Georgia
Standards of Excellence
Curriculum Frameworks

Mathematics

Accelerated GSE Algebra I/Geometry A

Unit 2: Reasoning with Linear Equations and Inequalities

Georgia Department of Education

Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

Unit 2

Reasoning with Linear Equations and Inequalities

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OVERVIEW

In this unit students will:

- Solve linear equations in one variable.
- Justify the process of solving an equation.
- Rearrange formulas to highlight a quantity of interest.
- Solve linear inequalities in one variable.
- Solve a system of two equations in two variables by using multiplication and addition.
- Justify the process of solving a system of equations.
- Solve a system of two equations in two variables graphically.
- Graph a linear inequality in two variables.
- Analyze linear functions using different representations.
- Interpret linear functions in context.
- Investigate key features of linear graphs.
- Recognize arithmetic sequences as linear functions.

By the end of eighth grade students have had a variety of experiences creating equations. In this unit, students continue this work by creating equations to describe situations. By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to rearrange formulas to highlight a quantity of interest, analyze and explain the process of solving an equation, and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. Students explore systems of equations, find, and interpret their solutions. Students create and interpret systems of inequalities where applicable. For example, students create a system to define the domain of a particular situation, such as a situation limited to the first quadrant. The focus is not on solving systems of inequalities. Solving systems of inequalities can be addressed in extension tasks.

All this work is grounded on understanding quantities and on relationships between them. In earlier grades, students define, evaluate, and compare functions and use them to model relationships between quantities. In this unit, students expand their prior knowledge of functions, learn function notation, develop the concepts of domain and range, analyze linear functions using different representations, and understand the limitations of various representations Students investigate key features of linear graphs and recognize arithmetic sequences as linear functions. Some standards are repeated in units 3, 4, and 5 as they apply to quadratics and exponentials.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. This unit provides much needed content information and excellent learning activities. However, the intent of the framework is not to provide a comprehensive resource for the implementation of all standards in the unit. A variety of resources should be utilized to supplement this unit. The tasks in this unit framework illustrate the types of learning activities that should be utilized from a variety of sources. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the **"Strategies for Teaching and Learning"** in the Comprehensive Course Overview and the tasks listed under **"Evidence of Learning"** be reviewed early in the planning process.

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

KEY STANDARDS

Create equations that describe numbers or relationships

MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, ~~quadratic,~~ ~~simple rational,~~ ~~and exponential~~ functions ~~(integer inputs only).~~

MGSE9-12.A.CED.2 Create linear, ~~quadratic, and exponential~~ equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase "in two or more variables" refers to formulas like the compound interest formula, in which $A = P(1 + r/n)^{nt}$ has multiple variables.)

MGSE9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

MGSE9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. *Examples: Rearrange Ohm's law V = IR to highlight resistance R;* ~~*Rearrange area of a circle formula A = π r² to highlight the radius r.*~~

Understand solving equations as a process of reasoning and explain the reasoning

MGSE9-12.A.REI.1 Using algebraic properties and the properties of real numbers, justify the steps of a simple, one-solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties.

Solve equations and inequalities in one variable

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable including equations with coefficients represented by letters. *For example, given ax + 3 = 7, solve for x.*

Solve systems of equations

MGSE9-12.A.REI.5 Show and explain why the elimination method works to solve a system of two-variable equations.

MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically

MGSE9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

MGSE9-12.A.REI.11 Using graphs, tables, or successive approximations, show that the solution to the equation f(x) = g(x) is the x-value where the y-values of f(x) and g(x) are the same.

MGSE9-12.A.REI.12 Graph the solution set to a linear inequality in two variables.

Build a function that models a relationship between two quantities

MGSE9-12.F.BF.1 Write a function that describes a relationship between two quantities.

> **MGSE9-12.F.BF.1a** Determine an explicit expression and the recursive process (steps for calculation) from context. *For example, if Jimmy starts out with $15 and earns $2 a day, the explicit expression "2x+15" can be described recursively (either in writing or verbally) as "to find out how much money Jimmy will have tomorrow, you add $2 to his total today."* $J_n = J_{n-1} + 2, J_0 = 15$

MGSE9-12.F.BF.2 Write arithmetic ~~and geometric~~ sequences recursively and explicitly, use them to model situations, and translate between the two forms. Connect arithmetic sequences to linear functions ~~and geometric sequences to exponential functions.~~

Understand the concept of a function and use function notation

MGSE9-12.F.IF.1 Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and f(x) is the output (an element of the range). Graphically, the graph is y = f(x).

MGSE9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MGSE9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. (Generally, the scope of high school math defines this subset as the set of natural numbers 1,2,3,4...) By graphing or calculating terms, students should be able to show how the recursive sequence $a_1=7$, $a_n=a_{n-1}+2$; the sequence $s_n = 2(n-1) + 7$; and the function f(x) = 2x + 5 (when x is a natural number) all define the same sequence.

Interpret functions that arise in applications in terms of the context

MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity.~~

MGSE9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

MGSE9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations

MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

> **MGSE9-12.F.IF.7a** Graph linear ~~and quadratic~~ functions and show intercepts, maxima, and minima (as determined by the function or by context).

MGSE9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.*

STANDARDS FOR MATHEMATICAL PRACTICE

Refer to the Comprehensive Course Overview for more detailed information about the Standards for Mathematical Practice.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ENDURING UNDERSTANDINGS

- Create linear equations and inequalities in one variable and use them in a contextual situation to solve problems.
- Create equations in two or more variables to represent relationships between quantities.
- Rearrange formulas to highlight a quantity of interest.
- Solve linear equations and inequalities in one variable.
- Graph linear equations and inequalities in two variables.
- Linear equations and inequalities can be represented graphically and solved using real-world context.
- Solve systems of linear equations in two variables exactly and approximately and explain why the elimination method works to solve a system of two-variable equations.
- Graph equations in two variables on a coordinate plane and label the axes and scales.
- Understand the concept of a function and be able to use function notation.
- Understand how to interpret linear functions that arise in applications in terms of the context.
- When analyzing linear functions, different representations may be used based on the situation presented.
- A function may be built to model a relationship between two quantities.
- Understand and interpret key features of functions.
- Understand how to interpret expressions for functions in terms of the situation they model.
- Understand that sequences are functions.
- Write recursive and explicit formulas for arithmetic sequences and understand the appropriateness of the use of each.

ESSENTIAL QUESTIONS

- How do I justify the solution to an equation?
- How do I solve an equation in one variable?
- How do I solve an inequality in one variable?
- How do I prove that a system of two equations in two variables can be solved by multiplying and adding to produce a system with the same solutions?
- How do I solve a system of linear equations graphically?
- How do I graph a linear inequality in two variables?
- How do I use graphs to represent and solve real-world equations and inequalities?
- Why is the concept of a function important and how do I use function notation to show a variety of situations modeled by functions?
- How do I interpret functions that arise in applications in terms of context?
- How do I use different representations to analyze linear functions?
- How do I build a linear function that models a relationship between two quantities?
- How do I interpret expressions for functions in terms of the situation they model?
- How do I interpret key features of graphs in context?
- Why are sequences functions?
- How do I write recursive and explicit formulas for arithmetic sequences?

CONCEPTS AND SKILLS TO MAINTAIN

Students may not realize the importance of unit conversion in conjunction with computation when solving problems involving measurement. Since today's calculating devices often display 8 to 10 decimal places, students frequently express answers to a much greater degree of precision than is required.

Measuring commonly used objects and choosing proper units for measurement are part of the mathematics curriculum prior to high school. In high school, students experience a broader variety of units through real–world situations and modeling, along with the exploration of the different levels of accuracy and precision of the answers.

An introduction to the use of variable expressions and their meaning, as well as the use of variables and expressions in real–life situations, is included in the Expressions and Equations Domain of Grade 7. Working with expressions and equations, including formulas, is an integral part of the curriculum in Grades 7 and 8. In high school, students explore in more depth the use of equations and inequalities to model real–world problems, including restricting domains and ranges to fit the problem's context, as well as rewriting formulas for a variable of interest.

Students are expected to have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre–assess to determine whether instructional time should be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Using the Pythagorean Theorem
- Understanding slope as a rate of change of one quantity in relation to another quantity
- Interpreting a graph
- Creating a table of values
- Working with functions
- Writing a linear equation
- Using inverse operations to isolate variables and solve equations
- Maintaining order of operations
- Understanding notation for inequalities
- Being able to read and write inequality symbols
- Graphing equations and inequalities on the coordinate plane
- Graphing points
- Choosing appropriate scales and label a graph
- Graphing points
- Choosing appropriate scales and label a graph
- Understanding algebraic properties, particularly, commutative, associative, and distributive properties.
- Knowing how to solve equations, using the distributive property, combining like terms and equations with variables on both sides.
- Knowing how to solve systems of linear equations.
- Understanding and be able to explain what a function is.
- Determining if a table, graphing or set of ordered pairs is a function.
- Distinguishing between linear and non-linear functions.
- Writing linear equations and using them to model real-world situations.

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for high school children. **Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**

http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary.

http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

- **Algebra:** The branch of mathematics that deals with relationships between numbers, utilizing letters and other symbols to represent specific sets of numbers, or to describe a pattern of relationships between numbers.

- **Arithmetic Sequence.** A sequence of numbers in which the difference between any two consecutive terms is the same.

- **Average Rate of Change.** The change in the value of a quantity by the elapsed time. For a function, this is the change in the y-value divided by the change in the x-value for two distinct points on the graph.

- **Coefficient.** A number multiplied by a variable in an algebraic expression.

- **Constant Rate of Change.** With respect to the variable x of a linear function $y = f(x)$, the constant rate of change is the slope of its graph.

- **Continuous.** Describes a connected set of numbers, such as an interval.

- **Discrete.** A set with elements that are disconnected.

- **Domain.** The set of x-coordinates of the set of points on a graph; the set of x-coordinates of a given set of ordered pairs. The value that is the input in a function or relation.

- **End Behaviors.** The appearance of a graph as it is followed farther and farther in either direction.

- **Equation:** A number sentence that contains an equals symbol.

- **Explicit Formula.** A formula that allows direct computation of any term for a sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$.

- **Expression:** A mathematical phrase involving at least one variable and sometimes numbers and operation symbols.

- **Expression.** Any mathematical calculation or formula combining numbers and/or variables using sums, differences, products, quotients including fractions, exponents, roots, logarithms, functions, or other mathematical operations.

- **Factor.** For any number x, the numbers that can be evenly divided into x are called factors of x. For example, the number 20 has the factors 1, 2, 4, 5, 10, and 20.

- **Inequality**: Any mathematical sentence that contains the symbols > (greater than), < (less than), ≤ (less than or equal to), or ≥ (greater than or equal to).

- **Interval Notation.** A notation representing an interval as a pair of numbers. The numbers are the endpoints of the interval. Parentheses and/or brackets are used to show whether the endpoints are excluded or included.

- **Linear Function.** A function with a constant rate of change and a straight line graph.

- **Linear Model.** A linear function representing real-world phenomena. The model also represents patterns found in graphs and/or data.

- **Ordered Pair**: A pair of numbers, (x, y), that indicate the position of a point on a Cartesian plane.

- **Parameter.** The independent variable or variables in a system of equations with more than one dependent variable.

- **Range.** The set of all possible outputs of a function.

- **Recursive Formula.** A formula that requires the computation of all previous terms to find the value of a_n.

- **Slope.** The ratio of the vertical and horizontal changes between two points on a surface or a line.

- **Substitution:** To replace one element of a mathematical equation or expression with another.

- **Term.** A value in a sequence--the first value in a sequence is the 1st term, the second value is the 2nd term, and so on; a term is also any of the monomials that make up a polynomial.

- **Variable:** A letter or symbol used to represent a number.

- ***X*-intercept.** The point where a line meets or crosses the x-axis

- ***Y*-intercept.** The point where a line meets or crosses the y-axis

The Properties of Operations

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$.
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$.
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

The Properties of Equality

Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

EVIDENCE OF LEARNING

By the conclusion of this unit, students should be able to demonstrate the following competencies:

- Justify the solution of a linear equation and inequality in one variable.
- Justify the solution to a system of 2 equations in two variables.
- Solve a system of linear equations in 2 variables by graphing.
- Graph a linear inequality in 2 variables.
- Explain what it means when two graphs $\{y = f(x)$ and $y = g(x)\}$ intersect.
- Define and use function notation, evaluate functions at any point in the domain, give general statements about how $f(x)$ behaves at different regions in the domain (as x gets very large or very negative, close to 0 etc.), and interpret statements that use function notation.
- Explain the difference and relationship between domain and range and find the domain and range of a function from a function equation, table or graph.
- Explain why sequences are functions.
- Interpret x and y intercepts, where the function is increasing or decreasing, where it is positive or negative, its end behaviors, given the graph, table or algebraic representation of a linear function in terms of the context of the function.
- Find and/or interpret appropriate domains and ranges for authentic linear functions.
- Calculate and interpret the average rate of change over a given interval of a function from a function equation, graph or table, and explain what that means in terms of the context of the function.
- Estimate the rate of change of a function from its graph at any point in its domain.
- Explain the relationship between the domain of a function and its graph in general and/or to the context of the function.
- Accurately graph a linear function by hand by identifying key features of the function such as the x- and y-intercepts and slope.
- Write recursive and explicit formulas for arithmetic sequences.

TEACHER RESOURCES

The following pages include teacher resources that teachers may wish to use to supplement instruction.

- Web Resources
- Methods for Solving Systems of Equations: Graphic Organizer

Web Resources

The following list is provided as a sample of available resources and is for informational purposes only. It is your responsibility to investigate them to determine their value and appropriateness for your district. GADOE does not endorse or recommend the purchase of or use of any particular resource.

- Same Solution Task
  http://www.illustrativemathematics.org/illustrations/613
  This task gives students 6 different equations and asks them to show whether they have the same solution.

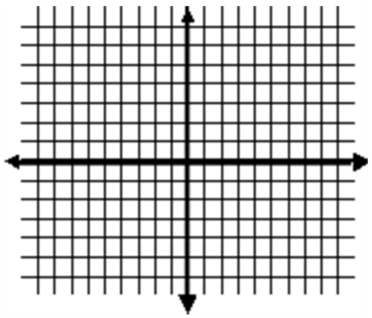
- Balancing Equations Applet
  http://illuminations.nctm.org/ActivityDetail.aspx?id=10
  This applet allows you to demonstrate how a point of intersection of two lines is the x–value where the two expressions have the same y–value. The applet shows the graph alongside a scale with algebraic expressions as the 'weights.'

- http://www.visualpatterns.org/
  This website provides many visual patterns/sequences for the purpose of helping students develop algebraic thinking through visual patterns.

Methods for Solving Systems of Equations

$$\begin{aligned} 2x + 3y &= 5 \\ 4x - y &= 17 \end{aligned}$$

GRAPHING



Intersecting Lines:

Parallel Lines:

Coinciding Lines:

SUBSTITUTION

ELIMINATION

What if both of the variables cancel out? Look at the resulting arithmetic equation.
* False statement indicates the lines are parallel so ___________________________.
* True statement indicates the lines coincide so ______________________________.

Adapted from Graphic Organizer by Dale Graham and Linda Meyer Thomas County Central High School; Thomasville GA

FORMATIVE ASSESSMENT LESSONS (FAL)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Overview.

SPOTLIGHT TASKS

A Spotlight Task has been added to each GSE mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit–level Georgia Standards of Excellence, and research–based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3–Act Tasks based on 3–Act Problems from Dan Meyer and Problem–Based Learning from Robert Kaplinsky.

3–ACT TASKS

A Three–Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.
More information along with guidelines for 3–Act Tasks may be found in the Comprehensive Course Overview.

TASKS

The following tasks represent the level of depth, rigor, and complexity expected of all Algebra I students. These tasks, or tasks of similar depth and rigor, should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as a performance task, they may also be used for teaching and learning (learning/scaffolding task).

Task Name	Task Type *Grouping Strategy*	Content Addressed	Standards
Acting Out *20-30 mi*	Scaffolding Task *Individual / Partner /Small Group*	Model and write an equation in one variable. Represent constraints with inequalities.	A.CED.1 A.CED.3 N.Q.1 N.Q.2 N.Q.3
Lucy's Linear Equations and Inequalities *30-45 min*	Practice Task *Individual / Partner*	Write linear equations and inequalities in one variable and solve problems in context.	A.CED.1 A.SSE.1 A.SSE.1a A.SSE.1b
Jaden's Phone Plan *30 min*	Scaffolding Task *Individual/Partner Task*	Model and write an equation in one variable. Represent constraints with inequalities.	A.REI.1 A.REI.3
Ivy Smith Grows Up *45–60 minutes* PDF / Word	Achieve CCSS–CTE Classroom Task Individual/Small Group	Model a situation with an equation and solve.	N.Q.1 A.CED.1 A.CED.3 A.REI.3
Forget the Formula *45 min*	Scaffolding Task *Partner/Small Group*	Create equations in two variables to represent relationships. Represent constraints. Rearrange formulas to highlight a quantity of interest.	A.CED.2 A.CED.3 A.CED.4
World Record Airbag Diving Spotlight Task *30-45 min*	Constructive Task *Small Group*	Decide on an appropriate formula to solve the freefall problem. Graph the diver's height (position) versus time during the fall. Graph the diver's velocity versus time.	A.CED.4 N.Q.1 N.Q.2 N.Q.3
The Largest Loser *2 days*	Formative Assessment Lesson	Understand constraints upon graphs in given	N.Q.1 N.Q.3

	Partner/ Small Group	contexts. Make sense of graph problems with differently-defined axes of measure.	A.CED.2
Cara's Candles Revisited *30 -45 min*	Scaffolding task *Individual/Partners*	Graph equations to represent linear relationships. Determine where a point is a solution to the equation f(x) = g(x) in context. Interpret whether the solution is viable in context.	A.CED.1 A.CED.2 A.CED.3 A.REI.11 F.IF.5 F.IF.6
Best Buy Task (Short Cycle Task) *20-30 min*	Short Cycle Task *Individual/Partner*	Use linear models to compare two purchasing options.	A.CED.2 A.REI.3 A.REI.11 F.BF.1 F.BF.1a F.BF.2
Solving Systems of Equations Algebraically: Part 1 and Part 2 *90 min*	Scaffolding Task *Individual/Partner Task/Whole Class*	Justify the solution to a system of equations by graphing and substituting values into the system. Create new equations by multiplying one or more equations by a constant. Solve systems of equations by elimination.	A.REI.5 A.REI.6
Dental Impressions *45–60 minutes* PDF / Word	Achieve CCSS–CTE Classroom Task Individual/Partner	Model a situation with a system of linear equations and solve.	N.Q.1 A.CED.1 A.REI.6
Ground Beef *45–60 minutes* PDF / Word	Achieve CCSS–CTE Classroom Task Partner/Small Group	Model a situation with a system of linear equations and solve.	N.Q.1 N.Q.3 A.CED.2 A.REI.6
Systems of Weights **(Spotlight Task)** *30–45 minutes*	Discovery Task *Partner Task*	Model a situation with a system of linear equations and solve.	A.REI.6 A.REI.12
Solving Linear Equations in Two Variables	Formative Assessment Lesson *Individual / Small Group*	Solve problems using a system of equations.	A.REI.5 A.REI.6

(FAL) *60–90 minutes* PDF			
Boomerangs (FAL) *60–90 minutes* PDF	Formative Assessment Lesson - Extension *Individual / Small Group*	Solve optimization tasks using a system of equations.	A.REI.5 A.REI.6 A.REI.12
Summer Job *90 min*	Scaffolding Task *Individual Partner/Small Group Task*	Model linear patterns. Create equations and inequalities in one and two variables to represent relationships.	A.REI.12
Graphing Inequalities *45 min*	Extension Task *Individual/Partner/Small Group Task*	Model with inequalities.	A.REI.12
Modeling Situations with Linear Equations (FAL) *60–90 minutes* PDF	Formative Assessment Lesson *Pairs*	Model everyday situations with algebra.	A.CED.4 A.REI.1 A.REI.10 F.IF.4
Family Outing *90 min*	Performance Task *Individual/Partner*	Graph equations on coordinate axes with labels and scales. Solve systems of equations. Solve systems of inequalities - Extension Determine constraints.	A.REI.1 A.REI.3 A.REI.5 A.REI.6 A.REI.12
Talk is Cheap! (Spotlight Task) *30-60 min* *This task has been revised as a Spotlight Task.*	Spotlight Task *Individual/Partner*	Graph linear functions. Use the graphing calculator to find the intersection of two linear functions. Interpret the intersection in terms of the problem situation. Compare functions represented algebraically, graphically, and in tables.	A.REI.10 A.REI.11 F.IF.2 F.IF.7 F.IF.9 F.BF.1

Functioning Well *30 min*	Practice Task *Individual/Partner*	Use function notation. Interpret statements that use function notation in terms of context. Recognize that sequences are functions.	F.IF.1 F.IF.2
Detention Hall Buy Out (Spotlight Task) *60 min*	Spotlight Task *Small Group*	Model and solve problems involving the intersection of two straight lines. Interpret the intersection in terms of the problem situation. Compare functions represented algebraically, graphically, and in tables.	A.REI.10 A.REI.11 F.IF.2. F.IF.7 F.IF.9 F.BF.1
Summing It Up: Putting the "Fun" in Functions *3-4 days*	Culminating Task *Individual/Partner*	Understand the concept of a function and use function notation. Interpret functions that arise in context. Analyze functions using different representations. Build new functions from existing functions. Construct and compare linear models and solve problems.	All

Acting Out (Scaffolding Task)

Back to Task Table

Introduction

In this task, students will use an inequality to find the distance between two homes. Students will also learn how to convert contextual information into mathematical notation. The second part has students determine how much water might a dripping faucet waste in a year.

Mathematical Goals

- Model and write an equation in one variable and solve a problem in context.
- Create one–variable linear equations and inequalities from contextual situations.
- Represent constraints with inequalities.
- Solve word problems where quantities are given in different units that must be converted to understand the problem.

Essential Questions

- How do I choose and interpret units consistently in formulas?
- How can I model constraints using mathematical notation?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, ~~quadratic,~~ ~~simple rational,~~ ~~and exponential~~ functions ~~(integer inputs only).~~

MGSE9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

RELATED STANDARDS

MGSE9–12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
b. Convert units and rates using dimensional analysis (English–to–English and Metric–to–Metric without conversion factor provided and between English and Metric with conversion factor);
c. Use units within multi–step problems and formulas; interpret units of input and resulting units of output.

MGSE9–12.N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. Given a situation, context, or problem, students will determine, identify, and use appropriate quantities for representing the situation.

MGSE9–12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers' precision is limited to the precision of the data given.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
   Students will have to determine different ways of measuring a fixed distance from a given point and will have to strategize the most accurate way to convert measurements.
5. Use appropriate tools strategically.
   Students will be given multiple tools to choose from to model the scenario in Part I.
6. Attend to precision.
   Students will have to know when to round their answers based on the units of measurement.

Background Knowledge
- Students can graph relationships.
- Students can write and interpret inequalities.
- Students can graph inequalities on a number line.
- Students can convert units (time and volume).

Common Misconceptions
- Students may only think of vertical and horizontal distances and not in terms of a radius.
- Students may naturally progress from days to weeks to months to years, rather than directly from days to years.

Materials
- colored pencils
- compass
- string
- graph paper

Grouping
- Part I: Small group / whole group
- Part II: Partner / Individual

Differentiation

Extension:
- Ask additional conversion questions: How old are you in minutes? How many inches long is a football field? If you are 1,000,000 seconds old, how old are you in years?

Intervention:
- Provide a conversion list.
- Review inequality symbols.

Formative Assessment Questions

- How are you choosing the units used throughout the problem?
- How do the units drive your conversions?
- How can we demonstrate limits algebraically?

Acting Out – Teacher Notes

Part I:

Erik and Kim are actors at a theater. Erik lives 5 miles from the theater and Kim lives 3 miles from the theater. Their boss, the director, wonders how far apart the actors live.

