Georgia
Standards of Excellence
Curriculum Frameworks

Mathematics

Accelerated GSE Algebra I/Geometry A

Unit 5: Comparing and Contrasting Functions

GaDOE
Georgia Department of Education
Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

Unit 5

Comparing and Contrasting Functions

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OVERVIEW

In this unit students will:

- Deepen their understanding of linear, quadratic, and exponential functions as they compare and contrast the three types of functions.
- Understand the parameters of each type of function in contextual situations.
- Interpret linear, quadratic, and exponential functions that arise in applications in terms of the context.
- Analyze linear, quadratic, and exponential functions and model how different representations may be used based on the situation presented.
- Construct and compare characteristics of linear, quadratic, and exponential models and solve problems.
- Distinguish between linear, quadratic, and exponential functions graphically, using tables, and in context.
- Recognize that exponential and quadratic functions have a variable rate of change while linear functions have a constant rate of change.
- Distinguish between additive and multiplicative change and construct and interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.
- Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. This unit provides much needed content information and excellent learning activities. However, the intent of the framework is not to provide a comprehensive resource for the implementation of all standards in the unit. A variety of resources should be utilized to supplement this unit. The tasks in this unit framework illustrate the types of learning activities that should be utilized from a variety of sources. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the **"Strategies for Teaching and Learning"** in the Comprehensive Course Overview and the tasks listed under **"Evidence of Learning"** be reviewed early in the planning process.

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

KEY STANDARDS ADDRESSED

Construct and compare linear, quadratic, and exponential models and solve problems
MGSE9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

> **MGSE9-12.F.LE.1a** Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. (This can be shown by algebraic proof, with a table showing differences, or by calculating average rates of change over equal intervals).
>
> **MGSE9-12.F.LE.1b.** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
>
> **MGSE9-12.F.LE.1c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

MGSE9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

MGSE9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Interpret expressions for functions in terms of the situation they model
MGSE9-12.F.LE.5 Interpret the parameters in a linear ($f(x) = mx + b$) and exponential ($f(x)=a\bullet d^x$) function in terms of context. (In the functions above, "m" and "b" are the parameters of the linear function, and "a" and "d" are the parameters of the exponential function.) In context, students should describe what these parameters mean in terms of change and starting value.

<u>Understand the concept of a function and use function notation</u>

MGSE9-12.F.IF.1 Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and f(x) is the output (an element of the range). Graphically, the graph is y = f(x).

MGSE9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

<u>Interpret functions that arise in applications in terms of the context</u>

MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity.~~

MGSE9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

MGSE9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

<u>Analyze functions using different representations</u>

MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

MGSE9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.*

<u>Build new functions from existing functions</u>

MGSE9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. ~~Include recognizing even and odd functions from their graphs and algebraic expressions for them.~~ *(Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.)*

STANDARDS FOR MATHEMATICAL PRACTICE

Refer to the Comprehensive Course Overview for more detailed information about the Standards for Mathematical Practice.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ENDURING UNDERSTANDINGS

- How to interpret linear and exponential functions that arise in applications in terms of the context.
- When analyzing linear and exponential functions, different representations may be used based on the situation presented.
- New functions can be created from existing functions.
- Understand how to construct and compare linear and exponential models and solve problems.
- The graph of any linear function is a straight line.
- The graph of any quadratic function is a parabola with a vertex that provides the maximum or minimum output value of the function and the input at which it occurs.
- The graph of any exponential function is a curve which either rises or falls sharply as the graph is read from left-to-right. Any exponential function will have a horizontal asymptote.
- Linear functions have a constant rate of change.
- Exponential functions have a constant percent of change.
- Quadratic functions have a variable rate of change that will be positive on one side of the graph and negative on the other side of the graph (divided by the vertical axis of symmetry).

ESSENTIAL QUESTIONS

- How do I use graphs to represent and solve real-world equations and inequalities?
- Why is the concept of a function important and how do I use function notation to show a variety of situations modeled by functions?
- How do I interpret functions that arise in applications in terms of context?
- How do I use different representations to analyze linear and exponential functions?
- How do I build a linear or exponential function that models a relationship between two quantities?
- How do I build new functions from existing functions?
- How can we use real-world situations to construct and compare linear and exponential models and solve problems?
- How do I interpret expressions for functions in terms of the situation they model?
- How is a relation determined to be linear, quadratic, or exponential?
- What are the specific features that distinguish the graphs of linear, quadratic, and exponential functions from one another?

CONCEPTS AND SKILLS TO MAINTAIN

In order for students to be successful, the following skills and concepts need to be maintained:

- Understand and be able to explain what a function is.
- Determine if a table, graph or set of ordered pairs is a function.
- Distinguish between linear and non-linear functions.
- Write linear and exponential equations and use them to model real-world situations.
- Understand and interpret key features of graphs.
- Solve linear equations, inequalities, and systems of equations.

- Graph the solution set to a linear inequality in two variables.
- Perform addition, subtraction, and multiplication of polynomials.
- Simplify radical expressions.
- Factor quadratic expressions.
- Solve quadratic equations in one variable.

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for high school children.
Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks
http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary.
http://intermath.coe.uga.edu/dictnary/homepg.asp

Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

- **Arithmetic Sequence.** A sequence of numbers in which the difference between any two consecutive terms is the same.
- **Average Rate of Change.** The change in the value of a quantity by the elapsed time. For a function, this is the change in the y-value divided by the change in the x-value for two distinct points on the graph.
- **Coefficient.** A number multiplied by a variable in an algebraic expression.
- **Constant Rate of Change.** With respect to the variable x of a linear function $y = f(x)$, the constant rate of change is the slope of its graph.

- **Continuous.** Describes a connected set of numbers, such as an interval.
- **Discrete.** A set with elements that are disconnected.
- **Domain.** The set of *x*-coordinates of the set of points on a graph; the set of *x*-coordinates of a given set of ordered pairs. The value that is the input in a function or relation.
- **End Behaviors.** The appearance of a graph as it is followed farther and farther in either direction.
- **Explicit Expression.** A formula that allows direct computation of any term for a sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$.
- **Exponential Function.** A nonlinear function in which the independent value is an exponent in the function, as in $y = ab^x$.
- **Exponential Model.** An exponential function representing real-world phenomena. The model also represents patterns found in graphs and/or data.
- **Expression.** Any mathematical calculation or formula combining numbers and/or variables using sums, differences, products, quotients including fractions, exponents, roots, logarithms, functions, or other mathematical operations.
- **Even Function.** A function with a graph that is symmetric with respect to the *y*-axis. A function is only even if and only if $f(-x) = f(x)$.
- **Factor.** For any number *x*, the numbers that can be evenly divided into *x* are called factors of *x*. For example, the number 20 has the factors 1, 2, 4, 5, 10, and 20.
- **Geometric Sequence.** A sequence of numbers in which the ratio between any two consecutive terms is the same. In other words, you multiply by the same number each time to get the next term in the sequence. This fixed number is called the common ratio for the sequence.
- **Horizontal shift.** A rigid transformation of a graph in a horizontal direction, either left or right.
- **Interval Notation.** A notation representing an interval as a pair of numbers. The numbers are the endpoints of the interval. Parentheses and/or brackets are used to show whether the endpoints are excluded or included.
- **Linear Function.** A function with a constant rate of change and a straight line graph.

- **Linear Model.** A linear function representing real-world phenomena. The model also represents patterns found in graphs and/or data.

- **Odd Function.** A function with a graph that is symmetric with respect to the origin. A function is odd if and only if $f(–x) = –f(x)$.

- **Parameter.** The independent variable or variables in a system of equations with more than one dependent variable.

- **Quadratic equation.** An equation of degree 2, which has at most two solutions.

- **Quadratic function.** A function of degree 2 which has a graph that "turns around" once, resembling an umbrella–like curve that faces either right–side up or upside down. This graph is called a parabola.

- **Root.** The *x*–values where the function has a value of zero.

- **Range.** The set of all possible outputs of a function.

- **Recursive Formula.** A formula that requires the computation of all previous terms to find the value of a_n.

- **Slope.** The ratio of the vertical and horizontal changes between two points on a surface or a line.

- **Term.** A value in a sequence--the first value in a sequence is the 1^{st} term, the second value is the 2^{nd} term, and so on; a term is also any of the monomials that make up a polynomial.

- **Vertical Translation.** A shift in which a plane figure moves vertically.

- ***X*-intercept.** The point where a line meets or crosses the *x*-axis

- ***Y*-intercept.** The point where a line meets or crosses the *y*-axis

EVIDENCE OF LEARNING

By the conclusion of this unit, students should be able to demonstrate the following competencies:

- Deepen their understanding of linear, quadratic, and exponential functions as they compare and contrast the three types of functions.
- Understand the parameters of each type of function in contextual situations.
- Interpret linear, quadratic, and exponential functions that arise in applications in terms of the context.
- Analyze linear, quadratic, and exponential functions and model how different representations may be used based on the situation presented.
- Construct and compare characteristics of linear, quadratic, and exponential models and solve problems.
- Recognize that exponential and quadratic functions have a variable rate of change while linear functions have a constant rate of change.
- Distinguish between additive and multiplicative change and construct and interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.
- Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- Define and use function notation, evaluate functions at any point in the domain, give general statements about how $f(x)$ behaves at different regions in the domain (as x gets very large or very negative, close to 0 etc.), and interpret statements that use function notation.
- Explain the difference and relationship between domain and range and find the domain and range of a linear, quadratic, or exponential functions from a function equation, table, or graph.
- Interpret x and y intercepts, where the function is increasing or decreasing, where it is positive or negative, its end behaviors, given the graph, table or algebraic representation of a linear, quadratic, or exponential function in terms of the context of the function.

- Find and/or interpret appropriate domains and ranges for authentic linear, quadratic, or exponential functions.

- Calculate and interpret the average rate of change over a given interval of a function from a function equation, graph or table, and explain what that means in terms of the context of the function.

- Explain the relationship between the domain of a function and its graph in general and/or to the context of the function.

- Accurately graph a linear function by hand by identifying key features of the function such as the *x*- and *y*-intercepts and slope.

- Discuss and compare different functions (linear, quadratic, and/or exponential) represented in different ways (tables, graphs or equations). Discussion and comparisons should include: identifying differences in rates of change, intercepts, and/or where each function is greater or less than the other.

- Write a function that describes a linear, quadratic, or exponential relationship between two quantities.

- Construct and compare linear, quadratic, and exponential models and solve problems.

TEACHER RESOURCES

The following pages include teacher resources that teachers may wish to use to supplement instruction.

- Web Resources
- Compare / Contrast: Linear, Quadratic, and Exponential Functions

The Georgia Online Formative Assessment Resource (GOFAR) accessible through SLDS contains test items related to content areas assessed by the Georgia Milestones Assessment System and NAEP. Teachers and administrators can utilize the GOFAR to develop formative and summative assessments, aligned to the state–adopted content standards, to assist in informing daily instruction. Students, staff, and classes are prepopulated and maintained through the State Longitudinal Data System (SLDS). Teachers and Administrators may view Exemplars and Rubrics in Item Preview. A scoring code may be distributed at a local level to help score constructed response items.

For GOFAR user guides and overview, please visit:
https://www.gadoe.org/Curriculum–Instruction–and–Assessment/Assessment/Pages/Georgia–Online–Formative–Assessment–Resource.aspx

Web Resources

The following list is provided as a sample of available resources and is for informational purposes only. It is your responsibility to investigate them to determine their value and appropriateness for your district. GADOE does not endorse or recommend the purchase of or use of any particular resource.

- Rate of Change Task
  http://www.nms.org/Portals/0/Docs/FreeLessons/Fill%20It%20Up,%20Please%20-%20Part%20III.pdf
  This task includes an extensive lesson plan with alignment to the standards

- Linear & Exponential Growth
  http://learnzillion.com/lessonsets/40-proving-how-linear-functions-grow
  This webpage includes short videos comparing linear and exponential functions.

