Georgia
Standards of Excellence
Curriculum Frameworks

Mathematics

Accelerated GSE Coordinate Algebra/Analytic Geometry A

Unit 6: Connecting Algebra and Geometry Through Coordinates

Georgia Department of Education

Richard Woods, Georgia's School Superintendent

"Educating Georgia's Future"

Unit 6

Connecting Algebra and Geometry Through Coordinates

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OVERVIEW

In this unit students will:

- prove the slope relationship that exists between parallel lines and between perpendicular lines and then use those relationships to write the equations of lines
- extend the Pythagorean Theorem to the coordinate plane
- develop and use the formulas for the distance between two points and for finding the point that partitions a line segment in a given ratio
- revisit definitions of polygons while using slope and distance on the coordinate plane
- use coordinate algebra to determine perimeter and area of defined figures

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. This unit provides much needed content information and excellent learning activities. However, the intent of the framework is not to provide a comprehensive resource for the implementation of all standards in the unit. A variety of resources should be utilized to supplement this unit. The tasks in this unit framework illustrate the types of learning activities that should be utilized from a variety of sources. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the **"Strategies for Teaching and Learning"** and the tasks listed under **"Evidence of Learning"** be reviewed early in the planning process.

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

KEY STANDARDS

Use coordinates to prove simple geometric theorems algebraically.

MGSE9–12.G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. For *example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; ~~prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2)~~. (Focus on quadrilaterals, right triangles, and ~~circles~~.)*

MGSE9–12.G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

MGSE9–12.G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

MGSE9–12.G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

RELATED STANDARDS

MGSE9–12.G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

MGSE9–12.A.REI.10 Understand that the graph of an equation in two variables is the set of its solutions plotted in the coordinate plane.

STANDARDS FOR MATHEMATICAL PRACTICE

Refer to the Comprehensive Course Overview for more detailed information about the Standards for Mathematical Practice.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ENDURING UNDERSTANDINGS

- Algebraic formulas can be used to find measures of distance on the coordinate plane.
- The coordinate plane allows precise communication about graphical representations.
- The coordinate plane permits use of algebraic methods to obtain geometric results.

ESSENTIAL QUESTIONS

- How can a line be partitioned?
- How can the distance between two points be determined?
- How are the slopes of lines used to determine if the lines are parallel, perpendicular, or neither?
- How do we write the equation of a line that goes through a given point and is parallel or perpendicular to another line?
- How can slope and the distance formula be used to determine properties of polygons and circles?
- How can slope and the distance formula be used to classify polygons?
- How do I apply what I have learned about coordinate geometry to a real–world situation?

CONCEPTS AND SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre–assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- approximating radicals
- calculating slopes of lines
- graphing lines
- writing equations for lines

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for high school. **Note – At the elementary level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**

http://www.teachers.ash.org.au/jeather/maths/dictionary.html
This web site has activities to help students more fully understand and retain new vocabulary (i.e. the definition page for *dice* actually generates rolls of the dice and gives students an opportunity to add them).

http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website because Intermath is geared towards middle and high school.

- **Distance Formula:** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

- **Formula for finding the point that partitions a directed segment AB at the ratio of *a* : *b* from A(x_1, y_1) to B(x_2, y_2):**

$$\left(x_1 + \frac{a}{a+b}(x_2 - x_1),\ \ y_1 + \frac{a}{a+b}(y_2 - y_1) \right)$$

or $$\left(\frac{a}{a+b}(x_2 - x_1) + x_1,\ \ \frac{a}{a+b}(y_2 - y_1) + y_1 \right)$$

or $$\left(\frac{bx_1 + ax_2}{b+a},\ \frac{by_1 + ay_2}{b+a} \right)$$ **← weighted average approach**

EVIDENCE OF LEARNING

At the conclusion of the unit, students should be able to:

- find the point that partitions a directed segment into a given ratio
- determine if a given pair of lines are parallel, perpendicular, or neither
- determine the equation of the line parallel or perpendicular to a given line and passing through a given point
- use distance and slope concepts to prove geometric theorems algebraically
- find perimeter of polygons and area of triangles and quadrilaterals

TEACHER RESOURCES

The following pages include teacher resources that teachers may wish to use to supplement instruction.

- Web Resources
- Graphic Organizer: Partitioning a Directed Line Segment
- Compare / Contrast: Two Methods for Finding Distance

Web Resources

The following list is provided as a sample of available resources and is for informational purposes only. It is your responsibility to investigate them to determine their value and appropriateness for your district. GaDOE does not endorse or recommend the purchase of or use of any particular resource.

- Distance Formula Applet
  http://www.mathwarehouse.com/algebra/distance_formula/interactive–distance–formula.php
  This applet shows the distance formula in action based on different points on grid. This resource is helpful for an introduction on the distance formula.

- Quadrilaterals Overview
  http://www.cut–the–knot.org/Curriculum/Geometry/Quadrilaterals.shtml
  This page has a helpful overview of quadrilaterals and an applet that names a quadrilateral as you move its vertices. The page includes a flow chart of quadrilaterals with inclusive definition of trapezoid.

Graphic Organizer: Partitioning a Directed Line Segment

MGSE9–12.G.GPE.6

EQ: How do you partition a directed line segment?

Problem: Given the points C (3, 4) and D(6, 10), find the coordinates of point P on a directed line segment $\overline{CD}$ that partitions $\overline{CD}$ in the ratio 1:2 (a:b).

Solution: Point P (,)

Steps to Use:

1. Subtract the *x*–coordinates (point D – point C).
2. Change the ratio a:b to $\frac{a}{a+b}$.
3. Multiply the answers from step 1 and step 2.
4. Add the beginning *x* coordinate (of C) to step 3's result.

Steps to Use:

1. Subtract the *y*–coordinates (point D – point C).
2. Change the ratio a:b to $\frac{a}{a+b}$.
3. Multiply the answers from step 1 and step 2.
4. Add the beginning *y* coordinate (of C) to step 3's result.

How does the step by step process above relate to the portioning formula below?

$$\left((x_2 - x_1)\frac{a}{a+b} + x_1, (y_2 - y_1)\frac{a}{a+b} + y_1\right)$$

Compare / Contrast: Two Methods for Finding Distance

Focus Question: How does the Pythagorean Theorem relate to the distance formula?	
Use the Pythagorean Theorem to find the distance between (2, 7) and (–1, –4)	**Use the distance formula to find the distance between (2, 7) and (–1, –4)**

FORMATIVE ASSESSMENT LESSONS (FAL)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Overview.

SPOTLIGHT TASKS

A Spotlight Task has been added to each GSE mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit–level Georgia Standards of Excellence, and research–based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3–Act Tasks based on 3–Act Problems from Dan Meyer and Problem–Based Learning from Robert Kaplinsky.

3–ACT TASKS

A Three–Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

More information along with guidelines for 3–Act Tasks may be found in the Comprehensive Course Overview.

TASKS

The following tasks represent the level of depth, rigor, and complexity expected of all Coordinate Algebra students. These tasks, or tasks of similar depth and rigor, should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as a performance task, they may also be used for teaching and learning (learning/scaffolding task).

Task Name	**Task Type** ***Grouping Strategy***	**Content Addressed**	**Standards**
Analyzing the Pentagon (Spotlight Task) *30–45 minutes*	Discovery Task *Partner/ Individual*	• Investigating area, perimeter and other properties of polygons	G.GPE.4, 7
New York City *90–120 minutes*	Learning Task *Partner / Small Gro up*	• Partition a line segment into a given ratio.	G.GPE.6
Slopes of Special Pairs of Lines *90–120 minutes*	Discovery Task *Partner / Individual*	• Show that the slopes of parallel lines are the same. • Show that the slopes of perpendicular lines are opposite reciprocals. • Given the equation of a line and a point not on the line, find the equation of the line that passes through the point and is parallel/perpendicular to the given line.	G.GPE.5
Geometric Properties in the Plane *90–120 minutes*	Performance Task *Partner Task*	• Use coordinates, slope relationships, and distance formula to prove simple geometric theorems algebraically. • Compute the perimeters of polygons using the coordinates of the vertices and the distance formula. • Find the areas of rectangles and triangles using the coordinates of the vertices and the distance formula.	G.GPE.4, 7

Equations of Parallel & Perpendicular Lines (FAL) *90–120 minutes* PDF	Formative Assessment Lesson *Individual / Small Group*	• Use equations of parallel and perpendicular lines to form geometric figures.	G.GPE.4, 5, 6, 7
Square *20–30 minutes* PDF	Short Cycle Task *Individual / Small Group*	• Use slope and length to determine whether a figure with given vertices is a square.	G.GPE.4, 5, 6, 7
Euler's Village *2 – 3 hours*	Culminating Task *Small Group / Part ner*	• Use coordinates, slope relationships, and distance formula to prove simple geometric theorems algebraically. • Compute the perimeters of polygons using the coordinates of the vertices and the distance formula. • Find the areas of rectangles and triangles using the coordinates of the vertices and the distance formula. • Given the equation of a line and a point not on the line, find the equation of the line that passes through the point and is parallel/perpendicular to the given line.	G.GPE.4, 5, 6, 7

Analyzing a Pentagon (Spotlight Task)

This spotlight task follows the 3 Act–Math task format originally developed by Dan Meyer. More information on these type tasks may be found at http://blog.mrmeyer.com/category/3acts/

Georgia Standards of Excellence
MGSE9–12.G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. For *example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; ~~prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2)~~. (Focus on quadrilaterals, right triangles, and ~~circles~~.)*
(Restrict contexts that use distance and slope.)
This standard is addressed through students assigning coordinates to key points on the picture and then using those coordinates to answer questions that they develop themselves. One example might be proving that all regular polygons with the same number of sides are similar. Students will use coordinates to explore properties and characteristics of polygons.

MGSE9–12.G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.
This standard is addressed through use of the distance formula to calculate the area and perimeter of the pentagon. Several other applications are possible, depending on the interests and questions of the students.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them. In this task, students will formulate their own problem to solve. They will need to decide on a reasonable question to answer using the mathematics they have available to them. Several possible investigations could require extend effort to solve. Students will need to persevere through the problem solving process in order to arrive at a solution.

5. Use appropriate tools strategically. Students will need to select appropriate tools (graph paper, calculator, formulas) in order to be successful at this task.

ESSENTIAL QUESTIONS
- How do I construct a mathematical question that can be answered?
- How do I calculate the distance between two points?
- How can I calculate the area and perimeter of a figure given only coordinates of vertices?

MATERIALS REQUIRED
- Copy of student handout (picture of the Pentagon)
- Graph paper

TIME NEEDED
- 30–45 minutes based on the depth of investigation

More information along with guidelines for 3–Act Tasks may be found in the Comprehensive Course Overview.

<u>Act One Description</u> (Dan Meyer http://blog.mrmeyer.com/2011/the–three–acts–of–a–mathematical–story/)
"Introduce the central conflict of your story/task clearly, visually, viscerally, using as few words as possible."

Act One:
Present the students with the aerial photograph of the Pentagon. Pose the question: "What do you wonder?"
It should be noted here that students will likely come up with many questions that are non–mathematical in nature. The teacher's task is to help them to refine their questions so that they can be answered using the mathematics that they know or can discover.
Here are some questions that students developed based on the picture:

- What is the perimeter of the building?
- What is the area of the building?
- What is the area of the courtyard?
- Are the outer pentagon and inner pentagon similar?

In order to introduce a more real–world feel to the questions, they could be modified to the following:

- How long would it take an average person to walk around the exterior of the Pentagon?
- How many acres does the Pentagon cover?
- How many football fields could fit in the courtyard?
- How many times bigger is the outer pentagon than the inner pentagon?

This is also a great opportunity to revisit unit conversions and other standards from Unit 1.
The important part is to honor the students' curiosity. They will engage with the activity more if it is their own questions that they are answering. If a student develops a question that is outside the scope of the course, be sure to honor that student's curiosity by pointing them in the right direction and encouraging them to continue on their own. Hopefully, students will begin the process of formulating mathematical questions and then using mathematical models to answer the questions.
Here are some questions to help guide the discussion, but be sure not to give too much away. The goal is to have students formulate the questions and the methods to answer them.