Comments

Students should understand that Erik and Kim could live anywhere on the circle with the theater as the center and the radius as the distance that they live from the theater.

1. On the given grid:

a. pick a point to represent the location of the theater.

b. Illustrate all of the possible places that Erik could live on the grid paper.

c. Using a different color, illustrate all of the possible places that Kim could live on the grid paper.

2. What is the smallest distance, *d*, that could separate their homes? How did you know?

Solution

5 – 3 = 2 miles

3. What is the largest distance, *d*, that could separate their homes? How did you know?

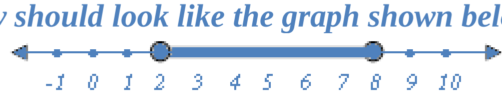
Solution

5 + 3 = 8 miles

4. Write and graph an inequality in terms of *d* to show their boss all of the possible distances that could separate the homes of the 2 actors.

Solution

An inequality that could represent this distance could be $2 \le d \le 8$ miles. Graphing this inequality should look like the graph shown below.



Students should understand that the solid dots on the graph represent the fact that Erik and Kim could live exactly 2 miles or exactly 8 miles apart. Should the situation have been different and they lived more than 2 miles or less than 8 miles apart, those dots would have been left open, or not filled in. The space shaded on the number line between the 2 and 8 means that they could live any of those distances apart.

Part II:

The actors are good friends since they live close to each other. Kim has a leaky faucet in her kitchen and asks Erik to come over and take a look at it.

1. Kim estimates that the faucet in her kitchen drips at a rate of 1 drop every 2 seconds. Erik wants to know how many times the faucet drips in a week. Help Erik by showing your calculations below.

Solution:

60 sec = 1 min
60 min = 1 hour
24 hours = 1 day
7 days = 1 week

(60) (60) (24) (7) = 604800
604800/2 = 302,400 drops

365 days = 1 year

$\frac{(60)(60)(24)(365)}{2}$***= 15,768,000 drops per year***

2. Kim estimates that approximately 575 drops fill a 100 milliliter bottle. Estimate how much water her leaky faucet wastes in a year.

Solution:

15,768,000/575 = 27,422.608

About 27,423 of 100 millimeter bottles would be filled.
Encourage students to write this answer in the most user–friendly measurement i.e. liters

27,423(100) = 2,742,300 milliliters or 2,742.3 m

Acting Out (Scaffolding Task)

Name_________________________________ **Date**__________________

Adapted from Shell Center Leaky Faucet Short Cycle Task
http://www.map.mathshell.org/materials/tasks.php?taskid=267&subpage=apprentice

Mathematical Goals

- Model and write an equation in one variable and solve a problem in context.
- Create one–variable linear equations and inequalities from contextual situations.
- Represent constraints with inequalities.
- Solve word problems where quantities are given in different units that must be converted to understand the problem.

Essential Questions

- How do I choose and interpret units consistently in formulas?
- How can I model constraints using mathematical notation?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear~~, quadratic,~~ ~~simple rational,~~ ~~and exponential~~ functions ~~(integer inputs only).~~

MGSE9–12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non–solution) under the established constraints.

RELATED STANDARDS

MGSE9–12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
b. Convert units and rates using dimensional analysis (English–to–English and Metric–to–Metric without conversion factor provided and between English and Metric with conversion factor);
c. Use units within multi–step problems and formulas; interpret units of input and resulting units of output.

MGSE9–12.N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. Given a situation, context, or problem, students will determine, identify, and use appropriate quantities for representing the situation.

MGSE9–12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers' precision is limited to the precision of the data given*.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
5. Use appropriate tools strategically.
6. Attend to precision.

<u>Acting Out (Scaffolding Task)</u>

Name________________________________ **Date**__________________

Adapted from Shell Center Leaky Faucet Short Cycle Task
http://www.map.mathshell.org/materials/tasks.php?taskid=267&subpage=apprentice

Part I:

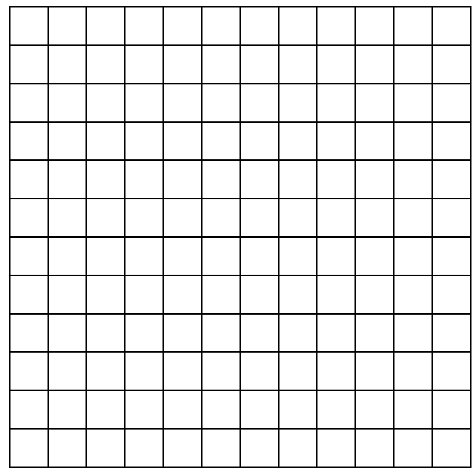
Erik and Kim are actors at a theater. Erik lives 5 miles from the theater and Kim lives 3 miles from the theater. Their boss, the director, wonders how far apart the actors live.

1. On the given grid:

a. pick a point to represent the location of the theater.

b. Illustrate all of the possible places that Erik could live on the grid paper.

c. Using a different color, illustrate all of the possible places that Kim could live on the grid paper.



2. What is the smallest distance, *d*, that could separate their homes? How did you know?

3. What is the largest distance, *d*, that could separate their homes? How did you know?

4. Write and graph an inequality in terms of *d* to show their boss all of the possible distances that could separate the homes of the 2 actors.

Part II:

The actors are good friends since they live close to each other. Kim has a leaky faucet in her kitchen and asks Erik to come over and take a look at it.

1. Kim estimates that the faucet in her kitchen drips at a rate of 1 drop every 2 seconds. Erik wants to know how many times the faucet drips in a week. Help Erik by showing your calculations below.

2. Kim estimates that approximately 575 drops fill a 100 milliliter bottle. Estimate how much water her leaky faucet wastes in a year.

Lucy's Linear Equations and Inequalities (Practice Task) Back to Task Table

Introduction

In this task, students will solve a series of linear equations and inequality word problems that Lucy has been assigned by her teacher. In order to help Lucy, students must explain in detail each step of the problem. It is a good idea to review key words that are associated with linear inequality word problems before beginning this task.

Keywords: fewer than, more than, at most, at least, less than, no less than

Mathematical Goals

- Create one–variable linear equations and inequalities from contextual situations.
- Solve and interpret the solution to multi–step linear equations and inequalities in context.

Essential Questions

- How do I interpret parts of an expression in terms of context?
- How do I create equations and inequalities in one variable and use them to solve problems arising from linear functions?
- How can I write, interpret and manipulate algebraic expressions, equations and inequalities?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear~~, quadratic~~, ~~simple rational, and exponential~~ functions~~ (integer inputs only).~~

RELATED STANDARDS

MGSE9–12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

MGSE9–12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.

MGSE9–12.A.SSE.1b Given situation which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.

STANDARDS FOR MATHEMATICAL PRACTICE

2. Reason abstractly and quantitatively.

4. Model with mathematics.
Students will define variables and create expressions and equations with those variables.

7. Look for and make use of structure.
Students will recognize patterns for modeling consecutive integers and expressing perimeters using algebraic expressions.

Background Knowledge

- Students can find the perimeter of figures.
- Students can solve inequalities and equations.
- Students know some key words for operations and inequalities (see above).

Common Misconceptions

- Students may not know to double the length and width to calculate perimeter or they may use the area formula to calculate.
- Students may not recognize how to model "consecutive integers" or "consecutive even/odd integers"

Materials

- None

Grouping

- Individual / Partner

Differentiation

Extension:

- Encourage "Resident Experts" to support struggling students.

Intervention:

- Supply students with key words.

Formative Assessment Questions

- How can we tell if a problem is an equation or an inequality?
- How do we solve equations differently than inequalities?

Lucy's Linear Equations and Inequalities – Teacher Notes

Lucy has been assigned the following linear equations and inequality word problems. Help her solve each problem below by using a five step plan.

- Drawing a Sketch(if necessary)
- Defining a Variable
- Setting up an equation or inequality
- Solve the equation or inequality
- Make sure you answer the question

1. The sum of 38 and twice a number is 124. Find the number.

   Solution

   Define a Variable: n = the number

   Equation: $38 + 2n = 124$
   $$2n = 86$$
   $$n = 43$$
   The number is 43.

   Check: $38 + 2(43) = 124$
   $$38 + 86 = 124$$
   $$124 = 124 \checkmark$$

2. The sum of two consecutive integers is less than 83. Find the pair of integers with the greatest sum.

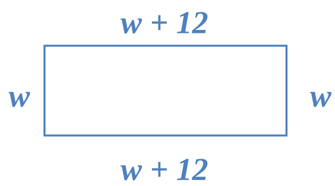
   Solution

   Define a Variable:
   x = the first consecutive number, so x +1 = the second consecutive number
   Equation: $x + x + 1 < 83$
   $$2x < 82$$
   $$x < 41$$
   The numbers are 40 and 41

   Check: $40 + 41 < 83$
   $$81 < 83 \checkmark$$

3. A rectangle is 12m longer than it is wide. Its perimeter is 68m. Find its length and width.
   Solution

   Sketch:



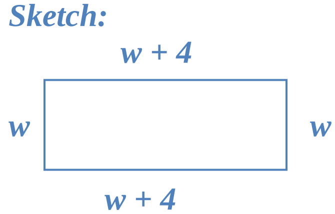
   Define a Variable:
   Width = w
   So length = w + 12

   Equation: $w + w + w + 12 + w + 12 = 68$
   $4w + 24 = 68$
   $4w = 44$
   $w = 11$
   Width = 11
   Length = 11 + 12 = 23
   So the width is 11m and the length is 23m.

   Check: $11 + (11+12) + (11) + (11+12) = 68$ ✓

4. The length of a rectangle is 4 cm more than the width and the perimeter is at least 48 cm. What are the smallest possible dimensions for the rectangle?
   Solution

   Sketch:



   Define a Variable:
   Width = w
   Length = w + 4

   Equation: $w + w + w + 4 + w + 4 \geq 48$
   $4w \geq 40$
   $w \geq 10$

   Width = 10
   Length = (10) + 4 = 14

So the width is 10cm and the length is 14cm.

Check: $10 + 10 + 14 + 14 \geq 48$
$48 \geq 48$ ✓

5. Find three consecutive integers whose sum is 171.
Solution

Define a Variable:
x ***= the first consecutive number***
so $x + 1$ ***= the second consecutive number***
and $x + 2$ ***= the third consecutive number***
Equation: $x + x + 1 + x + 2 = 171$
$3x + 3 = 171$
$3x = 168$
$x = 56$
56 = the first consecutive number
56 +1 = the second consecutive number
56 + 2 = the third consecutive number
The three consecutive numbers are 56, 57, and 58.
Check: $56 + 57 + 58 = 171$

6. Find four consecutive even integers whose sum is 244.
Solution

Define a Variable:
x ***= first number***
$x + 2$ ***= second number***
$x + 4$ ***= third number***
$x + 6$ ***= fourth number***
Equation: $x + x + 2 + x + 4 + x + 6 = 244$
$4x + 12 = 244$
$4x = 232$
$x = 58$
58 = first number
58 +2 = second number
58 + 4 = third number
58 + 6 = fourth number
The numbers are 58, 60, 62, and 64.

Check: $58 + 60 + 62 + 64 = 244$

7. Alex has twice as much money as Jennifer. Jennifer has $6 less than Shannon. Together they have $54. How much money does each have?

Solution

Define a Variable:
Shannon: s
Jennifer: s – 6
Alex: 2(s – 6)
Equation: $s + s - 6 + 2(s - 6) = 54$
$$2s - 6 + 2s - 12 = 54$$
$$4s - 18 = 54$$
$$4s = 72$$
$$s = 18$$
Shannon: $s = 18$
Jennifer: $s - 6 = 12$
Alex: $2(s - 6) = 24$
So, Shannon has $18, Jennifer has $12, and Alex has $24

Check: $18 + 12 + 24 = 54$

8. There are three exams in a marking period. A student received grades of 75 and 81 on the first two exams. What grade must the student earn on the last exam to get an average of no less than 80 for the marking period?
Solution

Define a Variable: x ***= last exam***

Equation:
$$\frac{75 + 81 + x}{3} \geq 80$$
$$75 + 81 + x \geq 240$$
$$x \geq 84$$
The student must receive an 84 or higher exam grade in order to have an average no less than 80 for the marking period.

Check: $\frac{75+81+x}{3} \geq 80$

Lucy's Linear Equations and Inequalities (Practice Task)

Name_________________________________ **Date**__________________

Mathematical Goals
- Create one–variable linear equations and inequalities from contextual situations.
- Solve and interpret the solution to multi–step linear equations and inequalities in context.

Essential Questions
- How do I interpret parts of an expression in terms of context?
- How do I create equations and inequalities in one variable and use them to solve problems arising from linear functions?
- How can I write, interpret and manipulate algebraic expressions, equations and inequalities?

GEORGIA STANDARDS OF EXCELLENCE
MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear~~, quadratic~~, ~~simple rational, and exponential~~ functions ~~(integer inputs only).~~

RELATED STANDARDS
MGSE9–12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

MGSE9–12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.

MGSE9–12.A.SSE.1b Given situation which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.

STANDARDS FOR MATHEMATICAL PRACTICE
2. Reason abstractly and quantitatively.
4. Model with mathematics.
7. Look for and make use of structure.

Lucy's Linear Equations and Inequalities (Practice Task)

Name________________________________ **Date**_________________

Lucy has been assigned the following linear equations and inequality word problems. Help her solve each problem below by using a five step plan.

- Drawing a Sketch (if necessary)
- Defining a Variable
- Setting up an equation or inequality
- Solve the equation or inequality
- Make sure you answer the question

1. The sum of 38 and twice a number is 124. Find the number.

2. The sum of two consecutive integers is less than 83. Find the pair of integers with the greatest sum.

3. A rectangle is 12m longer than it is wide. Its perimeter is 68m. Find its length and width.

4. The length of a rectangle is 4 cm more than the width and the perimeter is at least 48 cm. What are the smallest possible dimensions for the rectangle?

5. Find three consecutive integers whose sum is 171.

6. Find four consecutive even integers whose sum is 244.

7. Alex has twice as much money as Jennifer. Jennifer has $6 less than Shannon. Together they have $54. How much money does each have?

8. There are three exams in a marking period. A student received grades of 75 and 81 on the first two exams. What grade must the student earn on the last exam to get an average of no less than 80 for the marking period?

Jaden's Phone Plan (Scaffolding Task)

Back to Task Table

Introduction

In this task, students will solve a series of linear equations and inequality word problems to help Jaden choose a cell phone plan. In order to help Jaden, students must explain in detail each step of the problem and justify the answer.

Mathematical Goals

- Create one–variable linear equations and inequalities from contextual situations.
- Solve and interpret the solution to multi–step linear equations and inequalities in context.

Essential Questions

- How do I solve an equation in one variable?
- How do I justify the solution to an equation?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9–12.A.REI.1 Using algebraic properties and the properties of real numbers, justify the steps of a simple, one–solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties. *(Students should focus on and master linear equations and be able to extend and apply their reasoning to other types of equations in future courses.)*

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. *(For example, given $ax + 3 = 7$, solve for x.)*

STANDARDS FOR MATHEMATICAL PRACTICE

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
Students will justify their equations using reasoning.
4. Model with mathematics.
Students will us algebra to describe situations.

Background Knowledge

- Students can solve equations.
- Students know key words for modeling situations.

Common Misconceptions

- Students may mistake cents for dollars when modeling the situations.
- Students may confuse the input/output of the situation.
- Students may have issues substituting more than one variable.

Materials

- None

Grouping

- Individual / Partner

Differentiation

Extension:

- Determine what situations (number of texts and calls) would cause Plan A to be cheaper.
  Solution:
  $A = 0.15t + 0.10c; \quad B = 15 + 0.05(t + c); \quad A < B$
  $0.15t + 0.10c < 15 + 0.05(t + c)$
  $0.15t + 0.10c < 15 + 0.05t + 0.05c$
  $0.10t + 0.05c < 15$
  $10t + 5c < 1500$
  $2t + c < 300$
  If the value of $2t + c$ is less than 300, then Plan A is the cheaper choice.

Intervention:

- Review solving equations out of context.

Formative Assessment Questions

- How can linear equations model a real life situation? What key components should you look for to model a real life situation with a linear equation?
- How do you solve linear equations?

Jaden's Phone Plan – Teacher Notes

Jaden has a prepaid phone plan (Plan *A*) that charges 15 cents for each text sent and 10 cents per minute for calls.

***Comment:* As students solve equations throughout this task, have them explain each step using properties of operations or properties of equality.**

1. If Jaden uses only text, write an equation for the cost *C* of sending *t* texts.

   C = $.15t

   a. How much will it cost Jaden to send 15 texts? Justify your answer.

   C = $.15 * 15
   C = $ 2.25

   b. If Jaden has $6, how many texts can he send? Justify your answer.

   C = $.15t
   $6 = $.15t
   $6/$.15 = t
   t = 40 texts

2. If Jaden only uses the talking features of his plan, write an equation for the cost *C* of talking *m* minutes.

   C = $.10m

   a. How much will it cost Jaden to talk for 15 minutes? Justify your answer.

   C = $.10 * 15
   C = $1.50

   b. If Jaden has $6, how many minutes can he talk? Justify your answer.

   C= $.10m
   $6 = .10m
   $6/.10 = m
   m = 60 minutes

3. If Jaden uses both talk and text, write an equation for the cost *C* of sending t texts and talking *m* minutes.

C = $.15t + $.10m

a. How much will it cost Jaden to send 7 texts and talk for 12 minutes? Justify your answer.

C = $.15t + $.10m
C = $.15*7 + $.10*12
C = $1.05 + $1.20
C = $2.25

b. If Jaden wants to send 21 texts and only has $6, how many minutes can he talk? Will this use all of his money? If not, will how much money will he have left? Justify your answer.

C = $.15t + $.10m
$6 = $.15*21 + $.10m
$6 = $3.15 + .10m
$6 – $3.15 = .10m
$2.85 = .10m
$2.85/.10 = m
m = 28.5 minutes

Since most carriers will charge a full minute for any fraction of a minute, Jaden can talk for 28 minutes. He will have $0.05 left over if he talks for 28 minutes.

Jaden discovers another prepaid phone plan (Plan *B*) that charges a flat fee of $15 per month, then $.05 per text sent or minute used.

4. Write an equation for the cost of Plan *B*.

C = $15 + $.05n (Since the cost of text and talk are the same, the same variable can represent both.)

In an average month, Jaden sends 200 texts and talks for 100 minutes.

5. Which plan will cost Jaden the least amount of money? Justify your answer.
Plan A: C = $.15t + $.10m
C = $.15*200 + $.10*100
C = $30 + $10
C = $40
Plan B: C = 15 + .05n
C = $15 + $.05(200 + 100)

C = \$15 + \$.05(300)
C = \$15 + \$15
C = \$30

Based on Jaden's average usage, the cost for Plan A is \$40 per month and the cost for Plan B is \$30 per month. Therefore, Plan B will cost Jaden the least amount of money.

Jaden's Phone Plan (Scaffolding Task)

Name________________________________ **Date**_________________

Mathematical Goals

- Create one–variable linear equations and inequalities from contextual situations.
- Solve and interpret the solution to multi–step linear equations and inequalities in context.

Essential Questions

- How do I solve an equation in one variable?
- How do I justify the solution to an equation?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9–12.A.REI.1 Using algebraic properties and the properties of real numbers, justify the steps of a simple, one–solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties. *(Students should focus on and master linear equations and be able to extend and apply their reasoning to other types of equations in future courses.)*

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. *(For example, given $ax + 3 = 7$, solve for x.)*

STANDARDS FOR MATHEMATICAL PRACTICE

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

Scaffolding Task: Jaden's Phone Plan

Name________________________________ **Date**_________________

Jaden has a prepaid phone plan (Plan *A*) that charges 15 cents for each text sent and 10 cents per minute for calls.

1. If Jaden uses only text, write an equation for the cost C of sending t texts.

    a. How much will it cost Jaden to send 15 texts? Justify your answer.

    b. If Jaden has \$6, how many texts can he send? Justify your answer.

2. If Jaden only uses the talking features of his plan, write an equation for the cost C of talking m minutes.

    a. How much will it cost Jaden to talk for 15 minutes? Justify your answer.

    b. If Jaden has \$6, how many minutes can he talk? Justify your answer.

3. If Jaden uses both talk and text, write an equation for the cost C of sending t texts and talking m minutes.

    a. How much will it cost Jaden to send 7 texts and talk for 12 minutes? Justify your answer.

    b. If Jaden wants to send 21 texts and only has \$6, how many minutes can he talk? Will this use all of his money? If not, will how much money will he have left? Justify your answer.

Jaden discovers another prepaid phone plan (Plan *B*) that charges a flat fee of $15 per month, then $.05 per text sent or minute used.

4. Write an equation for the cost of Plan *B*.

In an average month, Jaden sends 200 texts and talks for 100 minutes.

5. Which plan will cost Jaden the least amount of money? Justify your answer.

Ivy Smith Grows Up (Career and Technical Education Task) Back to Task Table

Source: National Association of State Directors of Career Technical Education Consortium
PDF: **http://www.achieve.org/files/CCSS–CTE–Task–IvySmith–GrowsUp–FINAL.pdf**
Word: **http://www.achieve.org/files/CCSS–CTE–Task–IvySmith–GrowsUp–FINAL.doc**

Introduction

This task uses the growth of newborns and infants to help students understand conversion of units and determine a linear model for the data.

Mathematical Goals

- Write and use a linear model for data.
- Convert between standard and metric units.

Essential Questions

- How do you use real–life data to determine a linear model and use this model to approximate missing data?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear~~, quadratic,~~ ~~simple rational,~~ ~~and exponential~~ functions ~~(integer inputs only).~~

MGSE9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable including equations with coefficients represented by letters. *For example, given $ax + 3 = 7$, solve for x.*

RELATED STANDARDS

MGSE9–12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
b. Convert units and rates using dimensional analysis (English–to–English and Metric–to–Metric without conversion factor provided and between English and Metric with conversion factor);
c. Use units within multi–step problems and formulas; interpret units of input and resulting units of output.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
Students analyze givens, constraints, relationships, and goals. They must make conjectures about the form and meaning of the solution and plan a solution pathway.
2. Reason abstractly and quantitatively.
Students must attend to the meaning of the quantities throughout the problem.
4. Model with mathematics.
Students translate constraints into equations and extract information from graphs.
5. Use appropriate tools strategically.
Students use website, calculator, and the attached chart.
6. Attend to precision.
Students must use units, convert units, and perform calculations precisely.

Background Knowledge

- Students can convert units.
- Students can write the equation of a line given two points on a line.

Common Misconceptions

- Students may struggle to convert between standard and metric units.

Materials

- Graph paper
- Chart from website for #5

Grouping

- Individual / small group

Differentiation

- See extensions in task.

Forget the Formula (Scaffolding Task)

Back to Task Table

Introduction

In this task, students will develop a formula to convert temperatures from Celsius to Fahrenheit. Students will then manipulate this formula to create a Fahrenheit to Celsius formula. Students should develop meaning for each equation based on the context of the problem.

Mathematical Goals

- Rearrange formulas to highlight a quantity of interest.
- Create equations in two variables to represent relationships.
- Write and graph an equation to represent a linear relationship.
- Extend the concepts used in solving numerical equations to rearranging formulas for a particular variable.

Essential Questions

- How do I interpret parts of an expression in terms of context?
- How do I create equations in two variables to represent relationships between quantities?
- How can I rearrange formulas to highlight a quantity of interest?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.CED.2 Create linear~~, quadratic, and exponential~~ equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase "in two or more variables" refers to formulas like the compound interest formula, in which $A = P(1 + r/n)^{nt}$ has multiple variables.)

MGSE9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

MGSE9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. *Examples: Rearrange Ohm's law V = IR to highlight resistance R;* ~~*Rearrange area of a circle formula A = π r² to highlight the radius r.*~~

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
   Students will have to attempt several approaches to find the answer.
2. Reason abstractly and quantitatively.
   Students will recognize that the freezing and boiling points can be used to calculate slope.

