Georgia Standards of Excellence Curriculum Framework

Mathematics

GSE Foundations of Algebra

Module 1: Number Sense and Quantity

Georgia Department of Education

Richard Woods, Georgia's School Superintendent

"Educating Georgia's Future"

Module 1: Number Sense and Quantity

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FOUNDATIONS OF ALGEBRA REVISION SUMMARY

The Foundations of Algebra course has been revised based on feedback from teachers across the state. The following are changes made during the current revision cycle:

- Each module assessment has been revised to address alignment to module content, reading demand within the questions, and accessibility to the assessments by Foundations of Algebra teachers.
- All module assessments as well as the pre- and posttest for the course will now be available in GOFAR at the teacher level along with a more robust teacher's edition featuring commentary along with the assessment items.
- All modules now contain "Quick Checks" that will provide information on mastery of the content at pivotal points in the module (or prerequisite skills they will need to be successful). Both teacher and student versions of the "Quick Checks" will be accessible within the module.
- A "Materials List" can be found immediately after this page in each module. The list provides teachers with materials that are needed for each lesson in that module.
- A complete professional learning series with episodes devoted to the "big ideas" of each module and strategies for effective use of manipulatives will be featured on the Math Resources and Professional Learning page at https://www.gadoe.org/Curriculum-Instruction-and-Assessment/Curriculum-and-Instruction/Pages/Mathematics.aspx.
- Additional support such as Module Analysis Tables may be found on the Foundations of Algebra page on the High School Math Wiki at http://ccgpsmathematics9-10.wikispaces.com/Foundations+of+Algebra. This Module Analysis Table is NOT designed to be followed as a "to do list" but merely as ideas based on feedback from teachers of the course and professional learning that has been provided within school systems across Georgia.

MATERIALS LIST

Lesson	Materials
1. Building Number Sense Activities	• Ten Frames
Quick Check I	• Quick Check I student page
2. Fact Families	• Grid Paper • Colored pencils • Color tiles (optional) • Array-nging Our Fact Families recording sheet
3. Is it Reasonable?	• Compatible Numbers recording sheet
4. Yummy…Chocolate 3-Act Task	• Pictures of Ferrerro Rocher Chocolates
Problem Solving Assessment	• Assessment questions
Quick Check II	• Quick Check II student page
5. Birthday Cake	• Birthday Cake student recording sheet • Paper plates (optional) • Two sided counters (optional)
6. Fraction Clues	• Color Tiles • Fraction Clues recording sheet • Colored pencils
7. Multiplying Fractions	• Colored pencils • Area Model recording sheet • "How Many CC's ?" recording sheet
8. Birthday Cookout	• Birthday Cookout recording sheet
9. Chance of Surgery	• A Chance of Surgery recording sheet
10. Fractional Divisors	• Small bag (one per group) of chocolate candies • Base Ten blocks • Unit Cubes • Rulers

Lesson	Materials
11. Dividing Fractions With Models	• Freezer Pops (or paper-template in task) • Color Tiles • Grid paper
Quick Check III	• Quick Check III student page
12. Representing Powers of Ten Using Base Ten Blocks	• Base Ten blocks
13. Multiplying by Powers of Ten	• Internet connection
14. Patterns-R-Us	• Patterns R Us recording sheet • Calculators
15. Comparing Decimals	• Comparing Decimals recording sheet
16. Are These Equivalent?	• Grid paper (optional)
Quick Check IV	• Quick Check IV student page
17. Integers on a number line	• Number line • Two sided counters • Red and Yellow flags (or pieces of paper)
18. FAL: Using Positive and Negative Numbers in Context	• Temperature Change assessment lesson • Temperature Change (revisited) assessment lesson • City temperature cards • Changes in temperature cards
19. Deep Freeze	• Video for Deep Freeze • Recording sheet (attached)
Quick Check V	• Quick Check V student page

20. Penny Cube	• Video for Penny Cube • Recording sheet (attached)
21. Multiplying Rational Numbers	• Two sided counters (or two colors of counters)
22. Pattern of Multiplication and Division	• Patterns of Multiplication and Division lesson sheet • Colored pencils-red and yellow • Extra blank number lines (optional) • Two sided counters (or two colors of counters)
23. Rational or Irrational?	• Internet access
24. Estimating the Square Root of a Number	• Colored Tiles • Estimating the Square Root of a Number recording sheet
25. Decimal Approximation of Roots	• One inch Grid paper • Scissors • Decimal Approximation of Roots lesson • Open Middle recording sheet
26. Operations with Rational and Irrational Numbers	• Internet access
27. Debits and Credits	• Student ledger sheet

OVERVIEW

In this unit, the focus is not on algorithms but on building a conceptual understanding of basic mathematical ideas which will enhance the student's number sense. Number sense essentially refers to a student's ability to think fluidly and flexibly about numbers. To be successful in mathematics students need to have a sense that numbers are meaningful, i.e. what numbers mean and how they are related to one another. Additionally, they need to understand symbolic representations, use and understand numbers in real world contexts, and be able to perform mental math. Students who lack strong number sense do not have the foundation needed for simple arithmetic, much less more complex math. Building strong number sense is important because it promotes a sense of confidence in "making friends with numbers" (Carlyle and Mercado 2012). The unit begins primarily with the use of whole numbers followed by fractions, decimals, and integers through problem solving. Students will use a variety of strategies and manipulative tools such as quick number games/activities/apps, estimation, compatible numbers, colored tiles, number lines, and two-sided counters. Again the focus is on conceptual understanding; therefore, CALCULATORS SHOULD NOT BE USED!

STANDARDS FOR MATHEMATICAL CONTENT

Mathematical standards are interwoven and should be addressed throughout the year in as many different modules and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

KEY STANDARDS

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ1. Students will analyze number relationships.
a. Solve multi-step real world problems, analyzing the relationships between all four operations. For example, understand division as an unknown-factor problem in order to solve problems. Knowing that 50 x 40 = 2000 helps students determine how many boxes of cupcakes they will need in order to ship 2000 cupcakes in boxes that hold 40 cupcakes each. (MGSE3.OA.6, MGSE4.OA.3)
b. Understand a fraction a/b as a multiple of 1/b. (MGSE4.NF.4)
c. Explain patterns in the placement of decimal points when multiplying or dividing by powers of ten. (MGSE5.NBT.2)
d. Compare fractions and decimals to the thousandths place. For fractions, use strategies other than cross multiplication. For example, locating the fractions on a number line or using benchmark fractions to reason about relative size. For decimals, use place value. (MGSE4.NF.2; MGSE5.NBT.3,4)

MFANSQ2. Students will conceptualize positive and negative numbers (including decimals and fractions).
a. Explain the meaning of zero. (MGSE6.NS.5)
b. Represent numbers on a number line. (MGSE6.NS.5,6)
c. Explain meanings of real numbers in a real world context. (MGSE6.NS.5)

MFANSQ3. Students will recognize that there are numbers that are not rational, and approximate them with rational numbers.
a. Find an estimated decimal expansion of an irrational number locating the approximations on a number line. For example, for $\sqrt{2}$ show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue this pattern in order to obtain better approximations. (MGSE8.NS.1,2)
b. Explain the results of adding and multiplying with rational and irrational numbers. (MGSE9-12.N.RN.3)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
c. Interpret and solve contextual problems involving division of fractions by fractions. For example, how many 3/4-cup servings are in 2/3 of a cup of yogurt? (MGSE6.NS.1)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)
e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MGSE7.NS.3, MGSE7.EE.3, MGSE9-12.N.Q.3)

STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them. High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively. High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others. High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics. High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically. High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision. High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure. By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8. Look for and express regularity in repeated reasoning. High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in

mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.
The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. **Students who do not have an understanding of a topic may rely on procedures too heavily.** Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. **In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.**

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

*****Mathematical Practices 1 and 6 should be evident in EVERY lesson*****

ENDURING UNDERSTANDINGS

By the conclusion of this unit, the student should be able to:

- Perform the four basic mathematical operations with ease.
- Represent a number in a variety of ways (area models, expressions).
- Solve real world problems using a variety of strategies.
- Explore why dividing by zero is undefined.
- Determine the fractional amount of a given set and determine the entire set when given a fractional amount of the set.
- Find equivalent fractions.
- Represent multiplication of fractions using an array model.
- Complete a multi-step lesson.
- Multiply and divide by a power of ten.
- Identify, describe, and explain any patterns noticed when multiplying or dividing a number by 1000, 100, 10, 0, 1, 0.1, and 0.01.
- Understand the value of decimal numbers.
- Determine the equivalency of fractions and decimals.
- Identify the placement of integers on the number line.
- Understand and use positive and negative numbers in real world context.
- Use positive and negative numbers to describe opposites.
- Use a number line to multiply positive and negative numbers.

- Apply the patterns found in multiplying integers to division of positive and negative integers.
- Determine the relationship of integers when multiplied or divided.
- Identify rational and irrational numbers.
- Estimate the square root of a number using the number line or colored tiles.
- Identify the result as rational or irrational before working the problem.

ESSENTIAL QUESTIONS

How can you mentally compute mathematical operations?
How are multiplication and division related?
How can you model multiplication of whole numbers, fractions, decimals (base ten) and integers?
What are compatible numbers and how do they aid in multiplication and division?
What is the meaning of a remainder in a division problem?
What happens in division when there are zeroes in both the divisor and the dividend?
What does it mean to take the fraction portion of a whole number?
How do you solve a multi-step problem?
How can you represent a decimal using base ten blocks?
How do you compare decimals or decimals and fractions?
How can you represent integers on the number line?
How can you find the opposite of a number?
How can you use the number line to add/subtract/multiply integers?
What patterns in multiplication can you relate to division?
How are multiplication and division of integers related to one another?
How can you identify rational and irrational numbers?
How can you estimate the square root of a number?

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The terms below are for teacher reference only and are not to be memorized by the students. Teachers should present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

- Digit
- Array
- Fact Family
- Inverse Operation
- Factors
- Product
- Quotient
- Divisor

- Dividend
- Compatible Numbers
- Fraction
- Numerator
- Denominator
- Area Model
- Power of Ten
- Place Value
- Benchmark Fraction
- Integer
- Zero
- Opposite of a Number
- Rational Number
- Irrational Number
- Approximation
- Decimal Expansion
- Sum
- Difference
- Place Value
- Line Diagram

Again, discuss terminology as it naturally arises in discussion of the problems. Allow students to point out words or phrases that lead them to the model and solution of the problems. Words that imply mathematical operations vary based on context and should be delineated based on their use in the particular problem. A couple of suggested methods for students to record vocabulary are TIP Charts (https://www.youtube.com/watch?v=Rbts9h_ruu8) and Frayer Models (https://wvde.state.wv.us/strategybank/FrayerModel.html). The teacher and/or the students may find additional terms to include. The websites below are interactive and include a math glossary suitable for high school children. **Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**
http://www.amathsdictionaryforkids.com/ has activities to help students more fully understand and retain new vocabulary. http://intermath.coe.uga.edu/dictnary/homepg.asp. is geared towards middle and high school students.

INTERVENTION TABLE

Lesson Name	Name Of Intervention	Snapshot of summary or student I can statement …	Book, Page Or link
Building Number Sense Activities	Strategy Practice	Use a range of additive and simple multiplicative strategies with whole numbers, fractions, decimals, and percentages.	Strategy Practice
Fact Families	Bowl a Fact	Recall addition and subtraction facts to 20. Recall the multiplication and division facts for the multiples of 2, 3, 5, and 10. Recall multiplication to 10 x 10, and the corresponding division facts.	Bowl a Fact
Is It Reasonable?	Checking Addition and Subtraction by Estimation	Solve addition and subtraction problems by using place value	Checking Addition and Subtraction by Estimation Material Master 8-1
Birthday Cake	Birthday Cakes	Practice multiplying whole numbers by fractions	Birthday Cakes
Fraction Clues	Candy Creations Fraction Animals Fraction Strategies: Wafers	Find unit fractions of sets using addition facts Find unit fractions of sets using addition facts Find unit fractions of sets using addition facts	Candy Creations Fraction Animals Fraction Strategies: Wafers

Multiplying Fractions	A Fraction Times a Fraction	Work through some word problems to help increase fluency of multiplying fractions	A Fraction Times a Fraction
Representing Powers of Ten Using Base Ten Blocks	Powers of Powers	Use these activities to help your students develop knowledge of place value and powers of 10 to support multiplicative thinking	Powers of Powers
Multiplying By Powers of Ten	Powers of Powers	Use these activities to help your students develop knowledge of place value and powers of 10 to support multiplicative thinking	Powers of Powers
Pattern-R-Us	Powers of Powers	Use these activities to help your students develop knowledge of place value and powers of 10 to support multiplicative thinking	Powers of Powers
Comparing Decimals	Arrow Cards	Compare Decimals using decimal arrow cards and an understanding of place value.	Arrow Cards Material Master
Are These Equivalent?	Bead Strings	Find equivalents for decimals and fractions.	Bead Strings
Integers on the Number Line	Dropping and Rising Temperatures	Allow students to build fluency and automatize their use and operations with integers.	Dropping and Rising Temperatures
Deep Freeze	Hills and Valleys	Allow students to build fluency and automatize their use and operations with integers.	Hills and Valleys

Multiplying Rational Numbers	Multiplying Integers	Find a pattern in the multiplication facts of signed numbers.	Multiplying Integers
Rational or Irrational	Recurring and Termination Decimals	Solve problems by finding the prime factors of numbers.	Recurring and Terminating Decimals
Decimal Approximation of Roots	Powerful Numbers	Solve problems involving square roots	Powerful Numbers
Debits and Credits	Bucket Balance	Allow students to build fluency and automatize their use and operations with integers.	Bucket Balance

SCAFFOLDED INSTRUCTIONAL LESSONS

Lesson Name	Lesson Type/ *Grouping Strategy*	Content Addressed	Standard(s)
Building Number Sense Activities	*Partner/Whole Group*	Using Basic Number Sense with the Addition, Subtraction, Multiplication, and Division (Whole Numbers)	MFANSQ4.b
Quick Check I	Formative Assessment *Individual*	Prerequisite knowledge necessary to be successful with this first module	MFANSQ2
Fact Families	*Individual*	Building arrays and recognizing the fact family (Whole Numbers	MFANSQ1.a MFANSQ4.b,d
Is It Reasonable?	*Individual/Partner*	Multiplying and dividing in context problems through estimation, creating equal groups and building models.	MFANSQ1.a MFANSQ4.b,d
Yummy….Chocolate – 3 Act Task	*Individual/Partner/ Whole Group*	Solving real world problems.	MFANSQ1.a MFANSQ4.b,d
Problem Solving Assessments	*Individual*	Sampling of assessment questions focusing on multiplication and division and building models.	MFANSQ1.a MFANSQ4.b,d
Quick Check II	Formative Assessment *Individual*	Prerequisite knowledge necessary to be successful with fractions (May be used AFTER Birthday Cake and Fraction Clues depending on your students)	MFANSQ1.b MFANSQ4.b

Lesson Name	Lesson Type/ *Grouping Strategy*	Content Addressed	Standard(s)
Birthday Cake	*Partner/Group*	Determining the fraction of a set, and determine the set when given a fraction.	MFANSQ1.b MFANSQ4.b,c,d
Fraction Clues	*Partner*	Finding the fraction of a set, the set given a fraction, and equivalent fractions.	MFANSQ1.b MFANSQ4.b,c,d
Multiplying Fractions	*Individual/Partner*	Representing multiplication of fractions using a model.	MFANSQ1.b MFANSQ4.b,c,d
Birthday Cookout	*Individual/Partner*	Solving story problems that involve the multiplication of fractions.	MFANSQ1.b MFANSQ4.b,c,d
Chance of Surgery	*Individual*	Multiplying a whole number by a fraction using models.	MFANSQ1.b MFANSQ4.b,c,d,e
Fractional Divisors	*Individual/Partner/Small Group*	Dividing a fraction by a fraction using models.	MFANSQ4.b,c,d,e
Dividing Fractions with Models	*Individual/Partner*	Dividing a fraction by a fraction using models.	MFANSQ4.b,c,d,e
Quick Check III	Formative Assessment *Individual*	Fractions on a number line and Fraction Operations	MFANSQ1.b MFANSQ4.b,c,d,e
Representing Powers of Ten Using Base Ten Blocks	*Individual/Partner*	Multiplying and dividing by a power of ten.	MFANSQ1.c,d MFANSQ4.a,b,d
Multiplying By Powers of Ten	*Individual/Whole Group*	Multiplying and dividing by a power of ten.	MFANSQ1.c,d MFANSQ4.a,b,d
Pattern-R-Us	*Partner/Small Group*	Explaining patterns when multiplying or dividing by a power of ten.	MFANSQ1.c,d MFANSQ4.a,b,d
Comparing Decimals	*Individual/Partner*	Comparing decimals.	MFANSQ1.d MFANSQ4.a,b,d

Lesson Name	Lesson Type/ *Grouping Strategy*	Content Addressed	Standard(s)
Are These Equivalent?	*Individual/Partner*	Determining equivalency of fractions and decimals.	MFANSQ1.d MFANSQ4.a,b,d
Quick Check IV	Formative Assessment *Individual*	Equivalency of fractions	MFANSQ1.b MFANSQ2.b,c MFANSQ4.b,e
Integers on the Number Line	*Partner/Whole Group*	Identifying integers and their placement on the number line.	MFANSQ2.a,b,c MFANSQ4.a,b,d
FAL: Using Positive and Negative Numbers in Context	*Individual/Partner/ Whole Group*	Understanding and use integers in real world context.	MFANSQ2.a,b,c MFANSQ4.a,b,d
Deep Freeze	*Partner/Whole Group*	Using positive and negative numbers in real world context.	MFANSQ2.a,b,c MFANSQ4.a,b,d,e
Quick Check V	Formative Assessment *Individual*	Integers and Rational on a number line and in Real World contexts	MFANSQ2.a,b,c MFANSQ4.a,b,d,e
Penny Cube	*Partner/Whole Group*	Determining the value of a cube of pennies.	MFANSQ4.a,b,d,e
Multiplying Rational Numbers	*Partner/Whole Group*	Using the number line to multiply integers and apply the patterns to divide integers.	MFANSQ2.a.b.c MFANSQ4.a,b,d
Pattern of Multiplication and Division	*Individual/Partner*	Determining the relationship in integers when multiplied or divided.	MFANSQ2.a,b,c MFANSQ4.a,b,d
Rational or Irrational	*Individual/Partner*	Identifying rational and irrational numbers.	MFANSQ3.a,b MFANSQ4.a,b,d
Estimating the Square Root of a Number	*Individual/Partner*	Estimating the square root of a number using a number line or colored tiles.	MFANSQ3.a MFANSQ4.a,b,d

Lesson Name	Lesson Type/ *Grouping Strategy*	Content Addressed	Standard(s)
Decimal Approximation of Roots	*Individual/Partner*	Exploring why some square roots are irrational and determine the decimal approximation of these roots.	MFANSQ3.a MFANSQ4.a,b,d
Operations with Rational and Irrational Numbers	*Individual/Partner*	Identifying the result as rational or irrational before working a problem.	MFANSQ3.b MFANSQ4.b,d,e
Debits and Credits	*Individual/Partner*	Using a check register to record debits and credits and calculate a running total.	MFANSQ2.a,b,c MFANSQ4.a,b,d,e

The assessment for this module can be found through the Georgia Online Formative Assessment Resource (GOFAR). http://www.gadoe.org/Curriculum-Instruction-and-Assessment/Assessment/Pages/Georgia-Online-Formative-Assessment-Resource.aspx
This suggested assessment should be given as the pretest and posttest for this module.

Building Number Sense Activities

Adapted from Teaching Student-Centered Mathematics: Grades 3-5 by John A. Van de Walle.

The purpose of the activities in this lesson is to provide students the opportunity to build their conceptual understanding of whole numbers, look at computation in a different light and ultimately gain confidence in their number sense. ***These activities would be a great way to start class each day.***

SUGGESTED TIME FOR THIS LESSON:

5-10 minutes every day
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

> **MFANSQ4**. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
> b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4. Model with mathematics. High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify

important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7. Look for and make use of structure. By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8. Look for and express regularity in repeated reasoning. High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Recognize numbers quickly.
- Represent numbers in multiple ways.
- Fluently add and subtract whole numbers.
- Find products.
- Divide whole numbers.

Building Number Sense – Addition and Subtraction Activities

- Tell About It
- Battle of the Ten Frames (Game)
- 50 and Some More
- Compatible Pairs
- Sushi Monster (Game – Available on the App Store on iTunes)

ESSENTIAL QUESTIONS

- How can you represent a number in a variety of ways?
- How can you find the sum?
- How can you find the difference?

TEN FRAME ACTIVITIES

- Tell About It
- Battle of the Ten Frames

The objective of the ten frame activities is for students to build relationships with the anchor numbers of 5 and 10. The following video can be used to obtain a snapshot of a classroom implementing a Number Talk.

Video: Number Talks - **Math Perspectives: AMC Ten Frames Assessment**
https://www.youtube.com/watch?v=FGfj9oPaJW0

MATERIALS

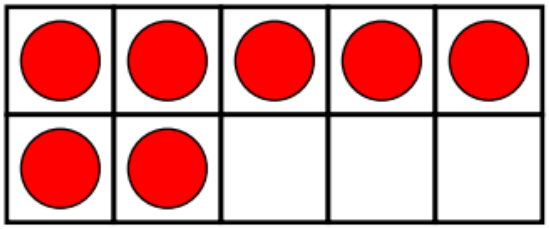
- Ten Frames (a set is provided at the end of this section)

TELL ABOUT IT

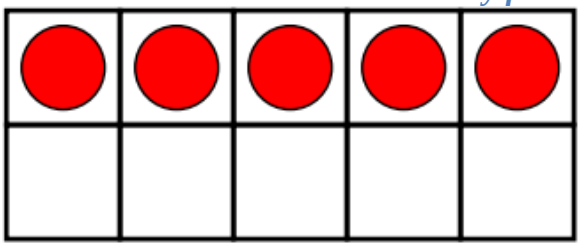
Grouping: Partners; Whole Group

Directions:
Give each student a card or project a card for the whole class to view. Ask the student(s), "What number do you see?" Then ask, "How could you write that number differently given this ten frame?"

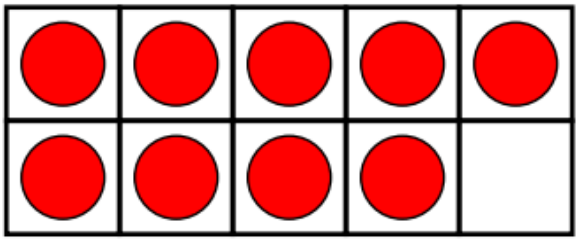
Possible answers: It is 7. It is 3 away from 10, 2 more than 5, or someone may see it as a group of 4 plus 3 more which is fine but encourage 5's and 10's. They are "nice numbers" which are easy to work with.



Possible answers: It is 5. It is 5 away from 10.



Possible answers: It is 9. It is 1 away from 10, 4 more than 5.

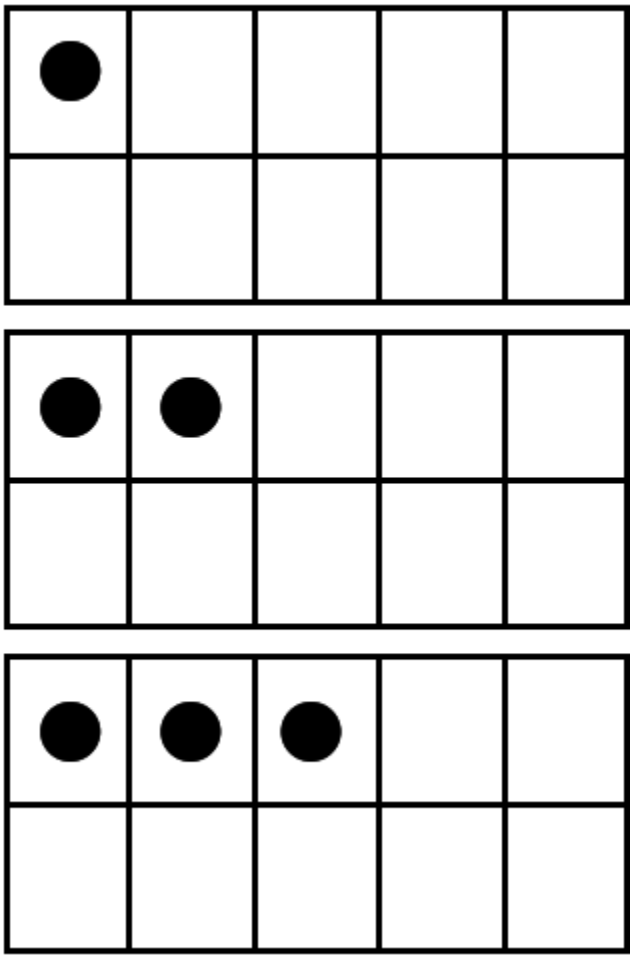


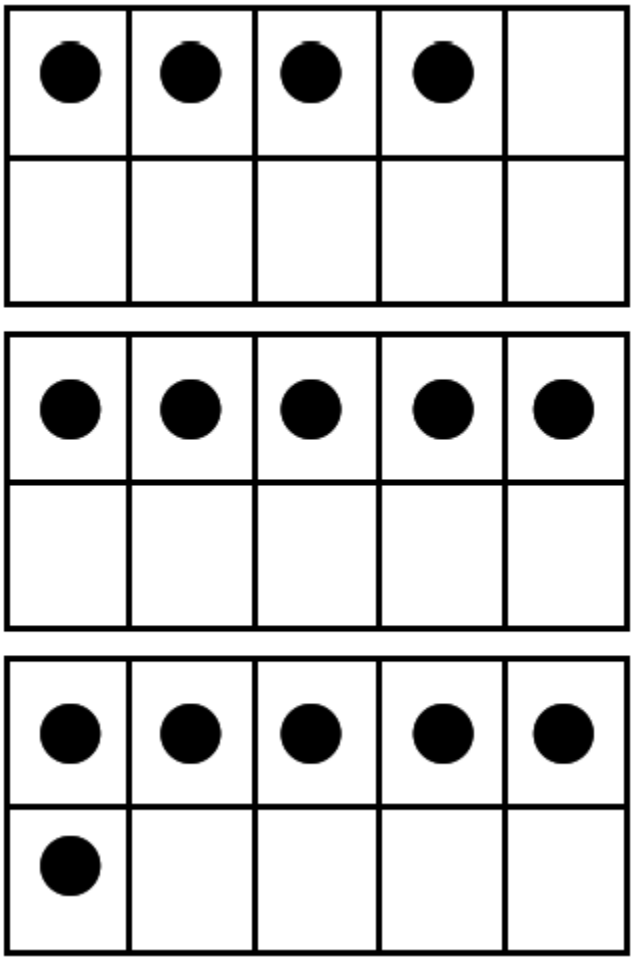
BATTLE OF THE TEN FRAMES: Adding and Subtracting

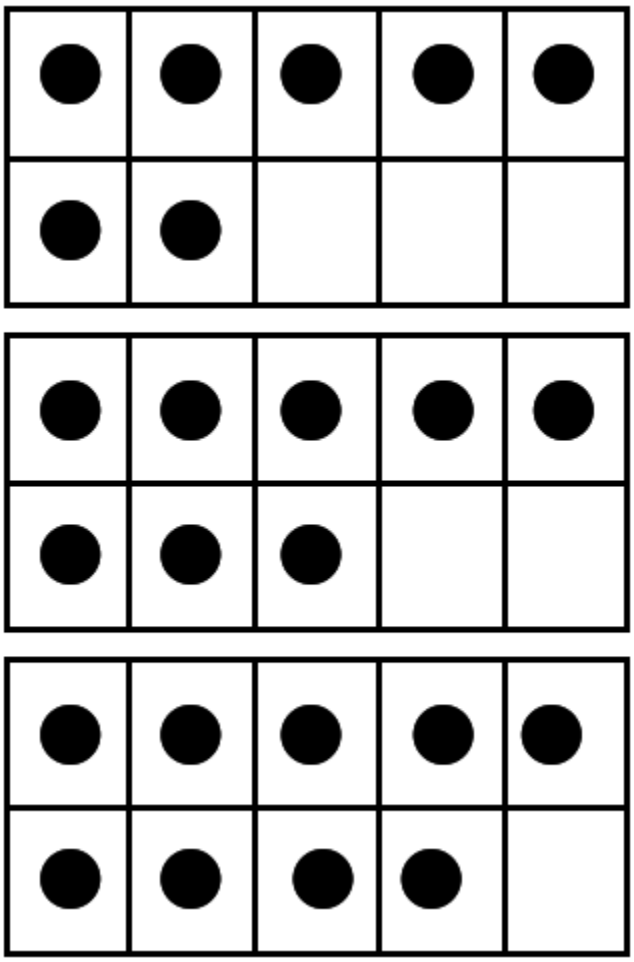
Grouping: Partners

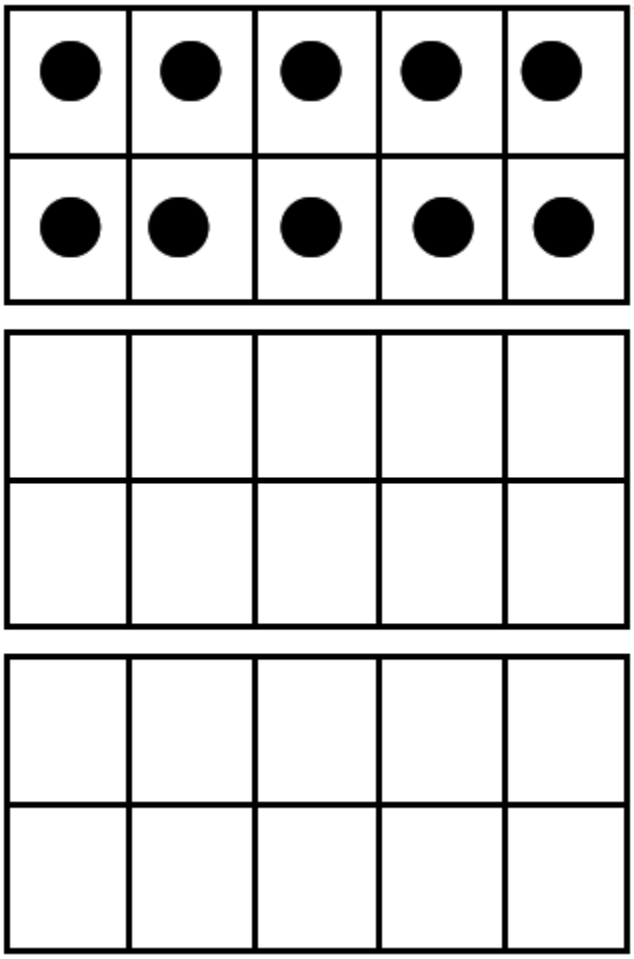
Directions:

Provide each student a deck of cards 1-10. Each student will place a card down. The student with correct sum first wins the pair. (Alternate Version: Subtraction)
Whole Class Addition Version: https://www.youtube.com/watch?v=AVjvswqL-Ow









50 AND SOME MORE

Grouping: Whole Group

Materials:

- none

Directions:
Say a number to the class. You can call on people or ask for volunteers. The student's response should be "50 and ___". For example, if the number is 72 they would say "50 and 22." If the number were, 39 then the student would say "50 and -11." You should probably start out with number greater than 50 then work to numbers less than 50 by the end of the course. This is a great time filler with a purpose. This activity could also work with 100. Again notice the anchor numbers of "5 and 10."

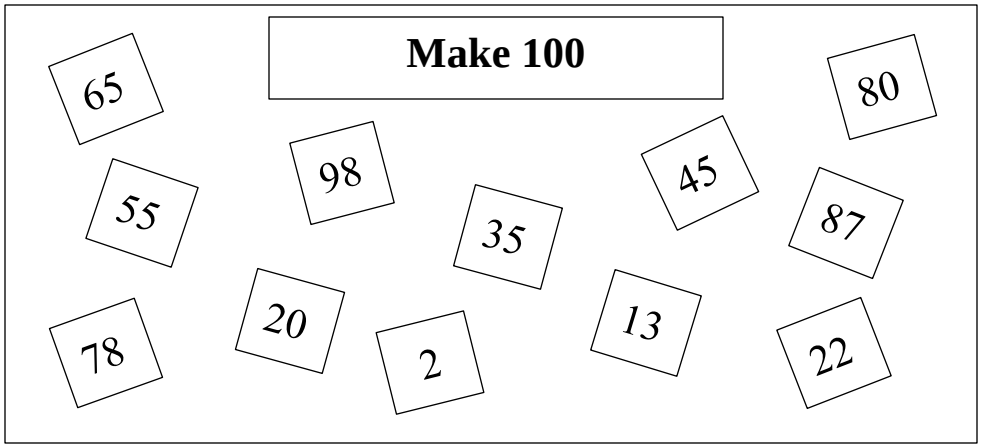
COMPATIBLE PAIRS

Grouping: Individual; Partners

Materials:

- WS/Card prepared by the teacher

Directions:
Provide students with a group of numbers. From the group of numbers given, students are to complete the lesson. See the example below where students are asked to make pairs that have a sum of 100. This is a great activity for **whole numbers, integers, fractions, or decimals**. The teacher can also vary the operation.



SUSHI MONSTER

Grouping: Individual

The Sushi Monster App can be found on the App Store on iTunes and is free. The app encourages practice in either addition or multiplication.

INTERVENTION

For extra help with addition and subtraction, please open the hyperlink Intervention Table.

Building Number Sense – Multiplication & Division

- Finding Factors
- Divide It Up
- Slice It Up
- Impoppable (Game – Available on the App Store on iTunes)
- Sushi Monster (Game – Available on the App Store on iTunes)

ESSENTIAL QUESTIONS

- How can you represent a number in a variety of ways?
- How can you find the factors of a given product?
- How can you find the product?

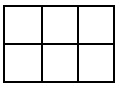
FINDING FACTORS

Grouping: Individual

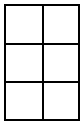
Materials

- Grid Paper
- Color Tiles

Directions:
Assign each student a number that has several factors. Have them create a variety of models for those factors. (For Example – arrays, use of the numbers, drawing sets)



2 by 3 is 6



3 by 2 is 6

DIVIDE IT UP

Grouping: Individual; Partners

Materials:

- Color Tiles

Directions:
Assign each student a number and that number of colored tiles. Tell the students how many equal sets you want or the number that should be in each. The students should then divide out the tiles to see if they can make equal sets. Have the student write the fact family if possible.
Example: Given the number 12. A student may make 3 equal sets with 4 tiles in each group. Fact family would be 3 x 4 = 12, 4 x 3 = 12, and 12 ÷ 3 = 4, 12 ÷ 4 = 3

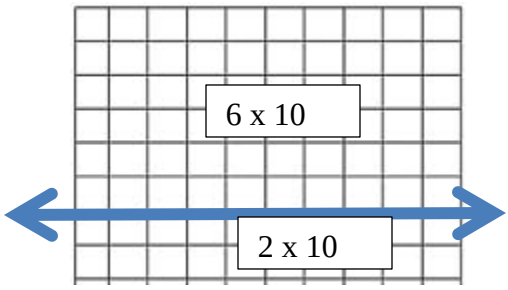
SLICE IT UP (Distributive Property)

Grouping: Individual; Partners

Materials:

- Grid Paper

Directions:
Give each student an array. Have them slice it up. For example, an 8 x 10 array may be sliced so that it shows a 6 x 10 and 2 x 10. 8 x 10 = (6 x 10) + (2 x 10)



IMPOPPABLE

Grouping: Individual

The Impoppable App can be found on the App Store on the iTunes and is free. The app encourages the use of fact families. Students are to quickly pop bubbles in order 3, 5, 15 (multiplication) or 15, 3, 5 (division). As you progress in the levels more fact families are added.

