Georgia
Standards of Excellence
Curriculum Frameworks

Mathematics

GSE Coordinate Algebra

Unit 2: Reasoning with Equations and Inequalities

Georgia Department of Education

Richard Woods, Georgia's School Superintendent

"Educating Georgia's Future"

Unit 2

Reasoning with Equations and Inequalities

Table of Contents

OVERVIEW

In this unit students will:

- solve linear equations in one variable
- solve linear inequalities in one variable
- solve a system of two equations in two variables by using multiplication and addition
- solve a system of two equations in two variables graphically
- graph a linear inequality in two variables

By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. The second unit of Coordinate Algebra expands the previously learned concepts of solving and graphing linear equations and inequalities, focusing on the reasoning and understanding involved in justifying the solution. Students are asked to explain and justify the mathematics required to solve both simple equations and systems of equations in two variables using both graphing and algebraic methods. Students explore systems of equations and inequalities, and they find and interpret their solutions. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. All of this work is grounded on understanding quantities and on relationships between them.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. This unit provides much needed content information and excellent learning activities. However, the intent of the framework is not to provide a comprehensive resource for the implementation of all standards in the unit. A variety of resources should be utilized to supplement this unit. The tasks in this unit framework illustrate the types of learning activities that should be utilized from a variety of sources. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the **"Strategies for Teaching and Learning"** and the tasks listed under **"Evidence of Learning"** be reviewed early in the planning process.

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

KEY STANDARDS

Understand solving equations as a process of reasoning and explain the reasoning
MGSE9-12.A.REI.1 Using algebraic properties and the properties of real numbers, justify the steps of a simple, one–solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties. *(Students should focus on and master linear equations and be able to extend and apply their reasoning to other types of equations in future courses.)*

Solve equations and inequalities in one variable
MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. *(For example, given $ax + 3 = 7$, solve for x.)*

Solve systems of equations
MGSE9–12.A.REI.5 Show and explain why the elimination method works to solve a system of two–variable equations. *(Limit to linear systems.)*
MGSE9–12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically
MGSE9-12.A.REI.12 Graph the solution set to a linear inequality in two variables.

RELATED STANDARDS

Reason quantitatively and use units to solve problems.

MGSE9-12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
b. Convert units and rates using dimensional analysis (English-to-English and Metric-to-Metric without conversion factor provided and between English and Metric with conversion factor);
c. Use units within multi-step problems and formulas; interpret units of input and resulting units of output.

Create equations that describe numbers or relationships

MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, ~~quadratic, simple rational,~~ and exponential functions (integer inputs only).

MGSE9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

STANDARDS FOR MATHEMATICAL PRACTICE

Refer to the Comprehensive Course Overview for more detailed information about the Standards for Mathematical Practice.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ENDURING UNDERSTANDINGS

- Solve linear equations and inequalities in one variable.
- Graph linear equations and inequalities in two variables.
- Solve systems of linear equations in two variables exactly and approximately.
- Create linear equations and inequalities in one variable and use them in a contextual situation to solve problems.
- Create equations in two or more variables to represent relationships between quantities.
- Graph equations in two variables on a coordinate plane and label the axes and scales.
- Write and use a system of equations and/or inequalities to solve real world problems.

ESSENTIAL QUESTIONS

- How do I justify the solution to an equation?
- How do I solve an equation in one variable?
- How do I solve an inequality in one variable?
- How do I prove that a system of two equations in two variables can be solved by multiplying and adding to produce a system with the same solutions?
- How do I solve a system of linear equations graphically?
- How do I graph a linear inequality in two variables?
- How do I graph a system of linear inequalities in two variables?

CONCEPTS AND SKILLS TO MAINTAIN

Students may not realize the importance of unit conversion in conjunction with computation when solving problems involving measurement. Since today's calculating devices often display 8 to 10 decimal places, students frequently express answers to a much greater degree of precision than is required.

Measuring commonly used objects and choosing proper units for measurement are part of the mathematics curriculum prior to high school. In high school, students experience a broader variety of units through real–world situations and modeling, along with the exploration of the different levels of accuracy and precision of the answers.

An introduction to the use of variable expressions and their meaning, as well as the use of variables and expressions in real–life situations, is included in the Expressions and Equations Domain of Grade 7.

Working with expressions and equations, including formulas, is an integral part of the curriculum in Grades 7 and 8. In high school, students explore in more depth the use of equations and inequalities to model real–world problems, including restricting domains and ranges to fit the problem's context, as well as rewriting formulas for a variable of interest.

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre–assess to determine whether instructional time should be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Using the Pythagorean Theorem
- Understanding slope as a rate of change of one quantity in relation to another quantity
- Interpreting a graph
- Creating a table of values
- Working with functions
- Writing a linear equation
- Using inverse operations to isolate variables and solve equations
- Maintaining order of operations
- Understanding notation for inequalities
- Being able to read and write inequality symbols
- Graphing equations and inequalities on the coordinate plane
- Understanding and use properties of exponents
- Graphing points
- Choosing appropriate scales and label a graph

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for high school children. **Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**

http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary.

http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

- **Algebra:** The branch of mathematics that deals with relationships between numbers, utilizing letters and other symbols to represent specific sets of numbers, or to describe a pattern of relationships between numbers.
- **Coefficient:** A number multiplied by a variable.
- **Equation:** A number sentence that contains an equals symbol.
- **Expression:** A mathematical phrase involving at least one variable and sometimes numbers and operation symbols.
- **Inequality**: Any mathematical sentence that contains the symbols > (greater than), < (less than), ≤ (less than or equal to), or ≥ (greater than or equal to).
- **Ordered Pair**: A pair of numbers, (x, y), that indicate the position of a point on a Cartesian plane.
- **Substitution:** To replace one element of a mathematical equation or expression with another.
- **Variable:** A letter or symbol used to represent a number.

The Properties of Operations

Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $–a$ so that $a + (–a) = (–a) + a = 0$.
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$.
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

The Properties of Equality

Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a – c = b – c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

EVIDENCE OF LEARNING

By the conclusion of this unit, students should be able to demonstrate the following competencies:

- justify the solution of a linear equation and inequality in one variable
- justify the solution to a system of 2 equations in two variables
- solve a system of linear equations in 2 variables by graphing
- graph a linear inequality in 2 variables
- graph a system of linear inequalities in 2 variables

TEACHER RESOURCES

The following pages include teacher resources that teachers may wish to use to supplement instruction.

Web Resources

The following list is provided as a sample of available resources and is for informational purposes only. It is your responsibility to investigate them to determine their value and appropriateness for your district. GaDOE does not endorse or recommend the purchase of or use of any particular resource.

- Same Solution Task
  http://www.illustrativemathematics.org/illustrations/613
  This task gives students 6 different equations and asks them to show whether they have the same solution.

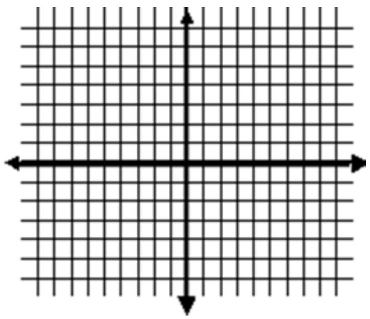
- Balancing Equations Applet
  http://illuminations.nctm.org/ActivityDetail.aspx?id=10
  This applet allows you to demonstrate how a point of intersection of two lines is the x–value where the two expressions have the same y–value. The applet shows the graph alongside a scale with algebraic expressions as the 'weights.'

This page intentionally left blank

Methods for Solving Systems of Equations

$$2x + 3y = 5$$
$$4x - y = 17$$

GRAPHING



Intersecting Lines:

Parallel Lines:

Coinciding Lines:

SUBSTITUTION

ELIMINATION

What if both of the variables cancel out? Look at the resulting arithmetic equation.
* False statement indicates the lines are parallel so ____________________________.
* True statement indicates the lines coincide so _______________________________.

Adapted from Graphic Organizer by Dale Graham and Linda Meyer Thomas County Central High School; Thomasville GA

This page intentionally left blank

FORMATIVE ASSESSMENT LESSONS (FAL)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Overview.

SPOTLIGHT TASKS

A Spotlight Task has been added to each GSE mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit–level Georgia Standards of Excellence, and research–based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3–Act Tasks based on 3–Act Problems from Dan Meyer and Problem–Based Learning from Robert Kaplinsky.

3–ACT TASKS

A Three–Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.
More information along with guidelines for 3–Act Tasks may be found in the Comprehensive Course Overview.

TASKS

The following tasks represent the level of depth, rigor, and complexity expected of all Coordinate Algebra students. These tasks, or tasks of similar depth and rigor, should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as a performance task, they may also be used for teaching and learning (learning/scaffolding task).

Task Name	Task Type *Grouping Strategy*	Content Addressed	Standards
Jaden's Phone Plan *30 min*	Scaffolding Task *Individual/Partner Task*	Model and write an equation in one variable Represent constraints with inequalities	A.REI.1, 3
Ivy Smith Grows Up *45–60 minutes* PDF / Word	Achieve CCSS–CTE Classroom Task	Model a situation with an equation and solve.	N.Q.1 A.CED.1, 3 A.REI.3
Solving Systems of Equations Algebraically: Part 1 and Part 2 *90 min*	Scaffolding Task *Individual/Partner Task*	Justify the solution to a system of equations by graphing and substituting values into the system Create new equations by multiplying one or more equations by a constant Solve systems of equations by elimination	A.REI.5,6
Dental Impressions *45–60 minutes* PDF / Word	Achieve CCSS–CTE Classroom Task	Model a situation with a system of linear equations and solve.	N.Q.1 A.CED.1 A.REI.6
Ground Beef *45–60 minutes* PDF / Word	Achieve CCSS–CTE Classroom Task	Model a situation with a system of linear equations and solve.	N.Q.1, 3 A.CED.2 A.REI.6
Systems of Weights (Spotlight Task) *30–45 minutes*	Discovery Task *Partner Task*	Model a situation with a system of linear equations and solve.	A.REI.6, 12
Solving Linear Equations in Two Variables (FAL) *60–90 minutes* PDF	Formative Assessment Lesson *Individual / Small Group*	Solve problems using a system of equations.	A.REI.5,6

Boomerangs (FAL) *60–90 minutes* PDF	Formative Assessment Lesson *Individual / Small Group*	Solve optimization tasks using a system of equations	A.REI.5, 6, 12
Summer Job *90 min*	Scaffolding Task *Partner/Small Group Task*	Modeling linear patterns Creating equation and inequalities in one and two variables to represent relationships	A.REI.12
Graphing Inequalities *45 min*	Extension Task *Partner/Small Group Task*	Modeling with inequalities	A.REI.12
Modeling Situations with Linear Equations (FAL) *60–90 minutes* PDF	Formative Assessment Lesson *Individual / Small Group*	Model everyday situations with algebra.	A.REI.12
Family Outing *90 min*	Culminating Task *Partner/Small Group Task*	Graph equations on coordinate axes with labels and scales Solve systems of equations Solve systems of inequalities Determining constraints	A.REI.1, 3, 5, 6, 12

Jaden's Phone Plan (Scaffolding Task)

Introduction

In this task, students will solve a series of linear equations and inequality word problems to help Jaden choose a cell phone plan. In order to help Jaden, students must explain in detail each step of the problem and justify the answer.

Mathematical Goals

- Create one–variable linear equations and inequalities from contextual situations.
- Solve and interpret the solution to multi–step linear equations and inequalities in context.

Essential Questions

- How do I solve an equation in one variable?
- How do I justify the solution to an equation?

Georgia Standards of Excellence

MGSE9–12.A.REI.1 Using algebraic properties and the properties of real numbers, justify the steps of a simple, one–solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties. *(Students should focus on and master linear equations and be able to extend and apply their reasoning to other types of equations in future courses.)*

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. *(For example, given $ax + 3 = 7$, solve for x.)*

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
Students will justify their equations using reasoning.
4. Model with mathematics.
Students will us algebra to describe situations.

Background Knowledge

- Students can solve equations.
- Students know key words for modeling situations.

Common Misconceptions

- Students may mistake cents for dollars when modeling the situations.
- Students may confuse the input/output of the situation.
- Students may have issues substituting more than one variable.

Materials

- None

Grouping

- Individual / Partner

Differentiation

Extension:

- Determine what situations (number of texts and calls) would cause Plan A to be cheaper.
  Solution:
  $A = 0.15t + 0.10c; \quad B = 15 + 0.05(t + c); \quad A < B$
  $0.15t + 0.10c < 15 + 0.05(t + c)$
  $0.15t + 0.10c < 15 + 0.05t + 0.05c$
  $0.10t + 0.05c < 15$
  $10t + 5c < 1500$
  $2t + c < 300$
  If the value of $2t + c$ is less than 300, then Plan A is the cheaper choice.

Intervention:

- Review solving equations out of context.

Formative Assessment Questions

- How can linear equations model a real life situation? What key components should you look for to model a real life situation with a linear equation?
- How do you solve linear equations?

Jaden's Phone Plan – Teacher Notes

Jaden has a prepaid phone plan (Plan *A*) that charges 15 cents for each text sent and 10 cents per minute for calls.

***Comment:* As students solve equations throughout this task, have them explain each step using properties of operations or properties of equality.**

1. If Jaden uses only text, write an equation for the cost *C* of sending *t* texts.

   C = $.15t

   a. How much will it cost Jaden to send 15 texts? Justify your answer.

   C = $.15 * 15
   C = $ 2.25

   b. If Jaden has $6, how many texts can he send? Justify your answer.

