

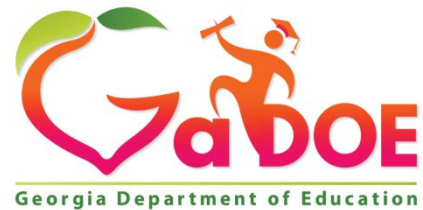


# Georgia Standards of Excellence Curriculum Frameworks

## Mathematics

GSE Coordinate Algebra

Unit 1: Relationships Between Quantities



Richard Woods, Georgia's School Superintendent  
*"Educating Georgia's Future"*

**Unit 1**

**Relationships Between Quantities**

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## **OVERVIEW**

In this unit students will:

- interpret units in the context of the problem
- convert units of measure in order to solve problems
- when solving a multi-step problem, use units to evaluate the appropriateness of the solution
- choose the appropriate units for a specific formula and interpret the meaning of the unit in that context
- choose and interpret both the scale and the origin in graphs and data displays
- determine and interpret appropriate quantities when using descriptive modeling
- determine the accuracy of values based on their limitations in the context of the situation
- identify the different parts of the expression and explain their meaning within the context of a problem
- decompose expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts
- create linear and exponential equations and inequalities in one variable and use them in a contextual situation to solve problems
- create equations in two or more variables to represent relationships between quantities
- graph equations in two variables on a coordinate plane and label the axes and scales
- write and use a system of equations and/or inequalities to solve a real world problem
- recognize that the equations and inequalities represent the constraints of the problem
- solve multi-variable formulas or literal equations, for a specific variable

The first unit of Coordinate Algebra involves relationships between quantities. Students will be provided with examples of real-world problems that can be modeled by writing an equation or inequality. The tasks begin with simple equations and inequalities and build up to equations in two or more variables. It is important to discuss using appropriate labels and scales on the axes when representing functions with graphs. Students will also explore examples illustrating when it is useful to rewrite a formula by solving for one of the variables in the formula.

In real-world situations, answers are usually represented by numbers associated with units. Units involve measurement and often require a conversion. Measurement involves both precision and accuracy. Estimation and approximation often precede more exact computations. Students need to develop sound mathematical reasoning skills and forms of argument to make reasonable judgments about their solutions. They should be able to decide whether a problem calls for an estimate, for an approximation, or for an exact answer. To accomplish this goal, teachers should provide students with a broad range of contextual problems that offer opportunities for performing operations with quantities involving units. These problems should be connected to science, engineering, economics, finance, medicine, or other career fields.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. This unit provides much needed content information and excellent learning activities. However, the intent of the framework is not to provide a comprehensive resource for the implementation of all standards in the unit. A variety of resources should be utilized to supplement this unit. The tasks in this unit framework illustrate the types of learning activities that should be utilized from a variety of sources. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the “**Strategies for Teaching and Learning**” and the tasks listed under “**Evidence of Learning**” be reviewed early in the planning process.

## **STANDARDS ADDRESSED IN THIS UNIT**

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

## **KEY STANDARDS**

### **Reason quantitatively and use units to solve problems.**

**MGSE9–12.N.Q.1** Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

- a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
- b. Convert units and rates using dimensional analysis (English-to-English and Metric-to-Metric without conversion factor provided and between English and Metric with conversion factor);
- c. Use units within multi-step problems and formulas; interpret units of input and resulting units of output.

**MGSE9–12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling. Given a situation, context, or problem, students will determine, identify, and use appropriate quantities for representing the situation.

**MGSE9–12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers’ precision is limited to the precision of the data given.*

### **Interpret the structure of expressions**

*Limit to linear expressions and to exponential expressions with integer exponents.*

**MGSE9–12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.

**MGSE9–12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients, in context.

**MGSE9–12.A.SSE.1b** Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.

**Create equations that describe numbers or relationships**

**MGSE9–12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, ~~quadratic, simple rational~~, and exponential functions (integer inputs only).

**MGSE9–12.A.CED.2** Create linear, ~~quadratic~~, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which  $A = P(1 + r/n)^{nt}$  has multiple variables.)

**MGSE9–12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

**MGSE9–12.A.CED.4** Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. *Examples: Rearrange Ohm’s law  $V = IR$  to highlight resistance  $R$ ; Rearrange area of a circle formula  $A = \pi r^2$  to highlight the radius  $r$ .*

**STANDARDS FOR MATHEMATICAL PRACTICE**

Refer to the Comprehensive Course Overview for more detailed information about the Standards for Mathematical Practice.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## **ENDURING UNDERSTANDINGS**

- Identify the vocabulary for the parts that make up the whole expression. Interpret their meaning in terms of a context.
- Solve word problems where quantities are given in different units that must be converted to understand the problem.
- Select appropriate units for a specific formula and interpret the meaning of the unit in that context.
- Create linear and exponential equations and inequalities in one variable and use them in a contextual situation to solve problems.
- Recognize that exponential functions can be used to model situations of growth, including the growth of an investment through compound interest.
- Create equations in two or more variables to represent relationships between quantities.
- Graph equations in two variables on a coordinate plane and label the axes and scales.
- Write and use a system of equations and/or inequalities to solve a real world problem.
- Solve multi-variable formulas or literal equations for a specific variable in a linear expression.

## **ESSENTIAL QUESTIONS**

- How do I choose and interpret units consistently in formulas?
- How do I interpret parts of an expression in terms of context?
- How do I create equations and inequalities in one variable and use them to solve problems arising from linear and exponential functions?
- How can I write, interpret and manipulate algebraic expressions, equations, and inequalities?
- How do I create equations in two or more variables to represent relationships between quantities?
- How do I graph equations on coordinate axes with the correct labels and scales?

- How can I rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations?
- How can I model constraints using mathematical notation?

## **CONCEPTS AND SKILLS TO MAINTAIN**

Students may not realize the importance of unit conversion in conjunction with computation when solving problems involving measurement. Since today’s calculating devices often display 8 to 10 decimal places, students frequently express answers to a much greater degree of precision than is required.

1 – 1

Measuring commonly used objects and choosing proper units for measurement are part of the mathematics curriculum prior to high school. In high school, students experience a broader variety of units through real–world situations and modeling, along with the exploration of the different levels of accuracy and precision of the answers.

An introduction to the use of variable expressions and their meaning, as well as the use of variables and expressions in real–life situations, is included in the Expressions and Equations Domain of Grade 7.

Working with expressions and equations, including formulas, is an integral part of the curriculum in Grades 7 and 8. In high school, students explore in more depth the use of equations and inequalities to model real–world problems, including restricting domains and ranges to fit the problem’s context, as well as rewriting formulas for a variable of interest.

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre–assess to determine whether instructional time should be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Using the Pythagorean Theorem
- Understanding slope as a rate of change of one quantity in relation to another quantity
- Interpreting a graph
- Creating a table of values
- Working with functions
- Writing a linear equation
- Using inverse operations to isolate variables and solve equations
- Maintaining order of operations
- Understanding notation for inequalities

- Being able to read and write inequality symbols
- Graphing equations and inequalities on the coordinate plane
- Understanding and using properties of exponents
- Graphing points
- Choosing appropriate scales and labeling a graph

## **SELECTED TERMS AND SYMBOLS**

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

**The definitions below are for teacher reference only and are not to be memorized by the students.** Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for high school children. **Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**

<http://www.amathsdictionaryforkids.com/>

This web site has activities to help students more fully understand and retain new vocabulary.

<http://intermath.coe.uga.edu/dictionary/homepg.asp>

Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

- **Algebra:** The branch of mathematics that deals with relationships between numbers, utilizing letters and other symbols to represent specific sets of numbers, or to describe a pattern of relationships between numbers.
- **Coefficient:** A number multiplied by a variable.
- **Domain:** The set of  $x$ -coordinates of the set of points on a graph; the set of  $x$ -coordinates of a given set of ordered pairs. The value that is the input in a function or relation.
- **Equation:** A number sentence that contains an equals symbol.



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- **Expression:** A mathematical phrase involving at least one variable and sometimes numbers and operation symbols.
- **Function:** A rule of matching elements of two sets of numbers in which an input value from the first set has only one output value in the second set.
- **Inequality:** Any mathematical sentence that contains the symbols  $>$  (greater than),  $<$  (less than),  $\leq$  (less than or equal to), or  $\geq$  (greater than or equal to).
- **Ordered Pair:** A pair of numbers,  $(x, y)$ , that indicate the position of a point on a Cartesian plane.
- **Perimeter:** The sum of the lengths of the sides of a polygon.
- **Pythagorean Theorem:** It is a theorem that states a relationship that exists in any right triangle. If the lengths of the legs in the right triangle are  $a$  and  $b$  and the length of the hypotenuse is  $c$ , we can write the theorem as the following equation:  $a^2 + b^2 = c^2$
- **Range:** The  $y$ -coordinates of the set of points on a graph. Also, the  $y$ -coordinates of a given set of ordered pairs. The range is the output in a function or a relation.
- **Substitution:** To replace one element of a mathematical equation or expression with another.
- **Variable:** A letter or symbol used to represent a number.

## **EVIDENCE OF LEARNING**

By the conclusion of this unit, students should be able to demonstrate the following competencies:

- create linear and exponential equations and inequalities in one variable and use them to solve problems.
- Create equations in two or more variables to represent relationships.
- Represent constraints by equations or inequalities, and by systems of equations and/or inequalities.
- Interpret solutions in modeling context.
- Rearrange linear formulas to highlight quantities of interest.

## **TEACHER RESOURCES**

The following pages include teacher resources that teachers may wish to use to supplement instruction.

### **Web Resources**

The following list is provided as a sample of available resources and is for informational purposes only. It is your responsibility to investigate them to determine their value and appropriateness for your district. GaDOE does not endorse or recommend the purchase of or use of any particular resource.

- [Scale of Universe](http://scaleofuniverse.com/)  
<http://scaleofuniverse.com/>  
Fantastic applet exploring the magnitudes of the universe.
- [Dimensional Analysis](http://www.alysion.org/dimensional/fun.htm)  
<http://www.alysion.org/dimensional/fun.htm>  
Helpful notes and examples on dimensional analysis
- [Algebraic Expressions](http://www.teachingchannel.org/videos/high-school-algebra-lesson?fd=1)  
<http://www.teachingchannel.org/videos/high-school-algebra-lesson?fd=1>  
Teacher Resource: Video of a lesson showing implementation of a FAL activity.
- [NASA Task](http://www.nasa.gov/audience/foreducators/exploringmath/algebra1/Prob_SuitYourself_detail.html)  
[http://www.nasa.gov/audience/foreducators/exploringmath/algebra1/Prob\\_SuitYourself\\_detail.html](http://www.nasa.gov/audience/foreducators/exploringmath/algebra1/Prob_SuitYourself_detail.html)  
In-depth lesson plan relating linear equations to space suit production.

## **FORMATIVE ASSESSMENT LESSONS (FAL)**

**Formative Assessment Lessons** are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Overview.

## **SPOTLIGHT TASKS**

A Spotlight Task has been added to each GSE mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Georgia Standards of Excellence, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky.

## **3-ACT TASKS**

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Overview.

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**TASKS**

The following tasks represent the level of depth, rigor, and complexity expected of all Coordinate Algebra students. These tasks, or tasks of similar depth and rigor, should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as a performance task, they may also be used for teaching and learning (learning/scaffolding task).

<b>Task Name</b> <i>Suggested Time</i>	<b>Task Type</b> <i>Grouping Strategy</i>	<b>Content Addressed</b>	<b>Standards</b>
Acting Out 20–30 min <i>(The second part of this task could be replaced with Leaky Faucet Task Below)</i>	Scaffolding Task <i>Individual / Partner</i>	<ul style="list-style-type: none"> <li>• Model and write an equation in one variable</li> <li>• Represent constraints with inequalities</li> </ul>	<ul style="list-style-type: none"> <li>• A.CED.1</li> <li>• A.CED.3</li> <li>• N.Q.1</li> <li>• N.Q.2</li> <li>• N.Q.3</li> </ul>
Leaky Faucet (Spotlight Task) 30–45 min <i>(This task could be used in place of the second part of Acting Out)</i>	Constructing Task <i>Individual / Partner</i>	<ul style="list-style-type: none"> <li>• Use units as a way to understand problems</li> <li>• Define appropriate quantities for the purpose of descriptive modeling.</li> <li>• Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.</li> </ul>	<ul style="list-style-type: none"> <li>• N.Q.1</li> <li>• N.Q.2</li> <li>• N.Q.3</li> </ul>
Yogurt Packaging 45–60 minutes <a href="#">PDF</a> / <a href="#">Word</a>	Achieve CCSS–CTE Classroom Task	<ul style="list-style-type: none"> <li>• Convert units in order to solve problems.</li> </ul>	<ul style="list-style-type: none"> <li>• N.Q.1</li> <li>• N.Q.3</li> </ul>
Dairy Barn 60 minutes <a href="#">PDF</a> / <a href="#">Word</a>	Achieve CCSS–CTE Classroom Task	<ul style="list-style-type: none"> <li>• Using a scale model and unit analysis to solve a problem in context.</li> </ul>	<ul style="list-style-type: none"> <li>• A.CED.2</li> <li>• N.Q.1</li> <li>• N.Q.3</li> </ul>
World Record Airbag (Spotlight Task) 30–45 min	Constructing Task <i>Individual / Partner</i>	<ul style="list-style-type: none"> <li>• Use units as a way to understand problems</li> <li>• Define appropriate quantities for the purpose of descriptive modeling.</li> <li>• Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.</li> </ul>	<ul style="list-style-type: none"> <li>• N.Q.1</li> <li>• N.Q.2</li> <li>• N.Q.3</li> </ul>
Framing a House 45–60 minutes <a href="#">PDF</a> / <a href="#">Word</a>	Achieve CCSS–CTE Classroom Task	<ul style="list-style-type: none"> <li>• Making a scale model and make conversions.</li> </ul>	<ul style="list-style-type: none"> <li>• N.Q.3</li> </ul>

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<b>Task Name</b> <i>Suggested Time</i>	<b>Task Type</b> <i>Grouping Strategy</i>	<b>Content Addressed</b>	<b>Standards</b>
Fences 45–60 minutes <a href="#">PDF</a> / <a href="#">Word</a>	Achieve CCSS– CTE Classroom Task	<ul style="list-style-type: none"> <li>• Choosing appropriate measures and making a scale model.</li> </ul>	<ul style="list-style-type: none"> <li>• N.Q.3</li> </ul>
Corn & Oats 60 minutes <a href="#">PDF</a> / <a href="#">Word</a>	Achieve CCSS– CTE Classroom Task	<ul style="list-style-type: none"> <li>• Making a scale model and make conversions.</li> </ul>	<ul style="list-style-type: none"> <li>• N.Q.1</li> <li>• N.Q.2</li> <li>• N.Q.3</li> </ul>
Lucy’s Linear Equations and Inequalities 30–45 min	Practice Task <i>Individual / Partner</i>	<ul style="list-style-type: none"> <li>• Write linear equations and inequalities in one variable and solve problems in context</li> </ul>	<ul style="list-style-type: none"> <li>• A.CED.1</li> <li>• A.SSE.1</li> </ul>
Forget the Formula 45 min	Scaffolding Task <i>Individual / Partner</i>	<ul style="list-style-type: none"> <li>• Creating equations in two variables to represent relationships</li> <li>• Represent constraints</li> <li>• Rearrange formulas to highlight a quantity of interest</li> </ul>	<ul style="list-style-type: none"> <li>• A.CED.2</li> <li>• A.CED.3</li> <li>• A.CED.4</li> </ul>
Cara’s Candles Revisited 45 min	Scaffolding Task <i>Individual / Partner</i>	<ul style="list-style-type: none"> <li>• Modeling equation to represent relationships</li> <li>• Determining constraints</li> </ul>	<ul style="list-style-type: none"> <li>• A.CED.1</li> <li>• A.CED.3</li> </ul>
The Shoe Problem 45 min	Constructing Task <i>Partner / Small Group</i>	<ul style="list-style-type: none"> <li>• Modeling linear patterns</li> <li>• Creating equation in one and two variables to represent relationships</li> </ul>	<ul style="list-style-type: none"> <li>• A.CED.1</li> <li>• A.CED.2</li> <li>• N.Q.3</li> </ul>
The Largest Loser	Formative Assessment Lesson <i>Individual / Small Group</i>	<ul style="list-style-type: none"> <li>• Understand Constraints upon graphs in given contexts and make sense of graph problems with differently–defined axes of measure.</li> </ul>	<ul style="list-style-type: none"> <li>• N.Q.1</li> <li>• N.Q.2</li> <li>• N.Q.3</li> <li>• A.CED.2</li> </ul>
Paper Folding 45 min	Constructing Task <i>Partner / Small Group</i>	<ul style="list-style-type: none"> <li>• Modeling with exponential functions</li> </ul>	<ul style="list-style-type: none"> <li>• A.CED.2</li> <li>• A.CED.3</li> <li>• A.SSE.1</li> </ul>
Growing by Leaps and Bounds <i>Parts I and II; 45 min</i> <i>Part III: 90 min</i>	Culminating Task <i>Partner / Small Group</i>	<ul style="list-style-type: none"> <li>• Graph equations on coordinate axes with labels and scales</li> <li>• Interpret expressions that represent a quantity in terms of its context</li> <li>• Determining constraints</li> <li>• Modeling with exponential functions</li> </ul>	<ul style="list-style-type: none"> <li>• All</li> </ul>

## **Acting Out (Scaffolding Task)**

### **Introduction**

In this task, students will use an inequality to find the distance between two homes. Students will also learn how to convert contextual information into mathematical notation. The second part has students determine how much water might a dripping faucet waste in a year.

