

# Georgia Standards of Excellence Curriculum Frameworks

# **Mathematics**

GSE Foundations of Algebra Module 2: Arithmetic to Algebra



Richard Woods, Georgia's School Superintendent "Educating Georgia's Future"

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Georgia Standards of Excellence Framework GSE Arithmetic to Algebra • Module 2

# Module 2: Arithmetic to Algebra

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# FOUNDATIONS OF ALGEBRA REVISION SUMMARY

The Foundations of Algebra course has been revised based on feedback from teachers across the state. The following are changes made during the current revision cycle:

- Each module assessment has been revised to address alignment to module content, reading demand within the questions, and accessibility to the assessments by Foundations of Algebra teachers.
- All module assessments, as well as the pre- and posttest for the course, will now be available in GOFAR at the teacher level along with a more robust teacher's edition featuring commentary along with the assessment items.
- All modules now contain "Quick Checks" that will provide information on mastery of the content at pivotal points in the module. Both teacher and student versions of the "Quick Checks" will be accessible within the module.
- A "Materials List" can be found immediately after this page in each module. The list provides teachers with materials that are needed for each lesson in that module.
- A complete professional learning series with episodes devoted to the "big ideas" of each module and strategies for effective use of manipulatives will be featured on the Math Resources and Professional Learning page at <a href="https://www.gadoe.org/Curriculum-Instruction-and-Assessment/Curriculum-and-Instruction/Pages/Mathematics.aspx">https://www.gadoe.org/Curriculum-Instruction/Pages/Mathematics.aspx</a>.

Additional support such as Module Analysis Tables may be found on the Foundations of Algebra page on the High School Math Wiki at <u>http://ccgpsmathematics9-</u>

<u>10.wikispaces.com/Foundations+of+Algebra</u>. This Module Analysis Table is NOT designed to be followed as a "to do list" but merely as ideas based on feedback from teachers of the course and professional learning that has been provided within school systems across Georgia.

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# MATERIALS LIST

Lesson	Materials
1. Arithmetic to Algebra	NA
2. Olympic Cola Display	<ul> <li>Act 1 picture -Olympic Cola Display</li> <li>Pictorial representations of the display</li> <li>Student recording sheet</li> </ul>
3. Distributing Using Area	<ul> <li>Student activity sheet</li> <li>Optional: Colored Sheets of paper cut into rectangles. These can be used to introduce the concepts found in this lesson and to create models of the rectangles as needed.</li> </ul>
4. Triangles and Quadrilaterals	<ul> <li>Student handout for <u>www.visualpatterns.org</u> activator</li> <li>Cut out triangles and quadrilaterals on template</li> <li>Envelopes</li> <li>Student activity sheet</li> <li>Match up cards for closing activity</li> <li>Template for Like Terms closing activity</li> </ul>
5. Tiling Lesson	<ul> <li>patty paper units for tiling</li> <li>(Teacher) 3 unit × 2 unit rectangle</li> <li>(Students) 5 large mystery rectangles lettered A- E (1 of each size per group)</li> <li>Student activity sheet</li> </ul>
<ol> <li>Conjectures about Properties</li> </ol>	<ul> <li>Student activity sheet</li> <li>Optional: manipulatives to show grouping (put 12 counters into groups of zero)</li> </ul>
7. Quick Check I	• Student sheet
8. Visual Patterns	<ul> <li>Various manipulatives such as two color counters</li> <li>Color tiles</li> <li>Connecting cubes</li> <li>Visual Patterns Handout</li> </ul>
9. Translating Math	• Sticky notes may be offered as a way to build tape diagrams (bar models)

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10. Exploring Expressions	<ul> <li>Optional: Personal white boards (or sheet protectors)</li> <li>Student lesson pages</li> <li>Marbles and a bag (if using the differentiation activity)</li> </ul>
11. A Few Folds	<ul> <li>Student activity sheet for each student/pair of students/or small group</li> <li>Paper for folding activity in Part 1</li> </ul>
12. Bacterial Growth	• Student activity sheet
13. Excursions with Exponents	• Student handout/note taking guide
14. Squares, Area, Cubes, Volume, RootsConnected?	<ul> <li>One box of Cheez-Its per team (algebra tiles or other squares may be substituted)</li> <li>One box of sugar cubes per team (average 200 cubes per one pound box) (algebra cubes, linking cubes, or other cubes may be substituted)</li> <li>Graphic Organizer for Squares</li> <li>Graphic Organizer for Cubes</li> <li>2 Large number lines (using bulletin board paper) to display in the class; one for square roots and one for cube roots</li> </ul>
15. Quick Check II	Student sheet
16. What's the "Hype" about Pythagoras?	<ul> <li>Student handout</li> <li>Calculators</li> <li>Sticky notes</li> <li>Link or download version of Robert Kaplinksy's "How Can We Correct the Scarecrow?" video <i>http://robertkaplinsky.com/work/wizard-of-oz</i></li> </ul>
17. Fabulous Formulas	<ul><li>Formula sheet</li><li>Application problems</li><li>Calculators</li></ul>
18. The Algebra of Magic	<ul> <li>Computer and projector or students with personal technology (optional)</li> <li>Directions for mathematical magic tricks</li> <li>Counters</li> <li>Sticky notes or blank pieces of paper-all the same size</li> </ul>

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# **OVERVIEW**

As the journey into Module 2 of Foundations of Algebra begins, the teacher is charged with a pretty daunting task: create a connection from arithmetic skills to operations in algebra. The teacher will build bridges between the "known" world of numbers and the "unknown" world of variables. Students, however, do not come to class as "blank slates" ready to absorb all that is attempted in order to make connections. Instead, many of them arrive as wounded survivors of a battle they have been fighting for over eight years. This battle has been against MATH. Many of them feel that they "can't do math" and that belief has been validated by peers ("How can you not understand that problem?.....It's easy"), parents ("I was never good at math either...We just don't have the math GENE"), teachers ("I have explained that to you a hundred times...what don't you get?"), and society ("Some people just aren't MATH people"). Research by such leaders as Jo Boaler from Stanford University has dispelled all of the "excuses" above to prove that ALL students CAN learn math.

In Module 2 of Foundations of Algebra, students will draw conclusions from arithmetic's focus on computation with specific numbers to build generalizations about properties that can be generalized to all sets of numbers. Students will look beyond individual problems to see patterns in mathematical relationships. They will apply properties of operations with emphasis on the commutative and distributive properties as they build and explore equivalent expressions with various representations. A critical emphasis on the area model for multiplication and the distributive property will provide a concrete connection to the multiplication of monomial and polynomial expressions. Students will experiment with composition and decomposition of numbers as they perform operations with variable expression. Students will connect concepts from Module 1's focus on operations with numbers to algebraic operations.

In Module 2 of Foundations of Algebra, students will also interpret and apply the properties of exponents. Students will explore and evaluate formulas involving exponents with attention to units of measure. Using concrete models, students will build area and volume models to explore square roots and cube roots. The module will conclude with specific applications of the Pythagorean Theorem.

This module supplies multiple opportunities for students to explore and experience behind the standards amalgamated for its development. Extra practice and review opportunities along with sample released assessment items are provided for course instruction. Teachers should select the materials most appropriate for his/her students as they journey toward the connection between what is mathematically true for specific cases (arithmetic) to what can be generalized for multiple situations (algebra). Module 2 also sets the essential groundwork for future modules on proportional reasoning, equations and inequalities, and functions.

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In this module, students will formally examine the connections between operations in arithmetic from elementary school to algebraic operations introduced in middle school in preparation for high school standards. The standards for mathematical content listed below will serve as the connection and focus for Module 2 of Foundations of Algebra.

### STANDARDS FOR MATHEMATICAL CONTENT

### Students will extend arithmetic operations to algebraic modeling.

# MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.

a. Apply properties of operations emphasizing when the commutative property applies. (MGSE7.EE.1)

b. Use area models to represent the distributive property and develop understandings of addition and multiplication (all positive rational numbers should be included in the models). (MGSE3.MD.7)

c. Model numerical expressions (arrays) leading to the modeling of algebraic expressions. (MGSE7.EE.1,2; MGSE9-12.A.SSE.1,3)

d. Add, subtract, and multiply algebraic expressions. (MGSE6.EE.3, MGSE6.EE.4, MC7.EE.1, MGSE9-12.A.SSE.3)

e. Generate equivalent expressions using properties of operations and understand various representations within context. For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between "3 more" and "3 times". (MGSE4.0A.2; MGSE6.EE.3, MGSE7.EE.1, 2, MGSE9-12.A.SSE.3)

f. Evaluate formulas at specific values for variables. For example, use formulas such as

A = 1 x w and find the area given the values for the length and width. (MGSE6.EE.2)

### MFAAA2. Students will interpret and use the properties of exponents.

a. Substitute numeric values into formulas containing exponents, interpreting units consistently. (MGSE6.EE.2, MGSE9-12.N.Q.1, MGSE9-12.A.SSE.1, MGSE9-12.N.RN.2)

b. Use properties of integer exponents to find equivalent numerical expressions. For example,  $3^2 x \ 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$ . (MGSE8.EE.1)

c. Evaluate square roots of perfect squares and cube roots of perfect cubes (MGSE8.EE.2)

d. Use square root and cube root symbols to represent solutions to equations of the form  $x^2 = p$  and  $x^3 = p$ , where p is a positive rational number. (MGSE8.EE.2)

e. Use the Pythagorean Theorem to solve triangles based on real-world contexts (Limit to finding the hypotenuse given two legs). (MGSE8.G.7)

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### STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

**1. Make sense of problems and persevere in solving them.** High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

**2. Reason abstractly and quantitatively.** High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

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**3.** Construct viable arguments and critique the reasoning of others. High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

**4. Model with mathematics.** High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

**5.** Use appropriate tools strategically. High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

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**6. Attend to precision.** High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure. By high school, students look closely to discern a pattern or structure. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8. Look for and express regularity in repeated reasoning. High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding (x - 1)(x + 1),  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

# Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. **Students who do not have an understanding of a topic may rely on procedures too heavily.** Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. **In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.** 

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for

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Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

### \*\*\*Mathematical Practices 1 and 6 should be evident in EVERY lesson\*\*\*

# ENDURING UNDERSTANDINGS

- The generating of equivalent, linear expressions with rational coefficients using the properties of operations relates to solving linear equation.
- The rewriting of expressions in different forms in a problem context leads to the understanding that the expressions are equivalent.
- The use of area models develops the relationship between addition and multiplication and the commutative property.
- The process of finding the area of a rectangle uncovers that the dimensions of the rectangle represent the factors in a multiplication problem.
- The realization that variables are used to represent numbers helps in any type of mathematical problem
- The using of the Commutative, Associative, Distributive, Identity, and Inverse Properties helps add, subtract, factor, and expand linear expressions with rational coefficients.
- The rewriting of an expression in a different form does not change the value of the expression.
- The discovery that formulas can represent set relationships between quantities represented by variables assists in evaluating specific values for these variables.
- The applying of the properties of integer exponents helps to generate equivalent numerical expressions.
- The reviewing of square roots and cube roots uncovers that these can be rational or irrational numbers.
- The realization that there are rational approximations of irrational numbers assists in comparing the size of irrational numbers, in locating them approximately on a number line, and in estimating the value of expressions.
- The Pythagorean Theorem is used both algebraically and geometrically in order to solve problems involving right triangles.
- The square root of a number is the inverse operation of squaring that number.
- The cube root of a number is the inverse operation of cubing that number.
- The ability to solve and explain real-life mathematical problems using numerical and algebraic expressions is important in the preparation for high school algebra.

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### **ESSENTIAL QUESTIONS**

- How can I apply properties of operations to generate equivalent expressions?
- How can I use the area model to represent the distributive property?
- How can I combine algebraic expressions using addition, subtraction, and multiplication?
- How can I translate verbal expressions into mathematical expressions given various contexts?
- How can I evaluate formulas at specific values for the variables contained within the formulas?
- How can I interpret and use the properties of exponents in numerical expressions?
- How can I use the symbols for cube and square roots to represent solutions to cube or square root equations?
- How can I apply the Pythagorean Theorem to find the hypotenuse of a right triangle given the two legs?

### SELECTED TERMS AND SYMBOLS

The following list of terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The terms below are for teacher reference only and are not to be memorized by the students. Teachers should present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

Equivalent expressions Distributive property Algebraic expression Numeric expression Area Model Commutative Property Associative Property **Identity Properties Inverse Operations** Variable Formula Square Number Square Root Pythagorean Theorem Hypotenuse Cubic Number Cube Root Rational Number Irrational Number Exponent

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The websites below are interactive and include a math glossary suitable for high school children. Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.

### http://www.amathsdictionaryforkids.com/

This web site has activities to help students more fully understand and retain new vocabulary.

### http://intermath.coe.uga.edu/dictnary/homepg.asp

Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

# **INTERNET INVESTIGATIONS FOR TEACHERS**

<u>http://bit.ly/1JidMqn</u> The Progressions for the State Standards in Mathematics (draft) for Expressions and Equations shows how the study of expressions and equations progress from grades 6 to 8. The progression of study and understanding that give rise to students solving real-life and mathematical problems using numerical and algebraic expressions and equations is presented in this document.

### **Internet Based Virtual Manipulatives**

For lessons that mention manipulatives that are not available in your classroom, consider using one or more of the virtual manipulatives sites below:

Math Playground http://www.mathplayground.com/math\_manipulatives.html

Think Central <a href="http://wwwk6.thinkcentral.com/content/hsp/math/mathinfocus/common/itools\_int\_97805476738">http://wwwk6.thinkcentral.com/content/hsp/math/mathinfocus/common/itools\_int\_97805476738</a>

Glencoe http://www.glencoe.com/sites/common\_assets/mathematics/ebook\_assets/vmf/VMF-Interface.html

Computing Technology for Math Excellence <a href="http://www.ct4me.net/math\_manipulatives\_2.htm">http://www.ct4me.net/math\_manipulatives\_2.htm</a>

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**INTERVENTION TABLE** 

Lesson Name	Name Of Intervention	Snapshot of summary or student I can statement	Book, Page Or link
Olympia Cala	Animal Arrays	I am learning to find other ways to solve problems.	Animal Arrays
Display	Array Games	The game allows students to practice their multiplication skills, and reinforces the 'array' concept of multiplication	Array Game
	Multiplication Smorgasbord	I am learning to solve multiplication problems using a variety of mental strategies	Multiplication Smorgasbord
Distributing and Factoring Using Area	Smiley Face	In this activity, students are encouraged to solve multiplication problems by deriving from known facts, looking for groupings and skip counting. Students are encouraged to explain and share their thinking.	Smiley Face
Conjectures About Properties	Conjectures About Properties	The purpose is to develop the students' deeper understanding of the way numbers behave, to enable them to use everyday language to make a general statement about these behaviors, and to understand the symbolic representation of these 'properties' of numbers and operations.	<u>Conjectures</u> <u>About</u> <u>Properties</u>
Translating Math	Writing Expressions	Apply algebra to the solution of a problem Devise and use problem solving strategies to explore situations mathematically	<u>Writing</u> Expressions
Exploring Expressions	The Algebra of Magic Part 1	Design and use models to solve measuring problems in practical contexts.	<u>The Algebra of</u> <u>Magic Part 1</u>
Squares, Area, Cubes, Volume, Connected?	Cube and Cube Roots	Calculate square and cube roots. Understand that squaring is the inverse of square rooting, and cubing is the inverse of cube rooting.	Cube and Cube Roots
What's the Hype about Pythagoras?	The Pythagorean Relationship	Find lengths of objects using Pythagoras' theorem.	<u>The</u> Pythagorean Relationship
	Pythagoras Plus	Explore Pythagoras' Theorem.	<u>Pythagoras</u> <u>Plus</u>

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SCAFFOLDED INSTRUCTIONAL LESSONS

T NT			
Lesson Name	Lesson Type/ Grouping Strategy	Content Addressed	<u>Standard(s)</u>
Arithmetic to	Review to be used	Strategies to bridge number	
Algebra	throughout unit	sense to algebra	
Olympic Cola	3 Act Task/	Distributive Property of	MFAAA1.b
Display	Individual/Partner and/or	Multiplication	
	Small Group		
Distributing and	Scaffolding Lesson/	Area models to represent	MFAAA1.b.d.e
Factoring Using	Individual/Partner	and discover the distributive	
Area		property	
Triangles and	Learning Lesson/	Expressions and equations	MFAAA1.a.c.d.e.
Quadrilaterals	Partner or Small Group	based on properties of	f
Quantitatorais		triangles and quadrilaterals	1
Tiling Lesson	Learning Lesson/	Area model to find the area	MEAAAlabcf
<u>Imig Lesson</u>	Whole group then Partner	of a rectangle with	WII 7 II II II II.u,0,0,0,1
	or Small Croup	fractional side longths and	
	or Small Group	avpross/simplify the	
		express/simplify the	
		perimeter of rectangles with	
		variable side lengths	
Conjectures	Formative Lesson		MFAAA1.a,c,e
About	Partner/Small Group	Properties of Numbers	
<u>Properties</u>			
Ouick Check I	Mini Assessment Lesson	Equivalent expressions,	MFAAA1.a,b, c,
	Individual	arrays to model expressions	d, e
Visual Patterns	Constructing Lesson	Expressions and formulas	MEAAAlcdef
<u>visual i atterns</u>	Individual/Partner	Expressions and formulas	WII 7 II II II II.e,u,e,i
Translating	Constructing Lasson	Verbal expressions into	
Moth	Individual/Dartner	symbolic representation	MFAAA1.e
	Individudi/1 driner	Simplify Expressions	
Exploring	Constructing Lesson	Properties of Numbers,	MFAAA1.a,c,d,e,
Expressions	Individual/Partner	Applications of Formulas,	f
		Evaluating Expressions	
A Few Folds	Learning Lesson/	Integer exponents	MFAAA2.b
	Partner or Small Group		
Bacterial	Formative Lesson	Integer exponents	MFAAA2.a.b
Growth	Partner/Small Group		<b>,</b> o
Excursions with	Learning Lesson/	Integer exponents	MFAAA2 h
Exponents	Partner or Small Group	meger experience	
Squares Area	Learning Lesson/	Square Roots Area Cube	MFAAA2 c.d
Cubes Volume	Partner or Small Group	Roots Volume	1711 / 1/ 1/ 1/2.0,0
Connected?			
Ouiok Chaok II			ΜΕΛΛΛΙαδα
	Mini Assessment Lesson	Expressions servers resta	f
	Individual	Expressions, square roots	
1			MFAAA2.C. d

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Lesson Name	Lesson Type/ Grouping Strategy	Content Addressed	<u>Standard(s)</u>
What's the	Constructing Lesson	Square Roots, Squares,	MFAAA2.c,d,e
Hype about	Individual/Partner	Pythagorean Theorem	
Pythagoras?			
<b>Fabulous</b>	Learning Lesson/	Formulas	MFAAA1.a,f
<u>Formulas</u>	Partner or Small Group		MFAAA2.a
The Algebra of	3 Act Task/	Simplification Review	MFAAA1
Magic	Individual/Partner and/or		
	Small Group		

The assessment for this module can be found through the Georgia Online Formative Assessment Resource (GOFAR). <u>http://www.gadoe.org/Curriculum-Instruction-and-Assessment/Assessment/Pages/Georgia-Online-Formative-Assessment-Resource.aspx</u> This suggested assessment should be given as the pretest and posttest for this module.