   C = $.15t
   $6 = $.15t
   $6/$.15 = t
   t = 40 texts

2. If Jaden only uses the talking features of his plan, write an equation for the cost *C* of talking *m* minutes.

   C = $.10m

   a. How much will it cost Jaden to talk for 15 minutes? Justify your answer.

   C = $.10 * 15
   C = $1.50

   b. If Jaden has $6, how many minutes can he talk? Justify your answer.

   C= $.10m
   $6 = .10m
   $6/.10 = m
   m = 60 minutes

3. If Jaden uses both talk and text, write an equation for the cost *C* of sending t texts and talking *m* minutes.

C = $.15t + $.10m

a. How much will it cost Jaden to send 7 texts and talk for 12 minutes? Justify your answer.

C = $.15t + $.10m
C = $.15*7 + $.10*12
C = $1.05 + $1.20
C = $2.25

b. If Jaden wants to send 21 texts and only has $6, how many minutes can he talk? Will this use all of his money? If not, will how much money will he have left? Justify your answer.

C = $.15t + $.10m
$6 = $.15*21 + $.10m
$6 = $3.15 + .10m
$6 – $3.15 = .10m
$2.85 = .10m
$2.85/.10 = m
m = 28.5 minutes

Since most carriers will charge a full minute for any fraction of a minute, Jaden can talk for 28 minutes. He will have $0.05 left over if he talks for 28 minutes.

Jaden discovers another prepaid phone plan (Plan *B*) that charges a flat fee of $15 per month, then $.05 per text sent or minute used.

4. Write an equation for the cost of Plan *B*.

C = $15 + $.05n (Since the cost of text and talk are the same, the same variable can represent both.)

In an average month, Jaden sends 200 texts and talks for 100 minutes.

5. Which plan will cost Jaden the least amount of money? Justify your answer.

Plan A: $C = \$.15t + \$.10m$
$C = \$.15*200 + \$.10*100$
$C = \$30 + \$10$
$C = \$40$

Plan B: $C = 15 + .05n$
$C = \$15 + \$.05(200 + 100)$
$C = \$15 + \$.05(300)$
$C = \$15 + \$15$
$C = \$30$

Based on Jaden's average usage, the cost for Plan A is $40 per month and the cost for Plan B is $30 per month. Therefore, Plan B will cost Jaden the least amount of money.

Scaffolding Task: Jaden's Phone Plan

Name________________________________ **Date**_________________

Mathematical Goals

- Create one–variable linear equations and inequalities from contextual situations.
- Solve and interpret the solution to multi–step linear equations and inequalities in context.

Essential Questions

- How do I solve an equation in one variable?
- How do I justify the solution to an equation?

Georgia Standards of Excellence

MGSE9–12.A.REI.1 Using algebraic properties and the properties of real numbers, justify the steps of a simple, one–solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties. *(Students should focus on and master linear equations and be able to extend and apply their reasoning to other types of equations in future courses.)*

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. *(For example, given* $ax + 3 = 7$*, solve for* x*.)*

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

Scaffolding Task: Jaden's Phone Plan

Name________________________________ **Date**__________________

Jaden has a prepaid phone plan (Plan *A*) that charges 15 cents for each text sent and 10 cents per minute for calls.

1. If Jaden uses only text, write an equation for the cost C of sending t texts.

   a. How much will it cost Jaden to send 15 texts? Justify your answer.

   b. If Jaden has \$6, how many texts can he send? Justify your answer.

2. If Jaden only uses the talking features of his plan, write an equation for the cost C of talking m minutes.

   a. How much will it cost Jaden to talk for 15 minutes? Justify your answer.

   b. If Jaden has \$6, how many minutes can he talk? Justify your answer.

3. If Jaden uses both talk and text, write an equation for the cost C of sending t texts and talking m minutes.

   a. How much will it cost Jaden to send 7 texts and talk for 12 minutes? Justify your answer.

   b. If Jaden wants to send 21 texts and only has \$6, how many minutes can he talk? Will this use all of his money? If not, will how much money will he have left? Justify your answer.

Jaden discovers another prepaid phone plan (Plan *B*) that charges a flat fee of $15 per month, then $.05 per text sent or minute used.

4. Write an equation for the cost of Plan *B*.

In an average month, Jaden sends 200 texts and talks for 100 minutes.

5. Which plan will cost Jaden the least amount of money? Justify your answer.

Ivy Smith Grows Up (Career and Technical Education Task)

Source: National Association of State Directors of Career Technical Education Consortium
PDF: **http://www.achieve.org/files/CCSS–CTE–Task–IvySmith–GrowsUp–FINAL.pdf**
Word: **http://www.achieve.org/files/CCSS–CTE–Task–IvySmith–GrowsUp–FINAL.doc**

Introduction

This task uses the growth of newborns and infants to help students understand conversion of units and determine a linear model for the data.

Mathematical Goals

- Write and use a linear model for data.
- Convert between standard and metric units.

Essential Questions

- How do you use real–life data to determine a linear model and use this model to approximate missing data?

Georgia Standards of Excellence

MGSE9-12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
b. Convert units and rates using dimensional analysis (English-to-English and Metric-to-Metric without conversion factor provided and between English and Metric with conversion factor);
c. Use units within multi-step problems and formulas; interpret units of input and resulting units of output.

MGSE9–12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions (integer inputs only).

MGSE9–12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non–solution) under the established constraints. *(Limit to linear equations and inequalities.)*

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. *(For example, given $ax + 3 = 7$, solve for x.)*

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
   Students analyze givens, constraints, relationships, and goals. They must make conjectures about the form and meaning of the solution and plan a solution pathway.
2. Reason abstractly and quantitatively.
   Students must attend to the meaning of the quantities throughout the problem.
4. Model with mathematics.
   Students translate constraints into equations and extract information from graphs.
5. Use appropriate tools strategically.
   Students use website, calculator, and the attached chart.
6. Attend to precision.
   Students must use units, convert units, and perform calculations precisely.

Background Knowledge

- Students can convert units.
- Students can write the equation of a line given two points on a line.

Common Misconceptions

- Students may struggle to convert between standard and metric units.

Materials

- Graph paper
- Chart from website for #5

Grouping

- Individual / small group

Differentiation

- See extensions in task.

Solving System of Equations Algebraically (Scaffolding Task)

Comment:

This task is written as self–guided instruction. Its questions are very similar to the questions a teacher would ask during direct instruction. It would best be used as a whole class task where teachers guide the students through the task. Students with higher level abilities could proceed ahead of the class.

Introduction

In this task, students justify the solution to a system of equations by both graphing and substituting values into the system. Students will then show that multiplying one or both equations in a system of equations by a constant creates a new system with the same solutions as the original. This task will lead into using the elimination method for solving a system of equations algebraically.

Mathematical Goals

- Model and write an equation in one variable and solve a problem in context.
- Create one–variable linear equations and inequalities from contextual situations.
- Represent constraints with inequalities.
- Solve word problems where quantities are given in different units that must be converted to understand the problem.

Essential Questions

- How do I solve an equation in one variable?
- How do I justify the solution to an equation?

Georgia Standards of Excellence

MGSE9–12.A.REI.5 Show and explain why the elimination method works to solve a system of two–variable equations. *(Limit to linear systems.)*

MGSE9–12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
Students will have to decipher the instructions and complete the described steps.

3. Construct viable arguments and critique the reasoning of others.
Students will have to justify the steps in the elimination process.

Background Knowledge

- Students can solve equations.
- Students can combine like terms.
- Students can distribute multiplication over addition.

Common Misconceptions

- Students may forget to multiply both sides of the equation to preserve equality.
- Students may have difficulty following the written instructions.

Materials

- Ruler
- Calculator

Grouping

- Individual / Whole class

Differentiation

Extension:

- Have students work through the task individually.

Intervention:

- Have students work through the task as a class.

Formative Assessment Questions

- Why does multiplying an equation by a constant not affect the solution(s) of the equation?
- What are the different pathways to solve a system of equations using the elimination method? What determines the pathway needed?

<u>Solving Systems of Equations Algebraically – Teacher Notes</u>

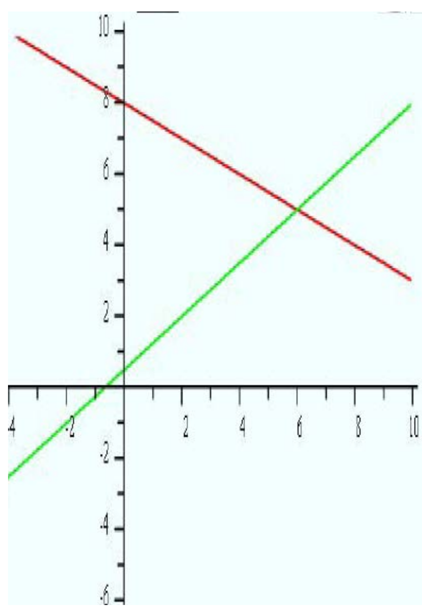
Comment:* *As students solve equations throughout this task, have them continue to explain each step using properties of operations or properties of equality.

Part 1:
You are given the following system of two equations: $x + 2y = 16$
$3x - 4y = -2$

1. What are some ways to prove that the ordered pair (6, 5) is a solution?

Graphing and direct substitution are two methods for proving that (6,5) is a solution.

a. Prove that (6, 5) is a solution to the system by graphing the system.



b. Prove that (6, 5) is a solution to the system by substituting in for both equations.

x + 2y = 16	***3x – 4y = –2***
6 + 2*5 = 16	***3*6 – 4*5 = –2***
6 + 10 = 16	***18 – 20 = –2***
16 = 16	***–2 = –2***

The solution (6, 5) works for both equations.

2. Multiply both sides of the equation $\boldsymbol{x + 2y = 16}$ by the constant '7'. Show your work.

$\mathbf{7}*(x + 2y) = \mathbf{7}*16$
$7*x + 7 * 2y = 112$

$\underline{7x + 14y = 112}$ New Equation

a. Does the new equation still have a solution of (6, 5)? Justify your answer.

$7x + 14y = 112$
$7*6 + 14*5 = 112$
$42 + 70 = 112$

b. Why do you think the solution to the equation never changed when you multiplied by the '7'?

Answers may vary.

3. Did it have to be a '7' that we multiplied by in order for (6, 5) to be a solution?

Answers may vary.

a. Multiply $\boldsymbol{x + 2y = 16}$ by three other numbers and see if (6, 5) is still a solution.

Students may pick any constant to multiply the original equation by. As long as the multiplication is correct, the solution (6, 5) will still work.

i. ________________________
ii. ________________________
iii. ________________________

b. Did it have to be the first equation $\boldsymbol{x + 2y = 16}$ that we multiplied by the constant for (6, 5) to be a solution? Multiply $\boldsymbol{3x – 4y = –2}$ by '7'? Is (6, 5) still a solution?
Use this exercise to help students discover that multiplying the equation by any constant will not change the solution.

$7(3x – 4y) = 7*–2$
$21x – 28y = –14$

$21*6 – 28 *5 = –14$
$126 – 140 = –14$
$–14 = –14$

c. Multiply $\mathbf{3x - 4y = -2}$ by three other numbers and see if (6, 5) is still a solution.

i. ________________________

ii. ________________________

iii. ________________________

4. Summarize your findings from this activity so far. Consider the following questions: What is the solution to a system of equations and how can you prove it is the solution?
Does the solution change when you multiply one of the equations by a constant?
Does the value of the constant you multiply by matter?
Does it matter which equation you multiply by the constant?

Answers will vary, but the purpose of this task is for students to discover that multiplying an equation by a constant does not change the solution to the equation, leading into the elimination method for solving a system of equations.

Let's explore further with a new system. $5x + 6y = 9$
$4x + 3y = 0$

5. Show by substituting in the values that (–3, 4) is the solution to the system.

$5x + 6y = 9$	$4x + 3y = 0$
$5(-3) + 6(4) = 9$	$4(-3) + 3(4) = 0$
$-15 + 24 = 9$	$-12 + 12 = 0$
$9 = 9$	$0 = 0$

6. Multiply $\mathbf{4x + 3y = 0}$ by '–5'. Then add your answer to $\mathbf{5x + 6y = 9}$. Show your work below.

$(-5)*(4x + 3y) = (-5)*0$ → ___$-20x - 15y = 0$___ Answer

\+ ___$5x + 6y = 9$___

___$-15x - 9y = 9$___ New Equation

7. Is (–3, 4) still a solution to the new equation? Justify your answer.

$$-15x - 9y = 9$$
$$-15(-3) - 9(4) = 9$$
$$45 - 36 = 9$$

8. Now multiply $4x + 3y = 0$ by '–2'. Then add your answer to $\mathbf{5x + 6y = 9}$. Show your work below.

$$\begin{array}{r} -8x - 6y = 0 \\ \underline{+\,5x + 6y = 9} \\ -3x + 0y = 9 \\ -3x = 9 \\ x = 9/(-3) \\ x = -3 \end{array}$$

a. What happened to the *y* variable in the new equation?

The y variable became 0y using the additive inverse property, therefore being eliminated from the equation.

b. Can you solve the new equation for *x*? What is the value of *x*? Does this answer agree with the original solution?

See work above.

The original solution was (–3, 4), so a value of x = –3 does agree.

c. How could you use the value of *x* to find the value of *y* from one of the original equations? Show your work below.