### **Mathematical Goals**

- Model and write an equation in one variable and solve a problem in context.
- Create one-variable linear equations and inequalities from contextual situations.
- Represent constraints with inequalities.
- Solve word problems where quantities are given in different units that must be converted to understand the problem.

### **Essential Questions**

- How do I choose and interpret units consistently in formulas?
- How can I model constraints using mathematical notation?

### **Georgia Standards of Excellence**

**MGSE9–12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, ~~quadratic, simple rational~~, and exponential functions (integer inputs only).

**MGSE9–12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

**MGSE9–12.N.Q.1** Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

- a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
- b. Convert units and rates using dimensional analysis (English-to-English and Metric-to-Metric without conversion factor provided and between English and Metric with conversion factor);
- c. Use units within multi-step problems and formulas; interpret units of input and resulting units of output.

**MGSE9–12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling. Given a situation, context, or problem, students will determine, identify, and use appropriate quantities for representing the situation.

**MGSE9–12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers' precision is limited to the precision of the data given.*

### **Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.  
*Students will have to determine different ways of measuring a fixed distance from a given point and will have to strategize the most accurate way to convert measurements.*
5. Use appropriate tools strategically.  
*Students will be given multiple tools to choose from to model the scenario in Part I.*
6. Attend to precision.  
*Students will have to know when to round their answers based on the units of measurement.*

### **Background Knowledge**

- Students can graph relationships.
- Students can write and interpret inequalities.
- Students can graph inequalities on a number line.
- Students can convert units (time and volume).

### **Common Misconceptions**

- Students may only think of vertical and horizontal distances and not in terms of a radius.
- Students may naturally progress from days to weeks to months to years, rather than directly from days to years.

### **Materials**

- colored pencils
- compass
- string
- graph paper

### **Grouping**

- Part I: Small group / whole group
- Part II: Partner / Individual

### **Differentiation**

#### **Extension:**

- Ask additional conversion questions: How old are you in minutes? How many inches long is a football field? If you are 1,000,000 seconds old, how old are you in years?

#### **Intervention:**

- Provide a conversion list.
- Review inequality symbols.

### **Formative Assessment Questions**

- How are you choosing the units used throughout the problem?
- How do the units drive your conversions?
- How can we demonstrate limits algebraically?

**Acting Out – Teacher Notes**

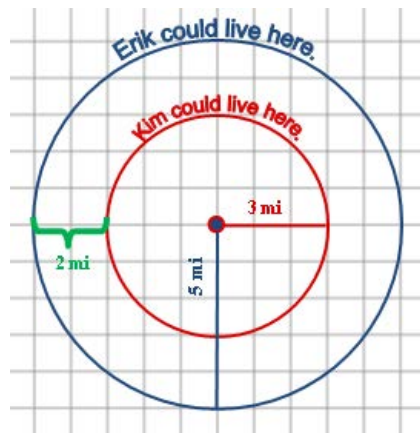
**Part I:**

Erik and Kim are actors at a theater. Erik lives 5 miles from the theater and Kim lives 3 miles from the theater. Their boss, the director, wonders how far apart the actors live.

Comments

*Students should understand that Erik and Kim could live anywhere on the circle with the theater as the center and the radius as the distance that they live from the theater.*

1. On the given grid:
  - a. pick a point to represent the location of the theater.
  - b. Illustrate all of the possible places that Erik could live on the grid paper.
  - c. Using a different color, illustrate all of the possible places that Kim could live on the grid paper.



2. What is the smallest distance,  $d$ , that could separate their homes? How did you know?

Solution

$5 - 3 = 2 \text{ miles}$

3. What is the largest distance,  $d$ , that could separate their homes? How did you know?

Solution

$5 + 3 = 8 \text{ miles}$

4. Write and graph an inequality in terms of  $d$  to show their boss all of the possible distances that could separate the homes of the 2 actors.

Solution

*An inequality that could represent this distance could be  $2 \leq d \leq 8 \text{ miles}$ .*

*Graphing this inequality should look like the graph shown below.*





*Students should understand that the solid dots on the graph represent the fact that Erik and Kim could live exactly 2 miles or exactly 8 miles apart. Should the situation have been different and they lived more than 2 miles or less than 8 miles apart, those dots would have been left open, or not filled in. The space shaded on the number line between the 2 and 8 means that they could live any of those distances apart.*

**Part II:**

The actors are good friends since they live close to each other. Kim has a leaky faucet in her kitchen and asks Erik to come over and take a look at it.

1. Kim estimates that the faucet in her kitchen drips at a rate of 1 drop every 2 seconds. Erik wants to know how many times the faucet drips in a week. Help Erik by showing your calculations below.

*Solution:*

*60 sec = 1 min*  
*60 min = 1 hour*  
*24 hours = 1 day*  
*7 days = 1 week*

$$(60)(60)(24)(7) = 604800$$
$$604800/2 = 302,400 \text{ drops}$$

$$365 \text{ days} = 1 \text{ year}$$

$$\frac{(60)(60)(24)(365)}{2} = 15,768,000 \text{ drops per year}$$

2. Kim estimates that approximately 575 drops fill a 100 milliliter bottle. Estimate how much water her leaky faucet wastes in a year.

*Solution:*

$$15,768,000/575 = 27,422.608$$

*About 27,423 of 100 millimeter bottles would be filled.*  
*Encourage students to write this answer in the most user-friendly measurement i.e. liters*

$$27,423(100) = 2,742,300 \text{ milliliters or } 2,742.3 \text{ m}$$

## **Scaffolding Task: Acting Out**

Name \_\_\_\_\_

Date \_\_\_\_\_

*Adapted from Shell Center Leaky Faucet Short Cycle Task*

<http://www.map.mathshell.org/materials/tasks.php?taskid=267&subpage=apprentice>

### **Mathematical Goals**

- Model and write an equation in one variable and solve a problem in context.
- Create one–variable linear equations and inequalities from contextual situations.
- Represent constraints with inequalities.
- Solve word problems where quantities are given in different units that must be converted to understand the problem.

### **Essential Questions**

- How do I choose and interpret units consistently in formulas?
- How can I model constraints using mathematical notation?

### **Georgia Standards of Excellence**

**MGSE9–12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, ~~quadratic, simple rational,~~ and exponential functions (integer inputs only).

**MGSE9–12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non–solution) under the established constraints.

**MGSE9–12.N.Q.1** Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

- a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
- b. Convert units and rates using dimensional analysis (English–to–English and Metric–to–Metric without conversion factor provided and between English and Metric with conversion factor);
- c. Use units within multi–step problems and formulas; interpret units of input and resulting units of output.

**MGSE9–12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling. Given a situation, context, or problem, students will determine, identify, and use appropriate quantities for representing the situation.

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**MGSE9–12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers' precision is limited to the precision of the data given.*

**Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.
5. Use appropriate tools strategically.
6. Attend to precision.

**Scaffolding Task: Acting Out**

Name \_\_\_\_\_

Date \_\_\_\_\_

*Adapted from Shell Center Leaky Faucet Short Cycle Task*

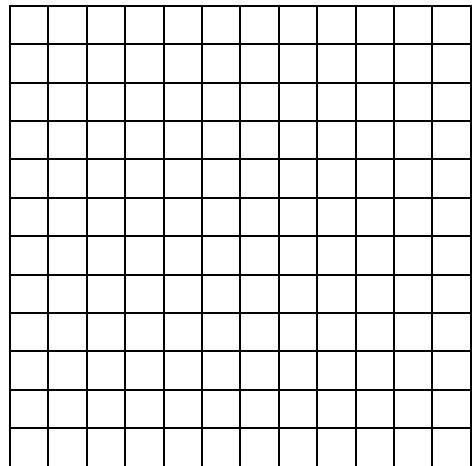
<http://www.map.mathshell.org/materials/tasks.php?taskid=267&subpage=apprentice>

**Part I:**

Erik and Kim are actors at a theater. Erik lives 5 miles from the theater and Kim lives 3 miles from the theater. Their boss, the director, wonders how far apart the actors live.

1. On the given grid:

- a. pick a point to represent the location of the theater.
- b. Illustrate all of the possible places that Erik could live on the grid paper.
- c. Using a different color, illustrate all of the possible places that Kim could live on the grid paper.



- 2. What is the smallest distance,  $d$ , that could separate their homes? How did you know?
- 3. What is the largest distance,  $d$ , that could separate their homes? How did you know?
- 4. Write and graph an inequality in terms of  $d$  to show their boss all of the possible distances that could separate the homes of the 2 actors.



### **TASK: Leaky Faucet (spotlight task)**

Task adapted from [Dan Meyer 3 Acts Math](http://threeacts.mrmeyer.com/leakyfaucet/) <http://threeacts.mrmeyer.com/leakyfaucet/> and GSE Coordinate Algebra Unit 1 task “Acting Out” (*second part of Acting Out could be replaced with this Spotlight task*)

#### **Georgia Standards of Excellence**

**MGSE9–12.N.Q.1** Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

- Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
- Convert units and rates using dimensional analysis (English-to-English and Metric-to-Metric without conversion factor provided and between English and Metric with conversion factor);
- Use units within multi-step problems and formulas; interpret units of input and resulting units of output.

**MGSE9–12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling. Given a situation, context, or problem, students will determine, identify, and use appropriate quantities for representing the situation.

**MGSE9–12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers’ precision is limited to the precision of the data given.*

#### **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Make sense of problems and persevere in solving them. *In this task, students will have to interpret the given situation to determine what information is needed to solve the problem. It is possible, and even desirable, to have students decide on the problem this video is posing. Students should be allowed to think independently then work in small groups to decide what problem is being presented.*
5. Use appropriate tools strategically. *Students should decide what “tools” are needed to solve the problem they posed based on the video. What information is needed and what tools will allow them to access that information.*
6. Attend to precision. *Students will have to decide the level of precision needed to answer the question(s) posed after watching and discussing the video clip.*

#### **ESSENTIAL QUESTIONS**

- How do I choose and interpret units consistently in solving application problems?
- How can I model constraints using mathematical notation?

## **MATERIALS REQUIRED**

- Video clip “leaky faucet” from 3 Acts Math
- Timing device (for first estimate; Act 2 provides exact amounts)
- Conversion factors for quantities mentioned in the problems posed after the video (*Act 2 will supply needed information but students could investigate independently before getting the information*)
- The facts about the dripping water in the video and the sink capacity, etc. *NOTE: the needed facts may vary based on the questions posed by students.*

## **TIME NEEDED**

- 30–45 minutes based on the depth of investigation

## **TEACHER NOTES**

*In this task, students will watch the video then discuss what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don’t have the information they need, and ask for it, it will be given to them*

*More information along with guidelines for 3–Act Tasks may be found in the Comprehensive Course Guide.*

## **Task Description**

Kim has a leaky faucet and asks Erik to come over and take a look at it.

### **ACT 1:**

Watch the video: <http://threeacts.mrmeyer.com/leakyfaucet/act1/act1.mov>

Think and wonder: What do you notice? What do you want to know after watching the video?

How can you come up with answers to your questions?

*Guiding questions to consider if the students don’t come up with them on their own might be:*

- 1. How long will it take the sink to fill up?
- 2. Write down a guess.
- 3. Write down an answer you know is too high. Too low.

### **ACT 2:**

What information would be useful to know here?

The links below are from Dan Meyer Leaky Faucet 3 Act Math site

- [video — drops per second](#)
- [video — ml per second](#)
- [image — the capacity of the sink](#)
- [image — the cost of water](#)

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**ACT 3**

Students will compare and share solution strategies.

- Reveal the answer. Discuss the theoretical math versus the practical outcome.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.
- [video — the sink fills up](#) Reveal the Actual Solution

**ACT 4 A Sequel Option:**

Describe two scenarios where a leaky faucet would take a week to fill something up.

**Extension:**

Suppose the sink is not plugged and the water leaks for a week before it is noticed. How much water would have leaked? How much would it cost?



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Task Title: \_\_\_\_\_

Name: \_\_\_\_\_

*Adapted from Andrew Stadel*

**ACT 1**

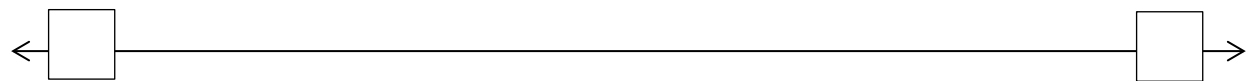
What did/do you notice?

What questions come to your mind?

**Main Question:** \_\_\_\_\_

Estimate the result of the main question? Explain?

*Place an estimate that is too high and too low on the number line*



Low estimate

*Place an “x” where your estimate belongs*

High estimate

**ACT 2**

What information would you like to know or do you need to solve the MAIN question?

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Record the given information (measurements, materials, etc...)

If possible, give a better estimate using this information: \_\_\_\_\_

**Act 2 (con't)**

Use this area for your work, tables, calculations, sketches, and final solution.

**ACT 3**

What was the result?

Which Standards for Mathematical Practice did you use?	
<input type="checkbox"/> Make sense of problems & persevere in solving them	<input type="checkbox"/> Use appropriate tools strategically.
<input type="checkbox"/> Reason abstractly & quantitatively	<input type="checkbox"/> Attend to precision.
<input type="checkbox"/> Construct viable arguments & critique the reasoning of others.	<input type="checkbox"/> Look for and make use of structure.
<input type="checkbox"/> Model with mathematics.	<input type="checkbox"/> Look for and express regularity in repeated reasoning.

