

College Readiness Mathematics

K-12 Mathematics Introduction

Georgia Mathematics focuses on actively engaging the student in the development of mathematical understanding by working independently and cooperatively to solve problems, estimating and computing efficiently, using appropriate tools, concrete models and a variety of representations, and conducting investigations and recording findings. There is a shift toward applying mathematical concepts and skills in the context of authentic problems and student understanding of concepts rather than merely following a sequence of procedures. In mathematics classrooms, students will learn to think critically in a mathematical way with an understanding that there are many different solution pathways and sometimes more than one right answer in applied mathematics. Mathematics is the economy of information. The central idea of all mathematics is to discover how knowing some things leads, via reasoning, to knowing more—without having to commit the information to memory as a separate fact. It is the reasoned, logical connections that make mathematics manageable. The implementation of Georgia Performance Standards in Mathematics places the expected emphasis on sense-making, problem solving, reasoning, representation, modeling, connections, and communication.

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College Readiness Mathematics is a fourth course option for students who have completed Algebra I or Coordinate Algebra, Geometry or Analytic Geometry, and Algebra II or Advanced Algebra, but are still struggling with high school mathematics standards essential for success in first year post-secondary mathematics courses required for non-STEM majors. The course is designed to serve as a bridge for high school students who will enroll in non-STEM post-secondary study and will serve to meet the high school fourth course graduation requirement. The course has been approved by the University System of Georgia as a fourth mathematics course beyond Algebra II or Advanced Algebra for non-STEM majors, so the course will meet the needs of college-bound seniors who will not pursue STEM fields.

College Readiness Mathematics focuses on key content and practice standards to ensure that students will be ready for post-secondary academic courses and career preparation in non-STEM fields. The course will revisit and expand the understanding of content standards introduced in earlier mathematics courses and will emphasize numeracy, algebra and functions, geometry, and statistics in a variety of contexts.

Instruction and assessment should include the appropriate use of manipulatives and technology. Mathematics concepts should be represented in multiple ways, such as concrete/pictorial, verbal/written, numeric/data-based, graphical, and symbolic. Concepts should be introduced and used, where appropriate, in the context of realistic experiences.

The Standards for Mathematical Practice will provide the foundation for instruction and assessment. The content standards selected are essential for post-secondary preparation in non-STEM study.

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Mathematics | Standards for Mathematical Practice

Mathematical Practices are listed with the course's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1 Make sense of problems and persevere in solving them.

High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments,

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distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision. High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure. By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y . High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

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8 Look for and express regularity in repeated reasoning.

High school students notice if calculations are repeated, and look both for general methods and for shortcuts.

Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. **Students who do not have an understanding of a topic may rely on procedures too heavily.** Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. **In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.**

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

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Quantities

N.Q

Reason quantitatively and use units to solve problems.

MGSE9-12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

- Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
- Convert units and rates using dimensional analysis (English-to-English and Metric-to-Metric without conversion factor provided and between English and Metric with conversion factor);
- Use units within multi-step problems and formulas; interpret units of input and resulting units of output.

MGSE9-12.N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. Given a situation, context, or problem, students will determine, identify, and use appropriate quantities for representing the situation.

Seeing Structure in Expressions

A.SSE

Interpret the structure of expressions

MGSE9-12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

MGSE9-12.A.SSE.2 Use the structure of an expression to rewrite it in different equivalent forms. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems

MGSE9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Creating Equations

A.CED

Create equations that describe numbers or relationships

MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, simple rational, and exponential functions (integer inputs only).

MGSE9-12.A.CED.2 Create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which $A = P(1 + r/n)^{nt}$ has multiple variables.)

MGSE9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

MGSE9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. *Examples: Rearrange Ohm’s law $V = IR$ to highlight resistance R ; Rearrange area of a circle formula $A = \pi r^2$ to highlight the radius r .*

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Reasoning with Equations and Inequalities A.REI

Understand solving equations as a process of reasoning and explain the reasoning

MGSE9-12.A.REI.1 Using algebraic properties and the properties of real numbers, justify the steps of a simple, one-solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties.