Substitute the value of x into one of the equations to find the value of 'y'.

$$5x + 6y = 9$$
$$5(3) + 6y = 9$$
$$-15 + 6y = 9$$
$$6y = 9 + 15$$
$$6y = 24$$
$$y = 24/6$$
$$y = 4$$

The method you have just used is called the Elimination Method for solving a system of equations. When using the Elimination Method, one of the original variables is eliminated from the system by adding the two equations together. Use the Elimination Method to solve the following system of equations:

9. $-3x + 2y = -6$
   $5x - 2y = 18$

   $-3x + 2y = -6$
   $\underline{5x - 2y = 18}$
   $2x + 0y = 12$
   $2x = 12$
   $x = 6$

   $5(6) - 2y = 18$
   $30 - 2y = 18$
   $-2y = -12$
   $y = 6$

   Solution :
   (6, 6)

   Check:
   $-3(6) + 2(6) = -6$
   $5(6) - 2(6) = 18$

10. $-5x + 7y = 11$
    $5x + 3y = 19$

    $-5x + 7y = 11$
    $\underline{5x + 3y = 19}$
    $0x + 10y = 30$
    $10y = 30$
    $y = 3$

    $-5x + 7(3) = 11$
    $-5x + 21 = 11$
    $-5x = -10$
    $x = 2$

    (2, 3)

    $-5(2) + 7(3) = 11$
    $5(2) + 3(3) = 19$

Part 2:

When using the Elimination Method, one of the original variables is eliminated from the system by adding the two equations together. Sometimes it is necessary to multiply one or both of the original equations by a constant. The equations are then added together and one of the variables is eliminated. Use the Elimination Method to solve the following system of equations:

1. $4x + 3y = 14$ (Equation 1)
   $-2x + y = 8$ (Equation 2)

   Choose the variable you want to eliminate.

a. To make the choice, look at the coefficients of the *x* terms and the *y* terms. The coefficients of *x* are '4' and '–2'. If you want to eliminate the *x* variable, you should multiply Equation 2 by what constant?
   Multiply the 2nd equation by the constant '2'.

i. Multiply Equation 2 by this constant. Then add your answer equation to Equation 1. What happened to the *x* variable?

2(–2x + y) = 2(8)
–4x + 2y = 16

–4x + 2y = 16 (New Equation 2)
+ 4x + 3y = 14 (Equation 1)
0x + 5y = 30
5y = 30
y = 6

The x variable was eliminated.

ii. Solve the equation for *y*. What value did you get for *y*?
See above.

iii. Now substitute this value for *y* in Equation 1 and solve for *x*. What is your ordered pair solution for the system?
x + 3y = 14
4x + 3(6) = 14
4x +18 = 14
4x = –4
x = –1

iv. Substitute your solution into Equation 1 and Equation 2 to verify that it is the solution for the system.
Solution: (–1, 6)

4x + 3y = 14	***4(–1) + 3(6) = 14***
–2x + y = 8	***–2(–1) + 6 = 8***

b. The coefficients of *y* are '3' and '1'. If you want to eliminate the *y* term, you should multiply Equation 2 by what constant?
Multiply Equation 2 by the constant (–3).

i. Multiply Equation 2 by this constant. Then add your answer equation to Equation 1. What happened to the *y* variable?

(–3)(–2x + y) = (–3)8
6x – 3y = –24 (New Equation 2)

6x – 3y = –24
+ 4x + 3y = 14

$10x + 0y = -10$
$10x = -10$
$x = -1$

ii. Solve the equation for x. What value did you get for x?
See above.

iii. Now substitute this value for x in Equation 1 and solve for y. What is your ordered pair solution for the system?
$4x + 3y = 14$
$4(-1) + 3y = 14$
$3y = 18$
$y = 6$

Solution: $(-1, 6)$

Use your findings to answer the following in sentence form:

c. Is the ordered pair solution the same for either variable that is eliminated? Justify your answer.
Answers may vary, but students should realize that the solution is the same for either variable that is eliminated.

d. Would you need to eliminate both variables to solve the problem? Justify your answer.
Answers may vary, but since either elimination yields the same answer, there is no need to eliminate both ways.

e. What are some things you should consider when deciding which variable to eliminate? Is there a wrong variable to eliminate?
Answers may vary, but there is no wrong variable to eliminate. Hopefully students have discovered that considering the coefficients of each variable will sometimes lessen the work involved in eliminating a particular variable.

f. How do you decide what constant to multiply by in order to make the chosen variable eliminate?
Answers may vary, but once the variable to be eliminated is chosen, the coefficients of that variable must be opposites so that the variable will eliminate.

Use the elimination method to solve the following systems of equations. Verify your solution by substituting it into the original system.

2. $3x + 2y = 6$ $-6x - 3y = -6$	3. $-6x + 5y = 4$ $7x - 10y = -8$	4. $5x + 6y = -16$ $2x + 10y = 5$
$(-2, 6)$	***$(0, \frac{4}{5})$***	***$(-5, \frac{3}{2})$***

Scaffolding Task: Solving Systems of Equations Algebraically

Name________________________________ **Date**_________________

Introduction

In this task, students justify the solution to a system of equations by both graphing and substituting values into the system. Students will then show that multiplying one or both equations in a system of equations by a constant creates a new system with the same solutions as the original. This task will lead into using the elimination method for solving a system of equations algebraically.

Mathematical Goals

- Model and write an equation in one variable and solve a problem in context.
- Create one–variable linear equations and inequalities from contextual situations.
- Represent constraints with inequalities.
- Solve word problems where quantities are given in different units that must be converted to understand the problem.

Essential Questions

- How do I solve an equation in one variable?
- How do I justify the solution to an equation?

Georgia Standards of Excellence

MGSE9–12.A.REI.5 Show and explain why the elimination method works to solve a system of two–variable equations. *(Limit to linear systems.)*

MGSE9–12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.

Scaffolding Task: Solving Systems of Equations Algebraically

Name________________________________ **Date**_________________

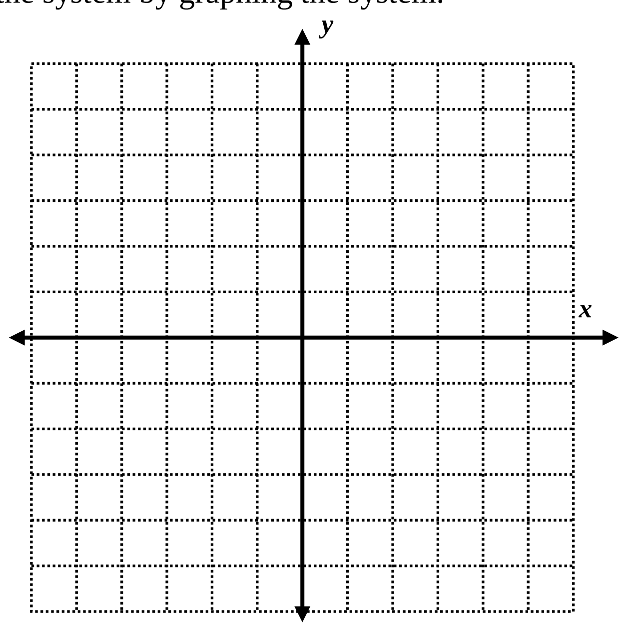
Part 1:

You are given the following system of two equations:

$$x + 2y = 16$$
$$3x - 4y = -2$$

1. What are some ways to prove that the ordered pair (6, 5) is a solution?

   a. Prove that (6, 5) is a solution to the system by graphing the system.



   b. Prove that (6, 5) is a solution to the system by substituting in for both equations.

2. Multiply both sides of the equation $x + 2y = 16$ by the constant '7'. Show your work.

   $7*(x + 2y) = 7*16$

   _______________ New Equation

   a. Does the new equation still have a solution of (6, 5)? Justify your answer.

   b. Why do you think the solution to the equation never changed when you multiplied by the '7'?

3. Did it have to be a '7' that we multiplied by in order for (6, 5) to be a solution?

   a. Multiply $x + 2y = 16$ by three other numbers and see if (6, 5) is still a solution.

      i. ________________________

      ii. ________________________

      iii. ________________________

   b. Did it have to be the first equation $x + 2y = 16$ that we multiplied by the constant for (6, 5) to be a solution? Multiply $3x - 4y = -2$ by '7'? Is (6, 5) still a solution?

c. Multiply $3x – 4y = –2$ by three other numbers and see if (6, 5) is still a solution.

i. ________________________

ii. ________________________

iii. ________________________

4. Summarize your findings from this activity so far. Consider the following questions:
What is the solution to a system of equations and how can you prove it is the solution?
Does the solution change when you multiply one of the equations by a constant?
Does the value of the constant you multiply by matter?
Does it matter which equation you multiply by the constant?

Let's explore further with a new system. $5x + 6y = 9$
$4x + 3y = 0$

5. Show by substituting in the values that (–3, 4) is the solution to the system.

6. Multiply $4x + 3y = 0$ by '–5'. Then add your answer to $5x + 6y = 9$. Show your work below.

$(–5)*(4x + 3y) = (–5)*0$ → ____________________ Answer

+ ____$5x + 6y = 9$______

_____________________ New Equation

7. Is (–3, 4) still a solution to the new equation? Justify your answer.

8. Now multiply $4x + 3y = 0$ by '–2'. Then add your answer to $5x + 6y = 9$. Show your work below.

    a. What happened to the y variable in the new equation?

    b. Can you solve the new equation for x? What is the value of x? Does this answer agree with the original solution?

    c. How could you use the value of x to find the value of y from one of the original equations? Show your work below.

The method you have just used is called the Elimination Method for solving a system of equations. When using the Elimination Method, one of the original variables is eliminated from the system by adding the two equations together. Use the Elimination Method to solve the following system of equations:

9. $-3x + 2y = -6$
   $5x - 2y = 18$

10. $-5x + 7y = 11$
    $5x + 3y = 19$

Part 2:

When using the Elimination Method, one of the original variables is eliminated from the system by adding the two equations together. Sometimes it is necessary to multiply one or both of the original equations by a constant. The equations are then added together and one of the variables is eliminated. Use the Elimination Method to solve the following system of equations:

1. $4x + 3y = 14$ (Equation 1)
   $-2x + y = 8$ (Equation 2)

Choose the variable you want to eliminate.

a. To make the choice, look at the coefficients of the *x* terms and the *y* terms. The coefficients of *x* are '4' and '–2'. If you want to eliminate the *x* variable, you should multiply Equation 2 by what constant?

   i. Multiply Equation 2 by this constant. Then add your answer equation to Equation 1. What happened to the *x* variable?

   ii. Solve the equation for *y*. What value did you get for *y*?

   iii. Now substitute this value for *y* in Equation 1 and solve for *x*. What is your ordered pair solution for the system?

   iv. Substitute your solution into Equation 1 and Equation 2 to verify that it is the solution for the system.

b. The coefficients of y are '3' and '1'. If you want to eliminate the y term, you should multiply Equation 2 by what constant?

i. Multiply Equation 2 by this constant. Then add your answer equation to Equation 1. What happened to the y variable?

ii. Solve the equation for x. What value did you get for x?

iii. Now substitute this value for x in Equation 1 and solve for y. What is your ordered pair solution for the system?

Use your findings to answer the following in sentence form:

c. Is the ordered pair solution the same for either variable that is eliminated? Justify your answer.

d. Would you need to eliminate both variables to solve the problem? Justify your answer.

e. What are some things you should consider when deciding which variable to eliminate? Is there a wrong variable to eliminate?

f. How do you decide what constant to multiply by in order to make the chosen variable eliminate?

Use the elimination method to solve the following systems of equations. Verify your solution by substituting it into the original system.

2. $3x + 2y = 6$
   $-6x - 3y = -6$

3. $-6x + 5y = 4$
   $7x - 10y = -8$

4. $5x + 6y = -16$
   $2x + 10y = 5$

Dental Impressions (Career and Technical Education Task)

Source: National Association of State Directors of Career Technical Education Consortium
PDF: **http://www.achieve.org/files/CCSS–CTE–Task–Dental–Impressions–FINAL.pdf**
Word: **http://www.achieve.org/files/CCSS–CTE–Task–Dental–Impressions–FINAL.doc**

Introduction

Students explore the supply needs of a dentist's office, determining plans for ordering materials. Students also use linear equations to determine the "break–even point" of two alternate plans.

Mathematical Goals

- Use units to plan and implement a solution strategy.
- Write linear equations and interpret their intersection as the "break–even point."

Essential Questions

- How can I use units and linear equations to answer questions about real–world situations?

Georgia Standards of Excellence

MGSE9-12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
b. Convert units and rates using dimensional analysis (English-to-English and Metric-to-Metric without conversion factor provided and between English and Metric with conversion factor);
c. Use units within multi-step problems and formulas; interpret units of input and resulting units of output.

MGSE9–12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

MGSE9–12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
   Students analyze givens, constraints, relationships, and goals. They must make conjectures about the form and meaning of the solution and plan a solution pathway.
2. Reason abstractly and quantitatively.
   Students make sense of quantities and their relationship in the problem situation.
3. Construct viable arguments and critique the reasoning of others.
   Students are asked for a written recommendation based on their mathematical findings.

4. Model with mathematics.
Students translate constraints into equations and extract information from both the algebraic solution and the graph.

6. Attend to precision.
Students must use units of measure, convert units, and perform calculations precisely.

Background Knowledge

- Students can use unit analysis to plan an approach multi–step problems.
- Students can convert units.
- Students understand the slope of a line as a rate of change and the *y*–intercept as an initial value.

Common Misconceptions

- In #3, students may round to the nearest whole number, 17, rather than rounding up to ensure they have enough gypsum for the last few impressions.
- In #4, students can show that the technology will be cheaper after two years simply by finding the cost for each after two years. Emphasize the instructions to “determine your break–even point” so students determine when they break even as opposed to the yes–no question of whether they break even within two years.
- When creating equations, students may confuse the slope (per–year rate) and the *y*–intercept (initial investment).

