# Georgia
# Standards of Excellence
# Curriculum Frameworks

## Mathematics

Accelerated GSE Coordinate Algebra/Analytic Geometry A

Unit 5: Transformations in the Coordinate Plane

Georgia Department of Education

Richard Woods, Georgia's School Superintendent
*"Educating Georgia's Future"*

# Unit 5

# Transformations in the Coordinate Plane

## Table of Contents

# OVERVIEW

In this unit students will:

- use and understand definitions of angles, circles, perpendicular lines, parallel lines, and line segments based on the undefined terms of point, line, distance along a line and length of an arc.
- describe and compare function transformations on a set of points as inputs to produce another set of points as outputs, including translations and horizontal or vertical stretching
- represent and compare rigid and size transformations of figures in a coordinate plane using various tools such as transparencies, geometry software, interactive whiteboards, waxed paper, tracing paper, mirrors and digital visual presenters.
- compare transformations that preserve size and shape versus those that do not.
- describe rotations and reflections of parallelograms, trapezoids or regular polygons that map each figure onto itself.
- develop and understand the meanings of rotation, reflection and translation based on angles, circles, perpendicular lines, parallel lines and line segments.
- transform a figure given a rotation, reflection or translation using graph paper, tracing paper, geometric software or other tools.
- create sequences of transformations that map a figure onto itself or to another figure.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. This unit provides much needed content information and excellent learning activities. However, the intent of the framework is not to provide a comprehensive resource for the implementation of all standards in the unit. A variety of resources should be utilized to supplement this unit. The tasks in this unit framework illustrate the types of learning activities that should be utilized from a variety of sources. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the **"Strategies for Teaching and Learning"** and the tasks listed under **"Evidence of Learning"** be reviewed early in the planning process.

## Webinar Information

A two–hour course overview webinar may be accessed at
http://www.gpb.org/education/common–core/2012/02/28/mathematics–9th–grade

The unit–by–unit webinars may be accessed at
https://www.georgiastandards.org/Common–Core/Pages/Math–PL–Sessions.aspx

## STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

## KEY STANDARDS

### Experiment with transformations in the plane

**MGSE9–12.G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

**MGSE9–12.G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

**MGSE9–12.G.CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

**MGSE9–12.G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

**MGSE9–12.G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

## STANDARDS FOR MATHEMATICAL PRACTICE

Refer to the Comprehensive Course Overview for more detailed information about the Standards for Mathematical Practice.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## ENDURING UNDERSTANDINGS

- The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation.
- Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes in general).
- Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes.

## ESSENTIAL QUESTIONS

- What are the undefined terms essential to any study of geometry?
- How do you determine the type of transformation that has occurred?
- What effects do transformations have on geometric figures?
- Can a transformation change an object's position, orientation, and / or size?
- How do we define and create geometric transformations?
- How do transformations of geometric figures and functions compare?
- What effects do transformations have on geometric figures?
- Which transformations create isometries?
- How do we know which transformations have created the mapping of an image?
- How do we translate geometric figures in the coordinate plane?
- How do we reflect points in a coordinate plane?
- How are reflections and rotations similar and different?
- How can we describe / represent a transformation (or series of transformations) that take place in the coordinate plane?
- How can the coordinate plane help me understand properties of reflections, translations and rotations?

- What is the relationship between reflections, translations and rotations?

- How do I apply what I've learned about transformations to figures in the coordinate plane?

## CONCEPTS AND SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre–assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- plotting points on a coordinate plane

- congruence of geometric figures and the correspondence of their vertices, sides, and angles

- recognizing line and rotational symmetry

- interpreting and sketching views from different perspectives

- calculate the perimeter and area of fundamental geometric plane figures

- use the concepts of ratio, proportion, and scale factor to demonstrate the relationships between similar plane figures

## SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

**The definitions below are for teacher reference only and are not to be memorized by the students.** Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for high school students. **Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**

http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary.

http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

- **Angle:** A figure created by two distinct rays that share a common endpoint (also known as a vertex). $\angle ABC$ or $\angle B$ or $\angle CBA$ indicate the same angle with vertex *B*.
- **Angle of Rotation:** The amount of rotation (in degrees) of a figure about a fixed point such as the origin.
- **Bisector:** A point, line or line segment that divides a segment or angle into two equal parts.
- **Circle:** The set of all points equidistant from a point in a plane.
- **Congruent:** Having the same size, shape and measure. $\angle A \cong \angle B$ indicates that angle A is congruent to angle B.
- **Corresponding angles:** Angles that have the same relative position in geometric figures.
- **Corresponding sides:** Sides that have the same relative position in geometric figures.
- **Endpoint:** The point at each end of a line segment or at the beginning of a ray.
- **Image:** The result of a transformation.
- **Intersection:** The point at which two or more lines intersect or cross.
- **Isometry:** a distance preserving map of a geometric figure to another location using a reflection, rotation or translation. $M \rightarrow M'$ indicates an isometry of the figure M to a new location M'. M and M' remain congruent.
- **Line:** One of the undefined terms of geometry that represents an infinite set of points with no thickness and its length continues in two opposite directions indefinitely. $\overleftrightarrow{AB}$ indicates a line that passes through points A and B.
- **Line segment:** A part of a line between two points on the line. $\overline{AB}$ indicates the line segment between points A and B.

- **Parallel lines:** Two lines are parallel if they lie in the same plane and do not intersect. $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ indicates that line AB is parallel to line CD.

- **Perpendicular lines:** Two lines are perpendicular if they intersect to form right angles. $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ indicates that line AB is perpendicular to line CD.

- **Point:** One of the basic undefined terms of geometry that represents a location. A dot is used to symbolize it and it is thought of as having no length, width or thickness.

- **Pre–image:** A figure before a transformation has taken place.

- **Ray:** A part of a line that begins at a point and continues forever in one direction. $\overrightarrow{AB}$ indicates a ray that begins at point A and continues in the direction of point B indefinitely.

- **Reflection:** A transformation of a figure that creates a mirror image, "flips," over a line.

- **Reflection Line (or line of reflection):** A line that acts as a mirror so that corresponding points are the same distance from the mirror.

- **Rotation:** A transformation that turns a figure about a fixed point through a given angle and a given direction, such as 90° clockwise.

- **Segment:** See line segment.

- **Transformation:** The mapping, or movement, of all points of a figure in a plane according to a common operation, such as translation, reflection or rotation.

- **Translation:** A transformation that slides each point of a figure the same distance in the same direction.

- **Vertex:** The location at which two lines, line segments or rays intersect.

## EVIDENCE OF LEARNING

By the conclusion of this unit, students should be able to demonstrate the following competencies:

- describe and compare function transformations on a set of points as inputs to produce another set of points as outputs, including translations and horizontal or vertical stretching

- represent and compare rigid and size transformations of figures in a coordinate plane using various tools such as transparencies, geometry software, interactive whiteboards, waxed paper, tracing paper, mirrors and digital visual presenters

- compare transformations that preserve size and shape versus those that do not.

- describe rotations and reflections of parallelograms, trapezoids or regular polygons that map each figure onto itself.

- develop and understand the meanings of rotation, reflection and translation based on angles, circles, perpendicular lines, parallel lines and line segments.

- transform a figure given a rotation, reflection or translation using graph paper, tracing paper, geometric software or other tools.

- create sequences of transformations that map a figure onto itself or to another figure.

## TEACHER RESOURCES

The following pages include teacher resources that teachers may wish to use to supplement instruction.

- Web Resources
- Guided Notes: Rotations
- Practice: Reflections

## Web Resources

The following list is provided as a sample of available resources and is for informational purposes only. It is your responsibility to investigate them to determine their value and appropriateness for your district. GaDOE does not endorse or recommend the purchase of or use of any particular resource.

- Transformational Geometry Applet
  http://www.howe–two.com/nctm/index.html
  This page contains animations helpful for presenting and visualizing transformations. For some of the applets on this page you may need a minute to familiarize yourself.

- Transformations in the Coordinate Plane
  http://www.virtualnerd.com/middle–math/integers–coordinate–plane/transformations
  This link has helpful short videos on the different rigid motion transformations.

- Learnist
  http://learni.st/users/S33572/boards/2623–transformations–in–the–coordinate–plane–common–core–standard–9–12–g–co–2
  This website links to many other helpful websites with notes on transformations.

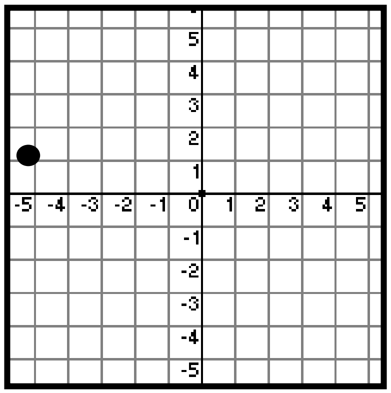
- Excerpt from Textbook on Transformations in the Coordinate Plane
  www.gaston.k12.nc.us/schools/cramerton/faculty/kllasky/Course%20Outline%20and%20Syllabus/Textbook/Ch%2010/Text%2010.8%20Transformations%20Coordinate%20Plane.pdf
  This link has a particular textbook's approach to the topic of geometric transformations.

- Escher – Tesselation Creation
  http://www.howe–two.com/nctm/eschersketch/eschersketch.html
  This webpage has a java applet which allows students to create intricate tessellations and geometric designs.

# Guided Notes: Rotations

Rotate around the origin.

Point A

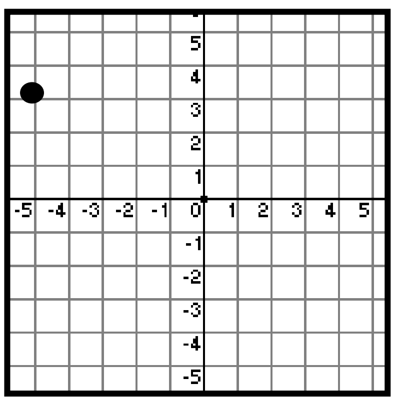


Rotate 90º counterclockwise
Point A’

Rotate 180º counterclockwise
Point A″

Rotate 270º counterclockwise
Point A″’

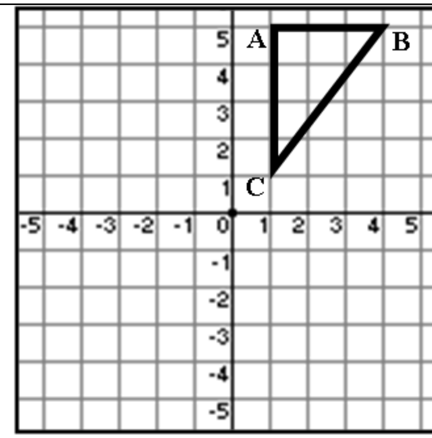
Point B



Rotate 90º counterclockwise
Point B’

Rotate 180º counterclockwise
Point B″

Rotate 270º counterclockwise
Point B″’




Rotate 90º counterclockwise.

Point A’ Point B’ Point C’

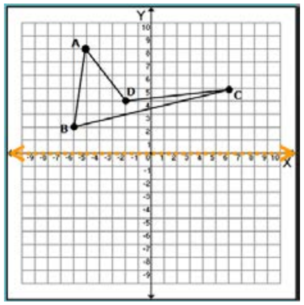
Rotate 180º counterclockwise.

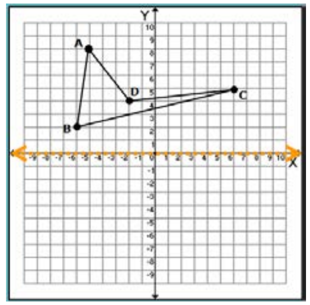
Point A″ Point B″ Point C″

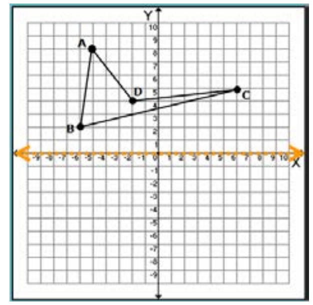
Rotate 270º counterclockwise.

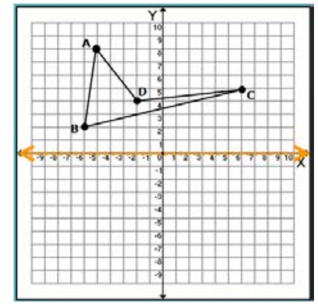
Point A″’ Point B″’ Point C″’

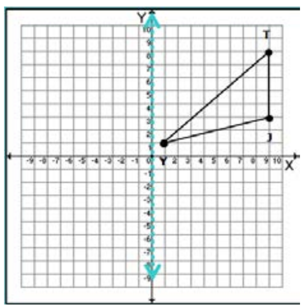
# Practice: Reflections

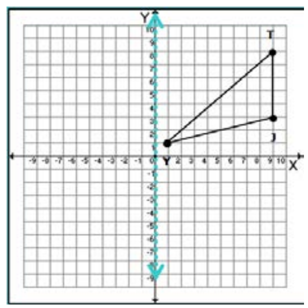


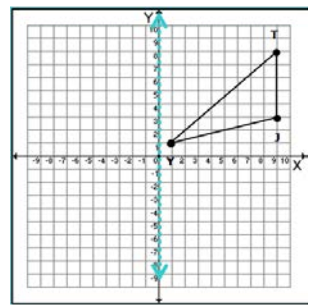



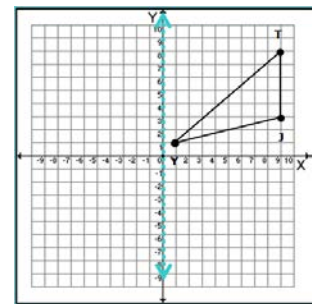



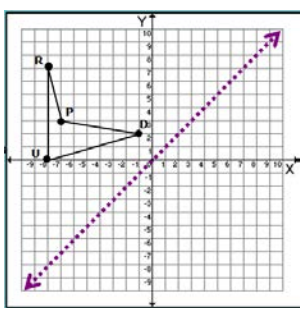



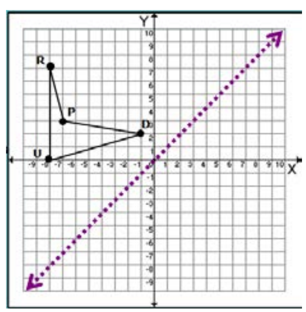



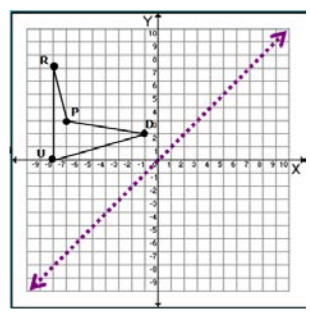



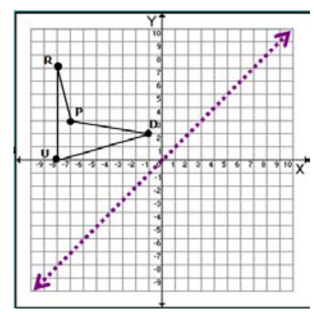



## FORMATIVE ASSESSMENT LESSONS (FAL)

**Formative Assessment Lessons** are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Overview.

## SPOTLIGHT TASKS

A Spotlight Task has been added to each GSE mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit–level Georgia Standards of Excellence, and research–based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3–Act Tasks based on 3–Act Problems from Dan Meyer and Problem–Based Learning from Robert Kaplinsky.

