Georgia
Standards of Excellence
Curriculum Frameworks

Mathematics

GSE Grade 8
Unit 5: Linear Functions

Georgia Department of Education

Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

Unit 5
LINEAR FUNCTIONS

TABLE OF CONTENTS

OVERVIEW

In this unit students will:

- graph proportional relationships;
- interpret unit rate as the slope;
- compare two different proportional relationships represented in different ways;
- use similar triangles to explain why the slope is the same between any two points on a non-vertical line;
- derive the equation $y = mx$ for a line through the origin;
- derive the equation $y = mx + b$ for a line intercepting the vertical axis at b; and
- interpret equations in $y = mx + b$ form as linear functions.

This unit focuses on extending the understanding of ratios and proportions. Unit rates have been explored in Grade 6 as the comparison of two different quantities with the second unit a unit of one, (unit rate). In seventh grade unit rates were expanded to complex fractions and percents through solving multi-step problems such as: discounts, interest, taxes, tips, and percent of increase or decrease. Proportional relationships were applied in scale drawings, and students should have developed an informal understanding that the steepness of the graph is the slope or unit rate. Now unit rates are addressed formally in graphical representations, algebraic equations, and geometry through similar triangles.

Distance time problems are notorious in mathematics. In this unit, they serve the purpose of illustrating how the rates of two objects can be represented, analyzed, and described in different ways: graphically and algebraically. Students create representative graphs and the meaning of various points. They then compare the same information when represented in an equation.

By using coordinate grids and various sets of three similar triangles, students prove that the slopes of the corresponding sides are equal, thus making the unit rate of change equal. After proving with multiple sets of triangles, students generalize the slope to $y = mx$ for a line through the origin and $y = mx + b$ for a line through the vertical axis at b.

In Grade 8, the focus is on linear functions, and students begin to recognize a linear function from its form $y = mx + b$. Students also need experiences with nonlinear functions, including functions given by graphs, tables, or verbal descriptions but for which there is no formula for the rule, such as a girl's height as a function of her age. Students learn that proportional relationships are part of a broader group of linear functions, and they are able to identify whether a relationship is linear. Nonlinear functions are included for comparison. Later, in high school, students use function notation and are able to identify types of nonlinear functions.

In the elementary grades, students explore number and shape patterns (sequences), and they use rules for finding the next term in the sequence. At this point, students describe sequences both by rules relating one term to the next and also by rules for finding the *n*th term directly. (In high school, students will call these recursive and explicit formulas.) Students express rules in both words and in symbols. Instruction focuses on additive and multiplicative sequences as well as sequences of square and cubic numbers, considered as areas and volumes of cubes, respectively. Students compute the area and perimeter of different-size squares and identify that one relationship is linear while the other is not by either looking at a table of value or a graph in which the side length is the independent variable (input) and the area or perimeter is the dependent variable (output).

When plotting points and drawing graphs, students develop the habit, based upon the context, of determining whether it is reasonable to "connect the dots" on the graph. In some contexts, the inputs are discrete, and connecting the dots can be misleading.

Students examine the graphs of linear functions and use graphing calculators or computer software to analyze or compare at least two functions at the same time. Illustrate with a slope triangle where the run is "1" that slope is the "unit rate of change." Compare this in order to compare two different situations and identify which is increasing/decreasing at a faster rate.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under "Evidence of Learning" be reviewed early in the planning process. A variety of resources should be utilized to supplement this unit. This unit provides much needed content information, but excellent learning activities as well. The tasks in this unit illustrate the types of learning activities that should be utilized from a variety of sources.

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

STANDARDS FOR MATHEMATICAL PRACTICE

Refer to the Comprehensive Course Overview for more detailed information about the Standards for Mathematical Practice.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

CONTENT STANDARDS

Understand the connections between proportional relationships, lines, and linear equations.

MGSE8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

MGSE8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Define, evaluate, and compare functions.

MGSE8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

BIG IDEAS

- Patterns and relationships can be represented graphically, numerically, and symbolically.
- Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems.
- Functions are a special type of relationship that uniquely associates members of one set with members of another set.
- The understanding of functions is strengthened when they are explored across representations because each one provides a different view of the same relationship.

ESSENTIAL QUESTIONS

- How can patterns, relations, and functions be used as tools to best describe and help explain real-life relationships?
- How can the same mathematical idea be represented in a different way? Why would that be useful?
- What is the significance of the patterns that exist between the triangles created on the graph of a linear function?
- When two functions share the same rate of change, what might be different/the same about their each of their representations? W
- What does the slope of the function line tell me about the unit rate?
- What does the unit rate tell me about the slope of the function line?

CONCEPTS/SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- determining unit rate
- applying proportional relationships
- recognizing a function in various forms
- plotting points on a coordinate plane
- understanding of writing rules for sequences and number patterns
- differences in graphing of discrete and continuous data
- attributes of similar figures

FLUENCY

It is expected that students will continue to develop and practice strategies to build their capacity to become fluent in mathematics and mathematics computation. The eventual goal is automaticity with math facts. This automaticity is built within each student through strategy development and practice. The following section is presented in order to develop a common understanding of the ideas and terminology regarding fluency and automaticity in mathematics:

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number**.**

Deep Understanding: Teachers teach more than simply "how to get the answer" and instead support students' ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students:

- **flexibly use a combination of deep understanding, number sense, and memorization.**
- **are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.**
- **are able to articulate their reasoning.**
- **find solutions through a number of different paths.**

For more about fluency, see: http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and: http://joboaler.com/timed-tests-and-the-development-of-math-anxiety

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for middle school students. **Note – Different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.** The definitions below are from the Common Core State Standards Mathematics Glossary and/or the Common Core GPS Mathematics Glossary when available.

Visit http://intermath.coe.uga.edu or http://mathworld.wolfram.com to see additional definitions and specific examples of many terms and symbols used in grade 8 mathematics.

- **Intersecting Lines:**
- **Origin:**
- **Proportional Relationships:**
- **Slope:**
- **Unit Rate:**

FORMATIVE ASSESSMENT LESSONS (FAL)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Guide.

SPOTLIGHT TASKS

A Spotlight Task has been added to each CCGPS mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for

Mathematical Practice, appropriate unit-level Common Core Georgia Performance Standards, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky.

3-ACT TASKS

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

TASKS

Task Name	**Task Type** *Grouping Strategy*	Content Addressed	Standard(s)
UP (Spotlight Task)	Performance Task *Partner*	Identifying independent and dependent variables. Relating the unit rate to the slope of a function line.	MGSE8.EE.5 MGSE8.EEE.6 MGSE8.F.3
Starburst Wrapper Dress (Spotlight Task)	Performance Task *Individual/Partner*	Identifying independent and dependent variables. Relating the unit rate to the slope of a function line.	MGSE8.EE.5 MGSE8.EEE.6 MGSE8.F.3
By the Book	Performance Task *Partner*	Identifying independent and dependent variables. Relating the unit rate to the slope of a function line.	MGSE8.EE.5 MGSE8.EE.6 MGSE8.F.3
Lines and Linear Equations (FAL)	Formative Assessment Lesson *Partner/Small Group*	Translate between equations and graphs and interpret speed as the slope of a linear graph.	MGSE8.F.4 MGSE8.F.5
What's My Line?	Performance Task *Individual/Partner*	Relating similar triangles to slope using different points on a graph.	MGSE8.EE.5 MGSE8.EE.6 MGSE8.F.3
Ditch Diggers (Spotlight Task)	Performance Task *Partner/Small Group*	Interpret a situation to determine the rate of change.	MGSE8.EE.5 MGSE8.F.3
Analyzing Linear Functions (FAL)	Formative Assessment Lesson *Partner/Small Group*	Understand how slope-intercept form and standard form relate to the graph and table of values.	MGSE8.F.2 MGSE8.F.3
Solving Real-Life Problems: Baseball Jerseys (FAL)	Formative Assessment Lesson *Partner/Small Group*	Interpret a situation and represent the variables mathematically.	MGSE8.EE.5
Culminating Task: Filling the Tank	Performance Task *Individual/Partner*	Understanding the unit rate/slope relationship in equations, graphs, and tables. Understanding the impact on graphs, tables, and equations of a starting point other than one.	MGSE8.EE.5 MGSE8.EE.6 MGSE8.F.3
APPENDIX			

Up (Spotlight Task)

Back to Task Table

Task adapted from http://www.101qs.com/2069-up

STANDARDS FOR MATHEMATICAL CONTENT

Understand the connections between proportional relationships, lines, and linear equations.

MGSE8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

MGSE8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Define, evaluate, and compare functions.

MGSE8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTIONS

- How can I find the constant rate of change?
- How can I use the constant rate of change to find the output for any input?

MATERIALS

- Latex Balloons filled with Helium
- Washers
- Scales
- Internet access to find the weight of a battleship
- Copy of generic student work sheet which can be found at the end of this task

TIME NEEDED
- 1-2 day

TEACHER NOTES
Show the students the picture of the war ship lifted by helium balloons and ask what they notice and then what they are curious about *(Act 1)*. Record their questions on the board with the student's name. You will receive a wide variety of questions. This is just a sampling from the website above:

1. Dan Meyer: How many balloons would that take, really?
2. Laurie Townshend: Is this possible?
3. Jen Guckenberger: How much does the ship weigh? How many balloons are there?
4. Michelle Ferrin: Why?

While all of them do not address our standards above many of them do. We are looking for a rate of change so whether we are looking at calories, money, or size of the garment our goal can be met. Students will then use mathematics to answer their own questions *(Act 2)*. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them. Once students have made their discoveries, it is time for the great reveal *(Act 3)*. Teachers should support good student dialogue and take advantage of comments and questions to help guide students into correct mathematical thinking.

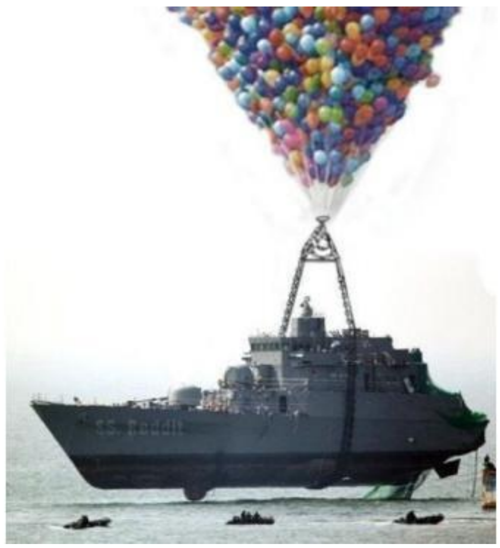
TASK DESCRIPTION

The following 3-Act Task can be found at: http://www.101qs.com/2069-up

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:

Show students this picture:



ACT 2:
Work session. Provide students with materials/information they need as they ask for it. This is the time for students to make sense of the problem and determine the tools/information they need to solve it (SMP 1 and 5). Students may ask for helium balloons and small items like washers to determine how much a balloon will lift. You may need to provide students with a helium latex balloon(s) and washers to determine how much a balloon will lift.

