Georgia Standards of Excellence Course Curriculum Overview
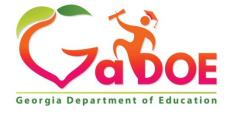
Mathematics

Accelerated GSE Analytic Geometry B/ Advanced Algebra

GaDOE

Georgia Department of Education

Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

Georgia Department of Education

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These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units.
All units will include the Mathematical Practices and indicate skills to maintain.

NOTE: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics.

Grade 9-12 Key:

Number and Quantity Strand: RN = The Real Number System, Q = Quantities, CN = Complex Number System, VM = Vector and Matrix Quantities

Algebra Strand: SSE = Seeing Structure in Expressions, APR = Arithmetic with Polynomial and Rational Expressions, CED = Creating Equations,   REI = Reasoning with Equations and Inequalities

Functions Strand: IF = Interpreting Functions, LE = Linear and Exponential Models, BF = Building Functions, TF = Trigonometric Functions

Geometry Strand: CO = Congruence, SRT = Similarity, Right Triangles, and Trigonometry, C = Circles, GPE = Expressing Geometric Properties with Equations, GMD = Geometric Measurement and Dimension, MG = Modeling with Geometry

Statistics and Probability Strand: ID = Interpreting Categorical and Quantitative Data, IC = Making Inferences and Justifying Conclusions, CP = Conditional Probability and the Rules of Probability, MD = Using Probability to Make Decisions

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The Comprehensive Course Overviews are designed to provide access to multiple sources of support for implementing and instructing courses involving the Georgia Standards of Excellence.

GSE Algebra II/Advanced Algebra

Accelerated Analytic Geometry B/Advanced Algebra is the second in a sequence of mathematics courses designed to ensure that students are prepared to take higher-level mathematics courses during their high school career, including Advanced Placement Calculus AB, Advanced Placement Calculus BC, and Advanced Placement Statistics.

The standards in the three-course high school sequence specify the mathematics that all students should study in order to be college and career ready. Additional mathematics content is provided in fourth credit courses and advanced courses including, calculus, advanced statistics, discrete mathematics, and mathematics of finance courses. High school course content standards are listed by conceptual categories including Number and Quantity, Algebra, Functions, Geometry, and Statistics and Probability. Conceptual categories portray a coherent view of high school mathematics content; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus. Standards for Mathematical Practice provide the foundation for instruction and assessment.

GSE Algebra II/Advanced Algebra: Unit Descriptions

It is in Accelerated Analytic Geometry B / Advanced Algebra that students pull together and apply the accumulation of learning that they have from their previous course, with content grouped into nine critical areas, organized into units. Quadratic expressions, equations, and functions are developed; comparing their characteristics and behavior to those of linear and exponential relationships from Accelerated Coordinate Algebra / Analytic Geometry A. Circles return with their quadratic algebraic representations on the coordinate plane. The link between probability and data is explored through conditional probability. Students expand their repertoire of functions to include quadratic (with complex solutions), polynomial, rational, and radical functions. And, finally, students bring together all of their experience with functions to create models and solve contextual problems. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Unit 1: Students will analyze quadratic functions. Students will investigate key features of graphs, solve quadratic equations by taking the square roots, factoring ($x^2 + bx + c$ AND $ax^2 + bx + c$), completing the square, and using the quadratic formula. Students will compare and contrast graphs in standard, vertex, and intercept forms. Students will only work with real number solutions.

Unit 2: Students will verify algebraically geometric relationships of circles in the coordinate plane. Students will derive the equation of a circle and model real-world objects using geometric shapes and concepts.

Unit 3: Students will understand independence and conditional probability and use them to interpret data. Building on standards from middle school, students will formalize the rules of

probability and use the rules to compute probabilities of compound events in a uniform probability model.

Unit 4: Students will revisit solving quadratic equations in this unit. Students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows x+1 = 0 to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. Students will perform operations with complex numbers and solve quadratic equations with complex solutions. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system. Students will also extend the laws of exponents to rational exponents and use those properties to evaluate and simplify expressions containing rational exponents.