- Can your question be answered using mathematics?
- How could you model an answer to your questions with an equation?
- Do you have all the information you need to answer your question?

Act Two Description (Dan Meyer http://blog.mrmeyer.com/2011/the–three–acts–of–a–mathematical–story/)
The protagonist/student overcomes obstacles, looks for resources, and develops new tools.

During Act Two, students will discuss the question in Act One and decide on the facts that are needed to answer the question. Students will also look for formulas and conversions that are needed to solve the problem. When students decide what they need to solve the problem, they should ask for the facts or use technology to find them.
Note: It is pivotal to the problem solving process that students decide what is needed without being given the information up front. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin.
The main content intended for use in this task is calculating areas and perimeters of polygons using the distance formula. Students may choose to try and measure the sides using a ruler. Instead, suggest using graph paper. Students were introduced to the distance formula in middle school so it should not be too much of a jump to apply the formula to the situation.
Students can be supplied with the following information when they ask for it:

- Typical graph paper is scaled 4 squares per inch.
- The scale of the photo is 1 inch = 263.14 feet
- 1 acre contains 43,560 square feet

Use your discretion on what other information students may look up on the internet. Calculating the area of a regular pentagon could be done using a formula, but resist the temptation to reduce it to that. Encourage the students to use other methods for calculating the area, such as decomposing the figure into triangles or trapezoids. This could even extend into students developing their own formula for the area of a polygons.

ACT 3
Students will compare and share solution strategies.

- Reveal the answer.
    - Each side of the Pentagon is 921 feet.
    - It covers 28.7 acres, and the interior courtyard is 5 acres
- Discuss the theoretical math versus the practical outcome.
- How close was your answer to the actual answer?
- What could account for the difference?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.

The answers that students come up with will vary based on their estimates for the coordinates of the vertices on the graph paper. Discuss the role of estimation and variation in their answers. The teacher also needs to be flexible and adapt the lesson to the curiosity of the class. Use this activity as a guide, but do not be afraid to deviate from it if the mathematics dictates that you do so

The Sequel: "The goals of the sequel task are to a) challenge students who finished quickly so b) I can help students who need my help. It can't feel like punishment for good work. It can't seem like drudgery. It has to entice and activate the imagination." Dan Meyer http://blog.mrmeyer.com/2013/teaching–with–three–act–tasks–act–three–sequel/

For a sequel, allow students to look up other aerial photographs of other famous buildings or landmarks. They could then proceed through a similar process. Google Earth or Google maps could be a good resource for this sequel.

Analyzing the Pentagon (Spotlight Task)
Image taken from www.googlemapsmania.blogsplot.com

What do you wonder?

New York City (Learning Task)

Introduction

This task provides a guided discovery of the procedure for partitioning a segment into a given ratio.

Mathematical Goals

- Find the point on a line segment that separates the segments into a given ratio.

Essential Questions

- How can a line be partitioned?

Georgia Standards of Excellence

MGSE9–12.G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
Students must make sense of the layout of the town and represent it in a way that they can answer questions about the town.
2. Reason abstractly and quantitatively.
When partitioning pathways, students must apply their reasoning in both specific and general situations.
4. Model with mathematics.
Students must represent the problem situation mathematically and use their model to answer questions about the situation.

Background Knowledge

- Students understand fractions as representing part of a whole.
- Students recognize the relationship between ratios and fractions. e.g., breaking a segment into two pieces with a ratio of 3:5 means the two pieces are $^3/_8$ and $^5/_8$ of the whole.

Common Misconceptions

- Students may have difficulty representing the situation graphically. Focus on the difference between Avenues and Streets in the problem, and relate this to (x, y) coordinates.
- Students may use the fraction $^3/_5$ instead of $^3/_8$ when partitioning into a ratio of 3:5. Remind students that fractions represent a part of the *whole*. 3 and 5 are both parts.
- Conversely, if the fraction “part / whole” (rather than the ratio “part : part”) is given, then students don’t need to change the denominator.

Materials

- Graph paper

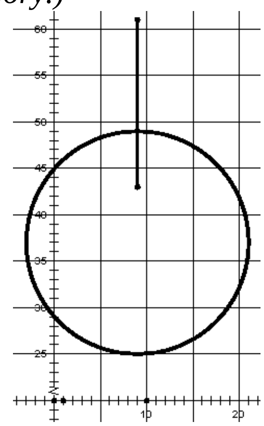
Grouping

- Partner / small group

Differentiation

Extension:

- Emily (at work) and Gregory (at his hotel) want to walk to a location so that each person walks the same distance. The corner restaurant is halfway *between* them, but there are other locations that are also equidistant from each of them. Describe all points that are equidistant from Emily and Gregory. *(Solution: Points on the perpendicular bisector of the segment connecting Emily and Gregory.)*
- Describe all locations that are twice [half, three times, etc.] as far from Emily as they are from Gregory.
  Solution will be a circle centered at (9, b) for some value of b. The solution here shows all points that are twice as far from Emily as they are from Gregory.



Intervention:

- Students can highlight the right triangles they form to help them focus on them only. This helps motivate the idea of using the Pythagorean Theorem for distance, and the idea of partitioning horizontal and vertical components separately for partitioning.
- The partitioning formula may be challenging to remember and/or apply for many students. Encourage students to think of partitioning as a multi–step process rather than a complicated formula.

Formative Assessment Questions

- Describe how you would find the point Z that partitions the directed line segment XY in the ratio of 4:3 using the points $X(–5, 7)$ and $Y(6, 13)$.

New York City – Teacher Notes

The streets of New York City are laid out in a rectangular pattern, with all blocks approximately square and approximately the same size. **Avenues** run in a north–south direction, and the numbers increase as you move west. **Streets** run in an east–west direction, and the numbers increase as you move north.

Emily works at a building located on the corner of 9th Avenue and 61st Street in New York City. Her brother, Gregory, is in town on business. He is staying at a hotel at the corner of 9th Avenue and 43rd Street.

1. Gregory called Emily at work, and they agree to meet for lunch. They agree to meet at a corner half way between Emily's work and Gregory's hotel. Then Gregory's business meeting ends early so he decides to walk to the building where Emily works.

   a. How many blocks does he have to walk? Justify your answer using a diagram on grid paper.

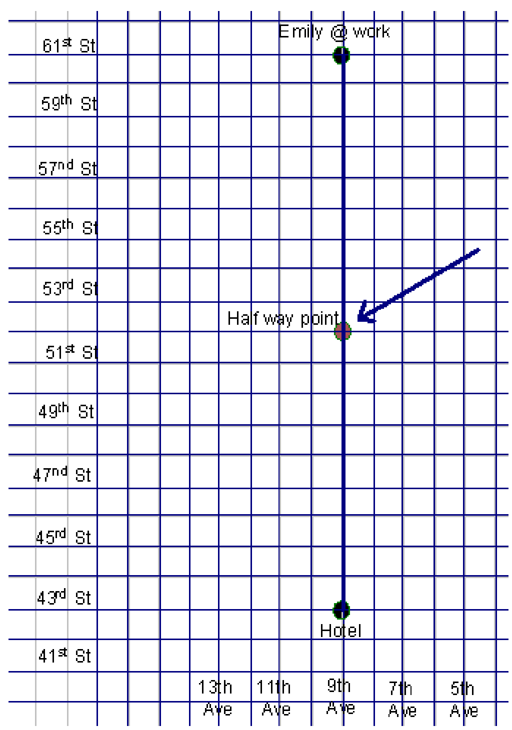
   b. After meeting Emily's coworkers, they walk back toward the corner restaurant. How many blocks must they walk? Justify your answer using your diagram.\

Comments:

Watch students carefully as they begin drawing a picture for this. Make sure they understand what is meant by North–South and East–West Avenues and Streets. It may be necessary to address this as a class so students understand the lay–out of the city streets.

Solutions:

The locations are 18 blocks apart. If each person walks 9 blocks they can meet at 9th Avenue and 52nd Street.



2. After lunch, Emily has the afternoon off so she walks back to the hotel with Gregory before turning to go to her apartment. Her apartment is three blocks north and four blocks west of the hotel.

   a. At what intersection is her apartment building located?

   Solution:
   Her apartment is located at 13th Avenue and 46th Street

   b. How many blocks south of the restaurant will they walk before Emily turns to go to her apartment?

   Solution:
   They will walk 6 blocks south of the restaurant.

   c. When Emily turns, what fraction of the distance from the restaurant to the hotel have the two of them walked? Express this fraction as a ratio of distance walked to distance remaining for Gregory.

   Solution:
   They will walk 6 blocks south of the restaurant which is $\frac{6}{9}$ or $\frac{2}{3}$ of the total distance Gregory will walk. This is a 6:3 or 2:1 ratio.

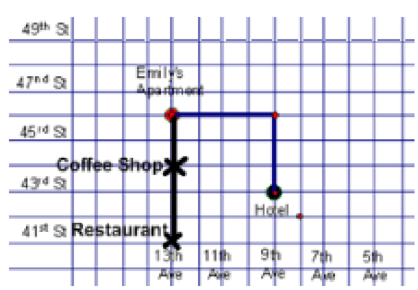
3. Gregory and Emily are going to meet for dinner at a restaurant 5 blocks south of her apartment.

   a. At which intersection is the restaurant located?

   Solution:
   The restaurant is at the corner of 13th avenue and 41st street.

   b. After dinner, they walk back towards her apartment, but stop at a coffee shop that is located three–fifths of the distance to the apartment. What is the location of the coffee shop?

   Solution:
   Since the restaurant is 5 blocks south of the apartment, $\frac{3}{5}$ of the distance back to the apartment means they will walk from 41st up to 44th. The coffee shop is located at 13th and 44th.



By investigating the situations that follow, you will determine a procedure for finding a point that partitions a segment into a given ratio.

4. Here, you will find a point that partitions a directed line segment from $C(4, 3)$ to $D(10, 3)$ in a given ratio.
   Comments:
   The task begins with fractions of horizontal and vertical segments so that students can reason through the step, clarifying distinctions between fractions of the whole and ratios of parts as well as direction of the partition.

   a. Plot the points on a grid. (Notice that the points lie on the same horizontal line.) What is the distance between the points?
   Solution:
   Distance from C to D is |10 – 4| = 6

   b. Use the fraction of the total length of CD to determine the location of Point A which partitions the segment from C to D in a ratio of 5:1. What are the coordinates of A?
   Solution:
   $$\boldsymbol{A\left(\left(\frac{5}{6}\right)6+4,3\right)=A(9,3)}$$

   c. Find point B that partitions a segment from C to D in a ratio of 1:2 by using the fraction of the total length of CD to determine the location of Point B. What are the coordinates of B?
   Solution:
   $$\boldsymbol{B\left(\left(\frac{1}{3}\right)6+4,3\right)=B(6,3)}$$

5. Find the coordinates of Point X along the directed line segment YZ.
   a. If Y(4, 5) and Z(4, 10), find X so the ratio is of YX to XZ is 4:1.
   Solution:
   $$\boldsymbol{X\left(4,\left(\frac{4}{5}\right)5+5\right)=X(4,9)}$$

   b. If Y(4, 5) and Z(4, 10), find X so the ratio is of YX to XZ is 3:2.
   Solution:
   $$\boldsymbol{X\left(4,\left(\frac{3}{5}\right)5+5\right)=X(4,8)}$$

So far, the situations we have explored have been with directed line segments that were either horizontal or vertical. Use the situations below to determine how the procedure used for Questions 4 and 5 changes when the directed line segment has a defined, nonzero slope.

Comments:
Students must treat the x and y values separately. Encourage them to plot the points on a grid and construct the vertical and horizontal components.