Materials

- None

Grouping

- Individual / partner

Differentiation

- See extensions in task.

Ground Beef (Career and Technical Education Task)

Source: National Association of State Directors of Career Technical Education Consortium
PDF: **http://www.achieve.org/files/CCSS–CTE–Ground–Beef–FINAL.pdf**
Word: **http://www.achieve.org/files/CCSS–CTE–Ground–Beef–FINAL.docx**

Introduction

Students use systems of equations to model mixture problems relating to a grocer's need to mix different formulations of ground beef.

Mathematical Goals

- Model and solve mixture problems using systems of equations.
- Calculate and compare profits.

Essential Questions

- How can I use systems of equations to model and solve real–world mixture problems?

Georgia Standards of Excellence

MGSE9–12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

MGSE9–12.A.CED.2 Create linear and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. *(The phrase "in two or more variables" refers to formulas like the compound interest formula, in which $A = P(1 + {}^{r}/_{n})^{nt}$ has multiple variables.)*

MGSE9-12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
b. Convert units and rates using dimensional analysis (English-to-English and Metric-to-Metric without conversion factor provided and between English and Metric with conversion factor);
c. Use units within multi-step problems and formulas; interpret units of input and resulting units of output.

MGSE9–12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *(For example, money situations are generally reported to the nearest cent (hundredth). Also, an answer's precision is limited to the precision of the data given.)*

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
   Students analyze givens, constraints, relationships, and goals. They must make conjectures about the form and meaning of the solution and plan a solution pathway.
2. Reason abstractly and quantitatively.
   Students make sense of quantities and their relationships throughout the problem.
4. Model with mathematics.
   Students translate constraints into a system of equations and use them to calculate the amounts of various types of meat needed.
6. Attend to precision.
   Students must be precise in establishing their equations and in performing calculations and they round solutions to appropriately represent money or decimal measures of weight.

Background Knowledge

- Students can set up and solve systems of linear equations in two variables.
- Students understand profit and percentages.

Common Misconceptions

- Students may need clarification to understand that different mixtures of boneless round and lean trim beef are used to create the three types of beef listed at the beginning of the task.
- Students may look at “per–pound” profit instead of overall profit in #4–5.

Materials

- None

Grouping

- Partner / small group

Differentiation

- See extensions in task.

Systems of Weights (Spotlight Task)

This spotlight task follows the 3 Act–Math task format originally developed by Dan Meyer. More information on these type tasks may be found at http://blog.mrmeyer.com/category/3acts/

Georgia Standards of Excellence
MGSE9–12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. *This standard is addressed through students setting up and solving systems of equations based on a scenario featured in a video. They can solve using a graph or other algebraic method.*

MGSE9–12.A.REI.12 Graph the solution set to a linear inequality in two variables.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them. In this task, students will formulate their own problem to solve. They will work to develop a method to solve the systems of equations.

4. Model with Mathematics. In this task, students will use equations to model the weights of different combinations of toys. They will use mathematics to model equations and inequalities

5. Use appropriate tools strategically. Students will need to select appropriate tools (graph paper, calculator, etc) in order to be successful at this task.

ESSENTIAL QUESTIONS
- How do I model a situation involving two unknown quantities?
- How do I solve a system of linear equations?
- How can I find the solutions of a system of linear inequalities?

MATERIALS REQUIRED
- Video "Systems of Weights" http://real.doe.k12.ga.us/vod/gso/math/videos/Coordinate–Algebra–Unit–2–Spotlight–opening.mp4
- Video "Systems of Weights Revealed" http://real.doe.k12.ga.us/vod/gso/math/videos/Coordinate–Algebra–Unit–2–Spotlight–reveal.mp4
- Graph paper (optional)
- Other Materials for a sequel could include a scale and other objects to weigh

It should also be noted that there is no "student version" of this task, as it involves showing a short video and formulating and solving a problem.

TIME NEEDED

- 30–45 minutes

It is recommended that this task be done towards the beginning of lessons involving systems of linear equations and linear inequalities

More information along with guidelines for 3–Act Tasks may be found in the Comprehensive Course Guide.

Act One Description (Dan Meyer http://blog.mrmeyer.com/2011/the–three–acts–of–a–mathematical–story/**)**
"Introduce the central conflict of your story/task clearly, visually, viscerally, using as few words as possible."

Act One:
Show the video "Systems of Weights". http://real.doe.k12.ga.us/vod/gso/math/videos/Coordinate–Algebra–Unit–2–Spotlight–opening.mp4

Pose the question: "How much does each type of candy weigh?"

This should launch the students into an investigation including setting up and solving a system of equations. Coordinate Algebra students will have experienced systems of equations (simultaneous equations) in 8th grade. Encourage students to develop methods to solve the system if they are unable to recall specific methods (substitution, elimination, graphing) from middle school.

Students could be grouped in pairs or work individually, according to the teacher's discretion.

Encourage students to be clear in defining their variables.

Sample Answer:

Let x be the weight in grams of one chocolate candy
Let y be the weight in grams of one peanut butter cup
Then the system of equations based on the video will be as follows:

$$4x + 5y = 63.8$$
$$6x + 6y = 82.3$$

There are several ways to solve this system, including substitution, elimination or graphing. As a part of the wrap up discussion, you may choose to have students discuss the merits of using the different methods.

Act Two Description (Dan Meyer http://blog.mrmeyer.com/2011/the–three–acts–of–a–mathematical–story/**)**
The protagonist/student overcomes obstacles, looks for resources, and develops new tools. During Act Two, students will discuss the question in Act One and decide on the facts that are needed to answer the question. Students will also look for formulas and conversions that are needed to solve the problem. When students decide what they need to solve the problem, they should ask for the facts or use technology to find them. To solve this problem, they will need to make the assumption that all chocolate candies of this type weigh the same and all peanut butter cups of this type weigh the same.

Note: It is pivotal to the problem solving process that students decide what is needed without being given the information up front. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin.

Here are some questions to help guide the discussion, but be sure not to give too much away. The goal is to have students formulate the questions and the methods to answer them.

- How is solving a system of equations different from solving one equation with one variable? How is it similar?
- How can you use both equations together to solve the system?

A sample solution is below:

Elimination Method:

$$\begin{array}{lll} 4x+5y=63.8 & 3(4x+5y=63.8) & 12x+15y=191.4 \\ 6x+6y=82.3 & -2(6x+6y=82.3) & -12x-12y=-164.6 \end{array}$$

$$\begin{array}{lll} 3y=26.8 & y=8.93333 & 4x+5(8.93333)=63.8 \end{array}$$

$$\begin{array}{ll} 4x=19.13333 & x=4.7833 \end{array}$$

Each chocolate candy weighs approximately 4.7833 grams and each peanut butter cup weighs approximately 8.933 grams.

ACT 3

Students will compare and share solution strategies.

- Reveal the answer. http://real.doe.k12.ga.us/vod/gso/math/videos/Coordinate–Algebra–Unit–2–Spotlight–reveal.mp4
- Discuss the theoretical math versus the practical outcome.
- Share student solution paths. Start with most common strategy.

The teacher also needs to be flexible and adapt the lesson to the curiosity of the class. Use this activity as a guide, but do not be afraid to deviate from it if the mathematics dictates that you do so.

The weights in the video do not match the weights from the equations. (The video shows that a chocolate candy weighs 4.9 grams and the peanut butter cup weighs 8.5 grams) Ask students to think about why this is. (As it turns out, not every chocolate candy is the same weight, and not every peanut butter cup is the same weight.) This can be an excellent discussion about theoretical versus practical outcomes.

Following this lesson, students will need the opportunity to practice setting up and solving systems of equations through the different methods.

The Sequel: "The goals of the sequel task are to a) challenge students who finished quickly so b) I can help students who need my help. It can't feel like punishment for good work. It can't seem like drudgery. It has to entice and activate the imagination." Dan Meyer http://blog.mrmeyer.com/2013/teaching–with–three–act–tasks–act–three–sequel/

For a sequel, present this situation: Hershey's and Reese's are teaming up to create a mixed bag of candies. The packaging can hold no more than 40 ounces of candy. In order to satisfy both chocolate and peanut butter lovers, the ratio of Hershey's to Reese's should be no more than 3 Hershey Kisses for every 2 Reese's Miniatures. How many of each should be included in the packaging. Justify your answer.

Another possible sequel could involve students weighing other objects and making their own systems of equations problems.

Solving Linear Equations in 2 Variables (Formative Assessment Lesson)

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=669

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Solving Linear Equations in 2 Variables*, is a Formative Assessment Lesson (FAL) that can be found at the website: http://map.mathshell.org/materials/lessons.php?taskid=209&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=669

Mathematical Goals

- Solving a problem using two linear equations with two variables.
- Interpreting the meaning of algebraic expressions.

Essential Questions

- Can I solve systems of equations using various methods: graphing, elimination, and substitution?
- What do the points on a line represent in relation to the situation they model?

Georgia Standards of Excellence

MGSE9–12.A.REI.5 Show and explain why the elimination method works to solve a system of two–variable equations. *(Limit to linear systems.)*

MGSE9–12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.
Students will interpret and compare various methods of solving the same system of equations.

3. Construct viable arguments and critique the reasoning of others.
Students will perform multiple error analyses and describe the patterns they see in student work.

Background Knowledge

- Students understand how to interpret parts of equations & expressions in relation to real life situations.
- Students understand use of variables in modeling real life situations.

Common Misconceptions

- Student assumes that the letter stands for an object not a number
- Student produces unsystematic guess and check work

Materials

- See FAL website.

Grouping

- Small group

Boomerangs (Formative Assessment Lesson)

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1241

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website: http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Optimizations Problems: Boomerangs*, is a Formative Assessment Lesson (FAL) that can be found at the website: http://map.mathshell.org/materials/lessons.php?taskid=207&subpage=problem

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1241

Mathematical Goals

- Interpret a situation and represent the constraints and variables mathematically.
- Select appropriate mathematical methods to use.
- Explore the effects of systematically varying the constraints.
- Interpret and evaluate the data generated and identify and confirm the optimum case.

Essential Questions

- How can I create a table, graph, or equation to represent a given scenario?
- How do I interpret systems of equations and their point of intersection in context?

Georgia Standards of Excellence

MGSE9–12.A.REI.5 Show and explain why the elimination method works to solve a system of two–variable equations. *(Limit to linear systems.)*

MGSE9–12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

MGSE9–12.A.REI.12 Graph the solution set to a linear inequality in two variables.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
   Students must work on an extended problem.
2. Reason abstractly and quantitatively
   Students must work with a real life scenario and its numerical, graphical, and algebraic representations.
3. Construct viable arguments and critique the reasoning of others.
   Students must analyze student work, identifying different approaches to the same problem.
4. Model with mathematics.
   Students model real life scenarios using equations.

Background Knowledge

- Students should know how to graph linear equations.
- Students should know how to create equations in two variables given a situation.

Common Misconceptions

- Students may make an incorrect interpretation of the constraints and variables.
- Student may have technical difficulties when using graphs.
- The student may present the work as a series of unexplained numbers and/or calculations, or as a table without headings.

Materials

- See FAL website.

Grouping

- Small group

Summer Job (Scaffolding Task)

Introduction

In this task, students will write a model for an inequality from the context of a word problem using real life situations. The students will then graph the inequality in two variables and analyze the solution. Students will reason quantitatively and use units to solve problems.

Mathematical Goals

- Model and write an inequality in two variables and solve a problem in context.
- Create two–variable linear equations and inequalities from contextual situations.
- Solve word problems involving inequalities.
- Represent constraints with inequalities.

Essential Questions

- How do I graph a linear inequality in two variables?
- How do I justify a solution to an equation?

Georgia Standards of Excellence

MGSE9–12.A.REI.12 Graph the solution set to a linear inequality in two variables.

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.
4. Model with mathematics.
Students will model their income from their summer job using linear equations and inequalities.

Background Knowledge

- Students can graph linear equations.
- Students can read and interpret graphs.
- Students can graph inequalities in two variables.

Common Misconceptions

- Students may be confused about the scale of the graphs and how to graph the solution.
- Students may question how to graph the constraints of the problem in terms of the appropriate quadrant.

Materials

- Graph paper
- Ruler
- Colored Pencils
- Graphing Calculator (optional)

Grouping

- Individual / Partner

Differentiation

Intervention:

- Partner struggling students with resident experts.

Formative Assessment Questions

- How do we display linear equations differently from linear inequalities? Why is it necessary to do so?
- How do you know when a real life situation should be modeled with a linear inequality and not a linear equation?

<u>Summer Job – Teacher Notes</u>

In order to raise money, you are planning to work during the summer babysitting and cleaning houses. You earn $10 per hour while babysitting and $20 per hour while cleaning houses. You need to earn at least $1000 during the summer.

1. Write an expression to represent the amount of money earned while babysitting. Be sure to choose a variable to represent the number of hours spent babysitting.

   Let b represent the number of hours spent babysitting
   $10b represents the amount of money earned while babysitting

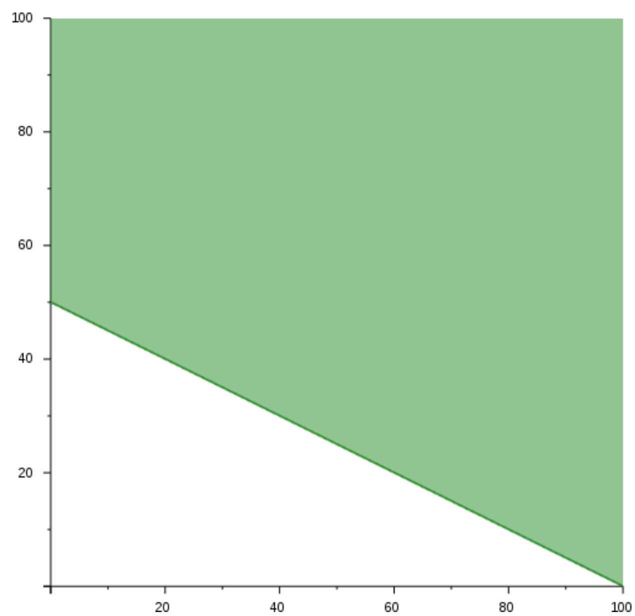
2. Write an expression to represent the amount of money earned while cleaning houses.

