Georgia
Standards of Excellence

Mathematics

Standards

Accelerated GSE Pre-Calculus

Georgia Department of Education
Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

K-12 Mathematics Introduction

Georgia Mathematics focuses on actively engaging the student in the development of mathematical understanding by working independently and cooperatively to solve problems, estimating and computing efficiently, using appropriate tools, concrete models and a variety of representations, and conducting investigations and recording findings. There is a shift toward applying mathematical concepts and skills in the context of authentic problems and student understanding of concepts rather than merely following a sequence of procedures. In mathematics classrooms, students will learn to think critically in a mathematical way with an understanding that there are many different solution pathways and sometimes more than one right answer in applied mathematics. Mathematics is the economy of information. The central idea of all mathematics is to discover how knowing some things leads, via reasoning, to knowing more—without having to commit the information to memory as a separate fact. It is the reasoned, logical connections that make mathematics coherent. The implementation of the Georgia Standards of Excellence in Mathematics places the expected emphasis on sense-making, problem solving, reasoning, representation, modeling, representation, connections, and communication.

Accelerated Pre-Calculus

Accelerated Pre-Calculus is the third in a sequence of mathematics courses designed to ensure that students are prepared to take higher-level mathematics courses during their high school career, including Advanced Placement Calculus AB, Advanced Placement Calculus BC, and Advanced Placement Statistics.

The standards in the three-course high school sequence specify the mathematics that all students should study in order to be college and career ready. Additional mathematics content is provided in fourth credit courses and advanced courses including, calculus, advanced statistics, discrete mathematics, and mathematics of finance courses. High school course content standards are listed by conceptual categories including Number and Quantity, Algebra, Functions, Geometry, and Statistics and Probability. Conceptual categories portray a coherent view of high school mathematics content; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus. Standards for Mathematical Practice provide the foundation for instruction and assessment.

Mathematics | Standards for Mathematical Practice

Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.

High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision. High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure. By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8 Look for and express regularity in repeated reasoning.

High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. **Students who do not have an understanding of a topic may rely on procedures too heavily.** Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. **In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.**

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Accelerated Pre-Calculus | Content Standards

The Complex Number System N.RN

Use properties of rational and irrational numbers.

MGSE9-12.N.CN.3 Find the conjugate of a complex number; use the conjugate to find the absolute value (modulus) and quotient of complex numbers.

Represent complex numbers and their operations on the complex plane.

MGSE9-12.N.CN.4 Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

MGSE9-12.N.CN.5 Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}i)^3 = 8$ *because* $(-1 + \sqrt{3}i)$ *has modulus 2 and argument 120°.*

MGSE9-12.N.CN.6 Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Vector and Matrix Quantities N.VM

Represent and model with vector quantities.

MGSE9-12.N.VM.1 Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\mathbf{v}$, $|\mathbf{v}|$, $||\mathbf{v}||$, v).

MGSE9-12.N.VM.2 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

MGSE9-12.N.VM.3 Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

MGSE9-12.N.VM.4 Add and subtract vectors.

MGSE9-12.N.VM.4a Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

MGSE9-12.N.VM.4b Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

MGSE9-12.N.VM.4c Understand vector subtraction $v - w$ as $v + (-w)$, where $(-w)$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

MGSE9-12.N.VM.5 Multiply a vector by a scalar.

MGSE9-12.N.VM.5a Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

MGSE9-12.N.VM.5b Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when $|c|v = 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).

Perform operations on matrices and use matrices in applications.

MGSE9-12.N.VM.6 Use matrices to represent and manipulate data, e.g., transformations of vectors.

MGSE9-12.N.VM.7 Multiply matrices by scalars to produce new matrices.

MGSE9-12.N.VM.8 Add, subtract, and multiply matrices of appropriate dimensions.

MGSE9-12.N.VM.9 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

MGSE9-12.N.VM.10 Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

MGSE9-12.N.VM.11 Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

MGSE9-12.N.VM.12 Work with 2 X 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Reasoning with Equations and Inequalities — A.REI

Solve systems of equations

MGSE9-12.A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ *and the circle* $x^2 + y^2 = 3$.