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# **Arithmetic to Algebra**

The following problem sets serve as a bridge between the number sense and quantity focus of Module 1 to the connection between arithmetic and algebra in Module 2. The problem sets are grouped around central strategies and build on patterns of mathematical thought. These sets may be used as warm ups or activators, math burst sets between activities within a class, as exit tickets, or as needed based on your students. Ideas for development are featured as needed prior to a set, and all solutions are provided at the conclusion of the sets. The strategies that promote mathematical understanding should be welcomed. Many problems are provided for each set to allow multiple samples for practice. All problems do not need to be completed at the same time or even within the same lesson. Strategies may be revisited as needed.

These additional resources for consideration as Number/Strategy Talks based on Numeracy Stages from the IKAN screener are provided below. These resources along with many others may be found on the New Zealand Numeracy website at <a href="http://nzmaths.co.nz/resource">http://nzmaths.co.nz/resource</a>.

### **Addition Strategies**

= 52

Adding in Parts: add numbers by splitting them into parts that are easier to combine

Sometimes problems can be done by splitting up one of the numbers so that the other number can be made into a "tidy number".

e.g. 19 + 7Split 7 into 1 + 6 and make 19 into a tidy number by adding 1 19 + 7 = 19 + 1 + 6= 20 + 6= 26e.g. 34 + 18 = 32 + 2 + 18= 32 + 20

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### **Exercise 1**

- 1) Use the strategy of splitting a number into parts to do these additions.
- 2) Do the problems in your head first.
- 3) Check you are correct by writing them down. Show them like the examples above.

1)	28 + 14	(2)	76 + 9	(3)	37 + 15
4)	46 + 17	(5)	68 + 24	(6)	48 + 37
7)	29 + 62	(8)	18 + 63	(9)	55 + 17
10)	29 + 54	(11)	38 + 17	(12)	37 + 54
13)	69 + 73	(14)	78 + 45	(15)	27 + 57
16)	19 + 64	(17)	27 + 46	(18)	38 + 75
19)	47 + 86	(20)	34 + 68	(21)	58 + 86
22)	74 + 38	(23)	95 + 29	(24)	88 + 36

### Using larger numbers

146 + 38 = 146 + 4 + 34	or	146 + 38 = 144 + 2 + 38
= 150 + 34		= 144 + 40
= 184		= 184

### **Exercise 2 larger numbers:**

- 1) Use the strategy of splitting a number into parts to do these additions.
- 2) Do the problems in your head first.
- 3) Check you are correct by writing them down. Show them like the examples above.

1)	294 + 87	(2)	392 + 118	(3)	698 + 77
4)	247 + 45	(5)	329 + 68	(6)	488 + 36
7)	539 + 83	(8)	495 + 126	(9)	597 + 363
10)	296 + 438	(11)	794 + 197	(12)	899 + 73
13)	998 + 115	(14)	724 + 89	(15)	1098 + 89
16)	3996 + 257	(17)	5997 + 325	(18)	2798 + 275

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Decimals can be added in a similar way. Make one number into a whole number and adjust the other number.

32.8 + 24.7e.g. 2 tenths or 0.2 is needed to make 32.8 into a whole number and 2 tenths or 0.2 taken from 24.7 gives 24.5 so 32.8 + 24.7 = 33 + 24.5 = 57.5

### **Exercise 3: adding tenths**

Knowledge Check – What number goes in the  $\Box$  to make the decimal into a whole number?

1)	$4.8 + \Box = 5$	(2)	$7.6 + \Box = 8$	(3)	$2.9 + \Box = 3$
4)	12.7 + 🗆 = 13	(5)	15.9 + <b>□</b> = 16	(6)	$23.5 + \Box = 24$
7)	$32.8 + \Box = 33$	(8)	74.7 + <b>□</b> = 75	(9)	42.6 + 🗆 = 43

### **Exercise 4: using decimals**

- Use the strategy of splitting a number into parts to do these additions. 1)
- Do the problems in your head first. 2)
- 3) Check you are correct by writing them down. Show them like the examples above.

1)	3.9 + 6.7	(2)	4.8 + 7.3	(3)	5.9 + 8.4
4)	7.4 + 1.8	(5)	8.8 + 7.6	(6)	10.6 + 7.8
7)	2.8 + 0.9	(8)	16.7 + 22.8	(9)	34.9 + 12.6
10)	52.7 + 16.8	(11)	21.9 + 17.8	(12)	63.8 + 34.7
13)	42.6 + 16.7	(14)	14.5 + 33.9	(15)	27.8 + 31.7
16)	42.9 + 35.6	(17)	54.9 + 22.7	(18)	36.9 + 52.7
19)	54.8 + 23.4	(20)	83.7 + 12.5	(21)	86.8 + 23.2

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### **SAMPLE for Exercise 5**: Jane knows 34 + 18 = 32 + 20

How does she know this without working out the answer?

She know this is true by thinking of the 34 as 32+2 and then adding the 2 onto the 18 to get 20 so 32+20 is the same.

### **Exercise 5:**

- 1) Decide whether each statement is True or False.
- 2) Do this without working out the answer.
- 3) Provide an explanation of your reasoning.
- 1) 68 + 34 = 70 + 32 (2) 36 + 57 = 39 + 54 (3) 87 + 15 = 90 + 18
- 4) 74 + 18 = 72 + 20 (5) 95 + 37 = 98 + 40 (6) 65 + 17 = 68 + 14
- 7) 153 + 19 = 154 + 20 (8) 325 + 216 = 329 + 220
- 9) 274 + 38 = 272 + 40 (10) 73.8 + 15.4 = 74 + 15.6
- 11) 45.7 + 11.6 = 46 + 11.3 (12) 82.3 + 17.7 = 82 + 18

**Exercise 6:** What number goes in the  $\Box$  to make a true statement?

1)	$26 + 35 = 30 + \square$	(2)	$18 + 77 = 20 + \square$	(3)	$37 + 18 = 40 + \Box$
4)	$44 + 78 = \Box + 80$	(5)	$54 + 28 = \Box + 30$	(6)	$93 + 79 = \Box + 80$
7)	46 + 48 = 47 + <b>□</b>	(8)	$47 + 85 = \Box + 88$	(9)	$54 + 28 = 58 + \square$
10)	$56 + 38 = \Box + 35$	(11)	74 + 49 = 77 + 🗖	(12)	73 + 45 = 70 + <b>□</b>
13)	26 + 57 = 23 + <b>□</b>	(14)	$56 + 17 = \Box + 13$	(15)	93 + 79 = 90 + <b>□</b>

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### Exercise 7

Find two numbers that can be placed in the  $\Box$  and the  $\triangle$  to make a true statement. Do each problem in three different ways.

1)	$38 + \Box = 36 + \bigtriangleup$	$38 + \Box = 36 + \bigtriangleup$	$38 + \Box = 36 + \bigtriangleup$
	Describe the relation	This is a share the $\Box$ and the $\Box$ and the the second se	nd the $\triangle$ .

- 2)  $26 + \Box = 29 + \triangle$   $26 + \Box = 29 + \triangle$   $26 + \Box = 29 + \triangle$ Describe the relationship between the  $\Box$  and the  $\triangle$ .
- 3)  $51 + \Box = 56 + \triangle$   $51 + \Box = 56 + \triangle$   $51 + \Box = 56 + \triangle$ Describe the relationship between the  $\Box$  and the  $\triangle$ .
- 4)  $75 + \Box = 72 + \triangle$   $75 + \Box = 72 + \triangle$   $75 + \Box = 72 + \triangle$ Describe the relationship between the  $\Box$  and the  $\triangle$ .
- 5)  $87 + \Box = 83 + \bigtriangleup \quad 87 + \Box = 83 + \bigtriangleup \quad 87 + \Box = 83 + \bigtriangleup$ Describe the relationship between the  $\Box$  and the  $\bigtriangleup$ .
- 6)  $93 + \Box = 90 + \bigtriangleup$   $93 + \Box = 90 + \bigtriangleup$   $93 + \Box = 90 + \bigtriangleup$ Describe the relationship between the  $\Box$  and the  $\bigtriangleup$ .
- 7)  $86 + \Box = 100 + \triangle$   $86 + \Box = 100 + \triangle$   $86 + \Box = 100 + \triangle$ Describe the relationship between the  $\Box$  and the  $\triangle$ .
- 8)  $148 + \Box = 150 + \bigtriangleup \quad 148 + \Box = 150 + \bigtriangleup \quad 148 + \Box = 150 + \bigtriangleup$ Describe the relationship between the  $\Box$  and the  $\bigtriangleup$ .
- 9)  $574 + \Box = 600 + \triangle \quad 574 + \Box = 600 + \triangle \quad 574 + \Box = 600 + \triangle$ Describe the relationship between the  $\Box$  and the  $\triangle$ .
- 10)  $423 + \Box = 450 + \bigtriangleup 423 + \Box = 450 + \bigtriangleup 423 + \Box = 450 + \bigtriangleup$ Describe the relationship between the  $\Box$  and the  $\bigtriangleup$ .

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### **Exercise 8: Equations and Inequalities**

Without working out the answer how do you know that each of the following are true? Write an explanation and then discuss with other members of your group.

1) 0.7 + 0.5 > 1

$$2) \qquad 54 + 28 = 52 + 30$$

- 3) 246 + 238 > 480
- 4)  $49 + 51 + 52 > 3 \times 50$
- 5) 28 + 33 + 38 = 33 + 33 + 33
- 6) 16 + 4 = 12 + 0
- 7)  $\triangle + \mathbf{O} = (\triangle + 2) + (\mathbf{O} 2)$  where  $\triangle$  and  $\mathbf{O}$  are any numbers

### **Exercise 9: generalizing the relationship**

- 1) Fill in the parenthesis to make a true statement.
- 2) The letter stands for *any number*.

Example:  $34 + n = 40 + (\dots)$  the numbers 34 and 40 are 6 units apart so to make this equation true the value in the parenthesis would have to be n-6.

- 1)  $27 + n = 30 + (\dots)$  (2)  $68 + e = 70 + (\dots)$
- 3)  $89 + a = 100 + (\dots)$  (4)  $46 + (\dots) = 50 + y$
- 5)  $37 + (\dots) = 60 + b$  (6)  $34 + w = 30 + (\dots)$
- 7)  $14.6 + m = 15 + (\dots)$  (8)  $48 + g = 47.9 + (\dots)$
- 9)  $65 + (\dots) = 50 + b$  (10)  $36 + (\dots) = 50 + p$
- 11)  $n+37 = (\dots)+50$  (12)  $(\dots)+113 = c+100$
- 13)  $(\dots) + 388 = h + 400$  (14)  $s + 227 = (\dots) + 250$

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### Answers to Numeracy Strategies

Exer	cise 1				
<i>1</i> )	<i>42</i>	(2)	85	(3)	52
<i>4</i> )	<i>63</i>	(5)	<i>92</i>	(6)	85
7)	<i>91</i>	(8)	<b>81</b>	(9)	72
<i>10</i> )	<i>83</i>	(11)	55	(12)	<i>91</i>
<i>13</i> )	<i>142</i>	(14)	<i>123</i>	(15)	<b>84</b>
<i>16</i> )	<i>83</i>	(17)	<i>73</i>	(18)	<i>113</i>
<i>19</i> )	<i>133</i>	(20)	<i>102</i>	(21)	<i>144</i>
22)	112	(23)	124	(24)	124
Exer	cise 2				
<i>1</i> )	<i>381</i>	(2)	<i>510</i>	(3)	775
4)	<i>292</i>	(5)	<i>397</i>	(6)	524
7)	<i>622</i>	(8)	<i>621</i>	(9)	960
10)	734	(11)	<i>991</i>	(12)	<i>972</i>
13)	<i>1113</i>	(14)	<i>813</i>	(15)	<i>1187</i>
<i>16</i> )	4253	(17)	6322	(18)	3073
Exer	cise 3				
<i>1</i> )	0.2	(2)	0.4	(3)	<i>0.1</i>
<i>4</i> )	0.3	(5)	<i>0.1</i>	(6)	0.5
7)	0.2	(8)	0.3	<b>(9</b> )	0.4
Exer	cise 4				
<i>1</i> )	10.6	(2)	<i>12.1</i>	(3)	<i>14.3</i>
<i>4</i> )	<i>9.2</i>	(5)	<i>16.4</i>	(6)	<i>18.4</i>
7)	3.7	(8)	<i>39.5</i>	(9)	47.5
<i>10</i> )	<i>69.5</i>	(11)	<i>39.7</i>	(12)	<i>98.5</i>
<i>13</i> )	<i>59.3</i>	(14)	<i>48.4</i>	(15)	<i>59.5</i>
<i>16</i> )	78.5	(17)	<b>77.6</b>	(18)	<b>89.6</b>
<b>19</b> )	78.2	(20)	<i>96.2</i>	(21)	<i>110</i>
Exer	cise 5				
<i>1</i> )	True	(2)	True	(3)	False
4)	True	(5)	False	(6)	True
7)	False	(8)	False	<b>(9</b> )	True
<i>10</i> )	False	(11)	True	(12)	True

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### Exercise 6

<i>1</i> )	<i>31</i>	(2)	75	(3)	<i>15</i>
<i>4</i> )	<i>42</i>	(5)	<i>52</i>	(6)	<i>92</i>
7)	47	(8)	44	(9)	24
<i>10</i> )	<i>59</i>	(11)	<b>46</b>	(12)	<b>48</b>
<i>13</i> )	<i>60</i>	(14)	<i>60</i>	(15)	<b>82</b>

### Exercise 7

Statements for each question will vary.

- 1) the number in the  $\triangle$  is 2 more than the number in the  $\square$
- 2) the number in the  $\triangle$  is 3 less than the number in the  $\square$
- 3) the number in the  $\triangle$  is 5 less than the number in the  $\square$
- 4) the number in the  $\triangle$  is 3 more than the number in the  $\square$
- 5) the number in the  $\triangle$  is 4 more than the number in the  $\square$

6) the number in the  $\Delta$  is 3 more than the number in the  $\square$ 

7) the number in the  $\triangle$  is 14 less than the number in the  $\square$ 

8) the number in the  $\triangle$  is 2 less than the number in the  $\square$ 

9) the number in the  $\triangle$  is 26 less than the number in the  $\square$ 

10) the number in the  $\triangle$  is 27 less than the number in the  $\square$ 

Exercise 8

Answers will vary.

- 1) 0.7 > 0.5 so total will be more than 0.5 + 0.5
- 2) Adding 2 to 28 gives 30 and subtracting 2 from 54 gives 52
- 3) 2 lots of 240 gives 480 and 246 is further from 240 than 238
- 4) 49 is one below 50, other two numbers are both above 50
- 5) 28 is five below 33 and 38 is five above 33 so their total is the same as 33 + 33
- 6) <sup>-4</sup> is increased by 4 and 16 is decreased by 4 so total is unchanged
- 7)  $\triangle$  is increased by 2 and O is decreased by 2 so total is unchanged

#### **Exercise** 9

<i>1</i> )	n - 3	(2)	<i>e</i> - 2	(3)	a - 11
<i>4</i> )	<i>y</i> + <i>4</i>	(5)	<i>b</i> + 23	(6)	<i>w</i> + <i>4</i>
7)	m - 0.4	(8)	<i>g</i> + <i>0.1</i>	(9)	<b>b - 15</b>
<i>10</i> )	<i>p</i> + 14	(11)	n - 13	(12)	<i>c</i> - <i>13</i>
13)	h + 12	(14)	s - 23		

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# **Olympic Cola Display**

Lesson adapted from: <u>http://mikewiernicki3act.wordpress.com/olympic-display/</u>

## **SUGGESTED TIME FOR THIS LESSON:**

### 60-90 minutes

The suggested time for the lesson will vary depending upon the needs of the students.

In this lesson, students will use their understanding of area models to represent the distributive property to solve problems associated with an Olympic cola display.