Unit 5: This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students will find inverse functions and verify by composition that one function is the inverse of another function.

Unit 6: In this unit, students continue their study of polynomials by identifying zeros and making connections between zeros of a polynomial and solutions of a polynomial equation. Students will see how the Fundamental Theorem of Algebra can be used to determine the number of solutions of a polynomial equation and will find all the roots of those equations. Students will graph polynomial functions and interpret the key characteristics of the function.

Unit 7: Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. Similarly, radical expressions follow the rules governed by irrational numbers.

Unit 8: Students extend their work with exponential functions to include solving exponential equations with logarithms. They analyze the relationship between these two functions.

Unit 9: In this unit students synthesize and generalize what they have learned about a variety of function families. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. They determine whether it is best to model with multiple functions creating a piecewise function. Students will also explore finite the sum of finite geometric series. The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions and statistics is applied in a modeling context.

Flipbooks

These "FlipBooks" were developed by the Kansas Association of Teachers of Mathematics (KATM) and are a compilation of research, "unpacked" standards from many states, instructional strategies and examples for each standard at each grade level. The intent is to show the connections to the Standards of Mathematical Practices for the content standards and to get detailed information at each level. The **High School Flipbook** is an interactive document arranged by the content domains listed on the following pages. The links on each domain and standard will take you to specific information on that standard/domain within the Flip Book.

Mathematics | High School – Number and Quantity

Numbers and Number Systems: During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3... Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers. With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings. Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5. Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities: In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an

important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

The Real Number System	**N.RN**

Extend the properties of exponents to rational exponents.

MGSE9-12.N.RN.1. Explain how the meaning of rational exponents follows from extending the properties of integer exponents to rational numbers, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{(1/3)}$ to be the cube root of 5 because we want $[5^{(1/3)}]^3 = 5^{[(1/3) \times 3]}$ to hold, so $[5^{(1/3)}]^3$ must equal 5.*

MGSE9-12.N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

The Complex Number System	**N.CN**

Perform arithmetic operations with complex numbers.

MGSE9-12.N.CN.1 Understand there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi where a and b are real numbers.

MGSE9-12.N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

MGSE9-12.N.CN.3 Find the conjugate of a complex number; use the conjugate to find the ~~absolute value (modulus) and~~ quotient of complex numbers.

Use complex numbers in polynomial identities and equations.

MGSE9-12.N.CN.7 Solve quadratic equations with real coefficients that have complex solutions by (but not limited to) square roots, completing the square, and the quadratic formula.

MGSE9-12.N.CN.8 Extend polynomial identities to include factoring with complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.*

MGSE9-12.N.CN.9 Use the Fundamental Theorem of Algebra to find all roots of a polynomial equation.

Mathematics | High School – Algebra

Expressions: An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities: An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions. Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x2 - 2 = 0$ are real numbers, not rational numbers; and the solutions of $x2 + 2 = 0$ are complex numbers, not real numbers. The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b1+b2)/2)h$, can be solved for h using the same deductive process. Inequalities can be solved by reasoning about the properties of

inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling: Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Seeing Structure in Expressions	A.SSE

Interpret the structure of expressions

MGSE9-12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

> **MGSE9-12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients, in context.

> **MGSE9-12.A.SSE.1b** Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.

MGSE9-12.A.SSE.2 Use the structure of an expression to rewrite it in different equivalent forms. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems

MGSE9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

> **MGSE9-12.A.SSE.3a** Factor any quadratic expression to reveal the zeros of the function defined by the expression.

> **MGSE9-12.A.SSE.3b** Complete the square in a quadratic expression to reveal the maximum and minimum value of the function defined by the expression.

> **MGSE9-12.A.SSE.3c** Use the properties of exponents to transform expressions for exponential functions. *For example, the expression 1.15^t, where t is in years, can be rewritten as $[1.15^{(1/12)}]^{(12t)} \approx 1.012^{(12t)}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

MGSE9-12.A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

Arithmetic with Polynomials and Rational Expressions	A.APR

Perform arithmetic operations on polynomials

MGSE9-12.A.APR.1 Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations.