## TASKS

The following tasks represent the level of depth, rigor, and complexity expected of all Coordinate Algebra students. These tasks, or tasks of similar depth and rigor, should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as a performance task, they may also be used for teaching and learning (learning/scaffolding task).

| Task Name | Task Type<br>*Grouping Strategy* | Content Addressed | Standards |
|---|---|---|---|
| **Introduction to Reflections, Rotations, and Translations**<br>*50–60 minutes* | Learning Task<br>*Individual / Partner / Whole Group* | Describe properties of translations, reflections and rotations. | G.CO.1, 2 |
| **Exploring Reflections and Rotations**<br>*50–60 minutes* | Learning Task<br>*Individual / Partner* | Compare reflections and rotations. | G.CO.1, 2, 3, 4, 5 |
| **Mirrored Mappings**<br>*20–30 minutes* | Performance Task<br>*Partner / Small Group* | Understand reflections in the coordinate plane. | G.CO.1, 2, 5 |
| **Programming Transformations (Spotlight Task)**<br>*60–90 minutes* | Learning Task<br>*Partner/ Small Group* | Program points to generate transformations | G.CO.1, 2, 5 |
| **Coordinating Translations**<br>*20–30 minutes* | Homework Task<br>*Individual / Partner* | Translate objects and use rules to map an input with an output. | G.CO.1, 2, 4, 5 |
| **Transformations in the Coordinate Plane**<br>*50–60 minutes* | Learning Task<br>*Partner / Small Group* | Find general rules to describe transformations in the plane. | G.CO.1, 2, 5 |
| **Transformations**<br>(FAL) | Formative Assessment Lesson<br>*Partner / Small Group* | Rotating, reflecting, or translating a figure in the coordinate plane | G.CO.1, 2, 5 |
| **Representing and Combining Transformations**<br>(FAL)<br>*90–120 minutes*<br>PDF | Formative Assessment Lesson<br>*Partner / Small Group* | Recognize and visualize transformations of 2D shapes.<br>Translate, reflect, and rotate shapes, and combine these transformations. | G.CO.1, 2, 3, 4, 5 |
| **Transforming Shapes**<br>*≈ 2 hours* | Culminating Task<br>*Individual / Partner* | Explain and generalize transformations that maintain congruence. | G.CO.1, 2, 3, 4, 5 |

# Introduction to Reflections, Translations, and Rotations (Learning Task)

## Introduction

Students will use the available resources to explore reflections, transformations and rotations and report to the class what they have learned from the activities on the computer. Teachers can use this task to teach the vocabulary associated with transformations as students share their discoveries. Teachers may opt to have students do each transformation separately or all three together. Students should be somewhat familiar with transformations from eighth grade so each exploration of a transformation and discussion should only last about 30 minutes.

## Mathematical Goals

- Explore transformations in the coordinate plane using appropriate tools.

## Essential Questions

- What effects do transformations have on geometric figures in a coordinate plane?

## Georgia Standards of Excellence

**MGSE9-12.G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

**MGSE9–12.G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle measures to those that do not (e.g., translation versus horizontal stretch).

## Standards for Mathematical Practice

4. Model with mathematics.
   *Students represent transformations graphically.*
5. Use appropriate tools strategically.
   *Students can use a variety of tools to help them perform the transformations.*

## Background Knowledge

- Students need to have knowledge of basic transformations.
- Previous knowledge of using patty paper and MIRA's is helpful but not necessary.

## Common Misconceptions

- Students sometimes confuse the terms "transformation" and "translation."

## Materials

- Computer and presenter (or multiple computers)
- Optional variations include the use of: MIRAs, patty paper, graph paper, Geometer's Sketchpad or another software

## Grouping

- Individual / partner / whole group task

## Differentiation

**Extension:**

- Challenge students to “stump” each other with their transformations so their partner cannot determine the steps to get back to the original polygon.

**Intervention:**

- Resources such as patty paper or Miras can help students perform transformations.
- Folding paper can help with reflections:
  Trace over the preimage with a graphite pencil, fold on the axis of reflection, and press with your fingernail to transfer the graphite to the other side.
- Using a small scrap of paper can help with rotations:
  Trace the image onto a scrap of paper, and draw a small arrow pointing upward. Place your pencil on the center of rotation, and spin your paper until the arrow is pointing the new direction you want (e.g., pointing to the right for a 90º clockwise rotation).
  *Note:* If students trace the preimage onto the back *and* the front of the scrap of paper at the beginning of this process, they can rub with their fingernail to transfer the new image onto the paper in its new orientation.

## Formative Assessment Questions

- Describe two different ways to use transformations to map a geometric figure back onto itself.

# Introduction to Reflections, Translations, and Rotations – Teacher Notes

## Task Comments:

*Teachers may want to introduce their students to transformation using computer applets if they have access. The activities at the National Library of Virtual Manipulatives offer opportunities for students to experiment with various transformations.*

### *Reflections*

*Have students visit the National Library of Virtual Manipulatives to explore and describe properties of reflection. Use the manipulative named, 9–12 Geometry "Transformations–Reflections" and click on "Activities" to access the following:*

- *Playing with Reflections*
- *Hitting the Target*
- *Describing Reflection*

*Direct Link: http://nlvm.usu.edu/en/nav/frames_asid_298_g_4_t_3.html?open=activities*
*Home Page: http://nlvm.usu.edu/en/nav/vlibrary.html*

*The introduction to reflections using this website may be done as a whole group with a projector or in a computer lab individually or in pairs. After students explore with the applet, they should be prompted to check the "Axes" boxes and make observations about the coordinates of the vertices of objects and their reflected images. Students can move the line of reflection on top of either the horizontal or vertical axes. If the students work on the tasks as individuals or in pairs, prepare a list of questions for them to answer while exploring the website, such as "What happens to a shape as the reflection line is moved?" or "What happens when a shape is positioned so that is intersected by a line of reflection?" Teachers should also prompt students to justify their answers to the questions provided on the site.*

*At the end of this session, whether the activity is done as a whole group, individually, or in pairs, students should report to the whole class what they have learned. The purpose of these activities is to provoke class discussion.*

### *Translations*

*Have students visit the National Library of Virtual Manipulatives to explore and describe properties of translations. Use the manipulative named, 9–12 Geometry "Transformations– Translations" and click on "Activities" to access the following:*

- *Playing with Translations*
- *Hitting the Target*
- *Describing Translations.*

*Direct Link: http://nlvm.usu.edu/en/nav/frames_asid_302_g_4_t_3.html?open=activities*
*Home Page: http://nlvm.usu.edu/en/nav/vlibrary.html*

*The introduction to translations using this website may be done as a whole group with a projector or in a computer lab individually or in pairs. If the students work on the tasks as individuals or in pairs, prepare a list of questions for them to answer while exploring the website. For example, "What affects the location of the translated image?" "What patterns do you notice in the coordinates of the vertices when a polygon has a horizontal translation?" and "What would happen if you connect the corresponding vertices of the origin polygon and its image?" At the end of this session, whether the activity is done as a whole group, individually, or in pairs, students should report to the whole class what they have learned.*

*Teachers should offer guiding questions or prompt discussion of the parallel lines that are imbedded in translations of polygons (a side and its translated image are parallel to one another; the translation vectors of each vertex are also parallel to one another). Teachers may choose to introduce the term vector however students are not expected to formally know these terms until later in the curriculum.*

***Rotations***

*Have students visit the National Library of Virtual Manipulatives to explore and describe properties of rotation. Use the manipulative named, 9–12 Geometry "Transformations–Rotations" and click on "Activities" to access the following:*

- *Playing with Rotations*
- *Hitting a Target*
- *Describing Rotations*

*Direct Link: http://nlvm.usu.edu/en/nav/frames_asid_300_g_4_t_3.html?open=activities*
*Home Page: http://nlvm.usu.edu/en/nav/vlibrary.html*

*The introduction to rotations using this website may be done as a whole group with a projector or in a computer lab individually or in pairs. If the students work on the tasks as individuals or in pairs, prepare a list of questions for them to answer while exploring the web site. For example, "What determines the location of the image of a rotation?" or "If a rectangle is rotated 90º counterclockwise, what happens to the coordinates of its vertices?" At the end of this session, whether the activity is done as a whole group, individually, or in pairs, students should report to the whole class what they have learned.*

*Teachers familiar with Geometer's Sketchpad may also choose to utilize many published activities that will introduce transformations to their students.*

*Teachers may also choose to investigate the TransmoGrapher 2 at Interactivate Activities: http://www.shodor.org/interactivate/activities/transform2/index.html*

## Task Solutions:

**Part I**

1. On your graph paper draw and label a square. Describe its original position and size.

   ***Answers will vary***

2. Rotate it 90 degrees clockwise around any point.

3. Translate it so that it is in the 4th quadrant.

4. Reflect it over a line *y* = "a number" so that the square is in the 1st quadrant.

5. Write 2 distinctly different ways that you can get the shape back in its original position.
   ***Comment:***
   ***Students may want to use patty paper or a Mira to help with the transformations of their figure.***
   ***Answers will vary.***

**Part II**

6. On your graph paper draw and label a triangle. Describe its original position and size.

   ***Answers will vary***

7. Rotate, Translate, and Reflect the triangle so that the one side is touching an original side in such a way that it forms a parallelogram. List your steps here:

   ***Answers will vary***

**Part III**

8. On your graph paper draw and label a parallelogram. Describe its original position and size.

   ***Answers will vary***

9. Rotate, Translate, and Reflect the parallelogram several times, listing your steps here:

   ***Answers will vary***

10. Now, challenge a friend to get the parallelogram back into its original position! Are the steps that your friend used the reverse of your steps, or are they different?

    ***Answers will vary***

# Learning Task: Introduction to Reflections, Translations, and Rotations

**Name**________________________________ **Date**__________________

## Mathematical Goals

- Explore transformations in the coordinate plane using appropriate tools.

## Essential Questions

- What effects do transformations have on geometric figures in a coordinate plane?

## Georgia Standards of Excellence

**MGSE9–12.G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle measures to those that do not (e.g., translation versus horizontal stretch).

## Standards for Mathematical Practice

**4.** Model with mathematics.
**5.** Use appropriate tools strategically.

# Learning Task: Introduction to Reflections, Translations, and Rotations

**Name**________________________________ **Date**__________________

## Part I

1. On your graph paper draw and label a square. Describe its original position and size.

2. Rotate it 90º clockwise around any point.

3. Translate it so that it is in the $4^{th}$ quadrant.

4. Reflect it over a line *y* = "a number" so that the square is in the 1st quadrant.

5. Write **two** different ways that you can get the shape back in its original position.

## Part II

6. On your graph paper draw and label a triangle. Describe its original position and size.

7. Rotate, translate, and/or reflect the triangle so that the two triangles create a parallelogram. List your steps here:

## Part III

**8.** On your graph paper draw and label a parallelogram. Describe its original position and size.

**9.** Rotate, translate, and/or reflect the parallelogram several times, listing your steps here:

**10.** Now, challenge a friend to get the parallelogram back into its original position! (Don't reveal your steps.)

**11.** Record your friend's steps here:

**12.** Are the steps that your friend used the reverse of your steps, or are they different?

# Exploring Reflections and Rotations (Learning Task)

## Introduction

Students will utilize Geometer's Sketchpad (or another geometry software program) to determine the definitions of rotations and reflections. They will investigate patterns and develop a general rule for mapping an image with a reflection or rotation. In the extension, students will determine when a reflection and a rotation can be the same and how to map one figure to another using a reflection and/or a rotation.

## Mathematical Goals

- Develop and demonstrate an understanding of reflections and rotations of figures in general and on a coordinate plane.

## Essential Questions

- How are reflections and rotations similar and different?

## Georgia Standards of Excellence

**MGSE9-12.G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

**MGSE9–12.G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle measure to those that do not (e.g., translation versus horizontal stretch).

**MGSE9–12.G.CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

**MGSE9–12.G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

**MGSE9–12.G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

## Standards for Mathematical Practice

5. Use appropriate tools strategically.
   *Students use computer software to demonstrate transformations.*
7. Look for and make use of structure.
   *Students use the software to perform transformations, then try to generalize and understand the transformation that has taken place.*

## Background Knowledge

- Students need to know how to use Geometer's Sketchpad (or a similar program)

## Common Misconceptions

- When using notation like $(x, y) \rightarrow (-x, y)$, students may believe that $-x$ must always be negative, when in fact it simply means that it has the opposite sign of $x$.
- Students sometimes confuse the terms "transformation" and "translation."

## Materials

- Wireless laptops for each student or student group.
- LCD projector linked to teacher's computer
- Paper/Pencil
- Geometer's Sketchpad installed on each computer (or other Geometry software)
- Student Worksheet or Alternative Version (see below)
- translation_sketch.gsp file on each laptop (or available for students' use) http://www.georgiastandards.org/Common–Core/Documents/translation_sketch.gsp

## Grouping

- Individual / partner

## Differentiation

**Extension:**

- See #19–20 on student worksheet (or #17–18 on alternative version). In this extension, students look for situations where rotation(s) are equivalent to reflection(s).

**Intervention:**

- For rotations, connecting a vertex to the center of rotation can help motivate the connection between rotations and circles. Consider the connector segment as the "spoke" of a wheel. Drawing the circle itself may be helpful as well for students who have trouble visualizing this.

## Formative Assessment Questions

- Have students write two paragraphs on how reflections and rotations affect the position of an object on a coordinate plane. Ask students to give examples to justify their explanations.

# Exploring Reflections and Rotations – Teacher Notes

**Procedures:**

1. ***Prior to lesson, be sure to have the translation_sketch.gsp file available for each student or student group.***
***(If you do not have Geometer's Sketchpad, you can create the same file in Geogebra or Cabri or any other Geometry software. You need the lines x=2and y=0.5x – 5 drawn on the coordinate plane, along with a square with the coordinates (3, 0), (4, 1), (5,0) and (4, –1). You will also need the point B located at (2, 0). You could also create a couple of files with different coordinates of the square for students to use as a way to differentiate or check for individual learning.) An alternative version is provided to be used if you do not have access to the technology.***
2. ***Ask students what they believe the word "transformation" means in mathematics. After a brief discussion give students the "official" definition of what a transformation is in mathematics.***
3. ***Tell students that they will be experimenting with two different types of transformations today. Be sure to note to students that these two transformations are not the ONLY ones (or they might believe they are). Introduce the terms rotation and reflection. Ask students what they think each transformation would do to an object and how someone would know what specifically to do.***
4. ***Discuss with students the concept of a line of reflection. Be careful not to go into too much detail as the activity will allow them to discover specifics about this concept.***
5. ***Discuss with students the concept of a point of rotation and rotating a certain number of degrees. Inform students that "counter–clockwise" is the positive direction of rotation and "clockwise" is the negative direction of rotation.***
6. ***If this is the first time students have used Geometer's Sketchpad, have them open it and go through some of the basics on how to use the program. Otherwise proceed to the next step.***
7. ***Pass out the student worksheets and tell students to open the translation_sketch.gsp file. Tell students to follow the directions on the worksheet in working with the file they have.***
8. ***Allow students 20 – 30 minutes to work with the file and to answer the questions on the worksheet.***
9. ***After students have finished, bring the class together and have students discuss the following concepts:***
   a) ***What is a transformation?***
   b) ***How does one perform a reflection?***
   c) ***How does one perform a rotation?***
   d) ***How is a reflection similar to a rotation?***

## Task Solutions:

**Reflections:**

1. Select the vertices of the **square** and go to Display → Label Points. Start with C so that your **square**'s vertices are labeled C, D, E, & F. Find the coordinates of each point: C = (___,___); D = (___,___); E = (___,___); F = (___,___)

   ***Solution***
   ***C = (3,0); D = (4,1); E = (5,0); F = (4,–1) Student points will be the same but may be in different order depending on the point on which they start with C and if they continue clockwise or counterclockwise.***

2. Double click the line *x*=2. You should see an animation on the line. This makes *x*=2 the line of reflection (also called the line of symmetry).
3. Select the interior & the vertices of your **square**. Go to Transform → Reflect.
4. Select the vertices of the new **square** and go to Display → Show Labels.
5. How have the new points changed?
   ***Solution:***
   ***The points have moved to the left of the line x=2. The y–coordinates are still the same.***

6. Double click on the y–axis to change it to the line of reflection. Select the interior & vertices of your **square**. Go to Transform → Reflect.
7. What has happened?
   ***Solution:***
   ***The points have moved to the left of the line x=0 (y–axis). The x–coordinates are now the opposite and the y–coordinates are still the same.***

   Why is this reflection further away than the last one

   ***Solution:***
   ***The line x=2 was closer to the points so they moved the same distance from the line. Since the y–axis was further away from the square, they points moved the same distance away which was much further than the previous reflection.***

   What effect did changing the reflection line have?

