# Georgia Standards of Excellence Curriculum Frameworks

## Mathematics

GSE Grade 8
Unit 4: Functions

Georgia Department of Education
Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

# Unit 4
# Functions

## TABLE OF CONTENTS

## OVERVIEW

In this unit students will:

- recognize a relationship as a function when each input is assigned to exactly one unique output;
- reason from a context, a graph, or a table, after first being clear which quantity is considered the input and which is the output;
- produce a counterexample: an "input value" with at least two "output values" when a relationship is not a function;
- explain how to verify that for each input there is exactly one output; and
- translate functions numerically, graphically, verbally, and algebraically.

The "vertical line test" should be avoided because (1) it is too easy to apply without thinking, (2) students do not need an efficient strategy at this point, and (3) it creates misconceptions for later mathematics, when it is useful to think of functions more broadly, such as whether $x$ might be a function of $y$.

"Function machine" representations are useful for helping students imagine input and output values, with a rule inside the machine by which the output value is determined from the input.

Notice that the standards explicitly call for exploring functions numerically, graphically, verbally, and algebraically (symbolically, with letters). This is sometimes called the "rule of four." For fluency and flexibility in thinking, students need experiences translating among these.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under "Evidence of Learning" be reviewed early in the planning process. A variety of resources should be utilized to supplement this unit. This unit provides much needed content information, but excellent learning activities as well. The tasks in this unit illustrate the types of learning activities that should be utilized from a variety of sources.

## STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

## STANDARDS FOR MATHEMATICAL PRACTICE

Refer to the Comprehensive Course Overview for more detailed information about the Standards for Mathematical Practice.

**1. Make sense of problems and persevere in solving them.** Students solve real world problems through the application of algebraic and geometric concepts. They seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, ―What is the most efficient way to solve the problem?, ―Does this make sense?, and ―Can I solve the problem in a different way?

**2. Reason abstractly and quantitatively.** Students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

**3. Construct viable arguments and critique the reasoning of others.** Students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays. They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like ―How did you get that?, ―Why is that true?‖ ―Does that always work?‖ They explain their thinking to others and respond to others' thinking.

**4. Model with mathematics.** Students model problem situations symbolically, graphically, tabularly, and contextually. They form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students solve systems of linear equations and compare properties of functions provided in different forms. Students use scatterplots to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.

**5. Use appropriate tools strategically.** Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 8 may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal.

**6. Attend to precision.** Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays.

**7. Look for and make use of structure.** Students routinely seek patterns or structures to model and solve problems. In grade 8, students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.

**8. Look for and express regularity in repeated reasoning.** Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students use iterative processes to determine more precise rational approximations for irrational numbers. During multiple opportunities to solve and model problems, they notice that the slope of a line and rate of change are the same value. Students flexibly make connections between covariance, rates, and representations showing the relationships between quantities.

## STANDARDS FOR MATHEMATICAL CONTENT

### Define, evaluate, and compare functions.

**MGSE8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

**MGSE8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, give a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

## BIG IDEAS

- A function is a specific type of relationship in which each input has a unique output.
- A function can be represented in an input-output table, graphically (using ordered pairs that consist of the input and the output of the function in the form (input, output), and with an algebraic rule

## ESSENTIAL QUESTIONS

- What is a function?
- What are the characteristics of a function?
- How do you determine if relations are functions?
- How is a function different from a relation?
- Why is it important to know which variable is the independent variable?
- How can a function be recognized in any form?
- What is the best way to represent a function?

- How do you represent relations and functions using tables, graphs, words, and algebraic equations?
- What strategies can I use to identify patterns?
- How does looking at patterns relate to functions?
- How are sets of numbers related to each other?
- How can you use functions to model real-world situations?
- How can graphs and equations of functions help us to interpret real-world problems?

## CONCEPTS/SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- computation with whole numbers and decimals, including application of order of operations
- plotting points in a four quadrant coordinate plan
- understanding of independent and dependent variables
- characteristics of a proportional relationship
- writing algebraic equations

## FLUENCY

**It is expected that students will continue to develop and practice strategies to build their capacity to become fluent in mathematics and mathematics computation. The eventual goal is automaticity with math facts. This automaticity is built within each student through strategy development and practice. The following section is presented in order to develop a common understanding of the ideas and terminology regarding fluency and automaticity in mathematics:**

**Fluency:** Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number**.**

**Deep Understanding:** Teachers teach more than simply “how to get the answer” and instead support students’ ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

**Memorization:** The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

**Number Sense:** Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

**Fluent students:**

- **flexibly use a combination of deep understanding, number sense, and memorization.**
- **are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.**
- **are able to articulate their reasoning.**
- **find solutions through a number of different paths.**

For more about fluency, see: **http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf** and: **http://joboaler.com/timed-tests-and-the-development-of-math-anxiety/**

## SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

**The definitions below are for teacher reference only and are not to be memorized by the students.** Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for middle school students. **Note – Different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.** The definitions below are from the Common Core State Standards Mathematics Glossary and/or the Common Core GPS Mathematics Glossary when available.

Visit http://intermath.coe.uga.edu or http://mathworld.wolfram.com to see additional definitions and specific examples of many terms and symbols used in grade 8 mathematics.

- **Domain:**
- **Function:**
- **Graph of a Function:**
- **Range of a Function:**

## FORMATIVE ASSESSMENT LESSONS (FAL)

**Formative Assessment Lessons** are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Guide.

## SPOTLIGHT TASKS

A Spotlight Task has been added to each CCGPS mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Common Core Georgia Performance Standards, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky.

# TASKS

| Task Name | Task Type *Grouping Strategy* | Content Addressed | Standard(s) |
|---|---|---|---|
| **Secret Codes and Number Rules** | Performance Task *Individual/Partner* | Unique relationships of input and output. | MGSE8.F.1 MGSE8.F.2 |
| **Foxes and Rabbits (Spotlight Task)** | Learning Task *Individual/Partner* | Define, evaluate, compare functions. | MGSE8.F1 |
| **The Vending Machine** | Performance Task *Individual/Partner* | Differentiating between functions and relations. | MGSE8.F.1 MGSE8.F.2 |
| **Order Matters** | Performance Task *Partner* | Relation of independent variable and functions. | MGSE8.F.1 MGSE8.F.2 |
| **Battery Charging (Spotlight Task)** | Learning Task *Individual/Partner* | Compare properties of two functions, each expressed in different ways | MGSE9.F2 |
| **Which is Which?** | Performance Task *Individual/Partner* | Using mathematics vocabulary to explain how real-life experiences | MGSE8.F.1 MGSE8.F.2 |
| **Modeling Situations with Linear Equations** | Formative Assessment Lesson *Partner/Small* | Explore relationships between variables in everyday situations. | MGSE8.F.1 MGSE8.F.2 |
| **Create Matching Function Cards** | Formative Assessment Lesson *Partner/Small* | Create equivalent representations of functions | MGSE8.F.1 MGSE8.F.2 |
| **Culminating Task: Sorting Functions** | Performance Task *Individual/Partner/ Small Group* | Recognizing functions in various forms. | MGSE8.F.1 MGSE8.F.2 |
| **Appendix** | | | |

## ***COMMON MISCONCEPTIONS***

*Some students will mistakenly think of a straight line as horizontal or vertical only. Some students will mix up the x- and y- axes on the coordinate plane. Emphasizing that the first value is plotted on the horizontal axes (usually x, with positive to the right) and the second is the vertical axis (usually called y, with positive up) point out that this is merely a convention. It could have been otherwise, but it is very useful for people to agree on a standard customary practice.*

## Secret Codes and Number Rules

***Back to Task Table***

In this task, students will define, evaluate, and compare the unique relationships of input and output.

## STANDARDS FOR MATHEMATICAL CONTENT:

**Define, evaluate, and compare functions.**

**MGSE8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

**MGSE8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

## STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## BACKGROUND KNOWLEDGE:

Students are surrounded by patterns in the world around them. Human beings, by nature, are pattern seeking. In this task, students are asked to do what they do almost through instinct. The difference here is that we are asking students to not only look for the pattern, but also explain in using the language of mathematics and the formats of data representations for functions.

Students should regularly be asked to work with growth patterns such as those in Teaching Student Centered Mathematics, by Van de Walle, Lovin, Bay-Williams & Karp. Another resource for growth patterns can be found on the following website: **http://www.visualpatterns.org/** (there are enough growth patterns on this site for students to investigate one for each day of the school year).

## ***COMMON MISCONCEPTIONS:***

*Students may confuse the input and output values/variables. This could result in the inverse of the function. This misconception can be a tremendous teaching tool. If you observe students showing evidence of this misconception, take an example (anonymously) and use it as a teachable moment. Ask students to discuss the work, discover the error, and compare it to the actual function. Students benefit from this type of exploration, not just to correct a misconception, but also gain ownership of the knowledge of inverse functions in the process.*

## ESSENTIAL QUESTIONS:

- What strategies can I use to identify patterns?
- What is a function?
- How does looking at patterns relate to functions?
- How are sets of numbers related to each other?
- How can you use functions to model real-world situations?
- How can graphs and equations of functions help us to interpret real-world problems?

## MATERIALS:

- Straightedge
- Graph paper http://incompetech.com/graphpaper/
- Copies of task for students

## GROUPING:

- Individual/Partner

## TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will complete a function table by applying a rule, determine a rule based on the relationship between the input and output within a function table and/or a graph, and will create and use function tables in order to solve real world problems. The terms input and output should be familiar to students, but if they are not, they should be easily understood in the context of the task. Input-output tables are sometimes used as a method to represent patterns.

***Comments:* Below are some additional resources for teachers.**
http://www2.edc.org/mathpartners/pdfs/6-8%20Patterns%20and%20Functions.pdf
http://mathandteaching.org/uploads/SampleLesson_PDFs/Lesson_12.1.pdf

## DIFFERENTIATION:

### Extension

- Partners: Each person should work individually to create your own cipher in which letters are replaced with other letters, not numbers. Determine a scheme for replacement in which letters are shifted. You must create an original cipher. Write a mathematically relevant message in code that you can trade with your partner and have them decode.

  Decoding: Decode the message your partner created. Determine the rule for the code your partner created.

### Intervention/Scaffolding

- Provide a table for students to write the code for the second graph (to be used for Part 1, problems 3 & 4). Students may need to review reading input/output tables and plotting points on a coordinate grid before doing Part 2. As an example, discuss the first input/output table in Part 2, and determine the rule as a class.

## Secret Codes and Number Rules

**Part 1**

Encryption is used by spies, secret societies, and other organizations to transfer information without other people reading their messages.

Secret codes can be created by a simple scheme of replacing letters with numbers. More complex codes replace letters with other letters.

The code scheme is called a cipher and can be written as a list or table.

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 |

Use the cipher above to decode these messages.

***Question:***

23 8 1 20   4 9 4   20 8 5   26 5 18 15   19 1 25   20 15   20 8 5   5 9 7 8 20?

***Solution:***

***What did the zero say to the eight?***

***Answer:***

14 9 3 5   2 5 12 20

***Solution***

***Nice belt***

***Question:***

23 8 5 14   4 15 5 19   8 1 12 6   15 6   5 9 7 8 20   5 17 21 1 12   20 8 18 5 5 ?

***Solution***

***When does half of eight equal three?***

***Answer:***

23 8 5 14 25 15 21 3 21 20 9 20 22 5 18 20 9 3 1 12 12 25.

***Solution***

***When you cut it vertically!***

Your turn! Write, find or share a joke or riddle! Write it in code and share with your classmates.

Ciphers can be written in which letters are replaced with other letters, not numbers. A scheme is determined for replacement. Often letters are shifted. For example, a letter could be replaced with the letter three down the alphabet.