Background Knowledge

- Students can graph linear equations.
- Students can write the equation of a line given two points on the line.

Common Misconceptions

- Students may be confused as to which temperature measure should be their x or y coordinate.
- Students may have trouble with the multiplicative inverse.
- Students may not realize that C=F is suggesting that they substitute one variable with the other.

Materials

- None

Grouping

- Partner / small group

Differentiation

Extension:

- Introduce Kelvin. ($K = C – 273$)

Intervention:

- Provide a drawing of a thermometer with Celsius and Fahrenheit marked for reference.

Formative Assessment Questions

- How can you derive a formula based on data gathered?

Forget the Formula – Teacher Notes

Temperature can be measured with many different systems, the most commonly used are Fahrenheit and Celsius. The relationship between the two systems is linear and therefore can be determined using any two equivalent measurements.

1. What is the boiling point of water in Fahrenheit and Celsius?

   Solution:
   The boiling point of water is 100 degrees Celsius and 212 degrees Fahrenheit.

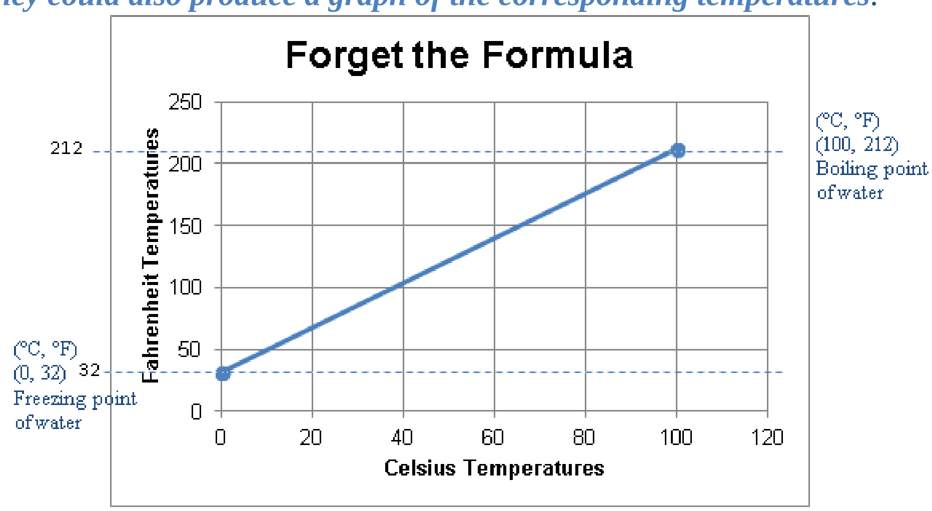
2. What is the freezing point of water in Fahrenheit and Celsius?

   Solution:
   The freezing point of water is 0 degrees Celsius and 32 degrees Fahrenheit.

3. Using these two points, create an equation that convert Celsius to Fahrenheit.

   Solution:
   One method of finding this formula is to use (0, 32) and (100, 212) as two points on a line. To find the equation of the line only the slope is needed, since the y–intercept, (0, 32), is already given. Calculating the slope, $\frac{212-32}{100-0}$, the students should simplify $\frac{180}{100}$ to $\frac{9}{5}$. Substituting this for m in y = mx + b, the equation becomes $y = \frac{9}{5}x + 32$. Because y represent the Fahrenheit temperature and x represents the Celsius temperature, the formula would be more appropriately written $F = \frac{9}{5}C + 32$.

   They could also produce a graph of the corresponding temperatures.



Rearrange the equation found in number 3 to solve for Celsius.

Students could solve this equation for C to produce the other form expressing the relationship:

$$F = \frac{9}{5}C + 32$$
$$F - 32 = \frac{9}{5}C + 32 - 32$$
$$(F - 32)\left(\frac{5}{9}\right) = \left(\frac{9}{5}\right)\left(\frac{5}{9}\right)C$$
$$(F - 32)\left(\frac{5}{9}\right) = C$$
$$C = \left(\frac{5}{9}\right)(F - 32)$$

or

$$C = \left(\frac{5}{9}\right)F - 17\frac{7}{9}$$

4. What does the constant represent in each equation? What does the slope represent in each equation?

32 is the y–intercept that means when it is 0 degrees Celsius it is 32 degrees Fahrenheit. The slope 9/5 shows that as the Celsius temperature increases or decreases five degrees that Fahrenheit will increase or decrease 9 degrees respectively.

$-17\frac{7}{9}$ ***is the y–intercept that means when it is 0 degrees Fahrenheit it is*** $-17\frac{7}{9}$ ***degrees Celsius. The slope 5/9 shows that as the Fahrenheit temperature increases or decreases nine degrees that Celsius will increase or decrease 5 degrees respectively.***

5. At what temperature is the degrees Celsius equal to the degrees Fahrenheit?

Possible solution. Set C=F and substitute into either equation

$$C = \frac{9}{5}C + 32$$
$$-\frac{4}{5}C = 32$$
$$C = -40$$

Scaffolding Task: Forget the Formula

Name__________________________________ **Date**__________________

Mathematical Goals

- Rearrange formulas to highlight a quantity of interest.
- Create equations in two variables to represent relationships.
- Write and graph an equation to represent a linear relationship.
- Extend the concepts used in solving numerical equations to rearranging formulas for a particular variable.

Essential Questions

- How do I interpret parts of an expression in terms of context?
- How do I create equations in two variables to represent relationships between quantities?
- How can I rearrange formulas to highlight a quantity of interest?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.CED.2 Create linear~~, quadratic, and exponential~~ equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase "in two or more variables" refers to formulas like the compound interest formula, in which $A = P(1 + r/n)^{nt}$ has multiple variables.)

MGSE9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

MGSE9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. *Examples: Rearrange Ohm's law V = IR to highlight resistance R;* ~~*Rearrange area of a circle formula $A = \pi r^2$ to highlight the radius r.*~~

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.

Forget the Formula (Scaffolding Task)

Name_________________________________ **Date**__________________

Temperature can be measured with many different systems, the most commonly used are Fahrenheit and Celsius. The relationship between the two systems is linear and therefore can be determined using any two equivalent measurements.

1. What is the boiling point of water in Fahrenheit and in Celsius?

2. What is the freezing point of water in Fahrenheit and in Celsius?

3. Using these two points, create an equation that convert Celsius to Fahrenheit.

4. Rearrange the equation found in the previous problem to solve for Celsius.

5. What does the constant represent in each equation? What does the slope represent in each equation?

6. At what temperature is the degrees Celsius equal to the degrees Fahrenheit?

World Record Airbag Diving (Spotlight Task)

Back to Task Table

This spotlight task is based on the work featured at https://docs.google.com/spreadsheet/ccc?key=0AjIqyKM9d7ZYdEhtR3BJMmdBWnM2YWxWYVM1UWowTEE#gid=0 ***and follows the 3 Act–Math task format originally developed by Dan Meyer. More information on these type tasks may be found at*** http://blog.mrmeyer.com/category/3acts/

GEORGIA STANDARDS OF EXCELLENCE

MGSE9–12.A.CED.4 Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. *Examples: Rearrange Ohm's law V = IR to highlight resistance R;* ~~*Rearrange area of a circle formula* $A = \pi r^2$ *to highlight the radius r.*~~

RELATED STANDARDS

MGSE9–12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
b. Convert units and rates using dimensional analysis (English–to–English and Metric–to–Metric without conversion factor provided and between English and Metric with conversion factor);
c. Use units within multi–step problems and formulas; interpret units of input and resulting units of output.

MGSE9–12.N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.

MGSE9–12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. ***In this task, students will have to interpret the given situation to determine what information is needed to solve the problem. It is possible, and even desirable, to have students decide on the problem this video is posing and other questions that interest them in the context of the given information. Students should be allowed to think independently then work in small groups to decide what problem is being presented.***

5. Use appropriate tools strategically. ***Students should decide what "tools" are needed to solve the problem they posed based on the video. Students must ask for or find (via technology resources) information that is needed and what tools will allow them to access that information.***

6. Attend to precision. ***Students will have to decide the level of precision needed to answer the question(s) posed after watching and discussing the video clip. This task also opens the door for a discussion of air friction or other factors that might influence the final result. Teachers should welcome this discussion in the context of class and encourage students to investigate other related factors outside of class.***

Essential Questions

- How do I choose and interpret units consistently in solving application problems?
- What are the constraints of this situation and how can I model them using mathematical notation?
- What factors are important when trying to solve a free fall problem?

Materials Required

- Video clip "world record airbag" from 3 Acts Math
- Conversion factors for quantities mentioned in the problems posed after the video (Act 2 will supply needed information but students could investigate independently before getting the information)
- The formulas for calculating an object's position and velocity relative to time. (Supplied in Act 2 unless students have technology resources)
- The facts about the airbag dive in the video NOTE: the needed facts may vary based on the questions posed by students.
- Graph paper for The Sequel

Times Needed
- 30–45 minutes based on the depth of investigation

More information along with guidelines for 3–Act Tasks may be found in the Comprehensive Course Guide.

Act One Description (Dan Meyer http://blog.mrmeyer.com/2011/the–three–acts–of–a–mathematical–story/**) "Introduce the central conflict of your story/task clearly, visually, viscerally, using as few words as possible."**

Act One: Show the video of the Guinness World Record for airbag diving. Note: this is a zipped file in its original source at https://s3.amazonaws.com/threeacts/worldrecordairbag.zip
During Act One students will watch the video (several times in most cases) and think independently and in small groups about what questions arise from the video. Based on the filming, all students will most likely want to know how fast he is traveling when he hits the air bag. Welcome and invite other questions.

Think and wonder: What do you notice? What do you want to know after watching the video? How can you come up with answers to your questions?
Guiding steps to consider if the students don't come up with them on their own might be:

1. How fast was the skydiver traveling when he made impact with the airbag? (This might open up the conversation to the concept of speed versus velocity.)
2. Write down a guess.
3. Write down an answer you know is too high. Too low.

Act Two Description (Dan Meyer http://blog.mrmeyer.com/2011/the–three–acts–of–a–mathematical–story/**) The protagonist/student overcomes obstacles, looks for resources, and develops new tools.**

During Act Two students will discuss the question in Act One and decide on the facts that are needed to answer the question. Students will also look for formulas and conversions that are needed to solve the problem. When students decide what they need to solve the problem, they should ask for the facts or use technology to find them.
Note: It is pivotal to the problem solving process that students decided what is needed without being given the information up front. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin.

Suggested Scaffolding Questions:
1. What is the problem you are trying to solve?
2. What do you think affects the situation?
3. Does it matter how high the skydiver is? What matters?

Act Two facts to be supplied upon request by students:
Note: It is suggested that teachers give all of the information below upon request for the formula to calculate speed/velocity at impact. It is very important to call attention the units used in the formulas/information below. Student will also need to make conversions based on the way the information was given in the video. Supply the conversion listed below the fact box upon request.
By giving all of this information, students must filter through related but non–essential facts to find the formula of interest. This filtering skill is crucial in the problem solving process and offers teachers another opportunity to scaffold the task for all levels of learners.

Free fall / falling speed equations source: http://www.angio.net/personal/climb/speed
The calculator (available from the website link above) uses the standard formula from Newtonian physics to figure out how long before the falling object goes splat:

- The force of gravity, $g = 9.8 \text{ m/s}^2$
  Gravity accelerates you at 9.8 meters per second *per second*. After one second, you're falling 9.8 m/s. After two seconds, you're falling 19.6 m/s, and so on.
- Time to splat: $\sqrt{(2 * \text{height} / 9.8)}$
  It's the square root because you fall faster the longer you fall;
  Examine the following formula: $h = \frac{1}{2} gt^2$
  The more interesting question is why it's times two: If you accelerate for 1 second, your *average* speed over that time is increased by only 9.8 / 2 m/s.
- Velocity at splat time: $\sqrt{(2 * g * height)}$

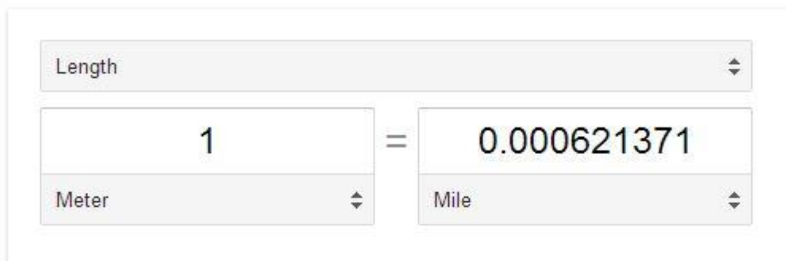
Examine the following formula: $V^2 = 2gh$

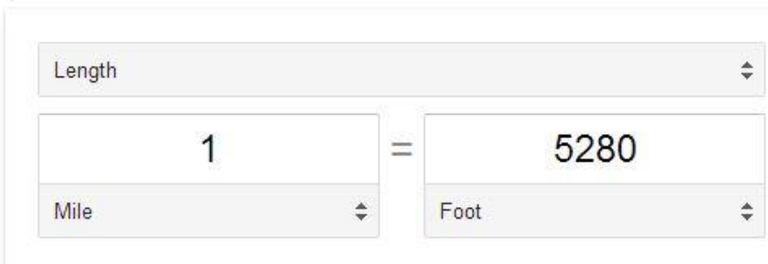
This is why falling from a higher height hurts more.

- Energy at splat time: 1/2 * mass * velocity^2 = mass * g * height

For Students who ask about customary as opposed to metric measure: g= 32 ft/sec² Students should be able to calculate that there are 3600 sec in one hour when making their conversion to miles per hour.

Conversion Factor: Source
https://support.google.com/websearch/answer/3284611?hl=en#unitconverter





ACT 3

Students will compare and share solution strategies.

- Reveal the answer. Discuss the theoretical math versus the practical outcome. ***In this case the calculation based on free fall will NOT match the actual speed of the sky diver due to the air resistance.***
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.
- Reveal the Actual Solution by showing the Act Three video from https://s3.amazonaws.com/threeacts/worldrecordairbag.zip

The Sequel: "The goals of the sequel task are to a) challenge students who finished quickly so b) I can help students who need my help. It can't feel like punishment for good work. It can't seem

like drudgery. It has to entice and activate the imagination." Dan Meyer
http://blog.mrmeyer.com/2013/teaching–with–three–act–tasks–act–three–sequel/
Challenge students to graphically represent the sky diver's position versus the time he is in the air.
Challenge students to graphically represent the sky diver's velocity versus the time he is in the air.
Challenge students to research how much air had to be in the air bag he landed on to keep him safe.
Challenge students to discuss and research ways the sky diver could increase his velocity…or decrease his velocity.
Challenge students to discuss and investigate other parts of this situation that interest them.

Information about graphing position versus time and velocity versus time for objects in free fall may be found at http://www.physicsclassroom.com/class/1DKin/Lesson–5/Representing–Free–Fall–by–Graphs

***Teacher's notes and suggestions*: make notes for yourself or to share with your colleagues about this 3 Act Task. What went well? What would you do differently? Were there things you did not think of in advance that came up during the unfolding of the task?**

Possible Solution: Please note that your students may come up with other questions. The solution below represents "typical" questions/answers. Welcome other questions and solutions for the class to discuss and validate.

How fast was the skydiver going upon impact?

Using V= $\sqrt{(2 * \text{height} / 9.8)}$ ***with a height of 342 feet using g=32 ft/sec² the velocity would be approximately 147.946 ft/sec. Students should then use unit conversion to arrive at approximately 100.87 miles/hr.***

If the student used g=9.8 m/ sec² along with other conversions given under Act 2, the solution would be approximately 101.1 miles/hr.

The actual speed at impact was given as 90 miles/hr. This discrepancy opens the discussion up for sources of error or factors that influence the speed (such as air resistance).

The Largest Loser (Formative Assessment Lesson)

Source: Georgia Mathematics Design Collaborative

This lesson is intended to help you assess how well students are able to:

- Utilize what they already know about linear equations in the context of different graphs
- Reasoning quantitatively, choose and interpret the appropriate scale and rate of change from graphs
- Understand Constraints upon graphs in given contexts and make sense of graph problems with differently-defined axes of measure
- Reason abstractly and compare graphs of linear equations with different scales of measure

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.CED.2 Create linear~~, quadratic, and exponential~~ equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which $A = P(1 + r/n)^{nt}$ has multiple variables.)

RELATED STANDARDS

MGSE9–12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
b. Convert units and rates using dimensional analysis (English–to–English and Metric–to–Metric without conversion factor provided and between English and Metric with conversion factor);
c. Use units within multi–step problems and formulas; interpret units of input and resulting units of output.

MGSE9-12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers’ precision is limited to the precision of the data given.*

STANDARDS FOR MATHEMATICAL PRACTICE:

This lesson uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Georgia Mathematics Design Collaborative are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding.

The task, *The Largest Loser*, is a Formative Assessment Lesson (FAL) that can be found on the Georgia Mathematics online professional learning community, edWeb.net. Once logged in, the task can be found directly at: https://www.edweb.net/?14@@.5ad26830

Cara's Candles Revisited (Scaffolding Task)

Back to Task Table

Introduction

In this task, students will create a table of values from a given scenario. After answering the question posed students will interpret whether the solution is viable or non–viable in modeling context. Students will also graph the equations to represent linear relationships.

Mathematical Goals

- Determine whether a point is a solution to an equation.
- Determine whether a solution has meaning in a real–world context.
- Interpret whether the solution is viable from a given model.
- Write and graph equations and inequalities representing constraints in contextual situations.

Essential Questions

- How do I graph equations on coordinate axes with the correct labels and scales?
- How do I create equations in two or more variables to represent relationships between two quantities?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear~~, quadratic,~~ ~~simple rational,~~ ~~and exponential~~ functions~~ (integer inputs only).~~

MGSE9-12.A.CED.2 Create linear~~, quadratic, and exponential~~ equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase "in two or more variables" refers to formulas like the compound interest formula, in which $A = P(1 + r/n)^{nt}$ has multiple variables.)

MGSE9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

MGSE9-12.A.REI.11 Using graphs, tables, or successive approximations, show that the solution to the equation f(x) = g(x) is the x-value where the y-values of f(x) and g(x) are the same.

MGSE9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

MGSE9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

STANDARDS FOR MATHEMATICAL PRACTICE

2. Reason abstractly and quantitatively.
4. Model with mathematics.
Students will represent the height of the candle using algebra.
8. Look for and express regularity in repeated reasoning.
Students will use patterns to fill in the table.

Background Knowledge

- Students understand that linear equations have a constant slope.
- Students can represent constraints (domain) with inequalities.

Common Misconceptions

- Students may have difficulty graphing a decimal slope.
- Students may not recognize the dependent and independent variable.

Materials

- colored pencils
- graphing calculators
- graph paper

Grouping

- Individual / partners

Differentiation

Extension:

- Ask the students to determine if the relationship is continuous or discrete and to explain why.
  (The relationship is continuous because the candles continuously burn.)

Intervention:

- Give students a blank graph.

Formative Assessment Questions:

- How can we model real–life situations with tables, graphs, or equations?
- What limiting factors are present in this situation?

Cara's Candles Revisited – Teacher Notes

Cara likes candles. She also likes mathematics and was thinking about using algebra to answer a question that she had about two of her candles. Her taller candle is 16 centimeters tall. Each hour it burns makes the candle lose 2.5 centimeters in height. Her short candle is 12 centimeters tall and loses 1.5 centimeters in height for each hour that it burns.

Cara started filling out the following table to help determine whether these two candles would ever reach the same height at the same time if allowed to burn the same length of time. Finish the table for Cara. Use the data in the table to determine what time the two candles will be at the same height.

Also, she wants to know what height the two candles would be at that time. If it is not possible, she wants to know why it could not happen and what would need to be true in order for them to be able to reach the same height. To help Cara understand what you are doing, justify your results. You will explain your thinking using the table and create a graphical representation of the situation.

Time (hours)	16 cm candle height (cm)	12 cm candle height (cm)
0	16	12
1	13.5	10.5
2	***11***	***9***
3	***8.5***	***7.5***
4	***6***	***6***
5	***3.5***	***4.5***
6	***1***	***3***
7	***–1.5***	***1.5***

1. Complete the table, and use it to write an equation for the height of each candle in terms of the number of hours it has burned. Be sure to include any constraints for the equation.

Solution

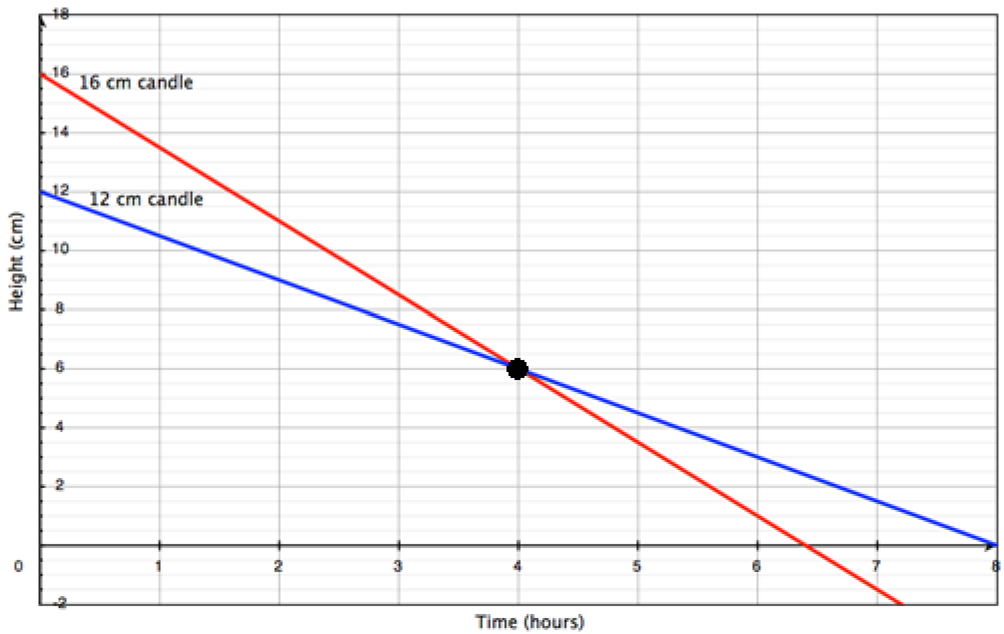
Students will use the table above to justify their solution. The candles will be the same height (6cm) in 4 hours.

Taller Candle
$y = -2.5x + 16;\ 0 \leq x \leq 6.4$ ***hours***

Shorter Candle
$y = -1.5x + 12;\ 0 \leq x \leq 8$ ***hours***

2. Create a graphical representation of your data, taking into account natural restrictions on domain, range, etc.

Solution:
Use the opportunity to bring out the concept of the natural restrictions. For instance, when x = 7 in the first function, the candle would have a negative height, which is impossible.



3. Cara has another candle that is 15 cm tall. How fast must it burn in order to also be 6 cm tall after 4 hours? Explain your thinking.

Solution:

The candle would need to lose 9 cm in four hours so it would have to burn at the rate of 2.25 cm per hour. The slope of its linear equation would be –2.25

4. If Cara had a candle that burned 3 cm every hour, how tall would it need to be to also reach the same height as the other three candles after 4 hours? Explain your thinking.

Solution:

The candle would have burned 12 cm in 4 hours. Its initial height would have been 18 cm tall. The y–intercept of its linear equation would be 18.

Cara's Candles Revisited (Scaffolding Task)

Name_________________________________ **Date**__________________

Mathematical Goals

- Determine whether a point is a solution to an equation.
- Determine whether a solution has meaning in a real–world context.
- Interpret whether the solution is viable from a given model.
- Write and graph equations and inequalities representing constraints in contextual situations.

Essential Questions

- How do I graph equations on coordinate axes with the correct labels and scales?
- How do I create equations in two or more variables to represent relationships between two quantities?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear~~, quadratic,~~ ~~simple rational,~~ ~~and exponential~~ functions~~ (integer inputs only).~~

MGSE9-12.A.CED.2 Create linear~~, quadratic, and exponential~~ equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase "in two or more variables" refers to formulas like the compound interest formula, in which $A = P(1 + r/n)^{nt}$ has multiple variables.)

