole number
c.) Every natural number is a whole number
d.) 3 is an element of the rational numbers

EXAMPLE: Express the following rational numbers as decimals:

a.) $\frac{5}{16}$ b.) $\frac{10}{11}$ c.) $-\frac{19}{10000}$

The set of **Irrational Numbers**:

Give three examples of irrational numbers: ____ ____ ____

The set of **Real Numbers**: Consists of all the rational and irrational numbers.

EXAMPLE: Determine which numbers in the set are natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers. Then graph each number on the real number line.

$$\left\{-\frac{2}{3}, \frac{\pi}{2}, 0.7, \sqrt{5}, \sqrt{16}, 0, 3.\overline{89}, -4\right\}$$

NATURAL NUMBERS:

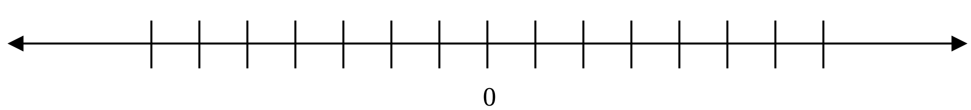
WHOLE NUMBERS:

INTEGERS:

RATIONAL NUMBERS:

IRRATIONAL NUMBERS:

REAL NUMBERS:



Give an example of a number that is a real number but not rational: ____________

EXAMPLE: Place a < , > , or = symbol between the numbers to make a true statement:

a.) $\frac{3}{5} \quad \frac{3}{4}$ b.) $-19 \quad -30$ c.) $\frac{\pi}{2} \quad 2.1$ d.) $\frac{1}{7} \quad \frac{11}{77}$

***Final Product*:** Create a graphic organizer to show the relationship between the subsets of the real number system that you have discovered in this extension. Include at least three examples of each.

DIFFERENTIATION

Intervention/Scaffolding:

- Have students mark each sentence in Robin's answer as either correct or incorrect. On a separate sheet of paper, have students re-write each incorrect statement, label it as a misconception, and explain why the statement is incorrect. Have students include correct and incorrect examples with their explanations. Consider this the study guide. You may want to guide the class through explaining the first misconception as an example.

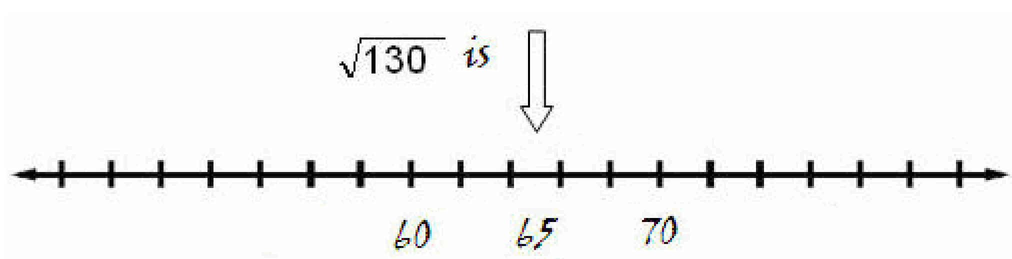
Rational Or Irrational Reasoning?

Analyze Robin's reasoning in her answer to the test question about rational or irrational numbers. Does she have a deep understanding of rational and irrational numbers? Does her reasoning make sense? If not, what misconceptions does she have about this topic? Create a study guide with explanations, examples, and graphics to help clear up any misconceptions students might have over these topics.

3rd Block — Robin Radical

1. Is $\sqrt{130}$ rational or irrational? Where would $\sqrt{130}$ be located on a number line? Explain your reasoning.

$\sqrt{130}$ is a rational number because 130 is even. All rational numbers are even and irrational numbers are odd when the numbers are under the square root sign. The square root sign is the opposite of squaring a number. Squaring a number is the same as raising it to the 2nd power. So, to find the value of a number under the square root sign, you divide it by 2. So, $130 \div 2$ is 65. The answer is 65.



Repeating Decimals – (FAL)

Back to Tasks Table

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1237

In this FAL, students will translate between decimal and fraction notation (particularly when the decimals are repeating), create and solve simple linear equations to find the fractional equivalent of a repeating decimal, and understand the effect of multiplying a decimal by a power of 10.

STANDARDS FOR MATHEMATICAL CONTENT:

MGSE8.EE.7 Solve linear equations in one variable.

MGSE8.EE.7a Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, *or* $a = b$ results (where a and b are different numbers).

MGSE8.EE.7b Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

MGSE8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

MGSE8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of* $\sqrt{2}$ *(square root of 2), show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
6. Attend to precision.

BACKGROUND KNOWLEDGE:

In order for students to be successful, the following skills and concepts need to be maintained:

- **MGSE7.NS.2**

COMMON MISCONCEPTIONS:

- *Some students may think that 0.3 and* $0.\overline{3}$ *represent the same fraction.*
- *Additional misconceptions can be found within the FAL Teacher Guide.*

ESSENTIAL QUESTIONS:

- What is the difference between rational and irrational numbers?
- When are rational approximations appropriate?
- Why do we approximate irrational numbers?

MATERIALS:

- **See FAL**

GROUPING:

- Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Repeating Decimals*, is a Formative Assessment Lesson (FAL) that can be found at the website: http://map.mathshell.org/materials/lessons.php?taskid=421&subpage=concept
The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1237

A Few Folds

Back to Task Table

Source: GPS 8th Grade Framework Unit 2

In this task, students will explore and continue patterns, and represent findings with integer outcomes.

STANDARDS FOR MATHEMATICAL CONTENT:

Work with radicals and integer exponents.

MGSE8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{(-5)} = 3^{(-3)} = \frac{1}{3^3} = \frac{1}{27}$.

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

In order for students to be successful, the following skills and concepts need to be maintained:

- **MGSE4.MD.1**

COMMON MISCONCEPTIONS:

- *The students may incorrectly state $2^0 = 0$. This task explores this property of exponents extremely well.*

ESSENTIAL QUESTIONS:

- When are exponents used and why are they important?
- How do I simplify and evaluate numeric expressions involving integer exponents?

MATERIALS:

- Copies of task for each student/pair of students/or small group
- Paper for folding activity in Part 1

GROUPING:

- Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will operate with integer exponents to describe and continue patterns. Students may want to create and use a table to organize their work and findings. Allow students to explore, discover, and generalize the properties of exponents and practice simplifying expressions with integer exponents.

DIFFERENTIATION:

Extension:

Students will investigate exponential growth.

http://www.mathwarehouse.com/exponential-growth/graph-and-equation.php

Intervention/Scaffolding:

Part 1: Set up the table for the students to mark their observations. Discuss the first fold as a class and how to mark that on the table.

Part 2: Remind students of liquid measurement conversions, perhaps by providing liquid measuring containers and filling in a reference chart. Remind students that these patterns are easier to see when noted in tables.

A Few Folds

Part 1:

Repeatedly fold one piece of paper in half, recording the number of folds and the resulting number of layers of paper. Assuming that you could continue the pattern, how many layers of paper would there be for 10 folds, 100 folds, n folds? How do you know?

Solution

Number of folds	*1*	*2*	*3*	*4*	*5*	*…*	*10*	*…*	*n*
Number of layers of paper	*2*	*4*	*8*	*16*	*32*	*…*	*1024*	*…*	2^n
Number of layers of paper written using integer exponents	2^1	2^2	2^3	2^4	2^5	*…*	2^{10}	*…*	2^n

Students should see that each fold resulted in twice as many layers of paper as the previous fold.

Part 2:

Explain the process that is used to generate the following patterns.

- Penny, dime, dollar, ten dollars, one hundred dollars, and so on.

Solution

Penny	*Dime*	*Dollar*	*Ten Dollars*	*One Hundred Dollars*
0.01	*0.10*	*1.00*	*10.00*	*100.00*
10^{-2}	10^{-1}	10^0	10^1	10^2

In the sequence of money, each amount is 10 times the previous one, giving 1 penny, ten pennies, 100 pennies, and so on.

- Cup, pint, quart, half gallon, and gallon, and so on.

Solution

Cup	*Pint*	*Quart*	*Half Gallon*	*Gallon*
1 cup	*2 cups*	*4 cups*	*8 cups*	*16 cups*
2^0	2^1	2^2	2^3	2^4

In terms of cups, 1 cup, 2 cups, 4 cups, 8 cups, 16 cups. Each measure is twice the previous one.

A Few Folds

Part 1:

Repeatedly fold one piece of paper in half, recording the number of folds and the resulting number of layers of paper. Assuming that you could continue the pattern, how many layers of paper would there be for 10 folds, 100 folds, n folds? How do you know?

Part 2:

Explain the process that is used to generate the following patterns.

- Penny, dime, dollar, ten dollars, one hundred dollars, and so on.

- Cup, pint, quart, half gallon, and gallon, and so on.

Got Cubes? Revisited (Spotlight Task)

Back to Task Table

Task adapted from www.gfletchy.wordpress.com

STANDARDS FOR MATHEMATICAL CONTENT

MGSE8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTIONS

- How do I simplify and evaluate numeric expressions involving integer exponents?

MATERIALS REQUIRED

- Act 1 video http://vimeo.com/98507175
- Act 3 Video http://vimeo.com/98507762
- Isometric Dot Paper
- Connecting cubes or colored blocks

TEACHER NOTES

In this task, students will watch the video, then ask what they noticed. They will then be asked to discuss what they wonder or are curious about. The questions in this activity are based on sequences of powers, which increase at a constant rate. Students will gain an appreciation of what is meant by an amount increasing “exponentially”. Encourage your students to think about the strategies they might use as they solve these problems.

Students might use a table to organize the data. They could also use a systematic tree diagram, although it may start to become too congested to be manageable. Nevertheless, a tree diagram could be used in the initial exploration, especially if it is done on a large piece of paper.
Some students will be able to use the knowledge they have gained from previous activities to make the connection between the months and the appropriate power. This may prompt them to use the calculator constant in conjunction with the table.

Students who are not calling on their previous knowledge to solve the question efficiently should be prompted by questions such as: "Is there a quicker way to calculate the tree growth?" "Can you remember a shorter way of writing that equation?" This will help them to recall the links between addition, multiplication, and exponents.

TASK DESCRIPTION

The following task can be found at: http://gfletchy3act.wordpress.com/got-cubes-revisited/

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:

Watch the video: http://vimeo.com/98507175

- What do you notice and what do you wonder?
- How many cubes will be in the structure at the 7th stage?
- Make an estimate you know is too high. Too low.
- Draw a sketch using the Isometric Dot Paper of what the last structure will look like.

ACT 2:

- Number of cubes in shown structures are 2, 8, and 32
- 7 structures in total

ACT 3:

Watch the video: http://vimeo.com/98507762

Students will compare and share solution strategies.

- Reveal the answer. Discuss the theoretical math versus the practical outcome.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.
- How many cubes are there in all 7 structures combined? Write 2 numerical expressions to that identify the total number of cubes?