ACT 3
Students will compare and share solution strategies.

- Reveal the answer. Discuss the theoretical math versus the practical outcome.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.

ACT 4

Extension:
Have students determine to number of balloons it would take to lift them. How much would that cost?

Intervention:
Provide the student with a table to complete.

# of Balloons	Weight Lifted
1	
2	
3	
4	
5	
6	
n	

Up Name:________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate the result of the main question? Explain?

Place an estimate that is too high and too low on the number line

Low estimate *Place an "x" where your estimate belongs* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:_______________________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Starburst Wrapper Dress (Spotlight Task) *Back to Task Table*

Task adapted from http://www.101qs.com/721-starburst-wrapper-dress

STANDARDS FOR MATHEMATICAL CONTENT

Understand the connections between proportional relationships, lines, and linear equations.

MGSE8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

MGSE8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Define, evaluate, and compare functions.

MGSE8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTIONS

- How can I find the constant rate of change?
- How can I use the constant rate of change to find the output for any input?

MATERIALS

- Picture of Starburst Dress
- Starburst Wrappers
- Copy of generic student work sheet which can be found at the end of this task

TIME NEEDED

- 1-2 day

<u>TEACHER NOTES</u>

Show the students the picture of the Starburst Dress and ask what they notice and then what they are curious about *(Act 1)*. Record their questions on the board with the student's name. You will receive a wide variety of questions. This is just a sampling from the website above:

1. Nathan Kraft: How much money did she save by making that dress?
2. Max Ray: How many starburst wrappers to make that shirt?
3. Andrew Stadel: Was the Starburst dress cheaper than buying a regular dress?
4. Jake Jouppi: how many starburst?
5. David Cox: How many Starbursts did it take to make the dress?
6. John Scammell: How many candies? And, if her date gets hungry, what will she wear?
7. Patti Charron: How long did it take to make?
8. statler hilton: Does she have diabetes yet?
9. Bob Lochel: How many wrappers for that dress?
10. Wesley Peacock: how long did that take to make?
11. Cam MacDonald: How many calories did that top cost?

While all of them do not address our standards above many of them do. We are looking for a rate of change so whether we are looking at calories, money, or size of the garment our goal can be met. Students will then use mathematics to answer their own questions *(Act 2)*. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them. Once students have made their discoveries, it is time for the great reveal *(Act 3)*. Teachers should support good student dialogue and take advantage of comments and questions to help guide students into correct mathematical thinking.

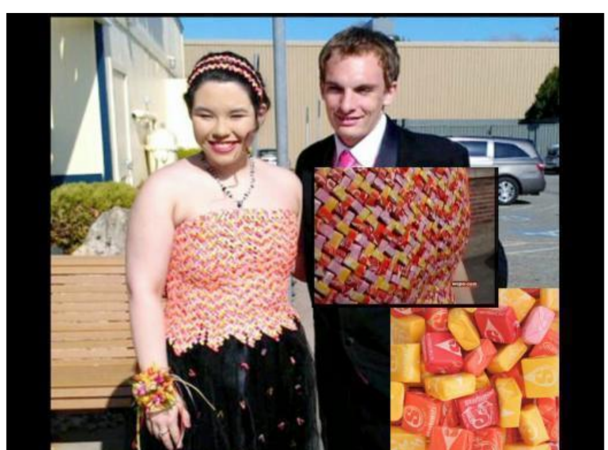
Task Description

The following 3-Act Task can be found at: http://www.101qs.com/721-starburst-wrapper-dress

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:

Show students this picture:



ACT 2:

Work session. You may need to provide students with Starburst wrappers so that they can recreate a swatch of "material". Students may also need caloric or price information.

ACT 3
Students will compare and share solution strategies.

- Reveal the answer. Discuss the theoretical math versus the practical outcome.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.

ACT 4

Extension:
Have students figure out the number of wrappers per yard, then have students convert a bodice pattern to see how many wrappers are needed per size.

Intervention:
Have a swatch of the "material" already created. Provide the student with a table to complete.

Square Inch(es)	**# of Wrappers**
1	
2	
3	
4	
5	
6	
n	

Starburst Wrapper Dress ____ Name:________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate the result of the main question? Explain?
Place an estimate that is too high and too low on the number line
Low estimate *Place an "x" where your estimate belongs* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:_______________________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

By the Book

Back to Task Table

In this task, students will identify independent and dependent variables. Students will also relate the unit rate to the slope of a function line.

STANDARDS FOR MATHEMATICAL CONTENT

Understand the connections between proportional relationships, lines, and linear equations.

MGSE8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

MGSE8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Define, evaluate, and compare functions.

MGSE8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE:

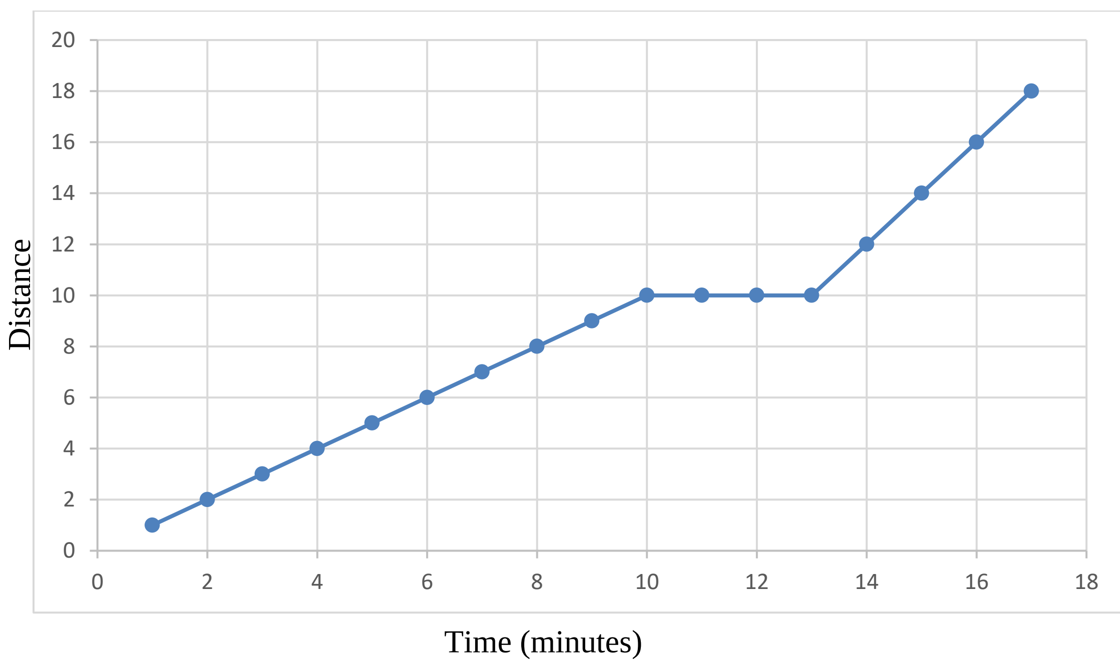
The explorations of rate of change in the previous tasks as well as explorations from grades 6 and 7 are purposeful in developing the concept of slope, which is the numeric value that describes the rate of change for a linear function. Conceptually, slope signifies how much y increases when x increases by 1. If a line contains two points, say 3, 5 and 4, -6, you can see that as x increases by 1, y decreases by 11. So, the rate of change or slope is -11.

Exploring slope first through reasoning is important if the goal is for them to make sense of and remember the formula for calculating slope when given two points. As students engage in further experiences and explorations involving slope, they will begin to generalize that you can find the rate of change (or slope) by finding the difference in the *y* values and dividing by the difference in the *x* values.

Experiences involving these simple contexts can also be very useful in exploring and making sense of zero slope and no slope. These two ideas are easily confused, but a simple context of walking rates can be extremely helpful:

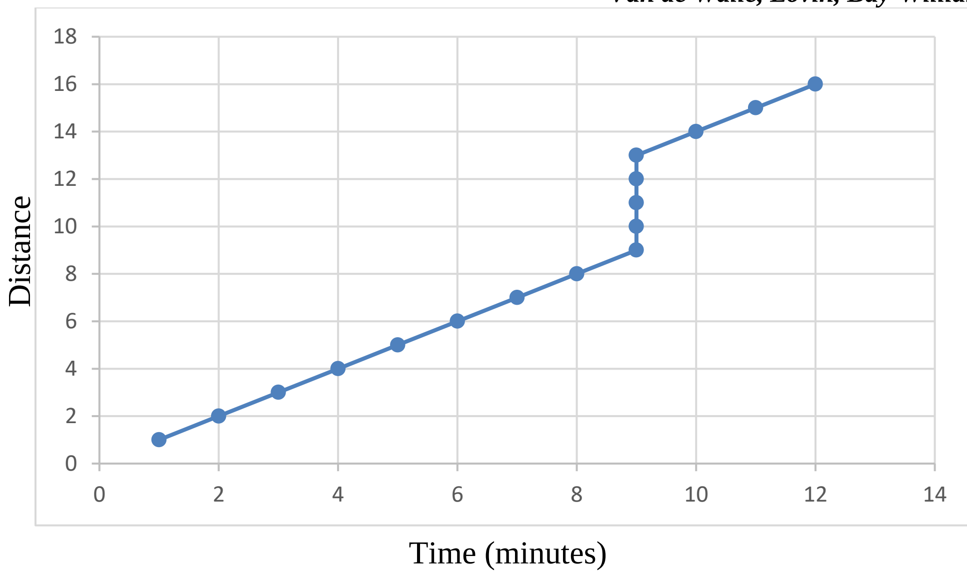
> You walk for 10 minutes at a rate of 1 mile per hour, then stop for 3 minutes to watch some puppies playing in the snow, then walk for 4 more minutes at 2 miles per hour.

What will the graph look like for the 3 minutes when you stop? What is your rate when you stop? Your rate is 0 because you are at the same distance for 3 minutes. The graph will be a horizontal line. (See Graph Below) Note: Students should create a graph like this from a context like the one above.



What would happen if your walking story included a vertical line (a line with no slope)? It would mean that you traveled some distance while no time passed (See sample graph below)! Rate is based on a change of 1 in the x value (in this case the x value is time).