### STANDARDS FOR MATHEMATICAL CONTENT

# MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.

b. Use area models to represent the distributive property and develop understandings of addition and multiplication (all positive rational numbers should be included in the models). (MGSE3.MD.7)

### Common Misconceptions:

A common misconception is that students should learn their multiplication tables 0-12 in order, and if the student has not internalized their multiplication facts before middle grades (and especially before high school), they will never "learn" them. Van de Walle states that students need to see multiplication as patterns and use different strategies in determining the product of factors such as the base-ten model and partial products. Students who are not fluent in multiplication facts benefit from looking for patterns and relationships as opposed to memorization. Using the distributive property in relation to partial products helps students gain confidence in other methods for multiplication. For example, if a student does not quickly know the product of 6 and 7, they could think of (5x7) and (1x7). Understanding patterns and relationships between numbers translates into a stronger foundation for algebraic reasoning.

### STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students must make sense of the problem by identifying what information they need to solve it.

2. **Reason abstractly and quantitatively.** Students are asked to make an estimate (high and low).

3. **Construct viable arguments and critique the reasoning of others.** After writing down their own questions, students discuss their question with partners, creating the opportunity to construct the argument of why they chose their question, as well as critiquing the questions that others came up with.

4. **Model with mathematics.** Once given the information, the students use that information to develop a mathematical model to solve their question.



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5. Use appropriate tools strategically. Students write their best estimate and two more estimates – one that is too low and one that is too high to establish a range in which the solution would occur.

6. Attend to precision. Students use clear and precise language when discussing their strategies and sharing their own reasoning with others.

7. Look for and make sense of structure. Students use their understanding of properties of operations and area models to make sense of the distributive property.

### **EVIDENCE OF LEARNING/LEARNING TARGETS**

By the conclusion of this lesson, students should be able to:

- Analyze the relationship between the concepts of area, multiplication, and addition.
- Solve word problems involving area of rectangular figures.
- Use models to represent the context of an area problem.

### **MATERIALS for Coca Cola LESSON**

- Act 1 picture -Olympic Cola Display
- Pictorial representations of the display
- Student recording sheet

### **ESSENTIAL QUESTIONS**



- Which strategies do we have that can help us understand how to multiply a two-digit number?
- How does understanding partial products (using the distributive property) help us multiply larger numbers?

In order to maintain a student-inquiry-based approach to this lesson, it may be beneficial to wait until Act 2 to share the Essential Questions (EQ's) with your students. By doing this, students will be allowed the opportunity to be very creative with their thinking in Act 1. By sharing the EQ's in Act 2, you will be able to narrow the focus of inquiry so that the outcome results in student learning directly related to the content standards aligned with this lesson.

### KEY VOCABULARY

The following terms should be reviewed/discussed as they arise in dialogue during the lesson:

- distributive property
- commutative property
- area model

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### SUGGESTED GROUPING FOR THIS LESSON

Individual/Partner and or Small Group

### LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this lesson, students will view the picture and tell what they notice. Next, they will be asked to discuss what they wonder about or are curious about. These questions will be recorded on a class chart or on the board and on the student recording sheet. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on **need**. When they realize they do not have the information they need, and ask for it, the information will be given to them.

In this lesson, students should build upon what they already know about arrays and area models to answer their questions. Specifically, in finding the total number of 12-packs in the display, students should construct strategies for decomposing the display into smaller areas (the distributive property). Note: Students should not be expected to find the total number of 12-packs by multiplying 14 x 23. They should use the area model with partial products to find the product. The Opener/Activator focuses on the area model with partial products as a way to review that strategy.

Although many of the students will want to use a calculator, **withhold the use of a calculator** to build the need for the distributive property in the area model.

This lesson follows the 3-Act Math Task format originally developed by Dan Meyer. More information on this type of lesson may be found at <u>http://blog.mrmeyer.com/category/3acts/</u>.

Students need **multiple** experiences with arrays to build their understanding of multiplication. Students should also understand how arrays and multiplication are connected to the concept of area, and how their flexibility with number can help them develop strategies for solving complex problems such as the one in this lesson.

Students need to have a good understanding of basic multiplication facts. They should also understand the various ways that multiplication number sentences can be written using an x, a dot, or parentheses.

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### **OPENER/ACTIVATOR**

Post the following problem on the board and ask students to find the answer as many ways as they can. Suggest that each student comes up with at least three different ways to find the product.

7 x 23 17 x 34 14 x 43

Ask for students to share in small groups or as a class. Look for strategies to highlight during the class discussion. Make sure to demonstrate the area/array model using partial products as shown below:

7 x 23 can be modeled as shown below:



Students should see the problem as 7(20 + 3) which is easier to evaluate as 7(20) + 7(3) to get 140 +21 or 161.

This same process can be applied to larger multiplication problems such as 17x34.

	30	4
10	300	40
7	210	28

You can represent this model as 10(30 + 4) + 7(30 + 4) to get a final answer of 578. Use of this strategy can have long term effects on understanding and visualizing multiplication of algebraic expressions as well as factoring algebraic expressions.

Further development of this model will happen in upcoming lessons (Distributing Using Area and Tiling Lesson)

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Access the Learnzillion video on area model at <u>https://learnzillion.com/lessons/1879-use-an-area-model-for-multiplication-of-two-digit-numbers-by-two-digit-numbers</u> for more practice or instruction. A guided practice video is also included after the lesson video at <u>https://learnzillion.com/resources/54500</u>

Interactives to examine the area model for multiplication may be found on the Annenberg learning site at

http://www.learner.org/courses/learningmath/number/session4/part\_b/multiplication.html

### **Lesson Directions:**

Act 1 – Whole Group - Pose the conflict and introduce students to the scenario by showing Act I picture. (Dan Meyer <u>http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/</u>) "Introduce the central conflict of your story/lesson clearly, visually, viscerally, using as few words as possible."

Show Act 1 picture to students.

• Ask students what they noticed in the picture, what they wonder about, and what questions they have about what they saw in the picture. Do a think-pair-share so that students have an opportunity to talk with each other before sharing questions with the whole group.



• Share and record students' questions. The teacher may need to guide students so that the questions generated are math-related. Consider using the Concept Attainment strategy to place those questions and responses that are accountable to the 3-Act Task in the "Yes, Let's Consider" circle and those that are not in the "Not Now" circle. Examples of "Yes, Let's Consider" and "Not Now" are listed below. Students may come up with additional ideas, and ideas that fall in the "Not Now" category should still be validated and are not to be considered "bad ideas".

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**Anticipated questions students may ask and wish to answer:** (\*Main question(s) to be investigated)

### "Yes, Let's Consider"

### "Not Now"

How many 12 packs of Coke are there?	How tall is it?
*How many 12 packs are there in the display?	How wide is it?
*How many cans of soda is that?	What is the area of the front of the display?
How many cans of each kind of soda are in the	How much time did it take to make that
How many cans of each kind of soda are in the display?	How much time did it take to make that display?
How many cans of each kind of soda are in the display? What are the dimensions of the display?	How much time did it take to make that display? Where is it?

Once students have their question, ask the students to estimate answers to their questions (think-pair-share). Students will write their best estimate, then write two more estimates – one that is too low and one that is too high so that they establish a range in which the solution should occur.

Students should plot their three estimates on an empty number line. To facilitate the discussion of possible solutions, each student could be given three post it notes to record their three estimates on. Then each student could post their estimates on the open number line to arrive at a class set of data to consider.

If you feel the students do not feel comfortable displaying their own estimates, you could have students "ball up" their three estimates in "snow balls" and throw them around the room for a minute, then end up with three "snow balls" at the end to post on the open number line.

Note: As the facilitator, you may choose to allow the students to answer their own posed questions, one question that a fellow student posed, or a related question listed above. For students to be completely engaged in the inquiry-based problem solving process, it is important for them to experience ownership of the questions posed.

<u>Important note:</u> Although students may only investigate the main question(s) for this lesson, it is important for the teacher to not ignore student generated questions. Additional questions may be answered after they have found a solution to the main question, or as homework or extra projects.

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Act 2 – Student Exploration - Provide additional information as students work toward solutions to their questions. (Dan Meyer <a href="http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/">http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/</a>) "The protagonist/student overcomes obstacles, looks for resources, and develops new tools."

During Act 2, students decide on the facts, tools, and other information needed to answer the question(s) (from Act1). When students decide what they need to solve the problem, they should ask for those things. It is pivotal to the problem solving process that students decide what is needed without being given the information up front.

Students need the opportunity to work with manipulatives on their own or with a partner in order to develop the understanding of multiplication. From the manipulatives, students will be able to move to pictorial representations of the display (attached), then more abstract representations (such as sketches), and finally to abstract representation of multiplication using numbers. It is important to remember that this progression begins with concrete representations using manipulatives.

The goal of the lesson is to use area models to represent the distributive property and develop understandings of addition and multiplication. By sectioning the model, students can apply the distributive property to solve the problem. For example, using concrete manipulatives such as color counters to represent the different types of soda and combine like terms by grouping. Or, students could section the model into components noting parts that are equal such as the top left and top right ring. Others may see the model as a large rectangle with three partial rings above it. Listed below are some suggestions to guide students to make connections between adding all the units of soda to multiplication to simplify the process to applying the distributive property. These are only sample ideas and students should not be limited to the one provided below.

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Suggested Questions (if needed) to guide the discussion toward area model and distributive property (CAUTION: Using too many direct questions can stifle the different ways students may solve the problem. These questions are only designed to give insight to the instructor.)

1. When looking at the drawing of the display, how can you divide it into sections to make computation easier? (note: a unit is defined as a 12 pack of soda)

Sample responses might include "a large rectangle with three partial circles above it"

2. How can you break apart those sections to calculate the total number of containers?

Sample responses might include "the large rectangle is 23 units long and 14 units tall so I could decompose that into (20+3)(10+4) which could be shown as



Next students will need to consider the top portion of the display which could be modeled as:

3 (7 + 5 + 3) since there are three congruent regions containing sections of 7,5, and 3 units of soda for a total of 3(7) + 3(5) + 3(3) or 21+15+9 or 45 units of soda for the top portion of the display.

Now students must combine the two sections for a grand total of 322 + 45 or 367 units of soda.

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The teacher provides guidance as needed during this phase. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin. Questioning is an effective strategy that can be used. Consider using these questions to help guide students during the problem solving phase. The questions are based on Polya's Problem Solving Approach.

### Stage 1: Understand the Problem

- What are you asked to find or show?
- Can you restate the problem in your own words?
- What is the problem you are trying to solve?
- What do you think affects the situation?
- How can the picture or diagram help you understand the problem?
- What are the knowns? What are the unknowns?
- What information do you obtain from the problem?
- What information, if any, is missing or not needed?

### Stage 2: Devise a Plan (Sample provided above)

- What strategy or strategies will you use to solve the problem?
- What strategies are you using?
- What assumptions are you making?
- What tools or models may help you?

### Stage 3: Carry out the Plan

- Can you explain what you've done so far?
- Can you explain clearly your steps to solve the problem?
- Can you use an array to represent your mathematics?
- Can you use partial products to represent your mathematics?

### Stage 4: Looking Back

- Can you check your result?
- Does the answer make sense? Is it reasonable?
- Why is that true?
- Does that make sense?

### Additional Information for Act 2

It is during Act 2 that you may provide the students with the pictorial representation of the display that is attached.

### Act 3 – Whole Group – Share solutions and strategies.

- Students to present their solutions and strategies and compare them.
- Teacher facilitates discussion to compare these strategies and solutions, asking questions such as:
  - How reasonable was your estimate?
  - Which strategy was most efficient?
  - Can you think of another method that might have worked?
  - What might you do differently next time?

### **OPTIONAL** (Act 4) – Not a Required Part of the 3 Act Task

Act 4, The Sequel – The following is a quote for Dan Meyer about the purpose of the OPTIONAL 4<sup>th</sup> Act: "The goals of the sequel lesson are to a) challenge students who finished quickly so b) I can help students who need my help. It can't feel like punishment for good work. It cannot seem like drudgery. It has to entice and activate the imagination." Dan Meyer http://blog.mrmeyer.com/2013/teaching-with-three-act-LESSONs-act-three-sequel/

Act 4: Share ideas (see extensions) or reference other student-generated questions that could be used for additional classwork, projects or homework. Students may choose to revisit one of the questions posed at the beginning but not investigated during Act 2. Allow students who complete the first TWO acts of the process to move on to Act 4 if they are ready. They may join the group discussion/debriefing during Act 3 when all students are ready.

### FORMATIVE ASSESSMENT QUESTIONS (These may be used during Act 3)

What partial products did you create? What organizational strategies did you use? What are the dimensions of your array(s)? What product/area does your model represent?

### **DIFFERENTIATION**

### Extension

Give students a base-ten block array or a drawing of an array and have them determine the product and its factors.

Have students create their own display, build it with base 10 blocks or connecting cubes, and then trade seats with a neighbor to determine the factors and find the product. Have students use an array to write/solve division problems.

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### Intervention

Begin with much smaller arrays, such as  $2 \times 3$ ,  $3 \times 4$ , and  $2 \times 6$ . Have students describe the dimensions and area of each array. Then connect dimensions and area to the actual multiplication sentence.

Use grid paper and allow students to place the base-ten blocks onto the grid paper first and then to count the grid squares as part of their calculations.

If necessary, allow students to use a times table chart or other cueing device if full mastery of the basic multiplication facts has not yet been attained.

For extra help with multiplication, please open the hyperlink Intervention Table.

### **CLOSING/SUMMARIZER**

Use the "Math Mistakes" activity from <u>http://mathmistakes.org/what-is-the-distributive-property/</u> to review the distributive property.

The Math Mistakes site is about compiling, analyzing and discussing the mathematical errors that students make. The site is edited by **Michael Pershan**, a middle school and high school math teacher from NYC.

Using student work for error analysis can be an effective strategy to increase understanding of a standard. This site (Math Mistakes) provides a way to analyze mistakes without using work from your class or school. You could assign them the role of "teacher" and ask them the following question:

### If this is your class, how do you respond to these student responses?

After students have time to respond to the prompt in small groups, allow several groups to share their ideas.
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## Act 1 Picture:



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Pictorial Representation of the Display:



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## LESSON Title: \_\_\_\_\_

Adapted from Andrew Stadel

Name: \_\_\_\_\_

## <u>ACT 1</u>

What did/do you notice?

What questions come to your mind?

Main Question: \_\_\_\_\_

What is your 1<sup>st</sup> estimate and why?

On an empty number line, record an estimate that is too low and an estimate that is too high.

## <u>ACT 2</u>

What information would you like to know or need to solve the MAIN question?

Record the given information (measurements, materials, etc...)

If possible, give a better estimation with this information:

Act 2 (con't)

Use this area for your work, tables, calculations, sketches, and final solution.

## <u>ACT 3</u>

What was the result?

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The following problems may be used as additional practice after completing the 3 Act Task. These problems may be modified to meet the needs of your students. Additionally, you may select problems based on student understanding for a differentiated assignment.

(*Teacher's Edition*) Sample Practice Problems to consider after completion of the Olympic Coca Cola 3 Act LESSON and class discussion.

Use area models to represent the distributive property and develop understandings of addition and multiplication (all positive rational numbers should be included in the models). (MGSE3.MD.7)

1. The 9th grade class at Georgia High School wanted to go on a field trip to a soda factory. The trip will cost \$100. The students decided to write a class newspaper and sell it to the kids at their school. Each of the 20 students will be given a 16 inch square for his/her article in the newspaper. (adapted from **Read All About It** lesson 3<sup>rd</sup> grade Unit 3)

How many pages long will the newspaper be if they used paper that was  $8\frac{1}{2} \times 11$  inches?

Suggest students draw a diagram of a page in the newspaper. Questions might arise about the layout of the page such as:

- Is there a margin?
- *How wide is the margin?*
- Is there a header or footer?
- How much space will that take up?

Depending on student ideas on margins and header/footer, they may decide that you can fit four of those blocks on a page. Thus, we would need 5 pages for each student to have one article.

Will there be enough room for additional graphics on the pages once the articles have been written? What do you need to know in order to answer this question? Since the total area of the page is 93.5 in<sup>2</sup> and we will put 4 squares at 16 in<sup>2</sup> each for a total amount of 64 in<sup>2</sup>, there will be 29.5 in<sup>2</sup> of space left for graphics.

How did you determine your solution to part b?

I had to compare the total amount of area available on the page to the amount of area that the articles would take up on the page. \*NOTE: Discussion should address the fact that there will be 29.5 in<sup>2</sup> left on the page which is greater than the area of one square. Why can we not add one more to the page?

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Questions 2-4 originally found at <u>http://www.orglib.com/home.aspx</u> used under a creative commons license

2. Which distributive property represents the area of the following rectangle?



How did you make your decision?

I could tell that the missing side of the blue rectangle would be 6 cm. I also know that to find the area I would multiply the width (3cm) by the length for each rectangle separated (6cm + 3cm). That means the area would be (3x3) + (3x6).

3. The diagram below (under part c) is a model of Samantha's kitchen table.

a) What is the area of Samantha's table? The area would be 3ft x  $4\frac{2}{3}$  ft or 14 ft<sup>2</sup>.

b) Show a way to use the distributive property to make the problem easier to solve.

 $3(4 + \frac{2}{3}) = (3x4) + (3x\frac{2}{3}) = 12 + 2 = 14 ft^2$ 

c) If you have a table cloth that is 2 yd<sup>2</sup>, will it cover the table? Justify your response.



\*Note: part c of the problem requires understanding of unit conversions. This part could be used as an extension.

One square yard covers 9 square feet  $(3ft \times 3ft = 9ft^2)$  so two square feet cover  $18ft^2$  thus YES the table cloth will cover the table.