   ***Solution:***
   ***It changed the distance from the points and thus the distance that the points had to move.***

8. Write out the coordinates of each square.

| Original Square | | Reflection over $x$=2 | | Reflection over y–axis | |
|---|---|---|---|---|---|
| C | (___,___) | C’ | (___,___) | C’ | (___,___) |
| D | (___,___) | D’ | (___,___) | D’ | (___,___) |
| E | (___,___) | E’ | (___,___) | E’ | (___,___) |
| F | (___,___) | F’ | (___,___) | F’ | (___,___) |

***Solution:***

| Original Square | | Reflection over $x$=2 | | Reflection over y–axis | |
|---|---|---|---|---|---|
| C | **(3,0)** | C’ | **(1,0)** | C’ | **(–3,0)** |
| D | **(4,1)** | D’ | **(0,1)** | D’ | **(–4,1)** |
| E | **(5,0)** | E’ | **(–1,0)** | E’ | **(–5,0)** |
| F | **(4,–1)** | F’ | **(0,–1)** | F’ | **(–4,–1)** |

How far apart are the original square and the first reflection? ***Solution: 2 spaces***

The original square and the second reflection? ***Solution: 6 spaces***

How far is the original square from $x$=2 and how far is the first reflection from $x$=2?
***Solution: 1 space***

How far is the original square from the y–axis and the second reflection and the $y$–axis?
***Solution: 3 spaces***

9. Delete the two reflections and their vertices.

10. Double click the line $y = 0.5x – 5$ to make it the new line of reflection. If you were to reflect the **square** over $y = 0.5x – 5$, **predict** where would the new vertices be?

| Original Square | | **Prediction** of Reflection over $y = 0.5x – 5$ | |
|---|---|---|---|
| C | (___,___) | C’ | (___,___) |
| D | (___,___) | D’ | (___,___) |
| E | (___,___) | E’ | (___,___) |
| F | (___,___) | F’ | (___,___) |

***Solution: Student responses will vary.***

11. ***After*** you have made your prediction, select the vertices and interior of the **square**. Go to Transform → Reflect. Select the vertices of the new **square** and go to Display → Show Labels. How does your prediction compare with the actual reflection?
***Solution:***
***Student responses will vary based upon their predictions. The reflection should be C’ (5.79, –5.63), D’ (7.24, –5.39),E’ (7.01, –3.99), F’ (5.60,–4.17).***

12. Make a general conclusion about what happens to coordinates of a point when they are reflected over a line.
***Solution:***
***Students should see that the distance from the point to the line remains the same for the reflection over the line.***

**Rotations:**

13. Delete the reflection so only the original square remains.
14. Using the Selection Arrow Tool, double click the point B. A brief animation should show. Point B is now your point of rotation.
15. Select the interior & the vertices of your **square**. Go to Transform → Rotate. A box labeled "Rotate" should come up. Move the box over to the side so you can see your sketch. You should see a lighter colored square that has appeared on your sketch. Change the number of degrees **BUT DON'T CLICK ANY BUTTONS**. The image should change where it goes. Try different numbers of degrees (less than 360). Explain what is happening to the square and the points?
***Solution:***
***The image of the square is being rotated about the point B like it is moving on a circle.***

16. Try negative degrees (once again **without clicking any buttons**). What do you notice about 90 and –270? 180 and –180? 45 and –315?
***Solution:***
***These pairs are creating the same image of the square at the same location.***

Why do you think this is?
***Solution:***
***This is a great preview for the unit circle! These measures take different directions in a circle and arrive at the same location.***

Are there any other pairs of measures that have the same phenomenon? How could we predict additional pairs?
***Solution:***
***There are infinite pairs of these rotation measures. If Angle A is a positive angle less than 360, then Angle A and Angle (A–360) will create these pairs.***

17. Type in 270 for the degrees and click the button "rotate". Write down the new coordinates and compare them to the old coordinates.

| Original Square | | Rotation around B | |
|---|---|---|---|
| C | (___,___) | C' | (___,___) |
| D | (___,___) | D' | (___,___) |
| E | (___,___) | E' | (___,___) |
| F | (___,___) | F' | (___,___) |

***Solution:***

| Original Square | | Rotation around B | |
|---|---|---|---|
| C | **(3, 0)** | C' | **(2, –1)** |
| D | **(4, 1)** | D' | **(3, –2)** |
| E | **(5, 0)** | E' | **(2, –3)** |
| F | **(4, –1)** | F' | **(1, –2)** |

What relationship is there between the points of the two figures?
***Solution:***
***The coordinates of the points are either 1 or 3 less, depending on their location at the start. All of the mapped points are the same distance from B, the point of rotation.***

18. What conclusions can you make about what happens to coordinates when rotated?
***Solution:***
***The points are rotated about a fixed point like points on a circle compared the center.***

**EXTENSION**

19. Is there ever a time when a rotation is the same as a reflection? Explain.
***Solution:***
***Students should experiment and find that a reflection about a horizontal or vertical line and a rotation of 180° about a point closest to the object on the line of reflection produces identical images. There are other rotations and reflections that come up with the same image as well.***

20. Create an animation in a new Geometer's Sketchpad window to support your answer to number 19. Email your animation to your teacher.

# Learning Task: Exploring Reflections and Rotations

**Name**__________________________________ **Date**___________________

## Mathematical Goals

- Develop and demonstrate an understanding of reflections and rotations of figures in general and on a coordinate plane.

## Essential Questions

- How are reflections and rotations similar and different?

## Georgia Standards of Excellence

**MGSE9-12.G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

**MGSE9–12.G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle measure to those that do not (e.g., translation versus horizontal stretch).

**MGSE9–12.G.CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

**MGSE9–12.G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

**MGSE9–12.G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

## Standards for Mathematical Practice

**5.** Use appropriate tools strategically.
**7.** Look for and make use of structure.

## Learning Task: Exploring Reflections and Rotations

**Name**_________________________________ **Date**__________________

**Reflections:**

1. Select the vertices of the **square** and go to Display → Label Points. Start with C so that your **square**'s vertices are labeled C, D, E, & F. Find the coordinates of each point:

   C (_____,_____); D (_____,_____); E (_____,_____); F (_____,_____)

2. Double click the line $x = 2$. You should see an animation on the line. This makes $x = 2$ the line of reflection (also called the line of symmetry).

3. Select the interior & the vertices of your **square**. Go to Transform → Reflect.

4. Select the vertices of the new **square** and go to Display → Show Labels.

5. How have the new points changed? ___________________________________________

   ________________________________________________________________________

6. Double click on the *y*–axis to change it to the line of reflection. Select the interior & vertices of your **square**. Go to Transform → Reflect.

7. What has happened? ______________________________________________________

   Why is this reflection further away than the last one? ____________________________

   ________________________________________________________________________

   What effect did changing the reflection line have? ______________________________

   ________________________________________________________________________

**8.** Write out the coordinates of each square.

| Original Square | Reflection over $x$ = 2 | Reflection over $y$–axis |
|---|---|---|
| C (___,___) | C' (___,___) | C' (___,___) |
| D (___,___) | D' (___,___) | D' (___,___) |
| E (___,___) | E' (___,___) | E' (___,___) |
| F (___,___) | F' (___,___) | F' (___,___) |

How far apart are the original square and the first reflection? ______________________

The original square and the second reflection? __________________________________

How far is the original square from $x$ = 2 and how far is the first reflection from $x$ = 2?

_____________________________________________________________________

How far is the original square from the $y$–axis and the second reflection from $y$–axis?

______________________________________________________________________

**9.** Delete the two reflections so that only your original square remains.

**10.** Double click the line $y$ = 0.5$x$ – 5 to make it the new line of reflection. If you were to reflect the **square** over $y$ = 0.5$x$ – 5, **predict** where would the new vertices be?

| Original Square | **Prediction** of Reflection over $y$ = 0.5$x$ – 5 |
|---|---|
| C (___,___) | C' (___,___) |
| D (___,___) | D' (___,___) |
| E (___,___) | E' (___,___) |
| F (___,___) | F' (___,___) |

**11.** ***<u>After</u>*** you have made your prediction, select the vertices and interior of the **square**. Go to Transform → Reflect. Select the vertices of the new **square** and go to Display → Show Labels. How does your prediction compare with the actual reflection?

________________________________________________________________

________________________________________________________________

**12.** Make a general conclusion about what happens to coordinates of a point when they are reflected over a line.

________________________________________________________________

________________________________________________________________

**Rotations:**

**13.** Delete the reflection so only the original square remains.

**14.** Using the Selection Arrow Tool, double click the point B. A brief animation should show. Point B is now your center of rotation.

**15.** Select the interior and the vertices of your **square**. Go to Transform → Rotate. A box labeled “Rotate” should come up. (Move the box over to the side so you can still see your sketch.) You should see a lighter colored square that has appeared on your sketch. Change the number of degrees **<u>BUT DON’T CLICK ANY BUTTONS</u>**. The image should change where it goes. Try different angles (between 0º and 360º). Explain what is happening to the square and the points.

________________________________________________________________

________________________________________________________________

**16.** Try negative angles (once again **without clicking any buttons**). What do you notice about 90º and –270º? 180º and –180º? 45º and –315º?

________________________________________________________________

Why do you think this is? ______________________________________________

________________________________________________________________

Are there any other pairs of measures that have the same phenomenon? How could we predict additional pairs?

______________________________________________________________________

**17.** Type in 270º (clockwise) for the angle and click the button "rotate". Write down the new coordinates and compare them to the old coordinates.

| Original Square | Rotation around B |
|---|---|
| C (___,___) | C' (___,___) |
| D (___,___) | D' (___,___) |
| E (___,___) | E' (___,___) |
| F (___,___) | F' (___,___) |

What relationship is there between the vertices of the two figures?

______________________________________________________________________

**18.** What conclusions can you make about what happens to coordinates when rotated?

______________________________________________________________________

______________________________________________________________________

**EXTENSION**

**19.** Is there ever a time when a rotation is the same as a reflection? Explain.

______________________________________________________________________

______________________________________________________________________

**20.** Create an animation in a new Geometer's Sketchpad window to support your answer to number 19. Email your animation to your teacher.

## Learning Task: Exploring Reflections and Rotations (Alternative)

**Name**_________________________________ **Date**__________________

**Reflections:**

1. On a piece of graph paper, graph the following points to create square CDEF
   C (3, 0); D (4, 1); E (5, 0); F (4, –1)

2. Draw the line: $x = 2$.

3. Using either Mira, patty paper or a transparency reflect the square over the $x = 2$ line.

4. How have the new points changed? ___________________________________________

   ________________________________________________________________________

5. Using the original square, now reflect it over the $y$–axis.

6. What has happened? ______________________________________________________

   Why is this reflection further away than the last one? ____________________________

   ________________________________________________________________________

   What effect did changing the reflection line have? _______________________________

   ________________________________________________________________________

7. Write out the coordinates of each square.

| Original Square | Reflection over $x = 2$ | Reflection over $y$–axis |
|---|---|---|
| C (___,___) | C' (___,___) | C' (___,___) |
| D (___,___) | D' (___,___) | D' (___,___) |
| E (___,___) | E' (___,___) | E' (___,___) |
| F (___,___) | F' (___,___) | F' (___,___) |

How far apart are the original square and the first reflection? ______________________

The original square and the second reflection?__________________________________

How far is the original square from $x = 2$ and how far is the first reflection from $x = 2$?

_______________________________________________________________________

How far is the original square from the $y$–axis and the second reflection and the $y$–axis?

________________________________________________________________________

**8.** Draw the line $y = 0.5x - 5$. If you were to reflect the **original square** over $y = 0.5x - 5$, **predict** where the new vertices would be.

| Original Square | **Prediction** of Reflection over $y = 0.5x - 5$ |
|---|---|
| C (___,___) | C' (___,___) |
| D (___,___) | D' (___,___) |
| E (___,___) | E' (___,___) |
| F (___,___) | F' (___,___) |

**9.** ***After*** you have made your prediction, use the Mira, patty paper, or transparency to reflect the original square over the $y = 0.5x - 5$ line. How does your prediction compare with the actual reflection?

________________________________________________________________________

________________________________________________________________________

**10.** Make a general conclusion about what happens to coordinates of a point when it is reflected over a line.

________________________________________________________________________

________________________________________________________________________

**Rotations:**

**11.** Return to the original square.

**12.** Plot the point B(2,0). Point B is now your center of rotation.

**13.** Experiment with rotating the square about point B using either patty paper or a transparency. Try different numbers of degrees (less than 360) counter clockwise. Explain what is happening to the square and the points.

______________________________________________________________________

______________________________________________________________________

**14.** Try rotating negative degrees (clockwise). What do you notice about 90º and –270º? 180º and –180º? 45º and –315º?

______________________________________________________________________

Why do you think this is?______________________________________________

______________________________________________________________________

Are there any other pairs of measures that have the same phenomenon? How could we predict additional pairs?

______________________________________________________________________

**15.** Rotate the square about point B, 270º clockwise. Write down the new coordinates and compare them to the old coordinates.

| Original Square | Rotation around B |
|---|---|
| C (___,___) | C' (___,___) |
| D (___,___) | D' (___,___) |
| E (___,___) | E' (___,___) |
| F (___,___) | F' (___,___) |

What relationship is there between the points of the two figures?

______________________________________________________________________

**16.** What conclusions can you make about what happens to coordinates when rotated?

______________________________________________________________________

______________________________________________________________________

**EXTENSION**

**17.** Is there ever a time when a rotation is the same as a reflection? Explain.

______________________________________________________________________

______________________________________________________________________

**18.** Create a sequence of transformations to support your answer to number 17.

# Mirrored Mappings (Performance Task)

## Introduction

This task is designed to develop a more formal definition of reflection. Students should develop the understanding that a reflection is not merely an image, but an action that maps an object to another location on the plane. This should build upon previous activities from earlier grades that introduced and discussed symmetry, and teachers may find it helpful to use additional activities utilizing MIRA's or Patty Paper™.

## Mathematical Goals

- Develop and demonstrate an understanding of reflections of figures on a coordinate plane.

## Essential Questions

- How do we reflect points in a coordinate plane?

## Georgia Standards of Excellence

**MGSE9-12.G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

**MGSE9–12.G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle measure to those that do not (e.g., translation versus horizontal stretch).

**MGSE9–12.G.CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

**MGSE9–12.G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

**MGSE9–12.G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

## Standards for Mathematical Practice

**5.** Use appropriate tools strategically.
*Students use a variety of tools to perform and understand reflections.*

**7.** Look for and make use of structure.
*Students generalize what they see with reflections into a formal definition of this transformation.*

**Background Knowledge**

- Previous knowledge of using patty paper and MIRA's is helpful but not necessary.

**Common Misconceptions**

- When using notation like $(x, y) \rightarrow (-x, y)$, students may believe that $-x$ must always be negative, when in fact it simply means that it has the opposite sign of $x$.
- Students sometimes confuse the terms "transformation" and "translation."