Source: Teaching Student Centered Mathematics, vol. 3, Grades 6-8, Van de Walle, Lovin, Bay-Williams, Karp, 2014



COMMON MISCONCEPTIONS:

- *Students may confuse the input and output values/variables. This could result in the inverse of the function. This misconception can be a tremendous teaching tool. If you observe students showing evidence of this misconception, take an example (anonymously) and use it as a teachable moment. Ask students to discuss the work, discover the error, and compare it to the actual function. Students benefit from this type of exploration, not just to correct a misconception, but also gain ownership of the knowledge of inverse functions in the process.*
- *Students may have trouble writing a general rule for these situations. They tend to confuse the dependent and independent variable. Contextual situations will help students correct this misconception.* ***http://www.visualpatterns.org/***
- *Some student may confuse the x- and y- axes on the coordinate plane. Emphasizing that the first value is plotted on the horizontal axes (usually x, with positive to the right) and the second is the vertical axis (usually called y, with positive up) point out that this is merely a convention. It could have been otherwise, but it is very useful for people to agree on a standard customary practice.*
- *Some students may mistakenly think only of a straight line as horizontal or vertical. This misconception may be corrected through classroom discussion and determining a tool that could be used to check if lines are straight (such as a ruler).*

ESSENTIAL QUESTIONS:

- How can patterns, relations, and functions be used as tools to best describe and help explain real-life relationships?
- How can the same mathematical idea be represented in a different way? Why would that be useful?
- What does the slope of the function line tell me about the unit rate?
- What does the unit rate tell me about the slope of the function line?

MATERIALS:

- Copies of task for students
- Straightedge
- Graph paper http://incompetech.com/graphpaper or https://www.desmos.com/calculator

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will determine the variables in a relationship, and then identify the independent variable and the dependent variable. Students will complete a function table by applying a rule, within a function table and/or a graph, and will create and use function tables in order to solve real world problems.

DIFFERENTIATION:

Extension:

- Using a textbook, determine the thickness of a page using the same process you used in the task. Develop the equation for the thickness of the pages in your textbook. Create a table and a graph and then explain your process in a detailed paragraph.

Intervention/Scaffolding:

- Have students review independent and dependent variables prior to beginning this task. For problem 5, have students measure out a stack of paper 2 cm in height, and then count the number of sheets of paper in the stack (round to the nearest 10 sheets). (This may help them appreciate why an equation for estimating this number is needed.) Some students may get overwhelmed when it comes to answering questions about Option A and Option B. Those students may only need to choose one or the other to complete.

By the Book

The thickness of book manuscripts depends on the number of pages in the manuscript. On your first day working at the publishing house, you are asked to create tools editors can use to estimate the thickness of proposed manuscripts. After experimenting with the different weights of paper, you discover that 100 sheets of paper averages 1.25 cm thick.

Your task is to create a table of possible pages and the corresponding thickness. This information should also be presented in a graph. Since you cannot include all the possible number of pages, you also need a formula that editors can use to determine the thickness of any manuscript.

Solutions

1. Based on the request, determine the variables in the relationship, then identify the independent variable and the dependent variable. Add them to the table below.
   Independent variable - ***number of pages in the manuscript***

   Explain your reasoning. ***The number of pages is the independent variable in this scenario because the number of pages in the manuscript determines the thickness of the manuscript.***

   Dependent variable - ***thickness of manuscript in centimeters***

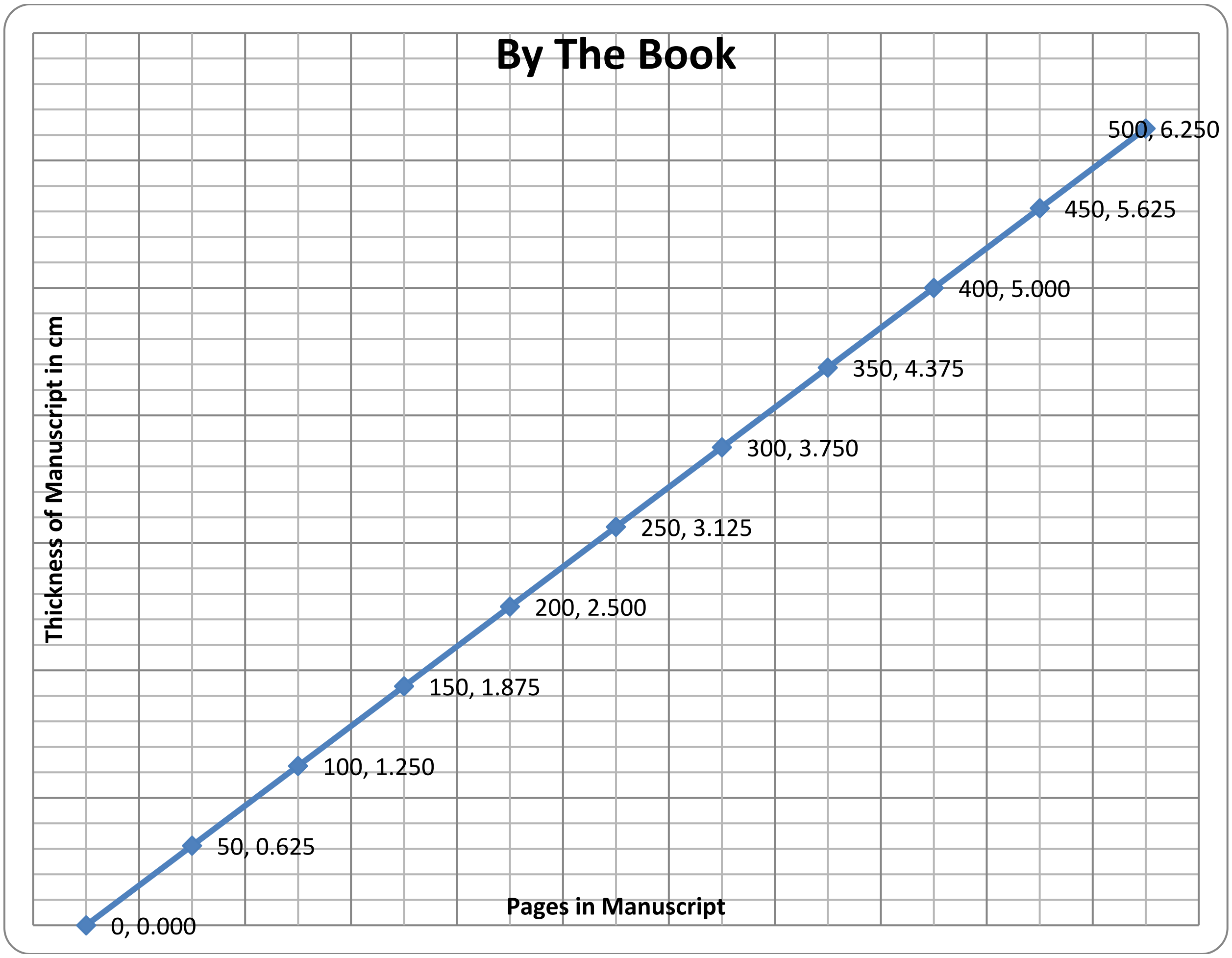
   Explain your reasoning. ***The thickness of the manuscript is determined by the number of pages in the manuscript.***

2. Complete the table for manuscripts from zero to 500 in 50 page intervals.

Number of pages (p)	*Thickness in cm (T)*
0	*0.000*
50	*0.625*
100	*1.250*
150	*1.875*
200	*2.500*
250	*3.125*
300	*3.750*
350	*4.375*
400	*5.000*
450	*5.625*
500	*6.250*

3. Use the information from the table in problem 2 to create a graph.

Solution



4. Write an equation for the thickness of a manuscript given any number of pages it might contain.

Solution

$T = 0.0125p$ where T is thickness of the manuscript and p is the number of pages in the manuscript.

The copyrighters for the publisher's website also asked that the information be represented for them so they can tell how many pages are in a book based on its thickness.

5. Based on the request, determine the variables in the relationship, the independent variable, and the dependent variable.

   Independent variable - ***thickness of manuscript in centimeters***

   Explain your reasoning. ***The thickness of the book is the independent variable because it determines how many pages are in the manuscript.***

   Dependent variable - ***pages in the manuscript***

   Explain your reasoning. ***The number of pages is determined by the thickness of the manuscript.***

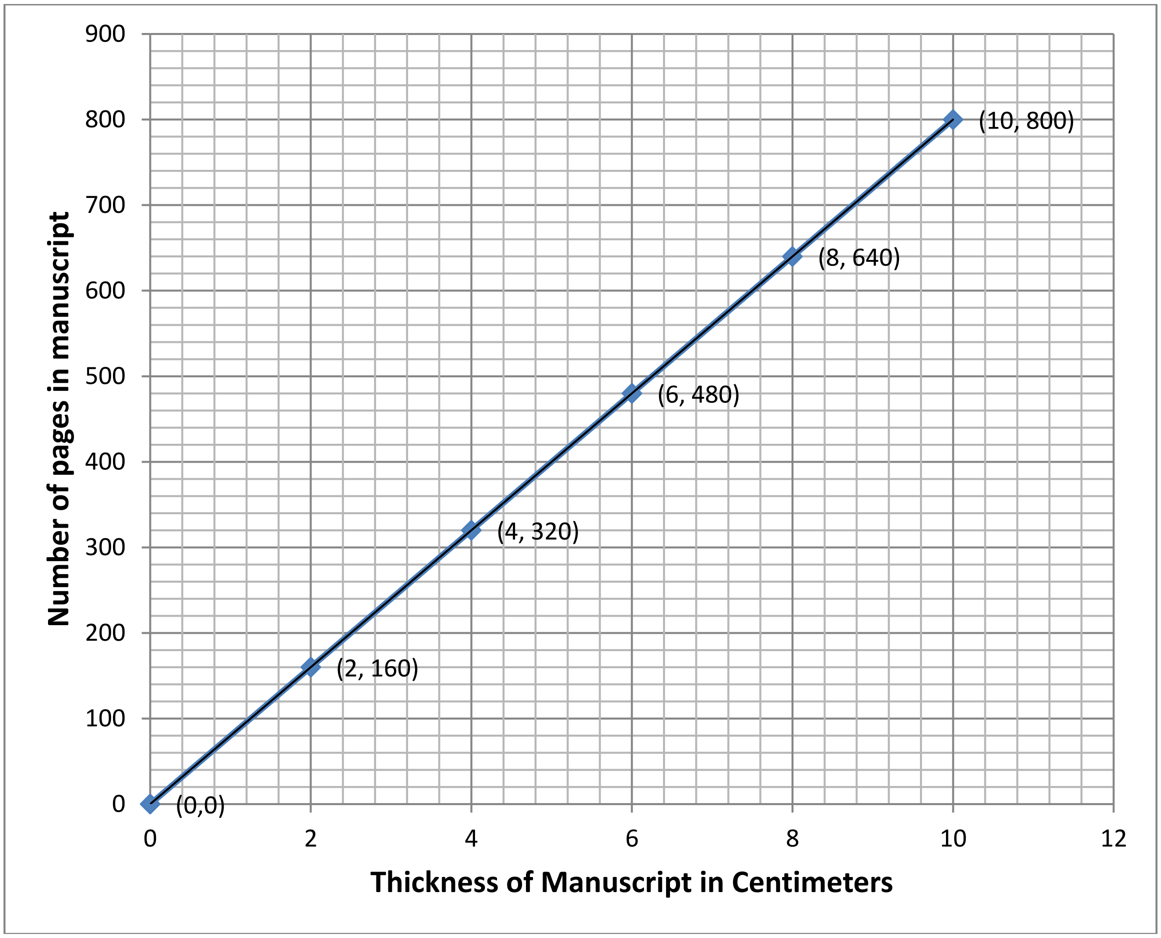
6. Using the information from problem 5, Create a table for manuscripts from zero to 10 in 2 cm intervals.