- Distinguishing between Linear & Exponential
  http://learnzillion.com/lessonsets/35-distinguishing-between-linear-functions-and-exponential-functions
  Further video resources for exponential & linear functions.

Compare / Contrast: Linear, Quadratic, and Exponential Functions

Show similarities and differences between linear, quadratic, and exponent functions:
What things are being compared? How are they similar? How are they different?

Attribute	Linear Functions	Quadratic Functions	Exponential Functions
Rate of change			
Domain & Range			
Intercepts			
Asymptotes			
End Behavior			

Functions to Graph and Discuss:

$f(x) = 2x + 3$ $f(x) = 2x^2 + 3$ $f(x) = 2^x + 3$

FORMATIVE ASSESSMENT LESSONS (FAL)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Overview.

SPOTLIGHT TASKS

A Spotlight Task has been added to each CCGPS mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Georgia Standards of Excellence, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky.

3-ACT TASKS

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Overview.

TASKS

The following tasks represent the level of depth, rigor, and complexity expected of all Algebra I students. These tasks, or tasks of similar depth and rigor, should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as a performance task, they may also be used for teaching and learning (learning/scaffolding task).

Task Name ***Suggested Time***	**Task Type** ***Grouping Strategy***	**Content Addressed**	**Standards**
Having Kittens (FAL) *≈ 2 hours*	Formative Assessment Lesson *Individual/Partner / Small Group*	Interpret a situation and represent the constraints and variables mathematically. Select appropriate mathematical methods to use. Make sensible estimates and assumptions. Investigate an exponentially increasing sequence.	F.LE.1 F.LE.1a F.LE.1b F.LE.1c F.LE.2 F.LE.3
Community Service, Sequences, and Functions *≈ 2 hours*	Performance Task *Individual/Partner*	Use technology to graph and analyze functions. Construct linear and exponential function (including reading these from a table). Observe the difference between linear and exponential functions	F.IF.5 F.LE.1b F.LE.1c F.LE.2 F.LE.3
Birthday Gifts and Turtle Problem *75-90 minutes*	Formative Assessment Lesson *Individual / Small Group*	Understand the rates of change of linear functions are constant, while the rates of change of exponential functions are not constant.	F.LE.1 F.LE.1a F.LE.1b F.LE.1c F.LE.2 F.LE.3
Exploring Paths *75-90 minutes*	Formative Assessment Lesson *Individual / Small Group*	Reason qualitatively. Compare linear and exponential models verbally, numerically, algebraically, and graphically.	F.BF.1 F.LE.1 F.LE.1a F.LE.1b F.LE.1c

Task Name ***Suggested Time***	**Task Type** ***Grouping Strategy***	**Content Addressed**	**Standards**
Comparing Investments *≈ 2 hours*	Formative Assessment Lesson *Individual/Partner*	Translate between descriptive, algebraic, and tabular data, and graphical representations of a function. Recognize how and why a quantity changes per unit interval.	F.LE.1 F.LE.1a F.LE.1b F.LE.1c F.LE.2 F.LE.3 F.LE.5 F.IF.6
Comparing Linear, Quadratic, and Exponential Models Graphically *50-60 minutes*	Learning Task *Individual/Partner*	Recognize the differences between the graphs of linear, quadratic, and exponential functions.	F.LE.2 F.LE.3 F.IF.1 F.IF.4 F.IF.7 F.IF.9
Paula's Peaches: The Sequel (Extension Task) *60-75 minutes*	Learning Task *Individual/Partner Task*	Analyze quadratic functions. Interpret key features of quadratic functions.	F.LE.3 F.IF.1 F.IF.2 F.IF.4 F.IF.5
Fences and Functions *50-60 minutes*	Culminating Task *Individual/ Partner Task*	Compare and contrast linear, quadratic, and exponential functions. Interpret key features of functions in context.	F.LE.1 F.LE.5 F.IF.7 F.IF.9

Having Kittens (Formative Assessment Lesson)

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Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1204

Task Comments and Introduction
Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Modeling: Having Kittens*, is a Formative Assessment Lesson (FAL) that can be found at the website: http://map.mathshell.org/materials/lessons.php?taskid=407&subpage=problem

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1204

Mathematical Goals
- Interpret a situation and represent the constraints and variables mathematically.
- Select appropriate mathematical methods to use.
- Make sensible estimates and assumptions.
- Investigate an exponentially increasing sequence.

Essential Questions
- How can I use mathematical models to determine whether the poster's claim that one cat can have 2000 descendants in just 18 months is reasonable?

GEORGIA STANDARDS OF EXCELLENCE
MGSE9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

MGSE9-12.F.LE.1a Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. (This can be shown by algebraic proof, with a table showing differences, or by calculating average rates of change over equal intervals).

MGSE9-12.F.LE.1b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

MGSE9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

MGSE9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

MGSE9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically

Background Knowledge

- The background knowledge required for this task is quite general. There are many entry points to this problem, all of which build from different types of background knowledge.

Common Misconceptions

- Students may forget that each new kitten can also have litters of its own after 4 months.
- Students must make assumptions in order to approach the problem. See discussion in "Solutions" of the FAL.

Materials

- see FAL website

Grouping

- Individual / Partner/ Small group

Community Service, Sequences, and Functions (Performance Task)

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Introduction
In this task, students will explore the relationship between arithmetic and geometric sequences and exponential functions.

Mathematical Goals
- Use technology to graph and analyze functions
- Construct arithmetic and geometric sequences (including reading these from a table)
- Observe the difference between linear and exponential functions

Essential Questions
- How can I construct linear and exponential functions?
- How do I compare linear and exponential functions?

GEORGIA STANDARDS OF EXCELLENCE
MGSE9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

MGSE9-12.F.LE.1b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

MGSE9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

MGSE9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

MGSE9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

STANDARDS FOR MATHEMATICAL PRACTICE
4. Model with mathematics.
Students will create sequences from context and model them with tables and equations.
7. Look for and make use of structure.
Students will use tables to formulate equations.
8. Look for and express regularity in repeated reasoning.
Students will recognize patterns within sequences.

Background Knowledge

- Students understand sequences as functions.
- Students can use and write explicit and recursive formulas for sequences.

Common Misconceptions

- Students may confuse explicit and recursive formulas and the parts that make them up.

Materials

- None

Grouping

- Partner / Individual

Differentiation

Extension:

- Have students write explicit and recursive formulas for the amount of money collected.

#7: $a_n = 5n$; $a_n = a_{n-1} + 5$
#8: $a_n = 5(2)^{n-1}$; $a_n = 2a_n$

Intervention:

- Provide students with two rows in the table
- Give formulas for the sequences.

Formative Assessment Questions

- How are sequences related to functions?
- What types of real-life situations can be modeled with functions?

Community Service, Sequences, and Functions (Performance Task) – Teacher Notes

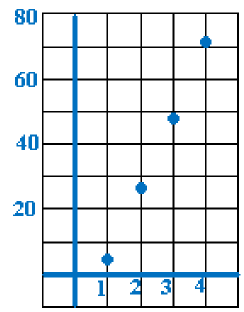
Comment:
Activities that require students to practice completing geometric and arithmetic sequences and generate an explicit and recursive formula from those sequences should occur prior to completing this task.

Larry, Moe, and Curly spend their free time doing community service projects. They would like to get more people involved. They began by observing the number of people who show up to the town cleanup activities each day. The data from their observations is recorded in the given table for the Great Four Day Cleanup.

x	y
1	5
2	27
3	49
4	71

1. Give a verbal description of what the domain and range presented in the table represents.
Solution:
The domain is the number of days. The range represents the number of people that showed up each day.

2. Sketch the data on the grid below.



3. Determine the type of function modeled in the graph above and describe key features of the graph.
Solution:
Answers may vary. The graph models a linear function. The sequence represents discrete data. Looking at the graph, the pattern appears to be increasing at a constant rate of change.

4. Based on the pattern in the data collected, what recursive process could Larry, Curly, and Moe write?
Solution:
$a_1 = 5, a_n = a_{n-1} + 22$

5. Write a linear equation to model the function.
Solution:
Students could answer in the form of the explicit formula, $a_n = 5 + 22(n - 1)$, or in slope-intercept form $f(x) = 22x - 17$. Students should understand the relationship between these two forms.

6. How would Larry, Curly, and Moe use the explicit formula to predict the number of people who would help if the cleanup campaign went on for 7 days?

Solution:
By evaluating f(x) = 22x – 17 (or $a_n = 5 + (n - 1)22$) substituting 7 for the x value in the explicit formula, they could predict that 22(7) –17 or 137 people will show up on day 7 if the cleanup campaign continued.

Excited about the growing number of people participating in community service, Larry, Curly, and Moe decide to have a fundraiser to plant flowers and trees in the parks that were cleaned during the Great Four Day cleanup. It will cost them $5,000 to plant the trees and flowers. They decide to sell some of the delicious pies that Moe bakes with his sisters. For every 100 pies sold, it costs Moe and his sisters $20.00 for supplies and ingredients to bake the pies. Larry, Curly, and Moe decide to sell the pies for $5.00 each.

7. Complete the following table to find the total number of pies sold and the amount of money the trio collects.

a. On Day 1, each customer buys the same number of pies as his customer number. In other words the first customer buys 1 pie, the second customer buys 2 pies. Fill in the table showing the number of pies and the amount collected on Day 1. Then calculate the total number of pies sold and dollars collected.

b. Write a recursive and explicit formula for the pies sold on Day 1. Explain your thinking.

Customer Number	***Number of Pies Sold***	***Amount Collected***
1	***1***	***$5***
2	***2***	***$10***
3	***3***	***$15***
4	***4***	***$20***
5	***5***	***$25***
6	***6***	***$30***
7	***7***	***$35***
8	***8***	***$40***
9	***9***	***$45***
10	***10***	***$50***
Total	***55***	***$275***

Solution:
Since the number of pies sold to each customer is the same as the customer number, we have the explicit formula that an = n , where an is the number of pies and n is the customer number. We can also notice that the number of pies increases by one each time so an = an 1 + 1, where an is the number of pies, is the recursive formula for the number of pies sold.

Extension:
To obtain the explicit formula for the amount collected, we can multiply the number of pies sold by 5. This gives us an = 5n, where an is the cost of the pies sold and n is the customer number.

c. On Day 2, the first customer buys 1 pie, the second customer buys 2 pies, the third customer buys 4 pies, the fourth customer buys 8 pies, and so on. Complete table based on the pattern established. Then calculate the total number of pies sold and dollars collected.

Customer Number	Number of Pies Sold	Amount Collected
1	1	$5
2	2	$10
3	4	$20
4	8	$40
5	16	$80
6	32	$160
7	64	$320
8	128	$640
9	256	$1280
10	512	$2560
Total	1023	$5115

d. Write a recursive and explicit formula for the pies sold on Day 2. Explain your thinking.

Solution:
Since the number of pies sold to each customer doubles each time, we have the explicit formula that an = 2n-1, where an¬ is the number of pies and n is the customer number. We also have that the recursive formula is an = 2an-1, where an is the number of pies.
Extension: To obtain the explicit formula for the amount collected, we can multiply the number of pies sold by 5. This gives us an = 5(2)n-1, where an is the price of the pies and n is the customer number. Looking at a recursive pattern, we notice that the price column still doubles each time. This gives a recursive formula of an = 2an-1 for the price of pies, where an is the cost of the pies.

8. Compare the rates of change on Day 1 and Day 2 for the number of pies sold.
Solution:
Answers may vary. Possible answer:
When the change in x is 1, Day 1 (linear) the change in y is a constant (slope).
On day two, (exponential) the y-values are multiplied by a constant ratio to get the succeeding y-value.