6. Find the coordinates of Point A along a directed line segment from C(1, 1) to D(9, 5) so that A partitions CD in a ratio of 3:1. Since CD is neither horizontal nor vertical, the *x* and *y* coordinates have to be considered distinctly.

   a. Find the *x*–coordinate of A using the fraction of the horizontal component of the directed line segment (i.e., the horizontal distance between C and D).
   Solution:
   Horizontal distance |9 – 1| = 8

   b. Find the *y*–coordinate of A using the fraction of the vertical component of the directed line segment (i.e., the vertical distance between C and D).
   Solution:
   Vertical distance |5 – 1| = 4

   c. What are the coordinates of A?
   Solution:
   $$A\left(\left(\frac{3}{4}\right)8+1,\left(\frac{3}{4}\right)4+1\right)=A(7,4)$$

7. Find the coordinates of Point A along a directed line segment from C(3, 2) to D(5, 8) so that A partitions CD in a ratio of 1:1. Since CD is neither horizontal nor vertical, the *x* and *y* coordinates have to be considered distinctly.

   a. Find the *x*–coordinate of A using the fraction of the horizontal component of the directed line segment (i.e., the horizontal distance between C and D).
   Solution:
   Horizontal distance |5 – 3| = 2

   b. Find the *y*–coordinate of A using the fraction of the vertical component of the directed line segment (i.e., the vertical distance between C and D).
   Solution:
   Vertical distance |8 – 2| = 6

   c. What are the coordinates of A?
   Solution:
   $$A\left(\left(\frac{1}{2}\right)2+3,\left(\frac{1}{2}\right)6+2\right)=A(4,5)$$

8. Now try a few more …

   a. Find Point Z that partitions the directed line segment XY in a ratio of 5:3.
   X(–2, 6) and Y(–10, –2)
   Solution:
   Horizontal distance – 10 – –2 = –8 Vertical distance –2 – 6= –8
   $$Z\left(\left(\frac{5}{8}\right)(-8)+-2,\left(\frac{5}{8}\right)(-8)+6\right)=Z(-7,1)$$

   b. Find Point Z that partitions the directed line segment XY in a ratio of 2:3.
   X(2,–4) and Y(7,2)
   Solution:
   Horizontal distance 7– 2=5 Vertical distance 2 – –4 = 6
   $$Z\left(\left(\frac{2}{5}\right)5+2,\left(\frac{2}{5}\right)6+\ -4\right)=Z(4,-1\tfrac{3}{5})$$

   c. Find Point Z that partitions the directed line segment YX in a ratio of 1:3.
   X(–2, –4) and Y(–7, 5) (Note the direction change in this segment.)
   Solution:
   Horizontal distance –2—7=5 Vertical distance –4– 5=–9
   $$Z\left(\left(\frac{1}{4}\right)5+\ -7,\left(\frac{1}{4}\right)(-9)+\ 5\right)=Z(-5\tfrac{3}{4},2\tfrac{3}{4})$$

Comments:

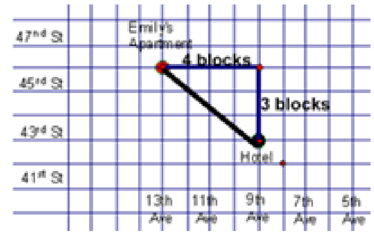
The next section of this task addresses using the Pythagorean Theorem to find the distance between two points in a coordinate system.

Back to Gregory and Emily….

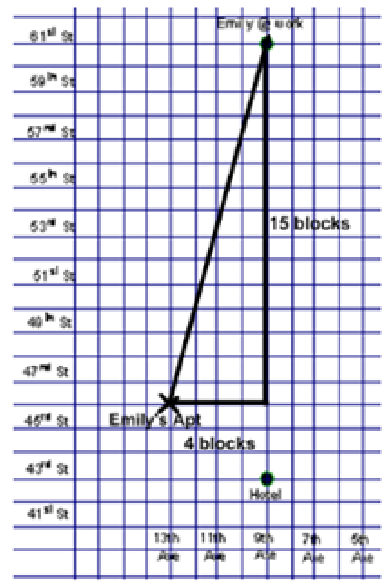
9. When they finished their coffee, Gregory walked Emily back to her apartment, and then walked from there back to his hotel.

   a. How many blocks did he walk?
   Solution:
   Gregory walked 3 + 4 = 7 blocks.

   b. If Gregory had been able to walk the direct path to the hotel from Emily's apartment, how far would he have walked? Justify your answer using your diagram.



Solution:
Gregory walked 7 blocks from Emily's apartment back to his hotel. If he had been able to walk the most direct route, he would walk 5 blocks.
$a^2 + b^2 = c^2$
$4^2 + 3^2 = c^2$
$5 = c$



b. What is the distance Emily walks to work from her apartment?
Solution:
Emily walks 19 blocks to work from her apartment.

c. What is the length of the direct path between Emily's apartment and the building where she works? Justify your answer using your diagram.
Solution:
If she had been able to walk the most direct route, he would walk approximately15.5 blocks.
$a^2 + b^2 = c^2$
$15^2 + 4^2 = c^2$
$c = \sqrt{241} \approx 15.5$

Determine a procedure for determining the distance between points on a coordinate grid by investigating the following situations.

10. What is the distance between 5 and 7? 7 and 5? –1 and 6? 5 and –3?
Comments:
This question is intended to get students thinking about using a formula to find the distance between two points. Students can easily draw a number line and count to find the distance between the given points. Help them recall that in 8th grade they learned how to find the distance between two points on a number line using $d = |a-b|$

Solutions:
Distance between 5 and 7 is 2. This can be found be simply subtracting 5 from 7. It can also be found by subtracting 7 from 5. The difference is whether the answer is positive or negative. Since distance should always be positive, taking the absolute value of the difference between the numbers will give you the distance between the two points.

$|7 - 5| = 2$ *or* $|5 - 7| = |-2| = 2$
$|6 - -1| = |6 + 1| = 7$ *or* $|-1 - 6| = |-7| = 7$
$|5 - -3| = |5 + 3| = 8$ *or* $|-3 - 5| = |-8| = 8$

11. Can you find a formula for the distance between two points, a and b, on a number line?

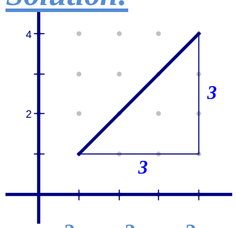
Comments:
At this point, students need to formalize their findings from above.

Solutions:
Distance between a and b is |a – b| or |b – a|

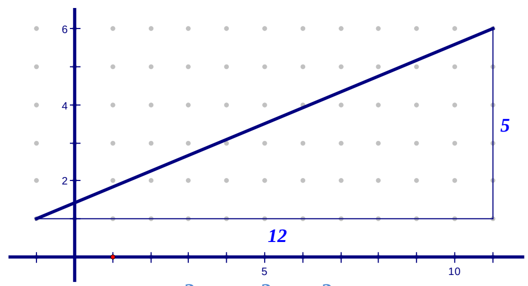
12. Using the same graph paper, find the distance between:

(1, 1) and (4, 4) (–1, 1) and (11, 6) (–1, 2) and (2, –6)

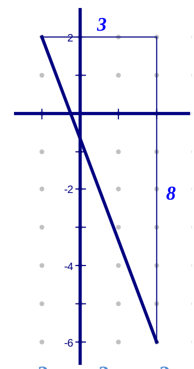
Solution:



$h^2 = 3^2 + 3^2$
$h^2 = 9 + 9$
$h^2 = 18$
$h = \sqrt{18} = 3\sqrt{2} \approx 4.2$

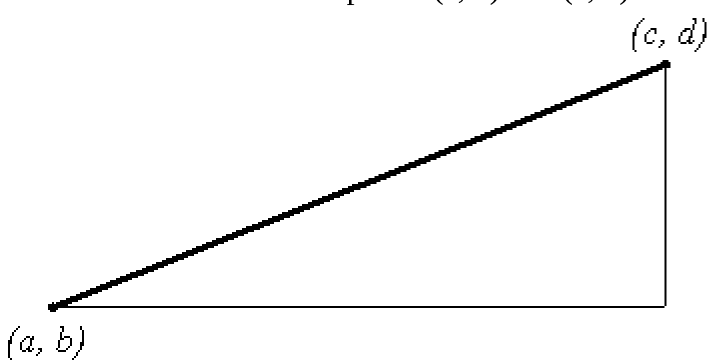


$h^2 = 12^2 + 5^2$
$h^2 = 144 + 25$
$h^2 = 169$
$h = \sqrt{169} = 13$



$h^2 = 3^2 + 8^2$
$h^2 = 9 + 64$
$h^2 = 73$
$h = \sqrt{73} \approx 8.5$

13. Find the distance between points (a, b) and (c, d) shown below.



Solution:

$\sqrt{(c-a)^2 + (d-b)^2}$

Comments:

Students need to look at the three problems from #13 to determine how they can find the distance between these points. Labeling the points and lengths on the earlier problems can help students see the pattern that is developing.

In the examples above, one leg of the right triangle is always parallel to the x–axis while the other leg is always parallel to the y–axis. Using the coordinates of the given points, the vertical length is always the difference of the x–coordinates of the points while the horizontal length is always the difference of the y–coordinates of the points. Help students relate this to #11.

14. Using your solutions from 13, find the distance between the point (x_1, y_1) and the point (x_2, y_2). Solutions written in this generic form are often called formulas.

<u>Comments:</u>

Encourage students to write one simple formula that will work all the time. To help students understand why the absolute value signs are not needed, discuss what happens to a number when you square it. Since the value, when squared, is always positive, it's not necessary to keep the absolute value signs.

<u>Solution:</u>

Groups may come up with slightly different solutions to this problem. All of the answers below are correct. Students should discuss the similarities and differences and why they are all valid formulas. Make sure to include a discussion of the role of mathematical properties.

$$\boldsymbol{d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} \qquad \boldsymbol{d=\sqrt{(y_1-y_2)^2+(x_1-x_2)^2}}$$
$$\boldsymbol{d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}} \qquad \boldsymbol{d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}}$$
$$\boldsymbol{d=\sqrt{(x_1-x_2)^2+(y_2-y_1)^2}} \qquad \boldsymbol{d=\sqrt{(y_1-y_2)^2+(x_2-x_1)^2}}$$
$$\boldsymbol{d=\sqrt{(x_2-x_1)^2+(y_1-y_2)^2}} \qquad \boldsymbol{d=\sqrt{(y_2-y_1)^2+(x_1-x_2)^2}}$$

15. Do you think your formula would work for any pair of points? Why or why not?

<u>Solution:</u>

Answers will vary. The formula from #14 should work for any pair of points.

Learning Task: New York City

Name_________________________________ **Date**__________________

Mathematical Goals

- Find the point on a line segment that separates the segments into a given ratio.

Essential Questions

- How can a line be partitioned?

Georgia Standards of Excellence

MGSE9–12.G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.

Learning Task: New York City

Name_________________________________ **Date**__________________

The streets of New York City are laid out in a rectangular pattern, with all blocks approximately square and approximately the same size. **Avenues** run in a north–south direction, and the numbers increase as you move west. **Streets** run in an east–west direction, and the numbers increase as you move north.

Emily works at a building located on the corner of 9th Avenue and 61st Street in New York City. Her brother, Gregory, is in town on business. He is staying at a hotel at the corner of 9th Avenue and 43rd Street.

1. Gregory calls Emily at work, and they agree to meet for lunch. They agree to meet at a corner half way between Emily's work and Gregory's hotel. Then Gregory's business meeting ends early so he decides to walk to the building where Emily works.

   a. How many blocks does he have to walk? Justify your answer using a diagram on grid paper.

   b. After meeting Emily's coworkers, they walk back toward the corner restaurant halfway between Emily's work and Gregory's hotel. How many blocks must they walk? Justify your answer using your diagram.

2. After lunch, Emily has the afternoon off, so she walks back to the hotel with Gregory before turning to go to her apartment. Her apartment is three blocks north and four blocks west of the hotel.

   a. At what intersection is her apartment building located?

   b. How many blocks south of the restaurant will they walk before Emily turns to go to her apartment?

   c. When Emily turns, what fraction of the distance from the restaurant to the hotel have the two of them walked? Express this fraction as a ratio of distance walked to distance remaining for Gregory.

3. Gregory and Emily are going to meet for dinner at a restaurant 5 blocks south of her apartment.

   a. At which intersection is the restaurant located?

   b. After dinner, they walk back towards her apartment, but stop at a coffee shop that is located three–fifths of the distance to the apartment. What is the location of the coffee shop?

By investigating the situations that follow, you will determine a procedure for finding a point that partitions a segment into a given ratio.