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C |

Therefore ***TWINKLE TWINKLE LITTLE STAR*** becomes

***WZLQOH WZLQOH OLWWOH VWDU***

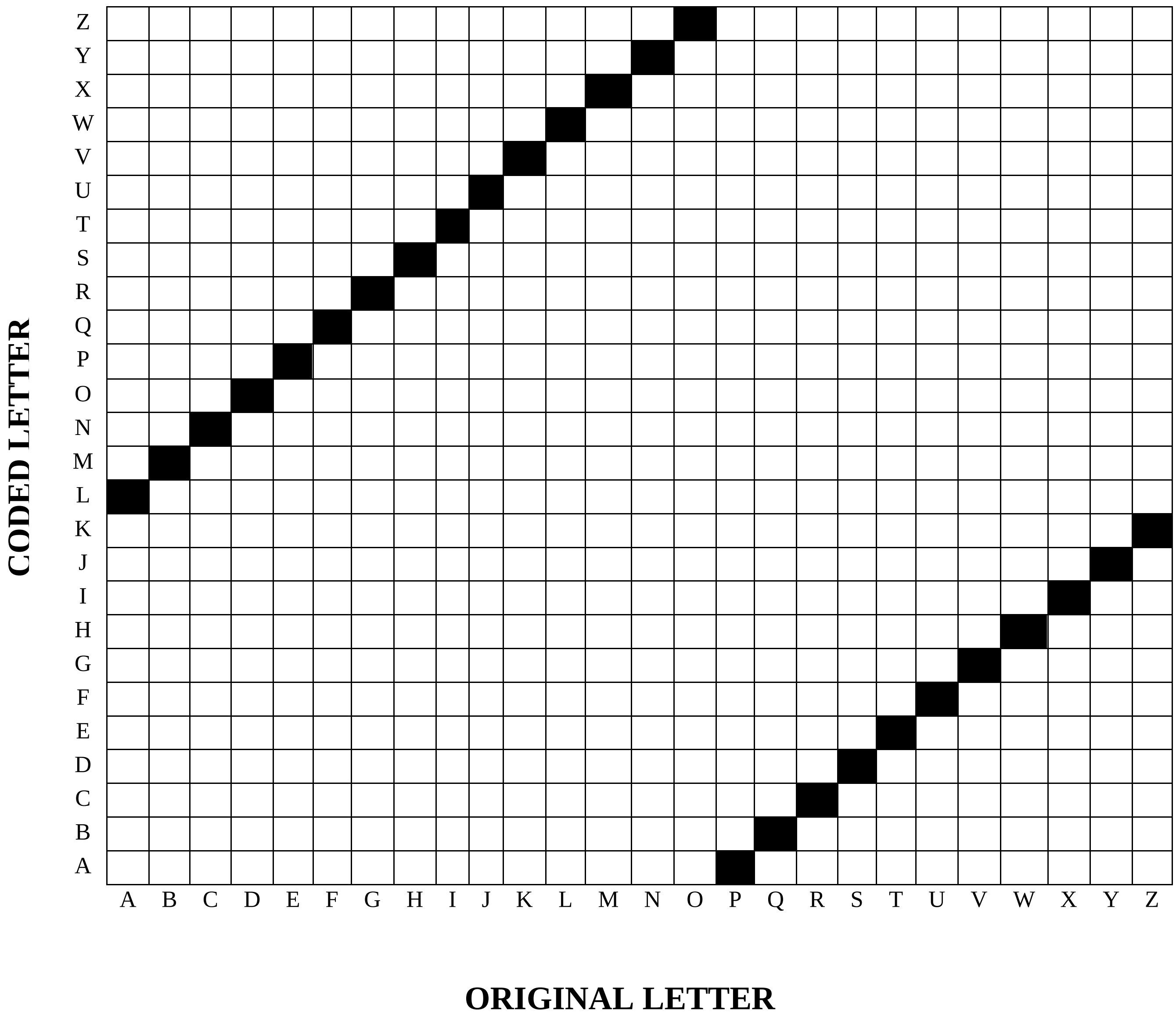
The next line is ***HOW I WONDER WHAT YOU ARE.***

Use the cipher to code it!

***Solution***

***KRZ L ZRQGHU ZKDW BRX DUH.***

A letter to letter code can also be represented in a graph. Determine the cipher represented in the graph below.




| | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
| L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K |

1. Use the code cipher from the preceding page to code the message *"Math is fun."*

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***Solution 1***

***X LES TD QFY***

2. Use the cipher to decode: *T SLGP L DPNCPE*

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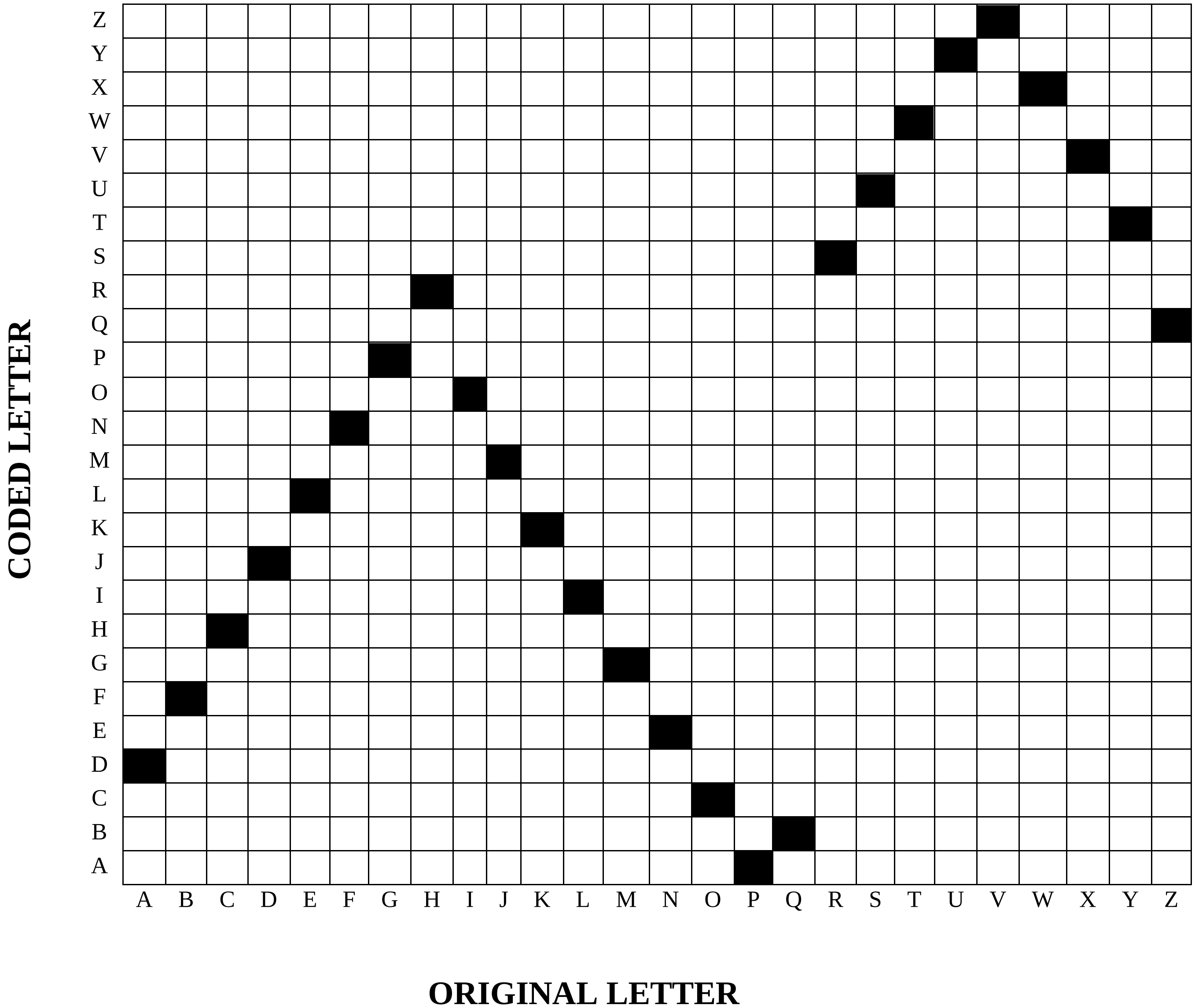
***Solution 2***

***I HAVE A SECRET***




3. Use the code cipher above to code the message: *The homework is on page fifty.*

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***Solution 3***

***WRL RCGL XCSK OU CE ADPL NONWT***

4. Use the cipher above to decode: *SLJ JCP FDSKU DW GOJEOPRW*

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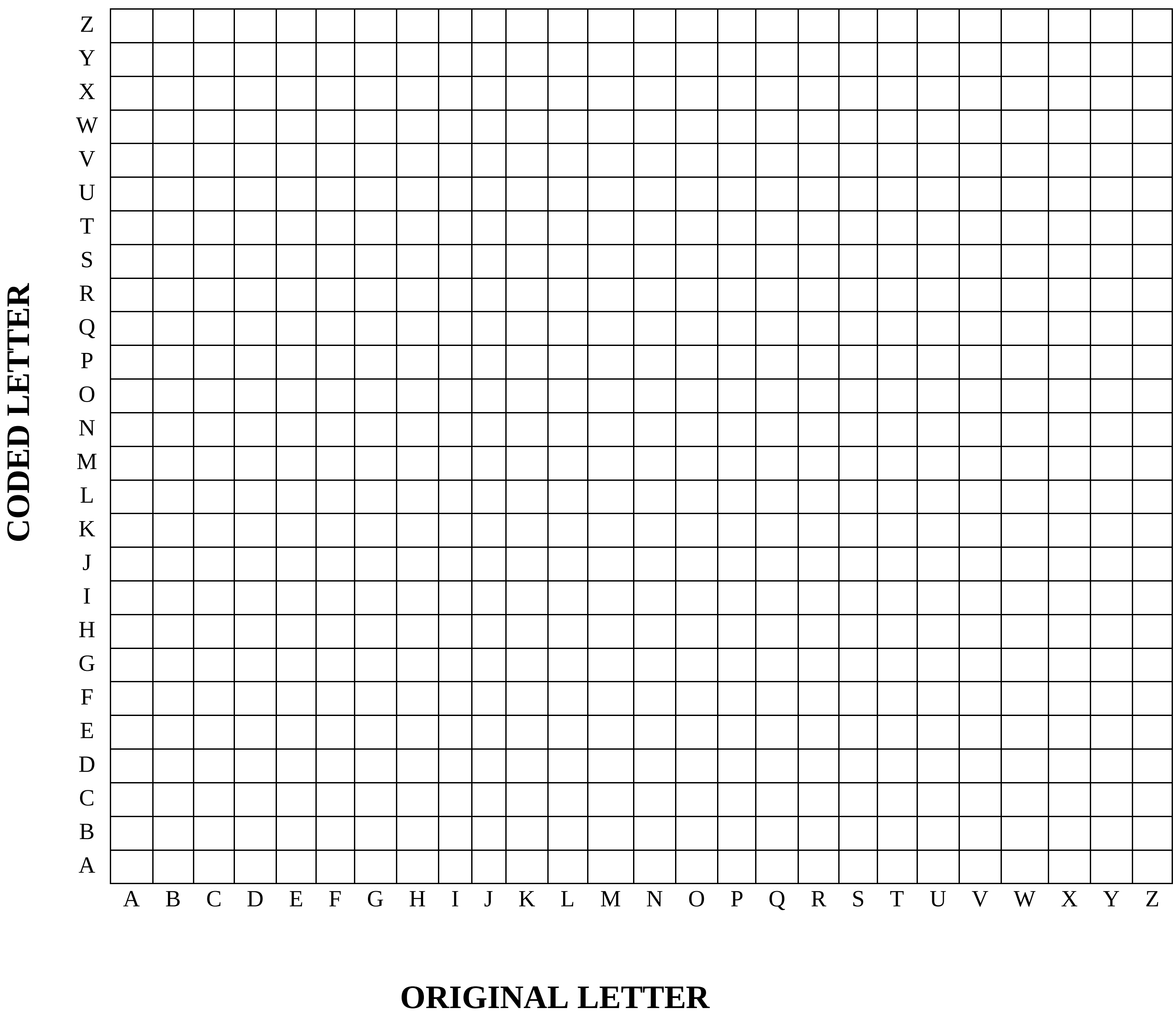
***Solution 4***

***RED DOG BARKS AT MIDNIGHT***

Create a cipher code similar to the ones on the preceding pages so that each letter is replaced by a letter that comes four positions later in the alphabet.