ACT 4

- Can you identify a rule that would tell you how many cubes there would be in the 10th structure? How about the 50 Structure? Write an expression to solve for any stage.
- Is there a pattern with the shape of each structure? If so what shape would the 83rd structure be?
- What would the stage before the 1st stage look like? How many cubes would it have? Can it be expressed as an exponent? (Answer: Act 4 Sequel)

Got Cubes? Revisited

Name:________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate the result of the main question? Explain?
Place an estimate that is too high and too low on the number line
Low estimate *Place an "x" where your estimate belongs* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:_______________________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Alien Attack

Back to Task Table

In this task, students will discover and generalize the properties of exponents.

STANDARDS FOR MATHEMATICAL CONTENT

Work with radicals and integer exponents.

MGSE8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{(-5)} = 3^{(-3)} = \frac{1}{3^3} = \frac{1}{27}$.

STANDARDS FOR MATHEMATICAL PRACTICE

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE:

In order to truly learn mathematics, students must make sense of it from their own point of understanding. This task allows students to do just that. Students are asked to reconstruct the properties of integer exponents based on examples with no explanation. Once engaged in this task, students are expected to demonstrate through their interactions the 8 standards for mathematical practice as they re-write the "forgotten" properties in their own words.

In order for students to be successful, the following skills and concepts need to be maintained:

MGSE7.NS.2

COMMON MISCONCEPTIONS:

After discovery the properties of exponents, students may confuse the properties with one another. This often happens when students are asked to memorize rules rather than make sense of the rules after looking at examples and searching for patterns. Allowing students to re-discover the properties of integer exponents engages them in sense making as they are writing the properties from scratch.

ESSENTIAL QUESTIONS:

- How do I simplify and evaluate numeric expressions involving integer exponents?

MATERIALS:

- Copies of task for each student/pair of students/or small group
- Copies of the Standards for Mathematical Practice

GROUPING:

Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

After an alien attack on mathematics, students must recreate the integer product rules by examining the examples left behind using the Practice Standards. Students should explore, discover, and generalize the properties of exponents. To complete the task students should write a rule and create a name for the rule.

FORMATIVE ASSESSMENT QUESTIONS:

- What pattern did you notice?
- How do you know?
- Why does your property work?
- Would your property apply to all types of numbers?
- Is there any number that your rule would not hold for?
- Create another problem and apply your property. Does it work? Can you find an example that will not work?
- What if the numbers were replaced with variables?

DIFFERENTIATION:

Extension:

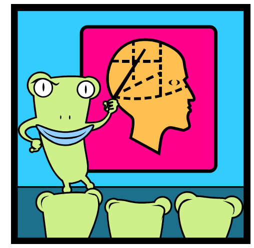
- Students will solve problems using properties of exponents.

  http://www.youtheducationservices.ca/secure/subjects/number_concepts/pdfs/9_Expo_Law_1_Wksht.pdf

Intervention/Scaffolding:

- Be sure to prompt struggling students by asking guiding questions. Group the students in a way that promotes cooperative learning (less observant/less pattern-oriented students with more observant/pattern-oriented students).
- Create a space for students needing support, where they may come to you at a table. Or pull students a group at a time for about 5 minutes each. Begin with the group(s) needing the most support to get them on track, then circulate.

<u>Alien Attack!</u>



Aliens from the Outer Space Galactic Task Force have been watching the recent gains in mathematical understanding developing in human brains from Planet Earth. They have become alarmed that Planet Earth might soon develop the capabilities to discover that life exists on other planets with this increased mathematical knowledge. To slow the progress, they have organized an attack on the World Wide Web and all other forms of math textbooks. All words have been eradicated from the mathematics examples! Humans will now be forced to study the patterns of numbers and previously worked examples to rediscover the properties of mathematics! They are confident that humans will not persevere in the challenge of making sense of problems, reasoning abstractly and quantitatively, constructing viable arguments, looking for and making use of structure, and using repeated reasoning to recreate the language of mathematics. They have already declared MISSION ACCOMPLISHED!

The next standard in your mathematics class requires knowledge of the properties of integer exponents. Can your team recreate and name the properties by using the Standards for Mathematical Practice to examine the examples that remain?

Problem continues on next page. →

Problem continued from previous page.

Solutions

Product of Powers Property $7^3 \bullet 7^2 = (7 \bullet 7 \bullet 7) \bullet (7 \bullet 7) = 7^{3+2} = 7^5$ $5^3 \bullet 5^3 = (5 \bullet 5 \bullet 5) \bullet (5 \bullet 5 \bullet 5) = 5^{3+3} = 5^6$ $3 \bullet 3^4 = (3) \bullet (3 \bullet 3 \bullet 3 \bullet 3) = 3^5$	***To multiply powers with the same base, add the exponents and keep the common base.***
Power of a Power Property $(2^2)^3 = (2^2) \bullet (2^2) \bullet (2^2)$ $= (2 \bullet 2) \bullet (2 \bullet 2) \bullet (2 \bullet 2) = 2^{2 \bullet 3} = 2^6$ $(6^4)^2 = (6^4) \cdot (6^4)$ $= (6 \bullet 6 \bullet 6 \bullet 6) \bullet (6 \bullet 6 \bullet 6 \bullet 6) = 6^{4 \bullet 2} = 6^8$	***To raise a product to a power, raise each factor to the power.***
Power of a Product Property $3^5 \bullet 2^5 = (3 \bullet 2)^5 = 6^5$ $4^3 \bullet 5^3 = (4 \bullet 5)^3 = 20^3$ $(2 \bullet 3)^6 = 2^6 \bullet 3^6$	***To find a power of a product, find the power of each factor and multiply.***

Problem continues on next page. →

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<table>
<tr><th>Exponent</th><th>Repeated Mult</th><th>Value</th><td rowspan="15">Zero Exponent

Any number raised to the zero power is equal to 1.

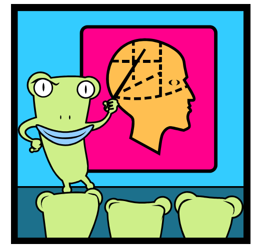
Negative Exponent

Negative exponents indicate reciprocation, with the exponent of the reciprocal becoming positive.</td></tr>
<tr><td>2^3</td><td>$2 \cdot 2 \cdot 2$</td><td>8</td></tr>
<tr><td>2^2</td><td>$2 \cdot 2$</td><td>4</td></tr>
<tr><td>2^1</td><td>2</td><td>2</td></tr>
<tr><td>2^0</td><td></td><td></td></tr>
<tr><td>2^{-1}</td><td></td><td></td></tr>
<tr><td>2^{-2}</td><td></td><td></td></tr>
<tr><td>2^{-3}</td><td></td><td></td></tr>
<tr><td>5^3</td><td>$5 \cdot 5 \cdot 5$</td><td>125</td></tr>
<tr><td>5^2</td><td>$5 \cdot 5$</td><td>25</td></tr>
<tr><td>5^1</td><td>5</td><td>5</td></tr>
<tr><td>5^0</td><td></td><td></td></tr>
<tr><td>5^{-1}</td><td></td><td></td></tr>
<tr><td>5^{-2}</td><td></td><td></td></tr>
<tr><td>5^{-3}</td><td></td><td></td></tr>
<tr><td colspan="3">Quotient of Like Bases

$\frac{5^5}{5^2} = 5^{5-2} = 5^3 = 125$

$\frac{2^3}{2^5} = 2^{3-5} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$</td><td>To divide powers with the same base, subtract the exponents and keep the common base.</td></tr>
</table>

Alien Attack!



Aliens from the Outer Space Galactic Task Force have been watching the recent gains in mathematical understanding developing in human brains from Planet Earth. They have become alarmed that Planet Earth might soon develop the capabilities to discover that life exists on other planets with this increased mathematical knowledge. To slow the progress, they have organized an attack on the World Wide Web and all other forms of math textbooks. All words have been eradicated from the mathematics examples! Humans will now be forced to study the patterns of numbers and previously worked examples to rediscover the properties of mathematics! They are confident that humans will not persevere in the challenge of making sense of problems, reasoning abstractly and quantitatively, constructing viable arguments, looking for and making use of structure, and using repeated reasoning to recreate the language of mathematics. They have already declared MISSION ACCOMPLISHED!

The next standard in your mathematics class requires knowledge of the properties of integer exponents. Can your team recreate and name the properties by using the Standards for Mathematical Practice to examine the examples that remain?

Problem continues on next page. →

____________________ $7^3 \bullet 7^2 = (7 \bullet 7 \bullet 7) \bullet (7 \bullet 7) = 7^5$ $5^3 \bullet 5^3 = (5 \bullet 5 \bullet 5) \bullet (5 \bullet 5 \bullet 5) = 5^6$ $3 \bullet 3^4 = (3) \bullet (3 \bullet 3 \bullet 3 \bullet 3) = 3^5$	
____________________ $(2^2)^3 = (2^2) \bullet (2^2) \bullet (2^2) = 2^6$ $(6^4)^2 = (6^4) \cdot (6^4) = 6^8$	
____________________ $3^5 \bullet 2^5 = (3 \bullet 2)^5 = 6^5$ $4^3 \bullet 5^3 = (4 \bullet 5)^3 = 20^3$ $(2 \bullet 3)^6 = 2^6 \bullet 3^6$	

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____________	____________	____
2^3	$2 \cdot 2 \cdot 2$	8
2^2	$2 \cdot 2$	4
2^1	2	2
2^0		
2^{-1}		
2^{-2}		
2^{-3}		
5^3	$5 \cdot 5 \cdot 5$	125
5^2	$5 \cdot 5$	25
5^1	5	5
5^0		
5^{-1}		
5^{-2}		
5^{-3}		

____

____

____

$$\frac{5^5}{5^2} = 5^{5-2} = 5^3 = 125$$

$$\frac{2^3}{2^5} = 2^{3-5} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

Nesting Dolls

Back to Task Table

Source: GPS 8th Grade Framework Unit 2

In this task, students will use the properties of exponents to extend and generalize a pattern as applied to a real world problem.

NOTE TO TEACHER: ***This task may be challenging for some students; therefore, teachers should consider the levels of ability among their students prior to assigning.***

STANDARDS FOR MATHEMATICAL CONTENT

Work with radicals and integer exponents.

MGSE8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{(-5)} = 3^{(-3)} = \frac{1}{3^3} = \frac{1}{27}$.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and lxl $\leq$ 25) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and lxl $\leq$ 10) has one solution. Evaluate square roots of perfect squares $\leq$ 625 and cube roots of perfect cubes $\geq$ -1000 and $\leq$ 1000.

STANDARDS FOR MATHEMATICAL PRACTICE

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
6. Attend to precision.

BACKGROUND KNOWLEDGE:

In order for students to be successful, the following skills and concepts need to be maintained:

- **MGSE7.NS.2**

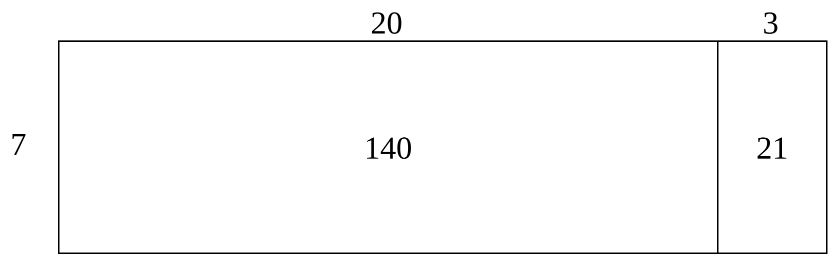
COMMON MISCONCEPTIONS:

Students often mix "rules" for operations with fractions due to a lack of conceptual understanding. Give every opportunity to students to develop this conceptual understanding through the use of manipulatives and models.