4. Here, you will find a point that partitions a directed line segment from *C*(4, 3) to *D*(10, 3) in a given ratio.

   a. Plot the points on a grid. What is the distance between the points?

   b. Use the fraction of the total length of *CD* to determine the location of Point *A* which partitions the segment from *C* to *D* in a ratio of 5:1. What are the coordinates of *A*?

   c. Find point *B* that partitions a segment from *C* to *D* in a ratio of 1:2 by using the fraction of the total length of *CD* to determine the location of Point *B*. What are the coordinates of *B*?

5. Find the coordinates of Point *X* along the directed line segment *YZ*.

   a. If *Y*(4, 5) and *Z*(4, 10), find *X* so the ratio is of *YX* to *XZ* is 4:1.

   b. If *Y*(4, 5) and *Z*(4, 10), find *X* so the ratio is of *YX* to *XZ* is 3:2.

So far, the situations we have explored have been with directed line segments that were either horizontal or vertical. Use the situations below to determine how the procedure used for Questions 4 and 5 changes when the directed line segment has a defined, nonzero slope.

6. Find the coordinates of Point *A* along a directed line segment from *C*(1, 1) to *D*(9, 5) so that *A* partitions *CD* in a ratio of 3:1. *NOTE:* Since *CD* is neither horizontal nor vertical, the *x* and *y* coordinates have to be considered distinctly.

   a. Find the *x*–coordinate of *A* using the fraction of the horizontal component of the directed line segment (i.e., the **horizontal** distance between *C* and *D*).

   b. Find the *y*–coordinate of *A* using the fraction of the vertical component of the directed line segment (i.e., the **vertical** distance between *C* and *D*).

   c. What are the coordinates of *A*?

7. Find the coordinates of Point *A* along a directed line segment from *C*(3, 2) to *D*(5, 8) so that *A* partitions *CD* in a ratio of 1:1. *NOTE:* Since *CD* is neither horizontal nor vertical, the *x* and *y* coordinates have to be considered distinctly.

   a. Find the *x*–coordinate of *A* using the fraction of the horizontal component of the directed line segment (i.e., the **horizontal** distance between *C* and *D*).

   b. Find the *y*–coordinate of *A* using the fraction of the vertical component of the directed line segment (i.e., the **vertical** distance between *C* and *D*).

   c. What are the coordinates of *A*?

8. Now try a few more …

   a. Find Point Z that partitions the directed line segment *XY* in a ratio of 5:3.
   X(–2, 6) and *Y*(–10, –2)

   b. Find Point *Z* that partitions the directed line segment *XY* in a ratio of 2:3.
   X(2, –4) and *Y*(7, 2)

   c. Find Point *Z* that partitions the directed line segment *YX* in a ratio of 1:3.
   X(–2, –4) and *Y*(–7, 5) (Note the direction change in this segment.)

Back to Gregory and Emily….

9. When they finished their coffee, Gregory walked Emily back to her apartment, and then walked from there back to his hotel.

   a. How many blocks did he walk?

   b. If Gregory had been able to walk the direct path (“as the crow flies”) to the hotel from Emily’s apartment, how far would he have walked? Justify your answer using your diagram.

   c. What is the distance Emily walks to work from her apartment?

   d. What is the length of the direct path between Emily’s apartment and the building where she works? Justify your answer using your diagram.

Determine a procedure for determining the distance between points on a coordinate grid by investigating the following situations.

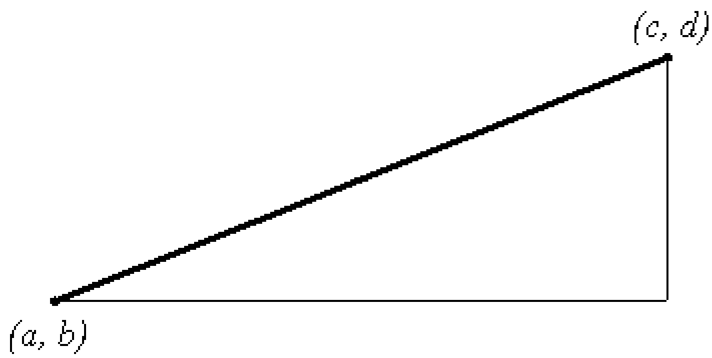
10. What is the distance between 5 and 7? 7 and 5? –1 and 6? 5 and –3?

11. Find a formula for the distance between two points, *a* and *b*, on a number line.

12. Using the same graph paper, find the distance between:

    (1, 1) and (4, 4)   (–1, 1) and (11, 6)   (–1, 2) and (2, –6)

13. Find the distance between points (a, b) and (c, d) shown below.



14. Using your solutions from #13, find the distance between the point (x_1, y_1) and the point (x_2, y_2). Solutions written in this generic form are often called formulas.

15. Do you think your formula would work for any pair of points? Why or why not?

Slopes of Special Pairs of Lines (Discovery Task)

Introduction

This task provides a guided discovery of the relationship between the slopes of parallel lines and the slopes of perpendicular lines.

Mathematical Goals

- Show that the slopes of parallel lines are the same.
- Show that the slopes of perpendicular lines are opposite reciprocals.
- Given the equation of a line and a point not on the line, find the equation of the line that passes through the point and is parallel/perpendicular to the given line.

Essential Questions

- How do we write the equation of a line that goes through a given point and is parallel or perpendicular to another line?

Georgia Standards of Excellence

MGSE9–12.G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.
Students interpret the meaning of parallel and perpendicular lines graphically and numerically, and they generalize their findings.

5. Use appropriate tools strategically.
Students use straightedges, protractors

Background Knowledge

- Students know the graphical definition of parallel and perpendicular lines.
- For the proof about slopes of parallel lines, scaffolded in #4, students need background knowledge of similar triangles. (This proof can be changed to eliminate the need for similar triangles. See “Intervention,” second bullet, below.)
- Students need to know the meaning of slope and how to calculate it.

Common Misconceptions

- The phrase “negative reciprocal” can be confusing for students if the slope is already negative. Using the phrase “opposite reciprocal” instead can mitigate this issue.
- Students sometimes think “perpendicular” means only that the lines intersect. Emphasize that they must form right angles. Additionally, *segments* can be perpendicular, forming an “X,” a “T”, or an “L” shape; they do not have to cross through each other.

Materials

- Graph paper
- Protractor
- Ruler

Grouping

- Partner / individual

Differentiation

Extension:

- Create another proof for the relationship of slopes of parallel and perpendicular lines. *Solution: See intervention notes about using transformations of right triangles.*
- If a line is written in standard form $Ax + By = C$, what would be similar / different for a line that was parallel / perpendicular to it?
  Solution: $Ax + By = C_2$ *for parallel;* $-Bx + Ay = C_2$ *or* $Bx - Ay = C_2$ *for perpendicular*

Intervention:

- Students may need remediation in writing equations of lines given a point and the slope of the line.
- Students can cut out triangles to perform transformations to serve as an entry point to proving the properties. Cut out a right triangle and label its legs appropriately as "rise" and "run." Translating the triangle and extending the hypotenuse creates a parallel line. (The fact that it's a translation means the "rise" and "run" sides haven't changed orientation.) Rotating the triangle 90º and extending the hypotenuse creates a perpendicular line. (The rotation causes the side labeled "rise" to now become horizontal, and the side labeled "run" to now become vertical. Visually, students can also see that the slope has changed from positive to negative or vice–versa.)

Formative Assessment Questions

- Describe how to write the equation of a line <u>parallel</u> to $2x + 3y = 15$ that passes through (8, –3).
- Describe how to write the equation of a line <u>perpendicular</u> to $2x + 3y = 15$ that passes through (8, –3).

Slopes of Special Pairs of Lines – Teacher Notes

Parallel Lines

1. On an *xy*–plane, graph lines ℓ_1, ℓ_2, and ℓ_3, containing the given points. ℓ_1 contains points *A* (0, 7) and *B* (8, 9); ℓ_2 contains points *C* (0, 4) and *D* (8, 6); ℓ_3 contains points *E* (0, 0) and *F* (8, 2). Make sure to carefully extend the lines past the given points.
Solutions are below.
   a. Find the distance between points *A* and *C* and between points *B* and *D*. What do you notice?

      What word describes lines ℓ_1 and ℓ_2?

   b. Find the distance between points *C* and *E* and between points *D* and *F*. What do you notice?

      What word describes lines ℓ_2 and ℓ_3?

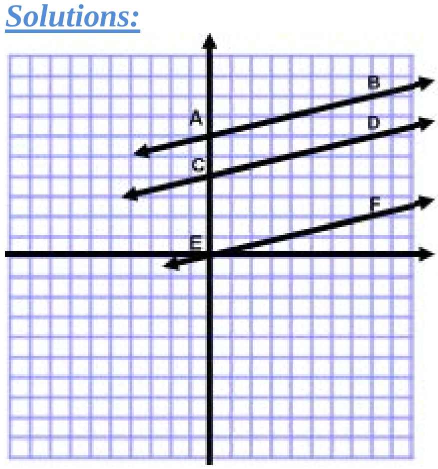
   c. Find the distance between points *A* and *E* and between points *B* and *F*. What do you notice?

      What word describes lines ℓ_1 and ℓ_3?

   d. Now find the slopes of ℓ_1, ℓ_2, and ℓ_3.

      What do you notice?

Solutions:



a. *A and C are 2 units apart, as are B and D.*
l_1 and l_2 are parallel.

b. *C and E are 4 units apart, as are D and F.*
l_2 and l_3 are parallel.

c. *A and E are 6 units apart, as are B and F.*
l_1 and l_3 are parallel.

d. *Slope of* $l_1 = \frac{2}{8} = \frac{1}{4}$
Slope of $l_2 = \frac{2}{8} = \frac{1}{4}$
Slope of $l_3 = \frac{2}{8} = \frac{1}{4}$

All slopes are the same.

2. Now plot line ℓ_4 through points W (–1, 3) and X(–3, 6) and line ℓ_5 through points Y (–2, 1) and Z (–4, 4) carefully extending the lines across the y–axis.

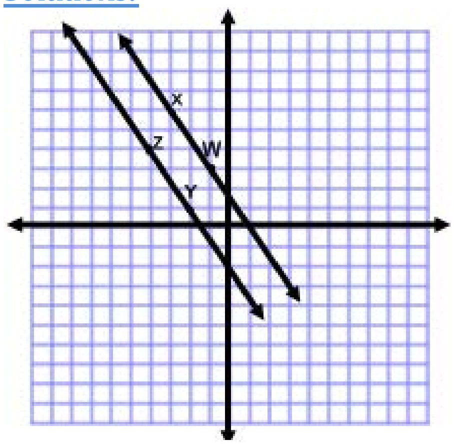
   Solutions are below.

   a. Use a ruler to measure the distance from W vertically to ℓ_5. Then measure the distance from X vertically to ℓ_5. What do you notice?

   b. What word describes these lines?

   c. Find the slope of each line. What do you notice?

   Solutions:



   a. The distances are the same

   b. parallel

   c. $-\frac{3}{2}$ & $-\frac{3}{2}$***; they are the same.***

3. What appears to be true about the slopes of parallel lines?
   Solution:
   Parallel lines have the same slope.

4. Follow the steps below to prove this true for all pairs of parallel lines.

   a. Let the straight lines ℓ and m be parallel. Sketch these on grid paper.

   b. Plot any points U and V on line ℓ and the point W so that WV is the rise and UW is the run of the slope of line ℓ. (A straight line can have only one slope.)

   That is, slope of line ℓ is $\frac{WV}{UW}$.

   c. Draw the straight line UW so that it intersects line m at point X and extends to include Point Z such that segment YZ is perpendicular to UW.

   d. What is the slope of line m?

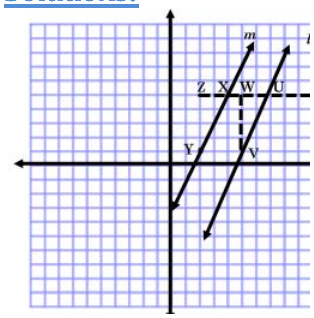
e. Line UZ is the ____________________ of the lines ℓ and m, so $\angle VUW$ and $\angle YXZ$ are _____________________________________ angles, so $\angle VUW$ ____ $\angle YXZ$.

f. Why is it true that $\angle UWV \cong \angle YXZ$?

g. Now, ΔUWV and ΔYXZ are similar, so the ratio of their sides is proportional. Write the proportion that relates the vertical leg to the horizontal leg of the triangles.

h. Note that this proportion shows the slope of line ℓ is the same as the slope of line m. Therefore, parallel lines have the same slope.