MGSE9-12.A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable including equations with coefficients represented by letters. *For example, given $ax + 3 = 7$, solve for x .*

MGSE9-12.A.REI.4 Solve quadratic equations in one variable.

Solve systems of equations

MGSE9-12.A.REI.5 Show and explain why the elimination method works to solve a system of two-variable equations.

MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

MGSE9-12.A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*

Represent and solve equations and inequalities graphically

MGSE9-12.A.REI.11 Using graphs, tables, or successive approximations, show that the solution to the equation $f(x) = g(x)$ is the x -value where the y -values of $f(x)$ and $g(x)$ are the same.

MGSE9-12.A.REI.12 Graph the solution set to a linear inequality in two variables.

Interpreting Functions

F.IF

Understand the concept of a function and use function notation

MGSE9-12.F.IF.1 Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and $f(x)$ is the output (an element of the range). Graphically, the graph is $y = f(x)$.

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Interpret functions that arise in applications in terms of the context

MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

MGSE9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

MGSE9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations

MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

MGSE9-12.F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

MGSE9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.*

Building Functions

F.BF

Build a function that models a relationship between two quantities

MGSE9-12.F.BF.1 Write a function that describes a relationship between two quantities.

MGSE9-12.F.BF.2 Write arithmetic and geometric sequences recursively and explicitly, use them to model situations, and translate between the two forms. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.

Build new functions from existing functions

MGSE9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Linear, Quadratic, and Exponential Models

F.LE

Construct and compare linear, quadratic, and exponential models and solve problems

MGSE9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

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MGSE9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

MGSE9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Interpret expressions for functions in terms of the situation they model

MGSE9-12.F.LE.5 Interpret the parameters in a linear ($f(x) = mx + b$) and exponential ($f(x) = a \cdot d^x$) function in terms of context. (In the functions above, “m” and “b” are the parameters of the linear function, and “a” and “d” are the parameters of the exponential function.) In context, students should describe what these parameters mean in terms of change and starting value.

Use coordinates to prove simple geometric theorems algebraically

MGSE9-12.G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.* (Focus on quadrilaterals, right triangles, and circles.)

MGSE9-12.G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Geometric Measurement and Dimension

G.GMD

Explain volume formulas and use them to solve problems

MGSE9-12.G.GMD.1 Give informal arguments for geometric formulas.

- Give informal arguments for the formulas of the circumference of a circle and area of a circle using dissection arguments and informal limit arguments.
- Give informal arguments for the formula of the volume of a cylinder, pyramid, and cone using Cavalieri’s principle.

MGSE9-12.G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Modeling with Geometry

G.MG

Apply geometric concepts in modeling situations

MGSE9-12.G.MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

MGSE9-12.G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

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Interpreting Categorical and Quantitative Data

S.ID

Summarize, represent, and interpret data on a single count or measurement variable

MGSE9-12.S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

MGSE9-12.S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, mean absolute deviation, standard deviation) of two or more different data sets.

MGSE9-12.S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Summarize, represent, and interpret data on two categorical and quantitative variables

MGSE9-12.S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

MGSE9-12.S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

Interpret linear models

MGSE9-12.S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

MGSE9-12.S.ID.8 Compute (using technology) and interpret the correlation coefficient “ r ” of a linear fit. (For instance, by looking at a scatterplot, students should be able to tell if the correlation coefficient is positive or negative and give a reasonable estimate of the “ r ” value.) After calculating the line of best fit using technology, students should be able to describe how strong the goodness of fit of the regression is, using “ r ”.

MGSE9-12.S.ID.9 Distinguish between correlation and causation

Making Inferences and Justifying Conclusions

S.IC

Understand and evaluate random processes underlying statistical experiments

MGSE9-12.S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

MGSE9-12.S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.