## **Yogurt Packaging (Career and Technical Education Task)**

*Source: National Association of State Directors of Career Technical Education Consortium*

PDF: <http://www.achieve.org/files/CCSS-CTE-Yogurt-Packaging-FINAL.pdf>

Word: <http://www.achieve.org/files/CCSS-CTE-Yogurt-Packaging-FINAL.docx>

### **Introduction**

Students use unit analysis to answer questions in the context of yogurt production.

### **Mathematical Goals**

- Use unit analysis to answer questions.
- Students will use percent increase and decrease.
- Students will turn amounts (grams and fl. oz.) to unit rates (grams per fl. oz.)

### **Essential Questions**

- How can I use unit analysis to answer questions in context?

### **Georgia Standards of Excellence**

**MGSE9–12.N.Q.1** Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

- a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
- b. Convert units and rates using dimensional analysis (English-to-English and Metric-to-Metric without conversion factor provided and between English and Metric with conversion factor);
- c. Use units within multi-step problems and formulas; interpret units of input and resulting units of output.

**MGSE9–12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling. Given a situation, context, or problem, students will determine, identify, and use appropriate quantities for representing the situation.

**MGSE9–12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers' precision is limited to the precision of the data given.*

### **Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.  
*Students must make conjectures about the form and meaning of the solution pathway. The task requires multi-step problem solving.*
2. Reason abstractly and quantitatively.  
*Students make sense of quantities and their relationships in the problem situation.*

**6. Attend to precision.**

*Students need to attend to units as they perform calculations. Rounding and estimation are a key part*

**Background Knowledge**

- Students can work with percentages, including percent increase and percent decrease.
- Students can apply unit analysis to answer questions.

**Common Misconceptions**

- When converting units, students often divide when they should multiply (or vice-versa). Writing units throughout the problem, rather than only in the answer, can help with this issue, as students can ensure that units “cancel” appropriately.
- For percent increase and decrease, students may forget to add to 1 or subtract from 1. Remind students that they are multiplying by a fraction that compares the *entire* new amount to the *original* amount.

**Materials**

- For the first extension, students need paper with which to construct a yogurt tub.

**Grouping**

- Individual / partner

**Differentiation**

- See extensions in task.

## **Dairy Barn (Career and Technical Education Task)**

*Source: National Association of State Directors of Career Technical Education Consortium*

PDF: <http://www.achieve.org/files/CCSS-CTE-Dairy-Barn-FINAL.pdf>

Word: <http://www.achieve.org/files/CCSS-CTE-Dairy-Barn-FINAL.docx>

### **Introduction**

Students find the volume of sand needed to fill an area to a specified depth, then use unit analysis and given constraints to determine the cheaper provider of sand.

### **Mathematical Goals**

- Calculate area of polygons and volume of prisms.
- Use unit analysis and proportional reasoning to determine cost.

### **Essential Questions**

- How can I use units and cost analysis to help me choose the better supplier?

### **Georgia Standards of Excellence**

**MGSE9–12.A.CED.2** Create linear and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which  $A = P(1 + r/n)^{nt}$  has multiple variables.)

**MGSE9–12.N.Q.1** Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

- a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
- b. Convert units and rates using dimensional analysis (English-to-English and Metric-to-Metric without conversion factor provided and between English and Metric with conversion factor);
- c. Use units within multi-step problems and formulas; interpret units of input and resulting units of output.

**MGSE9–12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers’ precision is limited to the precision of the data given.*

### **Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.  
*Students analyze givens, constraints, relationships, and goals. They must make conjectures about the form and meaning of the solution and plan a solution pathway.*

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2. Reason abstractly and quantitatively.  
*Students make sense of quantities and their relationships in the problem situation. Questions 3 and 4 ask them to write equations that represent a given situation and then solve those equations.*
4. Model with mathematics.  
*Students translate constraints into equations and extract information from a geometric model (diagram).*
6. Attend to precision.  
*Rounding and estimation are a key part of the thinking that students must use to accurately answer the questions. Students express answers with a degree of precision appropriate for the problem.*

**Background Knowledge**

- Students can use unit analysis to solve multi-step problems.
- Students can convert three-dimensional units (e.g.,  $\text{ft}^3$  to  $\text{yd}^3$ )
- Students can find the area of a geometric figure using dissection strategies.

**Common Misconceptions**

- In #3, students may forget that *each trip* (i.e., each truckload) adds to delivery costs.

**Materials**

- None

**Grouping**

- Partner / small group

**Differentiation**

**Extension:**

- What changes could be made to Supplier A's constraints to make them the cheaper provider? Be specific, and justify your answer.  
*(Possible solution: If Supplier A charged less per cubic yard—under \$2.43—or if they charged less per mile—under \$3.33 per mile—they would be cheaper than Supplier B.)*

**Intervention:**

- Breaking down calculations (especially in #3 and #4) into separate pieces can help students approach these multi-step processes. In #3, find the costs of sand and delivery separately, then add these costs together. Or find the cost of a single trip and multiply this by the number of trips.

## **World Record Airbag Diving (Spotlight Task)**

*This spotlight task is based on the work featured*

*at <https://docs.google.com/spreadsheets/cc?key=0AjIqyKM9d7ZYdEhtR3BJMmdBWnM2YWxWYVM1UWowTEE#gid=0> and follows the 3 Act–Math task format originally developed by Dan Meyer. More information on these type tasks may be found at <http://blog.mrmeyer.com/category/3acts/>*

### **Georgia Standards of Excellence**

**MGSE9–12.N.Q.1** Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

- Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
- Convert units and rates using dimensional analysis (English–to–English and Metric–to–Metric without conversion factor provided and between English and Metric with conversion factor);
- Use units within multi–step problems and formulas; interpret units of input and resulting units of output.

*This standard is addressed as students have to decide on an appropriate formula to solve the freefall problem. Students will have to use units to guide their calculations making necessary conversions in the process. As students are asked to graph the diver’s height (position) versus time during the fall and to graph the diver’s velocity versus time, they must consider appropriate scales for these graphs.*

**MGSE9–12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling. *This standard is addressed as students have to decide on appropriate quantities to use to model the skydiving situation.*

**MGSE9–12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers’ precision is limited to the precision of the data given. This standard is addressed as students have to decide on an appropriate level of accuracy for reporting their results. Discussion should include steps in their calculations where rounding might have occurred and how that affected their results. Encourage students to examine the concept of accuracy in the context of this problem.*

### **STANDARDS FOR MATHEMATICAL PRACTICE**

**1.** Make sense of problems and persevere in solving them. *In this task, students will have to interpret the given situation to determine what information is needed to solve the problem. It is possible, and even desirable, to have students decide on the problem this video is posing and other questions that interest them in the context of the given information. Students should be allowed to think independently then work in small groups to decide what problem is being presented.*

5. Use appropriate tools strategically. *Students should decide what “tools” are needed to solve the problem they posed based on the video. Students must ask for or find (via technology resources) information that is needed and what tools will allow them to access that information.*

6. Attend to precision. *Students will have to decide the level of precision needed to answer the question(s) posed after watching and discussing the video clip. This task also opens the door for a discussion of air friction or other factors that might influence the final result. Teachers should welcome this discussion in the context of class and encourage students to investigate other related factors outside of class.*

### ESSENTIAL QUESTIONS

- How do I choose and interpret units consistently in solving application problems?
- What are the constraints of this situation and how can I model them using mathematical notation?
- What factors are important when trying to solve a free fall problem?

### MATERIALS REQUIRED

- Video clip “world record airbag” from 3 Acts Math
- Conversion factors for quantities mentioned in the problems posed after the video (Act 2 will supply needed information but students could investigate independently before getting the information)
- The formulas for calculating an object’s position and velocity relative to time. (Supplied in Act 2 unless students have technology resources)
- The facts about the airbag dive in the video NOTE: the needed facts may vary based on the questions posed by students.
- Graph paper for The Sequel

### TIME NEEDED

- 30–45 minutes based on the depth of investigation

*More information along with guidelines for 3–Act Tasks may be found in the Comprehensive Course Guide.*

**Act One Description (Dan Meyer <http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/>)** “Introduce the central conflict of your story/task clearly, visually, viscerally, using as few words as possible.”

Act One: Show the video of the Guinness World Record for airbag diving. Note: this is a zipped file in its original source at <https://s3.amazonaws.com/threeacts/worldrecordairbag.zip>

*During Act One students will watch the video (several times in most cases) and think independently and in small groups about what questions arise from the video. Based on the filming, all students will most likely want to know how fast he is traveling when he hits the air bag. Welcome and invite other questions.*



Think and wonder: What do you notice? What do you want to know after watching the video?

How can you come up with answers to your questions?

Guiding steps to consider if the students don't come up with them on their own might be:

1. How fast was the skydiver traveling when he made impact with the airbag? (This might open up the conversation to the concept of speed versus velocity.)
2. Write down a guess.
3. Write down an answer you know is too high. Too low.

**Act Two Description (Dan Meyer <http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/>)** The protagonist/student overcomes obstacles, looks for resources, and develops new tools.

*During Act Two students will discuss the question in Act One and decide on the facts that are needed to answer the question. Students will also look for formulas and conversions that are needed to solve the problem. When students decide what they need to solve the problem, they should ask for the facts or use technology to find them.*

*Note: It is pivotal to the problem solving process that students decided what is needed without being given the information up front. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin.*

Suggested Scaffolding Questions:

1. What is the problem you are trying to solve?
2. What do you think affects the situation?
3. Does it matter how high the skydiver is? What matters?

Act Two facts to be supplied upon request by students:

*Note: It is suggested that teachers give all of the information below upon request for the formula to calculate speed/velocity at impact. It is very important to call attention the units used in the formulas/information below. Student will also need to make conversions based on the way the information was given in the video. Supply the conversion listed below the fact box upon request.*

*By giving all of this information, students must filter through related but non-essential facts to find the formula of interest. This filtering skill is crucial in the problem solving process and offers teachers another opportunity to scaffold the task for all levels of learners.*

**Free fall / falling speed equations** Source <http://www.angio.net/personal/climb/speed>  
*The calculator (available from the website link above) uses the standard formula from Newtonian physics to figure out how long before the falling object goes splat:*

- The force of gravity,  $g = 9.8 \text{ m/s}^2$   
Gravity accelerates you at 9.8 meters per second *per second*. After one second, you're falling 9.8 m/s. After two seconds, you're falling 19.6 m/s, and so on.
- Time to splat:  $\sqrt{(2 * \text{height} / 9.8)}$   
It's the square root because you fall faster the longer you fall.  
The more interesting question is why it's times two: If you accelerate for 1 second, your *average* speed over that time is increased by only  $9.8 / 2 \text{ m/s}$ .
- Velocity at splat time:  $\sqrt{(2 * g * \text{height})}$   
This is why falling from a higher height hurts more.
- Energy at splat time:  $1/2 * \text{mass} * \text{velocity}^2 = \text{mass} * g * \text{height}$

*For Students who ask about customary as opposed to metric measure:  $g = 32 \text{ ft/sec}^2$  Students should be able to calculate that there are 3600 sec in one hour when making their conversion to miles per hour.*

Conversion Factor:

Source <https://support.google.com/websearch/answer/3284611?hl=en#unitconverter>

The image shows three separate unit conversion calculator interfaces. Each interface has a dropdown menu for the unit being converted from, a text input field with the number '1', an equals sign, a text output field with the conversion factor, and a dropdown menu for the unit being converted to.

- Calculator 1: From Length (Foot) to Meter. Input: 1. Output: 0.3048.
- Calculator 2: From Length (Meter) to Mile. Input: 1. Output: 0.000621371.
- Calculator 3: From Length (Mile) to Foot. Input: 1. Output: 5280.

### ACT 3

#### Students will compare and share solution strategies.

- Reveal the answer. Discuss the theoretical math versus the practical outcome. *In this case the calculation based on free fall will NOT match the actual speed of the sky diver due to the air resistance.*
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.
  
- Reveal the Actual Solution by showing the Act Three video from <https://s3.amazonaws.com/threeacts/worldrecordairbag.zip>

**The Sequel:** “The goals of the sequel task are to a) challenge students who finished quickly so b) I can help students who need my help. It can't feel like punishment for good work. It can't seem like drudgery. It has to entice and activate the imagination.” Dan

Meyer <http://blog.mrmeyer.com/2013/teaching-with-three-act-tasks-act-three-sequel/>

Challenge students to graphically represent the sky diver's position versus the time he is in the air.

Challenge students to graphically represent the sky diver's velocity versus the time he is in the air.

Challenge students to research how much air had to be in the air bag he landed on to keep him safe.

Challenge students to discuss and research ways the sky diver could increase his velocity...or decrease his velocity.

Challenge students to discuss and investigate other parts of this situation that interest them.

**Information about graphing position versus time and velocity versus time for objects in free fall may be found at <http://www.physicsclassroom.com/class/1DKin/Lesson-5/Representing-Free-Fall-by-Graphs>**

*Teacher's notes and suggestions: make notes for yourself or to share with your colleagues about this 3 Act Task. What went well? What would you do differently? Were there things you did not think of in advance that came up during the unfolding of the task?*

Possible Solution: Please note that your students may come up with other questions. The solution below represents “typical” questions/answers. Welcome other questions and solutions for the class to discuss and validate.

**How fast was the skydiver going upon impact?**

*Using  $V = \sqrt{(2 * g * height)}$  with a height of 342 feet using  $g = 32 \text{ ft/sec}^2$  the velocity would be approximately 147.946 ft/sec. Students should then use unit conversion to arrive at approximately 100.87 miles/hr.*

*If the student used  $g = 9.8 \text{ m/sec}^2$  along with other conversions given under Act 2, the solution would be approximately 101.1 miles/hr.*

*The actual speed at impact was given as 90 miles/hr. This discrepancy opens the discussion up for sources of error or factors that influence the speed (such as air resistance).*

## **Framing a House (Career and Technical Education Task)**

Source: *National Association of State Directors of Career Technical Education Consortium*

PDF: <http://www.achieve.org/files/CCSS-CTE-Task-FramingHouse-FINAL.pdf>

Word: <http://www.achieve.org/files/CCSS-CTE-Task-FramingHouse-FINAL.docx>

### **Introduction**

This task uses the process of framing a house to help students change units, calculate cost of materials, and determine the amount of wasted material to frame parts of a house.

### **Mathematical Goals**

- Convert units.
- Calculate and use perimeter.
- Use percentages.

### **Essential Questions**

- How do we use appropriate units in real-life situations to determine actual lengths of geometric figures?

### **Georgia Standards of Excellence**

**MGSE9–12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answer's precision is limited to the precision of the data given.*

**MGSE9–12.G.MG.3** Apply geometric methods to solve design problems.

### **Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.  
*Students analyze givens, constraints, relationships, and goals. They must make conjectures about the form and meaning of the solution and plan a solution pathway.*
2. Reason abstractly and quantitatively.  
*Students must attend to the meaning of the quantities and pay attention to units as they represent and operate with the quantities and measures.*
5. Use appropriate tools strategically.  
*Students are asked to use an architecture scale (or ruler) for their scale drawing.*
6. Attend to precision.  
*Students are required to convert inches to feet and back again. Rounding is a key part of the measurement computations.*

### **Background Knowledge**

- Students can convert units (e.g., feet to inches)
- Students can work with percentages.
- Students can calculate perimeter of figures.