Students also have trouble using the distributive property over exponents. This can be made explicit with students by making sure the foundational understanding of the distributive property is developed. This can be accomplished through daily number talks as well as encouraging students to model what these mathematical ideas might look like.

For example:

23 x 7 = (20 + 3) 7 = (20 x 7) + (3 x 7) = 140 + 21 = 161



ESSENTIAL QUESTIONS

- When are exponents used and why are they important?

MATERIALS:

- Copies of task for each student/pair of students/or small group
- *Optional:* Set of Nesting Dolls (also called matryoshka, babushka or stacking dolls)

GROUPING:

- Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will employ the properties of exponents to extend and generalize a pattern as applied to a real-world problem. It's important that students see the value in organizing their thinking. Tasks like this can be very frustrating for students who are not used to organizing their work in a list or a table. The value of these organizational tools for problem solving is extremely high when searching for patterns and thinking algebraically.

Students will be using the relative sizes of nesting dolls to find a rule that explains how the height of each successive doll relates to the largest doll. Students will then be asked to write a general rule for the size of the n^{th} doll.

DIFFERENTIATION:

Intervention/Scaffolding:

- Remind students what $\frac{2}{3}$ of something means, both visually and algebraically. Set up a table through a class discussion to help students organize their thinking so they are given the opportunity to discover the pattern on their own.

Nesting Dolls

The first Russian nesting dolls were made over a hundred years ago. They were brightly painted in classic Russian designs and made so that the finished doll had many similar dolls that fit one inside the other. This type of doll is called a Matroyshka which is derived from the Latin word for mother and is a common name for women in Russian villages.

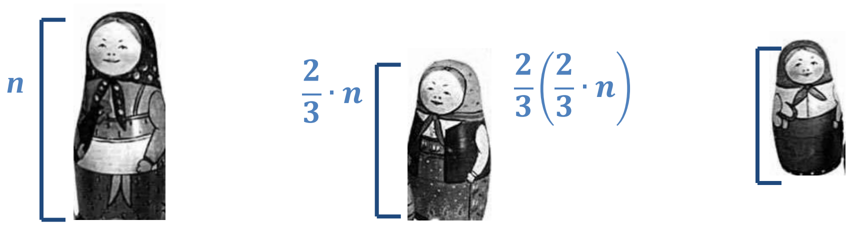
Suppose there are a total of eight dolls. The finished doll is n centimeters tall with seven smaller dolls that fit inside one another. Each doll is $\frac{2}{3}$ the height of the next larger doll.

a. What is the relationship of the heights of the third largest doll and the largest doll?

Solution

The third largest doll would have a height of $\left[\left(\frac{2}{3}\right)^2 \bullet n\right] cm$ ***or*** $\left(\frac{4}{9}n\right) cm$

where n represents the height of the largest doll.



largest doll ***2nd largest*** ***3rd largest***

b. What is the relationship of the smallest doll and the largest doll?

Solution

The smallest doll would have a height of $\left[\left(\frac{2}{3}\right)^7 \bullet n\right] cm$ ***or*** $\left(\frac{128}{2187}n\right) cm$

where n represents the height of the largest doll.

As Russia has dropped the Iron Curtain and opened doors, many of their production facilities have more technology. Instead of hand carving the Matroyshka, factories use computerized robots to make them. You have been asked to develop the general statement to generalize the height of each doll regardless of the number of dolls in the finished product.

Solution

Listing the heights of the dolls in descending order would be n, $\frac{2}{3}$ ***n cm,*** $\frac{4}{9}$ ***n cm,*** $\frac{8}{27}$ ***n cm,*** $\frac{16}{81}$ ***n cm,*** $\frac{32}{243}$ ***n cm,*** $\frac{64}{729}$ ***n cm,*** $\frac{128}{2187}$ ***n cm long.***

Doll	Height
First (largest doll)	***N***
Second	$\left(\frac{2}{3}\right)^{1}$ n cm
Third	$\left(\frac{2}{3}\right)^{2}$ n cm
Fourth	$\left(\frac{2}{3}\right)^{3}$ n cm
Fifth	$\left(\frac{2}{3}\right)^{4}$ n cm
Sixth	$\left(\frac{2}{3}\right)^{5}$ n cm
Seventh	$\left(\frac{2}{3}\right)^{6}$ n cm
Eighth (smallest doll)	$\left(\frac{2}{3}\right)^{7}$ n cm

Students should recognize a pattern developing. One possible student response could be

$$\left(\frac{2}{3}\right)^{n-1} \cdot n\ cm$$

Using the properties of exponents, they should realize that this is the same thing as

$$\left(\frac{2}{3}\right)^{n-1} = \left(\frac{2}{3}\right)^{n} \cdot \left(\frac{2}{3}\right)^{-1} = \left(\frac{3}{2}\right)\left(\frac{2}{3}\right)^{n}$$

Nesting Dolls

The first Russian nesting dolls were made over a hundred years ago. They were brightly painted in classic Russian designs and made so that the finished doll had many similar dolls that fit one inside the other. This type of doll is called a Matroyshka which is derived from the Latin word for mother and is a common name for women in Russian villages.

Suppose there are a total of eight dolls. The finished doll is *n* centimeters tall with seven smaller dolls that fit inside one another. Each doll is $\frac{2}{3}$ the height of the next larger doll.

a. What is the relationship of the heights of the third largest doll and the largest doll?

b. What is the relationship of the smallest doll and the largest doll?

As Russia has dropped the Iron Curtain and opened doors, many of their production facilities have more technology. Instead of hand carving the Matroyshka, factories use computerized robots to make them. You have been asked to develop the general statement to generalize the height of each doll regardless of the number of dolls in the finished product.

Exponential Exponents

Back to Tasks Table

In this task, students will apply the properties of integer exponents to generate equivalent numerical expressions.

STANDARDS FOR MATHEMATICAL CONTENT

Work with radicals and integer exponents.

MGSE8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{(-5)} = 3^{(-3)} = \frac{1}{3^3} = \frac{1}{27}$.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

STANDARDS FOR MATHEMATICAL PRACTICE

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

In order for students to be successful, the following skills and concepts need to be maintained:

- **MGSE7.NS.2**

COMMON MISCONCEPTIONS:

Students may, after discovery the properties of exponents, mix up the properties. Students will need many opportunities to complete tasks like this, discuss strategies, and make sense of the mathematics they're using before misconceptions are undone. Keep an eye out for students confusing properties of exponents and ask questions like "How did you decide to use that property?" or "Tell me what your reasoning is for this." Asking students to explain their reasoning (right or wrong) helps students make sense of mathematics.

ESSENTIAL QUESTIONS:

- How can I apply the properties of integer exponents to generate equivalent numerical expressions?

MATERIALS:

- cards cut apart – one card per group
- copy of Exponential Exponents handout per group
- colored pencils

GROUPING

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Divide students into groups to generate equivalent numerical expressions to the one assigned. Students should write an explanation of the strategies that were used to generate equivalent expressions. Groups then extend and evaluate the work of other groups.

TEACHER NOTES

- Opening: How many different ways can you express the number 128? (Many students have experience with this in elementary school. Sample answers: 130 – 2, 2 • 64, 128/1, 100 + 28, 256 ÷ 2.) Today we are going to extend this to the properties of integer exponents.

- Work time: Pass out a copy of the activity sheet and a card to each group. Students should write the numeric expression in the box in the center of the page.

  With your group use the properties of integer exponents to generate equivalent expressions to the expressions assigned. Each property of exponents should be represented.

  Write an explanation of the strategies that you used to generate the equivalent expressions.

  Circulate during work time asking students questions similar to the closing questions below.

  Give each group a different color pencil. Rotate the pages around the room clockwise. Using the color pencil for your group, add additional expressions to the page and correct any errors in the expressions listed. Continue this rotation process until the groups have had evaluated at least two other groups.

- Closing:

  What strategies enable you to create equivalent expressions?

  What challenges did your group encounter?

  Which property of exponents did you use the most?

  Which property was the most challenging to incorporate?

  After examining the work of other groups, what was the most unique expression?

  Post these charts around the room. Challenge your students to continue adding equivalent expressions throughout the unit.

DIFFERENTIATION:

Extension:

- Additional cards can be created to differentiate for your students.

Intervention/Scaffolding:

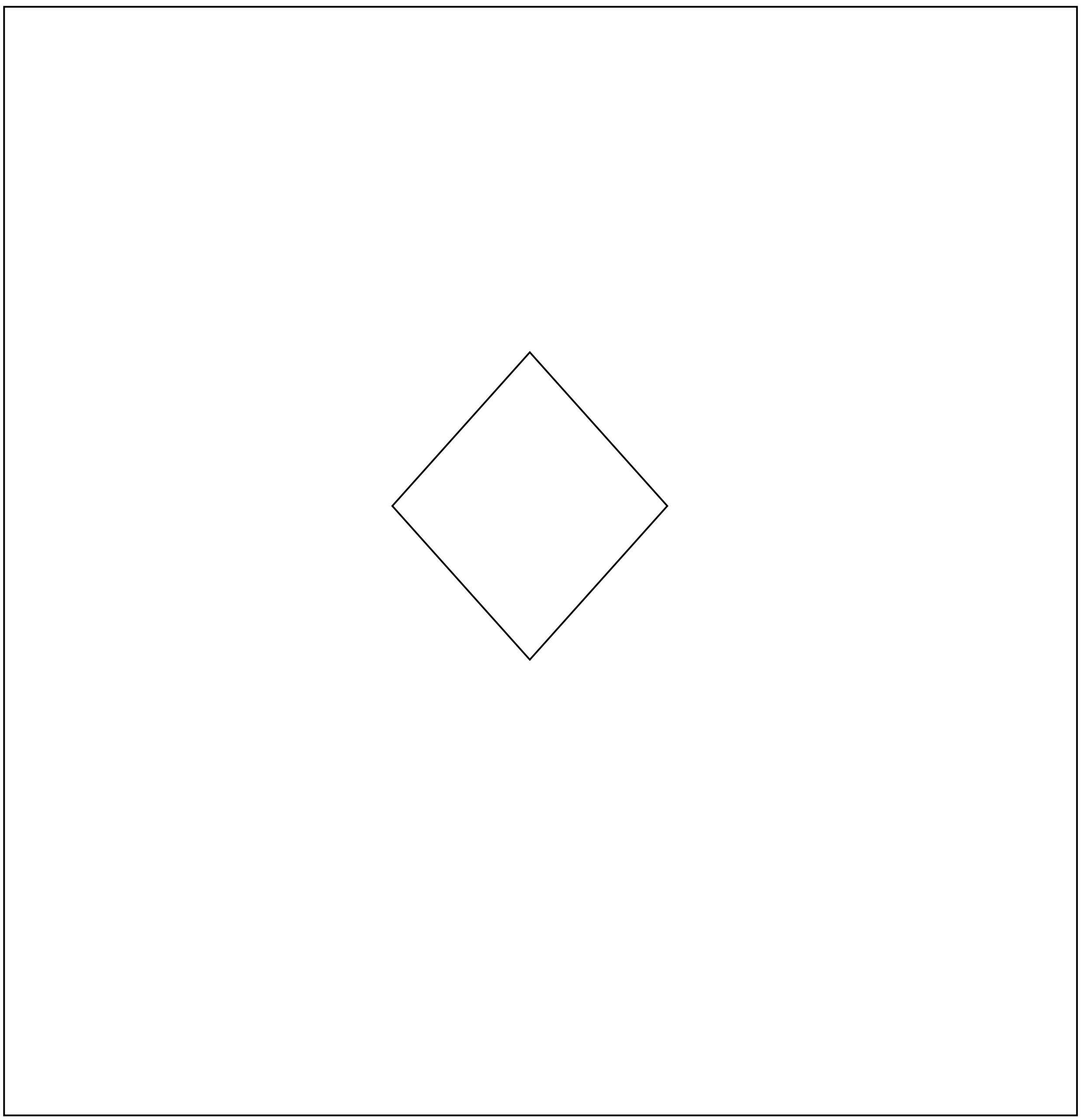
- Use class or small group discussion to work through an example.
- Additional cards can be created to differentiate for your students.