   Let c represent the number of hours spent cleaning
   $20c represents the amount of money earned while cleaning houses.

3. Write a mathematical model (inequality) representing the total amount of money earned over the summer from babysitting and cleaning houses.

   $10b + $20c ≥ $1000

4. Graph the mathematical model. Graph the hours babysitting on the *x*–axis and the hours cleaning houses on the *y*–axis.



5. Use the graph to answer the following:

    a. Why does the graph only fall in the 1st Quadrant?

    Neither the hours spent babysitting nor the hours cleaning houses can be negative.

    b. Is it acceptable to earn exactly $1000? What are some possible combinations of outcomes that equal exactly $1000? Where do all of the outcomes that total $1000 lie on the graph?

    Yes, it is possible to earn exactly $1000. Some possibilities include (100, 0), (20, 40), and (80, 10), but answers will vary. All of the outcomes totaling exactly $1000 lie on the line.

    c. Is it acceptable to earn more than $1000? What are some possible combinations of outcomes that total more than $1000? Where do all of these outcomes fall on the graph?

    Yes, it is acceptable to earn more than $1000. Some possibilities are (10, 60), and (70, 70), but answers will vary. All of the outcomes totaling more than $1000 are above the line.

    d. Is it acceptable to work 10 hours babysitting and 10 hours cleaning houses? Why or why not? Where does the combination of 10 hours babysitting and 10 hours cleaning houses fall on the graph? Are combinations that fall in this area a solution to the mathematical model? Why or why not?

    No it is not acceptable to work 10 hours babysitting and 10 hours cleaning houses. This combination would result in earnings of only $300 for the summer (10*10 + 20*10). Since you needed $1000 this is not acceptable. This combination falls below the line. Any combination that falls in the area below the line is not a solution because it would result in earnings less than $1000.

6. How would the model change if you could only earn more than $1000? Write a new model to represent needing to earn more than $1000. How would this change the graph of the model? Would the line still be part of the solution? How would you change the line to show this? Graph the new model.

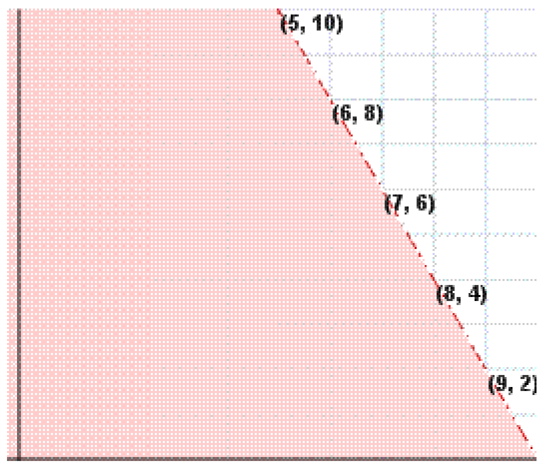
New model: $10x + $20y > $1000
The line on the graph would no longer be part of the solution, therefore it would be broken and not solid.

You plan to use part of the money you earned from your summer job to buy jeans and shirts for school. Jeans cost $40 per pair and shirts are $20 each. You want to spend less than $400 of your money on these items.

7. Write a mathematical model representing the amount of money spent on jeans and shirts.

   $40j + $20s < $400

8. Graph the mathematical model. Graph the number of jeans on the *x*–axis and shirts on the *y*–axis.



   a. Why does the graph only fall in the 1st Quadrant?

      Neither he number of pairs of jeans nor the number of shirts purchased can be negative.

   b. Is it acceptable to spend less than $400? What are some possible combinations of outcomes that total less than $400? Where do all of these outcomes fall on the graph?

It is acceptable to spend less than $400. All of the possible combinations totaling less than $400 fall below the line.

c. Is it acceptable to spend exactly $400? How does the graph show this?

It is not acceptable to spend exactly $400, therefore the line is broken.

d. Is it acceptable to spend more than $400? Where do all of the combinations that total more than $400 fall on the graph?

It is not acceptable to spend more than $400. All of the combinations totaling more than $400 are above the line on the graph.

Summarize your knowledge of graphing inequalities in two variables by answering the following questions in sentence form:

Answers to these questions will vary, but should demonstrate student understanding of the reasoning behind graphing inequalities.

9. Explain the difference between a solid line and a broken line when graphing inequalities. How can you determine from the model whether the line will be solid or broken? How can you look at the graph and know if the line is part of the solution?

Answers will vary, but a solid line indicates these combinations are part of the solution and the inequality contains an equal sign. A broken line indicates the line is not part of the solution and the inequality does not contain an equal sign.

10. How do you determine which area of the graph of an inequality to shade? What is special about the shaded area of an inequality? What is special about the area that is not shaded?

The area that contains solutions to the inequality is shaded. The area that is not shaded does not contain solutions to the inequality.

Scaffolding Task: Summer Job

Name________________________________ **Date**__________________

Mathematical Goals

- Model and write an inequality in two variables and solve a problem in context.
- Create two–variable linear equations and inequalities from contextual situations.
- Solve word problems involving inequalities.
- Represent constraints with inequalities.

Essential Questions

- How do I graph a linear inequality in two variables?
- How do I justify a solution to an equation?

Georgia Standards of Excellence

MGSE9–12.A.REI.12 Graph the solution set to a linear inequality in two variables.

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.
4. Model with mathematics.

Scaffolding Task: Summer Job

Name________________________________ **Date_________________**

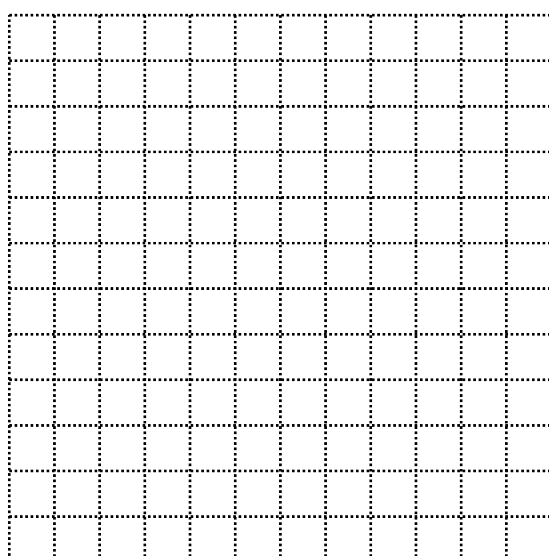
In order to raise money, you are planning to work during the summer babysitting and cleaning houses. You earn $10 per hour while babysitting and $20 per hour while cleaning houses. You need to earn at least $1000 during the summer.

1. Write an expression to represent the amount of money earned while babysitting. Be sure to choose a variable to represent the number of hours spent babysitting.

2. Write an expression to represent the amount of money earned while cleaning houses.

3. Write a mathematical model (inequality) representing the total amount of money earned over the summer from babysitting and cleaning houses.

4. Graph the mathematical model. Graph the hours babysitting on the *x*–axis and the hours cleaning houses on the *y*–axis.



5. Use the graph to answer the following:

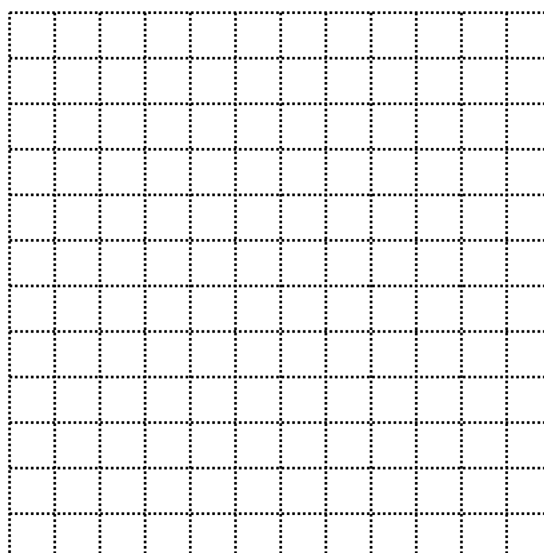
   a. Why does the graph only fall in the 1st Quadrant?

   b. Is it acceptable to earn exactly \$1000? What are some possible combinations of outcomes that equal exactly \$1000? Where do all of the outcomes that total \$1000 lie on the graph?

   c. Is it acceptable to earn more than \$1000? What are some possible combinations of outcomes that total more than \$1000? Where do all of these outcomes fall on the graph?

   d. Is it acceptable to work 10 hours babysitting and 10 hours cleaning houses? Why or why not? Where does the combination of 10 hours babysitting and 10 hours cleaning houses fall on the graph? Are combinations that fall in this area a solution to the mathematical model? Why or why not?

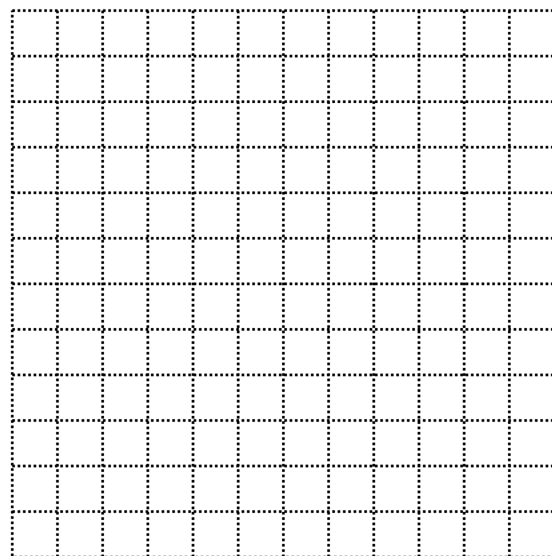
6. How would the model change if you could only earn more than \$1000? Write a new model to represent needing to earn more than \$1000. How would this change the graph of the model? Would the line still be part of the solution? How would you change the line to show this? Graph the new model.



You plan to use part of the money you earned from your summer job to buy jeans and shirts for school. Jeans cost $40 per pair and shirts are $20 each. You want to spend less than $400 of your money on these items.

7. Write a mathematical model representing the amount of money spent on jeans and shirts.

8. Graph the mathematical model. Graph the number of jeans on the *x*–axis and shirts on the *y*–axis.



   a. Why does the graph only fall in the 1st Quadrant?

   b. Is it acceptable to spend less than $400? What are some possible combinations of outcomes that total less than $400? Where do all of these outcomes fall on the graph?

   c. Is it acceptable to spend exactly $400? How does the graph show this?

d. Is it acceptable to spend more than $400? Where do all of the combinations that total more than $400 fall on the graph?

Summarize your knowledge of graphing inequalities in two variables by answering the following questions in sentence form:

9. Explain the difference between a solid line and a broken line when graphing inequalities. How can you determine from the model whether the line will be solid or broken? How can you look at the graph and know if the line is part of the solution?

10. How do you determine which area of the graph of an inequality to shade? What is special about the shaded area of an inequality? What is special about the area that is not shaded?

Graphing Inequalities (Extension Task)

Introduction

In this task, students will graph two separate inequalities in two variables and analyze the graph for solutions to each. The students will then graph the two inequalities in two variables on the same coordinate system to show that the solution to both inequalities is the area where the graphs intersect.

Mathematical Goals

- Solve word problems involving inequalities.
- Represent constraints with inequalities.
- Rearrange and graph inequalities.

Essential Questions

- How do I graph a linear inequality in two variables?
- How do I justify a solution to an equation?
- How do I graph a system of linear inequalities in two variables.

Georgia Standards of Excellence

MGSE9–12.A.REI.12 Graph the solution set to a linear inequality in two variables.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
   Students will need to work through problems and find multiple solutions to the problems.
5. Use appropriate tools strategically.
   Students will need to choose the tools to best model these equations based on the tools available.
6. Attend to precision.
   Students will need to be cautious of scale and constraints displayed in their graphs.

Background Knowledge

- Students can graph linear equations.
- Students can graph linear inequalities.

Common Misconceptions

- Students may be confused about the scale of the graphs and how to graph the solution.
- Students may question how to graph the constraints of the problem in terms of the appropriate quadrant.

Materials

- Colored pencils
- Calculator
- Graph Paper
- Ruler

Grouping

- Individual / Partner

Differentiation

Extension:

- Students should show algebraically that the solutions work

Intervention:

- Students should be given graphs of sample inequalities

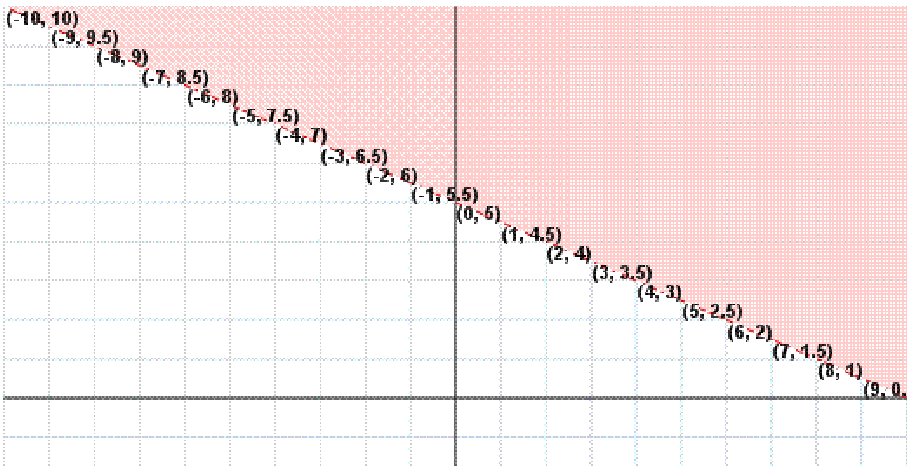
Formative Assessment Questions

- How do we display linear equations differently from linear inequalities? Why is it necessary to do so?
- When displaying multiple linear equations or inequalities, what additional considerations must we take into account?