5. Use your cipher to code: *April showers bring May flowers.*

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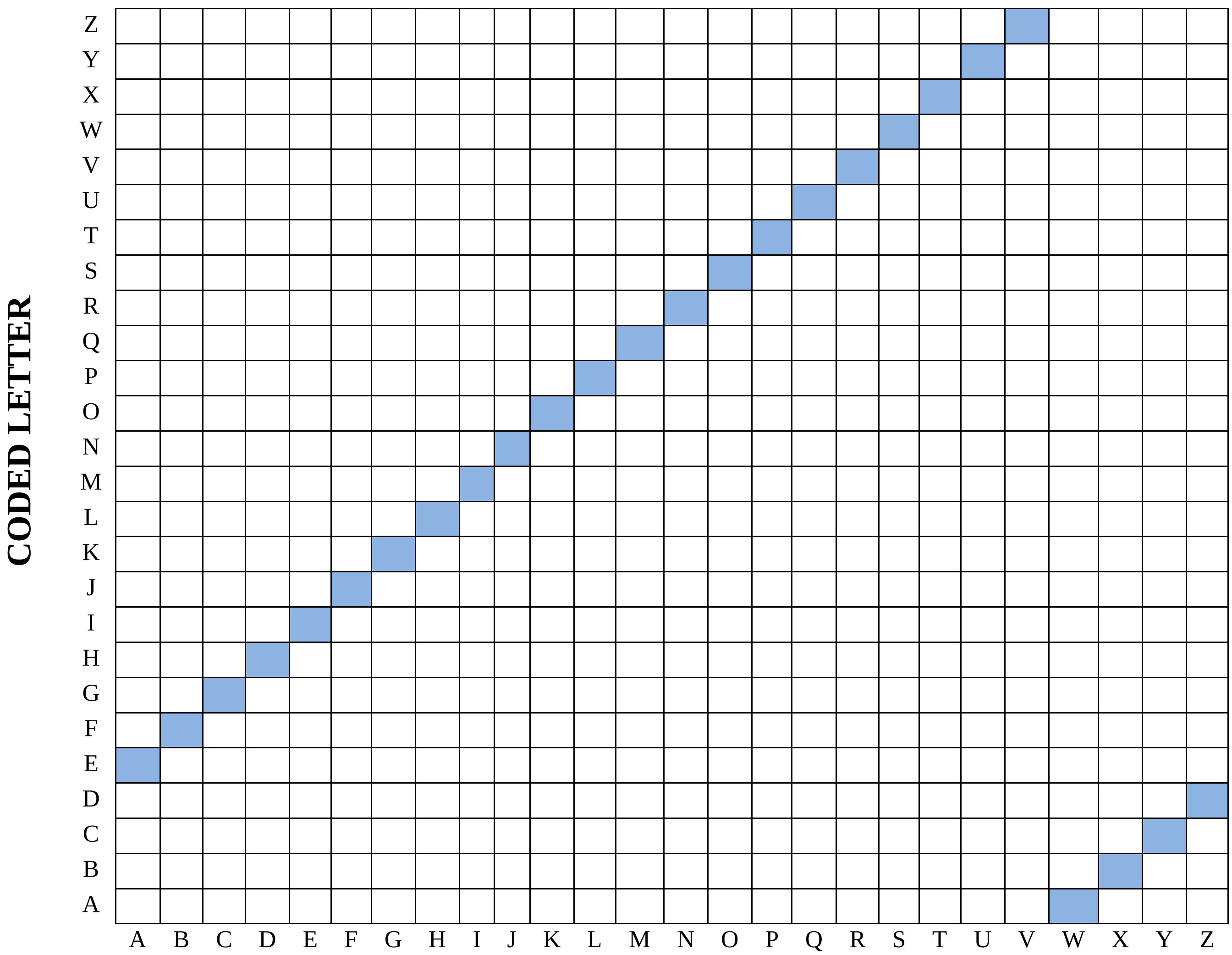
## Solution 5




***ESVMP WLSAIVW FVMRK QEC JPSAIVW***

## Comment

***Students could exchange papers to solve ciphers for code accuracy.***

Determine the rule for the codes represented below.

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 |

***Solution***

***Output = original letter order number plus 5***

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 31 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |

***Solution***

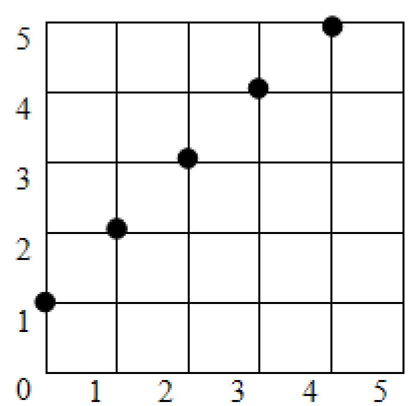
***Output = original letter order number plus 13***

## <u>Secret Codes and Number Rules</u>

**Part 2**

As rules can be created to code a message in letters, you can write a rule that changes numbers into other numbers. These rules can be represented by words, equations, tables and graphs.

The rule in words “add one to each number” can be represented in the other forms.

<table>
<tr><th>Form</th><th>Representation</th></tr>
<tr><td>Words – verbal expression</td><td>“add one to each number”</td></tr>
<tr><td>Equation</td><td>$y = x + 1$</td></tr>
<tr><td>Table</td><td><table><tr><td>$x$</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>$y$</td><td>1</td><td>2</td><td>3</td><td>4</td></tr></table></td></tr>
<tr><td>Graph</td><td>
</td></tr>
</table>

Consider these tables of values. Determine the rule. Express each function verbally and algebraically.

***Comment***

***It may be necessary for some students to plot the points in order to "see" the rule.***

| **Input** | -4 | 3 | 5 | 16 | 25 | 31 |
|---|---|---|---|---|---|---|
| **Output** | -7 | 7 | 11 | 33 | 51 | 63 |

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***Solutions***

***Output equals two times input plus one and $y = 2x + 1$***

| **Input** | -4 | -1 | 0 | 2 | 3 | 5 |
|---|---|---|---|---|---|---|
| **Output** | 15 | 0 | -1 | 3 | 8 | 24 |

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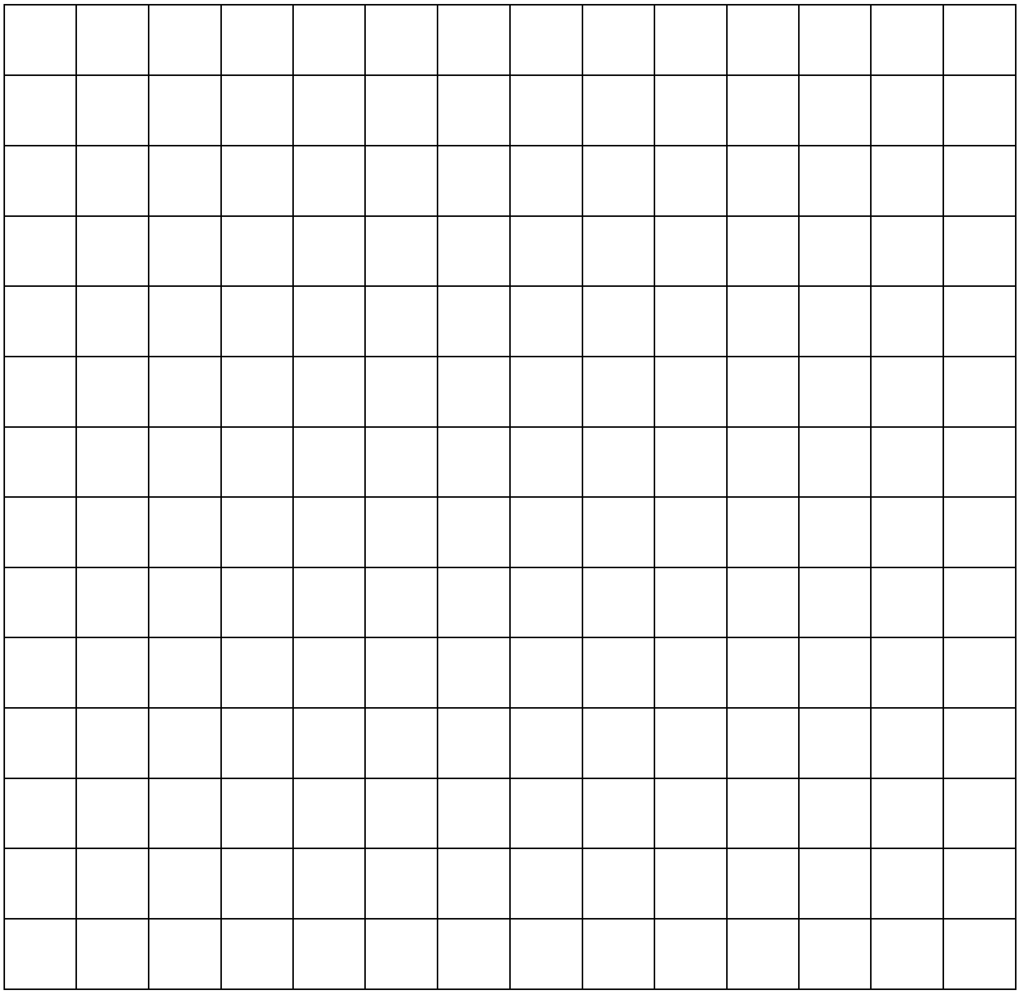
***Solutions***

***Output equals input squared minus one and $y = x^2 - 1$***

Create a table and graph for $y = x^2 + 1$

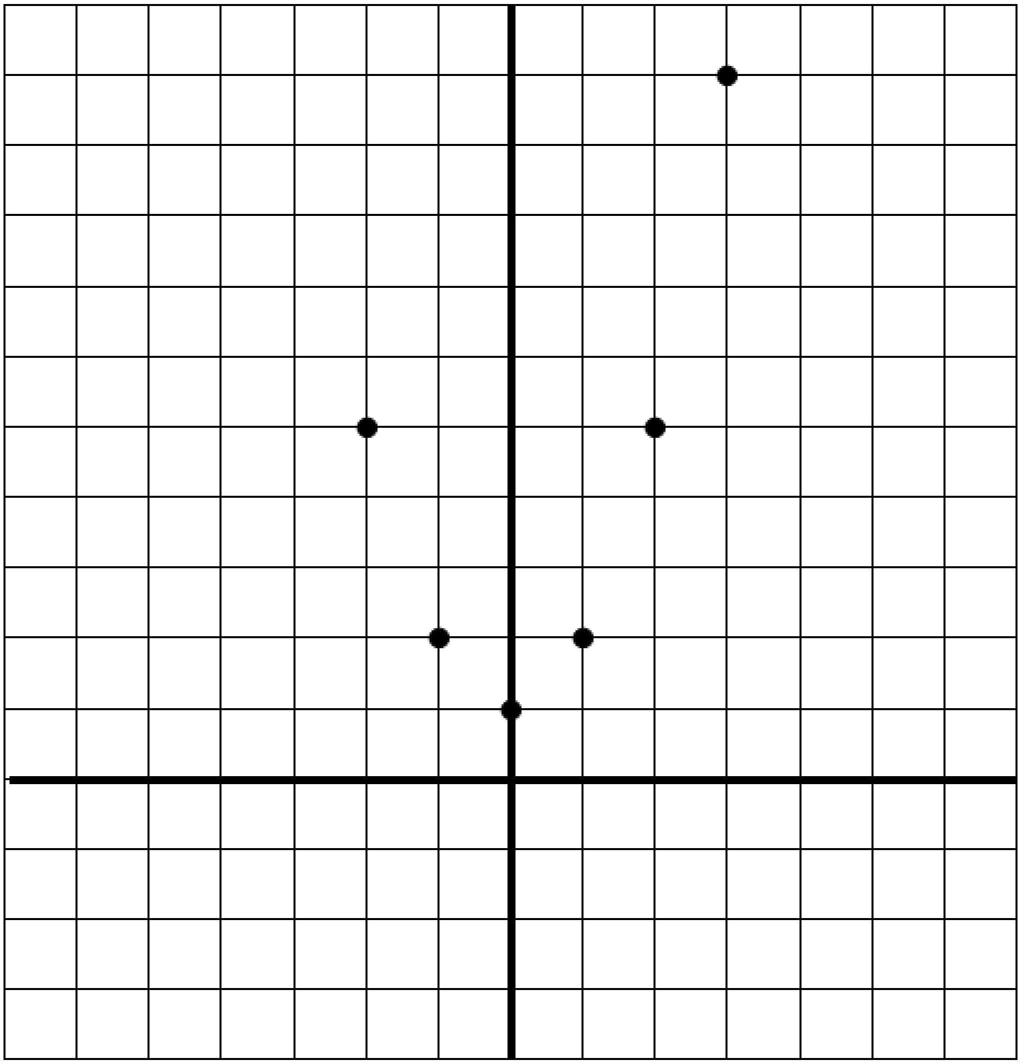
| **Input** | | | | | | |
|---|---|---|---|---|---|---|
| **Output** | | | | | | |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | |



***Solutions***

| *Input* | *-2* | *-1* | *0* | *1* | *2* | *3* |
|---|---|---|---|---|---|---|
| *Output* | *5* | *2* | *1* | *2* | *5* | *10* |



***Use the equation $y = x^2 + 1$ and the table and graph which you just created, to help you as you reason the following questions.***

Is it possible for two different inputs to give you the same output?  Justify your answer.