Solutions:



d. The slope of line m is $\frac{YZ}{XZ}$.

e. transversal; corresponding; $\cong$

f. The angles are right angles.

g. $\frac{WV}{UW} = \frac{YZ}{XZ}$.

5. Write equations of two lines that are parallel to the line. $y = \frac{2}{3}x + 4$

Solution:
Answers will vary, but all should have a slope of $^2/_3$

6. Determine which of the following lines is / are parallel to $2x - 3y = 21$. Explain why.

a. $y = -\frac{2}{3}x + 2$ b. $-6x + 9y = 12$ c. $\frac{1}{3}x + y = 6$
d. $2x + 3y = 7$ e. $3y = 2x + 1$

Comments:
A review of writing equations in slope–intercept form may be necessary prior to this problem.

Solution:
The given equation shows a slope of $\frac{2}{3}$. When each of the others are written in slope–intercept form, their equations are:
a. $y = -\frac{2}{3}x + 2$ *b.* $y = \frac{2}{3}x + \frac{4}{3}$ *c.* $y = -\frac{1}{3}x + 6$ *d.* $y = -\frac{2}{3}x + \frac{7}{3}$ *e.* $y = \frac{2}{3}x + \frac{1}{3}$
So, only choices b and e are parallel to the given line.

7. Line *m* is parallel to the line $y = -\frac{1}{2}x + 2$ and contains the point (–6, 1). What is the equation of line *m* in slope–intercept form?

Solution:

$y = mx + b$
$1 = (-\frac{1}{2})(-6) + b$
$1 = 3 + b$
$b = -2$

The slope of the given line is $-\frac{1}{2}$ Since line m is parallel, it has the same slope but a different y–intercept. By substituting a point known to lie on line m and the slope of line m into the slope–intercept form for the equation of the line, b can be found. Then the equation can be written using the slope and the newly found y–intercept: $y = -\frac{1}{2}x - 2$

8. What is the equation of the line that passes through (5, 2) and is parallel to the line that passes through (0, 5) and (–4, 8)?

Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = -\frac{3}{4}$$

$y = mx + b$
$2 = (-\frac{3}{4})(5) + b$
$2 = -\frac{15}{4} + b$
$b = \frac{23}{4}$

The slope of the given line is found using the slope formula.. Since line m is parallel, it has the same slope but a different y–intercept. By substituting a point known to lie on line m and the slope of line m into the slope–intercept form for the equation of the line, b can be found. Then the equation can be written using the slope and the newly found y–intercept, $y = -\frac{3}{4}x + \frac{23}{4}$.

Perpendicular Lines

1. On a coordinate grid, graph the following pairs of lines. For each pair, answer: Do these lines intersect? If so, describe the angles formed at their intersection. Use a protractor if necessary. If not, describe the lines.

   a. $y = -\frac{3}{4}x + 5$ and $y = \frac{4}{3}x + 1$ b. $y = 3x - 1$ and $y = -\frac{1}{3}x - 1$

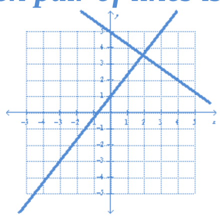
   c. $y = -7x + 2$ and $y = \frac{1}{7}x - 3$ d. $y = x$ and $y = -x - 8$

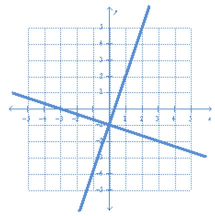
Comments:

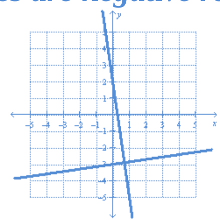
Expect students to see the relationship between the slopes of perpendicular lines as negative reciprocals, but not necessarily see that the product of the slopes is –1. In addition, the proof of the relationship will be challenging for many students. One proof based on transformations is given, but there are other ways to prove the statement.

Solutions:

Each pair of lines is perpendicular. The slopes are negative reciprocals.

a.


b.


c.


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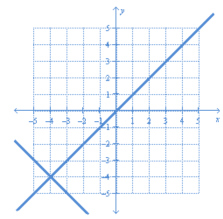


2. Create two equations that have the same type relationship as the lines in Question 1. Draw the lines on a grid to show this relationship. What characteristics do the equations of these lines possess?

Solution:

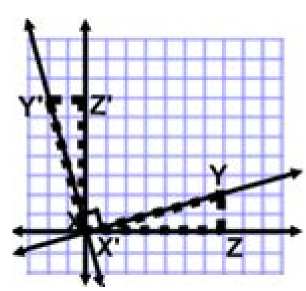
Student answers will vary, but slopes of the lines should be opposite reciprocals.

3. Will all lines with these characteristics have the same graphical relationship? If so, prove it. If not, give a counterexample.

Solution:

Yes, all perpendicular lines will have slopes that are negative reciprocals. Proofs will vary. A sample proof is below.

Sample Proof:
On a coordinate grid, use a protractor to draw two lines l and m perpendicular to each other at the origin. Lines l and m should be neither horizontal nor vertical.
Locate Points Y and Z such that the slope of Line l is $\frac{YZ}{XZ}$.
Rotate ΔXYZ around Point X 90°.
Name the new triangle ΔX'Y'Z'.
X' and Y' lie on Line m so that the slope of Line m is $-\frac{X'Z'}{Y'Z'}$.
Since the lengths of the sides of the figure do not change in a rotation, we have: $-\frac{X'Z'}{Y'Z'} \times \frac{YZ}{XZ} = -1$.



4. Use the relationship between slopes of perpendicular lines to answer the following questions.

   a. Line *m* has the equation $y = \frac{5}{4}x + 1$. What is the slope of a line perpendicular to line *m*?

   Solution:
   Since line m has the slope of $\frac{5}{4}$, the slope of the new line is $-\frac{4}{5}$, the opposite reciprocal.

   b. Write the equation of the line perpendicular to $y = -2x + 5$ whose y–intercept is 12.

   Solution:
   Since the given line has the slope of –2, the slope of the new line is $\frac{1}{2}$, the opposite reciprocal. Substituting the slope and y–intercept of the new line into the slope–intercept form of a line gives $= \frac{1}{2}x + 12$.

   c. Write the equation of the line perpendicular to $y = \frac{1}{5}x - 6$ which passes through the point (1, –3).

   Solution:
   Since the given line has the slope of $\frac{1}{5}$, the slope of the new line is–5, the opposite reciprocal. Substituting the slope and given point that lies on the new line into the slope–intercept form of a line gives:
   $-3 = -5(1) + b \rightarrow -3 = -5 + b \rightarrow b = 2$
   Substituting the slope and y–intercept into slope–intercept form yields, $y = -5x+2$.

   d. What is the equation of the line that passes through (5, 2) and is perpendicular to the line that passes through (0, 5) and (–4, 8)?

   Solution:

   $$m = \frac{y_2 - y_1}{x_2 - x_1} = -\frac{3}{4}$$

   $$2 = (^4/_3)(5) + b$$

   $$b = -\,^{14}/_3$$

   $$y = \frac{4}{3}x - \frac{14}{3}$$

   The slope of the given line is found using the slope formula. Then the opposite reciprocal of the result is found. The new slope and the given point on the line is substituted into slope–intercept form, so that the y–intercept can be determined. The slope and y–intercept are then put into slope–intercept form.

Discovery Task: Slopes of Special Pairs of Lines

Name________________________________ **Date**__________________

Mathematical Goals

- Show that the slopes of parallel lines are the same.
- Show that the slopes of perpendicular lines are opposite reciprocals.
- Given the equation of a line and a point not on the line, find the equation of the line that passes through the point and is parallel/perpendicular to the given line.

Essential Questions

- How do we write the equation of a line that goes through a given point and is parallel or perpendicular to another line?

Georgia Standards of Excellence

MGSE9–12.G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.
5. Use appropriate tools strategically.

Discovery Task: Slopes of Special Pairs of Lines

Name_________________________________ **Date**_________________

Parallel Lines

1. On an *xy*–plane, graph lines ℓ_1, ℓ_2, and ℓ_3, containing the given points. ℓ_1 contains points *A* (0, 7) and *B* (8, 9); ℓ_2 contains points *C* (0, 4) and *D* (8, 6); ℓ_3 contains points *E* (0, 0) and *F* (8, 2). Make sure to carefully extend the lines past the given points.

   a. Find the distance between *A* and *C* and between *B* and *D*. What do you notice?

   What word describes lines ℓ_1 and ℓ_2?

   b. Find the distance between *C* and *E* and between *D* and *F*. What do you notice?

   What word describes lines ℓ_2 and ℓ_3?

   c. Find the distance between *A* and *E* and between *B* and *F*. What do you notice?

   What word describes lines ℓ_1 and ℓ_3?

   d. Now find the slopes of ℓ_1, ℓ_2, and ℓ_3.

   What do you notice?

2. Now plot line ℓ_4 through points W (–1, 3) and X(–3, 6) and line ℓ_5 through points Y (–2, 1) and Z (–4, 4) carefully extending the lines across the y–axis.

   a. Use a ruler to measure the distance from W vertically to ℓ_5. Then measure the distance from X vertically to ℓ_5. What do you notice?

   b. What word describes these lines?

   c. Find the slope of each line. What do you notice?

3. What appears to be true about the slopes of parallel lines?

4. Follow the steps below to prove this true for all pairs of parallel lines.

   a. Let the straight lines ℓ and m be parallel. Sketch these on grid paper.

   b. Plot any points U and V on line ℓ and the point W so that WV is the rise and UW is the run of the slope of line ℓ. (A straight line can have only one slope.)

   That is, the slope of line ℓ is $\dfrac{WV}{UW}$.

   c. Draw the straight line UW so that it intersects line m at point X and extends to include Point Z such that segment YZ is perpendicular to UW.

   d. What is the slope of line m?

   e. Line UZ is the ______________________ of the lines ℓ and m, so $\angle VUW$ and $\angle YXZ$

   are ____ angles, so $\angle VUW$ ____ $\angle YXZ$.

f. Why is it true that $\angle UWV \cong \angle YXZ$?

g. Now, ΔUWV and ΔYXZ are similar, so the ratio of their sides is proportional.
Write the proportion that relates the vertical leg to the horizontal leg of the triangles.

h. Note that this proportion shows the slope of line ℓ is the same as the slope of line m.
Therefore, parallel lines have the same slope.

5. Write equations of two lines that are parallel to the line. $y = \frac{2}{3}x + 4$

6. Determine which of the following lines is / are parallel to $2x - 3y = 21$. Explain why.

a. $y = -\frac{2}{3}x + 2$ **b.** $-6x + 9y = 12$ **c.** $\frac{1}{3}x + y = 6$

d. $2x + 3y = 7$ **e.** $3y = 2x + 1$

7. Line m is parallel to the line $y = -\frac{1}{2}x + 2$ and contains the point (–6, 1). What is the equation of line m in slope–intercept form?

8. What is the equation of the line that passes through (5, 2) and is parallel to the line that passes through (0, 5) and (–4, 8)?

Perpendicular Lines

1. On a coordinate grid, graph the following pairs of lines. For each pair, answer:
   Do these lines intersect? If so, describe the angles formed at their intersection.
   Use a protractor if necessary. If not, describe the lines.

   a. $y = -\frac{3}{4}x + 5$ and $y = \frac{4}{3}x + 1$

   b. $y = 3x - 1$ and $y = -\frac{1}{3}x - 1$

   c. $y = -7x + 2$ and $y = \frac{1}{7}x - 3$

   d. $y = x$ and $y = -x - 8$

2. Create two equations that have the same type relationship as the lines in Question 1. Draw the lines on a grid to show this relationship. What characteristics do the equations of these lines possess?

3. Will all lines with these characteristics have the same graphical relationship?
   If so, prove it. If not, give a counterexample.