**Materials**

- Graph Paper
- Pencil
- MIRA's
- Patty Paper

**Grouping**

- Partners / small group

**Differentiation**

**Extension:**

- Investigate reflections across non–horizontal and non–vertical lines.
- Determine a general rule for reflecting a path (polygon) across any line.

**Intervention:**

- Encourage struggling students to fold their paper to see if the image and pre–image line up. Discuss the crease as the axis of reflection / line of symmetry, and use symmetry to derive properties of reflections.
- Folding paper can help draw reflections as well: Trace over the preimage with a graphite pencil, fold on the axis of reflection, and press with your fingernail to transfer the graphite to the other side.

**Formative Assessment Questions**

- How would you reflect the following points over the $x$–axis, the $y$–axis, and the line $x = 3$? Explain your process. Points: $A\ (2, 4)$ $B\ (5, 2)$ $C\ (3, -1)$

# Mirrored Mappings – Teacher Notes

Antonio and his friend Brittany were at summer math camp where the counselors had drawn a large coordinate plane on the gym floor. Antonio challenged Brittany to mirror him as he walked in the first quadrant. Map both of their travels on the same coordinate plane.

Antonio began at (2, 1) and walked to (3, 5); Brittany decided to begin at (–2, 1), then tried to mirror Antonio by walking to (–3, 5). Antonio jumped to (5, 5) and side–stepped to (4, 3); Brittany jumped to (–5, 5) then side–stepped to (–4, 3). Antonio returned to (2, 1) and Brittany returned to (–2, 1).

1. Did Brittany mirror Antonio?

   ***Comments:***

   ***Teachers should guide the discussion with questions such as: What does it mean to mirror? What exactly is being mirrored? What happens to your image in a mirror if you walk toward it? Or away from it? Do mirrored objects always have a reflection line? How could you determine where the reflection line is?***

   ***Solution:***

   ***Yes, Brittany mirrored Antonio. The line of symmetry (line of reflection) is the y–axis. Students should recognize through a class discussion that the y–axis is a "mirror" or reflection line. Some may argue that Brittany should have moved initially to (–1,5), which is moving the same distance and direction as Antonio and results in a translation.***

2. If Brittany had instead begun at (–2,1), walked to (–4,3), side–stepped to (–5,5), jumped to (–3,5) and then returned to (–2,1), could she claim that she created a mirror image of Antonio's path? Justify your answer.

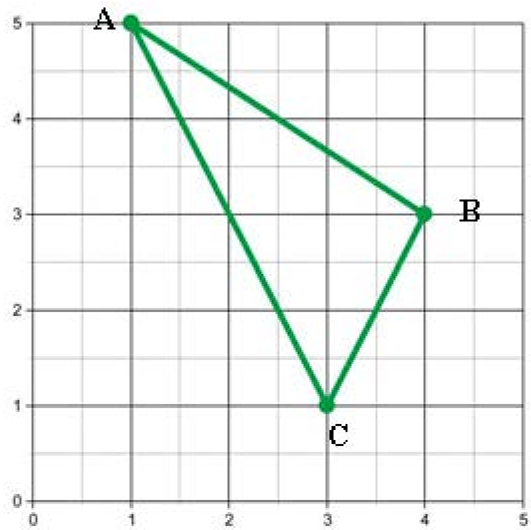
   ***Solution:***

   ***Yes, the completed path is a mirror image. Students should provide a justification for their answer that can help them develop the definition of reflections. During whole group discussions, teachers should use student justifications and debates about the questions to help students come to a consensus about a definition that is not dependent upon the particular movement of Brittany. Instead it is dependent upon creating a set of corresponding points that are reflected across the line of reflection.***

***Comments:***

***Students should recall from previous learning in earlier grades that they can fold the page along the x–axis to check their work. Students should discuss strategies for determining if points are reflected, including folding papers along the line of reflection and verifying the distance of corresponding points from the reflection line. It is critical to discuss that corresponding points with non–integer coordinates are still equidistant from the reflection line.***

Antonio and Brittany decided to change the game and use some lettered blocks to mark points they visited on the grid. Antonio placed blocks A, B and C as indicated by the points below, then drew a chalk line between them.




3. Label the coordinates Antonio used, and then construct the graph of where Brittany would place her blocks if she correctly reflected Antonio's figure across the x–axis.

   ***Solution:***

   ***The point (1, 5) would be mapped to (1, –5), the point (4, 3) would be mapped to (4, –3), and the point (3, 1) would be mapped to the point (3, –1).***

4. Describe how you determined where to place Brittany's blocks.

   ***Solution:***

   ***Flipping the blocks (points) over the x–axis means that they y coordinate now is the negative of the point flipped or reflected.***

5. Each block Brittany placed corresponds to one that Antonio placed. List each pair of coordinates that correspond.

   ***Solution:***

   ***(3, 1) → (3, –1) and (4, 3) → (4, –3)***

6. What can you observe about the distances between each of Antonio's blocks and the corresponding block Brittany placed?

   ***Solution:***

   ***The distance is twice the y coordinate of the block/point being reflected.***

7. If Antonio walked 2 feet from his block A toward his block C, and Brittany mirrored his movement by walking 2 feet from the blocks corresponding to A and C, would Brittany and Antonio be the same distance from the reflection line? How can you be certain?

   ***Solution:***

   ***Yes, Brittany and Antonio would be the same distance from the reflection line (the x–axis) because the actions are the same and would move both people closer to the x–axis by a little less than 2 feet. The same movement maps a mirror image when they start at mirrored locations.***

8. How would you define a reflection now that you have analyzed some of the properties of reflected images using the coordinate plane?

   ***Solution:***

   ***A reflection flips a point over the line of reflection so that the result looks like a mirrored image, with the line of reflection being the mirror.***

# Performance Task: Mirrored Mappings

**Name**__________________________________ **Date**__________________

**Mathematical Goals**

- Develop and demonstrate an understanding of reflections of figures on a coordinate plane.

**Essential Questions**

- How do we reflect points in a coordinate plane?

**Georgia Standards of Excellence**

**MGSE9-12.G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

**MGSE9–12.G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle measure to those that do not (e.g., translation versus horizontal stretch).

**MGSE9–12.G.CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

**MGSE9–12.G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

**MGSE9–12.G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

**Standards for Mathematical Practice**

**5.** Use appropriate tools strategically.
**7.** Look for and make use of structure.

## Performance Task: Mirrored Mappings

**Name**________________________________ **Date**_________________

Antonio and his friend Brittany were at summer math camp where the counselors had drawn a large coordinate plane on the gym floor. Antonio challenged Brittany to mirror him as he walked in the first quadrant. Map both of their travels on the same coordinate plane.

Antonio began at (2, 1) and walked to (3, 5); Brittany decided to begin at (–2, 1), then tried to mirror Antonio by walking to (–3, 5). Antonio jumped to (5, 5) and side–stepped to (4, 3); Brittany jumped to (–5, 5) then side–stepped to (–4, 3). Antonio returned to (2, 1) and Brittany returned to (–2, 1).

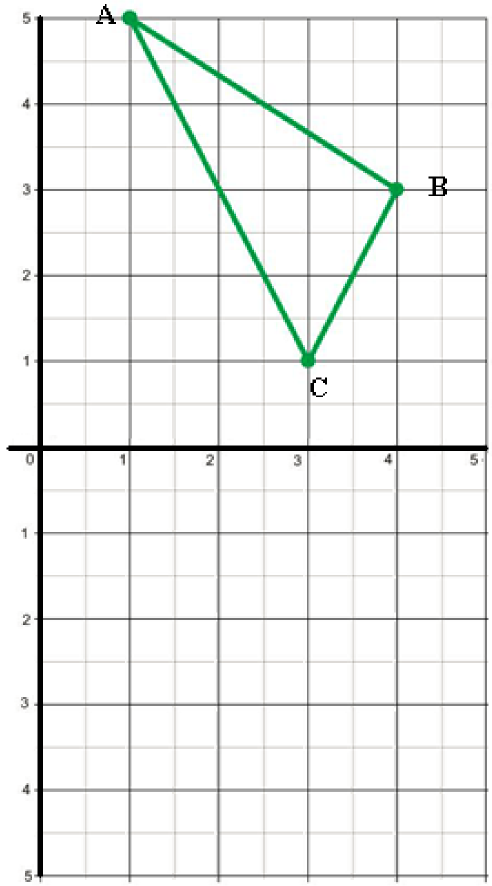
**1.** Did Brittany mirror Antonio?

   **a.** If you answered no, identify the incorrect coordinates Brittany used and find the correct coordinates. Explain your decision and identify the line of symmetry she should have used as a mirror. How did you know that this should have been the line of symmetry?

   **b.** If you answered yes, identify the line of symmetry Brittany used as a mirror. How did you know it was the line of symmetry?

**2.** If Brittany had instead begun at (–2,1), walked to (–4,3), side–stepped to (–5,5), jumped to (–3,5) and then returned to (–2,1), could she claim that she created a mirror image of Antonio's path? Justify your answer.

Antonio and Brittany decided to change the game and use some lettered blocks to mark points they visited on the grid. Antonio placed blocks A, B and C as indicated by the points below, then drew a chalk line between them.




**3.** Label the coordinates Antonio used, and then construct the graph of where Brittany would place her blocks if she correctly reflected Antonio's figure across the *x*–axis.

**4.** Describe how you determined where to place Brittany's blocks.

**5.** Each block Brittany placed corresponds to one that Antonio placed. List each pair of coordinates that correspond.

**6.** What can you observe about the distances between each of Antonio's blocks and the corresponding block Brittany placed?

**7.** If Antonio walked 2 feet from his block A toward his block C, and Brittany mirrored his movement by walking 2 feet from the blocks corresponding to A and C, would Brittany and Antonio be the same distance from the reflection line? How can you be certain?

**8.** How would you define a reflection now that you have analyzed some of the properties of reflected images using the coordinate plane?

# Programming Transformations (Spotlight Task)

## Georgia Standards of Excellence

**MGSE9-12.G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

**MGSE9–12.G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
<u>This standard is addressed by having students model transformations using coordinates. They will take a given point or set of points and write a function that will transform the points according to the translation, rotation or reflection.</u>

**MGSE9–12.G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
<u>This standard is addressed through use of the tools to map the sequence that will map a figure to a new location</u>

## STANDARDS FOR MATHEMATICAL PRACTICE

**1.** Make sense of problems and persevere in solving them. In this task, students will need to make sense of the geometric transformations of shapes by applying algebraic strategies. Students will need to persevere through the problem solving process in order to arrive at a solution.

**4.** Model with Mathematics. It will be critical in this task to make sense of a geometric concepts by modeling them with algebraic tools. By applying coordinates to a geometric transformation, students will be able to generalize what is happening to the shapes.

**5.** Use appropriate tools strategically. Students will need to select appropriate tools (graph paper, Desmos, Geogebra) in order to be successful at this task.

## ESSENTIAL QUESTIONS

- How can I model a geometric transformation on the coordinate plane?
- How can I combine transformations to map a figure to another location?

## MATERIALS REQUIRED

- Copy of student handout
- Graph paper
- Desmos Graphing App or Website (could be done without, but this site helps visualize the transformations)

**TIME NEEDED**

- 60–90 minutes based on the depth of investigation

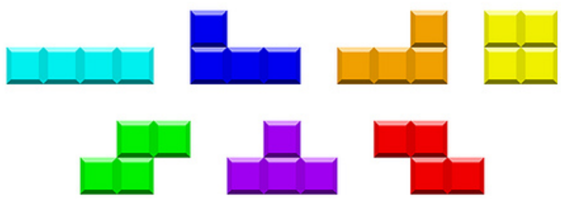
This task should be done near the beginning of unit 5 when talking about transformations of geometric figures. The later parts of this task can be challenging, but encouraging students to preserve will help them to better understand the material.

Introduction:

Begin the lesson by asking students if they have ever played the video game "Tetris". You can even show a video showing how the game was played on the side of a skyscraper:

http://www.nytimes.com/video/technology/100000002809966/the–worlds–biggest–tetris–game.html

In the game "Tetris", blocks are rotated and translated to complete horizontal lines. Blocks are made up of different arrangements of 4 squares.



When a horizontal line is completed, the line disappears and the rest of the blocks shift down. The blocks drop from the top of the screen and players must decide where to position the pieces as to clear the most lines. The only moves allowed in the game are horizontal shifts and rotations around the center of the figure. For our purposes, we will mark the lower left hand corner of each piece as a center for rotation. We will also explore reflections about horizontal and vertical lines in order to meet all of the standards.

The goal of this lesson is for students to write in generic, variable terms the formulas for transforming a figure. The context is stated as writing the computer code for a new variation of Tetris, one that includes translations, horizontal and vertical reflections, and rotations. The task is 3 fold:

1. Write the code that translates the point (a, b) horizontally in both directions and down vertically.
2. Write the code that reflects the point (a, b) over a vertical line and reflects down over a horizontal line.

3. Write the code that rotates the point (a, b) clockwise around another point (h, k) in increments of 90 degrees.

To begin, the students should define the corners of a brick. The lower left corner of the brick is defined as (a, b).

For example, the brick  would have the coordinates:

(a, b); (a +3, b); (a +3, b +1); (a +2, b + 1); (a +2, b +2); (a +1, b + 2); (a +1, b +1); (a, b + 1)

**Translations:**

In this portion, help students become familiar with using variables to represent points. The "code" writing is not difficult for this section, but take the time to solidify prerequisite skills such as defining variables and operations with variables and numbers. Also help students make the connection that performing the same translation on each vertex of the figure translates the entire figure.

Their final code should include defined variables and the operations necessary to complete the translation.

Sample answer:

Let (a, b) be a point on the coordinate plane.

Let h and k be integers

Translating (a, b) in a horizontal direction: (a+h, b) if h is positive, the point will translate h units to the right. If h is negative, the point will translate h units to the left.

Translating (a, b) in a vertical direction: (a, b+k) k can only be negative, so the point will translate down k units.

**Reflections:**

In this portion, students will write "code" to reflect a figure over a horizontal or vertical line. Encourage students to begin with a point and reflect it over either the x or y axis and see if they observe a pattern. They should observe that a reflection over a horizontal line preserves the x–coordinate, and a similar fact for the reflection over a vertical line.

In fact, the reflections over the axes are straightforward. (a, b) reflected over the x–axis becomes (a, –b). (a, b) reflected over the y–axis becomes (–a, b)

The challenge becomes guiding students to write the code for the reflection over a line other than an axis. The key idea is to think about translating the line being reflected over back to the x or y axis, and perform the reflection, then translate back to the original line.

For example, in order to reflect over the vertical line x = 2, translate the graph 2 units to the left (a–2, b), and then reflect (–(a–2), b), then translate the graph back to its original location (–(a–2)+2, b).

Do not spoil the students' discovery by revealing this relationship too soon. Allow them to experiment and discover on their own. Ask questions to guide them if they become stuck.

Sample questions:

What pattern do you observe by reflecting over the x or y axis?

How can you slide the graph to make it easier to reflect?

Could using a translation help?

