Georgia
Standards of Excellence
Curriculum Frameworks

Mathematics

GSE Grade 8
Unit 6: Linear Models and Tables

Georgia Department of Education

Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

Unit 6
Linear Models and Tables

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OVERVIEW

In this unit students will:

- identify the rate of change and the initial value from tables, graphs, equations, or verbal descriptions;
- write a model for a linear function;
- sketch a graph when given a verbal description of a situation;
- analyze scatter plots;
- informally develop a line of best fit;
- use bivariate data to create graphs and linear models; and
- recognize patterns and interpret bivariate data.

Students are given opportunities and examples to figure out the meaning of $y = mx + b$. They will be able to "see" m and b in graphs, tables, and formulas or equations, and they need to be able to interpret those values in contexts.

Using graphing calculators and web resources, students explore linear functions within context to build understanding of slope and y-intercept in a graph, especially for those patterns that do not start with an initial value of 0.

Students gather their own data or graphs in contexts they understand. Students measure, collect data, graph data, and look for patterns, then generalize and symbolically represent the patterns. They draw graphs (qualitatively, based upon experience) representing real-life situations with which they are familiar.

Students take a function in symbolic form and create a problem situation in words to match the function. Given a graph, students create a scenario that would fit the graph. Students sort a set of "cards" to match graphs, tables, equations, and problem situations and explain their reasoning to each other.

Building on the study of statistics using univariate data in Grades 6 and 7, students are now ready to study bivariate data. Students will extend their descriptions and understanding of variation to the graphical displays of bivariate data.

Scatter plots are the most common form of displaying bivariate data in Grade 8. Students practice informally finding the line of best fit using a scatter plot. Students create and interpret scatter plots, focusing on outliers, positive or negative association, linearity or curvature. By changing the data slightly, students have a rich discussion about the effects of the change on the graph. Students use a graphing calculator to determine a linear regression and discuss how this relates to the graph. Students informally draw a line of best fit for a scatter plot and informally assess the fit of a function to their data.

The study of the line of best fit ties directly to the algebraic study of slope and intercept. Students interpret the slope and intercept of the line of best fit in the context of the data. Then students make predictions based on the line of best fit. Student will construct and interpret two-way tables in order to summarize two categorical variables.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under "Evidence of Learning" be reviewed early in the planning process. A variety of resources should be utilized to supplement this unit. This unit provides much needed content information, but excellent learning activities as well. The tasks in this unit illustrate the types of learning activities that should be utilized from a variety of sources.

STANDARDS ADDRESSED IN THIS UNIT

STANDARDS FOR MATHEMATICAL PRACTICE

Refer to the Comprehensive Course Overview for more detailed information about the Standards for Mathematical Practice.

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

STANDARDS FOR MATHEMATICAL CONTENT

Use functions to model relationships between quantities.

MGSE8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

MGSE8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Investigate patterns of association in bivariate data.

MGSE8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

MGSE8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

MGSE8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

MGSE8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

a. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.
b. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

BIG IDEAS

- Collecting and examining data can sometimes help one discover patterns in the way in which two quantities vary.
- Changes in varying quantities are often related by patterns which, once discovered, can be used to predict outcomes and solve problems.

- Written descriptions, tables, graphs, and equations are useful in representing and investigating relationships between varying quantities.
- Different representations (written descriptions, tables, graphs, and equations) of the relationships between varying quantities may have different strengths and weaknesses.
- Linear functions may be used to represent and generalize real situations.
- Slope and *y*-intercept are keys to solving real problems involving linear relationships.

ESSENTIAL QUESTIONS

- What strategies can I use to help me understand and represent real situations involving linear relationships?
- How can the properties of lines help me to understand graphing linear functions?
- What can I infer from the data?
- How can functions be used to model real-world situations?
- How does a change in one variable affect the other variable in a given situation?
- Which tells me more about the relationship I am investigating – a table, a graph or an equation? Why?
- How can you construct and interpret two-way tables?
- How can I determine if there is an association between two given sets of data?
- How can I find the relative frequency using two-way tables?

CONCEPTS/SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- identifying and calculating slope
- identifying the *y*-intercept
- creating graphs using given data
- analyzing graphs
- making predictions from a graph

FLUENCY

It is expected that students will continue to develop and practice strategies to build their capacity to become fluent in mathematics and mathematics computation. The eventual goal is automaticity with math facts. This automaticity is built within each student through strategy development and practice. The following section is presented in order to develop a common understanding of the ideas and terminology regarding fluency and automaticity in mathematics:

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply "how to get the answer" and instead support students' ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students:

- **flexibly use a combination of deep understanding, number sense, and memorization.**
- **are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.**
- **are able to articulate their reasoning.**
- **find solutions through a number of different paths.**

For more about fluency, see: http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and: http://joboaler.com/timed-tests-and-the-development-of-math-anxiety/

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for middle school students. **Note – Different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.** The definitions below are from the Common Core State Standards Mathematics Glossary and/or the Common Core GPS Mathematics Glossary when available.

Visit http://intermath.coe.uga.edu or http://mathworld.wolfram.com to see additional definitions and specific examples of many terms and symbols used in grade 8 mathematics.

- **Model:** A mathematical representation of a process, device, or concept by means of a number of variables.
- **Interpret**:
- **Initial Value:** *y*-intercept.
- **Qualitative Variables:**
- **Linear:**
- **Non-linear:**
- **Slope:**
- **Rate of Change:**
- **Bivariate Data:** The following website has a short powerpoint (the 2nd one) that may be helpful. http://www.sophia.org/packets/bivariate-data-two-variables--2
- **Quantitative Variables:**
- **Scatter Plot:**
- **Line of Best Fit:**
- **Clustering:** The partitioning of a data set into subsets (clusters), so that the data in each subset (ideally) share some common trait - often similarity or proximity for some defined distance measure.
- **Outlier:**

FORMATIVE ASSESSMENT LESSONS (FAL)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Guide.

SPOTLIGHT TASKS

A Spotlight Task has been added to each CCGPS mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Common Core Georgia Performance Standards, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky.

3-ACT TASKS

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

TASKS

Task Name	Task Type *Grouping Strategy*	Content Addressed	Standard(s)
Sugar Prices	Short Cycle Task *Partner/Small Group*	Use a graph showing the prices and weights of bags of sugar to find the bag offering the best value	MGSE8.F.4 MGSE8.F.5
Winter Is Over …	Performance Task *Partner/Small Group*	Graphing data, creating a table, and writing a function.	MGSE8.F.4 MGSE8.F.5 MGSE8.SP.1
Heartbeats	Performance Task *Individual/Partner*	Analyzing data via a table, graph, and an equation.	MGSE8.F.4 MGSE8.F.5 MGSE8.SP.1
Styrofoam Cups (Spotlight Task)	Learning Task Small Group	Constructing a linear function from collected data.	MGSE8.F.4
Walk the Graph	Learning Task *Partner/Small Group*	Understanding of graphing linear functions and rates of change.	MGSE8.F.4 MGSE8.F.5 MGSE8.SP.2 MGSE8.SP.3 MGSE8.SP.4
Interpreting Distance-Time Graphs	Formative Assessment Lesson *Partner/Small Group*	This lesson unit is intended to help you assess how well students are able to interpret distance–time graphs.	MGSE8.F.4 MGSE8.F.5
Forget the Formula	Performance Task *Individual/Partner*	Writing a function given two points.	MGSE8.F.4 MGSE8.F.5 MGSE8.SP.4
Heartbeats Too!	Learning Task *Small Group*	Properties of Linear Graphs.	MGSE8.F.4 MGSE8.F.5 MGSE8.SP.1 MGSE8.SP.2 MGSE8.SP.4
Mineral Samples	Performance Task *Partner/Small Group*	Determining Line of Best Fit.	MGSE8.SP.1 MGSE8.SP.2 MGSE8.SP.4
Walking Race & Making Money	Learning Task *Partner/Small Group*	Introduces the concept of average rate of change as a tool for describing and understanding characteristics of functions.	MGSE8.F.4 MGSE8.F.5 MMC8.SP.1 MGSE8.SP.2 MGSE8.SP.3 MGSE8.SP.4
Mini Questions	Performance Task *Individual/Partner*	Creating a table and developing an equation.	MGSE8.F.4 MGSE8.F.5 MMC8.SP.1

			MGSE8.SP.2 MGSE8.SP. 3 MGSE8.SP.4
My Cotton Boll Data (with permission from Carrie Pierce, Lee County Schools)	Performance Task *Small Group*	Gathering bivariate data, creating a scatter plot, and analyzing data.	MGSE8.F.4 MGSE8.F.5 MMC8.SP.1 MGSE8.SP.2 MGSE8.SP.3 MGSE8.SP.4
Outdoor Theater	Performance Task *Small Group*	Use bivariate data to develop a function.	MGSE8.F.4 MGSE8.F.5 MMC8.SP.1 MGSE8.SP.2 MGSE8.SP.3 MGSE8.SP.4
How Long Should Shoe Laces Really Be?	Performance Task *Small Group*	Creating a scatter plot and determining a line of best fit.	MGSE8.F.4 MGSE8.F.5 MMC8.SP.1 MGSE8.SP.2 MGSE8.SP 3 MGSE8.SP.4
25 Billion Apps (Spotlight Task)	Learning Task Small Group	Determining and evaluating the appropriateness of a linear model.	MGSE8.F.4 MGSE8.SP.2
Sports and Musical Instruments	Learning Task *Whole Class/Individual*	Collect bivariate data to create two way tables & analyze patterns of association using relative frequencies.	MGSE8.SP.4
Two Way Tables	Learning & Performance Tasks *Individual/Partner/ Small Group*	Construct & interpret two-way tables. Summarize data on two categorical variables collected from the same subjects. Use relative frequencies to describe associations.	MGSE8.SP.4
Culminating Task: Is the Data Linear?	Performance Task *Individual/Partner Task*	Demonstrate understanding of linear relationships and generalizing the relationships through writing and graphing linear equations and inequalities	MGSE8.F.4 MGSE8.F.5 MMC8.SP.1 MGSE8.SP.2 MGSE8.SP.3 MGSE8.SP.4
Technology Resources			

<u>Sugar Prices</u> (Short Cycle Task)

Back to Task Table

Source: Balanced Assessment Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/tasks.php?taskid=257&subpage=apprentice

In this task, students must use a graph showing the prices and weights of bags of sugar to find the bag offering the best value for money.

STANDARDS FOR MATHEMATICAL CONTENT:

Investigate patterns of association in bivariate data.

MGSE8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

MGSE8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

MGSE8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

MGSE8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

a. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.
b. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

This rich task involves several aspects of mathematics and is structured to ensure that all students have access to the problem. Students are guided through of increasing challenges, enabling them to show the levels of performance. All of the mathematical practices are integrated throughout this task, but this task focuses on SMP 2 (Reason abstractly and quantitatively), SMP 6 (Attend to precision), SMP 3 (Construct viable arguments and critique the reasoning of others), and SMP 7 (Look for and make use of structure).

COMMON MISCONCEPTIONS:

Students may graph incorrectly. Students should investigate multiple contextual situations to gain an understanding of the roles of independent and dependent variables and how they are related. The convention that x usually represents the independent variable and y represents the dependent variable should be emphasized as students make sense of the relationships in each investigation. The reason for the convention is to facilitate to mathematical communication.

ESSENTIAL QUESTIONS:

- How can I describe a situation given a graph?
- What can I infer from the data?

MATERIALS:

- Copies of task for students

GROUPING:

- Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the

lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *Sugar Prices*, is a Mathematics Assessment Project Assessment Task that can be found at the website:
http://www.map.mathshell.org/materials/tasks.php?taskid=257&subpage=apprentice

The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=766

The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=767

DIFFERENTIATION:

Extension:

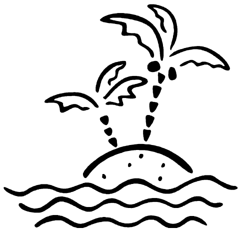
- Write a statement relating the weight of a bag of sugar and its price. Does the data appear to be correlated? If so, is it a strong or weak correlation? If you were to draw a line of best fit, which point would come closest to the line? What would the points below the line of best fit represent? What would the points above the line of best fit represent?

Intervention/Scaffolding:

- Be sure to help students make sense of the context of the problem before moving on. The context allows for students at all levels to access this task. Also, if needed, students may need to review plotting points on a coordinate plane prior to beginning this task.

Winter Is Over

Back to Task Table



In this task, students will graph data, create tables, and write a function.

STANDARDS FOR MATHEMATICAL CONTENT:

Use functions to model relationships between quantities.

MGSE8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

MGSE8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Investigate patterns of association in bivariate data.

MGSE8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

This task is designed to allow students at multiple levels to have access to this problem. There are 3 versions of the same problem with different levels of support for these three levels.

COMMON MISCONCEPTIONS:

Students often confuse a recursive rule with an explicit formula for a function. For example, after identifying that a linear function shows an increase of 2 in the values of the output for every change of 1 in the input, some students will represent the equation as $y = x + 2$ instead of realizing that this means $y = 2x + b$. When tables are constructed with increasing consecutive integers for input values, then the distinction between the recursive and explicit formulas is about whether you are reasoning vertically or horizontally in the table. Both types of reasoning – and both types of formulas – are important for developing proficiency with functions. Knowing the difference and when each type of reasoning is applicable is an important goal.

ESSENTIAL QUESTIONS:

- What strategies can I use to help me understand and represent real situations involving linear relationships?
- What can I infer from the data?
- How can functions be used to model real-world situations?
- How can I determine if there is an association between two given sets of data?

MATERIALS:

- Each group will need a different set of four congruent boxes or four of the same color Cuisenaire Rods (i.e., one group gets the green, another group gets orange, etc.)
- Rulers or Cuisenaire Rods (When using the Cuisenaire Rods, students should measure in centimeters.)
- Graph paper http://incompetech.com/graphpaper or an online, free graphing calculator such as Desmos
- Copies of task for students

GROUPING:

- Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

The students should find the surface area of one, two, three, and four boxes. Through inquiry, students will discover if they should be consistent or not in the stacking method and whether or not this makes a difference in the results. If students from group to group are using different Cuisenaire Rod lengths or boxes of different sizes, answers will vary.

DIFFERENTIATION:

Extension:

- Multiple Versions of the Task Provided

Intervention/Scaffolding:

- Appropriate level task versions:

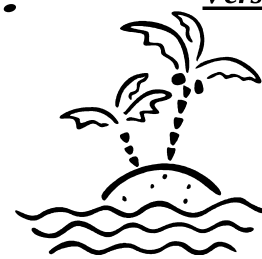
  - Version A - provides no hints or suggestions. Use with high achieving or accelerated students to provide experiences in problem-solving strategies.
  - Version B - provides hints as to the direction the group may take. This group understands the mathematics required, they just may need help in getting started. Students may also need guidance in recalling how to find surface area and volume.
  - Version C - provides hints along with a premade table and graph. This group may need additional hints and assistance in recalling how to find surface area and volume.

Comments

- ***Let the students determine whether they want to stack the boxes end to end or on top of one another.***
- ***You may want to review how to find surface area and volume before you begin.***
- ***You may want to complete the surface area part in class and assign the volume section for homework.***

Winter Is Over...

Version A



Solutions

Now is the time to shed cold weather clothes and heavy winter jackets for shorts, t-shirts, and flip flops. But first, we must pack the cold weather apparel away before the fun in the sun can begin.

I bought four storage boxes to help us with this task. They are all congruent so that they will stack neatly away in the closet. Depending on how much gear you have, we may not need all of them.

In order to keep out the dust and unwanted pests, we are going to wrap the boxes together. How much wrap will it take to cover all of the sides of one box? Two boxes? Three boxes? Four boxes?

The students should find the surface area of one, two, three, and four boxes. They should be consistent in the stacking method - or should they? You should let this group explore. Answers will vary depending on the length of the Cuisenaire Rod given to the group.

Is there a relationship between the number of boxes and the area needed? How can you prove whether or not a relationship exists?

Students should make a table and graph the data they collected. From the previous unit the students should notice that a function exists.

How could you find the area needed for any given number of boxes?

Students should create a rule or equation from the data.

Are there any outliers? Why or why not?

No, this is a linear function. If there are any outliers the students have made a calculation error.

Now, just how much will one box hold? Two boxes? Three boxes? Four boxes?

The students should find the volume of one, two, three, and four boxes. They should be consistent in the stacking method - or should they? You should let this group explore. Answers will vary depending in the length of the Cuisenaire Rod given to the group.

Is there a relationship between the number of boxes and the volume they hold? Show me.

Students should make a table and graph the data they collect. From the previous unit the students should notice that a function exists.

How could you find the volume of any given number of boxes?

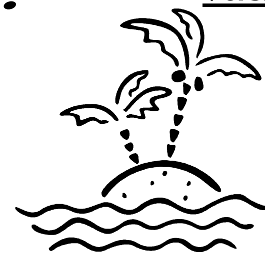
Students should create a rule or equation from the data.

Are there any outliers? Why or why not?

No, this is a linear function. If there are any outliers the students have made a calculation error.

Winter Is Over...

Version B



Solutions

Now is the time to shed cold weather clothes and heavy winter jackets for shorts, t-shirts, and flip flops. But first, we must pack the cold weather apparel away before the fun in the sun can begin.

I bought four storage boxes to help us with this task. They are all congruent so that they will stack neatly away in the closet. Depending on how much gear you have, we may not need all of them.

In order to keep out the dust and unwanted pests, we are going to wrap the boxes together. How much wrap will it take to cover all of the sides of one box? Two boxes? Three boxes? Four boxes? (Hint: Make a table)

The students should find the surface area of one, two, three, and four boxes. They should be consistent in the stacking method - or should they? You may let this group explore. Answers will vary depending on the length of the Cuisenaire Rod given to the group.

Is there a relationship between the number of boxes and the area needed? How can you prove whether or not a relationship exists? (Hint: Create a graph.)

Students should make a table and graph the data they collected. From the previous unit the students should notice that a function exists.

How could you find the area needed for any given number of boxes? (Hint: Write an equation.)

Students should create a rule or equation from the data.

Are there any outliers? Why or why not?

No, this is a linear function. If there are any outliers the students have made a calculation error.

Now, just how much will one box hold? Two boxes? Three boxes? Four boxes?
(Hint: Make a table.)

The students should find the volume of one, two, three, and four boxes. They should be consistent in the stacking method - or should they? You may let this group explore. Answers will vary depending in the length of the Cuisenaire Rod given to the group.

Is there a relationship between the number of boxes and the volume they hold? Show me. (Hint: Create a graph.)

Students should make a table and graph the data they collected. From the previous unit the students should notice that a function exists.

How could you find the volume of any given number of boxes? (Hint: Write an equation.)

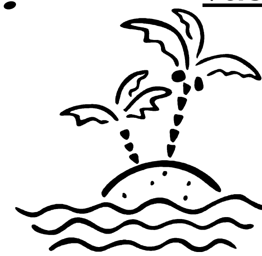
Students should create a rule or equation from the data.

Are there any outliers? Why or why not?

No, this is a linear function. If there are any outliers the students have made a calculation error.

Winter Is Over...

Version C



Solutions

Now is the time to shed cold weather clothes and heavy winter jackets for shorts, t-shirts, and flip flops. But first, we must pack the cold weather apparel away before the fun in the sun can begin.

I bought four storage boxes to help us with this task. They are all congruent so that they will stack neatly away in the closet. Depending on how much gear you have, we may not need all of them.

In order to keep out the dust and unwanted pests, we are going to wrap the boxes together. How much wrap will it take to cover all of the sides of one box? Two boxes? Three boxes? Four boxes?

Number of Boxes	Total Surface Area
1	
2	
3	
4	

(Remember, to find surface area you must find the area of each side of the box then add all of the sides together. How many sides are there to a box?)

The students should find the surface area of one, two, three, and four boxes. They should be consistent in the stacking method or should they? You may let this group explore, but they may need your guidance. Answers will vary depending on the length of the Cuisenaire Rod given to the group.

Is there a relationship between the number of boxes and the area needed? Demonstrate by making a table and using it to create a graph.

Students should make a table and graph the data they collected. From the previous unit the students should notice that a function exists.

How could I find out the area needed for any given number of boxes? (Hint: Write an equation of the line.)

a. Calculate the rate of change using the values in the table.

b. Find the y-intercept using the graph above.

c. Write the equation in slope-intercept form ($y = mx + b$)

Students should create a rule or equation from the data.

Are there any outliers? Why or why not?

No, this is a linear function. If there are any outliers the students have made a calculation error.

Now, just how much will one box hold? Two boxes? Three boxes? Four boxes?

Number of Boxes	Total Volume
1	
2	
3	
4	

(Remember – volume is how much an object can hold inside. In order to find the volume of a box multiply the 3 dimensions using length × width × height)

The students should find the volume of one, two, three, and four boxes. They should be consistent in the stacking method or should they? You may let this group explore, but they may need your guidance. Answers will vary depending on the length of the Cuisenaire Rod given to the group.

Is there a relationship between the number of boxes and the volume they hold? Show me by creating a graph.

Students should make a table and graph the data they collected. From the previous unit the students should notice that a function exists.

How could you find the volume of any given number of boxes? (Hint: Write an equation of the line.)

a. Calculate the rate of change using the values in the table.

b. Find the *y*-intercept using the graph above.

c. Write the equation in slope-intercept form ($y = mx + b$)

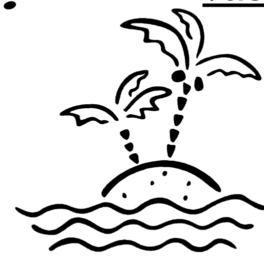
Students should create a rule or equation from the data.

Are there any outliers? Why or why not?

No, this is a linear function. If there are any outliers the students have made a calculation error.

Winter Is Over...

Version A



Now is the time to shed cold weather clothes and heavy winter jackets for shorts, t-shirts, and flip flops. But first, we must pack the cold weather apparel away before the fun in the sun can begin.

I bought four storage boxes to help us with this task. They are all congruent so that they will stack neatly away in the closet. Depending on how much gear you have, we may not need all of them.

In order to keep out the dust and unwanted pests, we are going to wrap the boxes together. How much wrap will it take to cover all of the sides of one box? Two boxes? Three boxes? Four boxes?

Is there a relationship between the number of boxes and the area needed? How can you prove whether or not a relationship exists?

How could you find the area needed for any given number of boxes?

Are there any outliers? Why or why not?

Now, just how much will one box hold? Two boxes? Three boxes? Four boxes?

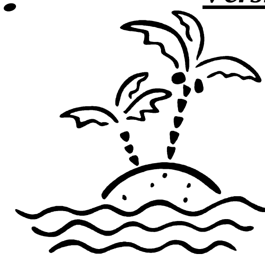
Is there a relationship between the number of boxes and the volume they hold? Show me.

How could you find the volume of any given number of boxes?

Are there any outliers? Why or why not?

Winter Is Over...

Version B



Now is the time to shed cold weather clothes and heavy winter jackets for shorts, t-shirts, and flip flops. But first, we must pack the cold weather apparel away before the fun in the sun can begin.

I bought four storage boxes to help us with this task. They are all congruent so that they will stack neatly away in the closet. Depending on how much gear you have, we may not need all of them.

In order to keep out the dust and unwanted pests, we are going to wrap the boxes together. How much wrap will it take to cover all of the sides of one box? Two boxes? Three boxes? Four boxes? (Hint: Make a table)

Is there a relationship between the number of boxes and the area needed? How can you prove whether or not a relationship exists? (Hint: Create a graph.)

How could you find the area needed for any given number of boxes? (Hint: Write an equation.)

Are there any outliers? Why or why not?

Now, just how much will one box hold? Two boxes? Three boxes? Four boxes?
(Hint: Make a table.)

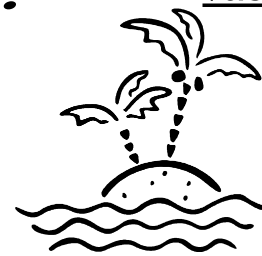
Is there a relationship between the number of boxes and the volume they hold? Show me.
(Hint: Create a graph.)

How could you find the volume of any given number of boxes? (Hint: Write an equation.)

Are there any outliers? Why or why not?

Winter Is Over...

Version C



Now is the time to shed cold weather clothes and heavy winter jackets for shorts, t-shirts, and flip flops. But first, we must pack the cold weather apparel away before the fun in the sun can begin.

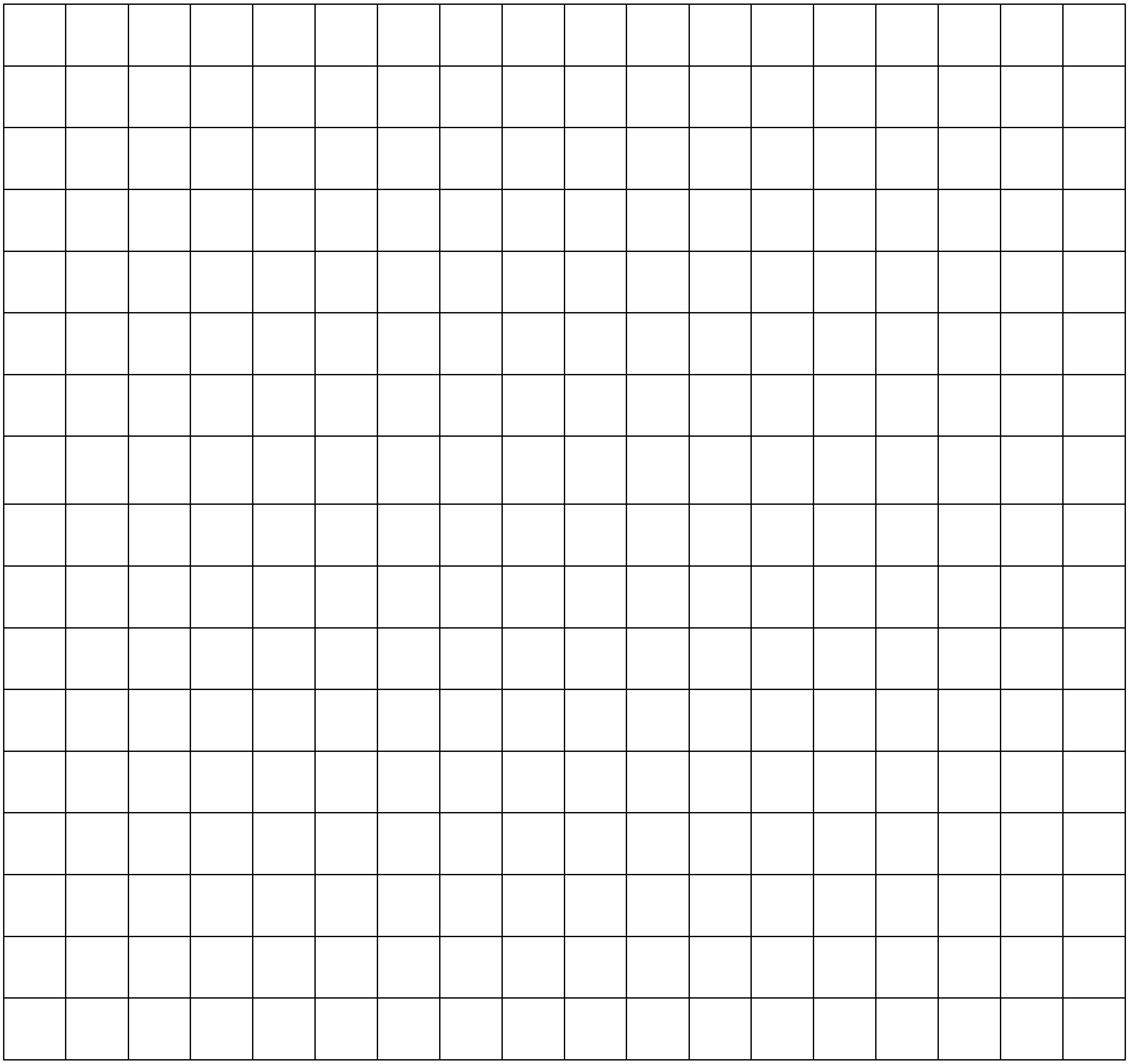
I bought four storage boxes to help us with this task. They are all congruent so that they will stack neatly away in the closet. Depending on how much gear you have, we may not need all of them.