9. Did Larry, Curly, and Moe earn enough in two days to fund their project? Consider costs incurred to bake the pies. Justify your reasoning.
Solution:
There are two ways students might answer this question. a) Students assume that $20 per 100 pies is really $0.20 per pie OR b)students assume that they must purchase pies in 100 pie increments.
Therefore…
On the first day, they sold 55 pies and made $275. a) At a cost of $20 per 100 pies, they spent $11 on ingredients. This yields a profit of $264. OR b) They spent $20 on 100 pies, of which they sold 55. They yield a profit $255.

On the second day, they sold 1023 pies and made $5115. a) At a cost of $20 per 100 pies, they spent $204.60 on ingredients. Their profit on day two was $4910.40. OR b) They spent $220 on 1100 pies, of which they sold 1023 pies. They yield a profit of $4895.

Combining their profit from day 1 and day 2 yields a total of a) $5174.40 OR b) $5150. Therefore, the trio reached their project goal of $5000.

Community Service, Sequences, and Functions (Performance Task)

Name__________________________________ **Date**___________________

Mathematical Goals
- Use technology to graph and analyze functions
- Construct linear and exponential sequences (including reading these from a table)
- Observe the difference between linear and exponential functions

Essential Questions
- How can I construct linear and exponential functions?
- How do I compare linear and exponential functions?

GEORGIA STANDARDS OF EXCELLENCE
MGSE9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

MGSE9-12.F.LE.1b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

MGSE9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

MGSE9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

MGSE9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

STANDARDS FOR MATHEMATICAL PRACTICE
4. Model with mathematics.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Community Service, Sequences, and Functions (Performance Task)

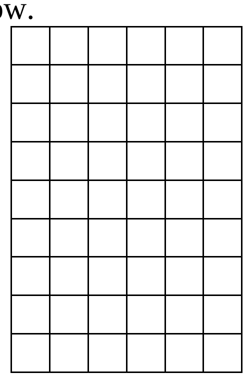
Name__________________________________ **Date**__________________

Larry, Moe, and Curly spend their free time doing community service projects. They would like to get more people involved. They began by observing the number of people who show up to the town cleanup activities each day. The data from their observations is recorded in the given table for the Great Four Day Cleanup.

x	y
1	5
2	27
3	49
4	71

1. Give a verbal description of what the domain and range presented in the table represents.

2. Sketch the data on the grid below.



3. Determine the type of function modeled in the graph above and describe key features of the graph.

4. Based on the pattern in the data collected, what recursive process could Larry, Curly, and Moe write?

5. Write a linear equation to model the function.

6. How would Larry, Curly, and Moe use the explicit formula to predict the number of people who would help if the cleanup campaign went on for 7 days?

Excited about the growing number of people participating in community service, Larry, Curly, and Moe decide to have a fundraiser to plant flowers and trees in the parks that were cleaned during the Great Four Day cleanup. It will cost them $5,000 to plant the trees and flowers. They decide to sell some of the delicious pies that Moe bakes with his sisters. For every 100 pies sold, it costs Moe and his sisters $20.00 for supplies and ingredients to bake the pies. Larry, Curly, and Moe decide to sell the pies for $5.00 each.

7. Complete the following table to find the total number of pies sold and the amount of money the trio collects.

a. On Day 1, each customer buys the same number of pies as his customer number. In other words the first customer buys 1 pie, the second customer buys 2 pies. Fill in the table showing the number of pies and the amount collected on Day 1. Then calculate the total number of pies sold and dollars collected.

Customer Number	***Number of Pies Sold***	***Amount Collected***
1	***1***	***$5***
2	***2***	***$10***
Total		

b. Write a recursive and explicit formula for the pies sold on Day 1. Explain your thinking.

c. On Day 2, the first customer buys 1 pie, the second customer buys 2 pies, the third customer buys 4 pies, the fourth customer buys 8 pies, and so on**.** Complete table based on the pattern established. Then calculate the total number of pies sold and dollars collected.

Customer Number	***Number of Pies Sold***	***Amount Collected***
1	***1***	***$5***
2	***2***	***$10***
Total		

d. Write a recursive and explicit formula for the pies sold on Day 2. Explain your thinking.

8. Compare the rates of change on Day 1 and Day 2 for the number of pies sold.

9. Did Larry, Curly, and Moe earn enough in two days to fund their project? Consider costs incurred to bake the pies. Justify your reasoning.

Birthday Gifts and Turtle Problem (Formative Assessment Lesson)

Back to Task Table

Source: Georgia Mathematics Design Collaborative

This lesson is intended to help you assess how well students are able to:

- Write linear and exponential functions from verbal sentences
- Understand the rates of change of linear functions are constant, while the rates of change of exponential functions are not constant

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

MGSE9-12.F.LE.1a Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. (This can be shown by algebraic proof, with a table showing differences, or by calculating average rates of change over equal intervals).

MGSE9-12.F.LE.1b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

MGSE9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

MGSE9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

MGSE9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

STANDARDS FOR MATHEMATICAL PRACTICE:

This lesson uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
4. Model with mathematics
6. Attend to precision

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Georgia Mathematics Design Collaborative are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding.

The task, *Birthday Gifts and Turtle Problem*, is a Formative Assessment Lesson (FAL) that can be found on the Georgia Mathematics online professional learning community, edWeb.net. Once logged in, the task can be found directly at: https://www.edweb.net/?14@@.5ad26830

Exploring Paths (Formative Assessment Lesson)

Back to Task Table

Source: Georgia Mathematics Design Collaborative

This lesson is intended to help you assess how well students are able to:

- Utilize what they already know about linear functions and exponential functions in the context of different graphs
- Reason qualitatively, compares linear and exponential models verbally, numerically, algebraically, and graphically

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.F.BF.1 Write a function that describes a relationship between two quantities.

MGSE9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

MGSE9-12.F.LE.1a Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. (This can be shown by algebraic proof, with a table showing differences, or by calculating average rates of change over equal intervals).

MGSE9-12.F.LE.1b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

MGSE9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

STANDARDS FOR MATHEMATICAL PRACTICE:

This lesson uses all of the practices with emphasis on:

4. Model with mathematics
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Georgia Mathematics Design Collaborative are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding.

The task, *Exploring Paths*, is a Formative Assessment Lesson (FAL) that can be found on the Georgia Mathematics online professional learning community, edWeb.net. Once logged in, the task can be found directly at: https://www.edweb.net/?14@@.5ad26830

Comparing Investments (Formative Assessment Lesson) Back to Task Table

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project

http://map.mathshell.org/materials/download.php?fileid=1250

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Comparing Investments*, is a Formative Assessment Lesson (FAL) that can be found at the website: http://map.mathshell.org/materials/lessons.php?taskid=426&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1250

Mathematical Goals

- Translate between descriptive, algebraic, and tabular data, and graphical representation of a function.
- Recognize how, and why, a quantity changes per unit interval.

Essential Questions

- How do I relate real-life problems to linear or exponential models?
- How can I use linear models to decide which of the two investment models is more lucrative?
- How do I apply my knowledge of linear and exponential functions to making investment decisions?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

MGSE9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

MGSE9-12.F.LE.1a Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. (This can be shown by algebraic proof, with a table showing differences, or by calculating average rates of change over equal intervals).

MGSE9-12.F.LE.1b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

MGSE9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

MGSE9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

MGSE9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

MGSE9-12.F.LE.5 Interpret the parameters in a linear ($f(x) = mx + b$) and exponential ($f(x) = a \cdot d^x$) function in terms of context. (In the functions above, "m" and "b" are the parameters of the linear function, and "a" and "d" are the parameters of the exponential function.) In context, students should describe what these parameters mean in terms of change and starting value.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
4. Model with mathematics.
7. Look for and make use of structure.

Background Knowledge

- Students can compute simple and compound interest.
- Students understand linear and exponential models.

Common Misconceptions

- Students may confuse the formulas and meanings of simple and compound interest.

Materials

- see FAL website

Grouping

- Individual / partners

Comparing Linear, Quadratic, and Exponential Models Graphically (Learning Task)

Back to Task Table

Mathematical Goals:

- Compare and contrast linear, quadratic, and exponential functions.
- Recognize the differences between the graphs of linear, quadratic, and exponential functions.

Essential Questions:

- What are unique characteristics of linear, quadratic, and exponential functions?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

MGSE9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

MGSE9-12.F.IF.1 Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and f(x) is the output (an element of the range). Graphically, the graph is y = f(x).

MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity.~~

MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

MGSE9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.*

STANDARDS FOR MATHEMATICAL PRACTICE

4. Model with mathematics by expecting students to apply the mathematics concepts they know in order to solve problems arising in everyday situations, and reflect on whether the results are sensible for the given scenario.

5. Use appropriate tools strategically by expecting students to consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a compass, a calculator, software, etc.

6. Attend to precision by requiring students to calculate efficiently and accurately; and to communicate precisely with others by using clear mathematical language to discuss their reasoning.

7. Look for and make use of structure by expecting students to apply rules, look for patterns and analyze structure.

8. Look for and express regularity in repeated reasoning by expecting students to understand broader applications and look for structure and general methods in similar situations.

Materials Needed:

- Graph Paper

Grouping:

- Individual/Partner

Time Needed:

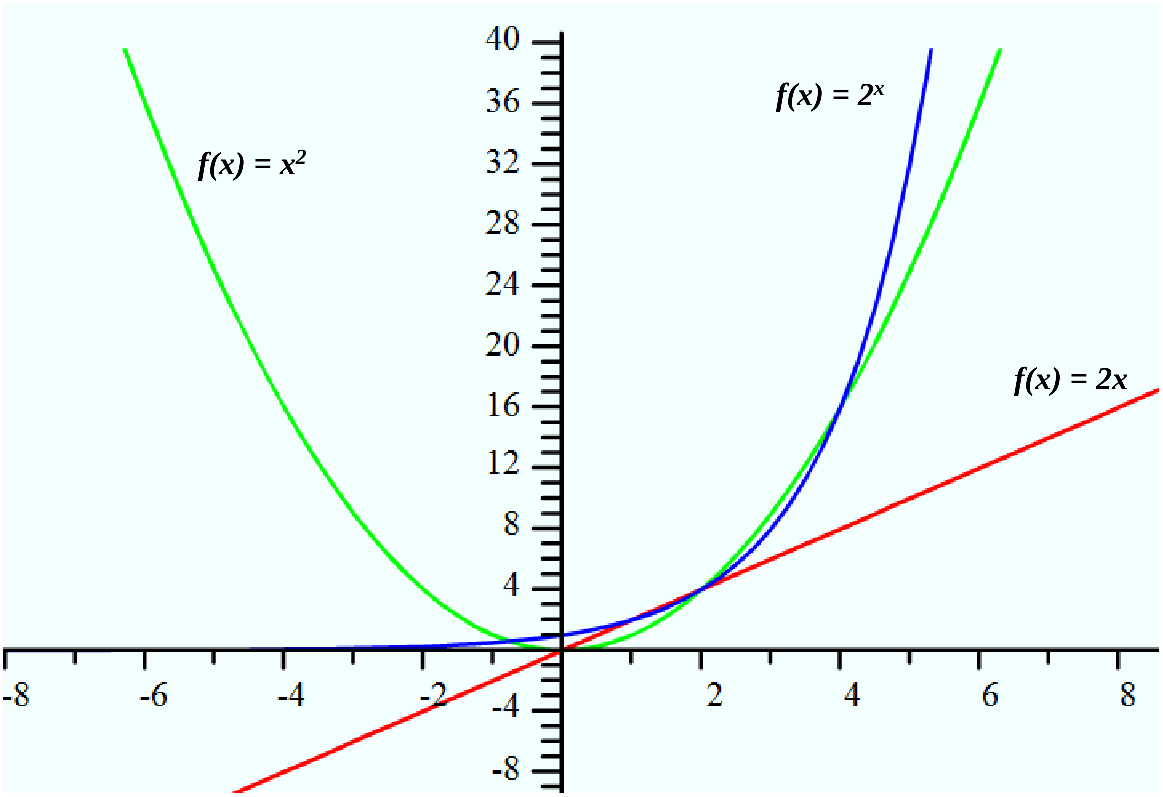
- 50-60 minutes

<u>Comparing Linear, Quadratic, and Exponential Models Graphically (Learning Task) – Teacher Notes</u>

