

Engineering Calculus

K-12 Mathematics Introduction

The Georgia Mathematics Curriculum focuses on actively engaging the students in the development of mathematical understanding by using manipulatives and a variety of representations, working independently and cooperatively to solve problems, estimating and computing efficiently, and conducting investigations and recording findings. There is a shift towards applying mathematical concepts and skills in the context of authentic problems and for the student to understand concepts rather than merely follow a sequence of procedures. In mathematics classrooms, students will learn to think critically in a mathematical way with an understanding that there are many different ways to a solution and sometimes more than one right answer in applied mathematics. Mathematics is the economy of information. The central idea of all mathematics is to discover how knowing some things well, via reasoning, permit students to know much else—without having to commit the information to memory as a separate fact. It is the reasoned, logical connections that make mathematics manageable. The implementation of the Georgia Standards of Excellence in Mathematics places a greater emphasis on sense making, problem solving, reasoning, representation, connections, and communication.

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Engineering Calculus is a fourth-year mathematics option for students who have completed AP Calculus BC. The course provides students with opportunities to develop an understanding of multivariable calculus as it applies to engineering systems, the history of engineering and its contributions to society. The course includes three-dimensional coordinate geometry; matrices and determinants; limits and continuity of functions with two independent variables; partial differentiation; multiple integration; the gradient; the divergence; the curl; Theorems of Green, Stokes, and Gauss; line integrals; integrals independent of path; and linear first-order differential equations. (*Prerequisite: Successful completion of AP Calculus BC*)

Instruction and assessment should include the appropriate use of technology. Topics should be presented in multiple ways, such as verbal/written, numeric/data-based, algebraic, and graphical. Concepts should be introduced and used, where appropriate, in the context of realistic phenomena.

Mathematics | Standards for Mathematical Practice

Mathematical Practices are listed with each grade/course mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical

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proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1 Make sense of problems and persevere in solving them.

High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and— if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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4 Model with mathematics.

High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y . High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

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8 Look for and express regularity in repeated reasoning.

High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. □ The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. **Students who do not have an understanding of a topic may rely on procedures too heavily.** Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. **In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.**

In this respect, those content standards that set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Process Standards for Engineering Calculus

The following process standards are essential to mastering each of the mathematics content standards. They emphasize critical dimensions of the mathematical proficiency that all students need.

MEC.PS.1 Students will solve engineering-based calculus problems (using appropriate technology).

- a. Build new mathematical knowledge through problem solving.
- b. Solve problems that arise in mathematics and in other contexts.

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- c. Apply and adapt a variety of appropriate strategies to solve problems, such as considering realistic constraints relevant to the design of a system, component, or process.
- d. Monitor and reflect on the process of mathematical problem solving and interpret problem solutions.

MEC.PS.2 Students will use visual and written communication to express basic design elements and will communicate mathematically.

- a. Organize and consolidate their mathematical thinking through communication.
- b. Communicate and use the language of mathematics to articulate their mathematical thinking coherently.
- c. Present a technical design, using computer-generated model, for an assigned design project.

MEC.PS.3 Students will describe the history of technological advancement and make connections among mathematical ideas and to other disciplines.

- a. Recognize and use connections among mathematical and engineering ideas.
- b. Recognize and apply mathematics in contexts outside of mathematics.
- c. Explain the impact of key persons and historical events and their impact on engineering and society.
- d. Describe the issues of necessity that have influenced innovation and technological development.

MEC.PS.4 Students will represent mathematics in multiple ways.

- a. Create and use representations to organize, record, and communicate mathematical ideas.
- b. Select, apply, and translate among mathematical representations to solve problems.
- c. Use representations to model and interpret physical, and mathematical phenomena.

MEC.PS.5 Students will develop vocabulary and communication skills by reading text material and problem descriptions associated with engineering and technology education.

- a. Reading in all curriculum areas.
- b. Discussing articles or books.
- c. Building vocabulary knowledge.

MEC.PS.6 Students will explore and summarize the educational requirements and professional expectations associated with engineering career paths.

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Advanced Engineering Calculus | Content Standards

Multi-Dimensional Engineering Analysis

Students will learn the basics of algebra and functions in two or more dimensions to prepare for the study of differentiation and integration of functions of several variables. In addition, students will explore and summarize the educational requirements and professional expectations associated with engineering career paths and solve problems related to various engineering disciplines.

MEC.A.1 Students will apply knowledge of mathematics, science, and engineering design to solve problems.

- a. Determine the equations of lines and surfaces using vectors and 3D graphing.
- b. Apply dot and cross products of vectors to express equations of planes, parallelism, perpendicularity, angles.
- c. Describe the role of vectors in engineering applications, such as modeling the velocity of moving objects or static forces on structures and objects.

MEC.A.2 Students will learn to evaluate matrices and apply their properties to solve engineering problems.

- a. Calculate determinants of matrices.
- b. Express systems of linear equations in matrix equation form.
- c. Use Gaussian elimination to compute solution sets of linear systems.

MEC.A.3 Students will investigate functions of two and three independent variables to model engineering systems.

- a. Compute limits of scalar and vector-valued functions.
- b. Identify and graph level curves of multivariate functions.
- c. Calculate regions of continuity of such functions.

MEC.A.4 Students will use visual and written communication to express basic design elements in the appropriate mathematics notation.

- a. Demonstrate fundamentals of technical sketching using computer-generated visuals by using the appropriate mathematics scale.
- b. Present a technical design, using computer-generated model, for an assigned design project utilizing the appropriate scientific units (US standards and SI units).

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Differentiation in Engineering

MEC.D.1 Students will evaluate and apply partial differentiation of multivariable functions with two or more independent variables.

- a. Compute the first and second partial derivatives of a function.
- b. Use the general chain rule to determine the partial derivatives of composite functions.
- c. Compute and apply the gradient of multivariable functions.
- d. Solve engineering optimization problems by applying partial differentiation or Lagrange multipliers.
- e. Utilize partial derivatives in developing the appropriate system balances (ex: mass balance) in engineering problems.

Multidimensional Integration in Engineering Systems

MEC.I.1 Students will apply the techniques of double and triple integration to multivariable scalar- and vector-valued functions.

- a. Manipulate integrals by changing the order of integration, introducing variable substitutions, or changing to curvilinear coordinates.
- b. Evaluate and apply line integrals that are independent of path.
- c. Apply properties of integrals to calculate and represent area, volume, or mass.
- d. Use integrals of vectors to define and apply the gradient, divergence, or the curl
- e. Interpret the theorems of Green, Stokes, or Gauss and apply them to the study of real-world phenomena.