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#### **Practice Problems**

Name

1. The 9th grade class at Georgia High School wanted to go on a field trip to a soda factory. The trip will cost \$100. The students decided to write a class newspaper and sell it to the kids at their school. Each of the 20 students will be given a 16 inch square for his/her article in the newspaper. (adapted from **Read All About It** lesson 3<sup>rd</sup> grade Unit 3)

How many pages long will the newspaper be if they used paper that was  $8\frac{1}{2} \times 11$  inches?

Will there be enough room for additional graphics on the pages once the articles have been written? What do you need to know in order to answer this question?

How did you determine your solution to part b?

*Questions 2-3 originally found at <u>http://www.orglib.com/home.aspx</u> used under a creative commons license* 

2. Which distributive property represents the area of the following rectangle?

○ (3 x 3) + (3 x 4)	3 cm	
○ (3 x 3) + (3 x 9)		3 cm
○ (9 x 3) + (6 x 9)		
○ (3 x 3) + (3 x 6)	9 cm	

How did you make your decision?

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- 3. The diagram below is a model of Samantha's kitchen table.
- a) What is the area of Samantha's table?
- b) Show a way to use the distributive property to make the problem easier to solve.
- c) If you have a table cloth that is 2 yd<sup>2</sup>, will it cover the table? Justify your response.



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## **Distributing Using Area**

#### **Adapted from NCTM: Illuminations**

In this lesson, students will use area models to represent and discover the distributive property. Students will be using rectangles whose sides may be variables in order to further their understanding of the distributive property.

### SUGGESTED TIME FOR THIS LESSON

Approximately 60-120 minutes

The suggested time for the class will vary depending upon the needs of the students.

## STANDARDS ADDRESSED IN THIS LESSON

# MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.

b. Use area models to represent the distributive property and develop understandings of addition and multiplication (all positive rational numbers should be included in the models). (MGSE3.MD.7)

d. Add, subtract, and multiply algebraic expressions. (MGSE6.EE.3, MGSE6.EE.4, MC7.EE.1, MGSE9-12.A.SSE.3)

e. Generate equivalent expressions using properties of operations and understand various representations within context. For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between "3 more" and "3 times". (MGSE4.0A.2; MGSE6.EE.3, MGSE7.EE.1,2,MGSE9-12.A.SSE.3)

## COMMON MISCONCEPTIONS

As students begin to build and work with expressions containing more than two operations, students tend to set aside the order of operations. For example, having a student rewrite an expression like 8 + 4(2x - 5) + 3x can bring to light several misconceptions. Do the students immediately add the 8 and 4 before distributing the 4? Do they only multiply the 4 and the 2x and not distribute the 4 to both terms in the parenthesis? Do they collect all like terms 8 + 4 - 5, and 2x + 3x? Each of these shows gaps in students' understanding of how to rewrite numerical expressions with multiple operations.

Students have difficulties understanding equivalent forms of numbers, their various uses and relationships, and how they apply to a problem. Make sure to expose students to multiple examples and in various contexts.

Students usually have trouble remembering to distribute to both parts of the parenthesis. They also try to multiply the two terms created after distributing instead of adding them.

Students also want to add the two final terms together whether they are like terms or not since they are used to a solution being a single term.

Finally, students need to be careful to make sure negative signs are distributed properly.

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## STANDARDS FOR MATHEMATICAL PRACTICE

2. **Reason abstractly and quantitatively.** In this LESSON students will address the dimensions of a rectangle as they relate to its area in the application of the distributive property.

4. **Model with mathematics.** Students will explore the area model for multiplication to compare the concepts of area as a sum and area as a product.

## **EVIDENCE OF LEARNING/LEARNING TARGETS**

By the conclusion of this lesson, students should be able to:

- Use the properties of operations to simplify linear expressions
- Apply the area representation of the distributive property.

### **MATERIALS**

• Activity Sheet

\*Optional: Colored Sheets of paper cut into rectangles. These can be used to introduce the concepts found in this lesson and to create models of the rectangles as needed.

### **ESSENTIAL QUESTIONS**

- What are strategies for finding the area of figures with side lengths that are represented by variables?
- How can area models be used to represent the distributive property?

### **KEY VOCABULARY**

The following terms should be reviewed/discussed as they arise in dialogue with the LESSON:

- distributive property
- commutative property

### **GROUPING**

Individual/Partner

### **LESSON DESCRIPTION**

This lesson is designed to help students understand the distributive property using area models. It is important to allow students time to develop their own solutions for how the distributive property can be used to solve problems.

#### Area Representation of the Distributive Property

The first part of this activity can be used to help students recall information regarding area as a precursor for the distributive property. The following examples could be featured as anchor charts and filled in as the students work through the parts. Anchor charts could then serve as a visual reminder during work time in the lesson.

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The first section introduces students to the idea of writing the area of a rectangle as an expression of the length  $\times$  width, even when one or more dimensions may be represented by a variable.



The next section demonstrates how two measurements of a segment can be added together to represent the sum of the entire length of the segment.



**The key section is next**, having students represent the area of each rectangle *two ways* to distribute the common factor among all parts of the expression in parentheses.



#### **OPENER/ACTIVATOR**

To get students thinking about area in a non-traditional way, use the site Estimation180. Open the link http://www.estimation180.com/day-166.html to access a problem where students will be asked to complete the estimation activity "How much area of the paper does my *wife's* hand cover?"

This activity serves two purposes:

- 1. engages the students in dialogue about area
- 2. engages the students in estimation

Andrew Stadel provides daily estimation challenges that get kids thinking about numbers and how they are used to quantify "real world things". His activities foster number sense and problem solving accessible to students at all levels of mathematical abilities.

If you are able to access the estimation activity via the internet, students will be able to enter their estimate and compare to others all over the world. If you are unable to access the site, the activity is provided below for you to use in class on either a display board or handout for students.

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http://www.estimation180.com/uploads/1/3/8/8/13883834/estimation\_180\_handout\_2013-08-01\_font.pdf provides access to a handout students could use to keep track of their estimation activities.

By deliberately planning estimation activities into your math class, students should grow in their ability to assess the reasonableness of their responses to all types of problems. According to NCTM, "Estimation is critical to building number sense, mental math skill, and computational fluency. NCTM's *Principles and Standards for School Mathematics* assert that the development of computational fluency requires students to make a "connection between conceptual understanding and computational proficiency" (NCTM 2000, p. 35).

For more Estimation 180 activities, go to http://www.estimation180.com/

A Printable copy of the activity appears on the next page.

How much area of the paper does my Wife'S hand cover?



Your estimate. *	
<b>Your reasoning. *</b> Do better than "I quessed.	

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How much *area of the paper* does my Wife'S hand cover?



Use the space below to explain your strategy or make computations.

What's too LOW? \*

What's too HIGH? \*

Your estimate. \*

Your reasoning. \*

Do better than "I guessed."

Your name. \*

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## **Teacher's Edition Distributing Using Area**

Write the expression that represents the area of each rectangle.



It is very important that students are encouraged to show their work on all activities of this lesson. The role of reviewing the step by step calculations will gauge the level of procedural fluency and conceptual fluency that is embedded within the work. During the formative review of the student work, prepare to intervene with strategies to address the procedural or computational error.

Find the area of each box in the pair.



Area is found by multiplying the length and width of a figure together.

This section allows students to begin to piece together some of the fundamental concepts for the distributive property.

We suggest that students write the areas of each of the figures within the corresponding boxes. 5) Area of first box: 4(x)=4x Area of second box: 4(3)=12\*Note: Students will need to recognize that the width of both figures is the same.

6) Area of first box: 7(a)=7a Area of second box: 7(9)=63 \*Note: Students will need to recognize that the width of both figures is the same.

7) Area of first box: 3(x)=3x Area of second box: (3)(2)=6

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Write the expression that represents the total length of each segment.



#### Solution:

8) The total length of this segment can be written as an expression: x+9 units
9) The total length of this segment can be written as an expression: x+4 units
10) The total length of this segment can be written as an expression: a+2 units
\*Teachers can scaffold this section and demonstrate the ways in which to measure the total length of a segment. Other representations of these types of segments can be added in order to help students think about the same concept in multiple ways.

Write the area of each rectangle as the product of  $length \times width$  and also as a sum of the areas of each box.



#### Solution:

The problems in this section demonstrate the multiple ways to represent area, which leads to the fundamental concepts of the <u>distributive property</u>.

- The area as a product section requires students to think about how to represent the area of the entire rectangle without using the area of each of the individual rectangles. Area of a rectangle can be found by multiplying the length (of the entire rectangle) and width of the rectangle.
- The area as a sum section requires students to think about how to represent the area of the rectangle by using the area of each individual rectangle and taking the sum of the areas to find the area of the whole rectangle.

11) <u>Area as a Product:</u> The length of the figure can be written as an expression x+7 (this has been referenced in 8-10 and teachers can use the previous questions to help students come to this realization). The width of this figure is 5 units. The area is found by multiplying (x + 7)5 which is equivalent to 5(x + 7) by the commutative property.

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<u>Area as a Sum:</u> The area of the first (left) rectangle can be found by multiplying the length, x, and the width, 5. Thus, the area of the first rectangle is x(5) or 5x by the commutative property. The area of the second rectangle can be found by multiplying the length, 7, by the width, 5. Thus the area of the first rectangle is 7(5)=35.

In order to find the total combined area, students must add together the areas of both figures. Therefore, the total combined area is found as the expression 5x+35.

<u>12) Area as a Product:</u> The length of the figure can be written as an expression x+12 (this has been referenced in 8-10 and teachers can use the previous questions to help students come to this realization). The width of this figure is 3 units. The area is found by multiplying (x + 12)3 which is equivalent to 3(x + 12) by the commutative property.

<u>Area as a Sum:</u> The area of the first (left) rectangle can be found by multiplying the length, x, and the width 3. Thus, the area of the first rectangle is 3(x) = 3x

The area of the second rectangle can be found by multiplying the length, 12, by the width, 3. Thus the area of the second rectangle is 12(3) = 36.

In order to find the total combined area, students must add together the areas of both figures. Therefore, the total combined area is found as the expression 3x + 36.

13) Area as a Product: The length of the figure can be written as an expression a+8. The width of this figure is 5 units. The area is found by multiplying (a + 8)5 which is equivalent to 5(a + 8) by the commutative property.

<u>Area as a Sum:</u> The area of the first left rectangle can be found by multiplying the length, a, and the width, 5. Thus, the area of the first rectangle is (a) = 5a.

The area of the second rectangle can be found by multiplying the length, 8, by the width, 5. Thus the area of the second rectangle is 8(5) = 40.

In order to find the total combined area, students must add together the areas of both figures. Therefore, the total combined area is found as the expression 5a + 40.

After finishing these questions, teachers need to help students come to the realization that the two expressions that they generated from these questions are <u>equivalent</u> and represent the same information in different ways.

*Use the distributive property to find sums that are equivalent to the following expressions. (You may want to use a rectangle to help you)* 

14. $4(x + 7) = 4x + 28$	15. $7(x-3) = \underline{7x-21}$
162(x+4) = -2x-8 *	17. $3(x+9) = 3x+27$
18. $4(a-1) = 4a-4$	19. $3(m+2) = 3m+6$
204(a-4) = -4a+16 *	$21.\frac{1}{2}(a-12) = \frac{1/2}{2}a - 6$

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For 14-21, teachers can ask students to use rectangles to solve the problems. This helps students recognize how to solve the problems while using the area model generated from questions 11-13.

\*Note: This section also gives students practice with negative numbers. Students may struggle with the idea of a "negative area." Please see below for Illumination's description for how to simplify for the total area, especially taking note of their suggestion for handling the subtraction problems or the negative signs. Adapted from NCTM: Illuminations http://illuminations.nctm.org/LessonDetail.aspx?id=L744

### FORMATIVE ASSESSMENT QUESTIONS

These questions can be used to help further develop understanding of the distributive property.

- What is the relationship between the *product* and *sum* representation of the area model?
- How does the area model help to explain the distributive property?
- Why do you think this property was named the distributive property?

## **DIFFERENTIATION**

#### Extension

Have students create and explain models to demonstrate the sum of four or more positive and negative numbers.

### Intervention

Have students use models other than those suggested in the lesson to add positive and negative numbers, for example, the stack or row model and hot air balloon model.

For extra help with multiplication problems, please open the hyperlink Intervention Table.

### **CLOSING**

Ask students to create a multiplication problem that would be easier to solve using the distributive property. Allow students to share with their neighbor. Select a few to pose to the class.

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## **Student Edition Learning Lesson: Distributing Using Area**

Write the expression that represents the area of each rectangle.





Use the distributive property to find sums that are equivalent to the following expressions. (You *may want to use a rectangle to help you)* 



20. -4(a - 4) =\_\_\_\_\_



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## **Triangles and Quadrilaterals**

Adapted from Engage NY A Story of Ratios

In this lesson, students will work with variables, variable expressions, combining like terms, and evaluating variable expressions for defined values.

#### SUGGESTED TIME FOR THIS LESSON

60 - 90 minutes

The suggested time for the lesson will vary depending upon the needs of the students.

#### STANDARDS FOR MATHEMATICAL CONTENT

## MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.

a. Apply properties of operations emphasizing when the commutative property applies. (MGSE7.EE.1)

c. Model numerical expressions (arrays) leading to the modeling of algebraic expressions. (MGSE7.EE.1,2; MGSE9-12.A.SSE.1,3)

d. Add, subtract, and multiply algebraic expressions. (MGSE6.EE.3, MGSE6.EE.4, MC7.EE.1, MGSE9-12.A.SSE.3)

e. Generate equivalent expressions using properties of operations and understand various representations within context. For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between "3 more" and "3 times". (MGSE4.0A.2; MGSE6.EE.3, MGSE7.EE.1,2,MGSE9-12.A.SSE.3)

f. Evaluate formulas at specific values for variables. For example, use formulas such as A = l x w and find the area given the values for the length and width. (MGSE6.EE.2)

## COMMON MISCONCEPTIONS

Students tend to make the following mistakes when working with variable expressions:

- Not recognizing that a single letter variable has a coefficient of 1 (r = 1r)
- Adding the coefficients of unlike terms
- Failing to identify all like terms in a sum
- *Making sign errors (failing to recognize that* -n = + n)

Additionally, students confuse application of the distributive property when the multiplier is written to the right (behind) of the parenthesis instead of to the left (in front) of the parenthesis. Clarify this misunderstanding with several examples based on student need. Additionally, students may ask if it is OK to rearrange the terms so that the multiplier is in front. Take this opportunity to explore the commutative property with the class.

Call attention to examples such as 3(2x - 4) being the same as (2x - 4)3.

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## STANDARDS FOR MATHEMATICAL PRACTICE

2. **Reason abstractly and quantitatively**. Students make sense of how quantities are related within a given context and formulate algebraic equations to represent this relationship.

4. **Model with mathematics**. Throughout the module, students use equations and inequalities as models to solve mathematical and real-world problems. Students test conclusions with a variety of objects to see if the results hold true, possibly improving the model if it has not served its purpose.

6. Attend to precision. Students are precise in defining variables. They understand that a variable represents one number. They use appropriate vocabulary and terminology when communicating about expressions and equations

7. Look for and make use of structure. Students recognize the repeated use of the distributive property as they write equivalent expressions. Students recognize how equations leading to the form px + q = r and (x + q) = r are useful in solving variety of problems.

#### **EVIDENCE OF LEARNING/LEARNING TARGETS**

By the conclusion of this lesson, students should be able to:

- Use the properties of operations to simplify linear expressions
- Combine like terms to generate equivalent expressions.

## MATERIALS REQUIRED

- Envelopes containing triangles and quadrilaterals
- Match up cards for closing activity
- Template for Like Terms closing activity

### ESSENTIAL QUESTIONS

- How can we represent values using variables?
- How can we determine which terms may be combined when adding or subtracting variable expressions?
- How can we evaluate variable expressions when the variable is assigned a value?

### **KEY VOCABULARY**

The following terms should be reviewed/discussed as they arise in dialogue within the LESSON:

- Variable expression
- commutative property

## **ACTIVATOR/OPENER**

To get students thinking about properties of shapes (specifically the number of sides) use the following pattern from the website <u>www.visualpatterns.org</u>.

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<u>http://www.visualpatterns.org/41-60.html</u> will show the following pattern: PATTERN 51 is shown below:



A **student handout** is provided on the next page to give students a place to gather their thoughts about the pattern they see. The goal is to figure out the pattern being used to create the next stage and to come up with a formula or equation to find the number of hexagons at any stage. Then, students are asked to answer the pattern (how many hexagons) for the 43 stage. In the image above, we would note that stage one has 4 hexagons, stage 2 has 7 hexagons, stage 3 has 10 hexagons.

The purpose of this introduction to the Visual Patterns Site is to get students analyzing patterns and making predictions based on the patterns and eventually to express the pattern using algebraic methods. Students will need time and repeated exposure to these activities to gain confidence and fluency. For the purpose of activating this lesson, the goal is to have dialogue about the number of sides in the shape and how many sides total are present in each stage. It is **not essential that students complete the entire visual pattern activity prior to the "Triangles and Quadrilaterals" lesson.** 

#### URL: http://visualpatterns.org

Name *KEY* Pattern <u>#: 51</u>



Teacher's Edition of a Sample Pattern

Student Page Follows with Open Space to show work

#### *Complete this table:*

Step	
n	# of <u>Hexagons</u>
1	4
2	7
3	10
4	13
5	16
6	19
10	31
37	112

What do you notice? What do you wonder? Number of hexagons increases by 3 each stage Write the equation: Total number of hexagons = 3n+1

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Name: \_\_\_\_\_ Pattern #: \_\_\_\_\_ Draw the next step: Complete this table:

Step	
п	# of
1	
2	
3	
4	
5	
6	
10	
37	

What do you notice?