SUSHI MONSTER

Grouping: Individual

The Sushi Monster App can be found on the App Store on the iTunes and is free. The app encourages practice in either addition or multiplication.

INTERVENTIONS

For extra help with operations, please open the hyperlink Intervention Table.

Quick Check I

STANDARDS FOR MATHEMATICAL CONTENT

MFANSQ2. Students will conceptualize positive and negative numbers (including decimals and fractions).
a. Explain the meaning of zero. (MGSE6.NS.5)
b. Represent numbers on a number line. (MGSE6.NS.5,6)
c. Explain meanings of real numbers in a real world context. (MGSE6.NS.5)

Instructional Tip

Quick Check I and Quick Check II are placed in this module as formative assessments pieces to assist the teacher in determining whether their students are ready for the upcoming lessons. If students struggle with these Quick Checks it is recommended to devote more instructional time on the Building Number Sense Activities before moving further into the lessons within Module I. As you do so listen carefully to students conjectures to adjust the instructions as needed, determining whether students are ready to work with more complex ideas.

Quick Checks III, IV, and V are placed in this module as formative assessments pieces to assist the teacher in determining whether their students have mastered the basic concepts from the lessons previously taught.

Quick Check I - Formative Assessment

Give a short answer for problems 1 – 4.

1. What are the digits? (Hint: we use them to make any number)
   0 thru 9 are the digits
2. What number does the set of **whole** numbers start with?
   The whole numbers start with zero.

3. If two numbers are **opposites** and one number is 32, what is the other number?
   The other number would be (-32)

4. What is an integer?
   An integer is a positive or negative whole number

Finish the following math sentences. Remember to use your number line.

5. 4 + (-3) = 1

6. (-2) + 9 = 7

7. (-4) + (-4) = -8

8. 3 + (-3) = 0

9. 5 – (-3) = 8

10. (-6) – (-4) = -2

Quick Check I - Formative Assessment

1. What are the digits? (Hint: we use them to make any number)

2. What number does the set of **whole** numbers start with?

3. If two numbers are **opposites** and one number is 32, what is the other number?

4. What is an integer?

Finish the following math sentences. Remember to use your number line.

5. $4 + (-3) =$

6. $(-2) + 9 =$

7. $(-4) + (-4) =$

8. $3 + (-3) =$

9. $5 – (-3) =$

10. $(-6) – (-4) =$

Fact Families

SUGGESTED TIME FOR THIS LESSON:

60-90 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ1. Students will analyze number relationships.
a. Solve multi-step real world problems, analyzing the relationships between all four operations. For example, understand division as an unknown-factor problem in order to solve problems. Knowing that 50 x 40 = 2000 helps students determine how many boxes of cupcakes they will need in order to ship 2000 cupcakes in boxes that hold 40 cupcakes each. (MGSE3.OA.6, MGSE4.OA.3)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

Common Misconceptions

- *Students may have difficulty seeing multiplication and division as inverse operations. In order to develop an understanding of this relationship, students need to have ample opportunities to explore these two operations simultaneously.*
- *When listing multiples of numbers, students may not list the number itself. Emphasize that the smallest multiple is the number itself.*
- *Some students may think that larger numbers have more factors. Having students share all factor pairs and how they found them will clear up this misconception.*
- *Some students may need to start with numbers that only have one pair of factors, then those with two pairs of factors before finding factors of numbers with several factor pairs.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

Represent multiplication and division using a rectangular array model.

- Represent a number in a variety of mathematical sentences.
- Solve real world problems using strategies.

MATERIALS

- Grid paper
- Color pencils
- "Array-nging Our Fact Families" recording sheet
- Color Tiles (optional for tactile learners)

ESSENTIAL QUESTIONS

- How are multiplication and division related?
- How can the same array represent both multiplication and division?

Grouping: Individual/Partner

OPENER/ACTIVATOR

Have students look at the picture of chocolates. Ask the students "How many do you see? How do you see them?"

Students may respond: 6 chocolates

3 rows of 2 in each row
2 columns of 3 in each column
2 groups of 3
3 groups of 2
3 x 2
2 x 3

Again have the students view the second picture of chocolate. Ask the students "How many chocolates are in this picture? How do you know?"

Students may respond: 12 chocolates

3 rows of 4 in each row
4 columns of 3 in each column
4 groups of 3
3 groups of 4
3 x 4
4 x 3

Student Page: Opener/Activator

How many do you see? How do you see them?

How many chocolates are in this picture? How do you know?

WORK SESSION

PART 1

Using the grid paper and colored pencils, have students create as many arrays possible using 12 squares. Is there a relationship between of the arrays drawn? (i.e. 3 by 4 array is the same as 4 by 3 array)

Have the students write a multiplication sentence for each array. Ask students to write a division sentence with the dividend represented by the total area of the array. For example, a student may make a 4 x 3 array. The dividend (area of 12) can be divided by 4 or 3 both factors of 12. Both dimensions are utilized, one as the divisor and the other as the quotient.

3 x 4 = 12
4 x 3 = 12
12 ÷ 3 = 4
12 ÷ 4 = 3

Another array may be a 2 by 6:
2 x 6 = 12
6 x 2 = 12
12 ÷ 2 = 6
12 ÷ 6 = 2

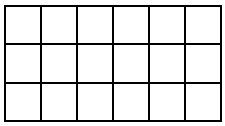
PART 2

Teacher Key

Students will follow the directions below from the "Array-nging Our Fact Families" recording sheet.

1. Draw the following arrays:

6 by 3

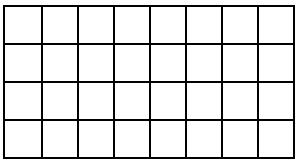


3

6

6 x 3 = 18
3 x 6 = 18
18 ÷ 3 = 6
18 ÷ 6 = 3

4 by 8

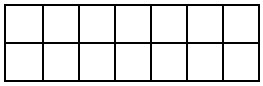


4

8

8 x 4 = 32
4 x 8 = 32
32 ÷ 8 = 4
32 ÷ 4 = 8

2 by 7



2

7

2 x 7 = 14
7 x 2 = 14
14 ÷ 2 = 7
14 ÷ 7 = 2

2. Use the example to complete the following for each array:

- Label the dimensions and total area. ***See answers above.***
- Write a multiplication sentence and label the factors and the product. ***See answers above.***
- Write a division sentence and label the divisor, dividend, and quotient. ***See answers above.***

3. Select one of your arrays and write two story problems that can be modeled with the array, one for multiplication and one for division.

Answers may vary.

Strategies for Teaching and Learning: The use of mathematical vocabulary can shy students away from story problems, so encourage students to use the appropriate vocabulary when discussing the fact families. For example, "the factors of 4 and 3 result in a product of 12" or "3 is the quotient of '12 ÷ 4'".

Differentiation: Allow students to build an array of their choice, but limit them to not larger than a 3 digit by 1 digit.

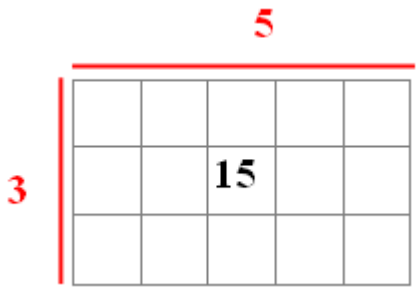
Technology: Impoppable
The Impoppable App can be found on the App Store on the iTunes and is free. The app encourages the use of fact families. Students are to quickly pop bubbles in order 3, 5, 15 (multiplication) or 15, 3, 5 (division). As you progress in the levels more fact families are added.

INTERVENTIONS

For extra help with number facts, please open the hyperlink Intervention Table.

Name _____ Date _____

Array-nging Our Fact Families



This 3 by 5 array has a total of 15 square units.
3 x 5 = 15.

3 and 5 are **factors**.
15 is the **product**.

Fifteen divided by three equals five.
15 ÷ 3 = 5
15 is the **dividend.**
3 is the **divisor.**
5 is the **quotient.**

1. Draw the following arrays listed in the table below.
2. Following the example above, complete the following for each array:
   - ☑ Label the dimensions and total area.
   - ☑ Write a multiplication sentence and label the factors and the product.
   - ☑ Write a division sentence and label the divisor, dividend, and quotient.

6 by 3	4 by 8	2 by 7

3. Select one of your arrays. On the back of this paper, write two story problems that can be modeled with the array, one for multiplication and one for division.

CLOSING/SUMMARIZER

Given the picture of Swiss Miss, write the fact family and a story problem.

Student Work: CLOSING/SUMMARIZER

Given the picture of Swiss Miss, write the fact family and a story problem.

Additional Practice Problems:

1. The vending machine has been loaded up with Mrs. Lane's favorite candy, Peanut Butter Cups. If there are 9 sections of Peanut Butter Cups and each one holds 12, how many days can you keep Mrs. Lane happy if you give her one each day?

   9 sections x 12 Peanut Butter Cups = (9 x 10) + (9 x 2) = 90 + 18 = 108 days

2. The time has come to purchase your parking pass for school. The student parking lot consists of 6 rows which hold 34 cars each. There are 200 students at your school that can drive. How many will not be able to park at school?

   6 rows x 34 cars = (6 x 30) + (6 x 4) = 180 + 24 = 204 spaces in the lot
   So there are actually 4 extra spaces.
   Zero – everyone will be able to park.

3. Your teacher wants to make sure everyone has a pencil for class. She has 104 students in all of her classes. The office gave her 4 boxes of pencils. How many pencils were in each box if each of her students receives 1 pencil?

   104students ÷4boxes of pencils = (100÷4) + (4÷4) = 25 + 1 = 26 pencils in each box

4. Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each pack. Sarah wheels in 6 packs with 12 bottles in each pack. About how many bottles of water still need to be collected?

   Max 3 packs x 6 bottles/pack – 18 bottles
   Sarah 6 packs x 12 bottles/pack = 72
   18 + 72 = 90 bottles (Mentally the student may think (10+70) = (8+2) = 80+10 = 90)
   300 – 90 = 210 bottles(Mentally using compatible numbers 300 – 100 = 200 + 10)

Student Edition: Additional Practice Problems:

1. The vending machine has been loaded up with Mrs. Lane's favorite candy, Peanut Butter Cups. If there are 9 sections of Peanut Butter Cups and each one holds 12, how many days can you keep Mrs. Lane happy if you give her one each day?

2. The time has come to purchase your parking pass for school. The student parking lot consists of 6 rows which hold 34 cars each. There are 200 students at your school that can drive. How many will not be able to park at school?

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Is It Reasonable?

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ1. Students will analyze number relationships.
a. Solve multi-step real world problems, analyzing the relationships between all four operations. For example, understand division as an unknown-factor problem in order to solve problems. Knowing that 50 x 40 = 2000 helps students determine how many boxes of cupcakes they will need in order to ship 2000 cupcakes in boxes that hold 40 cupcakes each. (MGSE3.OA.6, MGSE4.OA.3)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

Common Misconceptions:

- *Students tend to not ask themselves if the answer fits the situation. They feel the result of their calculations must be true for any given situation.*
- *Students do not realize there are situations when you must round up even if the decimal is less than five-tenths.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.

3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Estimate and find the product of a 2- digit number multiplied by a 2-digit number.
- Represent multiplication and division using a rectangular area model.
- Understand that multiplication may be used in problem contexts involving equal groups, rectangular arrays/area models, or rate.
- Solve division problems using strategies.
- Divide whole-numbers quotients and remainders with up to four-digit dividends and remainders with up to four-digit dividends and one-digit divisors.

MATERIALS

- "Compatible Numbers" Recording Sheet

ESSENTIAL QUESTIONS

- How are multiplication and division related to each other?
- What are some simple methods for solving multiplication and division problems?
- What patterns of multiplication and division can assist us in problem solving?
- How can you mentally compute a division problem?
- What are compatible numbers and how do they aid in dividing whole numbers?

Grouping: Individual/Partner

OPENER/ACTIVATOR

Pose this question: (No Paper/Pencil Allowed)

There are 592 students participating in Field Day. They are put into teams of 8 for the competition. How many teams are there?

Comments
The understanding of the relationship that exists between multiplication and division is critical for students to know as well as the strong relationship between the dividend, divisor, and quotient. This lesson is designed to allow students to further explore these relationships.

Possible answers:

Student 1	*Student 2*	*Student 3*
592 divided by 8 *There are 70 8's in 560* *592 - 560 = 32* *There are 4 8's in 32* *70 + 4 = 74*	*592 divided by 8* *I know that 10 8's is 80* *If I take out 50 8's that is 400* *592 - 400 = 192* *I can take out 20 more 8's which is 160* *192 - 160 = 32* *8 goes into 32 4 times* *I have none left* *I took out 50, then 20 more, then 4 more* *That is 74*	*I want to get to 592* *8 x 25 = 200* *8 x 25 = 200* *8 x 25 = 200* *200 + 200 + 200 = 600* *600 - 8 = 592* *I had 75 groups of 8 and took one away, so there are 74 teams*

WORK SESSION:

NUMBER TALKS

Several Number Talks strategies can help student build a stronger understanding of division. Repeated subtraction, partial quotients, multiplying up and proportional reasoning are all valuable strategies that students can explore through number talks. For more information refer to pages 286-299 in *Number Talks.* (Number Talks, 2010, Sherry Parrish)

LESSON DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Comments

Ask students how they could estimate the number of small prizes each of Mr. Wong's 9 students would receive if he had exactly 893 prizes to give away. If no one mentions compatible numbers, remind the class that they can estimate the answer to a problem by replacing the numbers in the problem with numbers that are easier to calculate with. Such easier numbers are called compatible numbers. You might show this example of compatible numbers:

- To estimate 3,456 ÷ 7, students might recognize 3,456 is close to 3,500 and choose compatible numbers 3,500 and 7. So, 3,456 ÷ 7 is about 3,500 ÷ 7, or 500.

Lesson Directions/Answers

Students will follow the directions below from the "Compatible Numbers" recording sheet.

1. Mr. Wong has between 300 and 1,000 small prizes to divide evenly among his 9 students over the course of the school year. He will give away as many prizes as possible. Estimate the number of small prizes each of Mr. Wong's 9 students would receive if he had exactly 893 prizes to give away.

   893 ÷ 9 ≈ 900 ÷ 9 ≈ 100 prizes/student

2. At Hatfield Elementary School, there are 504 students in 7 classes. Each class has the same number of students. What is a good estimate of the number of students in each class? Explain your reasoning.

   504 ÷ 7 = (490÷7) + (14÷7) = 70 students + 2 students = 72 students

3. Marcel worked 9 hours and earned $232. What is a good estimate of the amount that he earned each hour? Explain your reasoning.

   $232÷9
   $180÷9 = 20 leaving $52÷9 ≈ $54÷9 = 6 so $20 + $6 = about $26/hr

   Or

   Estimating $230 ÷ 10 = $23 which is low; therefore, we could estimate about $25/hr

FORMATIVE ASSESSMENT QUESTIONS

- What compatible numbers are you using?
- How did these compatible numbers make solve the problem easier?
- Do you think that is a reasonable estimate? Why?

DIFFERENTIATION

Extension

Have students solve the following problem with an estimate which fits the context. Mr. Wong has between 300 and 1,000 small prizes to divide evenly among his 9 students over the course of the school year. He will give away as many prizes as possible. What is the greatest number of prizes that could be left over? Is it possible for each student to get 200 prizes?

Intervention

Have students link basic division facts to identifying compatible numbers. You can begin with 35 ÷ 7, then 350 ÷ 7. Make explicit the connection of the compatibility between 35 and 7 and how it can be applied to 350 and 7.

For extra help with estimation, please open the hyperlink Intervention Table.

TECHNOLOGY

http://www.bbc.co.uk/schools/ks1bitesize/numeracy/division/index.shtml provides differentiated activities for 3 levels and can be used for remediation or additional practice.

http://www.thinkingblocks.com/mathplayground/tb_md/tb_md5.html Watch a short video that explains how to use thinking blocks to interpret remainders when solving division word problems. Then, practice modeling and solving problems. The video can be used to show other strategies for dividing or as an introduction to the lesson.

http://illuminations.nctm.org/activity.aspx?id=4197 is a game which can be used to practice the concept of division.

Name _____ Date _____

Compatible Numbers

Use compatible numbers to help you estimate the answers for the following problems.

1. Mr. Wong has between 300 and 1,000 small prizes to divide evenly among his 9 students over the course of the school year. He will give away as many prizes as possible. Estimate the number of small prizes each of Mr. Wong's 9 students would receive if he had exactly 893 prizes to give away.

2. At Hatfield Elementary School, there are 504 students in 7 classes. Each class has the same number of students. What is a good estimate of the number of students in each class? Explain your reasoning.

3. Marcel worked 9 hours and earned $232. What is a good estimate of the amount that he earned each hour? Explain your reasoning.

CLOSING/SUMMARIZER

Journal Ideas:

- Explain how compatible numbers are used to solve problems efficiently.
- Write a story problem in which one might use compatible numbers to solve.

Additional Practice:

1. There are 128 students going on a field trip. If each bus held 30 students, how many buses are needed?

   128÷ 30 ≈ 120 ÷ 30 ≈ 4 buses with 8 students leftover; therefore, 5 buses are needed

2. Chris bought clothes for school. She bought 3 shirts for $12 each and a skirt for $15. How much money did Chris spend on her new school clothes?

   3 shirts x $12 = $36 on shirts
   $36 + $15 = ($30 + $10) + ($6 + $5) = $40 + $11 = $51 on new clothes

3. Kim is making candy bags. There will be 5 pieces of candy in each bag. She had 53 pieces of candy. She ate 14 pieces of candy. How many candy bags can Kim make now?

   53 – 14 = 39 pieces of candy left to make bags
   (Mentally the student may have thought 54 – 14 = 40 – 1 = 39)
   39÷5 is close to 40÷5 which is 8 but we rounded up so we can only make 7 bags.

4. Write different word problems involving 44 ÷ 6 = ? where the answers are best represented as:
   a. Problem A: 7
   b. Problem B: 7 r 2
   c. Problem C: 8
   d. Problem D: 7 or 8
   e. Problem E: $7\frac{2}{6}$

   Answers may vary.

Student Edition: Additional Practice:

1. There are 128 students going on a field trip. If each bus held 30 students, how many buses are needed?

2. Chris bought clothes for school. She bought 3 shirts for $12 each and a skirt for $15. How much money did Chris spend on her new school clothes?

3. Kim is making candy bags. There will be 5 pieces of candy in each bag. She had 53 pieces of candy. She ate 14 pieces of candy. How many candy bags can Kim make now?

4. Write different word problems involving 44 ÷ 6 = ? where the answers are best represented as:
    a. Problem A: 7
    b. Problem B: 7 r 2
    c. Problem C: 8
    d. Problem D: 7 or 8
    e. Problem E: $7\frac{2}{6}$

Yummy….Chocolate – 3 Act Task

SUGGESTED TIME FOR THIS LESSON:

60-90 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ1. Students will analyze number relationships.
a. Solve multi-step real world problems, analyzing the relationships between all four operations. For example, understand division as an unknown-factor problem in order to solve problems. Knowing that 50 x 40 = 2000 helps students determine how many boxes of cupcakes they will need in order to ship 2000 cupcakes in boxes that hold 40 cupcakes each. (MGSE3.OA.6, MGSE4.OA.3)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.

6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to solve real world problems.

MATERIALS

Pictures of Ferrero Rocher Chocolates

ESSENTIAL QUESTIONS

How can you solve real world problems?

Grouping: Individual/Partner/Whole Group

OPENER/ACTIVATOR

ACT ONE:

Provide students with this information:

At Sam's Club, 21.1 ounces of Ferrero Rocher Chocolates costs $13.46.
Grade Level: 526 students

Teacher Notes: Brainstorm questions one might ask given this information.

- ***How many boxes must you buy to purchase one chocolate for every student?***
- ***How many ounces would each student get?***
- ***What is the cost per student?***

WORK SESSION:

ACT TWO:

Given the strategies to find compatible numbers, build arrays, and groups, show how you could solve this problem:

Can you justify this cost to treat each of your students one Ferrero Rocher Chocolate to celebrate the end of testing?

The box of chocolates is a 4 by 6 array; therefore there are 24 chocolates per box.

526÷24 is about 525÷25 (thinking of money) = There are 20 quarters in $5 and 1 quarter in 25 cents. This is 21 boxes; however, you need 1 more box because there are only 24 in the box not 25. You need 22 boxes.

Task Title:_____ Name:_____

Adapted from Andrew Stadel

ACT 1

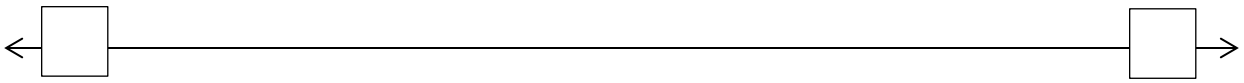
What did/do you notice?

What questions come to your mind?

Main Question:_____

Estimate the result of the main question? Explain?

Place an estimate that is too high and too low on the number line



Low estimate *Place an "x" where your estimate belongs* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:_____

Act 2 (con't)

Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

CLOSING/SUMMARIZER

ACT THREE:

Have students share their solutions via a gallery walk. Have the students reflect on their solutions and the solutions they viewed. Ask the students: "Were there any strategies solutions, or arguments that stick out in your mind? Why?" Have the students write about them in their journals.

Problem Solving Assessments

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ1. Students will analyze number relationships.
a. Solve multi-step real world problems, analyzing the relationships between all four operations. For example, understand division as an unknown-factor problem in order to solve problems. Knowing that 50 x 40 = 2000 helps students determine how many boxes of cupcakes they will need in order to ship 2000 cupcakes in boxes that hold 40 cupcakes each. (MGSE3.OA.6, MGSE4.OA.3)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
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8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to solve real-world problems involving number operations.

MATERIALS

- Assessment Questions

ESSENTIAL QUESTIONS

- How can you solve real world problems?

Grouping: Individual

OPENER/ACTIVATOR

Given the Problem:
The 9^{th} grade class at Jones County 9^{th} Grade Campus is going on their annual field trip to Savannah, GA. The teachers reserved two buses to take the 123 students and 14 chaperones. Each bus has 32 seats that can hold up to three people per seat. Will they need to sit in groups of 3 in each seat or can they have more room and only sit with a partner?

Analyze Dustin's Response Below:

If each bus has 32 seats, then 2 buses x 32 seats means there are 64 seats total. If there are 64 seats with 2 people on each seat, the two buses can carry 128 students comfortably. Since we only have 123 students, we will have plenty of room and no one will have to sit 3 to a seat.

How would you grade Dustin's answer?

A great rubric can be found at http://bit.ly/1eNEEBN

WORK SESSION:

Below are sample assessment questions. Pick and choose the question(s) you deem appropriate for your class. Questions not utilized in this activity could be placed on your End of Module Assessment.
Additional problems along with a rubric can be found at http://bit.ly/1eNEEBN

QUESTION #1
Kameron went to the arcade in Panama City Beach and played a game that involved throwing beanbags at clowns. She had one minute to hit as many clowns as she could, and she hit 33 clowns before time ran out. When she was done, she earned 2 points for each clown she hit.

At the prize counter, Kameron was able to buy one prize for every 7 points she earned.

Kameron bought as many prizes as she could with her points. How many prizes was Kameron able to buy? Use the space to below to show your thinking as you figure out the number of prizes Kameron bought. Then, fill in the blank to show your answer.

Answer: Kameron bought 9 prizes.

QUESTION #2
Landon and Grant were both selling candles for a school fundraiser.

Landon sold 69 candles and was awarded a prize point for every 4 candles he sold.

Grant's school only awarded a prize point for every 3 candles sold. However, Grant ended up earning the same number of points as Landon.

How many candles could Jaylen have sold? Use the space below to show your thinking, and then fill in the blank with your answer.

Answer: Grant could have sold 51, 52, or 53 tickets to earn the same number of points as Landon. Landon earned 17 points (69 ÷ 4). Since Grant's school awarded a point for every 3 tickets sold, then to earn the same amount of tickets- 17 x 3= 51 tickets. He could have also sold 52 or 53 tickets to earn the same 17 points.

QUESTION #3

Sophie worked hard to save for a senior trip. The trip costs $1500. She earned $9 an hour babysitting and $4 an hour raking leaves. In October, she worked 52 hours babysitting and 12 hours raking leaves. Then, in November, she babysat 2 times as much as she had in October. Each month she set aside $50 to spend and saved the rest. Did she make enough money to pay for her trip? If not, how much more money does she need to pay off the trip in full?

Solve using numbers, pictures and/or words.

Answer: Total of 156 babysitting hours at $9/hr = $1,404 and a total of 21 hours raking leaves at $4/hr is $84. Their sum gives you a total of $1,488. Take out the $100 ($50/month for 2 months of spending) gives you $1,388. $1,500 - $1,388 = $112. Sophie needs to earn $112 more dollars to attend her Senior Class Trip.

Student Edition: Assessment Questions Practice

QUESTION #1
Kameron went to the arcade in Panama City Beach and played a game that involved throwing beanbags at clowns. She had one minute to hit as many clowns as she could, and she hit 33 clowns before time ran out. When she was done, she earned 2 points for each clown she hit.

At the prize counter, Kameron was able to buy one prize for every 7 points she earned.

Kameron bought as many prizes as she could with her points. How many prizes was Kameron able to buy? Use the space to below to show your thinking as you figure out the number of prizes Kameron bought. Then, fill in the blank to show your answer.

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Landon and Grant were both selling candles for a school fundraiser.

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Grant's school only awarded a prize point for every 3 candles sold. However, Grant ended up earning the same number of points as Landon.

How many candles could Jaylen have sold? Use the space below to show your thinking, and then fill in the blank with your answer.

QUESTION #3
Sophie worked hard to save for a senior trip. The trip costs $1500. She earned $9 an hour babysitting and $4 an hour raking leaves. In October, she worked 52 hours babysitting and 12 hours raking leaves. Then, in November, she babysat 2 times as much as she had in October. Each month she set aside $50 to spend and saved the rest. Did she make enough money to pay for her trip? If not, how much more money does she need to pay off the trip in full?

Solve using numbers, pictures and/or words.

CLOSING/SUMMARIZER

Using the rubric at http://bit.ly/1eNEEBN *, have students rate themselves based on how they feel they did.*

Quick Check II

STANDARDS FOR MATHEMATICAL CONTENT

MFANSQ1. Students will analyze number relationships.
b. Understand a fraction a/b as a multiple of 1/b. (MGSE4.NF.4)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)

Instructional Tip

Quick Check I and Quick Check II are placed in this module as formative assessments pieces to assist the teacher in determining whether their students are ready for the upcoming lessons. If students struggle with these Quick Checks it is recommended to devote more instructional time on the Building Number Sense Activities before moving further into the lessons within Module I.
As you do so, listen carefully to students conjectures to adjust the instructions as needed, determining whether students are ready to work with more complex ideas.

Quick Checks III, IV, and V are placed in this module as formative assessments pieces to assist the teacher in determining whether their students have mastered the basic concepts from the lessons previously taught.

Quick Check II - Formative Assessment

1. Complete the following sentence. An improper fraction b) has a value more than 1
a) has a 1 in the numerator b) has a value more than 1
c) has a value less than 1 d) is equal to 1

2. What is a numerator? a
a) the top part of a fraction b) the bottom part of a fraction
c) the value of a fraction d) a number equal to 1

3. If you have $\frac{3}{8}$, how much do you need to make a whole (Hint: a whole is equal to 1)? d

a) 5 b) $\frac{3}{8}$ c) $\frac{8}{8}$ d) $\frac{5}{8}$

4. If the numerator is larger than the denominator, then the value of the fraction is c
a) equal to 1 b) less than 1 c) more than 1 d) equal to 0

5. Fill in the blank. *A fraction is equal to 1 (one) when* numerator and denominator are the same

6. a) Add the following fractions. $\frac{3}{5}+\frac{4}{5}=?$ 7/5 = 1 2/5
b) Draw a picture of your problem.
Pictures will vary, but 3/5 and 4/5 added together to equal 1 and 2/5 needs to be clear

7. Fill in the blank. $\frac{5}{11}+\quad\frac{6}{11}\quad=\frac{11}{11}=1$
6/11 is the answer

8. Subtract the following fractions. $\frac{5}{8}-\frac{2}{8}=?$ Answer should be 3/8

9. Add the following fractions. Give the answer as an improper fraction and a mixed numeral.
$\frac{2}{3}+\frac{6}{3}=$ Answer should be 8/3 and 2 2/3

10. Given, $\frac{4}{9}+\frac{5}{9}=\frac{9}{9}=1$, what fraction would you add to your answer to make it equal to 2? 9/9

Quick Check II - Formative Assessment

1. Complete the following sentence. An improper fraction _____
a) has a 1 in the numerator b) has a value more than 1
c) has a value less than 1 d) is equal to 1

2. What is a numerator?
a) the top part of a fraction b) the bottom part of a fraction
c) the value of a fraction d) a number equal to 1

3. If you have $\frac{3}{8}$, how much do you need to make a whole (Hint: a whole is equal to 1)?
a) 5 b) $\frac{3}{8}$ c) $\frac{8}{8}$ d) $\frac{5}{8}$

4. If the numerator is larger than the denominator, then the value of the fraction is
a) equal to 1 b) less than 1 c) more than 1 d) equal to 0

5. Fill in the blank. *A fraction is equal to 1 (one) when* _____.

6. a) Add the following fractions. $\frac{3}{5}+\frac{4}{5}=?$
b) Draw a picture of your problem.

7. Fill in the blank. $\frac{5}{11}+$ $-$ $=\frac{11}{11}=1$

8. Subtract the following fractions. $\frac{5}{8}-\frac{2}{8}=?$

9. Add the following fractions. Give the answer as an improper fraction and a mixed numeral.
$\frac{2}{3}+\frac{6}{3}=$

10. Given, $\frac{4}{9}+\frac{5}{9}=\frac{9}{9}=1$, what fraction would you add to your answer to make it equal to 2?

Birthday Cake

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ1. Students will analyze number relationships.
b. Understand a fraction a/b as a multiple of 1/b. (MGSE4.NF.4)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
c. Interpret and solve contextual problems involving division of fractions by fractions. For example, how many 3/4-cup servings are in 2/3 of a cup of yogurt? (MGSE6.NS.1)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

Common Misconceptions

- *Many students do not have a firm understanding of the part to whole concept. Visual representations are needed to build this foundation.*
- *Students seem to feel the need to use cross products any time they are working with fractions. While cross-products is not a method taught in the standards, many students have been exposed to this method along the way.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.

5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to determine the fractional part of a given set and determine the whole set given the fractional part.

MATERIALS

- "Birthday Cake" student recording sheet
- Optional Manipulatives:
  - Paper plates of large circles either cut out or drawn
  - Two sided counters, base ten units, or some other small counter

ESSENTIAL QUESTIONS

- What does it mean to take a fraction portion of a whole number?
- How is multiplication of fractions similar to division of whole numbers?
- How do we determine the whole amount when given a fractional value of the whole?
- How do we determine a fractional value when given the whole number?

Grouping: Group/Partner Lesson

OPENER/ACTIVATOR

Before asking students to work on this lesson, be sure students are able to:

- Use repeated addition to add fractions with the same denominator.
- Be able to decompose fraction, for example ⁴⁄₄ = ½ + ½ or ¼ + ¾.
- Have a strong understanding that the whole can be any number/size and the fractions always depend on taking a portion of this whole.

- Given 24 students in the class, how could the candy bar be divided so that everyone in the class receives an equal part? How much would each student receive?
- Give 3 expressions which are equivalent to 12÷12.

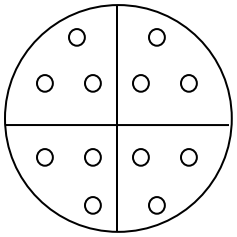
WORK SESSION

BACKGROUND KNOWLEDGE

For this activity students will be asked to determine the given number of candles on each piece of birthday cake when given a total number of candles on the cake. All problems assume that the cake pieces will be equal and everyone will always receive the same number of candles. The first part of the assignment provides students with whole to part problems. In other words, the students receive the whole amount in the question but need to produce the part of the whole to determine their answer. In the second part of the lesson, students are given the number of candles on just one piece of cake or one fraction of the cake and have to then determine how many candles were on the entire cake. The second part of the assignment provides students with part to whole problems where they receive a part or fraction in the question but need to produce the whole amount to determine the answer.

Whole to Part

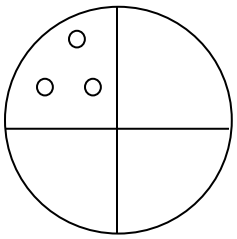
The four people at Tanya's birthday will get one-quarter (one-fourth) of the cake each. Tanya puts 12 candles on the cake so that each person gets the same number of candles on their piece of cake. How many candles will each person get on their piece of cake?



In the problem above the students need to determine the "part" when given the whole of 12 candles (i.e. 3 candles)

Part to Whole

Ricardo put enough candles on his birthday cake so that everyone would have the same number of candles. He then cut the cake into fourths. If each slice has three candles, how many candles did Ricardo put on his cake?



In the problem above, the students need to determine the "whole" when given only the part (i.e. ¼ of the cake had 3 candles, therefore the whole must be 12 candles). Although not always the case, these types of problems often pose a greater challenge to students.

Many of these could be completed by using simple division or multiplication without fractions at all. However, students need to understand that the / or slash in any fraction really means the division operation. By completing lessons such as these, students will begin to see a pattern and develop different strategies for multiplying and dividing denominators in order to solve problems that involve fractions.

LESSON DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this lesson, students will use a pie model to multiply a whole number by a fraction. Students will gain experience solving both part-to-whole and whole-to-part word problems that ask them to multiply a fraction by a whole number or multiply a whole number by a fraction

Teacher's Notes

Paper plates and counters should be made available for students to act out each of these problems. If paper plates are not available, a large circle drawn on an 11 x 8.5 inch paper will work just as well. Students could also color or draw their candles, or use glue and die cuts.

This lesson could be introduced by bringing in a cake and showing students how to distribute the candles in such a way that everyone receiving cake would get the same number of candles.

Lesson Directions

Students will follow directions below from the "Birthday Cake!" student recording sheet.

- Obtain a set of counters and paper plates.
- Work with a partner or small group to make a fraction cake and record it on your lesson sheet.
- Be ready to articulate your reasoning.

FORMATIVE ASSESSMENT QUESTIONS

- How do you know how many pieces of cake there are?
- Can you write an equivalent fraction for your answer? (for the example above, $\frac{1}{4} = \frac{3}{12}$)
- Are the candles evenly distributed or fairly distributed?
- In what other situations do we need to share evenly?