1. Complete the tables below.

Linear		**Quadratic**		**Exponential**	
$f(x) = 2x$		$g(x) = x^2$		$h(x) = 2^x$	
x	$f(x)$	x	$g(x)$	x	$h(x)$
–5	*–10*	**–5**	*25*	**–5**	*1/32*
–4	*–8*	**–4**	*16*	**–4**	*1/16*
–3	*–6*	**–3**	*9*	**–3**	*1/8*
–2	*–4*	**–2**	*4*	**–2**	*1/4*
–1	*–2*	**–1**	*1*	**–1**	*1/2*
0	*0*	**0**	*0*	**0**	*1*
1	*2*	**1**	*1*	**1**	*2*
2	*4*	**2**	*2*	**2**	*4*
3	*6*	**3**	*3*	**3**	*8*
4	*8*	**4**	*4*	**4**	*16*
5	*10*	**5**	*5*	**5**	*32*

2. Draw and label each graph on the same set of axes.



3. Identify the following features of each function.
(a) Domain and Range

- *$f(x) = 2x$* — ***Domain and Range are All Real Numbers (-∞, ∞)***
- *$g(x) = x^2$* — ***Domain is All Real Numbers (-∞, ∞); Range is g(x) ≥ 0 [0, ∞)***
- *$h(x) = 2^x$* — ***Domain is All Real Numbers (-∞, ∞); Range is h(x) > 0 [0, ∞)***

(b) Description of Shape

Answers may vary; some options:

- ***f(x) is a straight line that rises on the right and falls on the left.***
- ***g(x) is a parabola (curve) which opens up***
- ***h(x) is a curve that is almost flat on the left side and then rises sharply on the right as the x-values increase***

(c) Any characteristics unique to each function

Answers may vary. Guide students to think about topics such as…

- ***which quadrants are occupied by each function;***
- ***how the output values are all positive for g(x) and h(x); would that ALWAYS be the case;***
- ***end behaviors and how this connects to even and odd functions;***
- ***any maxima and minima***
- ***when are the outputs the same for these functions (if ever);***
- ***etc.---encourage students to look for as many similarities and differences as possible.***

Comparing Linear, Quadratic, and Exponential Models Graphically (Learning Task)

Name________________________ Date___________________

Mathematical Goals:

- Compare and contrast linear, quadratic, and exponential functions.
- Recognize the differences between the graphs of linear, quadratic, and exponential functions.

Essential Questions:

- What are unique characteristics of linear, quadratic, and exponential functions?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

MGSE9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

MGSE9-12.F.IF.1 Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and f(x) is the output (an element of the range). Graphically, the graph is y = f(x).

MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity.~~

MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

MGSE9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.*

STANDARDS FOR MATHEMATICAL PRACTICE

4. Model with mathematics by expecting students to apply the mathematics concepts they know in order to solve problems arising in everyday situations, and reflect on whether the results are sensible for the given scenario.

5. Use appropriate tools strategically by expecting students to consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a compass, a calculator, software, etc.

6. Attend to precision by requiring students to calculate efficiently and accurately; and to communicate precisely with others by using clear mathematical language to discuss their reasoning.

7. Look for and make use of structure by expecting students to apply rules, look for patterns and analyze structure.

8. Look for and express regularity in repeated reasoning by expecting students to understand broader applications and look for structure and general methods in similar situations.

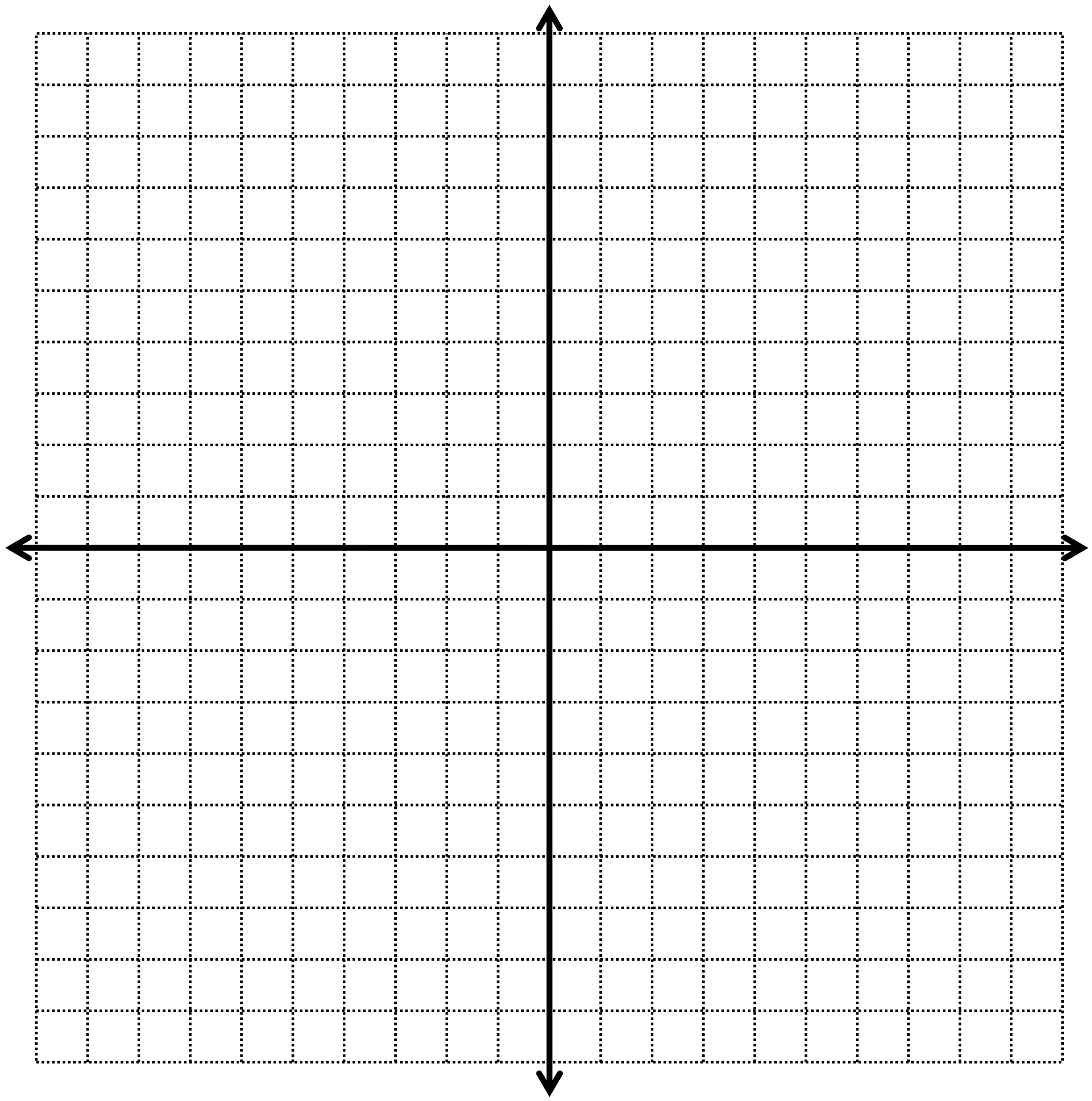
Comparing Linear, Quadratic, and Exponential Models Graphically (Learning Task)

Name_______________________________ **Date__________________**

1. Complete the tables below.

Linear			**Quadratic**			**Exponential**	
$f(x) = 2x$			$g(x) = x^2$			$h(x) = 2^x$	
x	$f(x)$		x	$g(x)$		x	$h(x)$
–5			–5			–5	
–4			–4			–4	
–3			–3			–3	
–2			–2			–2	
–1			–1			–1	
0			0			0	
1			1			1	
2			2			2	
3			3			3	
4			4			4	
5			5			5	

2. Draw and label each graph on the same set of axes.



3. Identify the following features of each function.

   (a) Domain and Range

   (b) Description of Shape

   (c) Any characteristics unique to each function

Paula's Peaches: The Sequel (Extension Task)

Back to Task Table

Mathematical Goals

- Analyze quadratic functions
- Interpret key features of functions

Essential Questions

- How can I interpret quadratic functions?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

MGSE9-12.F.IF.1 Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and f(x) is the output (an element of the range). Graphically, the graph is y = f(x).

MGSE9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity.~~

MGSE9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them by requiring students to interpret and make meaning of a problem and find a logical starting point, and to monitor their progress and change their approach to solving the problem, if necessary.

2. Reason abstractly and quantitatively by requiring students to make sense of quantities and their relationships to one another in problem situations.

4. Model with mathematics by expecting students to apply the mathematics concepts they know in order to solve problems arising in everyday situations, and reflect on whether the results are sensible for the given scenario.

5. Use appropriate tools strategically by expecting students to consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a compass, a calculator, software, etc.

7. Look for and make use of structure by expecting students to apply rules, look for patterns and analyze structure.

8. Look for and express regularity in repeated reasoning by expecting students to understand broader applications and look for structure and general methods in similar situations.

Materials Needed:

- Graph Paper

Grouping:

- Individual/Partner

Time Needed:

- 60-75 minutes

Paula's Peaches: The Sequel (Extension Task) – Teacher Notes

Common Student Misconceptions

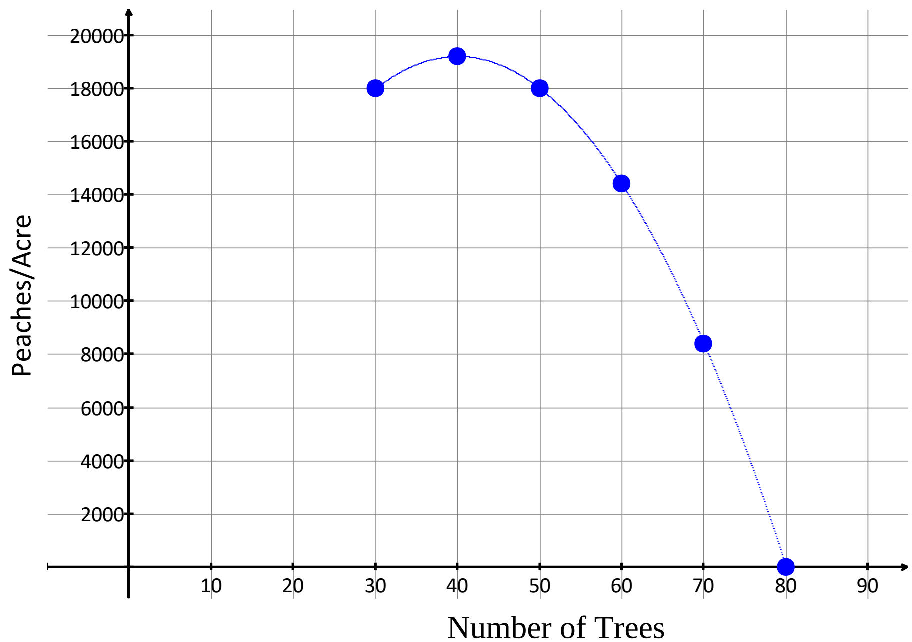
1. Students may believe that the use of algebraic expressions is merely the abstract manipulation of symbols. Use of real–world context examples to demonstrate the meaning of the parts of algebraic expressions is needed to counter this misconception.
2. Students may also believe that an expression cannot be factored because it does not fit into a form they recognize. They need help with reorganizing the terms until structures become evident.
3. Students will often combine terms that are not like terms. For example, $2 + 3x = 5x$ or $3x+ 2y= 5xy$.
4. Students sometimes forget the coefficient of 1 when adding like terms. For example, $x + 2x + 3x = 5x$ rather than $6x$.
5. Students will change the degree of the variable when adding/subtracting like terms. For example, $2x+ 3x= 5x^2$ rather than $5x$.
6. Students will forget to distribute to all terms when multiplying. For example, $6(2x + 1) = 12x + 1$ rather than $12x+ 6$.
7. Students may not follow the Order of Operations when simplifying expressions. For example, $4x^2$ when $x = 3$ may be incorrectly evaluated as $4•3^2 = 12^2 = 144$, rather than $4•9 = 36$. Another common mistake occurs when the distributive property should be used prior to adding or subtracting. For example, $2 + 3(x – 1)$ incorrectly becomes $5(x – 1) = 5x – 5$ instead of $2 + 3(x – 1) = 2 + 3x – 3 = 3x – 1$.
8. Students fail to use the property of exponents correctly when using the distributive property. For example, $3x(2x – 1) = 6x – 3x = 3x$ instead of simplifying as $3x(2x – 1) = 6x^2–3x$.
9. Students fail to understand the structure of expressions. For example, they will write $4x$ when $x = 3$ is 43 instead of $4x = 4•x$ so when $x = 3$, $4x = 4•3 = 12$. In addition, students commonly misevaluate $–3^2 = 9$ rather than $–3^2 = –9$. Students routinely see $–3^2$ as the same as $(–3)^2 = 9$. A method that may clear up the misconception is to have students rewrite as $–x^2 = –1•x^2$ so they know to apply the exponent before the multiplication of –1.
10. Students frequently attempt to "solve" expressions. Many students add "= 0" to an expression they are asked to simplify. Students need to understand the difference between an equation and an expression.
11. Students commonly confuse the properties of exponents, specifically the product of powers property with the power of a power property. For example, students will often simplify $(x^2)^3 = x^5$ instead of x^6.
12. Students will incorrectly translate expressions that contain a difference of terms. For example, 8 less than 5 times a number is often incorrectly translated as $8 – 5n$ rather than $5n – 8$.
13. Students may believe that equations of linear, quadratic and other functions are abstract and exist on "in a math book," without seeing the usefulness of these functions as modeling real–world phenomena.

14. Additionally, they believe that the labels and scales on a graph are not important and can be assumed by a reader and that it is always necessary to use the entire graph of a function when solving a problem that uses that function as its model.
15. Students may interchange slope and y–intercept when creating equation. For example, a taxi cab cost \$4 for a dropped flag and charges \$2 per mile. Students may fail to see that \$2 is a rate of change and is slope while the \$4 is the starting cost and incorrectly write the equation as $y = 4x + 2$ instead of $y = 2x + 4$.
16. Given a graph of a line, students use the x–intercept for b instead of the y–intercept.
17. Given a graph, students incorrectly compute slope as run over rise rather than rise over run. For example, they will compute slope with the change in x over the change in y.
18. Students do not know when to include the "or equal to" bar when translating the graph of an inequality.
19. Students do not correctly identify whether a situation should be represented by a linear, quadratic, or exponential function.
20. Students often do not understand what the variables represent. For example, if the height h in feet of a piece of lava t seconds after it is ejected from a volcano is given by $h(t) = –16t^2 + 64t + 936$ and the student is asked to find the time it takes for the piece of lava to hit the ground, the student will have difficulties understanding that $h = 0$ at the ground and that they need to solve for t.
21. Students may believe that all functions have a first common difference and need to explore to realize that, for example, a quadratic function will have equal second common differences in a table.
22. Students may also believe that the end behavior of all functions depends on the situation and not the fact that exponential function values will eventually get larger than those of any other polynomial functions.

This task allows students to further explore quadratic functions by factoring and studying more about the vertex. Students also will look at quadratic inequalities (extension of standards).

In this task, we revisit Paula, the peach grower who wanted to expand her peach orchard. In the established part of her orchard, there are 30 trees per acre with an average yield of 600 peaches per tree. We saw that the number of peaches per acre could be modeled with a quadratic function. The function *f(x)* where *x* represents the number of trees and *f(x)* represents the number of peaches per acre. This is given below.

$$\boldsymbol{f(x) = -12x^2 + 960x}$$



1. Remember that Paula wants to average more peaches. Her current yield with 30 trees is 18,000 peaches per acre and is represented by $f(30) = 18{,}000$. (Do you see this point on the graph?)

   a. Use the function above to write an **inequality** to express the average yield of peaches per acre to be at least 18,000.
   Comment(s):
   This item begins the introduction to quadratic inequalities (extension of standards), but students will need additional work with quadratic inequalities beyond the introduction in this task. A homework assignment should include quadratic inequalities in which "greater than", "less than", "less than or equal to", and "greater than or equal to" inequalities are investigated. In the task students are asked to solve inequalities graphically and to relate the solution of the corresponding equation to the solution. This work gives the basis for meeting the standard of solving inequalities graphically and algebraically. There are several methods for solving quadratic inequalities algebraically. These include: (i) finding points of equality on a number line and testing values between these points to determine their truth values and(ii) graphing points of equality on a number line and using positive vs. negative values of the expression's products to determine the truth value. Most students can be successful with the first algebraic method; teachers should decide the extent to which to pursue the other method with their students. The simplest method for solving quadratic inequalities involves

combining algebra and geometry to find the points of equality and use these to create subintervals of the number line and then consider the shape of the graph, concave up or concave downward, to determine which of the subintervals to include in the solution.

Solution(s):
In the established part of the orchard, the yield is (600 peaches per tree)(30 tress per acre) = 18000 peaches per acre.
The inequality is: $960x - 12x^2 \geq 18000$, ***or an equivalent version***

b. Since Paula desires at least 18,000 peaches per acre, draw a horizontal line showing her goal. Does this **line** represent her goal of **at least** 18,000 peaches per acre? Why or why not?

c. Shade the region that represents her goal of **at least** 18,000 peaches per acre.

d. Use the graph above to answer the following question: How many trees can Paula plant in order to yield at least 18,000 peaches? Write your solution as a compound inequality.
Comment(s):
Since the focus here is solving quadratic inequalities (extension of standards), the fact that there are only integer values in the domain does not need to be emphasized. The solution below shows an inequality with a notation that x is an integer. If students list the possible values of x, they are not wrong, but should be asked to write the inequality version of the solution since a larger finite set of values would make it impractical to list all of the solutions and since using an inequality is consistent with solutions in the continuous case. Students should have the correct endpoint values since they involve points whose coordinates were determined exactly in the earlier task and are likely labeled on students graphs.

Solution(s):
Examination of the graph shows that the solution consists of the domain values for points on or above the horizontal line y = 18000. Thus,

x is an integer and $30 \leq x \leq 50$,

or (listing the solutions as a finite set) x is an element of the set **{30, 31, 32, 33, 34, 35, 36, 37. 38, 39, 40}.**

e. What is the domain of *f(x)*? In the context of Paula's Peaches, is your answer representative of the domain of *f(x)*?
See above.

2. Now let's find the answer to question 1 algebraically as opposed to graphically.

a. Rewrite your inequality from 1a as an equation by replacing the inequality with an equal sign.

b. The equation you just wrote is known as a **corresponding equation.** Solve the corresponding equation.
Comment(s):
Students are asked to solve the equation as review.

Solution(s):

$$18000 = 960x - 12x^2$$

$$12x^2 - 960x + 18000 = 0$$

$$12(x^2 - 80x + 1500) = 0$$

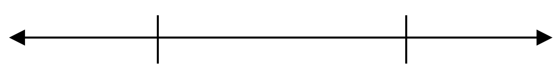
$$x^2 - 80x + 1500 = 0$$

$$(x - 30)(x - 50) = 0$$

$$x = 30 \text{ or } x = 50$$

These solutions give the exact values for the endpoints of the inequality.

c. When solving an inequality the solutions of the corresponding equation are called **critical values**. These values determine where the inequality is true or false. Place the critical values on the number line below. From your original inequalities, use an open circle if the value is not included and closed circle if value is included.



d. You can now test your inequality by substituting any value from a given interval of the number line into the inequality created in 1a (original inequality). All intervals that test true are your solutions. Test each interval (left, middle, right) from your number line above. Then indicate your testing results by shading the appropriate intervals. Write your solution as a compound inequality.
$30 \le x \le 50$

e. Compare your test in 2d to your answer in 1d. What do you notice?
They are the same.

3. Paula must abide by a government regulation that states any orchard that produces more than 18,432 peaches will be taxed.

a. Write an inequality to express when she will not be taxed.

$$-12x^2 + 960x \leq 18432$$

b. Write a *corresponding equation* and solve, finding the critical values.

$-12x^2 + 960x = 18432$

$-12x^2 + 960x - 18432 = 0$

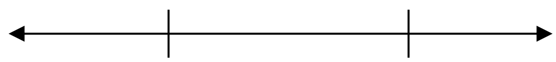
$-12(x^2 - 80x + 1536) = 0$

$x^2 - 80x + 1536 = 0$

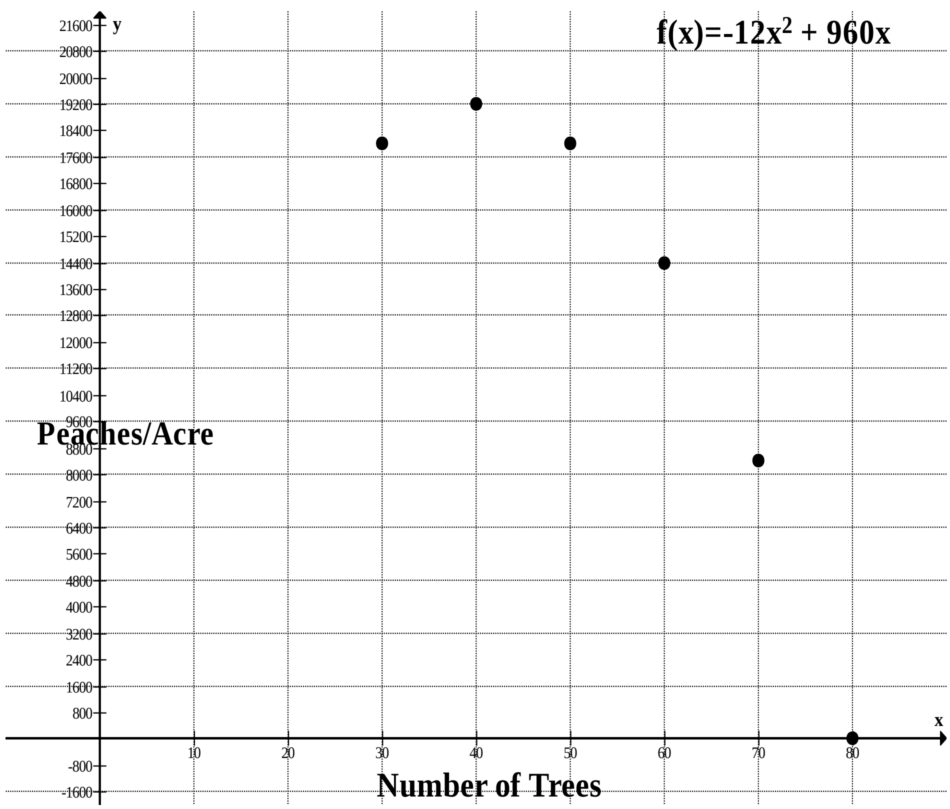
$(x - 32)(x - 48) = 0$

Critical values are 32 and 48

c. Now use a number line to solve the inequality as in part 2d. Write your answer as an inequality.