What do you wonder?

Write the expression to get the next stage:



Graph the relationship below.

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#### **ACTIVITY for Triangles and Quadrilaterals Lesson**

#### PART I

Each student is given an envelope containing triangles and quadrilaterals.

How might expressions be generated based on the contents of your envelope? What expressions could you write? First work individually. Next, compare with a partner. Finally, share as a whole class.

#### PART II

Each envelope contains a number of triangles and a number of quadrilaterals. For this exercise, let t represent the number of triangles, and let q represent the number of quadrilaterals.

a. Write an expression, using t and q, that represents the total number of sides in your envelope. Explain what the terms in your expression represent.

#### 3t+4q

Triangles have **3** sides, so there will be **3** sides for each triangle in the envelope. This is represented by **3t**. Quadrilaterals have **4** sides, so there will be **4** sides for each quadrilateral in the envelope. This is represented by **4q**. The total number of sides will be the number of triangle sides and the number of quadrilateral sides together.

b. You and your partner have the same number of triangles and quadrilaterals in your envelopes. Write an expression that represents the total number of sides that you and your partner have. If possible, write more than one expression to represent this total.

*3t*+*4q*+*3t*+*4q* 2(*3t*+*4q*) *6t*+*8q* 

Discuss the variations of the expression in part (b) and whether those variations are equivalent. This discussion helps students understand what it means to combine like terms; some students have added their number of triangles together and number of quadrilaterals together, while others simply doubled their own number of triangles and quadrilaterals since the envelopes contain the same number. This discussion further shows how these different forms of the same expression relate to each other. Students then complete part (c).

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c. Each envelope in the class contains the same number of triangles and quadrilaterals. Write an expression that represents the total number of sides in the room.

Answer depends on the seat size of the classroom. For example, if there are **12** students in the class, the expression would be 12(3t+4q), or an equivalent expression. Next, discuss any variations (or possible variations) of the expression in part (c), and discuss whether those variations are equivalent. Are there as many variations in part (c), or did students use multiplication to consolidate the terms in their expressions? If the latter occurred, discuss the students' reasoning.

Choose one student to open his/her envelope and count the numbers of triangles and quadrilaterals. Record the values of t and q as reported by that student for all students to see. Next, students complete parts (d), (e), and (f).

d. Use the given values of t and q, and your expression from part (a), to determine the number of sides that should be found in your envelope.

t=4 and q = 2 3t + 4q 3(4) + 4(2) 12 + 8 20There should be 20 sides contained in my envelope.

e. Use the same values for t and q, and your expression from part (b), to determine the number of sides that should be contained in your envelope and your partner's envelope combined.

Variation #1	Variation #2	Variation #3
2(3t+4q)	3t + 4q + 3t + 4q	6t + 8q
2(3(4) + 4(2))	3(4) + 4(2) + 3(4) + 4(2)	6(4) + 8(2)
2(12+8)	12 + 8 + 12 + 8	24 + 16
2(20)	20 + 12 + 8	40
40	40	

My partner and I have a combined total of 40 sides.

# f. Use the same values for t and q, and your expression from part (c), to determine the number of sides that should be contained in all of the envelopes combined

Variation 1	Variation 2	Variation 3
12(3t + 4q)	$\overline{3t+4q} + \overline{3t+4q} + \dots + \overline{3t+4q}$	36t + 48q
12(3(4)+4(2))	$\overline{3(4) + 4(2)} + \overline{3(4) + 4(2)} + \dots + \overline{3(4) + 4(2)}$	36(4) + 48(2)
12(12 + 8)	$\overline{3(4)+4(2)}$ + $\overline{3(4)+4(2)}$ + $\cdots$ + $\overline{3(4)+4(2)}$	144 + 96
12(20)	$\overline{12+8} + \overline{12+8} + \dots + \overline{12+8}$	240
240	$\frac{1}{20} + \frac{2}{20} + \dots + \frac{12}{20}$	240
For a class size of 12 st	udents, there should be 240 sides in all of the envelopes comb	ined.

Have all students open their envelopes and confirm that the number of triangles and quadrilaterals matches the values of t and q recorded after part (c). Then, have students count the number of sides contained on the triangles and quadrilaterals from their own envelope and confirm with their answer to part (d). Next, have partners count how many sides they have combined and confirm that number with their answer to part (e). Finally, total the number of sides reported by each student in the classroom and confirm this number with the answer to part (f).

### g. What do you notice about the various expressions in parts (e) and (f)?

The expressions in part (e) are all equivalent because they evaluate to the same number: 40. The expressions in part (f) are all equivalent because they evaluate to the same number: 240. The expressions themselves all involve the expression 3t + 4q in different ways. In part (e), 3t + 3t is equivalent to 6t, and 4q + 4q is equivalent to 8q. There appear to be several relationships among the representations involving the commutative, associative, and distributive properties.

When finished, have students return their triangles and quadrilaterals to their envelopes for use by other classes.

#### **Extension:**

Numbers of triangles and quadrilaterals can vary among students. Students can set up equivalent expressions (equations) using their available triangles and quadrilaterals.

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#### Intervention:

Envelopes can have adjusted numbers of triangles and quadrilaterals to make computation less cumbersome for students who need support with the concept of writing expressions using variables, and combining them.

## **PART III:**

Using shapes other than triangles and quadrilaterals, generate and model expressions and have a partner recreate your expressions given clues and instructions (from their partner).

For example, I am going to put squares and hexagons in an envelope (or get pattern blocks or draw models). Then, my partner will ask "yes or no" questions to decide what two shapes I am have chosen and how many of each I have.

### **CLOSING**

Two options for the closing are discussed below. One would be for students who still need additional practice on combining like terms. For students who are ready to combine like terms in expressions involving distributive property, a second option is available.

### Like Terms Closing Activity

**Pass out index cards (one per student) or cards from template at the end of this lesson, and tell students to find two other people who have a term that can be added to theirs.** They may not talk as they get up and silently look for their partners. Once the teams have been formed give them the following questions to answer:

1. What is the sum of your terms? Do you notice anything? Why do you think that happened? 2. If the value for your variable is -3, what is the value for your term? Each team member has an answer.

3. Based on the values found in #2, what is the difference between double the largest and double the smallest? What was the difference between the largest and smallest before you doubled them? What do you notice? Why do you think that happened?

NOTE: 24 cards are being provided. Modify the number of cards as needed. Since some of the values will get VERY Large/Small when making the substitution in for the variable, calculators can be available for student use.

### Match Up Card Closing Activity

Students will review the concept of equivalent expressions by playing the "Match Up" game in pairs. Each team will need a set of "Match Up" cards, a rule sheet, and a recording sheet. Two different versions (of equal difficulty) are provided.

Allow students to play the Match up game for 10 -15 minutes. Students who finish early may play again with the other set of cards provided.

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**\*NOTE:** Printing the cards on card stock and laminating them will prolong their use. Using different colors of paper (or gluing them to different colors of index cards) for set A and set B will help keep the sets together. It is useful to have the cards cut and put in envelopes prior to playing the game.

Cards, Rule Sheet, and Recording Sheet are provided at the conclusion of this LESSON.

#### **Other suggested lessons and activities**

Join the Club: Identifying and Combining Like Terms from NCTM Illuminations <u>http://illuminations.nctm.org/Lesson.aspx?id=3642</u> In this lesson, students learn the definition of like terms and gain practice in identifying key features to sort and combine them.

Teaching Channel Video (for teachers) on the distributive property with variables <u>https://www.teachingchannel.org/videos/teaching-the-distributive-property</u>

Like Terms Game from Math Warehouse <a href="http://www.mathwarehouse.com/games/our-games/like-terms-games/matching-action/">http://www.mathwarehouse.com/games/our-games/like-terms-games/matching-action/</a>

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**TEMPLATE** for Triangles and Quadrilaterals \*Use these templates to create triangles and quadrilaterals for the envelopes in the activity. Or, use pattern block as the triangles and quadrilaterals.



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## **LESSON: Triangles and Quadrilaterals**

Adapted from Engage NY

Name \_\_\_\_\_

#### PART I

Each student is given an envelope containing triangles and quadrilaterals.

How might expressions be generated based on the contents of your envelope? What expressions could you write?

#### PART II

Each envelope contains a number of triangles and a number of quadrilaterals. For this exercise, let t represent the number of triangles, and let q represent the number of quadrilaterals.

a. Write an expression, using t and q, that represents the total number of sides in your envelope. Explain what the terms in your expression represent.

b. You and your partner have the same number of triangles and quadrilaterals in your envelopes. Write an expression that represents the total number of sides that you and your partner have. If possible, write more than one expression to represent this total.

c. Each envelope in the class contains the same number of triangles and quadrilaterals. Write an expression that represents the total number of sides in the room.

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d. Use the given values of t and q, and your expression from part (a), to determine the number of sides that should be found in your envelope.

e. Use the same values for t and q, and your expression from part (b), to determine the number of sides that should be contained in your envelope and your partner's envelope combined.

f. Use the same values for t and q, and your expression from part (c), to determine the number of sides that should be contained in all of the envelopes combined

g. What do you notice about the various expressions in parts (e) and (f)?

### PART III:

Using shapes other than triangles and quadrilaterals, generate and model expressions and have a partner recreate your expressions given clues and instructions (from their partner).

## **TEMPLATE** for Like Terms Closing Activity

Cut the cards below out for use in the closing activity. You may print on cardstock or tape to index cards for more durability.

$3x^2$	$\frac{1}{3}x^2$	$-3x^2$
$3x^3$	$\frac{1}{3}x^3$	$-3x^{3}$
$3x^4$	$\frac{1}{3}x^4$	$-3x^4$
$3x^5$	$\frac{1}{3}x^5$	$-3x^{5}$
$3x^6$	$\frac{1}{3}x^6$	$-3x^{6}$
$3x^7$	$\frac{1}{3}x^7$	$-3x^{7}$
$3x^8$	$\frac{1}{3}x^8$	$-3x^{8}$
3 <i>x</i>	$\frac{1}{3}x$	-3x

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## Match Up Card Game Rules

Identifying Equivalent Expressions

#### Materials:

One set of 16 Match Up cards (one set for each pair of students);Two options provided Match Up Recording Sheet (one for each student)

Paper/pencil

2 players

### **Rules:**

Place the "Match Up" game cards face down on a flat surface.

- 2. The younger of the two players will go first (Player 1).
- 3. Player 1 will turn over two memory match cards from the set of game cards.
- 4. Player 1 will determine if the two cards flipped over are a "match". \*A "match" is made when two expressions are equivalent.

5. If the cards are equivalent expressions both players will record the equivalent expressions on the "Match Up" recording sheet under Player 1.

# Important: Player 2 will need to confirm that it is a match by using the corresponding properties to identify if they are equivalent.

6. The equivalent expression cards that were "matched" are placed in a discard pile.

7. If a match is made, Player 1 will be able to take a second turn during that round by turning over two more cards. If a match is not made, then that player's cards must be turned back over exactly where they were located and his/her turn is over. (**Do not** make notes on where cards are located)

\*This is an important step as it allows the other player to get a peek of the cards, so that they can try to remember the card for future rounds.

8. Player 2 will take his/her turn after Player 1 is completely finished and all matches are recorded on the recording sheet by both players.

9. Player 2 will repeat the same procedures for taking his/her turn as Player 1.

10. Continue playing until all equivalent expression cards have been matched. The winner of the game is the player with the most matches on the recording sheet.

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Match Up Ca	ard set A		
8( -5 - 6x)	-8(9x - 3)	-6(8x + 3) - 4x	12x +7
6x - 12	-48x -40	5(-7+3x)	-9 + 2(8 + 6x)
15x -35	-27x +38	9(5 - 3x) - 7	4(-x – 7)
-52x -18	-72x +24	3( -9x - 6) - 7x	-34x -18

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Match Up Ca	ard Set B		
-8(5x - 2) + 6	-4x - 28	-54x +36	30x +63
-9(6x - 4)	-27x + 38	19x -12	-20x +10
-2(-5+6x)-8x	-40x +22	9(5 - 3x) - 7	-3(4 - 2x)
6x - 8(9 - 4x)	-6x + 9(4x + 7)	38x -72	-3(4 - 7x) - 2x

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Match Up Recording Sheet Player 1\_\_\_\_\_

Record the cards that match in the table below (for each player) Use the Open Area on the Right for Scratch Work

Ш	
=	
=	
=	
=	
=	
=	
=	

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Record the cards that match in the table below (for each player) Player 2\_\_\_\_\_

## Use the Open Area on the Right for Scratch Work

=	
=	
=	
=	
=	
=	
=	
 =	
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## **Tiling Lesson**

Adapted from EngageNY A Story of Units This lesson extends the area model to find the area of a rectangle with fractional side lengths.

#### SUGGESTED TIME FOR THIS LESSON

60-90 minutes

The suggested time for the class will vary depending upon the needs of the students.

\*Also note that the teacher will need to create rectangles in advance for this lesson for the student activity (see notes in Materials Section).

#### STANDARDS FOR MATHEMATICAL CONTENT

# MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.

a. Apply properties of operations emphasizing when the commutative property applies. (MGSE7.EE.1)

b. Use area models to represent the distributive property and develop understandings of addition and multiplication (all positive rational numbers should be included in the models). (MGSE3.MD.7)

c. Model numerical expressions (arrays) leading to the modeling of algebraic expressions. (MGSE7.EE.1,2; MGSE9-12.A.SSE.1,3)

f. Evaluate formulas at specific values for variables. For example, use formulas such as A = l x w and find the area given the values for the length and width. (MGSE6.EE.2)

#### COMMON MISCONCEPTIONS

Students have difficulties understanding equivalent forms of numbers, their various uses and relationships, and how they apply to a problem. Make sure to expose students to multiple examples and in various contexts.

Students usually have trouble remembering to distribute to both parts of the parenthesis. They also try to multiply the two terms created after distributing instead of adding them. Students can struggle with operations with fractions. Students should be reminded of the activities with fractions used in Module 1 as needed.

#### STANDARDS FOR MATHEMATICAL PRACTICE

1. **Reason abstractly and quantitatively**. Students make sense of quantities and their relationships when they analyze a geometric shape or real life scenario and identify, represent, and manipulate the relevant measurements. Students decontextualize when they represent geometric figures symbolically and apply formulas

4. **Model with mathematics**. Students model with mathematics as they make connections between addition and multiplication as applied to area. They represent the area of geometric figures with equations and models, and represent fraction products with rectangular areas.

7. Look for and make use of structure. Students discern patterns and structures as they apply additive and multiplicative reasoning to determine area through application of the distributive property.

#### **EVIDENCE OF LEARNING/LEARNING TARGET**

By the conclusion of this lesson, students should be able to:

- Find the area of rectangles with whole-by-mixed and whole-by-fractional number side lengths by tiling, record by drawing, and relate to fraction multiplication.
- Find the area of rectangles with mixed-by-mixed and fraction-by-fraction side lengths by tiling, record by drawing, and relate to fraction multiplication.
- Measure to find the area of rectangles with fractional side lengths and variable side length.
- Multiply mixed number factors, and relate to the distributive property and the area model.
- Solve real world problems involving area of figures with fractional side lengths using visual models and the area formula.

#### MATERIALS

- patty paper units for tiling
- (Teacher) 3 unit  $\times$  2 unit rectangle
- (Students) 5 large mystery rectangles lettered A–E (1 of each size per group)

**LESSON Note**: The lesson is written such that the length of one standard patty paper  $(5\frac{1}{2}")$  by  $5\frac{1}{2}")$  is one unit. Hamburger patty paper (available from craft stores in boxes of 1,000) is the ideal square unit for this lesson due to its translucence and size. Measurements for the mystery rectangles are given in generic units so that any size square unit may be used to tile, as long as the tiling units can be folded. Any square paper (such as a square post it notes) may be used if patty paper is not available.

Consider color-coding Rectangles A-E for easy reference.

**Preparation**: Each group needs one copy of Rectangles A–E. The most efficient way of producing these rectangles is to use the patty paper to measure and trace the outer dimensions of one rectangle. Then use that rectangle as a template to cut the number required for the class. Rectangles should measure as follows:

Demo Rectangle A: 3 units  $\times$  2 units (See image) Rectangle B: 3 units  $\times$  2 ½ units Rectangle C: 1 ½ units  $\times$  5 units Rectangle D: 2 units  $\times$  1 ¾ units Rectangle E: ¾ unit  $\times$  5 units **ESSENTIAL QUESTIONS** 



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- What are strategies for finding the area of figures with side lengths that are represented by fractions or variables?
- How can area models be used to represent the distributive property?
- How is the commutative property of multiplication evident in an area model?
- How can the distributive property help me with computation?
- How can perimeter be used to find area in a rectangle?

#### SUGGESTED GROUPING THIS LESSON

This lesson begins as a whole group activity but transitions into a small group or partner activity. The first two problems will be completed as a group before students breaking into partners or small groups.

#### **KEY VOCABULARY**

The vocabulary terms listed below should be addressed (along with any others you feel students need support with) in the context of the LESSON and class discussion.

- Distributive Property: The sum of two addends multiplied by a number equals the sum of the product of each addend and that number.
- Algebraic expression: An expression consisting of at least one variable and also consisting of numbers and operations.
- Numerical expression: An expression consisting of numbers and operations.