DIFFERENTIATION

Extension

- Once students have completed the lesson above, this lesson could be extended to use larger numbers of candles and larger fractions.
- Students could solve problems where the numerator is a number other than 1. For example, $\frac{5}{6}$ of 30.
- Students could also extend this lesson by exploring how the lesson would change if you had 2 or 3 cakes rather than just one whole cake.

Interventions

- Students may use repeated addition to solve these problems.
- Students may be given cakes already "cut" or drawn in parts to help them realize what the denominator will be.
- Initially students can start with a smaller task such as 4 candles on a cake cut into ¼ then move up gradually to 8 candles on a cake cut into ¼ and eventually 12 candles on a cake.
- For extra help with multiplying who numbers and fractions please open the hyperlink Intervention Table.

TECHNOLOGY

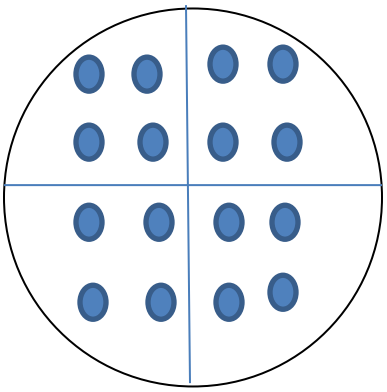
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L342 In this lesson, students identify fractions in real-world contexts from a set of items that are not identical. It can be used as an extension of the concept.
- http://www.visualfractions.com/Identify_sets.html This site would be best utilized with a lot of guidance from the teacher. It can be used to reinforce the concept within a small group or extend understanding of the concept.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L339 In this lesson, students identify fractions in real-world contexts from a set of items that are not identical. It can be used as an extension of the concept.

Name _____ Date _____

Birthday Cake!
Part 1

-Act out the problem using circles and counters.
-Draw your answer using the circle.
-Explain your answer using words.
-Lastly, write a number sentence for each problem

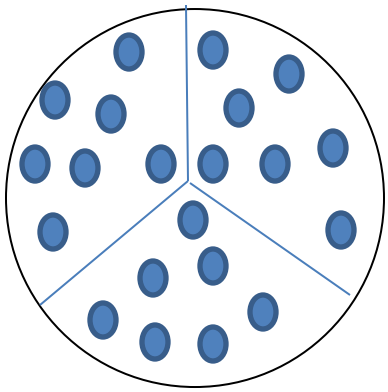
1. The four people at Carla's birthday will get one-quarter (one-fourth) of the cake each. Carla puts 16 candles on the cake so that each person gets the same number of candles on their piece of cake. How many candles will each person get on their piece of cake?



Explanation and Number Sentence

¼ x 16 = 4 candles/section

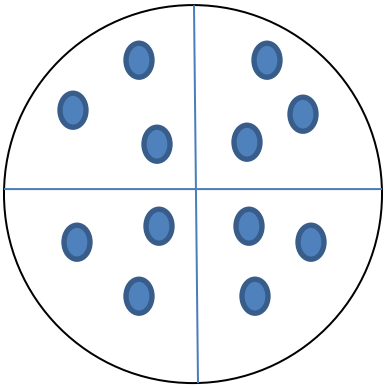
2. Three people are at Emmanuel's birthday party. Emmanuel puts 21 candles on the cake and cuts it into thirds so that each person gets the same number of candles on their piece of cake. How many candles will each person get on their piece of cake? (***The picture may not look to scale; the pieces should be thought of as the same size.)***



Explanation and Number Sentence

$\frac{1}{3}$ *x 21 = 7 candles/section*

3. At the party, the cake is cut into quarters (fourths). Twelve candles are put on the cake. Greedy Greg eats three-quarters of the cake. How many candles does he get?



Explanation and Number Sentence

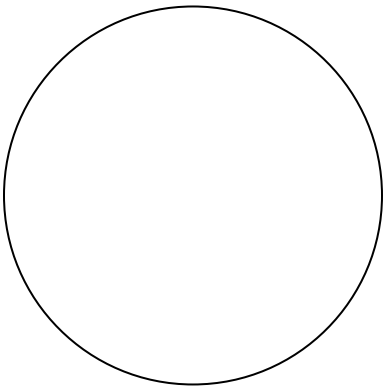
¼ x 12 =3 candles/section

Name _____ Date _____

Birthday Cake!
Part 1

-Act out the problem using circles and counters.
-Draw your answer using the circle.
-Explain your answer using words.
-Lastly, write a number sentence for each problem

1. The four people at Carla's birthday will get one-quarter (one-fourth) of the cake each. Carla puts 16 candles on the cake so that each person gets the same number of candles on their piece of cake. How many candles will each person get on their piece of cake?



Explanation and Number Sentence

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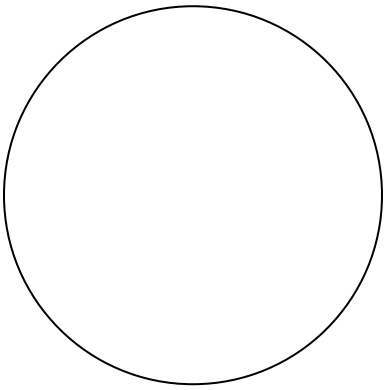
2. Three people are at Emmanuel's birthday party. Emmanuel puts 21 candles on the cake and cuts it into thirds so that each person gets the same number of candles on their piece of cake. How many candles will each person get on their piece of cake?



Explanation and Number Sentence

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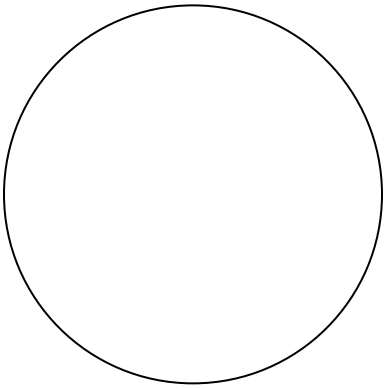
3. At the party, the cake is cut into quarters (fourths). Twelve candles are put on the cake. Greedy Greg eats three-quarters of the cake. How many candles does he get?



Explanation and Number Sentence

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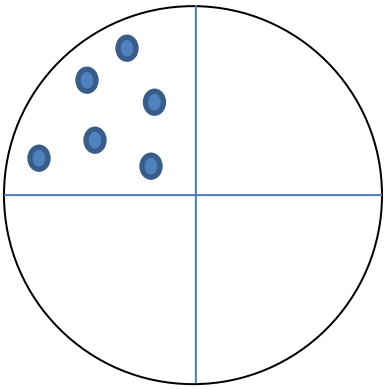
Name _____ Date _____

Birthday Cake!
Part 2

-Act out the problem using circles and counters.
-Draw your answer using the circle.
-Explain your answer using words.
-Lastly, write a number sentence for each problem.

1. Stan put enough candles on his birthday cake so that everyone would have the same number of candles. He then cut the cake into fourths. If each slice has six candles, how many candles did Stan put on his cake?

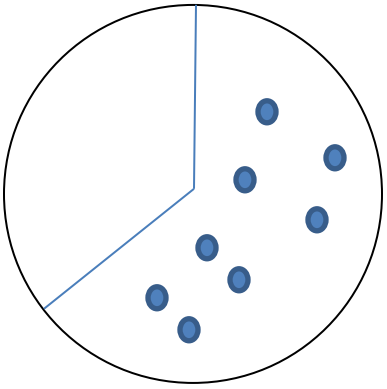


Explanation and Number Sentence

$\frac{1}{4}$ of the whole cake had 6 candles; therefore, the whole cake had 24 candles.

6 candles x 4 sections = 24 candles on the whole cake

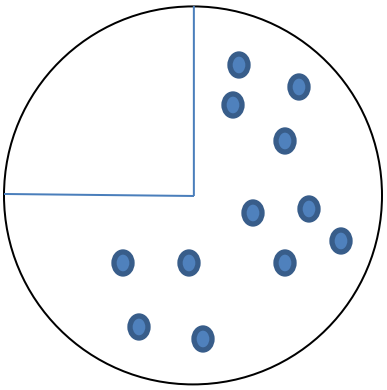
2. Pedro put enough candles on his birthday cake so that everyone would have the same number of candles. After cutting himself a large slice he noticed that two-thirds of the cake has eight candles on it. How many candles are on the whole cake?



Explanation and Number Sentence

$\frac{2}{3}$ of the whole cake had 8 candles; therefore, $\frac{1}{3}$ of the whole cake had 4 candles. Since $\frac{2}{3} + \frac{1}{3} = 1$ whole cake, the whole cake had 8 + 4 = 12 candles.

3. Priya and her father made a cake for her birthday and put enough candles on it so that everyone would have the same number of candles. Priya's father cut the cake into fourths and gave Priya the first slice. He then noticed that the three-fourths of the cake that was left had twelve candles on it. How many candles were on the whole cake?



Explanation and Number Sentence

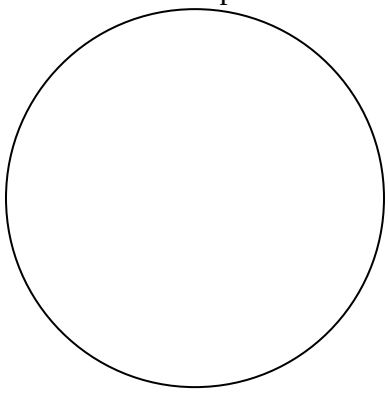
If $\frac{3}{4}$ of the cake had 12 candles then $\frac{1}{4}$ of the cake has 4 candles. Since $\frac{3}{4} + \frac{1}{4} = 1$ whole cake, 12 candles (from the $\frac{3}{4}$ section of the cake) + 4 candles (from the $\frac{1}{4}$ section of the cake) = 16 candles on the whole cake

Name _____ Date _____

Birthday Cake!
Part 2

-Act out the problem using circles and counters.
-Draw your answer using the circle.
-Explain your answer using words.
-Lastly, write a number sentence for each problem.

1. Stan put enough candles on his birthday cake so that everyone would have the same number of candles. He then cut the cake into fourths. If each slice has six candles, how many candles did Stan put on his cake?



Explanation and Number Sentence

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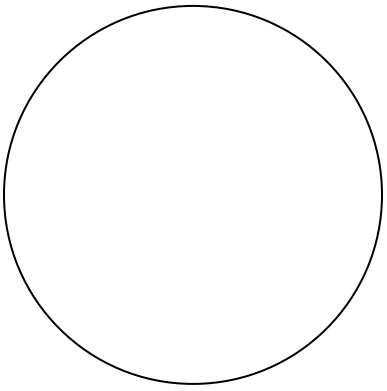
2. Pedro put enough candles on his birthday cake so that everyone would have the same number of candles. After cutting himself a large slice he noticed that two-thirds of the cake has eight candles on it. How many candles are on the whole cake?



Explanation and Number Sentence

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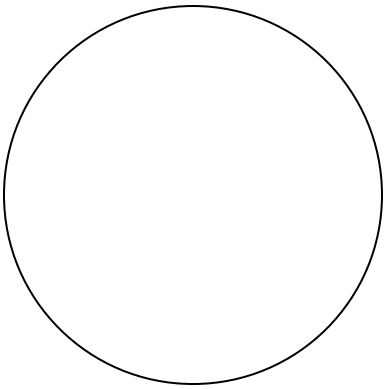
3. Priya and her father made a cake for her birthday and put enough candles on it so that everyone would have the same number of candles. Priya's father cut the cake into fourths and gave Priya the first slice. He then noticed that the three-fourths of the cake that was left had twelve candles on it. How many candles were on the whole cake?



Explanation and Number Sentence

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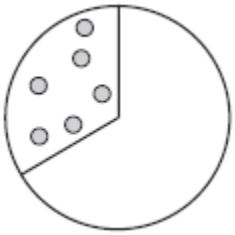
CLOSING/SUMMARIZER

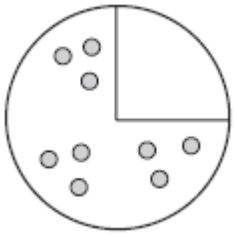
TE: TICKET OUT THE DOOR – SOLUTIONS

The cake below was cut into thirds. How many candles were on the whole cake?



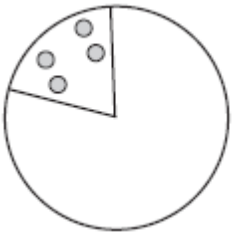
____*6 candles x 3 sections = 18 total candles*_____

The cake below was cut into fourths. How many candles were on the whole cake?



_____*3 candles x 4 sections = 12 total candles* _____

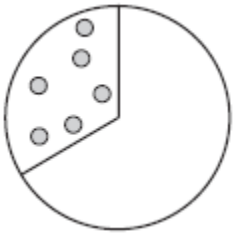
The cake below was cut into fifths. How many candles were on the whole cake?



_____*4 candles x 5 sections = 20 total candles* _____

Student Edition: TICKET OUT THE DOOR

The cake below was cut into thirds. How many candles were on the whole cake?



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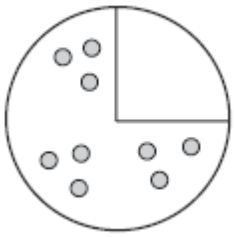
The cake below was cut into fourths. How many candles were on the whole cake?



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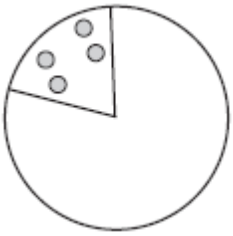
The cake below was cut into fifths. How many candles were on the whole cake?



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Fraction Clues

SUGGESTED TIME FOR THIS LESSON:

60-90 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ1. Students will analyze number relationships.
b. Understand a fraction a/b as a multiple of 1/b. (MGSE4.NF.4)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
c. Interpret and solve contextual problems involving division of fractions by fractions. For example, how many 3/4-cup servings are in 2/3 of a cup of yogurt? (MGSE6.NS.1)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

Common Misconceptions

- *Many students do not have a firm understanding of the part to whole concept. Visual representations are needed to build this foundation.*
- *Students seem to feel the need to use cross products any time they are working with fractions. While cross-products is not a method taught in the standards, many students have been exposed to it along the way.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.

4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

Determine the fractional part of a given set.
Find the whole set given a fractional part.
Find equivalent fractions.

MATERIALS

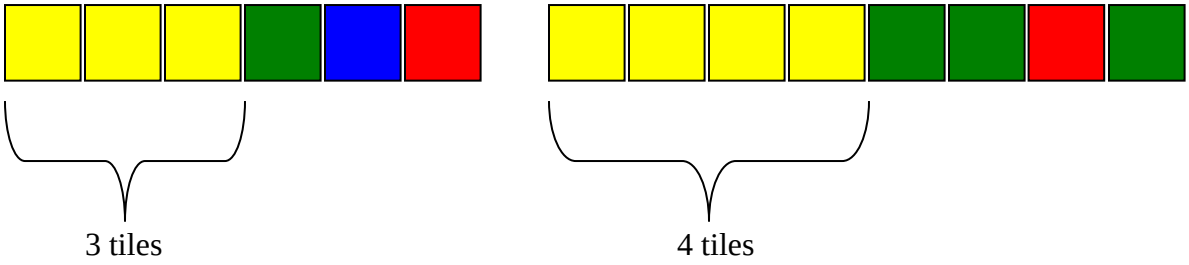
- Colored Tiles
- Fraction Clues recording sheet
- Colored pencils

ESSENTIAL QUESTIONS

- How can a fraction represent parts of a set?
- How can you represent fractions in different ways?
- How can you find equivalent fractions?
- How can you multipy a set by a fraction?

BACKGROUND INFORMATION

This activity is valuable because students start to realize that a different number of tiles in a different fraction bar can still be represented by the same fraction. For example



In the first bar, three yellow tiles represent ½ and in the second bar four tiles represent ½. Students will gain further understanding that the number of tiles being used (numerator) is always dependent on its relationship to the total number of tiles (denominator).

Before asking students to work on this lesson, be sure students are able to:

- identify the number of equal pieces needed to cover one whole as the denominator
- show equivalent fractions with an area model
- record on the student sheet equivalent fractions or fraction sets (either by coloring or gluing die cut squares)
- write an equation which shows the clues and verify their answer.

Grouping: Partner

OPENING/ACTIVATOR

To introduce this activity display these two fraction bars made from Color Tiles.

Ask students to find out what portion of the whole a tile in the first bar represents and what portion of the whole a tile in the second bar represents. Students should be able to determine that each tile in the first bar represents ¼ of the whole and each tile in the second bar represents ⅙ of the whole.

Ask students to explain what fractional part each color represents in each fraction bar. Give the following set of fraction clues that describe one of the fraction bars. Stop after each clue and ask students which fraction bar is the solution and how they know.

- The fraction bar is one-half green
- The fraction bar is one-third red
- The fraction is one-sixth blue

Many students will not need all three clues to determine the solution however, they should be comfortable arguing and verifying their answers. They may need all three clues to conclude that the solution is the second bar.

WORK SESSION

Lesson Directions ***(Note: Answers will vary.)***

Students will follow directions below from the Fraction Clues activity sheet.

- Obtain a set of colored tiles.
- Work with a partner to make a fraction bar and record it on their activity sheet.
- Write at least 3 clues that describe your fraction bar
- Exchange only your clues with another group
- Represent your answer with a number sentences (for example: if you have 10 tiles and ½ are red, then write the number sentence $^{10}/_{2} = 10 \times ½$ which is 5 tiles)
- Attempt to build another group's fraction bar as they attempt to build yours.
- Discuss results with each other.

Extension

- Once students have completed the lesson above, the lesson can be extended to have two pairs of students combine their fraction bars to make a larger fraction bar, then continue the activity writing clues for another group to solve.
- Students could also be encouraged to work with larger fraction bars as well as write more clues for determining those fraction bars. Most color tiles only have red, blue, green and yellow tiles; so, the activity will never have more than four fractions to represent.
- Often the clue with the largest denominator tells you how many tiles can be used. However, students could be challenged to use only 2 clues and therefore force them into situations where they need to find common denominators. For example, my fractions are ¼ red and ⅓ green. They will then need to build several bars that have 12 or 24 tiles.

Intervention

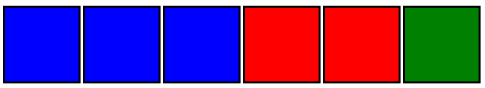
- If necessary students could begin this activity with a smaller set, such as using only four tiles.
- If students are struggling, they could attempt with activity with only three colors instead of using all four colored tiles.
- For extra help with unit fractions, please open the hyperlink Intervention Table.

TECHNOLOGY

- http://illuminations.nctm.org/LessonDetail.aspx?ID=L342 helps students identify fractions in real-world contexts from a set of items that are not identical. It can be used as an extension of the concept.
- http://www.visualfractions.com/Identify_sets.html would be best utilized with a lot of guidance from the teacher. It can be used to reinforce the concept within a small group or extend understanding of the concept.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L339 assists students in identifying fractions in real-world contexts from a set of items that are not identical. It can be used as an extension of the concept.

Name _____ Date _____

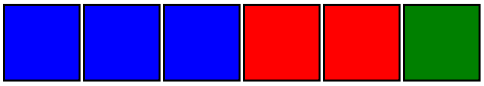
Fraction Clues
(Part 1)



Make a Color Tile fraction bar and then write a set of clues so that someone else could build it.

• Work with a partner. Choose 6 Color Tiles and arrange them in any way to form a fraction bar.

• Decide what fractional part of the whole bar is represented by each color you used. For example:



Blue: $\frac{3}{6}$ or $\frac{1}{2}$
Red: $\frac{2}{6}$ or $\frac{1}{3}$
Green: $\frac{1}{6}$

• Record your fraction bar on grid paper. Beneath the grid paper, write several clues that describe the fractional parts of your bar. For example: *My bar is* _____ *blue.*

• Exchange lists with another pair. Be careful not to peek at the back of the list! Follow the clues to try to build the other pair's fraction bar.

• Represent your answer with a number sentences (for example: if you have 10 tiles and ½ are red, then write the following: Half of 10 = $\frac{10}{2}$ = 10 x ½ = 5 tiles)

• When you have finished making the fraction bar, turn the list of clues over and compare what you built to the recording.

• Discuss your results with the other pair.

Clue 1: _____

_____

_____

_____

Clue 2: _____

_____

_____

_____

Clue 3: _____

_____

_____

_____

Clue 4: _____

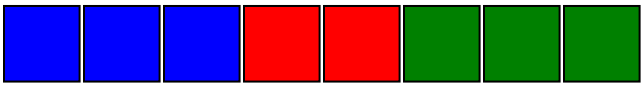
_____

_____

_____

• Work with a partner. Choose 8 color tiles and arrange them in any way to form a fraction bar.

• Decide what fractional part of the whole bar is represented by each color you used. For example:



Blue: ⅜
Red: ⅜ or ¼
Green: ⅜

• Record your fraction bar on grid paper. Beneath the grid paper, write several clues that describe the fractional parts of your bar. For example: *My bar is _____ blue.*

• Exchange lists with another pair. Be careful not to peek at the back of the list! Follow the clues to try to build the other pair's fraction bar.

• Represent your answer with a number sentences (for example: if you have 10 tiles and ½ are red then write the number sentence $^{10}/_{2} = 10 \div 2 = 5$ tiles)

• When you have finished making the fraction bar, turn the list of clues over and compare what you built to the recording.

• Discuss your results with the other pair.

Clue 1: _____

_____

_____

_____

Clue 2: _____

_____

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_____

Clue 3: _____

_____

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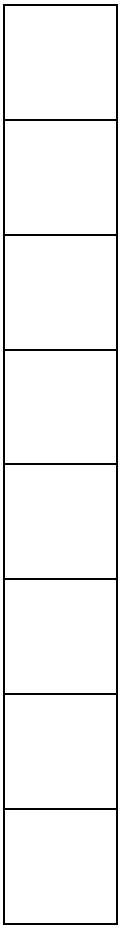
Clue 4: _____

_____

_____

_____



• Work with a partner. Choose 12 Color Tiles and arrange them in any way to form a fraction bar.

• Decide what fractional part of the whole bar is represented by each color you used. For example:

Blue: $\frac{4}{12}$ or $\frac{1}{3}$
Red: $\frac{2}{12}$ or $\frac{1}{6}$
Green: $\frac{6}{12}$ or $\frac{1}{2}$

• Record your fraction bar on grid paper. Beneath the grid paper, write several clues that describe the fractional parts of your bar. For example: *My bar is* _____ *blue.*

• Exchange lists with another pair. Be careful not to peek at the back of the list! Follow the clues to try to build the other pair's fraction bar.

• Represent your answer with a number sentences (for example: if you have 10 tiles and ½ are red then write the number sentence $\frac{10}{2} = 10 \div 2 = 5$ tiles)

• When you have finished making the fraction bar, turn the list of clues over and compare what you built to the recording.

• Discuss your results with the other pair.

Clue 1: _____

_____

_____

_____

Clue 2: _____

_____

_____

_____

Clue 3: _____

_____

_____

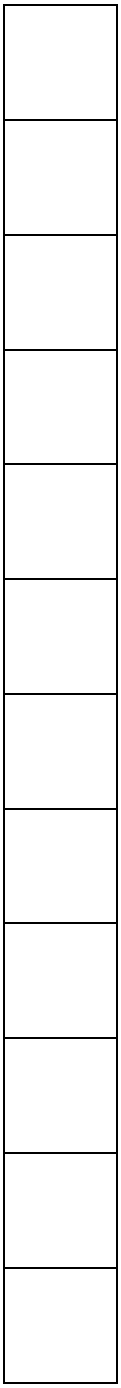
_____

Clue 4: _____

_____

_____

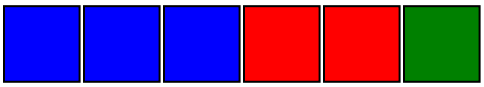
_____



CLOSING/SUMMARIZER ***(Note: Answers will vary.)***

Name _____ Date _____

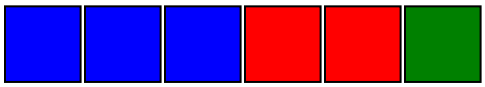
Fraction Clues
(Part 2)



Make a Color Tile fraction bar and then write a set of clues so that someone else could build it

• Work with a partner. Choose any number of Color Tiles and arrange them in any way to form a fraction bar.

• Decide what fractional part of the whole bar is represented by each color you used. For example:



Blue: $\frac{3}{6}$ or $\frac{1}{2}$
Red: $\frac{2}{6}$ or $\frac{1}{3}$
Green: $\frac{1}{6}$

• Record your fraction bar on grid paper. Beneath the grid paper, write several clues that describe the fractional parts of your bar. For example: *My bar is* _____ *blue.*

• Exchange lists with another pair. Be careful not to peek at the back of the list! Follow the clues to try to build the other pair's fraction bar.

• Represent your answer with a number sentences (for example: if you have 10 tiles and ½ are red then write the number sentence $\frac{10}{2} = 10 \div 2 = 5$ tiles)

• When you have finished making the fraction bar, turn the list of clues over and compare what you built to the recording.

• Discuss your results with the other pair.

Clue 1: _____

_____

_____

_____

Clue 2: _____

_____

_____

_____

Clue 3: _____

_____

_____

_____

Clue 4: _____

_____

_____

_____

Multiplying Fractions

SUGGESTED TIME FOR THIS LESSON:

60-90 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ1. Students will analyze number relationships.
b. Understand a fraction a/b as a multiple of 1/b. (MGSE4.NF.4)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
c. Interpret and solve contextual problems involving division of fractions by fractions. For example, how many 3/4-cup servings are in 2/3 of a cup of yogurt? (MGSE6.NS.1)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

Common Misconceptions:
Students often have the mindset that the operation of multiplication produces a larger product and the operation of division produces a smaller quotient. Students should develop an understanding that when a number is multiplied by a number less than 1, the product is less than the original number, and when a number is divided by a decimal number less than 1, the quotient will be greater than the dividend. This is important, yet often difficult for students to understand because it is counterintuitive based on students' previous experiences with multiplication and division.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:
Represent multiplication of fractions using an area model.
Determine various fractions of whole numbers.

MATERIALS

- Colored pencils
- Area Model recording sheet
- "How many CC's?" recording sheet

ESSENTIAL QUESTIONS

- What strategies can be used for finding products when multiplying a whole number by a fraction?
- How can you model the multiplication of a whole number by a fraction?
- How do you mupltiply a fraction by a whole number?

Grouping: Individual/Partner

OPENER/ACTIVATOR

Lesson Directions

Have students follow the directions on the area model recording sheet. Use the square below to draw an area model to represent the following multiplication problems. Use your area model to help you compute the answer to each problem.

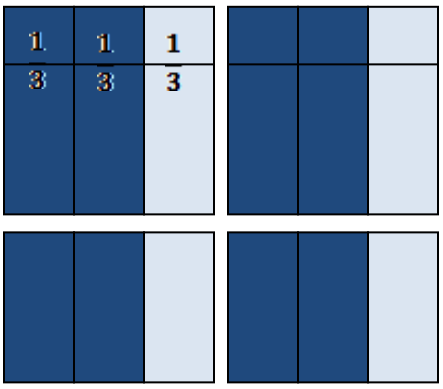
SOLUTIONS:

Draw an area model to represent each of the following operations. Use your area model to help you compute the answer to each problem.

$6 \cdot \frac{2}{3}$ ***Answers will vary.***

Possible Solution

A possible solution for $6 \cdot \frac{2}{3}$ ***is below. This model shows six rectangles with each having*** $\frac{2}{3}$ ***of their area shaded. The results show*** $\frac{12}{3}$ ***shaded which is equivalent to 4 whole rectangles.***

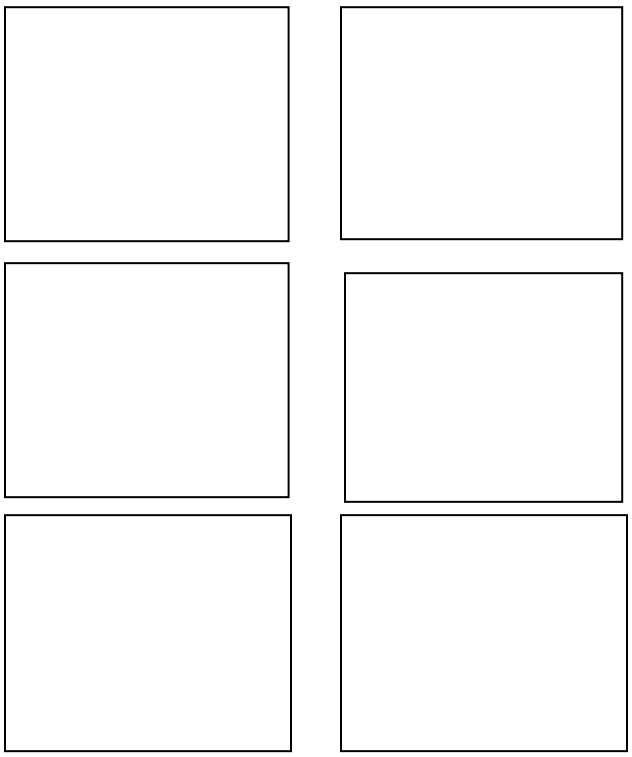


Name _____ Date _____

Area Models: Multiplication

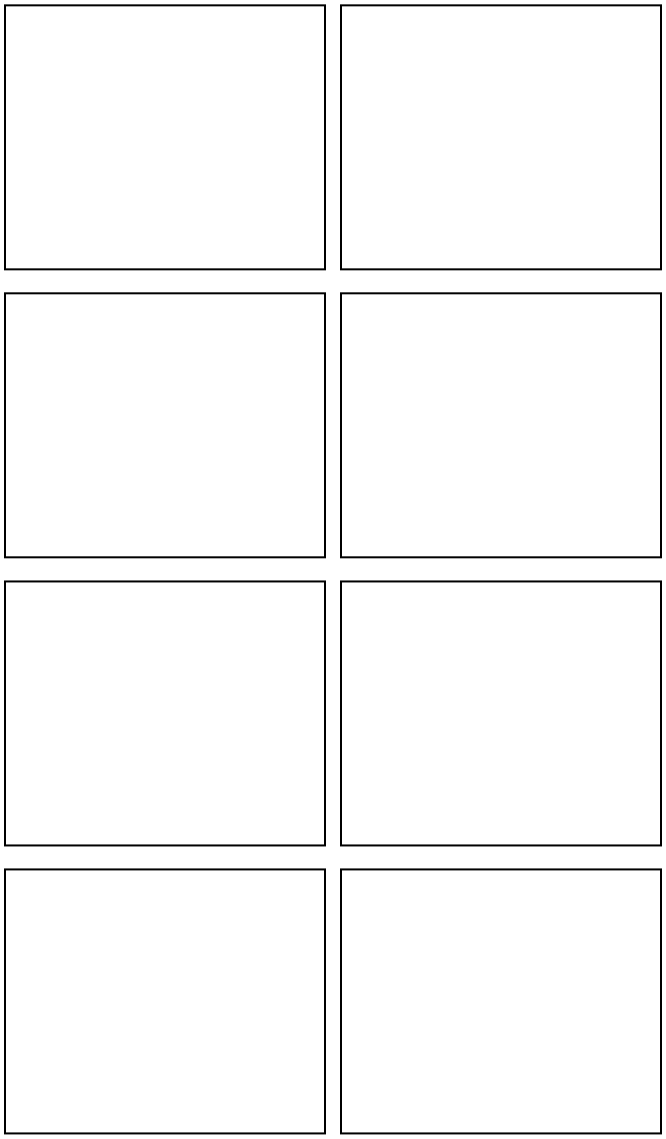
Use the squares below to draw an area model to represent the following multiplication problems. Use your area model to help you compute the answer to each problem.

$6 \cdot \frac{2}{3}$



Explanation:

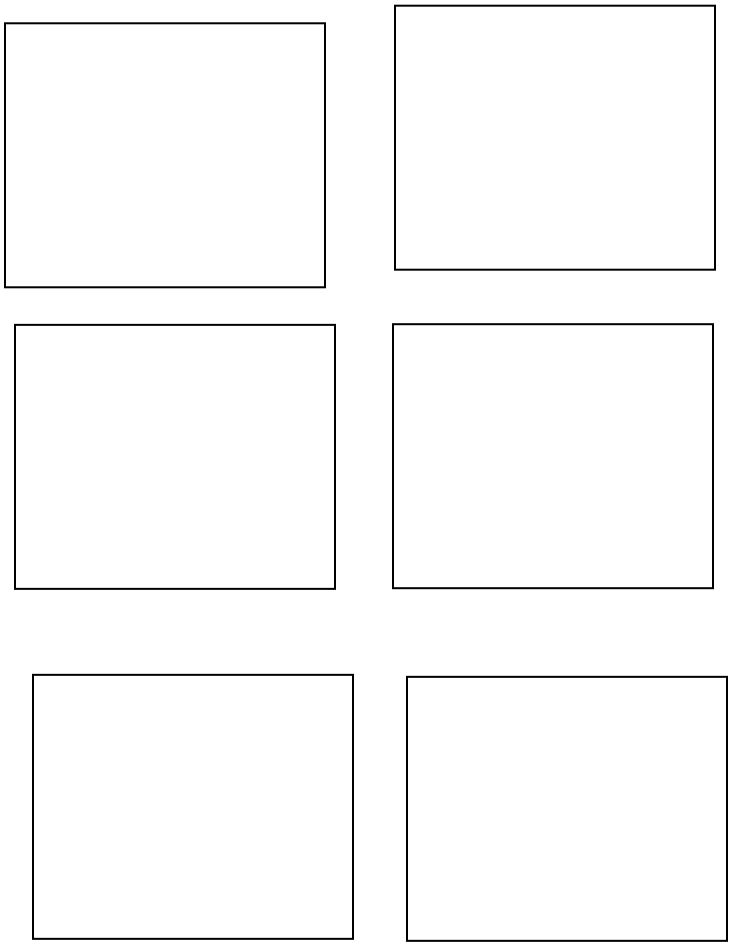
$8 \cdot \frac{3}{4}$



Explanation:

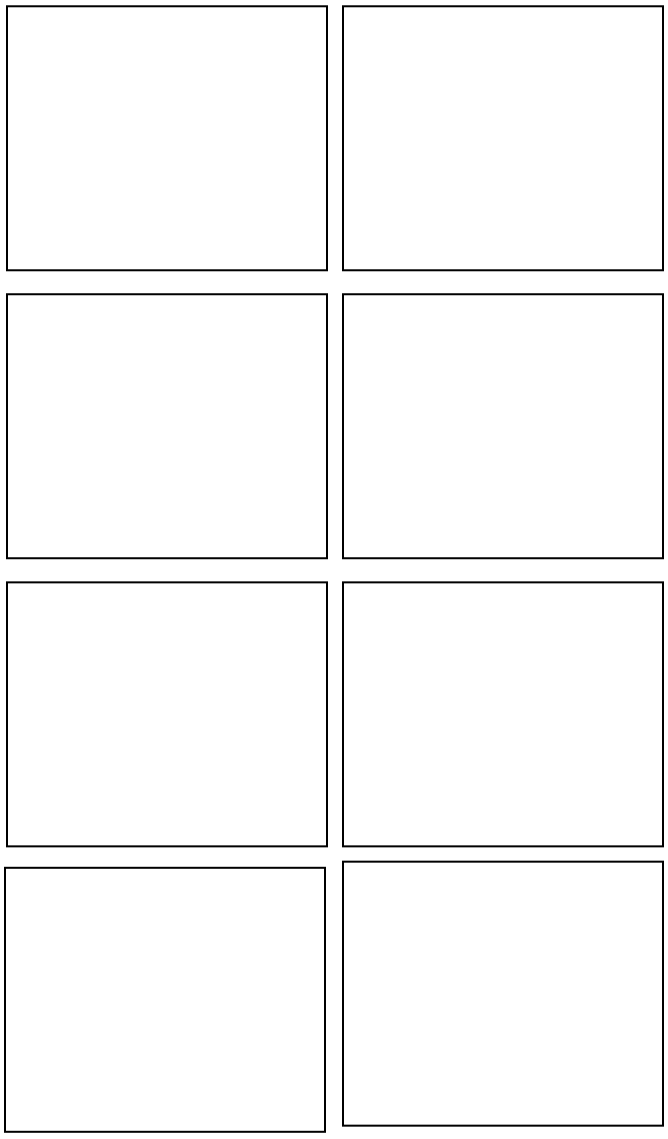
6 · $\frac{2}{5}$

Explanation:



Use the squares below to draw an area model to represent the following operations. Use your area model to help you compute the answer to each problem. What happens if you switch the equation around to read ½ times 8?