4. Use the relationship between slopes of perpendicular lines to answer the following questions.

   a. Line *m* has the equation $y = \frac{5}{4}x + 1$. What is the slope of a line perpendicular to line *m*?

   b. Write the equation of the line perpendicular to $y = -2x + 5$ whose *y*–intercept is 12.

   c. Write the equation of the line perpendicular to $y = \dfrac{1}{5}x - 6$ which passes through the point (1, –3).

   d. What is the equation of the line that passes through (5, 2) and is perpendicular to the line that passes through (0, 5) and (–4, 8)?

Geometric Properties in the Plane (Performance Task)

Introduction
This task provides students an opportunity to apply the algebraic concepts of slope, line segment partitioning, distance formula and the Pythagorean Theorem to geometric figures constructed on the coordinate plane. Students will also use their knowledge of the properties of various polygons to justify their solution of the problems.

Mathematical Goals
- Use coordinates, slope relationships, and distance formula to prove simple geometric theorems.
- Compute the perimeters of polygons using the coordinates of the vertices and the distance formula.
- Find the areas of rectangles and triangles using the coordinates of the vertices and the distance formula.

Essential Questions
- **How can slope and distance be used to determine properties of polygons?**

Georgia Standards of Excellence
MGSE9–12.G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. For *example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; ~~prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2)~~. (Focus on quadrilaterals, right triangles, and ~~circles~~.)*

MGSE9–12.G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Standards for Mathematical Practice
6. Attend to precision.
7. Look for and make use of structure.
Students use distance and slope to prove properties of various geometric figures.

Background Knowledge
- Students know some basic geometric definitions (scalene, isosceles, equilateral, trapezoid, kite, parallelogram, rhombus, rectangle, square, midpoint, diagonal).
- Students can find perimeter of polygons and area of triangles and rectangles.
- Students can work with radicals: $(\sqrt{3})^2 = 3$ and $\sqrt{9} = 3$ (not $\sqrt{9} = \sqrt{3}$, a common misconception)

Common Misconceptions

- Students may stop after finding one description of a quadrilateral without checking to see if a more specific quadrilateral better describes it. For example, in #4, Set 2, the quadrilateral is a parallelogram, but further calculations show it is better described as a rhombus (but not a square).

Materials

- Graph paper

Grouping

- Partners

Differentiation

Extension:

- The segment from (1, 1) to (1, 3) is a leg of an isosceles trapezoid. Give possible coordinates for the other vertices of the trapezoid.
  (Possible solution: (5, 5); (3, 5); (1, 3))

Intervention:

- Encourage students to *predict* answers visually, but emphasize the importance of justifying mathematically.
- Putting slopes and lengths in a table (as in solutions, below) can help students keep work organized.

Formative Assessment Questions

- Describe the similarities between a parallelogram and a rectangle referencing slope and distance of the sides.

Geometric Properties in the Plane – Teacher Notes

1. Determine the coordinates of a scalene triangle. Support your answer mathematically and justify with a drawing on a coordinate grid.

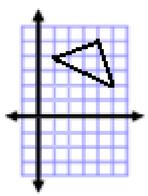
   Answers will vary

2. Classify the triangle as scalene, isosceles, or equilateral. Determine if it is also a right triangle. Then find the perimeter and area.

   a. (1, 4), (4, 5), (5, 2)

   Comments:
   For perimeter and area, students do not have the skills (yet) to add, subtract, and multiply radicals. Decimal approximations are appropriate for these problems.

Solution:



Points	*Slope*	*Distance*
(1, 4) *(4, 5)*	$\frac{1}{3}$	$\sqrt{10}$
(4, 5) *(5, 2)*	-3	$\sqrt{10}$
(5, 2) *(1, 4)*	$-\frac{1}{2}$	$\sqrt{20}$

Two sides have the same length, so the triangle is isosceles.

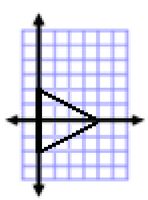
Two sides have slopes that are opposite reciprocals, so the triangle is a right triangle.

Perimeter: $\sqrt{10}+\sqrt{10}+\sqrt{20}\approx$ ***10.8 units***

Area: $\frac{\sqrt{10}\cdot\sqrt{10}}{2}=5$ ***square units***

   b. (0, –2), (0, 2), (4, 0)

Solution:



Points	*Slope*	*Distance*
(0, –2) *(0, 2)*	*undef.*	*4*
(0, 2) *(4, 0)*	$-\frac{1}{2}$	$\sqrt{20}$
(4, 0) *(0, –2)*	$\frac{1}{2}$	$\sqrt{20}$

Two sides have the same length, so the triangle is isosceles.

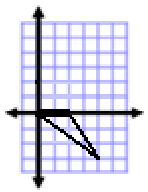
No two sides have slopes that are opposite reciprocals, so the triangle is not a right triangle.

Perimeter: $4+\sqrt{20}+\sqrt{20}\approx$ ***12.9 units***

Area: $\frac{4\cdot 4}{2}=8$ ***square units***

c. (0, 0), (2, 0), (4, –3)

Solution:



Points	*Slope*	*Distance*
(0, 0) *(2, 0)*	*0*	*2*
(2, 0) *(4, –3)*	$-\frac{3}{2}$	$\sqrt{13}$
(4, –3) *(0, 0)*	$-\frac{3}{4}$	*5*

No two sides have the same length, so the triangle is not isosceles.

No two sides have slopes that are opposite reciprocals, so the triangle is not a right triangle.

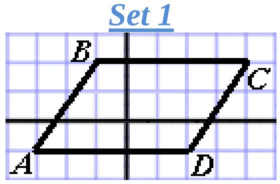
Perimeter: $7 + \sqrt{13} \approx 10.6$ ***units***
Area: $\frac{2 \cdot 3}{2} = 3$ ***square units***

3. Find the following information for each set of points below.
   a. Plot points and connect to form a quadrilateral.
   b. Determine whether the quadrilateral is a trapezoid, kite, parallelogram, rhombus, rectangle, or square. Justify with math.
   c. Find the midpoints of the diagonals. What do you notice?
   d. Find the slope of the diagonals. Are the diagonals perpendicular?
   e. Find the perimeter of each.

Set 1: *A* (–3, –1) *B* (–1, 2) *C* (4, 2) *D* (2, –1)
Set 2: *E* (1, 2) *F* (2, 5) *G* (4, 3) *H* (5, 6)

Solutions:

Set 1



b.

Side	*Slope*	*Length*
$\overline{AB}$	*0*	*5*
$\overline{BC}$	$\frac{3}{2}$	$\sqrt{13}$
$\overline{CD}$	*0*	*5*
$\overline{DA}$	$\frac{3}{2}$	$\sqrt{13}$

Since opposite sides are parallel (same slope), ABCD is a parallelogram.

Not all sides are the same length, so the shape cannot be a rhombus.

Adjacent sides do not have slopes that are opposite reciprocals, so there are no right angles and the shape cannot be a rectangle.

c. Midpoint (ratio 1:1)

$\overline{AC}$: $\left(\frac{-3+4}{2}, \frac{-1+2}{2}\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$

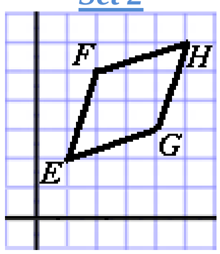
$\overline{BD}$: $\left(\frac{-1+2}{2}, \frac{2+-1}{2}\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$

The midpoints are the same.

d. Diagonals' slopes are –1 and $\frac{3}{7}$
They are not perpendicular, since the slopes are not opposite reciprocals.

e. Perimeter is $10 + 2\sqrt{13} \approx$ ***17.2 units.***

Set 2



b.

Side	***Slope***	***Length***
$\overline{EF}$	3	$\sqrt{10}$
$\overline{FH}$	$\frac{1}{3}$	$\sqrt{10}$
$\overline{HG}$	3	$\sqrt{10}$
$\overline{GE}$	$\frac{1}{3}$	$\sqrt{10}$

Since all sides are the same length, EFHG is a rhombus.

Adjacent sides do not have slopes that are opposite reciprocals, so there are no right angles and the shape cannot be a square.

c. Midpoint (ratio 1:1)

$\overline{EH}$: $\left(\frac{1+5}{2},\frac{2+6}{2}\right)=\left(\frac{6}{2},\frac{8}{2}\right)=(3,4)$

$\overline{FG}$: $\left(\frac{2+4}{2},\frac{3+5}{2}\right)=\left(\frac{6}{2},\frac{8}{2}\right)=(3,4)$

The midpoints are the same.

d. Diagonals' slopes are –1 and 1. They are perpendicular, since the slopes are opposite reciprocals.

e. Perimeter is $4\sqrt{10}$ ***≈ 12.6 units.***

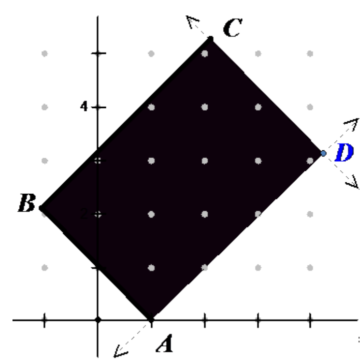
4. Plot points $A = (1, 0)$, $B = (-1, 2)$, and $C = (2, 5)$.

   a. Find the coordinates of a fourth point D that would make $ABCD$ a rectangle. Justify that $ABCD$ is a rectangle.

   b. Find the area of the rectangle.

Comments:

One way to solve this problem is to create lines that are perpendicular to segments AB and BC. The point of intersection of the two lines is the 4th point.



Solution:

a. D is located at (4, 3).
This makes ABCD a rectangle because:
The slope of $\overline{AB}$ ***= the slope of*** $\overline{CD}$ ***= –1.***
The slope of $\overline{BC}$ ***= the slope of*** $\overline{AD}$ ***= 1.***
So all pairs of adjacent sides are perpendicular.

b. $AB = CD = \sqrt{8}$, $CB = AD = \sqrt{18}$
$AB \perp AD$, ***so area is*** $\sqrt{8} \cdot \sqrt{18} = 12$ ***square units.***

Performance Task: Geometric Properties in the Plane

Name________________________________ **Date**_________________

Mathematical Goals

- Use coordinates, slope relationships, and distance formula to prove simple geometric theorems.
- Compute the perimeters of polygons using the coordinates of the vertices and the distance formula.
- Find the areas of rectangles and triangles using the coordinates of the vertices and the distance formula.

Essential Questions

- How can slope and distance be used to determine properties of polygons?

Georgia Standards of Excellence

MGSE9–12.G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. For *example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle;* ~~*prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2)*~~*. (Focus on quadrilaterals, right triangles, and* ~~*circles*~~*.)*

MGSE9–12.G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

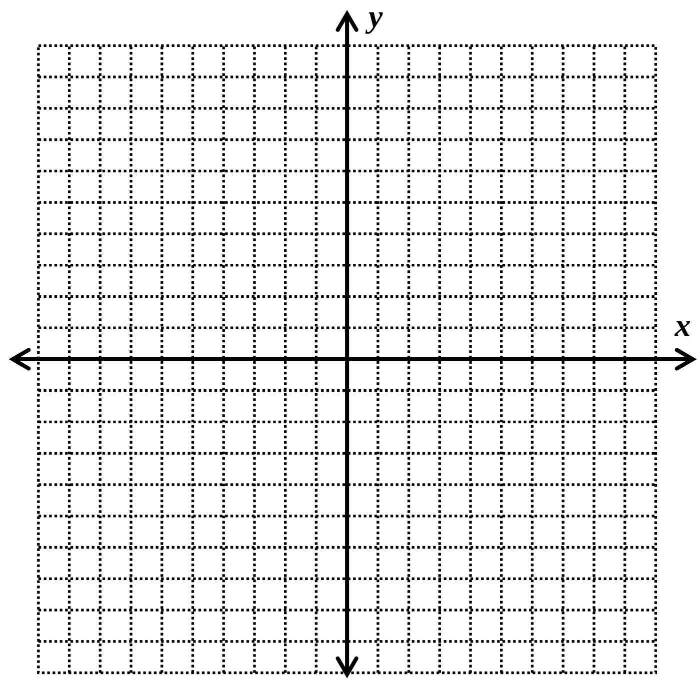
Standards for Mathematical Practice

6. Attend to precision.
7. Look for and make use of structure.

Performance Task: Geometric Properties in the Plane

Name________________________________ **Date**__________________

1. Determine the coordinates of a scalene triangle.
   Support your answer mathematically and justify with a drawing on a coordinate grid.