MGSE9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

MGSE9-12.A.REI.11 Using graphs, tables, or successive approximations, show that the solution to the equation f(x) = g(x) is the x-value where the y-values of f(x) and g(x) are the same.

MGSE9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

MGSE9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

STANDARDS FOR MATHEMATICAL PRACTICE

2. Reason abstractly and quantitatively.
4. Model with mathematics.
8. Look for and express regularity in repeated reasoning.

Cara's Candles Revisited (Scaffolding Task)

Name_________________________________ **Date**__________________

Cara likes candles. She also likes mathematics and was thinking about using algebra to answer a question that she had about two of her candles. Her taller candle is 16 centimeters tall. Each hour it burns makes the candle lose 2.5 centimeters in height. Her short candle is 12 centimeters tall and loses 1.5 centimeters in height for each hour that it burns.

Cara started filling out the following table to help determine whether these two candles would ever reach the same height at the same time if allowed to burn the same length of time. Finish the table for Cara. Use the data in the table to determine what time the two candles will be at the same height.

Also, she wants to know what height the two candles would be at that time. If it is not possible, she wants to know why it could not happen and what would need to be true in order for them to be able to reach the same height. To help Cara understand what you are doing, justify your results. You will explain your thinking using the table and create a graphical representation of the situation.

Time (hours)	16 cm candle height (cm)	12 cm candle height (cm)
0	16	12
1	13.5	10.5
2		
3		
4		
5		
6		
7		

1. Complete the table, and use it to write an equation for the height of each candle in terms of the number of hours it has burned. Be sure to include any constraints for the equation.

2. Create a graphical representation of your data, taking into account natural restrictions on domain, range, etc.

3. Cara has another candle that is 15 cm tall. How fast must it burn in order to also be 6 cm tall after 4 hours? Explain your thinking.

4. If Cara had a candle that burned 3 cm every hour, how tall would it need to be to also reach the same height as the other three candles after 4 hours? Explain your thinking.

Best Buy Tickets (Short Cycle Task)

Back to Task Table

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=824

Task Comments and Introduction
Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *Best Buy Tickets*, is a Mathematics Assessment Project Assessment Task that can be found at the website:
http://www.map.mathshell.org/materials/tasks.php?taskid=286&subpage=expert
The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=824
The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=825

Mathematical Goals

- Students can use linear models to compare two purchasing options.

Essential Questions

- How can I use linear models to decide which of the two payment models is cheaper?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.CED.2 Create linear~~, quadratic, and exponential~~ equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase "in two or more variables" refers to formulas like the compound interest formula, in which $A = P(1 + r/n)^{nt}$ has multiple variables.)

MGSE9-12.F.BF.1 Write a function that describes a relationship between two quantities.

MGSE9-12.F.BF.1a Determine an explicit expression and the recursive process (steps for calculation) from context. *For example, if Jimmy starts out with $15 and earns $2 a day, the explicit expression "2x+15" can be described recursively (either in writing or verbally) as "to find out how much money Jimmy will have tomorrow, you add $2 to his total today."* $J_n = J_{n-1} + 2, J_0 = 15$

MGSE9-12.F.BF.2 Write arithmetic ~~and geometric~~ sequences recursively and explicitly, use them to model situations, and translate between the two forms. Connect arithmetic sequences to linear functions ~~and geometric sequences to exponential functions.~~

MGSE9-12.A.REI.11 Using graphs, tables, or successive approximations, show that the solution to the equation f(x) = g(x) is the x-value where the y-values of f(x) and g(x) are the same.

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable including equations with coefficients represented by letters. *For example, given ax + 3 = 7, solve for x.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

Background Knowledge

- Students understand the meaning of slope and *y*-intercept in context when writing linear equations.
- Students may need to know how to solve a system of linear equations, depending on the solution path they follow.

Common Misconceptions

- Students may confuse the slope and the *y*-intercept of a linear equation.
- Students may fail to realize that the answer to the "which is the better buy" question depends on the number of people who attend.

Materials

- see FAL website

Grouping

- Individual / partners

Solving Systems of Equations Algebraically (Scaffolding Task) Back to Task Table

Comment:

This task is written as self–guided instruction. Its questions are very similar to the questions a teacher would ask during direct instruction. It would best be used as a whole class task where teachers guide the students through the task. Students with higher level abilities could proceed ahead of the class.

Introduction

In this task, students justify the solution to a system of equations by both graphing and substituting values into the system. Students will then show that multiplying one or both equations in a system of equations by a constant creates a new system with the same solutions as the original. This task will lead into using the elimination method for solving a system of equations algebraically.

Mathematical Goals

- Model and write an equation in one variable and solve a problem in context.
- Create one–variable linear equations and inequalities from contextual situations.
- Represent constraints with inequalities.
- Solve word problems where quantities are given in different units that must be converted to understand the problem.

Essential Questions

- How do I solve an equation in one variable?
- How do I justify the solution to an equation?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.REI.5 Show and explain why the elimination method works to solve a system of two-variable equations.

MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
Students will have to decipher the instructions and complete the described steps.

3. Construct viable arguments and critique the reasoning of others.
Students will have to justify the steps in the elimination process.

Background Knowledge
- Students can solve equations.
- Students can combine like terms.
- Students can distribute multiplication over addition.

Common Misconceptions
- Students may forget to multiply both sides of the equation to preserve equality.
- Students may have difficulty following the written instructions.

Materials
- Ruler
- Calculator

Grouping
- Individual / Partners/Whole class

Differentiation

Extension:
- Have students work through the task individually.

Intervention:
- Have students work through the task as a class.

Formative Assessment Questions
- Why does multiplying an equation by a constant not affect the solution(s) of the equation?
- What are the different pathways to solve a system of equations using the elimination method?
- What determines the pathway needed?

<u>Solving Systems of Equations Algebraically – Teacher Notes</u>

<u>Comment:</u>
As students solve equations throughout this task, have them continue to explain each step using properties of operations or properties of equality.

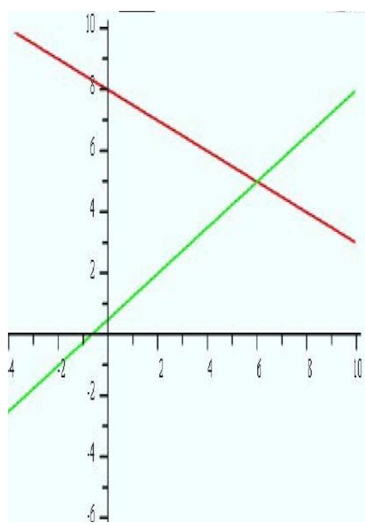
Part 1:

You are given the following system of two equations: $x + 2y = 16$
$3x - 4y = -2$

1. What are some ways to prove that the ordered pair (6, 5) is a solution?

   Graphing and direct substitution are two methods for proving that (6,5) is a solution.

   a. Prove that (6, 5) is a solution to the system by graphing the system.



   b. Prove that (6, 5) is a solution to the system by substituting in for both equations.

x + 2y = 16	***3x – 4y = –2***
6 + 2*5 = 16	***3*6 – 4*5 = –2***
6 + 10 = 16	***18 – 20 = –2***
16 = 16	***–2 = –2***

   The solution (6, 5) works for both equations.

2. Multiply both sides of the equation $\boldsymbol{x + 2y = 16}$ by the constant '7'. Show your work.

7*(x + 2y) = **7***16
7*x + 7 * 2y = 112

7x + 14y = 112 New Equation

a. Does the new equation still have a solution of (6, 5)? Justify your answer.

7x + 14y = 112
7*6 + 14*5 = 112
42 + 70 = 112

b. Why do you think the solution to the equation never changed when you multiplied by the '7'?

Answers may vary.

3. Did it have to be a '7' that we multiplied by in order for (6, 5) to be a solution?

Answers may vary.

a. Multiply ***x* + 2*y* = 16** by three other numbers and see if (6, 5) is still a solution.

Students may pick any constant to multiply the original equation by. As long as the multiplication is correct, the solution (6, 5) will still work.

i. __________________________
ii. __________________________
iii. __________________________

b. Did it have to be the first equation ***x* + 2*y* = 16** that we multiplied by the constant for (6, 5) to be a solution? Multiply **3*x* – 4*y* = –2** by '7'? Is (6, 5) still a solution?
Use this exercise to help students discover that multiplying the equation by any constant will not change the solution.

7(3x – 4y) = 7*–2
21x – 28y = –14

21*6 – 28 *5 = –14
126 – 140 = –14
–14 = –14

c. Multiply **3*x* – 4*y* = –2** by three other numbers and see if (6, 5) is still a solution.

i. ________________________
ii. ________________________
iii. ________________________

4. Summarize your findings from this activity so far. Consider the following questions:
   What is the solution to a system of equations and how can you prove it is the solution?
   Does the solution change when you multiply one of the equations by a constant?
   Does the value of the constant you multiply by matter?
   Does it matter which equation you multiply by the constant?

   Answers will vary, but the purpose of this task is for students to discover that multiplying an equation by a constant does not change the solution to the equation, leading into the elimination method for solving a system of equations.

Let's explore further with a new system. $5x + 6y = 9$
$4x + 3y = 0$

5. Show by substituting in the values that (–3, 4) is the solution to the system.

$5x + 6y = 9$	$4x + 3y = 0$
$5(-3) + 6(4) = 9$	$4(-3) + 3(4) = 0$
$-15 + 24 = 9$	$-12 + 12 = 0$
$9 = 9$	$0 = 0$

6. Multiply **4*x* + 3*y* = 0** by '–5'. Then add your answer to **5*x* + 6*y* = 9**. Show your work below.

   $(-5)*(4x + 3y) = (-5)*0$ → $\underline{\quad -20x - 15y = 0 \quad}$ Answer

   $+$ $\underline{\quad 5x + 6y = 9 \quad}$

   $\underline{\quad -15x - 9y = 9 \quad}$ New Equation

7. Is (–3, 4) still a solution to the new equation? Justify your answer.

   $-15x - 9y = 9$
   $-15(-3) - 9(4) = 9$
   $45 - 36 = 9$

8. Now multiply $4x + 3y = 0$ by '–2'. Then add your answer to **5*x* + 6*y* = 9**. Show your work below.

   $-8x - 6y = 0$

$$\underline{+\,5x + 6y = 9}$$
$$-3x + 0y = 9$$
$$-3x = 9$$
$$x = 9/(-3)$$
$$x = -3$$

a. What happened to the *y* variable in the new equation?

It canceled out (became 0y), therefore being eliminated from the equation.

b. Can you solve the new equation for *x*? What is the value of *x*? Does this answer agree with the original solution?

See work above.
The original solution was (–3, 4), so a value of x = –3 does agree.

c. How could you use the value of *x* to find the value of *y* from one of the original equations? Show your work below.

Substitute the value of x into one of the equations to find the value of 'y'.

$$5x + 6y = 9$$
$$5(3) + 6y = 9$$
$$-15 + 6y = 9$$
$$6y = 9 + 15$$
$$6y = 24$$
$$y = 24/6$$
$$y = 4$$

The method you have just used is called the Elimination Method for solving a system of equations. When using the Elimination Method, one of the original variables is eliminated from the system by adding the two equations together. Use the Elimination Method to solve the following system of equations:

9. $-3x + 2y = -6$
$5x - 2y = 18$

$$-3x + 2y = -6$$
$$\underline{5x - 2y = 18}$$
$$2x + 0y = 12$$
$$2x = 12$$
$$x = 6$$

10. $-5x + 7y = 11$
$5x + 3y = 19$

$$-5x + 7y = 11$$
$$\underline{5x + 3y = 19}$$
$$0x + 10y = 30$$
$$10y = 30$$
$$y = 3$$

$5(6) - 2y = 18$
$30 - 2y = 18$
$-2y = -12$
$y = 6$
Solution :
(6, 6)

$-5x + 7(3) = 11$
$-5x + 21 = 11$
$-5x = -10$
$x = 2$
(2, 3)

Check:
$-3(6) + 2(6) = -6$
$5(6) - 2(6) = 18$

$-5(2) + 7(3) = 11$
$5(2) + 3(3) = 19$

Part 2:

When using the Elimination Method, one of the original variables is eliminated from the system by adding the two equations together. Sometimes it is necessary to multiply one or both of the original equations by a constant. The equations are then added together and one of the variables is eliminated. Use the Elimination Method to solve the following system of equations:

1. $4x + 3y = 14$ (Equation 1)
   $-2x + y = 8$ (Equation 2)

Choose the variable you want to eliminate.

a. To make the choice, look at the coefficients of the *x* terms and the *y* terms. The coefficients of *x* are '4' and '–2'. If you want to eliminate the *x* variable, you should multiply Equation 2 by what constant?

Multiply the 2nd equation by the constant '2'.

i. Multiply Equation 2 by this constant. Then add your answer equation to Equation 1. What happened to the *x* variable?

$2(-2x + y) = 2(8)$
$-4x + 2y = 16$

$-4x + 2y = 16$ (New Equation 2)
$\underline{+\ 4x + 3y = 14}$ (Equation 1)
$0x + 5y = 30$
$5y = 30$
$y = 6$

The x variable was eliminated.

ii. Solve the equation for y. What value did you get for y?

See above.

iii. Now substitute this value for y in Equation 1 and solve for x. What is your ordered pair solution for the system?

$x + 3y = 14$
$4x + 3(6) = 14$
$4x + 18 = 14$
$4x = -4$
$x = -1$

iv. Substitute your solution into Equation 1 and Equation 2 to verify that it is the solution for the system.

Solution: (–1, 6)

$4x + 3y = 14$ $\qquad$ $4(-1) + 3(6) = 14$
$-2x + y = 8$ $\qquad$ $-2(-1) + 6 = 8$

b. The coefficients of y are '3' and '1'. If you want to eliminate the y term, you should multiply Equation 2 by what constant?

Multiply Equation 2 by the constant (–3).

i. Multiply Equation 2 by this constant. Then add your answer equation to Equation 1. What happened to the y variable?

$(-3)(-2x + y) = (-3)8$
$6x - 3y = -24$ ***(New Equation 2)***

$$\begin{array}{r} 6x - 3y = -24 \\ \underline{+\ 4x + 3y = 14} \\ 10x + 0y = -10 \\ 10x = -10 \\ x = -1 \end{array}$$

ii. Solve the equation for x. What value did you get for x?

See above.

iii. Now substitute this value for x in Equation 1 and solve for y. What is your ordered pair solution for the system?

$4x + 3y = 14$

$$4(-1) + 3y = 14$$
$$3y = 18$$
$$y = 6$$

Solution: (–1, 6)

Use your findings to answer the following in sentence form:

c. Is the ordered pair solution the same for either variable that is eliminated? Justify your answer.

Answers may vary, but students should realize that the solution is the same for either variable that is eliminated.

d. Would you need to eliminate both variables to solve the problem? Justify your answer.

Answers may vary, but since either elimination yields the same answer, there is no need to eliminate both ways.

e. What are some things you should consider when deciding which variable to eliminate? Is there a wrong variable to eliminate?

Answers may vary, but there is no wrong variable to eliminate. Hopefully students have discovered that considering the coefficients of each variable will sometimes lessen the work involved in eliminating a particular variable.

f. How do you decide what constant to multiply by in order to make the chosen variable eliminate?

Answers may vary, but once the variable to be eliminated is chosen, the coefficients of that variable must be opposites so that the variable will eliminate.

Use the elimination method to solve the following systems of equations. Verify your solution by substituting it into the original system.

2. $3x + 2y = 6$
   $-6x - 3y = -6$

   (–2, 6)

3. $-6x + 5y = 4$
   $7x - 10y = -8$

   (0, 4/5)

4. $5x + 6y = -16$
   $2x + 10y = 5$

   (–5, 3/2)

Solving Systems of Equations Algebraically (Scaffolding Task)

Name________________________________ **Date**_________________

Introduction

In this task, students justify the solution to a system of equations by both graphing and substituting values into the system. Students will then show that multiplying one or both equations in a system of equations by a constant creates a new system with the same solutions as the original. This task will lead into using the elimination method for solving a system of equations algebraically.

Mathematical Goals

- Model and write an equation in one variable and solve a problem in context.
- Create one–variable linear equations and inequalities from contextual situations.
- Represent constraints with inequalities.
- Solve word problems where quantities are given in different units that must be converted to understand the problem.

Essential Questions

- How do I solve an equation in one variable?
- How do I justify the solution to an equation?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.REI.5 Show and explain why the elimination method works to solve a system of two-variable equations.

MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.

Solving Systems of Equations Algebraically (Scaffolding Task)

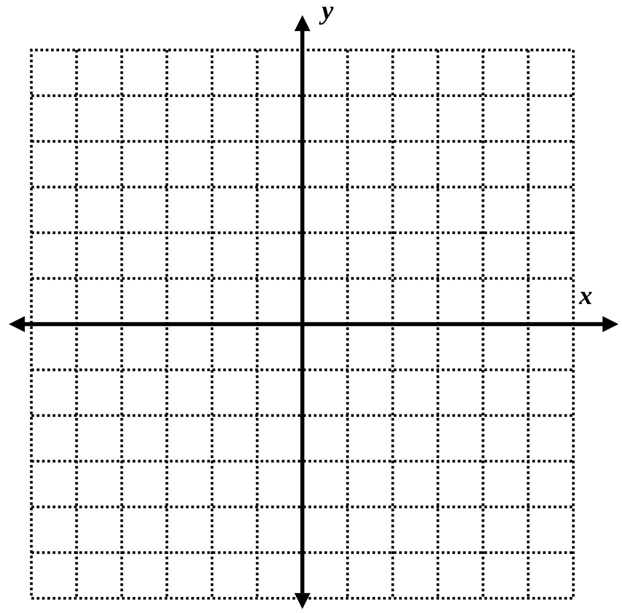
Name__________________________________ **Date**___________________

Part 1:

You are given the following system of two equations: $x + 2y = 16$
$3x - 4y = -2$

1. What are some ways to prove that the ordered pair (6, 5) is a solution?

   a. Prove that (6, 5) is a solution to the system by graphing the system.



   b. Prove that (6, 5) is a solution to the system by substituting in for both equations.

2. Multiply both sides of the equation $\boldsymbol{x + 2y = 16}$ by the constant '7'. Show your work.

   $\mathbf{7}*(x + 2y) = \mathbf{7}*16$

   ________________ New Equation

a. Does the new equation still have a solution of (6, 5)? Justify your answer.

b. Why do you think the solution to the equation never changed when you multiplied by the '7'?

3. Did it have to be a '7' that we multiplied by in order for (6, 5) to be a solution?

a. Multiply $\boldsymbol{x + 2y = 16}$ by three other numbers and see if (6, 5) is still a solution.

i. ________________________

ii. ________________________

iii. ________________________

b. Did it have to be the first equation $\boldsymbol{x + 2y = 16}$ that we multiplied by the constant for (6, 5) to be a solution? Multiply $\boldsymbol{3x - 4y = -2}$ by '7'? Is (6, 5) still a solution?

c. Multiply $\boldsymbol{3x - 4y = -2}$ by three other numbers and see if (6, 5) is still a solution.

i. ________________________

ii. ________________________

iii. ________________________

4. Summarize your findings from this activity so far. Consider the following questions: What is the solution to a system of equations and how can you prove it is the solution? Does the solution change when you multiply one of the equations by a constant?

Does the value of the constant you multiply by matter?
Does it matter which equation you multiply by the constant?

Let's explore further with a new system. $5x + 6y = 9$
$4x + 3y = 0$

5. Show by substituting in the values that (–3, 4) is the solution to the system.

6. Multiply $\mathbf{4x + 3y = 0}$ by '–5'. Then add your answer to $\mathbf{5x + 6y = 9}$. Show your work below.

$(–5)*(4x + 3y) = (–5)*0 \rightarrow$ ____________________ Answer

$+$ ______$5x + 6y = 9$______

____________________ New Equation

7. Is (–3, 4) still a solution to the new equation? Justify your answer.

8. Now multiply $4x + 3y = 0$ by '–2'. Then add your answer to $\mathbf{5x + 6y = 9}$. Show your work below.

   a. What happened to the *y* variable in the new equation?

b. Can you solve the new equation for x? What is the value of x? Does this answer agree with the original solution?

c. How could you use the value of x to find the value of y from one of the original equations? Show your work below.

The method you have just used is called the Elimination Method for solving a system of equations. When using the Elimination Method, one of the original variables is eliminated from the system by adding the two equations together. Use the Elimination Method to solve the following system of equations:

9. $-3x + 2y = -6$
$5x - 2y = 18$

10. $-5x + 7y = 11$
$5x + 3y = 19$

Part 2:

When using the Elimination Method, one of the original variables is eliminated from the system by adding the two equations together. Sometimes it is necessary to multiply one or both of the original equations by a constant. The equations are then added together and one of the variables is eliminated. Use the Elimination Method to solve the following system of equations:

1. $4x + 3y = 14$ (Equation 1)
   $-2x + y = 8$ (Equation 2)

Choose the variable you want to eliminate.

a. To make the choice, look at the coefficients of the x terms and the y terms. The coefficients of x are '4' and '–2'. If you want to eliminate the x variable, you should multiply Equation 2 by what constant?

i. Multiply Equation 2 by this constant. Then add your answer equation to Equation 1. What happened to the x variable?

ii. Solve the equation for y. What value did you get for y?

iii. Now substitute this value for y in Equation 1 and solve for x. What is your ordered pair solution for the system?

iv. Substitute your solution into Equation 1 and Equation 2 to verify that it is the solution for the system.

b. The coefficients of y are '3' and '1'. If you want to eliminate the y term, you should multiply Equation 2 by what constant?

   i. Multiply Equation 2 by this constant. Then add your answer equation to Equation 1. What happened to the y variable?

   ii. Solve the equation for x. What value did you get for x?

   iii. Now substitute this value for x in Equation 1 and solve for y. What is your ordered pair solution for the system?

Use your findings to answer the following in sentence form:

c. Is the ordered pair solution the same for either variable that is eliminated? Justify your answer.

d. Would you need to eliminate both variables to solve the problem? Justify your answer.

e. What are some things you should consider when deciding which variable to eliminate? Is there a wrong variable to eliminate?

f. How do you decide what constant to multiply by in order to make the chosen variable eliminate?

Use the elimination method to solve the following systems of equations. Verify your solution by substituting it into the original system.

2. $3x + 2y = 6$
   $-6x - 3y = -6$

3. $-6x + 5y = 4$
   $7x - 10y = -8$

4. $5x + 6y = -16$
   $2x + 10y = 5$

Dental Impressions (Career and Technical Education Task) Back to Task Table

Source: National Association of State Directors of Career Technical Education Consortium
PDF: **http://www.achieve.org/files/CCSS–CTE–Task–Dental–Impressions–FINAL.pdf**
Word: **http://www.achieve.org/files/CCSS–CTE–Task–Dental–Impressions–FINAL.doc**

Introduction

Students explore the supply needs of a dentist's office, determining plans for ordering materials. Students also use linear equations to determine the "break–even point" of two alternate plans.

Mathematical Goals

- Use units to plan and implement a solution strategy.
- Write linear equations and interpret their intersection as the "break–even point."

Essential Questions

- How can I use units and linear equations to answer questions about real–world situations?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear~~, quadratic,~~ ~~simple rational,~~ ~~and exponential~~ functions ~~(integer inputs only).~~

MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

RELATED STANDARDS

MGSE9–12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
b. Convert units and rates using dimensional analysis (English–to–English and Metric–to–Metric without conversion factor provided and between English and Metric with conversion factor);
c. Use units within multi–step problems and formulas; interpret units of input and resulting units of output.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
Students analyze givens, constraints, relationships, and goals. They must make conjectures about the form and meaning of the solution and plan a solution pathway.
2. Reason abstractly and quantitatively.
Students make sense of quantities and their relationship in the problem situation.
3. Construct viable arguments and critique the reasoning of others.
Students are asked for a written recommendation based on their mathematical findings.