$\mathbf{8 \cdot \frac{1}{2}}$



Explanation:

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

To complete this lesson correctly, students must understand that the amount of drainage fluid and Albumin are equivalent for each reading. Clues are provided within the table to help the students determine the cc of fluid for each. On one occasion, students use the amount of Albumin to determine the drainage fluid.

You can explain to the students "cc" are the cubic centimeters commonly used to measure fluids in the medical field. It is not necessary to attempt to convert cc to another unit of measure.

LESSON SOLUTIONS:

Registered Nurse Molly has a patient with a drainage tube in her abdomen. The tube was inserted after a recent surgery. The volume measured in the tube has to be replaced by a blood product called albumin through an IV line. Nurse Molly checks the drainage every 6 hours. The chart below shows the amount of fluid measured in the drainage tube at the checked times. Help Nurse Molly determine how much Albumin to give the patient after each checkpoint.

Encourage the students to think of money.

Time	**Drainage Fluid**	**Albumin**
6:00am	100cc	***Whatever comes out must be replaced*** ***100cc***
12 noon	¾ of the volume at 6am	***¾ x 100 = 75cc***
6:00pm	***If 75 cc of Albumin was added the patient must have lost*** ***75cc***	75cc
12 midnight	$\frac{2}{3}$ of the volume at 6pm	$\frac{2}{3}$ ***x 75 = 50cc*** ***(Think of money – there are 3 quarters in 75 cents then double it.)***
6:00am	$\frac{4}{8}$ of 100cc	$\frac{4}{8}$ ***x 100 = *See Note Below*** ***½ x 100 = 50cc***
12 noon	***If 25cc of Albumin was added the patient must have lost 25cc***	½ of 50cc = ***25cc***

NOTE: ***Students may run into difficulty when trying to determine $\frac{4}{8}$ of 100cc. Allow students to struggle, which will provide an opportunity to apply understanding of equivalent fractions. An easy equivalent would be ½. Again, allow students to discover this through struggle.***

DIFFERENTIATION

Extension

- The Albumin is administered from a syringe containing 100cc of the fluid using an automatic pump. If the nurse sets the pump for 30 minutes, the fluid will dispense 100cc over the course of the 30 minutes. If the nurse sets it for 15 minutes, how many cc of Albumin will be dispensed? How much time is needed to dispense 25cc? 75cc?

Intervention

- Have students determine the fraction of drainage fluid using unit fractions first. For example, ¼ of 6am before figuring ¾ of 6am.
- For extra help with multiplying fractions, please open the hyperlink Intervention Table.

TECHNOLOGY

- http://illuminations.nctm.org/LessonDetail.aspx?ID=U123 This four part lesson can be used for additional practice or to extend understanding of the concepts.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L341 This lesson reinforces fractional parts. It can be used to extend understanding of the concept.

Name: _____ Date:_____

How Many CCs?

Registered Nurse Molly has a patient with a drainage tube in her abdomen. The tube was inserted after a recent surgery. The volume measured in the tube has to be replaced by a blood product called Albumin through an iv line. Nurse Molly checks the drainage every 6 hours. The chart below shows the amount of fluid measured in the drainage tube at the checked times. Help Nurse Molly determine how much Albumin to give the patient after each checkpoint.

Time	**Drainage Fluid**	**Albumin**
6:00am	100cc	
12 noon	¾ of the volume at 6am	
6:00pm		75cc
12 midnight	$\frac{2}{3}$of the volume at 6pm	
6:00am	$\frac{4}{8}$ of 100cc	
12 noon		½ of 50cc =

CLOSING/SUMMARIZER

Possible Ticket Out the Door Question: (This is a 2 part question.)

Lanny rode his bike to school which was $\frac{1}{3}$of a mile away. He completed this trip 4 times a day. What is the total distance the Lanny rode? Draw a picture to represent your thinking.

Abi is preparing for a ½ marathon. She runs 4 $\frac{2}{3}$of a mile every day after school. Who travels the most distance a day? Create a model of your thinking.

SOLUTION:

Distance ridden by Lanny		
Distance ridden by Lanny		
Distance ridden by Lanny		
Distance ridden by Lanny		

From the model above, Lanny rode $\frac{1}{3}$of a mile 4 times which is $\frac{4}{3}$or 1 $\frac{1}{3}$miles.

Abi rode 4 $\frac{2}{3}$miles. See representation below

If you remove Lanny's 1 $\frac{1}{3}$miles (noted by the x in the boxes), you will find that Abi rode 3 $\frac{1}{3}$ miles more.

X	*X*	*X*
X		

Other possible questions can be found at:
http://bit.ly/1Qn8Yih lesson 7

Birthday Cookout

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ1. Students will analyze number relationships.
b. Understand a fraction a/b as a multiple of 1/b. (MGSE4.NF.4)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
c. Interpret and solve contextual problems involving division of fractions by fractions. For example, how many 3/4-cup servings are in 2/3 of a cup of yogurt? (MGSE6.NS.1)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.

6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to solve word problems that involve the multiplication of a whole number and a fraction.

MATERIALS

- "Birthday Cookout" recording sheet

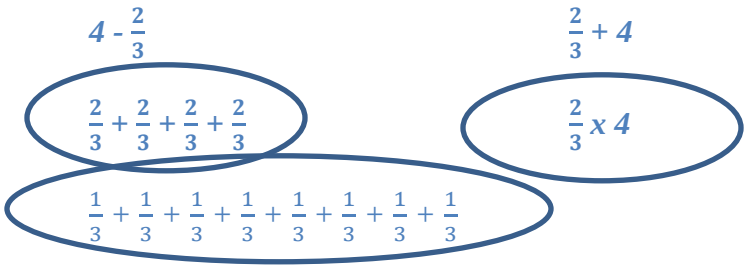
ESSENTIAL QUESTIONS

- How can we use fractions to help us solve problems?
- How can we model answers to fraction problems?
- How can we write equations to represent our answers when solving word problems?

Grouping: Individual/Partner

OPENER/ACTIVATOR:

It is a hot day in Atlanta, GA. There are not many people at Six Flags. The Batman rollercoaster is only $\frac{2}{3}$ full. It ran 4 times at that capacity. Circle the expressions that model this scenario.



Student Edition:

It is a hot day in Atlanta, GA. There are not many people at Six Flags. The Batman rollercoaster coaster is only $\frac{2}{3}$ full. It ran 4 times at that capacity. Circle the expressions you could use to find how many full loads that would equal.

$4 - \frac{2}{3}$ $\quad\quad$ $\frac{2}{3} + 4$

$\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$ $\quad\quad$ $\frac{2}{3} \times 4$

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

Additional problems can be found at: http://bit.ly/1Qn8Yih

WORK SESSION

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

This lesson asks students to use problem solving strategies and their knowledge of fractions to solve a real-world problem involving food for a birthday party.

Comments

The setting of this lesson is likely a familiar one for students. You may want to begin with a discussion of how math is used when planning a birthday party. The discussion may include a wide range of mathematical ideas such as number of invitations, amount of food, and the amount of money needed to purchase food.

Solutions are given below:

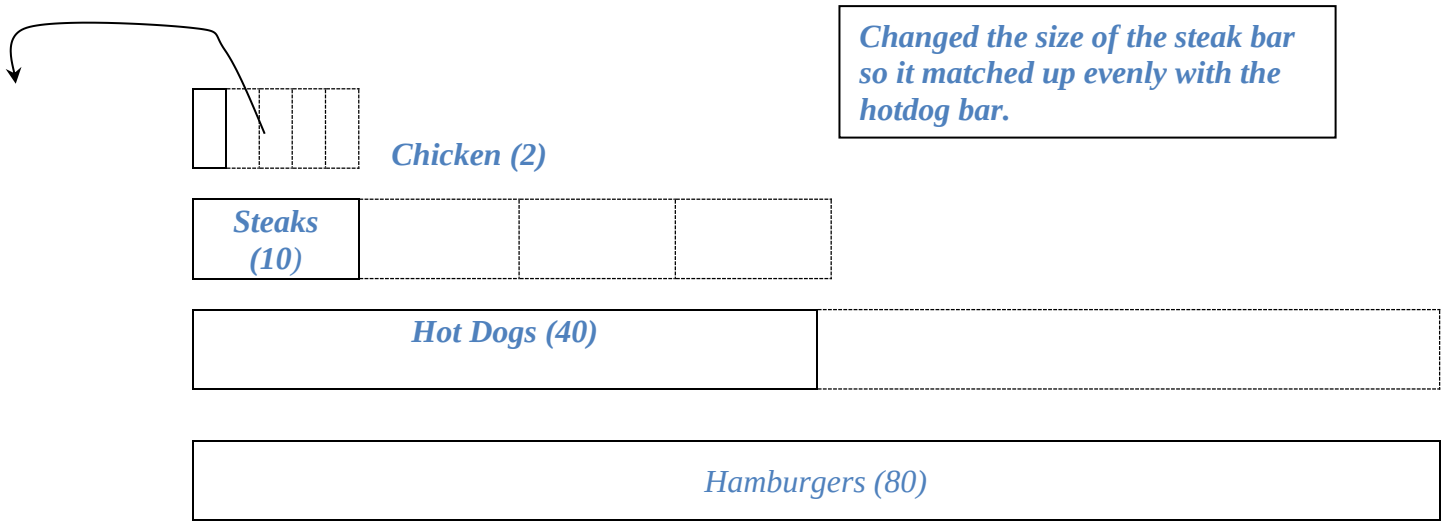
- How many people asked for chicken? ($\frac{1}{5}$ of 10 is 2)
- How many people asked for steak? ($\frac{1}{4}$ of 40 is 10)
- How many asked for hot-dogs? ($\frac{1}{2}$ of 80 is 40)

Lesson Directions

Have students follow the directions on the "Birthday Cookout" student recording sheet.

BACKGROUND KNOWLEDGE and SOLUTION

You may want to review problem solving strategies with your students as they begin work on this lesson. Strategies such as making a table and working backward are two approaches to this lesson. Another suggestion for solving fraction word problems such as this is to utilize the Singapore Math strategy of drawing bars that are proportionate to the values in the problem. For example, we know that 80 people ordered hamburgers so we can draw a large bar to represent the hamburgers. We can then draw a bar ½ the size of our "hamburger" bar to represent the number of people that want hotdogs. Next, we can draw a bar that is ¼ the size of our "hot dog" bar to represent the number of people that want steak. Finally, we can draw a bar ⅕ the size of our "steak" bar to represent the number of people that want chicken.



DIFFERENTIATION

Extension

- Have students research and determine the cost of the items the chef needs.
- Have students create their own menu and create a new problem involving fractions.
- Have students determine the percentage of guests who chose each menu item.
- Students could explore this problem with larger numbers such as 320 hamburgers and then look for patterns.
- Students could be given different information to begin with other than the number of hamburgers. How would the problem change if we only knew that 40 people asked for steak?

Intervention

- Use smaller numbers, for example instead of 80 hamburgers, use 40 hamburgers.
    - How many people asked for chicken? ($\frac{1}{5}$ of 5 is 1)
    - How many people asked for steak? ($\frac{1}{4}$ of 20 is 5)
    - How many asked for hot-dogs? ($\frac{1}{2}$ of 40 is 20)
- These lessons can always be physically performed with fraction tiles for the more kinesthetic learners. Many fraction bars are labeled and students can turn them over to assign them the value of the hamburgers, hot dogs, etc.

TECHNOLOGY

- http://illuminations.nctm.org/LessonDetail.aspx?ID=L342 In this lesson, students identify fractions in real-world contexts from a set of items that are not identical. It can be used as an extension of the concept.
- http://www.visualfractions.com/Identify_sets.html This site would be best utilized with a lot of guidance from the teacher. It can be used to reinforce the concept within a small group or extend understanding of the concept.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L339 In this lesson, students identify fractions in real-world contexts from a set of items that are not identical. It can be used as an extension of the concept.

Name _____ Date _____

Birthday Cookout

Bob turned 60 this year! His family celebrated by having a cookout. Marcy took orders and found one fifth as many people wanted chicken as wanted steaks, one fourth as many people wanted steaks as wanted hot dogs, and one half as many people wanted hot dogs as wanted hamburgers. She gave her son-in-law, the chef, an order for 80 hamburgers.

The chef needs more information. He has to know:

- How many people asked for chicken?
- How many people asked for steak?
- How many asked for hot-dogs?

Use words, pictures and numbers to tell the chef what he needs to know. Be prepared to share!

CLOSING/SUMMARIZER

Share your results of "Birthday Cookout"
Journal Entry – Describe an "Ah-Ha!" moment you had during this activity. It may have been in your approach to the problem or an idea shared by a classmate.

Chance of Surgery

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ1. Students will analyze number relationships.
b. Understand a fraction a/b as a multiple of 1/b. (MGSE4.NF.4)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
c. Interpret and solve contextual problems involving division of fractions by fractions. For example, how many 3/4-cup servings are in 2/3 of a cup of yogurt? (MGSE6.NS.1)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)
e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MGSE7.NS.3, MGSE7.EE.3, MGSE9-12.N.Q.3)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
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6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:
Use models to multiply a whole number by a fraction.
Complete a multi-step lesson.

MATERIALS

- "A Chance of Surgery" recording sheet

ESSENTIAL QUESTIONS

- How can you model the multiplication of a whole number and a fraction?
- What fractional understanding do you need to mupltiply a fraction by a whole number?
- How do you solve a multi-step problem?

Grouping: Individual/Partner

OPENER/ACTIVATOR:

Students will need to determine $\frac{2}{3}$ and $\frac{1}{3}$ of 15 in order to correctly complete the multi-step lesson.

WORK SESSION

BACKGROUND KNOWLEDGE

Students need to be able to apply their understanding of mathematical concepts with multi-step problems. In accordance with SMP 1, students need to make sense of the problems in an attempt to solve them. For this lesson, it is essential that students make sense of the information provided as students will need to apply their fractional understanding in order to multiply accurately. The operation of multiplication is applied to various kinds of numbers.

LESSON with SOLUTIONS

Dr. Clifton is a surgeon at Children's Healthcare of Atlanta Egleston. In 2012, he performed 15 surgeries to treat biliary atresia. Studies have shown that $\frac{2}{3}$of the patients treated for biliary

atresia eventually need a liver transplant. Of Dr. Clifton's patients last year, how many will eventually need a liver transplant?

$$\frac{2}{3} x\ 15 = 10$$

According to the statistic, how many patients will not need a transplant?

If $\frac{2}{3}$ need a liver transplant later, then $\frac{1}{3}$ have success and do NOT need the transplant.

$$\frac{1}{3} x\ 15 = 5$$

Or

15 surgeries – 10 who later need the transplant = 5 find success with a transplant

If it typically takes Dr. Clifton 4 hours to complete a biliary atresia surgery, about how many hours did he perform this operation last year?

4 hour x 15 surgeries = 60 hours in surgery

If Dr. Clifton's case load doubles in 2013, how many patients can he expect to have successful surgery and possibly not need a liver transplant?

If he did 15 surgeries in 2012 and doubles the number is 2013, he will do 30 surgeries in 2013.
2 x 15 = 30

How many hours should he anticipate conducting the surgery in 2013?

At 4 hours a surgery for 30 surgeries, the result is 120 hours.
4 x 30 = 120

DIFFERENTIATION

Extension

- Determine the amount of patients who will not need a liver transplant and who will need a liver transplant if Dr. Clifton's caseload quadrupled. How many hours would he be in surgery?
- Create a function table that shows the effect of the caseload on the amount of hours in surgery. Include at least 5 data points within the table.

Intervention

- Allow students to use manipulatives such as counters, beans, bears, etc. to represent the 15 patients.

TECHNOLOGY

- http://illuminations.nctm.org/LessonDetail.aspx?ID=U123 This four part lesson can be used for additional practice or to extend understanding of the concepts.
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L341 This lesson reinforces fractional parts. It can be used to extend understanding of the concept.

Name _____ Date_____

A Chance Surgery

Dr. Clifton is a surgeon at Children's Healthcare of Atlanta Egleston. In 2012, he performed 15 surgeries to treat biliary atresia. Studies have shown that $\frac{2}{3}$ of the patients treated for biliary atresia eventually need a liver transplant. Of Dr. Clifton's patients last year how many will eventually need a liver transplant? According to the statistic, how many patients will not need a transplant?

If it typically takes Dr. Clifton 4 hours to complete a biliary atresia surgery, about how many hours did he perform this operation last year?

If Dr. Clifton's case load doubles in 2013, how many patients can he expect to have successful surgery and possibly not need a liver transplant? How many hours should he anticipate conducting the surgery in 2013?

CLOSING/SUMMARIZER

Kamron answers $\frac{4}{5}$ of her Physical Science questions correctly. There were 35 questions total. How many questions did she answer correctly?

Fractional Divisors

SUGGESTED TIME FOR THIS LESSON:

90-120 minutes
Exact timings will depend on the needs of yours class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
c. Interpret and solve contextual problems involving division of fractions by fractions. For example, how many 3/4-cup servings are in 2/3 of a cup of yogurt? (MGSE6.NS.1)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)
e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MGSE7.NS.3, MGSE7.EE.3, MGSE9-12.N.Q.3)

Common Misconceptions:
Students often misapply the invert-and-multiply procedure for dividing by a fraction because they are missing the conceptual understanding of the procedure.

- *not inverting either fraction; for example, a student may solve the problem* $\frac{2}{3} \div \frac{4}{5}$ *by multiplying the fractions without inverting* $\frac{4}{5}$ *(e.g., writing that* $\frac{2}{3} \div \frac{4}{5} = \frac{8}{15}$*)*
- *inverting the wrong fraction (e.g.,* $\frac{2}{3} \div \frac{4}{5} = \frac{3}{2} x \frac{4}{5}$*)*
- *inverting both fractions (*$\frac{2}{3} \div \frac{4}{5} = \frac{3}{2} x \frac{5}{4}$*)*

Such errors generally reflect a missing conceptual understanding of why the invert-and-multiply procedure produces the correct quotient. The invert-and-multiply procedure translates a multi-step calculation into a more efficient procedure.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.

3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson students should be able to:

- Model division of fractions.
- Explain how to divide fractions.

MATERIALS

- Small bag (1 per group) of chocolate candies ***(any bag of candy where students can receive 17 pieces each, or students can be provided with a zip-lock of 17 small paper circles instead of candy)***
- Base Ten Blocks (If base 10 blocks are not available, you can print your own representation. See the sample https://www.illustrativemathematics.org/content-standards/5/NBT/A/2/LESSONs/1620. ***Another option would be to have students create their own base 10 blocks.***
- Unit Cubes
- Rulers

ESSENTIAL QUESTIONS

- Why does the process of invert and multiply work when dividing fractions?
- When you divide one number by another number, do you always get a quotient smaller than your original number?
- When you divide a fraction by a fraction, what do the dividend, quotient and divisor represent?
- What kind of models can you use to show solutions to word problems involving fractions?

Grouping: Individual/Partner/Groups of 3-4

OPENING/ACTIVATOR:

What does it mean to divide?
Provide each group of 3-4 students with a small bag of candy ***(any bag of candy where students can receive 17 pieces each, or students can be provided with a zip-lock of 17 small paper circles instead of candy)).*** Have the students "divide" the candies evenly among the group. Ask each group what they did to complete this process or if they encountered any obstacles. If a group did encounter an obstacle, ask what it was and how they persevered through the obstacle.

WORK SESSION:

Teacher Notes
"Multiply" by the reciprocal is probably one of the most mysterious rules in mathematics. Van de Walle urges us to avoid this mystery by allowing students to come to their own conclusions about why we multiply by the reciprocal. (Van de Walle, John. Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades 3 – 5. Pearson: 2013.)This lesson allows students to explore partitive and measurement interpretations of fractions with fractional divisors. Keeping in mind that the partitive interpretation asks the question, "How much is one?" while measurement is equal subtraction and can have remainders.

Before the lesson, review the student work from the previous task with a focus on partitive and measurement interpretations of fractions with fractional divisors.

When students finish, get answers from the class for each problem. If more than one answer is offered, simply record them and offer no evaluation.

Have students explain their strategies for thinking about the problem either on the board, with a document camera, etc. You may need to ask questions about drawings or explanations to make sure everyone in the class follows the rationale. Encourage the class to comment or ask questions about the student's representation or thinking. Ask if others used a different representation or solved the problem in a different way. If so, have the students come forward to share their solutions If there are different answers, the class should evaluate the solution strategies and decide which answer is correct and why.

It is important to have students compare and contrast the two problems and the methods for solving them.

In what ways are the two problems similar?

In what ways are they different?

What does the quotient represent in each of the problems?

What does the divisor represent in each of the problems?

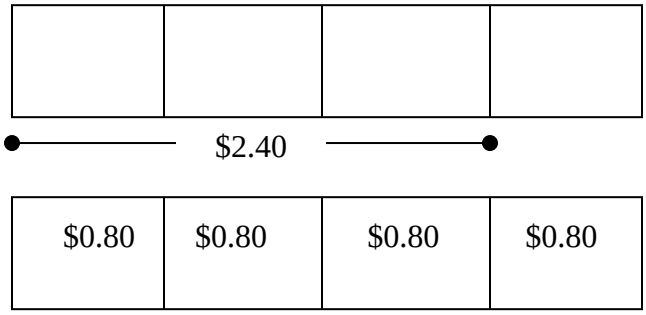
What does the dividend represent in each of the problems?

SOLUTIONS

Partitive Interpretation of Division with Fractional Divisors

Use a model (e.g., manipulative materials, pictures, number line) to find the answers to the problems below. Be sure to write a number sentence to illustrate each situation.

1. Michael's mom paid \$2.40 for a $\frac{3}{4}$ - pound box of cereal. How much is that per pound?



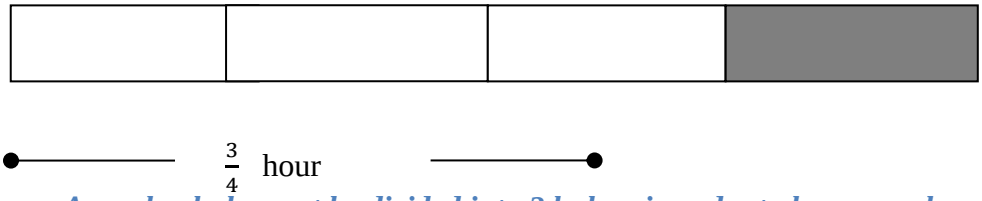
One-fourth of a pound is \$0.80(\$2.40 ÷3), so one pound will be 4 x \$0.80 = \$3.20 per pound.

2. Melitta found out that if she walks really fast during her morning exercise, she can cover $2\frac{1}{2}$ miles in $\frac{3}{4}$ of an hour. How fast is she walking in miles per hour?

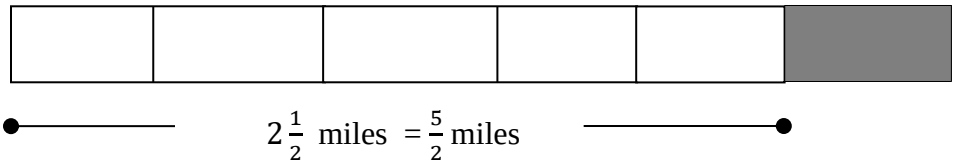
Diagram A

Melitta walks $2\frac{1}{2}$ miles in $\frac{3}{4}$ of an hour. In order to model this situation with tape diagrams, the student must understand that $2\frac{1}{2}$ is being divided into three quarters. Both diagrams are important because you have three fourths of an hour and need four fourths to determine miles per hour. You will have to find the distance walked in one quarter hour (one box) in order to determine the distance traveled in one whole hour.

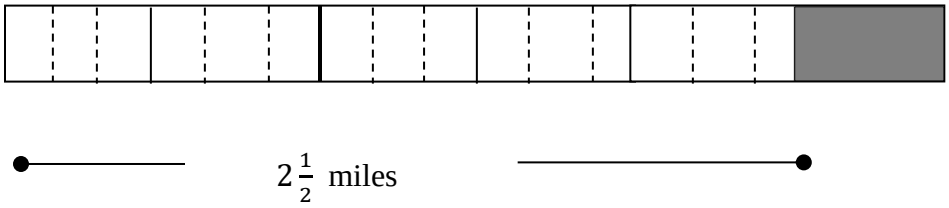
Diagram B



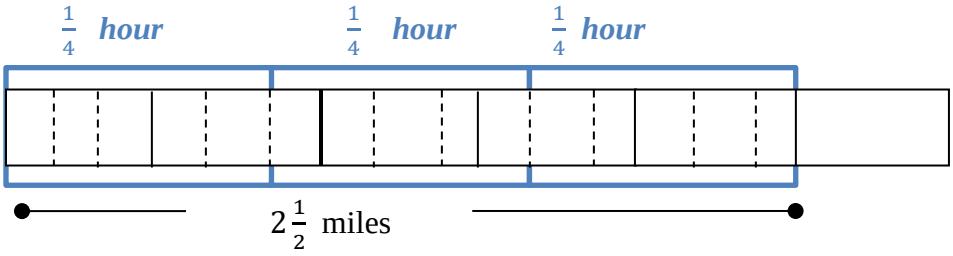
In Diagram A, each whole must be divided into 2 halves in order to have equal parts. Now there are 5 halves.



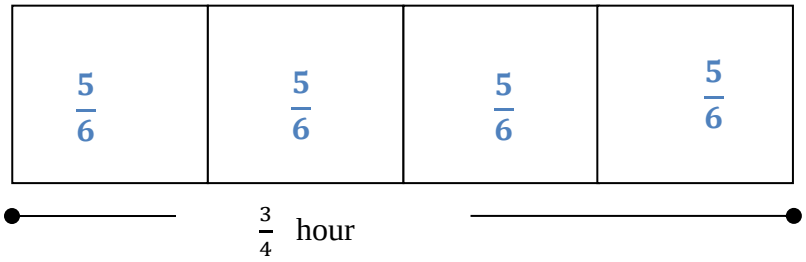
Each box of Diagram A now needs to be subdivided into three equal sections because there are five parts that cannot currently be divided into 3 equal parts ($\frac{3}{4}$ hour). Since you cannot divide 5 evenly into three boxes, divide each one-half piece into three parts. Now you have 15 even pieces.



Each piece now represents $\frac{1}{6}$ of an hour. NOW, divide the 15 one-sixth pieces into 3 groups. This represents 3quarters of an hour.



Each quarter hour has 5 pieces. Each piece is $\frac{1}{6}$ of an hour. Another way to say this is that each quarter hour she walks $\frac{5}{6}$ mile.



So, Melitta is walking $\frac{20}{6}$ or 3 $\frac{1}{3}$ miles per hour.

Measurement Interpretation of Division with Fractional Divisors

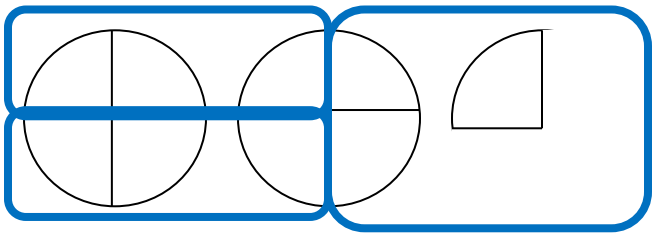
Use a model (e.g., manipulative materials, pictures, number line) to find the answers to the problems below. Be sure to write a number sentence to illustrate each situation.

3. It is your birthday and you are going to have a party. From the grocery store you get 6 pints of ice cream. If you serve $\frac{3}{4}$ of a pint of ice cream to each of your guests, how many guests can be served?

Typically students draw pictures of six items divided into fourths and count out how many servings of ¾ can be found. The difficulty is in seeing this as $6 \div \frac{3}{4}$, and that requires some guidance on the teacher's part. Try to compare the problem to one involving whole numbers (6 pints, 2 per guest).

4. Sam is a landscaper. He found that he had $2\frac{1}{4}$ gallons of liquid fertilizer concentrate. It takes $\frac{3}{4}$ gallon to make a tank of mixed fertilizer. How many tankfuls can he mix?

This question asks, "How many sets of three-fourths are in a set of 9 fourths?" Sam can mix 3 tankfuls.



Name_____

TASK: Fractional Divisors

Partitive Interpretation of Division with Fractional Divisors

Use a model (e.g., manipulative materials, pictures, number line) to find the answers to the problems below. Be sure to write a number sentence to illustrate each situation.

1. Michael's mom paid $2.40 for a $\frac{3}{4}$ - pound box of cereal. How much is that per pound?

2. Melitta found out that if she walks really fast during her morning exercise, she can cover $2\frac{1}{2}$ miles in $\frac{3}{4}$ of an hour. How fast is she walking in miles per hour?

Measurement Interpretation of Division with Fractional Divisor

Use a model (e.g., manipulative materials, pictures, number line) to find the answers to the problems below. Be sure to write a number sentence to illustrate each situation.

3. It is your birthday and you are going to have a party. From the grocery store you get 6 pints of ice cream. If you serve $\frac{3}{4}$ of a pint of ice cream to each of your guests, how many guests can be served?

4. Sam is a landscaper. He found that he had $2\frac{1}{4}$ gallons of liquid fertilizer concentrate. It takes $\frac{3}{4}$ gallon to make a tank of mixed fertilizer. How many tankfuls can he mix?

CLOSING:

Journal Entry: When you divide a fraction by a fraction what do the dividend, quotient and divisor represent?

Additional Practice: Dividing Fractions in Context

1. Suppose you have $2\frac{1}{2}$ apples. If a student serving consists of $\frac{3}{4}$ of an apple, how many student servings (including parts of a serving) can you make?

   $3\frac{1}{3}$ *student servings*

2. Suppose instead that you have $1\frac{1}{2}$ apples. If this is enough to make $\frac{3}{5}$ of an adult serving, how many apples (and parts of an apple) make up one adult serving?

   $2\frac{1}{2}$ *apples*

3. Emma is making posters by hand to advertise her school play, but her posters are not the same length as a standard sheet of paper (the width is the same, though). She has $3\frac{1}{2}$ sheets of paper left over, which she says is enough to make $2\frac{1}{3}$ posters. How many sheets of paper (and parts of a sheet) does each poster use?

   $1\frac{1}{2}$ *sheets of paper*

4. If Connor is also making posters, but his posters only use $\frac{2}{3}$ of a sheet of paper, how many of Connor's posters will those $3\frac{1}{2}$ sheets of paper make?

   $5\frac{1}{4}$ *posters*

5. Laura is tying ribbons in bows on boxes. She uses $2\frac{1}{4}$ feet of ribbon on each box. If she has $7\frac{1}{2}$ feet of ribbon left, how many bows (or parts of a bow) can she make?

   $3\frac{1}{3}$ *bows*

6. Audrey is also tying ribbons into bows. Audrey sees the same $7\frac{1}{2}$ feet of ribbon measured out and says, "Since my bows are bigger than Lura's, that's only enough for me to make $2\frac{1}{4}$ bows." How much ribbon does Audrey use on each bow?

   $3\frac{1}{3}$ *feet of ribbon*

7. Alex has been serving $\frac{2}{3}$ cup of lemonade to each student. If he has $1\frac{1}{2}$ cups of lemonade left, how many students can still get lemonade? How much of a serving will the last student get?

   2 *students,* $\frac{1}{4}$ *of a serving*

SE: Dividing Fractions in Context

1. Suppose you have $2\frac{1}{2}$ apples. If a student serving consists of $\frac{3}{4}$ of an apple, how many student servings (including parts of a serving) can you make?

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7. Alex has been serving $\frac{2}{3}$ cup of lemonade to each student. If he has $1\frac{1}{2}$ cups of lemonade left, how many students can still get lemonade? How much of a serving will the last student get?

Dividing Fractions with Models

SUGGESTED TIME FOR THIS LESSON:

60-90 minutes
Exact timings will depend on the needs of yours class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
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Common Misconceptions:
Students often misapply the invert-and-multiply procedure for dividing by a fraction because they are missing the conceptual understanding of the procedure.

- *not inverting either fraction; for example, a student may solve the problem* $\frac{2}{3} \div \frac{4}{5}$ *by multiplying the fractions without inverting* $\frac{4}{5}$ *(e.g., writing that* $\frac{2}{3} \div \frac{4}{5} = \frac{8}{15}$*)*
- *inverting the wrong fraction (e.g.,* $\frac{2}{3} \div \frac{4}{5} = \frac{3}{2} x \frac{4}{5}$*)*
- *inverting both fractions (*$\frac{2}{3} \div \frac{4}{5} = \frac{3}{2} x \frac{5}{4}$*)*

Such errors generally reflect a missing conceptual understanding of why the invert-and-multiply procedure produces the correct quotient. The invert-and-multiply procedure translates a multi-step calculation into a more efficient procedure.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.

3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
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8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson students should be able to:

- Model division of fractions.
- Explain how to divide fractions.

MATERIALS

- Freezer Pops (paper – see template in task)
- Chocolate Bar Models (paper – see template in task)
- Color Tiles
- Grid Paper

ESSENTIAL QUESTIONS

- Why does the process of invert and multiply work when dividing fractions?
- When I divide one number by another number, do I always get a quotient smaller than my original number?
- When I divide a fraction by a fraction what do the dividend, quotient and divisor represent?
- What kind of models can I use to show solutions to word problems involving fractions?

Grouping: Individual/Partner

OPENING/ACTIVATOR:

TEACHER NOTES

In this lesson students represent division of fractions using manipulatives, such as freezer pops, candy bars, and models such as drawing squares. Students develop an algorithm from these examples and solve problems using fractions.

The concept of division of fractions has been greatly misunderstood. Developing an understanding of what happens when you divide by a fraction prior to development of the algorithm is essential in the thought process. This is accomplished by having the student visually see and understand what dividing by a fraction means with physical examples.

This task was adapted from
http://ims.ode.state.oh.us/ODE/IMS/Lessons/Content/CMA_LP_S01_BH_L06_I08_01.pdf

Activity

Ask five volunteers to come to the front of the classroom. Give each student a freezer pop (use pops with two sticks) and ask if they have ever eaten one. Then ask if they had eaten the entire freezer pop or split it in half. Because of the two sticks, one student may answer that he/she splits the freezer pop in half. Ask students to split the pops in half and have a student count the total number of halves. Use *Frozen Juice Pops*, as a visual representation for the situation.