   Solution

Thickness in cm (t)	*Pages in book (P)*
0	*0*
2	*160*
4	*320*
6	*480*
8	*640*
10	*800*

7. Use the information from your table in problem 6 to create a graph.

Solution



8. Write an equation for the number of pages a manuscript will contain given its thickness.

P = 80t where P is the number of pages in the manuscript and t is thickness of manuscript.

Based on your findings, answer the following questions.

9. What is the unit rate for sheets of paper per centimeter? Explain how you know.

There are 80 pages per centimeter of thickness of the manuscript. This is found by setting up a proportion of pages to centimeters and simplifying.

$$\frac{100\ pages}{1.25\ cm} = \frac{80\ pages}{1\ cm}$$

10. What is the unit rate for height for sheet of paper? Explain how you know.

Each page is 0.0125 cm thick. This is found by setting up a proportion of centimeters to pages and simplifying.

$$\frac{1.25\ cm}{100\ pages} = \frac{0.0125\ cm}{1\ pages}$$

11. How many sheets of paper will be in a manuscript 4 cm thick? Explain how you know.

Using a proportion will show that 4 centimeters of manuscript will have 320 pages.

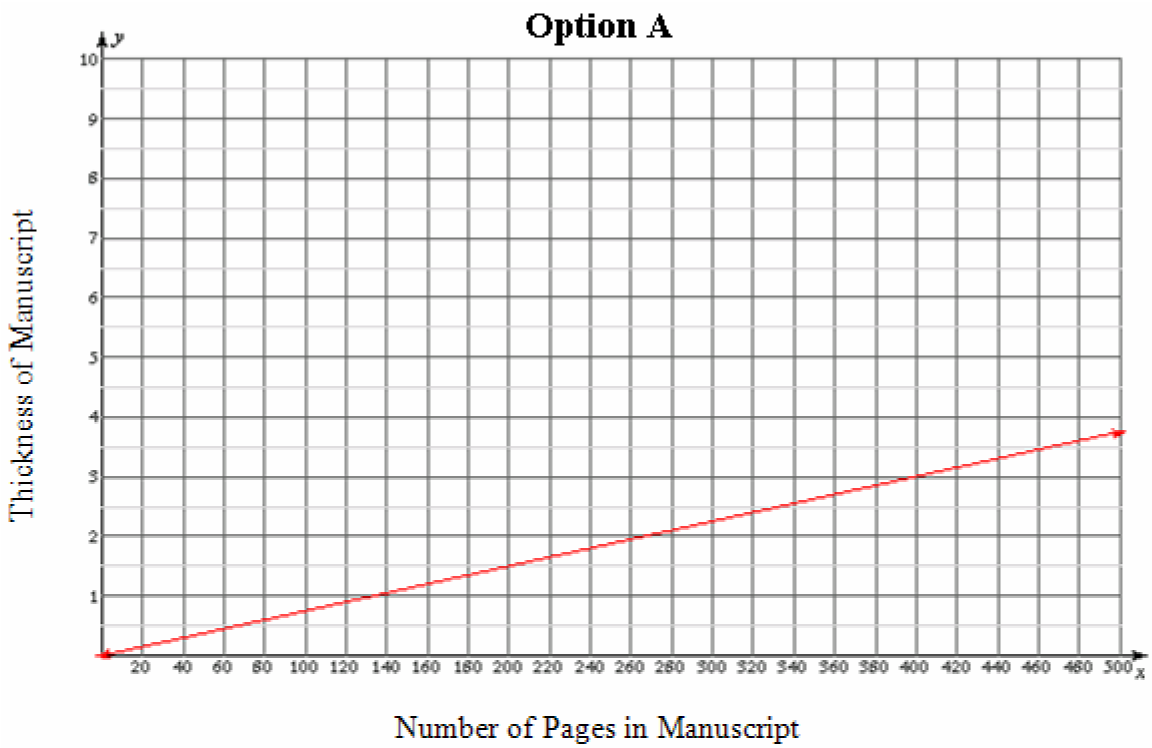
$$\frac{100\ pages}{1.25\ cm} = \frac{320\ pages}{4\ cm}$$

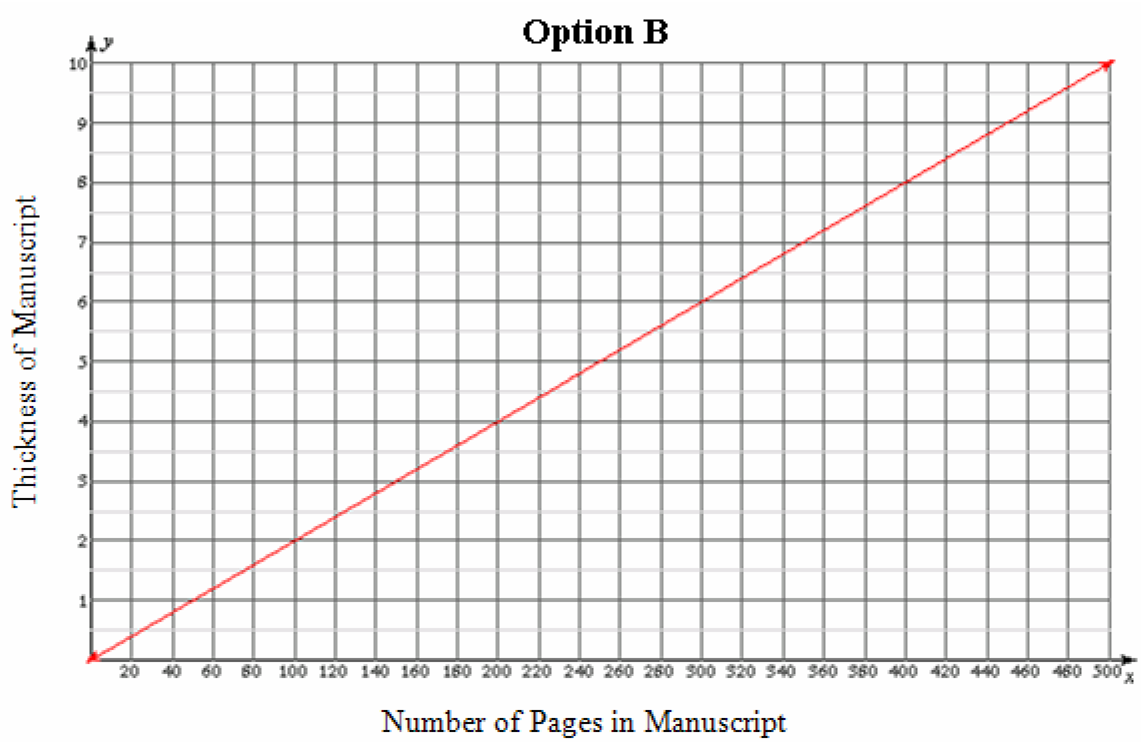
12. How thick will a 300 page manuscript be? Explain how you know.

Using a proportion will show that a 300 page manuscript will be 3.75 cm thick. .

$$\frac{1.25\ cm}{100\ pages} = \frac{3.75\ cm}{300\ pages}$$

The publishing company offers two other types of paper for printing. The graphs below show the thickness of manuscripts based on the number of pages for two different options of paper.





13. Use the graphs to determine the slope of the line for each option.

Solutions

Option A: ***0.0075***

Option B: ***0.02***

14. Express each of these slopes as a unit rate for the thickness of the manuscripts.

Option A: ***1 page = 0.0075cm or*** $^{0.0075\ cm}/_{1\ page}$

Option B: ***1 page = 0.02cm or*** $^{0.02\ cm}/_{1\ page}$

15. Write an equation for each paper option.

Option A: ***T = 0.0075p***

Option B: ***T = 0.02p***

16. Explain how the unit rate/slope is evident in each equation and graph.

Option A: ***Option A is thinner than the original papers because the line is less steep. The coefficient of the independent variable is less for Option A.***

Option B: ***Option B is thicker than the original. The graph of the line is steeper. The slope is greater than any other option. The coefficient of the independent variable is greater for Option B.***

By the Book

The thickness of book manuscripts depends on the number of pages in the manuscript. On your first day working at the publishing house, you are asked to create tools editors can use to estimate the thickness of proposed manuscripts. After experimenting with the different weights of paper, you discover that 100 sheets of paper averages 1.25 cm thick.

Your task is to create a table of possible pages and the corresponding thickness. This information should also be presented in a graph. Since you cannot include all the possible number of pages, you also need a formula that editors can use to determine the thickness of any manuscript.

1. Based on the request, determine the variables in the relationship, then identify the independent variable and the dependent variable. Add them to the table below.
   Independent variable -

   Explain your reasoning.

   Dependent variable -

   Explain your reasoning.

2. Complete the table for manuscripts from zero to 500 in 50 page intervals.

3. Use the information from the table in problem 2 to create a graph.

4. Write an equation for the thickness of a manuscript given any number of pages it might contain.

The copyrighters for the publisher's website also asked that the information be represented for them so they can tell how many pages are in a book based on its thickness.

5. Based on the request, determine the variables in the relationship, the independent variable, and the dependent variable.

   Independent variable -

   Explain your reasoning.

   Dependent variable -

   Explain your reasoning.

6. Using the information from problem 5, Create a table for manuscripts from zero to 10 in 2 cm intervals.

7. Use the information from your table in problem 6 to create a graph.

8. Write an equation for the number of pages a manuscript will contain given its thickness.

Based on your findings, answer the following questions.