In order to keep out the dust and unwanted pests, we are going to wrap the boxes together. How much wrap will it take to cover all of the sides of one box? Two boxes? Three boxes? Four boxes?

Number of Boxes	Total Surface Area
1	
2	
3	
4	

(Remember, to find surface area you must find the area of each side of the box then add all of the sides together. How many sides are there to a box?)

Is there a relationship between the number of boxes and the area needed? Demonstrate by making a table and using it to create a graph.



How could I find out the area needed for any given number of boxes? (Hint: Write an equation of the line.)

a. Calculate the rate of change using the values in the table.

b. Find the *y*-intercept using the graph above.

c. Write the equation in slope-intercept form ($y = mx + b$)

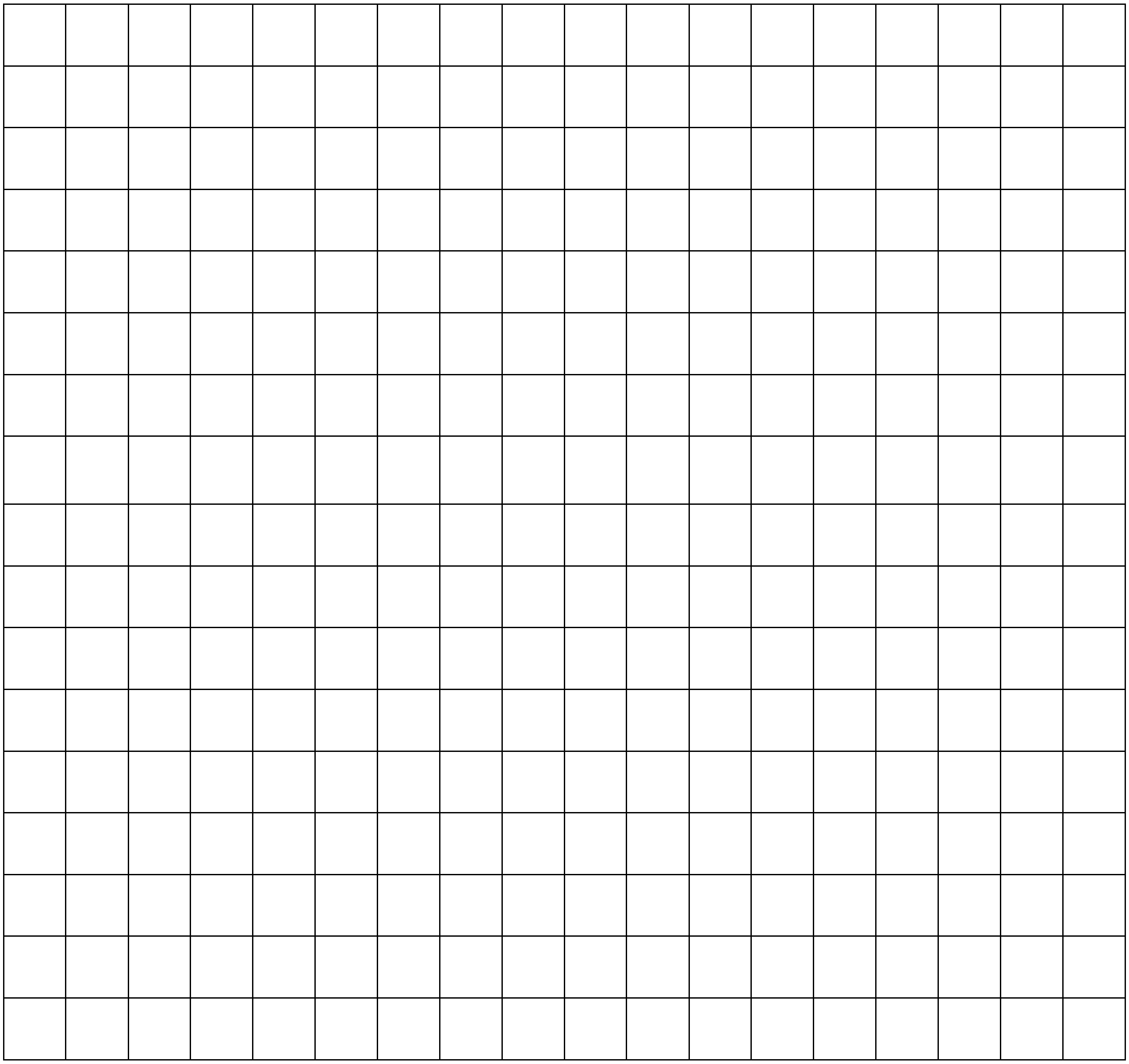
Are there any outliers? Why or why not?

Now, just how much will one box hold? Two boxes? Three boxes? Four boxes?

Number of Boxes	Total Volume
1	
2	
3	
4	

(Remember – volume is how much an object can hold inside. In order to find the volume of a box multiply the 3 dimensions using length × width × height)

Is there a relationship between the number of boxes and the volume they hold? Show me by creating a graph.



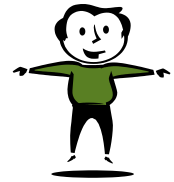
How could you find the volume of any given number of boxes? (Hint: Write an equation of the line.)

a. Calculate the rate of change using the values in the table.

b. Find the y-intercept using the graph above.

c. Write the equation in slope-intercept form ($y = mx + b$)

Are there any outliers? Why or why not?



Heartbeats

Back to Task Table

In this task, students will analyze data via a table, graph, and an equation.

STANDARDS FOR MATHEMATICAL CONTENT:

Use functions to model relationships between quantities.

MGSE8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

MGSE8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Investigate patterns of association in bivariate data.

MGSE8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE:

This task has a lot of potential for deep mathematical discussion of results.

COMMON MISCONCEPTIONS:

- *Students may infer a cause and effect between independent and dependent variables, but this is often not the case. Helping students build an understanding based on the context of problems like this one will help students correct this misconception.*
- *In developing a function rule using the collected data, some students may experience the fact that not all of their data will be linear. Helping students realize that data collected from the real world to solve problems in the real world, is what mathematics is all about. Since data from the real world is hardly ever "neat" or "clean" we need to find nice approximations for this data with tools and strategies such as lines of best fit. Some of the best mathematical discussions come from these types of investigations, because of the un-textbook-like data.*
- *Some students may feel that $y = mx + b$, where b is not 0 is considered a direct proportion. This is difficult for students to understand because of the idea of a constant. Since the constant is just staying the same (and it's not really "with" the other part of the equation, students often make the assumption that it is a direct proportion. To correct this, again go back to the contexts of the tasks and investigations. Allow students to explain what is happening within the context and what would have to happen if it were a direct proportion.*

ESSENTIAL QUESTIONS:

- How can functions be used to model real-world situations?
- How does a change in one variable affect the other variable in a given situation?
- Which tells me more about the relationship I am investigating – a table, a graph or an equation? Why?

MATERIALS:

- stopwatches or clocks with minute hands
- large grid paper or large grid transparencies
- Straightedge
- Copies of task for students

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students should solidify the connection between direct proportions and linear functions. Teachers should support good student dialogue and take advantage of comments and questions to help guide students into correct mathematical thinking. Initial explanations regarding the number of heartbeats being directly proportional to the number of seconds could center on such arguments as, “If I checked my pulse three times as long, then the number of beats should be three times as many.”

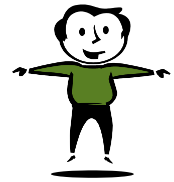
DIFFERENTIATION:

Extension:

- To extend the heartbeat activity, pose the question of how to interpret $y = 72x$ if it represents a function describing the kind of data involved in this problem situation. (If x is the number of minutes and the heart beats 72 times per minute, then the total number of beats y is $72x$. Emphasize the meaning of the slope as the rate of change: for each additional minute the number of heartbeats increases by 72. Pose the hypothetical situation that another person has $y = 120x$ as the function for her number of heartbeats. Ask the students to interpret what this means. The discussion would certainly include the fact that this woman’s heart rate is 120 beats per minute, which is much greater than the other rate of 72 beats per minute.

Intervention/Scaffolding

- Before beginning the task, help students build and understanding of the context of the problem and what they are trying to figure out. Reviewing skills such as plotting points on a coordinate plane and writing a function rule prior to beginning this task may also be helpful.



Heartbeats

Solutions for Parts 1-3

Students will use a variety of different methods to determine the number of heartbeats for 25, 60, and 120 seconds. Some may find the number of heartbeats for 25 seconds by adding half of the number of heartbeats collected for 10 seconds to the number of heartbeats collected for 20 seconds. Others may add the average of the number of heartbeats collected for 10 and 20 seconds to the number of heartbeats for 10 seconds. To find the number of heartbeats for 60 seconds, some students may triple the number of heartbeats for 20 seconds while others may add the number of heartbeats for 20 and 40 seconds. There will also be students who graph the data and use the graph to make predictions. As long as the students can estimate the number of heartbeats using correct mathematical thinking, encourage them to be creative.

Part 1:

In the sixth and seventh grade, you studied proportional relationships. Do you think that the number of heartbeats you can count is proportional to the number of seconds that you check your pulse? Explain why or why not.

Part 2:

- Work with your partner to take measurements and test your conjecture. One of you will be the timer and the other will count their own number of heartbeats per period of time.
- First, count and record the number of beats in 10 seconds, and repeat the experiment counting the number of beats in 20 seconds and 40 seconds.

Heartbeat Data for ___________________________

Number of Seconds	Number of Heartbeats
10	
20	
40	

- Predict how many times your heart would beat in 25 seconds, in 60 seconds, and in 120 seconds. Explain how you made your predictions.

Part 3:

After gathering this data, change jobs. The person who kept the time now checks his/her pulse rate for 10 seconds, 20 seconds, and 40 seconds.

Heartbeat Data for ___________________________

Number of Seconds	Number of Heartbeats
10	
20	
40	

- Predict how many times your heart would beat in 25 seconds, in 60 seconds, and in 120 seconds. Explain how you made your predictions.

Part 4:

- Develop a function rule (equation) to represent your pulse rate. How does your function rule compare with your partner's function rule? Explain to your partner why your function rule is valid.

Solution – Part 4

In developing a function rule using the collected data, some may experience the fact that not all of their data will be linear. This could take the class into a discussion of irregular heartbeats and using the data collected to determine if their heartbeats were regular. Later in the unit, the teacher may elect to return to this task and ask students to find a line of best fit for any non-linear data. Students could come up with possible explanations for variations in heart-rate, such as the effect of exercise, health conditions (e.g. thyroid problems), etc.

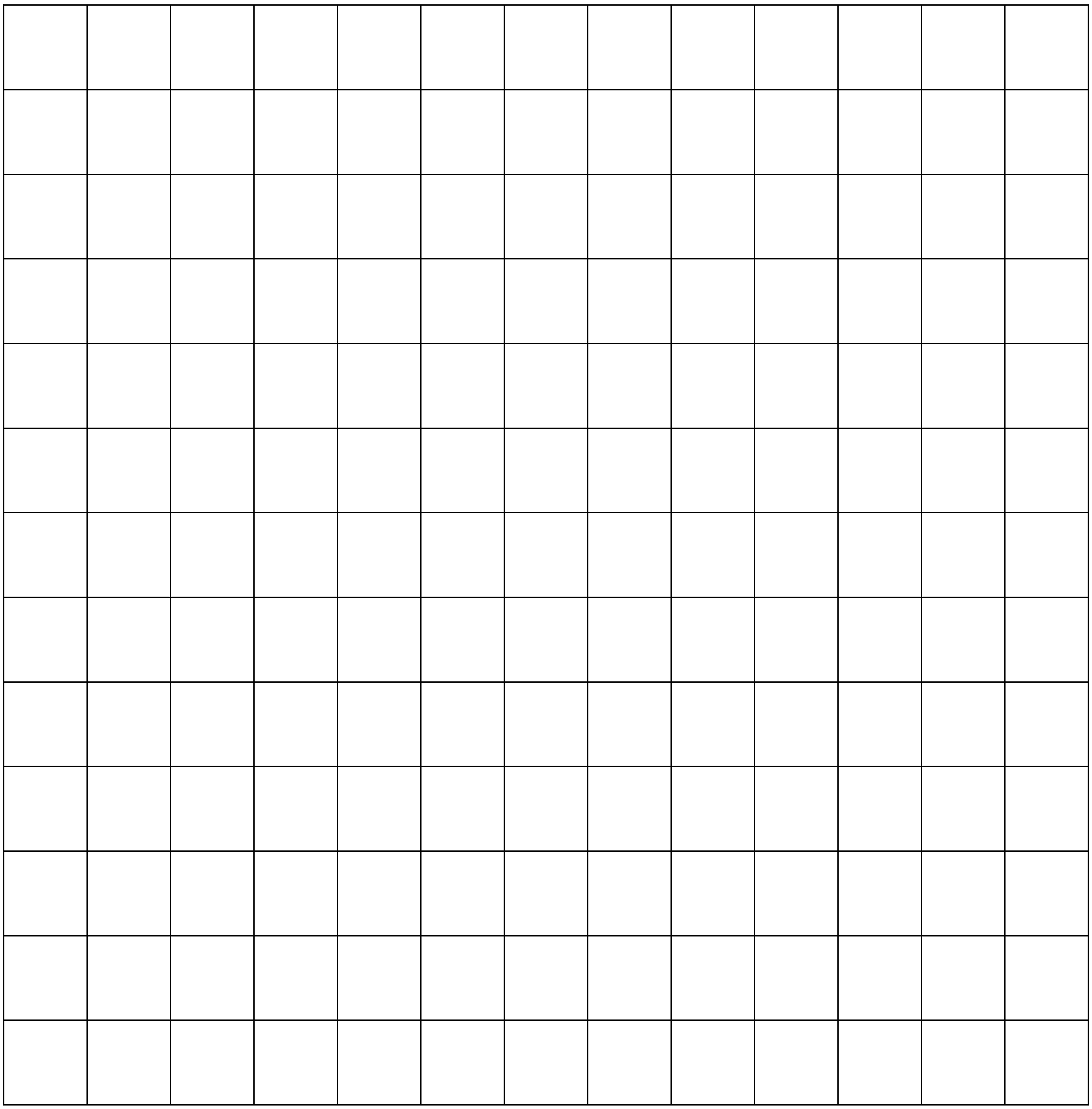
Comments

Direct proportions are one class of linear functions. In a direct proportion, the y-intercept is 0. Thus, direct proportions are of the form $y = kx$ as studied in Grade 6 and Grade 7 mathematics.

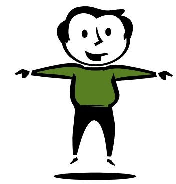
Problem continues on next page. →

Part 5:

Create a scatter plot to represent your pulse rate. How does your graph compare with your partner's graph? Explain to your partner why your graph is valid.



Heartbeats



Part 1:

In the sixth and seventh grade, you studied proportional relationships. Do you think that the number of heartbeats you can count is proportional to the number of seconds that you check your pulse? Explain why or why not.

Part 2:

- Work with your partner to take measurements and test your conjecture. One of you will be the timer and the other will count their own number of heartbeats per period of time.
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Heartbeat Data for ______________________________

Number of Seconds	Number of Heartbeats
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Part 3:

After gathering this data, change jobs. The person who kept the time now checks his/her pulse rate for 10 seconds, 20 seconds, and 40 seconds.

Heartbeat Data for ____________________________

Number of Seconds	Number of Heartbeats
10	
20	
40	

- Predict how many times your heart would beat in 25 seconds, in 60 seconds, and in 120 seconds. Explain how you made your predictions.

Part 4:

- Develop a function rule (equation) to represent your pulse rate. How does your function rule compare with your partner's function rule? Explain to your partner why your function rule is valid.

Part 5:

Create a scatter plot to represent your pulse rate. How does your graph compare with your partner's graph? Explain to your partner why your graph is valid.

Styrofoam Cups (Spotlight Task)

Back to Task Table

From Estimation 180 **http://www.estimation180.com/styrofoamcups.html**

STANDARDS FOR MATHEMATICAL CONTENT

MGSE8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve real world problems through the application of algebraic concepts, seek the meaning of a problem and look for efficient ways to represent and solve it.
2. Reason abstractly and quantitatively. Students demonstrate quantitative reasoning by representing and solving real world situations using visuals, equations, inequalities and linear relationships into real world situations
3. Construct viable arguments and critique the reasoning of others Students will apply their knowledge of functions to support their arguments and critique the reasoning of others while supporting their own position.
6. Attend to precision. Students will use appropriate algebraic language to describe and represent functions.

ESSENTIAL QUESTIONS

- How can functions be used to model real-world situations?
- How does a change in one variable affect the other variable in a given situation?

MATERIALS REQUIRED

- Access to Estimation 180 videos and pictures, possibly Styrofoam cups for concrete modeling

TIME NEEDED

- 1 class

TEACHER NOTES

This task can be found at: http://www.estimation180.com/styrofoamcups.html
In this task, students will predict the number of cups needed to reach the top of the door. Students should determine what information they need in order to make the calculation, and use that information to write relevant functions.

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

TASK DESCRIPTION

Styrofoam cups are stacked side-by-side. How many cups will it take to reach the top of the door? Write a guess that is too high, one that is too low, and your best guess. How do you know?

ACT 1:
Watch the video **http://www.estimation180.com/styrofoamcups.html**
Think and wonder: What do you notice? What do you want to know after watching the video? How can you come up with answers to your questions?
Guiding questions to consider if the students don't come up with them on their own might be:

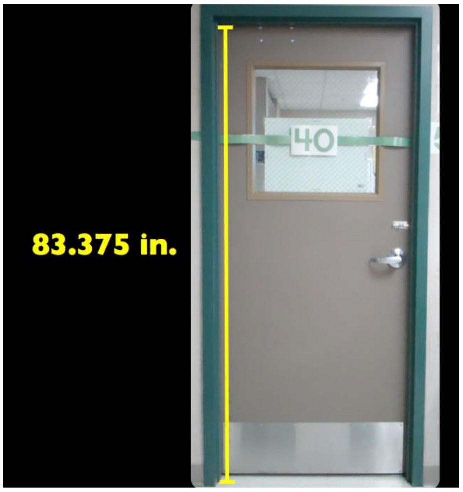
- 1. How many cups will it take to reach the top of the door?
- 2. Write down a guess.

ACT 2:
What information would be useful to know here?

With the information from Act 2, find out how many cups will be needed to reach the door top.

- Students may work individually for 5-10 minutes to come up with a plan, test ideas, and problem-solve.
- After individual time, students may work in groups (3-4 students) to collaborate and solve the task.

Info 1



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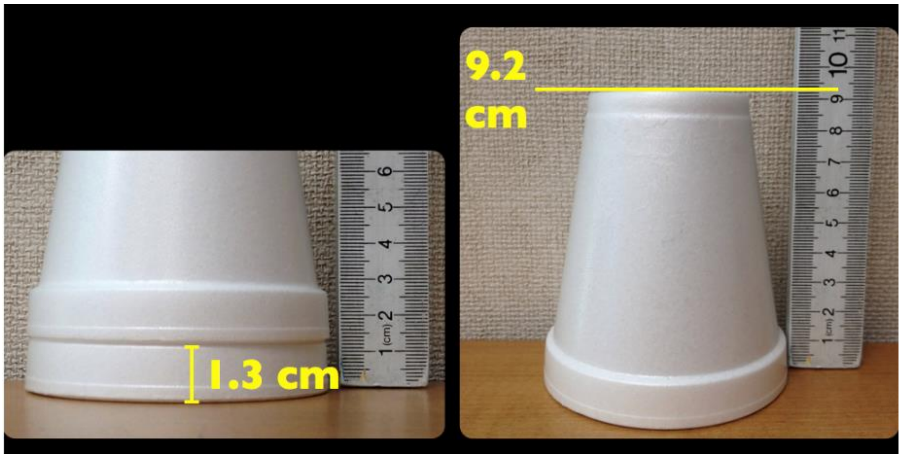

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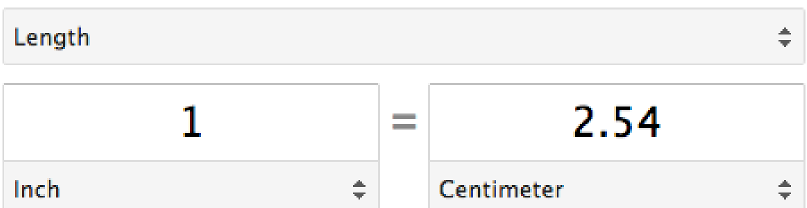
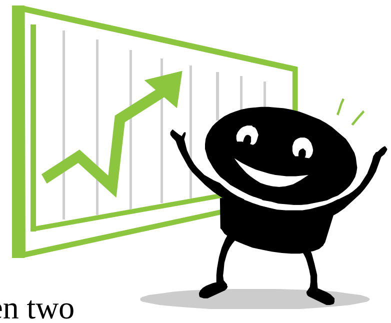


ACT 3
If we weren't exactly right, what could account for the error?

EXTENSIONS:
Design a cup so that 100 of the cups would stack to the top of the door.
How should we title this lesson so it captures **the math we used** and **where we used it**?
Possible: Use Linear Equations to Stack Cups to the Height of a Door

Walk the Graph

Back to Task Table



In this task, students will develop a deeper understanding of graphing linear functions and rates of change.

STANDARDS FOR MATHEMATICAL CONTENT:

Use functions to model relationships between quantities.

MGSE8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

MGSE8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Investigate patterns of association in bivariate data.

MGSE8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

MGSE8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

MGSE8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

a. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.
b. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task is intended to engage students in the mathematical practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

Students need multiple experiences with graphing "stories" such as this. These stories can become part of a daily or weekly routine that takes just a few minutes and may lead to some rich mathematical discussions and corrections of misconceptions such as the one below. A nice resource for stories for students to graph is Graphing Stories.

COMMON MISCONCEPTIONS:

- *Students may see a negative slope and interpret that it means that a student is walking down hill. This can be corrected through discussions of graph contexts. As in learning to read fiction and non-fiction, students need to be taught how to use context clues. These clues, found on titles and labels of a graph, in the context of how the data was collected and for what purpose, as well as the shape of the graph itself all tell a story about that data. Students need to be guided through reading a graph for understanding. Comprehension is the goal, so context and the clues they present must be addressed.*

- *Students may infer a cause and effect between independent and dependent variables, but this is often not the case. Helping students build an understanding based on the context of problems like this one will help students correct this misconception.*

- *Students may graph incorrectly. Students should investigate multiple contextual situations to gain an understanding of the roles of independent and dependent variables and how they are related. The convention that x usually represents the independent variable and y represents the dependent variable should be emphasized as students make sense of the relationships in each investigation. The reason for the convention is to facilitate to mathematical communication.*

ESSENTIAL QUESTIONS:

- What strategies can I use to help me understand and represent real situations involving linear relationships?
- What can I infer from the data?
- How can functions be used to model real-world situations?
- How can I determine if there is an association between two given sets of data?
- How can the properties of lines help me to understand graphing linear functions?

MATERIALS:

- calculator based ranger
- motion detector *(optional)*
- Graph paper http://incompetech.com/graphpaper or an online, free graphing calculator such as Desmos
- Straightedge
- Copies of task for students

GROUPING:

- Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will explore slopes of linear functions through the relationship of distance and time. It is important that students are provided an opportunity to perform a comparative analysis on the different slopes and their corresponding graphs to help students develop a deeper understanding of graphing linear functions and rates of change. Students should also be encouraged to create a table in addition to the graph for each given line.

DIFFERENTIATION:

Extension:

- In the task, Eddie was attached to a motion detector to create the desired slopes and lines. Your job is to create a real-world situation in which Eddie does various things to create the desired slope or line. You'll need to decide on a place that is considered "home base" or (0,0) as it relates to time and distance. Write a story about Eddie and his adventures. Then identify when his actions would produce each type slope or line.

Intervention/Scaffolding:

- For kinesthetic learners, it could benefit them to actually walk the graph. Construct a coordinate grid (with masking tape on the floor of your classroom or with chalk outside) and have the students physically walk each graph, explaining what is happening as they move (for the first graph, as students move along the graph creating the negative slope, students may say, “My distance from the motion detector is decreasing as time increases.”)

Walk the Graph

Eddie's teacher used a motion detector connected to an overhead graphing calculator to show graphs of how far Eddie was from the motion detector for several seconds. Eddie was asked to walk in such a way as to produce graphs with certain characteristics. Explain how Eddie needs to walk to produce a graph which is:

1. A line with a negative slope.

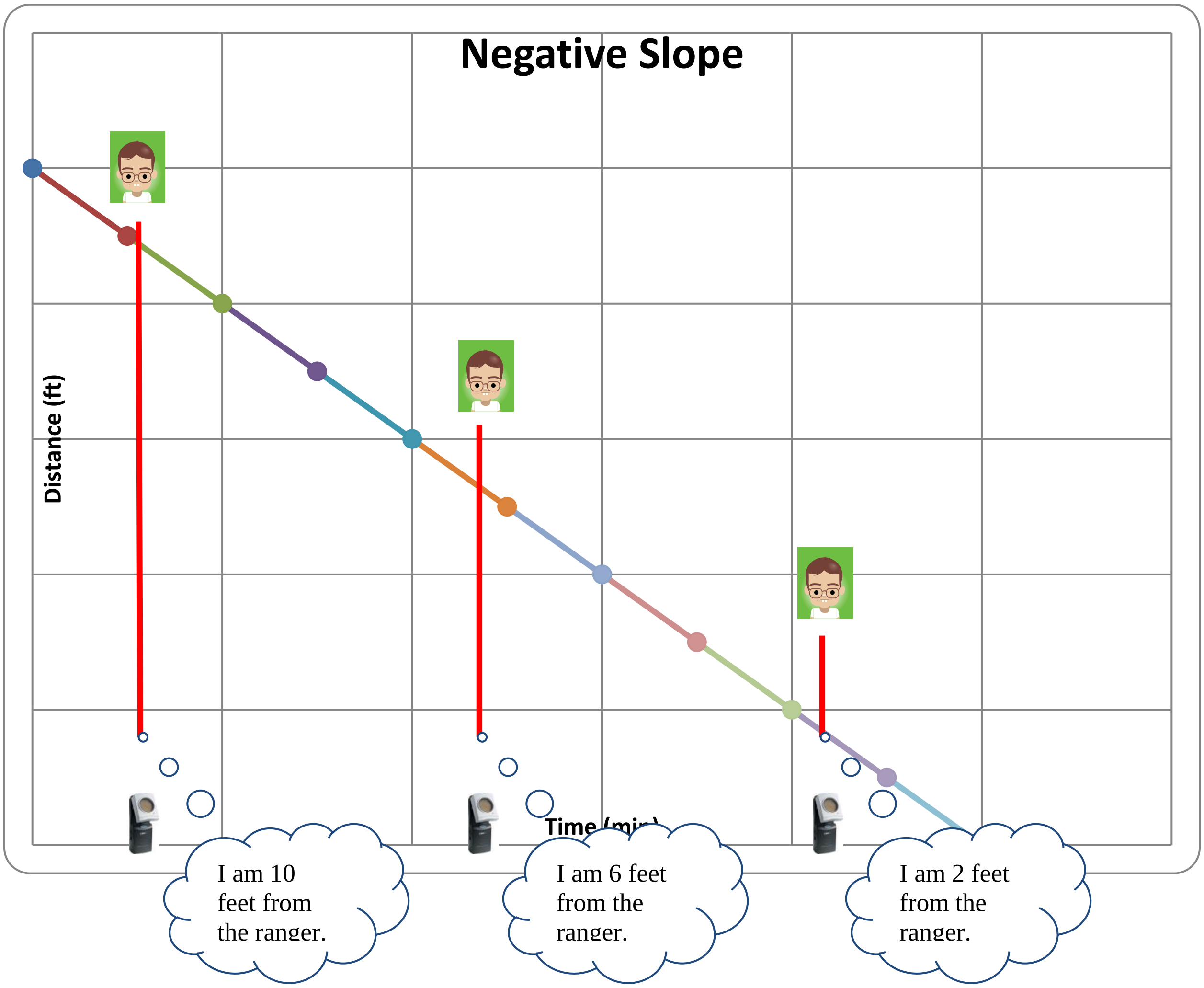
Comments

Students may benefit from sketching a line with a negative slope and creating a corresponding table. It is important that students analyze the graph and make connections between the graph, the table, and the context of the problem.