Sample Answer:

Let (a,b) be a point on a graph and the line x = h be a vertical line of reflection

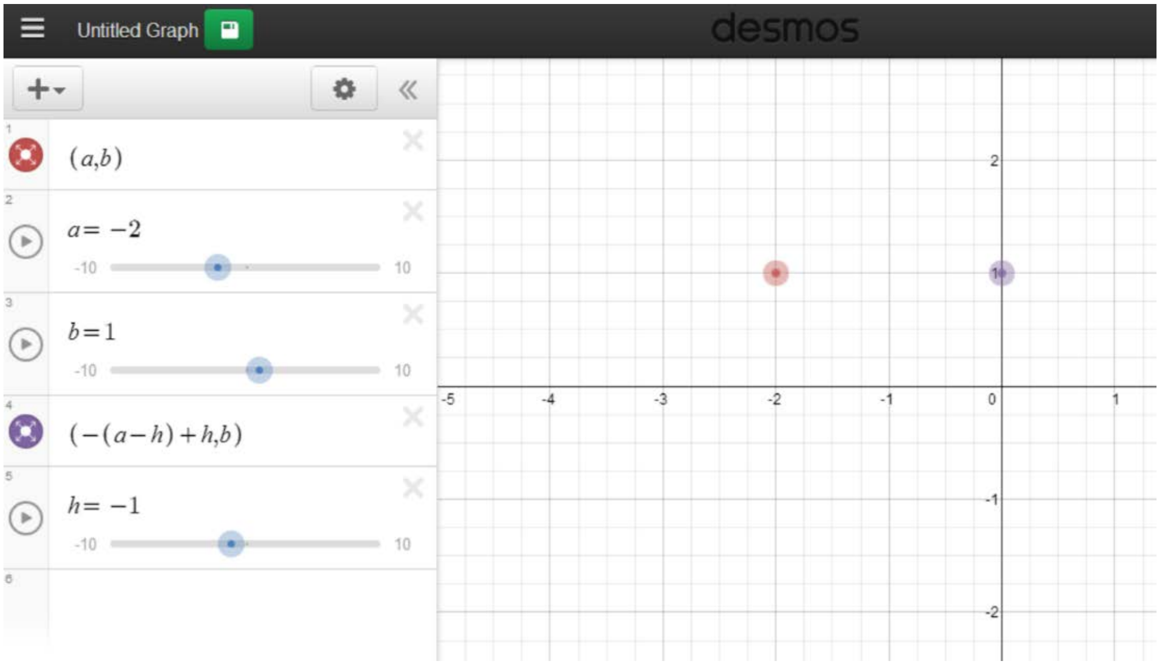
Then (–(a–h)+h, b) is the reflection over the line.

Let (a,b) be a point on a graph and line y = k be a horizontal line of reflection

Then (a, –(b–k)+k) is the reflection over the line.

Desmos.com option:

Have the students input the points and experiment with sliders to test their code:




**Rotations:**

This is the most challenging aspect of the task. Students must recognize the relationship between rotated points. Students have been exposed to this idea in 8th grade, but it needs to be solidified and deepened in Coordinate Algebra. The core understanding behind rotations is that every rotation of a point lies on a circle centered at the center of rotation.

Allow students to demonstrate what they already know and build on their previous knowledge. If they have difficulty getting started, give them a point on the x–axis and have them draw a circle centered at the origin and passing through that point. Then have them list the other 3 points that intersect the axes. Ask if they observe any patterns.

Follow this same line of inquiry by picking a point in the first quadrant and have them draw a circle centered at the origin and passing through that point. What other points does the circle pass through? What pattern exists between the points?

–For example, the circle centered at the origin and passing through (3, 4) also passes through (4, –3), (3, –4) and (–4, 3). (It passes through other points, but these points match the pattern discovered in the previous set of questions.)

Continue this investigation until the students are comfortable with the pattern of a generic point rotating around the origin.

(a, b) rotated around the origin 90 degrees in a clockwise direction is the point (b, –a) (a, b) rotated around the origin 180 degrees in a clockwise direction is the point (–a, –b) (this is equivalent to reflecting over the x – axis and then the y – axis.)

(a, b) rotated 270 degrees in a clockwise direction is the point (–b, a)

To finish the rotation code, students will need to consider the case of rotating around a point other than the origin. As with the reflections, the key is translating the center of rotation from the origin to the given center of rotation.

Here is an example of the reasoning required to write the code: (do not spoil it for the student, allow them to do the discovery)

Consider the point (2,4) rotated 90 degrees about the point (–1, 2). First, translate both points so that the center of the rotation is the origin. (2 –(–1), 4 – 2) rotated about the point (0, 0). Now use the rules from rotating about the origin: ( 4 – 2, –(2 – (–1))). Then translate it back to the original point of rotation. ((4 – 2) + (–1),–( 2 –(–1)) + 2). Thus the rotated point is (1, –1). Now generalize it for point (a, b) around point (h, k)

Thus the final code should look something like this:

Let (a, b) be a point rotated around another point (h, k).

Then (a, b) rotated clockwise 90 degrees is (b –k +h, –(a – h) +k)

(a, b) rotated clockwise 180 degrees is (–(a – h)+h, –(b – k) +k)

(a, b) rotated clockwise 270 degrees is (–(b – k)+h, a – h +k)

At this point, it is necessary to emphasize to the teacher and the student that this exercise is not designed to memorize formulas or steps. It is designed to have students model the relationships so that they will understand the relationships better. By associating the rotation around any point as the rotation around a point translated from the origin, it will help the student to find the rotated coordinates.

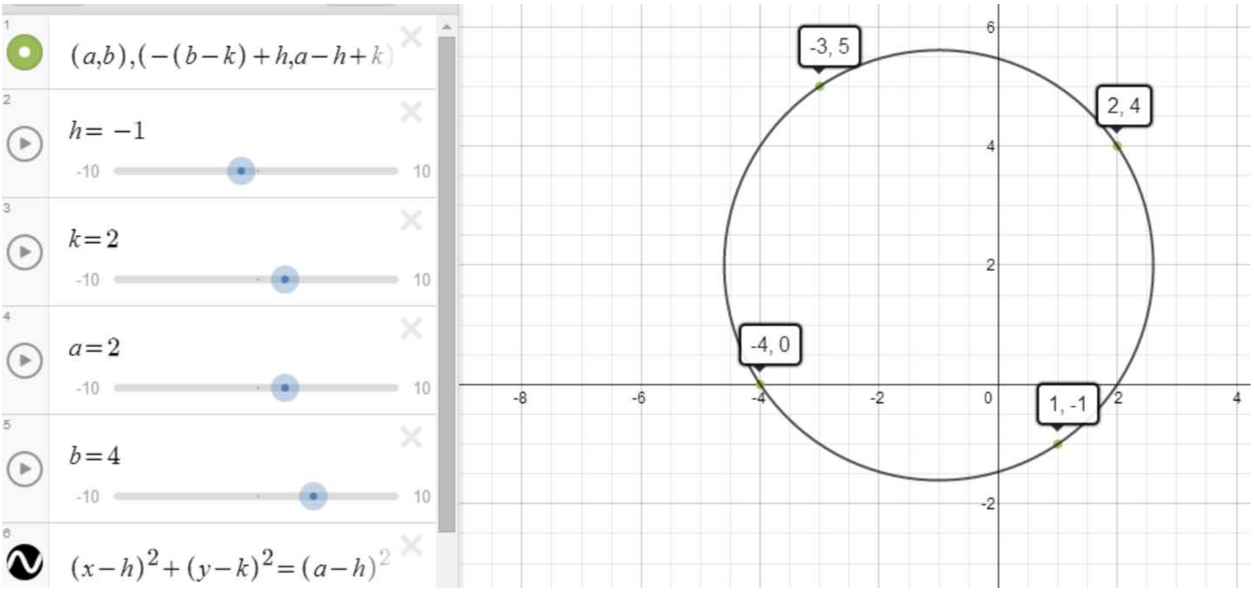
Now that the students have written the code for the moves, it is time to test their codes with other students.

Have each pair of students team up with another pair of students to “play” Tetris. Each pair chooses a piece from one of the seven Tetris pieces for the other pair to maneuver down the “board” (graph paper). The other team then chooses transformations to get the piece to the bottom. They should keep a list of the transformations performed and the input points and output points that resulted from using their “code”.

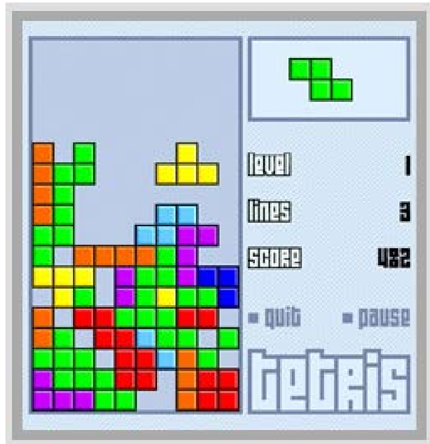
Students can try several pieces until they feel comfortable with all the transformations.

Extension:

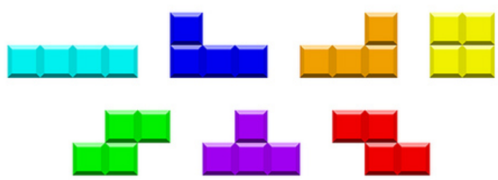
For students who master the content early in the lesson, allow them to investigate the rotations further. There is a wealth of mathematics available involving equations of circles, the distance formula, Pythagorean theorem, etc. that could be investigated. Here is a graph from Desmos.com involving the rotations discussed above.



# Programming Transformations Spotlight Task



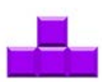

Tetris is a computer game that drops bricks in vertical columns. The player then uses rotations and translations to place the bricks in order to complete horizontal lines. There are 7 different bricks in the game Tetris.



Write a code for each of the following transformations

The code to represent any two of the above pieces, beginning with the bottom left corner of the piece as point (a, b). The points should only be the corners of each piece.

***The following is one example of possible answers. There are many possible correct answers.***

***For example, the brick***  ***would have the coordinates:***

***(a, b); (a +3, b); (a +3, b +1); (a +2, b + 1); (a +2, b +2); (a +1, b + 2); (a +1, b +1); (a, b + 1)***

Translate down a given number of units

***{Let c be the given number of units; then: (a, b – c) translates c units down}***

Translate left or right a given number of units

***{Let c be the given number of units; then: (a + c , b) translates c units right, and (a – c, b) translates c units left}***

Reflect over a given vertical line

***Let (a, b) be a point on a graph and the line x = h be a vertical line of reflection***

***Then (–(a–h)+h, b) is the reflection over the line.***

Reflect down over a given horizontal line

***Let (a, b) be a point on a graph and line y = k be a horizontal line of reflection***

***Then (a, –(b–k)+k) is the reflection over the line.***

Rotate 90 degrees clockwise around the point (h, k)

***Let (a, b) be a point rotated around another point (h, k).***

***Then (a, b) rotated clockwise 90 degrees is (b –k +h, –(a – h) +k)***

Rotate 180 degrees clockwise around the point (h, k)

***Let (a, b) be a point rotated around another point (h, k).***

***Then (a, b) rotated clockwise 180 degrees is (–(a – h)+h, –(b – k) +k)***

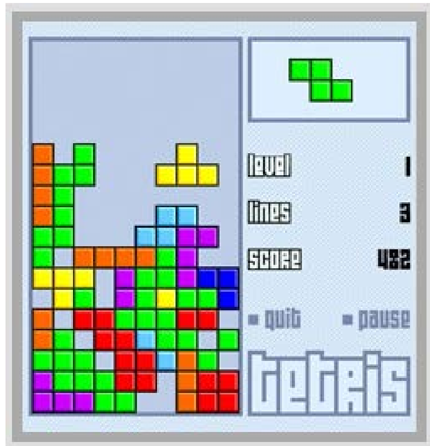
Rotate 270 degrees clockwise around the point (h, k)

***Let (a, b) be a point rotated around another point (h, k).***

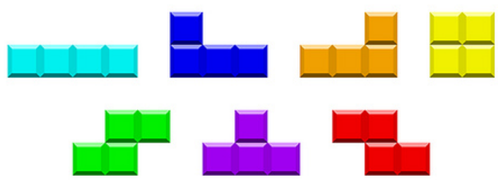
***(a, b) rotated clockwise 270 degrees is (–(b – k)+h, a – h +k)***

Be sure to clearly define any variables that you use.

## Programming Transformations (Spotlight Task) Student Version




Tetris is a computer game that drops bricks in vertical columns. The player then uses rotations and translations to place the bricks in order to complete horizontal lines. There are 7 different bricks in the game Tetris.



Write a code for each of the following transformations

The code to represent any two of the above pieces, beginning with the bottom left corner of the piece as point (a, b). The points should only be the corners of each piece.

Translate down a given number of units

Translate left or right a given number of units

Reflect over a given vertical line

Reflect down over a given horizontal line

Rotate 90 degrees clockwise around the point (h, k)

Rotate 180 degrees clockwise around the point (h, k)

Rotate 270 degrees clockwise around the point (h, k)

Be sure to clearly define any variables that you use.

# Coordinating Translations (Homework Task)

### Introduction

In this lesson, students will work on their own to create a polygon, complete three translations and answer questions that demonstrate their understanding of translations.

### Mathematical Goals

- Translate a geometric figure based on given directions and then explain the general rule.

### Essential Questions

- How do we translate a geometric figure in the coordinate plane?

### Georgia Standards of Excellence

**MGSE9-12.G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

**MGSE9–12.G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle measure to those that do not (e.g., translation versus horizontal stretch).

**MGSE9–12.G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

**MGSE9–12.G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

### Standards for Mathematical Practice

**2.** Reason abstractly and quantitatively.
*Students connect graphical representations with the function (symbolic) representation of translations.*

**8.** Look for and express regularity in repeated reasoning.
*Students look for patterns in their results to develop a general rule for the symbolic representation of translations.*

### Background Knowledge

- Students need to understand that transformations can be represented using notation like $(x + 4, y – 3)$

### Common Misconceptions

- Students sometimes confuse the terms “transformation” and “translation.”

## Materials

- Graph paper
- Pencil
- Ruler (optional)
- Directions for students either on a hand–out or using a projector

## Grouping

- Individual / partner

## Differentiation

**Extension:**

- See #6–9 on student worksheet. These problems ask students to generalize the rules for translations.
- Compare series of translations to single equivalent translations. Use function representations to prove the two are equivalent. For example: "right 3, down 4, left 2, up 1, right 5" is equivalent to "right 6, down 3" because $(x, y) \rightarrow (x + 3, y) \rightarrow$ $(x + 3, y - 4) \rightarrow (x + 3 - 2, y - 4) \rightarrow (x + 3 - 2, y - 4 + 1) \rightarrow (x + 3 - 2 + 5, y - 4 + 1)$ $= (x + 6, y - 3)$, which is a shift right 6 and down 3.

**Intervention:**

- This task is already heavily scaffolded for students.

## Formative Assessment Questions

- Describe the transformation that takes place for each of the following rules:
  $(x, y) \rightarrow (x - 5, y + 7)$
  $(x, y) \rightarrow (x, y - 4)$
  $(x, y) \rightarrow (x + 2, y)$

## Coordinating Translations – Teacher Notes

Create any polygon you want on the coordinate plane, and then create polygons congruent to the one you designed using the three directions described below. Use the same coordinate plane for all transformations.

1. For each vertex of your original polygon in the form $(x, y)$, create its image at the coordinates $(x + 4, y)$.
2. For each vertex of your original polygon in the form $(x, y)$, create its image at the coordinates $(x, y – 3)$.
3. For each vertex of your original polygon in the form $(x, y)$, create its image at the coordinates $(x – 4, y + 1)$.
4. What kind of transformations are these?
5. Can you create a translation $(x + 2, y + 2)$? Is it necessary that the same number is added or subtracted to the $x$ and $y$ coordinates of the polygon? Why or Why not?