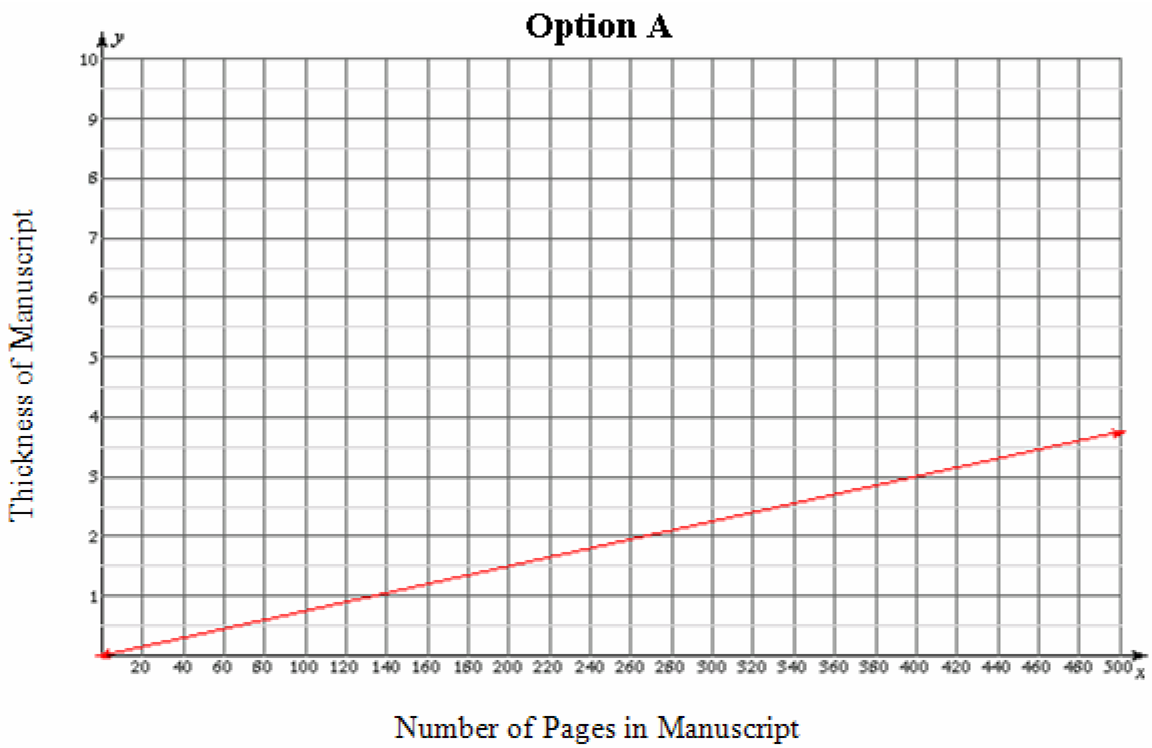
9. What is the unit rate for sheets of paper per centimeter? Explain how you know.

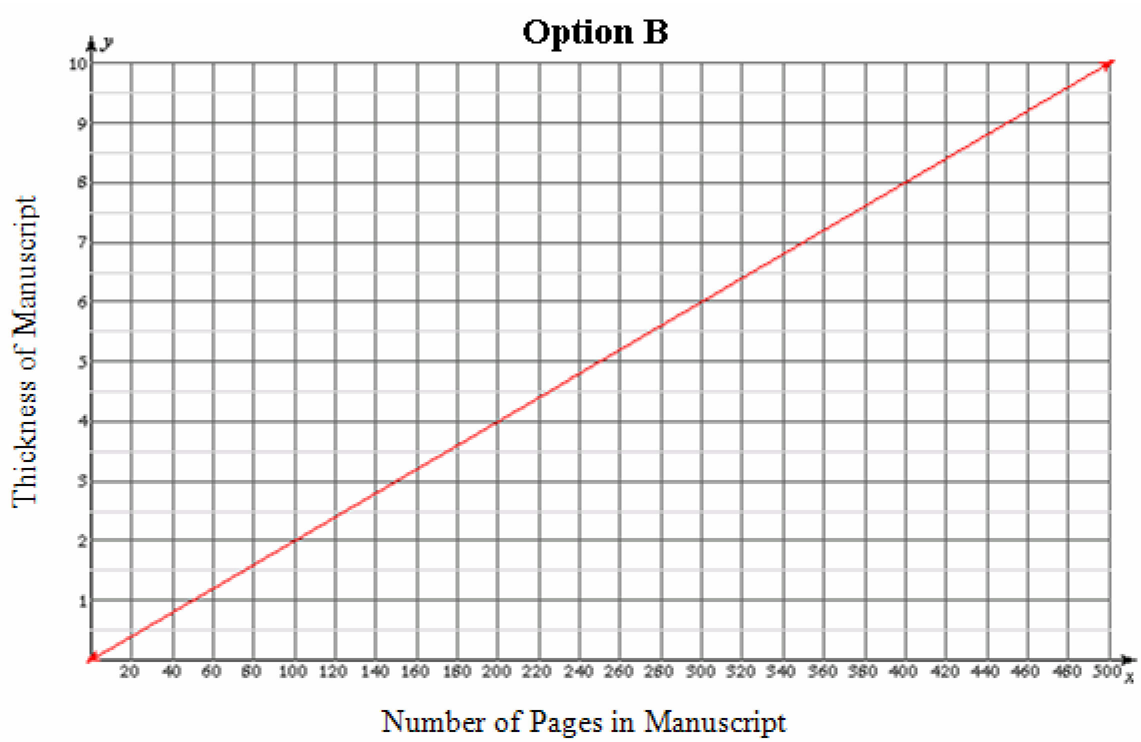
10. What is the unit rate for height for sheet of paper? Explain how you know.

11. How many sheets of paper will be in a manuscript 4 cm thick? Explain how you know.

12. How thick will a 300 page manuscript be? Explain how you know.

The publishing company offers two other types of paper for printing. The graphs below show the thickness of manuscripts based on the number of pages for two different options of paper.





13. Use the graphs to determine the slope of the line for each option.

    Option A:

    Option B:

14. Express each of these slopes as a unit rate for the thickness of the manuscripts.

    Option A:

    Option B:

15. Write an equation for each paper option.

    Option A:

    Option B:

16. Explain how the unit rate/slope is evident in each equation and graph.

    Option A:

    Option B:

Lines and Linear Equations – FAL

Back to Task Table

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1282

In this task, students will translate between equations and graphs and interpret speed as the slope of a linear graph.

STANDARDS FOR MATHEMATICAL CONTENT:

Use functions to model relationships between quantities.

MGSE8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

MGSE8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

STANDARDS FOR MATHEMATICAL PRACTICE:

This lesson uses all of the practices with emphasis on:

2. Reason abstractly and quantitatively.
4. Model with mathematics.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

This task builds on the previous task in that the concept of slope is based on the unit rate. In the context of this task, the unit rate is speed in meters per second. It is intended to help you assess how well students are able to:

- Interpret speed as the slope of a linear graph.
- Translate between the equation of a line and its graphical representation.

COMMON MISCONCEPTIONS:

- *Students may confuse the input and output values/variables. This could result in the inverse of the function. This misconception can be a tremendous teaching tool. If you observe students showing evidence of this misconception, take an example (anonymously) and use it as a teachable moment. Ask students to discuss the work, discover the error, and compare it to the actual function. Students benefit from this type of exploration, not just to correct a misconception, but also gain ownership of the knowledge of inverse functions in the process.*
- *Students may have trouble writing a general rule for these situations. They tend to confuse the dependent and independent variable. Contextual situations will help students correct this misconception.* ***http://www.visualpatterns.org/***
- *Some student may confuse the x- and y- axes on the coordinate plane. Emphasizing that the first value is plotted on the horizontal axes (usually x, with positive to the right) and the second is the vertical axis (usually called y, with positive up) point out that this is merely a convention. It could have been otherwise, but it is very useful for people to agree on a standard customary practice.*
- *Some students may mistakenly think only of a straight line as horizontal or vertical. This misconception may be corrected through classroom discussion and determining a tool that could be used to check if lines are straight (such as a ruler).*
- *Some students may misinterpret the scale, as the scale is in increments of 5, rather than 1.*

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Lines and Linear Equations*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=440&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1282

What's My Line?

Back to Task Table

STANDARDS FOR MATHEMATICAL CONTENT:

Understand the connections between proportional relationships, lines, and linear equations.

MGSE8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

MGSE8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Define, evaluate, and compare functions.

MGSE8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE:

This task builds on the previous two tasks, building on the idea of slope as a unit rate. Students should continue to explore the interrelatedness of slope and unit rate.

COMMON MISCONCEPTIONS:

- *Students may confuse the input and output values/variables. This could result in the inverse of the function. This misconception can be a tremendous teaching tool. If you observe students showing evidence of this misconception, take an example (anonymously) and use it as a teachable moment. Ask students to discuss the work, discover the error, and compare it to the actual function. Students benefit from this type of exploration, not* *just to correct a misconception, but also gain ownership of the knowledge of inverse functions in the process.*

- *Students may have trouble writing a general rule for these situations. They tend to confuse the dependent and independent variable. Contextual situations will help students correct this misconception.* ***http://www.visualpatterns.org/***
- *Some student may confuse the x- and y- axes on the coordinate plane. Emphasizing that the first value is plotted on the horizontal axes (usually x, with positive to the right) and the second is the vertical axis (usually called y, with positive up) point out that this is merely a convention. It could have been otherwise, but it is very useful for people to agree on a standard customary practice.*
- *Some students may mistakenly think only of a straight line as horizontal or vertical. This misconception may be corrected through classroom discussion and determining a tool that could be used to check if lines are straight (such as a ruler).*

ESSENTIAL QUESTIONS:

- How can patterns, relations, and functions be used as tools to best describe and help explain real-life relationships?
- How can the same mathematical idea be represented in a different way? Why would that be useful?
- What is the significance of the patterns that exist between the triangles created on the graph of a linear function?
- When two functions share the same rate of change, what might be different about their tables, graphs and equations? What might be the same?
- What does the slope of the function line tell me about the unit rate?
- What does the unit rate tell me about the slope of the function line?

MATERIALS:

- Copies of task for students
- Straightedge
- Graph paper http://incompetech.com/graphpaper or https://www.desmos.com/calculator

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will investigate the relationship patterns that exist between the triangles created on the graph of a linear function.

DIFFERENTIATION:

Extension:

- Students may research a career in which they may be interested. Find the average salary/wage per hour for the career. Use the information to create a table, equation, and a graph for the job based on a 40-hour work week. Compare and contrast the pros and cons of the chosen careers with other students.