4. Model with mathematics.
Students translate constraints into equations and extract information from both the algebraic solution and the graph.

6. Attend to precision.
Students must use units of measure, convert units, and perform calculations precisely.

Background Knowledge

- Students can use unit analysis to plan an approach multi–step problems.
- Students can convert units.
- Students understand the slope of a line as a rate of change and the *y*–intercept as an initial value.

Common Misconceptions

- In #3, students may round to the nearest whole number, 17, rather than rounding up to ensure they have enough gypsum for the last few impressions.
- In #4, students can show that the technology will be cheaper after two years simply by finding the cost for each after two years. Emphasize the instructions to "determine your break–even point" so students determine when they break even as opposed to the yes–no question of whether they break even within two years.
- When creating equations, students may confuse the slope (per–year rate) and the *y*–intercept (initial investment).

Materials

- None

Grouping

- Individual / partner

Differentiation

- See extensions in task.

Ground Beef (Career and Technical Education Task) [Back to Task Table]

Source: National Association of State Directors of Career Technical Education Consortium
PDF: **http://www.achieve.org/files/CCSS–CTE–Ground–Beef–FINAL.pdf**
Word: **http://www.achieve.org/files/CCSS–CTE–Ground–Beef–FINAL.docx**

Introduction

Students use systems of equations to model mixture problems relating to a grocer's need to mix different formulations of ground beef.

Mathematical Goals

- Model and solve mixture problems using systems of equations.
- Calculate and compare profits.

Essential Questions

- How can I use systems of equations to model and solve real–world mixture problems?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

MGSE9-12.A.CED.2 Create linear~~, quadratic, and exponential~~ equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase "in two or more variables" refers to formulas like the compound interest formula, in which $A = P(1 + r/n)^{nt}$ has multiple variables.)

RELATED STANDARDS

MGSE9–12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
b. Convert units and rates using dimensional analysis (English–to–English and Metric–to–Metric without conversion factor provided and between English and Metric with conversion factor);
c. Use units within multi–step problems and formulas; interpret units of input and resulting units of output.

MGSE9-12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers' precision is limited to the precision of the data given.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
Students analyze givens, constraints, relationships, and goals. They must make conjectures about the form and meaning of the solution and plan a solution pathway.

2. Reason abstractly and quantitatively.
Students make sense of quantities and their relationships throughout the problem.

4. Model with mathematics.
Students translate constraints into a system of equations and use them to calculate the amounts of various types of meat needed.

6. Attend to precision.
Students must be precise in establishing their equations and in performing calculations and they round solutions to appropriately represent money or decimal measures of weight.

Background Knowledge

- Students can set up and solve systems of linear equations in two variables.
- Students understand profit and percentages.

Common Misconceptions

- Students may need clarification to understand that different mixtures of boneless round and lean trim beef are used to create the three types of beef listed at the beginning of the task.
- Students may look at "per–pound" profit instead of overall profit in #4–5.

Materials

- None

Grouping

- Partner / small group

Differentiation

- See extensions in task.

Systems of Weights (Spotlight Task)

Back to Task Table

This spotlight task follows the 3 Act–Math task format originally developed by Dan Meyer. More information on these type tasks may be found at http://blog.mrmeyer.com/category/3acts/

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

MGSE9-12.A.REI.12 Graph the solution set to a linear inequality in two variables.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. In this task, students will formulate their own problem to solve. Students will work to develop a method to solve the systems of equations.

4. Model with Mathematics. In this task, students will use equations to model the weights of different combinations of toys. Students will use mathematics to model equations and inequalities

5. Use appropriate tools strategically. Students will need to select appropriate tools (graph paper, calculator, etc) in order to be successful at this task.

Essential Questions

- How do I model a situation involving two unknown quantities?
- How do I solve a system of linear equations?

Materials Required

- Video "Systems of Weights" http://real.doe.k12.ga.us/vod/gso/math/videos/Coordinate–Algebra–Unit–2–Spotlight–opening.mp4
- Video "Systems of Weights Revealed" http://real.doe.k12.ga.us/vod/gso/math/videos/Coordinate–Algebra–Unit–2–Spotlight–reveal.mp4
- Graph paper (optional)
- Other Materials for a sequel could include a scale and other objects to weigh

It should also be noted that there is no "student version" of this task, as it involves showing a short video and formulating and solving a problem.

Time Needed

- 30–45 minutes

It is recommended that this task be done towards the beginning of lessons involving systems of linear equations and linear inequalities

More information along with guidelines for 3–Act Tasks may be found in the Comprehensive Course Guide.

Act One Description (Dan Meyer http://blog.mrmeyer.com/2011/the–three–acts–of–a–mathematical–story/**)**
"Introduce the central conflict of your story/task clearly, visually, viscerally, using as few words as possible."

Act One:
Show the video "Systems of Weights".
http://real.doe.k12.ga.us/vod/gso/math/videos/Coordinate–Algebra–Unit–2–Spotlight–opening.mp4

Pose the question: "How much does each type of candy weigh?"

This should launch the students into an investigation including setting up and solving a system of equations. Coordinate Algebra students will have experienced systems of equations (simultaneous equations) in 8^{th} grade. Encourage students to develop methods to solve the system if they are unable to recall specific methods (substitution, elimination, graphing) from middle school.

Students could be grouped in pairs or work individually, according to the teacher's discretion.

Encourage students to be clear in defining their variables.

Sample Answer:

Let x be the weight in grams of one chocolate candy
Let y be the weight in grams of one peanut butter cup
Then the system of equations based on the video will be as follows:

$$4x + 5y = 63.8$$
$$6x + 6y = 82.3$$

There are several ways to solve this system, including substitution, elimination or graphing. As a part of the wrap up discussion, you may choose to have students discuss the merits of using the different methods.

Act Two Description (Dan Meyer http://blog.mrmeyer.com/2011/the–three–acts–of–a–mathematical–story/**)**
The protagonist/student overcomes obstacles, looks for resources, and develops new tools.
During Act Two, students will discuss the question in Act One and decide on the facts that are needed to answer the question. Students will also look for formulas and conversions that are needed to solve the problem. When students decide what they need to solve the problem, they should ask for the facts or use technology to find them. To solve this problem, they will need to make the assumption that all chocolate candies of this type weigh the same and all peanut butter cups of this type weigh the same.

Note: It is pivotal to the problem solving process that students decide what is needed without being given the information up front. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin.

Here are some questions to help guide the discussion, but be sure not to give too much away. The goal is to have students formulate the questions and the methods to answer them.

- How is solving a system of equations different from solving one equation with one variable? How is it similar?
- How can you use both equations together to solve the system?

A sample solution is below:

Elimination Method:

$4x+5y=63.8$	$3(4x+5y=63.8)$	$12x+15y=191.4$
$6x+6y=82.3$	$-2(6x+6y=82.3)$	$-12x-12y=-164.6$
$3y=26.8$	$y=8.93333$	$4x+5(8.93333)=63.8$
$4x=19.13333$	$x=4.7833$	

Each chocolate candy weighs approximately 4.7833 grams and each peanut butter cup weighs approximately 8.933 grams.

ACT 3

Students will compare and share solution strategies.

- Reveal the answer. http://real.doe.k12.ga.us/vod/gso/math/videos/Coordinate–Algebra–Unit–2–Spotlight–reveal.mp4
- Discuss the theoretical math versus the practical outcome.
- Share student solution paths. Start with most common strategy.

The teacher also needs to be flexible and adapt the lesson to the curiosity of the class. Use this activity as a guide, but do not be afraid to deviate from it if the mathematics dictates that you do so.

The weights in the video do not match the weights from the equations. (The video shows that a chocolate candy weighs 4.9 grams and the peanut butter cup weighs 8.5 grams) Ask students to think about why this is. (As it turns out, not every chocolate candy is the same weight, and not every peanut butter cup is the same weight.) This can be an excellent discussion about theoretical versus practical outcomes.

Following this lesson, students will need the opportunity to practice setting up and solving systems of equations through the different methods.

The Sequel: "The goals of the sequel task are to a) challenge students who finished quickly so b) I can help students who need my help. It can't feel like punishment for good work. It can't seem like drudgery. It has to entice and activate the imagination." Dan Meyer http://blog.mrmeyer.com/2013/teaching–with–three–act–tasks–act–three–sequel/

For a sequel, present this situation: Hershey's and Reese's are teaming up to create a mixed bag of candies. The packaging can hold no more than 40 ounces of candy. In order to satisfy both chocolate and peanut butter lovers, the ratio of Hershey's to Reese's should be no more than 3 Hershey Kisses for every 2 Reese's Miniatures. How many of each should be included in the packaging. Justify your answer.

Another possible sequel could involve students weighing other objects and making their own systems of equations problems.

Solving Linear Equations in 2 Variables (Formative Assessment Lesson)

Back to Task Table

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=669

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Solving Linear Equations in 2 Variables*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=209&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=669

Mathematical Goals

- Solving a problem using two linear equations with two variables.
- Interpreting the meaning of algebraic expressions.

Essential Questions

- Can I solve systems of equations using various methods: graphing, elimination, and substitution?
- What do the points on a line represent in relation to the situation they model?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.REI.5 Show and explain why the elimination method works to solve a system of two-variable equations.

MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

STANDARDS FOR MATHEMATICAL PRACTICE

2. Reason abstractly and quantitatively.
Students will interpret and compare various methods of solving the same system of equations.

3. Construct viable arguments and critique the reasoning of others.
Students will perform multiple error analyses and describe the patterns they see in student work.

Background Knowledge

- Students understand how to interpret parts of equations & expressions in relation to real life situations.
- Students understand use of variables in modeling real life situations.

Common Misconceptions

- Student assumes that the letter stands for an object not a number
- Student produces unsystematic guess and check work

Materials

- See FAL website.

Grouping

- Individual/Small group

Boomerangs (Formative Assessment Lesson) - Extension

Back to Task Table

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1241

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website: http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Optimizations Problems: Boomerangs*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=207&subpage=problem

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1241

Mathematical Goals

- Interpret a situation and represent the constraints and variables mathematically.
- Select appropriate mathematical methods to use.
- Explore the effects of systematically varying the constraints.
- Interpret and evaluate the data generated and identify and confirm the optimum case.

Essential Questions

- How can I create a table, graph, or equation to represent a given scenario?
- How do I interpret systems of equations and their point of intersection in context?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.REI.5 Show and explain why the elimination method works to solve a system of two-variable equations.

MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

MGSE9-12.A.REI.12 Graph the solution set to a linear inequality in two variables.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
   Students must work on an extended problem.
2. Reason abstractly and quantitatively
   Students must work with a real life scenario and its numerical, graphical, and algebraic representations.
3. Construct viable arguments and critique the reasoning of others.
   Students must analyze student work, identifying different approaches to the same problem.
4. Model with mathematics.
   Students model real life scenarios using equations.

Background Knowledge

- Students should know how to graph linear equations.
- Students should know how to create equations in two variables given a situation.

Common Misconceptions

- Students may make an incorrect interpretation of the constraints and variables.
- Student may have technical difficulties when using graphs.
- The student may present the work as a series of unexplained numbers and/or calculations, or as a table without headings.

Materials

- See FAL website.

Grouping

- Individual/Small group

Summer Job (Scaffolding Task)

Back to Task Table

Introduction

In this task, students will write a model for an inequality from the context of a word problem using real life situations. The students will then graph the inequality in two variables and analyze the solution. Students will reason quantitatively and use units to solve problems.

Mathematical Goals

- Model and write an inequality in two variables and solve a problem in context.
- Create two–variable linear equations and inequalities from contextual situations.
- Solve word problems involving inequalities.
- Represent constraints with inequalities.

Essential Questions

- How do I graph a linear inequality in two variables?
- How do I justify a solution to an equation?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9–12.A.REI.12 Graph the solution set to a linear inequality in two variables.

STANDARDS FOR MATHEMATICAL PRACTICE

2. Reason abstractly and quantitatively.
4. Model with mathematics.
Students will model their income from their summer job using linear equations and inequalities.

Background Knowledge

- Students can graph linear equations.
- Students can read and interpret graphs.
- Students can graph inequalities in two variables.

Common Misconceptions

- Students may be confused about the scale of the graphs and how to graph the solution.
- Students may question how to graph the constraints of the problem in terms of the appropriate quadrant.

Materials

- Graph paper
- Ruler
- Colored Pencils
- Graphing Calculator (optional)

Grouping

- Individual/Partner/Small Group

Differentiation

Intervention:

- Partner struggling students with resident experts.

Formative Assessment Questions

- How do we display linear equations differently from linear inequalities? Why is it necessary to do so?
- How do you know when a real life situation should be modeled with a linear inequality and not a linear equation?

Summer Job – Teacher Notes

In order to raise money, you are planning to work during the summer babysitting and cleaning houses. You earn \$10 per hour while babysitting and \$20 per hour while cleaning houses. You need to earn at least \$1000 during the summer.

1. Write an expression to represent the amount of money earned while babysitting. Be sure to choose a variable to represent the number of hours spent babysitting.

   Let b represent the number of hours spent babysitting
   \$10b represents the amount of money earned while babysitting

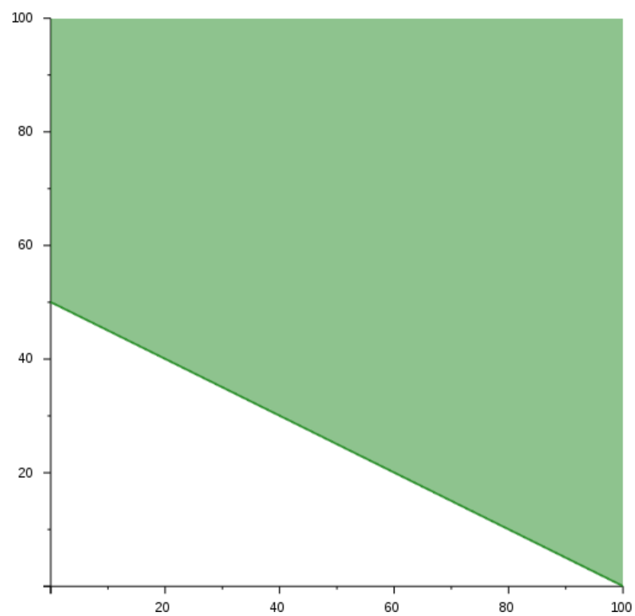
2. Write an expression to represent the amount of money earned while cleaning houses.

   Let c represent the number of hours spent cleaning
   \$20c represents the amount of money earned while cleaning houses.

3. Write a mathematical model (inequality) representing the total amount of money earned over the summer from babysitting and cleaning houses.

   $\$10b + \$20c \geq \$1000$

4. Graph the mathematical model. Graph the hours babysitting on the *x*–axis and the hours cleaning houses on the *y*–axis.



5. Use the graph to answer the following:

a. Why does the graph only fall in the 1st Quadrant?

Neither the hours spent babysitting nor the hours cleaning houses can be negative.

b. Is it acceptable to earn exactly \$1000? What are some possible combinations of outcomes that equal exactly \$1000? Where do all of the outcomes that total \$1000 lie on the graph?

Yes, it is possible to earn exactly \$1000. Some possibilities include (100, 0), (20, 40), and (80, 10), but answers will vary. All of the outcomes totaling exactly \$1000 lie on the line.

c. Is it acceptable to earn more than \$1000? What are some possible combinations of outcomes that total more than \$1000? Where do all of these outcomes fall on the graph?

Yes, it is acceptable to earn more than \$1000. Some possibilities are (10, 60), and (70, 70), but answers will vary. All of the outcomes totaling more than \$1000 are above the line.

d. Is it acceptable to work 10 hours babysitting and 10 hours cleaning houses? Why or why not? Where does the combination of 10 hours babysitting and 10 hours cleaning houses fall on the graph? Are combinations that fall in this area a solution to the mathematical model? Why or why not?

No it is not acceptable to work 10 hours babysitting and 10 hours cleaning houses. This combination would result in earnings of only \$300 for the summer (10*10 + 20*10). Since you needed \$1000 this is not acceptable. This combination falls below the line. Any combination that falls in the area below the line is not a solution because it would result in earnings less than \$1000.

6. How would the model change if you could only earn more than \$1000? Write a new model to represent needing to earn more than \$1000. How would this change the graph of the model? Would the line still be part of the solution? How would you change the line to show this? Graph the new model.

New model: $\$10x + \$20y > \$1000$

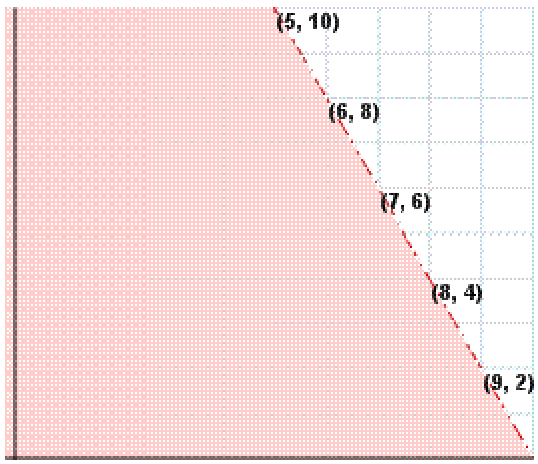
The line on the graph would no longer be part of the solution, therefore it would be broken and not solid.

You plan to use part of the money you earned from your summer job to buy jeans and shirts for school. Jeans cost $40 per pair and shirts are $20 each. You want to spend less than $400 of your money on these items.

7. Write a mathematical model representing the amount of money spent on jeans and shirts.

   $40j + $20s < $400

8. Graph the mathematical model. Graph the number of jeans on the *x*–axis and shirts on the *y*–axis.



   a. Why does the graph only fall in the 1st Quadrant?

      Neither he number of pairs of jeans nor the number of shirts purchased can be negative.

   b. Is it acceptable to spend less than $400? What are some possible combinations of outcomes that total less than $400? Where do all of these outcomes fall on the graph?

      It is acceptable to spend less than $400. All of the possible combinations totaling less than $400 fall below the line.

   c. Is it acceptable to spend exactly $400? How does the graph show this?

It is not acceptable to spend exactly $400, therefore the line is broken.

d. Is it acceptable to spend more than $400? Where do all of the combinations that total more than $400 fall on the graph?

It is not acceptable to spend more than $400. All of the combinations totaling more than $400 are above the line on the graph.

Summarize your knowledge of graphing inequalities in two variables by answering the following questions in sentence form:

Answers to these questions will vary, but should demonstrate student understanding of the reasoning behind graphing inequalities.

9. Explain the difference between a solid line and a broken line when graphing inequalities. How can you determine from the model whether the line will be solid or broken? How can you look at the graph and know if the line is part of the solution?

Answers will vary, but a solid line indicates these combinations are part of the solution and the inequality contains an equal sign. A broken line indicates the line is not part of the solution and the inequality does not contain an equal sign.

10. How do you determine which area of the graph of an inequality to shade? What is special about the shaded area of an inequality? What is special about the area that is not shaded?

The area that contains solutions to the inequality is shaded. The area that is not shaded does not contain solutions to the inequality.

Summer Job (Scaffolding Task)

Name__________________________________ **Date**___________________

Mathematical Goals

- Model and write an inequality in two variables and solve a problem in context.
- Create two–variable linear equations and inequalities from contextual situations.
- Solve word problems involving inequalities.
- Represent constraints with inequalities.

Essential Questions

- How do I graph a linear inequality in two variables?
- How do I justify a solution to an equation?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9–12.A.REI.12 Graph the solution set to a linear inequality in two variables.

STANDARDS FOR MATHEMATICAL PRACTICE

2. Reason abstractly and quantitatively.
4. Model with mathematics.

Summer Job (Scaffolding Task)

Name_________________________________ **Date__________________**

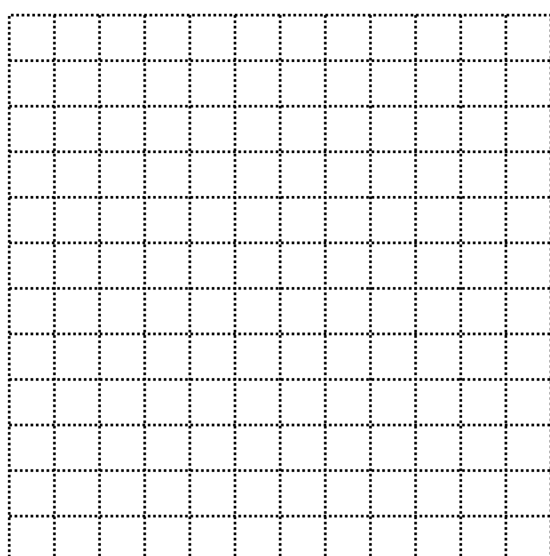
In order to raise money, you are planning to work during the summer babysitting and cleaning houses. You earn \$10 per hour while babysitting and \$20 per hour while cleaning houses. You need to earn at least \$1000 during the summer.

1. Write an expression to represent the amount of money earned while babysitting. Be sure to choose a variable to represent the number of hours spent babysitting.

2. Write an expression to represent the amount of money earned while cleaning houses.

3. Write a mathematical model (inequality) representing the total amount of money earned over the summer from babysitting and cleaning houses.

4. Graph the mathematical model. Graph the hours babysitting on the x–axis and the hours cleaning houses on the y–axis.



5. Use the graph to answer the following:

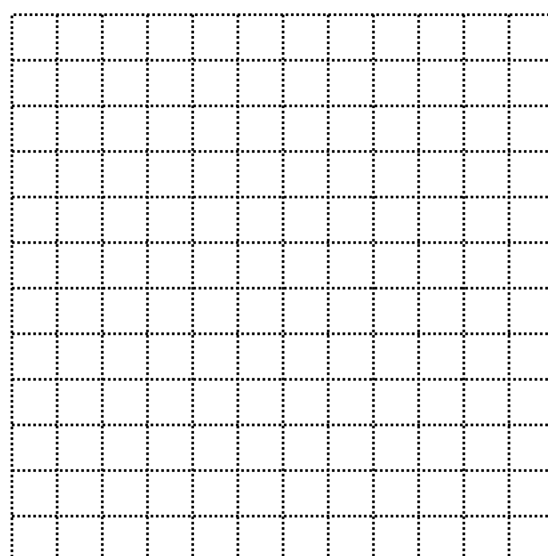
a. Why does the graph only fall in the 1^{st} Quadrant?

b. Is it acceptable to earn exactly \$1000? What are some possible combinations of outcomes that equal exactly \$1000? Where do all of the outcomes that total \$1000 lie on the graph?

c. Is it acceptable to earn more than \$1000? What are some possible combinations of outcomes that total more than \$1000? Where do all of these outcomes fall on the graph?

d. Is it acceptable to work 10 hours babysitting and 10 hours cleaning houses? Why or why not? Where does the combination of 10 hours babysitting and 10 hours cleaning houses fall on the graph? Are combinations that fall in this area a solution to the mathematical model? Why or why not?

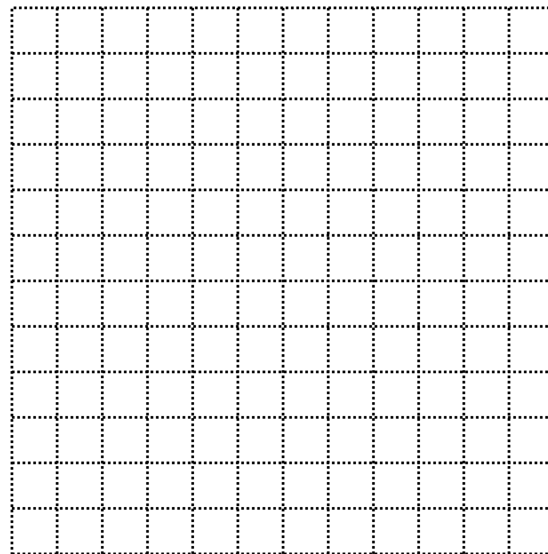
6. How would the model change if you could only earn more than \$1000? Write a new model to represent needing to earn more than \$1000. How would this change the graph of the model? Would the line still be part of the solution? How would you change the line to show this? Graph the new model.