Graphing Inequalities (Extension Task) – Teacher Notes

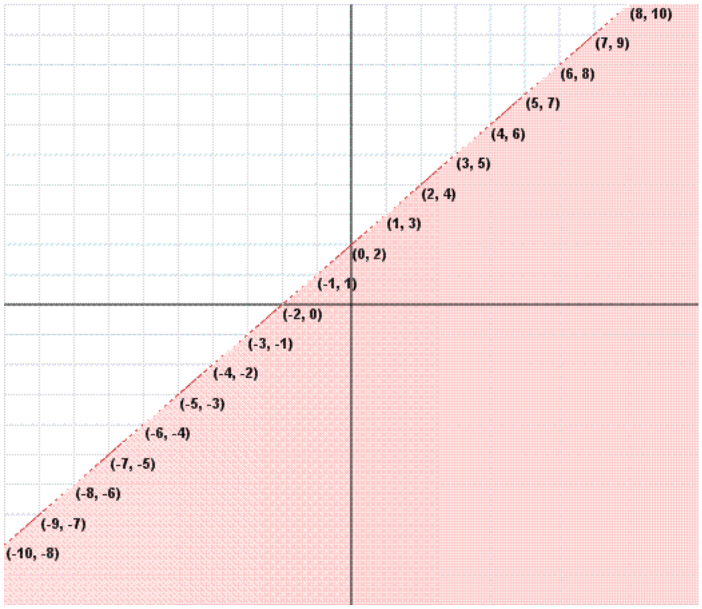
1. Graph the inequality $y > -\frac{1}{2}x + 5$. What are some solutions to the inequality?

NOTE: The points shown in the graph are the boundary points but not actually part of the solution set. Students should label points in the shaded region.



2. Graph the inequality $y < x + 2$. What are some solutions to the inequality?

NOTE: The points shown in the graph are the boundary points but not actually part of the solution set. Students should label points in the shaded region.



3. Look at both graphs.

   The main purpose of this exercise is to allow students to discover visually and conceptually where the solutions to the inequalities lie on the graph.

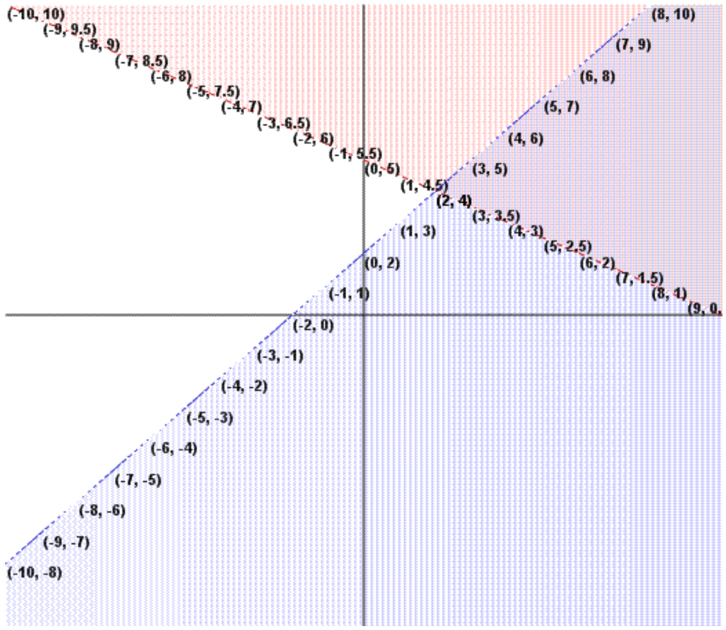
   a. Are there any solutions that work for both inequalities? Give 3 examples.

      There are many solutions that work for both, including: (–2, 7), (4, 4), (7, 3)

   b. Are there any solutions that work for 1 inequality but not the other? Give 3 examples and show which inequality it works for.

      There are many solutions that work for one inequality but not the other.

4. Graph both inequalities on the same coordinate system, using a different color to shade each.
   NOTE: Students do not need to label points in this problem. However, IF they do, they should only be labeling points in the region shaded by both graphs.



a. Look at the region that is shaded in both colors. What does this region represent?

The region shaded in both colors represents the solutions to the system.

b. Look at the regions that are shaded in only 1 color. What do these regions represent?

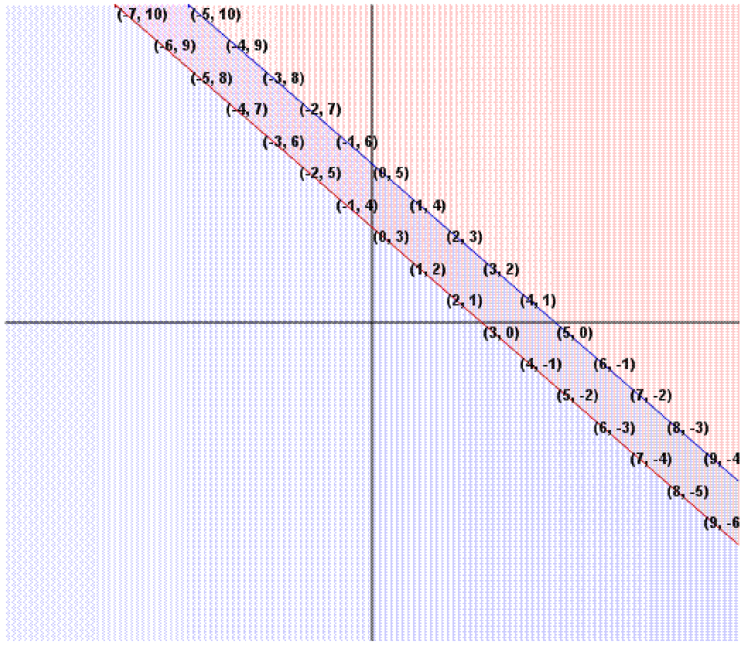
The regions shaded in one color represent solutions that work for one inequality, but not the other.

c. Look at the region that is not shaded. What does this region represent?

The region that is not shaded represents combinations that are not solutions to either inequality.

5. Graph the following system on the same coordinate grid. Use different colors for each.

$$x + y \geq 3$$
$$y \leq -x + 5$$



a. Give 3 coordinates that are solutions to the system.

Answers may vary.

b. Give 3 coordinates that are not solutions to the system.

Answers may vary.

c. Is a coordinate on either line a solution?

Yes, coordinates on the line are solutions to the system.

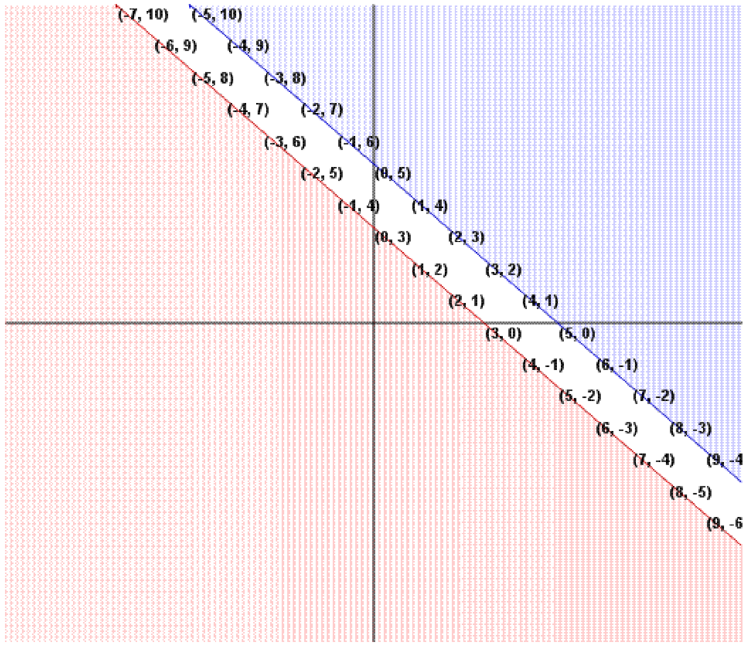
d. How would you change the inequality $x + y \geq 3$ so that it would shade below the line?

If you change the $\geq$ to $\leq$, the graph will shade below.

e. How would you change the inequality $y \leq -x + 5$ so that it would shade above the line?

If you change $\leq$ to $\geq$, the graph will shade above the line.

6. Graph the new equations from 'd' and 'e' above on the same coordinate grid. Use blue for one graph and red for the other.



a. What do the coordinates in blue represent?

Each color represents solutions to one inequality, but not the other.

b. What do the coordinates in red represent?

See above.

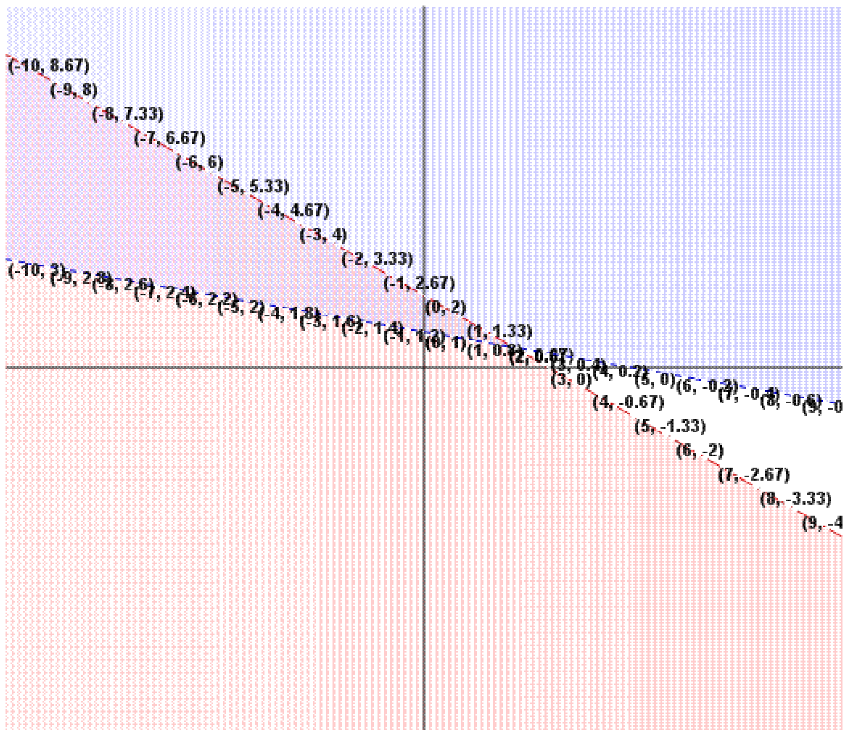
c. Why do the colors <u>not</u> overlap this time?

There is no coordinate that is a solution to both inequalities. Therefore, the system has no solution.

Graph the following on the same coordinate grid and give 3 solutions for each.

7. $2x + 3y < 6$
   $x + 5y > 5$

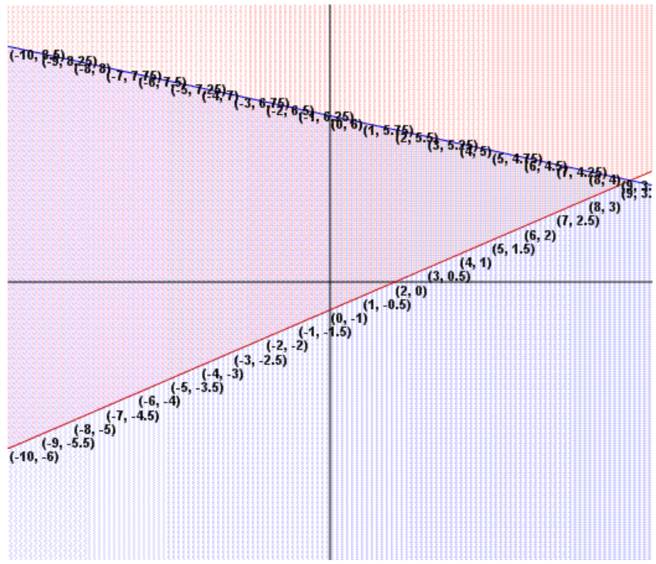
NOTE: The points shown in the graph are the boundary points but not actually part of the solution set. Students should provide three points in the area shaded by both regions.