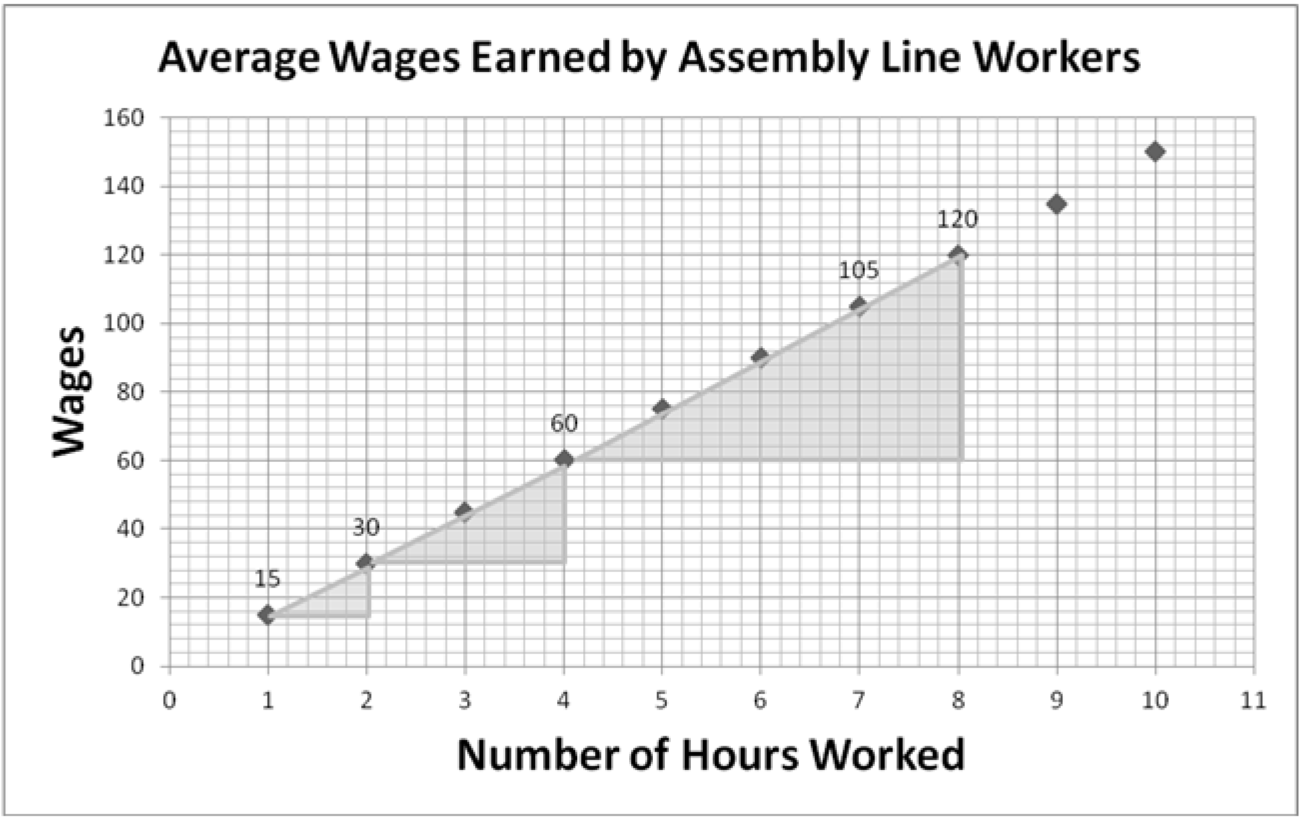
Intervention/Scaffolding:

- Encourage students with misconceptions/narrow understandings of slope to build ratio tables, unit rates, or scale factors to make sense of problems involving proportions in order to develop their understanding over time, rather than following procedures blindly.

What's My Line?

Part 1: Average Wages

The data shown in the graph below reflects average wages earned by assembly line workers across the nation.



Solutions

a. What hourly rate is indicated by the graph? Explain how you determined your answer.

The average assembly line worker is paid $15 per hour. The hour is the independent variable and the wages earned is the dependent variable. As the independent variable increases by one, the dependent variable increases by fifteen.

b. What is the ratio of the height to the base of the small, medium and large triangles? What patterns do you observe? What might account for those patterns?

$$\frac{\$15}{1\ hour} = \frac{\$30}{2\ hours} = \frac{\$60}{4\ hours}$$

All of these ratios reduce to $\frac{15}{1}$ ***which is the unit rate. $15 per hour***

c. The slope of a line is found by forming the ratio of the change in *y* to the change in *x* between any two points on the line. What is the slope of the line formed by the data points in the graph above? Explain how you know.

$$\frac{\$15}{1\ hour} = \frac{\$30}{2\ hours} = \frac{\$60}{4\ hours}$$

All of these ratios simplify to a unit rate of $\frac{\$15}{1\ hour}$.
This gives a slope of 15.

d. Write an equation for the earnings of the average assembly line worker.
W = Wages earned and h = hours worked
W = $15h

e. According to the graph and equation, in a 40-hour week, how much will the average assembly line worker earn? How do you know?

W = $15h
W = 600
The average assembly line worker will earn $600 per week. This can be determined by substituting 40 for h in the equation W = $15h. You can also either add more points to the graph or use the existing point for 8 hours and multiply this by 5 days to arrive at the same solution of $600 per week.

f. With changes in the economy, the average wages can change. How would a decrease of $2 in the average change the equation and graph?

g. ***The coefficient of the independent variable, hours worked, would change from $15 to $13. The slope of the line for this function would decrease so the line would be less steep. Both would still start at (0, 0) because if no hours are worked no wages are earned.***

h. How would an increase $5 in the average change the equation and graph?

The coefficient of the independent variable, hours worked, would change from $15 to $20. The slope of the line for this function would increase so the line would be steeper. Both would still start at (0, 0) because if no hours are worked no wages are earned.

Part 2: Comparing Wages

The average hourly wages of different jobs are presented below. Using the information provided for each of the different jobs, compare the average hourly wage of these jobs with that of the assembly line worker in Part 1. For each job, include the hourly wage (unit rate), wages earned for 40 hours, and number of hours worked needed to earn $100.

Plumber

$W = \$20h$, where W represents the wages earned and h represents the hours worked.

Machinist

Hours worked	Wages Earned
1	$ 18.50
2	$ 37.00
3	$ 55.50
4	$ 74.00
5	$ 92.50
6	$ 111.00
7	$ 129.50
8	$ 148.00
9	$ 166.50
10	$ 185.00

Comment

If students need additional practice with graphing lines, consider adding this to the task.

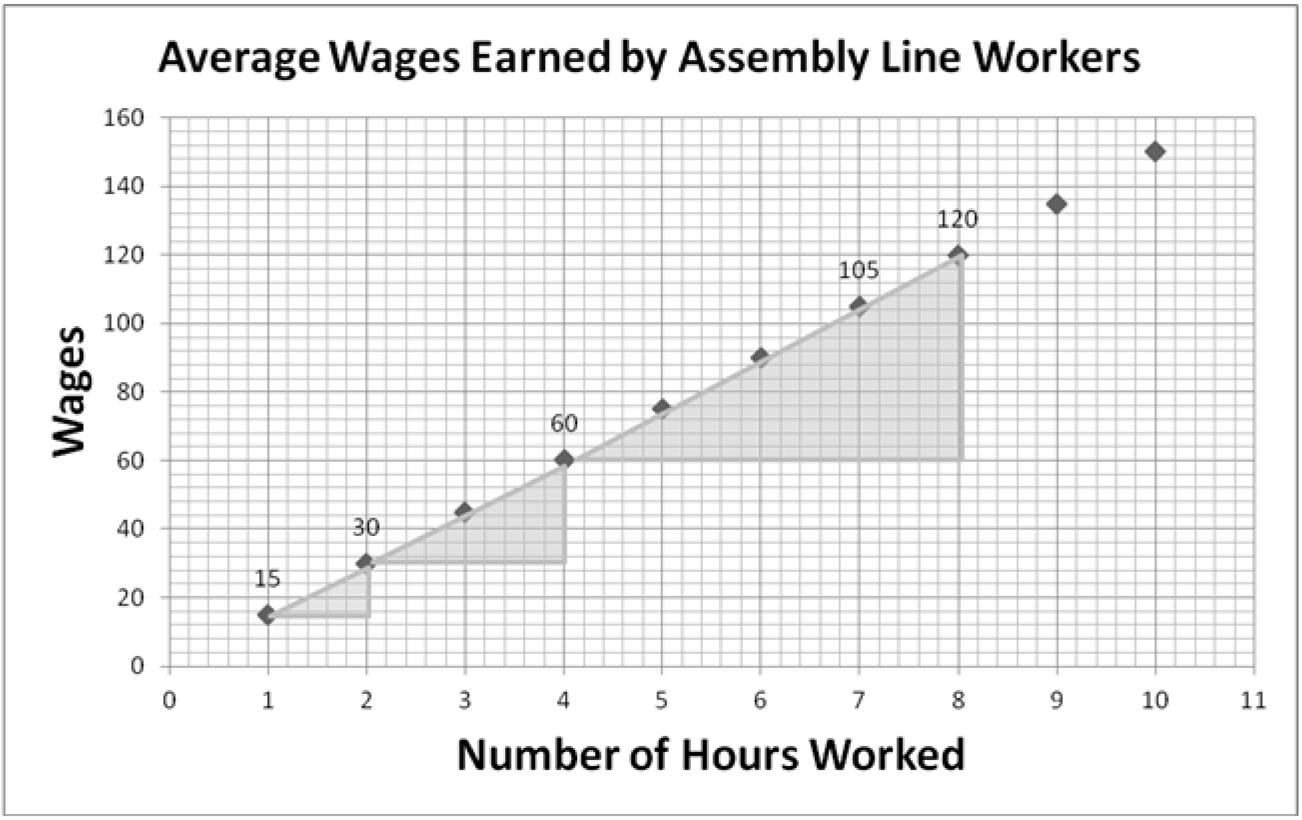
Solution

The lines for both function rules would be steeper than graph of the function of the wages earned by the average assembly line worker because the constant rate of change is greater. The plumber earns $20 per hour and the machinist earns $18.50 per hour. The plumber makes $800 in a 40 hour work week and the machinist makes $740 while the assembly line worker makes $600. It would take the plumber 5 hours to earn $100, the machinist $5\frac{1}{2}$ ***hours and the assembly line worker*** $6\frac{2}{3}$ ***hours.***

What's My Line?

Part 1: Average Wages

The data shown in the graph below reflects average wages earned by assembly line workers across the nation.



a. What hourly rate is indicated by the graph? Explain how you determined your answer.

b. What is the ratio of the height to the base of the small, medium and large triangles? What patterns do you observe? What might account for those patterns?

c. The slope of a line is found by forming the ratio of the change in y to the change in x between any two points on the line. What is the slope of the line formed by the data points in the graph above? Explain how you know.

d. Write an equation for the earnings of the average assembly line worker.

e. According to the graph and equation, in a 40-hour week, how much will the average assembly line worker earn? How do you know?

f. With changes in the economy, the average wages can change. How would a decrease of $2 in the average change the equation and graph?

g. How would an increase $5 in the average change the equation and graph?