***Comments:***

***Teachers should encourage students to use fairly simple polygons at first, but then move to more complicated designs. Students should also recognize through class discussion that all points, not merely integer coordinates would be translated using the notation (x+h, y+k). As an extension, teachers can use a variety of rational number coordinates.***

***Solution:*** *Answers will vary.*

**EXTENSION:**

6. $(x + c, y)$

***Solution:* If c is positive, this moves each point c units to the right (parallel to the x–axis).**

7. $(x, y – c)$

***Solution:* If c is positive, this moves each point c units down (parallel to the y–axis).**

8. $(x – c, y)$

***Solution:* If c is positive, this moves each point c units to the left.**

9. $(x , y + c)$

***Solution:* If c is positive, this moves each point c units up.**

# Homework Task: Coordinating Translations

**Name**________________________________ **Date**__________________

## Mathematical Goals

- Translate a geometric figure based on given directions and then explain the general rule.

## Essential Questions

- How do we translate a geometric figure in the coordinate plane?

## Georgia Standards of Excellence

**MGSE9-12.G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

**MGSE9–12.G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle measure to those that do not (e.g., translation versus horizontal stretch).

**MGSE9–12.G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

**MGSE9–12.G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

## Standards for Mathematical Practice

**2.** Reason abstractly and quantitatively.
**8.** Look for and express regularity in repeated reasoning.

## Homework Task: Coordinating Translations

**Name**________________________________ **Date**__________________

Create any polygon you want on the coordinate plane, and then create polygons congruent to the one you designed using the three directions described below. Use the same coordinate plane for all transformations.

**1.** For each vertex of your original polygon in the form $(x, y)$, create its image at the coordinates $(x + 4, y)$. Record your answers in the table below.

**2.** For each vertex of your original polygon in the form $(x, y)$, create its image at the coordinates $(x, y – 3)$. Record your answers in the table below.

**3.** For each vertex of your original polygon in the form $(x, y)$, create its image at the coordinates $(x – 4, y + 1)$. Record your answers in the table below.

| **Original polygon's vertices** $(x, y)$ | **#1** $(x + 4, y)$ | **#2** $(x, y – 3)$ | **#3** $(x – 4, y + 1)$ |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

**4.** What kind of transformations are these?

**5.** Can you create a translation $(x + 2, y + 2)$? Is it necessary that the same number is added or subtracted to the $x$ and $y$ coordinates of the polygon? Why or why not?

**EXTENSION:**

Provide a description of each of the following translations, where $c$ represents a positive number.

**6.** $(x + c, y)$

**7.** $(x, y - c)$

**8.** $(x - c, y)$

**9.** $(x , y + c)$

# Transformations in the Coordinate Plane (Learning Task)

## Introduction

In this activity, students will create six figures using ordered pairs on different coordinate planes and compare the last five figures to the first figure. They will have to decide which figures are congruent, similar or neither and create rules of how the coordinate changed. Then students will take the first figure and create a list of transformations to move it from its pre–image to entirely located in Quadrant III, drawing a picture of each step and writing the rule for the coordinates.

## Mathematical Goals

- Find rules to describe transformations in the coordinate plane.

## Essential Questions

- How do we describe a transformation or series of transformations that take place in the coordinate plane?

## Georgia Standards of Excellence

**MGSE9-12.G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

**MGSE9–12.G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

**MGSE9–12.G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

## Standards for Mathematical Practice

**1.** Make sense of problems and persevere in solving them.
*Students must visually make sense of the transformations that have taken place and represent those transformations mathematically.*

**4.** Make sense of problems and persevere in solving them.
*Students mathematically describe the transformations that have occurred.*

## Background Knowledge

- Students should know the difference between translations, rotations, reflections, and horizontal / vertical stretches.
- Students need to draw transformations described in mathematical terms.
- Students need to describe (in mathematical terms) transformations that map one given figure onto another.

**Common Misconceptions**

- Students sometimes confuse the terms "transformation" and "translation."

**Materials**

- Graph paper (provided after student worksheet)
- Hand–out
- Pencil
- Straight–edge

**Grouping**

- Partners / Small Group

**Differentiation**

**Extension:**

- Provide an already–transformed figure and ask students to describe the transformations that carried Figure 1 onto the given figure. These can vary in difficulty.

**Intervention:**

- If students are having difficulty identifying transformations visually, encourage them to keep an organized table of coordinates to help them identify the types of transformations that have occurred.
- For #4 and #5, using fewer transformations will make it easier for students to transform Figure 1 onto their new figure. Start by using just one or two transformations (perhaps just translations), but be sure students go further than this stepping stone.

**Formative Assessment Questions**

- Describe the transformation that takes place when the following rules are applied to a point or a figure:
  $(x, y) \rightarrow (4x, 4y)$
  $(x, y) \rightarrow (x, 2y)$
  $(x, y) \rightarrow (3x, y)$

# Transformations in the Coordinate Plane – Teacher Notes

Plot the ordered pairs given in the table to make six different figures. Draw each figure on a separate sheet of graph paper. Connect the points with line segments as follows:

- For Set 1, connect the points in order.
  Connect the last point in the set to the first point in the set.
- For Set 2, connect the points in order.
  Connect the last point in the set to the first point in the set.
- For Set 3, connect the points in order.
  Do **not** connect the last point in the set to the first point in the set.
- For Set 4, make a dot at each point (don't connect the points).

| **FIGURE 1** | **FIGURE 2** | **FIGURE 3** | **FIGURE 4** | **FIGURE 5** | **FIGURE 6** |
|---|---|---|---|---|---|
| **Set 1** | | | | | |
| For Set 1, connect the points in order, and connect the last point to the first point in the set. | | | | | |
| (6, 4) | (12, 8) | (18, 4) | (18, 12) | (6, 12) | (8, 6) |
| (6, –4) | (12, –8) | (18, –4) | (18, –12) | (6, –12) | (8, –2) |
| (–6, – 4) | (–12, –8) | (–18, –4) | (–18, –12) | (–6, –12) | (–4, –2) |
| (–6, 4) | (–12, 8) | (–18, 4) | (–18, 12) | (–6, 12) | (–4, 6) |
| **Set 2** | | | | | |
| For Set 2, connect the points in order, and connect the last point to the first point in the set. | | | | | |
| (1, 1) | (2, 2) | (3, 1) | (3, 3) | (1, 3) | (3, 3) |
| (1, –1) | (2, –2) | (3, –1) | (3, –3) | (1, –3) | (3, 1) |
| (–1, –1) | (–2, –2) | (–3, –1) | (–3, –3) | (–1, –3) | (1, 1) |
| (–1, 1) | (–2, 2) | (–3, 1) | (–3, 3) | (–1, 3) | (1, 3) |
| **Set 3** | | | | | |
| For Set 3, connect the points in order, but DO NOT connect the last point to the first point. | | | | | |
| (4, –2) | (8, –4) | (12, –2) | (12, –6) | (4, –6) | (6, 0) |
| (3, –3) | (6, –6) | (9, –3) | (9, –9) | (3, –9) | (5, –1) |
| (–3, –3) | (–6, –6) | (–9, –3) | (–9, –9) | (–3, –9) | (–1, –1) |
| (–4, –2) | (–8, –4) | (–12, –2) | (–12, –6) | (–4, –6) | (–2, 0) |
| **Set 4** | | | | | |
| For Set 4, don't connect the points at all. | | | | | |
| (4, 2) | (8, 4) | (12, 2) | (12, 6) | (4, 6) | (6, 4) |
| (–4, 2) | (–8, 4) | (–12, 2) | (–12, 6) | (–4, 6) | (–2, 4) |

After drawing the six figures, **compare Figure 1 to each of the other figures** and answer the following questions.

***Comments:***

***Source: Adapted from Stretching and Shrinking: Similarity, Connected Mathematics, Dale Seymour Publications***

***Students will find rules to describe transformations in the coordinate plane. Rules of the form (nx, ny) transform a figure in the plane into a similar figure in the plane. This transformation is called a dilation with the center of dilation at the origin. The coefficient of x and y is the scale factor. Adding a number to x or y results in a translation of the original figure but does not affect the size. Thus, a more general rule for dilations centered at the origin is (nx + a, ny + b).***

***Students will also observe that congruence is a special case of similarity (n=1). Congruent figures have the same size and shape. As students learned in previous tasks, transformations that preserve congruence are translations, reflections, and rotations.***