***Possible Solution***

***Yes, since the square of a number is always positive, a pair of opposites will have the same output.***

Is it possible to derive two different outputs from the same input?  Justify your answer.

***Possible Solution***

***No, for every possible input number there is a rule that determines an output number.***

## Secret Codes and Number Rules

### Part 1

Encryption is used by spies, secret societies, and other organizations to transfer information without other people reading their messages.

Secret codes can be created by a simple scheme of replacing letters with numbers. More complex codes replace letters with other letters.

The code scheme is called a cipher and can be written as a list or table.

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 |

Use the cipher above to decode these messages.

***Question:***

23 8 1 20 4 9 4 20 8 5 26 5 18 15 19 1 25 20 15 20 8 5 5 9 7 8 20?

***Answer:***

14 9 3 5 2 5 12 20

***Question:***

23 8 5 14 4 15 5 19 8 1 12 6 15 6 5 9 7 8 20 5 17 21 1 12 20 8 18 5 5 ?

***Answer:***

23 8 5 14 25 15 21 3 21 20 9 20 22 5 18 20 9 3 1 12 12 25.

Your turn! Write, find or share a joke or riddle! Write it in code and share with your classmates.

__________________________________________________________

__________________________________________________________

Ciphers can be written in which letters are replaced with other letters, not numbers. A scheme is determined for replacement. Often letters are shifted. For example, a letter could be replaced with the letter three down the alphabet.

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C |

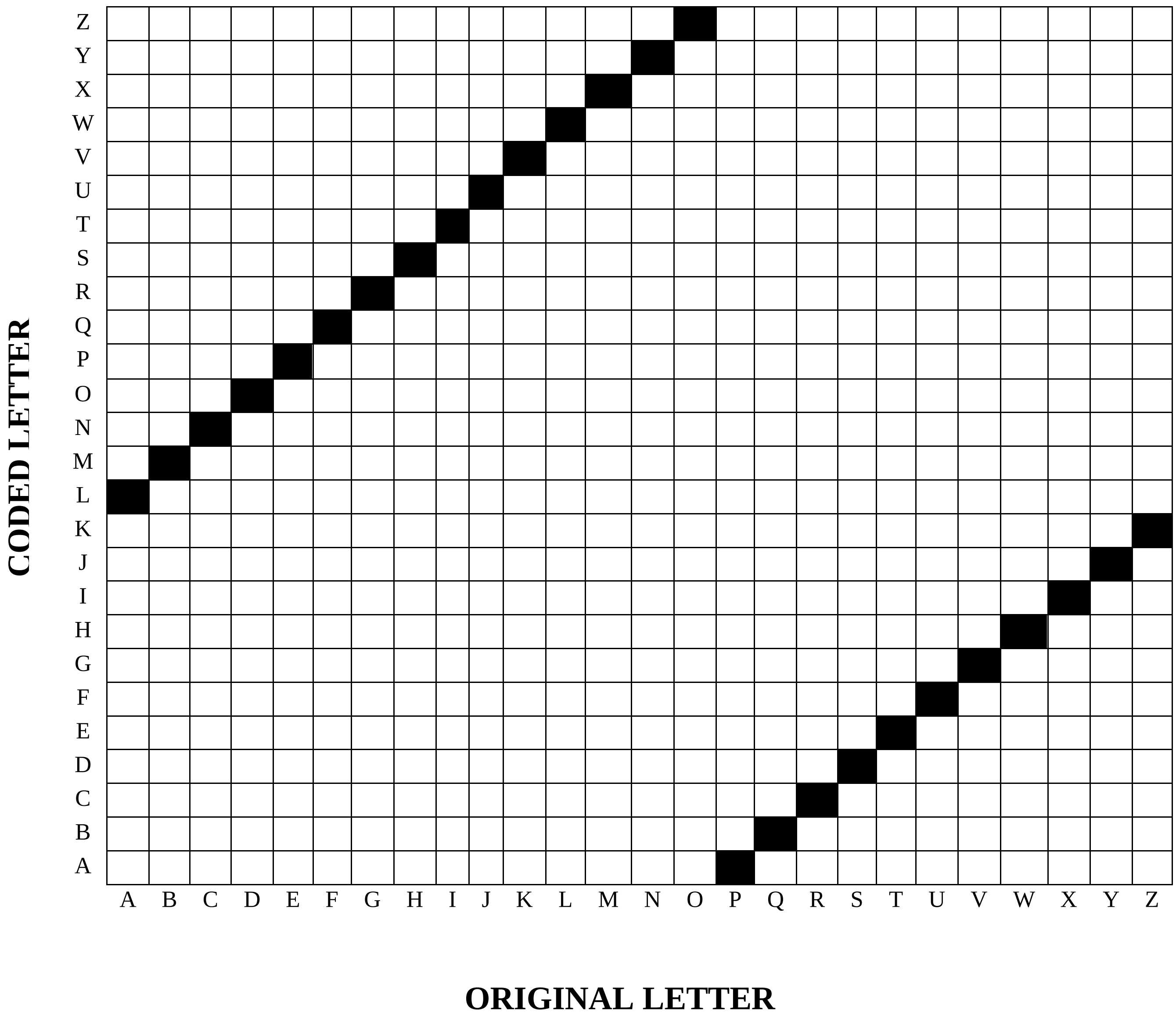
Therefore ***TWINKLE TWINKLE LITTLE STAR*** becomes

***WZLQOH WZLQOH OLWWOH VWDU***

The next line is ***HOW I WONDER WHAT YOU ARE.***

Use the cipher to code it!

A letter to letter code can also be represented in a graph. Determine the cipher represented in the graph below.



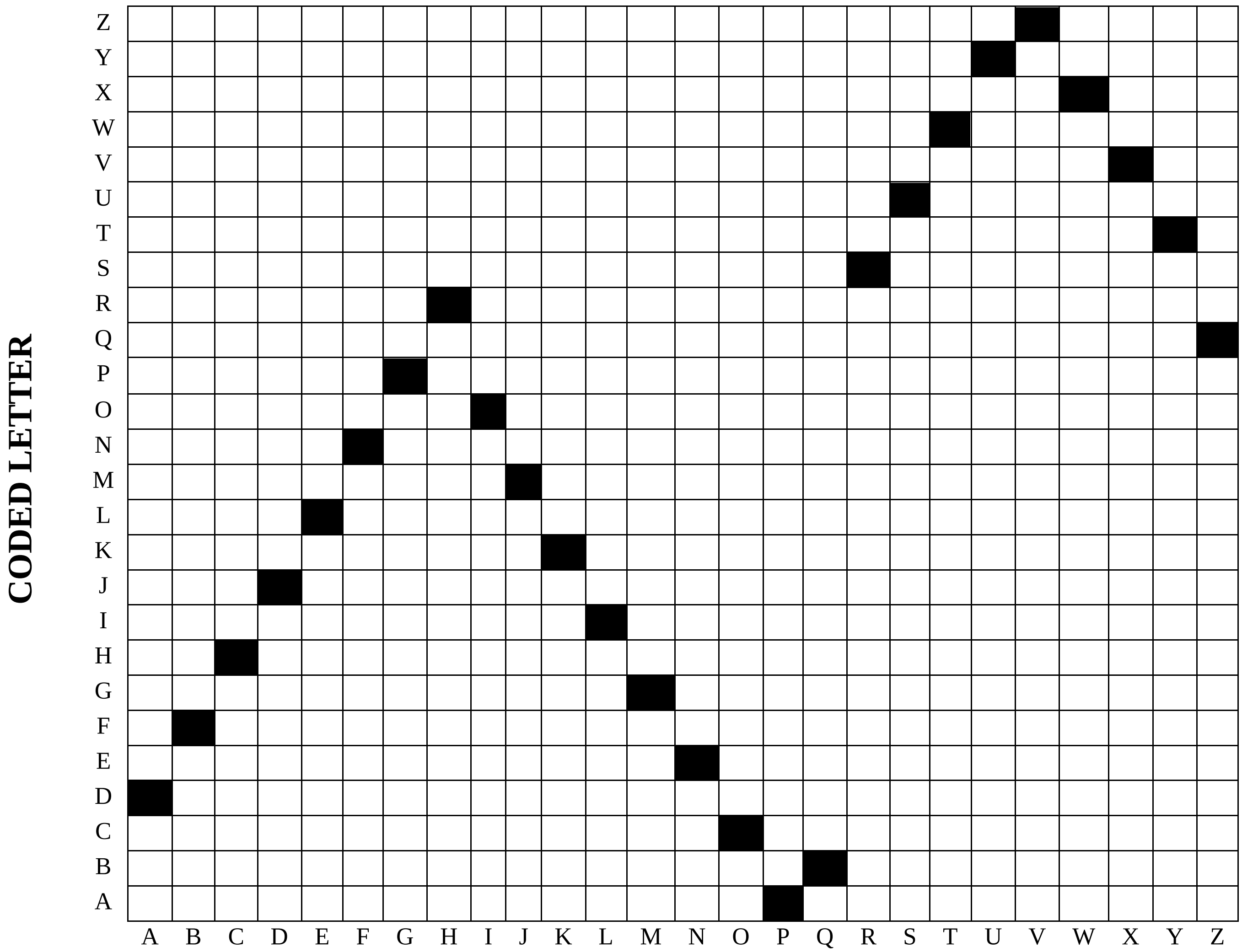

| | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
| L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | A | B | C | D | E | F | G | H | I | J | K |

1. Use the code cipher from the preceding page to code the message *"Math is fun."*

______________________________________________________________________________

2. Use the cipher to decode: *T SLGP L DPNCPE*

______________________________________________________________________________




## ORIGINAL LETTER

3. Use the code cipher above to code the message: *The homework is on page fifty.*

______________________________________________________________________

4. Use the cipher above to decode: *SLJ JCP FDSKU DW GOJEOPRW*

______________________________________________________________________

Create a cipher code similar to the ones on the preceding pages so that each letter is replaced by a letter that comes four positions later in the alphabet.

**CODED LETTER**

| | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Z | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Y | | | | | | | | | | | | | | | | | | | | | | | | | | |
| X | | | | | | | | | | | | | | | | | | | | | | | | | | |
| W | | | | | | | | | | | | | | | | | | | | | | | | | | |
| V | | | | | | | | | | | | | | | | | | | | | | | | | | |
| U | | | | | | | | | | | | | | | | | | | | | | | | | | |
| T | | | | | | | | | | | | | | | | | | | | | | | | | | |
| S | | | | | | | | | | | | | | | | | | | | | | | | | | |
| R | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Q | | | | | | | | | | | | | | | | | | | | | | | | | | |
| P | | | | | | | | | | | | | | | | | | | | | | | | | | |
| O | | | | | | | | | | | | | | | | | | | | | | | | | | |
| N | | | | | | | | | | | | | | | | | | | | | | | | | | |
| M | | | | | | | | | | | | | | | | | | | | | | | | | | |
| L | | | | | | | | | | | | | | | | | | | | | | | | | | |
| K | | | | | | | | | | | | | | | | | | | | | | | | | | |
| J | | | | | | | | | | | | | | | | | | | | | | | | | | |
| I | | | | | | | | | | | | | | | | | | | | | | | | | | |
| H | | | | | | | | | | | | | | | | | | | | | | | | | | |
| G | | | | | | | | | | | | | | | | | | | | | | | | | | |
| F | | | | | | | | | | | | | | | | | | | | | | | | | | |
| E | | | | | | | | | | | | | | | | | | | | | | | | | | |
| D | | | | | | | | | | | | | | | | | | | | | | | | | | |
| C | | | | | | | | | | | | | | | | | | | | | | | | | | |
| B | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |

**ORIGINAL LETTER**

5. Use your cipher to code: *April showers bring May flowers.*

______________________________________________________________

Determine the rule for the codes represented below.