Part 2: Comparing Wages

The average hourly wages of different jobs are presented below. Using the information provided for each of the different jobs, compare the average hourly wage of these jobs with that of the assembly line worker in Part 1. For each job, include the hourly wage (unit rate), wages earned for 40 hours, and number of hours worked needed to earn $100.

<u>Plumber</u>

$W = \$20h$, where W represents the wages earned and h represents the hours worked.

<u>Machinist</u>

Hours worked	Wages Earned
1	$ 18.50
2	$ 37.00
3	$ 55.50
4	$ 74.00
5	$ 92.50
6	$ 111.00
7	$ 129.50
8	$ 148.00
9	$ 166.50
10	$ 185.00

Ditch Diggers (Spotlight Task)

Back to Task Table

Adapted from Dan Meyer *(http://www.101qs.com/1823-ditch-diggers)*

STANDARDS FOR MATHEMATICAL CONTENT

Understand the connections between proportional relationships, lines, and linear equations.

MGSE8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

Define, evaluate, and compare functions.

MGSE8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTIONS

- What information do you need to make sense of this problem?
- How can you use estimation strategies to find out possible solutions to the questions you generated based on the video provided?

MATERIALS REQUIRED

- Access to videos for each Act
- Student Recording Sheet
- Pencil
- Graph paper, graphing calculator, or online graphing calculator https://www.desmos.com/calculator

TIME NEEDED

- 1 day

TEACHER NOTES

Task Description
In this task, students will watch the video, generate questions that they would like to answer, make reasonable estimates, and then justify their estimates mathematically. This is a student-centered task that is designed to engage learners at the highest level in learning the mathematics content. During Act 1, students will be asked to discuss what they wonder or are curious about after watching the quick video. These questions should be recorded on a class chart or on the board. Students will then use mathematics, collaboration, and prior knowledge to answer their own questions. Students will be given additional information needed to solve the problem based on need. When they realize they don't have a piece of information they need to help address the problem and ask for it, it will be given to them.

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:
Watch the video:

http://101qs-media.s3.amazonaws.com/videos/1823-ditch-diggers.mp4

Ask students what they notice. Write these on the board/chart paper.

Ask students what they wonder (what they are curious about). Write these on the board/chart paper.

Below are some possible student "wonderings":

- ➢ Will the two ditch diggers meet?
- ➢ How long before the two diggers meet?
- ➢ Is one digger digging faster than the other?
- ➢ How fast is each digger digging? (What are their rates?)
- ➢ Are the two diggers using the same tools?

Give students adequate "think time" between the two acts to discuss what they want to know. Focus in on one of the questions generated by the students, i.e. Will the two ditch diggers meet? If so, how long will it take and ask students to use the information from the video in the first act to figure it out.

Circulate throughout the classroom and ask probing questions, as needed.

Have students come up with a reasonable estimate in their group. This can be facilitated by first asking students for a too low and a too high estimate. Using the recording sheet below may also provide useful formative instructional information based on student responses and the placements of all three estimates on the empty number line.

ACT 2:

Work session. Share the Act 2 Video when the students request to know more and express a need for additional information.

http://101qs-media.s3.amazonaws.com/videos/_70_the-coordinates.mp4
Ask students how they would use this information to further refine their answer to the original question.

Give students time to work in groups to refine their estimates using the provided information in Act 2.

Circulate throughout the classroom and ask probing questions, as needed.

Have students share their results with other groups. They should engage in critiquing the reasoning of others and justifying their estimates using mathematics talk.

ACT 3

Students will compare and share solution strategies.

- Reveal the answer. Discuss the mathematics of why the diggers did not meet.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.
- Does this reveal prompt further questions from students? Continue the task as students ask more probing questions into the context of the problem:
    - When are the two diggers closest?
    - When should they both stop and change directions in order to meet?

Show the Act 3 video reveal.

http://101qs-media.s3.amazonaws.com/videos/_71_answer-no.mp4

Ditch Diggers Name:________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate the result of the main question? Explain?
Place an estimate that is too high and too low on the number line
Low estimate *Place an "x" where your estimate belongs* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:________________________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Analyzing Linear Functions - FAL

Back to Task Table

Source: Georgia Mathematics Design Collaborative

This lesson is intended to help you assess how well students are able to:

- Understand how slope-intercept form relates to the graph and table of values
- Understand how standard form relates to the graph and table of values

STANDARDS FOR MATHEMATICAL CONTENT:

MGSE8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

MGSE8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points* $(1,1)$, $(2,4)$ *and* $(3,9)$, *which are not on a straight line.*

STANDARDS FOR MATHEMATICAL PRACTICE:

This lesson uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Georgia Mathematics Design Collaborative are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding.

The task, *Analyzing Linear Functions*, is a Formative Assessment Lesson (FAL) that can be found on the Georgia Mathematics online professional learning community, edWeb.net. Once logged in, the task can be found directly at: https://www.edweb.net/?14@@.5abfa3bd.

Solving Real Life Problems: Baseball Jerseys - FAL

Back to Task Table

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1265

In this task, students will interpret a situation and represent the variables mathematically.

CONTENT STANDARDS:

Understand the connections between proportional relationships, lines, and linear equations.

MGSE8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

MGSE8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Define, evaluate, and compare functions.

MGSE8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

STANDARDS FOR MATHEMATICAL PRACTICE:

This lesson uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

BACKGROUND KNOWLEDGE:

This task builds on the understanding of slope as a unit rate developed in the previous tasks. Students should continue to engage in explorations of slope in contextual problems and tasks such as these in order to build a conceptual understanding of slope, rather than a procedural understanding. Students should continue to represent rates and proportional relationships with tables, graphs, and equations so they can build an understanding that these representations are all different views of the same idea.

<u>COMMON MISCONCEPTIONS:</u>

- *Students may confuse the input and output values/variables. This could result in the inverse of the function. This misconception can be a tremendous teaching tool. If you observe students showing evidence of this misconception, take an example (anonymously) and use it as a teachable moment. Ask students to discuss the work, discover the error, and compare it to the actual function. Students benefit from this type of exploration, not just to correct a misconception, but also gain ownership of the knowledge of inverse functions in the process.*
- *Students may have trouble writing a general rule for these situations. They tend to confuse the dependent and independent variable. Contextual situations will help students correct this misconception. http://www.visualpatterns.org/*
- *Some student may confuse the x- and y- axes on the coordinate plane. Emphasizing that the first value is plotted on the horizontal axes (usually x, with positive to the right) and the second is the vertical axis (usually called y, with positive up) point out that this is merely a convention. It could have been otherwise, but it is very useful for people to agree on a standard customary practice.*
- *Some students may mistakenly think only of a straight line as horizontal or vertical. This misconception may be corrected through classroom discussion and determining a tool that could be used to check if lines are straight (such as a ruler).*
- *8th grade students may plot slope as run over rise instead of rise over run. This misconception can be corrected and even prevented by teaching slope through contextual problems (as in many of the previous tasks) and allowing students to make sense of slope as a ratio or relationship between two variables (miles per hour, tiles per stage, paint powder per water, etc).*

<u>ESSENTIAL QUESTIONS:</u>

- How can patterns, relations, and functions be used as tools to best describe and help explain real-life relationships?
- How can the same mathematical idea be represented in a different way? Why would that be useful?
- When two functions share the same rate of change, what might be different about their tables, graphs and equations? What might be the same?

<u>MATERIALS</u>:

- See the task links.

<u>GROUPING</u>:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Solving Real Life Problems: Baseball Jerseys*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=433&subpage=problem

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.
The PDF version of the task can be found here:
http://map.mathshell.org/materials/download.php?fileid=1265

Culminating Task: Filling the Tank

Back to Task Table

In this task, students will understand the unit rate/slope relationship in equations, graphs, and tables. Students will also understand the impact on graphs, tables, and equations of a starting point other than one.

STANDARDS FOR MATHEMATICAL CONTENT:

Understand the connections between proportional relationships, lines, and linear equations.

MGSE8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

Define, evaluate, and compare functions.

MGSE8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE:

This task builds on the understanding of slope as a unit rate developed in the previous tasks. Students should continue to engage in explorations of slope in contextual problems and tasks such as these in order to build a conceptual understanding of slope, rather than a procedural understanding. Students should continue to represent rates and proportional relationships with tables, graphs, and equations so they can build an understanding that these representations are all different views of the same idea.

COMMON MISCONCEPTIONS:

- *Students may confuse the input and output values/variables. This could result in the inverse of the function. This misconception can be a tremendous teaching tool. If you observe students showing evidence of this misconception, take an example (anonymously) and use it as a teachable moment. Ask students to discuss the work, discover the error, and compare it to the actual function. Students benefit from this type of exploration, not just to correct a misconception, but also gain ownership of the knowledge of inverse functions in the process.*
- *Students may have trouble writing a general rule for these situations. They tend to confuse the dependent and independent variable. Contextual situations will help students correct this misconception. http://www.visualpatterns.org/*
- *Some student may confuse the x- and y- axes on the coordinate plane. Emphasizing that the first value is plotted on the horizontal axes (usually x, with positive to the right) and the second is the vertical axis (usually called y, with positive up) point out that this is merely a convention. It could have been otherwise, but it is very useful for people to agree on a standard customary practice.*
- *Some students may mistakenly think only of a straight line as horizontal or vertical. This misconception may be corrected through classroom discussion and determining a tool that could be used to check if lines are straight (such as a ruler).*
- *8th grade students may plot slope as run over rise instead of rise over run. This misconception can be corrected and even prevented by teaching slope through contextual problems (as in many of the previous tasks) and allowing students to make sense of slope as a ratio or relationship between two variables (miles per hour, tiles per stage, paint powder per water, etc).*

ESSENTIAL QUESTIONS:

- How can patterns, relations, and functions be used as tools to best describe and help explain real-life relationships?
- How can the same mathematical idea be represented in a different way? Why would that be useful?
- What does the slope of the function line tell me about the unit rate?
- What does the unit rate tell me about the slope of the function line?

MATERIALS:

- Copies of task for students
- Straightedge
- Graph paper http://incompetech.com/graphpaper or https://www.desmos.com/calculator
- Calculator *(optional)*

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will define, evaluate, and compare functions represented in different ways. The task is presented as a real-world example of problem-solving in a job situation.

DIFFERENTIATION:

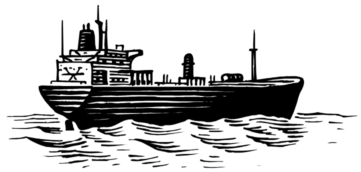
Extension:

- Imagine that instead of pumping petroleum product into the tanker, you want to pump it out of a full tanker holding 1,785,000 gallons at a rate of 50,000 gallons per hour. Write the equation, create a table and graph the situation. Explain how your equation, table and graph are different from those in the task. Be sure to explain fully.