2. Classify the triangle with the given vertices as scalene, isosceles, or equilateral.
   Determine if it is also a right triangle. Then find the perimeter and area.

   a. (1, 4) (4, 5) (5, 2)

   b. (0, –2) (0, 2) (4, 0)

   c. (0, 0) (2, 0) (4, –3)

3. Find the following information for each set of points below.
   a. Plot points and connect to form a quadrilateral.
   b. Determine whether the quadrilateral is a trapezoid, kite, parallelogram, rhombus, rectangle, or square. Justify mathematically.
   c. Find the midpoints of the diagonals. What do you notice?
   d. Find the slope of the diagonals. Are the diagonals perpendicular?
   e. Find the perimeter of each quadrilateral.

   Set 1: $A(-3, -1)$ $B(-1, 2)$ $C(4, 2)$ $D(2, -1)$

   Set 2: $E(1, 2)$ $F(2, 5)$ $G(4, 3)$ $H(5, 6)$

4. Plot points $A(1, 0)$ $B(-1, 2)$ $C(2, 5)$.

   a. Find the coordinates of a fourth point *D* that would make *ABCD* a rectangle. Justify that *ABCD* is a rectangle.

   b. Find the area of the rectangle.

Equations of Parallel & Perpendicular Lines (Formative Assessment Lesson)

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=703

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Finding Equations of Parallel and Perpendicular Lines*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=226&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=703

Mathematical Goals

- Find, from their equations, lines that are parallel and perpendicular.
- Identify and use intercepts.

Essential Questions

- How do we use equations of parallel and perpendicular lines to form geometric figures?

Georgia Standards of Excellence

MGSE9–12.G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. For *example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; ~~prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2)~~. (Focus on quadrilaterals, right triangles, and ~~circles~~.)*

MGSE9–12.G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

MGSE9–12.G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

MGSE9–12.G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
Students must determine what each question is asking and how to approach it.

3. Construct viable arguments and critique the reasoning of others.
Students must justify why two lines are parallel, perpendicular, or neither.

7. Look for and make use of structure.
Students use patterns relating the slopes of parallel and perpendicular lines to generalize to form rules about these pairs of lines.

Background Knowledge

- Students know how to find the slope, *x*–intercept, and *y*–intercept of a line.
- Students know how to write the equation of a line.
- Students know the definition of a rectangle and connect this to the coordinate plane.

Common Misconceptions

- Students may believe a rectangle needs only to have two pairs of congruent sides, but this is only sufficient to show the figure is a parallelogram. Students must show that adjacent sides are perpendicular.
- The phrase “negative reciprocal” can be confusing for students if the slope is already negative. Using the phrase “opposite reciprocal” instead can mitigate this issue.

Materials

- See FAL website.

Grouping

- Individual / small group

Square (Short Cycle Task)

Source: Balanced Assessment Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=792

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *Square*, is a Mathematics Assessment Project Assessment Task that can be found at the website:
http://www.map.mathshell.org/materials/tasks.php?taskid=270&subpage=apprentice

The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=792

The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=793

Mathematical Goals

- Use slope and length to determine whether a figure with given vertices is a square.

Essential Questions

- How do you use slope and distance to classify a geometric figure?

Georgia Standards of Excellence

MGSE9–12.G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. For *example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; ~~prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2)~~. (Focus on quadrilaterals, right triangles, and ~~circles~~.)*

MGSE9–12.G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

MGSE9–12.G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

MGSE9–12.G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles.

Standards for Mathematical Practice

3. Construct viable arguments and critique the reasoning of others.
Students must mathematically justify whether the shape is a square.

7. Look for and make use of structure.
Students use the properties of squares to determine what to look for in their calculations.

Background Knowledge

- Students know the definition of a square.
- Students know how to find and interpret slope and length of segments.

Common Misconceptions

- Students may stop after finding one description of a quadrilateral without checking to see if a more specific quadrilateral better describes it.

Materials

- See FAL website.

Grouping

- Partner / small group

Euler's Village (Culminating Task)

Introduction

This task provides students an opportunity to apply the algebraic concepts of slope, intersection of two lines, distance from a point to line, and the distance formula.

Mathematical Goals

- Find the point on a line segment that separates the segments into a given ratio.
- Given the equation of a line and a point not on the line, find the equation of the line that passes through the point and is parallel/perpendicular to the given line.
- Use coordinates to compute perimeter and area of a quadrilateral using the distance formula.
- Use coordinates, slope relationships, and distance formula to prove simple geometric theorems.

Essential Questions

- How can a line be partitioned?
- How do we write the equation of a line that goes through a given point and is parallel or perpendicular to another line?
- How do I use coordinates, slope relationships, and the distance formula to classify a quadrilateral?
- How do I compute the perimeter and area of a quadrilateral in the coordinate plane?

Georgia Standards of Excellence

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MGSE9–12.G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Standards for Mathematical Practice

4. Model with mathematics.
Students use mathematical properties to represent and answer questions about the problem situation.

6. Attend to precision.
Students must decide on an appropriate answer form (radical, decimal, etc.) and level of accuracy.

Background Knowledge

- The distance between a point and a line is the length of the *shortest* segment connecting the point and the line. This segment is always perpendicular to the line.
- Students can write equations of lines and solve systems of linear equations.
- Students know the definition of a parallelogram.
- Students know how to find and interpret slope and length of segments.

Common Misconceptions

- Students might measure the distance from a point to a line only vertically or horizontally, rather than perpendicular to the line.

Grouping

- Partners / small group

Differentiation

Extension:

- Find another way (without using systems of equations) to find the point on the road that is closest to the well.
  (Possible solution: Draw three segments from the well to the road: one that is perpendicular to the road, one that is horizontal, and another that is vertical. This forms similar right triangles—two small and one large. The lengths of the large triangle's sides can be found, and proportional reasoning can be used to find other lengths to locate the house.)
  (Alternate solution: Draw three segments as described above. This happens to be an isosceles triangle, so the best location for the house is the midpoint of the large triangle's hypotenuse.)
- What if the road were curved? While we can't use calculations to determine the best location of the house, we can use tools like a compass and straightedge to estimate its location. Describe your method.
  (Possible solution: Draw a series of concentric circles centered at the well, increasing the radius until a circle intersects the road in one point. This intersection is the best location of the house.)

Intervention:

- Scaffolding may be necessary to find the minimum distance from the well to the road.

Formative Assessment Questions

- Describe how you can find the distance between the line $y = \frac{1}{3}x + 7$ and the point (–4, 5)?
- How do you use slope and distance to classify a figure?
- How do you interpret perimeter and area in real-world situations?

Euler's Village – Teacher Notes

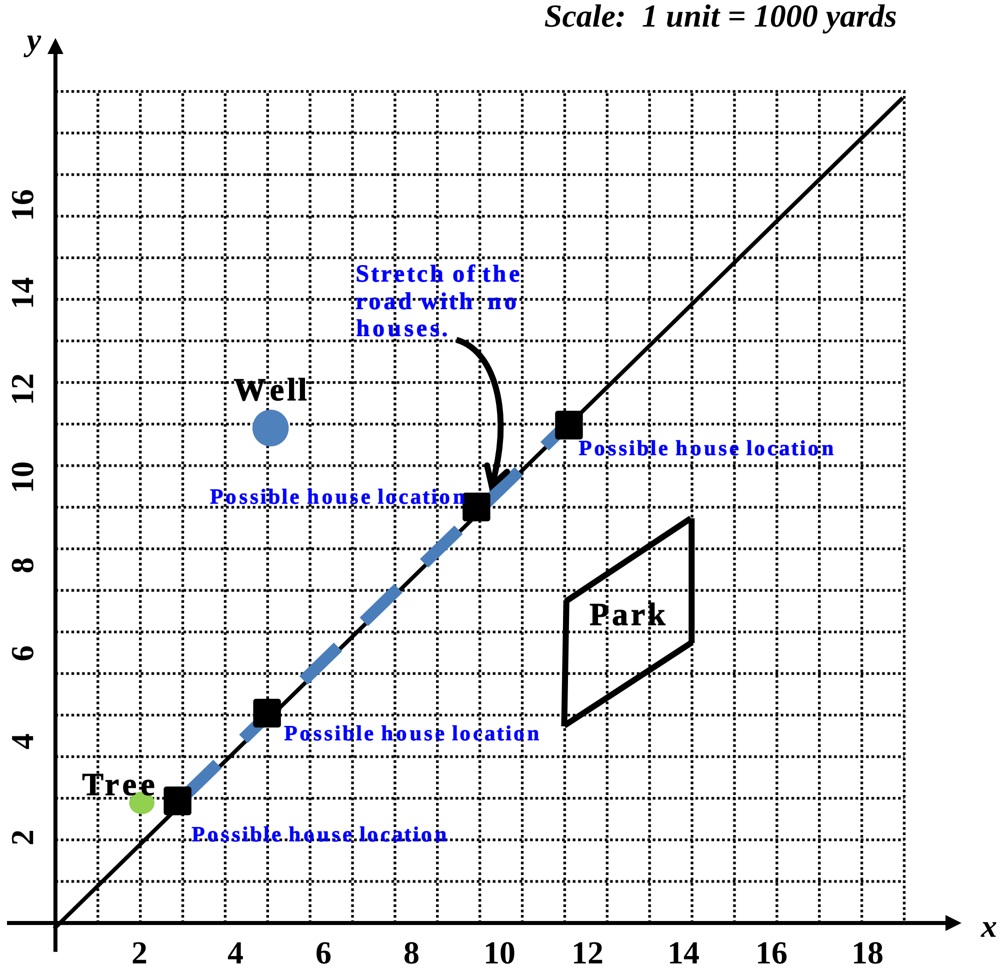
You would like to build a house close to the village of Euler. There is a beautiful park just outside the village, and the road you would like to build your house on begins right at the town square and goes by this park.

The road follows an approximately north east direction as you leave town and continues for 3,000 feet. It passes right by a large shade tree located approximately 200 yards east and 300 yards north of the town square. There is a stretch of the road, between 300 and 1200 yards to the east of town, which currently has no houses. This stretch of road is where you would like to locate your house. All water supplies are linked to town wells, and the closest well to this part of the road is 500 yards east and 1200 yards north of the town square.

Comments:

The first thing students will need to do is make a sketch of the town, the road, the tree and the well. The idea of a road traveling 'approximately north east' may need to be discussed. The easiest way to model this is to use the line f(x) = x.