#### **INSTRUCTIONAL STRATEGIES**

Strategies for this lesson will build on the area model and distributive property from 3<sup>rd</sup> grade. Students will draw a rectangular grid to model the distributive property as it relates to fractional components to see the correlation to their work with whole number dimensions. Video support for this strategy may be found at <u>https://learnzillion.com/lessons/1544-find-the-area-of-a-rectangle-by-multiplying-a-fraction-and-a-whole-number</u>

Help students to gain a fundamental understanding that the *distributive property* works "on the right" as well as "on the left," in addition to "forwards" as well as "backwards" by sharing examples that lend to a discussion of these facets of the distributive property. Examples such as: 3(2x + 4), (2x + 4)3, 6x + 12 could be used to show equivalent expressions with the distributive property.

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#### **OPENER/ACTIVATOR**

The following Number Talk activity could be used prior to the introduction to the Tiling Lesson. Using this Number Talk can open dialogue about mental strategies and number sense that will flow into the Tiling Lesson.

The purposes of a "Number Talk" include giving students opportunities to think and reason with numbers, helping students develop place value, number, and operation concepts, and helping students develop computational fluency. The activity could take anywhere from five to 15 minutes. Using a timer can help with keeping the Number Talk from extending too long. Teachers can explain the process and tell students at the onset about the time allowed for the given Number Talk.

In a Number Talk the teacher will give the class a problem to solve mentally. Students may use pencil and paper to keep track of the steps as they do the mental calculations but the goal is to avoid using traditional algorithms and encourage the use of student-led strategies. After all students have had time to think about the problem, students' strategies are shared and discussed to help all students think more flexibly as they work with numbers and operations.

Video footage from a class Number Talk may be seen at

http://www.insidemathematics.org/classroom-videos/number-talks/3rd-grade-math-one-digit-bytwo-digit-multiplication. Although many classrooms featured in Number Talk segments may be elementary grade level, middle and high school students can benefit from the flexible thinking patterns that number talks promote.

#### **Directions for this Number Talk:**

Write a multiplication expression **horizontally** on the board. (For example: 12 x 25)

Give students a minute to estimate the answer and record a few estimates on the board. High school students may be hesitant to do this out loud until they become more comfortable with the process. To still allow this important step in a "safe" manner, students could record their estimate on a sticky note or slip of paper WITHOUT putting their names and the teacher could quickly sort the estimates to share some with the class. This will allow the teacher to see how students are developing their number sense and operational use of strategies in solving problems.

Ask students to **mentally** find the solution using a strategy that makes sense to them. After all students have had a chance to think about the problem, they can then share their strategy (or strategies) with a partner or small group.

As students discuss their strategies, listen to their explanations and find a few strategies you want to share with the whole class. For example: Using  $10 \times 25$  to get 250 and  $2\times 25$  to get 50 then combining the results to get 300.

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Ask a student to fully explain the steps he/she followed to solve the problem and record the steps carefully while asking for clarification as needed. This is your chance as the teacher to point out the "big ideas" of the strategy used by the student. In the example above, this student used partial product to make the problem easier to solve.

Ask other students to share different methods they used for solving the equation and ask questions about why their strategies work.

Repeat the process with other similar problems as time allows.

Other problems to consider in the same nature as the one used in the example include: 16 x 25 32 x 50 13 x 12 102 x 9 12 x 25 19 x 99

More Number Talk information and examples may be found at: <u>http://schoolwires.henry.k12.ga.us/Page/37071</u>, <u>http://www.mathsolutions.com/documents/numbertalks\_sparrish.pdf</u> <u>https://sites.google.com/site/get2mathk5/home/number-talks</u>

#### **INTERVENTION/EXTENSION**

Use this link <u>http://www.raftbayarea.org/readpdf?isid=604</u> to access a card game called "Algebra Rummy". This game reviews key terminology such as coefficient, term, like terms, and expressions and also provides extension opportunities for students who need more of a challenge. Students will try to get "three of a kind" with like terms and combine them. Other options for game play are explained on the RAFT site. Teams of 3-4 students should be able to play the game in 10-15 minutes. Teachers may set a timer and declare the winner after a set number of minutes.

The NRICH article on multiplication using arrays is a supporting document found at <a href="http://nrich.maths.org/8773">http://nrich.maths.org/8773</a>

#### Teacher's Edition: <u>Tiling LESSON</u> (student pages follow teacher pages)

Directions for 1-5:

- Sketch the rectangles and your tiling.
- Write the dimensions and the units you counted in the blanks.
- Show the steps you take to solve the problems and justify your calculations.
- Use multiplication to confirm the area

Class Note: We will do Rectangles A and B together

\*Snapshot solutions to 1-5 courtesy of EngageNY

#### **Rectangle A: Sketch Below Showing Tiles**



**Rectangle B: Sketch Below Showing Tiles** 

#### 2. Rectangle B:



Question students as you draw the model about partitioning into sections to make the multiplication easier and apply the distributive property to solve the model.

$$3\left(2\frac{1}{2}\right) = 3\left(2+\frac{1}{2}\right) = 3\left(2+\frac{1}{2}\right) = 3(2) + 3\left(\frac{1}{2}\right) = 6 + \frac{3}{2} = 6 + 1\frac{1}{2} = 7\frac{1}{2}$$

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#### **Rectangle C: Sketch Below Showing Tiles**



**Rectangle D: Sketch below Showing Tiles** 

4. Rectangle D:



Rectangle D is

$$2$$
 units long  $1\frac{3}{4}$  units wide  
Area =  $3\frac{1}{2}$  units<sup>2</sup>

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**Rectangle E: Sketch Below Showing Tiles** 



6. You found a rectangle on the floor that had been split into an array as shown below.



a) Write an expression to find the area of this rectangle.

 $2(2 + \frac{1}{2}) \text{ or } 2(2) + 2(\frac{1}{2}) \text{ or } 2(2\frac{1}{2})$ 

b) Find the area of this rectangle and explain mathematically how you arrived at your solution.

The area of this rectangle is 5 units<sup>2</sup>. I used the distributive property to evaluate the expression for the

area represented by length x width or 2  $(2 + \frac{1}{2})$  which simplifies to  $2(2) + 2\left(\frac{1}{2}\right)$  and can be evaluated as 4 + 1 for a total of 5 units<sup>2</sup>.

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7. Draw a rectangle whose dimensions are  $5\frac{1}{3}$  units by 6 units.

a) Construct an array model within that rectangle to make computing its area easier and compute the area.

b) Justify your solution and model.

In this model I can multiply the width of 6 by the sum of the parts of the length of 5 and 1/3 to make the computation easier. I can use the distributive property to compute 6(5 + 1/3) as 6(5) + 6(1/3) to arrive at 30 + 2 or 32 units<sup>2</sup>.

8. A rectangle has dimensions  $2\frac{1}{2}$  units  $\times 4\frac{1}{2}$  units.

a) Draw an array showing where you would divide it to make your computation of the area of the rectangle easier.

b) Find the area and justify your answer. c) Find the perimeter of this rectangle.





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9. A rectangle has dimensions of a+2 by  $3\frac{1}{2}$ .

a) Draw an array showing where you would divide it to make your computation of the area of the rectangle easier.



b) Find the area and justify your answer.

 $(a + 2) (3 + \frac{1}{2}) = a (3 + \frac{1}{2}) + 2 (3 + \frac{1}{2}) = (a*3) + (a*1/2) + (2*3) + (2*1/2) = 3a + \frac{1}{2}a + 6 + 1 = (3 \frac{1}{2}a + 7) units^2$ 

I found this expression for the area using the distributive property based on my partial product area model.

c) Find the perimeter of this rectangle. The perimeter of this rectangle would represent the distance around the rectangle: length + length + width + width or 2\* length + 2\* width would be 2(a+2) + 2(3+1/2) = 2\*a + 2\*2 + 2\*3 + 2\*1/2 = 2a + 4 + 6 + 1 = (2a + 11) units.

10. You have a rectangle whose length is twice as much as its width. The perimeter of this rectangle is 36 units.

\*Note: Answers may vary based on the dimension chosen to be the variable side.

a) Draw a rectangle to model this relationship and label the sides based on the relationship between the length and the width. Use a single variable to represent this relationship but do not find the actual dimensions of the rectangle.



b) Which dimension, length or width, did you decide to make your primary variable? Students may choose the width as w since the length is twice the width. Or, students may choose the length as L since it is twice as much as the width.

c) How can you represent the other dimension using the primary variable you have chosen and the relationship stated in the problem?

If the student chose the width as the variable then the length would be 2w. If the student chose the length as the variable then the width would be  $\frac{1}{2}$  L since the width is half as much as the length.

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d) Write a variable expression to represent the perimeter of this rectangle.

P = 2(w+2w) which could simplify to 2(3w) or 2w + 4w which could simplify to 6wOr P = 2(1/2L + L) which could simplify to 2(3/2L) or 2(1/2L) + 2L which could simplify to 3L.

e) Using any method you choose, find the dimensions of the rectangle. Show all your steps to support your solution.

Students could use a variety of methods to find the dimensions. A few options are shown below but are not to be considered a complete list. Allow students to solve and justify their work.

One option would be to solve the equation 36=6w or 36=3L (depending on the choice of variable) to yield w=6 units and L=12 units.

Another option could be the use of a tape diagram where the unit could be w so L would get 2 units (since it is twice as much as w) and then the other side (width) would be another unit and the final side (length) would also be 2 units. The tape would then reveal that all 6 units total 36 giving each a value of 6.

W	W	W	W	W	W
width	Ler	igth	width	Le	ength

So 6w = 36 units with each bar equal to 6 units

Students might have worked with tape diagrams (also called bar models) in elementary grades. For more information about tape diagrams (bar models) please visit one or more of the following suggested websites:

https://www.engageny.org/sites/default/files/resource/attachments/04-

introduction\_to\_tape\_diagrams.pdf

https://learnzillion.com/lessons/589-solve-ratio-problems-using-a-tape-diagram

http://www.mathplayground.com/tb\_ratios/thinking\_blocks\_ratios.html

http://www.thesingaporemaths.com/

http://www.utahsmathfuture.com/usj.pdf

f) Write a variable expression to represent the area of this rectangle. Students could represent area as either w(2w) or  $2w^2$  or as  $L(\frac{1}{2}L)$  or  $\frac{1}{2}L^2$ .

g) Based on the dimensions you found, what is the area of this rectangle? Since students found the width to be 6 units and the length to be 12 units, the area of the rectangle would be 72 units<sup>2</sup>.

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#### **EXTENSION**

Challenge students to investigate different ways to break apart the arrays in this lesson. Students can summarize ways they find which make the computation easier or just different. Students could also look for area of composite figures using the distributive property as appropriate or consider problems where dimensions such as perimeter and one side length are given and they are asked to find the area.

#### Sample Extension Problems:

- Rachel made a mosaic from different color rectangular tiles. Three tiles measured  $3\frac{1}{2}$  inches × 3 inches. Six tiles measured 4 inches ×  $3\frac{1}{4}$  inches. What is the area of the whole mosaic in square inches?
- A rectangle has a perimeter of  $35\frac{1}{2}$  feet. If the length is 12 feet, what is the area of the rectangle?

#### **REVIEW/INTERVENTION**

For students who are struggling with the concept of area and the distributive property, provide more opportunities to count the actual units of the models to better understand the connection between the formula for area, multiplication using the distributive property, and the actual area of rectangular regions divided into arrays.

#### **CLOSING/SUMMARIZER**

To close or summarize this LESSON, please revisit the essential questions listed at the beginning. Have students discuss and explain in multiple ways the concepts listed in the essential questions.

#### **Possible Internet Activities**:

The **NRICH Project** aims to enrich the mathematical experiences of all learners. To support this aim, members of the NRICH team work in a wide range of capacities, including providing professional development for teachers wishing to embed rich mathematical LESSONs into everyday classroom practice and is part of the family of activities in the <u>Millennium</u> <u>Mathematics Project</u> from the University of Cambridge.

Perimeter Possibilities http://nrich.maths.org/9691 allows for further investigation between area and perimeter and whole number versus fractional dimensions.

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#### **Illustrative Mathematics**

7.EE.1– Equivalent Expressions: The purpose of this lesson is to directly address a common misconception held by many students who are learning to solve equations. Because a frequent strategy for solving an equation with fractions is to multiply both sides by a common denominator (so all the coefficients are integers), students often forget why this is an "allowable" move in an equation and try to apply the same strategy when they see an expression. Two expressions are equivalent if they have the same value no matter what the value of the variables in them. After learning to transform expressions and equations into equivalent expressions and equations, it is easy to forget the original definition of equivalent expressions and mix up which transformations are allowed for expressions and which are allowed for equations.

Engage NY Curriculum 7<sup>th</sup> grade Module 3 – Expressions and Equations Expressions

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#### **LESSON:** Tiling

Name \_\_\_\_\_

Directions for 1-5:

- Sketch the rectangles and your tiling.
- Write the dimensions and the units you counted in the blanks.
- Show the steps you take to solve the problems and justify your calculations.
- **Use multiplication** to confirm the area.

\*We will do Rectangles A and B together

#### 1. Rectangle A: Sketch Below Showing Tiles

Rectangle A is	units long	units wide	Area =	units <sup>2</sup>
*Remember each til	le is one unit long and o	one unit wide		

2. Rectangle B: Sketch Below Showing Tiles

Rectangle B is \_\_\_\_\_ units long \_\_\_\_\_ units wide Area = \_\_\_\_\_ units<sup>2</sup>

**3. Rectangle C:** Sketch Below Showing Tiles

Rectangle C is \_\_\_\_\_ units long \_\_\_\_\_ units wide  $Area = \____ units^2$ 

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#### 4. Rectangle D: **Sketch Below Showing Tiles**

Rectangle D is \_\_\_\_\_ units long \_\_\_\_\_ units wide  $Area = \____ units^2$ **Sketch Below Showing Tiles** 

Rectangle E is \_\_\_\_\_ units long \_\_\_\_\_ units wide  $Area = \____ units^2$ 

#### 6. You found a rectangle on the floor that had been split into an array as shown below.



5. Rectangle E:

a) Write an expression to find the area of this rectangle.

b) Find the area of this rectangle and explain mathematically how you arrived at your solution.

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7. Draw a rectangle whose dimensions are  $5\frac{1}{3}$  units by 6 units.

a) Construct an array model within that rectangle to make computing its area easier.

b) Justify your solution and model.

8. A rectangle has dimensions  $2\frac{1}{2}$  units  $\times 4\frac{1}{2}$  units.

a) Draw an array showing where you would divide it to make your computation of the area of the rectangle easier.

b. Find the area and justify your answer.

c. Find the perimeter of this rectangle.

9. A rectangle has dimensions of a+2 by  $3\frac{1}{2}$ .

a) Draw an array showing where you would divide it to make your computation of the area of the rectangle easier.

- b) Find the area and justify your answer.
- c) Find the perimeter of this rectangle.

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10. You have a rectangle whose length is twice as much as its width. The perimeter of this rectangle is 36 units.

a) Draw a rectangle to model this relationship and label the sides based on the relationship between the length and the width. Use a single variable to represent the relationship but do not find the actual dimensions of the rectangle.

b) Which dimension, length or width, did you decide to make your variable?

c) How can you represent the other dimension using the variable you have chosen and the relationship stated in the problem?

d) Write a variable expression to represent the perimeter of this rectangle.

e) Using any method you choose, find the dimensions of the rectangle. Show all your steps to support your solution.

f) Write a variable expression to represent the area of this rectangle.

g) Based on the dimensions you found, what is the area of this rectangle?

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## **Conjectures About Properties**

This lesson is designed to connect properties of arithmetic to algebra. Students will examine properties in the context of variable expressions.

#### SUGGESTED TIME FOR THIS LESSON

#### 90 -120 minutes

The suggested time for the lesson will vary depending upon the needs of the students.

#### STANDARDS FOR MATHEMATICAL CONTENT

# MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.

a. Apply properties of operations emphasizing when the commutative property applies. (MGSE7.EE.1)

c. Model numerical expressions (arrays) leading to the modeling of algebraic expressions. (MGSE7.EE.1,2; MGSE9-12.A.SSE.1,3)

e. Generate equivalent expressions using properties of operations and understand various representations within context. For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between "3 more" and "3 times". (MGSE4.0A.2; MGSE6.EE.3, MGSE7.EE.1,2,MGSE9-12.A.SSE.3)

#### COMMON MISCONCEPTIONS

Students are almost certainly not going to know or understand why division by zero is not possible. You will need to provide contexts for them to make sense of this property. To avoid an arbitrary rule pose problems to be modeled that involve dividing by zero: "Take thirty counters. How many sets of zero can be made?" or "Put twelve blocks in zero equal groups. How many are in each group?"

#### STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of properties by identifying a rule that matches a group of equations.

**3. Construct viable arguments and critique the reasoning of others.** Students construct explanations about properties.

6. **Attend to precision.** Students use the language of the Commutative, Associative, Identity, Zero Property of Multiplication, and Distributive properties.

7. Look for and make use of structure. Students apply properties to generate equivalent expressions.

8. Look for and express regularity in repeated reasoning. Students will identify properties and reason why that group is identified with a certain property.

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#### **EVIDENCE OF LEARNING/LEARNING TARGETS**

- Apply properties to simplify or evaluate expressions
- Apply properties to generate equivalent expressions

#### **MATERIALS**

- lesson sheet
- manipulative to show grouping (put 12 counters into groups of zero) if needed

#### **ESSENTIAL QUESTIONS**

- How can I tell if a group of equations satisfies a property? (Commutative, Associative, Identity, Zero Property of Multiplication, and Distributive)
- How are properties of numbers helpful in computation?