Understand the relationship between zeros and factors of polynomials

MGSE9-12.A.APR.2 Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).

MGSE9-12.A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems

MGSE9-12.A.APR.4 Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*

MGSE9-12.A.APR.5 Know and apply that the Binomial Theorem gives the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined using Pascal's Triangle.

Rewrite rational expressions

MGSE9-12.A.APR.6 Rewrite simple rational expressions in different forms using inspection, long division, or a computer algebra system; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x).

MGSE9-12.A.APR.7 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Creating Equations	A.CED

Create equations that describe numbers or relationships

MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, simple rational, and exponential functions ~~(integer inputs only)~~.

MGSE9-12.A.CED.2 Create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase "in two or more variables" refers to formulas like the compound interest formula, in which $A = P(1 + r/n)^{nt}$ has multiple variables.)

MGSE9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

MGSE9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. *Examples: Rearrange Ohm's law $V = IR$ to highlight resistance R; Rearrange area of a circle formula $A = \pi r^2$ to highlight the radius r.*

Reasoning with Equations and Inequalities	A.REI

Understand solving equations as a process of reasoning and explain the reasoning

MGSE9-12.A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable

MGSE9-12.A.REI.4 Solve quadratic equations in one variable.

> **MGSE9-12.A.REI.4a** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from $ax^2 + bx + c = 0$.

> **MGSE9-12.A.REI.4b** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation ~~(limit to real number solutions)~~.

Represent and solve equations and inequalities graphically

MGSE9-12.A.REI.11 Using graphs, tables, or successive approximations, show that the solution to the equation $f(x) = g(x)$ is the x-value where the y-values of $f(x)$ and $g(x)$ are the same.

Mathematics | High School – Functions

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates:
Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Interpret functions that arise in applications in terms of the context

MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity~~.

MGSE9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

MGSE9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations

MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

    MGSE9-12.F.IF.7a Graph ~~linear and~~ quadratic functions and show intercepts, maxima, and minima (as determined by the function or by context).

    MGSE9-12.F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

    MGSE9-12.F.IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

    MGSE9-12.F.IF.7d Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

    MGSE9-12.F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, ~~and trigonometric functions, showing period, midline, and amplitude.~~

MGSE9-12.F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

    MGSE9-12.F.IF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. *For example, compare and contrast quadratic functions in standard, vertex, and intercept forms.*

MGSE9-12.F.IF.8b Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)^t, y = (0.97)^t, y = (1.01)$^{(12t)}$, y = (1.2)$^{(t/10)}$, and classify them as representing exponential growth and decay.*

MGSE9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.*

Building Functions	F.BF

Build a function that models a relationship between two quantities

MGSE9-12.F.BF.1 Write a function that describes a relationship between two quantities.

MGSE9-12.F.BF.1a Determine an explicit expression and the recursive process (steps for calculation) from context. *For example, if Jimmy starts out with $15 and earns $2 a day, the explicit expression "2x+15" can be described recursively (either in writing or verbally) as "to find out how much money Jimmy will have tomorrow, you add $2 to his total today."* $J_n = J_{n-1} + 2, J_0 = 15$

MGSE9-12.F.BF.1b Combine standard function types using arithmetic operations in contextual situations (Adding, subtracting, and multiplying functions of different types).

MGSE9-12.F.BF.1c Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

Build new functions from existing functions

MGSE9-12.F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

MGSE9-12.F.BF.4 Find inverse functions.

MGSE9-12.F.BF.4a Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2(x^3) or f(x) = (x+1)/(x-1) for x ≠ 1.

MGSE9-12.F.BF.4b Verify by composition that one function is the inverse of another.

MGSE9-12.F.BF.4c Read values of an inverse function from a graph or a table, given that the function has an inverse.