You plan to use part of the money you earned from your summer job to buy jeans and shirts for school. Jeans cost \$40 per pair and shirts are \$20 each. You want to spend less than \$400 of your money on these items.

7. Write a mathematical model representing the amount of money spent on jeans and shirts.

8. Graph the mathematical model.
   Graph the number of jeans on the x–axis
   and shirts on the y–axis.



   a. Why does the graph only fall
      in the 1st Quadrant?

   b. Is it acceptable to spend less than \$400?
      What are some possible combinations of
      outcomes that total less than \$400?
      Where do all of these outcomes fall on the graph?

   c. Is it acceptable to spend exactly \$400? How does the graph show this?

   d. Is it acceptable to spend more than \$400? Where do all of the combinations that
      total more than \$400 fall on the graph?

Summarize your knowledge of graphing inequalities in two variables by answering the following questions in sentence form:

9. Explain the difference between a solid line and a broken line when graphing inequalities. How can you determine from the model whether the line will be solid or broken? How can you look at the graph and know if the line is part of the solution?

10. How do you determine which area of the graph of an inequality to shade? What is special about the shaded area of an inequality? What is special about the area that is not shaded?

Graphing Inequalities (Extension Task)

Back to Task Table

Introduction

In this task, students will graph two separate inequalities in two variables and analyze the graph for solutions to each.

Mathematical Goals

- Solve word problems involving inequalities.
- Represent constraints with inequalities.
- Rearrange and graph inequalities.

Essential Questions

- How do I graph a linear inequality in two variables?
- How do I justify a solution to an equation?
- How do I graph a system of linear inequalities in two variables?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9–12.A.REI.12 Graph the solution set to a linear inequality in two variables.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
   Students will need to work through problems and find multiple solutions to the problems.
5. Use appropriate tools strategically.
   Students will need to choose the tools to best model these equations based on the tools available.
6. Attend to precision.
   Students will need to be cautious of scale and constraints displayed in their graphs.

Background Knowledge

- Students can graph linear equations.
- Students can graph linear inequalities.

Common Misconceptions

- Students may be confused about the scale of the graphs and how to graph the solution.
- Students may question how to graph the constraints of the problem in terms of the appropriate quadrant.

Materials

- Colored pencils
- Calculator
- Graph Paper
- Ruler

Grouping

- Individual/Partner/Small Group

Differentiation

Extension:

- Students should show algebraically that the solutions work

Intervention:

- Students should be given graphs of sample inequalities

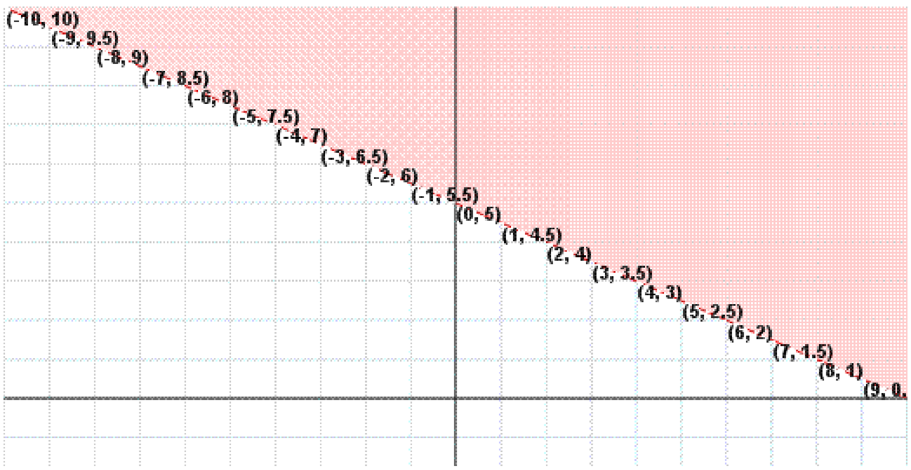
Formative Assessment Questions

- How do we display linear equations differently from linear inequalities? Why is it necessary to do so?
- When displaying multiple linear equations or inequalities, what additional considerations must we take into account?

<u>Graphing Inequalities (Extension Task) – Teacher Notes</u>

1. Graph the inequality $y > -\frac{1}{2}x + 5$. What are some solutions to the inequality?

NOTE: The points shown in the graph are the boundary points but not actually part of the solution set. Students should label points in the shaded region.



2. Graph the inequality $y < x + 2$. What are some solutions to the inequality?

NOTE: The points shown in the graph are the boundary points but not actually part of the solution set. Students should label points in the shaded region.

3. Look at both graphs.

   The main purpose of this exercise is to allow students to discover visually and conceptually where the solutions to the inequalities lie on the graph.

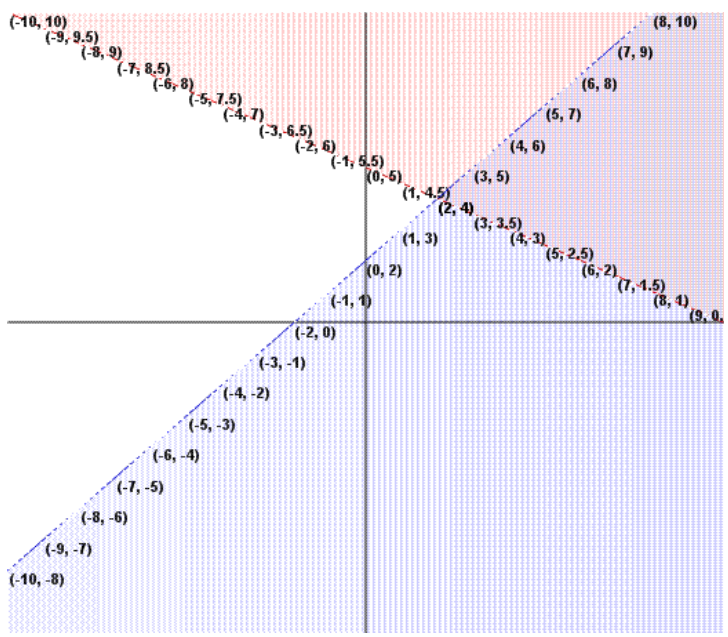
   a. Are there any solutions that work for both inequalities? Give 3 examples.

      There are many solutions that work for both, including: (–2, 7), (4, 4), (7, 3)

   b. Are there any solutions that work for 1 inequality but not the other? Give 3 examples and show which inequality it works for.

      There are many solutions that work for one inequality but not the other.

4. Graph both inequalities on the same coordinate system, using a different color to shade each.
   NOTE: Students do not need to label points in this problem. However, IF they do, they should only be labeling points in the region shaded by both graphs.



   a. Look at the region that is shaded in both colors. What does this region represent?

The region shaded in both colors represents the solutions to the system.

b. Look at the regions that are shaded in only 1 color. What do these regions represent?

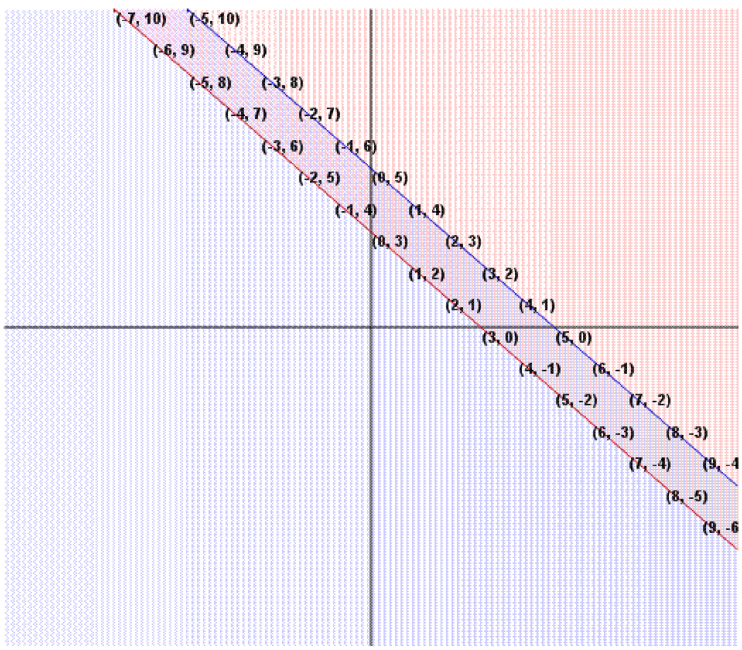
The regions shaded in one color represent solutions that work for one inequality, but not the other.

c. Look at the region that is not shaded. What does this region represent?

The region that is not shaded represents combinations that are not solutions to either inequality.

5. Graph the following system on the same coordinate grid. Use different colors for each.

$$x + y \geq 3$$
$$y \leq -x + 5$$



a. Give 3 coordinates that are solutions to the system.

Answers may vary.

b. Give 3 coordinates that are not solutions to the system.

Answers may vary.

c. Is a coordinate on either line a solution?

Yes, coordinates on the line are solutions to the system.

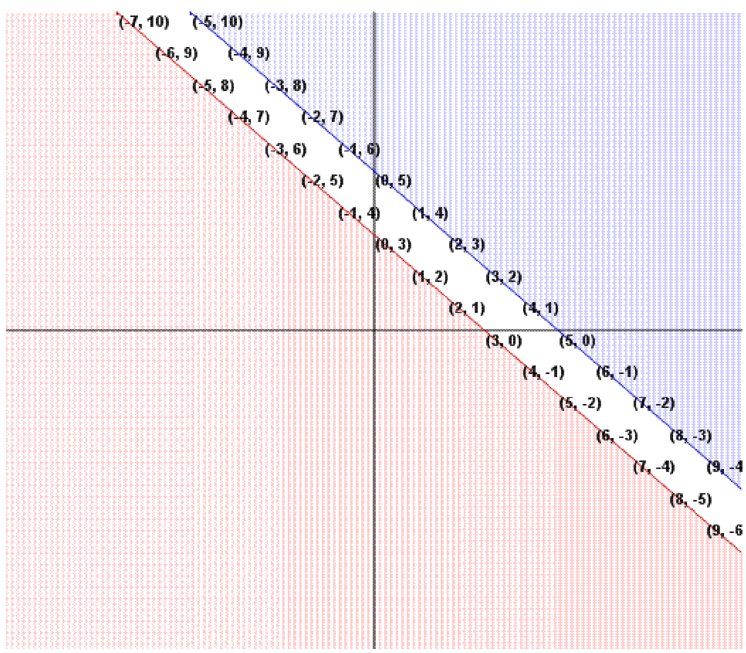
d. How would you change the inequality $x + y \geq 3$ so that it would shade below the line?

If you change the $\geq$ to $\leq$, the graph will shade below.

e. How would you change the inequality $y \leq -x + 5$ so that it would shade above the line?

If you change $\leq$ to $\geq$, the graph will shade above the line.

6. Graph the new equations from 'd' and 'e' above on the same coordinate grid. Use blue for one graph and red for the other.



a. What do the coordinates in blue represent?

Each color represents solutions to one inequality, but not the other.

b. What do the coordinates in red represent?

See above.

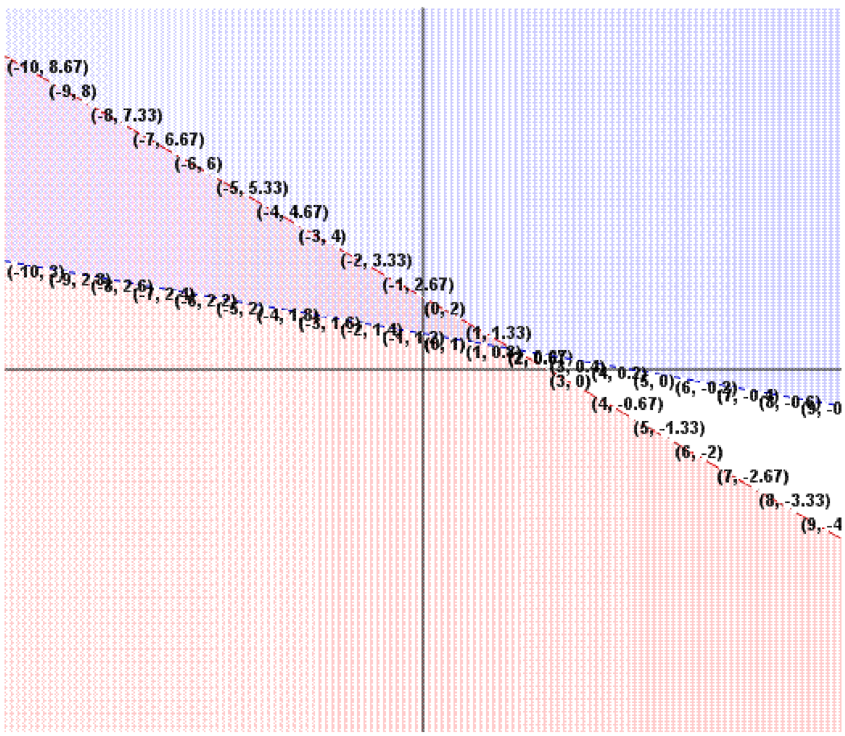
c. Why do the colors <u>not</u> overlap this time?

There is no coordinate that is a solution to both inequalities. Therefore, the system has no solution.

Graph the following on the same coordinate grid and give 3 solutions for each.

7. $2x + 3y < 6$
   $x + 5y > 5$

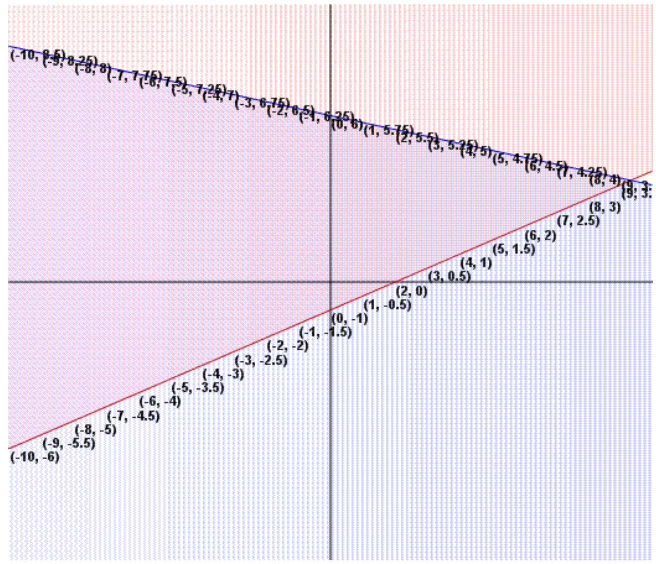
NOTE: The points shown in the graph are the boundary points but not actually part of the solution set. Students should provide three points in the area shaded by both regions.



8. $y \geq \frac{1}{2}x - 1$

$y \leq -\frac{1}{4}x + 6$

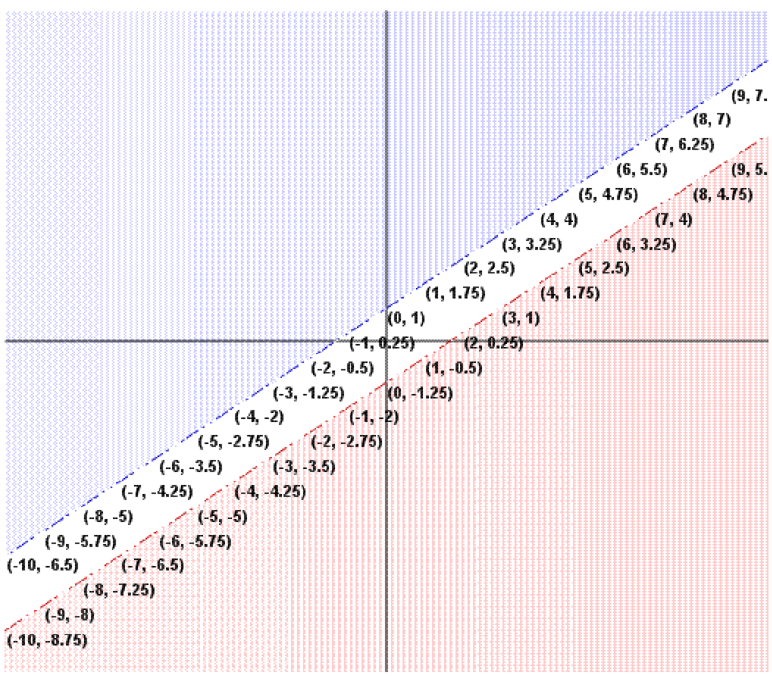
NOTE: Students should provide three points in the area shaded by both region OR on either of the boundary lines.



9. $3x - 4y > 5$

$y > \frac{3}{4}x + 1$

NOTE: The points shown in the graph are the boundary points but not actually part of the solution set. There are no solutions to this system of inequalities.



Extension Task: Graphing Inequalities

Back to Task Table

Name________________________________ **Date**_________________

Mathematical Goals

- Solve word problems involving inequalities.
- Represent constraints with inequalities.
- Rearrange and graph inequalities.

Essential Questions

- How do I graph a linear inequality in two variables?
- How do I justify a solution to an equation?
- How do I graph a system of linear inequalities in two variables.

GEORGIA STANDARDS OF EXCELLENCE

MGSE9–12.A.REI.12 Graph the solution set to a linear inequality in two variables.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
5. Use appropriate tools strategically.
6. Attend to precision.

Graphing Inequalities (Extension Task)

Name________________________________ **Date**_________________

1. Graph the inequality $y > -\frac{1}{2}x + 5$. What are some solutions to the inequality?

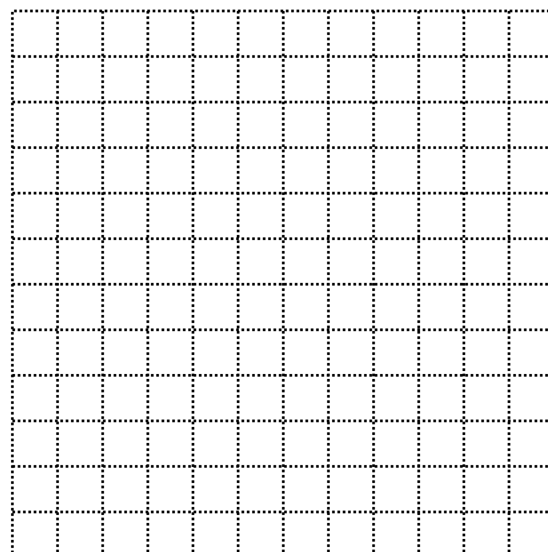
2. Graph the inequality $y < x + 2$. What are some solutions to the inequality?

3. Look at both graphs.

    a. Are there any solutions that work for both inequalities? Give 3 examples.

    b. Are there any solutions that work for 1 inequality but not the other? Give 3 examples and show which inequality it works for.

4. Graph both inequalities on the same coordinate system, using a different color to shade each.



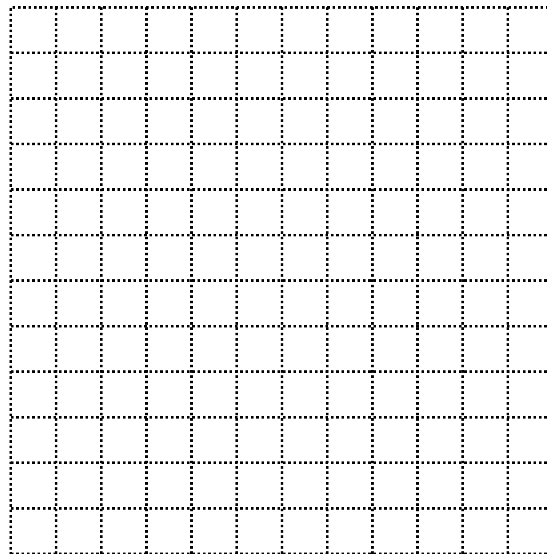
    a. Look at the region that is shaded in both colors. What does this region represent?

    b. Look at the regions that are shaded in only 1 color. What do these regions represent?

    c. Look at the region that is not shaded. What does this region represent?

5. Graph the following system on the same coordinate grid. Use different colors for each.

$$x + y \geq 3$$
$$y \leq -x + 5$$



a. Give 3 coordinates that are solutions to the system.

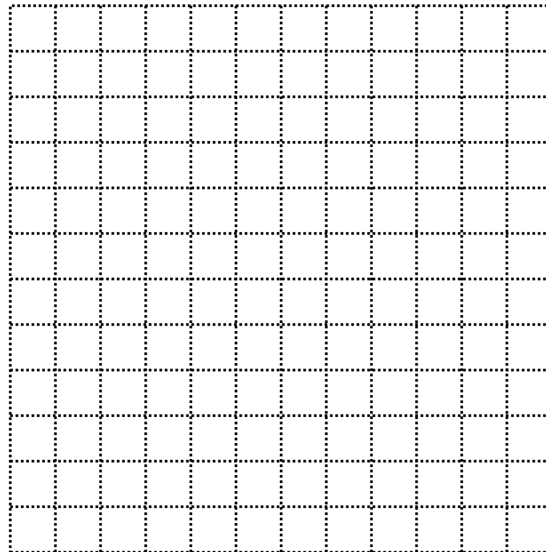
b. Give 3 coordinates that are not solutions to the system.

c. Is a coordinate on either line a solution?

d. How would you change the inequality $x + y \geq 3$ so that it would shade below the line?

e. How would you change the inequality $y \leq -x + 5$ so that it would shade above the line?

6. Graph the new equations from 'd' and 'e' above on the same coordinate grid. Use blue for one graph and red for the other.

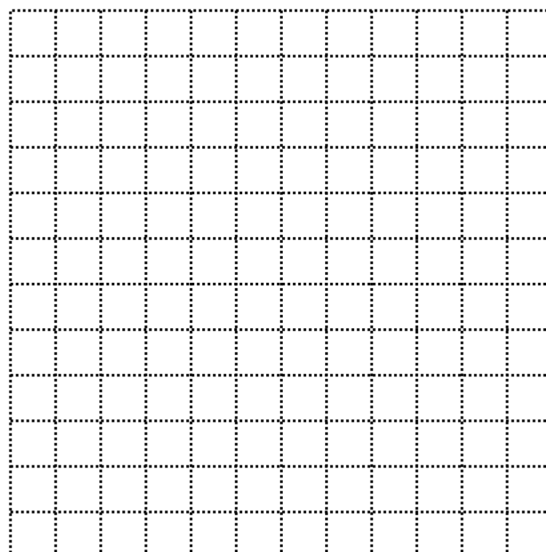


a. What do the coordinates in blue represent?

b. What do the coordinates in red represent?

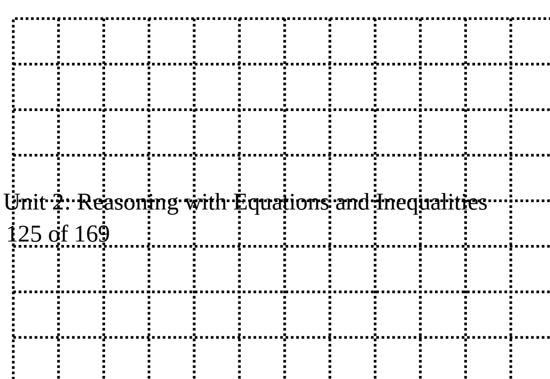
c. Why do the colors not overlap this time?

Graph the following on the same coordinate grid and give 3 solutions for each.