#### **KEY VOCABULARY**

The vocabulary terms listed below should be addressed in the context of the lesson and class discussion. The terms are defined and explained in the teacher notes section of the lesson in blue:

- Commutative Property
- Associative Property
- Identity Property
- Zero Property of Multiplication Property
- Distributive Property

#### **ACTIVATOR/OPENING**

The following problem was originally featured on Dan Meyers' Blog: <u>http://blog.mrmeyer.com/the-most-interesting-math-problems-to-me-right-now/</u>

Ask students to follow the steps listed below:

- Everybody pick a number.
- Multiply it by four.
- Add two.
- Divide by two.
- Subtract one.
- Divide by two again.
- Now subtract your original number.

On the count of three, everybody say the number you have.

- What do you notice?
- Why do you think that happened?
- Does it matter what number we pick?
- Can we prove/show that this will always be true?

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At this point you could lead a discussion of the problem where the chosen number is represented by a variable and make the parallel between operations with numbers and operations with variable.

\*Extension Opportunity for Number Patterns from <u>http://mathprojects.com/2012/10/05/number-tricks-student-sample/</u>

<u>Number Tricks</u> is a lesson that involves writing and simplifying expressions. It demands higher order thinking skills in several ways. 1) The students are to write a mathematical model for a trick given to them. 2) They are to create their own trick and offer the algebraic expression that represents it. 3) It presses the students to understand the concept of a variable; in this case, the variable represents the number originally chosen. 4) The students are asked to compare their simplified expression to the pattern generated by the various numbers tested. The lesson offers a great opportunity for a high level of critical thinking with a rather low level piece of content.

On this site you can find the following Error Analysis opportunity:

Here is an **erroneous** submission from my Algebra class. I want to analyze the mistake and discuss why this lesson was so very good for this student even though the "answer was wrong." This was Dewey's response to creating his own Number Trick, including 3 numbers to generate the pattern, and the algebraic expression it represents:

Pick a number	3	10	-7	X	
Add 4	7	14	-3	x + 4	
Multiply by 2	14	28	-6	2x + 4	
Subtract 3	11	25	-9	2x + 4 - 3	
Subtract the Original Number815-2 $2x + 4 - 3 - x$					
Simplified: x + 1					
Common Result: one more than the number picked					

Have students look for and explain the mistake made in the table above. Error analysis helps students see math in a different light and learn to be more evaluative of their own work.

Introducing variable operations and reviewing number operations sets a mental stage for the Properties Lesson.

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#### **Introduction to Lesson**

*Numerical expressions* are meaningful combinations of numbers and operation signs. A *variable* is a letter or other symbol that is a placeholder for an unknown number or a quantity that varies. An expression that contains at least one variable is called an *algebraic expression*.

When each variable in an algebraic expression is replaced by a number, the result is a numerical expression whose value can be calculated. This process is called *evaluating the algebraic expression*.

In this lesson the idea of substituting variables to represent numbers is introduced in the context of making conjectures about number properties.

**Before the Lesson:** Post the following expressions on the board. Ask students to decide if they are true or false. And, ask them to explain, model, show, or prove their response.

36 + 45 = 45 + 36 123 + 24 = 24 + 123 4 + 6 = 6 + 4

- Looking at these three number sentences, what do you notice?
- Do you think this is true for all numbers?
- Can you state this idea without using numbers?

By using variables we can make a number sentence that shows your observation that "order does not matter when we add numbers."

#### a + b = b + a

- Do you think this is also true for subtraction?
- Do you think it is true for multiplication?
- Do you think it is true for division?

Repeat with examples to consider for other operations as needed.

#### **Teacher Notes**

The full class should discuss the various conjectures, asking for clarity or challenging conjectures with counterexamples. Conjectures can be added to the class word wall with the formal name for the property as well as written in words and in symbols. The example provided above was for the commutative property. More properties will be explored in the following exploratory lesson.

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#### A Closer Look at Cluster 6.EE (part of MFAAA1 and 2)

In the standards for grades K–5, arithmetic is both a life skill and a thinking subject—a rehearsal

for algebra. Students in grades K–5 *calculated*, but they also *operated*. For example, students used the distributive property and other properties of operations as they came to learn the standard algorithms for multi-digit multiplication in grades 3 through 5. And students learned about the meanings of operations as they solved word problems with

**6.EE.A.** Apply and extend previous understandings of arithmetic to algebraic expressions.

the basic operations. (Note that the four operations mean the same thing, model the same quantitative relationships, and solve the same kinds of word problems regardless of whether the numbers involved are whole numbers, fractions, decimals, or any combination of these—or even a variable standing for any of these.) In grade 6, students use properties of operations and meanings of operations as a pivot from arithmetic to algebra.

For example, standard 6.EE.A.3 requires students to apply the properties of operations to generate equivalent expressions. This table highlights the coherence between the grade 6 expectations and the expectations of previous grades.

<b>Property of</b>	Previous Grade Expectation	Grade 6 Expectation
Operations		from 6.OA.A.3
Distributive	$8 \times (5+2) = (8 \times 5) + (8 \times 2)$ which is 56	24x + 18y = 6(4x + 3y)
	(3.OA.B.5)	
Associative	$3 \times 5 \times 2 = 15 \times 2 \text{ OR } 3 \times 10 $ (3.OA.B.5)	$3 \times r \times 5 = 15 \times r \text{ OR } 3r \times 5 \text{ OR}$
		$3 \times 5r$
Commutative	$4 \times 6 = 24$ , so $6 \times 4 = 24$ (3.OA.B.5)	$r \times 6 = 24$ , so $6r = 24$
Addition and	Interpret 5 × 7 as 5 groups of 7 objects	Interpret y + y + y of 3y
Multiplication	Interpret 5 × 7 as 5 groups of 7 objects	Interpret $y + y + y$ as $3y$

The cluster is not only about extending these skills, but also applying them.

#### **Additional Suggested Activities**

The Teaching Channel <u>https://www.teachingchannel.org/videos/think-pair-share-lesson-idea</u> Provides lesson plan and video support for a think-pair-share activity aligned to 7.EE.1 Simplifying Expressions

The Teaching Channel <u>https://www.teachingchannel.org/videos/class-warm-up-routine</u> Provides a warm up activity called "My Favorite No" which uses an example problem on simplifying expressions.

#### **EXPLORATION LESSON: CONJECTURE ABOUT PROPERTIES**

With a partner look at the following sets of number sentences and determine if what you observe would be true for all numbers. Create statements with words about what you observe in each set of number sentences then write the number sentences using variables to represent numbers.

Property articulations by students may vary. They do not have to be exact language but mathematical ideas must be sound and variable representations precise.

$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2	12 - 0 = 12 Y - 0 = 0 64 - 0 = 64	3	a * 1 = a 37 * 1 = 37 64 * 1 = 64	
Identity Property of Additi	on Ide	entity Property of Subt	raction	Identity Property of	
a + 0 = a		a - <b>0</b> = a		<b>Multiplication</b>	
				a x 1 = a	
$4  12 \div 1 = 12$	5	c * 0 = 0	6	$12 \div 0 = 12$	
$37 \div 1 = 37$		37 * 0 = 0		$f \div 0 = f$	
$b \div 1 = b$		64 * 0 = 0		$64 \div 0 = 64$	
Identity Property of Division	on Ze	ro Property of Multipli	ication	Undefined – See	
$a \div l = a \text{ or } \frac{a}{1} = a$		$a \ge 0$		summary notes ab	ove
7 12(4 + 3) = 48 +	36	8	12(	(4-3) = 48	
5(6+c) = 30+5		6(x	(x-2) = 6x - 12		
4(10 + 3) = 40 +		4(1	(0-3) = 40 - 12		

Distributive Property a(b + c) = ab + ac(This will need to be revisited during integer exploration.) Distributive Property a(b-c) = ab - ac

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#### **SUMMARY OF PROPERTIES**

<u>The Commutative Property</u>: Changing the order of the values you are adding or multiplying does not change the sum or product. 6+4=4+6  $a \bullet b = b \bullet a$ 

<u>The Associative Property</u>: Changing the grouping of the values you are adding or multiplying does not change the sum or product. (2+7)+3=2+(7+3) (ab)c = a(bc)

<u>The Identity Property</u>: The sum of any number and zero is the original number (additive identity). The product of any number and 1 is the original number (multiplicative identity).

The Distributive Property of Multiplication over Addition and Subtraction helps to evaluateexpressions that have a number multiplying a sum or a difference.Example:9(4+5) = 9(4) + 9(5)5(8-2) = 5(8) - 5(2)a(b+c) = ab + aca(b-c) = ab - ac

After completion of the Properties lesson, the following closing activity is suggested.

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#### **CLOSING ACTIVITY: Number Talks Activity**

Adapted from New Zealand Numeracy Project

http://nzmaths.co.nz/resource/homework-sheet-stage-6-7-revision-add-sub-and-mult-div-2

# While Module One spent a considerable amount of time developing operations with numbers, this Number Talks activity revisits those operations and connects to the properties examined in the "Conjecture about Properties" Lesson.

The following problems open discussion about how numbers relate to each other and how number properties allow efficient methods for solving problems mentally.

- Allow students independent working time on the following problem (approximately 10-30 minutes depending on the level of your students).
- Ask students to share ideas with a partner or small group (10 minutes or more depending on the level of your students and their attention to LESSON) about methods they used to solve the problems as you walk around the room noting methods you would like to highlight during full group discussion.
- Bring the group back together to highlight uses of number properties that you heard during small group discussion. Students should share ideas and strategies for solving the problems.

NOTE: The following problems could be given to the students in written form, posted around the room for review, or written on the board.

#### Mental Math

Work out the answers to these problems in your head. Use the quickest method for each problem, and then record your strategy on paper so that other people can understand how you have done the problem

1)	2 + 25 +8	2)	57 - 15 + 3	3)	5 x 37 x 2
4)	257 + 199	5)	478 - 125	6)	4 × 37
7)	140 ÷ 4	8)	198 - 74	9)	125 ÷ 10
10)	195 × 4	11)	235 - 77	12)	198 - 99
13)	179 + 56	14)	128 ÷ 5	15)	345 + 60
16)	63 × 10	17)	$26 \times 5$	18)	612 ÷ 6
19)	264 ÷ 6	20)	257 + 356	21)	374 - 189

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Note: There are multiple ways to look at each problem; therefore, solutions are not listed for all problems. A representative sample of problems is listed below. The strategies listed DO NOT include all options. More information about mental strategies may be found on the New Zealand Numeracy website <u>http://nzmaths.co.nz/</u> or in Number Talks resources with sample video clips at <u>http://www.insidemathematics.org/classroom-videos/number-talks</u>. Additional sample problems and strategies may be accessed at the end of this module. Use these strategy explorations as time permits each day in class to see strengthening of number sense in students.

1) 2 + 25 + 8 could be solved using the commutative property and associative property:
2 + 8 + 25 commutative property
(2+8) + 25 associative property
10 + 25
35

10) 195 x 4 could be solved by (200 x 4) - (5 x 4) 800 - 20 780

Or by partial products  $(100 \ x \ 4) + (90 \ x \ 4) + (5 \ x \ 4)$  400 + 360 + 20780 14) 128 ÷ 5 could be decomposed to  $(100 \div 5) + (20 \div 5) + (8 \div 5)$  to get 20 + 4 + 1 + 3/5 to get  $25 \frac{3}{5}$ 

**INTERVENTION** For extra help with properties, please open the hyperlink Intervention Table.

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#### Name\_\_\_\_\_ LESSON: CONJECTURES ABOUT PROPERTIES

With a partner look at the following sets of number sentences and determine if what you observe would be true for all numbers. Create statements with words about what you observe in each set of number sentences then write the number sentences using variables to represent numbers.

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#### Additional Practice on Simplifying Expressions

**TEACHER'S EDITION** 

Please use the following problems as needed for additional practice on simplifying variable expressions.

Create a like term for the given term (NOTE: answers will vary as there are unlimited options)

- 1.  $4x \quad 2x \quad 2. -7y \quad 3/4y \quad 3. 12ab 3ab$
- 4.  $\frac{2}{3}x^2 \frac{5x^2}{5x^2}$  5.  $-5ax^2 \frac{4ax^2}{4ax^2}$  6.  $14c \frac{-7c}{4ax^2}$

Simplify the expression if possible by combining like terms

- 6.  $7y + 10y \underline{17y}$  7.  $6x + 8y + 2x \underline{8x + 8y}$
- 8.  $5x + 2(x + 8) \frac{7x + 16}{9}$  9.  $5x 2(x 8) \frac{3x + 16}{9}$
- 10. 9(x + 5) + 7(x 3) 16x + 24 11. 8 (x 4)2 16 2x
- 12.  $\frac{2}{3}(12x-6) \frac{8x-4}{2}$  13.  $\frac{1}{2}(12x-6) + \frac{1}{2}(12x-6) \frac{12x-6}{2}$
- 14. 9y + 4y 2y + y 12y 15. x + 5x 17 + 12 + x 7x 5
- 16. Add your answer from #10 to your answer for  $#13\underline{28x + 18}$
- 17. Double your answer for #8 and add it to half of your answer for #13 20x + 29
- 18. Subtract your answer for #9 from triple your answer for #12 21x 28

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Simplifying Expressions		Name					
Create a like term for the	given term						
1.4x	27y	3. 12ab					
4. $\frac{2}{3}x^2$	55ax <sup>2</sup>	6. 14c					
Simplify the expression if	possible by con	abining like terms					
6. 7 <i>y</i> + 10 <i>y</i>	-	7. $6x + 8y + 2x$					
8. $5x + 2(x + 8)$		9. $5x - 2(x - 8)$					
10. 9(x + 5) + 7(x - 3) _		11. $8 - (x - 4)2$					
12. $\frac{2}{3}(12x-6)$		$13.\frac{1}{2}(12x-6) + \frac{1}{2}(12x-6)$					
14. 9 $y$ + 4 $y$ - 2 $y$ + $y$		15. $x + 5x - 17 + 12 + x$					
16. Add your answer from #10 to your answer for #13							
17. Double your answer for #8 and add it to half of your answer for #13							
18. Subtract your answer for #9 from triple your answer for #12							

# **Quick Check I**

#### STANDARDS FOR MATHEMATICAL CONTENT

# MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.

a. Apply properties of operations emphasizing when the commutative property applies. (MGSE7.EE.1)

b. Use area models to represent the distributive property and develop understandings of addition and multiplication (all positive rational numbers should be included in the models). (MGSE3.MD.7)

c. Model numerical expressions (arrays) leading to the modeling of algebraic expressions. (MGSE7.EE.1,2; MGSE9-12.A.SSE.1,3)

d. Add, subtract, and multiply algebraic expressions. (MGSE6.EE.3, MGSE6.EE.4, MC7.EE.1, MGSE9-12.A.SSE.3)

e. Generate equivalent expressions using properties of operations and understand various representations within context. For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between "3 more" and "3 times". (MGSE4.0A.2; MGSE6.EE.3, MGSE7.EE.1, 2, MGSE9-12.A.SSE.3)

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### **Quick Check I**

#### **TEACHER'S EDITION**

#### Simplify the expression

1. 2.3 + 0.5(3 + x) = 0.5x + 3.8 (or equivalent values)

2. 
$$\frac{8}{6} + \left(\frac{3}{8} + x\right)(2) = 2x + \frac{50}{24}$$
 (or equivalent values)

3.2y + 3x + 5y + 4(6x) = 27x + 7y

4. Circle all the equivalent expressions listed below: The correct answers are circled below:



5. Justify and explain how you know the expressions circled above are equivalent.

Students may explain this in a variety of ways. For example: "I distributed the 7, so  $7(b+5) + 3 = 7 \times b + (7 \times 5) + 3$ . Because  $7 \times b$  can be expressed as 7b, I know that 7(b+5) + 3 is equivalent to  $7b + (7 \times 5) + 3$ . Then, I found that the value of  $(7 \times 5) + 3$  is 38. So,  $7b + (7 \times 5) + 3$  is equivalent to 7b + 38. That means all three expressions are equivalent."

6. There is one mistake in the work shown below. Next to the first incorrect equation, write the correct result, and then, correct the rest of the problem.

P + P + 6(3P + 4) + 4P	=	2P + 6(3P + 4) + 4P	
	=	6(3P+4) + 2P + 4P	
	=	3P + 4 + 6 + 2P + 4P	18P + 24 + 2P + 4P
	=	3P + 4 + 6 + 6P	18P + 2P + 4P + 24
	=	9 <i>P</i> + 10	24P + 24

7. Write an expression for the perimeter of this triangle. k + 6 + 3 or 9 + k or equivalent



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8. Write an expression for the perimeter of this triangle.  $\_T + T + T \text{ or } 3T \text{ or equivalent}\_$ 



9. Draw a rectangular array in order to find the solution to  $2\frac{1}{3} \times 3\frac{1}{4}$ .



10. Explain your solution to #9 above.

Students may explain this in a variety of ways.

For example: 
$$2\frac{1}{3} \times 3\frac{1}{4} = (2 + \frac{1}{3}) \times (3 + \frac{1}{4})$$
  
 $= (2 \times 3) + (2 \times \frac{1}{4}) + (\frac{1}{3} \times 3) + (\frac{1}{3} \times \frac{1}{4})$   
 $= 6 + \frac{1}{2} + 1 + \frac{1}{12}$   
 $= 7 + \frac{7}{12}$   
 $= 7\frac{7}{12}$ 

I found the area of each rectangle and then added those areas together.