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 |

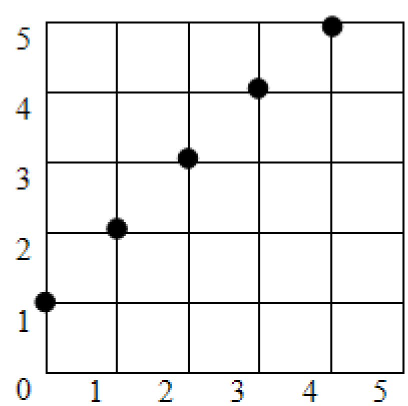
| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 31 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |

# <u>Secret Codes and Number Rules</u>

**Part 2**

As rules can be created to code a message in letters, you can write a rule that changes numbers into other numbers. These rules can be represented by words, equations, tables and graphs.

The rule in words "add one to each number" can be represented in the other forms.

<table>
<tr><th>Form</th><th>Representation</th></tr>
<tr><td>Words – verbal expression</td><td>"add one to each number"</td></tr>
<tr><td>Equation</td><td>$y = x + 1$</td></tr>
<tr><td>Table</td><td><table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>y</td><td>1</td><td>2</td><td>3</td><td>4</td></tr></table></td></tr>
<tr><td>Graph</td><td>
</td></tr>
</table>

Consider these tables of values. Determine the rule. Express each function verbally and algebraically.

| **Input** | -4 | 3 | 5 | 16 | 25 | 31 |
|---|---|---|---|---|---|---|
| **Output** | -7 | 7 | 11 | 33 | 51 | 63 |

______________________________________________________________

| **Input** | -4 | -1 | 0 | 2 | 3 | 5 |
|---|---|---|---|---|---|---|
| **Output** | 15 | 0 | -1 | 3 | 8 | 24 |

______________________________________________________________

Create a table and graph for $y = x^2 + 1$

| | | | | | | |
|---|---|---|---|---|---|---|
| **Input** | | | | | | |
| **Output** | | | | | | |

***Use the equation $y = x^2 + 1$ and the table and graph which you just created, to help you as you reason the following questions.***

Is it possible for two different inputs to give you the same output?  Justify your answer.

Is it possible to derive two different outputs from the same input?  Justify your answer.

## Foxes And Rabbits (Spotlight Task)

***Back to Task Table***

Adapted from Illustrative Mathematics
http://www.illustrativemathematics.org/illustrations/713

## STANDARDS FOR MATHEMATICAL CONTENT:

**Define, evaluate, and compare functions.**

**MGSE8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

## STANDARDS FOR MATHEMATICAL PRACTICE

3. Construct viable arguments and critique the reasoning of others
6. Attend to precision.
7. Look for and make use of structure.

## ESSENTIAL QUESTIONS

- What is a function?
- What are the characteristics of a function?
- How do you determine if relations are functions?
  How is a function different from a relation?
- Compare and contrast functions and relations.

## MATERIALS REQUIRED

- Student copy of the task

## TIME NEEDED

- Part of 1 class period

## TEACHER NOTES

There is a natural (and complicated!) predator-prey relationship between the fox and rabbit populations, since foxes thrive in the presence of rabbits, and rabbits thrive in the absence of foxes. However, this relationship, as shown in the given table of values, cannot possibly be used to present either population as a function of the other. This task emphasizes the importance of the "every input has exactly one output" clause in the definition of a function, which is violated in the table of values of the two populations. Noteworthy is that since the data is a collection of input-output pairs, no verbal description of the function is given, so part of the task is processing what the "rule form" of the proposed functions would look like.

The predator-prey example of foxes and rabbits is picked up again in F-IF Foxes and Rabbits 2 and 3 where students are asked to find trigonometric functions to model the two populations as functions of time.

This task could be used early on when functions are introduced. It illustrates examples of functions as well as relationships that are not functions. It could also be used as an assessment item.

This task is adapted from "Functions Modeling Change", Connally et al, Wiley 2007.

**PART 1**

www.warrenphotographic.co.uk

Students study the picture and write down three thoughts and/or possible mathematic problems on a note card.
With a partner, share ideas about the picture. What mathematical questions/problems could be posed related to the picture?

**PART 2**
Given below is a table that gives the populations of foxes and rabbits in a national park over a 12 month period. Note that each value of *t* corresponds to the beginning of the month.

| $t$, month | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $R$, number of rabbits | 1000 | 750 | 567 | 500 | 567 | 750 | 1000 | 1250 | 1433 | 1500 | 1433 |
| $F$, number of foxes | 150 | 143 | 125 | 100 | 75 | 57 | 50 | 57 | 75 | 100 | 125 |

Looking back at your list of comments, questions and mathematics problems, what additional questions might be asked regarding the rabbits and foxes?

According to the data in the table, is F a function of R?
Is R a function of F?
Are either R or F functions of t?

Explain.

**PART 3**
Students share and compare their answers to the questions in Part 2.

Were any additional questions or mathematics problems posed in Part 1 that can be answered or discussed now that were not part of the questions in Part 2?

***Solution: Each input has exactly one output***

***a.The key is understanding that a function is a rule that assigns to each input exactly one output, so we will test the relationships in question according to this criterion:***

***For the first part, that is, for F to be a function of R, we think of R as the input variable and F as the output variable, and ask ourselves the following question: Is there a rule, satisfying the definition of a function, which inputs a given rabbit population and outputs the corresponding fox population. The answer is no: We can see from the data that when R=1000, we have one instance where F=150, and another where F=50. Since this means that a single input value corresponds to more than one output value, F is not a function of R. In the language of the problem's context, this says that the fox population is not completely determined by the rabbit population; during two different months there are the same number of rabbits but different numbers of foxes.***

***Similarly, we can see that if we consider F as our input and R as our output, we have a case where F=100 corresponds to both R=500 and R=1500, two different outputs for the same input. So R is not a function of F: There are two different months which have the same number of foxes but two different numbers of rabbits.***

***b .Letting t, months, be the input, we can clearly see that there is exactly one output R for each value of t. That is, the rule which assigns to a month t the population of rabbits during that month fits our definition of a function, and so R is a function of t. By the same reasoning F is also a function of t. Again, in the context of the situation it makes sense that at any given point in time, there is a unique number of foxes and a unique number of rabbits in the park.***

## Foxes And Rabbits

Name ______________________________

**PART 1**

www.warrenphotographic.co.uk

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Looking back at your list of comments, questions and mathematics problems, what additional questions might be asked regarding the rabbits and foxes?

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Are either R or F functions of t?

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## Vending Machines

***Back to Task Table***

## STANDARDS OF MATHEMATICAL CONTENT:

**Define, evaluate, and compare functions.**

**MGSE8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

**MGSE8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

## STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## BACKGROUND KNOWLEDGE:

The context of this "real-world" problem is such that it connects functions to something that students are very familiar with – vending machines. Using this context of an input/output machine (we input money and the machine outputs a snack) to prompt a discussion of who is correct in their reasoning about functions and relations can be engaging and the discussions can be lively.

## *COMMON MISCONCEPTIONS:*

*Students may confuse the input and output values/variables. This could result in the inverse of the function. This misconception can be a tremendous teaching tool. If you observe students showing evidence of this misconception, take an example (anonymously) and use it as a teachable moment. Ask students to discuss the work, discover the error, and compare it to the actual function. Students benefit from this type of exploration, not just to correct a misconception, but also gain ownership of the knowledge of inverse functions in the process.*

## ESSENTIAL QUESTIONS:

- What is a function?
- What are the characteristics of a function?
- How is a function different from a relation?
- How can a function be recognized in any form?
- What is the best way to represent a function?
- How can you use functions to model real-world situations?

## MATERIALS:

- Copies of task for students

## GROUPING:

- Individual/Partner

## TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

This task has the potential to advance and strengthen students' abilities to think in terms of functions in order to solve real world problems. Functions have three components: inputs, outputs, and the rule that relates them. In order to think in terms of functions, students must have the ability to abstract from particular input-output pairs. Standard questions about the domain and ranges of functions are artificial and usually feel purposeless to students and as a consequence, do not prepare them for the process of abstraction. In the questions in this task, inputs and outputs appear in a concrete context that is meaningful in a real-world situation for students.

## DIFFERENTIATION:

**Extension**

- Function Machine Drawing:

    1. Create a *creative and original* drawing of a function machine. Label the input, the rule, and the output.
    2. A description of how the machine works and why it represents a function must accompany your drawing.
    3. Your description must include an explanation of each of the following terms: function, input, output, domain, and range

**Intervention/Scaffolding**

- Have students review/discuss the differences between relations and functions before starting this task. Also, spend time with students, discussing the various forms of input to the machine. Helping students make sense of the problem is often half the battle.

# Vending Machines

Marinathe, Hezza, and Samuel had been studying math all day. In the afternoon they took a break and went to the vending machine for a snack. After studying functions all afternoon, function vocabulary was on their minds. With money in hand, they stood in front of the machine. Samuel said the machine was like a function machine with input and output. Marinathe said while there was input and output, she thought the machine was more like a relation machine instead of a function machine. Hezza said she thought it could be either, depending on how one thought about it.

***Comment***

***The vending machine has two forms of input. The coins or amount of money put in the machine is one form. The letter buttons for selection are another form of input.***

1. Under what circumstance would the machine be a function machine?

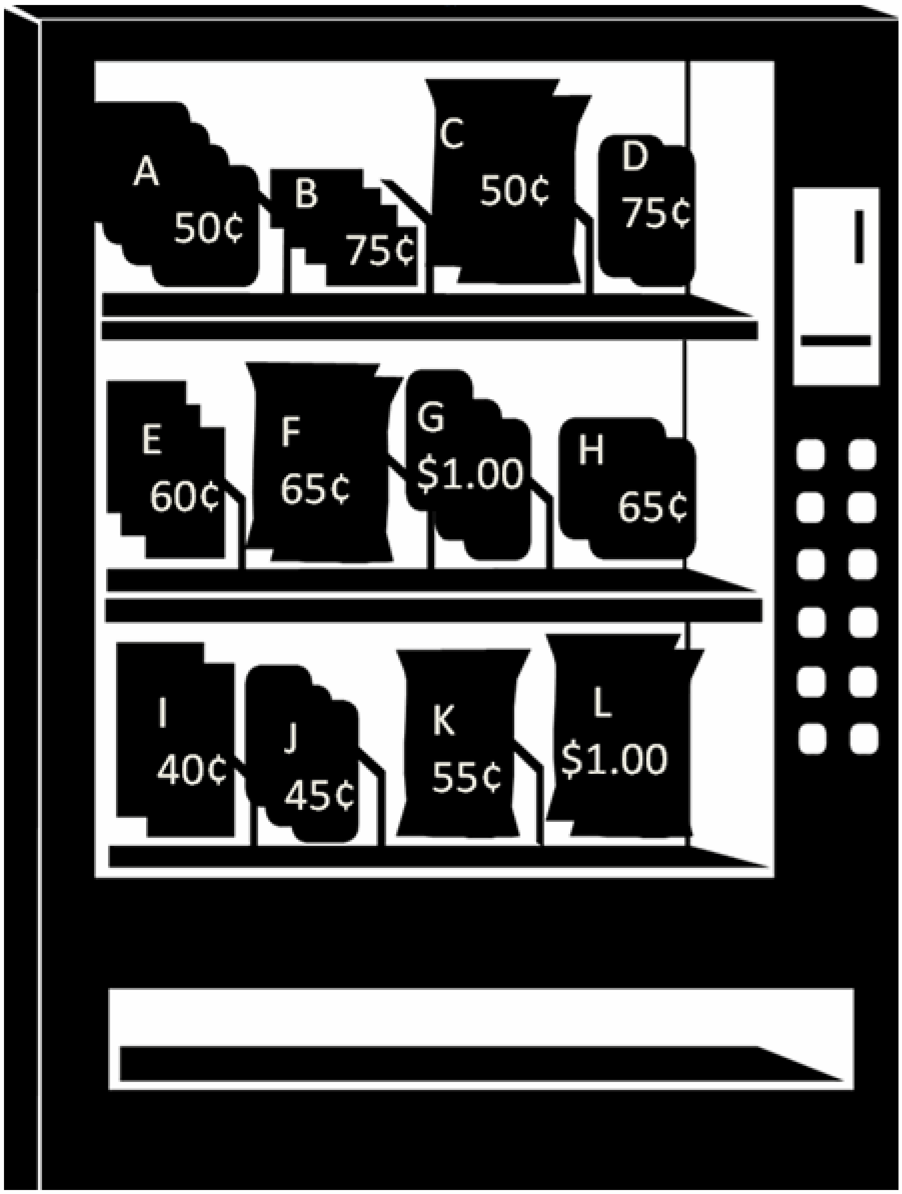
***Solution***

***A function is a rule that has a unique output for each input. If you consider the input of the letters, the vending machine is a function. Every time you input the letter F you should always get the same snack. Letter B would always give you the same output. There is a unique output for each unique input.***