Cards for Task: *Cut apart and distribute one to each group*

2^{27}	5^{12}	10^{30}
3^{18}	7^{36}	8^{24}
$\frac{1}{64}$	$\frac{1}{36}$	$\frac{125}{64}$

Exponential Exponents

- Write your numeric expression in the box in the center of the page.
- With your group, use the properties of integer exponents to generate expressions equivalent to your numeric expression. Each property of exponents should be represented.
- Write an explanation of the strategies that you used to generate the equivalent expressions.



Exploring Powers Of 10

Back to Task Table

Source: GPS 8th Grade Framework Unit 2. Adapted from Van de Walle (2004). *Elementary and Middle School Mathematics Teaching Developmentally* (5th ed.). Boston: Pearson.

In this task, students will develop a conceptual understanding of scientific notation through exploration.

STANDARDS FOR MATHEMATICAL CONTENT:

Work with radicals and integer exponents.

MGSE8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{(-5)} = 3^{(-3)} = \frac{1}{3^3} = \frac{1}{27}$.

MGSE8.EE.3 Use numbers expressed in scientific notation to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.*

MGSE8.EE.4 Add, subtract, multiply and divide numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Understand scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g. use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology (e.g. calculators).

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

In order for students to be successful, the following skills and concepts need to be maintained:

- **MGSE5.NBT.1**, **MGSE5.NBT.2**, **MGSE5.NBT.3**
- **MGSE7.NS.2**

COMMON MISCONCEPTIONS:

- *Students have the misconception of the decimal point moving to multiply and divide by powers of 10. This has no mathematical basis and should not be taught. Rather, students should use their understandings of place value and digit value to reason that the decimal point remains stationary and the values of the digits change by a factor of 10. Since the digits are changing by a factor of 10, the digits must shift.*
- *Students may also develop the misconception that the exponent tells the number of zeroes to add to the number. Address this explicitly in the classroom with questions addressing the number of zeroes for several different examples. Ask: "Is there a relationship between the exponent and the number of zeroes?" (No, it depends on how many non-zero digits are in the number.)*
- *The students may incorrectly state $2^0 = 0$. This task can be used to reinforce this idea which was introduced in an earlier task.*

ESSENTIAL QUESTIONS

- How can I represent very small and large numbers using integer exponents and scientific notation?
- How can I perform operations with numbers expressed in scientific notation?
- How can I interpret scientific notation that has been generated by technology?

MATERIALS:

- Calculators (scientific or graphing TI-83/TI-84)

GROUPING:

- Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will use exponents and scientific notation to represent very large and very small numbers. Completion of this task allows students to develop a conceptual understanding of scientific notation. Students may find the calculator very useful in exploring and drawing conclusions about how to represent very large and very small numbers.

DIFFERENTIATION:

Extension:

- Students may develop a deeper understanding of negative exponents through the study of different bases. Some of the more popular bases are 2, 8, 12, and 60 because of their everyday uses in various fields.

Intervention/Scaffolding:

- Work through Part 2, Question 1 with students to give them an example of pattern description. Set up tables for student use in Part 2, Questions 4 & 5.

Exploring Powers Of 10

This activity will help you to learn how to represent very large and very small numbers.

Part 1: Very Small Numbers (negative exponents)

Complete the following table. What patterns do you see?

Solution

10^6 = **1000000**
10^5 = **100000**
10^4 = **10000**
10^3 = **1000**
10^2 = **100**
10^1 = **10**
10^0 = **1**
$10^{-1} = \mathbf{0.1} = \frac{1}{10^1} = \frac{1}{10}$
$10^{-2} = \mathbf{0.01} = \frac{1}{10^2} = \frac{1}{100}$
$10^{-3} = \mathbf{0.001} = \frac{1}{10^3} = \frac{1}{1000}$

Comment

***Very Small Numbers**: In this sequence, the most obvious entry for 10^0 is 1, and that is the definition of 10^0. By definition, any nonzero number raised to the power 0 is 1. By continuing this pattern to 10^{-1}, the 1 should move to the right of the decimal. Notice how each of these numbers is written as a decimal. Students should be encouraged to explore these numbers on a calculator (see Using the TI-83 or TI-84: Exploring Powers of 10).*

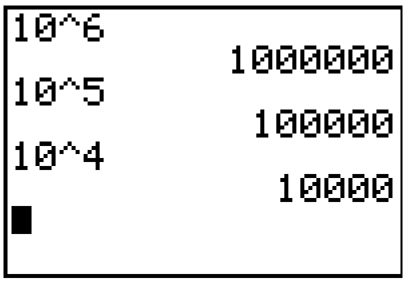
DIRECTIONS FOR USING THE TI-83 or TI-84: Exploring Powers of 10

It may be appropriate to use a calculator or computer when data may contain numbers that are difficult to work with using paper and pencil. Exploring very large and very small numbers can be done on the TI-83 or TI-84 graphing calculator with the following instructions.

1. Turn on the calculator ON.

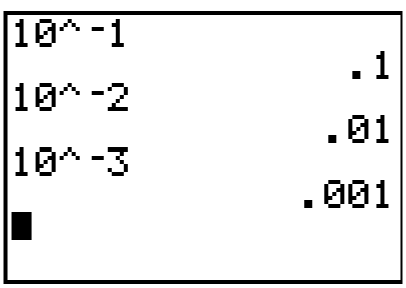
2. Enter the number 10, press the ^ key, enter the number 6, and press ENTER.

3. Continue step 2 for the positive exponents and zero.



4. Enter the number 10, press the ^ key, press the negation (−) key, enter the number 1, and press ENTER.

5. Continue step 4 for the negative exponents.



Part 2: Very Large Numbers (positive exponents)

Sometimes it can be cumbersome to say and write numbers in their common form (for example trying to display numbers on a calculator with limited screen space). Another option is to use exponential notation and the base-ten place-value system.

Using a calculator (with exponential capabilities), complete each of the following.

1. Explore 10^N for various values of N. What patterns do you notice?

   Solution

   The value of N determines the placement of the decimal point. As N increases by 1, the value of each digit increases by 10; this change in the values of the digits moves them each one place to the left. Students should see that multiplying by powers of 10 results in the digits shifting to the left. The decimal point never moves. Digits shift because their values are being changed by a power of 10 each time N is increased or decreased by one. Digits shift one place to the left for each positive power of 10 and one place to the right for each negative power of 10.

2. Enter 45 followed by a string of zeros. How many will your calculator permit? What happens when you press enter? What does 4.5E10 mean? What about 2.3E4? Can you enter this another way?

   Solution

   The number of digits permitted will depend on the calculator. When the screen is full, the calculator will automatically convert to the shorthand notation, 4.5E10 which represents $4.5 \times 10^{10} = 45000000000$.

   Note: 2.3E4 represents $2.3 \times 10^4 = 23000$
   Answers may include 2.3E4 = 23E3 = 230E2 =2300E1 = 23000

3. Try sums like $(4.5 \times 10^N) + (2 \times 10^K)$ for different values of N and K. Describe any patterns that you may notice.

   Solution

   Answers will vary. Students should discover that the only time the answer written in scientific notation could be 6.5×10^T is if $N = K = T$. In most cases, there is no major advantage for using scientific notation when adding or subtracting unless the powers of 10 are the same or at least close.

4. Try products like $(4.5 \times 10^N) \bullet (2 \times 10^K)$. Describe any patterns that you may notice.

Solution

$9 \times 10^{N+K}$. Students should be encouraged to look at positive, negative, and zero values of N and K.

4.5×10^N	2×10^K	$\left(4.5 \times 10^N\right) \cdot \left(2 \times 10^K\right)$
$\mathbf{4.5 \times 10^1}$	$\mathbf{2 \times 10^1}$	$\mathbf{9 \times 10^2 = 9 \times 10^{(1+1)}}$
$\mathbf{4.5 \times 10^2}$	$\mathbf{2 \times 10^2}$	$\mathbf{9 \times 10^4 = 9 \times 10^{(2+2)}}$
$\mathbf{4.5 \times 10^5}$	$\mathbf{2 \times 10^3}$	$\mathbf{9 \times 10^8 = 9 \times 10^{(5+3)}}$
$\mathbf{4.5 \times 10^0}$	$\mathbf{2 \times 10^{-2}}$	$\mathbf{9 \times 10^{-2} = 9 \times 10^{(0+(-2))}}$
$\mathbf{4.5 \times 10^{-3}}$	$\mathbf{2 \times 10^{-1}}$	$\mathbf{9 \times 10^{-4} = 9 \times 10^{(-3+(-1))}}$
$\mathbf{4.5 \times 10^{-4}}$	$\mathbf{2 \times 10^0}$	$\mathbf{9 \times 10^{-4} = 9 \times 10^{((-4)+0)}}$

5. Try quotients like $(4.5 \times 10^N) \div (2 \times 10)$. Describe any patterns that you may notice.

Solution

$2.25 \text{ x } 10^{N-K}$. Students should be encouraged to look at positive, negative, and zero values of N and K.