MGSE9-12.A.REI.8 Represent a system of linear equations as a single matrix equation in a vector variable.

MGSE9-12.A.REI.9 Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Interpreting Functions — F.IF

Interpret functions that arise in applications in terms of the context

MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

Analyze functions using different representations

MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

> **MGSE9-12.F.IF.7e** Graph ~~exponential and logarithmic functions, showing intercepts and end behavior, and~~ trigonometric functions, showing period, midline, and amplitude.

Building Functions — F.BF

Build new functions from existing functions

MGSE9-12.F.BF.4 Find inverse functions.

> **MGSE9-12.F.BF.4d** Produce an invertible function from a non-invertible function by restricting the domain.

Trigonometric Functions — F.TF

Extend the domain of trigonometric functions using the unit circle

MGSE9-12.F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

MGSE9-12.F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

MGSE9-12.F.TF.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π - x, π + x, and 2π - x in terms of their values for x, where x is any real number.

MGSE9-12.F.TF.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

<u>Model periodic phenomena with trigonometric functions</u>

MGSE9-12.F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

MGSE9-12.F.TF.6 Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

MGSE9-12.F.TF.7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

<u>Prove and apply trigonometric identities</u>

MGSE9-12.F.TF.8 Prove the Pythagorean identity $(\sin A)^2 + (\cos A)^2 = 1$ and use it to find sin A, cos A, or tan A, given sin A, cos A, or tan A, and the quadrant of the angle.

MGSE9-12.F.TF.9 Prove addition, subtraction, double, and half-angle formulas for sine, cosine, and tangent and use them to solve problems.

Similarity, Right Triangles, and Trigonometry — G.SRT

<u>Apply trigonometry to general triangles</u>

MGSE9-12.G.SRT.9 Derive the formula A = (1/2)ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

MGSE9-12.G.SRT.10 Prove the Laws of Sines and Cosines and use them to solve problems.

MGSE9-12.G.SRT.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Expressing Geometric Properties with Equations — G.GPE

<u>Translate between the geometric description and the equation for a conic section</u>

MGSE9-12.G.GPE.2 Derive the equation of a parabola given a focus and directrix.

MGSE9-12.G.GPE.3 Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Interpreting Categorical and Quantitative Data S.ID

Summarize, represent, and interpret data on a single count or measurement variable

MGSE9-12.S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, ~~mean absolute deviation~~, standard deviation) of two or more different data sets.

MGSE9-12.S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Making Inferences and Justifying Conclusions S.IC

Understand and evaluate random processes underlying statistical experiments

MGSE9-12.S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

MGSE9-12.S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0. 5. Would a result of 5 tails in a row cause you to question the model?*

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

MGSE9-12.S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

MGSE9-12.S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

MGSE9-12.S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

MGSE9-12.S.IC.6 Evaluate reports based on data. *For example, determining quantitative or categorical data; collection methods; biases or flaws in data.*

Conditional Probability and the Rules of Probability S.CP

Use the rules of probability to compute probabilities of compound events in a uniform probability model

MGSE9-12.S.CP.8 Apply the general Multiplication Rule in a uniform probability model, P(A and B) = [P(A)]x[P(B|A)] =[P(B)]x[P(A|B)], and interpret the answer in terms of the model.

MGSE9-12.S.CP.9 Use permutations and combinations to compute probabilities of compound events and solve problems.

Use Probability to Make Decisions S.MD

Calculate expected values and use them to solve problems

MGSE9-12.S.MD.1 Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

MGSE9-12.S.MD.2 Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

MGSE9-12.S.MD.3 Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*

MGSE9-12.S.MD.4 Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

Use probability to evaluate outcomes of decisions

MGSE9-12.S.MD.5 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

> **MGSE9-12.S.MD.5a** Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*
>
> **MGSE9-12.S.MD.5b** Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

MGSE9-12.S.MD.6 Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

MGSE9-12.S.MD.7 Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).