2. Under what circumstance would the machine be a relation machine, but not a function?

***Solution***

***A relation is just a rule for which there is an output for each input. They do not have to be unique. If you consider the input as the amount of money that is put in the machine, it is just a relation machine. An input of 65¢ has an output of F or H.***


A
50¢
B
75¢
C
50¢
D
75¢
E
60¢
F
65¢
G
$1.00
H
65¢
I
40¢
J
45¢
K
55¢
L
$1.00

## Vending Machines

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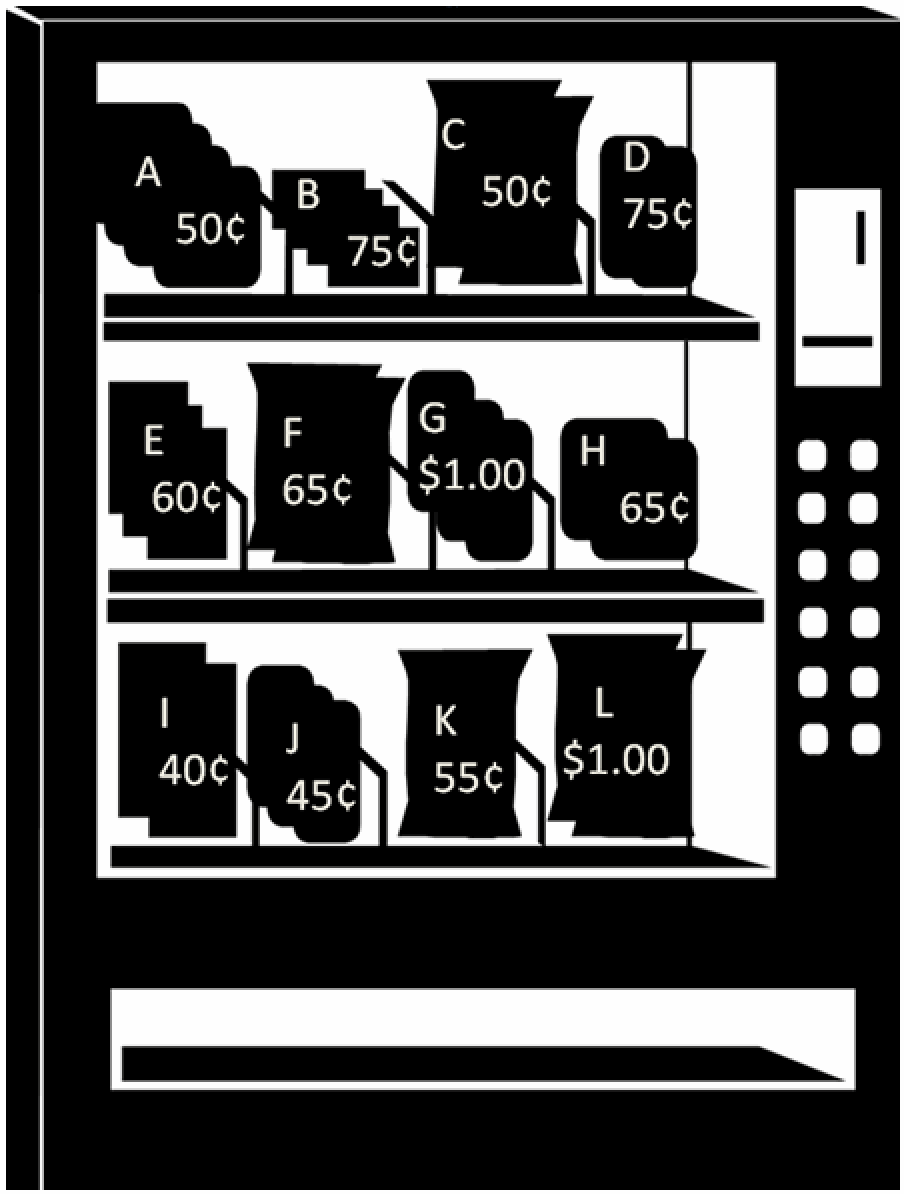
1. Under what circumstance would the machine be a function machine?

2. Under what circumstance would the machine be a relation machine, but not a function?


A
50¢
B
75¢
C
50¢
D
75¢
E
60¢
F
65¢
G
$1.00
H
65¢
I
40¢
J
45¢
K
55¢
L
$1.00

## Order Matters

***Back to Task Table***

## STANDARDS OF MATHEMATICAL CONTENT:

**Define, evaluate, and compare functions.**

**MGSE8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

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## STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
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3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## BACKGROUND KNOWLEDGE:

In this task, students must attempt to determine whether certain situations are functions. These situations do not have any numbers or variables, only relationships. Students must use their reasoning and their understandings of functions and relations to make sense of whether these relationships are functions or not. This task is all about sense making. Students should engage in this type of task consistently.

## *COMMON MISCONCEPTIONS:*

*Students may confuse the input and output values/variables. This could result in the inverse of the function. This misconception can be a tremendous teaching tool. If you observe students showing evidence of this misconception, take an example (anonymously) and use it as a teachable moment. Ask students to discuss the work, discover the error, and compare it to the actual function. Students benefit from this type of exploration, not just to correct a misconception, but also gain ownership of the knowledge of inverse functions in the process.*

Some students may also have trouble understanding that Questions 1 and 2 represent a function. These students are still incorrectly interpreting the mathematical definition of a function.

## ESSENTIAL QUESTIONS:

- What is a function?
- What are the characteristics of a function?
- How is a function different from a relation?
- Why is it important to know which variable is the independent variable?
- How can a function be recognized in any form?
- What is the best way to represent a function?
- How can you use functions to model real-world situations?

## MATERIALS:

- Copies of task for students

## GROUPING:

- Partner

## TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

This task provides an opportunity to introduce domain and range as vocabulary associated with relations and functions. Through diagrams and mapping of a real-world situation, students use reasoning skills to make connections between input/domain, and output/range as ordered pairs.

## DIFFERENTIATION:

**Extension**

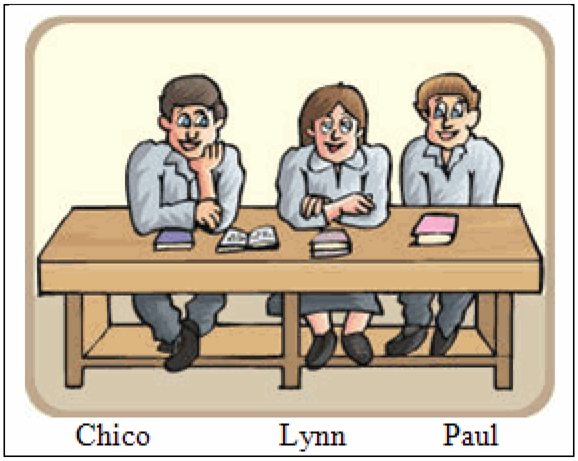
- Explain a situation using anything in the classroom, similar to the task you just completed, which could be represented as a function. Explain why your relation is a function (be sure to use the vocabulary!) and then show it as ordered pairs and as a mapping. If you reversed the domain and range, would it still be a function? Why or why not?

**Interventions/Scaffolding**

- Show this task to the students in color, not just a grayscale copy. Review/Discuss the differences between relations and functions, along with relation/function mapping prior to beginning this task.

# Order Matters

***Directions: Use the picture below to Questions 1-4.***
***Note: All shirts are the same color.***




1. Is (name, shirt color) a function? Why or why not?

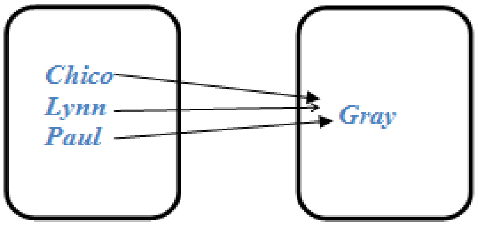
***Solution***

***Yes, this is a function because each input has a unique output. While the output is repeated for the unique input there is a unique output. The output, Gray, just happens to be repeated. From the mappings there is only one arrow leaving each member of the domain. Each member of the domain is associated with only one member of the range. There is only one y for each x.***

2. Prove your reasoning by showing the relation as ordered pairs and as a mapping.

***Solutions***

***(Chico, Gray)***
***(Lynn, Gray)***
***(Paul, Gray)***



3. Is (shirt color, name) a function? Why or why not?

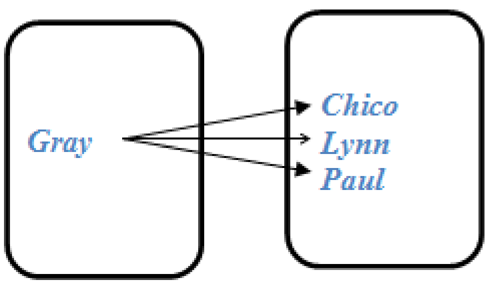
***Solutions***

***No, this is a relation not a function because each member of the domain is associated with more than one member of the range. There are multiple arrows coming from a member of the domain to multiple members of the range. The member of the domain, Gray, is associated with more than one member of the range.***

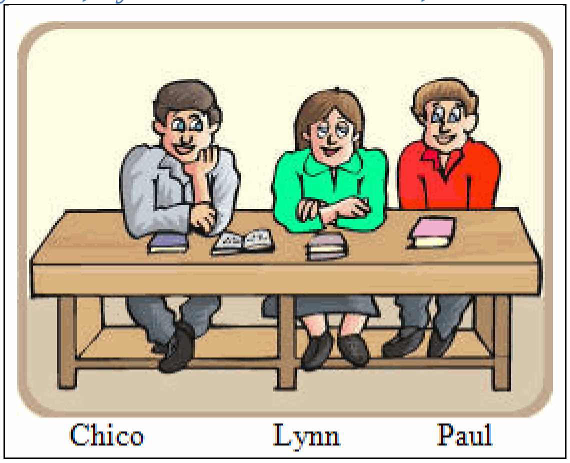
4. Prove your reasoning by showing the relation as ordered pairs and as a mapping.