4.5×10^N	2×10^K	$\left(4.5 \times 10^N\right) \div \left(2 \times 10^K\right)$
$\mathbf{4.5 \times 10^1}$	$\mathbf{2 \times 10^1}$	$\mathbf{2.25 \times 10^0 = 2.25 \times 10^{(1-1)}}$
$\mathbf{4.5 \times 10^2}$	$\mathbf{2 \times 10^2}$	$\mathbf{2.25 \times 10^0 = 2.25 \times 10^{(2-2)}}$
$\mathbf{4.5 \times 10^5}$	$\mathbf{2 \times 10^3}$	$\mathbf{2.25 \times 10^2 = 2.25 \times 10^{(5-3)}}$
$\mathbf{4.5 \times 10^0}$	$\mathbf{2 \times 10^{-2}}$	$\mathbf{2.25 \times 10^2 = 2.25 \times 10^{(0-(-2))}}$
$\mathbf{4.5 \times 10^{-3}}$	$\mathbf{2 \times 10^{-1}}$	$\mathbf{2.25 \times 10^{-2} = 2.25 \times 10^{(-3-(-1))}}$
$\mathbf{4.5 \times 10^{-4}}$	$\mathbf{2 \times 10^0}$	$\mathbf{2.25 \times 10^{-4} = 2.25 \times 10^{((-4)-0)}}$

TEACHER NOTES

When students use scientific or graphing calculators to display numbers with more digits than the display will hold, the calculator will display the number using scientific notation. Scientific notation is a decimal number between 1 and 10 times a power of 10. Have students move from the calculator to paper and pencil converting from large numbers to scientific notation and vice versa. Also, have students follow up addition/subtraction and multiplication/division in scientific notation mentally. Notice the advantages of scientific notation for multiplication and division. Here the significant digits can be multiplied mentally (4.5 × 2) and the exponents added to produce $9 \times 10^{N+K}$.

ADDITIONAL RESOURCES

- **http://www.edinformatics.com/math_science/scinot.htm**

Exploring Powers Of 10

This activity will help you to learn how to represent very large and very small numbers.

Part 1: Very Small Numbers (negative exponents)

Complete the following table. What patterns do you see?

$10^6 =$
$10^5 =$
$10^4 =$
$10^3 =$
$10^2 =$
$10^1 =$
$10^0 =$
$10^{-1} =$
$10^{-2} =$
$10^{-3} =$

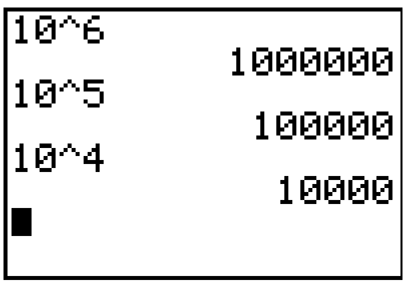
DIRECTIONS FOR USING THE TI-83 or TI-84: Exploring Powers of 10

It may be appropriate to use a calculator or computer when data may contain numbers that are difficult to work with using paper and pencil. Exploring very large and very small numbers can be done on the TI-83 or TI-84 graphing calculator with the following instructions.

1. Turn on the calculator ON.

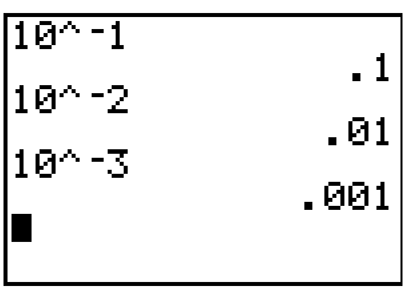
2. Enter the number 10, press the ^ key, enter the number 6, and press ENTER.

3. Continue step 2 for the positive exponents and zero.



4. Enter the number 10, press the ^ key, press the negation (−) key, enter the number 1, and press ENTER.

5. Continue step 4 for the negative exponents.



Part 2: Very Large Numbers (positive exponents)

Sometimes it can be cumbersome to say and write numbers in their common form (for example trying to display numbers on a calculator with limited screen space). Another option is to use exponential notation and the base-ten place-value system.

Using a calculator (with exponential capabilities), complete each of the following:

1. Explore 10^N for various values of N. What patterns do you notice?

2. Enter 45 followed by a string of zeros. How many will your calculator permit? What happens when you press enter? What does 4.5E10 mean? What about 2.3E4? Can you enter this another way?

3. Try sums like $(4.5 \times 10^N) + (2 \times 10^K)$ for different values of N and K. Describe any patterns that you may notice.

4. Try products like $(4.5 \times 10^N) \bullet (2 \times 10^K)$. Describe any patterns that you may notice.

5. Try quotients like $(4.5 \times 10^N) \div (2 \times 10)$. Describe any patterns that you may notice.

"Ponzi" Pyramid Scheme (Short Cycle Task)

Back to Task Table

Source: Balanced Assessment Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=808

In this task, students will experience a real-world application of exponents using a chain letter.

STANDARDS FOR MATHEMATICAL CONTENT:

Work with radicals and integer exponents.

MGSE8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{(-5)} = 3^{(-3)} = \frac{1}{3^3} = \frac{1}{27}$.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

MGSE8.EE.3 Use numbers expressed in scientific notation to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.*

MGSE8.EE.4 Add, subtract, multiply and divide numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Understand scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g. use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology (e.g. calculators).

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

In order for students to be successful, the following skills and concepts need to be maintained:

- **MGSE5.NBT.1**, **MGSE5.NBT.2**, **MGSE5.NBT.3**
- **MGSE7.NS.2**
- Understand the underlying principles of a Ponzi scheme/chain letter

COMMON MISCONCEPTIONS:

- *Students have the misconception of the decimal point moving to multiply and divide by powers of 10. This has no mathematical basis and should not be taught. Rather, students should use their understandings of place value and digit value to reason that the decimal point remains stationary and the values of the digits change by a factor of 10. Since the digits are changing by a factor of 10, the digits must shift.*
- *Students may also develop the misconception that the exponent tells the number of zeroes to add to the number. Address this explicitly in the classroom with questions addressing the number of zeroes for several different examples. Ask: "Is there a relationship between the exponent and the number of zeroes?" (No, it depends on how many non-zero digits are in the number.)*
- *The students may incorrectly state $2^0 = 0$. This task can be used to reinforce this idea which was introduced in an earlier task.*

ESSENTIAL QUESTIONS:

- How does a Ponzi scheme use the properties of exponents?
- How can the properties of exponents and knowledge of working with scientific notation help me interpret information?

MATERIALS:

- **See task**

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *"Ponzi" Pyramid Schemes*, is a Mathematics Assessment Project Assessment Task that can be found at the website:
http://www.map.mathshell.org/materials/tasks.php?taskid=278&subpage=expert

The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=808

The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=809

Estimating Length Using Scientific Notation (Formative Assessment Lesson)

Back to Tasks Table

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1221

Estimating lengths of everyday objects, converting between decimal and scientific notation, and making comparison of the size of numbers expressed in both decimal and scientific notation.

STANDARDS FOR MATHEMATICAL CONTENT:

MGSE8.EE.3 Use numbers expressed in scientific notation to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.*

MGSE8.EE.4 Add, subtract, multiply and divide numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Understand scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g. use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology (e.g. calculators).

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

The contexts in this FAL are designed to allow students to make sense of the mathematics in which they are engaged. Students should be allowed productive struggle throughout the tasks and asked formative assessment questions throughout and in closing.

In order for students to be successful, the following skills and concepts need to be maintained:

- **MGSE5.NBT.1**, **MGSE5.NBT.2**, **MGSE5.NBT.3**
- **MGSE7.NS.2**

COMMON MISCONCEPTIONS:

- *Students have the misconception of the decimal point moving to multiply and divide by powers of 10. This has no mathematical basis and should not be taught. Rather, students should use their understandings of place value and digit value to reason that the decimal point remains stationary and the values of the digits change by a factor of 10. Since the digits are changing by a factor of 10, the digits must shift.*
- *Students may also develop the misconception that the exponent tells the number of zeroes to add to the number. Address this explicitly in the classroom with questions addressing the number of zeroes for several different examples. Ask: "Is there a relationship between the exponent and the number of zeroes?" (No, it depends on how many non-zero digits are in the number.)*
- *The students may incorrectly state $2^0 = 0$. This task can be used to reinforce this idea which was introduced in an earlier task.*
- ***See FAL***

ESSENTIAL QUESTIONS:

- How can I represent very small and large numbers using integer exponents and scientific notation?
- How can I perform operations with numbers expressed in scientific notation?

MATERIALS:

- See FAL

GROUPING:

- Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Estimating Length Using Scientific Notation*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=414&subpage=concept
The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1221

E. Coli

Back to Task Table

Source: GPS 8th Grade Framework Task "It's a Big Universe (or is it small?)

In this task, students will apply properties of exponents to problems involving numbers expressed in scientific notation.

STANDARDS FOR MATHEMATICAL CONTENT:

MGSE8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{(-5)} = 3^{(-3)} = \frac{1}{3^3} = \frac{1}{27}$.

MGSE8.EE.3 Use numbers expressed in scientific notation to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.*

MGSE8.EE.4 Add, subtract, multiply and divide numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Understand scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g. use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology (e.g. calculators).

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

In order for students to be successful, the following skills and concepts need to be maintained:

- **MGSE5.NBT.1**, **MGSE5.NBT.2**, **MGSE5.NBT.3**
- **MGSE7.NS.2**

COMMON MISCONCEPTIONS:

- *Students may confuse the Product of Powers Property and Quotient of Like Bases Properties. Revisiting some previous tasks with these students may be helpful with students who have developed these misconceptions.*

- *Students will be unclear of when to divide and/or multiply the values.*

ESSENTIAL QUESTIONS:

- When are exponents used and why are they important?
- How do I simplify and evaluate numerical expressions involving integer exponents?
- What are some applications of using scientific notation?

MATERIALS:

- Copies of task for each student/pair of students/or small group
- Calculators (scientific or graphing TI-83/TI-84)

GROUPING:

- Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will apply properties of exponents to problems involving numbers expressed in scientific notation. In addition, students will perform operations with numbers written in scientific notation.

Using the context of the E. coli bacteria has a basis for real world application. It may be helpful to find some articles to share with students before engaging in this task.

DIFFERENTIATION:

Extension:

- CDC estimates that each year roughly 1 in 6 Americans get sick from foodborne diseases. Use the internet to roughly determine the population of the United States. Then, using the information provided, determine about how many people in America get sick from foodborne illnesses each year. Express your answer in standard form and in scientific notation.

Roughly 128,000 Americans are hospitalized due to foodborne illnesses each year. What fraction of Americans is hospitalized each year? Express as a fraction and in scientific notation.

Roughly 3,000 Americans die of foodborne diseases each year. What fraction of Americans dies from foodborne diseases each year? Express as a fraction and in scientific notation.

Intervention/Scaffolding:

- Do an example problem that involves finding the mass of a population. Possible example: The approximate mass of a water molecule is 3×10^{-26} kg (http://en.wikipedia.org/wiki/Molecular_mass). There are approximately 8.36×10^{24} molecules of water in a 250 mL glass (http://www.thenakedscientists.com/forum/index.php?topic=12443.0). What is the approximate mass of the water in the glass?

E. Coli

The Center for Disease Control (CDC) monitors public facilities to assure that there are no health risks to the people that visit these facilities. Occasionally, *Escherichia coli* will be discovered, which is also known as *E. coli*. This is a type of bacterium that can make people very sick. The mass of each *E. coli* bacterium is 2×10^{-12} gram.

The last time that *E. coli* was discovered in Georgia, the number of bacteria had been increasing for 24 hours! This caused each bacterium to be replaced by a population of 3.84×10^{8} bacteria.

a. What was the mass of the population of *E. coli* when it was discovered? Justify your answer.