The walker should stand as far away from the motion detector as possible but within the range of the motion detector (staying in front of the motion detector) and walk at a steady pace towards the motion detector.

Solution

Time (min)	***0***	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***
Distance (ft)	***10***	***9***	***8***	***7***	***6***	***5***	***4***	***3***	***2***	***1***	***0***



Students should recognize that the change in distance compared to the change in time is negative. The graph and table illustrates the following:

- ***the walker is moving toward the motion detector as time increases***
- ***distance decreases as time increases***

The slope of this line is -1. Therefore, the distance decreases one foot per minute.

$$\frac{\Delta\ distance}{\Delta\ time} = \frac{-1\ ft}{1\ min} = -1\ {}^{ft}/_{min}$$

Comments

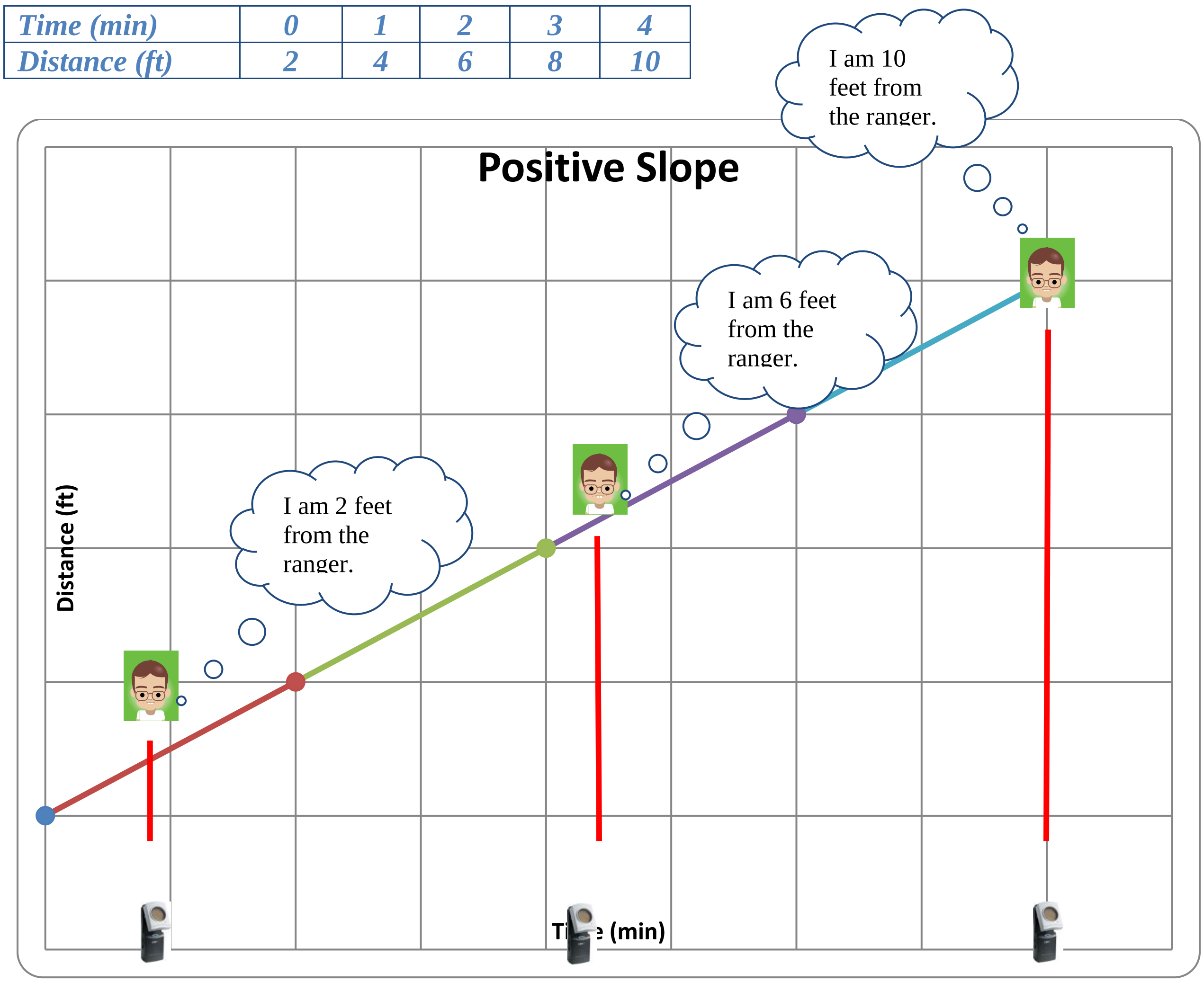
It is important for students to understand that to create a line representing a linear function they must walk at a steady pace; constant rate of change.

2. A line with a positive slope.

Comments
For this graph, the walker should stand close to the motion detector when the timing begins and walk at a steady pace away from the motion detector, yet staying in front of it.

Solution

Time (min)	*0*	*1*	*2*	*3*	*4*
Distance (ft)	*2*	*4*	*6*	*8*	*10*



Students should recognize that the change in distance compared to the change in time is positive. The graph and table illustrates the following:

- ***the walker is moving away from the motion detector as time increases***
- ***distance increases as time increases***

The slope for this line is 2. Therefore, the distance increases two feet per minute.

$$\frac{\Delta\, distance}{\Delta\, time} = \frac{2\, ft}{1\, min} = 2\ {}^{ft}/_{min}$$

<u>Comments</u>

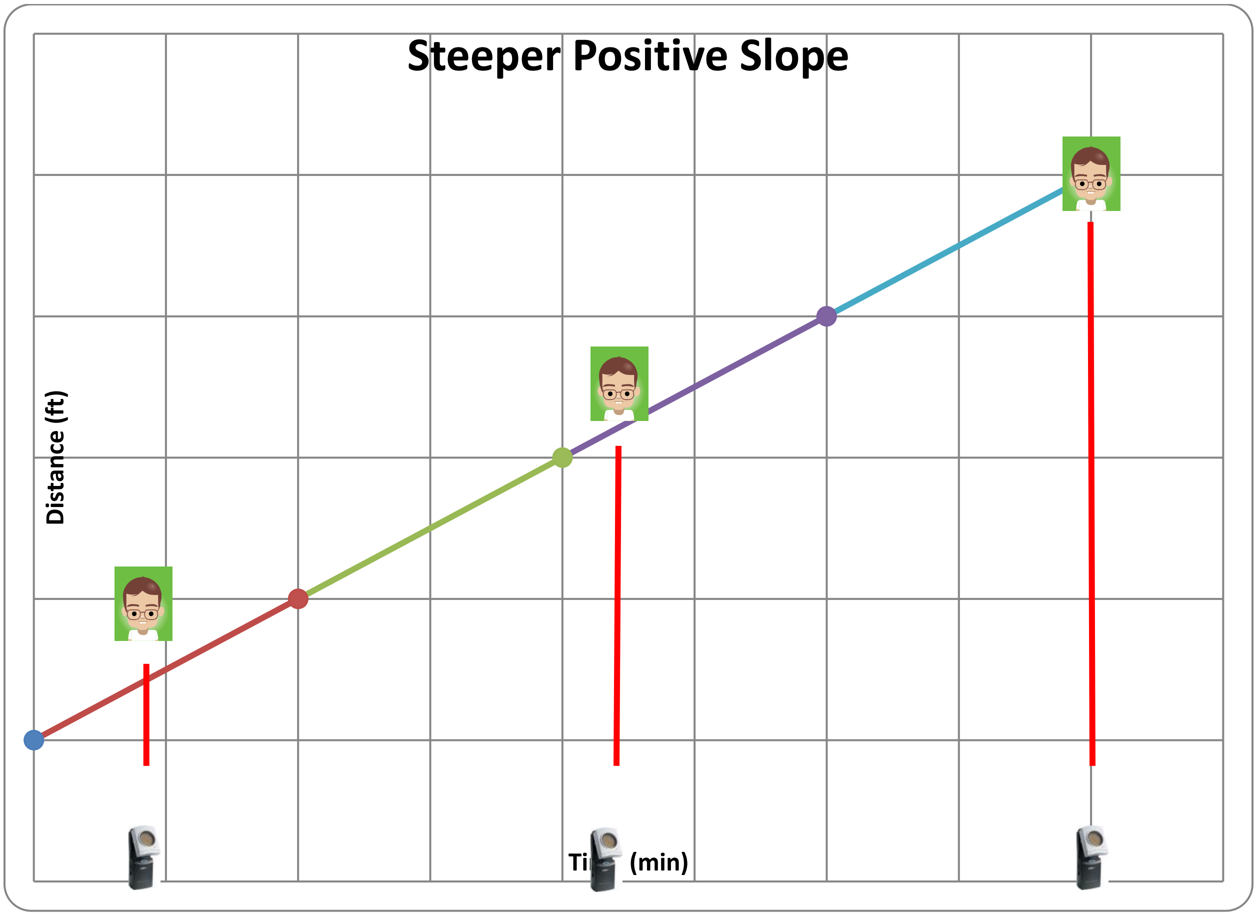
It is important for students to understand that to create a line representing a linear function they must walk at a steady pace; constant rate of change.

3. A line with a steeper slope than the one in problem 2.

<u>Comments</u>
For this graph, the walker repeats the process to produce graph (2), but walks at a faster pace. The distance away from the motion detector must increase more rapidly than in graph (2).

<u>Solution</u>

Time (min)	***0***	***1***	***2***	***3***	***4***
Distance (ft)	***5***	***10***	***15***	***20***	***25***



Students should recognize that the change in distance compared to the change in time is positive. The graph and table illustrates the following:

- ***the walker is moving away from the motion detector as time increases***
- ***distance increases as time increases***

The slope for this line is 5. Therefore, the distance increases five feet per minute.

$$\frac{\Delta\ distance}{\Delta\ time} = \frac{5}{1} = 5$$

Comments

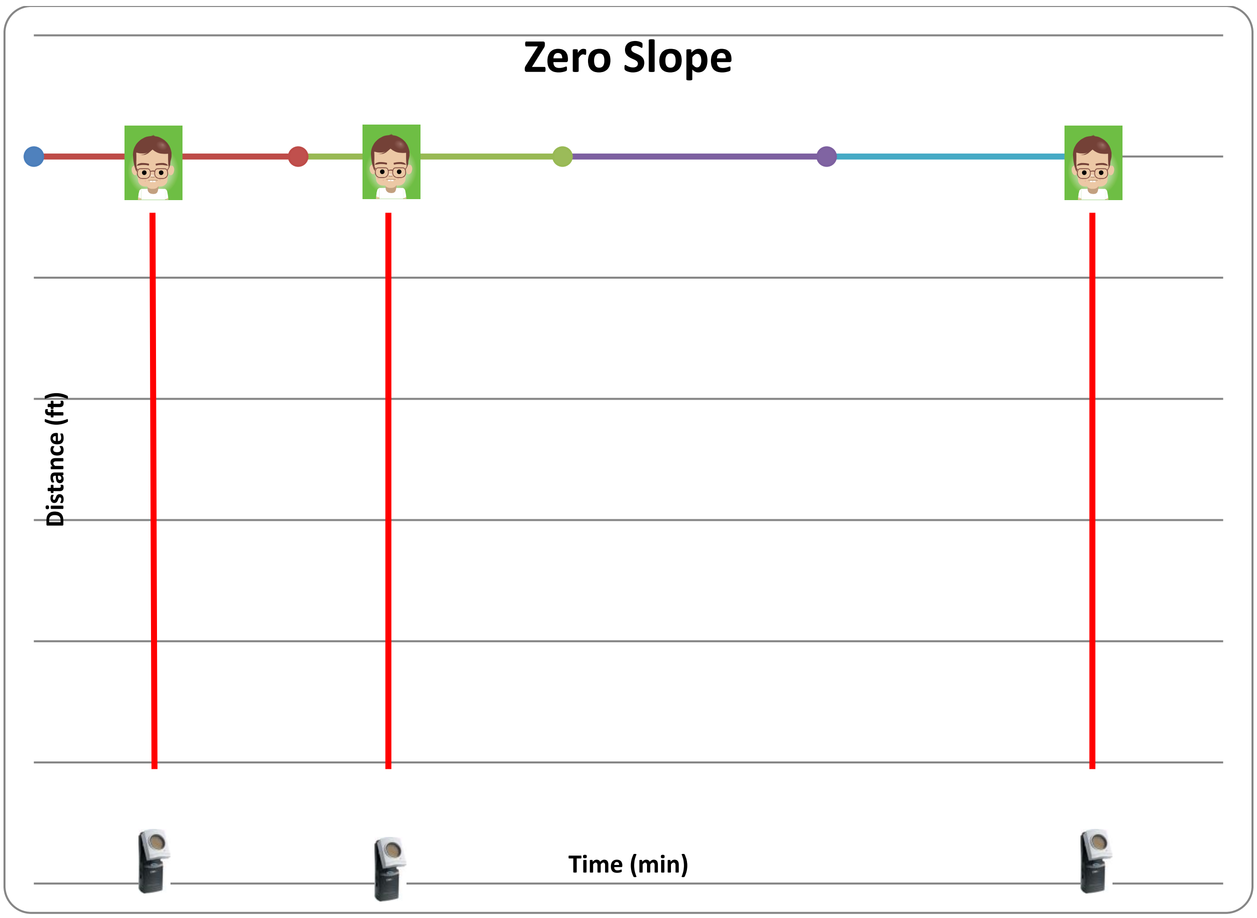
It is important for students to compare the rate of change in question (2), two feet per minute, to this rate of change, five feet per minute, and each corresponding graph.

4. A horizontal line.

Comments
For this graph, the walker may choose some distance away from the motion detector and remain in that position. This is an example of a constant function. The distance from the motion detector remains constant.

Solution

Time (min)	*0*	*1*	*2*	*3*	*4*
Distance (ft)	*3*	*3*	*3*	*3*	*3*



Students should recognize that the change in distance compared to the change in time is zero. The graph and table illustrates the following:

- ***the walker is three feet from the motion detector and is not moving as time increases***
- ***distance is constant as time increases***

The slope for this line is 0. Therefore, the distance does not change while time increases.

$$\frac{\Delta \text{ distance}}{\Delta \text{ time}} = \frac{0 \text{ ft}}{1 \text{ min}} = 0 \ {}^{\text{ft}}/_{\text{min}}$$

<u>Comments</u>

It is important for students to understand constant functions as they will continue to use linear functions throughout Mathematics 1.

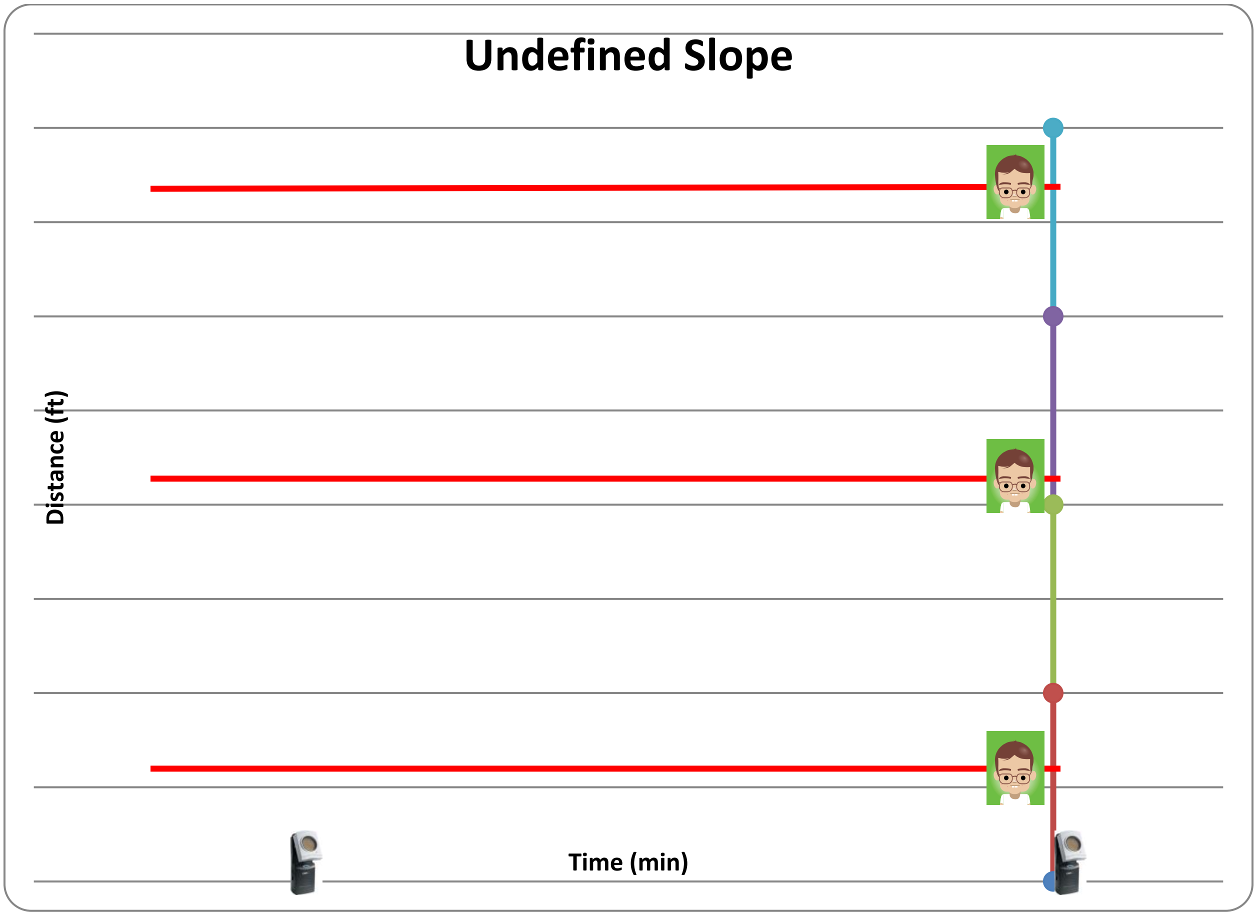
5. A vertical line.

<u>Comments</u>

For this graph, the walker should realize that time cannot remain constant while the distance is increasing. This graph is challenging for most students and may require guiding questions from the teacher.

<u>Solution</u>

Time (min)	***6***	***6***	***6***	***6***	***6***
Distance (ft)	***0***	***1***	***2***	***3***	***4***



Students should recognize that the change in distance compared to the change in time is not possible (undefined). The graph and table illustrates the following:

- ***the walker is moving away from the motion detector as time remains constant***
- ***distance increases as time remains constant***

The slope for this line is undefined. Therefore, distance increasing while time remains constant is impossible.

$$\frac{\Delta\, distance}{\Delta\, time} = \frac{1\, ft}{0\, min} = undefined$$

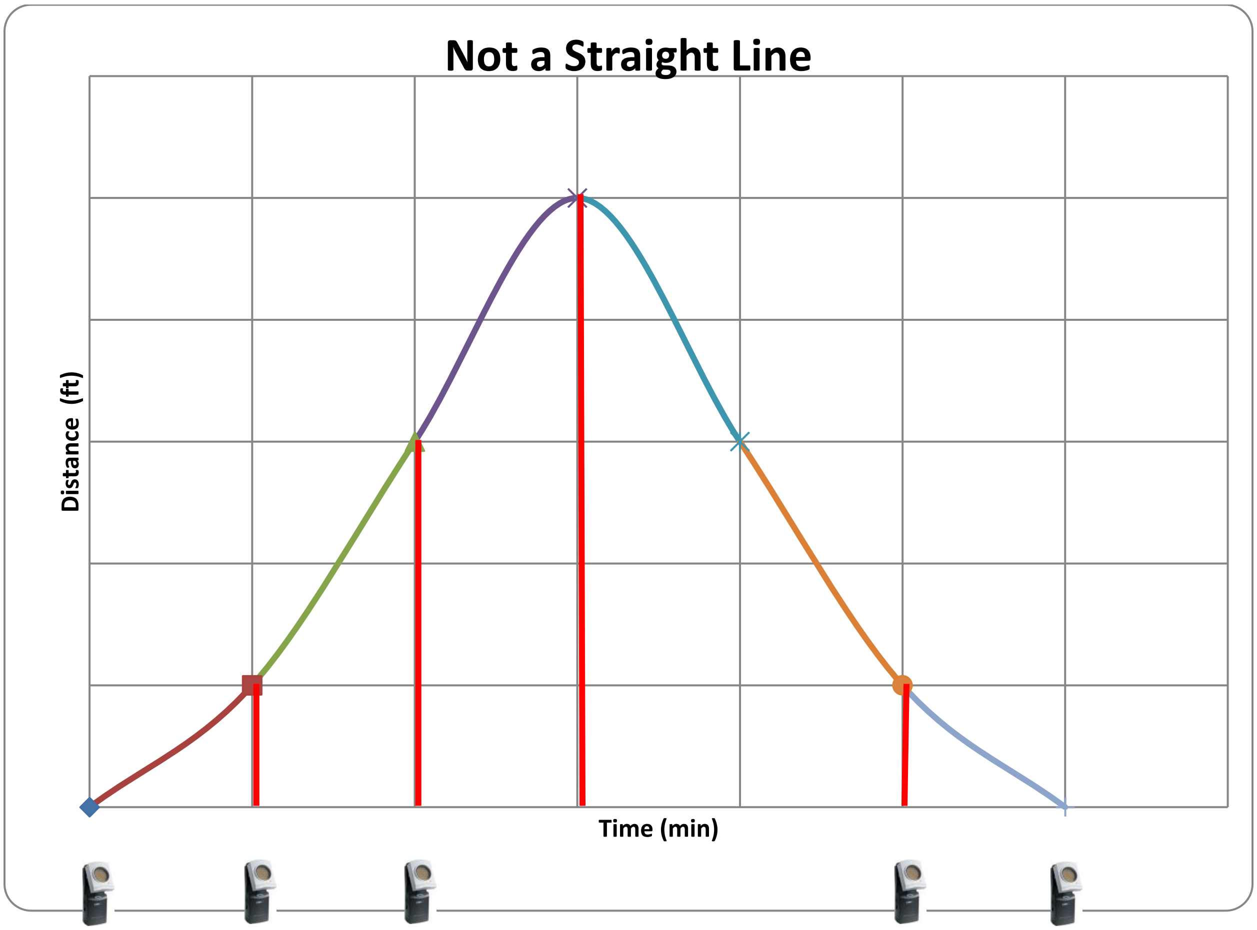
<u>Comments</u>

It is important for students to understand that to create a line representing a linear function they must walk at a steady pace; constant rate of change. Answers will vary.

6. Not a straight line.

Solution

Time (min)	*0*	*1*	*2*	*3*	*4*	*5*	*6*
Distance (ft)	*0*	*1*	*3*	*5*	*3*	*1*	*0*



Students should recognize that the change in distance compared to the change in time is not constant. The graph and table illustrates the following:

- ***the walker is moving away from the motion detector and after three minutes is walking towards the motion detector as time increases***
- ***as time increases , distances increases then decreases***

There is not a slope for this graph due to variable rates of change.

Comments

For this graph, the walker could vary the pace at which he walks. For example, the walker might start out as far away from the motion detector as possible, walk slowly at first, and then increase his or her speed until the walker gets very close to the motion detector. Then if the walker walks away from the detector, quickly at first, and then slowing down, the walker might even produce a graph that looks symmetrical. Teachers may have students predict what the graph will look like before they take this walk.

Walk the Graph

Eddie's teacher used a motion detector connected to an overhead graphing calculator to show graphs of how far Eddie was from the motion detector for several seconds. Eddie was asked to walk in such a way as to produce graphs with certain characteristics. Explain how Eddie needs to walk to produce a graph which is:

1. A line with a negative slope.

2. A line with a positive slope.

3. A line with a steeper slope than the one in problem 2.

4. A horizontal line.

5. A vertical line.

6. Not a straight line.

Interpreting Distance-Time - FAL

Back to Task Table

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/lessons.php?taskid=208&subpage=concept

In this task, students will use functions to model relationships between quantities.

STANDARDS FOR MATHEMATICAL CONTENT:

Use functions to model relationships between quantities.

MGSE8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

MGSE8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.

BACKGROUND KNOWLEDGE:

This task builds on the previous task, as students are tasked with graphing contexts or stories. Students need multiple experiences with graphing “stories” such as this. These stories can become part of a daily or weekly routine that takes just a few minutes and may lead to some rich mathematical discussions and corrections of misconceptions such as the one below. A nice resource for stories for students to graph is Graphing Stories.

COMMON MISCONCEPTIONS:

- *See FAL*

ESSENTIAL QUESTIONS:

- How can I describe a situation given a graph?
- What can I infer from the data?
- How can functions be used to model real-world situations?

MATERIALS:

- See FAL

GROUPING:

- Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, "Interpreting Distance-Time," is a Mathematics Assessment Project Assessment Task that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=208&subpage=concept

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1521

Forget the Formula

Back to Task Table

In this task, students will develop an understanding of graphing linear functions and rates of change.

STANDARDS FOR MATHEMATICAL CONTENT:

Use functions to model relationships between quantities.

MGSE8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

MGSE8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Investigate patterns of association in bivariate data.

MGSE8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

a. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.
b. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

Students should engage in this task collaboratively. The context of this problem can be misleading as many students may "know" the boiling points and freezing points of water for both measurement systems. This may provide a false sense of understanding to students and they may quickly get frustrated if they are working independently.
If students are working collaboratively, they can use their discussions as they work through the problem, to write their expository writing at the end of the task. The best thing about collaborative work for students is that the writing and reflecting is much richer due the discussions while working the task.

COMMON MISCONCEPTIONS:

- *Students often confuse a recursive rule with an explicit formula for a function. For example, after identifying that a linear function shows an increase of 2 in the values of the output for every change of 1 in the input, some students will represent the equation as $y = x + 2$ instead of realizing that this means $y = 2x + b$. When tables are constructed with increasing consecutive integers for input values, then the distinction between the recursive and explicit formulas is about whether you are reasoning vertically or horizontally in the table. Both types of reasoning – and both types of formulas – are important for developing proficiency with functions. Knowing the difference and when each type of reasoning is applicable is an important goal.*
- *When input values are not increasing consecutive integers (e.g., when the input values are decreasing, when some integers are skipped, or when some input values are not integers), some students have more difficulty identifying the pattern and calculating the slope. It is important that all students have experience with such tables, so as to be sure that they do not overgeneralize from the easier examples.*

ESSENTIAL QUESTIONS:

- What strategies can I use to help me understand and represent real situations involving linear relationships?
- What can I infer from the data?
- How can functions be used to model real-world situations?