MGSE9-12.F.BF.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models	F.LE

Construct and compare linear, quadratic, and exponential models and solve problems

MGSE9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

MGSE9-12.F.LE.4 For exponential models, express as a logarithm the solution to $ab^{(ct)} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Mathematics | High School – Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate that states that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.) During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shape in general). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence

criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations: The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Expressing Geometric Properties with Equations	G.GPE

Translate between the geometric description and the equation for a conic section

MGSE9-12.G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Use coordinates to prove simple geometric theorems algebraically

MGSE9-12.G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, ~~prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle;~~ prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0,2).*
(Focus on ~~quadrilaterals, right triangles, and~~ circles.)

Modeling with Geometry	G.MG

Apply geometric concepts in modeling situations

MGSE9-12.G.MG.1 Use geometric shapes, their measures, and their properties to describe objects
(e.g., modeling a tree trunk or a human torso as a cylinder).

MGSE9-12.G.MG.2 Apply concepts of density based on area and volume in modeling situations
(e.g., persons per square mile, BTUs per cubic foot).

MGSE9-12.G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Mathematics | High School – Statistics & Probability

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be

evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling: Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Interpreting Categorical and Quantitative Data   S.ID

Summarize, represent, and interpret data on two categorical and quantitative variables

MGSE9-12.S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

> **MGSE9-12.S.ID.6a** Decide which type of function is most appropriate by observing graphed data, charted data, or by analysis of context to generate a viable (rough) function of best fit. Use this function to solve problems in context. Emphasize ~~linear,~~ quadratic ~~and exponential~~ models.

Conditional Probability and the Rules of Probability   S.CP

Understand independence and conditional probability and use them to interpret data

MGSE9-12.S.CP.1 Describe categories of events as subsets of a sample space using unions, intersections, or complements of other events (*or, and, not*).

MGSE9-12.S.CP.2 Understand that if two events A and B are independent, the probability of A and B occurring together is the product of their probabilities, and that if the probability of two events A and B occurring together is the product of their probabilities, the two events are independent.

MGSE9-12.S.CP.3 Understand the conditional probability of A given B as P (A and B)/P(B). Interpret independence of A and B in terms of conditional probability; that is, the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

MGSE9-12.S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, use collected data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

MGSE9-12.S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

Use the rules of probability to compute probabilities of compound events in a uniform probability model

MGSE9-12.S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in context.

MGSE9-12.S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answers in context.

Mathematics | Standards for Mathematical Practice

Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction. The BLUE links will provide access to classroom videos on each standard for mathematical practice accessed on the [Inside Math](#) website.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 [Make sense of problems and persevere in solving them].
High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 [Reason abstractly and quantitatively.]
High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3 [Construct viable arguments and critique the reasoning of others](#).
High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and— if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 [Model with mathematics](#).
High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 [Use appropriate tools strategically](#).
High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 [Attend to precision](#).
High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 [Look for and make use of structure](#).

By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8 [Look for and express regularity in repeated reasoning.](#)
High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

More information of the Standards for Mathematical Standards may be found on the [Inside Math](#) website.

Connecting the Standards for Mathematical Practice to the Content Standards

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics. See [Inside Math](#) for more resources.

Classroom Routines

The importance of continuing the established classroom routines cannot be overstated. Daily routines must include such obvious activities as estimating, analyzing data, describing patterns, and answering daily questions. They should also include less obvious routines, such as how to select materials, how to use materials in a productive manner, how to put materials away, how to access classroom technology such as computers and calculators. An additional routine is to allow plenty of time for children to explore new materials before attempting any directed activity with these new materials. The regular use of routines is important to the development of students' number sense, flexibility, fluency, collaborative skills and communication. These routines contribute to a rich, hands-on standards based classroom and will support students' performances on the tasks in this unit and throughout the school year.

Strategies for Teaching and Learning

- Students should be actively engaged by developing their own understanding.
- Mathematics should be represented in as many ways as possible by using graphs, tables, pictures, symbols and words.
- Interdisciplinary and cross curricular strategies should be used to reinforce and extend the learning activities.
- Appropriate manipulatives and technology should be used to enhance student learning.
- Students should be given opportunities to revise their work based on teacher feedback, peer feedback, and metacognition which includes self-assessment and reflection.
- Students should write about the mathematical ideas and concepts they are learning.
- Consideration of all students should be made during the planning and instruction of this unit. Teachers need to consider the following:
    - What level of support do my struggling students need in order to be successful with this unit?
    - In what way can I deepen the understanding of those students who are competent in this unit?
    - What real life connections can I make that will help my students utilize the skills practiced in this unit?