8. $y \geq \frac{1}{2}x - 1$

$y \leq -\frac{1}{4}x + 6$

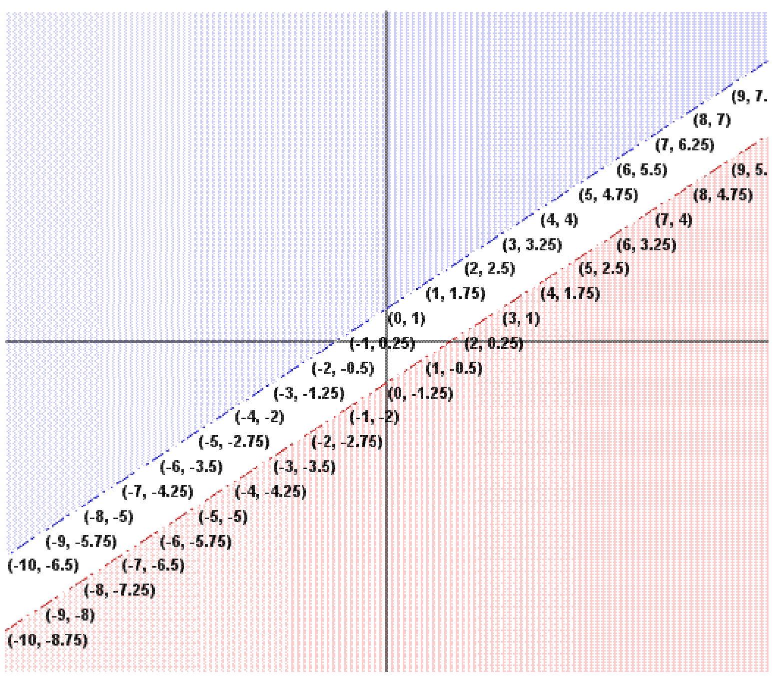
NOTE: Students should provide three points in the area shaded by both region OR on either of the boundary lines.



9. $3x - 4y > 5$

$y > \frac{3}{4}x + 1$

NOTE: The points shown in the graph are the boundary points but not actually part of the solution set. There are no solutions to this system of inequalities.



Extension Task: Graphing Inequalities

Name________________________________ **Date**_________________

Mathematical Goals

- Solve word problems involving inequalities.
- Represent constraints with inequalities.
- Rearrange and graph inequalities.

Essential Questions

- How do I graph a linear inequality in two variables?
- How do I justify a solution to an equation?
- How do I graph a system of linear inequalities in two variables?

Georgia Standards of Excellence

MGSE9–12.A.REI.12 Graph the solution set to a linear inequality in two variables.

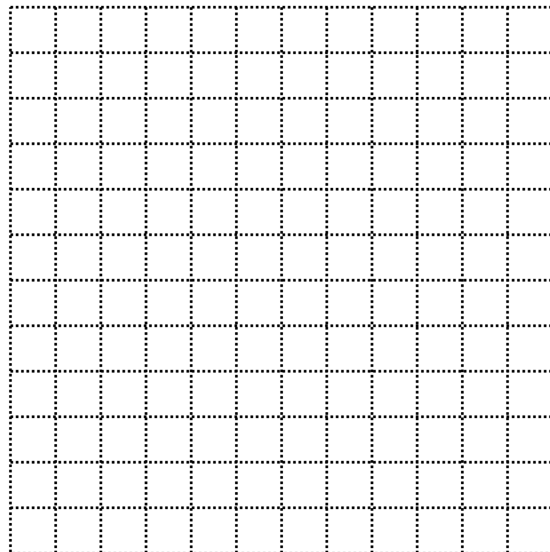
Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
5. Use appropriate tools strategically.
6. Attend to precision.

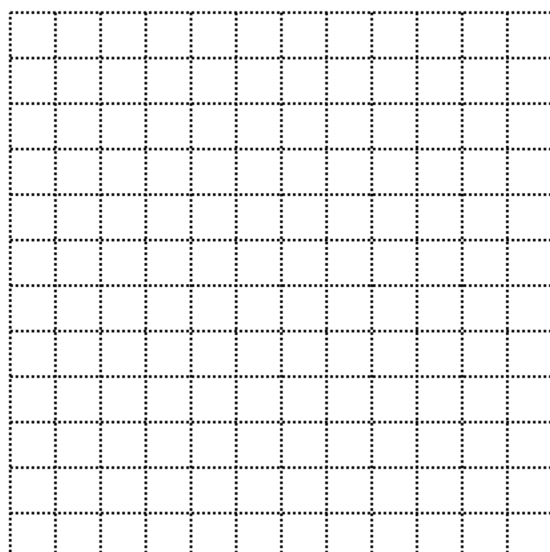
Extension Task: Graphing Inequalities

Name________________________________ **Date**_________________

1. Graph the inequality $y > -\frac{1}{2}x + 5$. What are some solutions to the inequality?



2. Graph the inequality $y < x + 2$. What are some solutions to the inequality?

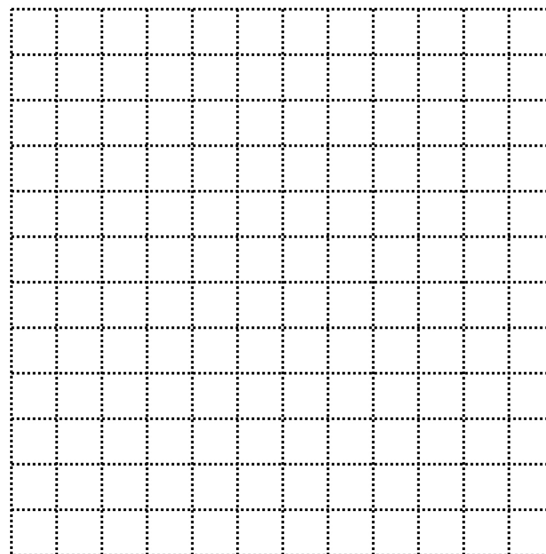


3. Look at both graphs.

   a. Are there any solutions that work for both inequalities? Give 3 examples.

   b. Are there any solutions that work for 1 inequality but not the other? Give 3 examples and show which inequality it works for.

4. Graph both inequalities on the same coordinate system, using a different color to shade each.



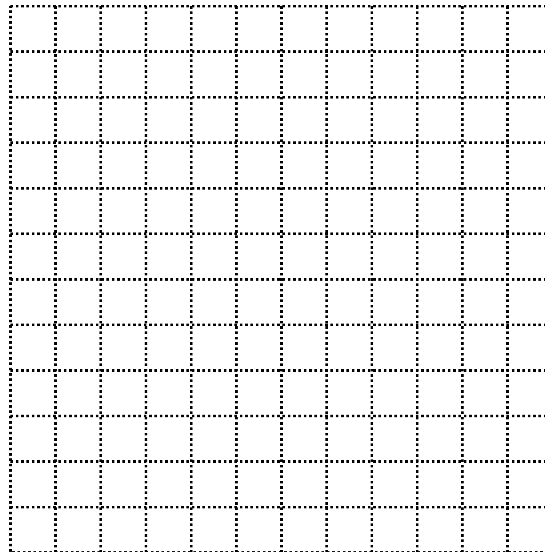
   a. Look at the region that is shaded in both colors. What does this region represent?

   b. Look at the regions that are shaded in only 1 color. What do these regions represent?

   c. Look at the region that is not shaded. What does this region represent?

5. Graph the following system on the same coordinate grid. Use different colors for each.

$$x + y \geq 3$$
$$y \leq -x + 5$$



a. Give 3 coordinates that are solutions to the system.

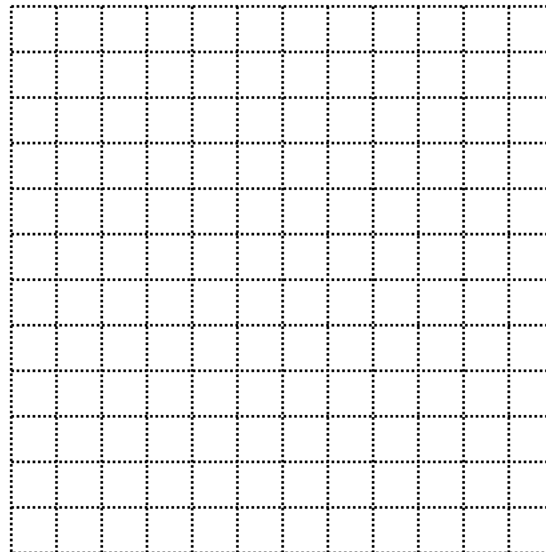
b. Give 3 coordinates that are not solutions to the system.

c. Is a coordinate on either line a solution?

d. How would you change the inequality $x + y \geq 3$ so that it would shade below the line?

e. How would you change the inequality $y \leq -x + 5$ so that it would shade above the line?

6. Graph the new equations from ‘d’ and ‘e’ above on the same coordinate grid. Use blue for one graph and red for the other.



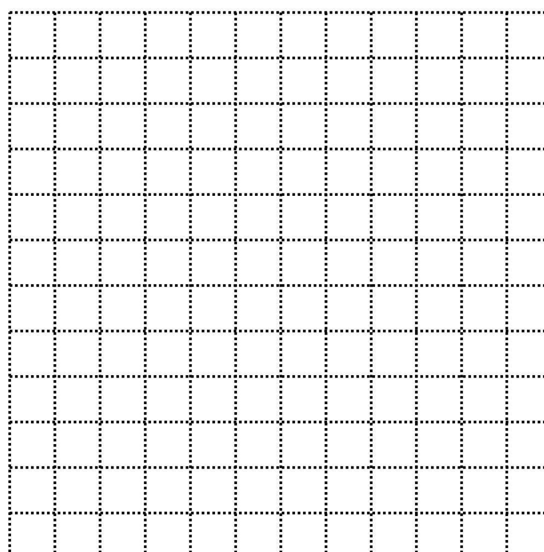
a. What do the coordinates in blue represent?

b. What do the coordinates in red represent?

c. Why do the colors not overlap this time?

Graph the following on the same coordinate grid and give 3 solutions for each.

7. $2x + 3y < 6$
   $x + 5y > 5$



8. $y \geq \frac{1}{2}x - 1$

   $y \leq -\frac{1}{4}x + 6$

9. $3x - 4y > 5$

   $y > \frac{3}{4}x + 1$

Modeling Situations with Linear Equations (Formative Assessment Lesson)

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=673

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website: http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Modeling Situations with Linear Equations*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=211&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=673

Mathematical Goals

- Explore relationships between variables in everyday situations.
- Find unknown values from known values.
- Find relationships between pairs of unknowns, and express these as tables and graphs.
- Find general relationships between several variables, and express these in different ways by rearranging formulas.

Essential Questions

- Can I interpret the different parts of an algebraic expression?
- Can I create a general equation using all variables from a specific scenario?

Georgia Standards of Excellence

MGSE9–12.A.REI.12 Graph the solution set to a linear inequality in two variables.

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.
4. Model with mathematics.
Students create equations relating time/distance/speed, money/time/units, etc.

Background Knowledge

- Students know how to create expressions using variables and operations
- Students know how to graph linear functions and interpret its characteristics

Common Misconceptions

- Student uses incorrect operation in equation
- Student does not explain or misinterprets the significance of the x–intercept

Materials

- See FAL website.

Grouping

- Pairs

Family Outing (Culminating Task)

Introduction

In this task, students will write a model for an inequality from the context of a word problem using real life situations. The students will then graph the inequality in two variables and analyze the solution. Students will reason quantitatively and use units to solve problems. An extension to this task involves graphing and analyzing a system of linear inequalities in context.

Mathematical Goals

- Model and write an inequality in two variables and solve a problem in context.
- Create two–variable linear equations and inequalities from contextual situations.
- Represent and solve word problems and constraints using inequalities.

Essential Questions

- How do I graph a system of linear equations or inequalities in two variables?
- How do I solve a system of linear equations or inequalities graphically or algebraically?
- How do I justify the solution to a system of equations or inequalities?

Georgia Standards of Excellence

MGSE9-12.A.REI.1 Using algebraic properties and the properties of real numbers, justify the steps of a simple, one–solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties. *(Students should focus on and master linear equations and be able to extend and apply their reasoning to other types of equations in future courses.)*

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. *(For example, given $ax + 3 = 7$, solve for x.)*

MGSE9–12.A.REI.5 Show and explain why the elimination method works to solve a system of two–variable equations. *(Limit to linear systems.)*

MGSE9–12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

MGSE9–12.A.REI.12 Graph the solution set to a linear inequality in two variables.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Students should use all eight SMPs when exploring this task.

Background Knowledge

- Students will apply everything they have learned in this unit.

Common Misconceptions

- Address misconceptions brought to light during the rest of the unit.

Materials

- Colored Pencils
- Ruler
- Calculator

Grouping

- Individual / Partner

Differentiation

Extension:

- Ask students to come up with their own situations that would be modeled by a system of equations or inequalities.

Intervention:

- Strategic grouping

Culminating Task: Family Outing – Teacher Notes

You and your family are planning to rent a van for a 1 day trip to Family Fun Amusement Park in Friendly Town. For the van your family wants, the Wheels and Deals Car Rental Agency charges $25 per day plus 50 cents per mile to rent the van. The Cars R Us Rental Agency charges $40 per day plus 25 cents per mile to rent the same type van.

1. Write a mathematical model to represent the cost of renting a van from the Wheels and Deals Agency for 1 day.

   C = $25 + $0.50m

   a. Do the units matter for this equation?