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- Students understand how to make a scaled drawing on graph paper.

**Common Misconceptions**

- Students may forget to change the percent to a decimal before multiplying to find waste.
- Remind students how to change from one unit to another and how to use appropriate units for the problem at hand.

**Materials**

- Graph paper
- Ruler

**Grouping**

- Individual / small group

**Differentiation**

- See extensions in task.

## **Fences (Career and Technical Education Task)**

*Source: National Association of State Directors of Career Technical Education Consortium*

PDF: <http://www.achieve.org/files/CCSS-CTE-Task-Fences-FINAL.pdf>

Word: <http://www.achieve.org/files/CCSS-CTE-Task-Fences-FINAL.docx>

### **Introduction**

Students design a fence, subject to various constraints, to surround a property. Students then determine and compare costs for supplies from two different lumber yards.

### **Mathematical Goals**

- Design and analyze a geometric figure subject to various constraints.
- Use units and proportional reasoning to determine subtotals and total cost for materials.

### **Essential Questions**

- How can I mathematically determine the less expensive materials supplier for building a fence that meets certain requirements?

### **Georgia Standards of Excellence**

**MGSE9–12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers' precision is limited to the precision of the data given.*

**MGSE9–12.G.MG.3** Apply geometric methods to solve design problems.

### **Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.  
*Students analyze givens, constraints, relationships, and goals. They must make conjectures about the form and meaning of the solution and plan a solution pathway.*
2. Reason abstractly and quantitatively.  
*This task requires that students make sense of quantities and their relationships in the problem situation.*
3. Construct viable arguments and critique the reasoning of others.  
*Students are required to justify their conclusions.*
4. Model with mathematics.  
*Students must create a mathematical representation (model) that can replace the situation described in the prompt.*
6. Attend to precision.  
*Students must communicate precisely, organizing their information, as they show their mathematical thinking. Students must also attend to appropriate level of precision in their calculations.*

**Background Knowledge**

- Students can make accurate scale drawings to represent problem situations.
- Students can use proportional reasoning / unit analysis to determine the required materials and related costs.

**Common Misconceptions**

- Understanding the construction of the fence itself may be challenging. The given diagram gives some information, but a physical model that includes posts, horizontal supports, and vertical cedar slats may be helpful.
- " means inches; ' means feet. Students need to pay close attention to these symbols to ensure they are using correct units in their calculations.

**Materials**

- Graph paper
- Ruler

**Grouping**

- Individual / partner

**Differentiation**

- See extensions in task.



## **Corn and Oats (Career and Technical Education Task)**

*Source: National Association of State Directors of Career Technical Education Consortium*

PDF: <http://www.achieve.org/files/CCSS-CTE-Corn%26Oats-FINAL.pdf>

Word: <http://www.achieve.org/files/CCSS-CTE-Corn%26Oats-FINAL.docx>

### **Introduction**

This task uses the process of planting corn and oats to help students convert units, calculate area, and work with percents to determine fertilization and planting needs for Producer Bob.

### **Mathematical Goals**

- Convert and use appropriate units.
- Find the area of a geometric figure.
- Solve problems using percents.

### **Essential Questions**

- How do I use appropriate units in real-life situations to determine area, fertilization, and planting needs?

### **Georgia Standards of Excellence**

**MGSE9–12.N.Q.1** Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

- a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
- b. Convert units and rates using dimensional analysis (English-to-English and Metric-to-Metric without conversion factor provided and between English and Metric with conversion factor);
- c. Use units within multi-step problems and formulas; interpret units of input and resulting units of output.

**MGSE9–12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.

**MGSE9–12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers' precision is limited to the precision of the data given.*

### **Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.  
*This task requires multi-step problem solving, sense making, and understanding of relationships.*
2. Reason abstractly and quantitatively.  
*This task requires a great deal of quantitative reasoning.*

4. Model with mathematics.  
*Students employ mathematics and interpret their results in the context of the situation.*
6. Attend to precision.  
*The quantitative demands of this task are high, and students need to pay careful attention to units and unit conversions. They need to calculate accurately and express numerical answers with a degree of precision appropriate for the problem context.*

### **Background Knowledge**

- Students can convert units.
- Students can work with percentages.
- Students can find the area of triangles and rectangles.
- Students can use proportional reasoning to solve problems.

### **Common Misconceptions**

- Students may forget how to convert from standard to metric units.
- Students must remember that all percents are out of 100 (“per cent”) when setting up a proportion.

### **Materials**

- None

### **Grouping**

- Small group

### **Differentiation**

#### **Intervention:**

- Many students are likely to need clarification on acreage and parceling land before beginning this task.

## **Lucy’s Linear Equations and Inequalities (Practice Task)**

### **Introduction**

In this task, students will solve a series of linear equations and inequality word problems that Lucy has been assigned by her teacher. In order to help Lucy, students must explain in detail each step of the problem. It is a good idea to review key words that are associated with linear inequality word problems before beginning this task.

Keywords: fewer than, more than, at most, at least, less than, no less than

### **Mathematical Goals**

- Create one–variable linear equations and inequalities from contextual situations.
- Solve and interpret the solution to multi–step linear equations and inequalities in context.

### **Essential Questions**

- How do I interpret parts of an expression in terms of context?
- How do I create equations and inequalities in one variable and use them to solve problems arising from linear functions?
- How can I write, interpret and manipulate algebraic expressions, equations and inequalities?

### **Georgia Standards of Excellence**

**MGSE9–12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions, and simple rational and~~ exponential functions (integer inputs only).

**MGSE9–12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.

**MGSE9–12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

**MGSE9–12.A.SSE.1b** Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.

### **Standards for Mathematical Practice**

2. Reason abstractly and quantitatively.
4. Model with mathematics.  
*Students will define variables and create expressions and equations with those variables.*
7. Look for and make use of structure.  
*Students will recognize patterns for modeling consecutive integers and expressing perimeters using algebraic expressions.*

**Background Knowledge**

- Students can find the perimeter of figures.
- Students can solve inequalities and equations.
- Students know some key words for operations and inequalities (see above).

**Common Misconceptions**

- Students may not know to double the length and width to calculate perimeter or they may use the area formula to calculate.
- Students may not recognize how to model “consecutive integers” or “consecutive even/odd integers”

**Materials**

- None

**Grouping**

- Individual / partners

**Differentiation**

**Extension:**

- Encourage “Resident Experts” to support struggling students.

**Intervention:**

- Supply students with key words.

**Formative Assessment Questions**

- How can we tell if a problem is an equation or an inequality?
- How do we solve equations differently than inequalities?

### **Lucy's Linear Equations and Inequalities – Teacher Notes**

Lucy has been assigned the following linear equations and inequality word problems. Help her solve each problem below by using a five step plan.

- Drawing a Sketch(if necessary)
- Defining a Variable
- Setting up an equation or inequality
- Solve the equation or inequality
- Make sure you answer the question

1. The sum of 38 and twice a number is 124. Find the number.

*Solution*

*Define a Variable:  $n = \text{the number}$*

*Equation:  $38 + 2n = 124$*

$$2n = 86$$

$$n = 43$$

*The number is 43.*

*Check:  $38 + 2(43) = 124$*

$$38 + 86 = 124$$

$$124 = 124 \checkmark$$

2. The sum of two consecutive integers is less than 83. Find the pair of integers with the greatest sum.

*Solution*

*Define a Variable:*

*$x = \text{the first consecutive number, so } x + 1 = \text{the second consecutive number}$*

*Equation:  $x + x + 1 < 83$*

$$2x < 82$$

$$x < 41$$

*The numbers are 40 and 41*

*Check:  $40 + 41 < 83$*

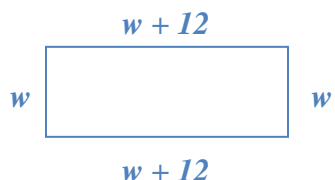
$$81 < 83 \checkmark$$

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3. A rectangle is 12m longer than it is wide. Its perimeter is 68m. Find its length and width.

**Solution**

*Sketch:*



*Define a Variable:*

*Width =  $w$*

*So length =  $w + 12$*

*Equation:  $w + w + w + 12 + w + 12 = 68$*

*$4w + 24 = 68$*

*$4w = 44$*

*$w = 11$*

*Width = 11*

*Length =  $11 + 12 = 23$*

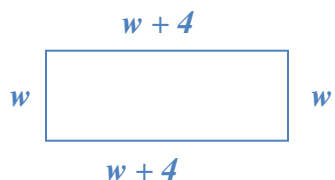
*So the width is 11m and the length is 23m.*

*Check:  $11 + (11+12) + (11) + (11+12) = 68$  ✓*

4. The length of a rectangle is 4 cm more than the width and the perimeter is at least 48 cm. What are the smallest possible dimensions for the rectangle?

**Solution**

*Sketch:*



*Define a Variable:*

*Width =  $w$*

*Length =  $w + 4$*

*Equation:  $w + w + w + 4 + w + 4 \geq 48$*

*$4w \geq 40$*

*$w \geq 10$*

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*Width = 10*

*Length = (10) + 4 = 14*

*So the width is 10cm and the length is 14cm.*

*Check:  $10 + 10 + 14 + 14 \geq 48$   
 $48 \geq 48 \checkmark$*

5. Find three consecutive integers whose sum is 171.

*Solution*

*Define a Variable:*

*$x$  = the first consecutive number*

*so  $x + 1$  = the second consecutive number*

*and  $x + 2$  = the third consecutive number*

*Equation:  $x + x + 1 + x + 2 = 171$*

$$3x + 3 = 171$$

$$3x = 168$$

$$x = 56$$

*56 = the first consecutive number*

*56 + 1 = the second consecutive number*

*56 + 2 = the third consecutive number*

*The three consecutive numbers are 56, 57, and 58.*

*Check:  $56 + 57 + 58 = 171$*

6. Find four consecutive even integers whose sum is 244.

*Solution*

*Define a Variable:*

*$x$  = first number*

*$x + 2$  = second number*

*$x + 4$  = third number*

*$x + 6$  = fourth number*

*Equation:  $x + x + 2 + x + 4 + x + 6 = 244$*

$$4x + 12 = 244$$

$$4x = 232$$

$$x = 58$$

*58 = first number*

*58 + 2 = second number*

*58 + 4 = third number*

*58 + 6 = fourth number*

*The numbers are 58, 60, 62, and 64.*

*Check:  $58 + 60 + 62 + 64 = 244$*

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7. Alex has twice as much money as Jennifer. Jennifer has \$6 less than Shannon. Together they have \$54. How much money does each have?

*Solution*

*Define a Variable:*

*Shannon:  $s$*

*Jennifer:  $s - 6$*

*Alex:  $2(s - 6)$*

*Equation:  $s + s - 6 + 2(s - 6) = 54$*

$$2s - 6 + 2s - 12 = 54$$

$$4s - 18 = 54$$

$$4s = 72$$

$$s = 18$$

*Shannon:  $s = 18$*

*Jennifer:  $s - 6 = 12$*

*Alex:  $2(s - 6) = 24$*

*So, Shannon has \$18, Jennifer has \$12, and Alex has \$24*

*Check:  $18 + 12 + 24 = 54$*

8. There are three exams in a marking period. A student received grades of 75 and 81 on the first two exams. What grade must the student earn on the last exam to get an average of no less than 80 for the marking period?

*Solution*

*Define a Variable:  $x = \text{last exam}$*

*Equation:*

$$\frac{75 + 81 + x}{3} \geq 80$$

$$75 + 81 + x \geq 240$$

$$x \geq 84$$

*The student must receive an 84 or higher exam grade in order to have an average no less than 80 for the marking period.*

*Check:  $\frac{75+81+84}{3} \geq 80$*



## **Practice Task: Lucy’s Linear Equations and Inequalities**

Name \_\_\_\_\_

Date \_\_\_\_\_

### **Mathematical Goals**

- Create one–variable linear equations and inequalities from contextual situations.
- Solve and interpret the solution to multi–step linear equations and inequalities in context.

### **Essential Questions**

- How do I interpret parts of an expression in terms of context?
- How do I create equations and inequalities in one variable and use them to solve problems arising from linear functions?
- How can I write, interpret and manipulate algebraic expressions, equations and inequalities?

### **Georgia Standards of Excellence**

**MGSE9–12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions, and simple rational and exponential functions~~ (integer inputs only).

**MGSE9–12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.

**MGSE9–12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

**MGSE9–12.A.SSE.1b** Given situation which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.

### **Standards for Mathematical Practice**

2. Reason abstractly and quantitatively.
4. Model with mathematics.
7. Look for and make use of structure.

## **Practice Task: Lucy's Linear Equations and Inequalities**

Name \_\_\_\_\_

Date \_\_\_\_\_

Lucy has been assigned the following linear equations and inequality word problems. Help her solve each problem below by using a five step plan.

- Drawing a Sketch (if necessary)
  - Defining a Variable
  - Setting up an equation or inequality
  - Solve the equation or inequality
  - Make sure you answer the question
1. The sum of 38 and twice a number is 124. Find the number.
  
  
  
  
  
  
  
  
  
  
  2. The sum of two consecutive integers is less than 83. Find the pair of integers with the greatest sum.
  
  
  
  
  
  
  
  
  
  
  3. A rectangle is 12m longer than it is wide. Its perimeter is 68m. Find its length and width.
  
  
  
  
  
  
  
  
  
  
  4. The length of a rectangle is 4 cm more than the width and the perimeter is at least 48 cm. What are the smallest possible dimensions for the rectangle?

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5. Find three consecutive integers whose sum is 171.
  
  
  
  
  
  
  
  
  
  
6. Find four consecutive even integers whose sum is 244.
  
  
  
  
  
  
  
  
  
  
7. Alex has twice as much money as Jennifer. Jennifer has \$6 less than Shannon. Together they have \$54. How much money does each have?
  
  
  
  
  
  
  
  
  
  
8. There are three exams in a marking period. A student received grades of 75 and 81 on the first two exams. What grade must the student earn on the last exam to get an average of no less than 80 for the marking period?

## **Forget the Formula (Scaffolding Task)**

### **Introduction**

In this task, students will develop a formula to convert temperatures from Celsius to Fahrenheit. Students will then manipulate this formula to create a Fahrenheit to Celsius formula. Students should develop meaning for each equation based on the context of the problem.

### **Mathematical Goals**

- Rearrange formulas to highlight a quantity of interest.
- Create equations in two variables to represent relationships.
- Write and graph an equation to represent a linear relationship.
- Extend the concepts used in solving numerical equations to rearranging formulas for a particular variable.

### **Essential Questions**

- How do I interpret parts of an expression in terms of context?
- How do I create equations in two variables to represent relationships between quantities?
- How can I rearrange formulas to highlight a quantity of interest?

### **Georgia Standards of Excellence**

**MGSE9–12.A.CED.2** Create linear and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which  $A = P(1 + r/n)^{nt}$  has multiple variables.)

**MGSE9–12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non–solution) under the established constraints.

**MGSE9–12.A.CED.4** Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. *Examples: Rearrange Ohm’s law  $V = IR$  to highlight resistance  $R$ ; Rearrange area of a circle formula  $A = \pi r^2$  to highlight the radius  $r$ .*

### **Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.  
*Students will have to attempt several approaches to find the answer.*
2. Reason abstractly and quantitatively.  
*Students will recognize that the freezing and boiling points can be used to calculate slope*

**Background Knowledge**

- Students can graph linear equations.
- Students can write the equation of a line given two points on the line.