Tasks

The framework tasks represent the level of depth, rigor, and complexity expected of all Coordinate Algebra students. These tasks, or tasks of similar depth and rigor, should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as a performance task, they may also be used for teaching and learning (learning/scaffolding task). The table below provides a brief explanation of the types of tasks that teachers will find in the frameworks units for Advanced Algebra/Algebra II.

Scaffolding Task	Tasks that build up to the learning task.
Learning Task	Constructing understanding through deep/rich contextualized problem solving tasks.
Practice Task	Tasks that provide students opportunities to practice skills and concepts.
Performance Task	Tasks which may be a formative or summative assessment that checks for student understanding/misunderstanding and or progress toward the standard/learning goals at different points during a unit of instruction.
Culminating Task	Designed to require students to use several concepts learned during the unit to answer a new or unique situation. Allows students to give evidence of their own understanding toward the mastery of the standard and requires them to extend their chain of mathematical reasoning.
Short Cycle Task	Designed to exemplify the performance targets that the standards imply. The tasks, with the associated guidance, equip teachers to monitor overall progress in their students' mathematics.
Formative Assessment Lesson (FAL) *more information on page 24	Lessons that support teachers in formative assessment which both reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards.
3-Act Task *more information on page 25	
Achieve CCSS- CTE Classroom Tasks	Designed to demonstrate how the CCSS and Career and Technical Education knowledge and skills can be integrated. The tasks provide teachers with realistic applications that combine mathematics and CTE content.

Formative Assessment Lessons (FALs)

The **Formative Assessment Lesson** is designed to be part of an instructional unit typically implemented approximately two-thirds of the way through the instructional unit. The results of the tasks should then be used to **inform** the instruction that will take place for the remainder of the unit.

Formative Assessment Lessons are intended to support teachers in formative assessment. They both reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

Videos of Georgia Teachers implementing FALs can be accessed HERE and a sample of a FAL lesson may be seen HERE

More information on types of Formative Assessment Lessons, their use, and their implementation may be found on the Math Assessment Project's guide for teachers.

Formative Assessment Lessons can also be found at the following sites:
Mathematics Assessment Project
Kenton County Math Design Collaborative
MARS Tasks by grade level

A **sample FAL** with extensive dialog and suggestions for teachers may be found HERE. This resource will help teachers understand the flow and purpose of a FAL.

The **Math Assessment Project** has developed Professional Development Modules that are designed to help teachers with the practical and pedagogical challenges presented by these lessons.

Module 1 introduces the model of *formative assessment* used in the lessons, its theoretical background and practical implementation. Modules 2 & 3 look at the two types of *Classroom Challenges* in detail. Modules 4 & 5 explore two crucial pedagogical features of the lessons: asking probing questions and collaborative learning.

Georgia RESA's may be contacted about professional development on the use of FALs in the classroom. The request should be made through the teacher's local RESA and can be referenced by asking for more information on the Mathematics Design Collaborative (MDC).

Spotlight Tasks

A Spotlight Task has been added to each GSE mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Georgia Standards of Excellence, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky.

3-Act Tasks

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

Guidelines for 3-Act Tasks and Patient Problem Solving (Teaching without the Textbook)
Adapted from Dan Meyer

Developing the mathematical Big Idea behind the 3-Act task:
- Create or find/use a clear visual which tells a brief, perplexing mathematical story. Video or live action works best. (See resource suggestions in the Guide to 3-Act Tasks)
- Video/visual should be real life and allow students to see the situation unfolding.
- Remove the initial literacy/mathematics concerns. Make as few language and/or math demands on students as possible. You are posing a mathematical question without words.
- The visual/video should inspire curiosity or perplexity which will be resolved via the mathematical big idea(s) used by students to answer their questions. You are creating an intellectual need or cognitive dissonance in students.