$32 \leq x \leq 48$



4. One year, a frost stunted production and the maximum possible yield was 14,400 peaches per acre.

   a. Write an inequality for this level of peach production using the function above.
   $-12x^2 + 960 \leq 14400$

   b. Since parabolas are symmetric, plot the reflective points on the graph above.

   c. Draw a horizontal line representing the maximum possible yield 14,400.

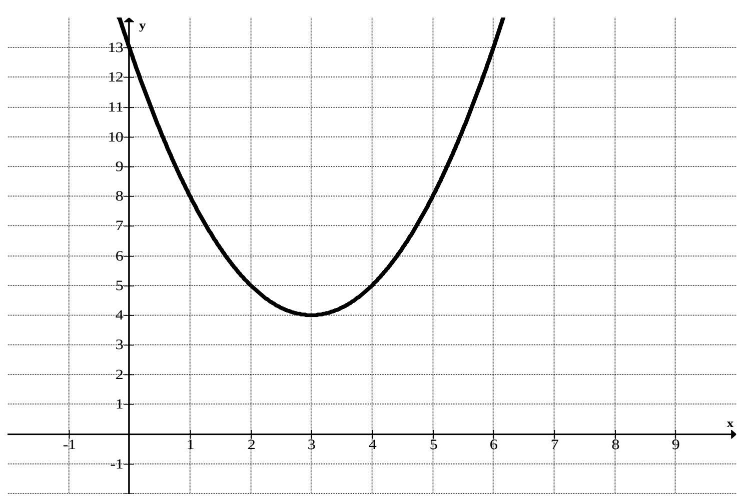
   d. Shade the region that represents her maximum yield of 14,400 peaches per acre.

   e. Are any of these values not in the original domain? Explain your answer and write your final solution as an inequality.
   The answer to the inequality is $20 \leq x \leq 60$, however, this is more extensive than the original domain, $30 \leq x \leq 40$, so the solution must be the original domain.

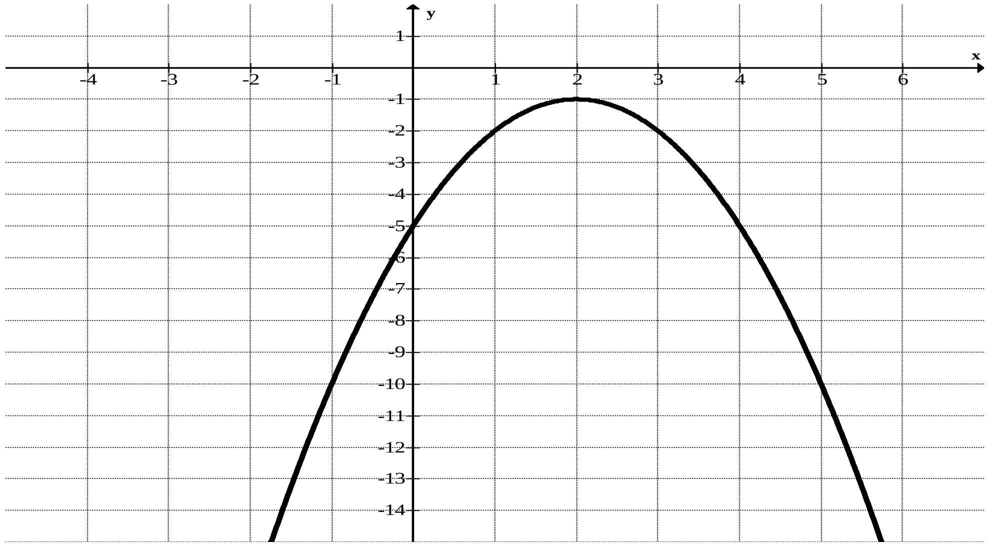
Practice Problems

For each graph below, solve the given inequality, writing your solution as an inequality. Each quadratic equation is given as *f(x).*

1. $f(x) \geq 8$

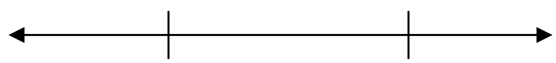


2. $f(x) \geq -10$



3. Solve the following inequalities algebraically.

a.) $x^2 - 4x - 2 < -5$

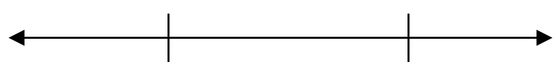


$x^2 - 4x + 3 < 0$

$(x-3)(x-1) < 0$

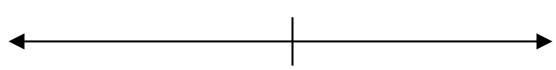
End points are 1 and 3. Graph is shaded between 1 and 3. Using 2 as a test point yields a correct inequality –6 < –5. Using 0 as a test point yields an incorrect inequality –2 < –5.

b.) $3x^2 - 5x - 8 > 4$



$3x^2 - 5x - 12 > 0$
$(3x+4)(x-3) > 0$
End points are –4/3 and 3. Graph is shaded to the left of –4/3 and to the right of 3. Using 0 as a test point yields an incorrect inequality –8>4.

c) $x^2 + 6x \geq -9$



$x^2 + 6x + 9 \geq 0$
$(x+3)(x+3) \geq 0$
The only end point is 3. The entire graph is shaded because any number will yield a correct inequality.

Paula's Peaches: The Sequel (Extension Task)

Mathematical Goals

- Analyze quadratic functions
- Interpret key features of functions

Essential Questions

- How can I interpret quadratic functions?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

MGSE9-12.F.IF.1 Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and f(x) is the output (an element of the range). Graphically, the graph is y = f(x).

MGSE9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity.~~

MGSE9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them by requiring students to interpret and make meaning of a problem and find a logical starting point, and to monitor their progress and change their approach to solving the problem, if necessary.

2. Reason abstractly and quantitatively by requiring students to make sense of quantities and their relationships to one another in problem situations.

4. Model with mathematics by expecting students to apply the mathematics concepts they know in order to solve problems arising in everyday situations, and reflect on whether the results are sensible for the given scenario.

5. Use appropriate tools strategically by expecting students to consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a compass, a calculator, software, etc.

7. Look for and make use of structure by expecting students to apply rules, look for patterns and analyze structure.

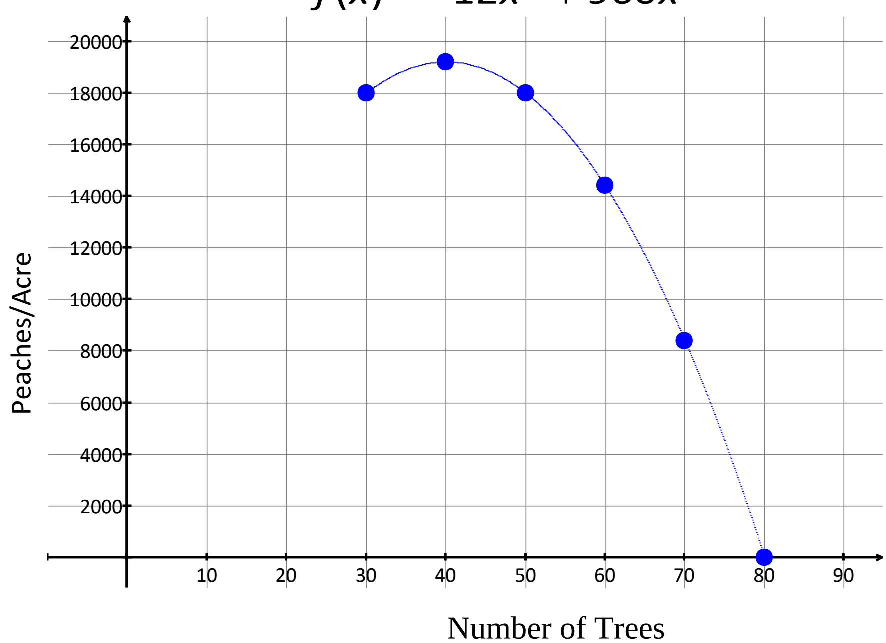
8. Look for and express regularity in repeated reasoning by expecting students to understand broader applications and look for structure and general methods in similar situations.

Paula's Peaches: The Sequel (Extension Task)

Name______________________________ **Date**__________________

In this task, we revisit Paula, the peach grower who wanted to expand her peach orchard. In the established part of her orchard, there are 30 trees per acre with an average yield of 600 peaches per tree. We saw that the number of peaches per acre could be modeled with a quadratic function. The function $f(x)$ where x represents the number of trees and $f(x)$ represents the number of peaches per acre. This is given below.

$$f(x) = -12x^2 + 960x$$



1. Remember that Paula wants to average more peaches. Her current yield with 30 trees is 18,000 peaches per acre and is represented by $f(30) = 18{,}000$. (Do you see this point on the graph?)

   a. Use the function above to write an **inequality** to express the average yield of peaches per acre to be at least 18,000.

   b. Since Paula desires at least 18,000 peaches per acre, draw a horizontal line showing her goal. Does this **line** represent her goal of **at least** 18,000 peaches per acre? Why or why not?

   c. Shade the region that represents her goal of **at least** 18,000 peaches per acre.

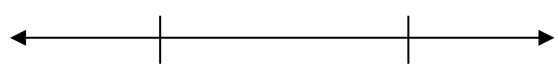
d. Use the graph above to answer the following question: How many trees can Paula plant in order to yield at least 18,000 peaches? Write your solution as a compound inequality.

e. What is the domain of $f(x)$? In the context of Paula's Peaches, is your answer representative of the domain of $f(x)$?

2. Now let's find the answer to question 1 algebraically as opposed to graphically.
   a. Rewrite your inequality from 1a as an equation by replacing the inequality with an equal sign.

   b. The equation you just wrote is known as a **corresponding equation.** Solve the corresponding equation.

   c. When solving an inequality the solutions of the corresponding equation are called **critical values**. These values determine where the inequality is true or false. Place the critical values on the number line below. From your original inequalities, use an open circle if the value is not included and closed circle if value is included.



   d. You can now test your inequality by substituting any value from a given interval of the number line into the inequality created in 1a (original inequality). All intervals that test true are your solutions. Test each interval (left, middle, right) from your number line above. Then indicate your testing results by shading the appropriate intervals. Write your solution as a compound inequality.

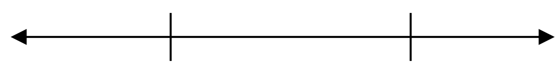
   e. Compare your test in 2d to your answer in 1d. What do you notice?

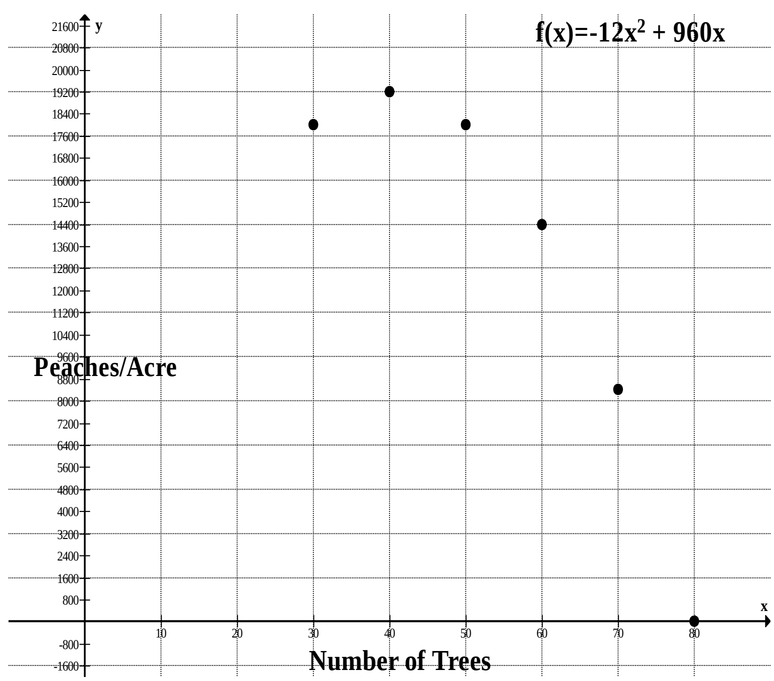
3. Paula must abide by a government regulation that states any orchard that produces more than 18,432 peaches will be taxed.

   a. Write an inequality to express when she will not be taxed.

   b. Write a *corresponding equation* and solve, finding the critical values.

   c. Now use a number line to solve the inequality as in part 2d. Write your answer as an inequality.