**Common Misconceptions**

- Students may be confused as to which temperature measure should be their x or y coordinate.
- Students may have trouble with the multiplicative inverse.
- Students may not realize that  $C=F$  is suggesting that they substitute one variable with the other.

**Materials**

- None

**Grouping**

- Partner / small group

**Differentiation**

**Extension:**

- Introduce Kelvin. ( $K = C - 273$ )

**Intervention:**

- Provide a drawing of a thermometer with Celsius and Fahrenheit marked for reference.

**Formative Assessment Questions**

- How can you derive a formula based on data gathered?

## Forget the Formula – Teacher Notes

Temperature can be measured with many different systems, the most commonly used are Fahrenheit and Celsius. The relationship between the two systems is linear and therefore can be determined using any two equivalent measurements.

1. What is the boiling point of water in Fahrenheit and Celsius?

Solution:

*The boiling point of water is 100 degrees Celsius and 212 degrees Fahrenheit.*

2. What is the freezing point of water in Fahrenheit and Celsius?

Solution:

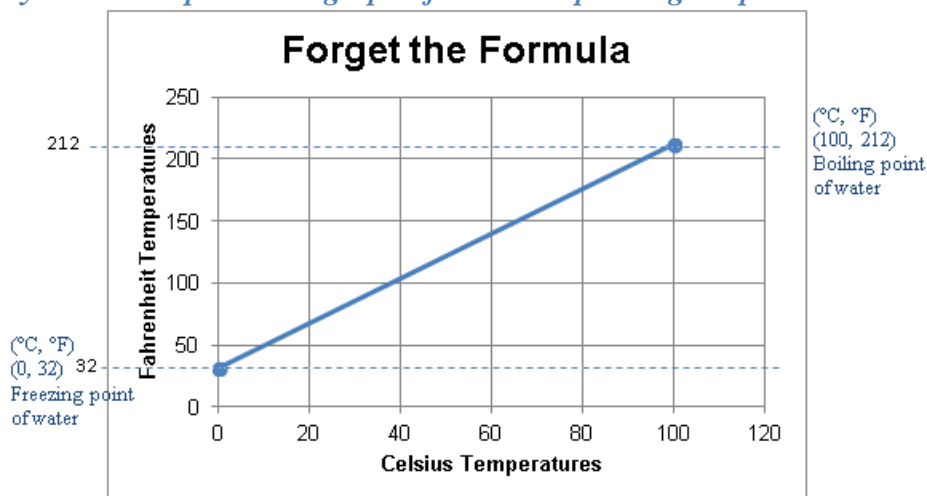
*The freezing point of water is 0 degrees Celsius and 32 degrees Fahrenheit.*

3. Using these two points, create an equation that convert Celsius to Fahrenheit.

Solution:

*One method of finding this formula is to use (0, 32) and (100, 212) as two points on a line. To find the equation of the line only the slope is needed, since the y-intercept, (0, 32), is already given. Calculating the slope,  $\frac{212-32}{100-0}$ , the students should simplify  $\frac{180}{100}$  to  $\frac{9}{5}$ . Substituting this for m in  $y = mx + b$ , the equation becomes  $y = \frac{9}{5}x + 32$ . Because y represent the Fahrenheit temperature and x represents the Celsius temperature, the formula would be more appropriately written  $F = \frac{9}{5}C + 32$ .*

*They could also produce a graph of the corresponding temperatures.*



Rearrange the equation found in number 3 to solve for Celsius.

*Students could solve this equation for C to produce the other form expressing the relationship:*

$$F = \frac{9}{5}C + 32$$

$$F - 32 = \frac{9}{5}C + 32 - 32$$

$$(F - 32) \left(\frac{5}{9}\right) = \left(\frac{9}{5}\right) \left(\frac{5}{9}\right) C$$

$$(F - 32) \left(\frac{5}{9}\right) = C$$

$$C = \left(\frac{5}{9}\right) (F - 32)$$

or

$$C = \left(\frac{5}{9}\right) F - 17\frac{7}{9}$$

4. What does the constant represent in each equation? What does the slope represent in each equation?

*32 is the y-intercept that means when it is 0 degrees Celsius it is 32 degrees Fahrenheit. The slope 9/5 shows that as the Celsius temperature increases or decreases five degrees that Fahrenheit will increase or decrease 9 degrees respectively.*

*$-17\frac{7}{9}$  is the y-intercept that means when it is 0 degrees Fahrenheit it is  $-17\frac{7}{9}$  degrees Celsius. The slope 5/9 shows that as the Fahrenheit temperature increases or decreases nine degrees that Celsius will increase or decrease 5 degrees respectively.*

5. At what temperature is the degrees Celsius equal to the degrees Fahrenheit?

*Possible solution. Set C=F and substitute into either equation*

$$C = \frac{9}{5}C + 32$$

$$-\frac{4}{5}C = 32$$

$$C = -40$$

## **Scaffolding Task: Forget the Formula**

Name \_\_\_\_\_

Date \_\_\_\_\_

### **Mathematical Goals**

- Rearrange formulas to highlight a quantity of interest.
- Create equations in two variables to represent relationships.
- Write and graph an equation to represent a linear relationship.
- Extend the concepts used in solving numerical equations to rearranging formulas for a particular variable.

### **Essential Questions**

- How do I interpret parts of an expression in terms of context?
- How do I create equations in two variables to represent relationships between quantities?
- How can I rearrange formulas to highlight a quantity of interest?

### **Georgia Standards of Excellence**

**MGSE9–12.A.CED.2** Create linear and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which  $A = P(1 + r/n)^{nt}$  has multiple variables.)

**MGSE9–12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non–solution) under the established constraints.

**MGSE9–12.A.CED.4** Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. *Examples: Rearrange Ohm’s law  $V = IR$  to highlight resistance  $R$ ; Rearrange area of a circle formula  $A = \pi r^2$  to highlight the radius  $r$ .*

### **Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.



**Scaffolding Task: Forget the Formula**

Name \_\_\_\_\_

Date \_\_\_\_\_

Temperature can be measured with many different systems, the most commonly used are Fahrenheit and Celsius. The relationship between the two systems is linear and therefore can be determined using any two equivalent measurements.

1. What is the boiling point of water in Fahrenheit and in Celsius?
  
2. What is the freezing point of water in Fahrenheit and in Celsius?
  
3. Using these two points, create an equation that convert Celsius to Fahrenheit.



## Cara's Candles Revisited (Scaffolding Task)

### Introduction

In this task, students will create a table of values from a given scenario. After answering the question posed students will interpret whether the solution is viable or non-viable in modeling context. Students will also graph the equations to represent linear relationships.

### Mathematical Goals

- Determine whether a point is a solution to an equation.
- Determine whether a solution has meaning in a real-world context.
- Interpret whether the solution is viable from a given model.
- Write and graph equations and inequalities representing constraints in contextual situations.

### Essential Questions

- How do I graph equations on coordinate axes with the correct labels and scales?
- How do I create equations in two or more variables to represent relationships between two quantities?

### Georgia Standards of Excellence

**MGSE9–12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear ~~and quadratic functions, and simple rational~~ and exponential functions (integer inputs only).

**MGSE9–12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

### Standards for Mathematical Practice

2. Reason abstractly and quantitatively.
4. Model with mathematics.  
*Students will represent the height of the candle using algebra.*
8. Look for and express regularity in repeated reasoning.  
*Students will use patterns to fill in the table.*

### Background Knowledge

- Students understand that linear equations have a constant slope.
- Students can represent constraints (domain) with inequalities.

### Common Misconceptions

- Students may have difficulty graphing a decimal slope.
- Students may not recognize the dependent and independent variable.

**Materials**

- colored pencils
- graphing calculators
- graph paper

**Grouping**

- Individual / partners

**Differentiation**

**Extension:**

- Ask the students to determine if the relationship is continuous or discrete and to explain why.  
*(The relationship is continuous because the candles continuously burn.)*

**Intervention:**

- Give students a blank graph.

**Formative Assessment Questions:**

- How can we model real-life situations with tables, graphs, or equations?
- What limiting factors are present in this situation?

### Cara's Candles Revisited – Teacher Notes

Cara likes candles. She also likes mathematics and was thinking about using algebra to answer a question that she had about two of her candles. Her taller candle is 16 centimeters tall. Each hour it burns makes the candle lose 2.5 centimeters in height. Her short candle is 12 centimeters tall and loses 1.5 centimeters in height for each hour that it burns.

Cara started filling out the following table to help determine whether these two candles would ever reach the same height at the same time if allowed to burn the same length of time. Finish the table for Cara. Use the data in the table to determine what time the two candles will be at the same height.

Also, she wants to know what height the two candles would be at that time. If it is not possible, she wants to know why it could not happen and what would need to be true in order for them to be able to reach the same height. To help Cara understand what you are doing, justify your results. You will explain your thinking using the table and create a graphical representation of the situation.

Time (hours)	16 cm candle height (cm)	12 cm candle height (cm)
0	16	12
1	13.5	10.5
2	<i>11</i>	<i>9</i>
3	<i>8.5</i>	<i>7.5</i>
4	<i>6</i>	<i>6</i>
5	<i>3.5</i>	<i>4.5</i>
6	<i>1</i>	<i>3</i>
7	<i>-1.5</i>	<i>1.5</i>

1. Complete the table, and use it to write an equation for the height of each candle in terms of the number of hours it has burned. Be sure to include any constraints for the equation.

#### Solution

*Students will use the table above to justify their solution. The candles will be the same height (6cm) in 4 hours.*

#### *Taller Candle*

$$y = -2.5x + 16; 0 \leq x \leq 6.4 \text{ hours}$$

#### *Shorter Candle*

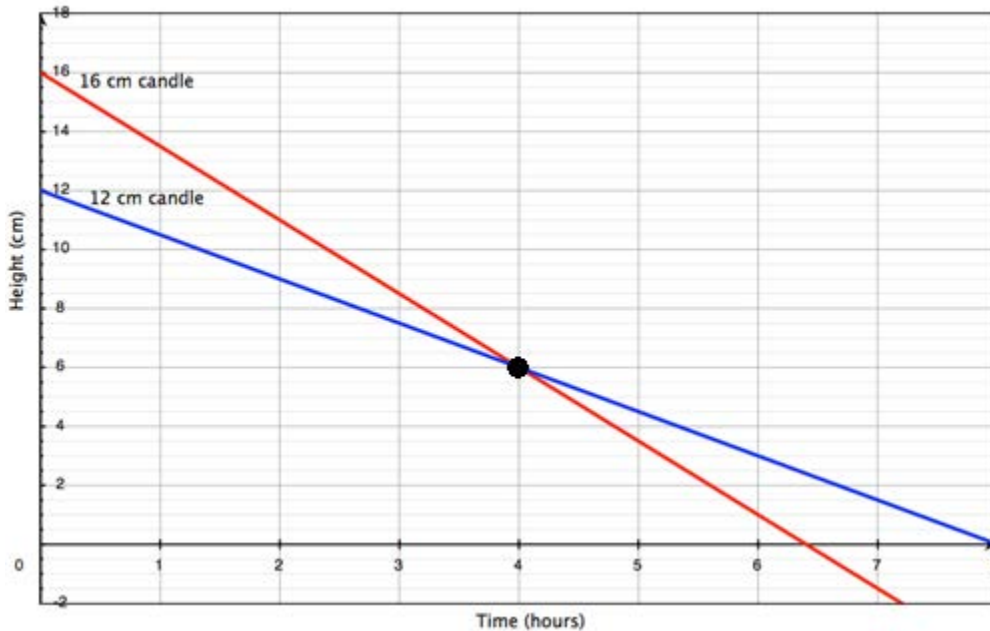
$$y = -1.5x + 12; 0 \leq x \leq 8 \text{ hours}$$

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2. Create a graphical representation of your data, taking into account natural restrictions on domain, range, etc.

**Solution:**

*Use the opportunity to bring out the concept of the natural restrictions. For instance, when  $x = 7$  in the first function, the candle would have a negative height, which is impossible.*



3. Cara has another candle that is 15 cm tall. How fast must it burn in order to also be 6 cm tall after 4 hours? Explain your thinking.

**Solution:**

*The candle would need to lose 9 cm in four hours so it would have to burn at the rate of 2.25 cm per hour. The slope of its linear equation would be  $-2.25$*

4. If Cara had a candle that burned 3 cm every hour, how tall would it need to be to also reach the same height as the other three candles after 4 hours? Explain your thinking.

**Solution:**

*The candle would have burned 12 cm in 4 hours. Its initial height would have been 18 cm tall. The  $y$ -intercept of its linear equation would be 18.*

## **Scaffolding Task: Cara’s Candles Revisited**

Name \_\_\_\_\_

Date \_\_\_\_\_

### **Mathematical Goals**

- Determine whether a point is a solution to an equation.
- Determine whether a solution has meaning in a real-world context.
- Interpret whether the solution is viable from a given model.
- Write and graph equations and inequalities representing constraints in contextual situations.

### **Essential Questions**

- How do I graph equations on coordinate axes with the correct labels and scales?
- How do I create equations in two or more variables to represent relationships between two quantities?

### **Georgia Standards of Excellence**

**MGSE9–12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear ~~and quadratic functions, and simple rational~~ and exponential functions (integer inputs only).

**MGSE9–12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

### **Standards for Mathematical Practice**

2. Reason abstractly and quantitatively.
4. Model with mathematics.
8. Look for and express regularity in repeated reasoning.

**Scaffolding Task: Cara’s Candles Revisited**

Name \_\_\_\_\_

Date \_\_\_\_\_

Cara likes candles. She also likes mathematics and was thinking about using algebra to answer a question that she had about two of her candles. Her taller candle is 16 centimeters tall. Each hour it burns makes the candle lose 2.5 centimeters in height. Her short candle is 12 centimeters tall and loses 1.5 centimeters in height for each hour that it burns.

Cara started filling out the following table to help determine whether these two candles would ever reach the same height at the same time if allowed to burn the same length of time. Finish the table for Cara. Use the data in the table to determine what time the two candles will be at the same height.

Also, she wants to know what height the two candles would be at that time. If it is not possible, she wants to know why it could not happen and what would need to be true in order for them to be able to reach the same height. To help Cara understand what you are doing, justify your results. You will explain your thinking using the table and create a graphical representation of the situation.

Time (hours)	16 cm candle height (cm)	12 cm candle height (cm)
0	16	12
1	13.5	10.5
2		
3		
4		
5		
6		
7		

1. Complete the table, and use it to write an equation for the height of each candle in terms of the number of hours it has burned. Be sure to include any constraints for the equation.





## **The Shoe Problem (Constructing Task)**

Adapted from PBS Mathline: <http://www.pbs.org/teachers/mathline/lessonplans/pdf/hsmp/yoyo.pdf>

### **Introduction**

The lesson starts with the presentation of the shoe problem. Students then complete a hands-on activity involving a design created with pennies that allows them to explore a linear pattern and express that pattern in symbolic form.

### **Mathematical Goals**

- Explore linear patterns.
- Create one variable and two variable linear equations.
- Graph equations on coordinate axes with labels and scales.

### **Essential Questions**

- How do I create equations in one or two variables to represent relationships between quantities?

### **Georgia Standards of Excellence**

**MGSE9–12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions, and simple rational and~~ exponential functions (integer inputs only).

**MGSE9–12.A.CED.2** Create linear and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which  $A = P(1 + r/n)^{nt}$  has multiple variables.)