***Solutions***

***(Gray, Chico)***
***(Gray, Lynn)***
***(Gray, Paul)***



***Directions: Use the picture below to answer Questions 5-8.***
***Note: Chico has on a gray shirt, Lynn has on a blue shirt, and Paul hos on a red shirt.***




***Comment***

***If a color copier is not available, teachers will need to have students write the shirt colors underneath the name of each "person."***

5. Is (name, shirt color) a function? Why or why not?

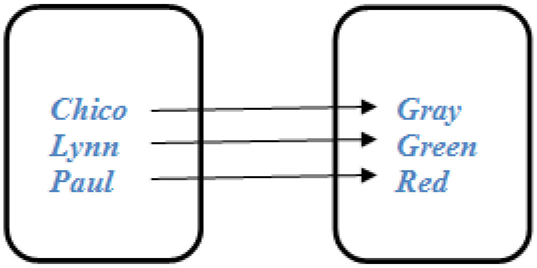
***Solutions***

***Yes, this is a function because each input has a unique output. From the mappings there is only one arrow leaving each member of the domain. Each member of the domain is associated with only one member of the range. There is only one y for each x.***

6. Prove your reasoning by showing the relation as ordered pairs and as a mapping.

***Solutions***

***(Chico, Gray)***
***(Lynn, Green)***
***(Paul, Red)***




7. Is (shirt color, name) a function? Why or why not?

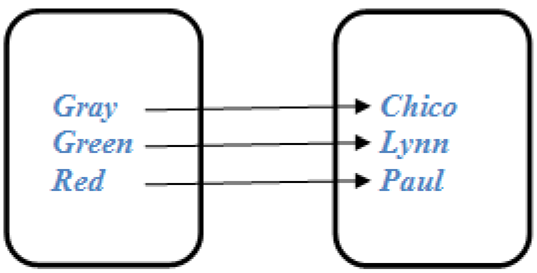
***Solutions***

***Yes, this is a function because each input has a unique output. From the mappings there is only one arrow leaving each member of the domain. Each member of the domain is associated with only one member of the range. There is only one y for each x.***

8. Prove your reasoning by showing the relation as ordered pairs and as a mapping.

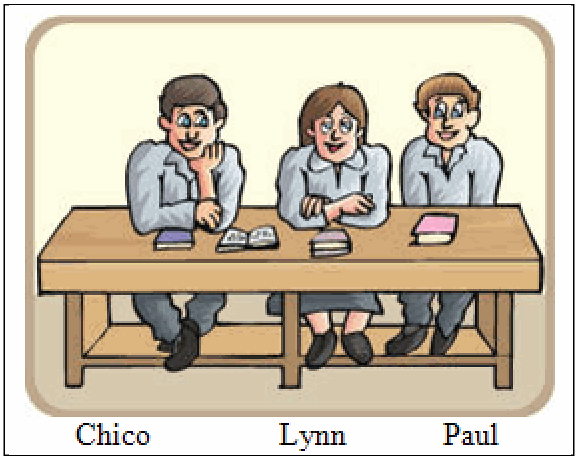
***Solutions***

***(Gray, Chico)***
***(Green, Lynn)***
***(Red, Paul)***



## Order Matters

***Directions: Use the picture below to Questions 1-4.***



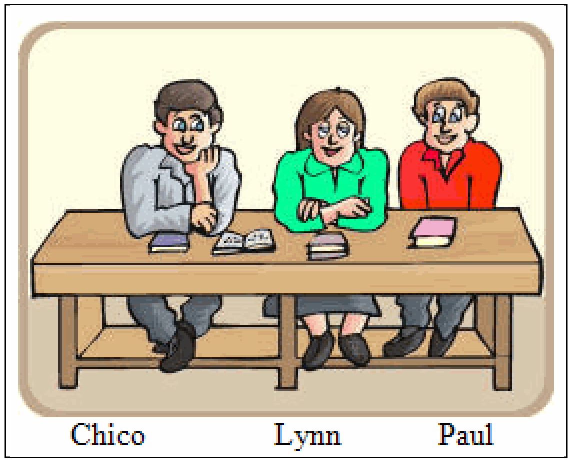

1. Is (name, shirt color) a function? Why or why not?

2. Prove your reasoning by showing the relation as ordered pairs and as a mapping.

3. Is (shirt color, name) a function? Why or why not?

4. Prove your reasoning by showing the relation as ordered pairs and as a mapping.

***Directions: Use the picture below to answer Questions 5-8.***
*Note: Chico has on a gray shirt, Lynn has on a blue shirt, and Paul hos on a red shirt.*




5. Is (name, shirt color) a function? Why or why not?

6. Prove your reasoning by showing the relation as ordered pairs and as a mapping.

7. Is (shirt color, name) a function? Why or why not?

8. Prove your reasoning by showing the relation as ordered pairs and as a mapping.

## <u>Battery Charging</u> (Spotlight Task)

***Back To Task Table***

Adapted from Illustrative Mathematics
http://www.illustrativemathematics.org/illustrations/641

## STANDARDS FOR MATHEMATICAL CONTENT

**MGSE8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions)

## STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

## ESSENTIAL QUESTIONS

- How can functions be recognized in any form?
- How can graphs and equations of functions help us to interpret real-world problems?

## TIME NEEDED

- 1 class

## TEACHER NOTES:

This task has students engaging in a simple modeling exercise, taking verbal and numerical descriptions of battery life as a function of time and writing down linear models for these quantities. To draw conclusions about the quantities, students have to find a common way of describing them. There are three solution techniques presented below:

1. Finding equations for both functions.
2. Using tables of values.
3. Using graphs.

There are also ample opportunities to talk about the role of modeling here, touching on mathematical practice standard MP4. How reasonable is it that the output units are reported as percents? Does the model hold for all time? In particular, note that the model predicts that the percent charged grows linearly for all time, even beyond 100%!

If the task is done in small groups, different groups would likely use different representations in their solutions. Having groups present their answers could lead to a rich discussion on connecting different representations of functions.

## <u>Battery Charging</u>

Name ________________________

**PART 1**

Sam wants to take his MP3 player and his video game player on a car trip. An hour before they plan to leave, he realized that he forgot to charge the batteries last night. At that point, he plugged in both devices so they can charge as long as possible before they leave.

What problem(s) is Sam facing? What math problems might this scenario be leading up to? What information do you need to know in order to help Sam?

**PART 2**

Sam knows that his MP3 player has 40% of its battery life left and that the battery charges by an additional 12 percentage points every 15 minutes.

His video game player is new, so Sam doesn't know how fast it is charging but he recorded the battery charge for the first 30 minutes after he plugged it in.

| time charging (minutes) | 0 | 10 | 20 | 30 |
|---|---|---|---|---|
| video game player battery charge (%) | 20 | 32 | 44 | 56 |

a. If Sam's family leaves as planned, what percent of the battery will be charged for each of the two devices when they leave? How do you know?

b. How much time would Sam need to charge the battery 100% on both devices? Explain how you know.

**PART 3**

Students reveal their answers and discuss the similarities and differences in what they found.

***Solution: Finding and using equations***

1. ***The battery charge of both devices can be modeled with linear functions. The wording describing the MP3 player suggests a linear function since it uses a constant rate of change. The table of values for the video game player shows a constant rate of change for the first 30 minutes. It is a reasonable assumption that the battery will continue to charge at the same rate. However, it is an assumption on our part. (Another possibility would be that as the battery charge approaches 100%, the rate of change decreases, but that would be much harder to model.)***

   ***The MP3 player charges at a rate of 12 percentage points every 15 minutes, which is equal to 0.8 percentage points per minute. If we let y be battery charge of the device (in percentage points) we have:***

   $$y=0.8t+40,$$

   ***where t is measured in minutes.***

   ***We know that the video game player is initially 20% charged and from the table we see that the charge increases by an additional 12 percentage points every 10 minutes, or 1.2 percentage points per minute. So for this function we get:***

   $$y=1.2t+20.$$

   ***Sam's family is planning to leave the house 60 minutes after Sam started charging his devices. We are looking for the charge when t=60:***

   ***MP3 player:*** $y=0.8\cdot 60+40=88$ ***% charged***

   ***video game player:*** $y=1.2\cdot 60+20=92$ ***% charged***

2. ***To answer this question, we need to find the values of t for which each function has output value 100.***

   ***MP3 player: Solving*** $100=0.8t+40$ ***for t we have,*** $t=75$***minutes.***

   ***video game player: Solving*** $100=1.2t+20$ ***for t we have*** $t=67$ ***minutes.***

   ***So if Sam's family could wait just 15 more minutes, Sam could have both devices fully charged for the car trip.***

***Solution: Using tables***

1. ***Since the video game player's battery charge is given in a table, we can extend the table and see what value it will give after 60 minutes. Note that the rate of change of the data in the table is constant: For every 10 minutes the charge increases by 12 percentage points. Assuming that this pattern continues, we have:***

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| ***time charging (minutes)*** | ***0*** | ***10*** | ***20*** | ***30*** | ***40*** | ***50*** | ***60*** |
| ***video game player battery charge (%)*** | ***20*** | ***32*** | ***44*** | ***56*** | ***68*** | ***80*** | ***92*** |

2. ***We can make a similar table for the MP3 player:***

| | | | | | |
|---|---|---|---|---|---|
| ***time charging (minutes)*** | ***0*** | ***15*** | ***30*** | ***45*** | ***60*** |
| ***MP3 player battery charge (%)*** | ***40*** | ***52*** | ***64*** | ***76*** | ***88*** |

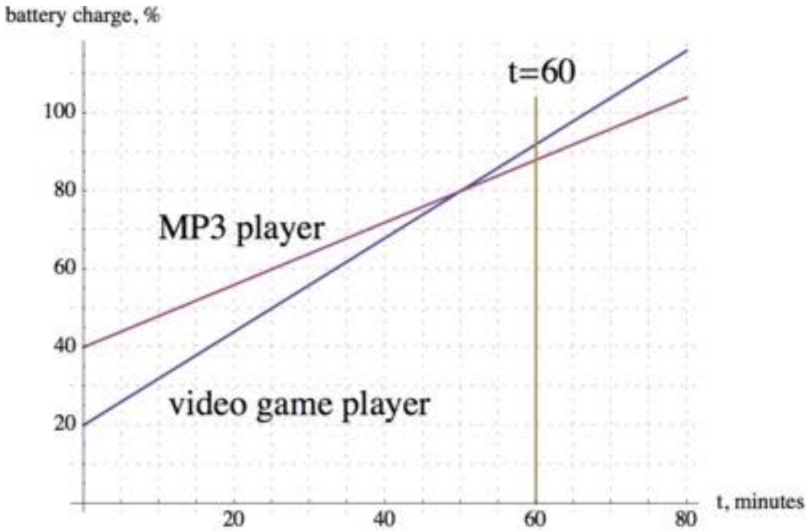
3. ***So after 60 minutes, the MP3 player's battery would be 88% charged and the video game player will be 92% charged.***
4. ***We can see from the table above that the MP3 player would be fully charged in another 15 minutes, we just have to add one more column to the table to find that answer.***

   ***The video game player will need less than 10 minutes to fully charge, since we are only missing 8 percentage points after 60 minutes. To be exact, using the rate of increase, we will need 2/3 of 10 minutes, which is just under 7 minutes.***

***Solution: Using graphs***

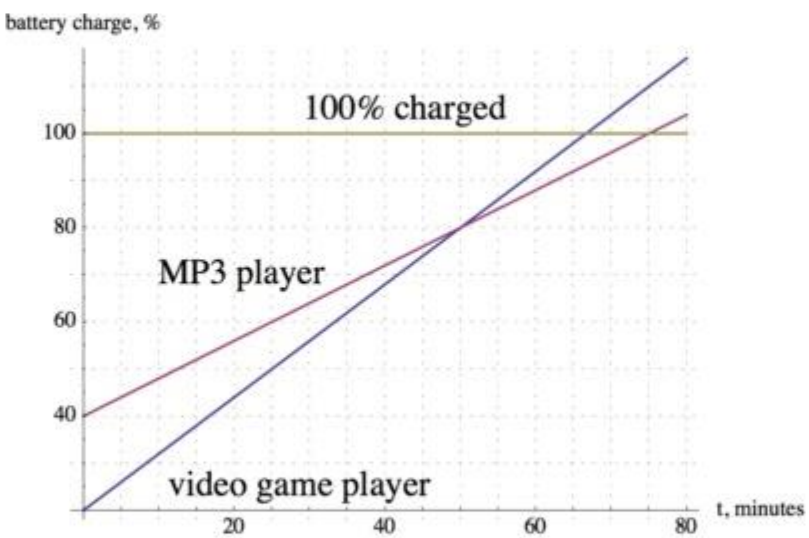
1. ***With the given information, it is quite straight-forward to graph the functions for both devices. For the MP3 player we have a starting value (i.e. vertical intercept) of 40% and a rate of change (i.e. slope) of 12/15 = 0.8 percentage points per minute.***

   ***For the video game player we have a starting value of 20% and the rate of change for the data in the table is constant at 12/10 = 1.2 percentage points per minute. Below are the two graphs.***




*We can estimate from the graph that after 60 minutes the MP3 player has a battery charge of just under 90% and the video game player has a battery charge of just over 90%. Zooming in on a graphing calculator or other graphing device would give us better estimates.*

2. *To find out how long it will take until both batteries are fully charged, we need to find values of t for which the output value is 100% for both functions.*




*From the graph we see that the MP3 player will take the longest to charge and it will take about 75 minutes total. So if Sam's family can wait an extra 15 minutes before they leave, Sam would have both devices fully charged.*

## <u>Battery Charging</u> (Spotlight Task)

Name ___________________________

**PART 1**

Sam wants to take his MP3 player and his video game player on a car trip. An hour before they plan to leave, he realized that he forgot to charge the batteries last night. At that point, he plugged in both devices so they can charge as long as possible before they leave.