4. One year, a frost stunted production and the maximum possible yield was 14,400 peaches per acre.

   a. Write an inequality for this level of peach production using the function above.

   b. Since parabolas are symmetric, plot the reflective points on the graph above.

   c. Draw a horizontal line representing the maximum possible yield 14,400.

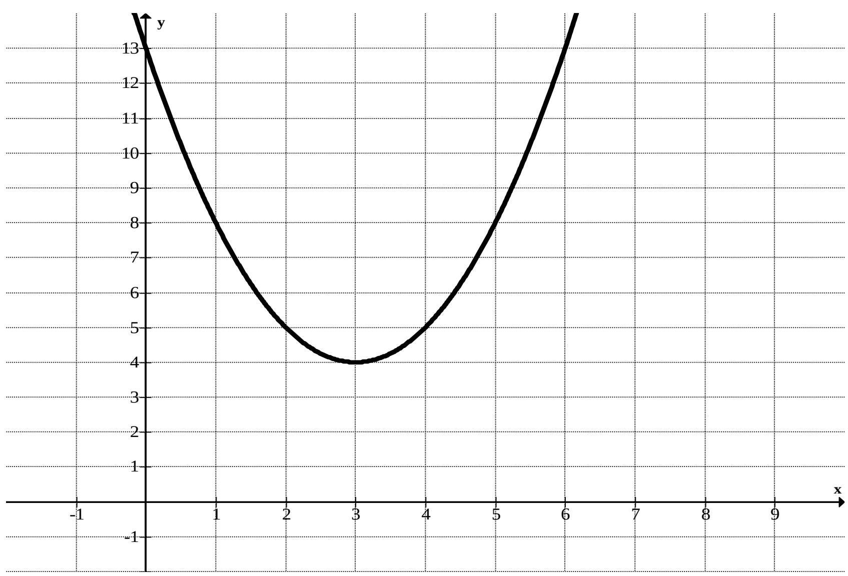
   d. Shade the region that represents her maximum yield of 14,400 peaches per acre.

   e. Are any of these values not in the original domain? Explain your answer and write your final solution as an inequality.

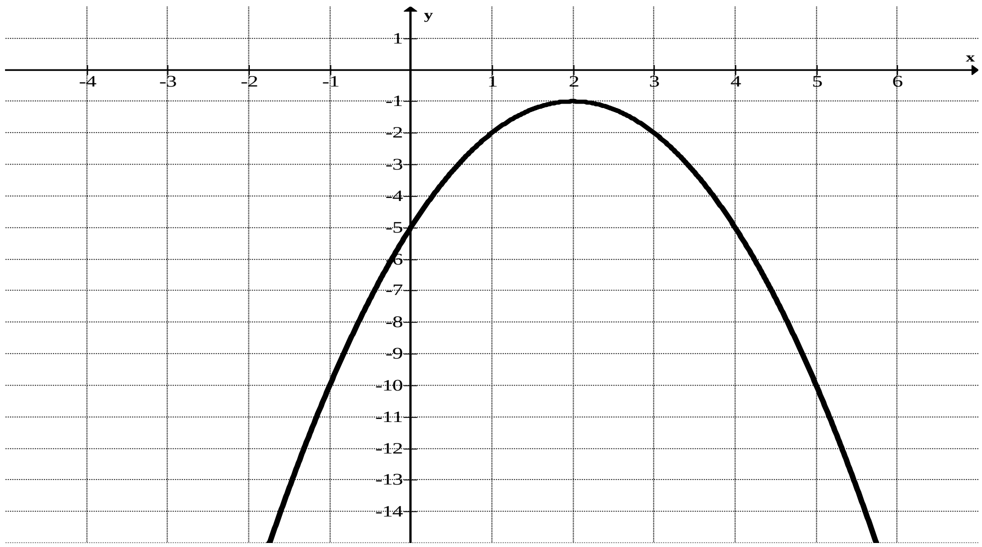
Practice Problems

For each graph below, solve the given inequality, writing your solution as an inequality. Each quadratic equation is given as $f(x)$.

1. $f(x) \geq 8$

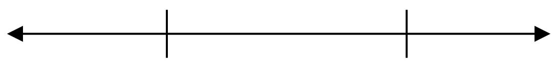


2. $f(x) \geq -10$

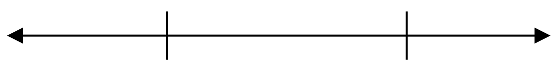


3. Solve the following inequalities algebraically.

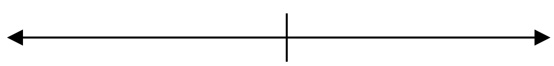
a.) $x^2 - 4x - 2 < -5$



b.) $3x^2 - 5x - 8 > 4$



c) $x^2 + 6x \geq -9$



Fences and Functions (Culminating Task)

Back to Task Table

Mathematical Goals:

- Compare and contrast linear, quadratic, and exponential functions
- Interpret key features of functions in context

Essential Questions:

- How do I distinguish between linear, quadratic, and exponential situations?
- How do I represent linear, quadratic, and exponential situations?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

MGSE9-12.F.LE.5 Interpret the parameters in a linear ($f(x) = mx + b$) and exponential ($f(x) = a \cdot d^x$) function in terms of context. (In the functions above, "*m*" and "*b*" are the parameters of the linear function, and "*a*" and "*d*" are the parameters of the exponential function.) In context, students should describe what these parameters mean in terms of change and starting value.

MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

MGSE9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions*). For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Materials Needed:

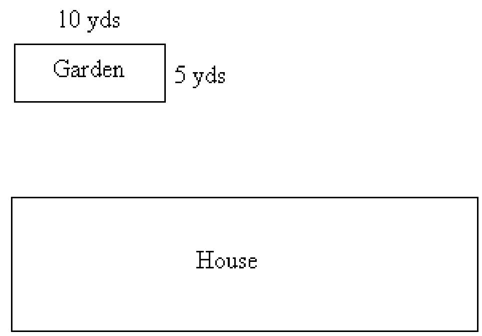
- Graph Paper

Grouping:
- Individual/Partner

Time Needed:
- 50-60 minutes

Fences and Functions (Culminating Task) – Teacher Notes

Claire decided to plant a rectangular garden in her back yard using 30 yards of chain-link fencing that were given to her by a friend. Claire wanted to determine the possible dimensions of her garden, assuming that she would use all of the fencing and make each side a whole number of yards. She began making a drawing that would give her a garden 5 yards by 10 yards.



1. Claire looked at the drawing and decided that she wanted to consider other possibilities for the dimensions of the garden. In order to organize her thoughts, she let x be the garden dimension parallel to the back of her house, measured in yards, and let y be the other dimension, perpendicular to the back of the house, measured in yards. She recorded the first possibility for the dimensions.

   a. Make a table showing the possible values for x and y.

   Comments: Guide students to see the pattern that develops when completing the table. Ask: "Why can't one side length be 15 yards?"

x	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*
y	*14*	*13*	*12*	*11*	*10*	*9*	*8*	*7*	*6*	*5*	*4*	*3*	*2*	*1*

   b. Find the perimeter of each of the possible gardens you listed in part a. What do you notice? Explain why this happens.

   Comments: You may choose to have students add another row to the table above. Remind students of the formula for finding the perimeter of a rectangle: $P = 2L + 2W$

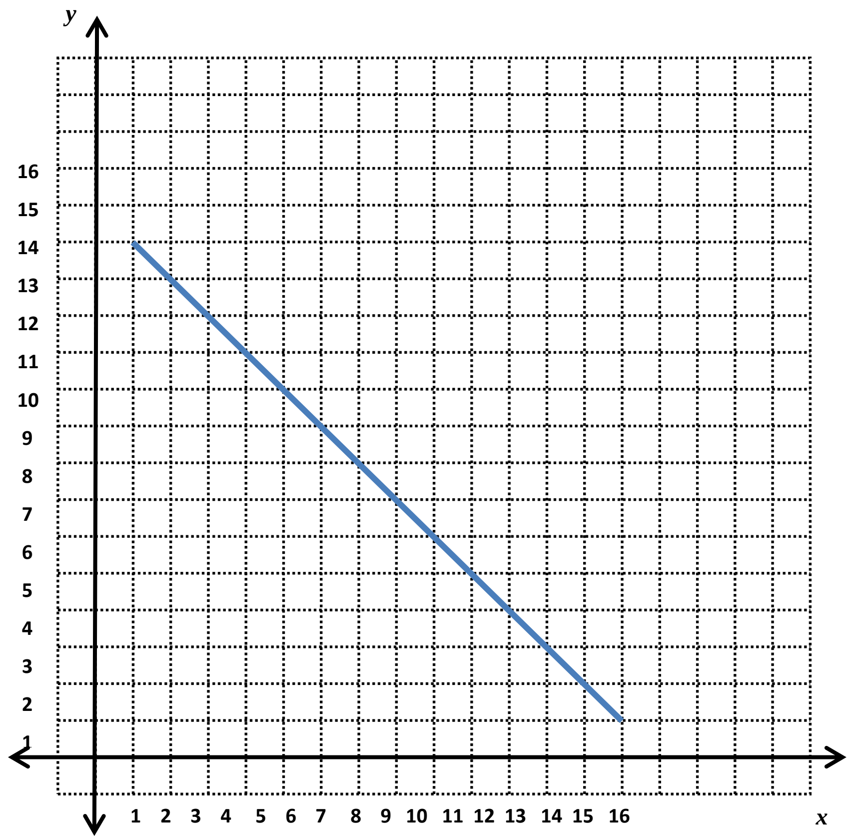
x	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*
y	*14*	*13*	*12*	*11*	*10*	*9*	*8*	*7*	*6*	*5*	*4*	*3*	*2*	*1*
P	*30*	*30*	*30*	*30*	*30*	*30*	*30*	*30*	*30*	*30*	*30*	*30*	*30*	*30*

   c. Write a formula relating the y-dimension of the garden to the x-dimension.

   Comments: Ask students to decide whether this table represents a linear, quadratic, or exponential function and to explain their reasoning. They should see that rate of change is –1, resulting in a linear relationship. $y = -x + 15$

   d. Make a graph of the possible dimensions of Claire's garden.

Comments: Students should recognize that the graph of this function will be constrained to quadrant 1 since the side lengths cannot be zero or negative. Discuss the fact that while Claire wanted the side lengths to be a whole number of yards, chain link fencing could be turned at any point. Therefore, the graph can be displayed as a continuous function.



2. After listing the possible rectangular dimensions of the garden, Claire realized that she needed to pay attention to the area of the garden, because area determines how many plants can be grown.

   a. Using the possible side lengths from your work above, make a table showing the possible areas. What do you notice? Explain why this happens.