Solution

The mass would be the product of the mass of each E. coli bacterium and the new population of the bacteria:

$$(2 \times 10^{-12})(3.84 \times 10^{8})$$

$$= (2 \times 3.84)\ (10^{-12} \times 10^{8})$$

$$= 7.68 \times 10^{-12+8}$$

$$= 7.68 \times 10^{-4}$$

The mass of the new population of E. coli was 0.000768 gram.

b. If the mass of a small paper clip is about 1 gram, how many times more is its mass than that of the discovered *E. coli* bacteria? Show how you know.

Solution

The quotient of the mass of the paper clip and the mass of the new population of the bacteria would be:

$(1 \text{ x } 10^{0}) \div (7.68 \text{ x } 10^{-4}) = (1 \div 7.68)\ (10^{0} \div 10^{-4})$ or $\frac{1 \times 10^{0}}{7.68 \times 10^{-4}} = \frac{1}{7.68} \times \frac{10^{0}}{10^{-4}}$

$\approx 0.13 \text{ x } 10^{0-(-4)}$ | $\approx 0.13 \times 10^{0-(-4)}$

$\approx 0.13 \text{ x } 10^{4}$ | $\approx 0.13 \times 10^{4}$

$\approx 1.3 \text{ x } 10^{5}$ | $\approx 1.3 \times 10^{5}$

The paper clip would have a mass about $1.3 \text{ x } 10^{5}$ larger than the mass of the new E. coli bacteria.

- Visit http://www.cdc.gov/ecoli for more information on E. coli.

E. Coli

The Center for Disease Control (CDC) monitors public facilities to assure that there are no health risks to the people that visit these facilities. Occasionally, *Escherichia coli* will be discovered, which is also known as *E. coli*. This is a type of bacterium that can make people very sick. The mass of each *E. coli* bacterium is 2×10^{-12} gram.

The last time that *E. coli* was discovered in Georgia, the number of bacteria had been increasing for 24 hours! This caused each bacterium to be replaced by a population of 3.84×10^{8} bacteria.

a. What was the mass of the population of *E. coli* when it was discovered? Justify your answer.

b. If the mass of a small paper clip is about 1 gram, how many times more is its mass than that of the discovered *E. coli* bacteria? Show how you know.

- Visit **http://www.cdc.gov/ecoli** for more information on E. coli.

Giantburgers (Short Cycle Task)

Back to Task Table

Source: Mathematics Assessment Project – Shell Center/MARS, University of Nottingham & UC Berkeley http://www.map.mathshell.org/materials/tasks.php?taskid=266&subpage=apprentice

In this task, the student will use properties of exponents and scientific notation.

STANDARDS FOR MATHEMATICAL CONTENT:

Work with radicals and integer exponents.

MGSE8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{(-5)} = 3^{(-3)} = \frac{1}{3^3} = \frac{1}{27}$.

MGSE8.EE.3 Use numbers expressed in scientific notation to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.*

MGSE8.EE.4 Add, subtract, multiply and divide numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Understand scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g. use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology (e.g. calculators).

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

In order for students to be successful, the following skills and concepts need to be maintained:

- **MGSE5.NBT.1**, **MGSE5.NBT.2**, **MGSE5.NBT.3**
- **MGSE7.NS.2**

COMMON MISCONCEPTIONS:

- *Students may confuse the Product of Powers Property and Quotient of Like Bases Properties.*
- *Students will be unclear of when to divide and/or multiply the values.*

Revisiting some previous tasks with these students may be helpful with students who have developed these misconceptions.

ESSENTIAL QUESTIONS:

- How can the properties of exponents and knowledge of working with scientific notation help me interpret information?

MATERIALS:

- Copies of task for each student

GROUPING:

- Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will use properties of exponents and scientific notation to evaluate a newspaper headline. This task can be very engaging and may be adapted to real fast food chains nearby. Through the mathematics exploration of this task, students evaluate whether a fast food chain's claims are reasonable. Working with your team literacy teacher may open up a cross-curricular project where students solve a real-world problem (Are the business's claims true, and if not, what should we do about it?) and then write about it. The writing project could include a letter to the business itself and explain the mathematics behind the claims and how accurate they are.

DIFFERENTIATION:

Extension:

- The percentage estimate you discovered should not be exactly 7%, but it should be close. Your job is to investigate what looks like a small percentage difference and determine if the difference is really significant. Using the percentage you came up with, determine about how many people eat at Giantburger each day. If 7% of Americans eat at Giantburger each day, how many people would that be? What is the difference in the two? Is Giantburger overestimating based on your answer or underestimating? How many people would they be overestimating/underestimating by in a week? In a month?

In a year? If you originally stated that their headline using 7% is true, do you still feel this way? Why or why not? Can a small difference in a percentage make a big difference in actual values? Explain. If you originally stated that their headline using 7% was not true, do you still feel that way? Do you now have additional data to back up your claims? Explain.

Intervention/Scaffolding:

- Be sure to prompt struggling students by asking guiding questions. Group the students in a way that promotes cooperative learning. It may also be helpful to meet with struggling students in a small group setting to provide the scaffolding needed.

<u>Giantburgers</u>

This headline appeared in a newspaper.

Every day 7% of Americans eat at Giantburger restaurants

Decide whether this headline is true using the following information.

- There are about 8×10^3 Giantburger restaurants in America.
- Each restaurant serves about 2.5×10^3 people every day.
- There are about 3×10^8 Americans.

Explain your reasons and show clearly how you figured it out.

Rubric

	Giantburgers	Rubric	
		Points	Section points
	Attempts to calculate the number of people who eat at Giantburger restaurants $8 \times 10^3 \times 25 \times 10^2 = 200 \times 10^5$ or equivalent *Partial credit* For partially correct solutions	2 3 (2) (1)	5
	Attempts to find 7% of 3×10^8 **= 21×10^6** Attempts to calculate 2×10^7 as a percentage of 3×10^8. **= 6.7%**	2 2 or 2 2	4
	States that the statement is true since: **6.7% is approximately equal to 7%** Accept alternative correct solutions	1	1
	Total Points		**10**

Student Work Sample – scored

Every day 7% of Americans eat at Giantburger restaurants

Decide whether this headline is true using the following information.

- There are about 8×10^3 Giantburger restaurants in America.
- Each restaurant serves about 2.5×10^3 people every day.
- There are about 3×10^8 Americans.

Explain your reasons and show clearly how you figured it out.

To figure out how many people Giantburger 2
serves per day, you would multiply 3
(8x(10)^3) by (2.5x(10)^3), totaling 20,000,000
Then you divide that number by how many
Americans there are, (3x(10)^8) or 300,000,000. 2
So, 20000,000 ⟌300,000,000 = .066. If you rounded 2
.066 to .07 or 7%, the statement is correct. 1

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10

- Additional student work samples available at http://www.map.mathshell.org/materials/tasks.php?taskid=266&subpage=apprentice

Giantburgers

This headline appeared in a newspaper.

Decide whether this headline is true using the following information.

- There are about 8×10^3 Giantburger restaurants in America.
- Each restaurant serves about 2.5×10^3 people every day.
- There are about 3×10^8 Americans.

Explain your reasons and show clearly how you figured it out.

<u>100 People</u> (Short Cycle Task)

Back to Task Table

Source: Balanced Assessment Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=1046

In this task, students will use scientific notation in a real-world application of using population.

STANDARDS FOR MATHEMATICAL CONTENT:

Work with radicals and integer exponents.

MGSE8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{(-5)} = 3^{(-3)} = \frac{1}{3^3} = \frac{1}{27}$.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

MGSE8.EE.3 Use numbers expressed in scientific notation to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.*

MGSE8.EE.4 Add, subtract, multiply and divide numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Understand scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g. use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology (e.g. calculators).

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

In order for students to be successful, the following skills and concepts need to be maintained:

- **MGSE5.NBT.1**, **MGSE5.NBT.2**, **MGSE5.NBT.3**
- **MGSE7.NS.2**

COMMON MISCONCEPTIONS:

- *Students may confuse the Product of Powers Property and Quotient of Like Bases Properties.*
- *Students will be unclear of when to divide and/or multiply the values.*
- *The number of zeros equals the exponent.*

Revisiting some previous tasks with these students may be helpful with students who have developed these misconceptions.

ESSENTIAL QUESTIONS:

- How do I perform operations with numbers expressed in scientific notation?
- What are some applications of using scientific notation?
- How can the properties of exponents and knowledge of working with scientific notation help me interpret information?

MATERIALS:

- See task

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

An interesting and engaging way to introduce this task might be to use the children's picture book: If the World Were a Village, by David J Smith. This book explores the lives of the 100 villagers, representing the whole world population (currently about 7 billion).

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *100 People*, is a Mathematics Assessment Project Assessment Task that can be found at the website:
http://www.map.mathshell.org/materials/tasks.php?taskid=359&subpage=apprentice

The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=1046

The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=1047

Geology Rocks Equations

Back to Task Table

Task Adapted from: *http://illuminations.nctm.org/LessonDetail.aspx?id=L786*

In this lesson, students explore linear equations with manipulatives and discover various steps used in solving equation problems. Students use blocks and counters as tactile representations to help them solve for unknown values of *x*. Students should work in groups or pairs. This will encourage discussion during the lesson, which will help with understanding the manipulative representation. Try to match students who are likely to understand the manipulatives with students who may have trouble initially.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE8.EE.7 Solve linear equations in one variable.

MGSE8.EE.7b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.

COMMON MISCONCEPTIONS

- *Students will just want to take pieces off the scale instead of attempting to keep it balanced.*
- *Many students have not ever seen a balance scale and therefore will need an explanation of what they look like, how they work, and how they are used in everyday life (science) or historically (in the past).*

ESSENTIAL QUESTIONS

- What strategies can be used to represent real situations using algebraic expressions and equations?

MATERIALS

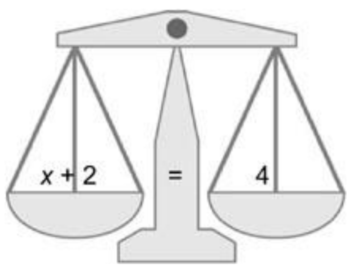
- Manipulative blocks (or similar manipulatives)
- Counter chips (or similar manipulatives)
- Geology Rocks Equations Activity Sheet

GROUPING

Individual / Partner

TASK DESCRIPTION

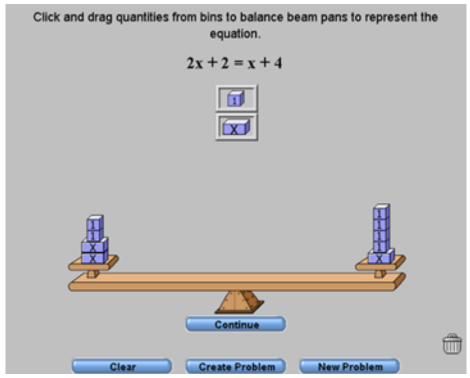
The teacher should present a scale with objects on both sides. Use a real scale and objects if possible. If not, use clip art or another picture of a scale as a visual, as shown here:

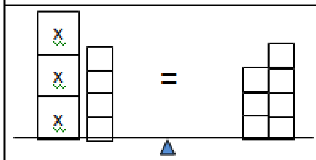
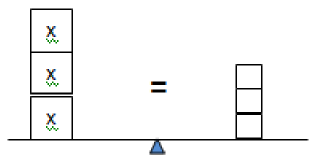
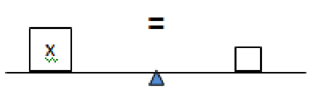


Students should then be allowed to build and balance equations using technology or hands-on manipulatives. The teacher should use discretion as to which type of modeling will be best for their own students for the introduction portion of this task.