Freezer pops could be made out of construction paper and craft sticks to illustrate this model. See template.

Ask students if they notice anything about the size of the 10 pieces compared to the original 5 freezer pops. Student should note that they are smaller. Elicit that they are half the size of the original freezer pops.

Ask a volunteer for an equation to represent the 5 freezer pops divided in half.

Answer 5÷ ½ = 10

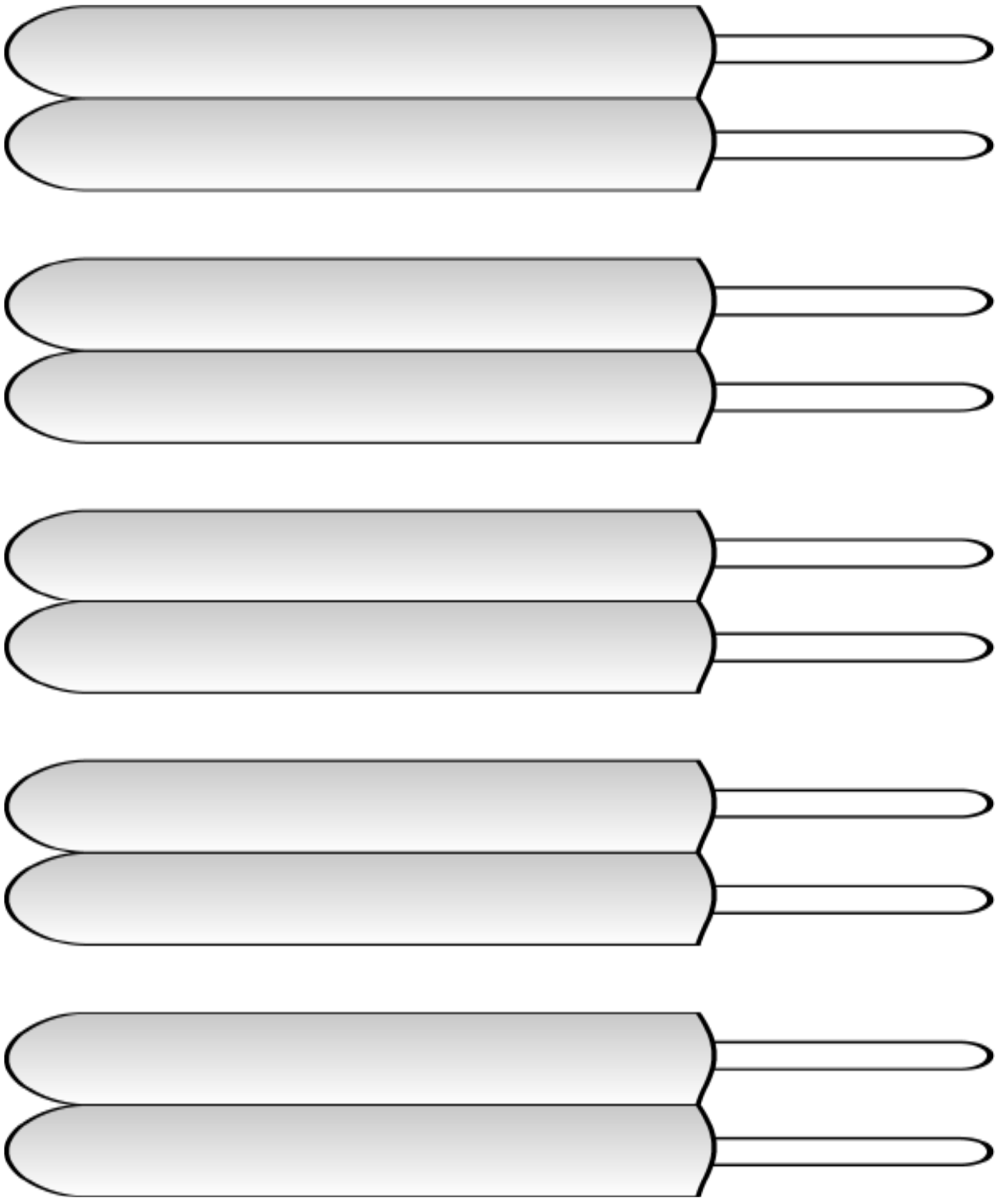
Write the equation on the board for the class to see. If students need help determining this equation, ask "How many half-size freezer pops were contained in the original 5 whole freezer pops?" Then, remind the class that when we ask how many of something there are in something else, that is a division situation (e.g., if we want to know how many 3's are in 12, we divide 12 by 3).

Distribute a variety of chocolate bars that are made with divided sections or use *Chocolate Bar Models*. Ask students to describe how the bars could be divided and give equations to represent this division. $5 \div \frac{1}{4} = 20$, $3 \div \frac{1}{12} = 36$

Again write the equations on the board. Discuss with the students that the size of the pieces desired is not the same as the original pieces. Note: Some students may come up with an equation such as $4 \div 16 = \frac{1}{4}$, which is also a correct way to model this situation. Encourage these students to find a second equation that also models the situation (How many little pieces are in the original big piece?).

Instructional Tip

Make sure students are able to recognize that even though they end with a greater number of pieces after completing the division, the size of the portion is less.

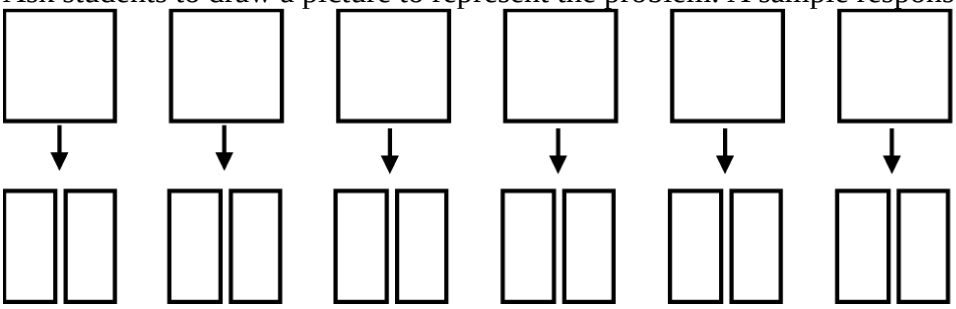


WORK SESSION:

Model Example #1:
Pose the following situation to the class.

I have six squares that I want to divide by one half. How many pieces would I have?

Ask students to draw a picture to represent the problem. A sample response should be



Ask the following guiding questions;

How many squares did I have? ***(6)*** What size did I want? ***(*** $\frac{1}{2}$ ***)***

How many pieces of that size do we have? ***(12)***

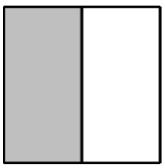
Ask students how this situation would be represented as an equation. Guide the discussion to obtain the equation $6 \div \frac{1}{2} = 12$.

Model Example #2:

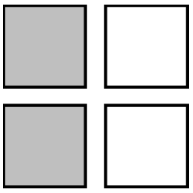
Place students in pairs and pose another situation. Ask them to model it and write an equation that represents the situation.

I have $\frac{1}{2}$ of a square and I want to divide it by $\frac{1}{4}$. How many pieces would I have?

Monitor the partners working on the task and ask the same type of guiding questions when students appear to be struggling with how to represent the situation. The solution should resemble the following example:



This represents having one half of the square.



This represents dividing the square into pieces whose size is one-fourth. The students then need to answer the question of how many pieces of size one-fourth do I have?

Ask the partners to write an equation for the problem. $\frac{1}{2} \div \frac{1}{4} = 2$.

Ask for a volunteer to provide the equation. Ask the student why he/she placed the numbers in that order.

Teacher Notes: Write the equations on the board after each situation, noting the relationships among the numbers in the equations. Have students look for any patterns or relationships they note in the equations.

Have partners make conjectures or descriptions as to what they believe is happening when they divide a number by a fraction. Ask partners to share their conjectures with the class. Record the conjectures and descriptions on the board or chart paper.

Possible conjectures include:

- ***When you divide by a fraction you get a whole number.***
- ***When you divide by a fraction you get a larger number.***
- ***When you divide by a fraction you multiply the whole number by the denominator.***

Instructional Tip

Use the conjectures to adjust the instructions as needed, determining whether students are ready to work with more complex fractions or dividing a fraction by a whole number. Students can test their conjectures and refine their descriptions. The goal is to enable students to determine the algorithm for dividing by a fraction.

Model Example #3:

Present the following situation:

Cierra has $2\frac{5}{8}$ meters of yarn that she wants to cut into $\frac{1}{2}$ -meter lengths. How many $\frac{1}{2}$ -meter lengths of yarn will Cierra have?

Instruct students to draw a model to solve the problem.

A sample model may be

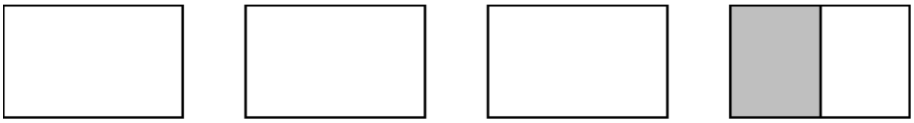
Divide the pieces in half.

There are 5 halves in $2\frac{5}{8}$. There is also $\frac{1}{8}$ left, which is $\frac{1}{4}$ of the remaining half of the whole. Therefore, there are $5\frac{1}{4}$ halves in $2\frac{5}{8}$. ***Cierra will have 5 complete*** $\frac{1}{2}$ ***meter lengths of yarn.***

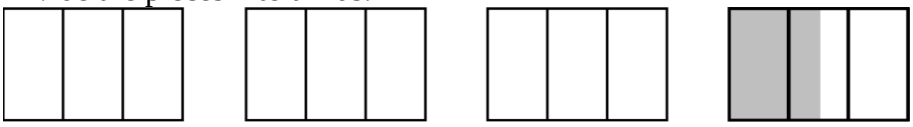
Select a student to model the problem on the board. Note: there will be a piece left over after finding 5 halves. If students are not sure what this represents, ask, "What is the relationship of $\frac{1}{8}$ to the remaining half?" ***(*** $\frac{1}{8}$ ***is*** $\frac{1}{4}$ ***of the remaining half.)***

Model Example #4:
Present a similar situation, such as:

Jonathan has $3\frac{1}{2}$ cups of chocolate chips to make cookies. The recipe uses $\frac{1}{3}$ cup of chips in each batch. How many batches of cookies can Jonathan make?



Divide the pieces into thirds.



Jonathan can make $10\frac{1}{2}$ ***batches of cookies.***

Instructional Tip

Students should see that once they divide the whole part(s) into the desired fractional part, the remaining part, which is a fractional part of another whole piece, needs to be divided also. Students may see a fractional piece left over and should determine the relationship of this piece to the original piece.

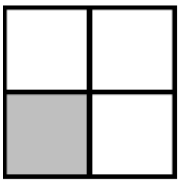
Model Example #5:
Present the following problem:

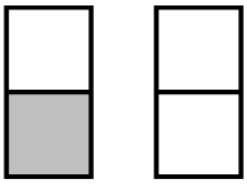
I have one fourth of a square and I want to divide it by one half. How many pieces would I have?

Have students work as partners to solve the problem. Remind students to draw a picture to represent the problem.

Ask guiding questions used with other situations such as, "What are you looking for (How many sets of $\frac{1}{2}$ are in $\frac{1}{4}$)?"

A sample response could be $\frac{1}{2}$ ***;*** $\frac{1}{4} \div \frac{1}{2} = \frac{1}{2}$ ***I have one half of the desired size.***





Have students compare this problem with the previous problem noting any differences and similarities. Discuss the meaning of the answer in each problem.

Have the partners test and refine the conjectures and descriptions by completing *Dividing By a Fraction.* Observe the partners working on the situations and provide intervention, reminding them of the guiding questions that were used with the other situations.

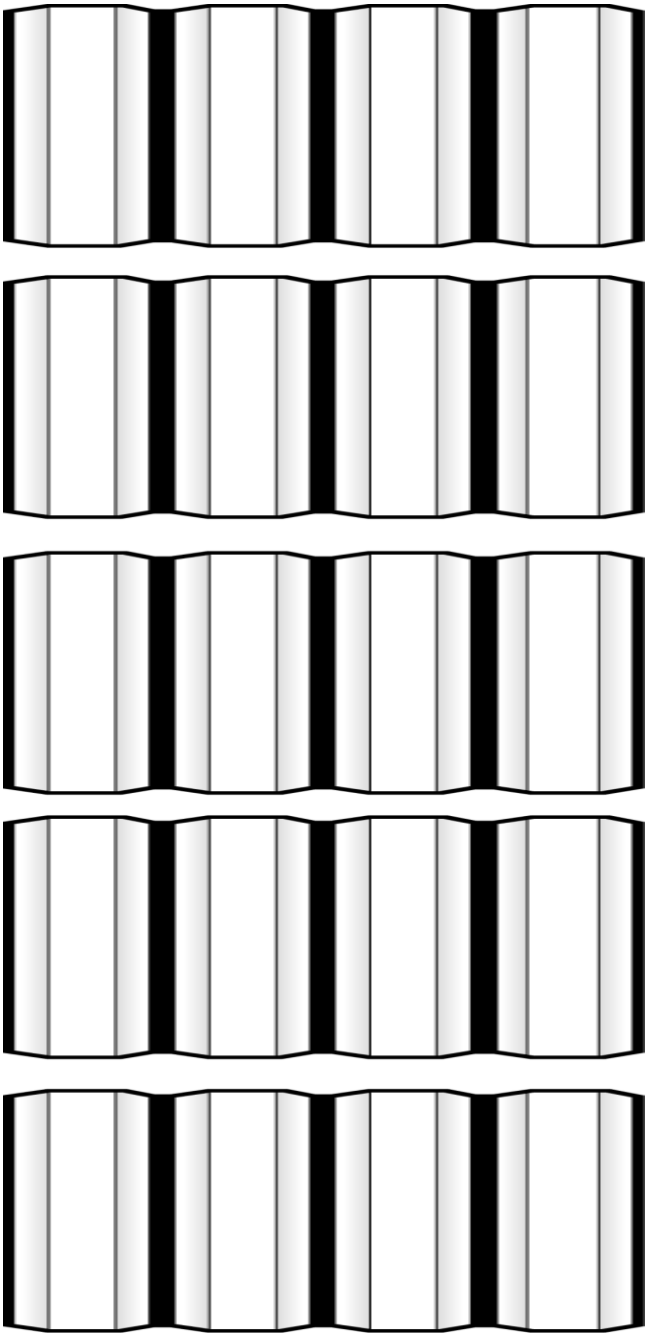
Put partners together to make groups of four. Have each group check the solutions obtained, then review answers as a class, asking each group to provide the solutions to the situations. Use questioning to modify incorrect models.

Ask groups to review the conjectures and descriptions based on the new information. Lead students through the revision process by looking at each conjecture or description and determining if any of the situations contradicted the statement or if clarification is needed.

Have students determine if the statements refer to the process or the results of the problem.

Partner/Individual Work
Give each student a copy of *Models for Dividing Fractions*. They should complete this individually or with a partner.

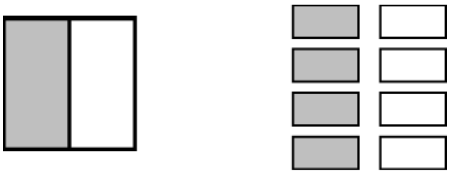
Chocolate Bar Models



TASK PART 1: Dividing by a Fraction

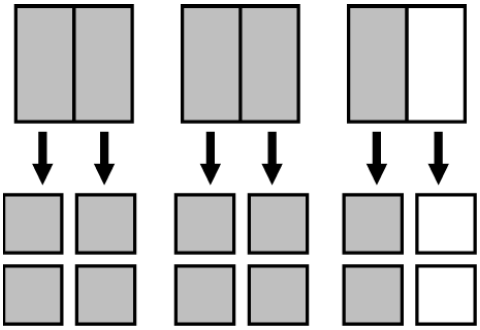
1. I have one-half of a square and I want to divide it by one-eighth. How many pieces would I have?

4 ; $\frac{1}{2} \div \frac{1}{8} = 4$ *I have 4 pieces that are one eighth of the square.*



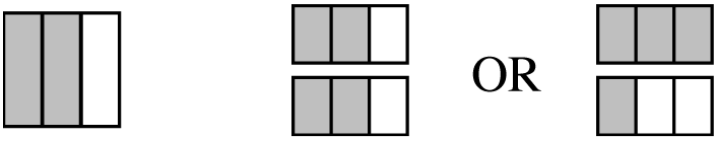
2. I have two and one-half squares and I want to divide them by one-fourth. How many pieces would I have?

10 ; $\frac{5}{2} \div \frac{1}{4} = 10$ *I have 10 pieces that are one fourth of a square.*



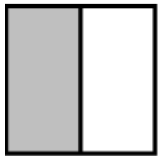
3. I have two- thirds of a square and I want to divide it by one-half. How many pieces would I have?

$\frac{4}{3} or 1\frac{1}{3}$*;* $\frac{2}{3} \div \frac{1}{2} = \frac{4}{3} or 1\frac{1}{3}$ *I have four thirds of the size one half of a square.*

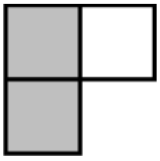


4. I have one-half of a square and I want to divide it by three-fourths. How many pieces would I have?

$\frac{2}{3}$; $\frac{1}{2} \div \frac{3}{4} = \frac{2}{3}$



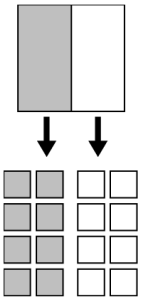
I have two thirds of the size three fourths of a square.



Name_____

TASK PART 2: Models for Dividing Fractions

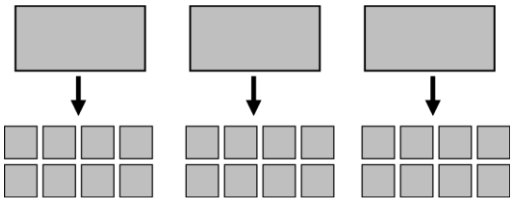
1. I have a one-half gallon container of ice cream and want to divide it into one-cup servings to share with the students in my class. A cup is one sixteenth of a gallon. How many serving dishes would I need?



Model the problem situation.

Write an equation and show how to solve the problem $\frac{1}{2} \div \frac{1}{16} = 8$

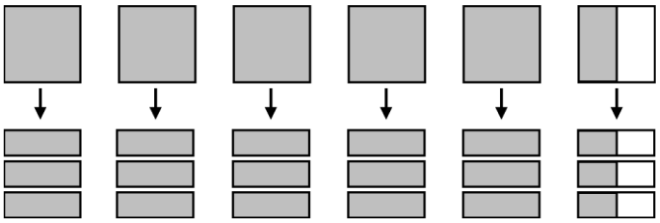
2. I also have three large chocolate candy bars that are perforated into eight sections each. If I divide the bars into these sections how many sections will I have altogether?



Model the problem situation.

Write an equation and show how to solve the problem $3 \div \frac{1}{8} = 24$

3. Becca works for the Humane Society and had to buy food for the dogs. She bought $5\frac{1}{2}$ pounds of dog food. She feeds each dog about one-third of a pound. How many dogs can she feed?



Model the problem situation.

Write an equation and show how to solve the problem. ***Answer:*** $5\frac{1}{2} \div \frac{1}{3} = \frac{33}{2} = 16\frac{1}{2}$

Name_____

TASK PART 1: Dividing by a Fraction

Directions: Model each situation using squares; write the equation for each.

1. I have one-half of a square and I want to divide it by one-eighth. How many pieces would I have?

2. I have two and one half squares and I want to divide them by one-fourth. How many pieces would I have?

3. I have two-thirds of a square and I want to divide it by one-half. How many pieces would I have?

4. I have one-half of a square and I want to divide it by three-fourths. How many pieces would I have?

TASK PART 2: Models for Dividing Fractions

Directions: Read the situation, draw a picture to represent the situation and then write an equation to represent the situation.

1. I have a one-half gallon container of ice cream and want to divide it into one-cup servings to share with the students in my class. A cup is one-sixteenth of a gallon. How many serving dishes would I need?

   Model the problem situation.

   Write an equation and show how to solve the problem.

2. I also have three large chocolate candy bars that are perforated into eight sections each. If I divide the bars into these sections how many sections will I have altogether?

   Model the problem situation.

   Write an equation and show how to solve the problem

3. Becca works for the Humane Society and had to buy food for the dogs. She bought $5\frac{1}{2}$ pounds of dog food. She feeds each dog about one-third of a pound. How many dogs can she feed?

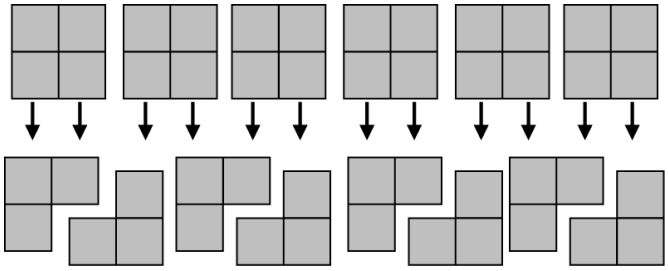
   Model the problem situation.

   Write an equation and show how to solve the problem.

CLOSING:

Ticket Out The Door:

Julie goes to the park across the street from her house several times a day and jogs a total of six miles every day. She jogs three-fourths of a mile at a time. How many times each day does she go to the park to run?



Model the problem situation.

Write an equation and show how to solve the problem. ***Solution:*** $6 \div \frac{3}{4} = 8$

Student Edition: TICKET OUT THE DOOR

Julie goes to the park across the street from her house several times a day and jogs a total of six miles every day. She jogs three-fourths of a mile at a time. How many times each day does she go to the park to run?

Model the problem situation.

Write an equation and show how to solve the problem.

Quick Check III

STANDARDS FOR MATHEMATICAL CONTENT

MFANSQ1. Students will analyze number relationships.
b. Understand a fraction a/b as a multiple of 1/b. (MGSE4.NF.4)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
c. Interpret and solve contextual problems involving division of fractions by fractions. For example, how many 3/4-cup servings are in 2/3 of a cup of yogurt? (MGSE6.NS.1)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)
e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MGSE7.NS.3, MGSE7.EE.3, MGSE9-12.N.Q.3)

Quick Check III - Formative Assessment

1. A recipe calls for $1\frac{1}{3}$ cups of flour to make cookies. If you only want to make half of the recipe, which of the following expressions would help you figure out how much flour to use?

a) $1\frac{1}{3} \div \frac{1}{2}$ b) $1\frac{1}{3} \times \frac{1}{2}$ c) $1\frac{1}{3} \times 2$ d) $\frac{1}{2} \div 1\frac{1}{3}$

The answer is (b).

2. Which of the following situations could be modeled by the expression $6 \div \frac{1}{4}$?

a) You have one-fourth of a pizza and want to split it with six friends
b) You want to know how much are six quarters is worth
c) You share six pizzas with your friends and they each get one-fourth of a pizza
d) You have six dollars and a quarter

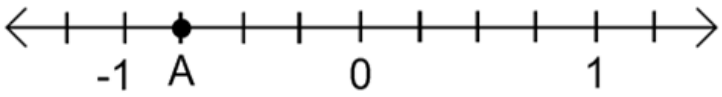
The answwer is (c).

3. Which of the following shows the fractions in order from **least to greatest**?

a) $\frac{1}{2}, \frac{3}{4}, \frac{1}{8}, \frac{3}{8}$ b) $\frac{1}{8}, \frac{3}{8}, \frac{1}{2}, \frac{3}{4}$ c) $\frac{1}{8}, \frac{3}{4}, \frac{1}{2}, \frac{3}{8}$ d) $\frac{3}{4}, \frac{1}{2}, \frac{3}{8}, \frac{1}{8}$

The answer is (b).

4. What is the value of point A on the number line?



a) -2 b) $-1\frac{1}{4}$ c) $-\frac{3}{4}$ d) $-\frac{1}{3}$

The answer is (c).

5. Your mom ordered three cookie cakes for a party. You want each person at the party to get one-fifth (1/5) of a cookie cake. How many people can be at the party? **Write an expression that would help you solve this problem.**

3 divided by 1/5 ($3 \div 1/5$)

Quick Check III - Formative Assessment

For problems 6-9 Simplify the expression completely (this includes changing an improper fraction to a mixed numeral when possible). **You must show all of your work to receive full credit**. Use scratch paper provided if needed. Remember, you get points for your work as well as your answer.

The work for these is shown below. Give credit for all work shown.

6. $3\frac{3}{4}\times\frac{2}{3}=$ $\frac{15}{4}\times\frac{2}{3}=$ $\frac{30}{12}=$ $\frac{5}{2}=$ $2\frac{1}{2}$

7. $3\frac{1}{3}\times1\frac{3}{5}=$ $\frac{10}{3}\times\frac{8}{5}=$ $\frac{80}{15}=$ $\frac{16}{3}=$ $5\frac{1}{3}$

8. $\frac{5}{6}\div\frac{4}{3}=$ $\frac{5}{6}\times\frac{3}{4}=$ $\frac{15}{24}=$ $\frac{5}{8}$

9. $7\frac{1}{2}\div1\frac{1}{4}=$ $\frac{15}{2}\div\frac{5}{4}=$ $\frac{15}{2}\times\frac{4}{5}=$ $\frac{60}{10}=$ 6

Quick Check III - Formative Assessment

1. A recipe calls for $1\frac{1}{3}$ cups of flour to make cookies. If you only want to make half of the recipe, which of the following expressions would help you figure out how much flour to use?

a) $1\frac{1}{3} \div \frac{1}{2}$ b) $1\frac{1}{3} \times \frac{1}{2}$ c) $1\frac{1}{3} \times 2$ d) $\frac{1}{2} \div 1\frac{1}{3}$

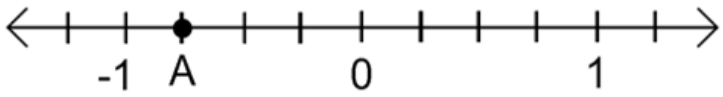
2. Which of the following situations could be modeled by the expression $6 \div \frac{1}{4}$?

a) You have one-fourth of a pizza and want to split it with six friends
b) You want to know how much are six quarters is worth
c) You share six pizzas with your friends and they each get one-fourth of a pizza
d) You have six dollars and a quarter

3. Which of the following shows the fractions in order from **least to greatest**?

a) $\frac{1}{2}, \frac{3}{4}, \frac{1}{8}, \frac{3}{8}$ b) $\frac{1}{8}, \frac{3}{8}, \frac{1}{2}, \frac{3}{4}$ c) $\frac{1}{8}, \frac{3}{4}, \frac{1}{2}, \frac{3}{8}$ d) $\frac{3}{4}, \frac{1}{2}, \frac{3}{8}, \frac{1}{8}$

4. What is the value of point A on the number line?



a) -2 b) $-1\frac{1}{4}$ c) $-\frac{3}{4}$ d) $-\frac{1}{3}$

5. Your mom ordered three cookie cakes for a party. You want each person at the party to get one-fifth (1/5) of a cookie cake. How many people can be at the party? **Write an expression that would help you solve this problem.**

Quick Check III - Formative Assessment

For problems 6-9 **you must show all of your work to receive full credit**. Use scratch paper provided if needed. Remember, you get points for your work as well as your answer.

Answer each of the following problems. Remember to use the rules for operations. SIMPLIFY completely for extra credit (this includes changing an improper fraction to a mixed numeral when possible).

6. $3\frac{3}{4} \times \frac{2}{3} =$

7. $3\frac{1}{3} \times 1\frac{3}{5} =$

8. $\frac{5}{6} \div \frac{4}{3} =$

9. $7\frac{1}{2} \div 1\frac{1}{4} =$

Representing Powers of Ten Using Base Ten Blocks

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ1. Students will analyze number relationships.
c. Explain patterns in the placement of decimal points when multiplying or dividing by powers of ten. (MGSE5.NBT.2)
d. Compare fractions and decimals to the thousandths place. For fractions, use strategies other than cross multiplication. For example, locating the fractions on a number line or using benchmark fractions to reason about relative size. For decimals, use place value. (MGSE4.NF.2;MGSE5.NBT.3,4)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1, 2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1, 2)

Common Misconception

Students may not think about the place value of the digits when multiplying.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.

3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Multiply a decimal by a power of ten.
- Divide a decimal by a power of ten.
- Explain the result on the product when multiplying by a power of ten.

MATERIALS

- Base Ten Blocks (If base 10 blocks are not available, you can print your own representation. See the sample https://www.illustrativemathematics.org/content-standards/5/NBT/A/2/LESSONs/1620. ***Another option is to have the students create the base 10 blocks.***

ESSENTIAL QUESTIONS

- How can you represent a decimal using base ten blocks?
- How can you multiply decimals by powers of ten?

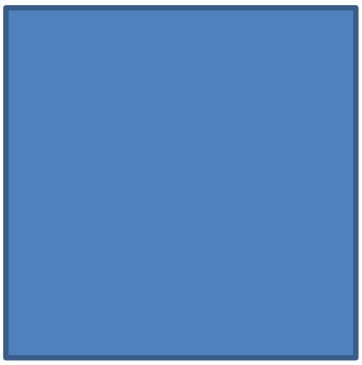
Grouping: Individual/Partner

OPENER/ACTIVATOR:

Teacher notes: Use this activity to teach students how to use base ten blocks to represent decimals.

1 hundredth

1 tenth



The large flat is 1 unit. It is comprised of 100 hundredths or 10 tenths.

https://www.illustrativemathematics.org/content-standards/5

WORK SESSION

Teacher notes: In this activity, use the key provided as to the representation of the ten blocks. The single square now represent one tenths. A key is provided in link.
https://www.illustrativemathematics.org/content-standards/5/NBT/A/2/LESSONs/1620

INTERVENTION

For extra help with Powers of 10, please open the hyperlink Intervention Table.

CLOSING/SUMMARIZER

Journal Entry: Given your explorations of multiplying decimals by the power of ten, create your own problem and illustrate how you would solve it.

Multiplying By Powers of Ten

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

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a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1, 2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1, 2)

Common Misconception

Students may not think about the place value of the digits when multiplying.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.

4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:
Multiply a decimal by a power of ten.
Divide a decimal by a power of ten.
Explain the result on the product when multiplying by a power of ten.

MATERIALS

- Internet Connection

ESSENTIAL QUESTIONS

- How can you represent a decimal using base ten blocks?
- How can you multiply decimals by powers of ten?

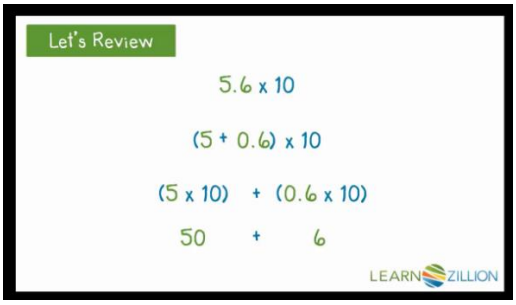
Grouping: Individual/Whole Group

OPENER/ACTIVATOR

Teacher Note: View the youtube video "Power of 10^n" (1977). You may not want to watch the entire 9 minutes, but at least view the first couple of minutes to get the idea.
https://www.youtube.com/watch?v=0fKBhvDjuy0

WORK SESSION

Teacher Notes: This video will revisit the use of base ten blocks and assist in making the connection as to why the decimal point moves.
https://learnzillion.com/lessons/2762 -explain-patterns-in-the-placement-of-the-decimal-point-when-a-decimal-is-multiplied-by-a-power-of-10



Following the video, allow students to work problems on whiteboards and to hold up their boards for a quick check or to submit their answers electronically.

Sample problems:

5.3 x 10	6.23 x 1000	83.5 x 100
2.05 x 10,000	903.85 x 10	0.3856 x 100

Technology Option: Zombies vs. Exponents in the App Store

TEACHER NOTE: Students should realize that the powers of 10 can be written exponentially such as $100 = 10^2$

INTERVENTION

For extra help with powers of 10, please open the hyperlink Intervention Table.

CLOSING/SUMMARIZER

Journal Entry: Ally claims that 3.4 x 100 = 3.400. Do you support her answer? If you do not support it, can you explain Ally's misunderstanding? Illustrate how you tell Ally to work the problem.

Adapted from Illustrative Mathematics, Marta's Multiplication Error
https://www.illustrativemathematics.org/content-standards/5/NBT/A/2/LESSONs/1524

Patterns-R-Us

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ1. Students will analyze number relationships.
c. Explain patterns in the placement of decimal points when multiplying or dividing by powers of ten. (MGSE5.NBT.2)
d. Compare fractions and decimals to the thousandths place. For fractions, use strategies other than cross multiplication. For example, locating the fractions on a number line or using benchmark fractions to reason about relative size. For decimals, use place value. (MGSE4.NF.2;MGSE5.NBT.3,4)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1, 2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1, 2)

Common Misconception

- *Students may not think about the place value of the digits when multiplying.*
- *Multiplication can increase or decrease a number. From previous work with computing whole numbers, students understand that the product of multiplication is greater than the factors. However, multiplication can have a reducing effect when multiplying a positive number by a decimal less than one or multiplying two decimal numbers together. We need to put the term multiplying into a context with which we can identify and which will then make the situation meaningful. Also, using the terms times and groups of interchangeably can assist with the contextual understanding.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:
Identify, describe, and explain any patterns noticed when multiplying or dividing numbers by 1000, 100, 10, 0.1, and 0.01.

MATERIALS

- "Patterns-R-Us" recording sheet
- Calculators

ESSENTIAL QUESTIONS

- How does multiplying or dividing by a power of ten affect the product?

Grouping: Small Group

BACKGROUND KNOWLEDGE

Students should develop an understanding that when a number is multiplied by a number less than 1, the product is less than the original number, and when a number is divided by a decimal number less than 1, the quotient will be greater than the dividend. This is important, yet often difficult for students to understand because it is counterintuitive based on students' previous experiences with multiplication and division.

Students have learned how to multiply and divide with decimals, but they could benefit from solving the problems in part one without a calculator. Students **may use a calculator** for the rest of the lesson. Students may also benefit from a discussion reviewing the powers of ten and discussing what patterns appear as they increase and decrease the power (exponent) each time.

OPENER/ACTIVATOR:

An introduction for this lesson could be a round of "What's My Rule?" The rule could be x1000 which is 10^3, x100 which is 10^2, x10 which is 10^1, x0.1 which is 10^{-1}, or x0.01 which is 10^{-2}. Also, the rule could be ÷1000 which is 10^3, ÷100 which is 10^2, ÷10 which is 10^1, ÷0.1 which is 10^{-1}, or ÷0.01 which is 10^{-2}.

For example:

Input	Output
5	5000
6	6000
23	23,000

Multiply by 1,000

Input	Output
700	70
80	8
10	1

Divide by 10

WORK SESSION:

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments: This lesson is designed to serve as a discovery opportunity for the students. Students should notice that a pattern is created when a number is multiplied or divided by a power of 10. While students may notice patterns in each individual part of the lesson, encourage them to look for a pattern when considering the overall lesson. Students should be able to explain and defend their solutions through multiple representations. For example, students should try several numbers for each part to verify that each number follows the same pattern. This activity lends itself to working in pairs for reinforcement.