MATERIALS:

- Straightedge
- Graph paper http://incompetech.com/graphpaper or an online, free graphing calculator such as Desmos
- Copies of task for students

GROUPING:

- Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

This task challenges students to make conversions from Celsius to Fahrenheit and vice versa. The task is written so that students must "discover" the formulas even though they may already know them. Students practice expository writing by explaining, in detail, how they arrived at the formulas.

DIFFERENTIATION:

Extension:

- Multiple Versions of the Task Provided

Intervention/Scaffolding:

- Intervention/Scaffolding is built into the task (Versions A, B, C, and D).

Forget the Formula

Mrs. Kersey overheard two of her students talking about how to convert temperatures from Celsius to Fahrenheit and vice versa. The students said they knew there was a formula, but they could not remember what it was. Mrs. Kersey remarked to you that if they just knew about the freezing point and boiling point of water for each temperature scale, the formula could easily be "rediscovered." Mrs. Kersey has asked you to write an explanation for how to find the formula, showing all your calculations.

Comments

Students will need to recall or research to find that the freezing point of water is 0 degrees Celsius and 32 degrees Fahrenheit. The boiling point of water is 100 degrees Celsius and 212 degrees Fahrenheit.

Solution

One method of finding this formula is to use (0, 32) and (100, 212) as two points on a line. To find the equation of the line only the slope is needed, since the y-intercept, (0, 32), is already given. Calculating the slope, $\frac{212-32}{100-0}$, the students should simplify $\frac{180}{100}$ to $\frac{9}{5}$. Substituting this value for the slope (m) in the equation y = mx + b, we get $y = \frac{9}{5}x + 32$. Because y represents the Fahrenheit temperature and x represents the Celsius temperature, the formula would be more appropriately written $F = \frac{9}{5}C + 32$.

Students could be encouraged to solve this equation for C to produce another form expressing the relationship:

$$F = \frac{9}{5}C + 32$$

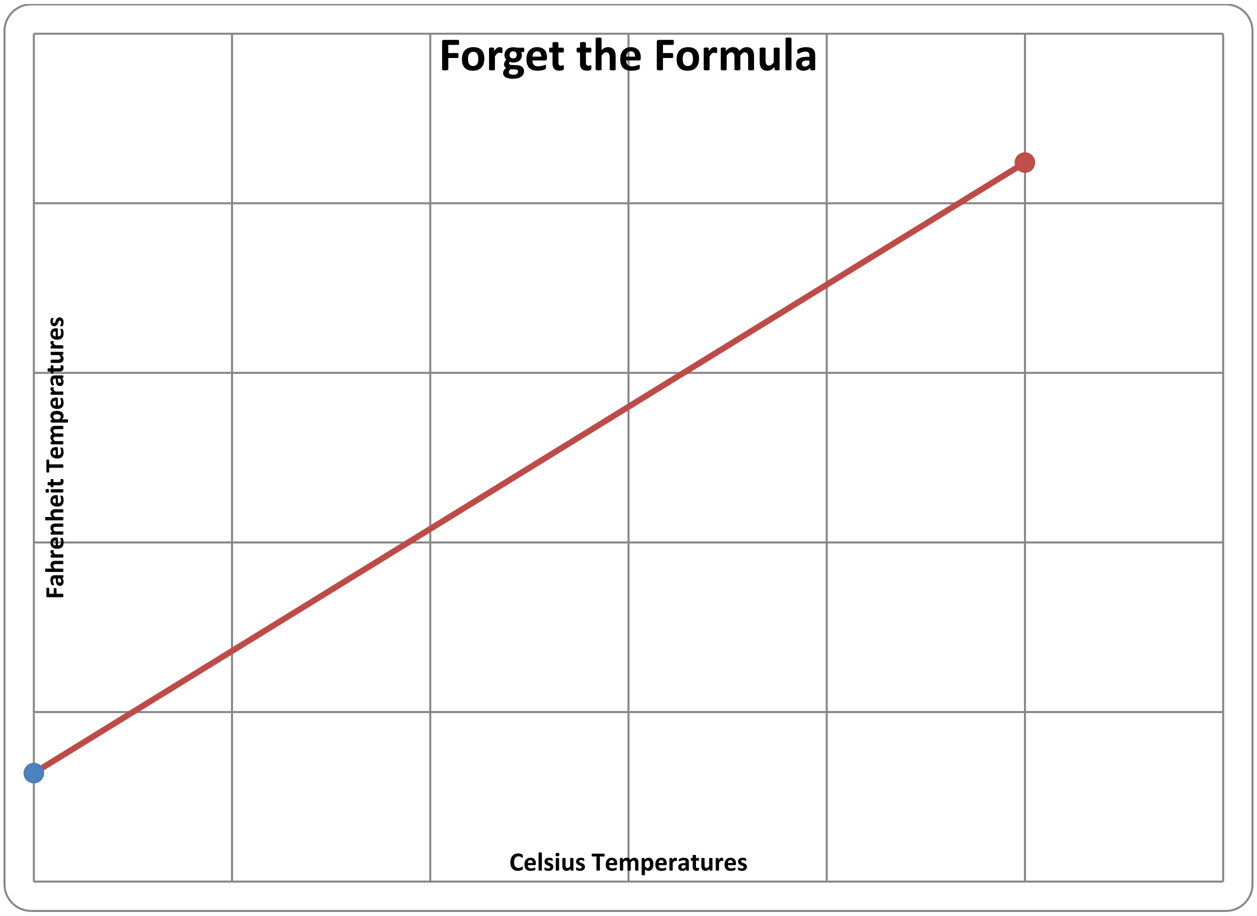
$$F - 32 = \frac{9}{5}C + 32 - 32$$

$$(F - 32)\left(\frac{5}{9}\right) = \left(\frac{9}{5}\right)\left(\frac{5}{9}\right)C$$

$$(F - 32)\left(\frac{5}{9}\right) = C$$

$$C = \left(\frac{5}{9}\right)(F - 32)$$

Students could also produce a graph of the corresponding temperatures.



"Forget the Formula" is representative of a variety of situations in which information is known about the graph and the students have to be able to find the equation from that information. In "Forget the Formula" the information used is two points on the given line; one of these happens to be the y-intercept if the Fahrenheit temperature is used as the y value. Be sure to include other variations on finding the equation of a line, such as given the slope and a point on the line which is not the y-intercept or two points on a line when one is not the y-intercept.

Should neither of the two points cross the y-axis, students would most likely use the point-slope form of an equation. This form is derived from finding the slope using two points on the line. From this, it is seen that $y - y_1 = m(x - x_1)$ ***and*** $y = m(x - x_1) + y_1$***. It is important for students to understand the connections between the various forms of linear equations.***

An interactive website may be used to help students understand the relationships between the various forms of linear equations:
http://zonalandeducation.com/mmts/functionInstitute/linearFunctions/lsif.html

Version A

<u>Forget the Formula</u>

Mrs. Kersey overheard two of her students talking about how to convert temperatures from Celsius to Fahrenheit and vice versa. The students said they knew there was a formula, but they could not remember what it was. Mrs. Kersey remarked to you that if they just knew about the freezing point and boiling point of water for each temperature scale, the formula could easily be "rediscovered." Mrs. Kersey has asked you to find the formula, showing all your calculations.

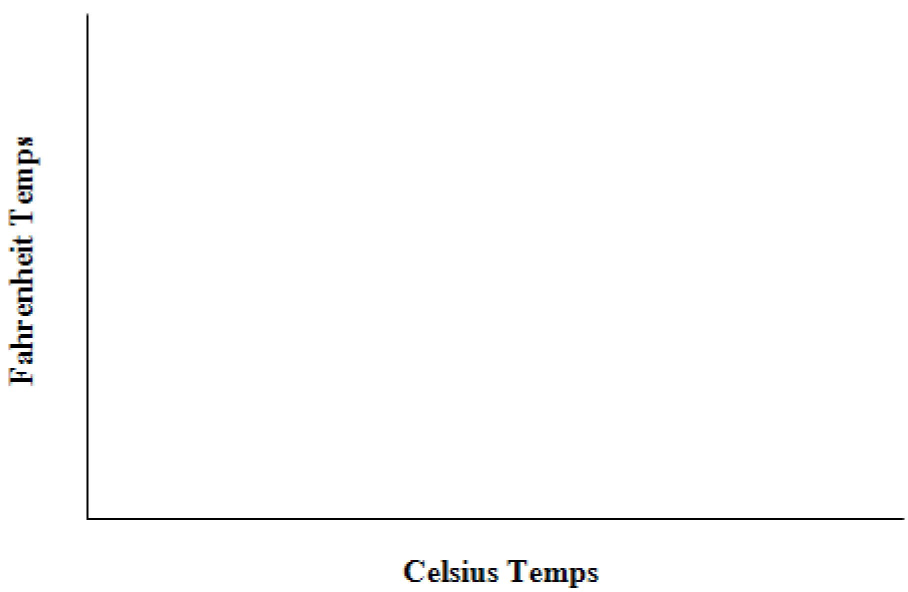
1. Define your variables to convert Celsius to Fahrenheit. Your known is ________; this will be the independent variable. You are trying to find _________, which will be your dependent variable.

2. Complete the table:

	Freezing	Boiling
Celsius		
Fahrenheit		

3. Write the data from the table as 2 ordered pairs. __________ and ___________
   (Remember, x is independent and y is dependent)

4. Sketch a graph and draw a line connecting the points.



5. Find the slope of the line you drew.

6. What is the *y*-intercept of the line? ______________

7. Write an equation for this relationship in slope-intercept form: _____________________

8. Use this information to convert the following Celsius temperatures into Fahrenheit temperatures.

Celsius Temps	Show work	Fahrenheit Temps
25		
−5		
30		

9. Now convert Fahrenheit to Celsius. Solve your formula for the other variable and write the equation below. *(Hint: undo the addition on the right and then divide.)*

10. Convert the following Fahrenheit temperatures to Celsius.

Fahrenheit Temps	Show work	Celsius Temps
50		
80		
100		

Version B

Forget the Formula

Mrs. Kersey overheard two of her students talking about how to convert temperatures from Celsius to Fahrenheit and vice versa. The students said they knew there was a formula, but they could not remember what it was. Mrs. Kersey remarked to you that if they just knew about the freezing point and boiling point of water for each temperature scale, the formula could easily be "rediscovered." Mrs. Kersey has asked you to find the formula, showing all your calculations.

1. Define your variables to convert Celsius to Fahrenheit:

   Independent: ___________________ Dependent: _____________________

2. Use the freezing and boiling points to write 2 ordered pairs.

3. Sketch a graph and draw a line connecting the points.

4. Use the slope and the y-intercept of your graph to write an equation for this relationship.

5. Use your equation to convert the following Celsius temperatures into Fahrenheit temperatures: 23^{o}, -5^{o}, 45^{o}

6. Now convert Fahrenheit to Celsius. Solve your formula for the other variable and write the equation here:

7. Convert the following Fahrenheit temperatures to Celsius: 53^{o}, 82^{o}, 98.6^{o}

Version C

Forget the Formula

Mrs. Kersey overheard two of her students talking about how to convert temperatures from Celsius to Fahrenheit and vice versa. The students said they knew there was a formula, but they could not remember what it was. Mrs. Kersey remarked to you that if they just knew the freezing point and boiling point of water for each temperature scale, the formula could easily be "rediscovered." Mrs. Kersey has asked you to write an explanation for how to find the formula, showing all your calculations.

Mrs. Kersey also wants you to use your written explanation to convert the following temperatures: -7^{o} C, 23^{o} C, 71^{o} C, -8^{o} F, 45^{o} F, 98.6^{o} F

Version D

Forget the Formula

Mrs. Kersey overheard two of her students talking about how to convert temperatures from Celsius to Fahrenheit and vice versa. The students said they knew there was a formula, but they could not remember what it was. Mrs. Kersey remarked to you that if they just knew about the freezing point and boiling point of water for each temperature scale, the formula could easily be "rediscovered." Mrs. Kersey has asked you to write an explanation for how to find the formula, showing all your calculations.

Heartbeats Too!

Back to Task Table

In this task, students are introduced to writing linear equations to fit data.

STANDARDS FOR MATHEMATICAL CONTENT:

Use functions to model relationships between quantities.

MGSE8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

MGSE8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Investigate patterns of association in bivariate data.

MGSE8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

MGSE8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

MGSE8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

a. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.
b. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

This task is an extension of the "Heartbeats" task earlier in this unit. In this task, students use what they learned in that task, as well as the other tasks leading to this, to find a line of best fit for the data they collect and write an equation to describe the data.

COMMON MISCONCEPTIONS:

- *Students may infer a cause and effect between independent and dependent variables, but this is often not the case. Helping students build an understanding based on the context of problems like this one will help students correct this misconception.*
- *In developing a function rule using the collected data, some students may experience the fact that not all of their data will be linear. Helping students realize that data collected from the real world to solve problems in the real world, is what mathematics is all about. Since data from the real world is hardly ever "neat" or "clean" we need to find nice approximations for this data with tools and strategies such as lines of best fit. Some of the best mathematical discussions come from these types of investigations, because of the un-textbook-like data.*
- *Some students may feel that $y = mx + b$, where b is not 0 is considered a direct proportion. This is difficult for students to understand because of the idea of a constant. Since the constant is just staying the same (and it's not really "with" the other part of the equation, students often make the assumption that it is a direct proportion. To correct this, again go back to the contexts of the tasks and investigations. Allow students to explain what is happening within the context and what would have to happen if it were a direct proportion.*
- *Some student may not realize that a data point may or may not be needed to find the line of best fit. Discussions that look at multiple lines (spaghetti (uncooked) works well for this) for a set of graphed points can help students determine that all points may not be needed.*

ESSENTIAL QUESTIONS:

- What strategies can I use to help me understand and represent real situations involving linear relationships?
- How can the properties of lines help me to understand graphing linear functions?
- What can I infer from the data?

MATERIALS:

- stopwatch or classroom timer
- uncooked spaghetti
- straightedge
- Graph paper http://incompetech.com/graphpaper or an online, free graphing calculator such as Desmos
- Copies of task for students

GROUPING:

- Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students are introduced to writing linear equations to fit data. Students will experience two different methods to determine a line of best. Teachers should support good student dialogue and take advantage of comments and questions to help guide students into correct mathematical thinking.

DIFFERENTIATION:

Extension:

- Method 2 serves as the Extension

Intervention/Scaffolding:

- Have students review creating scatter plots and lines of best fit, along with writing equations for lines of best fit before starting this task. Review box-and-whisker plots, lower and upper quartiles prior to beginning Method 2 of this task. Prompt struggling students with questions that guide them to build their own understandings. Group students in a way that promotes a collaborative learning situation.

Heartbeats Too!

Part ❶: Data Collection

- Work with your partner to take measurements and test your conjecture. One of you will be the timer and the other will count their own number of heart beats per period of time.

- First, count and record the number of beats in 10 seconds, then repeat the experiment counting the number of beats in 20 seconds, 30 seconds, 40 seconds, 50 seconds, and 60 seconds.

Heartbeat Data for ___________________________

Time (sec)	Number of Heartbeats
10	
20	
30	
40	
50	
60	
70	
80	
90	
100	
110	
120	

Part ❷: Data Representation

- Graph your data.

Part ❸: Fitting a Line to Data

<u>Method 1: Estimation</u>

- Use a piece of raw spaghetti to visualize and estimate a line of best fit.
- Choose two points on the line.
- Use your two points to write an equation for a line of best fit.

<u>Method 2: Lower and Upper Quartiles</u>

- Find the five-number summary for your *x*-values (time).

- Find the five-number summary for your *y*-values (number of heartbeats).

- Record your lower and upper quartiles for your *x*-values and your *y*-values.

	Time (sec)	Number of Heartbeats
Lower Quartile		
Upper Quartile		

- Draw a horizontal box-and-whisker plot using the five-number summary for your *x*-values (time.) Plot your box-and whisker plot under the *x*-axis on your graph.

- Draw a vertical box-and-whisker plot using the five-number summary for your *y*-values (number of heartbeats). Plot your box-and whisker plot next to the *y*-axis on your graph.

- Draw vertical lines from the lower and upper quartile values on the *x*-axis box-and whisker plot.

- Draw horizontal lines from the lower and upper quartile values on the *y*-axis box-and whisker plot.

- Find the coordinates of the vertices of the rectangle formed by the intersection of the vertical and horizontal lines. We will refer to these vertices as quartile points. **Note: Use only the quartile points (vertices) that follow the direction of the data.**

- Do the quartile points have to be actual data points? Why or why not?

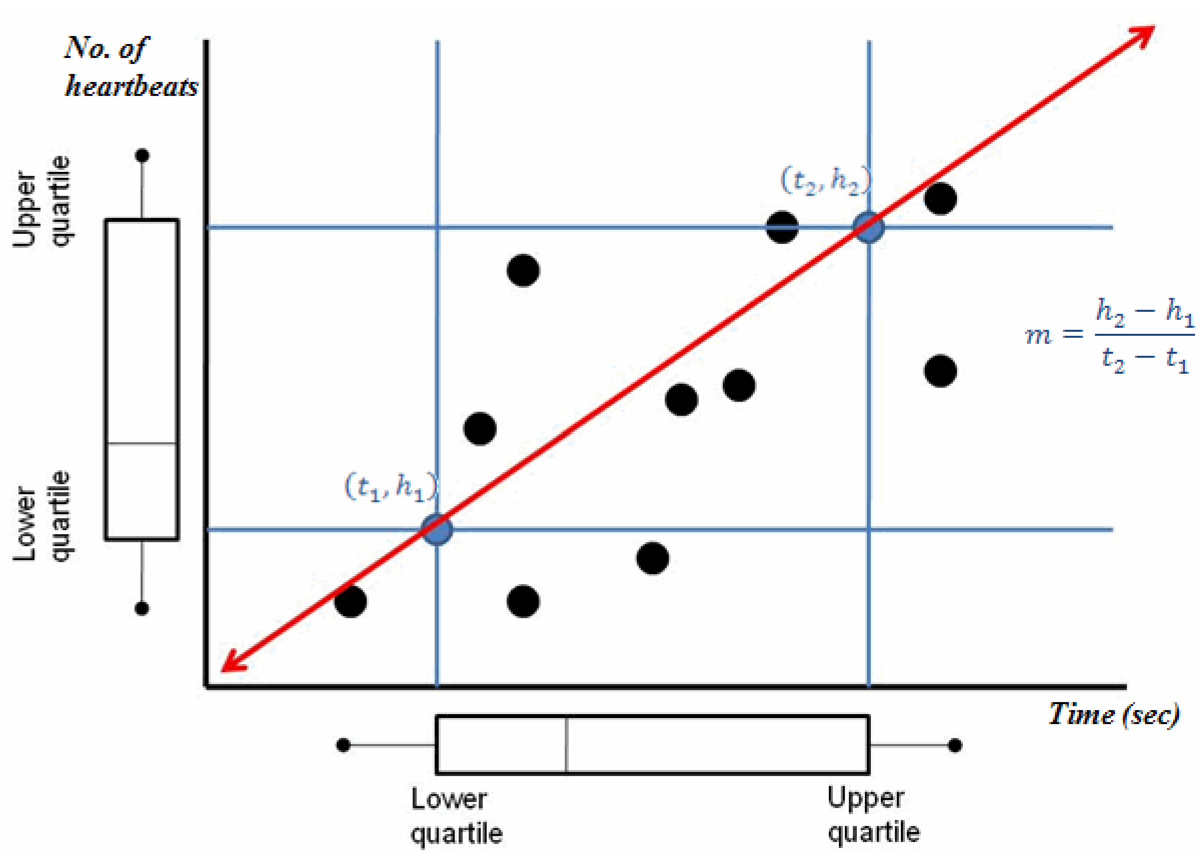
- Draw a line connecting the two quartile points.

- Write an equation for this line. This is a line of best fit for your data.

- Use one of your equations to predict how many times your heart would beat in 25 seconds, in 240 seconds, and in 3 minutes. Explain how you made your predictions.

  Comments

  Students should recognize that the variable representing time represents seconds and students will need to convert three minutes to seconds before using substitution.



Part ❹: Data Collection
After gathering this data, change jobs. The person who kept time now checks his/her pulse rate and repeat **Part ❸: Fitting a Line to Data.**

Part ❺: Analysis

- How does your line of best fit compare with your partner's line of best fit? Explain to your partner why your line of best fit is valid.

Comments

Students could come up with possible explanations for variations in heart-rate, such as the effect of exercise, health conditions (e.g., thyroid problems), etc.

Heartbeats Too!

Part ❶: Data Collection

- Work with your partner to take measurements and test your conjecture. One of you will be the timer and the other will count their own number of heart beats per period of time.

- First, count and record the number of beats in 10 seconds, then repeat the experiment counting the number of beats in 20 seconds, 30 seconds, 40 seconds, 50 seconds, and 60 seconds.

Heartbeat Data for ___________________________

Time (sec)	Number of Heartbeats
10	
20	
30	
40	
50	
60	
70	
80	
90	
100	
110	
120	

Part ❷: Data Representation

- Graph your data.

Part ❸: Fitting a Line to Data

Method 1: Estimation

- Use a piece of raw spaghetti to visualize and estimate a line of best fit.
- Choose two points on the line.
- Use your two points to write an equation for a line of best fit.

Method 2: Lower and Upper Quartiles

- Find the five-number summary for your x-values (time).

- Find the five-number summary for your y-values (number of heartbeats).

- Record your lower and upper quartiles for your x-values and your y-values.

	Time (sec)	Number of Heartbeats
Lower Quartile		
Upper Quartile		

- Draw a horizontal box-and-whisker plot using the five-number summary for your x-values (time.) Plot your box-and whisker plot under the x-axis on your graph.

- Draw a vertical box-and-whisker plot using the five-number summary for your *y*-values (number of heartbeats). Plot your box-and whisker plot next to the *y*-axis on your graph.

- Draw vertical lines from the lower and upper quartile values on the *x*-axis box-and whisker plot.

- Draw horizontal lines from the lower and upper quartile values on the *y*-axis box-and whisker plot.

- Find the coordinates of the vertices of the rectangle formed by the intersection of the vertical and horizontal lines. We will refer to these vertices as quartile points. **Note: Use only the quartile points (vertices) that follow the direction of the data.**

- Do the quartile points have to be actual data points? Why or why not?

- Draw a line connecting the two quartile points.

- Write an equation for this line. This is a line of best fit for your data.

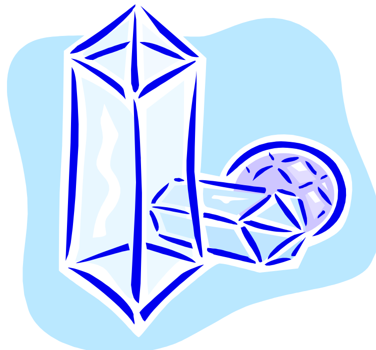
- Use one of your equations to predict how many times your heart would beat in 25 seconds, in 240 seconds, and in 3 minutes. Explain how you made your predictions.

Part ❹: **Data Collection**
After gathering this data, change jobs. The person who kept time now checks his/her pulse rate and repeat **Part ❸**: **Fitting a Line to Data.**

Part ❺: **Analysis**
- How does your line of best fit compare with your partner's line of best fit? Explain to your partner why your line of best fit is valid.

Mineral Samples

Back to Task Table



In this task, students will investigate the properties of linear graphs and develop their understanding of lines of best fit.

STANDARDS FOR MATHEMATICAL CONTENT:

Investigate patterns of association in bivariate data.

MGSE8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

MGSE8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

MGSE8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

a. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.
b. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE:

This task builds on the previous task, offering more practice with lines of best fit and writing equations for these lines to describe a set of data.

COMMON MISCONCEPTIONS:

- *Students may infer a cause and effect between independent and dependent variables, but this is often not the case. Helping students build an understanding based on the context of problems like this one will help students correct this misconception.*
- *Some student may not realize that a data point may or may not be needed to find the line of best fit. Discussions that look at multiple lines (spaghetti (uncooked) works well for this) for a set of graphed points can help students determine that all points may not be needed.*

ESSENTIAL QUESTIONS:

- How can I analyze a scatter plot?
- How can I use a linear model to solve problems?
- How can I use bivariate data to solve problems?
- How can functions be used to model real-world situations?
- What can I infer from the data?

MATERIALS:

- Straightedge
- Graph paper http://incompetech.com/graphpaper or an online, free graphing calculator such as Desmos
- Copies of task for students
- Graphing calculator *(optional)*

- **GROUPING:**

- Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will demonstrate their understanding of lines of best fit. Students will interpret and make inferences from mineral data by graphing and determining a line of best fit.

DIFFERENTIATION:

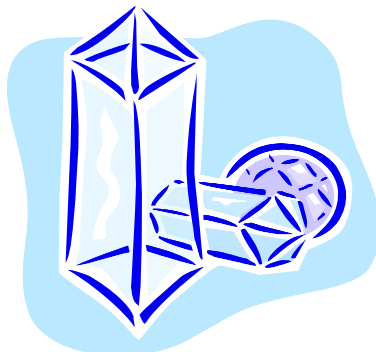
Extension:

- Method 2 serves as the Extension

Intervention/Scaffolding:

- Note – See comments in teacher's edition.

Mineral Samples



Last summer Ian went to the mountains and panned for gold. While he didn't find any gold, he did find some pyrite (fool's gold) and many other kinds of minerals. Ian's friend, who happens to be a geologist, took several of the samples and grouped them together. She told Ian that all of those minerals were the same. Ian had a hard time believing her, because they are many different colors. She suggested Ian analyze some data about the specimens. Ian carefully weighed each specimen in grams (g) and found the volume of each specimen in milliliters (ml).

Ian has asked you to be his science fair partner and help him analyze the data. Write your analysis of his data given in the table below:

Specimen Number	Mass or weight (g)	Volume (ml)
1	17	7
2	10	4
3	13	5
4	16	6
5	7	3
6	24	10
7	5	2

Comments

Prior to asking students to participate in this task, teachers may want to revisit Heartbeats Too and make sure that students understand the purpose and procedures for establishing a line of best fit for a set of data.

Students could graph the volume as the independent variable (x) and mass as the dependent variable (y). Their graphs could be produced either by hand on graph paper or as a STAT PLOT on a graphing calculator. Students could choose two points that are on their line of best fit that they determined by eye-balling the data, using the lower and upper quartiles, or they could use the linear regression feature of the calculator to obtain the regression equation $y = 2.41x + .40$.

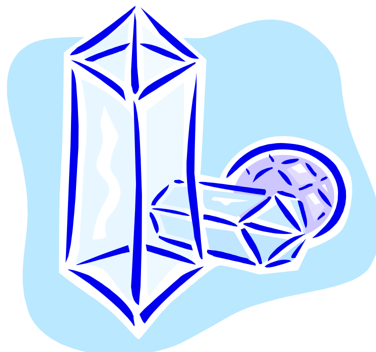
Students might divide the mass by the volume by hand for each specimen and then find the average of this value. They could also use the calculator to divide the values and find the average. The average value (2.49) is close to the coefficient of the x term in the regression equation. As students explore the meaning of the slope in this problem context, they should come to understand that it means every time the volume goes up by one ml the mass goes up by approximately 2.4 or 2.5 grams. This rate of mass in grams per milliliter of volume is the density of the mineral.

Research could be done to find lists of densities for particular minerals. While earth science references will list the density (also called "specific gravity") of quartz as 2.6, the samples used for the data above were quartz. This could lead to a discussion of the precision and accuracy of measurements, as well as to a discussion of impurities and other factors that could influence the results.