**MGSE9–12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers’ precision is limited to the precision of the data given.*

### **Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.  
*Students will have to make equations based on what’s given.*
6. Attend to precision.  
*Students will have to calculate precisely and know when to round appropriately.*
7. Look for and make use of structure.  
*Students will have to recognize patterns and sequences.*

**Background Knowledge**

- understanding of percentages
- recognizing patterns

**Common Misconceptions**

- Students may be confused about how to set up the penny pattern in a spoke pattern.
- Students may not understand how to calculate sales tax.

**Materials**

- Pennies (or some other type of marker)

**Grouping**

- Individual / partners

**Differentiation**

**Extension:**

- Ask students to make another shape with the pennies and determine the sequence. e.g. a central penny surround by 4, 5,  $n$  external pennies.
- Ask students to compare results for part one if the sales tax were different.

**Intervention:**

- Review percentages.

**Formative Assessment Questions**

- What are the components we need to take into account when saving money or purchasing items?
- What other patterns can be described by linear functions?

## **The Shoe Problem – Teacher Notes**

### **Part I: The Shoe Problem**

Andy wants to buy a new pair of shoes. His new kicks cost \$89.99 plus 6% sales tax. Andy has already saved \$17.25, and he is earning \$7.20 a week by doing odd jobs and chores. How many weeks will it take him to save enough money for the shoes?

1. How much sales tax will Andy have to pay?

*Solution:*  
*\$5.40*

2. What will be the total cost of the shoes, including tax?

*Solution:*  
*\$95.39*

3. Let  $w$  be the number of weeks that it will take Andy to save enough money to buy the shoes. Write an algebraic equation that will help you solve the problem.

*Solution:*  
 *$17.25 + 7.20w = 95.39$*

4. Solve your equation for  $w$ , and check your answer. Be prepared to present your solution to the class.

*Solution:*  
 *$w = 10.8528$  or approximately 11 weeks*

#### *Comments*

*Students need to be reminded when to approximate solutions in terms of the context of the problem.*

**Part II: The Penny Pattern**



1. Create a pattern using pennies. Stage one of the pattern is shown next to the title above—one penny surrounded by six additional pennies. To create each additional stage of the design, place more pennies extending out from the six that surround the center penny. Continue making this design until you have used up all of your pennies. On the back of this sheet, sketch the first four stages of the pattern.

2. Using your penny pattern or the sketches of your penny pattern, create a table of values.

Stage Number, $n$	1	2	3	4	5
Number of Pennies Required	7	13	19	25	31

3. How many pennies are needed to make stage 6, stage 7, and stage 8 of the penny pattern? How did you determine your answer?

**Solution:**  
*Stages 6, 7, 8 require 37, 43, and 49 pennies.*

4. Write an algebraic model that expresses the relationship between the stage number,  $n$ , and the number of pennies required to make that design,  $p$ .

**Solution:**  
 $p = 1 + 6n$

5. Use your model to determine how many pennies are needed to make stage 80, stage 95, and stage 100 of the penny pattern.

**Solution:**  
*Stages 80, 95, and 100 require 481, 571, and 601*

6. If you use 127 pennies to make the penny pattern, how many pennies will be in each spoke coming out from the center penny?

**Solution:**  
 $1 + 6n = 127$   
 $6n = 126$   
 $n = 21$   
*There will 21 pennies in each spoke coming out from the center penny.*

## **Constructing Task: The Shoe Problem**

Name \_\_\_\_\_

Date \_\_\_\_\_

Adapted from PBS Mathline: <http://www.pbs.org/teachers/mathline/lessonplans/pdf/hsmp/yoyo.pdf>

### **Introduction**

The lesson starts with the presentation of the shoe problem. Students then complete a hands-on activity involving a design created with pennies that allows them to explore a linear pattern and express that pattern in symbolic form.

### **Mathematical Goals**

- Explore linear patterns.
- Create one variable and two variable linear equations.
- Graph equations on coordinate axes with labels and scales.

### **Essential Questions**

- How do I create equations in one or two variables to represent relationships between quantities?

### **Georgia Standards of Excellence**

**MGSE9–12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions, and simple rational and~~ exponential functions (integer inputs only).

**MGSE9–12.A.CED.2** Create linear and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which  $A = P(1 + r/n)^{nt}$  has multiple variables.)

**MGSE9–12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers’ precision is limited to the precision of the data given.*

### **Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.
6. Attend to precision.
7. Look for and make use of structure.

## **Constructing Task: The Shoe Problem**

Name \_\_\_\_\_

Date \_\_\_\_\_

### **Part I: The Shoe Problem**

Andy wants to buy a new pair of shoes. His new kicks cost \$89.99 plus 6% sales tax. Andy has already saved \$17.25, and he is earning \$7.20 a week by doing odd jobs and chores. How many weeks will it take him to save enough money for the shoes?

1. How much sales tax will Andy have to pay?
  
  
  
  
  
  
  
  
  
  
2. What will be the total cost of the shoes, including tax?
  
  
  
  
  
  
  
  
  
  
3. Let  $w$  be the number of weeks that it will take Andy to save enough money to buy the shoes. Write an algebraic equation that will help you solve the problem.
  
  
  
  
  
  
  
  
  
  
4. Solve your equation for  $w$ , and check your answer. Be prepared to present your solution to the class.

**Part II: The Penny Pattern**



1. Create a pattern using pennies. Stage one of the pattern is shown above—one penny surrounded by six additional pennies. To create each additional stage of the design, place more pennies extending out from the six that surround the center penny. Continue making this design until you have used up all of your pennies. On another sheet, sketch the first four stages of the pattern.

2. Using your penny pattern or the sketches of your penny pattern, create a table of values.

Stage #, $n$	1	2	3	4	5
# of Pennies Required					

3. How many pennies are needed to make stage 6, stage 7, and stage 8 of the penny pattern? How did you determine your answer?

4. Write an algebraic model that expresses the relationship between the stage number,  $n$ , and the number of pennies required to make that design,  $p$ .

5. Use your model to determine how many pennies are needed to make stage 80, stage 95, and stage 100 of the penny pattern.

6. If you use 127 pennies to make the penny pattern, how many pennies will be in each spoke coming out from the center penny?



## **Formative Assessment Lesson: The Largest Loser**

*Source: Georgia Mathematics Design Collaborative*

This lesson is intended to help you assess how well students are able to:

- Utilize what they already know about linear equations in the context of different graphs
- Reasoning quantitatively, choose and interpret the appropriate scale and rate of change from graphs
- Understand Constraints upon graphs in given contexts and make sense of graph problems with differently-defined axes of measure
- Reason abstractly and compare graphs of linear equations with different scales of measure

### **STANDARDS ADDRESSED IN THIS TASK:**

**MGSE9-12.N.Q.1** Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

- a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
- b. Convert units and rates using dimensional analysis (English-to-English and Metric-to-Metric without conversion factor provided and between English and Metric with conversion factor);
- c. Use units within multi-step problems and formulas; interpret units of input and resulting units of output.

**MGSE9-12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers' precision is limited to the precision of the data given.*

**MGSE9-12.A.CED.2** Create linear, ~~quadratic~~, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which  $A = P(1 + r/n)^m$  has multiple variables.)

### **STANDARDS FOR MATHEMATICAL PRACTICE:**

This lesson uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision

**TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:**

Tasks and lessons from the Georgia Mathematics Design Collaborative are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding.

The task, *The Largest Loser*, is a Formative Assessment Lesson (FAL) that can be found at: <http://ccgpsmathematics9-10.wikispaces.com/Georgia+Mathematics+Design+Collaborative+Formative+Assessment+Lessons>

## **Paper Folding (Constructing Task)**

Adapted from PBS Mathline: <http://www.pbs.org/teachers/mathline/lessonplans/pdf/hsmp/rhinos.pdf>

### **Introduction**

Students will use paper folding to model exponential functions. Students will collect data, create scatterplots, and determine algebraic models that represent their functions. Students begin this lesson by collecting data within their groups. They fold a sheet of paper and determine the area of the smallest region after each fold. Next they draw a scatterplot of their data and determine by hand an algebraic model. This investigation allows students to explore the patterns of exponential models in tables, graphs, and symbolic form.

### **Mathematical Goals**

- Write and graph an equation to represent an exponential relationship.
- Model a data set using an equation.
- Choose the best form of an equation to model exponential functions.
- Use properties of exponents to solve and interpret the solution to exponential equations in context.
- Graph equations on coordinate axes with labels and scales.

### **Essential Questions**

- How do I create equations in two variables to represent relationships between quantities?
- How can I represent patterns algebraically? What types of patterns exist in exponential relationships?

### **Georgia Standards of Excellence**

**MGSE9–12.A.CED.2** Create linear and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which  $A = P(1 + r/n)^{nt}$  has multiple variables.)

**MGSE9–12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non–solution) under the established constraints.

**MGSE9–12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.

**MGSE9–12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

**MGSE9–12.A.SSE.1b** Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.

### **Standards for Mathematical Practice**

- 1.** Make sense of problems and persevere in solving them.  
*Students will have to make several attempts to fold the paper to get the appropriate folds.*
- 4.** Model with mathematics.  
*Students will create a table, graph, and equation representing the data collected.*
- 8.** Look for and express regularity in repeated reasoning.  
*Students will recognize a pattern in the number of sections created by each additional fold.*

### **Background Knowledge**

- Students can make a table, graph, and equation
- Students can choose an appropriate scale for a graph.
- Students have some knowledge of exponents (e.g.,  $x^0 = 1$  if  $x \neq 0$ )

### **Common Misconceptions**

- Students may not understand how to show a fractional “area” of a piece of paper.

### **Materials**

- Graph paper
- Paper for folding

### **Grouping**

- Individual or partner

### **Differentiation**

#### **Extension:**

- Try with different kinds of paper.
- Discuss discrete versus continuous functions.

#### **Intervention:**

- Pair with resident experts.
- Do a group demonstration.

### **Formative Assessment Questions**

- How might this problem change with a different kind of paper?
- What are the limiting factors to the activity?
- How might you fold the paper to form a linear sequence?

## **Paper Folding – Teacher Notes**

### **Part I: Number of Sections**

#### **Comments**

*Students will need guidance in determining a mathematical model that represents the data. This is their first exposure to modeling exponential functions.*

- 1.** Fold an 8.5” × 11” sheet of paper in half and determine the number of sections the paper has after you have made the fold.
- 2.** Record this data in the table and continue in the same manner until it becomes too hard to fold the paper.
- 3.** Make a scatter plot of your data in a separate sheet of graph paper.
- 4.** Determine a mathematical model that represents this data by examining the patterns in the table.

<b># of Folds</b>	<b># of Sections</b>
<b>0</b>	<b>1</b>
<b>1</b>	<b>2</b>
<b>2</b>	<b>4</b>
<b>3</b>	<b>8</b>
<b>4</b>	<b>16</b>
<b>5</b>	<b>32</b>
<b>6</b>	<b>64</b>

#### **Solution**

$$y = 2^x$$

- 5.** What might be different if you tried this experiment with an 8.5 x 11” sheet of wax paper or tissue paper?

#### **Solution:**

*The results would be exactly the same, but you would be able to make more folds and collect more data because the paper would be thinner.*

## Part II: Area of Smallest Section

### Comments

*Students will need guidance in determining a mathematical model that represents the data. This is their first exposure to modeling exponential functions.*

1. Fold an 8.5" × 11" sheet of paper in half and determine the area of *one* of the two sections after you have made the fold.
2. Record this data in the table and continue in the same manner until it becomes too hard to fold the paper.
3. Make a scatter plot of your data on a separate sheet of graph paper.
4. Determine a mathematical model that represents this data by examining the patterns in the table.

# of Folds	Area of Smallest Section
0	1
1	$1/2$
2	$1/4$
3	$1/8$
4	$1/16$
5	$1/32$
6	$1/64$

### Solution

$$y = (1/2)^x$$

## **Constructing Task: Paper Folding**

Name \_\_\_\_\_ Date \_\_\_\_\_

Adapted from PBS Mathline: <http://www.pbs.org/teachers/mathline/lessonplans/pdf/hsmp/rhinos.pdf>

### **Mathematical Goals**

- Write and graph an equation to represent an exponential relationship.
- Model a data set using an equation.
- Choose the best form of an equation to model exponential functions.
- Use properties of exponents to solve and interpret the solution to exponential equations in context.
- Graph equations on coordinate axes with labels and scales.

### **Essential Questions**

- How do I create equations in two variables to represent relationships between quantities?
- How can I represent patterns algebraically? What types of patterns exist in exponential relationships?

### **Georgia Standards of Excellence**

**MGSE9–12.A.CED.2** Create linear and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which  $A = P(1 + r/n)^{nt}$  has multiple variables.)

**MGSE9–12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non–solution) under the established constraints.

**MGSE9–12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.

**MGSE9–12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

**MGSE9–12.A.SSE.1b** Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.

**Standards for Mathematical Practice**

- 1.** Make sense of problems and persevere in solving them.
- 4.** Model with mathematics.
- 8.** Look for and express regularity in repeated reasoning.



**Constructing Task: Paper Folding**

Name \_\_\_\_\_ Date \_\_\_\_\_

**Part I: Number of Sections**

6. Fold an 8.5" × 11" sheet of paper in half and determine the number of sections the paper has after you have made the fold.

7. Record this data in the table and continue in the same manner until it becomes too hard to fold the paper.

8. Make a scatter plot of your data in a separate sheet of graph paper.

9. Determine a mathematical model that represents this data by examining the patterns in the table.

# of Folds	# of Sections
0	
1	
2	
3	
4	
5	
6	

10. What might be different if you tried this experiment with an 8.5 x 11" sheet of wax paper or tissue paper?

**Part II: Area of Smallest Section**

**5.** Fold an 8.5" × 11" sheet of paper in half and determine the area of *one* of the two sections after you have made the fold.

**6.** Record this data in the table and continue in the same manner until it becomes too hard to fold the paper.

**7.** Make a scatter plot of your data on a separate sheet of graph paper.

**8.** Determine a mathematical model that represents this data by examining the patterns in the table.

<b># of Folds</b>	<b>Area of Smallest Section</b>
<b>0</b>	<b>1</b>
<b>1</b>	
<b>2</b>	
<b>3</b>	
<b>4</b>	
<b>5</b>	
<b>6</b>	

## **Culminating Task: Growing by Leaps and Bounds**

### **Comments on the Task:**

The following task covers the majority of the material from the unit. When using this task you should be aware that there are some new material portions. Part I is very similar to the paper folding task creating the growth function  $y = 2^x$ . However in Part II new material is given about the parts of an exponential function. Given the length of the task, Parts I and II should be done separately from Part III. Additionally, it should be noted in Part III that interest can be modeled by a linear or exponential model based on simple or compound interest. If available, this task is a good opportunity to teach students about Excel programming.

### **Introduction**

This task introduces students to exponential functions. At this point in their study, students have extended their understanding of exponents to include all integer values but have not yet discussed rational or real number exponents. In Part I, students investigate a mathematical model of spreading a rumor in which the domain of the function is limited to a finite set of nonnegative integers. In Part II, students learn the definition of an exponential function and see the model from Part I as an example of such a function. The emphasis in Part II is the pattern for the formula of an exponential function and an introduction to the shape of the graph. In Part III, students work with the compound interest formula.