Interactive balance scales:
1. Hands-On Equations for IPADS http://www.edudemic.com/2012/12/a-free-interactive-ipad-app-to-teach-algebra/ (most schools have Hands-On Equations kits, Algebra Lab Gear, or some other hands-on manipulatives that would work for modeling equations)

2. Algebra Balance Scales: This virtual manipulative allows you to solve simple linear equations through the use of a balance.



Picture	Action Taken	Equation
	Original equation.	$3x+4=7$
	Subtract 4 units from both sides.	$3x+4=7$ $-4\ -4$ $3x=3$
	Divide both sides by 3.	$\frac{3x}{3}=\frac{3}{3}$ $x=1$

Above is a sample written assessment to use while students are interacting with Algebra Balance Scales. On a chart draw a picture of the balance beam for every step, write a description for every step, and write down the algebraic representation and show work algebraically for every step.

Adapted from :
http://nlvm.usu.edu/en/nav/frames_asid_324_g_3_t_2.html?open=instructions&from=category_g_3_t_2.html

Teacher Notes: Geology Rocks Task

1. Begin the lesson by distributing the Geology Rocks Equations activity sheet and the manipulatives. Have students read the opening paragraph and Question 1 on their own. Use the first question as a demonstration of what they will be doing with the questions that follow.
2. Tell students that the blocks represent the crates, and each chip counter represents a 1-pound rock. Ask students to create a representation of the problem on their desks using the manipulatives. Then, ask them to discuss in their groups ways to figure out how many rocks are in the crates using only this information.

3. You may want to allow students time to explore and share their thoughts within their groups about how they are using the manipulatives to help them solve the problem. After you have given students time to explore, ask the groups to share their answers with the class.
4. The correct answer to Question 1 is that there are 2 rocks in each crate. Have students who found the correct solution share their strategy with the class.

5. Some students may have used a guess-and-check strategy until they found out the weight of the crate, while others may have rewritten the equation. As an introduction to the problem, this is fine. As students begin to think-pair-share their strategies to balance the equation, encourage those who are using guess-and-check to collaborate with those using algebra. Guess and check is a valid strategy when beginning with these equations; it requires solving the problem mathematically using a step-by-step process. Students are performing the operations multiple times and finding solutions that do not work, but at the same time they are gaining practice until they find the solution that does work.
6. Allow students to work on their own to complete Questions 2 to 5. As you move through the class, ask students to explain to you how the equation they are building using the manipulatives represents the problem on the paper. Encourage students to use multiple approaches to solve each problem—this will help them build the connections between representations.
7. After students have finished Questions 2 to 5, have them share within their group how many rocks they think are in each crate. For each solution, ask the class if they agree. If all groups agree, move on to the next problem. If someone disagrees have the class re-create the problem using the manipulatives and walk through the solution steps together to agree on the solution. Use student volunteers where possible as you go through the answers.

   Activity Sheet Answers

   2. $x = 4$

   3. $x = 12$

   4. $x = 4$

   5. $x = 7$

8. Once the class has agreed on all the solutions, inform students that the solutions they created are actually linear equations and can be represented mathematically with equations, similar to the equations you showed earlier using boxes.
9. Ask students to look over each problem and write an equation they think mathematically represents the initial picture Mr. Anderson made. Some students may not immediately

make the connection that the boxes stand for unknown values of x and the rocks represent constants. You should stress this connection as students are working. To help them with the connection, you can have students use x's or boxes or both. Some students may have an easier time making the connection using the $\square$ instead of x.

Equations

2. $4x + 2 = 18$ or $4\square + 2 = 18$

3. $2x + 12 = 3x$ or $2\square + 12 = 3\square$

4. $2x + 8 = x + 12$ or $2\square + 8 = \square + 12$

5. $5x + 3 = 2x + 24$ or $5\square + 3 = 2\square + 24$

10. As soon as students have created the equations, discuss them as a class so they are agreed upon. When students have finished writing their equations, allow them to complete Questions 6 to 9 on the activity sheet. Allow students to solve these equations with or without manipulatives, depending on their own preference.

Answers (continued)

6. $x = 3$

7. $x = 6$

8. $x = 4$

9. $x = 3$

11. To summarize the lesson, have students share with the class any strategies they came up with that make it quicker to solve the problems without creating a manipulative representation each time. Students may have realized that you are subtracting or adding a constant, and then dividing by the coefficient of x. Discuss with students the steps involved in solving these equations. You are isolating the numbers on one side of the equation and the unknown on the other. You are grouping like terms and solving. Use an example not on the activity sheet, such as the one below in your discussion:

$$\begin{array}{rcl}
3x + 8 & = & 1x + 12 \\
-8 & & -8 \\
\hline
3x & = & 1x + 4 \\
-1x & & -1x \\
\hline
\dfrac{2x}{2} & = & \dfrac{4}{2} \\
\hline
x & = & 2
\end{array}$$

12. Ask students to write down on their activity sheet a list of steps to solve the problems and compare their lists with one another. Most solutions will require at least 2 steps to solve.

As students begin to more fully grasp the process of 2-step equations, ask them to solve the equation $5x - 8 = 1x + 2$. In solving this equation, students will be faced with using negative integers and accepting a solution that is not a whole number ($x = 2.5$). Students may find it tough to create manipulative representations of these harder equations. Allow them to explore and work it out on their own. This is truly an exploration for students.

DIFFERENTIATION

Extension

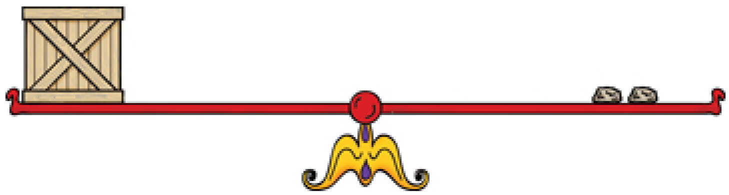
- The teacher could allow students to create their own representation of the situation with the chips, which will help ensure that each student understands the problem. After 30 seconds or so, they can pair-share to correct any problems with their representations and discuss their solutions.
- Present an altered form of Mr. Anderson's lab problems, in which he uses balloons that have exactly one pound of lift (–1). If a balloon is tied to a rock, together they weigh 0 pounds because they balance each other out. Each crate still represents *x*, the unknown number of rocks inside. Create similar pictures and equations for students and have them find the number of rocks in each crate.
- Extend this activity to include equations involving negative and non-integral unknowns. For example, $4x - 4 = x + 3$ or $-4x + 12 = x - 3$.
- Present this problem to students: A 40-pound rock is dropped on the ground, and it breaks into 4 pieces, each piece weighing an integral number of pounds. Using a balance scale, these 4 pieces can be used to measure any integer weight from 1 to 40 pounds. How much does each piece of the rock weigh?

Intervention

- If students do not find the answer to question 1, lead the class through the process:

Mr. Anderson realizes that he can remove 2 rocks from each side of the scale and still have a balanced scale. This leaves him with the following:

Then, Mr. Anderson realizes that he can remove a crate from each side of the scale and still remain balanced. This leaves him with the following:



Students can represent these situations algebraically. Use boxes instead of *x*'s:

$\square + \square + 2 = \square + 4$

This could then be rewritten to:

$2\square + 2 = \square + 4$

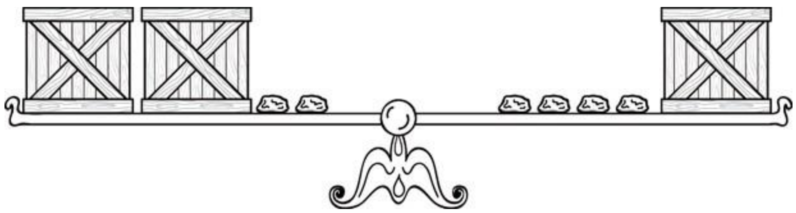
- Students can remove boxes and numbers just as they did on the scale. Teachers often give students manipulatives and expect them to complete the analogy on their own. However, quite often, students are not able to make the connection. The teaching is much more effective if you make the analogy explicit for students. That is, explain explicitly how the boxes and rocks represent variables and numbers.

Geology Rocks Equations

Name ________________________

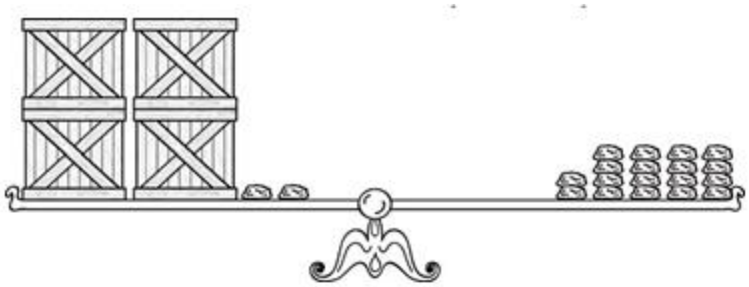
Mr. Anderson is a geologist and has a laboratory full of rocks. He knows that each rock weighs exactly one pound (+1), and he would like to figure out how many rocks are in each crate. To figure that out without opening the crates, Mr. Anderson places crates and rocks on a scale until they are balanced. Using his math skills, he is able to reason how many rocks are in each crate without having to look inside.

1. The following picture represents the first set of crates and rocks Mr. Anderson put on the balance. How many rocks are inside each crate?

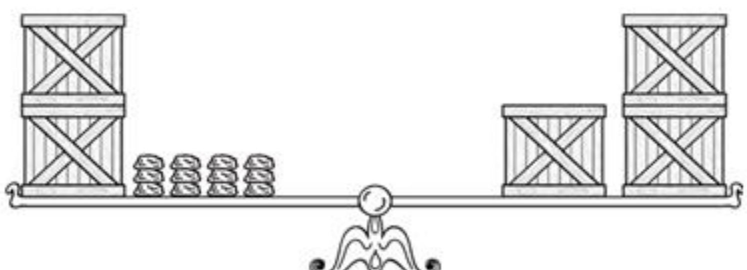


Mr. Anderson has made several picture representations on his clipboard of other combinations of crates and rocks that balanced. Can you figure out how many rocks are in each set of crates?

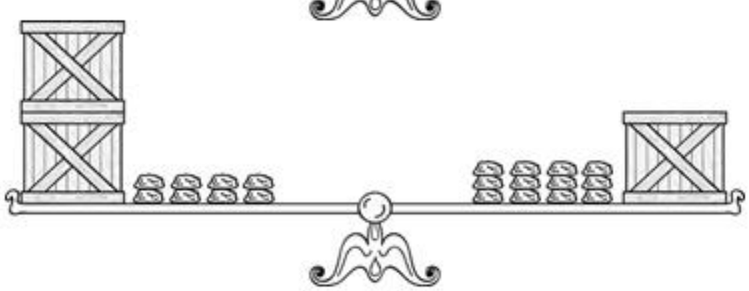
2.