The relationship between the mass and volume of the specimens might be described by some students as a proportional relationship. This could lead to the conclusion that a theoretical model for this relationship might be written as $y = 2.6x$. In other words, a hypothetical sample with a volume of zero would have a mass of zero, so the y-intercept should be zero.

As an extension, ask students where data points would be for samples of minerals that have a density greater than 2.6. These would be in the half-plane above the regression line for the quartz samples. This can provide a transition to graphing inequalities.

Mineral Samples



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Ian has asked you to be his science fair partner and help him analyze the data. Write your analysis of his data given in the table below:

Specimen Number	Mass or weight (g)	Volume (ml)
1	17	7
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7	5	2

Walking Race and Making Money

Back to Task Table

This task introduces the concept of average rate of change as a tool for describing and understanding characteristics of functions.

STANDARDS FOR MATHEMATICAL CONTENT:

Use functions to model relationships between quantities.

MGSE8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

MGSE8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Investigate patterns of association in bivariate data.

MGSE8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

MGSE8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

MGSE8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

MGSE8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

a. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.
b. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

This task is filled with content and should not be rushed. Students may need multiple days to complete this task and discuss the results. To help students persevere and build stamina for lengthy problem solving tasks such as this, it is important to guide them in breaking it into reasonable chunks and set times for completion of each chunk. This is an important life skill that happens to fit well in a mathematics classroom.

COMMON MISCONCEPTIONS:

- *Students may infer a cause and effect between independent and dependent variables, but this is often not the case. Helping students build an understanding based on the context of problems like this one will help students correct this misconception.*
- *In developing a function rule using the collected data, some students may experience the fact that not all of their data will be linear. Helping students realize that data collected from the real world to solve problems in the real world, is what mathematics is all about. Since data from the real world is hardly ever "neat" or "clean" we need to find nice approximations for this data with tools and strategies such as lines of best fit. Some of the best mathematical discussions come from these types of investigations, because of the un-textbook-like data*.

ESSENTIAL QUESTIONS:

- How can I create a linear model given a scatter plot?
- How can I use bivariate data to solve problems?
- What can I infer from the data?
- How can functions be used to model real-world situations?
- How does a change in one variable affect the other variable in a given situation?

MATERIALS:

- Graph paper http://incompetech.com/graphpaper or an online, free graphing calculator such as Desmos
- Copies of task for students
- Calculators *(optional)*

GROUPING:

- Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

This task introduces the concept of average rate of change as a tool for describing and understanding characteristics of functions.

Most students should have some basic notions of speed based on everyday experience with miles per hour when riding in vehicles and have previously worked with the basic formula, distance = rate × time. This task draws heavily on this prior knowledge. It begins with exploration of constant speed versus variable speed in a context that allows students, if desired, to role play and experience the context for themselves.

The last two items move students from the context of speed, where they have some background experience in discussing rates, to contexts where the concept of rate is new. However, the overall context involves straightforward ideas of buying and selling. Item 3 illustrates a negative average rate of change in a setting where students will anticipate the negative: the drop in the price of a product that accompanies selling more of the product. The last item gives another context where the average rate of change is variable, but here the rate involves change in revenue with respect to the number of items produced and sold. These last two items introduce some standard terminology and approaches used for mathematical analysis of basic business situations so that students can have access to a variety of examples involving supply, demand, price, cost, revenue, and profit.

Teacher Notes:

This task is adapted from the GPS Math 1 Frameworks.

The first question should be appropriate for all ability levels.

The second question might be better suited for acceleration. It does include vocabulary such as domain inequality notation, and function of p.

DIFFERENTIATION:

Extension:

- The second question is recommended as an Extension.

Intervention/Scaffolding:

- Have students stop before doing problem 2. (See comments in teacher's edition.) Prompt struggling students with questions that help students build their own understandings. Group students in a way that promote a collaborative learning environment.

Walking Race and Making Money

In previous mathematics courses, you studied the formula *distance* = *rate* × *time*, which is usually abbreviated $d = rt$. If you and your family take a trip and spend 4 hours driving 200 miles, then you can substitute 200 for *d*, 4 for *t*, and solve the equation 200 = *r* × 4 to find that *r* = 50. Thus, we say that the average speed for the trip was 50 miles per hour. In this task, we develop the idea of *average rate of change* of a function, and see that it corresponds to average speed in certain situations.

1. To begin a class discussion of speed, Dwain and Beth want to stage a walking race down the school hallway. After some experimentation with a stop watch, and using the fact that the flooring tiles measure 1 foot by 1 foot, they decide that the distance of the race should be 40 feet and that they will need about 10 seconds to go 40 feet at a walking pace. They decide that the race should end in a tie, so that it will be exciting to watch, and finally they make a table showing how their positions will vary over time. Your job is to help Dwain and Beth make sure that they know how they should walk in order to match their plans as closely as possible.

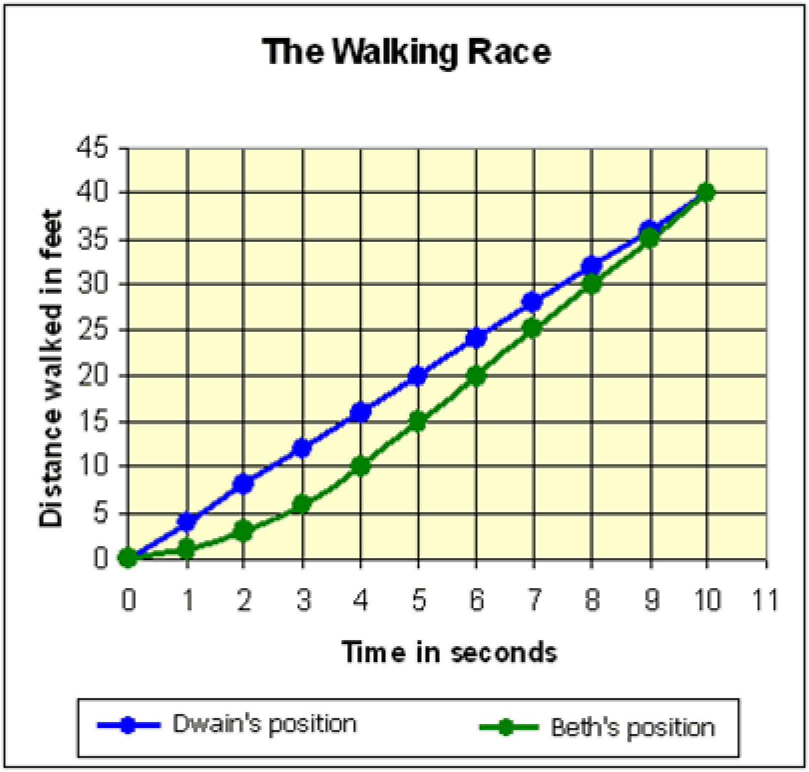
Time (sec.)	0	1	2	3	4	5	6	7	8	9	10
Dwain's position (ft.)	0	4	8	12	16	20	24	28	32	36	40
Beth's position (ft.)	0	1	3	6	10	15	20	25	30	35	40

a. Draw a graph for this data. Should you connect the dots? Explain.

Solution

See graph on next page →

The dots should be connected since Beth and Dwain will walk continuously throughout the 10 seconds.



b. How can you tell that the race is supposed to end in a tie? Provide two explanations.

Comments

There are many ways to phrase these ideas, but one explanation should reference the same table values for Dwain and Beth when t = 10, and the other should reference the common point on the right end of the graphs.

Solution

At the end of 10 seconds, both Beth and Dwain will be the same distance from the starting point, 40 feet. Explanation 1: The graphs end in the same point (10, 40) so both students will be 40 feet from the starting line after 10 seconds. Explanation 2: The table shows the same value for Beth's position and for Dwain's position when t = 10 so both will be at the same distance from the starting line.

c. Who is ahead 5 seconds into the race? Provide two explanations.

Solution

Dwain is ahead 5 seconds into the race. Explanation 1: The table shows that, after 5 seconds, Dwain should have walked 20 ft while Beth walked only 15 ft. Explanation 2: On the graphs, at t = 5, the point on Dwain's graph is higher than the point on Beth's graph, showing that Dwain's distance should be greater.

d. Describe how Dwain should walk in order to match his data. In particular, should Dwain's speed be constant or changing? Explain how you know, using observations from both the graph and the table.

Comments

Great care must be taken here to promote a reasonable interpretation of the data without making the mistake of claiming that the graph in question 1.a. is the only graph consistent with the data. One second provides sufficient time for more than one walking step. Dwain could adopt a walking style of alternating a slow step with a quick step and still move exactly 4 feet during every one second time segment. However, this idea is not likely to occur to students, and one second is a short enough time span that it is most reasonable to assume that Dwain moves at a constant speed. If the time intervals were longer, then it would not be reasonable to assume that Dwain moves at a constant speed, since we have the example of Beth's walk, which is done in the same 10 seconds as Dwain's walk. These considerations show why the very short time span of 10 seconds was selected for this item.

Solution

Dwain should walk at a constant speed. The table values indicate that, for each 1 second increase in the time, Dwain's position should be 4 feet farther from the starting point. These data imply a constant speed of 4 feet per second. The graph of Dwain's position as a function of time is a straight line segment with slope 4. The interpretation of this slope is that Dwain should move 4 feet further from the starting point for every second that passes.

e. Describe how Beth should walk in order to match his data. In particular, should Beth's speed be constant or changing? Explain how you know, using observations from both the graph and the table.

Comments

Observations similar to those made in question 1.d. apply to the interpretation for Beth's data. Beth could use alternating slow-quick steps to cover 5 feet per second for $4 \leq t \leq 10$, but given the short duration of one second, it is most reasonable to assume that Beth is walking at a constant rate for the last six seconds of the race.

Solution

Beth's should start slowly, speed up each second until she is walking at 5 feet per second, and keep this pace until the end of the race. In the table, Beth's position changes by 1 foot during the first second, and then by 2 feet, 3 feet, and 4 feet during the second, third, and fourth seconds, respectively. During the fifth through tenth seconds, her position changes by 5 feet for each second. In the graph, for the time interval $0 \leq t \leq 4$, the graph is curved and getting steeper over the interval showing that Beth should speed up. For the time interval $4 \leq t \leq 10$, the graph is a straight line with slope 5 showing that Beth should walk at a speed of 5 five per second.

f. In your answers above, sometimes you paid attention to the actual data in table. At other times, you looked at how the data change, which involved computing differences between values in the table. Give examples of each. How can you use the graph to distinguish between actual values of the data and differences between data values?

Comments

This question is very important for student understanding. It requires that students examine, in depth, how they read graphs and how measurements on the graph (relative to the given scale of the graph) represent numerical values.

Solution/Examples

The questions in parts b) and c) required looking at values in the table and comparing the value for Dwain to the value for Beth. The questions in parts d) and e) required looking at how the data for Dwain and Beth, respectively, change as the values of t change.

On the graph, actual values of the data correspond to coordinates of points. Each value of t in the first coordinate of a point is represented by the (horizontal) distance from the point to the y-axis. Any distance, for Dwain or Beth, in the second coordinate of a point, is represented by the (vertical) distance from the point to the x-axis. Thus, in answering the question for part c), a question about an actual value, we referred to the

fact that, at t = 5, the point for Dwain's position is higher than the point for Beth's position and implying that the corresponding distance is greater since the height of the point above the x-axis represents the distance.

On the graph, to see differences between data values, we examine differences in coordinates. In answering part d), we see that the successive points have x-coordinates that differ by one unit; this difference represents one second of time elapsed. The heights of successive points differ by 4; this difference represents four feet of distance covered. We summarized these observations in our statement about slope.

g. Someone asks, "What is Beth's speed during the race?" Kellee says that this question does not have a specific numeric answer. Explain what she means.

Solution

Kellee is expressing the fact that, when we calculate Beth's speed for each one second time interval of the walk, we do not get the same value for every interval.

h. Chris says that Beth went 40 feet in 10 seconds, so Beth's speed is 4 feet per second. But Kellee thinks that it would be better to say that Beth's *average speed* is 4 feet per second. Is Chris's calculation sensible? What does Kellee mean?

Solution

Chris's calculation is reasonable. Ten seconds is a short period of time. Covering 40 feet in 10 seconds will require a walking behavior that is close to walking at 4 feet per second. Kellee means that Beth will cover the same distance as someone walking at a constant speed of 4 feet per second but that, during the time of the walk, she may walk at speeds other than 4 feet per second.

i. Taylor explains that to compute average speed over some time interval, you divide the distance during the time interval by the amount of time. Compute Dwain's average speed over several time intervals (e.g., from 2 to 5 seconds; from 3 to 8 seconds). What do you notice? Explain the result.

Comments

Students should be encouraged to see that the distributive property is the key to how these computations work out, as follows: If a is the time at the beginning of the interval and b is the time at the time at the end of the interval, then the distance at time a is 4a and the distance at time b is 4b, so the average speed over the interval from time a to time b is $\frac{4b-4a}{b-a}=\frac{4(b-a)}{b-a}=4$.

Solution

Dwain's average speed over several example time intervals:

from 2 to 5 seconds: $\frac{20-8}{5-2}=\frac{12}{3}=4$ ***ft per second***

from 3 to 8 seconds: $\frac{32-12}{8-3}=\frac{20}{5}=4$ ***ft per second***

from 1 to 6 seconds: $\frac{24-4}{6-1}=\frac{20}{5}=4$ ***ft per second***

from 5 to 7 seconds: $\frac{28-20}{7-5}=\frac{8}{2}=4$ ***ft per second***

from 6 to 10 seconds: $\frac{40-24}{10-6}=\frac{16}{4}=4$ ***ft per second***

For every interval we use, the average speed is 4 ft per second. Since Dwain is walking at a constant speed of 4 ft per second, for any interval we choose, the distance walked during the interval will be 4 times the number of seconds in the interval, so the answer is also 4 ft per second.

j. What can you say about Beth's speed during the first five seconds of the race? What about the last five seconds? Explain.

Solution

During the first five seconds, Beth is supposed to walk 15 feet so we can say that during the first five seconds Beth will walk at an average speed of $\frac{15-0}{5-0}=\frac{15}{5}=3$ ***ft per second.***
During the last five seconds, from t = 5 to t =10, Beth walks with an average speed of $\frac{40-15}{10-5}=\frac{25}{5}=5$ ***ft per second and this should be her constant speed too since the graph shows a slope of 5 ft per second.***

k. Trey wants to race alongside Dwain and Beth. He wants to travel at a constant speed during the first five seconds of the race so that he will be tied with Beth after five seconds. At what speed should he walk? Explain how Trey's walking can provide an interpretation of Beth's average speed during the first five seconds.

Solution

To be tied with Beth at the end of five seconds, Trey needs to walk 15 feet in 5 seconds so he should walk 15/5 = 3 feet during each second, that is, at a constant speed of 3 feet per second. Trey's walking shows that, when Beth walks at an average speed of 3 feet per second for the first five seconds, in that five seconds he covers the same distance as someone walking at a constant speed of 3 feet per second for five seconds.

Average speed is a common application of the concept of average rate of change, but certainly not the only one. There are many applications, including analyzing the money a company can make from producing and selling products.

The rest of this task explores average rate of change for functions related to the Vee Company and its production and sale of a game called Zingo. The Vee Company is a small privately owned manufacturing company which sells to exclusively to a national chain of toy stores. Zingo games are packaged and sold in cartons holding 24 games each. Due to the size of the Vee Company work force, the maximum number of games per week that can be produced is 6000, which is enough to fill 250 cartons.

2. The table below shows data that the Vee Company has collected about the relationship between the wholesale price per game and the number of cartons of Zingo that the toy store chain will order each week. In the business world, it generally happens that lowering the price of a product increases the number that will be bought; this holds true for the Zingo sales data. Also, it may seem a bit backwards, but in business analysis, price is usually expressed as a function of the number sold, as indicated in the table.

Number of cartons ordered per week, *x*	10	20	30	40	50	60	70	80	90	100
Wholesale price per Zingo game in dollars, *y*	14.50	14.00	13.50	13.00	12.50	12.00	11.50	11.00	10.50	10.00

Comments

The units here are more complex than they may seem at first glance.

Values of the input variable give the number of cartons that the toy store chain will order. (It is appropriate for students to imagine something like KayBee Toy and Hobby, even ToysRUs; the toy store chain was not named in the text material to reduce the potential for confusion between the manufacturer and the retail outlet.)

However, values of the output give the price, in dollars, for a single Zingo game, not the price of a whole carton of 24 games. These units add to the realism of the context, but, more importantly, they give students experience with complex units in a situation that is familiar enough to allow them to manage the complexity. Success with complex units here lays a foundation for work less familiar complex units later. Students may need to be reminded as they work that number ordered is measured in cartons while price is per game.

a. When the number of cartons ordered increases from 10 per week to 20 per week, the price per game changes. If we subtract the 10 cartons-per-week price per game from the 20 cartons-per-week price per game, is the difference positive or negative? What is the difference? What does the positive or negative sign on the number tell you about how the price per game changes as the number of cartons ordered increases from 10 to 20?

Solution

14.00 – 14.50 = – 0.50 so the difference is negative and equals – 0.50. That the difference is negative tells us that the price per game decreases as the number of cartons ordered increases from 10 to 20.

b. Calculate the average rate of change of the price per game with respect to the number of cartons ordered as x increases from 10 to 20. What are the units of measure for this average rate of change?

Solution

$\frac{-0.50}{10} = -0.05$ ***The units are dollars per game per number of cartons ordered per week (and not dollars per carton ordered since the dollars per carton change is 24 times this number).***

Comments

Parts a) and b) are designed to guide students to understand a key condition of the formula for average rate of change that was not an issue in items 1 and 2. The change in y must be calculated for the same "direction" as the change in x. Specifically, if the change in x is considered to be a positive number (such as 10 for increasing the number of cartons by 10), then the change in y must be a positive or negative number depending on whether y increases or decreases, respectively, as x increases.

Side note

This condition was not an issue in items 1 and 2 because students were calculating speeds. By definition, speed is always positive or zero, that is, a nonnegative number. The concept of velocity is used in physics and higher mathematics to include speed and direction. For example, for a particle moving along a straight line, the velocity is a signed number so that the sign tells the direction in which the particle is moving and the absolute value of the number gives the speed.

c. Calculate the average rate of change of the price per game with respect to the number of cartons ordered for the increase from 20 to 40 cartons ordered per week, for the increase from 40 to 70 cartons ordered per week, and for the increase from 50 to 100 cartons ordered per week.

Solution

average rate of change of price per game with respect to number of cartons ordered per week

increase from 20 to 40 cartons ordered per week: $\dfrac{13.00-14.00}{40-20}=\dfrac{-1.00}{20}=-0.05$

increase from 40 to 70 cartons ordered per week: $\dfrac{11.00-13.00}{70-40}=\dfrac{-1.50}{30}=-0.05$

increase from 50 to 100 cartons ordered per week: $\dfrac{10.00-12.50}{100-50}=\dfrac{-2.50}{50}=-0.05$

d. Based on the data in the table, is the average rate of change of price per game constant for all these increases in the number of cartons ordered, or does it change depending on the particular numbers of cartons ordered?

Solution

Based on the data in the table, the average rate of change is constant.

e. Based on your calculations of average rate of change, determine whether the relationship between *x* and *y* is a linear relationship or a non-linear relationship. Explain your reasoning.

Solution

Based on the calculations, the relationship is linear. Since the average rate of change is always –0.05, each time x increases by 1, y will change by –0.05. When the output changes by the same amount, each time the input increases by 1, the relationship is linear.

f. How much does the Vee Company need to change the price of a Zingo game to sell one more carton per week? Does your answer depend on how many cartons are currently being sold? Explain.

Solution

They need to decrease the price per game by $0.05, or 5 cents. The answer does not depend on how many are currently being sold. Selling one more carton per week corresponds to increasing the value of x by 1. According to the average rate of change calculations, then y should change by –0.05. Since the units for y are the price per game in dollars, the price needs to decrease by 0.05 dollars, which is 5 cents.

g. The function p can be viewed as a finite sequence with 250 terms. Explain why and relate this observation to the domain of *p*. What does the value of the n^{th} term mean?

Comments

Given the phrasing of this question, most students are likely to omit consideration of whether 0 should be included in the domain of p. If students do bring up the issue of whether 0 should be in the domain, they should come to a conclusion similar to the one stated here: we omit 0 from the domain because, given the description of input and output values for this relation, an input of 0 would correspond to more than one output and we are told to view the price as a function of the number of crates ordered.

Solution

The domain of the function p consists of the numbers that can be used for the number of cartons ordered per week. The Vee Company can produce a maximum of 250 cartons of Zingo games per week, so 250 is the largest number in the domain. The number of cartons, x, must be a whole number; so the only positive numbers in the domain are 1, 2, 3, …, 250. Is 0 in the domain? It makes sense to have 0 cartons ordered per week, but this input would correspond to infinite outputs. Once we find a price high enough that no cartons are ordered, surely any higher price would also result in 0 cartons ordered per

week. So, to keep the relationship a function, we cannot include 0, and the domain of p is {1, 2, 250}. A function with this domain can be viewed as a sequence of 250 terms. The n^{th} term of this sequence gives the price per Zingo game when n cartons of games are ordered each week.

h. Write a formula to calculate the price per Zingo Game. What information from your answers above do you need in order to find this formula? To what values of *x* does the formula apply?

Comments

Based on their study of linear functions, students should have a variety of approaches that lead to a correct formula. Some of them may use data from the table to calculate the slope without making the connection that the average rate of change is the slope. Class discussion should be used to make sure that all students understand that the average rate of change is the same as slope for linear functions.

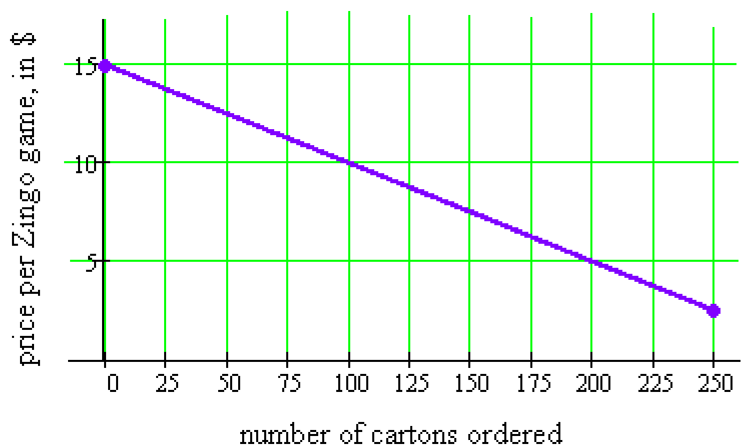
Solution

$y = -0.05x + 15.00$
From part e), we know that the relationship is linear so the formula is of the form $y = mx + b$. From part c), we know that the average rate of change of y with respect to x is –0.05, so the slope is –0.05. This formula applies for $x = 1, 2, \ldots, 250$.

i. Graph the equation for the domain $0 \le x \le 250$. Is this the graph of the function *p*? Explain why or why not.

Comments

On this graph, a solid line is drawn with the domain of all the positive integers from 1 through 250 and should have an open circle at (0, 15). This part of the item should encourage students to contemplate how they could draw the graph of a function with a large set of integers as domain.



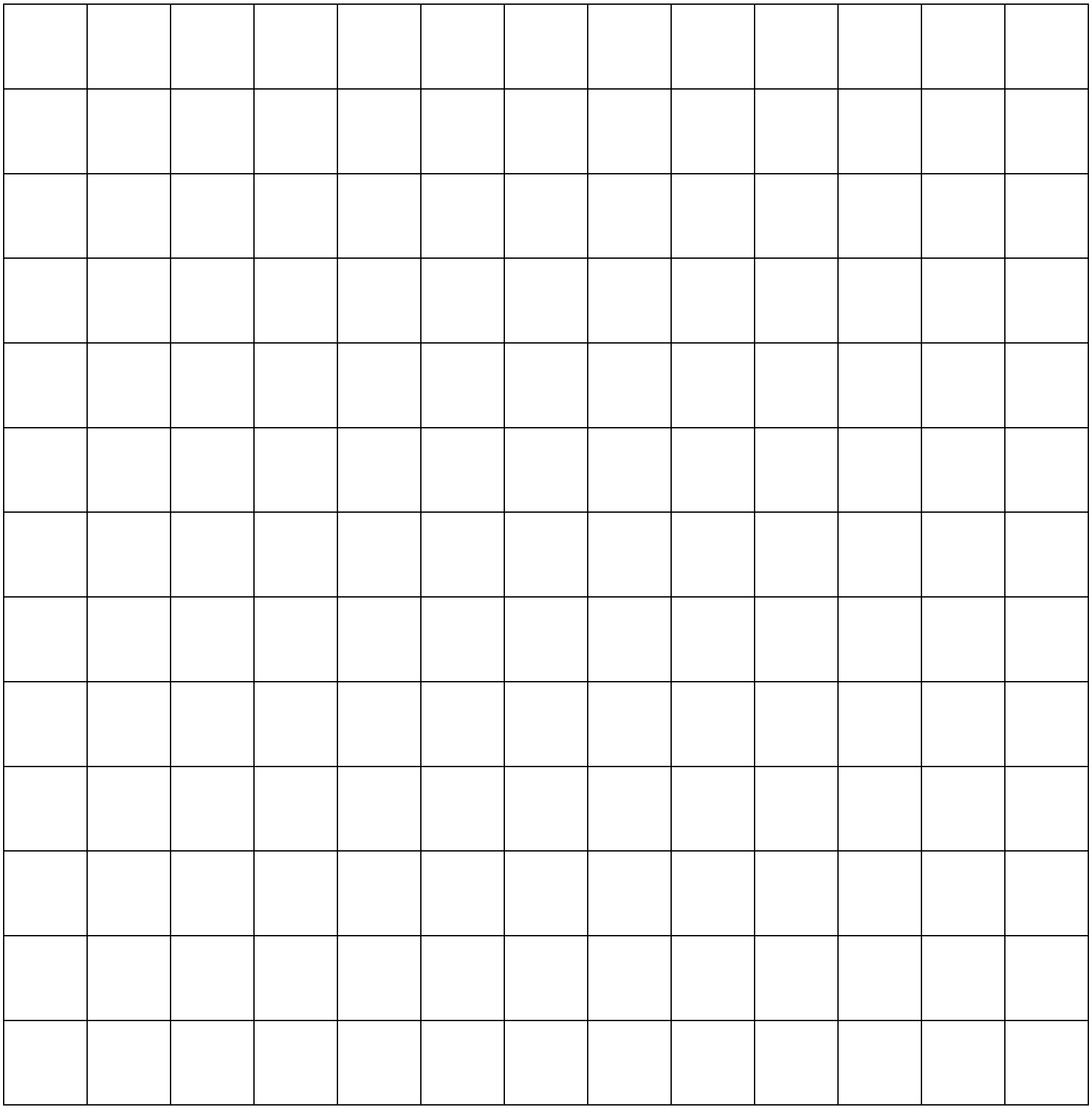
Walking Race and Making Money

In previous mathematics courses, you studied the formula *distance* = *rate* × *time*, which is usually abbreviated $d = rt$. If you and your family take a trip and spend 4 hours driving 200 miles, then you can substitute 200 for d, 4 for t, and solve the equation $200 = r \times 4$ to find that $r = 50$. Thus, we say that the average speed for the trip was 50 miles per hour. In this task, we develop the idea of *average rate of change* of a function, and see that it corresponds to average speed in certain situations.

1. To begin a class discussion of speed, Dwain and Beth want to stage a walking race down the school hallway. After some experimentation with a stop watch, and using the fact that the flooring tiles measure 1 foot by 1 foot, they decide that the distance of the race should be 40 feet and that they will need about 10 seconds to go 40 feet at a walking pace. They decide that the race should end in a tie, so that it will be exciting to watch, and finally they make a table showing how their positions will vary over time. Your job is to help Dwain and Beth make sure that they know how they should walk in order to match their plans as closely as possible.