### **Mathematical Goals**

- Create one–variable exponential equations from contextual situations.
- Solve and interpret the solution to exponential equations in context.
- Write and graph an equation to represent an exponential relationship.
- Graph equations on coordinate axes with labels and scales.
- Use technology to explore exponential graphs.

### **Essential Questions**

- How can I apply what I have learned about mathematical modeling to real–world situations?

### **Georgia Standards of Excellence**

**MGSE9–12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions, and simple rational and~~ exponential functions (integer inputs only).

**MGSE9–12.A.CED.2** Create linear and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which  $A = P(1 + r/n)^{nt}$  has multiple variables.)

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**MGSE9–12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non–solution) under the established constraints.

**MGSE9–12.A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *Examples: Rearrange Ohm’s law  $V = IR$  to highlight resistance  $R$ ; Rearrange area of a circle formula  $A = \pi r^2$  to highlight the radius  $r$ .*

**MGSE9–12.N.Q.1** Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

- a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
- b. Convert units and rates using dimensional analysis (English–to–English and Metric–to–Metric without conversion factor provided and between English and Metric with conversion factor);
- c. Use units within multi–step problems and formulas; interpret units of input and resulting units of output.

**MGSE9–12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling. Given a situation, context, or problem, students will determine, identify, and use appropriate quantities for representing the situation.

**MGSE9–12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answer’s precision is limited to the precision of the data given.*

**MGSE9–12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.

**MGSE9–12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

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**MGSE9–12.A.SSE.1b** Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.

**Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

*Students should use all eight SMPs when exploring this task.*

**Background Knowledge**

- Students will apply everything they have learned in this unit.

**Common Misconceptions**

- Address misconceptions brought to light during the rest of the unit.

**Materials**

- Graph paper
- Graphing utility
- Optional: spreadsheet software (e.g., Excel)

**Grouping**

- Individual / Partners

## **Growing by Leaps and Bounds – Teacher Notes**

### **Part I: Meet Linda**

Linda’s lifelong dream has been to open her own business. After working, sacrificing, and saving, she finally has enough money to open up an ice cream business. The grand opening of her business is scheduled for the Friday of Memorial Day weekend. She would like to have a soft opening for her business on the Tuesday before. The soft opening should give her a good idea of any supply or personnel issues and give her time to correct them before the big official opening.

A soft opening means that the opening of the business is not officially announced; news of its opening is just spread by word of mouth (see, not all rumors are bad!). Linda needs a good idea of when she should begin the rumor in order for it to spread reasonably well before her soft opening. She has been told that about 10% of the people who know about an event will actually attend it. Based on this assumption, if she wants to have about 50 people visit her store on the Tuesday of the soft opening, she will need 500 people to know about it.

1. Linda plans to tell one person each day and will ask that person to tell one other person each day through the day of the opening, and so on. Assume that each new person who hears about the soft opening is also asked to tell one other person each day through the day of the opening and that each one starts the process of telling their friends on the day after he or she first hears. When should Linda begin telling others about the soft opening in order to have at least 500 people know about it by the day it occurs?

#### **Comments**

*With the table in item 2 below, it is likely that many students will organize their work in a similar way. Whether or not they use such a table, they will need to count up from the first day that Linda begins to spread the news to find out how many days it will take for the number of people who know to reach 500 and then count backwards to determine the day Linda should start. The Memorial Day reference in the problem gives a convenient way to express the answer.*

#### **Solution**

*Linda should tell her first person about the soft opening on Monday two weeks before Memorial Day because. See →*

*The 9<sup>th</sup> day corresponds to the Tuesday of the soft opening. So, the 2<sup>nd</sup> day is the Tuesday one week before, and the 1<sup>st</sup> day is the Monday that is two weeks before Memorial Day.*

*1<sup>st</sup> day: Linda tells one other person, so 2 people know.*

*2<sup>nd</sup> day: Each of the two people who know tell another person, so 4 people know.*

*3<sup>rd</sup> day: Each of the four people who know tell another person, so 8 people know.*

*4<sup>th</sup> day: 16 people know*

*5<sup>th</sup> day: 32 people know*

*6<sup>th</sup> day: 64 people know*

*7<sup>th</sup> day: 128 people know*

*8<sup>th</sup> day: 256 people know*

*9<sup>th</sup> day: 512 people know*

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2. Let  $x$  represent the day number and let  $y$  be the number of people who know about the soft opening on day  $x$ . Consider the day before Linda told anyone to be Day 0, so that Linda is the only person who knows about the opening on Day 0. Day 1 is the first day that Linda told someone else about the opening.

a. Complete the following table.

Day	0	1	2	3	4	5
Number of people who know	1	2	4	8	16	32

Comments

*The table of values is limited to fewer than that needed to answer the question in item 1 so that the graph in part b below will show the  $y$ -intercept and the shape typical of an exponential function.*

Solutions

*The completed table is shown above.*

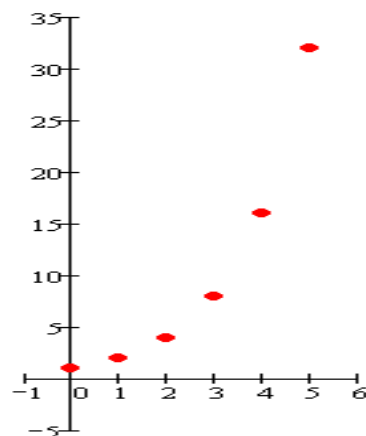
b. Graph the points from the table in part a.

Comments

*Since this is the students' first experience with an exponential graph, it is recommended that students draw this graph by hand on graph paper.*

Solutions

*The graph is shown at the right.*



3. Write an equation that describes the relationship between  $x$  (day) and  $y$  (number of people who know) for the situation of spreading the news about the soft opening of Linda's ice cream store.

Comments

*Students should easily see that the outputs of the function are powers of 2 and then note that the day number and the power of 2 are the same.*

Solutions

$$y = 2^x$$

4. Does your equation describe the relationship between day and number who know about Linda's ice cream store soft opening completely? Why or why not?

Comments

*The point of this question is that students realize that the domain is restricted in ways not implied by the equation. Since students have not yet studied a definition for non-integer exponents, they may believe that the equation makes sense only for integer exponents. However, they know about negative integer exponents and thus need to explicitly excluded these from the domain. They also need to exclude integers greater than 9 from the domain since Linda's method of spreading the news of the soft opening stops on the day of the opening. If students state the correct inequalities but do not explicitly state that the exponents should be integers, teachers need to explain that this restriction must be included since other numbers can be exponents, although they will not study other exponents explicitly until a later course.*

Solutions

*No, the equation does not describe the relationship completely because the domain needs to be restricted to the integers 0, 1, 2, . . . , 9, and this information is not included in the equation.*

**Part II: What if?**

The spread of a rumor or the spread of a disease can be modeled by a type of function known as exponential function; in particular, an exponential **growth** function. An **exponential function** has the form

$$f(x) = a \cdot b^x,$$

where  $a$  is a non-zero real number and  $b$  is a positive real number other than 1. An exponential growth function has a value of  $b$  that is greater than 1.



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In the case of Linda’s ice cream store, what values of  $a$  and  $b$  yield an exponential function to model the spread of the rumor of the soft store opening?

*Comments* *For the rumor model, the coefficient  $a$  has a value of 1. However, other exponential functions in this task have coefficients other than 1, so students are introduced to the general definition of an exponential function from the beginning.*

*Solutions*

*$a = 1$  and  $b = 2$*

1. In this particular case, what is an appropriate domain for the exponential function? What range corresponds to this domain?

*Comments*

*Students are asked to specify the domain for this particular case. There is overlap with item 4 of part I. The earlier question focused on whether all of the information is included in the equation. Here, the focus is explicitly to find the domain. Students may express the correct answer in a variety of ways. Two of these are shown in the solutions.*

*Solutions*

*The set of all nonnegative integers less than or equal to 9, or  $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$*

2. In part I, item 2, you drew a portion of the graph of this function. Does it make sense to connect the dots on the graph? Why or why not?

*Comments*

*The question of whether to connect the dots was prominent in students’ early formal study of models in middle school. It reminds students to think of the meaning of points on the graph and to consider what values of the independent variable are meaningful in the situation. The description here is that the output is total number of people who know on a given day. Fractional parts of a day are not meaningful. Note: It would be impossible to draw an accurate model of this situation with a continuous time domain since we do not know when during the day each person who knows tells another person.*

*Solutions*

*No, it does not make sense to connect the dots. Connecting the dots would imply time passing continuously. We do not know when during the day people hear about the soft opening. We just have a count of the total number of people who know on each day.*

3. How would the graph change if Linda had told two people each day rather than one and had asked that each person also tell two other people each day?

*Comments:* *This question asks students to think in terms of function values or points on the graph, but they will have to think through the situation in a similar manner to the*

*original. If students answer more generally here without being specific about new function values, then they will have more work to do in item 5 to find the new equation.*

**Solutions**

*The point (0, 1) would stay the same since on Day 0 Linda would still be the only person who knows about the opening. But for the other days, more people would know so the points for the other days would be higher.*

*In particular, on the first day, 3 people (Linda and the two people she tells) would know giving the point (1, 3). On the second day, 9 people would know, because each of the 3 who know will tell 2 others giving a total of  $3 + 2(3) = 9$ . So the point for Day 2 is (2, 9). We can continue in this way for the other points.*

4. How would the equation change if Linda had told two people each day rather than one and had asked that each person also tell two other people each day? What would be the values of  $a$  and  $b$  in this case?

**Comments**

*In addition to giving the formula, students must specify the values of  $a$  and  $b$  in order to reinforce the definition of exponential function.*

**Solutions**

*The equation would have a 3 as base for the exponent instead of a 2, that is, the equation would be  $y = 3^x$ . In this case,  $a = 1$  and  $b = 3$ .*

4. How long would it take for at least 500 people to find out about the opening if the rumor spread at this new rate?

**Comments**

*Students can use the graph or a table of values to determine the answer. If they draw a graph using a graphing utility, they should realize that the points of this function are only the integer valued points on the continuous graph shown. It is likely that most students will just count up to find the first power of 3 that is greater than 500.*

**Solutions**

*It would take 6 days for at least 500 people to find out*

<i>Day number, <math>x</math></i>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<i>No. people who know, <math>y = 3^x</math></i>	<b>1</b>	<b>3</b>	<b>9</b>	<b>27</b>	<b>81</b>	<b>243</b>	<b>729</b>

**Part III: The Beginning of a Business**

How in the world did Linda ever save enough to buy the franchise to an ice cream store? Her mom used to say, “That Linda, why she could squeeze a quarter out of a nickel!” The truth is that Linda learned early in life that patience with money is a good thing. When she was just about 9 years old, she asked her dad if she could put her money in the bank. He took her to the bank and she opened her very first savings account.

Each year until Linda was 16, she deposited her birthday money into her savings account. Her grandparents (both sets) and her parents each gave her money for her birthday that was equal to twice her age; so on her ninth birthday, she deposited \$54 (\$18 from each couple).

Linda’s bank paid her 3% interest, compounded quarterly. The bank calculated her interest using the following standard formula:

$$A = P \left( 1 + \frac{r}{n} \right)^{nt}$$

where  $A$  = final amount,  $P$  = principal amount,  $r$  = interest rate,  $n$  = number of times per year the interest is compounded, and  $t$  is the number of years the money is left in the account.

1. Verify the first entry in the following chart, and then complete the chart to calculate how much money Linda had on her 16<sup>th</sup> birthday. Do not round answers until the end of the computation, then give the final amount rounded to the nearest cent.

Age	Birthday \$	Amount from previous year plus Birthday	Total at year end
9	54	0	55.63831630
10			
11			
12			
.	.	.	.
.	.	.	.

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Comments

*Students will need exploration time to understand the compound interest formula. They may need to look up the term “principal amount” to understand that it refers to the amount deposited into the account. They will also need to realize that the interest rate  $r$  must be expressed in decimal form. If they enter numbers in their calculators following the formula exactly, they may need to be reminded about order of operations and that the calculator will not make the correct calculation unless the expression,  $nt$ , in the exponent is put in parentheses during calculation.*

*Some students may benefit from verifying the meaning of the compound interest formula by stepping through the compound interest calculation as four applications of simple interest using a rate of  $\frac{0.03}{4} = 0.0075$  for each quarter for four quarters of one year as shown in the table below.*

<i>Quarter number</i>	<i>Amount invested at beginning of quarter</i>	<i>Amount of interest paid</i>	<i>Amount at end of quarter</i>
1	54	$54(.0075) = 0.405$	54.405
2	54.405	0.4080375	54.8130375
3	54.8130375	0.4110977813	55.22413528
4	55.22413528	0.4141810146	55.63831630

*Advanced students may benefit from seeing how the compound interest formula is developed using calculations similar to the above but using  $P$  for the amount of money, as shown below. One quarter is one-fourth of a year, so the number of quarters is always 4 times the number of years.*

<i>#. of yrs</i>	<i># of qtrs</i>	<i>Amt. invested at beginning of quarter</i>	<i>Amount of interest paid</i>	<i>Amount at end of quarter</i>
1/4	1	$P$	$P(.0075)$	$P(1 + 0.0075)$
1/2	2	$P(1.0075)$	$[P(1.0075)](.0075)$	$P(1 + 0.0075)(1 + .0075) = P(1.0075)^2$
3/4	3	$P(1.0075)^2$	$[P(1.0075)^2](.0075)$	$[P(1.0075)^2](1 + .0075) = P(1.0075)^3$
1	4	$P(1.0075)^3$	$[P(1.0075)^3](.0075)$	$[P(1.0075)^3](1 + .0075) = P(1.0075)^4$
5/4	5	$P(1.0075)^4$	$[P(1.0075)^4](.0075)$	$[P(1.0075)^4](1 + .0075) = P(1.0075)^5$
3/2	6	$P(1.0075)^5$	$[P(1.0075)^5](.0075)$	$[P(1.0075)^5](1 + .0075) = P(1.0075)^6$

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Solutions

*For her deposit at age 9,  $P = 54$ ,  $r = 0.03$ ,  $n = 4$ ,  $t = 4$ .*

$$A = 54 \left( 1 + \frac{0.03}{4} \right)^{(4 \cdot 1)} = 54(1.0075)^4$$

*$A \approx 54(1.030339191) = 54.6383163 \approx 54.63832$ , as in the chart*

*On the day before her 16<sup>th</sup> birthday, a year after her 15<sup>th</sup>, Linda had \$563.35.*

Age	\$ received on this Birthday	Amt from previous year plus Birthday	Total at year end
8	N/A	N/A	0
9	54	54	55.63832
10	60	115.63832	$115.63832(1.0075)^4 = 119.14669$
11	66	185.14669	$185.14669(1.0075)^4 = 190.76389$
12	72	262.76389	$262.76389(1.0075)^4 = 270.73593$
13	78	348.73593	$348.73593(1.0075)^4 = 359.31630$
14	84	443.31630	$443.31630(1.0075)^4 = 456.76616$
15	90	546.76616	$546.76616(1.0075)^4 = 563.35460$

- On her 16<sup>th</sup> birthday, the budding entrepreneur asked her parents if she could invest in the stock market. She studied the newspaper, talked to her economics teacher, researched a few companies and finally settled on the stock she wanted. She invested all of her money in the stock and promptly forgot about it. When she graduated from college on her 22<sup>nd</sup> birthday, she received a statement from her stocks and realized that her stock had appreciated an average of 10% per year. How much was her stock worth on her 22<sup>nd</sup> birthday?