Intervention/Scaffolding:

- Encourage struggling students to use two different colors to graph the lines in problem 4, and two more colors for problem 9. Group students in a way that promotes cooperative learning. Prompt struggling students/groups with questions that promote reasoning over just answers.

Filling the Tank



On your first day as an apprentice at the oil refinery, you are asked to create usable graphs and tables for the other workers who fill the oil tankers.

You are given the following information to assist you with your assigned task at the oil refinery:

An oil tanker holds approximately 1,785,000 gallons of petroleum product. The refinery has two hoses that can be used to fill a tanker, a large one and a small one. On an average day, it takes 20 hours to fill an empty tank with the largest hose and 30 hours with the smallest hose.

Solutions

1. Determine the constant rate of change in the hold of the tanker for each hose.

   a. Constant rate of change using the large hose, Hose A

      89,250 gallons per hour

   b. Constant rate of change using the small hose, Hose B

      59,500 gallons per hour

2. Use these constant rates of change to write equations for rules for the content of the tanker after any given number of hours.

   a. Equation for Hose A

      G = 89,250h where G is the gallons in the tanker and h is the number of hours the tanker has been filling

   b. Equation for Hose B

      G = 59.500h where G is the gallons in the tanker and h is the number of hours the tanker has been filling

3. Using this information create a table of values from zero to thirty in five hour intervals showing how much petroleum product is in a tanker for each hose.

Hours	*Gallons in Tank Hose A*	*Gallons in Tank Hose B*
0	*0*	*0*
5	*446,250*	*297,500*
10	*892,500*	*595,000*
15	*1,338,750*	*892,500*
20	*1,785,000*	*1,190,000*
25	*2,231,250*	*1,487,500*
30	*2,677,500*	*1,785,000*

Problem continues on next page. →

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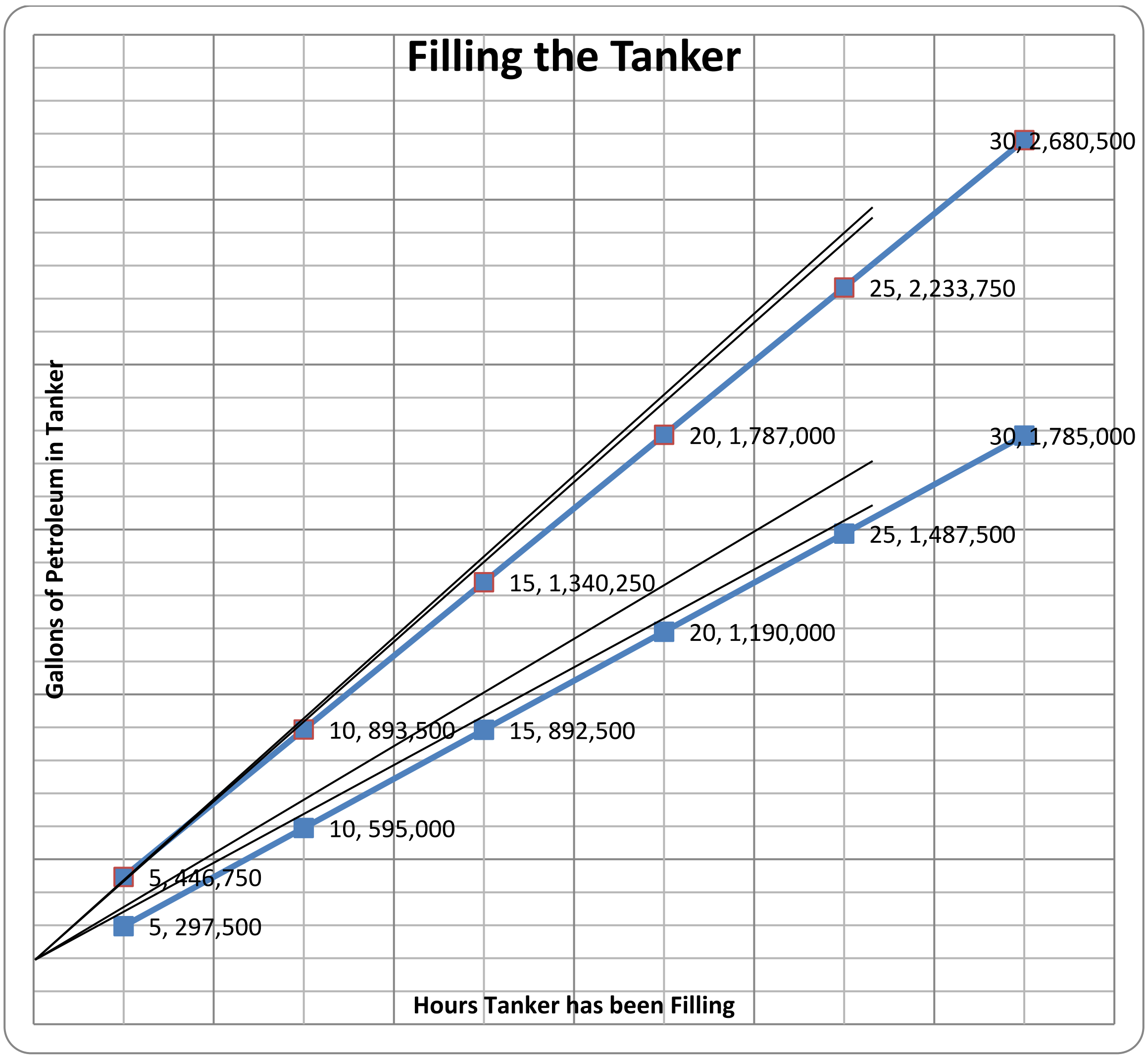
4. Represent the information from the table on <u>one</u> graph.

Solution/Comment

The graph also contains lines which will be added later in the activity (see question 9).



5. Describe the rate at which a tanker fills using the two different hoses. How are the differences evident in the equations, table, and graph?

Both lines start at (0, 0) because there is no oil in the tanker at hour zero. Hose A has a greater constant rate of change and fills the tanker faster than Hose B that has a smaller constant rate of change.

6. The refinery is introducing new hoses. Hose C will fill a tanker at a rate of 90,000 gallons per hour and Hose D will fill a tanker at 55,000 gallons per hour. Without making a table of values or graphing the information, describe how the equations and graphs for the two new hoses will compare to Hose A and Hose B. Explain your reasoning.

   The constant rate of change of Hose C is greater than the previously largest hose, Hose A. Therefore, it will fill the tanker faster than Hose A. The graph of the line for Hose C will increase faster. The slope will be greater/steeper.

   The constant rate of change for Hose D is smaller that the previously smallest hose, Hose B. Hose D will fill the tanker slower than all of the other hoses. The graph of the line for Hose D will be less steep than any of the others. It will have the least slope.

   One day a tanker that is not empty enters with filling port. The crew tells you that the tanker still has 60,000 gallons of petroleum product in the hold.

7. Write an equation showing the amount of oil in this tanker after any given number of hours of filling with each of the hoses.

   <u>Hose A</u>
   G = 89,250h + 60,000 where G is the gallons in the tanker and h is the number of hours it has been filling

   <u>Hose B</u>
   G = 59,500h + 60,000 where G is the gallons in the tanker and h is the number of hours it has been filling

   <u>Hose C</u>
   G = 90,000h + 60,000 where G is the gallons in the tanker and h is the number of hours it has been filling

   <u>Hose D</u>
   G = 55,000h + 60,000 where G is the gallons in the tanker and h is the number of hours it has been filling

Problem continues on next page. →
Problem continued from previous page.

8. Represent the information from question 7 in a table.

Hours	***Gallons in Tank Hose A***	***Gallons in Tank Hose B***	***Gallons in Tank Hose C***	***Gallons in Tank Hose D***
0	***60,000***	***60,000***	***60,000***	***60,000***
5	***506,250***	***357,500***	***510,000***	***335,000***
10	***952,500***	***655,000***	***960,000***	***610,000***
15	***1,398,750***	***952,500***	***1,410,000***	***885,000***
20	***1,845,000***	***1,250,000***	***1,860,000***	***1,160,000***
25	***2,291,250***	***1,547,500***	***2,310,000***	***1,435,000***
30	***2,737,500***	***1,845,000***	***2,760,000***	***1,710,000***

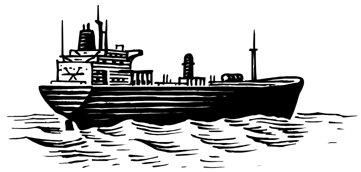
9. Add the graph of this information to the graph in question 4.

See graph for question 4.

10. How are these lines for this tanker different than the other lines? Why? How is this difference evident in the equation and table?

The new lines do not start at (0, 0) because the initial 60,000 gallons in the tanker make the starting point higher on the y-axis. When the independent variable is at zero, the dependent variable is at 60,000. That initial 60,000 is added to the product of the constant rate of change and the independent variable.

Filling the Tank



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1. Determine the constant rate of change in the hold of the tanker for each hose.

   a. Constant rate of change using the large hose, Hose A

   b. Constant rate of change using the small hose, Hose B

2. Use these constant rates of change to write equations for rules for the content of the tanker after any given number of hours.

   a. Equation for Hose A

   b. Equation for Hose B

3. Using this information create a table of values from zero to thirty in five hour intervals showing how much petroleum product is in a tanker for each hose.

Hours	***Gallons in Tank Hose A***	***Gallons in Tank Hose B***

4. Represent the information from the table on one graph.

5. Describe the rate at which a tanker fills using the two different hoses. How are the differences evident in the equations, table, and graph?

6. The refinery is introducing new hoses. Hose C will fill a tanker at a rate of 90,000 gallons per hour and Hose D will fill a tanker at 55,000 gallons per hour. Without making a table of values or graphing the information, describe how the equations and graphs for the two new hoses will compare to Hose A and Hose B. Explain your reasoning.

One day a tanker that is not empty enters with filling port. The crew tells you that the tanker still has 60,000 gallons of petroleum product in the hold.

7. Write an equation showing the amount of oil in this tanker after any given number of hours of filling with each of the hoses.

8. Represent the information from question 7 in a table.

9. Add the graph of this information to the graph in question 4.

   How are these lines for this tanker different than the other lines? Why? How is this difference evident in the equation and table?

Appendix – Technology Resources

Back to Task Table

Understand the connections between proportional relationships, lines, and linear equations.

MGSE8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

https://www.illustrativemathematics.org/content-standards/8/EE/B/5
https://www.desmos.com/calculator

MGSE8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

https://www.illustrativemathematics.org/content-standards/8/EE/B/6/tasks/1537
https://www.desmos.com/calculator

Define, evaluate, and compare functions.

MGSE8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

https://www.illustrativemathematics.org/content-standards/8/F/A/3/tasks/813

http://reasonandwonder.com/match-my-graph-linear-functions/ - for practice graphing linear functions.