Time (sec.)	0	1	2	3	4	5	6	7	8	9	10
Dwain's position (ft.)	0	4	8	12	16	20	24	28	32	36	40
Beth's position (ft.)	0	1	3	6	10	15	20	25	30	35	40

a. Draw a graph for this data. Should you connect the dots? Explain.



b. How can you tell that the race is supposed to end in a tie? Provide two explanations.

c. Who is ahead 5 seconds into the race? Provide two explanations.

d. Describe how Dwain should walk in order to match his data. In particular, should Dwain’s speed be constant or changing? Explain how you know, using observations from both the graph and the table.

e. Describe how Beth should walk in order to match his data. In particular, should Beth's speed be constant or changing? Explain how you know, using observations from both the graph and the table.

f. In your answers above, sometimes you paid attention to the actual data in table. At other times, you looked at how the data change, which involved computing differences between values in the table. Give examples of each. How can you use the graph to distinguish between actual values of the data and differences between data values?

g. Someone asks, "What is Beth's speed during the race?" Kellee says that this question does not have a specific numeric answer. Explain what she means.

h. Chris says that Beth went 40 feet in 10 seconds, so Beth's speed is 4 feet per second. But Kellee thinks that it would be better to say that Beth's *average speed* is 4 feet per second. Is Chris's calculation sensible? What does Kellee mean?

i. Taylor explains that to compute average speed over some time interval, you divide the distance during the time interval by the amount of time. Compute Dwain's average speed over several time intervals (e.g., from 2 to 5 seconds; from 3 to 8 seconds). What do you notice? Explain the result.

j. What can you say about Beth's speed during the first five seconds of the race? What about the last five seconds? Explain.

k. Trey wants to race alongside Dwain and Beth. He wants to travel at a constant speed during the first five seconds of the race so that he will be tied with Beth after five seconds. At what speed should he walk? Explain how Trey's walking can provide an interpretation of Beth's average speed during the first five seconds.

Average speed is a common application of the concept of average rate of change, but certainly not the only one. There are many applications, including analyzing the money a company can make from producing and selling products.

The rest of this task explores average rate of change for functions related to the Vee Company and its production and sale of a game called Zingo. The Vee Company is a small privately owned manufacturing company which sells to exclusively to a national chain of toy stores. Zingo games are packaged and sold in cartons holding 24 games each. Due to the size of the Vee Company work force, the maximum number of games per week that can be produced is 6000, which is enough to fill 250 cartons.

2. The table below shows data that the Vee Company has collected about the relationship between the wholesale price per game and the number of cartons of Zingo that the toy store chain will order each week. In the business world, it generally happens that lowering the price of a product increases the number that will be bought; this holds true for the Zingo sales data. Also, it may seem a bit backwards, but in business analysis, price is usually expressed as a function of the number sold, as indicated in the table.

Number of cartons ordered per week, *x*	10	20	30	40	50	60	70	80	90	100
Wholesale price per Zingo game in dollars, *y*	14.50	14.00	13.50	13.00	12.50	12.00	11.50	11.00	10.50	10.00

a. When the number of cartons ordered increases from 10 per week to 20 per week, the price per game changes. If we subtract the 10 cartons-per-week price per game from the 20 cartons-per-week price per game, is the difference positive or negative? What is the difference? What does the positive or negative sign on the number tell you about how the price per game changes as the number of cartons ordered increases from 10 to 20?

b. Calculate the average rate of change of the price per game with respect to the number of cartons ordered as x increases from 10 to 20. What are the units of measure for this average rate of change?

c. Calculate the average rate of change of the price per game with respect to the number of cartons ordered for the increase from 20 to 40 cartons ordered per week, for the increase from 40 to 70 cartons ordered per week, and for the increase from 50 to 100 cartons ordered per week.

d. Based on the data in the table, is the average rate of change of price per game constant for all these increases in the number of cartons ordered, or does it change depending on the particular numbers of cartons ordered?

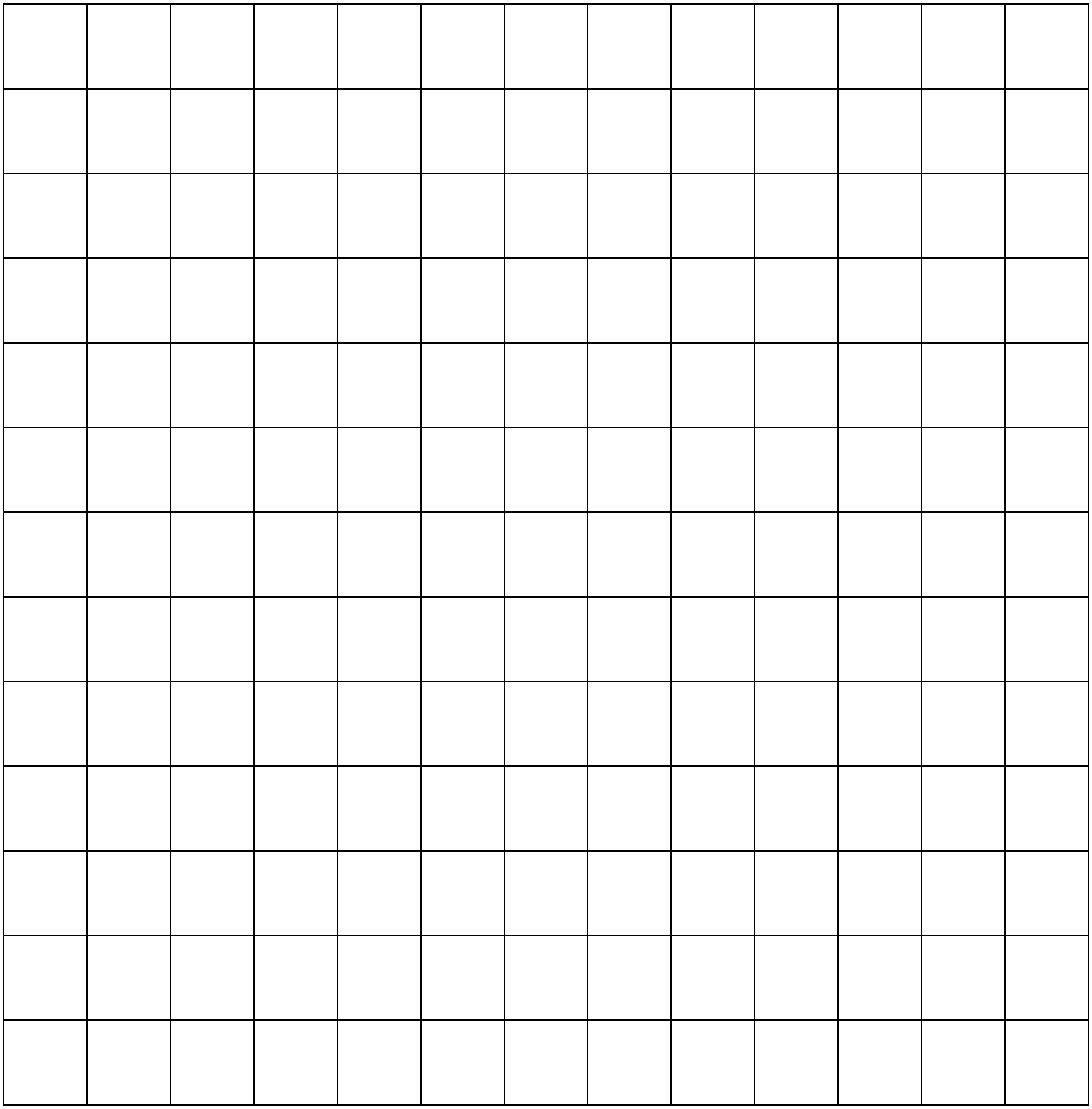
e. Based on your calculations of average rate of change, determine whether the relationship between x and y is a linear relationship or a non-linear relationship. Explain your reasoning.

f. How much does the Vee Company need to change the price of a Zingo game to sell one more carton per week? Does your answer depend on how many cartons are currently being sold? Explain.

g. The function p can be viewed as a finite sequence with 250 terms. Explain why and relate this observation to the domain of p. What does the value of the n^{th} term mean?

h. Write a formula to calculate the price per Zingo Game. What information from your answers above do you need in order to find this formula? To what values of x does the formula apply?

i. Graph the equation for the domain $0 \le x \le 250$. Is this the graph of the function p? Explain why or why not.



Mini-Problems

[Back to Task Table]

In this task, students will create a table and develop an equation.

STANDARDS OF MATHEMATICAL CONTENT:

Use functions to model relationships between quantities.

MGSE8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

MGSE8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Investigate patterns of association in bivariate data.

MGSE8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

MGSE8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

MGSE8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

MGSE8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

a. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.

b. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

This task is designed for students to show what they know. It can be used in multiple ways (see below) but no matter how it is used, the formative instructional data it will provide is extremely useful.

COMMON MISCONCEPTIONS:

- Some students may not pay attention to the scale on a graph, assuming that the scale units *are always "one." When making axes for a graph, some students may not use equal intervals to create the scale. This is a misconception that can be fixed by exposing students to Graphing Stories early in the year and consistently throughout the year. Briefly discussing the intervals and scales on the graphs used in these stories throughout the course of the year exposes students to real graphs of real situations with different scales and intervals. The graphs used and the discussions help students see the meaning behind these concepts, giving them purpose and making them less like "math vocabulary."*

- *Students often confuse a recursive rule with an explicit formula for a function. For example, after identifying that a linear function shows an increase of 2 in the values of the output for every change of 1 in the input, some students will represent the equation as $y = x + 2$ instead of realizing that this means $y = 2x + b$. When tables are constructed with increasing consecutive integers for input values, then the distinction between the recursive and explicit formulas is about whether you are reasoning vertically or horizontally in the table. Both types of reasoning – and both types of formulas – are important for developing proficiency with functions. Knowing the difference and when each type of reasoning is applicable is an important goal.*

- *Students may infer a cause and effect between independent and dependent variables, but this is often not the case. Helping students build an understanding based on the context of problems like this one will help students correct this misconception.*

ESSENTIAL QUESTIONS:

- How can I write a function to model a linear relationship?
- What strategies can I use to help me understand and represent real situations involving linear relationships?
- How can the properties of lines help me to understand graphing linear functions?
- How can functions be used to model real-world situations?
- How does a change in one variable affect the other variable in a given situation?
- Which tells me more about the relationship I am investigating – a table, a graph or an equation? Why?

MATERIALS:

- Graph paper http://incompetech.com/graphpaper or an online, free graphing calculator such as Desmos
- Straightedge
- Scientific or Graphing Calculator *(optional)*
- Copies of task for students

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

These Mini-Problems may be utilized in a variety of ways:

- use them together, assign a different one to each group and let them share or video their results
- small group, individually, take home assignments

DIFFERENTIATION:

Extension:

- Write your own mini-problem similar to the task problems. Be sure to create a situation that will lend itself nicely to the creation of a table, graph and equation in slope-intercept form. Create clear directions as to what you would like a classmate to do in order to analyze your problem situation and develop questions to go with your problem. You will be trading problems with a classmate.

Intervention/Scaffolding:

- Help struggling students set up the tables and graphs. Prompt struggling students with questions that help them build their own understanding. Group students in a way that promotes a collaborative learning environment.

Mini-Problems

1. Susan and Shumaq work at Philips Arena selling programs to the Hawks games. They get paid $25 per game plus $0.50 for each program they sell.

   a. Make a table showing the pay they can expect for any game as a function of the number of programs they sell. Include values for 0 to 100 programs in steps of 5.

   Solutions

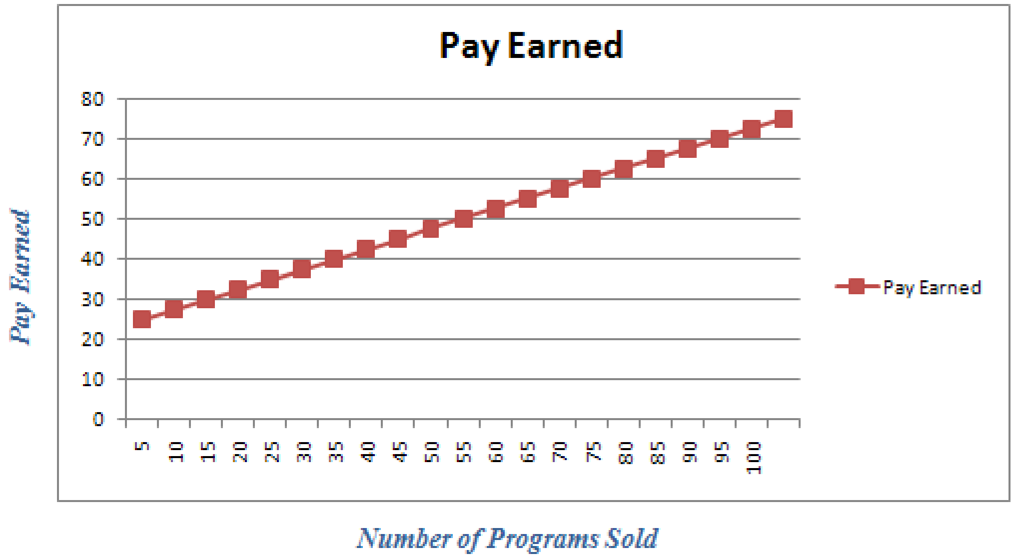
Programs Sold	*Pay Earned*
0	*25*
5	*$27.50*
10	*$30.00*
15	*$32.50*
20	*$35.00*
25	*$37.50*
30	*$40.00*
35	*$42.50*
40	*$45.00*
45	*$47.50*
50	*$50.00*
55	*$52.50*
60	*$55.00*
65	*$57.50*
70	*$60.00*
75	*$62.50*
80	*$65.00*
85	*$67.50*
90	*$70.00*
95	*$72.50*
100	*$75.00*

   b. Write an equation relating programs sold S and pay P for a game.

   $P = .50(S) + 25$

   c. Graph the relation between programs sold and pay (for 0 to 100 programs). Since pay depends on the number of programs sole, pay is the dependent (y-axis) variable. The number of programs sold is the independent variable (x-axis).

Solutions



d. How do the numbers from the pay rule of \$25 plus \$0.50 per program sold relate to the patterns in the table, the graph, and the equation?

   Lowest amount paid is \$25 then is increases in increments of \$2.50.

2. Amani and Greg work for a small production company creating music videos. Amani is a producer with several years of experience, so she is paid $20 per hour. This is Greg's first job in this line of work, so he earns only $8.50 per hour.

   a. Make a table showing how Amani's and Greg's earnings will grow over the summer as a function of hours worked.

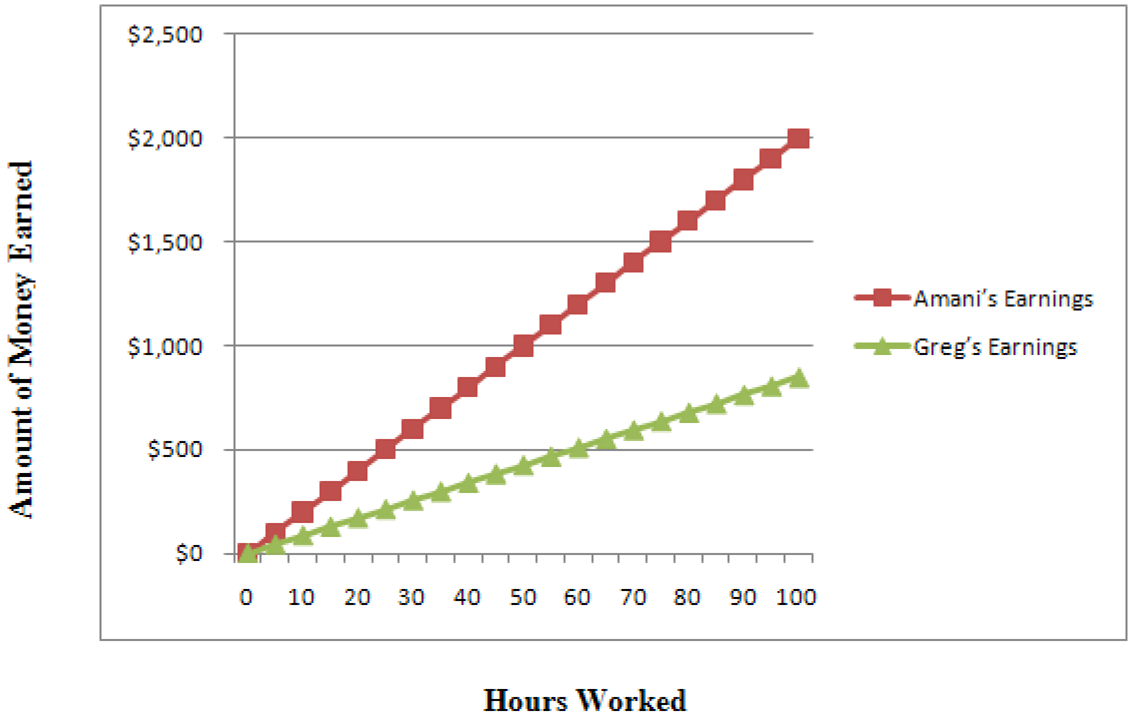
Solutions

Hours Worked	*Amani's Earnings*	*Greg's Earnings*
0	*$0*	*$0*
5	*$100*	*$42.50*
10	*$200*	*$85*
15	*$300*	*$127.5*
20	*$400*	*$170*
25	*$500*	*$212.50*
30	*$600*	*$255*
35	*$700*	*$297.50*
40	*$800*	*$340*
45	*$900*	*$382.50*
50	*$1000*	*$425*
55	*$1100*	*$467.50*
60	*$1200*	*$510*
65	*$1300*	*$552.50*
70	*$1400*	*$595*
75	*$1500*	*$637.50*
80	*$1600*	*$680*
85	*$1700*	*$722.50*
90	*$1800*	*$765*
95	*$1900*	*$807.50*
100	*$2000*	*$850*

   b. Write equations relating hours worked H and pay earned for each worker. Use *A* for Amani's earnings and *G* for Greg's earnings.

Equation for Amani's Earnings $A = 20(H)$
Equation for Greg's Earnings $G = 8.50(H)$

c. Sketch graphs of the two pay rules on the same coordinate system.

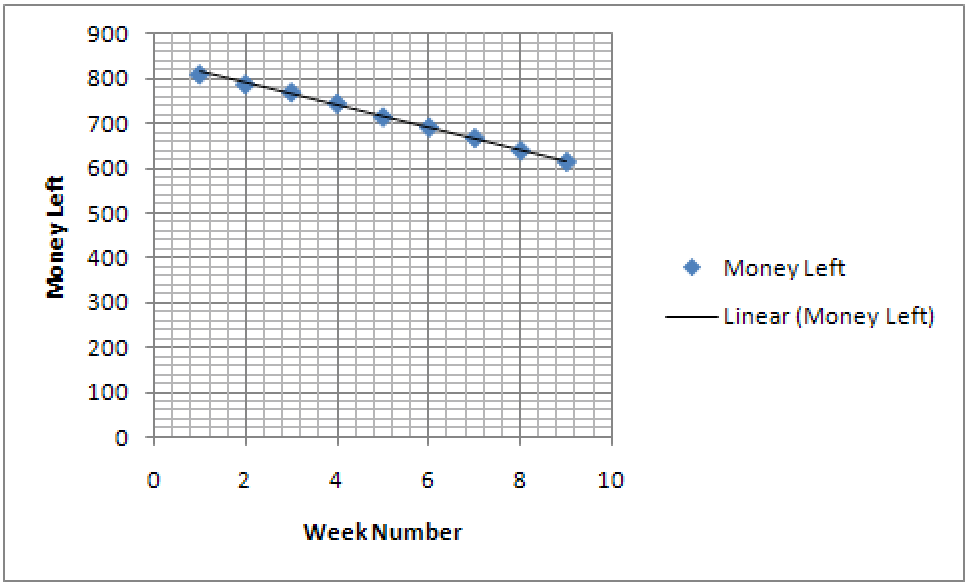


3. Laine had a babysitting service the summer before she left for Georgia Tech. She earned $1200. While she did spend some of it on new college clothes, she saved quite a bit to have some spending money while away at college. The following table shows her bank account balance over a 9-week period in the fall semester.

Week Number	1	2	3	4	5	6	7	8	9
Money Left	810	788	770	745	715	692	668	640	615

a. Make a scatter plot of Laine's bank balance data. Draw a line that models the trend in that plot.

Solutions



b. Write an equation for the linear model. Use W for weeks and M for money left. Remember that money left depends on the week number.

Answers may vary but should be in the neighborhood of M =840 – 24(W)

c. Predict Laine's bank balance after 10, 15, and 20 weeks.

Balance after:

Week #10	***Week #15***	***Week #20***
$M = 840 - 24\,(10)$	$M = 840 - 24\,(15)$	$M = 840 - 24(20)$
$M = 840 - 240$	$M = 840 - 360$	$M = 840 - 480$
$M = 600$	$M = 480$	$M = 360$

d. Explain how the patterns in the table, graph, and equation are related to each other. What do they say about Laine's spending habits?

The pattern indicates that Laine's money supply is dwindling. It appears to be decreasing by about $24 a day.

4. Eric is an umpire for a softball league. The job pays \$960 for the season. The league plays 3 nights a week for 8 weeks. Eric is already aware that he will need a substitute a few of those nights. He will have to pay the substitute from his salary.

   Solutions

   a. What should Eric pay a substitute for one night?

      3 nights per week for 8 weeks = 24 total nights to work

      960/24 = \$40 per night

   b. Use the letters *N* for nights a substitute works, *S* for pay to the substitute, and E for Eric's total earnings.

      - Write an equation relating *N* and *S*, beginning *S* =

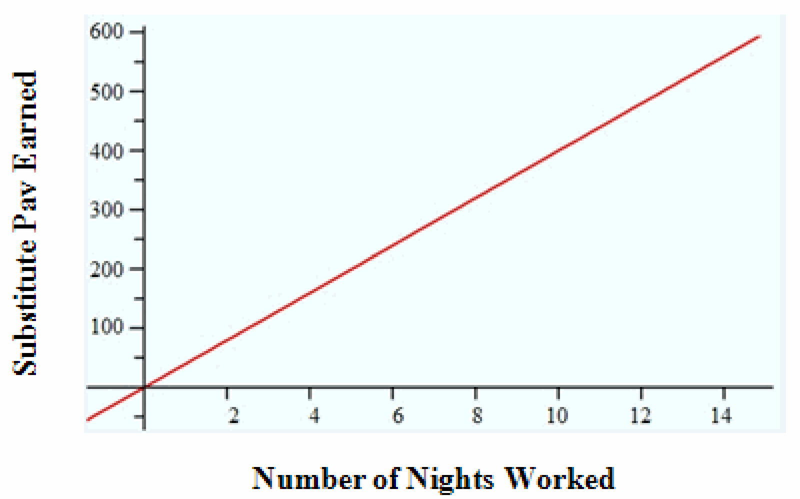
      S = 40 (N)

      - Write an equation relating *N* and *E*, beginning *E* =

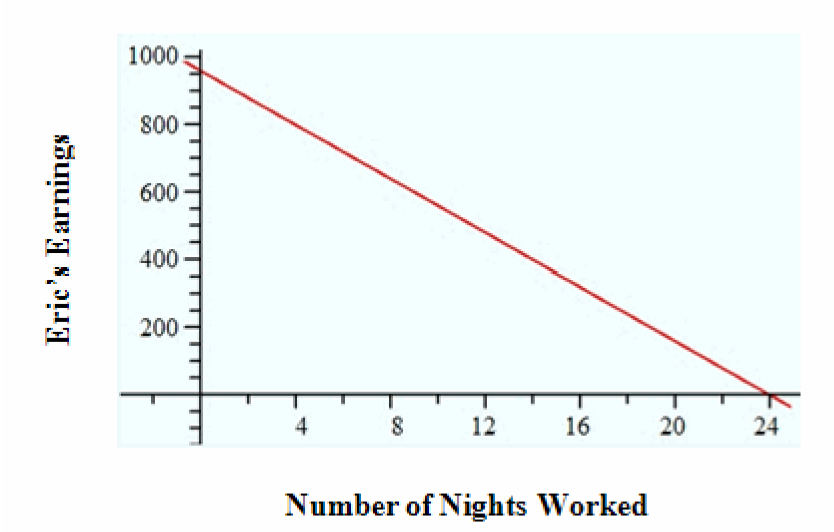
      E =960 – 40 (N)

   c. Create graphs of the equations in part b.

      Graph of S = 40 (N)



Graph of E = 960 – 40 (N)



d. How do the equations and the patterns of the graphs show Eric's earning prospects from two views?

From Eric's point of view which is represented by the equation E = 960 – 40(N), his earnings are decreasing by a rate of $40 every night that he cannot work.

From the substitute's point which is represented by the equations S = 40(N), the earnings are increasing every night he/she gets to go to work.

Mini-Problems

1. Susan and Shumaq work at Philips Arena selling programs to the Hawks games. They get paid \$25 per game plus \$0.50 for each program they sell.

   a. Make a table showing the pay they can expect for any game as a function of the number of programs they sell. Include values for 0 to 100 programs in steps of 5.

   b. Write an equation relating programs sold *S* and pay *P* for a game.

   c. Graph the relation between programs sold and pay (for 0 to 100 programs). Since pay depends on the number of programs sole, pay is the dependent (*y*-axis) variable. The number of programs sold is the independent variable (*x*-axis).

   d. How do the numbers from the pay rule of \$25 plus \$0.50 per program sold relate to the patterns in the table, the graph, and the equation?

2. Amani and Greg work for a small production company creating music videos. Amani is a producer with several years of experience, so she is paid \$20 per hour. This is Greg's first job in this line of work, so he earns only \$8.50 per hour.

   a. Make a table showing how Amani's and Greg's earnings will grow over the summer as a function of hours worked.

   b. Write equations relating hours worked H and pay earned for each worker. Use *A* for Amani's earnings and *G* for Greg's earnings.

   c. Sketch graphs of the two pay rules on the same coordinate system.

3. Laine had a babysitting service the summer before she left for Georgia Tech. She earned \$1200. While she did spend some of it on new college clothes, she saved quite a bit to have some spending money while away at college. The following table shows her bank account balance over a 9-week period in the fall semester.

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Money Left	810	788	770	745	715	692	668	640	615

Problem continues on next page. →

Problem continued from previous page.

a. Make a scatter plot of Laine's bank balance data. Draw a line that models the trend in that plot.

b. Write an equation for the linear model. Use *W* for weeks and *M* for money left. Remember that money left depends on the week number.

c. Predict Laine's bank balance after 10, 15, and 20 weeks.

d. Explain how the patterns in the table, graph, and equation are related to each other. What do they say about Laine's spending habits?

4. Eric is an umpire for a softball league. The job pays $960 for the season. The league plays 3 nights a week for 8 weeks. Eric is already aware that he will need a substitute a few of those nights. He will have to pay the substitute from his salary.

   a. What should Eric pay a substitute for one night?

   b. Use the letters *N* for nights a substitute works, *S* for pay to the substitute, and E for Eric's total earnings.

      - Write an equation relating *N* and *S*, beginning *S* =

      - Write an equation relating *N* and *E*, beginning *E* =

   c. Create graphs of the equations in part b

   d. How do the equations and the patterns of the graphs show Eric's earning prospects from two views?

My Cotton Boll Data

[Back to Task Table]

STANDARDS FOR MATHEMATICAL CONTENT:

Use functions to model relationships between quantities.

MGSE8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

MGSE8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Investigate patterns of association in bivariate data.