LESSON

Students will follow the directions below from the "Patterns-R-Us" Recording Sheet.

A statistician is interested in finding out what pattern is created, if any, under certain situations. Your mission is to help come up with concrete rules for certain mathematical situations. Record all of your work and explain your thinking in order to defend your answer. Good luck!

Part 1

1. Start with any whole number, for example 18.
2. Multiply that number by 10^3, 10^2, 10, 10^{-1}, and 10^{-2}.
   18,000; 1,800; 180; 1.8; 0.18
3. What is happening?
   Decimal moves; If multiplied by a number larger than 1 the product is larger. If multiplied by a factor smaller than 1, the product is smaller.
4. Is there a pattern?
   Yes – At this point, students may just say move it the number of zeros. If zeros are the zeros are on the right then move it to the right that number of zeros. Same idea exists for zero to the left then move to the left.
5. What do you think would happen if you multiplied your number by 10^6? 10^{-5}?
   18,000,000; 0.00018

Part 2

1. Pick any decimal as your number, for example 12.3.
2. Multiply that number by 10^3, 10^2, 10, 10^{-1}, and 10^{-2}.
   12,300; 1,230; 123; 1.23; 0.123
3. What is happening?
   Moving the decimal
4. Is there a pattern?
   Yes – The idea of the number of zeros should be evolving. Hopefully, students view the move in terms of PLACE VALUE.
5. What do you think would happen if you multiplied your number by 10^6? 10^{-5}?
   12,300,000; 0.000123

Part 3

1. Start with any whole number, for example 18.
2. Divide that number by 10^3, 10^2, 10, 10^{-1}, and 10^{-2}.
   0.018; 0.18; 1.8; 180; 1,800
3. What is happening?
   Moving the decimal
4. Is there a pattern?
   Yes – you are dividing now. So look at multiplying by the reciprocal
5. What do you think would happen if you divided your number by 10^6? 10^{-5}?

Part 4

1. Pick any decimal as your number, for example 10.8.
2. Predict what will happen when you divide that number by 10^3, 10^2, 10, 10^{-1}, and 10^{-2}.
   0.0108; 0.108; 1.08; 108; 1,080
3. After working out the problem, is your prediction correct? Why or why not?
   Answers will vary
4. Is there a similar pattern that you recognize?
   Yes – Inverse – Same principle as multiplication just moving the opposite direction

FORMATIVE ASSESSMENT QUESTIONS

- How do you know your answer is correct?
- What would happen if you started with a different number?
- What patterns are you noticing?
- Can you predict what would come next in the pattern?

DIFFERENTIATION

Extension

- Have students multiply a number by 0.1. Now ask them to divide that same number by 10. What happened? Repeat this with several numbers. Can a conjecture be made based on the results? Have students write their conjecture. Now, share their conjecture with a partner. Are the two conjectures the same? (You may also use 0.01 and 100 as another example.)

Intervention

- Pair students who may need additional time so that they will have time needed to process this lesson.
- For extra help with Powers of 10, please open the hyperlink Intervention Table.

Name_____ Date_____

Patterns-R-Us

A statistician is interested in finding out what pattern is created, if any, under certain situations. Your mission is to help come up with concrete rules for certain mathematical situations. Record all of your work and explain your thinking in order to defend your answer. Good luck!

PART ONE

1. Start with any whole number, for example 18.
2. Multiply that number by 10^3, 10^2, 10, 10^{-1}, and 10^{-2}.
3. What is happening?
4. Is there a pattern?
5. What do you think would happen if you multiplied your number by 10^6? 10^{-5}?

PART TWO

1. Pick any decimal as your number, for example 12.3.
2. Multiply that number by 10^3, 10^2, 10, 10^{-1}, and 10^{-2}.
3. What is happening?
4. Is there a pattern?
5. What do you think would happen if you multiplied your number by 10^6? 10^{-5}?

PART THREE

1. Start with any whole number, for example 18.
2. Divide that number by 10^3, 10^2, 10, 10^{-1}, and 10^{-2}.
3. What is happening?
4. Is there a pattern?
5. What do you think would happen if you divided your number by 10^6? 10^{-5}?

PART FOUR

1. Pick any decimal as your number, for example 10.8.
2. Predict what will happen when you divide that number by 10^3, 10^2, 10, 10^{-1}, and 10^{-2}.
3. After working out the problem, is your prediction correct? Why or why not?
4. Is there a similar pattern that you recognize?

CLOSING/SUMMARIZER

Journal Entry: Have students summarize their findings by writing a note to their partner. Have them exchange journals and edit each other's work.

Comparing Decimals

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ1. Students will analyze number relationships.
d. Compare fractions and decimals to the thousandths place. For fractions, use strategies other than cross multiplication. For example, locating the fractions on a number line or using benchmark fractions to reason about relative size. For decimals, use place value. (MGSE4.NF.2;MGSE5.NBT.3,4)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

Common Misconceptions

Students often allow the length of the decimal to determine which decimal value is larger.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.

4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to show understanding of decimal numbers.

MATERIALS

- "Comparing Decimals" recording sheet

ESSENTIAL QUESTIONS

- How can you compare decimals?

Grouping: Individual/Partner

OPENER/ACTIVATOR

Teacher Notes: Students will use their knowledge of modeling decimals with base ten blocks to compare decimals.
Lesson with explanations can be found at:
https://www.illustrativemathematics.org/content-standards/NBT/5/A/3/LESSONs/1801

a. Which is greater, 0.01 or 0.001? Explain. Draw a picture to illustrate your explanation.

b. Which is greater, 0.03 or 0.007? Explain. Draw a picture to illustrate your explanation.

c. Which is greater, 0.025 or 0.052? Explain. Draw a picture to illustrate your explanation.

d. Which is greater, 0.13 or 0.031? Explain. Draw a picture to illustrate your explanation.

e. Which is greater, 0.203 or 0.21? Explain. Draw a picture to illustrate your explanation.

WORK SESSION

Adapted from http://bit.ly/1AJrqzU

Teacher Notes: Encourage students to think of money as they work through this lesson.

Comparing Decimals

Place each of the decimals below in the correct box.

4.23 4.6 4.09 4.491 4.2 4.009 4.9

Numbers smaller than 4.5	Numbers larger than 4.5
4.23 *4.09* *4.491* *4.2* *4.009*	*4.6* *4.9*

Which number is nearest 4.5? Explain.

4.491
Students may draw pictures or explain with words that all of the numbers present have four whole units. The partial unit closest to 0.5 is 0.491. 0.5 is equivalent to 0.500 which is 500 tiny squares colored in a whole unit made of 1,000 tiny squares. 0.491 has 491 tiny squares colored in a whole unit made of 1,000 tiny squares. This is the closest.

Write the numbers in order from least to greatest.

4.009, 4.09, 4.2, 4.23, 4.491, 4.6, 4.9

Explain.

They all have four whole units and are equivalent in that regards so you must look at the partial unit. If you look at the models in terms of one thousand tiny squares in a whole unit, there are 9, 90, 200, 230, 491, 600, and then 900; hence, the answer above.

What is a number that is between 4.2 and 4.23?

Common answers may be: 4.21 and 4.22
You may have a student think is terms of thousandth as above. 4.201, 4.207, 4.229, etc

INTERVENTION

For extra help with comparing decimals, please open the hyperlink Intervention Table.

Student Edition: Comparing Decimals

Place each of the decimals below in the correct box.

4.23 4.6 4.09 4.491 4.2 4.009 4.9

Numbers smaller than 4.5	**Numbers larger than 4.5**

Which number is nearest 4.5? Explain.

Write the numbers in order from least to greatest.

Explain.

What is a number that is between 4.2 and 4.23?

CLOSING/SUMMARIZER

Journal Entry – Given two decimals 7.463 and 7.4063. Which is the greatest? Explain.

Are These Equivalent?

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

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MFANSQ1. Students will analyze number relationships.
d. Compare fractions and decimals to the thousandths place. For fractions, use strategies other than cross multiplication. For example, locating the fractions on a number line or using benchmark fractions to reason about relative size. For decimals, use place value. (MGSE4.NF.2;MGSE5.NBT.3,4)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
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8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to determine equivalency of fractions and decimals.

MATERIALS

- Grid Paper (optional)

ESSENTIAL QUESTIONS

- How can you compare decimals and fractions?

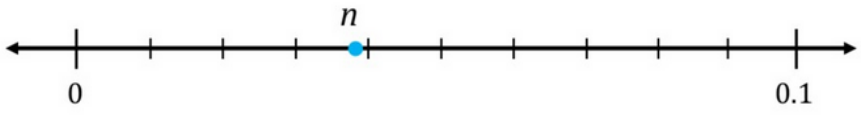
Grouping: Individual/Partner

OPENER/ACTIVATOR

TEACHER NOTE: Students will need to review rounding decimals.

Use Illustrative Math Activity: Rounding to Tenths and Hundredths at
https://www.illustrativemathematics.org/content-standards/5/NBT/A/4/tasks/1804

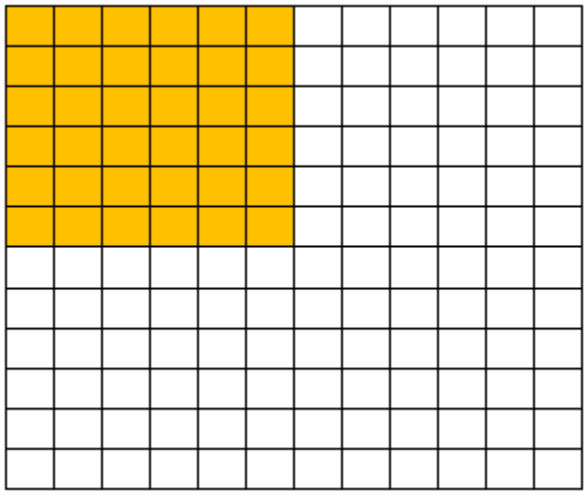
A number n is shown on the number line.



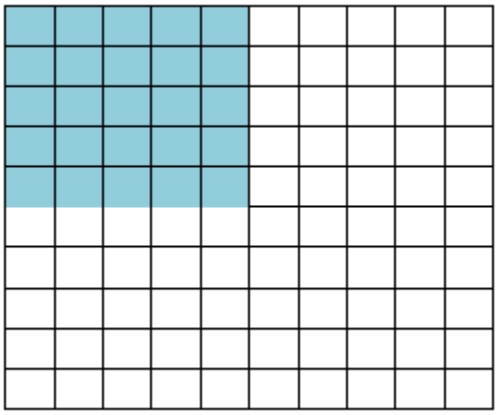
1. The tick marks are evenly spaced. Label them.
2. What is n rounded to the nearest hundredth?
3. What is n rounded to the nearest tenth?

WORK SESSION

Given Melissa's grid below, what fraction of the 12 x 12 grid did she shade? How do you know? How does Melissa's Grid compare to Joey's Grid? Why did they need to shade different numbers of grid squares?



Melissa's Grid



Joey's Grid

Melissa shaded 36 out of 144 squares which is ¼ of the grid.
We know this because ¼ of 144 is 36. Students may say they can make 4 equal groups of 36 out of the 144.

Joey shaded 25 out of his 100 block grid. This is also ¼ of his grid. So while they did not shade the same number of squares in their respective grids, they did shade the same fractional amount.

¼ of 144 is 36 while ¼ of 100 is 25. ¼ depends on the total amount in the whole.

Teacher Notes: Encourage the students to draw models. (For example – grid paper, decimal place value chart, base ten blocks) Once the models have been drawn have the students place them in order from least to greatest on a number line.

Jeremiah, Tatum, and Dustin had a disagreement. Jeremiah said that $4\frac{7}{100}$ is larger than .47 and 4.7. Tatum disagrees with him and claims that .47 is the largest number. Dustin promises they are both wrong and neither one knows what they are talking about because 4.7 is the largest number. Who is correct? Explain.

Dustin is correct in that 4.7 is larger than $4\frac{7}{100}$ and 0.47. First of all, 0.47 isn't even in the running because it is only a part of a whole unit while both 4.7 and $4\frac{7}{100}$ have 4 whole units plus part of another. If you compare the 0.7 and $\frac{7}{100}$, you will find that 0.7 is larger because 0.7 is equivalent to 0.70. There are 70 squares shaded in a 100 square grid while 7/100 has only 7 squares shaded in a 100 square grid. After looking at the models, it is easy to conclude that 4.7 is the larger number.

Teacher Notes: Encourage the students to draw models. (For example – grid paper, decimal place value chart, base ten blocks) Discuss the use of benchmark fractions – common fractions between 0 and 1 such as halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths, and hundredths.

There are two cakes on the counter that are the same size. The first cake has $\frac{1}{2}$ of it left. The second cake has $\frac{5}{12}$ left. Which cake has more left?

Again you could draw model or look at benchmark fractions as mentioned above.

In the first cake, ½ is left. The second cake has $\frac{5}{12}$ left. $\frac{5}{12}$ is less than the benchmark fraction of ½ ; therefore, the first cake has more left.

Student Edition:

QUESTION #1

Jeremiah, Tatum, and Dustin had a disagreement. Jeremiah said that $4\frac{7}{100}$ is larger than .47 and 4.7. Tatum disagrees with him and claims that .47 is the largest number. Dustin promises they are both wrong and neither one knows what they are talking about because 4.7 is the largest number. Who is correct? Explain.

QUESTION #2

There are two cakes on the counter that are the same size. The first cake has $\frac{1}{2}$ of it left. The second cake has $\frac{5}{12}$ left. Which cake has more left?

INTERVENTIONS

For extra help with equivalent fractions and decimals, please open the hyperlink Intervention Table.

CLOSING/SUMMARIZER

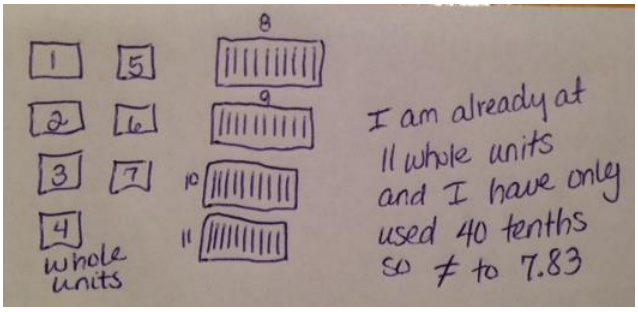
This lesson is adapted from https://www.illustrativemathematics.org/content-standards/NBT/5/A/3/LESSONs/1813

Have the students draw models to look for equivalency. The understanding after modeling all of these examples is POWERFUL!!!!

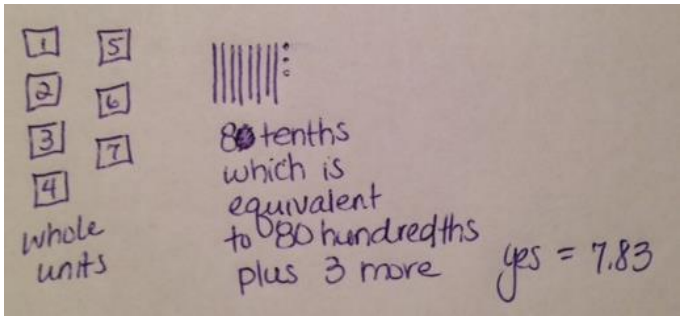
Answer: b, e, f

Mr. Mears is thinking of the number 7.83 in his head. Decide whether each of these has the same value as 7.83 and discuss your reasoning.

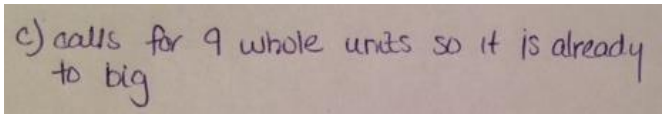
a) Seven and eighty-two tenths



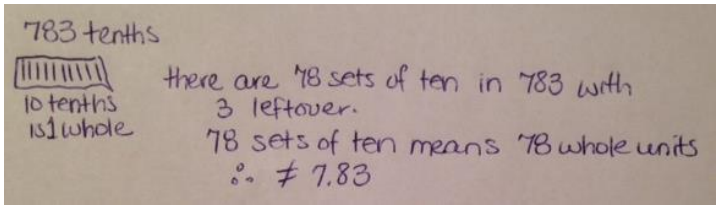
b) 7 + 0.8 + 0.03



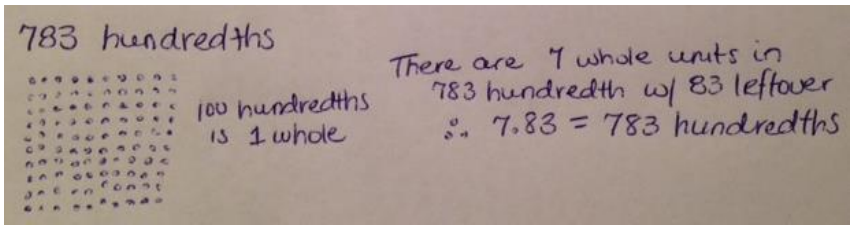
c) $9\frac{13}{25}$



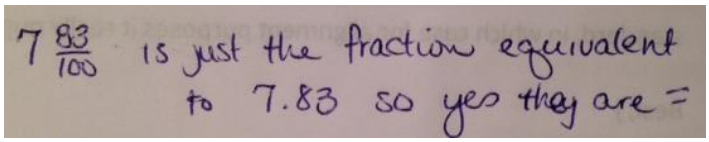
d) 783 tenths



e) 783 hundredths



f) $7\frac{83}{100}$



Student Edition:

Mr. Mears is thinking of the number 7.83 in his head. Decide whether each of these has the same value as 7.83 and discuss your reasoning.

a) Seven and eighty-two tenths

b) 7 + 0.8 + 0.03

c) $9\frac{13}{25}$

d) 783 tenths

e) 783 hundredths

f) $7\frac{83}{100}$

Quick Check IV

STANDARDS FOR MATHEMATICAL CONTENT

MFANSQ1. Students will analyze number relationships.
b. Understand a fraction a/b as a multiple of 1/b. (MGSE4.NF.4)

MFANSQ2. Students will conceptualize positive and negative numbers (including decimals and fractions).
b. Represent numbers on a number line. (MGSE6.NS.5,6)
c. Explain meanings of real numbers in a real world context. (MGSE6.NS.5)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
c. Interpret and solve contextual problems involving division of fractions by fractions. For example, how many 3/4-cup servings are in 2/3 of a cup of yogurt? (MGSE6.NS.1)
e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MGSE7.NS.3, MGSE7.EE.3, MGSE9-12.N.Q.3)

Quick Check IV - Formative Assessment

1. Which of these fractions is equivalent to $\frac{2}{3}$?

a) $\frac{1}{2}$ b) $\frac{3}{4}$ c) $\frac{6}{9}$ d) $\frac{2}{6}$

The answer is (c).

2. Are the following fractions equivalent? $\frac{4}{6}$ *and* $\frac{8}{12}$

a) no, the denominators are not the same
b) no, you add 6 to the denominator and 4 to the numerator
c) yes, you multiply the numerator and denominator by the same number
d) yes, you add 4 to the numerator and 6 to the denominator

The answer is (c).

3. What denominator would you use to add the fractions $\frac{5}{6}+\frac{1}{3}$

a) 3 b) 6 c) 9 d) 15

The answer is (b).

4. Subtract. $\frac{8}{10}-\frac{1}{5}=?$ (Hint: Simplify your answer - use equivalent fractions)

a) $\frac{3}{5}$ b) $\frac{4}{10}$ c) $\frac{7}{10}$ d) $\frac{7}{5}$

The answer is (a).

For problems 5 and 6, find **two** equivalent fractions for each of the given fractions.

5. $\frac{1}{3}$ Some examples are 2/6, 3/9, 4/12, 10/30

6. $\frac{3}{4}$ Some examples are 6/8, 9/12, 12/16, 30/40

7. Add the following fractions. **Show your work.** $\frac{7}{24}+\frac{4}{16}=$ The answer is 13/24

8. Subtract the following fractions. **Show your work.** $\frac{8}{28}-\frac{6}{21}=$ The answer is zero.

Quick Check IV - Formative Assessment

1. Which of these fractions is equivalent to $\frac{2}{3}$?

a) $\frac{1}{2}$ b) $\frac{3}{4}$ c) $\frac{6}{9}$ d) $\frac{2}{6}$

2. Are the following fractions equivalent? $\frac{4}{6}$ *and* $\frac{8}{12}$

a) no, the denominators are not the same
b) no, you add 4 to the numerator and 6 to the denominator
c) yes, you multiply the numerator and denominator by the same number
d) yes, you add 4 to the numerator and 6 to the denominator

3. What denominator would you use to add the fractions $\frac{5}{6}+\frac{1}{3}$

a) 3 b) 6 c) 9 d) 15

4. Subtract. $\frac{8}{10}-\frac{1}{5}=?$ (Hint: Simplify your answer - use equivalent fractions)

a) $\frac{3}{5}$ b) $\frac{4}{10}$ c) $\frac{7}{10}$ d) $\frac{7}{5}$

For problems 5 and 6, find **two** equivalent fractions for each of the given fractions.

5. $\frac{1}{3}$

6. $\frac{3}{4}$

7. Add the following fractions. **Show your work.** $\frac{7}{24}+\frac{4}{16}=$

8. Subtract the following fractions. **Show your work.** $\frac{8}{28}-\frac{6}{21}=$

Integers on the Number Line

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ2. Students will conceptualize positive and negative numbers (including decimals and fractions).
a. Explain the meaning of zero. (MGSE6.NS.5)
b. Represent numbers on a number line. (MGSE6.NS.5,6)
c. Explain meanings of real numbers in a real world context. (MGSE6.NS.5)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.

6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:
Identify the placement of integers on the number line.
Use positive and negative numbers in real world context.
Use positive and negative numbers to describe opposites.

MATERIALS

- Number Line
- Red and Yellow Flags (pieces of paper)

ESSENTIAL QUESTIONS

- How can you represent integers on the number line?
- How can you find the opposite of a number?
- How can you find the total distance between two locations on the number line?

Grouping: Whole Group

OPENER/ACTIVATOR

Teacher Notes: The teacher will share various scenarios. The students will identify these as positive (yellow flag) or negative (red flag) by raising the appropriate flag.

Situations:

You made $45.
Scuba diving at 60 feet below sea level
Lose 10 lbs.
3 candy bars
Spilling 2 cups of water
15 yard penalty
Made a 3 point shot
Docked 2 hours at work
Fell 2000 feet
Elevation of 1500 feet above sea level

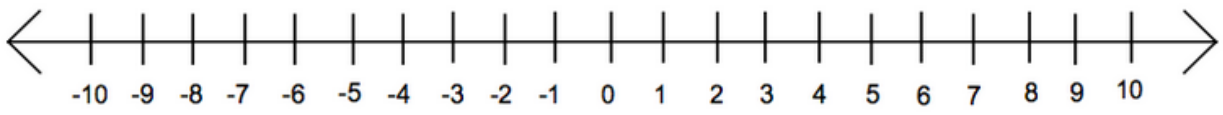
WORK SESSION

View the video: "The Number Line Dance" to hype them up.
https://www.youtube.com/watch?v=6EWq9EZmIKg
Discussion questions for after the video:

- How much did the little boy have after he went to the taco shop and got ice cream with his friends?
- What happened to the money he earned from washing cars?
- What makes -8 the opposite of 8?

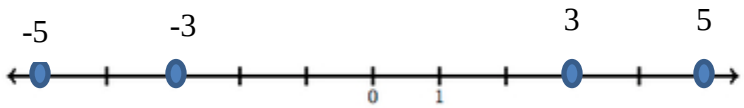
Have the students find the number 6 on the number line. Now ask the students "What is the opposite of 6?" and "Why?" Have the students identify it on the number line.

Teacher Note: Show students the mathematical way of saying "the opposite of 6" which is -6. Also, "the opposite of a -7" is – (- 7).



Lesson: Labeling Integers on the Number Line
Adapted from Integers on the Number Line from Illustrative Mathematics
https://www.illustrativemathematics.org/content-standards/6/NS/C/6/LESSONs/2009

On the number line below:



a) Find and label the integers -3 and -5 on the number line. Explain.

Negative numbers are located to the left of zero. i.e.: -3 is three units to the left of zero

b) Find and label the – (- 3) and – (- 5) on the number line. Explain.

The opposite of a negative 3 would place it on the other side zero; hence, a positive 3. Same for the opposite of a negative 5 would be a positive 5.

c) Find and label – 0 on the number line. Explain.

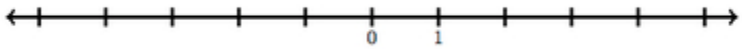
Zero is the balancing point, so, it is neither positive or negative. It is the point of equilibrium.

INTERVENTIONS

For extra help with operations with integers, please open the hyperlink Intervention Table.

Student Edition:

On the number line below:



a) Find and label the integers -3 and -5 on the number line. Explain.

b) Find and label the – (- 3) and – (- 5) on the number line. Explain.

c) Find and label – 0 on the number line. Explain.

CLOSING/SUMMARIZER

Journal Entry: Why are the numbers 8 and – 8 opposites? Write a story problem which would show this.

Using Positive and Negative Numbers in Context

A Formative Assessment Lesson by the Shell Center
http://map.mathshell.org/materials/lessons.php?LESSONid=453&subpage=concept

SUGGESTED TIME FOR THIS LESSON:

90 - 120 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ2. Students will conceptualize positive and negative numbers (including decimals and fractions).
a. Explain the meaning of zero. (MGSE6.NS.5)
b. Represent numbers on a number line. (MGSE6.NS.5,6)
c. Explain meanings of real numbers in a real world context. (MGSE6.NS.5)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.

5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to understand and use integers in real world context.

MATERIALS

- Temperature Change Assessment Lesson
- Temperature Change (Revisited) Assessment Lesson
- City Temperature Cards
- Changes in Temperature Cards

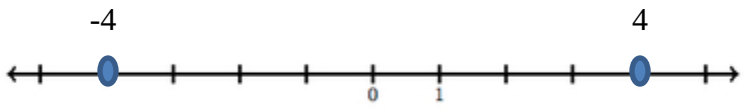
ESSENTIAL QUESTIONS

- How can you use real numbers in real world context?

Grouping: Individual/Partner/Whole Group

OPENER/ACTIVATOR

It was 4 degrees today in Anchorage, AK. How would you represent Anchorage's temperature on a number line? If it is – 4 degrees in Snow Hill, Antarctica, how many degrees difference is there? Justify your answer using a number line.



There is an 8 degree difference.

WORK SESSION

Teacher Notes: It is important that you read the lesson guide. It provides you with very explicit directions every step of the way. The lesson is very teacher-friendly.
Click this link to access all materials:
http://map.mathshell.org/materials/download.php?fileid=1304

CLOSING/SUMMARIZER

Temperature Change (Revisited) Assessment Lesson (See link above in Work Session)

Additional Problems:

One afternoon the temperature was -17 degrees in Lima, OH, and 62 degrees in Leesburg, GA. How many degrees warmer was it in Leesburg than in Lima on that afternoon?

There is a 79 degree difference.

The highest point on Earth is Mount Everest at 29,029 feet. The lowest point on Earth is Challenger Deep at the bottom of Mariana Trench. It is 36, 201 feet below sea level. What is the elevation difference between this two?

There is a 65,230 foot difference.

Deep Freeze

SUGGESTED TIME FOR THIS LESSON:

60-90 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ2. Students will conceptualize positive and negative numbers (including decimals and fractions).
a. Explain the meaning of zero. (MGSE6.NS.5)
b. Represent numbers on a number line. (MGSE6.NS.5,6)
c. Explain meanings of real numbers in a real world context. (MGSE6.NS.5)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)
e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MGSE7.NS.3, MGSE7.EE.3, MGSE9-12.N.Q.3)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.

4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Identify the placement of integers on the number line.
- Use positive and negative numbers in real world context.
- Use positive and negative numbers to describe opposites.

MATERIALS

- Videos for Deep Freeze – 3-Act Task
- Recording sheet (attached)

ESSENTIAL QUESTIONS

- How does a number line model addition or subtraction of rational numbers?
- How can models make sense of the real world application of rational numbers?
- How do I model addition and subtraction of integers on a vertical number line?
- What patterns are present when adding and subtracting integers?

Grouping: Whole Class

Teacher Notes
In this task, students will watch the video, then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they do not have the information they need, and ask for it, it will be given to them

Task Description
The following 3-Act Task can be found at: http://vimeo.com/94462885

OPENER/ACTIVATOR

ACT 1:

Watch the video and answer these questions on the student recording sheet:

- What's the temperature in Duluth, Minnesota? Estimate
- Write an estimate you know is too high. Write an estimate you know is too low.

WORK SESSION

ACT 2:

- It is 7 degrees in Atlanta, Georgia
- There is a 31 degree difference in the temperature between Atlanta and Duluth, Minnesota.

There's a 31 degree difference between Atlanta, Georgia and Duluth, Minnesota

.

INTERVENTIONS

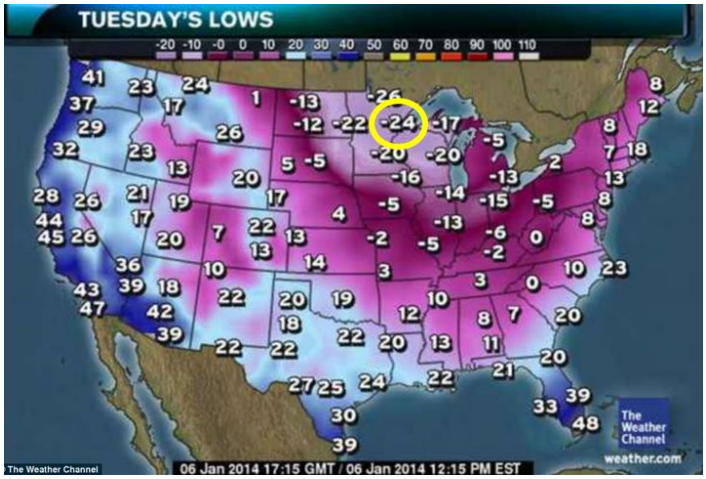
For extra help with integer operations, please open the hyperlink Intervention Table.

CLOSING/SUMMARIZER

ACT 3:

Students will compare and share solution strategies.

- Reveal the answer. Discuss the theoretical math versus the practical outcome.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.



ACT 4

- Have students identify the temperature difference between 5 different cities

Task Title:_____ Name:_____

Adapted from Andrew Stadel

ACT 1

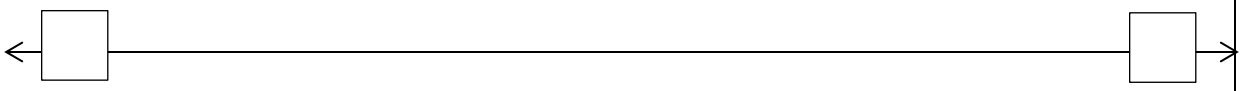
What did/do you notice?

What questions come to your mind?

Main Question:_____

Estimate the result of the main question? Explain?

Place an estimate that is too high and too low on the number line



Low estimate *Place an "x" where your estimate belongs* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:_____

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Quick Check V

STANDARDS FOR MATHEMATICAL CONTENT

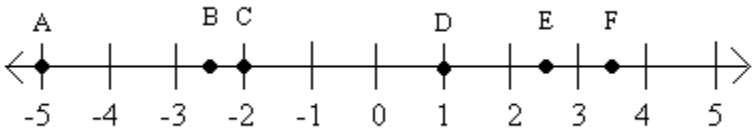
MFANSQ2. Students will conceptualize positive and negative numbers (including decimals and fractions).
a. Explain the meaning of zero. (MGSE6.NS.5)
b. Represent numbers on a number line. (MGSE6.NS.5,6)
c. Explain meanings of real numbers in a real world context. (MGSE6.NS.5)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)
e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MGSE7.NS.3, MGSE7.EE.3, MGSE9-12.N.Q.3)

Quick Check V - Formative Assessment

For problems 1-3, use the number line below.

1. Which of the following is the opposite of point C?



a) -5 b) -2 c) 1 d) 2 e) point F

The answer is (d).

2. When you simplify the expression $-2+3=?$ you will end up at which point?

a) *A* b) *B* c) *C* d) *D* e) *E*

The answer is (d).

3. Which letter best represents the number $-2\frac{1}{2}$?

a) B b) C c) E d) F

The answer is (c).

4. Steve was looking for the average ticket price for a movie in Athens. When he calculated it, he came up with 7.6582. How much money would this be (round to the neareest hundredth/cent)?

a) $7.00 b) $7.50 c) $7.66 d) $7.80

The answer is (c).

5. Tanya had three and a half dollars. She needed twice that much money to get into the football game. How much does it cost to go to the football game?

a) $1.75 b) $3.50 c) $5.50 d) $7.00

The answer is (d).

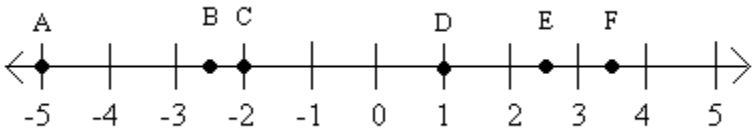
6. Beyonce has 50 pairs of shoes that she wants to take to her new house. The boxes she has to carry them in can hold 8 pairs of shoes each. How many boxes will she completely fill and how many pairs of shoes will be left over? **Show all of your work for credit.**

She will have 6 completely filled boxes with 2 pairs left over (50 divided by 8).

Quick Check V - Formative Assessment

For problems 1-3, use the number line below.

1. Which of the following is the opposite of point C?



a) -5 b) -2 c) 1 d) 2 e) point F

2. When you simplify the expression $-2+3=?$ you will end up at which point?

a) *A* b) *B* c) *C* d) *D* e) *E*

3. Which letter best represents the number $-2\frac{1}{2}$?

a) B b) C c) E d) F

4. Steve was looking for the average ticket price for a movie in Athens. When he calculated it, he came up with 7.6582. How much money would this be (round to the neareest hundredth/cent)?

a) $7.00 b) $7.50 c) $7.66 d) $7.80

5. Tanya had three and a half dollars. She needed twice that much money to get into the football game. How much does it cost to go to the football game?

a) $1.75 b) $3.50 c) 5.50 d) $7.00

6. Beyonce has 50 pairs of shoes that she wants to take to her new house. The boxes she has to carry them in can hold 8 pairs of shoes each. How many boxes will she completely fill and how many pairs of shoes will be left over? **Show all of your work for credit.**

Penny Cube

SUGGESTED TIME FOR THIS LESSON:

60-90 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)
e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MGSE7.NS.3, MGSE7.EE.3, MGSE9-12.N.Q.3)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.

6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Multiply decimals.
- Make sense of rational numbers in the real world.

MATERIALS

- Videos for Penny Cube – 3-Act Task
- Recording sheet (attached)

ESSENTIAL QUESTIONS

- How do you calculate the amount of money in a cube of pennies?