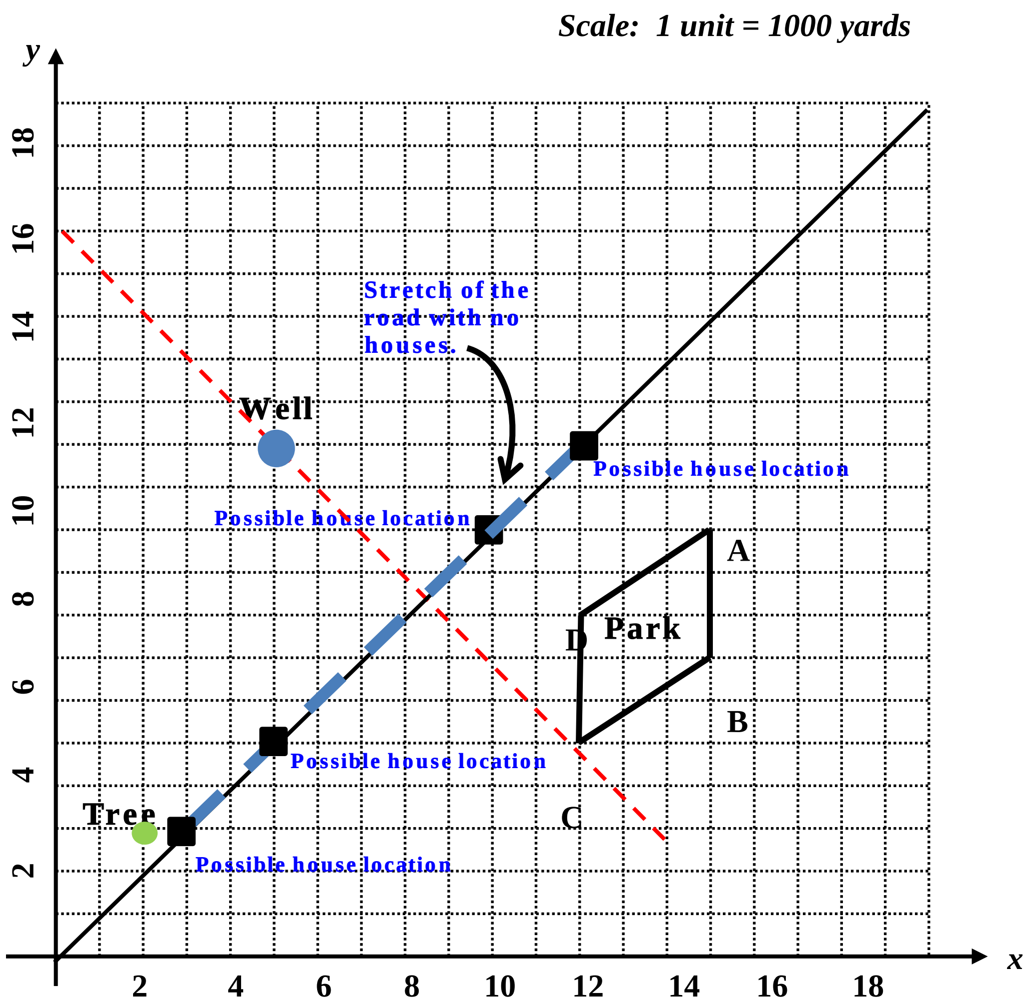
Scale: 1 unit = 1000 yards



1. How far from the well would it be if the house was located on the road 300 yards east of town? 500 yards east of town? 1,000 yards east of town? 1,200 yards east of town? (For the sake of calculations, do not consider how far the house is from the road, just use the road to make calculations)

Comments:
Students can use the distance formula to calculate these distances. The key to the problem is making sure they can find the coordinates for the well, houses, etc.

The calculation for the distance to the points at (500,500) and (1200, 1200) is shown below. It is not necessary for students to use the distance formula, but if they do, watch to see how they complete the calculations. This could provide an opportunity to focus on how the rule $\sqrt{x^2} = |x|$ makes the calculations simpler here.



Solutions:
The well is located at (500, 1200).

300 yards east of town would place the house at (300, 300)
$\sqrt{(\mathbf{500-300})^2+(\mathbf{1200-300})^2}=\sqrt{\mathbf{850000}}\approx \mathbf{921.95}$ → *922 yards*

500 yards east of town would place the house at (500, 500)
The distance to the well is 700 yards. This can be found directly from the graph because it is directly below the well.

1000 yards east of town would place the house at (1000, 1000)
$\sqrt{(\mathbf{500-1000})^2+(\mathbf{1200-1000})^2}=\sqrt{\mathbf{290000}}\approx \mathbf{538.52}$ → *539 yards*

1200 yards east of town would place the house at (1200, 1200)
The distance to the well is 700 yards. This can be found directly from the graph because this location is directly to the right of the well.

2. The cost of the piping leading from the well to the house is a major concern. Where should you locate your house in order to have the shortest distance to the well? (*Remember: the shortest distance between a line and a point is the length of the segment perpendicular to the line that passes through the point*). Justify your answer mathematically.

Comment:
The students need to determine the point on the line closest to the well. To do this they can find a line that is perpendicular to the road and goes through the point where the well is located. The point where the perpendicular line intersects the road would be the best place to locate the house. There are several ways the students can find the equation of the line: slope–intercept (shown), point–slope, etc.

Solution: ***The slope of the road is 1. A line perpendicular to the road would have a slope of –1. But it also needs to go through the point (500, 1200).***	
m = –1 ***y = –1x + b*** ***1200 = –1(500) + b*** ***1700 = b*** ***y = –x + 1700***	***Graphing this shows the point we are interested in for the location of the house. Now we need to calculate the point of intersection of the two lines.***

The house should be located at the point (850, 850) to be the shortest distance to the well.

Using elimination:	*Using substitution:*
$y = x$ $\underline{y = -x + 1700}$ $2y = 1700$ $y = 850$ $x = 850$	$y = x$ $y = -x + 1700$ $x = -x + 1700$ $2x = 1700$ $x = 850$ $y = 850$

3. If the cost of laying pipes is \$22.50 per linear yard, how much will it cost to connect your house to the well?

Comments:
The students should use the location they found in #2. To find the length of the piping they need to use the distance formula.

Solution:
First, we need to determine the distance from (500, 1200), the location of the well, to the point (850, 850) the location of the house. Using the distance formula we find:
$\sqrt{(500-850)^2+(1200-850)^2} = \sqrt{245000} \approx 494.97$

To calculate cost we need to multiply the cost of the pipes by the distance:
494.97 yds · \$22.50 / yd = \$11,136.83

4. You also want to install a swimming pool on the line with the pipes. You want the front edge of the pool to be $\frac{3}{5}$ the distance from the road to the well. What are the coordinates of the front corner of the swimming pool?

Solution: $\left(\frac{3}{5}(500-850)+850, \frac{3}{5}(1200-850)+850\right) = (640, 1060)$

5. The builder of your house is impressed by your calculations and wants to use the same method for placing other houses. Describe the method you used. Would you want him to place the other houses in the same manner?

Comment:
The students need to remember they were trying to find the point that would be closest to the well. If the builder wants to place other houses along the road, he will not need to calculate a perpendicular line. He will simply need to calculate the distance between the two points.

Solution:

Answers will vary. Sample answer:

The perpendicular distance from any point to a line is the shortest. So, we needed to find a line perpendicular to the road that would go through the point where the well was located. To do this we used the point slope formula. We knew the slope of the perpendicular line would be –1 and the point it had to go through was (500, 1200). Once we had the equation of the line, we had to find the point of intersection of the two lines, which was (850, 850). We then had to find the distance between that point and the well. Once we had that distance we could multiply it by the cost of the piping.

The same method (with different numbers, of course) would always yield the correct answer.

6. Write a formula that the builder could use to find the cost of laying pipes to any house along this road. How would you have to change your formula for another road?

Solution:

Let (a, b) represent the location of the house. The distance from the well to the house would be:

distance from (500, 1200) to (a, b): $\sqrt{(500-a)^2+(1200-b)^2}$

The builder would then need to multiply this distance by the cost of the piping, $22.50 / yard.

This equation would give him the cost of the piping to any point along the line.
Cost = 22.50 · $\sqrt{(500-a)^2+(1200-b)^2}$

7. One day you were wondering what geometric shape the park formed, so you looked at the map and took some measurements. Using those measurements, prove what specific geometric shape is formed by the park boundaries.

Solution:
Students will need to calculate the lengths of the sides using the distance formula. They will also need to find the slope of each side to test for parallel/perpendicular sides. To aid in communicating, the students will need to label each vertex.

SIDE	SLOPE	LENGTH
$\overline{AB}$	Undefined	3000
$\overline{BC}$	$\frac{2}{3}$	$\sqrt{2000^2+3000^2}\approx 3605$
$\overline{CD}$	Undefined	3000
$\overline{DA}$	$\frac{2}{3}$	$\sqrt{2000^2+3000^2}\approx 3605$

Since opposite sides of the figure have the same slope, they are parallel. Since opposite sides of the figure have the same length, they are congruent. Therefore, the figure is a PARALLELOGRAM.

8. Your service club noticed that the fence around the park needed to be replaced. How much fencing would be needed?

<u>Solution:</u>
Students should recognize that they are finding the perimeter of the park. So, adding the lengths found in #7, the amount of fencing needed is 13,210 yards.

9. While the fence was being replaced, some damage occurred to a portion the grass in the park. The entire park is covered with grass, but only ¼ of the total area needs to be replaced. Grass seed is sold in bags that cover 5000 square yards each. How many bags of grass seed would your club need to purchase to reseed the entire park?

<u>Solution:</u>
Using the formula A = bh, students will find the entire area of the park:

$$A = (3000)(3000)$$
$$A = 9{,}000{,}000\ yd^2$$

Since only ¼ of the grass needs to be replaced, the area to be reseeded is found by:

$$\frac{1}{4}(9{,}000{,}000) = 2{,}250{,}000\ yd^2$$

Each bag will cover 5000 yd², so the club will need to purchase:

$$\frac{2{,}250{,}000}{5000} = 450 \text{ bags of grass seed}$$

Culminating Task: Euler's Village

Name________________________________ **Date**__________________

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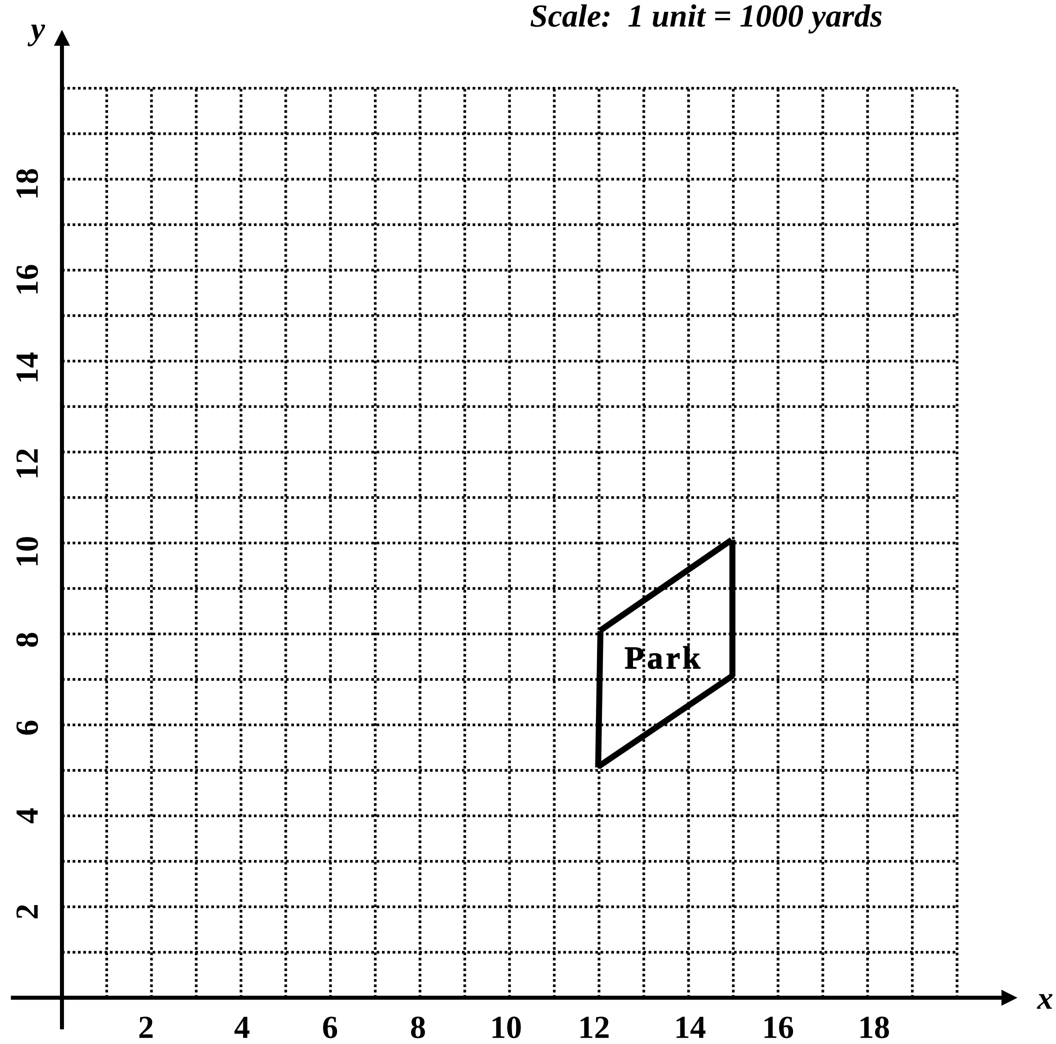

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