MGSE8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

MGSE8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

MGSE8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

MGSE8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

a. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.
b. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores.*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

COMMON MISCONCEPTIONS:

- *Students may infer a cause and effect between independent and dependent variables, but this is often not the case. Helping students build an understanding based on the context of problems like this one will help students correct this misconception.*
- *Students often confuse a recursive rule with an explicit formula for a function. For example, after identifying that a linear function shows an increase of 2 in the values of the output for every change of 1 in the input, some students will represent the equation as $y = x + 2$ instead of realizing that this means $y = 2x + b$. When tables are constructed with increasing consecutive integers for input values, then the distinction between the recursive and explicit formulas is about whether you are reasoning vertically or horizontally in the table. Both types of reasoning – and both types of formulas – are important for developing proficiency with functions. Knowing the difference and when each type of reasoning is applicable is an important goal.*

ESSENTIAL QUESTIONS:

- How can I write a function to model a linear relationship?
- How can I use a linear model to solve problems?
- How can I use bivariate data to solve problems?
- How can functions be used to model real-world situations?

MATERIALS:

- 1 boll of open cotton per student *(or any item that can be separated such as a peach and a peach pit, peanuts and shell, peas and pods)*
- Stopwatches
- Triple beam balance
- Graph paper http://incompetech.com/graphpaper or an online, free graphing calculator such as Desmos
- Copies of task for students

GROUPING:

- Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Great opportunity interdisciplinary unit with GEORGIA STUDIES!!!!

While it may be difficult for some regions to get their hands on cotton, for those of you that can - the data may surprise you. This activity is fun and educational at the same time.

Your student groups will be quite large (8-12 people per group). The must be large in order to obtain enough points to determine if it is a function.

Versions:

- ***A – need more structure and guidance***
- ***B – average ability***
- ***C – can take information and run***

DIFFERENTIATION:

Extension

- Multiple Versions of the Task Provided

Intervention/Scaffolding

- Note – Interventions/Scaffolding is built into the task (Versions A, B, & C).

Version A

My Cotton Boll Data

EQ- Is there a relationship between the weight of a cotton boll before the seeds are picked and the number of seeds in the boll?

1. Pick the cotton out of the boll.

2. Weigh the lint on the balance. Record the weight here. ______________ g

3. **Estimate** the number of seeds in your lint.   Write your guess here: __________

4. **Estimate** how long it will take you to pick out all of the seeds: ________ min.

5. Use the EQ above to write an if-then hypothesis.

6. List your variables: Independent______________________________

   Dependent_______________________________

   Control__________________________________

7. Now, using a stopwatch, time yourself while you pick out all the seeds from the lint. You must pick the seeds CLEAN! Record your time and # of seeds below.

   Time: _________ # of seeds: ______

8. Answer the EQ. Be specific and as detailed as possible.

9. Compile the data from your group. Use the space below to create a table which shows # of seeds and weight for each person. Use this table to create a mapping diagram of this relation.

Group Member	Number of Seeds	Weight of Cotton Boll

10. Use the grid to create a graph of your group's data.

Good graphs include the following…

- Equally spaced increments
- Labels on both axes
- Clearly written units on the axes
- A title that tells the reader what the graph is all about

11. Use your graph to make a prediction…estimate the weight of a boll that contains 100 seeds. Explain why you think so and show any math used to make this prediction.

12. For your group, is this relation a function? ________ Why or why not? Explain.

13. Is there a correlation? What kind? Is there a line of best fit? If so what would it look like?

14. Finally, one person from the group must create a poster that answers questions 9 & 10. The poster must include a table, a mapping, and a graph of the group's data.

Version B

My Cotton Boll Data

EQ- Is there a relationship between the weight of a cotton boll before the seeds are picked and the number of seeds in the boll?

1. Pick the cotton out of the boll.

2. Weigh the lint on the balance. Record the weight here. ______________ g

3. **Estimate** the number of seeds in your lint. Write your guess here: __________

4. **Estimate** how long it will take you to pick out all of the seeds: ________ min.

5. Use the EQ above to write an if-then hypothesis.

6. List your variables: Independent______________________________

   Dependent_______________________________

   Control__________________________________

7. Now, using a stopwatch, time yourself while you pick out all the seeds from the lint. You must pick the seeds CLEAN! Record your time and # of seeds below.

   Time: _________ # of seeds: ______

8. Answer the EQ. Be specific and as detailed as possible.

9. Compile the data from your group. Use the space below to create a table which shows the number of seeds and weight for each person. Use this table to create a mapping diagram of this relation.

10. Use the grid below to create a graph of your group's data.

Good graphs include the following…

- Equally spaced increments
- Labels on both axes
- Clearly written units on the axes
- A title that tells the reader what the graph is all about

11. Use your graph to make a prediction…estimate the weight of a boll that contains 100 seeds. Explain why you think so and show any math used to make this prediction.

12. For your group, is this relation a function? ________ Why or why not? Explain.

13. Is there a correlation? What kind? Is there a line of best fit? If so what would it look like?

14. Finally, one person from the group must create a poster that answers questions 9 & 10.
The poster must include a table, a mapping, and a graph of the group's data.

Version C

<u>My Cotton Boll Data</u>

EQ- Is there a relationship between the weight of a cotton boll before the seeds are picked and the number of seeds in the boll?

1. After picking the cotton out of the boll. Weigh the lint on the balance. Record the weight.

2. **Estimate** the number of seeds in your lint.

3. **About** how many minutes do you think it will take you to pick out all of the seeds?

4. Write an if-then hypothesis from the EQ above.

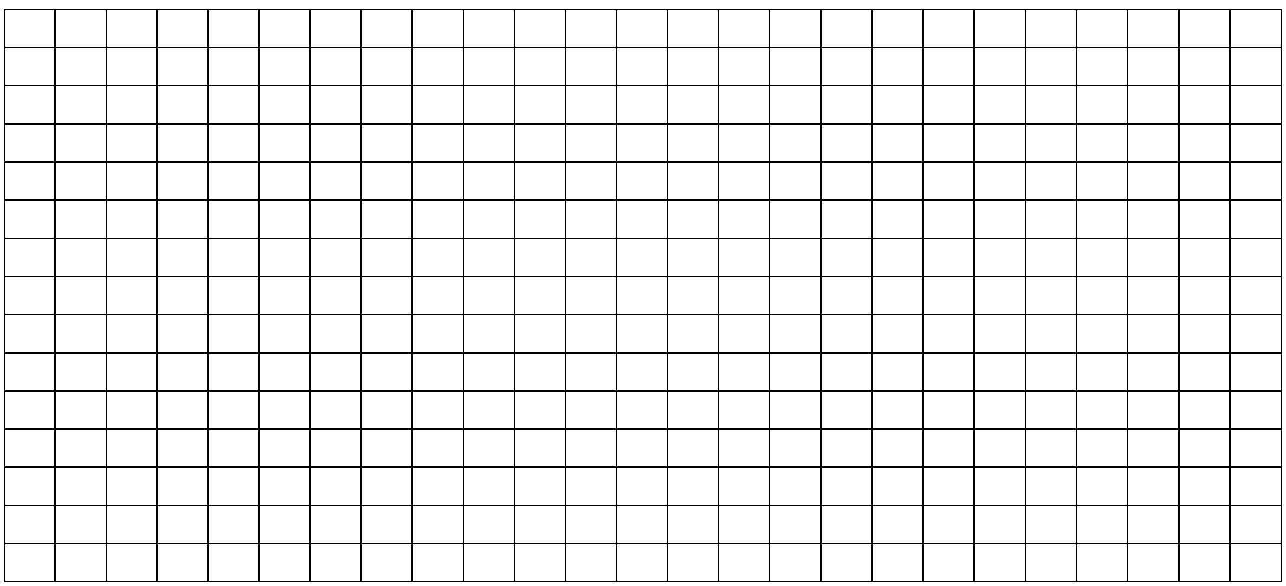
5. Identify the Independent, Dependent, and Control variables.

6. Now, using a stopwatch, time yourself while you pick out all the seeds from the lint. You must pick the seeds CLEAN! Record your time and # of seeds below.

7. Answer the EQ. Be specific and as detailed as possible.

8. Compile the data from your group. Use the space below to create a table which shows # of seeds and weight for each person. Use this table to create a mapping diagram of this relation.

9. Use the grid below to create a graph of your group's data.



10. Use your graph to make a prediction…estimate the weight of a boll that contains 100 seeds. Explain why you think so and show any math used to make this prediction.

11. For your group, is this relation a function? ________ Why or why not? Explain.

12. Is there a correlation? Is there a line of best fit? If so what would it look like?

13. Finally, one person from the group must create a poster that answers questions 9 & 10. The poster must include a table, a mapping, and a graph of the group’s data.

Version A

My Cotton Boll Data

EQ- Is there a relationship between the weight of a cotton boll before the seeds are picked and the number of seeds in the boll?

1. Pick the cotton out of the boll.
2. Weigh the lint on the balance. Record the weight here. _______________ g
3. **Estimate** the number of seeds in your lint. Write your guess here: ___________
4. **Estimate** how long it will take you to pick out all of the seeds: ________ min.
5. Use the EQ above to write an if-then hypothesis.

6. List your variables: Independent________________________________

   Dependent__________________________________

   Control_____________________________________

7. Now, using a stopwatch, time yourself while you pick out all the seeds from the lint. You must pick the seeds CLEAN! Record your time and # of seeds below.

   Time: _________ # of seeds: ______

8. Answer the EQ. Be specific and as detailed as possible.

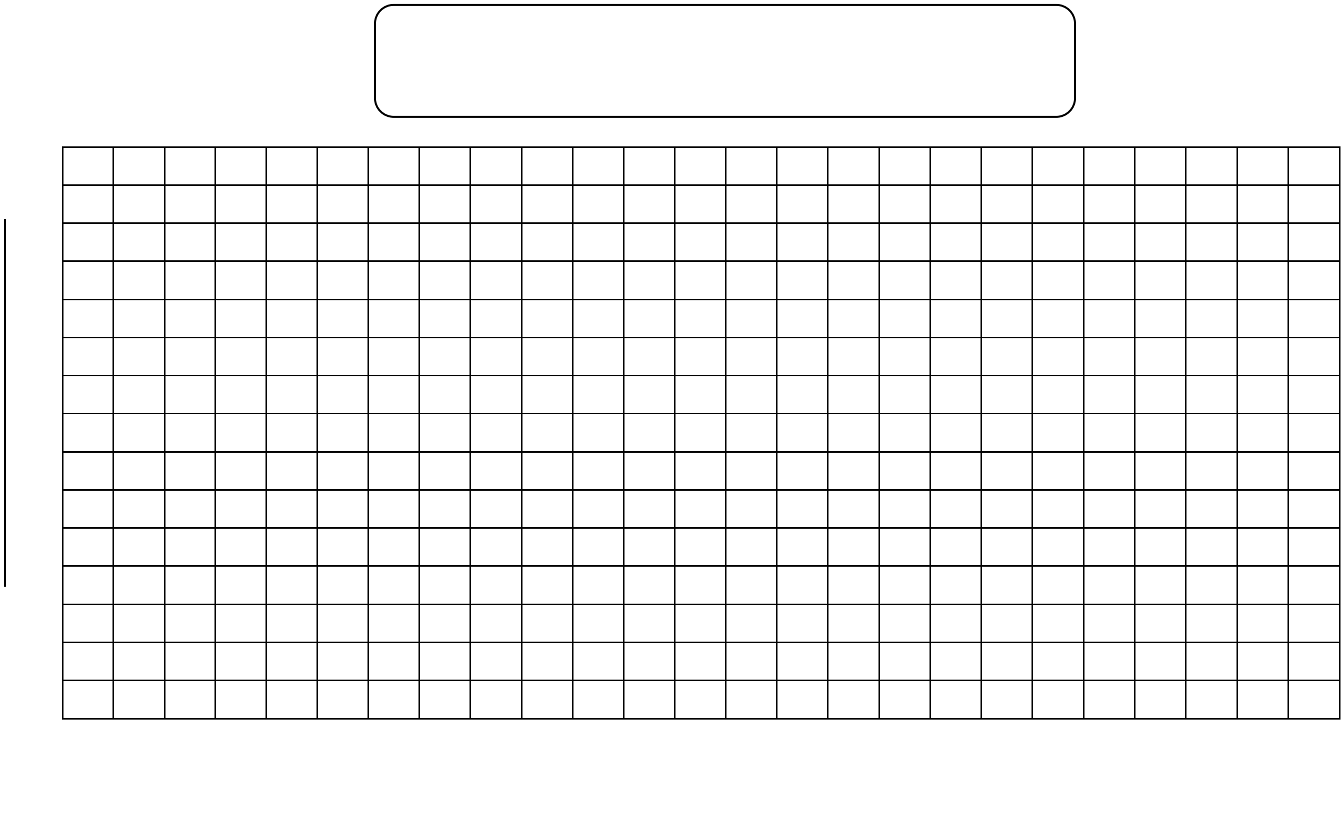
9. Compile the data from your group. Use the space below to create a table which shows # of seeds and weight for each person. Use this table to create a mapping diagram of this relation.

Group Member	Number of Seeds	Weight of Cotton Boll

10. Use the grid to create a graph of your group's data.

Good graphs include the following…

- Equally spaced increments
- Labels on both axes
- Clearly written units on the axes
- A title that tells the reader what the graph is all about



11. Use your graph to make a prediction…estimate the weight of a boll that contains 100 seeds. Explain why you think so and show any math used to make this prediction.

12. For your group, is this relation a function? ________ Why or why not? Explain.

13. Is there a correlation? What kind? Is there a line of best fit? If so what would it look like?

14. Finally, one person from the group must create a poster that answers questions 9 & 10. The poster must include a table, a mapping, and a graph of the group's data.

Version B

My Cotton Boll Data

EQ- Is there a relationship between the weight of a cotton boll before the seeds are picked and the number of seeds in the boll?

1. Pick the cotton out of the boll.

2. Weigh the lint on the balance. Record the weight here. _______________ g

3. **Estimate** the number of seeds in your lint. Write your guess here: ___________

4. **Estimate** how long it will take you to pick out all of the seeds: _________ min.

5. Use the EQ above to write an if-then hypothesis.

6. List your variables: Independent________________________________

   Dependent__________________________________

   Control_____________________________________

7. Now, using a stopwatch, time yourself while you pick out all the seeds from the lint. You must pick the seeds CLEAN! Record your time and # of seeds below.

   Time: __________ # of seeds: ______

8. Answer the EQ. Be specific and as detailed as possible.

9. Compile the data from your group. Use the space below to create a table which shows the number of seeds and weight for each person. Use this table to create a mapping diagram of this relation.

10. Use the grid below to create a graph of your group's data.

Good graphs include the following…

- Equally spaced increments
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12. For your group, is this relation a function? ________ Why or why not? Explain.

13. Is there a correlation? What kind? Is there a line of best fit? If so what would it look like?

14. Finally, one person from the group must create a poster that answers questions 9 & 10. The poster must include a table, a mapping, and a graph of the group's data.

Version C

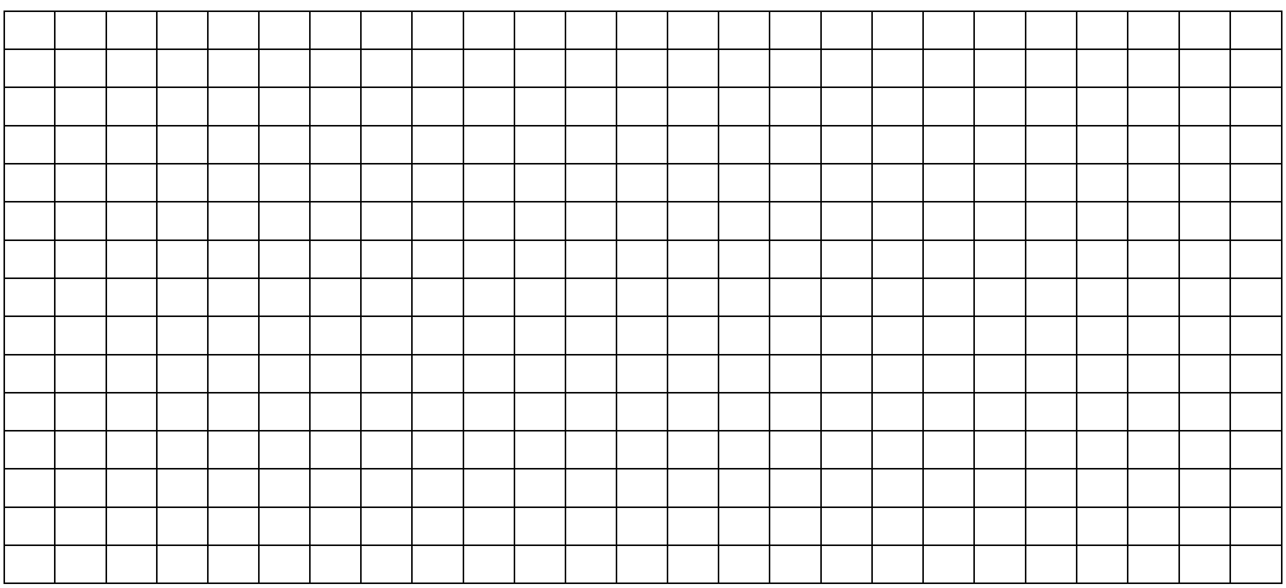
<u>My Cotton Boll Data</u>

EQ- Is there a relationship between the weight of a cotton boll before the seeds are picked and the number of seeds in the boll?

1. After picking the cotton out of the boll. Weigh the lint on the balance. Record the weight.
2. **Estimate** the number of seeds in your lint.
3. **About** how many minutes do you think it will take you to pick out all of the seeds?
4. Write an if-then hypothesis from the EQ above.
5. Identify the Independent, Dependent, and Control variables.
6. Now, using a stopwatch, time yourself while you pick out all the seeds from the lint. You must pick the seeds CLEAN! Record your time and # of seeds below.
7. Answer the EQ. Be specific and as detailed as possible.

8. Compile the data from your group. Use the space below to create a table which shows # of seeds and weight for each person. Use this table to create a mapping diagram of this relation.

9. Use the grid below to create a graph of your group's data.



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12. Is there a correlation? Is there a line of best fit? If so what would it look like?

13. Finally, one person from the group must create a poster that answers questions 9 & 10. The poster must include a table, a mapping, and a graph of the group's data.

Outdoor Theater

Back to Task Table

STANDARDS FOR MATHEMATICAL CONTENT:

Use functions to model relationships between quantities.

MGSE8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

MGSE8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Investigate patterns of association in bivariate data.

MGSE8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

MGSE8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

MGSE8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

MGSE8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

a. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.

b. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE:

This task was developed to help students make connections to mathematics standards in previous grades, specifically: MGSE7.G.1 and MGSE7.RP.2. The connections students may make will depend upon the experiences they are given and those that they have had with projections on a screen or wall using projectors or flashlights. This task is based on students having some prior experience with this type of activity. Students who lack these experiences may require more time to internalize what they observe before connecting it to previous and present mathematics.

COMMON MISCONCEPTIONS:

- *Students may infer a cause and effect between independent and dependent variables, but this is often not the case. Helping students build an understanding based on the context of problems like this one will help students correct this misconception.*
- *Students often confuse a recursive rule with an explicit formula for a function. For example, after identifying that a linear function shows an increase of 2 in the values of the output for every change of 1 in the input, some students will represent the equation as $y = x + 2$ instead of realizing that this means $y = 2x + b$. When tables are constructed with increasing consecutive integers for input values, then the distinction between the recursive and explicit formulas is about whether you are reasoning vertically or horizontally in the table. Both types of reasoning – and both types of formulas – are important for developing proficiency with functions. Knowing the difference and when each type of reasoning is applicable is an important goal.*

ESSENTIAL QUESTIONS:

- How can I use bivariate data to solve problems?

- What can I infer from the data?
- How can functions be used to model real-world situations?
- How does a change in one variable affect the other variable in a given situation?
- Which tells me more about the relationship I am investigating – a table, a graph or an equation? Why?

MATERIALS:

- Graph paper http://incompetech.com/graphpaper or an online, free graphing calculator such as Desmos
- Copies of task for students
- Moveable LCD projectors (1 per group of 3-4 students)

GROUPING:

- Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students investigate the relationship between dimensions when projected on a screen from various distances. Students will look for make predictions, look for patterns, plot data, and write equations based on the information they gather.

Teacher Notes:

- ***Depending on the availability of moveable LCD projectors you may have to gather the data as a class.***

- ***Groups of 3 or 4***

DIFFERENTIATION:

Extension:

- If the LCD Projector is placed 20 meters from the screen, what would be the area projected? Do you see any concerns placing the projector farther and farther away from the screen? What issues might you face?

Intervention/Scaffolding:

- If you have a large group of struggling learners in a class, you may want to do this task together as a whole class.

<u>Outdoor Theater</u>

Our school is sponsoring an outdoor movie for the community. They have held several outdoor movie events for various groups and noticed that the movie projected enlarges the further the LCD projector is away from the screen. You can discover more about that relation with some simple experiments using an LCD projector.

In this experiment, you will be checking to see how the dimensions increase when projected on a screen from various distances.

<u>Solutions</u>

Answers may vary. Inaccuracies may occur due to rounding, the projector not being perpendicular to the wall, inaccurate measurements, etc.

1. First step is to collect some data.

   a. Place the LCD projector so that its lens is 2 meters from a screen. Then collect data comparing the distance to the area projected.

   b. Move the LCD projector to a distance of 3 meters from the screen.

   c. Repeat this process to complete the table below

LCD Projector Distance (*m*)	1	2	3	4	5	6
Area Projected						

2. Plot the (LCD Projector Distance, Area Projected) data on a coordinate graph. Draw a linear model that you believe fits the trend in the data well.

3. Give a verbal description of the pattern of change in area projected *A* as the distance *D* changes. Explain how that pattern is shown in the data table in the modeling line.

4. Find an equation relating *D* and *A* that matches both the linear model and the data trend to help us know where to set up the projector for various size venues.

5. Since we want to utilize this equation to make predictions, let's test our equation.

   a. Use your linear model or equations to predict the area when the LCD projector is placed 2.5 meters from the screen. Also predict the projected area for a distance of 4.25 meters from the screen.

   b. Carefully make actual measurements to test both predictions. Then make a report assessing the accuracy of your predictions. If they were inaccurate, revise your model and equation.

Outdoor Theater

Our school is sponsoring an outdoor movie for the community. They have held several outdoor movie events for various groups and noticed that the movie projected enlarges the further the LCD projector is away from the screen. You can discover more about that relation with some simple experiments using an LCD projector.

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How Long Should Shoe Laces Really Be?

Back to Task Table

STANDARDS FOR MATHEMATICAL CONTENT:

Use functions to model relationships between quantities.

MGSE8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

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MGSE8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

MGSE8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

MGSE8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

a. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.
b. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on*

whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

STANDARDS OF MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

This task, like the previous task, has a lot to offer. Unlike the previous task, most, if not all students have experience with shoe laces. This makes it very easy for students to identify with from the very start.

COMMON MISCONCEPTIONS:

- *Students may infer a cause and effect between independent and dependent variables, but this is often not the case. Helping students build an understanding based on the context of problems like this one will help students correct this misconception.*

- *Students often confuse a recursive rule with an explicit formula for a function. For example, after identifying that a linear function shows an increase of 2 in the values of the output for every change of 1 in the input, some students will represent the equation as $y = x + 2$ instead of realizing that this means $y = 2x + b$. When tables are constructed with increasing consecutive integers for input values, then the distinction between the recursive and explicit formulas is about whether you are reasoning vertically or horizontally in the table. Both types of reasoning – and both types of formulas – are important for developing proficiency with functions. Knowing the difference and when each type of reasoning is applicable is an important goal.*

- *Students may graph incorrectly. Students should investigate multiple contextual situations to gain an understanding of the roles of independent and dependent variables and how they are related. The convention that x usually represents the independent variable and y represents the dependent variable should be emphasized as students make sense of the relationships in each investigation. The reason for the convention is to facilitate to mathematical communication.*

ESSENTIAL QUESTIONS:

- How can I write a function to model a linear relationship?
- How can I use bivariate data?
- How can I use bivariate data to solve problems?

MATERIALS:

- Graph paper http://incompetech.com/graphpaper or an online, free graphing calculator such as Desmos
- Copies of task for students
- Computer to create presentation *(optional)*
- Graphing calculators *(optional)*

GROUPING:

- Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will investigate whether there is a relationship between shoe lace length and the number of eyelets in a shoe. Students may wish to add their own data to the table to make the task more personal.

Teacher Comments

Students will take the data provided and create a scatter plot. From the scatter plot, the students will informally determine a line of best fit. Results may vary slightly.

Group Size: 2 to 4

Technology: Students could use graphing calculators to verify their results are in the ballpark.

DIFFERENTIATION:

Extension:

- If students are working in groups, ask them to add at least two more rows to the table with their own data using the lace lengths and eyelets from their own shoes. Then, complete the task with their data included.

Intervention/Scaffolding:

- Prompt struggling students with questions that guide students to build their own understandings. Group students in a way that promotes a collaborative learning environment.

How Long Should Shoe Laces Really Be?

Dear 8th Graders,

Have you ever bought a pair of shoes and the laces are too short? Doesn't that just drive you crazy when you have three inches of lace left to tie a tiny little bow? Or on the flip side you have a ton of lace left and the bow flops all down the sides of your shoes and on the ground? ARRGGGGHHHHH…..it makes me INSANE in the MEMBRANE!

So…my friends and I have decided to do something about it. We have conducted a survey and need your help to interpret the results. We would like to submit the results to all of the major shoe companies to see if they will listen.

Here is the data that we collected from people as they were shopping at the local mall. We asked people of various ages and gender for their opinions.

Person Polled	Lace Length (in inches)	Eyelets (numbers)		Person Polled	Lace Length (in inches)	Eyelets (numbers)
Ms. Roosevelt	45	8		Jamal	54	12
Mary Ellen	54	10		Neffi	24	4
Tom	26	4		Mr. Jones	72	14
Keshia	63	14		Amber	54	12
George	63	12		Rev. Smith	72	16
Mrs. Thomas	36	8		Jeremy	72	18

We were thinking that if we could organize this data and maybe create a graph then we could develop a rule/chart for the manufacturers to follow. Please help us to create a report/presentation to make our argument.

Thanks!
The 5th Graders

How Long Should Shoe Laces Really Be?

Dear 8th Graders,

Have you ever bought a pair of shoes and the laces are too short? Doesn't that just drive you crazy when you have three inches of lace left to tie a tiny little bow? Or on the flip side you have a ton of lace left and the bow flops all down the sides of your shoes and on the ground? ARRGGGGHHHHH…..it makes me INSANE in the MEMBRANE!

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Thanks!
The 5th Graders

25 Billion Apps (spotlight task)

Back to Task Table

Task adapted from http://threeacts.mrmeyer.com/25billionapps/

STANDARDS FOR MATHEMATICAL CONTENT

MGSE8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

MGSE8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students must make sense of the problem in order to find trends in the data and then make a model.
3. Construct viable arguments and critique the reasoning of others. In both small-group and whole-group discussions, students will need to defend their own approach to the problem, critique the reasoning of others' approaches, and if necessary, will provide useful questions to improve the arguments of others.
4. Model with mathematics. Students will develop a mathematical model to represent the situation and will use that model to make a prediction.
5. Use appropriate tools strategically. A variety of tools, including tables, graphs, equations, and graphing calculators, can be used to solve this problem. Students must be able to determine which tool is must helpful for this particular situation.
8. Look for and express regularity in repeated reasoning. Students will notice a pattern as they collect their data and will evaluate the appropriateness of their model throughout the process.