***Possible Solutions:***

***The scale used on the x– and y–axes in the figures below is 2 units. Each square is 4 square units (2 x 2).***

*Figure 1:*

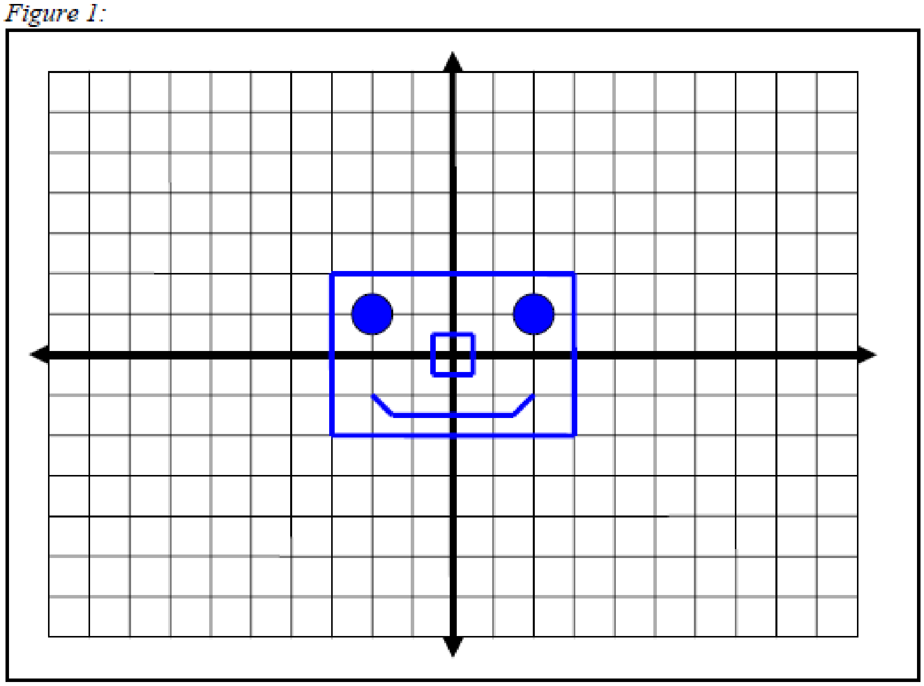


*Figure 2:*

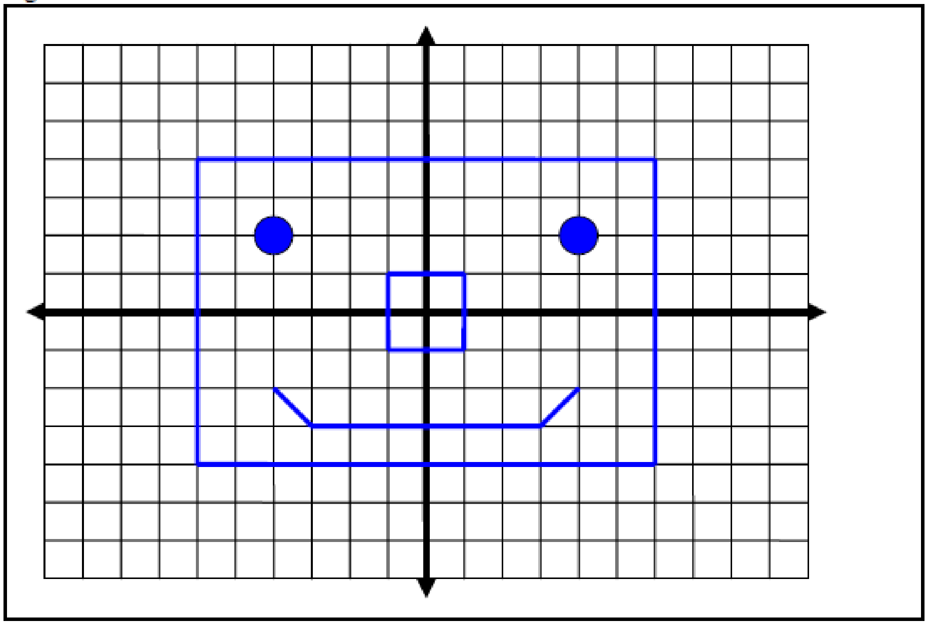


*Figure 3:*

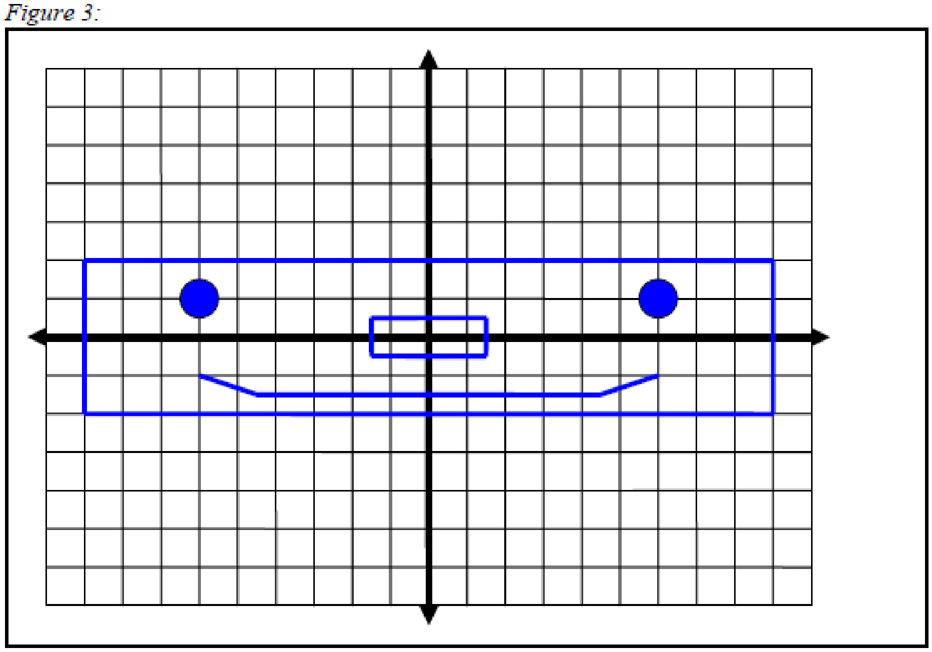


*Figure 4:*

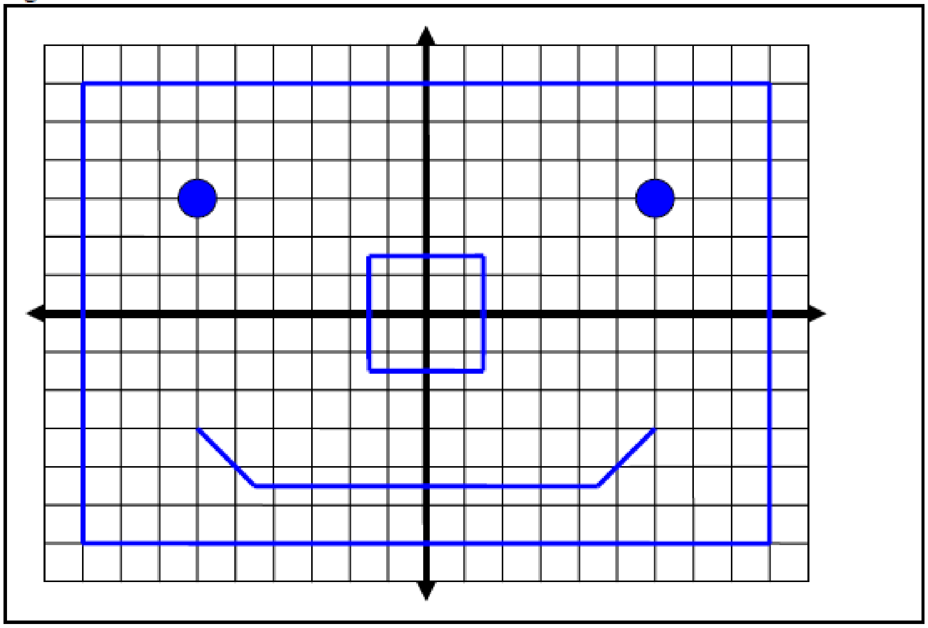


*Figure 5:*

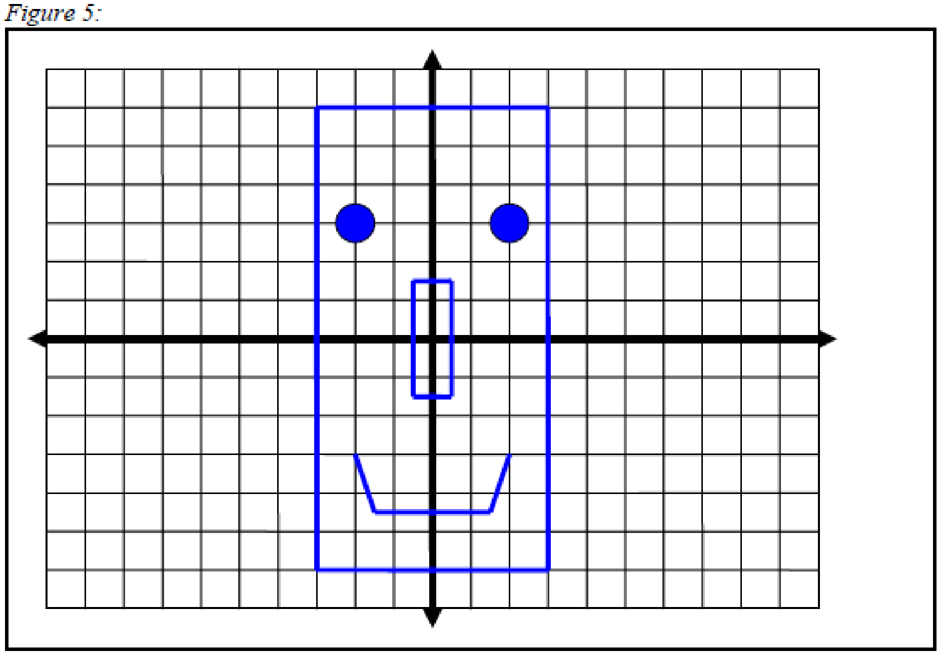
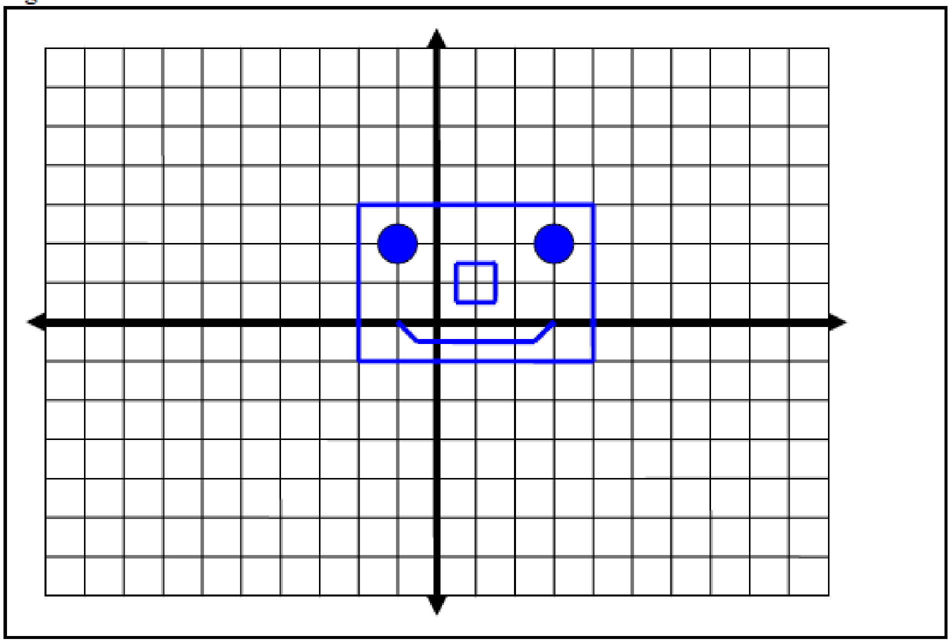


*Figure 6:*



1. Describe any similarities and/or differences between Figure 1 and each of the other figures.

   ***Solution:***

   ***Figure 2** is an enlargement of Figure 1. The figures have the same shape but different sizes. The ratio of the lengths of the corresponding sides is 1 to 2. The corresponding angles are equal in measure.*

   ***Figure 3** is wider or longer than Figure 1. The figures are different shapes and sizes. The ratio of the lengths of the corresponding sides is not constant. For one dimension, the ratio is 1 to 3; for the other dimension, the ratio is 1 to 1. The corresponding angles are equal in measure.*

   ***Figure 4** is an enlargement of Figure 1. The figures have the same shape but different sizes. The ratio of the lengths of the corresponding sides is 1 to 3. The corresponding angles are equal in measure.*

   ***Figure 5** is taller than Figure 1. The figures have different shapes and sizes. The ratio of the lengths of the corresponding sides is not constant. For one dimension, the ratio is 1 to 3; for the other dimension, the ratio is 1 to 1. The corresponding angles are equal in measure.*

   ***Figure 6** is the same shape and size as Figure 1. Figure 1 is shifted (i.e., translated) up and to the right to get Figure 6. The ratio of the lengths of the corresponding sides is 1 to 1. The corresponding angles are equal in measure.*

2. How do the coordinates of each figure compare to the coordinates of Figure 1? Write general rules for making Figures 2–6.

   ***Solution:***

   ***Figure 2: Both the x and y coordinates are multiplied by 2. (2x, 2y)***

   ***Figure 3: The x coordinates in Figure 3 are three times the corresponding x coordinates in Figure 1; the y coordinates are the same. (3x, y)***

   ***Figure 4: Both the x and y coordinates are multiplied by 3. (3x, 3y)***

   ***Figure 5: The x coordinates in Figure 5 are the same as the corresponding x coordinates in Figure 1. The y coordinates are three times the corresponding y coordinates in Figure 1. (x, 3y)***

   ***Figure 6: Two is added to both the x and y coordinates. (x + 2, y + 2)***

3. Translate, reflect, and rotate (between 0 and 180°) Figure 1 so that it lies entirely in Quadrant III on the coordinate plane. You may perform the transformations in any order that you choose. Draw a picture of the new figure at each step and explain the procedures you followed to get the new figure. Use coordinates to describe the transformations you used.

   ***Solution: Student responses will vary.***

4. Translate, reflect, and rotate (between 0 and 180°) Figure 1 so that it lies entirely in Quadrant IV on the coordinate plane. You may perform the transformations in any order that you choose. Draw a picture of the new figure at each step and explain the procedures you followed to get the new figure. Use coordinates to describe the transformations you used.

   ***Solution: Answers will vary depending on the transformations that students use.***

5. Describe the transformations necessary to take the figure you created in question #4 on to Figure 6.

   ***Solution: Student responses will vary.***

## Learning Task: Transformations in the Coordinate Plane

**Name**_________________________________ **Date**__________________

### Mathematical Goals

- Find rules to describe transformations in the coordinate plane.

### Essential Questions

- How do we describe a transformation or series of transformations that take place in the coordinate plane?

### Georgia Standards of Excellence

**MGSE9-12.G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

**MGSE9–12.G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

**MGSE9–12.G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

### Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.

# Learning Task: Transformations in the Coordinate Plane

**Name__________________________________** **Date__________________**

Plot the ordered pairs given in the table to make six different figures. Draw each figure on a separate sheet of graph paper. Connect the points with line segments as follows:

- For Set 1, connect the points in order.
  Connect the last point in the set to the first point in the set.
- For Set 2, connect the points in order.
  Connect the last point in the set to the first point in the set.
- For Set 3, connect the points in order.
  Do **not** connect the last point in the set to the first point in the set.
- For Set 4, make a dot at each point (don't connect the dots).

| **FIGURE 1** | **FIGURE 2** | **FIGURE 3** | **FIGURE 4** | **FIGURE 5** | **FIGURE 6** |
|---|---|---|---|---|---|
| **Set 1** | | | | | |
| For Set 1, connect the points in order, and connect the last point to the first point in the set. | | | | | |
| (6, 4) | (12, 8) | (18, 4) | (18, 12) | (6, 12) | (8, 6) |
| (6, –4) | (12, –8) | (18, –4) | (18, –12) | (6, –12) | (8, –2) |
| (–6, –4) | (–12, –8) | (–18, –4) | (–18, –12) | (–6, –12) | (–4, –2) |
| (–6, 4) | (–12, 8) | (–18, 4) | (–18, 12) | (–6, 12) | (–4, 6) |
| **Set 2** | | | | | |
| For Set 2, connect the points in order, and connect the last point to the first point in the set. | | | | | |
| (1, 1) | (2, 2) | (3, 1) | (3, 3) | (1, 3) | (3, 3) |
| (1, –1) | (2, –2) | (3, –1) | (3, –3) | (1, –3) | (3, 1) |
| (–1, –1) | (–2, –2) | (–3, –1) | (–3, –3) | (–1, –3) | (1, 1) |
| (–1, 1) | (–2, 2) | (–3, 1) | (–3, 3) | (–1, 3) | (1, 3) |
| **Set 3** | | | | | |
| For Set 3, connect the points in order, but DO NOT connect the last point to the first point. | | | | | |
| (4, –2) | (8, –4) | (12, –2) | (12, –6) | (4, –6) | (6, 0) |
| (3, –3) | (6, –6) | (9, –3) | (9, –9) | (3, –9) | (5, –1) |
| (–3, –3) | (–6, –6) | (–9, –3) | (–9, –9) | (–3, –9) | (–1, –1) |
| (–4, –2) | (–8, –4) | (–12, –2) | (–12, –6) | (–4, –6) | (–2, 0) |
| **Set 4** | | | | | |
| For Set 4, don't connect the dots at all. | | | | | |
| (4, 2) | (8, 4) | (12, 2) | (12, 6) | (4, 6) | (6, 4) |
| (–4, 2) | (–8, 4) | (–12, 2) | (–12, 6) | (–4, 6) | (–2, 4) |

After drawing the six figures, **compare Figure 1 to each of the other figures** and answer the following questions.

1. Describe any similarities and/or differences between Figure 1 and each of the other figures.

2. Consider how the coordinates of each figure compare to the coordinates of Figure 1. Write general rules for making Figures 2–6.

3. Translate, reflect, and rotate (between 0° and 180°) Figure 1 so that it lies entirely in Quadrant III on the coordinate plane. You may perform the transformations in any order that you choose. Draw a picture of the new figure at each step and explain the procedures you followed to get the new figure. Use coordinates to describe the transformations you used.

4. Translate, reflect, and rotate (between 0º and 180°) Figure 1 so that it lies entirely in Quadrant IV on the coordinate plane. You may perform the transformations in any order that you choose. Draw a picture of the new figure at each step and explain the procedures you followed to get the new figure. Use coordinates to describe the transformations you used.

5. Describe the transformations necessary to take the figure you created in question #4 on to Figure 6.

## Coordinating Translations: Graph Paper

**Name**__________________________________ **Date**__________________

Figure 1

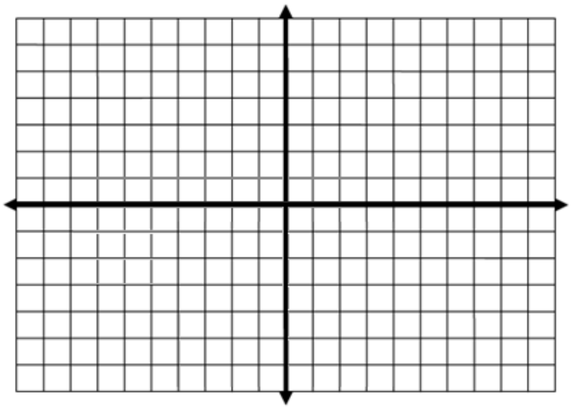


Figure 2

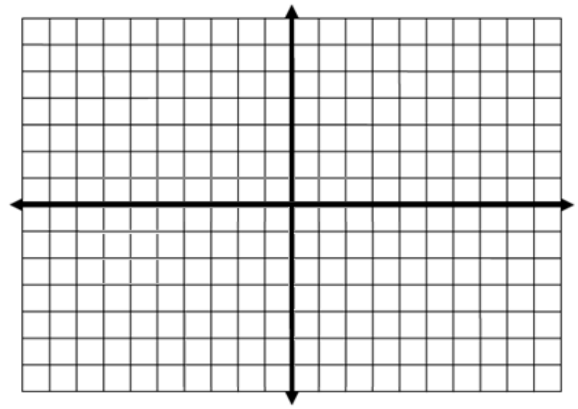


Figure 3

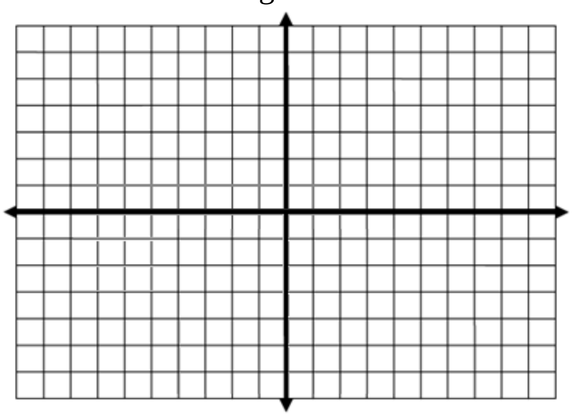


Figure 4

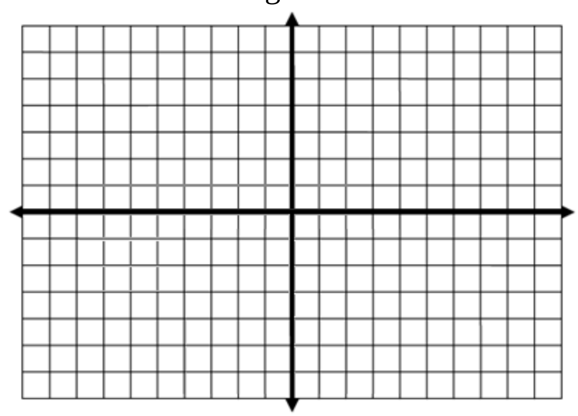


Figure 5

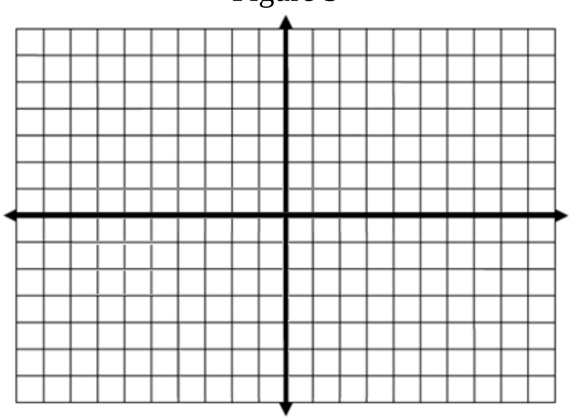
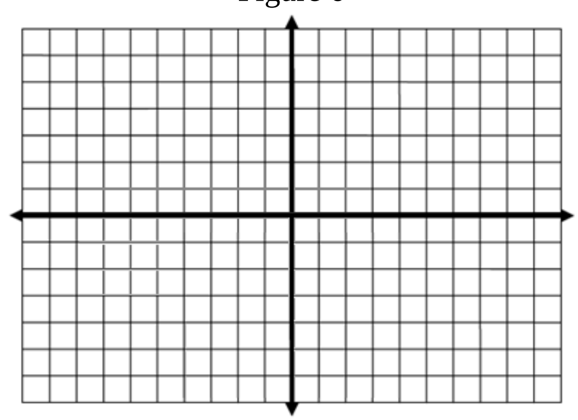


Figure 6



## Formative Assessment Lesson: Transformations

*Source: Georgia Mathematics Design Collaborative*

This lesson is intended to help you assess how well students are able to:

- Describe transformations that carry a figure onto itself
- Rotate, reflect, or translate a figure in the coordinate plane

### Georgia Standards of Excellence

**MGSE9-12.G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

**MGSE.9-12.G.CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

**MGSE.9-12.G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

### Standards for Mathematical Practice

This lesson uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically

### TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Georgia Mathematics Design Collaborative are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding.