Enacting the 3-Act in the Classroom

Act 1 (The Question):
Set up student curiosity by sharing a scenario:
- Teacher says, "I'm going show you something I came across and found interesting" or, "Watch this."
- Show video/visual.
- Teacher asks, "What do you notice/wonder?" and "What are the first questions that come to mind?"
- Students share observations/questions with a partner first, then with the class (Think-Pair-Share). Students have ownership of the questions because they posed them.
- Leave no student out of this questioning. Every student should have access to the scenario. No language or mathematical barriers. Low barrier to entry.

- Teacher records questions (on chart paper or digitally-visible to class) and ranks them by popularity.
- Determine which question(s) will be immediately pursued by the class. If you have a particular question in mind, and it isn't posed by students, you may have to do some skillful prompting to orient their question to serve the mathematical end. However, a good video should naturally lead to the question you hope they'll ask. You may wish to pilot your video on colleagues before showing it to students. If they don't ask the question you are after, your video may need some work.
- Teacher asks for estimated answers in response to the question(s). Ask first for best estimates, then request estimates which are too high and too low. Students are no defining and defending parameters for making sense of forthcoming answers.
- Teacher asks students to record their actual estimation for future reference.

Act 2 (Information Gathering):

Students gather information, draw on mathematical knowledge, understanding, and resources to answer the big question(s) from Act-1:

- Teacher asks, "What information do you need to answer our *main question*?"
- Students think of the important information they will need to answer their questions.
- Ask, "What mathematical tools do you have already at your disposal which would be useful in answering this question?"
- What mathematical tools might be useful which students don't already have? Help them develop those.
- Teacher offers smaller examples and asks probing questions.
  - What are you doing?
  - Why are you doing that?
  - What would happen if…?
  - Are you sure? How do you know?

Act 3 (The Reveal):

The payoff.

- Teacher shows the answer and validates students' solutions/answer.
- Teacher revisits estimates and determines closest estimate.
- Teacher compares techniques, and allows students to determine which is most efficient.

The Sequel:

- Students/teacher generalize the math to any case, and "algebrafy" the problem.
- Teacher poses an extension problem- best chance of student engagement if this extension connects to one of the many questions posed by students which were not the focus of Act 2, or is related to class discussion generated during Act 2.
- Teacher revisits or reintroduces student questions that were not addressed in Act 2.

Why Use 3-Act Tasks? A Teacher's Response

The short answer: It's what's best for kids!

If you want more, read on:

The need for students to make sense of problems can be addressed through tasks like these. The challenge for teachers is, to quote Dan Meyer, "be less helpful." (To clarify, being less helpful means to first allow students to generate questions they have about the picture or video they see in the first act, then give them information as they ask for it in act 2.) Less helpful does not mean give these tasks to students blindly, without support of any kind!

This entire process will likely cause some anxiety (for all). When jumping into 3-Act tasks for the first (second, third, . . .) time, students may not generate the suggested question. As a matter of fact, in this task about proportions and scale, students may ask many questions that are curious questions, but have nothing to do with the mathematics you want them to investigate. One question might be "How is that ball moving by itself?" It's important to record these and all other questions generated by students. This validates students' ideas. Over time, students will become accustomed to the routine of 3-act tasks and come to appreciate that there are certain kinds of mathematically answerable questions – most often related to quantity or measurement.

These kinds of tasks take time, practice, and patience. When presented with options to use problems like this with students, the easy thing for teachers to do is to set them aside for any number of "reasons." I've highlighted a few common "reasons" below with my commentary (in blue):

- This will take too long. I have a lot of content to cover. (Teaching students to think and reason is embedded in mathematical content at all levels - how can you **not** take this time?)
- They need to be taught the skills first, then maybe I'll try it. (An important part of learning mathematics lies in productive struggle and learning to persevere [SMP 1]. What better way to discern what students know and are able to do than with a mathematical context [problem] that lets them show you, based on the knowledge they already have - prior to any new information. To quote John Van de Walle, "Believe in kids and they will, flat out, amaze you!")
- My students can't do this. (Remember, whether you think they can or they can't, you're right!) (Also, this expectation of students persevering and solving problems is in every state's standards - and was there even before common core!)
- I'm giving up some control. (Yes, and this is a bit scary. You're empowering students to think and take charge of their learning. So, what can you do to make this *less* scary?
  Do what we expect students to do:
    - Persevere. Keep trying these and other open-beginning, -middle, and -ended problems. Take note of what's working and focus on it!
    - Talk with a colleague (work with a partner). Find that critical friend at school, another school, online. . .
    - Question (use #MTBoS on Twitter, or blogs, or Google: 3-act tasks).