Comments

*The challenge for students here is realizing that the information that Linda's stock had appreciated an average of 10% per year means that the money grew as if it were invested at 10% compounded annually for the 6 years.*

Solution

*Linda's stock was worth \$998.01 by application of the compound interest formula with  $P = 563.35$ ,  $r = 0.10$ ,  $n = 1$ , and  $t = 6$ :*

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$$A = 563.35 \left( 1 + \frac{0.10}{1} \right)^{(1 \cdot 6)} = 563.35(1.1)^6 = 563.35(1.771561) \approx 998.01$$

When Linda graduated from college, she received an academic award that carried a \$500 cash award. On her 22<sup>nd</sup> birthday, she used the money to purchase additional stock. She started her first job immediately after graduation and decided to save \$50 each month. On her 23<sup>rd</sup> birthday she used the \$600 (total of her monthly amount) savings to purchase new stock. Each year thereafter she increased the total of her savings by \$100 and, on her birthday each year, used her savings to purchase additional stock. Linda continued to learn about stocks and managed her accounts carefully. On her 35<sup>th</sup> birthday she looked back and saw that her stock had appreciated at 11% during the first year after college and that the rate of appreciation increased by 0.25% each year thereafter. At age 34, she cashed in enough stock to make a down payment on a bank loan to purchase her business. What was her stock worth on her 34<sup>th</sup> birthday? Use a table like the one below to organize your calculations.

Age	Amt from previous year	Amt Linda added from savings that year	Amount invested for the year	Interest rate for the year	Amt at year end
22	998.01	500	1498.01	11.00%	1662.79
23	1662.79	600		11.25%	
24		700		11.50%	
25		800		11.75%	
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.

Comments

*This item brings closure to this part of the learning task. The calculations are simple applications of the compound interest formula. If students have access to a spreadsheet program, having them set up the spreadsheet formulas is a possible extension of this activity.*

Solutions

*At age 34, Linda's stock was worth \$30,133.63.  
 The completed table is given on the next page as an Excel file.*

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<i>Age</i>	<i>Amt from previous year</i>	<i>Amt Linda added</i>	<i>Amt invested for the year</i>	<i>Interest rate for the year</i>	<i>Amt at year end</i>
22	998.01	500	1498.01	0.1100	1662.79
23	1662.79	600	2262.79	0.1125	2517.36
24	2517.36	700	3217.36	0.1150	3587.35
25	3587.35	800	4387.35	0.1175	4902.86
26	4902.86	900	5802.86	0.1200	6499.21
27	6499.21	1000	7499.21	0.1225	8417.86
28	8417.86	1100	9517.86	0.1250	10707.59
29	10707.59	1200	11907.59	0.1275	13425.81
30	13425.81	1300	14725.81	0.1300	16640.17
31	16640.17	1400	18040.17	0.1325	20430.49
32	20430.49	1500	21930.49	0.1350	24891.11
33	24891.11	1600	26491.11	0.1375	30133.63

*The same spreadsheet with formulas turned on is pasted in below.*

<i>Age</i>	<i>Amt from previous year</i>	<i>Amt Linda added</i>	<i>Amt invested for the year</i>	<i>Interest rate for the year</i>	<i>Amt at year end</i>
22	998.01	500	=B2+C2	0.11	=D2*(1+E2)
23	=F2	600	=B3+C3	0.1125	=D3*(1+E3)
24	=F3	700	=B4+C4	0.115	=D4*(1+E4)
25	=F4	800	=B5+C5	0.1175	=D5*(1+E5)
26	=F5	900	=B6+C6	0.12	=D6*(1+E6)
27	=F6	1000	=B7+C7	0.1225	=D7*(1+E7)
28	=F7	1100	=B8+C8	0.125	=D8*(1+E8)
29	=F8	1200	=B9+C9	0.1275	=D9*(1+E9)
30	=F9	1300	=B10+C10	0.13	=D10*(1+E10)
31	=F10	1400	=B11+C11	0.1325	=D11*(1+E11)
32	=F11	1500	=B12+C12	0.135	=D12*(1+E12)
33	=F12	1600	=B13+C13	0.1375	=D13*(1+E13)

## **Culminating Task: Growing by Leaps and Bounds**

Name \_\_\_\_\_ Date \_\_\_\_\_

### **Mathematical Goals**

- Create one–variable exponential equations from contextual situations.
- Solve and interpret the solution to exponential equations in context.
- Write and graph an equation to represent an exponential relationship.
- Graph equations on coordinate axes with labels and scales.
- Use technology to explore exponential graphs.

### **Essential Questions**

- How can I apply what I have learned about mathematical modeling to real–world situations?

### **Georgia Standards of Excellence**

**MGSE9–12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions, and simple rational and~~ exponential functions (integer inputs only).

**MGSE9–12.A.CED.2** Create linear and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase in two or more variables” refers to formulas like the compound interest formula, in which  $A = P(1 + r/n)^{nt}$  has multiple variables.)

**MGSE9–12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non–solution) under the established constraints.

**MGSE9–12.A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *Examples: Rearrange Ohm’s law  $V = IR$  to highlight resistance  $R$ ; Rearrange area of a circle formula  $A = \pi r^2$  to highlight the radius  $r$ .*

**MGSE9–12.N.Q.1** Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

- a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
- b. Convert units and rates using dimensional analysis (English–to–English and Metric–to–Metric without conversion factor provided and between English and Metric with conversion factor);
- c. Use units within multi–step problems and formulas; interpret units of input and resulting units of output.

**MGSE9–12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling. Given a situation, context, or problem, students will determine, identify, and use appropriate quantities for representing the situation.



**MGSE9–12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *For example, money situations are generally reported to the nearest cent (hundredth). Also, an answer’s precision is limited to the precision of the data given.*

**MGSE9–12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.

**MGSE9–12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

**MGSE9–12.A.SSE.1b** Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.

**Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

**Culminating Task: Growing by Leaps and Bounds**

Name \_\_\_\_\_ Date \_\_\_\_\_

**Part I: Meet Linda**

Linda’s lifelong dream has been to open her own business. After working, sacrificing, and saving, she finally has enough money to open up an ice cream business. The grand opening of her business is scheduled for the Friday of Memorial Day weekend. She would like to have a soft opening for her business on the Tuesday before. The soft opening should give her a good idea of any supply or personnel issues and give her time to correct them before the big official opening.

A soft opening means that the opening of the business is not officially announced; news of its opening is just spread by word of mouth (see, not all rumors are bad!). Linda needs a good idea of when she should begin the rumor in order for it to spread reasonably well before her soft opening. She has been told that about 10% of the people who know about an event will actually attend it. Based on this assumption, if she wants to have about 50 people visit her store on the Tuesday of the soft opening, she will need 500 people to know about it.

1. Linda plans to tell one person each day and will ask that person to tell one other person each day through the day of the opening, and so on. Assume that each new person who hears about the soft opening is also asked to tell one other person each day through the day of the opening and that each one starts the process of telling their friends on the day after he or she first hears. When should Linda begin telling others about the soft opening in order to have at least 500 people know about it by the day it occurs?

2. Let  $x$  represent the day number and let  $y$  be the number of people who know about the soft opening on day  $x$ . Consider the day before Linda told anyone to be Day 0, so that Linda is the only person who knows about the opening on Day 0. Day 1 is the first day that Linda told someone else about the opening.

a. Complete the following table.

Day	0	1	2	3	4	5
Number of people who know the rumor	1	2				

- b. Graph the points from the table in part a.
3. Write an equation that describes the relationship between  $x$  (day) and  $y$  (number of people who know) for the situation of spreading the news about the soft opening of Linda's ice cream store.
4. Does your equation describe the relationship between day and number who know about Linda's ice cream store soft opening completely? Why or why not?

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### Part II: What If?

The spread of a rumor or the spread of a disease can be modeled by a type of function known as exponential function; in particular, an exponential **growth** function. An **exponential function** has the form

$$f(x) = a \cdot b^x,$$

where  $a$  is a non-zero real number and  $b$  is a positive real number other than 1. An exponential growth function has a value of  $b$  that is greater than 1.

1. In the case of Linda's ice cream store, what values of  $a$  and  $b$  yield an exponential function to model the spread of the rumor of the soft store opening?

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2. In this particular case, what is an appropriate domain for the exponential function? What range corresponds to this domain?
3. In Part I, #2, you drew a portion of the graph of this function. Does it make sense to connect the dots on the graph? Why or why not?
4. How would the graph change if Linda had told two people each day rather than one and had asked that each person also tell two other people each day?
5. How would the equation change if Linda had told two people each day rather than one and had asked that each person also tell two other people each day? What would be the values of  $a$  and  $b$  in this case?
6. How long would it take for at least 500 people to find out about the opening if the rumor spread at this new rate?

**Part III: The Beginning of a Business**

How in the world did Linda ever save enough to buy the franchise to an ice cream store? Her mom used to say, “That Linda, why she could squeeze a quarter out of a nickel!” The truth is that Linda learned early in life that patience with money is a good thing. When she was just about 9 years old, she asked her dad if she could put her money in the bank. He took her to the bank and she opened her very first savings account.

Each year until Linda was 16, she deposited her birthday money into her savings account. Her grandparents (both sets) and her parents each gave her money for her birthday that was equal to twice her age; so on her ninth birthday, she deposited \$54 (\$18 from each couple).

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$$A = P \left( 1 + \frac{r}{n} \right)^{nt}$$

where  $A$  = final amount,  $P$  = principal amount,  $r$  = interest rate,  $n$  = number of times per year the interest is compounded, and  $t$  is the number of years the money is left in the account.

1. Verify the first two rows of the following chart, and then complete the chart to calculate how much money Linda had on her 16<sup>th</sup> birthday. Do not round answers until the end of the computation, then give the final amount rounded to the nearest cent.

Age	Birthday Money	Amount from previous year plus birthday money	Total at year end
8	N/A	N/A	0
9	54	$0 + 54 = 54$	55.63831630
10			
11			
12			
⋮	⋮	⋮	⋮

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2. On her 16<sup>th</sup> birthday, the budding entrepreneur asked her parents if she could invest in the stock market. She studied the newspaper, talked to her economics teacher, researched a few companies and finally settled on the stock she wanted. She invested all of her money in the stock and promptly forgot about it. When she graduated from college on her 22<sup>nd</sup> birthday, she received a statement from her stocks and realized that her stock had appreciated an average of 10% per year. How much was her stock worth on her 22<sup>nd</sup> birthday?

3. When Linda graduated from college, she received an academic award that carried a \$500 cash award. On her 22<sup>nd</sup> birthday, she used the money to purchase additional stock. She started her first job immediately after graduation and decided to save \$50 each month. On her 23<sup>rd</sup> birthday she used the \$600 (total of her monthly amount) savings to purchase new stock. Each year thereafter she increased her total of her savings by \$100 and, on her birthday each year, used her savings to purchase additional stock. Linda continued to learn about stocks and managed her accounts carefully. On her 35<sup>th</sup> birthday she looked back and saw that her stock had appreciated at 11% during the first year after college and that the rate of appreciation increased by 0.25% each year thereafter. At age 34, she cashed in enough stock to make a down payment on a bank loan to purchase her business. What was her stock worth on her 34<sup>th</sup> birthday? Use a table like the one below to organize your calculations.

Age	Amount from previous year	Amount Linda added from savings that year	Amount invested for the year	Interest rate for the year	Amount at year end
22	998.01	500	1498.01	11.00%	1662.79
23	1662.79	600		11.25%	
24		700		11.50%	
25		800		11.75%	
⋮	⋮	⋮	⋮	⋮	⋮

**Additional Resources and Supplemental Tasks**

The tasks featured in this table provide additional resources and supplemental tasks to be incorporated into unit 1 instruction as deemed appropriate by the instructor.

<b>UNIT 1: Linear Equations in One Variable</b>	<b>Standards Addressed in the Task</b>
<a href="#">KD vs. Lebron</a>  <a href="http://whenmathhappens.files.wordpress.com/2013/12/lebron-kd-warm-up.pdf">http://whenmathhappens.files.wordpress.com/2013/12/lebron-kd-warm-up.pdf</a>  When the Math Happens Task developed by Dane Ehlert	A.REI.1, 3, A.CED.1
<a href="#">Pepsi Points</a>  <a href="http://mrpiccmath.weebly.com/1/post/2012/07/3-acts-pepsi-points.html">http://mrpiccmath.weebly.com/1/post/2012/07/3-acts-pepsi-points.html</a>  Task Developed by Timon Piccini	A.REI.1, 3, A.CED.1
<a href="#">Gas Pump</a>  <a href="http://wmh3acts.weebly.com/gas-pump.html">http://wmh3acts.weebly.com/gas-pump.html</a>  When the Math Happens Task developed by Dane Ehlert	A.REI.1, 3, A.CED.1
<a href="#">Bottomless Coffee Mug</a>  <a href="http://www.101qs.com/1694-bottomless-mug-act-1">http://www.101qs.com/1694-bottomless-mug-act-1</a>  Task Developed by Andrew Stadel	A.REI.1, 3, A.CED.1
<a href="#">The Perfect Chocolate Milk Mix</a>  <a href="http://www.yummymath.com/2010/chocolate-milk-and-mixture-problems/">http://www.yummymath.com/2010/chocolate-milk-and-mixture-problems/</a>  Task Featured on Yummymath)	A.CED.1, 8.EE.7

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<a href="#">Styrofoam Cups</a>  <a href="http://mr-stadel.blogspot.com/2013/01/styrofoam-cups.html%20">http://mr-stadel.blogspot.com/2013/01/styrofoam-cups.html%20</a>  Task Developed by Andrew Stadel	A.CED.1, 2, 3, 4, A.REI.3
<a href="#">M&amp;M's</a>  <a href="http://wmh3acts.weebly.com/mms.html">http://wmh3acts.weebly.com/mms.html</a>  When the Math Happens Task developed by Dane Ehlert	A.CED.1, 2, A.REI.1
Traffic Jam Task from Illustrative Math <a href="https://www.illustrativemathematics.org/illustrations/84">https://www.illustrativemathematics.org/illustrations/84</a>	N.Q.1, N.Q.2, N.Q.3
Felicia's Drive Task from Illustrative Math <a href="https://www.illustrativemathematics.org/illustrations/80">https://www.illustrativemathematics.org/illustrations/80</a>	N.Q1, N.Q.3
Harvesting the Fields from Illustrative Math <a href="https://www.illustrativemathematics.org/illustrations/83">https://www.illustrativemathematics.org/illustrations/83</a>	N.Q.1, A.CED.1
How Much is a Penny Worth? Task from Illustrative Math <a href="https://www.illustrativemathematics.org/illustrations/473">https://www.illustrativemathematics.org/illustrations/473</a>	N.Q.1
Ice Cream Van Task from Illustrative Math <a href="https://www.illustrativemathematics.org/illustrations/85">https://www.illustrativemathematics.org/illustrations/85</a>	N.Q.1
Mixing Candies Task from Illustrative Math <a href="https://www.illustrativemathematics.org/illustrations/389">https://www.illustrativemathematics.org/illustrations/389</a>	A.SSE.1
Mixing Fertilizers Task from Illustrative Math <a href="https://www.illustrativemathematics.org/illustrations/88">https://www.illustrativemathematics.org/illustrations/88</a>	A.SSE.1
Animal Populations Task from Illustrative Math <a href="https://www.illustrativemathematics.org/illustrations/436">https://www.illustrativemathematics.org/illustrations/436</a>	A.SSE.1, 2