Grouping: Learning Partners; Whole Class

Teacher Notes
In this lesson, students will watch the video, then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they do not have the information they need, and ask for it, it will be given to them

LESSON Description
The following 3-Act Task can be found at: http://mikewiernicki.com/penny-cube/

OPENER/ACTIVATOR

ACT 1:

Watch the video and answer these questions on the student recording sheet:

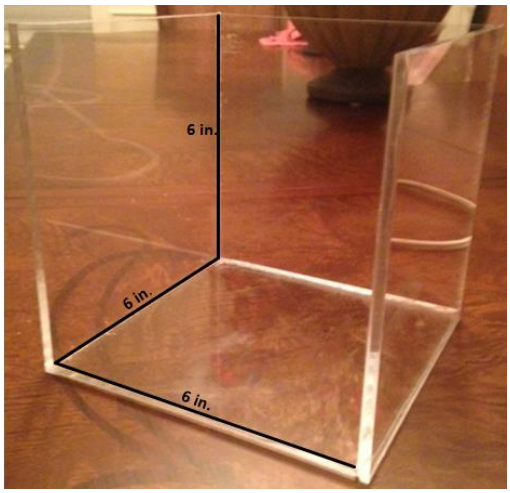
- Ask the students what they wonder after seeing the video.
  - How many pennies is that?
  - How much money is that?
- Have students record their estimates on the recording sheet. What would be an estimate that is too high? Too low?

WORK SESSION

ACT 2:

What do students need to know to answer the questions above? Below you will find some of the information they may ask for.

- Penny Cube Dimensions



- Coin Specifications

Denomination	Penny	Nickel	Dime	Quarter Dollar
Weight	About 0.09 oz	About 0.18 oz	About 0.08 oz	About 0.2 oz
Diameter	0.750 in	0.835 in	0.705 in.	0.955 in.
Thickness	About 0.06 in.	About 0.08 in.	About 0.05 in	About 0.07 in.

.

CLOSING/SUMMARIZER

ACT 3:

Students will compare and share solution strategies.

- Reveal the answer. ***Video solution is located on the website above.*** Discuss the theoretical math versus the practical outcome.
- How reasonable was your estimate?
- Share student solution paths. Start with most common strategy.
- What might you do differently if you were to do this again?
- Revisit any initial student questions that weren't answered.

ACT 4 - Extensions

- How much does this penny cube weigh?
- Quarters would fit nicely in this cube as well. Which would you rather have, a cube of pennies or a cube of quarters?
- If this cube was one cubic foot, how much money would it hold?

LESSON Title:_____
Name:_____

Adapted from Andrew Stadel

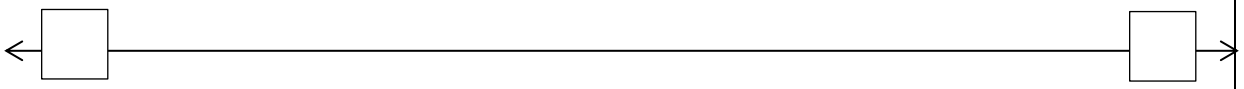
ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:_____

Estimate the result of the main question? Explain?
Place an estimate that is too high and too low on the number line



Low estimate *Place an "x" where your estimate belongs* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:_____

Act 2 (con't)

Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Multiplying Rational Numbers

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ2. Students will conceptualize positive and negative numbers (including decimals and fractions).
a. Explain the meaning of zero. (MGSE6.NS.5)
b. Represent numbers on a number line. (MGSE6.NS.5,6)
c. Explain meanings of real numbers in a real world context. (MGSE6.NS.5)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

Common Misconceptions

- *Visualizing multiplication is a very difficult concept for students to comprehend. You may have some learners that find the lesson more confusing than helpful depending on their understanding of how concrete examples apply to abstract concepts.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain

calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
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5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:
Use a number line to multiply positive and negative integers.
Apply the patterns found in multiplying integers to division of positive and negative integers.
Use a number line to show multiplication should be used.

MATERIALS

- Two color counters

ESSENTIAL QUESTIONS

- How do you use a number line to multiply rational numbers?
- What patterns in multiplication can you relate to division?
- How do multiplication and division of rational numbers relate to one another?

Grouping: Partner/Whole Group

OPENER/ACTIVATOR

If everyone in class owes you $2, what is the total debt? Explain.

WORK SESSION

Students should be asked to answer these questions and prove their responses.

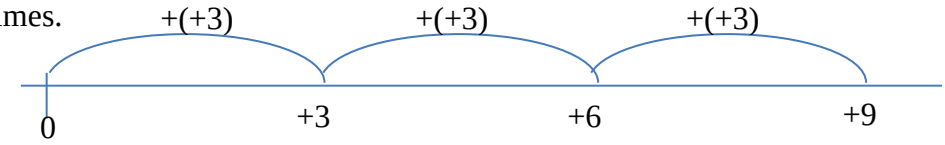
- ***Is it always true that multiplying a negative factor by a positive factor results in a negative product?***
- ***Does a positive factor times a positive factor always result in a positive product?***
- ***What is the sign of the product of two negative factors?***
- ***When three factors are multiplied, how is the sign of the product determined?***
- ***How is the numerical value of the product of any two numbers found?***

LESSON DESCRIPTION

To introduce the concept of multiplying integers, help students make the connection between this and multiplying positive whole numbers.

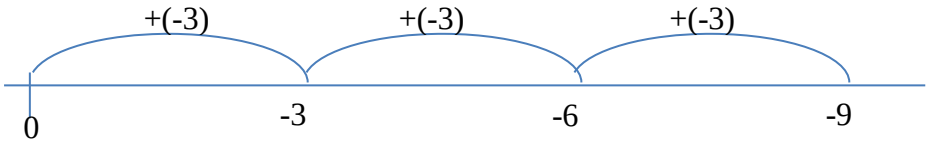
Let's begin with 3 x 3. Using counters, have students model 3 groups of positive 3. Ask a student to explain how they modeled the expression. Record their thinking on an empty number line. An empty number line is a line without designated numbers.

The student should explain they began with 0 counters and added 3 positive counters a total of 3 times.



Have students model 3 x -3 using the counters. Ask a student to explain how they modeled the expression. Record their thinking on an empty number line.

The student should explain they began with 0 counters and added 3 negative counters a total of 3 times.

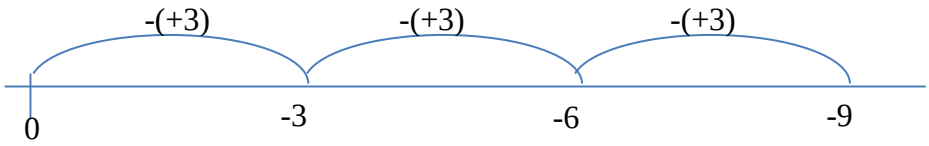


Have students model -3 x 3 using the counters. Circulate around the room to see how students are grappling with the idea of having negative 3 groups of 3. Look for students who:

- Apply the commutative property to make sense of the expression.
- Attempt to make zero pairs to make sense of the expression but become stuck.
- Attempt to make zero pairs and remove 3 groups of positive 3.

Explain to students, with the previous examples, they began with 0 and repeatedly added the quantities, so they used repeated addition when creating the positive groups. Since the groups are no longer positive, they are the opposite; students should perform the opposite operation, repeated subtraction. With this model, students should remove 3 groups of positive 3 from 0.

Allow students the opportunity to discuss with a partner how to model removing from 0.
0 can be created through the use of zero pairs, +a + (-a) = 0. Instruct students create enough zero pairs in which 3 groups of positive 3 can be removed. Once this is modeled, instruct students to remove the 3 groups of +3. Ask students what quantity is left over. With a total of 9 positive counters removed, there should be -9 counters left over.
Record this thinking on an empty number line.

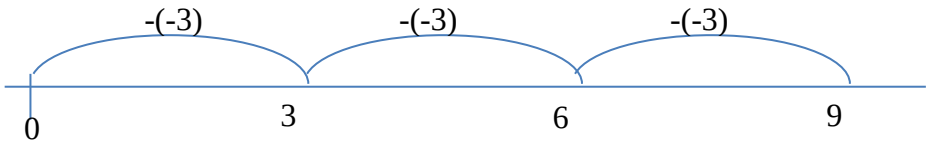


Have students model -3 x -3 using the counters. Circulate around the room to see how students are grappling with the idea of having negative 3 groups of -3. Look for students who:

- Attempt to apply a pattern determined from the previous models.
- Attempt to make zero pairs to make sense of the expression but become stuck.
- Attempt to make zero pairs and remove 3 groups of negative 3.

Allow students the opportunity to discuss with a partner how to model removing from 0.
0 can be created through the use of zero pairs, +a + (-a) = 0. Instruct students create enough zero pairs in which 3 groups of negative 3 can be removed. Once this is modeled, instruct students to remove the 3 groups of -3. Ask students what quantity is left over. With a total of 9 negative counters removed, there should be 9 counters left over.

Record this thinking on an empty number line.



Apply the conventions within a context.

1. Mr. Fletcher's bank account is assessing a $5 fee withdrawal for everyday it is under $500. It has been a total of 4 days under $500. What amount will the bank withdraw from his account?

   4 x -$5 = -$20

2. Billy is participating in a biggest Loser competition. His goal is to lose 3 pounds a week. If Billy meets his goal every week for 6 weeks, how much weight will he lose?

-3 x 6 = -18

3. Mike's son has to pay his dad $7 for every pound he loses. Mike has lost 5 pounds. How much money does his son owe him?

-$7 x -5 = $35
His son owes him $35.

Modeling the Multiplication of Integers

Try these problems on your own. Model each problem using counters or an empty number line. Record the equation and model on your paper.

1. Suppose the temperature outside is dropping 3 degrees each hour. How much will the temperature change in 8 hours?

-3 x 8 = -24

2. A computer stock gained 2 points each hour for 6 hours. Describe the total change in the stock after 6 hours.

2 x 6 = 12

3. A drought can cause the level of the local water supply to drop by a few inches each week. Suppose the level of the water supply drops 2 inches each week. How much will it change in 4 weeks?

-2 x 4 = -8

4. Mike's son has to pay his dad $8 for every pound he loses. Mike has lost 10 pounds. How much money does his son owe him?

-8 x -10 = 80

DIFFERENTIATION

Extension:

- Have students develop generalized conjectures about multiplying integers and explain them. For example, +a X –b = -c because a groups of –b added to 0 is –c.

Intervention:

- For students who struggle with the empty number line. Encourage them to continue modeling the problems using the two colored counters.
- For extra help with multiplication with signed numbers, please open the hyperlink Intervention Table.

Student Edition Learning Lesson: Multiplying Rational Numbers

Try these problems on your own. Model each problem using counters or an empty number line. Record the equation and model on your paper.

1. Suppose the temperature outside is dropping 3 degrees each hour. How much will the temperature change in 8 hours?

2. A computer stock gained 2 points each hour for 6 hours. Describe the total change in the stock after 6 hours.

3. A drought can cause the level of the local water supply to drop by a few inches each week. Suppose the level of the water supply drops 2 inches each week. How much will it change in 4 weeks?

4. Mike's son has to pay his dad $8 for every pound he loses. Mike has lost 10 pounds. How much money does his son owe him?

CLOSING/SUMMARIZER

Write a story problem which involves multiplying integers. Solve using a number line.

Pattern of Multiplication and Division

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ2. Students will conceptualize positive and negative numbers (including decimals and fractions).
a. Explain the meaning of zero. (MGSE6.NS.5)
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b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

Common Misconceptions:
The section on division of positive and negative numbers using two color counters can be confusing upon first glance. Encourage students to persevere in working through the examples in order to gain understanding.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
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8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to determine the relationship of integers when multiplied or divided.

MATERIALS

- Patterns of Multiplication and Division Lesson Sheet
- Colored pencils – red and yellow
- Extra blank number lines (optional)
- Two-color counters (red/yellow)

ESSENTIAL QUESTIONS

- How are multiplication and division of integers related to one another?
- How do I model division of integers on a number line?

Grouping: Individual/Partners

OPENER/ACTIVATOR

Complete these mathematical sentences:

3 + __ = 5	**10 + 6 = __**	**-4 + 2 = __**
2 + 3 = __	**6 + __ = 16**	**2 + -4 = __**
5 – 2 = __	**16 – 6 = __**	**-2 – (- 4) = __**
5 – 3 = __	**16 – __ = 6**	**-2 – 2 = __**

What is the relationship all 3 sets have in common?

WORK SESSION

LESSON COMMENTS

This lesson uses the number line model to illustrate division of integers. When teaching this concept, it is important to revisit number line models of multiplication and addition. Revisiting patterns that are found in multiplying integers is also recommended.

LESSON DESCRIPTION

To introduce the lesson, begin with student's knowledge of addition and subtraction facts and the relationship between the two processes. This same type of relationship occurs between multiplication and division. Point out that for any multiplication fact, we can write another multiplication fact and two different related division facts.

$2 \times 5 = 10$ $5 \times 2 = 10$ $10 \div 5 = 2$ $10 \div 2 = 5$

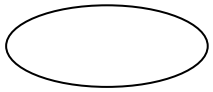
Revisit multiplication of positive and negative integers.

The following examples are adapted from Van de Walle, J., &Lovin, L.H. (2006). *Teaching student-centered mathematics: Grades 5-8*. Boston, MA: Pearson Education and Allyn& Bacon.
*******Show the examples with the color counters and the number line*******

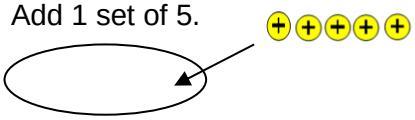
Example 1: 10 ÷ 5 = 2

How many sets of 5 will make a set of 10?

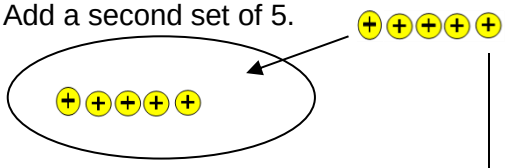
Begin with zero.



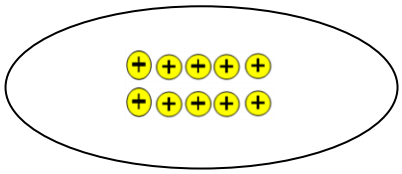
Add 1 set of 5.



Add a second set of 5.



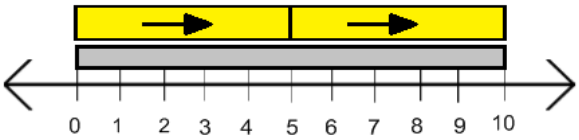
It took 2 sets of 5 to make 10.



To arrive at the answer of +2, notice that on the number line we are moving forward. We move forward 2 times.

10 ÷ 5

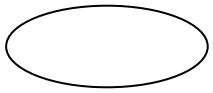
There are 2 fives in 10. Therefore, the answer is 2.



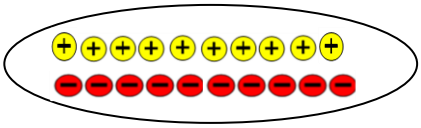
Example 2: (-10) ÷ 5 = (-2)

How many sets of +5 will make -10?

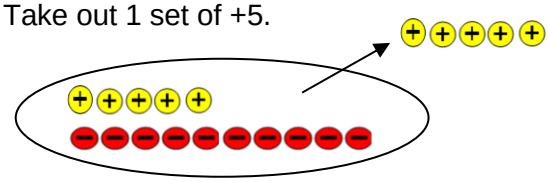
Begin with zero.



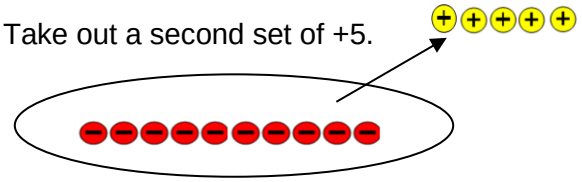
Change the representation. Add 10 neutral pairs.



Take out 1 set of +5.

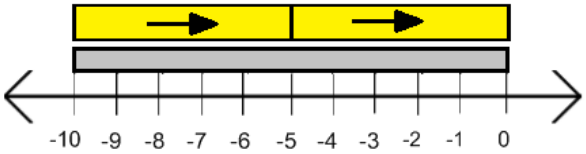


Take out a second set of +5.



2 sets of +5 were removed to make -10 or -2 sets of 5 were used to make -10.

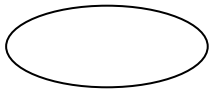
To arrive at the answer of (-2), notice that on the number line we are backing up. We back up 2 times.



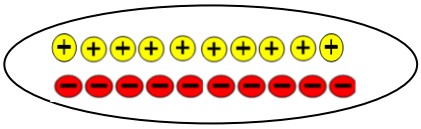
Example 3: 10 ÷ (-5) = (-2)

How many sets of -5 will make +10?

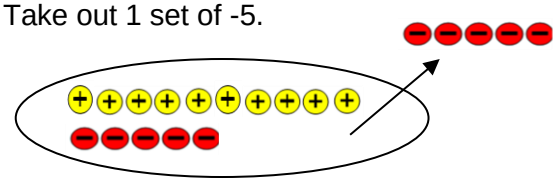
Begin with zero.



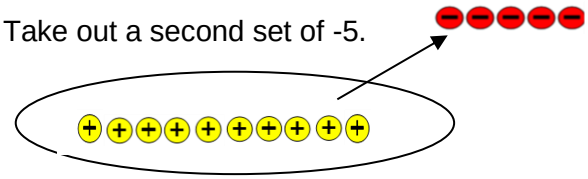
Change the representation. Add 10 neutral pairs.



Take out 1 set of -5.

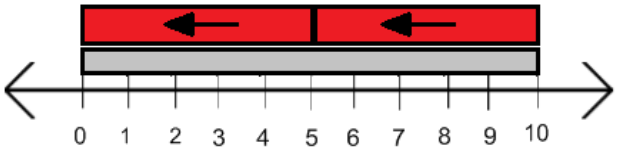


Take out a second set of -5.



2 sets of -5 were removed to make +10 or -2 sets of -5 were used to make +10.

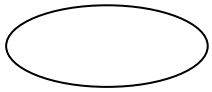
To arrive at the answer of (-2), notice that on the number line we are backing up (-5). We back up 2 times.



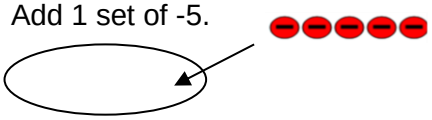
Example 4: (-10) ÷ (-5) = 2

How many sets of -5 will make a set of -10?

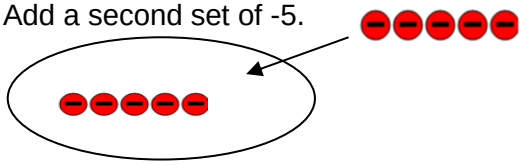
Begin with zero.



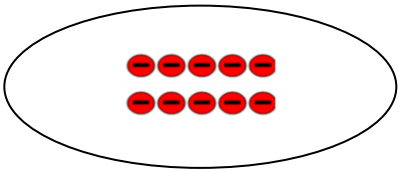
Add 1 set of -5.



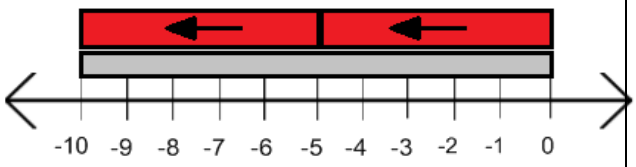
Add a second set of -5.



It took 2 sets of -5 to make -10.



To arrive at the answer of +2, notice that on the number line we are moving forward. We move (-5) forward 2 times.



As you teach the division model using counters, keep in mind these simple steps:

- Determine how many sets of the divisor are needed to make the dividend.
- Begin with zero or a representation of zero using neutral pairs.
- Remove or add sets of the divisor to make the dividend.
- The number of sets removed or added determines the answer.

As you teach the division model using a number line, keep in mind these simple steps:

- Identify the dividend on the number line.
- Look at the divisor, is it positive (yellow with right arrow) or negative (red with left arrow).
- Determine how many times the divisor will have to move forward (+) or backward (-) to equal the dividend.
- **The number of times it must move and the type of movement determine the answer.**

For further clarification of the division model using a number line, please watch the following video.
http://www.youtube.com/watch?v=Lh0tBKOTq8I

Have students model the problems.
LESSON DIRECTION

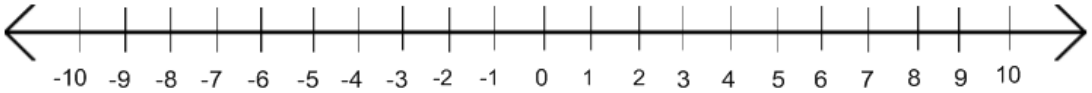
You have recently practiced multiplying positive and negative integers on a number line. It is now your turn to model how to divide. Below are "hints" to help you get started.

When you divide, keep in mind these simple steps:

- Identify the **dividend** on the number line.
- Look at the **divisor**, is it **positive (yellow with right arrow)** or **negative (red with left arrow)**.
- Determine how many times the **divisor** will have to **move forward (+)** or **backward (-) to equal the dividend**.
- The **number of times** it must move and the **type of movement determine the answer.**

Model the following on the number line.

8 ÷ 2

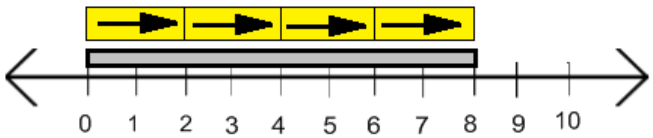


1. What is the dividend? _____

2. What is the divisor? _____

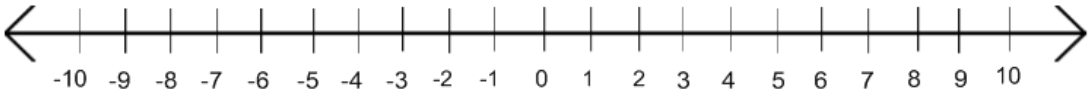
3. What is the solution and how did you find it?

Solution:
1. 8
2. 2
3. 4



The dividend is 8, the divisor is 2, so I counted by two's.
I counted 4 times forward.

$(-9) \div 3$

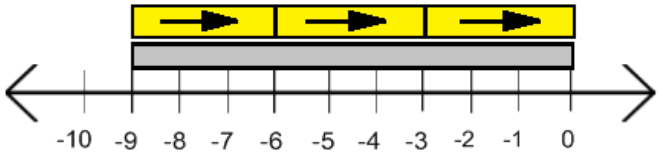


4. What is the dividend?_____

5. What is the divisor?_____

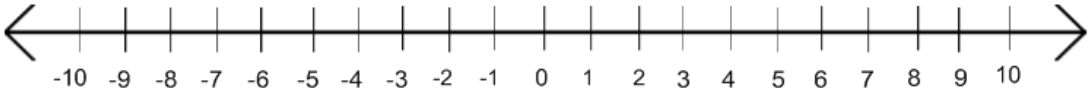
6. What is the solution and how did you find it?

Solution:
4. (-9)
5. 3
6. (-3)



The dividend is (-9), the divisor is 3, so I counted by three's. I counted 3 times backward.

$6 \div (-2)$

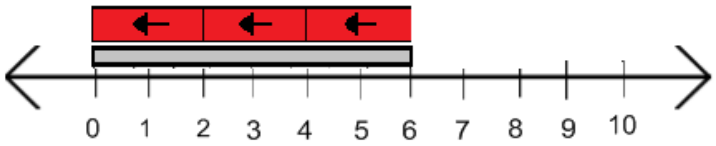


7. What is the dividend? _____

8. What is the divisor? _____

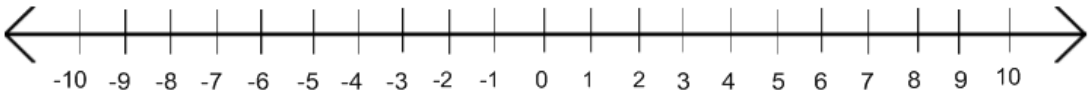
9. What is the solution and how did you find it?

Solution:
10. 6
11. (-2)
12. (-3)



The dividend is 6, the divisor is (-2), so I counted by two's. I counted 3 times backward.

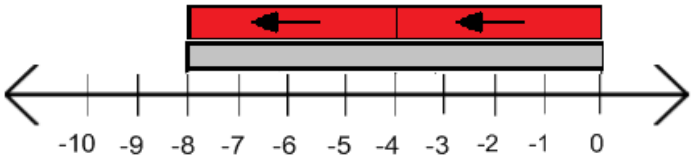
$(-8) \div (-4)$



10. What is the dividend?_____
11. What is the divisor?_____

12. What is the solution and how did you find it?

Solution:
16. (-8)
17. (-4)
18. 2



The dividend is (-8), the divisor is (-4), so I counted by four's. I counted 2 times forward.

CLOSING/SUMMARIZER

Let's look to see if there are any patterns.

1. When given a **positive integer** as the **dividend**…
   a. What was the result of **dividing** by a **positive integer**?

   b. What was the result of **dividing** by a **negative integer?**

2. When given a **negative integer** as the **dividend**…
   a. What was the result of **dividing** by a **positive integer**?

   b. What was the result of **dividing** by a **negative integer?**

Student Edition Learning LESSON: Patterns of Multiplication and Division

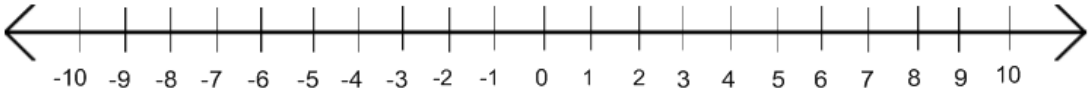
You have recently practiced dividing positive and negative integers on a number line. It is now your turn to model how to divide. Below are "hints" to help you get started.

When you divide, keep in mind these simple steps:

- Identify the **dividend** on the number line.
- Look at the **divisor**, is it **positive (yellow with right arrow)** or **negative (red with left arrow).**
- Determine how many times the **divisor** will have to **move forward (+)** or **backward (-) to equal the dividend.**
- The **number of times** it must move and the **type of movement determine the answer**.

Model the following on the number line.

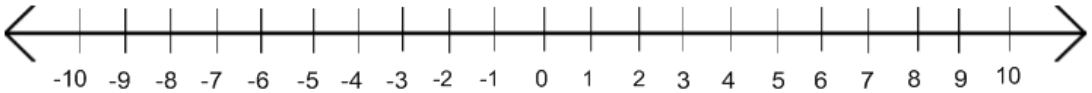
8 ÷ 2



1. What is the dividend?_____

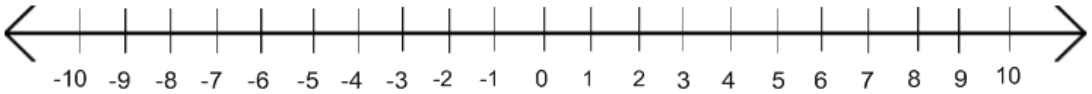
2. What is the divisor? _____
3. What is the solution and how did you find it?

(−9) ÷ 3



4. What is the dividend? _____
5. What is the divisor? _____
6. What is the solution and how did you find it?

$6 \div (-2)$

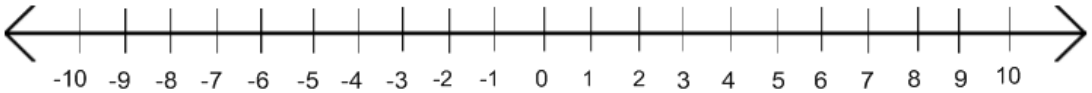


7. What is the dividend?_____

8. What is the divisor?_____

9. What is the solution and how did you find it?

$(-8) \div (-4)$



10. What is the dividend?_____

11. What is the divisor?_____

12. What is the solution and how did you find it?

CLOSING/SUMMARIZER

Let's look to see if there are any patterns.

1. When given a **positive integer** as the **dividend**…
    a. What was the result of **dividing** by a **positive integer**?

    b. What was the result of **dividing** by a **negative integer**?

2. When given a **negative integer** as the **dividend**…
    a. What was the result of **dividing** by a **positive integer**?

    b. What was the result of **dividing** by a **negative integer**?

Rational or Irrational

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ3. Students will recognize that there are numbers that are not rational, and approximate them with rational numbers.
a. Find an estimated decimal expansion of an irrational number locating the approximations on a number line. For example, for $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue this pattern in order to obtain better approximations. (MGSE8.NS.1,2)
b. Explain the results of adding and multiplying with rational and irrational numbers. (MGSE9-12.N.RN.3)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

Common Misconceptions:

- *Because we name so few irrational numbers, students sometimes conclude that there are not that many irrational numbers. Actually, there are more irrational than rational.*
- *Some students may believe that both terminating and repeating decimals are rational, without considering nonrepeating and nonterminating decimals as irrational numbers.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.

3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to identify rational and irrational numbers.

MATERIALS

- Internet Access

ESSENTIAL QUESTIONS

- How can you identify rational and irrational numbers?

Grouping: Individual/Partner

OPENER/ACTIVATOR

Video on Rational Numbers (optional): https://www.youtube.com/watch?v=m94WTZP14SA

Video on Rational vs. Irrational: https://www.youtube.com/watch?v=KKfoORhiSA0

WORK SESSION

Teacher Notes: After viewing the video, have the students complete the activity from Illustrative Mathematics: Identifying Rational and Irrational Numbers which can be found at this link: https://www.illustrativemathematics.org/content-standards/8/NS/A/1/LESSONs/334

INTERVENTIONS

For extra help with decimals, please open the hyperlink Intervention Table.

CLOSING/SUMMARIZER

Journal Entry: Create a Venn diagram comparing rational and irrational numbers.

Estimating the Square Root of a Number

SUGGESTED TIME FOR THIS LESSON:

60-90 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ3. Students will recognize that there are numbers that are not rational, and approximate them with rational numbers.
a. Find an estimated decimal expansion of an irrational number locating the approximations on a number line. For example, for $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue this pattern in order to obtain better approximations. (MGSE8.NS.1,2)
b. Explain the results of adding and multiplying with rational and irrational numbers. (MGSE9-12.N.RN.3)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

Common Misconceptions

- *Students often mistake the square root symbol for long division.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.

4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to estimate the square root of a number using the number line or colored tiles.

MATERIALS

- Colored Tiles
- "Estimating the Square Root of a Number" recording sheet

ESSENTIAL QUESTIONS

- How can you estimate the square root of a number?

Grouping: Individual/Partner/Whole Group

OPENER/ACTIVATOR

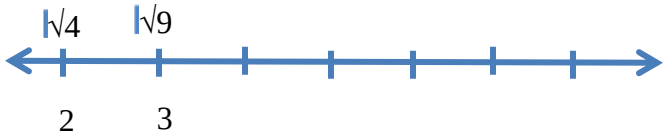
App: HOODAMATH Use the "Roots of Life" game. Great practice with perfect squares.
Teacher Notes: The teacher can project the game to the board and the class can yell out the answer or each student could load it on their own device. Fun quick game to get the brain pumping!!!

WORK SESSION

Teacher Notes: Read through these two options. Use the one you feel would best fit your students based on the manipulatives you have in your classroom.

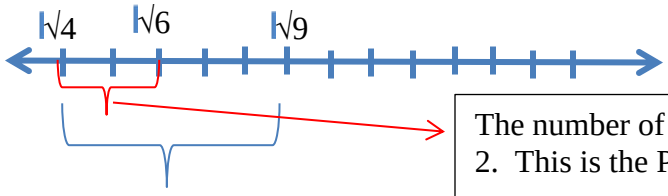
Most of us grew up narrowing down the two perfect squares that the number was between. For example, the √6 is between the √4 and the √9; therefore, the answer is 2 point something. We then used trial and error until we were satisfied. There is an easier way which helps to reinforce the part to whole concept.

NUMBER LINE APPROACH:
We begin by using a number line to estimate the square root of a number (6).



We can easily see the square root of the number 6 is located between the whole numbers of 2 and 3.

Let's take a closer look. This time zooming in and looking at the square root format.



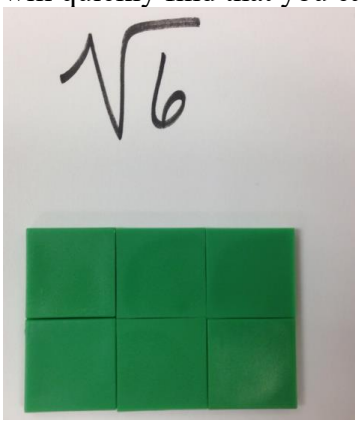
The number of units from √4 and the √6 is 2. This is the PART of the distance.

The number of units from √4 and the √9 is 5. This is the WHOLE distance.

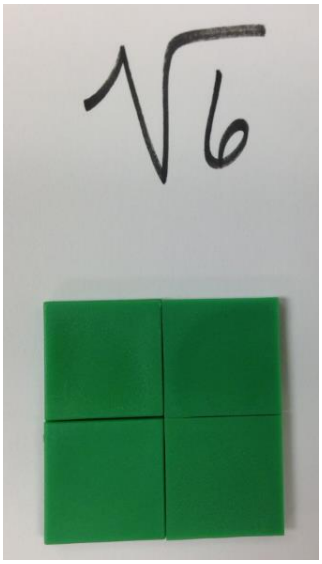
So the √6 is $\frac{2}{5}$ of the way to 3; therefore the estimated value of √6 is $2\frac{2}{5}$ which is about 2.4.

COLOR TILE APPROACH:

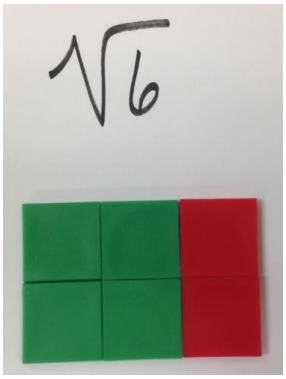
Let us take a different approach to estimating the √6, place 6 colored tiles in an array. Since we are trying to find the square root, the array should be in the form of a square; however, students will quickly find that you can't make a square out of 6 colored tiles.



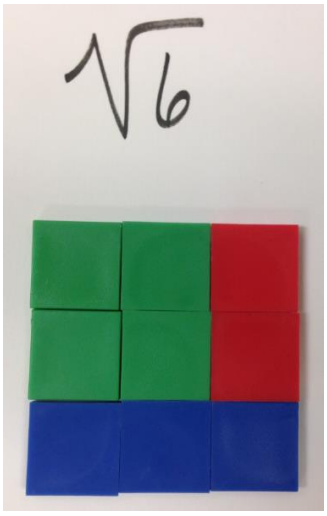
Ask the students, "What is the largest square that can be made with the 6 tiles?" The answer as you can see below is a 2 by 2 which uses 4 tiles. But….



We cannot just toss away the 2 extras after all we are looking for the √6. Again, take note that we know the √6 is 2 point something because √6 is greater than the √4. In order to remind ourselves of this, place the other 2 colored tiles down in a different color. Also remember that we are thinking in terms of part to whole. How many more color tiles do you need to add to get to the next square?



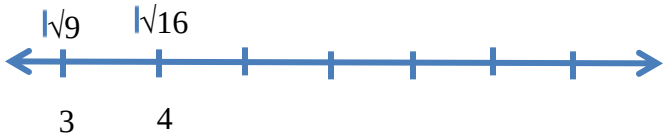
Using yet another color and by adding 3 more colored tiles, the next square has been created. Again focus on the part to whole, we had 2 tiles and it took a total of 5 tiles to get to the next size perfect square of 9. The estimated value of √6 is $2\frac{2}{5}$ which is about 2.4.



Let's look at another example, the √15 is between the √9 and the √16; therefore, the answer is 3 point something. We then used trial and error until we were satisfied. There is an easier way which helps to reinforce the part to whole concept.