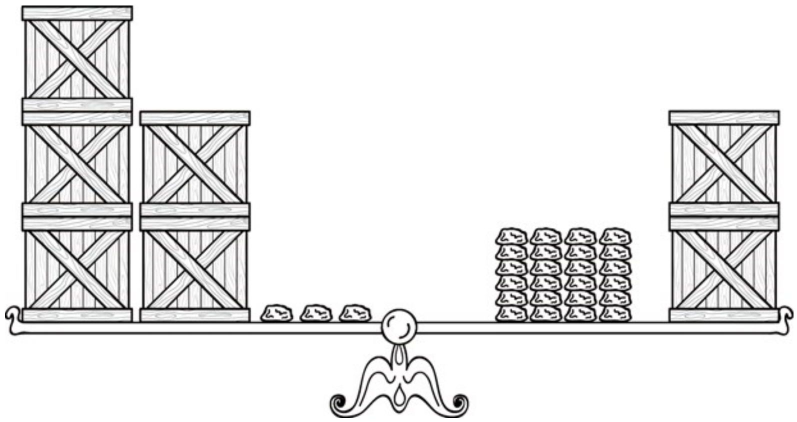


3.



4.



5. 

Mr. Anderson wrote down the following equations but did not draw any pictures. Can you find the value of x in each? (Hint: Think of each x as a crate of rocks.)

6. $7x = 6 + 5x$

7. $30 = 4x + 6$

8. $2(x + 4) = 16$

9. $7 + 5x = 3x + 13$

Solving Linear Equations In One Variable - (FAL)

Back to Tasks Table

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1286

This lesson unit is intended to help you assess how well students are able to:

- Solve linear equations in one variable with rational number coefficients.
- Collect like terms.
- Expand expressions using the distributive property.
- Categorize linear equations in one variable as having one, none, or infinitely many solutions.

STANDARDS FOR MATHEMATICAL CONTENT:

Understand the connections between proportional relationships, lines, and linear equations.

MGSE8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

MGSE8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Define, evaluate, and compare functions.

MGSE8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

In order for students to be successful, the following skills and concepts need to be maintained:

MGSE7.EE.4

COMMON MISCONCEPTIONS:

- **See FAL**

ESSENTIAL QUESTIONS:

- What strategies can I use to create and solve linear equations with one solution, infinitely many solutions, or no solutions?

MATERIALS:

- **See FAL**

GROUPING:

- Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

From the FAL document:

This lesson unit is structured in the following way:

• Before the lesson, students work individually on an assessment task that is designed to reveal their current understanding and difficulties. You then review their responses and create questions for students to consider when improving their work.s

• After a whole-class introduction, students work in small groups on a collaborative discussion task, categorizing equations based on the number of solutions. Throughout their work, students justify and explain their thinking and reasoning.

• In the same small groups, students critique the work of others and then discuss as a whole class what they have learned.

• Finally, students return to their original task and try to improve their own, individual responses.

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the

lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Solving Linear Equations in One Variable*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=442&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:

http://map.mathshell.org/materials/download.php?fileid=1286

Writing For A Math Website

Back to Tasks Table

In this task, students will write equations with various solution types and provide step-by-step solutions with justifications.

STANDARDS FOR MATHEMATICAL CONTENT:

Work with radicals and integer exponents.

MGSE8.EE.7 Solve linear equations in one variable.

MGSE8.EE.7a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

MGSE8.EE.7b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tolls strategically.
6. Attend to precision.

BACKGROUND KNOWLEDGE:

In order for students to be successful, the following skills and concepts need to be maintained:

- **MGSE7.EE.3**, **MGSE7.EE.4**

COMMON MISCONCEPTIONS:

- *Students will continue have trouble using inverse operations to solve equations.*

ESSENTIAL QUESTIONS:

- What strategies can I use to create and solve linear equations with one solution, infinitely many solutions, or no solutions?

MATERIALS:

- Copy of the task for each student

GROUPING:

- Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Students become designers of mathematics problems that meet the specs of the standards. They design 6 equations and provide step-by-step solutions with justifications as part of a community service project for the school. Students should follow the directions in the task, working with a partner or small group.

Teachers should end the lesson by using discussion questions like those that follow in order to help students make the mathematics more explicit.

Teacher Questions for Facilitating Student Discussion

- *Which categories of problems were the easiest to create?*
- *What strategy did you use to create the problems with infinitely many solutions?*
- *What challenges did your group encounter while completing this task?*
- *Did you notice any patterns? If so, describe the pattern(s).*
- *Would you like to be a writer for a website or textbook company to write math problems? Why or why not?*

DIFFERENTIATION:

Extension:

- Create three word problems that can be solved by writing linear equations. One problem must have one solution, one problem must have no solution, and one problem must have infinitely many solutions. Each problem must also meet one of the specs of the standard as listed below. You may not use a spec more than once.

- linear equation in one variable
- include rational number coefficients
- require use of the distributive property
- variables on both sides of the equation
- require collecting like terms

Write your word problems then solve them showing your solutions step-by-step.

Intervention/Scaffolding:

- Have students try working backwards. Create an example (meeting the specs of the standard) with the class, complete with justification for each step.

Writing For A Math Website

As a community service project your school has partnered with a national web based company to help develop a website that will provide math support to 8th graders with the Common Core Georgia Performance Standards. Your middle school has been assigned the standards below.

MGSE8.EE.7 Solve linear equations in one variable.

MGSE8.EE.7a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, *or* $a = b$ results (where a and b are different numbers).

MGSE8.EE.7b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Your help has been requested writing the problems and providing the solutions. The new program will be interactive. It must tell the students if the answer is correct or incorrect and show the problems completely worked out along with justification for each step.

1. Create 6 equations that can be sorted into the categories in the table below (2 per category) and provide solutions to the problem. The solutions must show all steps along with written justifications for each step. Each problem created must meet the specs of the standard as listed below:
   - linear equation in one variable
   - include rational number coefficients
   - require use of the distributive property
   - variables on both sides of the equation
   - require collecting like terms

One Solution	No Solution	Infinitely Many Solutions
Equations have one solution when the variables do not cancel out.	***Equations having no solution have variables that will cancel out and constants that are not equal. This means that there is not a value that can be substituted for x that will make the sides equal.***	***An equation with infinitely many solutions occurs when both sides of the equation are the same. Any value of x will produce a valid equation.***
$\frac{1}{4}(8y+4)-17=-4(\frac{1}{2}y-8)$ $y = 12$	$-(\frac{3}{4}x+7-\frac{1}{4}x)=19-\frac{1}{2}x$ $-7 \neq 19$	$-6(\frac{1}{2}a+3)+6=\frac{3}{4}(-16-4a)$ $-3a-12=-12-3a$

2. Provide a written explanation to help other 8th graders understand the meaning of having one solution, no solution, or infinitely many solutions.

Comment

Accept a variety of answers.

Teacher Questions for Discussion

- ***Which categories of problems were the easiest to create?***
- ***What strategy did you use to create the problems with infinitely many solutions?***
- ***What challenges did your group encounter while completing this task?***
- ***Did you notice any patterns? If so, describe the pattern(s).***
- ***Would you like to be a writer for a website or textbook company to write math problems? Why or why not?***

Writing For A Math Website

As a community service project your school has partnered with a national web based company to help develop a website that will provide math support to 8th graders with the Common Core Georgia Performance Standards. Your middle school has been assigned the standards below.

MGSE8.EE.7 Solve linear equations in one variable.

MGSE8.EE.7a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, *or* $a = b$ results (where a and b are different numbers).

MGSE8.EE.7b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Your help has been requested writing the problems and providing the solutions. The new program will be interactive. It must tell the students if the answer is correct or incorrect and show the problems completely worked out along with justification for each step.

1. Create 6 equations that can be sorted into the categories in the table below (2 per category) and provide solutions to the problem. The solutions must show all steps along with written justifications for each step. Each problem created must meet the specs of the standard as listed below:

    - linear equation in one variable
    - include rational number coefficients
    - require use of the distributive property
    - variables on both sides of the equation
    - require collecting like terms

ONE SOLUTION	NO SOLUTION	INFINITELY MANY SOLUTIONS

2. Provide a written explanation to help other 8^{th} graders understand the meaning of having one solution, no solution, or infinitely many solutions.

Culminating Task: Integer Exponents & Scientific Notation

Back to Tasks Table

Source: New York State Common Core
http://www.engageny.org/resource/grade-8-mathematics-module-1

STANDARDS FOR MATHEMATICAL CONTENT:

Work with radicals and integer exponents.

MGSE8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{(-5)} = 3^{(-3)} = \frac{1}{3^3} = \frac{1}{27}$.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

MGSE8.EE.3 Use numbers expressed in scientific notation to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.*

MGSE8.EE.4 Add, subtract, multiply and divide numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Understand scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g. use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology (e.g. calculators).

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

This culminating task is from the Engage NY website. This task is adaptable and grade level teams should take note of what the task involves and how students should engage in it. Teachers also have the flexibility to determine how the end product should look. The product does not have to look exactly as it does on the website. Allowing students some creativity in this task may prove beneficial.

COMMON MISCONCEPTIONS:

- *See prior Unit 2 Tasks*

ESSENTIAL QUESTIONS:

- How can I apply the properties of integer exponents to generate equivalent numerical expressions?
- How can I represent very small and large numbers using integer exponents and scientific notation?
- How can I perform operations with numbers expressed in scientific notation?
- How can I interpret scientific notation that has been generated by technology?
- Why is it useful for me to know the square root of a number?
- How do I simplify and evaluate numeric expressions involving integer exponents?
- How can the properties of exponents and knowledge of working with scientific notation help me interpret information?

MATERIALS:

- Copies of task for each student/pair of students/or small group
    - These can be found embedded in Module 1 at engageny.org
        - Use Grade 8 Mathematics Mid-Module and End-of-Module Assessments

GROUPING:

- Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

- Mid-Module Assessment and Rubric and End-of-Module Assessment and Rubric embedded in Module 1 at engageny.org

- NOTE: The document found at http://www.engageny.org/resource/grade-8-mathematics-module-1 is not meant to be utilized in its entirety. View the assessment modules found in the middle and end of the document. Pick and choose what you feel would best fit your students.

DIFFERENTIATION:

Extension:

- Create three word problems that can be solved by writing linear equations. One problem must have one solution, one problem must have no solution, and one problem must have infinitely many solutions. Each problem must also meet one of the specs of the standard as listed below. You may not use a spec more than once.

- linear equation in one variable
- include rational number coefficients
- require use of the distributive property
- variables on both sides of the equation
- require collecting like terms

Write your word problems then solve them showing your solutions step-by-step.

Intervention/Scaffolding:

- Have students try working backwards. Create an example (meeting the specs of the standard) with the class, complete with justification for each step.