The benefits of students learning to question, persevere, problem solve, and reason mathematically far outweigh any of the reasons (read excuses) above. The time spent up front, teaching through tasks such as these and other open problems, creates a huge pay-off later on. However, it is

important to note, that the problems themselves are worth nothing without teachers setting the expectation that students: question, persevere, problem solve, and reason mathematically on a daily basis. Expecting these from students, and facilitating the training of how to do this consistently and with fidelity is principal to success for both students and teachers.

Yes, all of this takes time. For most of my classes, mid to late September (we start school at the beginning of August) is when students start to become comfortable with what problem solving really is. It's not word problems - mostly. It's not the problem set you do after the skill practice in the textbook. Problem solving is what you do when you don't know what to do! This is difficult to teach kids and it does take time. But it is worth it! More on this in a future blog!

Tips:

One strategy I've found that really helps students generate questions is to allow them to talk to their peers about what they notice and wonder first (Act 1). Students of all ages will be more likely to share once they have shared and tested their ideas with their peers. This does take time. As you do more of these types of problems, students will become familiar with the format and their comfort level may allow you to cut the amount of peer sharing time down before group sharing.

 What do you do if they don't generate the question suggested? Well, there are several ways that this can be handled. If students generate a similar question, use it. Allowing students to struggle through their question and ask for information is one of the big ideas here. Sometimes, students realize that they may need to solve a different problem before they can actually find what they want. If students are way off, in their questions, teachers can direct students, carefully, by saying something like: "You all have generated some interesting questions. I'm not sure how many we can answer in this class. Do you think there's a question we could find that would allow us to use our knowledge of mathematics to find the answer to (insert quantity or measurement)?" Or, if they are really struggling, you can, again carefully, say "You know, I gave this problem to a class last year (or class, period, etc.) and they asked (insert something similar to the suggested question here). What do you think about that?" Be sure to allow students to share their thoughts.

After solving the main question, if there are other questions that have been generated by students, it's important to allow students to investigate these as well. Investigating these additional questions validates students' ideas and questions and builds a trusting, collaborative learning relationship between students and the teacher.

Overall, we're trying to help our students mathematize their world. We're best able to do that when we use situations that are relevant (no dog bandanas, please), engaging (create an intellectual need to know), and perplexing. If we continue to use textbook type problems that are too helpful, uninteresting, and let's face it, perplexing in all the wrong ways, we're not doing what's best for kids; we're training them to not be curious, not think, and worst of all . . . dislike math.

3-Act Task Resources:

- www.estimation180.com
- www.visualpatterns.org
- 101 Questions
- Dan Meyer's 3-Act Tasks
- 3-Act Tasks for Elementary and Middle School
- Andrew Stadel
- Jenise Sexton
- Graham Fletcher
- Fawn Nguyen
- Robert Kaplinsky
- Open Middle
- Check out the Math Twitter Blog-o-Sphere (MTBoS) - you'll find tons of support and ideas!

Assessment Resources and Instructional Support Resources

The resource sites listed below are provided by the GADOE and are designed to support the instructional and assessment needs of teachers. All BLUE links will direct teachers to the site mentioned.

- Georgiastandards.org provides a gateway to a wealth of instructional links and information. Select the ELA/Math tab at the top to access specific math resources for GSE.