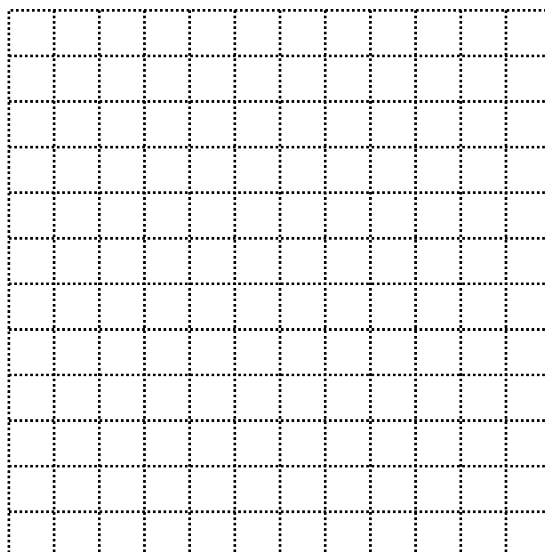
7. $2x + 3y < 6$
$x + 5y > 5$

8. $y \geq \frac{1}{2}x - 1$
$y \leq -\frac{1}{4}x + 6$



9. $3x - 4y > 5$

   $y > \frac{3}{4}x + 1$



Modeling Situations with Linear Equations (Formative Assessment Lesson)

Back to Task Table

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=673

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website: http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Modeling Situations with Linear Equations*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=211&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=673

Mathematical Goals

- Explore relationships between variables in everyday situations.
- Find unknown values from known values.
- Find relationships between pairs of unknowns, and express these as tables and graphs.
- Find general relationships between several variables, and express these in different ways by rearranging formulas.

Essential Questions

- Can I interpret the different parts of an algebraic expression?
- Can I create a general equation using all variables from a specific scenario?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. *Examples: Rearrange Ohm's law $V = IR$ to highlight resistance R;* ~~*Rearrange area of a circle formula $A = \pi r^2$ to highlight the radius r.*~~

MGSE9-12.A.REI.1 Using algebraic properties and the properties of real numbers, justify the steps of a simple, one-solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties.

MGSE9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity.~~

STANDARDS FOR MATHEMATICAL PRACTICE

2. Reason abstractly and quantitatively.
4. Model with mathematics.
Students create equations relating time/distance/speed, money/time/units, etc.

Background Knowledge

- Students know how to create expressions using variables and operations
- Students know how to graph linear functions and interpret its characteristics

Common Misconceptions

- Student uses incorrect operation in equation
- Student does not explain or misinterprets the significance of the x–intercept

Materials

- See FAL website.

Grouping

- Pairs

Family Outing (Performance Task)

Back to Task Table

Introduction
In this task, students will write a model for an inequality from the context of a word problem using real life situations. The students will then graph the inequality in two variables and analyze the solution. Students will reason quantitatively and use units to solve problems. An extension to this task involves graphing and analyzing a system of linear inequalities in context.

Mathematical Goals
- Model and write an inequality in two variables and solve a problem in context.
- Create two–variable linear equations and inequalities from contextual situations.
- Represent and solve word problems and constraints using inequalities.

Essential Questions
- How do I graph linear inequalities?
- How do I solve a system of linear equations graphically or algebraically?
- How do I justify the solution to a system of equations?

GEORGIA STANDARDS OF EXCELLENCE
MGSE9-12.A.REI.1 Using algebraic properties and the properties of real numbers, justify the steps of a simple, one-solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties.

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable including equations with coefficients represented by letters. *For example, given* $ax + 3 = 7$*, solve for* x*.*

MGSE9-12.A.REI.5 Show and explain why the elimination method works to solve a system of two-variable equations.

MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

MGSE9-12.A.REI.12 Graph the solution set to a linear inequality in two variables.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Students should use all eight SMPs when exploring this task.

Background Knowledge

- Students will apply everything they have learned in this unit.

Common Misconceptions

- Address misconceptions brought to light during the rest of the unit.

Materials

- Colored Pencils
- Ruler
- Calculator

Grouping

- Individual / Partner

Differentiation

Extension:

- Ask students to come up with their own situations that would be modeled by a system of equations or inequalities.

Intervention:

- Strategic grouping

Family Outing (Performance Task) – Teacher Notes

You and your family are planning to rent a van for a 1 day trip to Family Fun Amusement Park in Friendly Town. For the van your family wants, the Wheels and Deals Car Rental Agency charges $25 per day plus 50 cents per mile to rent the van. The Cars R Us Rental Agency charges $40 per day plus 25 cents per mile to rent the same type van.

1. Write a mathematical model to represent the cost of renting a van from the Wheels and Deals Agency for 1 day.

   C = $25 + $0.50m

   a. Do the units matter for this equation?

      Yes, the units matter. Both the cost per day and the cost per mile should be in the same unit.

   b. Use the equation to determine the cost for renting the van from this agency for 1 day and driving 40 miles.

      C = $25(1) + $0.50(40)
      C = $25 + $20
      C = $45

2. Write a mathematical model to represent the cost of renting from the Cars R Us Agency for 1 day.

   C = $40 + $.25m

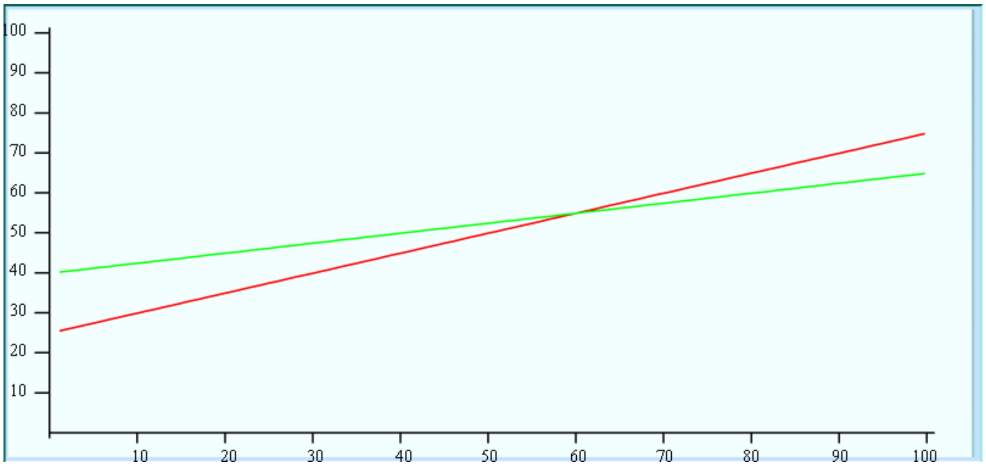
   a. Do the units for this equation match the units for the equation in problem 1? Does this matter when comparing the 2 equations?

      The units should be the same for both equations.

   b. Use the equation from '2a' to determine the cost for renting the van from Cars R Us for 1 day and driving 40 miles.

      C = $40(1) + $0.25(40)
      C = $50

3. Graph the 2 models on the same coordinate system. Be sure to extend the lines until they intersect.



a. Where do the 2 lines intersect?

(60, 55) After 60 miles, the cost for the rental will be $55.

b. What does the point of intersection represent?

The point represents the number of miles for which the cost of the rental will be the same for both agencies.

c. When is it cheaper to rent from Wheels and Deals?

It is cheaper to rent from Wheels and Deals when you are driving less than 60 miles.

d. When is it cheaper to rent from Cars R Us?

It is cheaper to rent from Cars R Us when you are driving more than 60 miles.

4. Friendly Town is approximately 80 miles from your home town. Which agency should you choose? Justify your answer.

You should choose the Cars R Us agency because the cost of renting from them would be approximately $60. The cost for renting from Wheels and Deals would be approximately $65.

When you leave the car rental agency, your father goes to the Fill 'er Up Convenience Store for gas. The gas hand indicates the van is on empty, so your father plans to fill the tank. Gas at the station is $3.49 per gallon.

5. If your father spends $78 on gas, approximately how many gallons did he purchase?

   $78 = $3.49*g
   g = $78/$3.49

   He purchased approximately 22 gallons of gas.

While in the store, your father purchases drinks for the six people in your van. Part of your family wants coffee and the rest want a soda.

6. Coffee in the store costs $0.49 per cup and sodas are $1.29 each. The cost of the drinks before tax was $6.14.

   a. Write a mathematical model that represents the total number of cups of coffee and sodas.

      $c + s = 6$

   b. Write a mathematical model that represents the cost of the coffee and soda.

      $\$0.49c + \$1.29s = \$6.14$

   c. Solve the system of equations using the elimination method.

$$
\begin{aligned}
-0.49(c + s) &= -0.49(6) \\
-0.49c - 0.49s &= -2.94 \\
\underline{+\ 0.49c + 1.29s} &\underline{= 6.14} \\
0.8s &= 3.2 \\
s &= 4 \\
\\
c + 4 &= 6 \\
c &= 2
\end{aligned}
$$

   Your father purchased 2 cups of coffee and 4 sodas.

EXTENSION: When you arrive in Friendly Town at the Family Fun Amusement Park, the 6 people in your family pair up to enter the park. You and your brother decide to enter and ride together. The cost to enter the park is $10, with each ride costing $2.

7. You bring \$55 to the park. You must pay to enter the park and you budget an additional \$10 for food. Write and solve an inequality to determine the maximum number of rides you can ride. Explain your answer.

   \$10 + \$10 + \$2r ≤ \$55

   \$2r ≤ \$35

   r ≤ 17.5

   The maximum number rides you can ride is 17, because you can't ride half of a ride.

8. Your brother brings \$70 to the park and budgets \$12 for food. How many more rides can he ride than you? Explain your answer.

   \$10 + \$12 + 2r ≤ \$70
   r ≤ 24 rides

   Your brother can ride up to 24 rides. You can ride up to 17. Therefore, he can ride 7 more rides than you.

Inside the park, there are 2 vendors that sell popcorn and cotton candy. Jiffy Snacks sells both for \$2.50 per bag. Quick Eats has cotton candy for \$4 per bag and popcorn for \$2 per bag.

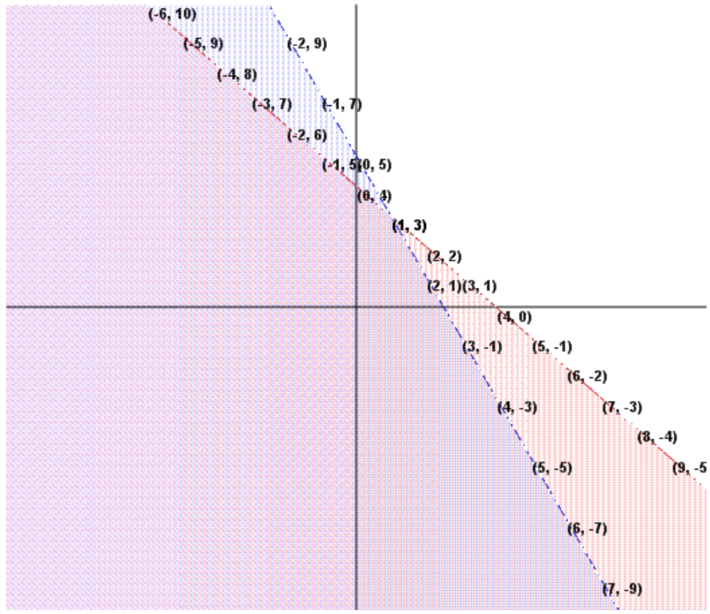
9. If you use the \$10 you budgeted for food, write an inequality to model the possible combinations of popcorn and cotton candy you can purchase from Jiffy Snacks.

   \$2.50c + \$2.50p ≤ \$10

10. Write an inequality to model the possible combinations of popcorn and cotton candy you can purchase from Quick Eats.

    \$4.00c + \$2.00p ≤ \$10

11. Graph the system of inequalities. Give two combinations that work for both vendors.



12. Assuming you purchase at least one of each, what is the maximum number of bags of cotton candy and popcorn that work for both equations?

The maximum that works for both equations is 1 bag of cotton candy and 3 bags of popcorn.

When you leave the park, your father notices that you have used ¾ of the tank of gas you purchased before you left.

13. Do you have enough gas to get home? Justify your answer.

The methods for answering this question may vary, but you do not have enough gas to get home. You have used approximately 17 of the 22 gallons you purchased earlier. You will need approximately 12 gallons of gas to get home.

14. Your father wants to purchase enough gas to get home, but not leave extra in the tank when the van is returned to the rental agency. Approximately how many more gallons should he purchase? Justify your answer.

See above.

Family Outing (Performance Task)

Name________________________________ **Date_________________**

Mathematical Goals

- Model and write an inequality in two variables and solve a problem in context.
- Create two–variable linear equations and inequalities from contextual situations.
- Represent and solve word problems and constraints using inequalities.

Essential Questions

- How do I graph linear inequalities?
- How do I solve a system of linear equations graphically or algebraically?
- How do I justify the solution to a system of equations?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.REI.1 Using algebraic properties and the properties of real numbers, justify the steps of a simple, one-solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties.

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable including equations with coefficients represented by letters. *For example, given* $ax + 3 = 7$, *solve for* x.

MGSE9-12.A.REI.5 Show and explain why the elimination method works to solve a system of two-variable equations.

MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

MGSE9-12.A.REI.12 Graph the solution set to a linear inequality in two variables.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Family Outing (Performance Task)

Name_________________________________ **Date**__________________

You and your family are planning to rent a van for a 1 day trip to Family Fun Amusement Park in Friendly Town. For the van your family wants, the Wheels and Deals Car Rental Agency charges $25 per day plus 50 cents per mile to rent the van. The Cars R Us Rental Agency charges $40 per day plus 25 cents per mile to rent the same type van.

1. Write a mathematical model to represent the cost of renting a van from the Wheels and Deals Agency for 1 day.

    a. Do the units matter for this equation?

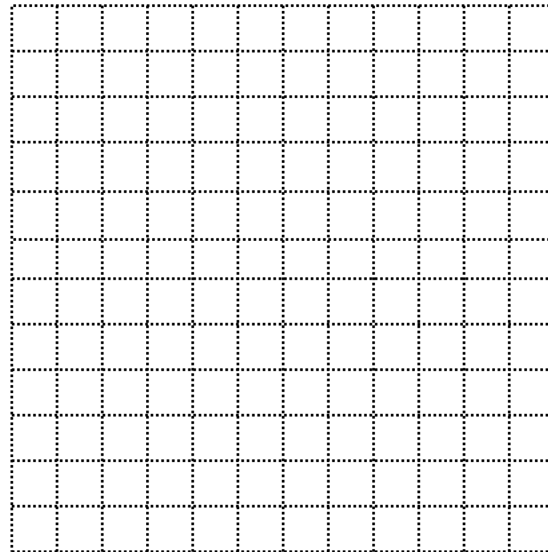
    b. Use the equation to determine the cost for renting the van from this agency for 1 day and driving 40 miles.

2. Write a mathematical model to represent the cost of renting from the Cars R Us Agency for 1 day.

    a. Do the units for this equation match the units for the equation in problem 1? Does this matter when comparing the 2 equations?

    b. Use the equation from ‘2a’ to determine the cost for renting the van from Cars R Us for 1 day and driving 40 miles.

3. Graph the 2 models on the same coordinate system.
   Be sure to extend the lines until they intersect.



   a. Where do the 2 lines intersect?

   b. What does the point of intersection represent?

   c. When is it cheaper to rent from Wheels and Deals?

   d. When is it cheaper to rent from Cars R Us?

4. Friendly Town is approximately 80 miles from your home town. Which agency should you choose? Justify your answer.

When you leave the car rental agency, your father goes to the Fill 'er Up Convenience Store for gas. The gas hand indicates the van is on empty, so your father plans to fill the tank. Gas at the station is $3.49 per gallon.

5. If your father spends $78 on gas, approximately how many gallons did he purchase?

While in the store, your father purchases drinks for the six people in your van. Part of your family wants coffee and the rest want a soda.

6. Coffee in the store costs \$0.49 per cup and sodas are \$1.29 each. The cost of the drinks before tax was \$6.14.

    a. Write a mathematical model that represents the total number of cups of coffee and sodas.

    b. Write a mathematical model that represents the cost of the coffee and soda.

    c. Solve the system of equations using the elimination method.

EXTENSION:
When you arrive in Friendly Town at the Family Fun Amusement Park, the 6 people in your family pair up to enter the park. You and your brother decide to enter and ride together. The cost to enter the park is \$10, with each ride costing \$2.

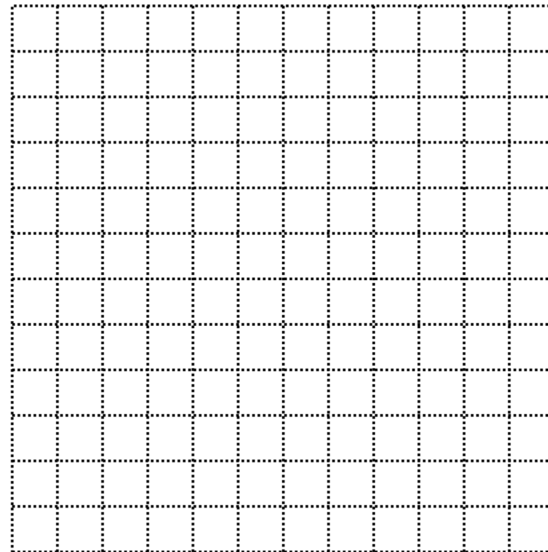
7. You bring \$55 to the park. You must pay to enter the park and you budget an additional \$10 for food. Write and solve an inequality to determine the maximum number of rides you can ride. Explain your answer.
8. Your brother brings \$70 to the park and budgets \$12 for food. How many more rides can he ride than you? Explain your answer.

Inside the park, there are 2 vendors that sell popcorn and cotton candy. Jiffy Snacks sells both for \$2.50 per bag. Quick Eats has cotton candy for \$4 per bag and popcorn for \$2 per bag.

9. If you use the \$10 you budgeted for food, write an inequality to model the possible combinations of popcorn and cotton candy you can purchase from Jiffy Snacks.

10. Write an inequality to model the possible combinations of popcorn and cotton candy you can purchase from Quick Eats.

11. Graph the system of inequalities. Give two combinations that work for both vendors.



12. Assuming you purchase at least one of each, what is the maximum number of bags of cotton candy and popcorn that work for both equations?

When you leave the park, your father notices that you have used ¾ of the tank of gas you purchased before you left.

13. Do you have enough gas to get home? Justify your answer.

14. Your father wants to purchase enough gas to get home, but not leave extra in the tank when the van is returned to the rental agency. Approximately how many more gallons should he purchase? Justify your answer.

Talk Is Cheap! (Spotlight Task)

Back to Task Table

The original Talk is Cheap task was featured in the CCGPS Coordinate Algebra Unit 3 Frameworks. This Spotlight version is designed to open the task to multiple levels of learners by allowing students to formulate their own questions based on a given bit of information. This task will allow easy access and high scalability for differentiation.

Introduction

This task can be used to introduce students to functions in a realistic setting—choosing a cell phone plan given certain conditions. Students gain experience working with decimals and translating among different representations of linear functions. They use the graphing calculator to find the intersection of two linear functions graphically and interpret the intersection in terms of the problem situation.

Essential Questions

- Why is the concept of a function important?
- How do I use function notation to show a variety of situations modeled by functions?
- How do I interpret expressions for functions in terms of the situation they model?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

MGSE9-12.A.REI.11 Using graphs, tables, or successive approximations, show that the solution to the equation f(x) = g(x) is the x-value where the y-values of f(x) and g(x) are the same.

MGSE9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

MGSE9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.*

MGSE9-12.F.BF.1 Write a function that describes a relationship between two quantities.

STANDARDS FOR MATHEMATICAL PRACTICE

1. **Make sense of problems and persevere in solving them.** Students will have to make multiple tables and graphs to demonstrate the relationship. Students must make sense of the problem by identifying what information they need to solve it.
2. **Reason abstractly and quantitatively**. Students were asked to make an estimate (high and low).
3. **Construct viable arguments and critique the reasoning of others.** After writing down their own question, students discussed their question with tablemates, creating the opportunity to construct the argument of why they chose their question, as well as critiquing the questions that others came up with.
4. Model with mathematics. Students will create linear functions representing payment plans for cell phones.
5. Use appropriate tools strategically. Students may find points of intersection using graphing calculators.

Materials Required

- Graph paper and/or graphing calculator

Time Needed

- 30 minutes to one hour depending on the depth of questioning

TEACHER NOTES

In this task, students will read the brief amount of information provided, and then discuss what they noticed. They will then be asked to formulate questions about what they wonder or are curious about. These questions will be recorded on the board and on student recording sheet. Students will then use mathematics to answer their own questions.
More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

Act 1: The Information:

- Talk Fast cellular phone service charges $0.10 for each minute the phone is used.
- Talk Easy cellular phone service charges a basic monthly fee of $18.00 plus $0.04 for each minute the phone is used.
- Both plans charge $5.00 per month for unlimited texting service.

NOTE: Students might ask if the $5.00 per month charge for unlimited texting is included in the Talk Easy cellular basic monthly fee of $18.00. This question opens the door for more mathematical discussion: What difference will that (including the $5.00) make on lining up the prices? For the purposes of the solution listed at the end of this task, the $5.00 fee was in addition to the $18.00 fee.

Act 2: The Investigation:

During Act 2 Students will come up with questions that interest them relative to the two cell phone carriers. They will then use various methods of solving systems of linear equations to answer the questions they posed. Recall in 8^{th} grade an in Unit 2 of CCGPS Coordinate Algebra students used graphing, substitution, and linear combination (elimination) methods to solve systems of linear equations. Students should be encouraged to develop the function notation for each of the cellular carriers (**MGSE9-12.F.IF.2)** and follow with the graphing of these functions (**MGSE9-12.F.IF.7).** This task lends well to discussion of parameters or feasible solutions and to compare the properties of two functions (**MGSE9-12.F.IF.9)** in a "real world" setting. Such questions as: "Does it make sense to consider values below zero for the number of minutes? And "How much would you (the student) think is "reasonable" to spend on a cell phone plan for a month?" should be part of small group or class discussions as deemed appropriate.

Act 3: The Reveal:

During Act 3 Students will discuss, present, or defend their solutions to the problem(s) they investigated in Act 2.

Note: The solution will depend on the question asked by the student. If they decided to find out when the plans cost the same, they would find that for 300 minutes each plan will cost $35.00. Prior to 300 minutes of talk, the plan with $0.10 per minute charge (Talk Fast) will be cheaper but after the 300 minute mark, the plan for $0.04 per minute (Talk Easy) will be the better buy.

A template for working through the task is presented on the next page.

Task Title:________________________ Name:________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate how

	Your estimate	Estimate (too low)	Estimate (too high)

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)
If possible, give a better estimate using this information:________________________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Functioning Well! (Practice Task)

Back to Task Table

Introduction

This task is designed to allow students to practice working with functions prior to completing rigorous integrated tasks that require them to interpret, analyze, build, construct, and compare linear and exponential functions to solve problems and model real-world situations.

Mathematical Goals

- Understand the domain and range, notation, and graph of a function
- Use function notation
- Interpret statements that use function notation in terms of context
- Recognize that sequences are functions

Essential Questions

- How do I represent real life situations using function notation?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.F.IF.1 Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and f(x) is the output (an element of the range). Graphically, the graph is y = f(x).

MGSE9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

STANDARDS FOR MATHEMATICAL PRACTICE

2. Reason abstractly and quantitatively.
Students must describe real life situations using function notation.
3. Construct viable arguments and critique the reasoning of others.
Students must defend why different representations of relations are functions.

Background Knowledge

- Students understand the difference between a relation and a function.
- Students can read and interpret graphs.
- Students can use function notation to relate inputs and outputs.

Common Misconceptions

- Students may confuse inputs and outputs. Students need to know how a function in defined in terms of inputs and outputs.
- Students may not see the distinction between adding a constant to the input or to the output of a function. They may not understand how that changes the application in a real-life situations.

Materials
- None

Grouping
- Partner / Individual

Differentiation

Extension:
- Create your own situations showing relations that are or are NOT functions. Show a verbal, graphical and numeric example.

Intervention:
- Use strategic grouping.
- Provide manipulatives.

Formative Assessment Questions
- What is a function and what are the different ways it can be expressed?
- How are arithmetic operations of functions similar/different to operations on real numbers?