What problem(s) is Sam facing? What math problems might this scenario be leading up to? What information do you need to know in order to help Sam?

**PART 2**

Sam knows that his MP3 player has 40% of its battery life left and that the battery charges by an additional 12 percentage points every 15 minutes.

His video game player is new, so Sam doesn't know how fast it is charging but he recorded the battery charge for the first 30 minutes after he plugged it in.

| time charging (minutes) | 0 | 10 | 20 | 30 |
|---|---|---|---|---|
| video game player battery charge (%) | 20 | 32 | 44 | 56 |

a. If Sam's family leaves as planned, what percent of the battery will be charged for each of the two devices when they leave? How do you know?

b. How much time would Sam need to charge the battery 100% on both devices? Explain how you know.

**PART 3**

Students reveal their answers and discuss the similarities and differences in what they found.

## Which is Which?

***Back to Task Table***

## STANDARDS FOR MATHEMATICAL CONTENT:

**Define, evaluate, and compare functions.**

**MGSE8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

**MGSE8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

## STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## BACKGROUND KNOWLEDGE:

This task builds on the work done in the previous three tasks, specifically the *Vending Machine* task. The main difference here is that students are required to use domain, and range in their reasoning/explanations.

## *COMMON MISCONCEPTIONS:*

*Students may confuse the input and output values/variables. This could result in the inverse of the function. This misconception can be a tremendous teaching tool. If you observe students showing evidence of this misconception, take an example (anonymously) and use it as a teachable moment. Ask students to discuss the work, discover the error, and compare it to the actual function. Students benefit from this type of exploration, not just to correct a misconception, but also gain ownership of the knowledge of inverse functions in the process.*

## ESSENTIAL QUESTIONS:

- What is a function?
- What are the characteristics of a function?
- How is a function different from a relation?
- How can a function be recognized in any form?
- What is the best way to represent a function?
- How can you use functions to model real-world situations?

## MATERIALS:

- Copies of task for students

## GROUPING:

- Individual/Partner

## TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students are presented with the opportunity to apply the vocabulary of domain, range, input, output, relation and function, by explaining, in writing, how each real life experience is like a function.

## DIFFERENTIATION:

**Extension**

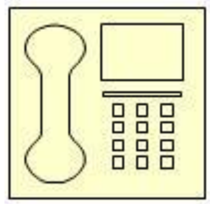
- Write a real-life situation problem, similar to those in the task you just completed, that could be represented as a function. Explain why your situation represents a function (be sure to use the vocabulary!) Then, write a real-life situation problem, similar to those in the task you just completed, that could be a relation but not a function. Explain why your situation is not a function (be sure to use the vocabulary!)

**Interventions/Scaffolding**

- Have students review the vocabulary of domain, range, input, output, relation and function prior to starting this task. Group students in a way that promotes cooperative learning (pairing less verbal students with more verbal students).

## Which is Which?

***Directions:*** Using the vocabulary of domain, range, input, output, relation and function, explain how each real life experience is like a function.



1. When you pick up a phone and dial (404) 624-9453, you will get Zoo Atlanta. When you pick up a phone and dial (404) 656-2846, you will get the Georgia State Capital Museum. When you pick up a phone and dial (478) 621-6701, you will get the Bibb County Tax Assessor's Office. When you pick up a phone and dial a specific number, you will get only one party.

***Solution***

***This represents a function. The domain is the set of inputs .The range is the set of all output or results. In the telephone example, the domain is the set of phone numbers. The range is the set of offices that are reached. Every time a specific number is dialed, the same office should always be reached. If the number for the Bibb County Tax Assessor's office is dial, the Georgia State Capital Museum should not answer.***

*Problems continue on the next page. →*

2. Malachi was going to make a surprise for his mother on her birthday. On the kitchen table, he had placed flour, sugar, vanilla, chocolate chips, butter, and eggs. Vanessa wanted to surprise their mother, too. On the kitchen counter, Vanessa assembled spaghetti sauce, noodles, ricotta cheese, oregano, and mozzarella cheese. While each of them was making a surprise for their mother, they each were creating a different surprise. Malachi could not use Vanessa's ingredients to make his surprise, nor could Vanessa use Malachi's ingredients to make her surprise.

***Solution***

***The domain or inputs is represented by the ingredients. The range or outputs is the surprise that is created. The relation between the ingredients and the surprise may be a function because the combination will always yield the same result. It may be a relation because the result of those ingredients does not always yield the same result. For example, Vanessa's ingredients can be used to create lasagna or stuffed manicotti. Malachi's ingredients can be used to create cookies or brownies.***

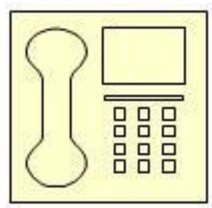
3. Suki babysat three nights last week. One night she babysat 3 hours and earned $45. The next night she babysat 2 hours and earned $30. On the third night she babysat five hours and earned $75. She knows if she can babysit for 10 hours she will earn $150.

***Solution***

***The domain is the set of numbers of hours she babysat. The range is the amount she is paid for any given number of hours worked. The relation is a function because there is only one payment amount for any given number of hours worked. An earned amount is always determined by the hours worked.***

## Which is Which?

***Directions:*** Using the vocabulary of domain, range, input, output, relation and function, explain how each real life experience is like a function.



1. When you pick up a phone and dial (404) 624-9453, you will get Zoo Atlanta. When you pick up a phone and dial (404) 656-2846, you will get the Georgia State Capital Museum. When you pick up a phone and dial (478) 621-6701, you will get the Bibb County Tax Assessor's Office. When you pick up a phone and dial a specific number, you will get only one party.

*Problems continue on the next page. →*

2. Malachi was going to make a surprise for his mother on her birthday. On the kitchen table, he had placed flour, sugar, vanilla, chocolate chips, butter, and eggs. Vanessa wanted to surprise their mother, too. On the kitchen counter, Vanessa assembled spaghetti sauce, noodles, ricotta cheese, oregano, and mozzarella cheese. While each of them was making a surprise for their mother, they each were creating a different surprise. Malachi could not use Vanessa's ingredients to make his surprise, nor could Vanessa use Malachi's ingredients to make her surprise.

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## Modeling Situations with Linear Equations – (FAL)

***Back to Task Table***

*Source: Balanced Assessment Materials from Mathematics Assessment Project*
http://map.mathshell.org/materials/download.php?fileid=673

In this FAL, students will explore relationships between variables in everyday situations, find unknown values from known values, find relationships between pairs of unknowns, and express these as tables and graphs, find general relationships between several variables, and express these in different ways by rearranging formulae.

## STANDARDS ADDRESSED IN THIS TASK:

**Define, evaluate, and compare functions.**

**MGSE8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

**MGSE8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

## STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

2. Reason abstractly and quantitatively.
4. Model with mathematics.

## *COMMON MISCONCEPTIONS:*

- *Students may confuse the input and output values/variables. This could result in the inverse of the function. This misconception can be a tremendous teaching tool. If you observe students showing evidence of this misconception, take an example (anonymously) and use it as a teachable moment. Ask students to discuss the work, discover the error, and compare it to the actual function. Students benefit from this type of exploration, not just to correct a misconception, but also gain ownership of the knowledge of inverse functions in the process.*
- *Students may have trouble writing a general rule for these situations because of their confusion about dependent and independent variables.*
- *Some students may confuse the x- and y- axes on the coordinate plane. Emphasizing that the first value is plotted on the horizontal axes (usually x, with positive to the right) and the second is the vertical axis (usually called y, with positive up) point out that this is merely a convention. It could have been otherwise, but it is very useful for people to agree on a standard customary practice.*
- *Additional misconceptions can be found within the FAL Teacher Guide.*

## ESSENTIAL QUESTIONS:

- How do you represent relations and functions using tables, graphs, words, and algebraic equations?
- How can graphs and equations of functions help us to interpret real-world problems?

## MATERIALS:

- See FAL

## GROUPING:

- Partner/Small Group

## TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, "Modeling Situations with Linear Equations," is a Mathematics Assessment Project Assessment Task that can be found at the website:
http://www.map.mathshell.org/materials/tasks.php?taskid=278&subpage=expert

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=673

## FAL: Create Matching Function Cards

*Back to Task Table*

*Source: Georgia Mathematics Design Collaborative*

This lesson is intended to help you assess how well students are able to:

- Create functions
- Change the form of how a function is represented

**STANDARDS ADDRESSED IN THIS TASK:**

**MGSE8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

**MGSE8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

**STANDARDS FOR MATHEMATICAL PRACTICE:**

This lesson uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them
6. Attend to precision.
7. Look for and make use of structure

**TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:**

Tasks and lessons from the Georgia Mathematics Design Collaborative are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding.

The task, *Create Matching Function Cards*, is a Formative Assessment Lesson (FAL) that can be found on the Georgia Mathematics online professional learning community, edWeb.net. Once logged in, the task can be found directly at: https://www.edweb.net/?14@@.5abfa3bd.

## Culminating Task: <u>Sorting Functions</u>

*Back to Task Table*

*Source: Inside Mathematics*
https://www.insidemathematics.org/sites/default/files/materials/sorting%20functions.pdf

In this culminating task, students will match equations to tables, graphs, and rules.

## STANDARDS OF MATHEMATICAL CONTENT:

### Define, evaluate, and compare functions.

**MGSE8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

**MGSE8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

## STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

### COMMON MISCONCEPTIONS:

*Students may confuse the input and output values/variables. This could result in the inverse of the function. This misconception can be a tremendous teaching tool. If you observe students showing evidence of this misconception, take an example (anonymously) and use it as a teachable moment. Ask students to discuss the work, discover the error, and compare it to the actual function. Students benefit from this type of exploration, not just to correct a misconception, but also gain ownership of the knowledge of inverse functions in the process.*

*Some student will mix up the x- and y- axes on the coordinate plane. Emphasizing that the first value is plotted on the horizontal axes (usually x, with positive to the right) and the second is the vertical axis (usually called y, with positive up) point out that this is merely a convention. It could have been otherwise, but it is very useful for people to agree on a standard customary practice.*

## ESSENTIAL QUESTIONS:

- What is a function?
- What are the characteristics of a function?
- How can a function be recognized in any form?
- How do you represent relations and functions using tables, graphs, words, and algebraic equations?
- How can graphs and equations of functions help us to interpret real-world problems?

## MATERIALS:

- Scissors
- Copies of diagrams and tables for each student or group of students.

## GROUPING:

- Individual/Partner/Small Group

## TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this culminating task, students will have the opportunity to solidify their understanding of the concept of functions, as well as to gain experience in recognizing and comparing functions in various forms.

## DIFFERENTIATION:

**Extension**
Why is it important to be able to move fluidly between the multiple representations of functions? Think of a situation in which you might encounter each representation.

**Intervention/Scaffolding**
Prompt struggling students by asking guiding questions. (see task)

## Appendix

*Back to Task Table*

**MGSE8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

https://www.illustrativemathematics.org/content-standards/8/F/A/1
http://www.visualpatterns.org/
http://illuminations.nctm.org/Unit.aspx?id=6526

**MGSE8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

https://www.illustrativemathematics.org/content-standards/8/F/A/2
http://illuminations.nctm.org/Unit.aspx?id=6526
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