      Yes, the units matter. Both the cost per day and the cost per mile should be in the same unit.

   b. Use the equation to determine the cost for renting the van from this agency for 1 day and driving 40 miles.
      C = $25(1) + $0.50(40)
      C = $25 + $20
      C = $45

2. Write a mathematical model to represent the cost of renting from the Cars R Us Agency for 1 day.
   C = $40 + $.25m

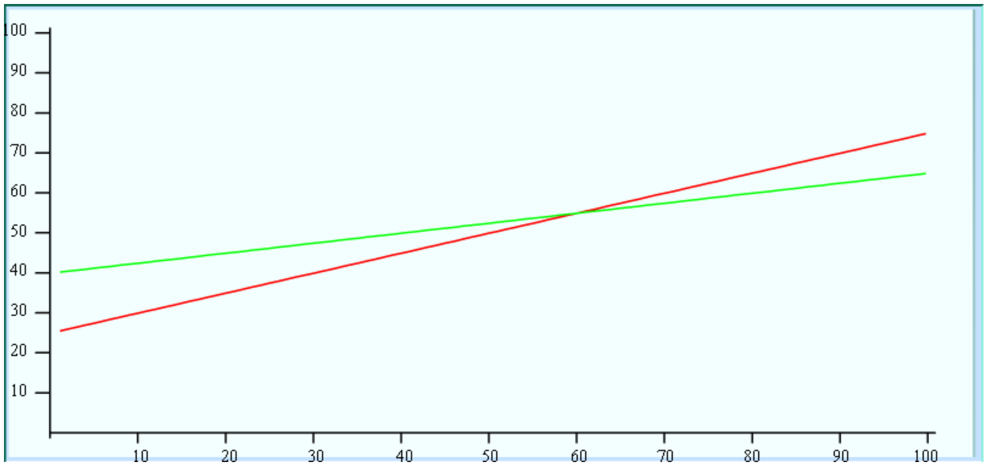
   a. Do the units for this equation match the units for the equation in problem 1? Does this matter when comparing the 2 equations?

      The units should be the same for both equations.

   b. Use the equation from '2a' to determine the cost for renting the van from Cars R Us for 1 day and driving 40 miles.

      C = $40(1) + $0.25(40)
      C = $50

3. Graph the 2 models on the same coordinate system. Be sure to extend the lines until they intersect.



   a. Where do the 2 lines intersect?

      (60, 55) After 60 miles, the cost for the rental will be $55.

   b. What does the point of intersection represent?

      The point represents the number of miles for which the cost of the rental will be the same for both agencies.

   c. When is it cheaper to rent from Wheels and Deals?

      It is cheaper to rent from Wheels and Deals when you are driving less than 60 miles.

   d. When is it cheaper to rent from Cars R Us?

      It is cheaper to rent from Cars R Us when you are driving more than 60 miles.

4. Friendly Town is approximately 80 miles from your home town. Which agency should you choose? Justify your answer.

   You should choose the Cars R Us agency because the cost of renting from them would be approximately $60. The cost for renting from Wheels and Deals would be approximately $65.

When you leave the car rental agency, your father goes to the Fill 'er Up Convenience Store for gas. The gas hand indicates the van is on empty, so your father plans to fill the tank. Gas at the station is $3.49 per gallon.

5. If your father spends $78 on gas, approximately how many gallons did he purchase?

   $78 = $3.49*g
   g = $78/$3.49

   He purchased approximately 22 gallons of gas.

While in the store, your father purchases drinks for the six people in your van. Part of your family wants coffee and the rest want a soda.

6. Coffee in the store costs $0.49 per cup and sodas are $1.29 each. The cost of the drinks before tax was $6.14.

   a. Write a mathematical model that represents the total number of cups of coffee and sodas.

   $c + s = 6$

   b. Write a mathematical model that represents the cost of the coffee and soda.

   $\$0.49c + \$1.29s = \$6.14$

   c. Solve the system of equations using the elimination method.

$$\begin{aligned}
-0.49(c+s) &= -0.49(6)\\
-0.49c - 0.49s &= -2.94\\
\underline{+\,0.49c + 1.29s} &\underline{= 6.14}\\
0.8s &= 3.2\\
s &= 4\\
\\
c + 4 &= 6\\
c &= 2
\end{aligned}$$

   Your father purchased 2 cups of coffee and 4 sodas.

EXTENSION: When you arrive in Friendly Town at the Family Fun Amusement Park, the 6 people in your family pair up to enter the park. You and your brother decide to enter and ride together. The cost to enter the park is $10, with each ride costing $2.

7. You bring $55 to the park. You must pay to enter the park and you budget an additional $10 for food. Write and solve an inequality to determine the maximum number of rides you can ride. Explain your answer.

$\$10 + \$10 + \$2r \leq \$55$

$\$2r \leq \$35$

$r \leq 17.5$

The maximum number rides you can ride is 17, because you can't ride half of a ride.

8. Your brother brings $70 to the park and budgets $12 for food. How many more rides can he ride than you? Explain your answer.

$\$10 + \$12 + 2r \leq \$70$
$r \leq 24$ rides

Your brother can ride up to 24 rides. You can ride up to 17. Therefore, he can ride 7 more rides than you.

EXTENSION: Inside the park, there are 2 vendors that sell popcorn and cotton candy. Jiffy Snacks sells both for $2.50 per bag. Quick Eats has cotton candy for $4 per bag and popcorn for $2 per bag.

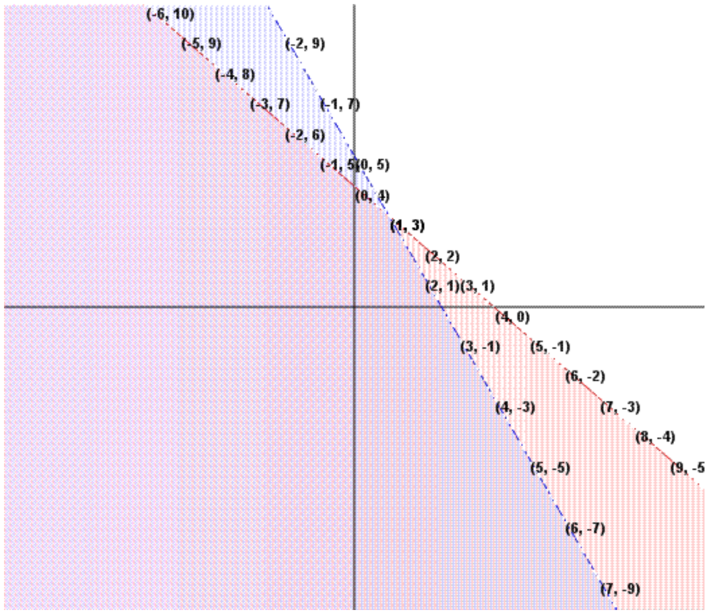
9. If you use the $10 you budgeted for food, write an inequality to model the possible combinations of popcorn and cotton candy you can purchase from Jiffy Snacks.

$\$2.50c + \$2.50p \leq \$10$

10. Write an inequality to model the possible combinations of popcorn and cotton candy you can purchase from Quick Eats.

$\$4.00c + \$2.00p \leq \$10$

11. Graph the system of inequalities. Give two combinations that work for both vendors.



12. Assuming you purchase at least one of each, what is the maximum number of bags of cotton candy and popcorn that work for both equations?

The maximum that works for both equations is 1 bag of cotton candy and 3 bags of popcorn.

When you leave the park, your father notices that you have used ¾ of the tank of gas you purchased before you left.

13. Do you have enough gas to get home? Justify your answer.

The methods for answering this question may vary, but you do not have enough gas to get home. You have used approximately 17 of the 22 gallons you purchased earlier. You will need approximately 12 gallons of gas to get home.

14. Your father wants to purchase enough gas to get home, but not leave extra in the tank when the van is returned to the rental agency. Approximately how many more gallons should he purchase? Justify your answer.

See above.

Culminating Task: Family Outing

Name________________________________ **Date**_________________

Mathematical Goals

- Model and write an inequality in two variables and solve a problem in context.
- Create two–variable linear equations and inequalities from contextual situations.
- Represent and solve word problems and constraints using inequalities.

Essential Questions

- How do I graph a system of linear equations or inequalities in two variables?
- How do I solve a system of linear equations or inequalities graphically or algebraically?
- How do I justify the solution to a system of equations or inequalities?

Georgia Standards of Excellence

MGSE9-12.A.REI.1 Using algebraic properties and the properties of real numbers, justify the steps of a simple, one–solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties. *(Students should focus on and master linear equations and be able to extend and apply their reasoning to other types of equations in future courses.)*

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. *(For example, given $ax + 3 = 7$, solve for x.)*

MGSE9–12.A.REI.5 Show and explain why the elimination method works to solve a system of two–variable equations. *(Limit to linear systems.)*

MGSE9–12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

MGSE9–12.A.REI.12 Graph the solution set to a linear inequality in two variables.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Culminating Task: Family Outing

Name________________________________ Date_________________

You and your family are planning to rent a van for a 1 day trip to Family Fun Amusement Park in Friendly Town. For the van your family wants, the Wheels and Deals Car Rental Agency charges $25 per day plus 50 cents per mile to rent the van. The Cars R Us Rental Agency charges $40 per day plus 25 cents per mile to rent the same type van.

1. Write a mathematical model to represent the cost of renting a van from the Wheels and Deals Agency for 1 day.

   a. Do the units matter for this equation?

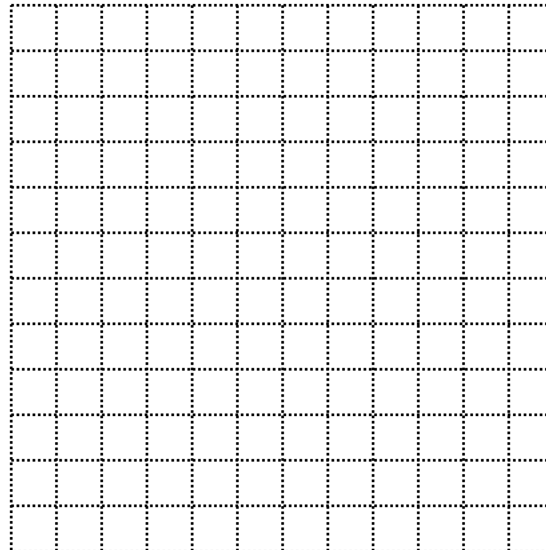
   b. Use the equation to determine the cost for renting the van from this agency for 1 day and driving 40 miles.

2. Write a mathematical model to represent the cost of renting from the Cars R Us Agency for 1 day.

   a. Do the units for this equation match the units for the equation in problem 1? Does this matter when comparing the 2 equations?

   b. Use the equation from '2a' to determine the cost for renting the van from Cars R Us for 1 day and driving 40 miles.

3. Graph the 2 models on the same coordinate system. Be sure to extend the lines until they intersect.



   a. Where do the 2 lines intersect?

   b. What does the point of intersection represent?

   c. When is it cheaper to rent from Wheels and Deals?

   d. When is it cheaper to rent from Cars R Us?

4. Friendly Town is approximately 80 miles from your home town. Which agency should you choose? Justify your answer.

When you leave the car rental agency, your father goes to the Fill 'er Up Convenience Store for gas. The gas hand indicates the van is on empty, so your father plans to fill the tank. Gas at the station is $3.49 per gallon.

5. If your father spends $78 on gas, approximately how many gallons did he purchase?

While in the store, your father purchases drinks for the six people in your van. Part of your family wants coffee and the rest want a soda.

6. Coffee in the store costs \$0.49 per cup and sodas are \$1.29 each. The cost of the drinks before tax was \$6.14.

   a. Write a mathematical model that represents the total number of cups of coffee and sodas.

   b. Write a mathematical model that represents the cost of the coffee and soda.

   c. Solve the system of equations using the elimination method.

EXTENSION:

When you arrive in Friendly Town at the Family Fun Amusement Park, the 6 people in your family pair up to enter the park. You and your brother decide to enter and ride together. The cost to enter the park is \$10, with each ride costing \$2.

7. You bring \$55 to the park. You must pay to enter the park and you budget an additional \$10 for food. Write and solve an inequality to determine the maximum number of rides you can ride. Explain your answer.
8. Your brother brings \$70 to the park and budgets \$12 for food. How many more rides can he ride than you? Explain your answer.

EXTENSION:

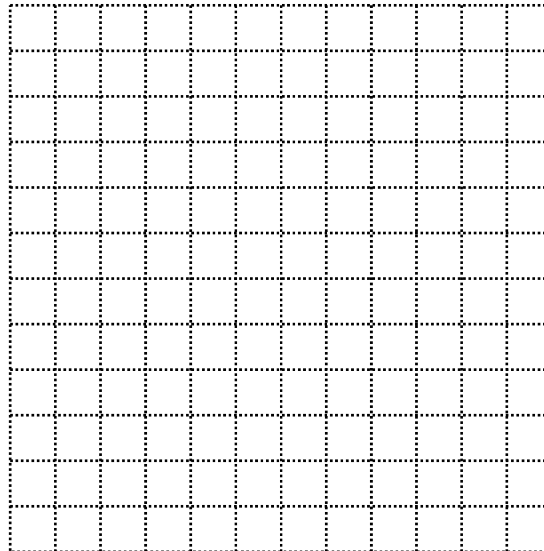
Inside the park, there are 2 vendors that sell popcorn and cotton candy. Jiffy Snacks sells both for \$2.50 per bag. Quick Eats has cotton candy for \$4 per bag and popcorn for \$2 per bag.

9. If you use the \$10 you budgeted for food, write an inequality to model the possible combinations of popcorn and cotton candy you can purchase from Jiffy Snacks.

10. Write an inequality to model the possible combinations of popcorn and cotton candy you can purchase from Quick Eats.

EXTENSION:

11. Graph the system of inequalities. Give two combinations that work for both vendors.



EXTENSION:

12. Assuming you purchase at least one of each, what is the maximum number of bags of cotton candy and popcorn that work for both equations?

When you leave the park, your father notices that you have used ¾ of the tank of gas you purchased before you left.

13. Do you have enough gas to get home? Justify your answer.

14. Your father wants to purchase enough gas to get home, but not leave extra in the tank when the van is returned to the rental agency. Approximately how many more gallons should he purchase? Justify your answer.