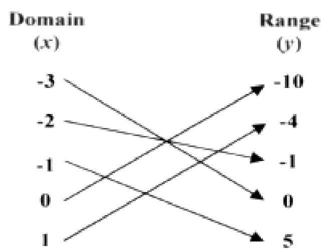
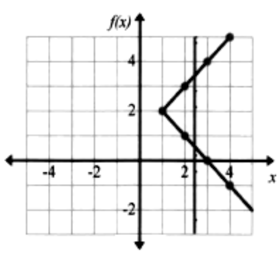
Functioning Well – Teacher Notes

Consider the definition of a function (A function is a *rule* that assigns each element of set A to a *unique* element of set B. It may be represented as a set of ordered pairs such that no two ordered pairs have the same first member, i.e. each element of a set of inputs (the domain) is associated with a unique element of another set of outputs (the range)).

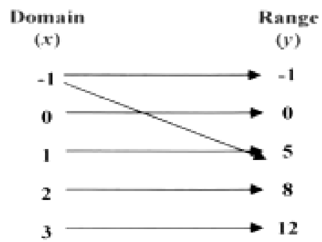
Part I – Function or Not

Determine whether or not each of the following is a function or not. Write "function" or "not a function" and explain why or why not.

Comment:
Make sure students explain their reasoning.

	Relation	Answer and Explanation
1.		***Solution:*** ***Function. By following each arrow from an x (domain) to each y (range), you can see this is a function. Each x has only one y it connects to, which by definition is a function.***
2.		***Solution:*** ***Not a function. This graph fails the vertical line test. Also, some of the x's (2, 3, 4) are connected to more than one y each. The graph demonstrates one input is generating more than one output.***

3.



Solution:
Not a function. An input of –1 sometimes gives an output of -1 and other times gives an output of 5. Therefore, there is no consistent rule and cannot be a function.

4. (x, y) = (student's name, student's shirt color)

Solution:
(x, y) = (student's name, shirt color) Function. Students may explain that each name is paired with a unique shirt color.

Part II – Function Notation

Suppose a restaurant has to figure the number of pounds of fresh fish to buy given the number of customers expected for the day. Let $p = f(E)$ where p is the pounds of fish needed and E is the expected number of customers.

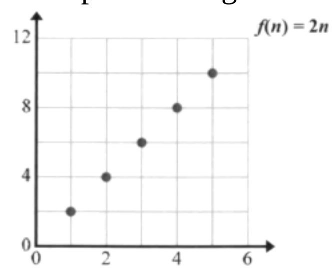
5. What would the expressions $f(E + 15)$ and $f(E) + 15$ mean?
   Solution:
   These two expressions are similar in that they both involve adding 15. However, for f(E + 15), the 15 is added on the inside, so 15 is added to the number of customers expected. Therefore, f(E + 15) gives the number of pounds of fish needed for 15 extra customers. The expression f(E) + 15 represents an outside change. We are adding 15 to f(E), which represents pounds of fish, not expected number of customers. Therefore, f(E) + 15 means that we have 15 more pounds of fish than we need for E expected customers.

6. The restaurant figured out how many pounds of fish needed and bought 2 extra pounds just in case. Use function notation to show the relationship between domain and range in this context.
   Solution:
   p = f(E) + 2

7. On the day before a holiday when the fish markets are closed, the restaurant bought enough fish for two nights. Using function notation, illustrate how the relationship changed.
   Solution:
   p = 2f(E)

8. The owner of the restaurant planned to host his 2 fish-loving parents for dinner at the restaurant. Illustrate using function notation
Solution:
p = f(E + 2)

Part III – Graphs are Functions
Write each of the points using function notation.
9.



Solution
f (1) = 1; f (2) = 2; f (3) = 3;
f (4) = 4; f (5) = 5

References: Jordan-Granite Consortium (2012).
http://secmathccss.wordpress.com

Functioning Well (Practice Task)

Name________________________________ **Date**_________________

Mathematical Goals

- Understand the domain and range, notation, and graph of a function
- Use function notation
- Interpret statements that use function notation in terms of context
- Recognize that sequences are functions

Essential Questions

- How do I represent real life situations using function notation?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.F.IF.1 Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and f(x) is the output (an element of the range). Graphically, the graph is y = f(x).

MGSE9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

STANDARDS FOR MATHEMATICAL PRACTICE

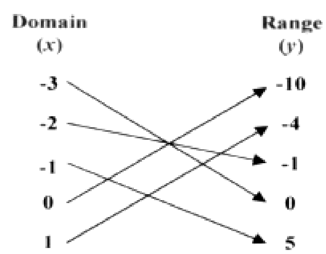
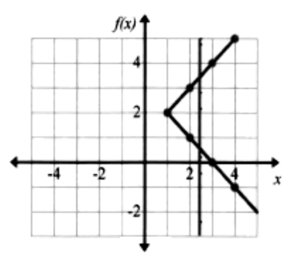
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

<u>Functioning Well (Practice Task)</u>

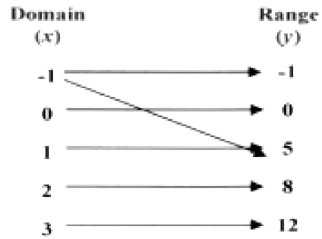
Name___________________________________ **Date**___________________

Consider the definition of a function (A function is a *rule* that assigns each element of set A to a *unique* element of set B. It may be represented as a set of ordered pairs such that no two ordered pairs have the same first member, i.e. each element of a set of inputs (the domain) is associated with a unique element of another set of outputs (the range)).

Part I – Function or Not
Determine whether or not each of the following is a function or not. Write "function" or "not a function" and explain why or why not.

	Relation	**Answer and Explanation**
1.		
2.		

3.



4. (x, y) = (student's name, student's shirt color)

Part II – Function Notation

Suppose a restaurant has to figure the number of pounds of fresh fish to buy given the number of customers expected for the day. Let $p = f(E)$ where p is the pounds of fish needed and E is the expected number of customers.

5. What would the expressions $f(E+15)$ and $f(E) + 15$ mean?

6. The restaurant figured out how many pounds of fish needed and bought 2 extra pounds just in case. Use function notation to show the relationship between domain and range in this context.

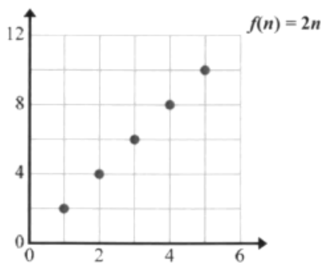
7. On the day before a holiday when the fish markets are closed, the restaurant bought enough fish for two nights. Using function notation, illustrate how the relationship changed.

8. The owner of the restaurant planned to host his 2 fish-loving parents in addition to his expected customers for dinner at the restaurant. Illustrate using function notation

Part III – Graphs are Functions

Write each of the points using function notation.

9.



The Detention Buy-Out (Spotlight Task)

Back to Task Table

"Detention Hall Buy Out" originally accessed at http://tapintoteenminds.com/real-world-math/exploring-linear-relationships-and-patterning/ involves setting up and solving a linear system in an engaging context. This task includes video support along with a handout for students support.

Estimated Task Time: One Hour

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

MGSE9-12.A.REI.11 Using graphs, tables, or successive approximations, show that the solution to the equation f(x) = g(x) is the x-value where the y-values of f(x) and g(x) are the same.

MGSE9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

MGSE9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.*

MGSE9-12.F.BF.1 Write a function that describes a relationship between two quantities.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them: analyze a real-world situation and make a connection to prior knowledge.
4. Model with mathematics: deconstruct the problem and use prior knowledge to create a solution to the problem.
5. Use appropriate tools strategically: show adequate steps to clearly demonstrate understanding using a variety of methods.

The Task is listed below and has been adapted from http://tapintoteenminds.com/real-world-math/exploring-linear-relationships-and-patterning/.

Part 1: Students will watch a video called The Detention Buy-Out. In the video, three administrators from Tecumseh Vista Academy K-12 School are interviewed and propose individual options for students to avoid serving detentions by paying the administrators according to their buy-out offers.

Part 2: After watching the video Students will then be split into groups of 2 or 3, to determine which administrator should each student buy-out from.

Encourage students to show their solution in any way they would like or you can assign certain methods to particular groups.
The exploring linear relationships problem can be solved in a number of ways which increase the scalability of this task and provide opportunities for multiple methods including:

- Trial and error / guess and check
- Table of values
- Graphing to find point of intersection
- Creating equations and substitute different values of $\boldsymbol{x}$
- Solving a system of equations using elimination
- Solving the system of equations using substitution

Part 3: Students should report the results of their exploration with supporting evidence from their method of solving the problem.

Advertisement Picture for Detention Hall Buy Out
http://i0.wp.com/tapintoteenminds.com/wp-content/uploads/2014/05/The-Detention-Buy-Out-Which-administrator-should-they-buy-out-from.png

Link to actual PDF
https://dl.dropboxusercontent.com/u/7108182/TVAMathletes/mfm1p/TVADetentionBuy-Out.pdf

Putting the "Fun" in Functions (Culminating Task)

Back to Task Table

Mathematical Goals

- Interpret linear models that represent real-life situations
- Understand the concept of a function and use function notation
- Analyze functions using different representations
- Building new functions from existing functions
- Construct and compare linear models and solve problems

Essential Questions

- How can I use and apply what I have learned about linear functions?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear~~, quadratic, simple rational, and exponential~~ functions~~ (integer inputs only).~~

MGSE9-12.A.CED.2 Create linear~~, quadratic, and exponential~~ equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase "in two or more variables" refers to formulas like the compound interest formula, in which $A = P(1 + r/n)^{nt}$ has multiple variables.)

MGSE9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

MGSE9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. *Examples: Rearrange Ohm's law V = IR to highlight resistance R;* ~~*Rearrange area of a circle formula A = π r² to highlight the radius r.*~~

MGSE9-12.A.REI.1 Using algebraic properties and the properties of real numbers, justify the steps of a simple, one-solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties.

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable including equations with coefficients represented by letters. *For example, given ax + 3 = 7, solve for x.*

MGSE9-12.A.REI.5 Show and explain why the elimination method works to solve a system of two-variable equations.

MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
MGSE9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

MGSE9-12.A.REI.11 Using graphs, tables, or successive approximations, show that the solution to the equation f(x) = g(x) is the x-value where the y-values of f(x) and g(x) are the same.

MGSE9-12.A.REI.12 Graph the solution set to a linear inequality in two variables.

MGSE9-12.F.BF.1 Write a function that describes a relationship between two quantities.

> **MGSE9-12.F.BF.1a** Determine an explicit expression and the recursive process (steps for calculation) from context. *For example, if Jimmy starts out with $15 and earns $2 a day, the explicit expression "2x+15" can be described recursively (either in writing or verbally) as "to find out how much money Jimmy will have tomorrow, you add $2 to his total today."* $J_n = J_{n-1} + 2, J_0 = 15$

MGSE9-12.F.BF.2 Write arithmetic ~~and geometric~~ sequences recursively and explicitly, use them to model situations, and translate between the two forms. Connect arithmetic sequences to linear functions ~~and geometric sequences to exponential functions~~.

MGSE9-12.F.IF.1 Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and f(x) is the output (an element of the range). Graphically, the graph is y = f(x).

MGSE9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MGSE9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. (Generally, the scope of high school math defines this subset as the set of natural numbers 1,2,3,4...) By graphing or calculating terms, students should be able to show how the recursive sequence $a_1=7$, $a_n=a_{n-1}+2$; the sequence $s_n = 2(n-1) + 7$; and the function f(x) = 2x + 5 (when x is a natural number) all define the same sequence.

MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity.~~

MGSE9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

MGSE9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

MGSE9-12.F.IF.7a Graph linear ~~and quadratic~~ functions and show intercepts, maxima, and minima (as determined by the function or by context).

MGSE9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Putting the "Fun" in Functions (Culminating Task) -Teacher Notes

In this unit you have learned the concept of a function and how to use function notation, interpret functions that arise in applications in terms of the context, analyze functions using different representations, building new functions from existing functions, and construct and compare linear models and solve problems.

Using the guide provided, you will construct a function booklet or create a webpage for students who will learn about linear functions next year. Before designing your booklet or webpage, use the guide to plan your pages or links. Make sure your use the graphing calculator to test all of your models prior to adding them to the booklet or webpage. Use the checklist to ensure that all parts of the task have been addressed.

Comment:
Walk through the guide/checklist with the students. Model using examples and discuss the use of technology (such as spreadsheets to create graphs and graphing calculators to check multiple representations of a function) in completing this culminating task. Remind students to label all parts, tables, the scale and axis for all graphs. Also, remind them to use complete thoughts. This task may take about three days to complete.

Examples for various parts of the assignment are given below.

Summing it Up: Putting the "Fun" in Functions Booklet / Webpage Planning Guide and Checklist

- ☐ **Booklet Cover/Home link on webpage: (1 point)**
  - ☐ Give your booklet/page a title
  - ☐ Use a mathematical symbol or symbols that are unique to learning about linear functions on your cover or home link
  - ☐ Include your name, date, and class period
- ☐ **Table of Contents Page or Link: (1 point)**
  - ☐ Page number for unit Definitions or link to Definitions
  - ☐ Page number or link for Function Notation
  - ☐ Page number or link for Interpreting Linear Functions Arising in Applications
  - ☐ Page number or link for Analyzing Linear Functions
  - ☐ Page number or link for Constructing and Comparing Linear Models
  - ☐ Page number or link for Unit Reflection Summary
  - ☐ Page number or link for Works Cited
- ☐ **Definitions Page or Link: (8 points)**
  - ☐ Choose at least 10 important vocabulary words from the unit to define
  - ☐ Provide a model or example of each vocabulary word. (You may use symbols, graphs, tables, or pictures.)
- ☐ **Systems of Equations/Inequalities Page or Link: (10 points)**
  - ☐ Create a contextual situation that would illustrate the application of systems of equations in context. Explain constraints as they apply to the context.
  - ☐ Create a contextual situation that would illustrate a linear inequality in two variables. Graph the solution set to the linear inequality.
- ☐ **Function Notation Page or Link: (10 points)**
  - ☐ Provide at least one example of a domain and range that illustrates a function and explain why it is a function.
  - ☐ Provide at least one example of a domain and range that is not a function and explain why.
  - ☐ Create one real world scenario in which function notation may be used to model a linear function. Show how the function might be evaluated for inputs in the domain based on the context of the scenario.
    Example: Marcus currently owns 200 songs in his iTunes collection. If he added 15 new songs each month, how many songs will he own in a year? The initial value for his function is 200 and the rate of change is 15 per month. With this information, we can write $f(x) = 15x + 200$. To show how to evaluate this function for the number of songs that he would have in one year, we would input 12.
  - ☐ Use the scenarios to create a recursive formula
- ☐ **Interpreting Linear Functions Arising in Applications: (20 point)**
  - ☐ Create a story that would generate a linear function and describe the meaning of key features (intercepts, intervals where the function is increasing, decreasing, positive, or negative; end behaviors) of the graph as they relate to the story.

- ☐ Show the graph of your function and relate the domain to the quantitative relationship it describes. Describe the rate of change for a linear function or the rate a change over an interval.
  Example for the quantitative part of the story: You are hoping to make a profit on the school play and have determined the function describing the profit to be f(t)= 8t - 2654 where t is the number of tickets sold. What is a reasonable domain for this function? Explain.

☐ **Analyzing Linear Functions: (10 points)**

- ☐ Create one linear function expressed symbolically. Graph the function using technology (print for booklet or paste on web)
- ☐ Create two different linear functions. Show one algebraically and the other using a verbal description. Compare the two functions.
  Example: Which has a greater slope?
  f(x) = x + 5 or a function representing the number of bottle caps in a shoebox where 5 are added each time

☐ **Building Functions: (10 points)**

- ☐ Explain how to find an explicit expression, a recursive process, or steps for calculation to complete a sequence/pattern. Write the sequence both recursively and with an explicit formula.
  Example: Find the number of objects (squares, toothpicks, etc.) needed to make the next three patterns in a series. Show the recursive and explicit formula for the pattern created.

☐ **Constructing and Comparing Linear Models (20 points)**

- ☐ Design a word problem that involves a linear model. Use a table or sequence to illustrate the relationships described in the models.
  Example: What's the better deal, earning $1000 a day for the rest of your life or earning $.01 the first day, and doubling it every day for the rest of your life? How do you know? Do you think an 80-year-old would make the same choice? Should she?
- ☐ Explain the constant rate per unit interval relative to another for the word problem that you designed.
- ☐ Construct the graphs for each model in the word problem that you designed.
- ☐ Compare the linear models from your word problem. Interpret the parameters.

☐ **Reflection / Summary: (8 points)**

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- ☐ Any other insight you would like to share about Unit 2.

☐ **Works Cited: (2 points)**

- ☐ Use MLA format to cite any books, websites, and any other references used to create your booklet or webpage.
  Comment:
  Point students toward online citation tools to assist them.

Putting the "Fun" in Functions (Culminating Task)

Name________________________________ **Date**_________________

Mathematical Goals

- Interpret linear models that represent real-life situations
- Understand the concept of a function and use function notation
- Analyze functions using different representations
- Building new functions from existing functions
- Construct and compare linear models and solve problems

Essential Questions

- How can I use and apply what I have learned about linear functions?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear~~, quadratic,~~ ~~simple rational,~~ ~~and exponential~~ functions ~~(integer inputs only).~~

MGSE9-12.A.CED.2 Create linear~~, quadratic, and exponential equations~~ in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase "in two or more variables" refers to formulas like the compound interest formula, in which $A = P(1 + r/n)^{nt}$ has multiple variables.)

MGSE9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

MGSE9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. *Examples: Rearrange Ohm's law V = IR to highlight resistance R;* ~~*Rearrange area of a circle formula $A = \pi r^2$ to highlight the radius r.*~~

MGSE9-12.A.REI.1 Using algebraic properties and the properties of real numbers, justify the steps of a simple, one-solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties.

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable including equations with coefficients represented by letters. *For example, given $ax + 3 = 7$, solve for x.*

MGSE9-12.A.REI.5 Show and explain why the elimination method works to solve a system of two-variable equations.

MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

MGSE9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

MGSE9-12.A.REI.11 Using graphs, tables, or successive approximations, show that the solution to the equation f(x) = g(x) is the x-value where the y-values of f(x) and g(x) are the same.

MGSE9-12.A.REI.12 Graph the solution set to a linear inequality in two variables.

MGSE9-12.F.BF.1 Write a function that describes a relationship between two quantities.

> **MGSE9-12.F.BF.1a** Determine an explicit expression and the recursive process (steps for calculation) from context. *For example, if Jimmy starts out with $15 and earns $2 a day, the explicit expression "2x+15" can be described recursively (either in writing or verbally) as "to find out how much money Jimmy will have tomorrow, you add $2 to his total today."* $J_n = J_{n-1} + 2, J_0 = 15$

MGSE9-12.F.BF.2 Write arithmetic ~~and geometric~~ sequences recursively and explicitly, use them to model situations, and translate between the two forms. Connect arithmetic sequences to linear functions ~~and geometric sequences to exponential functions.~~

MGSE9-12.F.IF.1 Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and f(x) is the output (an element of the range). Graphically, the graph is y = f(x).

MGSE9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MGSE9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. (Generally, the scope of high school math defines this subset as the set of natural numbers 1,2,3,4...) By graphing or calculating terms, students should be able to show how the recursive sequence $a_1=7$, $a_n=a_{n-1}+2$; the sequence $s_n = 2(n-1) + 7$; and the function f(x) = 2x + 5 (when x is a natural number) all define the same sequence.

MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity.~~

MGSE9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

MGSE9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

> **MGSE9-12.F.IF.7a** Graph linear ~~and quadratic~~ functions and show intercepts, maxima, and minima (as determined by the function or by context).

MGSE9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Putting the "Fun" in Functions (Culminating Task)

Name_________________________________ **Date**__________________

In this unit you have learned the concept of a function and how to use function notation, interpret functions that arise in applications in terms of the context, analyze functions using different representations, building new functions from existing functions, and construct and compare linear models and solve problems.

Using the guide provided, you will construct a function booklet or create a webpage for students who will learn about linear functions next year. Before designing your booklet or webpage, use the guide to plan your pages or links. Make sure you use the graphing calculator to test all of your models prior to adding them to the booklet or webpage. Use the checklist to ensure that all parts of the task have been addressed.

Booklet/ Webpage Planning Guide and Checklist

- ☐ **Booklet Cover/Home link on webpage: (1 point)**
  - ☐ Give your booklet/page a title
  - ☐ Use a mathematical symbol or symbols that are unique to learning about linear functions on your cover or home link
  - ☐ Include your name, date, and class period
- ☐ **Table of Contents Page or Link: (1 point)**
  - ☐ Page number for unit Definitions or link to Definitions
  - ☐ Page number or link for Function Notation
  - ☐ Page number or link for Interpreting Linear Functions Arising in Applications
  - ☐ Page number or link for Analyzing Linear Functions
  - ☐ Page number or link for Constructing and Comparing Linear Models
  - ☐ Page number or link for Unit Reflection Summary
  - ☐ Page number or link for Works Cited
- ☐ **Definitions Page or Link: (8 points)**
  - ☐ Choose at least 10 important vocabulary words from the unit to define
  - ☐ Provide a model or example of each vocabulary word. (You may use symbols, graphs, tables, or pictures.)
- ☐ **Systems of Equations/Inequalities Page or Link: (10 points)**
  - ☐ Create a contextual situation that would illustrate the application of systems of equations in context. Explain constraints as they apply to the context.
  - ☐ Create a contextual situation that would illustrate a linear inequality in two variables. Graph the solution set to the linear inequality.
- ☐ **Function Notation Page or Link: (10 points)**
  - ☐ Provide at least one example of a domain and range that illustrates a function and explain why it is a function.
  - ☐ Provide at least one example of a domain and range that is not a function and explain why.

- ☐ Create one real world scenario in which function notation may be used to model a linear function. Show how the function might be evaluated for inputs in the domain based on the context of the scenario.
- ☐ Use the scenarios to create a recursive formula

☐ **Interpreting Linear Functions Arising in Applications: (20 point)**
- ☐ Create a story that would generate a linear function and describe the meaning of key features (intercepts, intervals where the function is increasing, decreasing, positive, or negative; end behaviors) of the graph as they relate to the story.
- ☐ Show the graph of your function and relate the domain to the quantitative relationship it describes. Describe the rate of change for a linear function or the rate a change over an interval.

☐ **Analyzing Linear Functions: (10 points)**
- ☐ Create one linear function expressed symbolically. Graph the function using technology (print for booklet or paste on web)
- ☐ Create two different linear functions. Show one algebraically and the other using a verbal description. Compare the two functions.

☐ **Building Functions: (10 points)**
- ☐ Explain how to find an explicit expression, a recursive process, or steps for calculation to complete a sequence/pattern. Write the sequence both recursively and with an explicit formula.

☐ **Constructing and Comparing Linear Models (20 points)**
- ☐ Design a word problem that involves a linear model. Use a table or sequence to illustrate the relationships described in the models.
- ☐ Explain the constant rate per unit interval relative to another for the word problem that you designed.
- ☐ Construct the graphs for each model in the word problem that you designed.
- ☐ Compare the linear models from your word problem. Interpret the parameters.

☐ **Reflection / Summary: (8 points)**
- ☐ Describe your learning journey throughout the unit. Reflect on topics that you found easy to learn and those that were most difficult.
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ADDITIONAL TASKS

The tasks featured in this table provide additional resources and supplemental tasks to be incorporated into unit 2 instruction as deemed appropriate by the instructor.

UNIT2: Linear Functions	Standards Addressed in the Task
More Graphing Stories http://graphingstories.com Site Developed by Dan Meyer (1-3 Stories)	F-BF, F-IF
Points on a Graph Task from Illustrative Math https://www.illustrativemathematics.org/illustrations/630	F.IF.1
The Customers Task from Illustrative Math https://www.illustrativemathematics.org/illustrations/624	F.IF.1
Using Function Notation Task from Illustrative Math https://www.illustrativemathematics.org/illustrations/598	F.IF.1
Warming and Cooling Task from Illustrative Math https://www.illustrativemathematics.org/illustrations/639	F.IF.4
The Random Walk Task from Illustrative Math https://www.illustrativemathematics.org/illustrations/626	F.IF.2