ESSENTIAL QUESTIONS

- How can an equation be written from data to model a situation?
- How can a model be used to make predictions?

MATERIALS REQUIRED

- 3-Act Task http://threeacts.mrmeyer.com/25billionapps/
- Act-1 Video http://threeacts.mrmeyer.com/25billionapps/act1/act1.mov
- Act-2 video http://threeacts.mrmeyer.com/25billionapps/act2/act2.mov
- Act-2 calendar photo http://threeacts.mrmeyer.com/25billionapps/act2/feb2012.png
- Act-2 time zone photo http://threeacts.mrmeyer.com/25billionapps/act2/timezone.png
- Act-3 Picture http://threeacts.mrmeyer.com/25billionapps/act3/answer.png
- 3-Act Task Recording Sheet

TIME NEEDED

- 1-2 days

TEACHER NOTES

In this task, students will watch a short video, collect data, and develop a model in order to make a prediction. A discussion on the appropriateness of the model is of the utmost importance here as students should realize that the number of apps downloaded over an extended period of time does not follow a linear pattern.

TASK DESCRIPTION

The following Three Act task can be found at: http://threeacts.mrmeyer.com/25billionapps/

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:
Watch the video of Act 1 (link: http://threeacts.mrmeyer.com/25billionapps/act1/act1.mov). Using a think-pair-share, ask students the questions: *What did you notice? What questions do you have?* Record students' questions on the board for everyone to see and as a class, decide on the main question. Students should write down an estimate they think is too high and one that is too low.

Main Question: *When will the 25 billionth app be downloaded?*

Important Note: Although the MAIN QUESTION of this lesson is "*When will the 25 billionth app be downloaded?* it is important for the teacher to not ignore student-generated questions and try to answer as many of them as possible (before, during, and after the lesson as a follow up).

ACT 2:
Ask students to think about other information they might need to know in order to answer the main question. Provide this information to students as they ask for it.

Required information:

- The video was captured on February 24, 2012 (picture provided of this month shows 2012 is a leap year. (link: http://threeacts.mrmeyer.com/25billionapps/act2/feb2012.png)
- The video began recording at 6:26 p.m. PST (link: http://threeacts.mrmeyer.com/25billionapps/act2/timezone.png)

Students may work individually for 5-10 minutes to come up with a plan, test ideas, and problem solve. After individual time, students may work in groups (3-4 students) to collaborate and solve this task together.

The Act 1 video only lasts about 9 seconds. A 16-minute video is provided in Act 2 that will allow students the opportunity to collect data on their own. If the technology is available, you may give each group a computer so that they can watch the video as they wish. If not, the 16-minute video could be displayed to the entire class.

ACT 3:
Students should share their predictions with the class and explain how they arrived at their answer. Students may realize that an increasing number of apps will be downloaded as the ticker approaches 25 billion and will adjust their answer accordingly. Reveal the answer http://threeacts.mrmeyer.com/25billionapps/act3/answer.png and allow the group with the closest answer to share their strategy. This is an opportunity to discuss sources of error and the reliability of our model.

Additionally, Act 3 should include time for:

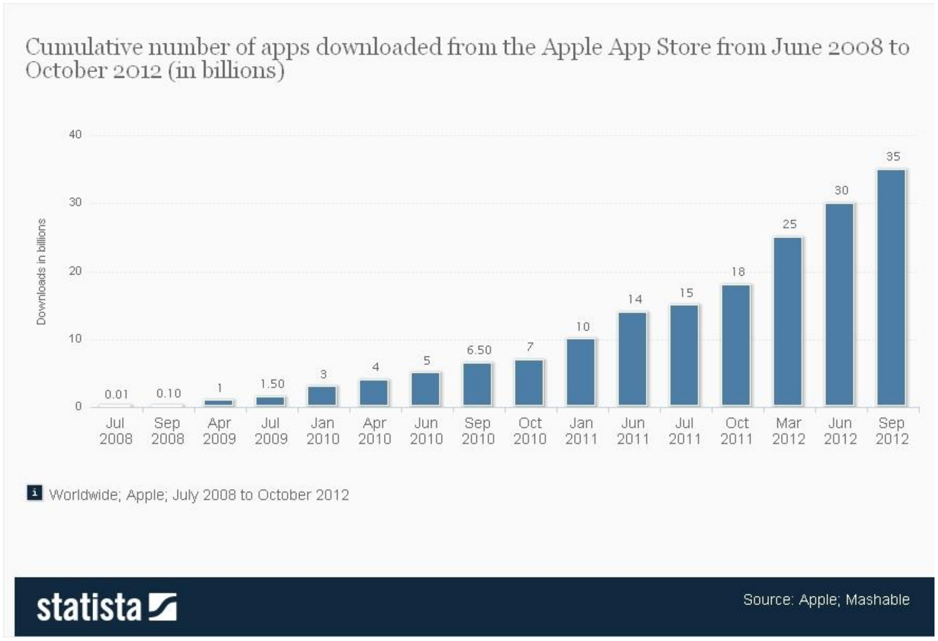
- Students to revisit initial estimates.
- The teacher to revisit questions from Act 1.
- Students to decide on a title for the lesson.

Extension: It turns out, that a linear model does not work best for this situation. Although data gathered from the 16-minute video appears to be almost perfectly linear, the video shows the number of apps being downloaded in a very short period of time. Looking at the number of apps downloaded from the iTunes store since 2008, it is clear that a linear model is not appropriate.

Cumulative number of apps downloaded from the Apple App Store from July 2008 to September 2012 (in billions)

	Values
Jul 2008	0.01
Sep 2008	0.10
Apr 2009	1.00
Jul 2009	1.50
Jan 2010	3.00
Apr 2010	4.00
Jun 2010	5.00
Sep 2010	6.50
Oct 2010	7.00
Jan 2011	10.00
Jun 2011	14.00
Jul 2011	15.00
Oct 2011	18.00
Mar 2012	25.00
Jun 2012	30.00
Sep 2012	35.00

Downloads in Billions



In order to extend the lesson and expose students to non-linear functions (MGSE8.F.3), provide students with the table of data above and ask them to explain why their linear model may not have been appropriate. If students have access to a graphing calculator, this data could be displayed in a scatter plot. The data is best modeled by a quadratic function, but in 8th grade, students just need to distinguish between linear and non-linear functions.

Intervention: Struggling students may need access to the data provided here in still shots http://threeacts.mrmeyer.com/25billionapps/act2/timer.pdf or in a table format. Additionally, the bar chart above for app downloads from 2008-2012 may be provided for students to allow a discussion on non-linear functions to take place without students being required to graph the data in a scatterplot.

25 Billion Apps

Name:________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate the result of the main question. Explain.
Place an estimate that is too high and too low on the number line
Low estimate *Place an "x" where your estimate belongs* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:_______________________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Sports and Musical Instruments

[Back to Task Table]

STANDARDS FOR MATHEMATICAL CONTENT:

Investigate patterns of association in bivariate data.

MGSE8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.

Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

STANDARDS OF MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE:

This task is the first task in this unit designed to specifically address two-way tables and relative frequency.

COMMON MISCONCEPTIONS:

- *Some students may calculate a relative frequency incorrectly by using the wrong numbers or by writing it as a percentage instead of a decimal. If students show evidence of this misconception, it may be helpful to allow students to discuss what relative frequency means and how to determine the relative frequency.*

ESSENTIAL QUESTIONS:

- How can you construct and interpret two-way tables?
- How can I determine if there is an association between two given sets of data?
- How can I find the relative frequency using two-way tables?

MATERIALS:

- Copies of task for students

GROUPING:

- Whole Class/Individual

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will be collecting bivariate data to create two way tables and analyze patterns of association using relative frequencies.

DIFFERENTIATION:

This is intended to be a whole class learning task. Some students my need to be paired with a partner to help them record data in the appropriate places and follow along as you collect the data and create the table together.

Sports and Musical Instruments

Is there an association between whether a student plays a sport and whether he or she plays a musical instrument? To investigate this, each student in your class should answer the following two questions as you collect the data as a whole class:

Do you play a sport? (yes or no)   Do you play a musical instrument? (yes or no)

Record the answers in the table below.

Student	Sport?	Musical Instrument?
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		
25		
26		
27		
28		
29		
30		
31		

32		
33		
34		
35		

Summarize the data into a clearly labeled table.

		Sport		
		YES	NO	Total
Instrument	YES			
	NO			
	Total			

a. Of those students who play a sport, what proportion plays a musical instrument?

b. Of those students who do not play a sport, what proportion plays a musical instrument?

c. Based on the class data, do you think there is an association between playing a sport and playing an instrument? Justify your reasoning.

d. Create another visual representation that could help you determine the association, if any, between playing a sport and playing a musical instrument.

Two Way Tables

[Back to Task Table]

Develop, Solidify, Practice, Assess

In the four tasks presented here, students will understand how to create a two way table when given data on two categorical variables taken from the same subjects. They will then be able to convert the numbers into relative frequencies and be able to interpret the data.

STANDARDS ADDRESSED IN THIS TASK:

Investigate patterns of association in bivariate data.

MGSE8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

a. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.
b. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

STANDARDS OF MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE:

This task, like the previous task, offers students a chance to extend their learning of two-way tables and relative frequency in a context. This task may also be used as formative assessment.

The four tasks within this task are specifically designed to help students develop, solidify, practice, and assess their understanding.

COMMON MISCONCEPTIONS:

Some students may calculate a relative frequency incorrectly by using the wrong numbers or by writing it as a percentage instead of a decimal. If students show evidence of this misconception, it may be helpful to allow students to discuss what relative frequency means and how to determine the relative frequency.

ESSENTIAL QUESTIONS:

- How can you construct and interpret two-way tables?
- How can I determine if there is an association between two given sets of data?
- How can I find the relative frequency using two-way tables?

MATERIALS:

- Copies of task for students

GROUPING:

- Individual/Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will be collecting bivariate data to create two way tables and analyze patterns of association using relative frequencies.

DIFFERENTIATION:

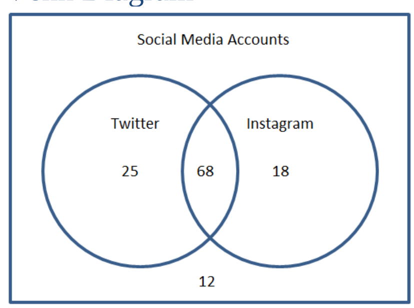
Based on their level of understanding, all students may not need to complete all four tasks (Develop, Solidify, Practice, Assess) included here. Use your judgment for each individual student.

Two Way Tables

Develop, Solidify, Practice, Assess

Teacher Notes:

<table>
<tr><th>Tasks:</th><th>Misconceptions & Recommendations</th><th>Anticipated Responses</th></tr>
<tr><td>Develop:</td><td>Discuss with students the pros and cons of each type of chart. Point out that with Venn diagrams you are limited to how many categories there can be. A list can be hard to use to compare data. A bar graph can also be hard to read if there is more data.

Prompt: could the numbers be shown as a percentage? How would you do that? Introduce relative frequency: (occurrences/possible)</td><td>Two Way Table
<table><tr><td></td><td>Instagram</td><td>No Instagram</td></tr><tr><td>Twitter</td><td>68</td><td>25</td></tr><tr><td>No Twitter</td><td>18</td><td>12</td></tr></table>
Venn Diagram


List
Bar Graph</td></tr>
</table>

Solidify:

Students will experiment with completing a two-way table and calculating relative frequencies for data found in a two-way table. (Questioning should occur during and after the activity. Questioning should relate back to the develop task for further understanding).

Questioning:

- If you were not given all of the data needed to complete a two-way table can they be calculated?
- How can you change from raw data to relative frequencies?
- How?

Reading the wrong line, especially on large tables
Calculating a relative frequency wrong – use wrong numbers or write as a percentage instead of leaving as a decimal.
Answers
1a. 25 1b. 195 1c. 1 pet 1d. 8th grade
2.

	Five	Not a Five	Total
Head	18	75	93
Tail	19	88	107
Total	37	163	200

3.

	Popcorn	Hot Dog	Total
Boys	8	53	61
Girls	38	27	65
Total	46	80	126

4a/b.

	Under 18	18 or older	Total
Boys	22	35	57
Girls	30	25	55
Total	52	60	112

5a/b.

	At most 2	More than 2	Total
Boys	15	25	40
Girls	36	12	48
Total	51	37	88

5c. .75 or 75%
5d. .625 or 62.5%

Practice:

Halloween Survey

- The amount of data may be overwhelming for some, so correct construction of the table is critical.
- Students may find the percent of students who dress up out of the total population surveyed instead of out of the grade level.
- All data has been rounded
- The data does show a relationship.
- It may be helpful for students to use a calculator.

Answers:

a.

	Grade 6	Grade 7	Grade 8	Total
Dress up	28	19	7	54
Not Dress up	10	16	20	46
Total	38	35	27	100

b.

	Grade 6	Grade 7	Grade 8
Dress up	74%	54%	26%
Not Dress up	26%	46%	74%

c. Students in higher grades are less likely to dress up.

Assessment:

Summer Camp

- Students may look at the task and be overwhelmed with the idea of creating a table from scratch.
- Students may request assistance designing the table.
- Students may confuse “total” values with “yes” values and therefore have difficulties with the table.
- Students may fail to use deductive reasoning and become “stuck”.

	Swimming	No Swimming	Total
Canoeing	28	34	62
No Canoeing	43	45	88
Total	71	79	150

2. Some *possible* responses:
More students prefer canoeing and swimming over doing neither.
Almost half (28 out of 62 or 45%) of the students who canoe also swim.
About 3/7ths (28 out of 71, or 39%) of the students who swim also canoe.
45 students do not pick either (30%, or 45 out of 150)

<u>Data Organization Task</u> (Develop)

Given the following information, design another way (other than in a paragraph) to organize the information:

Mario surveyed students at his school. From his survey he was able to determine the following information: Sixty-eight students have both a Twitter account and an Instagram account. Eighteen of the students have an Instagram account, but do not have a Twitter account. Twenty-five of the students have a Twitter account but do not have an Instagram account. Twelve of the students do not have either type of account.

<u>Two Way Tables</u> (Solidify)

Question 1:
There are 250 students in each of the grades at Chamblee Middle School; 6th Grade, 7th Grade, and 8th Grade. A survey was conducted to find how many pets the students at Chamblee Middle School owned. The results are shown in the table below.

Number of pets	**6th Grade**	**7th Grade**	**8th Grade**
0	5	25	160
1	145	110	55
2	70	95	30
3	25	20	5
4	5	0	0

a. How many 7th graders own no pets?
b. How many of the students own exactly two pets?
c. What is the most common number of pets owned by students?
d. Which grade level of students owns the least number of pets?

Question 2:
Megan rolls a number cube and tosses a coin 200 times as part of an experiment. From her experiment, she records that a five was rolled 37 times and the coin landed on tails 107 times. On 88 occasions, neither a five was rolled nor did the coin land on heads. Complete the table.

	Five	Not a Five	**Total**
Head			
Tail			
Total			

Question 3:
In an attempt to increase attendance, a local movie theatre is running a promotion in which they offer moviegoers either a free hot dog or a free box of popcorn with the cost of admission. On Saturday, 126 people attended the movies. Complete the table indicating the type of free snack selected by the patrons on Saturday.

	Popcorn	Hot Dog	**Total**
Boys		53	
Girls	38	27	
Total			

Question 4:
A swim club has 112 members. Fifty-seven of these members are boys. Thirty of the members are girls who are under the age of 18 and 35 of the members are boys who are over the age of 18.

	Under 18	18 or Older	**Total**
Boys		35	57
Girls	30		
Total			112

a) Complete the two-way table.
b) Determine how many members of the swim club are girls over the age of 18.

Question 5:
Mr. Smith keeps a log of students who attend his after school tutorial. He divides these students into two categories; those who attend at most two tutorials in a month and those who attend more than two tutorials in a month.

a) Design a table, using Mr. Smith's categories, which he could use to show how many boys and how many girls attended his after school tutorials.

b) In one month 36 girls and 15 boys attended at most two of Mr. Smith's tutorials. In the same month 12 girls and 25 boys attended more than two of Mr. Smith's tutorials. Use Mr. Smith's monthly data to complete your table.

c) What is the relative frequency of girls who attended at most two of Mr. Smith's tutorials for this month?

d) What is the relative frequency of boys who attended more than two of Mr. Smith's tutorials for this month?

Halloween Survey (Practice)

You randomly survey students in a school about whether they will dress up for Halloween this year.

6^{th} Grade students: 28 dress up and 10 do not dress up
7^{th} Grade students: 19 dress up and 16 not dress up
8^{th} Grade students: Seven dress up and 10 not dress up

a. Make a two-way table including totals of the rows and columns.

b. For each grade level, what percent of the students in the survey will dress up? What percent of the students will not dress up? Organize the results in a two-way table. Explain what one of the entries means.

c. Does the table in part b show a relationship between grade level and willingness to dress up for Halloween? Explain your thinking.

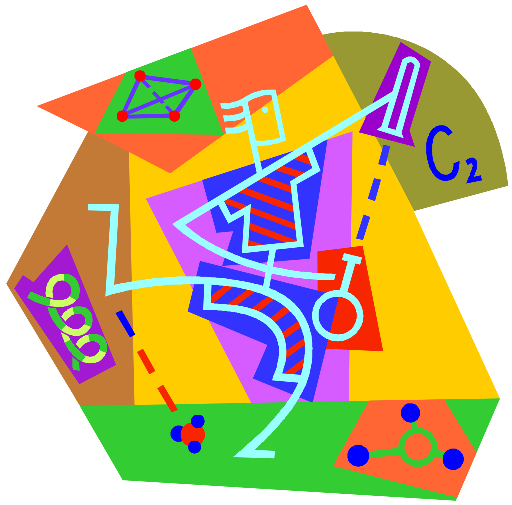
<u>Summer Camp</u> (Assess)

1) One hundred and fifty children attended summer camp. Seventy-one of the 150 children signed up for swimming and 62 of the 150 children signed up for canoeing. Twenty-eight of the 62 children who signed up for canoeing also signed up for swimming. Construct a two-way table summarizing the data.

2) Give at least three statements that provide an interpretation of the data based on your table.

Culminating Task: Is the Data Linear?

Back to Task Table



STANDARDS FOR MATHEMATICAL CONTENT:

Use functions to model relationships between quantities.

MGSE8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

MGSE8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Investigate patterns of association in bivariate data.

MGSE8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

MGSE8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

MGSE8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

MGSE8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

a. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.
b. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

<u>STANDARDS FOR MATHEMATICAL PRACTICE</u>:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

<u>BACKGROUND KNOWLEDGE:</u>

Every skill from this unit may not be addressed in this task, however this task requires students to show mastery of the big ideas of this unit.

<u>*COMMON MISCONCEPTIONS:*</u>

- *Students may infer a cause and effect between independent and dependent variables, but this is often not the case. Helping students build an understanding based on the context of problems like this one will help students correct this misconception.*

- *Students often confuse a recursive rule with an explicit formula for a function. For example, after identifying that a linear function shows an increase of 2 in the values of the output for every change of 1 in the input, some students will represent the equation as $y = x + 2$ instead of realizing that this means $y = 2x + b$. When tables are constructed*

  with increasing consecutive integers for input values, then the distinction between the recursive and explicit formulas is about whether you are reasoning vertically or horizontally in the table. Both types of reasoning – and both types of formulas – are important for developing proficiency with functions. Knowing the difference and when each type of reasoning is applicable is an important goal.

- *Students may graph incorrectly. Students should investigate multiple contextual situations to gain an understanding of the roles of independent and dependent variables and how they are related. The convention that x usually represents the independent variable and y represents the dependent variable should be emphasized as students make sense of the relationships in each investigation. The reason for the convention is to facilitate to mathematical communication.*

ESSENTIAL QUESTIONS:

- What can I infer from the data?
- How does a change in one variable affect the other variable in a given situation?
- Which tells me more about the relationship I am investigating, a table, a graph or a formula?
- What strategies can I use to help me understand and represent real situations involving linear relationships?
- How can the properties of lines help me to understand graphing linear functions?

MATERIALS:

- refer to each experiment

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will collect and model real world data. Students will also write and graph equations of data that represent a linear relationship. Completion of this task will allow students to demonstrate their understanding of linear relationships and generalizing the relationships through writing and graphing linear equations and inequalities.

DIFFERENTIATION:

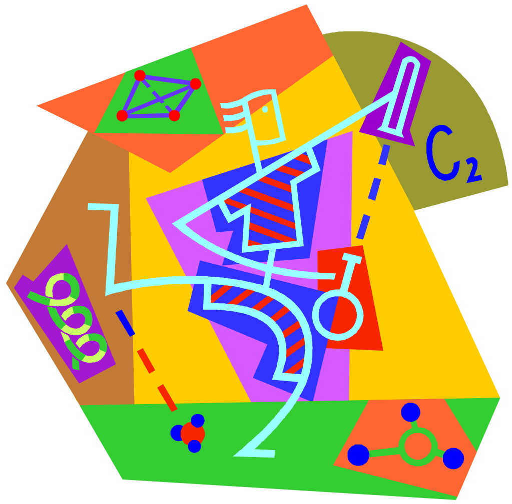
Extension:

- None – several different experiments included to choose form

Intervention/Scaffolding:

- Use your own judgment (based on your students) about how many / which of these experiments to conduct and analyze.

Is the Data Linear?



Several experiments are described below.
Choose as many of them as time permits for collection and analysis of data.

Before performing each experiment, make a conjecture as to whether you believe the data collected will represent a linear relationship between the variables. After collecting your data, use more than one method to analyze whether a linear model is a good fit. For experiments which you believe to exhibit a linear trend, find the equation for the line of best fit and interpret the meaning of the coefficients in the problem context, if possible. Also use your predictor equations to answer questions you pose dealing with each context. For each experiment that does not follow a linear pattern, explain as much as you can about the relationship between the variables.

❶ Rolling Cars

Make an incline (ramp) using a stack of books. Choose a particular car to use from the assortment of small toy cars. Release the car at the top of the ramp. Measure how many centimeters the car rolls when released from various heights. You could measure the height in terms of the number of books used or measure the height in centimeters.

❷ Spring Experiment

For this experiment you will need a spring and several weights the same size. Attach the spring to something stationery (overhead projector handle, doorknob, hook, etc.). On the other end of the spring attach a large paper clip or other device for hanging the weights. Measure the length of the spring with 0, 1, 2, 3, and 4 weights attached.

❸ Candy Experiment

On a paper napkin or paper plate pour a supply of candies that have a letter marked on one side. Count the beginning number of candies. Shake the candies in a bag and pour them back on the napkin or plate. Remove (and eat) any of the candies that have the letter showing on the top side. Count how many are left and record your data. Repeat the shaking, eating, counting, and recording steps until you have one or zero candies left. Your data will be comparing the number of candies left to the number of shakes.

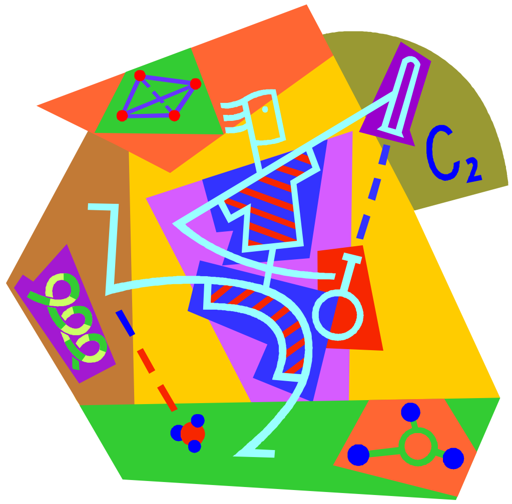
❹ Bouncing Ball

Your group will need one ball and a meter stick. Record for various heights that the ball is dropped, how many centimeters the ball bounces back up. For example, one group member may hold the meter stick and drop the ball from a position that is 90 centimeters from the floor. Another group member would watch closely to measure the height to which the ball bounces.

❺ Meter Stick Experiment

Your group will need three meter sticks. One meter stick is to be placed in various positions with one end against a wall, so that it reaches different heights up the wall. For example, the group may place the meter stick so that it reaches 60 centimeters up the wall. Next the group measures how far the other end of the meter stick (which is against the floor) is from the wall. Record the data for comparing corresponding measurements for the distance the meter stick is from the wall and the distance the meter stick is from the floor.

CULMINATING TASK: <u>Is the Data Linear</u>?



Several experiments are described below.
Choose as many of them as time permits for collection and analysis of data.

Before performing each experiment, make a conjecture as to whether you believe the data collected will represent a linear relationship between the variables. After collecting your data, use more than one method to analyze whether a linear model is a good fit. For experiments which you believe to exhibit a linear trend, find the equation for the line of best fit and interpret the meaning of the coefficients in the problem context, if possible. Also use your predictor equations to answer questions you pose dealing with each context. For each experiment that does not follow a linear pattern, explain as much as you can about the relationship between the variables.

❶ Rolling Cars

Make an incline (ramp) using a stack of books. Choose a particular car to use from the assortment of small toy cars. Release the car at the top of the ramp. Measure how many centimeters the car rolls when released from various heights. You could measure the height in terms of the number of books used or measure the height in centimeters.

❷ Spring Experiment

For this experiment you will need a spring and several weights the same size. Attach the spring to something stationery (overhead projector handle, doorknob, hook, etc.). On the other end of the spring attach a large paper clip or other device for hanging the weights. Measure the length of the spring with 0, 1, 2, 3, and 4 weights attached.

❸ Candy Experiment

On a paper napkin or paper plate pour a supply of candies that have a letter marked on one side. Count the beginning number of candies. Shake the candies in a bag and pour them back on the napkin or plate. Remove (and eat) any of the candies that have the letter showing on the top side. Count how many are left and record your data. Repeat the shaking, eating, counting, and recording steps until you have one or zero candies left. Your data will be comparing the number of candies left to the number of shakes.

❹ Bouncing Ball

Your group will need one ball and a meter stick. Record for various heights that the ball is dropped, how many centimeters the ball bounces back up. For example, one group member may hold the meter stick and drop the ball from a position that is 90 centimeters from the floor. Another group member would watch closely to measure the height to which the ball bounces.

❺ Meter Stick Experiment

Your group will need three meter sticks. One meter stick is to be placed in various positions with one end against a wall, so that it reaches different heights up the wall. For example, the group may place the meter stick so that it reaches 60 centimeters up the wall. Next the group measures how far the other end of the meter stick (which is against the floor) is from the wall. Record the data for comparing corresponding measurements for the distance the meter stick is from the wall and the distance the meter stick is from the floor.

Technology Resources:

Back to Task Table

Use functions to model relationships between quantities.

MGSE8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

http://nzmaths.co.nz/resource/balancing-acts
https://teacher.desmos.com/activities
https://www.illustrativemathematics.org/content-standards/8/F/B/4
http://graphingstories.com/

MGSE8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

https://www.illustrativemathematics.org/content-standards/8/F/B/5
http://graphingstories.com/

Investigate patterns of association in bivariate data.

MGSE8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

https://www.illustrativemathematics.org/content-standards/8/SP/A/1
http://nzmaths.co.nz/resource/stork-delivery
http://nzmaths.co.nz/resource/suspect-foot
http://nzmaths.co.nz/resource/data-squares-level-3

MGSE8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

https://www.illustrativemathematics.org/content-standards/8/SP/A/2
http://nzmaths.co.nz/resource/stork-delivery
http://nzmaths.co.nz/resource/suspect-foot

MGSE8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

https://www.illustrativemathematics.org/content-standards/8/SP/A/3
http://nzmaths.co.nz/resource/rates-change

MGSE8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

a. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.
b. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

https://www.illustrativemathematics.org/content-standards/8/SP/A/4
http://nzmaths.co.nz/resource/data-squares-level-3

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