The task, *Transformations*, is a Formative Assessment Lesson (FAL) that can be found at: **http://ccgpsmathematics9-10.wikispaces.com/Georgia+Mathematics+Design+Collaborative+Formative+Assessment+Lessons**

## Representing and Combining Transformations (Formative Assessment Lesson)

*Source: Formative Assessment Lesson Materials from Mathematics Assessment Project*
**http://map.mathshell.org/lessons.php?unit=8310&collection=8**

### Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Representing and Combining Transformations*, is a Formative Assessment Lesson (FAL) that can be found at the
website: http://map.mathshell.org/lessons.php?unit=8310&collection=8

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/lessons.php?unit=8310&collection=8

### Mathematical Goals

- Recognize and visualize transformations of 2–D shapes.
- Translate, reflect, and rotate shapes, and combine these transformations.

### Essential Questions

- What effects do transformations have on geometric figures?
- What is the relationship between reflections, translations and rotations?
- How do you determine the type of transformation that has occurred?

## Georgia Standards of Excellence

**MGSE9-12.G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

**MGSE9–12.G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle measure to those that do not (e.g., translation versus horizontal stretch).

**MGSE9–12.G.CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

**MGSE9–12.G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

**MGSE9–12.G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

## Standards for Mathematical Practice

**1.** Make sense of problems and persevere in solving them.
*Students must understand and apply the various types of transformations.*

**3.** Construct viable arguments and critique the reasoning of others.
*Students must justify their choices in the card sort, matching transformations' visual representations to their verbal descriptions.*

**5.** Use appropriate tools strategically.
*Students use transparencies, pins, etc., to help them physically model transformations.*

## Background Knowledge

- Students should know how to apply translations, reflections, and rotations in the coordinate plane.
- Students should know how to describe transformations that map one figure onto another.

## Common Misconceptions

- Students may confuse the various types of transformations.
- Students may struggle to use the correct center of rotation, especially if they have simply memorized a rule (e.g., $(x, y) \rightarrow (y, -x)$ for a 90º clockwise rotation works only if the center of rotation is the origin.)

## Materials

- See FAL page.

## Grouping

- Individual / partner / small group

## Transforming Shapes (Culminating Task)

### Introduction

In this culminating task, students will create their own figure and describe the transformation of a given rule. It is optional to allow students to use graph paper to assist them as they complete this and would be a good support for students who need it. Next students will explain which transformations create similar or congruent figures and generalize this to all transformations. Students will also generate a rule for a rotation of 90º and explain how they reached their conclusion. Upon accurate completion of this task, students will have demonstrated a good understanding of transformations in the coordinate plane.

### Mathematical Goals

- Demonstrate knowledge of transformations according to rules given and create their own rule for a transformation.

### Essential Questions

- How do I apply what I've learned about transformations to figures in the coordinate plane?

### Georgia Standards of Excellence

**MGSE9-12.G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

**MGSE9–12.G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle measure to those that do not (e.g., translation versus horizontal stretch).

**MGSE9–12.G.CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

**MGSE9–12.G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

**MGSE9–12.G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

## Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

*Students should use all eight SMPs when exploring this task.*

## Background Knowledge

- Students will apply everything they have learned in this unit.

## Common Misconceptions

- Address misconceptions brought to light during the rest of the unit.

## Materials

- Pencil
- Handout
- Graph paper (optional)
- Straight–edge (optional)
- Patty paper (optional)

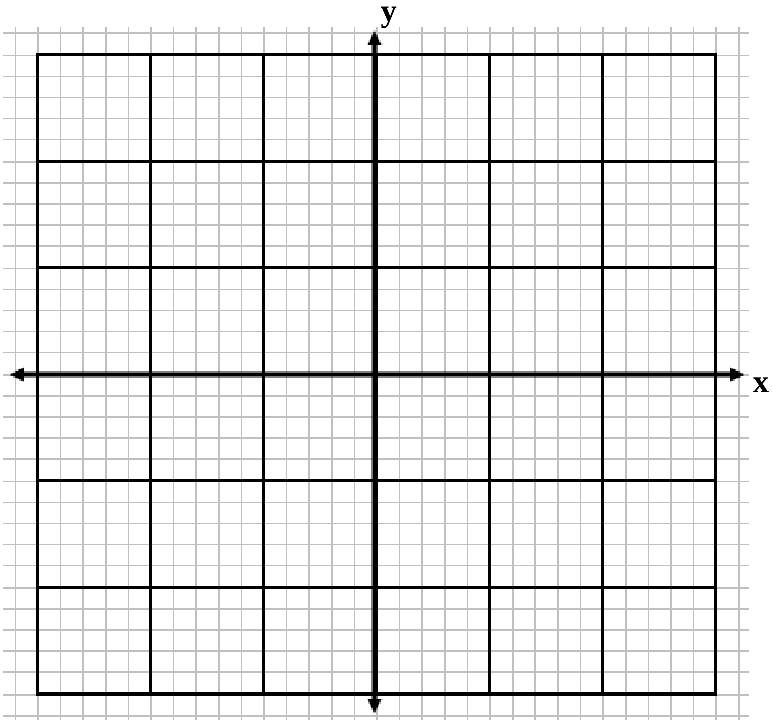
## Grouping

- Individual / partner

## Transforming Shapes – Teacher Notes

***Comments***
***This task assesses students' ability to identify the effects of transforming a figure according to a rule involving dilations and/or translations. Students will have different quadrilaterals but all of the answers to the questions should still be the same.***

Draw a non–square rectangle or parallelogram or an isosceles trapezoid in the coordinate plane so that portions of the shape are in each of the four quadrants. Explain what would happen to your shape if you transformed it using each of the given rules.




1. $(-x, y)$
   ***Reflect across the y–axis***
2. $(x, -y)$
   ***Reflect across x–axis***
3. $(x + 3, y)$
   ***Shift (translate) right 3 units***
4. $(x, y - 2)$
   ***Shift (translate) down 2 units***
5. $(x - 1, y + 4)$
   ***Shift (translate) left 1 unit, up 4 units***
6. $(2x, 2y)$
   ***Dilate from the origin by a scale factor of 2***
7. $(-x, -y)$
   ***Rotate 180º around the origin***
   ***(or reflect across both axes)***
8. $(3x + 2, y - 1)$
   ***Horizontal stretch and shift,***
   ***and shift down 1 unit***

***Other acceptable solutions follow.***

***Possible Alternate Solutions to #1–8:***

1. ***Reflection over the y–axis, the line x = 0***
2. ***Reflection over the x–axis, the line y = 0***
3. ***Translation to the right 3 spaces***
4. ***Translation down 2 spaces***
5. ***Translation left 1 space and up 4 spaces***
6. ***Dilation to create a similar shape that is twice each dimension of the figure. The area inside is 4 times as large as the original shape but it is still the same shape because all dimensions were multiplied by the same amount.***
7. ***Reflection over the x–axis and y–axis which is also a rotation through 180°.***
8. ***The x–coordinates are enlarged three times. The new figure is then translated right 2 spaces and down 1 space. The area is three times the original area and the figures are not the same shape any longer. Thus it is stretched horizontally three times.***

9. Which of the transformed figures are congruent to the original figure? Explain.

***Solution:***

***All of the figures are congruent except for number 6 and number 8. When figures are reflected, rotated, or translated, they retain their shape which makes these rigid transformations.***

10. Which of the transformed figures are **NOT** congruent to the original figure? Are any of these figures similar? Explain.

***Solution:***

***Number 6 and number 8 changes the original shape so are not congruent to the original. Since all dimensions are multiplied by the same amount in number 6, this creates a similar figure because the sides are still in the same proportion and corresponding angles are still congruent. In number 8, the angles have changed and some of the side lengths have changed making it a different shape and not similar to the original shape.***

11. If you make a new figure by adding 2 to the *x*– and *y*–coordinates of each side of your shape, will the two figures be congruent or similar? Explain.

    ***Solution:***

    ***Adding units to the coordinates of points translates the figures but doesn't change the size or shape. Therefore, adding 2 units to the x and y coordinates will maintain congruence to the original.***

12. Create a rule or set of rules to make a 90° rotation of your figure. Explain how you decided the rules for both *x* and *y* coordinates.
    Point in Quadrant I: (_________, _________)→ (_________, _________)
    Point in Quadrant II: (_________, _________)→ (_________, _________)
    Point in Quadrant III: (_________, _________)→ (_________, _________)
    Point in Quadrant IV: (_________, _________)→ (_________, _________)
    In general, $(x, y) \rightarrow$ (_________, _________)

    ***Solution:***

    ***Students should see a pattern from the four points of their quadrilaterals to generalize that the rotation of 90° (counterclockwise) will map $(x,y) \rightarrow (-y, x)$.***

13. Will a 90° rotation of any shape create congruent or similar figures? Explain.

    ***Solution:***

    ***Any rotation will create congruent figures because the size and shape of the original figure is maintained.***

14. What general conclusion can be made determining which transformations will produce congruent figures in the plane?

    ***Solution:***

    ***Any rotation, translation or reflection will produce a congruent figure because the size or shape of the figure is not changed.***

15. Describe at least one transformation (or a series of transformations) that will transform your figure onto itself. Explain how you know this will transform your figure onto itself.

***Solution:***

***Answers will vary. Possible answers:***
***Non–Rectangular Parallelogram: no reflections will work***
***rotation 180º around "center" of parallelogram***
***several series of transformations possible***
***Non–Square Rectangle: two possible reflections will work***
***rotate 180º around "center" of rectangle***
***several series of transformations possible***
***Isosceles Trapezoid: one reflection will work***
***no rotations will work***
***several series of transformations possible***

16. Describe a transformation that will make each side of the image parallel to the corresponding side of the pre–image. Explain why this results in parallel lines.

***Solution:***

***Answers will vary. Possible answers:***
***Non–Rectangular Parallelogram: any translation***
***180º rotation around any point***
***reflection across any line that is parallel to a side***
***Non–Square Rectangle: any translation***
***180º rotation around any point***
***reflection across any line that is parallel to a side or parallel to an angle bisector***
***Isosceles Trapezoid: any translation***
***180º rotation around any point***
***no reflections***

17. Describe a transformation that will make each side of the image perpendicular to the corresponding side of the pre–image. Explain why this results in parallel lines.

***Solution:***

***Answers will vary. Possible answers:***

| | |
|---|---|
| ***Non–Rectangular Parallelogram:*** | ***rotation by 90º or 270º around any point*** |
| ***Non–Square Rectangle:*** | ***rotation by 90º or 270º around any point*** <br> ***reflection across any line that is parallel to an angle bisector*** |
| ***Isosceles Trapezoid:*** | ***rotation by 90º or 270º around any point*** <br> ***no reflections*** |

# Culminating Task: Transforming Shapes

**Name**__________________________________ **Date**__________________

### Mathematical Goals

- Demonstrate knowledge of transformations according to rules given and create their own rule for a transformation.

### Essential Questions

- How do I apply what I've learned about transformations to figures in the coordinate plane?

### Georgia Standards of Excellence

**MGSE9-12.G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

**MGSE9–12.G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle measure to those that do not (e.g., translation versus horizontal stretch).

**MGSE9–12.G.CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

**MGSE9–12.G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

**MGSE9–12.G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

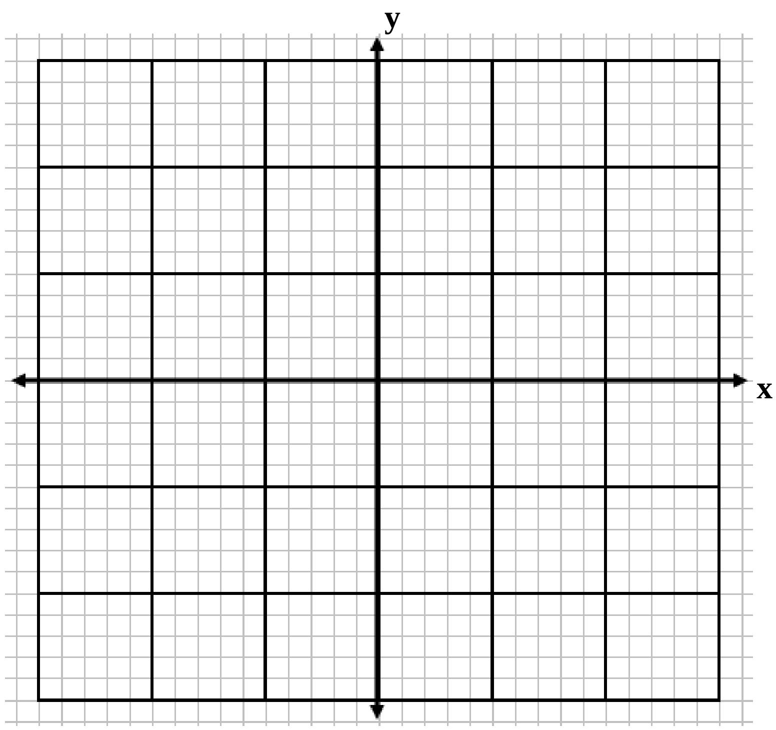
### Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## Culminating Task: Transforming Shapes

**Name**__________________________________ **Date**__________________

Draw a rectangle, parallelogram, square, or isosceles trapezoid in the coordinate plane so that portions of the shape are in each of the four quadrants. Explain what would happen to your shape if you transformed it using each of the given rules.




**1.** $(-x, y)$

**2.** $(x, -y)$

**3.** $(x + 3, y)$

**4.** $(x, y - 2)$

**5.** $(x - 1, y + 4)$

**6.** $(2x, 2y)$

**7.** $(-x, -y)$

**8.** $(3x + 2, y - 1)$

**9.** Which of the transformed figures are congruent to the original figure? Explain.

**10.** Which of the transformed figures are **NOT** congruent to the original figure? Are any of these figures similar? Explain.

**11.** If you make a new figure by adding 2 to the *x*– and *y*–coordinates of each side of your shape, will the two figures be congruent, similar, or neither? Explain.

**12.** Create a rule or set of rules to make a 90° rotation of your figure. Explain how you decided the rules for both *x* and *y* coordinates.

Point in Quadrant I: (_________, _________) → (_________, _________)

Point in Quadrant II: (_________, _________) → (_________, _________)

Point in Quadrant III: (_________, _________) → (_________, _________)

Point in Quadrant IV: (_________, _________) → (_________, _________)

In general: $(x, y)$ → (_________, _________)

**13.** Will a 90° rotation of any shape create congruent or similar figures? Explain.

**14.** What general conclusion can be made determining which transformations will produce congruent figures in the plane?

**15.** Describe a transformation (or a series of transformations) that will transform your figure onto itself. Explain how you know this will transform your figure onto itself.

**16.** Describe a transformation that will make each side of the image parallel to the corresponding side of the pre–image. Explain why this results in parallel lines.

**17.** Describe a transformation that will make each side of the image perpendicular to the corresponding side of the pre–image. Explain why this results in parallel lines.