- MGSE Frameworks are "models of instruction" designed to support teachers in the implementation of the Georgia Standards of Excellence (GSE). The Georgia Department of Education, Office of Standards, Instruction, and Assessment has provided an example of the Curriculum Map for each grade level and examples of Frameworks aligned with the GSE to illustrate what can be implemented within the grade level. School systems and teachers are free to use these models as is; modify them to better serve classroom needs; or create their own curriculum maps, units and tasks. http://bit.ly/1AJddmx

- The Teacher Resource Link (TRL) is an application that delivers vetted and aligned digital resources to Georgia's teachers. TRL is accessible via the GADOE "tunnel" in conjunction with SLDS using the single sign-on process. The content is pushed to teachers based on course schedule.
- Georgia Virtual School content available on our Shared Resources Website is available for anyone to view. Courses are divided into modules and are aligned with the Georgia Standards of Excellence.

- The Georgia Online Formative Assessment Resource (GOFAR) accessible through SLDS contains test items related to content areas assessed by the Georgia Milestones Assessment System and NAEP. Teachers and administrators can utilize the GOFAR to develop formative and summative assessments, aligned to the state-adopted content standards, to assist in informing daily instruction.

The Georgia Online Formative Assessment Resource (GOFAR) provides the ability for Districts and Schools to assign benchmark and formative test items/tests to students in order to obtain information about student progress and instructional practice. GOFAR allows educators and their students to have access to a variety of test items – selected response and constructed response – that are aligned to the State-adopted content standards for Georgia's elementary, middle, and high schools.

Students, staff, and classes are prepopulated and maintained through the State Longitudinal Data System (SLDS). Teachers and Administrators may view Exemplars and Rubrics in Item Preview. A scoring code may be distributed at a local level to help score constructed response items.

For GOFAR user guides and overview, please visit: https://www.gadoe.org/Curriculum-Instruction-and-Assessment/Assessment/Pages/Georgia-Online-Formative-Assessment-Resource.aspx

- Georgia Milestones Assessment System resources can be found at: http://www.gadoe.org/Curriculum-Instruction-and-Assessment/Assessment/Pages/Georgia-Milestones-Assessment-System.aspx
  Features the Georgia Milestones Assessment System include:
    o Open-ended (constructed-response) items
    o Norm-referenced to complement the criterion-referenced information and to provide a national comparison;
    o Transition to online administration over time, with online administration considered the primary mode of administration and paper-pencil as back-up until the transition is complete.

Internet Resources

The following list is provided as a sample of available resources and is for informational purposes only. It is your responsibility to investigate them to determine their value and appropriateness for your district. GADOE does not endorse or recommend the purchase of or use of any particular resource.

General Resources

[Illustrative Mathematics](#)
Standards are illustrated with instructional and assessment tasks, lesson plans, and other curriculum resources.

[Mathematics in Movies](#)
Short movie clips related to a variety of math topics.

[Mathematical Fiction](#)
Plays, short stories, comic books and novels dealing with math.

[The Shodor Educational Foundation](#)
This website has extensive notes, lesson plans and applets aligned with the standards.

[NEA Portal Arkansas Video Lessons on-line](#)
The NEA portal has short videos aligned to each standard. This resource may be very helpful for students who need review at home.

[Learnzillion](#)
This is another good resource for parents and students who need a refresher on topics.

[Math Words](#)
This is a good reference for math terms.

[National Library of Virtual Manipulatives](#)
Java must be enabled for this applet to run. This website has a wealth of virtual manipulatives helpful for use in presentation. The resources are listed by domain.

[Geogebra Download](#)
Free software similar to Geometer's Sketchpad. This program has applications for algebra, geometry, and statistics.

[Utah Resources](#)
Open resource created by the Utah Education Network.

Resources for Problem-based Learning

[Dan Meyer's Website](#)
Dan Meyer has created many problem-based learning tasks. The tasks have great hooks for the students and are aligned to the standards in this [spreadsheet.](#)

[Andrew Stadel](#)
Andrew Stadel has created many problem-based learning tasks using the same format as Dan Meyer.

[Robert Kaplinsky](#)
Robert Kaplinsky has created many tasks that engage students with real life situations.

[Geoff Krall's Emergent Math](#)
Geoff Krall has created a curriculum map structured around problem-based learning tasks.