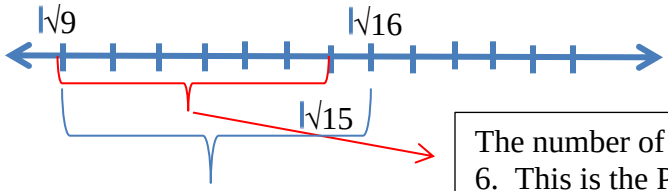
NUMBER LINE APPROACH:

We begin by using a number line to estimate the square root of a number.



We can easily see the number is located between the whole numbers of 3 and 4.

Let's take a closer look. This time zooming in and looking at the square root format.



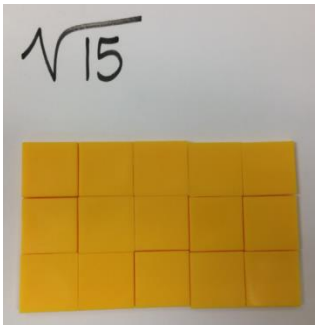
The number of units from √9 and the √15 is 6. This is the PART of the distance.

The number of units from √9 and the √16 is 7. This is the WHOLE distance.

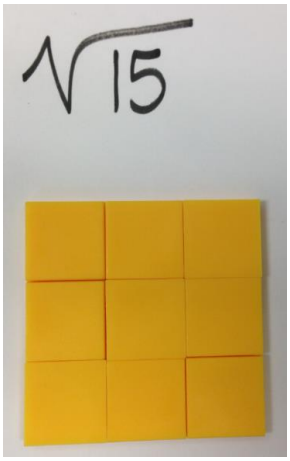
So the √15 is $\frac{6}{7}$ of the way to 4; therefore the estimated value of √15 is $3\frac{6}{7}$ which is about 3.9.

COLOR TILE APPROACH:

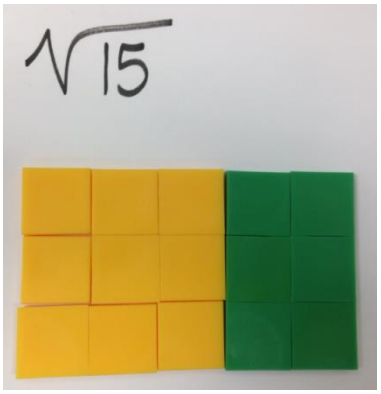
To estimate the √15, place 15 colored tiles in an array. Since we are trying to find the square root, the array should be in the form of a square; however, once again the students will quickly find that you cannot make a square out of 15 colored tiles



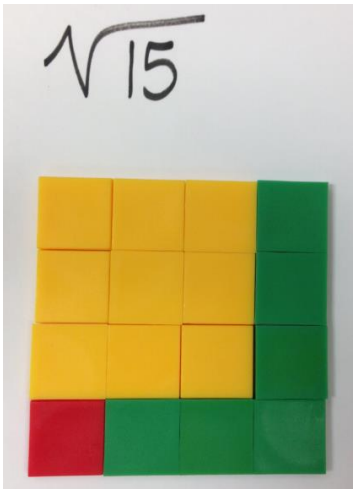
Again ask the students, "What is the largest square that can be made with the 15 tiles?" The answer as you can see below is a 3 by 3 which uses 9 tiles. But….



We cannot just toss away the 6 extras after all we are looking for the √15. Again, take note that we know the √15 is 3 point something because √15 is greater than the √9. In order to remind ourselves of this, place the other 6 colored tiles down in a different color. Also remember that we are thinking in terms of part to whole. How many more color tiles do you need to add to get to the next square?



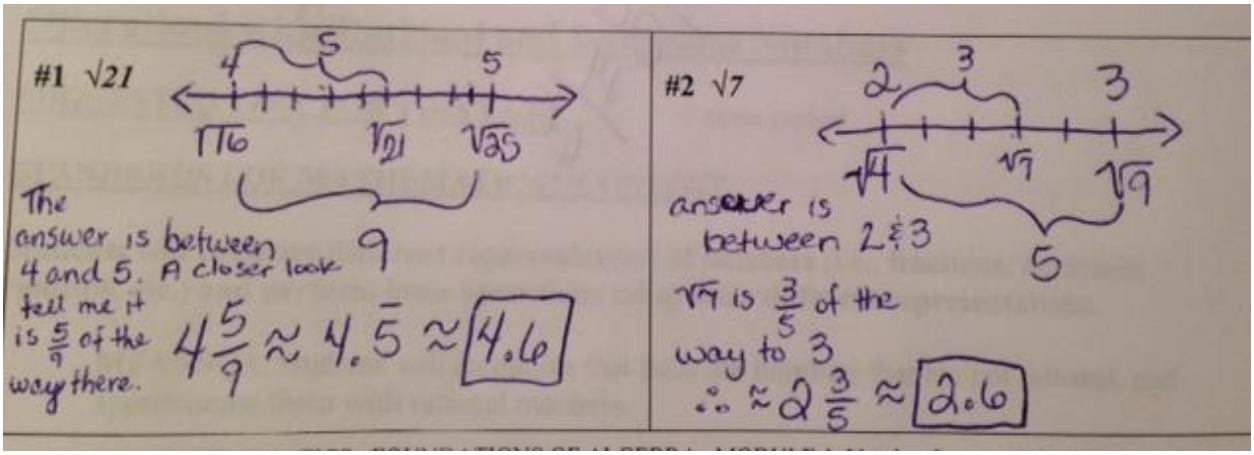
Using yet another color and by adding just 1 more colored tile, the next square has been created. Again focus on the part to whole, we had 6 tiles and it took a total of 7 tiles to get to the next size perfect square of 16. The estimated value of $\sqrt{15}$ is $3\frac{6}{7}$ which about 3.9 .

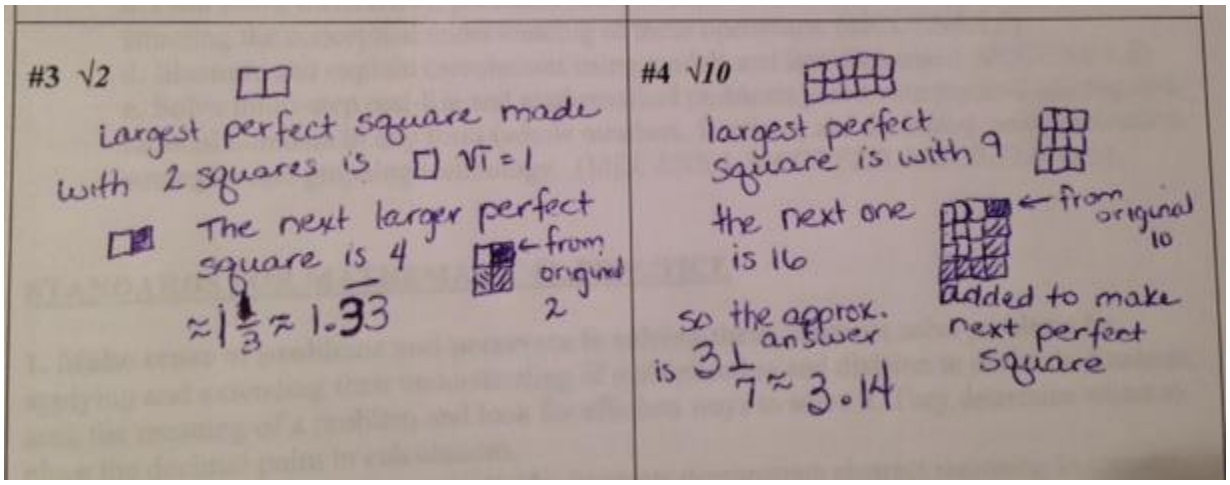


After working a few examples with the class, have the students work in pair to model estimating a few square roots of their own. Working with a partner, they both work the same problem and then share the results with each other. The students should continue this method as the teacher walks around the room checking progress.

Teacher Edition: ESTIMATING THE SQUARE ROOT OF A NUMBER

Directions: Find the estimate of the square root of a number. Draw a model to illustrate your findings.





Student Edition: ESTIMATING THE SQUARE ROOT OF A NUMBER

Directions: Find the estimate of the square root of a number. Draw a model to illustrate your findings.

#1 $\sqrt{21}$	**#2** $\sqrt{7}$
#3 $\sqrt{2}$	**#4** $\sqrt{10}$

CLOSING/SUMMARIZER

Journal – Estimate the $\sqrt{27}$. Use a model to explain your estimation.

Decimal Approximation of Roots

http://www.openmiddle.com/decimal-approximations-of-roots/#prettyPhoto
Source: Bryan Anderson

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ3. Students will recognize that there are numbers that are not rational, and approximate them with rational numbers.
a. Find an estimated decimal expansion of an irrational number locating the approximations on a number line. For example, for $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue this pattern in order to obtain better approximations. (MGSE8.NS.1,2)
b. Explain the results of adding and multiplying with rational and irrational numbers. (MGSE9-12.N.RN.3)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)

Common Misconceptions:

Students may only focus on the idea that square roots are related to multiplication, so they may just divide by the number in the radical. For example, students may see $\sqrt{2}$ *and think "what can I do to a number to get a 2 in the square root?" Often, students will just divide by the number, in this case 2, and get* $\sqrt{2} = 1$. *To address this misconception, students need to experience square numbers as actual squares, built and/or drawn, and then find the root of the square to be the length of a side of that square.*
The symbol we use for square, cube, or any other root is just that, a symbol to represent how two quantities are related, not a concept in itself. In order to help students understand this relationship, give them the following pattern:

1, 4, 9, 16, . .

Ask students to find the next 3 numbers in the sequence. When they find the next three, ask them to explain how they determined the next 3 numbers. Many students will likely say that

they found the pattern 1 + 3 = 4, 4 + 5 = 9, 9 + 7 = 16, so they kept going with + 9, + 11, + 13, to get each successive number in the pattern.

Now, ask students to build rectangles for each of the numbers in the pattern using square tiles. Students will find that all of the numbers create squares and from there, students can be guided to the idea that the root of the squares is the length of one side of the square. This also connects back to the original pattern in that the square root is the number of the term in the sequence, i.e., the square root of 1 is 1 - 1st term of the sequence. The square root of 4 is 2 – the second term of the sequence. The square root of 9 is 3 – the third term of the sequence, etc.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.
6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to explain why some square roots are irrational and determine decimal approximations for these roots.

MATERIALS

- One inch grid paper (multi-weight grid lines-see attached)
- Scissors
- Decimal Approximations of Roots Lesson
- Open Middle Recording Sheet

ESSENTIAL QUESTIONS

- How can we determine approximations of square roots that are irrational?
- How do we know if our approximations are reasonable?

Grouping: Individual/Partner

OPENER/ACTIVATOR

Give students the following pattern:

1, 4, 9, 16, . .

Ask them to determine the next three terms in the sequence and justify how they found them (identify a pattern). Most will find the pattern mentioned in the Common Misconceptions section above.

Now, ask students to build rectangles using the number of square tiles indicated by the number in the sequence (i.e. use 4 square tiles to build a rectangle for the number 4).

Discuss the results. Ask students questions about the relationship between the side length and the square. The side length or square root, is the number of the term in the sequence. The square root of 1 is 1 – the first term in the sequence. The square root of 4 is 2 – the second term in the sequence, etc.

BACKGROUND KNOWLEDGE

When students memorize procedures for square roots without making sense of them, misconceptions develop. The opener, above, is a first step in making sure this does not happen. Students need to make sense of the conceptual idea of a square root. Once students conceptualize the idea of a square root, using the square root symbol to communicate their ideas mathematically is all that is needed.

NUMBER TALKS

Several Number Talks strategies can help student build a stronger understanding of square roots. Repeated subtraction, partial quotients, multiplying up and proportional reasoning are all valuable strategies that students can explore through number talks. For more information refer to pgs. 286-299 in *Number Talks.* (Number Talks, 2010, Sherry Parrish)

LESSON DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Part I:
Start this lesson with a whole group discussion regarding square roots. Guide the discussion so that the conceptual understanding is the focus.

Students will work with materials to determine approximations for square roots of numbers that are not perfect squares.

Lesson Directions
Give pairs of students a sheet of the multi-weight grid paper (copy provided on following pages) and ask them to cut out two large squares (1"). Ask students to discuss how the squares are divided, then share with the group. Each square is divided into tenths (columns/rows) and hundredths (there are 100 small squares within each large square.

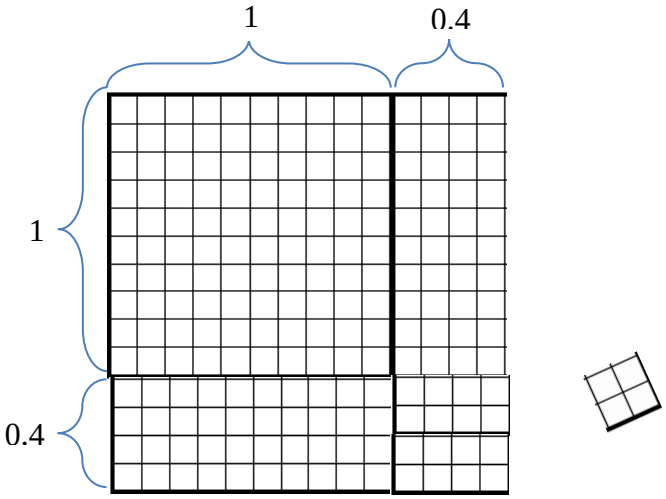
The lesson for the students is to approximate the square root of a square that has an area of two square units. Begin by asking students what this means. This may be the time for students to think about what the square root of a square with an area of 4 square units would be, then pose the lesson question again.

Once students have the idea that they need to find out what the a square with an area of 2 square units might look like, then determine the approximate length of the side of that square to find the square root, they can begin the lesson.

Begin by estimating.
The following questions should be discussed by students before proceeding to the lesson. Have pairs share their reasoning for each.

- Which square number is 2 close to? (1).
- Will the side of the square (the square root) of 2 be more than the square root of 1 or less?

This part may take a full class period. Many students may arrive at a different approximations. Some with different approximations. One possible student solution is shown below.



Leftover hundredths need to be shared with the whole square.

Students' approximations should be close to 1.4 and a little bit more (1.41).

Try this lesson again with another number close to a perfect square, such as 5.

Begin by estimating.

The following questions should be discussed by students before proceeding to the lesson. Have pairs share their reasoning for each.

- Which square number is 5 close to? (4).
- Will the side of the square (the square root) of 5 be more than the square root of 4 or less?

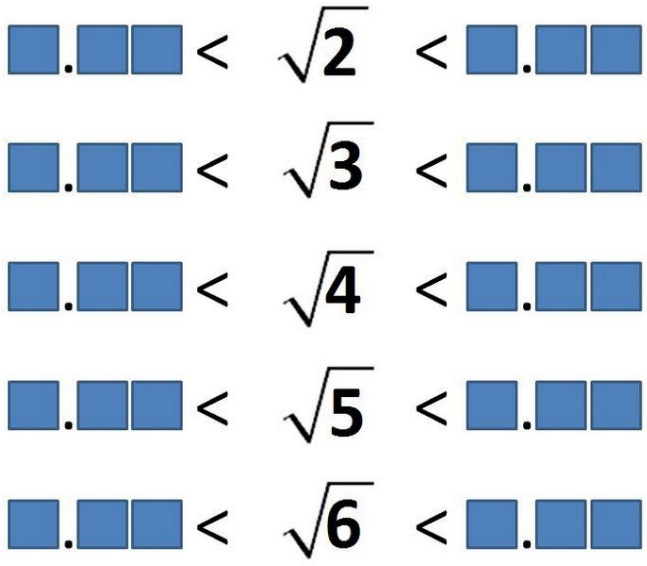
Allow students to cut their squares and make approximations based on the cuts.

Tip: As students discover their squares, have them keep a record of them in their notebook/journal as evidence of their progression of understanding square roots and the approximations.

Part II
Students will complete an Open Middle Problem from www.openmiddle.com. A copy of the **student worksheets** are included in this lesson.

Lesson Directions
Students will use their understandings of approximating square roots of decimals to complete the following problem:

Directions: Using only numbers 1-6 (and only once per inequality), fill in the boxes to create a true statement with the smallest possible interval:



Give students time to make sense of the problem and discuss what it means with their partners, then share with the class. Making sense of the problem is a huge part of any problem solving lesson, and students need to learn how to do this by sharing their ideas during the learning process before being asked to do this independently.

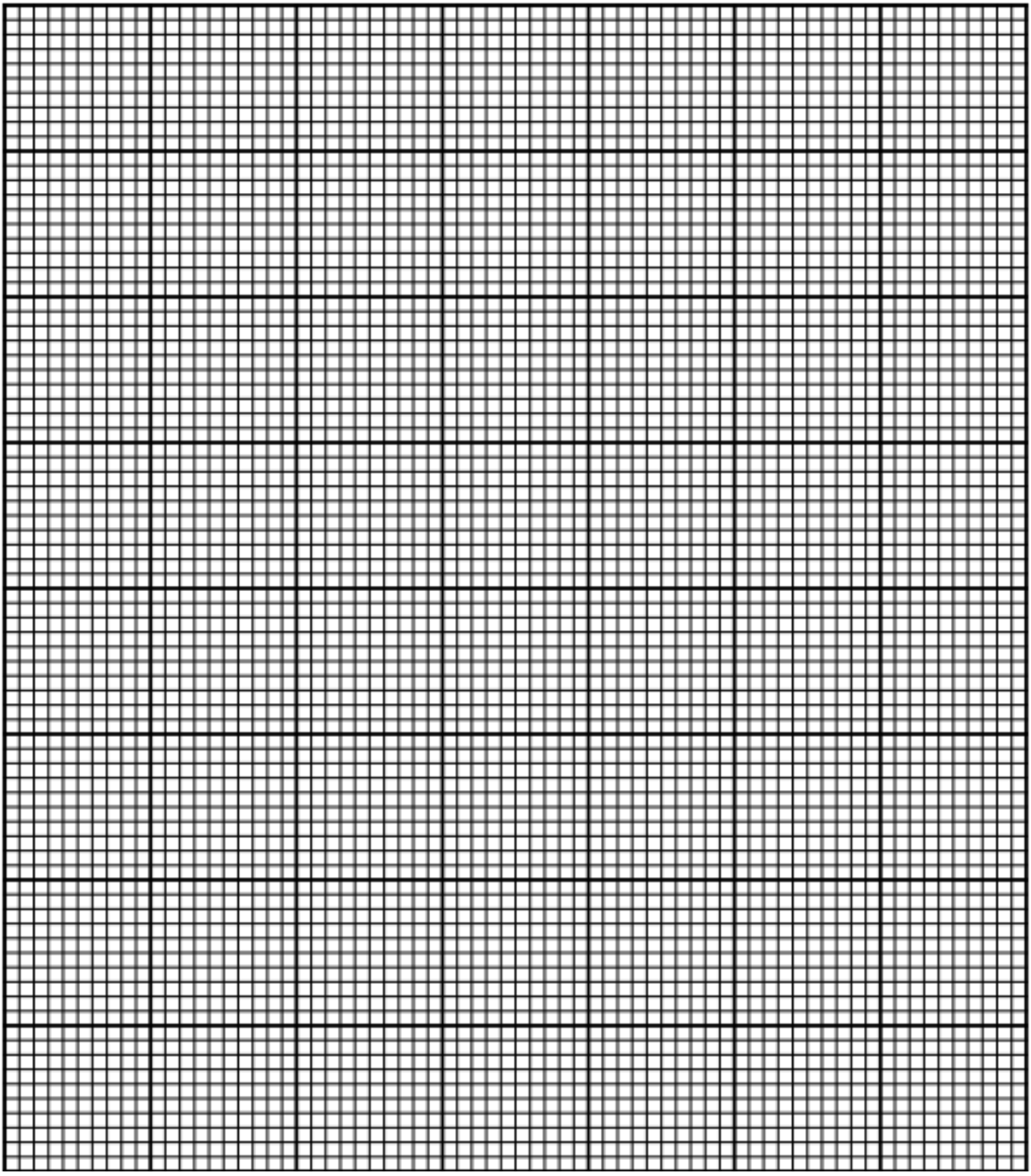
FORMATIVE ASSESSMENT QUESTIONS

- What place value will impact the interval the most?
- How can you decide which numbers to place in the units (ones) place?
- For each of the problems, which approximation is closest? How can you tell?

DIFFERENTIATION

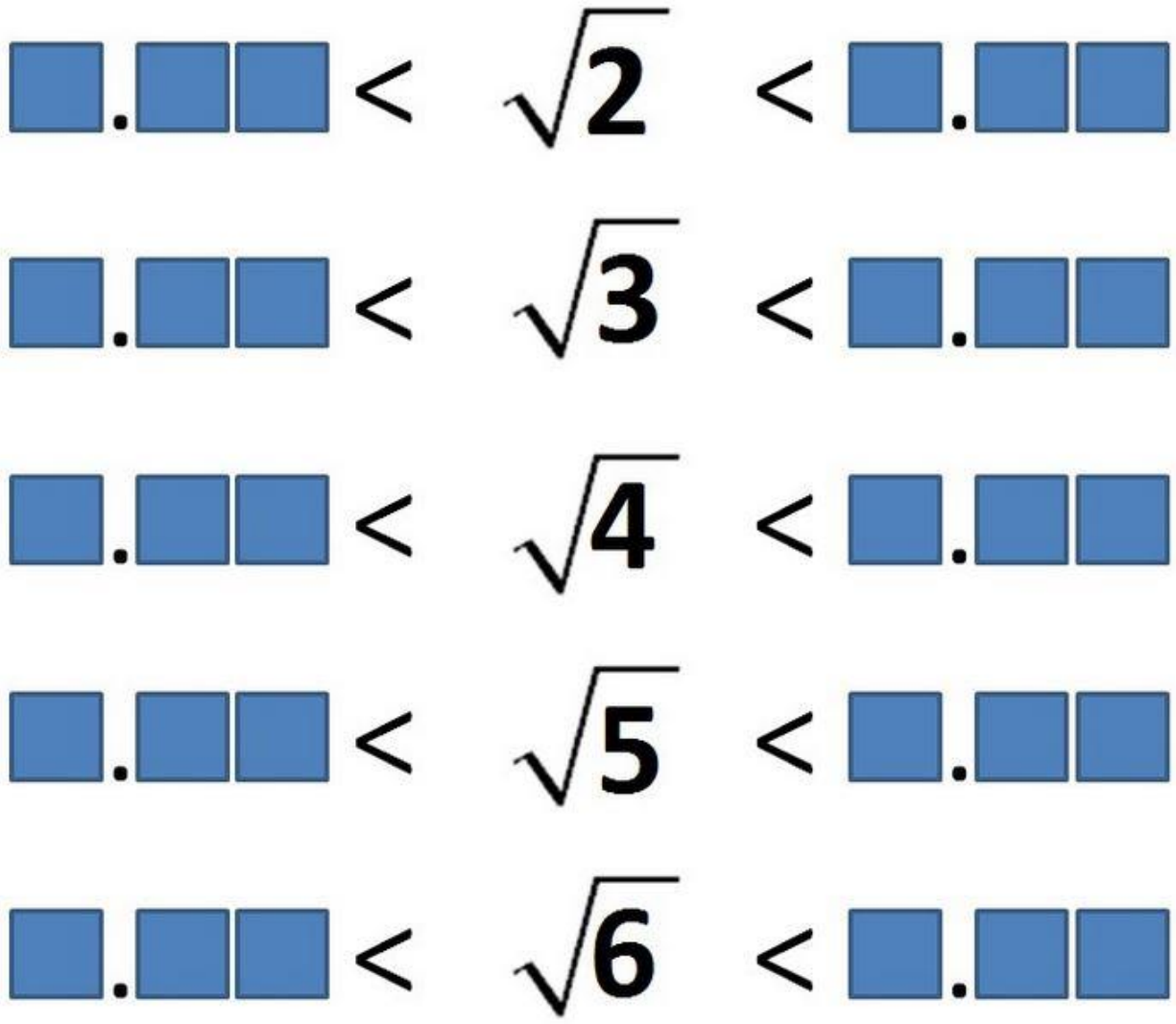
Intervention

- http://nzmaths.co.nz/resource/square-and-cube-roots
- For extra help with square roots, please open the hyperlink Intervention Table.



Decimal Approximations of Roots

Directions: Using only numbers 1-6 (and only once per inequality), fill in the boxes to create a true statement with the smallest possible interval:



Student Worksheet

Name: _____ Period: _____ Date: _____

First attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Second attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Third attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Fourth attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Fifth attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Sixth attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Operations with Rational and Irrational Numbers

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ3. Students will recognize that there are numbers that are not rational, and approximate them with rational numbers.
b. Explain the results of adding and multiplying with rational and irrational numbers. (MGSE9-12.N.RN.3)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)
e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MGSE7.NS.3, MGSE7.EE.3, MGSE9-12.N.Q.3)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning to connect decimal quantities to fractions, and to compare relative values of decimal numbers. Students round decimal numbers using place value concepts.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations and placement of the decimal point, based upon models and rules that generate patterns. They explain their thinking to others and respond to others' thinking.
4. Model with mathematics. Students use base ten blocks, drawings, number lines, and equations to represent decimal place value, multiplication and division. They determine which models are most efficient for solving problems.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.

6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and properties of operations to fluently multiply and divide decimals.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to identify the results of a problem as rational or irrational before working the problem.

MATERIALS

- Internet Access

ESSENTIAL QUESTIONS

- How do you know what type of number will be the result from a given situation?

Grouping: Individual/Partner

OPENER/ACTIVATOR

Identify 3 rational numbers between 6 and 7. Identify 3 irrational numbers between 6 and 7

WORK SESSION

Teacher Notes: Have the students complete the activity from Illustrative Mathematics: Operations with Rational and Irrational Numbers which can be found at this link: https://www.illustrativemathematics.org/content-standards/LESSONs/690

CLOSING/SUMMARIZER

Have the students complete the activity from Illustrative Mathematics: Operations with Rational and Irrational Numbers which can be found at this link:
https://www.illustrativemathematics.org/content-standards/HSN/RN/B/3/LESSONs/1817

Debits and Credits

SUGGESTED TIME FOR THIS LESSON:

50-60 minutes
Exact timings will depend on the needs of your class.

STANDARDS FOR MATHEMATICAL CONTENT

Students will compare different representations of numbers (i.e., fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ2. Students will conceptualize positive and negative numbers (including decimals and fractions).
a. Explain the meaning of zero. (MGSE6.NS.5)
b. Represent numbers on a number line. (MGSE6.NS.5,6)
c. Explain meanings of real numbers in a real world context. (MGSE6.NS.5)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.
a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)
e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MGSE7.NS.3, MGSE7.EE.3, MGSE9-12.N.Q.3)

Common Misconceptions:
Visual representations may be helpful as students begin this work. If they do not have a visual to illustrate what is happening when they are adding and subtracting integers, they will get lost in the symbols and will not know how to combine the absolute value of the integers.
Students may struggle with the vocabulary since there are many different ways to debit your account.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students solve problems by applying and extending their understanding of multiplication and division to decimals. Students seek the meaning of a problem and look for efficient ways to solve it. They determine where to place the decimal point in calculations.
5. Use appropriate tools strategically. Students select and use tools such as graph or grid paper, base ten blocks, and number lines to accurately solve multiplication and division problems with decimals.

6. Attend to precision. Students use clear and precise language, (math talk) in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to decimal place value and use decimal points correctly.
7. Look for and make use of structure. Students use properties of operations as strategies to multiply and divide with decimals. Students utilize patterns in place value and powers of ten to correctly place the decimal point.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to use a check register to record debits and credits and calculate a running total balance.

MATERIALS

- Student Ledger Sheet

ESSENTIAL QUESTIONS

- When keeping a ledger, why are the rules for adding and subtracting rational numbers important?

Grouping: Partner/Individual

OPENER/ACTIVATOR

Discuss and define:

Suppose you have been given a checkbook. Your checkbook has a ledger for you to record your transactions. There are two types of transactions that may take place, (1) deposits (money placed in the account) and (2) debits/ payments (money out of the account). The difference between debits/payments and deposits tells the value of the account. If there are more credits than debits, the account is positive, or "in the black." If there are more debits than credits, the account is in debt, shows a negative cash value, or is "in the red."

Vocabulary key –
Transaction = debit or credit from an account
Debit (withdrawal) = Check or debit card usage written out of the checking account
Credit= Deposit of money put in the account

Suggestion- Bring an actual check register to show and demonstrate the described situations in the lesson.

WORK SESSION
LESSON DESCRIPTION

Situation #1:

Use the ledger to record the information and answer the questions.
Note: On August 12, your beginning balance is $0.00 ***(This will be the first line in the ledger.)***
1. On August 16, you receive a check from your Grandmother for $40 for your birthday.
2. On August 16, you receive a check from your Parents for $100 for your birthday.
3. On August 17, you purchase a pair of pants from Old Navy for $23.42.
4. On August 18, you find $5.19 in change during the day.
5. On August 19, you purchase socks from Wal-Mart for $12.76.

Comment
The ledger below shows the transactions.

DATE	TRANSACTION	PAYMENT (-)	DEPOSIT (+)	BALANCE
8.12	*Beginning Balance*			*0.00*
8.16	*Money from Grandma*		*$40.00*	*$40.00*
8.16	*Money from Parents*		*$100.00*	*$140.00*
8.17	*Old Navy*	*$23.42*		*$116.58*
8.18	*Found Change*		*$5.19*	*$121.77*
8.19	*Wal-Mart*	*$12.76*		*$109.01*

A. What is your balance after five transactions?
Solution
$109.01

B. How much money did you deposit (show as a positive value)?
Solution
$145.19

C. How much money did you pay or withdraw (show as a negative value)?
Solution
$-36.18

Situation #2:
Use the ledger to record the information and answer the questions.
Note: On May 5, your beginning balance is $8.00
1. On May 6, you spent $4.38 on a gallon of ice cream at Marty's Ice Cream Parlor.
2. On May 7, you spent $3.37 on crackers, a candy bar, and a coke from Circle H convenience store.
3. On May 8, you received $10 for cutting the neighbor's grass.
4. On May 8, you spent $14.80 on a downloaded book for your Kindle.

Comment
The ledger below shows the transactions.

DATE	TRANSACTION	PAYMENT (-)	DEPOSIT (+)	BALANCE
5.5	*Beginning Balance*			*$8.00*
5.6	*Marty's Ice Cream Parlor*	*$4.38*		*$3.62*
5.7	*Circle H Convenience*	*$3.37*		*$.25*
5.8	*Cutting Grass*		*$10.00*	*$10.25*
5.8	*Book for Kindle*	*$14.80*		*$-4.55*

A. What is your balance after four transactions?
Solution
There is no money left. There is a negative balance of $4.55.

B. How much money did you deposit (show as a positive value)?
Solution
$10.00

C. How much money did you pay or withdraw (show as a negative value)?
Solution
$-22.55
D. Can you really afford to spend $14.80 on a book for your Kindle? If not, how much money do you need to earn to have an account balance of $0?

Solution
No. I need to deposit another $4.55 to have an account balance of $0.

Situation #3:
Use the ledger to record the information and answer the questions.
Note: On July 4, your beginning balance is (-$40).
Show, using at least eight transactions, a way you can have an ending account balance of more than $145. You must include debit and credit amounts that have cents in at least five of your transactions. Your ledger must show both credits and debits. Be sure to fill out the ledger as you go.

Comment

Answers will vary. An example is given below.

DATE	TRANSACTION	PAYMENT (-)	DEPOSIT (+)	BALANCE
7.4	*Beginning balance*			*$-40.00*
7.5	*Allowance*		*$20.50*	*$-19.50*
7.12	*Allowance*		*$20.50*	*$1.00*
7.14	*Cutting Grass*		*$10.00*	*$11.00*
7.15	*Yard Sale: Video Game*	*$8.25*		*$2.75*
7.17	*Birthday money*		*$100.00*	*$102.75*
7.19	*Wal-Mart*	*$8.43*		*$94.32*
7.20	*Allowance*		*$20.50*	*$114.82*
7.27	*Allowance*		*$20.50*	*$135.32*
7.31	*Money for Extra Chore*		*$15.50*	*$150.82*

DIFFERENTIATION

Extensions:
Situation #3 could be used as an extension activity. Also, you could extend the assignment to include a monthly budget. Use newspapers or technology so students find an apartment they can afford, a grocery budget, entertainment, and within a given income (e.g. $1000 a month).

Intervention:
Rewrite: The standards associated with this task are addressed by completing situations 1 and 2. For extra help with integer operations, please open the hyperlink Intervention Table.

Student Edition Performance Lesson: Debits and Credits

Suppose you have been given a checkbook. Your checkbook has a ledger for you to record your transactions. There are two types of transactions that may take place, (1) deposits (money placed in the account) and (2) debits/ payments (money –which you spend and it comes out of the account). The difference between debits and the deposits tells the value of the account. If there are more credits than debits, the account is positive, or "in the black". "in the black." If there are more debits than credits, the account is in debt, shows a negative cash value, or is "in the red."

Vocabulary key –
Transaction = debit or credit from an account
Debit (withdrawal) = Check or debit card usage written out of the checking account
Credit= Deposit of money put in the account

Situation #1:
Use the ledger to record the information and answer the questions.
Note: On August 12, your beginning balance is $0.00
1. On August 16, you receive a check from your Grandmother for $40 for your birthday.
2. On August 16, you receive a check from your Parents for $100 for your birthday.
3. On August 17, you purchase a pair of pants from Old Navy for $23.42.
4. On August 18, you find $5.19 in change during the day.
5. On August 19, you purchase socks from Wal-Mart for $12.76.

DATE	TRANSACTION	PAYMENT (-)	DEPOSIT (+)	BALANCE
8/16	Beginning balance			$0.00

A. What is your balance after five transactions?

B. How much money did you deposit (show as a positive value)?

C. How much money did you pay or withdraw (show as a negative value)?

Situation #2:

Use the ledger to record the information and answer the questions.
Note: On May 5, your beginning balance is $8.00

1. On May 6, you spent $4.38 on a gallon of ice cream at Marty's Ice Cream Parlor.
2. On May 7, you spent $3.37 on crackers, a candy bar, and a coke from Circle H convenience store.
3. On May 8, you received $10 for cutting the neighbor's grass.
4. On May 8, you spent $14.80 on a downloaded book for your Kindle.

DATE	TRANSACTION	PAYMENT (-)	DEPOSIT (+)	BALANCE

A. What is your balance after four transactions?

B. How much money did you deposit (show as a positive value)?

C. How much money did you pay or withdraw (show as a negative value)?

D. Can you really afford to spend $14.80 on a book for your Kindle? If not, how much money do you need to earn to have an account balance of $0?

Situation #3:

Use the ledger to record the information and answer the questions.
Note: On July 4, your beginning balance is (-$40).

Requirements:

- Use at least eight transactions, four of which are debits and four are credits.
- You must have an ending balance of $145.
- You must include debit and credit amounts that have cents in at least five of your transactions.

Be sure to fill out the ledger as you go.

DATE	TRANSACTION	PAYMENT (-)	DEPOSIT (+)	BALANCE

CLOSING/SUMMARIZER

Journal: George had $230 in his checking account. His car payment is due for $250. After he makes that payment, what is the balance in his account? What might be some of the consequences?