Comments: You may choose to have students add another row to the table from 1b. Remind students of the formula for finding the area of a rectangle: A = bh.

x	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*
y	*14*	*13*	*12*	*11*	*10*	*9*	*8*	*7*	*6*	*5*	*4*	*3*	*2*	*1*
P	*30*	*30*	*30*	*30*	*30*	*30*	*30*	*30*	*30*	*30*	*30*	*30*	*30*	*30*
A	*14*	*26*	*36*	*44*	*50*	*54*	*56*	*56*	*54*	*50*	*44*	*36*	*23*	*14*

b. Write a formula describing the area of the garden in terms of *x* and *y*.

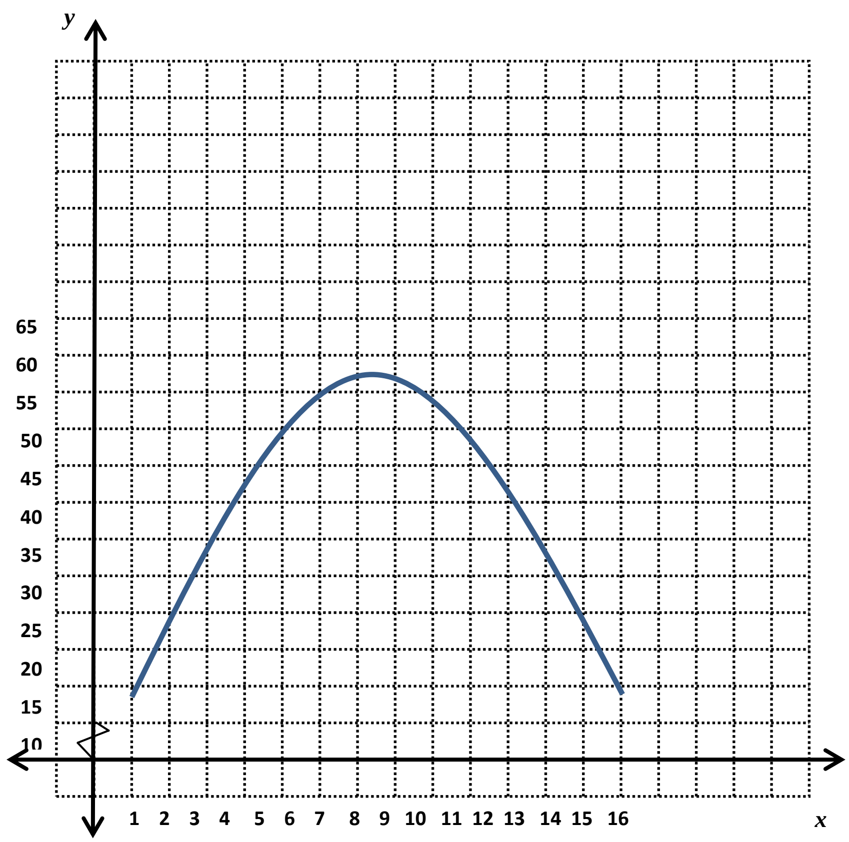
Comments: Ask students to decide whether this function represents a linear, quadratic, or exponential function. They should recognize that the rate of change has neither a common first difference or common ratio. Guide them to check the second difference, which will allow them to see that this is a quadratic function. After examining the table, students will recognize that the vertex of the function is somewhere between 7 and 8. Guide students to realize that a square with side lengths of 7.5 will maximize the area, thus giving the vertex of the graph as (7.5, 56.25). From this information the students can write the equation of the function in vertex form:

$$y = -(x - 7.5)^2 + 56.25$$

You may want the students to convert to standard form: $y = -x^2 + 15x$

c. Make a graph showing the relationship between the x-dimension and the area of the garden.

Comments: Notice that a different scale is used on the vertical axis so that all of the data values will fit on the coordinate plane and clearly show that this is a quadratic function. The graph is constructed as a continuous function with the same constraints as mentioned in 1d.



3. Later in the summer as the garden plants were fading, Claire decided that she would raise rabbits. She pulled out the dead plants and cleaned up the area. Her research showed that each rabbit needs 2 square feet of space in a pen, and that rabbits reproduce every

month, having litters of about 6 kits. She started with 2 rabbits (one male and one female). Claire began tracking the number of rabbits at the end of each month and displayed her data in the table:

# of months	0	1	2
# of rabbits	2	8	32

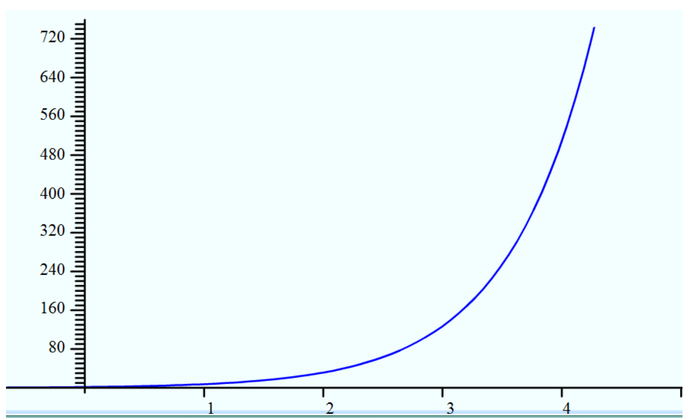
a. Write a formula relating the # of months and the # of rabbits.

Comments: Ask students to decide if the table represents a linear, quadratic, or exponential function. They should quickly see that this is neither a linear nor an quadratic function. Guide the students to recognize that the number of rabbits is increasing by a common percent showing that this is an exponential function.

$$y = 2(4)^x$$

b. Make a graph showing the relationship between the # of months and the # of rabbits.

Comments: Students will need to adjust the scale on the vertical axis in order to accommodate the rapidly increasing number of rabbits. The graph is constructed as a continuous function even though the domain will be only the set of integers greater than or equal to 2. This should help the students see how the population changes quickly.



c. The dimensions of the rabbit pen (formerly the garden) are 7 yards by 8 yards. When will Claire run out of room for her rabbits? Explain.

Comments: Students will need to use unit analysis to convert square yards into square feet. Then they may use their equation from part 3a to extend the table.
Claire's pen has 504 ft^2 of space. After 4 months, Claire will have 512 rabbits and will be out of space.

Fences and Functions (Culminating Task)

Name_____________________________ **Date___________**

Mathematical Goals:

- Compare and contrast linear, quadratic, and exponential functions
- Interpret key features of functions in context

Essential Questions:

- How do I distinguish between linear, quadratic, and exponential situations?
- How do I represent linear, quadratic, and exponential situations?

GEORGIA STANDARDS OF EXCELLENCE

MGSE9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

MGSE9-12.F.LE.5 Interpret the parameters in a linear ($f(x) = mx + b$) and exponential ($f(x) = a \cdot d^x$) function in terms of context. (In the functions above, "*m*" and "*b*" are the parameters of the linear function, and "*a*" and "*d*" are the parameters of the exponential function.) In context, students should describe what these parameters mean in terms of change and starting value.

MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

MGSE9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions*). For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.*

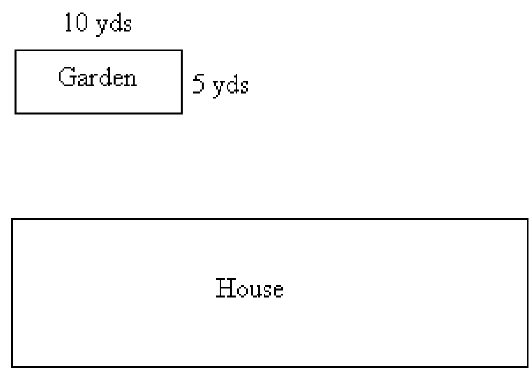
STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Fences and Functions (Culminating Task)

Name______________________________ **Date______________**

Claire decided to plant a rectangular garden in her back yard using 30 yards of chain-link fencing that were given to her by a friend. Claire wanted to determine the possible dimensions of her garden, assuming that she would use all of the fencing and make each side a whole number of yards. She began making a drawing that would give her a garden 5 yards by 10 yards.



1. Claire looked at the drawing and decided that she wanted to consider other possibilities for the dimensions of the garden. In order to organize her thoughts, she let *x* be the garden dimension parallel to the back of her house, measured in yards, and let *y* be the other dimension, perpendicular to the back of the house, measured in yards. She recorded the first possibility for the dimensions.

   a. Make a table showing the possible values for *x* and *y*.

   b. Find the perimeter of each of the possible gardens you listed in part a. What do you notice? Explain why this happens.

   c. Write a formula relating the *y*-dimension of the garden to the *x*-dimension.

   d. Make a graph of the possible dimensions of Claire's garden.

2. After listing the possible rectangular dimensions of the garden, Claire realized that she needed to pay attention to the area of the garden, because area determines how many plants can be grown.

   a. Using the possible side lengths from your work above, make a table showing the possible areas. What do you notice? Explain why this happens.

   b. Write a formula relating the *y*-dimension of the garden to the *x*-dimension.

   c. Make a graph showing the relationship between the *x*-dimension and the area of the garden.

3. Later in the summer as the garden plants were fading, Claire decided that she would raise rabbits. She pulled out the dead plants and cleaned up the area. Her research showed

that each rabbit needs 2 square feet of space in a pen, and that rabbits reproduce every month, having litters of about 6 kits. She started with 2 rabbits (one male and one female). Claire began tracking the number of rabbits at the end of each month and displayed her data in the table:

# of months	0	1	2
# of rabbits	2	8	32

a. Write a formula relating the # of months and the # of rabbits.

b. Make a graph showing the relationship between the # of months and the # of rabbits.

c. The dimensions of the rabbit pen (formerly the garden) are 7 yards by 8 yards. When will Claire run out of room for her rabbits? Explain.

Additional Tasks

Back to Task Table

The tasks featured in this table provide additional resources and supplemental tasks to be incorporated into unit 5 instruction as deemed appropriate by the instructor.

UNIT 5: Comparing and Contrasting Functions	Standards Addressed in the Task
Population and Food Supply Task from Illustrative Math https://www.illustrativemathematics.org/illustrations/645	A.REI.11, F.LE.2, F.LE.3
Visual Patterns www.visualpatterns.org	F.LE.1, F.LE.2, F.LE.3
Graphing Stories http://graphingstories.com Site Developed by Dan Meyer (1-3 Stories)	F-BF, F-IF
Double Sunglasses http://threeacts.mrmeyer.com/doublesunglasses Task Developed by Dan Meyer	F-LE.1
Relation Stations http://musingmathematically.blogspot.ca/2013/02/relation-stations.html Task Developed by Nat Banting	F-IF.1, F-BF.1, 1a, F-LE.1,1a,1b
Pixel Patterns http://threeacts.mrmeyer.com/pixelpattern/ Task Developed by Dan Meyer	F-LE.2
Math Taboo http://threeacts.mrmeyer.com/pixelpattern/ Task Idea Developed by Fawn Nguyen	F-LE 1,1a,1b

Moving on Up http://emergentmath.com/2011/05/11/u-haul-linear-systems-problem-updated-and-improved/ Task Developed by Geoff Krall on Emergent Math	F-BF.1a, c; F-LE.2, F-IF.6,7a
Points on a Graph Task from Illustrative Math https://www.illustrativemathematics.org/illustrations/630	F.IF.1
The Customers Task from Illustrative Math https://www.illustrativemathematics.org/illustrations/624	F.IF.1
Using Function Notation Task from Illustrative Math https://www.illustrativemathematics.org/illustrations/598	F.IF.1
Do Two Points Always Determine a Straight Line? Task from Illustrative Math https://www.illustrativemathematics.org/illustrations/377	F.IF.1, LE.2
Warming and Cooling Task from Illustrative Math https://www.illustrativemathematics.org/illustrations/639	F.IF.4