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 Quick Check 1
 Name: \_\_\_\_\_

Simplify the expression 1. 2.3 + 0.5(3 + x) = \_\_\_\_x + \_\_\_\_ 2.  $\frac{8}{6} + (\frac{3}{8} + x)(2) = ____x + ____$  $3. 2y + 3x + 5y + 4(6x) = ____x + ____y$ 4. Circle all the equivalent expressions listed below:

- 7(b+5)+3 b+38  $7b+7\times8$  

   7b+38  $7b+(7\times5)+3$
- 5. Justify and explain how you know the expressions circled above are equivalent.

6. There is one mistake in the work shown below. Next to the first incorrect equation, write the correct result, and then, correct the rest of the problem.

P + P + 6(3P + 4) + 4P	=	2P + 6(3P + 4) + 4P	
	=	6(3P+4) + 2P + 4P	
	=	3P + 4 + 6 + 2P + 4P	
	=	3P + 4 + 6 + 6P	
	Ξ	9 <i>P</i> + 10	

7. Write an expression for the perimeter of this triangle.



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8. Write an expression for the perimeter of this triangle.



9. Draw a rectangular array in order to find the solution to  $2\frac{1}{3} \times 3\frac{1}{4}$ .

10. Explain your solution to #9 above.

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## Visual Patterns

Original Source for lesson idea: <u>http://www.visualpatterns.org/</u>

This lesson uses visual patterns to provide a connection from concrete or pictorial representations to abstract algebraic relationships and to help students to think algebraically. Unfortunately, students are often directed to make a table to find a pattern, rather than building strong figural reasoning.

#### SUGGESTED TIME FOR THIS LESSON

120 minutes

The suggested time for the lesson will vary depending upon the needs of the students.

Initially, 120 minutes (depending on your students) should be planned for the activity outlined below. Continued time (throughout this module and into future modules) should be set aside for students to investigate these types of patterns. Students should be given multiple chances to engage in these investigations with different visual patterns.

#### STANDARDS FOR MATHEMATICAL CONTENT

# **MFAAA1.** Students will generate and interpret equivalent numeric and algebraic expressions.

c. Model numerical expressions (arrays) leading to the modeling of algebraic expressions. (MGSE7.EE.1,2; MGSE9-12.A.SSE.1,3)

d. Add, subtract, and multiply algebraic expressions. (MGSE6.EE.3, MGSE6.EE.4, MC7.EE.1, MGSE9-12.A.SSE.3)

e. Generate equivalent expressions using properties of operations and understand various representations within context. For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between "3 more" and "3 times". (MGSE4.0A.2; MGSE6.EE.3, MGSE7.EE.1,2,MGSE9-12.A.SSE.3)

f. Evaluate formulas at specific values for variables. For example, use formulas such as A = 1 x w and find the area given the values for the length and width. (MGSE6.EE.2)

#### COMMON MISCONCEPTIONS

When working with visual patterns to predict future stages and generalize for all stages, students confuse parts of the patterns that remain and parts that change. Using color coded tiles of models can help students arrive at the pattern more readily. Also, as students graph the stage versus pattern (i.e. number of squares or triangles), they want to connect the points on the graph. This data is not continuous data (it is discrete) so points should not be connected. There is no way to have "half a stage".

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#### STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will make sense of the problems by building, extending, creating and explaining visual patterns.

2. **Reason abstractly and quantitatively.** Students will reason with quantities of objects in the patterns, and then create abstract generalizations to write expressions and equations that explain the patterns.

3. Construct viable arguments and critique the reasoning of others. Students will share their expressions/equations with other groups and students and discuss the validity of each. Students will likely discover equivalent expressions/equations within the class.

4. **Model with mathematics.** Students will model the patterns they are studying using materials, diagrams and tables as well as equations.

5. Use appropriate tools strategically. Students will choose appropriate tools to solve the visual patterns.

6. Attend to precision. Students will attend to precision through their use of the language of mathematics as well as their computations.

7. Look for and make use of structure. Students will apply properties to generate equivalent expressions. They interpret the structure of an expression in terms of a context. Students will identify a "term" in an expression.

8. Look for and express regularity in repeated reasoning. The repeated reasoning required to explain patterns is evident in this lesson. Students will express this regularity through their conversations with one another and the class.

#### EVIDENCE OF LEARNING/LEARNING TARGETS

By the conclusion of this lesson, students should be able to:

- Represent the next stage in a pattern.
- Generalize a pattern to represent future stages.
- Recognize equivalent representations of algebraic expressions.

#### MATERIALS REQUIRED

- Various manipulatives such as two color counters
- Color tiles
- Connecting cubes
- Visual Patterns Handout (proceeds teacher notes)

#### ESSENTIAL QUESTIONS

- What strategies can I use to help me understand and represent real situations using algebraic expressions?
- How are the properties (Identify, Associative and Commutative) used to evaluate, simplify and expand expressions?
- How can I represent a pattern using a variable expression?
- How can I tell if two expressions used to generate a pattern are equivalent?
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#### **OPENING/ACTIVATOR**

Display the following pattern on the board and ask students to generate statements about what they notice and what they wonder. Encourage them to list as many things as they can for each part (notice and wonder).



After students have time to think and list a few items, make a list of things the class noticed and what they are curious about. Use their ideas as a launch into the visual patterns below.

#### **TEACHER NOTES**

The use of visual patterns to concretize algebraic relationships and teach students to think algebraically is not uncommon. Unfortunately, students are directed to make a table to find a pattern, rather than building strong figural reasoning. This often oversimplifies the lesson meaning that students merely count to complete a table and ignore the visual model. This creates iterative thinking such as (I just need to add 5 each time), guessing and checking, or the application of rote procedures, rather than an understanding of the structural relationships within the model.

This lesson was created to introduce teachers and students to a resource filled with visual patterns. Teachers can assign 2 or 3 patterns per pair of students, initially. As students become more confident in their ability, visual patterns can be assigned individually.

Fawn Nguyen, a middle school math teacher in California, has put together a library of visual patterns <u>here</u>. In the teacher section on this site is a helpful <u>tool for assigning random visual</u> <u>patterns</u> to students as well as <u>a form for students to use</u> to help them organize their thinking.

Students need multiple experiences working with and explaining patterns. Giving students a visual pattern to build concretely, allows students to experience the growth of the pattern and explain it based on that experience.

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As students begin investigating visual patterns, ask them to look at how it grows from one stage (or step) to the next. When they build the pattern using manipulatives, have them use one color to begin with. Does any part of the pattern seem to be staying the same? Whatever students "see" as staying the same have them replace those pieces with a different color.

For example: **Pattern 1**:



Students may be investigating the visual pattern on the left. Initially, students will build the pattern with materials such as color tiles and use all blue (for example) tiles to build it.

Next, after students look for parts that stay the same as the pattern grows, they use a different color tile to show what parts stay the same. These students thought that the "part that sticks out on the right" stay the same. Another group may say that "the bottom two of each step stay the same. Either way, students are encouraged to go with it.

Step 1 Step 2 Step 3

Once students have identified the part that stays the same (the constant) and the part that changes (the variable), students can begin organize their thinking. One way to do this is through the use of an expanded t-table. This provides a place to organize all parts and the whole of the visual pattern based on the how students see it growing.

One version of this kind of table can be seen below. It's important that students determine what information is important to keep up with in the table.

Step	Sketch	Stays	Changes	Total
(Stage)		Same		(tiles in
				this case)
1	This column allows students to keep a record of their	1	3	4
2	thinking. If they run out of time, they can pick up	1	4	5
3	where they left off later without having to start from	1	5	6
4	scratch.	1	6	7
5		1	7	8
13		1	15	16
37		1	39	40
Ν		1	n +2	n+2+1

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As students reason about the visual patterns, be sure to ask questions to check their understandings and address misconceptions.

To help students move away from iterative reasoning (I just need to add 5 each time) to explicit reasoning, it is best to ask students to build the next two steps or stages (4 and 5 in the table above), then skip some. Choose a number that is not a multiple of 1 through 5, but fairly close. Some students will continue with their iterative reasoning to fill in the table for this stage. Next, choose a larger number, again not a multiple of any of the previous stage numbers. This gives students a subtle nudge to begin thinking about finding relationships within the data that has already been collected. When students determine a rule, they should check to see that it works for all of the stages. This is important because some rules or expressions may work for 2 or three stages, but not the rest. This is an act of being precise with the mathematics.

Students should also represent the patterns in other ways, such as on a coordinate grid. This gives students a preview of mathematics to come later on. It's also a nice way for students to attach the equations they create to a series of points on a line that represent the growth of the pattern. **IMPORTANT NOTE:** When students plot the points on the coordinate grid, they should not connect the points. In this context, it does not make sense to connect the points, since we can't have a fraction of a stage or a fraction of a square tile.

Continue to practice developing expressions to generate specific stages of visual patterns using samples listed below or developed on your own. A sheet of the patterns given as examples is provided as a handout after the activity.

# Size 1 Size 2 Size 3

#### Sample Patterns adapted from <a href="http://www2.edc.org/mathpartners">http://www2.edc.org/mathpartners</a>

Pattern 2:

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#### Pattern 3:



#### Pattern 4:



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# Number Challenge Extension Opportunity

After students have had experience with visual patterns, offer the extension opportunities listed below. Students are asked to build patterns numerically. Have students generate a rule for the pattern, draw a model (or use manipulatives), and ask others to solve their pattern challenge. You could have each student put one pattern on their desk and then have the class move around the room to solve as many as they can in a set period of time.

1. Build growing or shrinking patterns that have the number 20 as the fourth term.

2. Start at 100. Build as many shrinking patterns as possible that end exactly at zero.



# FORMATIVE ASSESSMENT QUESTION

1. How many tiles are needed for a model at design 5?

2. How many tiles are needed for a model at design 11?

3. Explain how you determined the number needed for design 11.

4. Determine an expression for the number of tiles in a model of any design, n.

# **CLOSING**

During the closing of the lesson, students should share their expressions/equations using precise mathematical language. Pairs of students who investigate similar patterns should discuss the expressions/equations they create – especially if they look different. For example – with the "L" shaped pattern mentioned above (pattern 1), depending on what students "see" as staying the same, the following expressions or equations may be derived:

1 + n + 2

3 + n

4 + n - 1

Students should determine whether or not these expressions work for the pattern, then determine why they all work, since some are very different looking. **Extension:** 

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Students should be encouraged to create their own growth patterns. This will allow for student creativity. Students should not only create the pattern, but also find the expression/equation that explains it with several examples as proof that their equation is true. Also, some patterns are much more challenging than others. Using these patterns can provide students the challenge they need. Finally, looking at the quantity of squares in the pattern is only one possible pattern to explore. Another option would be to look at perimeter for the same pattern. Are the patterns for these two different ideas similar or very different? Why? This gets even more interesting when the visual patterns are three dimensional!

#### **Intervention:**

Students needing support can be given a graphic organizer like the one above. Using this can help students make sense of the patterns they are investigating based on how they see the pattern growing. Also, some patterns are easier to explain than others. Patterns using different shapes (such as pattern blocks) can be helpful to students needing support since the differing shapes can help them focus on what is changing and staying the same.

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#### VISUAL PATTERNS PRACTICE

Look at the visual patterns below. Choose any two to investigate. Write an algebraic expression to explain the pattern. Using your expression, write an equation for the total: t = \_\_\_\_\_ Graph your visual pattern on a coordinate grid.



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# HANDOUT OF PATTERNS FEATURED IN THE LESSON



PATTERN 2





Size 2



Size 1

Size 3



PATTERN 4



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# ADDITIONAL PRACTICE PROBLEMS

Please select problems from the set below or generate your own problems based on the performance of your students in the lesson above. Some students will not need as much practice as others. You may scaffold the assignment to increase the rigor or provide additional cues to open the problems to learners who might need more direction.

1. Consider the following pattern constructed by connecting toothpicks: (adapted from NCTM Illuminations)



a) How can you represent the number of triangles in each stage of the pattern?

b) How can you represent the number of toothpicks needed to construct each stage?

c) Based on you representation of the toothpicks, how many toothpicks would you need for stage 8?

2. Consider the following pattern:



a) How can you represent the number of squares in each stage of the pattern?

b) How can you represent the number of toothpicks needed to construct each stage?

c) Based on you representation of the toothpicks, how many toothpicks would you need for stage 8?

For more visual patterns to try with your students, go to the source: <u>www.visualpatterns.org</u>

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<u>Additional Internet Resources</u> Patterns and Functions, Grades 6–8

http://www2.edc.org/MathPartners/pdfs/6-8%20Patterns%20and%20Functions.pdf

#### **Investigating Growing Patterns, Mathwire**

http://mathwire.com/algebra/growingpatterns.html

# **Inside the Math TOY TRAINS**

## http://bit.ly/1ETe2UN

This lesson challenges a student to use algebra to represent, analyze, and generalize a variety of functions including linear relationships. A student must be able to:

- relate and compare different forms of representation for a relationship including words, tables, graphs, and writing an equation to describe a functional pattern
- use rules of operations to extend a pattern and use its inverse

The lesson was developed by the Mathematics Assessment Resource Service (MARS) and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the lesson, the number of core points, and the percent of students that scored at standard on the lesson. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the LESSON, are included in the LESSON packet.

# **Beads Under the Clouds Lesson**

# http://education.ky.gov/educational/diff/Documents/BeadsUnderTheCloud.pdf

This problem solving lesson is intended to help you assess how well students are able to identify patterns in a realistic context: the number of beads of different colors that are hidden behind the cloud. In particular, this problem solving lesson aims to identify and help students who have difficulties with:

- Choosing an appropriate, systematic way to collect and organize data.
- Examining the data and looking for patterns.
- Describing and explaining findings clearly and effectively.

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# **Translating Math**

This lesson is designed to connect students' knowledge of arithmetic expressions to writing algebraic expressions. The lesson is based on the lesson from Illustrative Math <u>https://www.illustrativemathematics.org/content-standards/4/OA/A/2/LESSONs/263</u> and <u>https://www.illustrativemathematics.org/content-standards/LESSONs/541</u>

# SUGGESTED TIME FOR THIS LESSON

60 to 90 minutes The suggested time for the class will vary depending upon the needs of the students.

# STANDARDS FOR MATHEMATICAL CONTENT

# MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.

e. Generate equivalent expressions using properties of operations and understand various representations within context. *For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between "3 more" and "3 times".* (MGSE4.0A.2; MGSE6.EE.3, MGSE7.EE.1, 2; MGSE9-12.A.SSE.3)

# COMMON MISCONCEPTIONS

As students translate verbal mathematical expressions into symbolic expressions, they often confuse the order of operations. Special attention should be called to the order in which symbolic math is written. For example, students often translate the phrase "Sam (s) has three less than Fred (f) as 3 - f instead of f - 3. Problem #2 in the LESSON below calls attention to ordering concerns.

# STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of properties by applying properties to generate equivalent expressions.

7. Look for and make use of structure. Students apply properties to generate equivalent expressions. Students will interpret and apply formulas.

# EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this LESSON, students should be able to:

- Interpret and translate verbal expressions into equivalent algebraic expressions.
- Model and interpret comparative expressions.

# **MATERIALS**

• Sticky notes may be offered as a way to build tape diagrams (bar models)

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#### **ESSENTIAL QUESTIONS**

- How can you determine which mathematical operation to apply?
- How can you model and interpret mathematical expressions?

# SUGGESTED GROUPING FOR THIS LESSON

Small group or pairs of students for the opening activity followed by independent practice

# KEY VOCABULARY

In this LESSON, discuss terminology as it naturally arises in discussion of the problems. Allow students to point out words or phrases that lead them to the model and solution of the problems. Words that imply mathematical operations vary based on context and should be delineated based on their use in the particular problem.

# **OPENER/ACTIVATOR**

- The opening activity sets the stage for translating verbal expressions into mathematical expressions along with reviewing methods of organizing mathematical thoughts such as the use of tables or charts. The opener is based on a Number Strings activity at http://numberstrings.com/2014/04/01/moving-straight-ahead/
- The dialogue below **should not be interpreted as a script** to be read to your class. It is an example of how the number string could be played in your classroom. Please modify the situation to engage your class in the activity. This number string activity will get students thinking and discussing math operations in context. Many different strategies (including tape diagram/bar model) might be used in solving this problem.
- "Suppose you are on a trip outside of New York City, riding along in a car, going at a steady rate with no traffic whatsoever. Want to roll down the window and feel the wind in your hair? You are cruising. After 45 minutes in the car, your dad says you've

Time (in hours)	Distance (miles)
.75	36

traveled 36 miles. I'm going to record the situation so far on this table."

• **NOTE**: you could model this situation with a bar model with the whole being one hour and the part being .25 or <sup>1</sup>/<sub>4</sub> of an hour. The bar model will help students with a pictorial

representation of the situation and can be used to complete the remaining parts.

• Some students might also benefit from looking at a clock and discussion where .75 or <sup>3</sup>/<sub>4</sub> of an hour would be. These students benefit from a concrete representation of the problem. Others will be able to abstract the situation without a model or representation.

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- "Turn and talk to your partner about whether this table captures the situation so far or whether I should have written something different. Remember you've driven 36 miles in 45 minutes. Does this table capture what's happened so far? Why or why not? Who could convince us?"
- You may need to pose this question if kids are not convinced that 45 minutes is equivalent to .75 hour.

"What's the same and what's	different about these tables?"
-----------------------------	--------------------------------

Time (in hours) Distance (miles)		Time (in minutes)	Distance (miles)
.75	36	45	36

Time (in hours)	Distance (miles)
.75	36
1.5	

• "Let's say you keep going at this rate. How far have you traveled now (after 1.5 hours)? How do you know?"

• **NOTE**: You may revisit the clock model or the bar model to progress to this and future parts.

• "What about now (after 3.0 hours)? How far have you traveled? And how do you know?"

Time (in hours)	Distance (miles)
.75	36
1.5	72
3.0	

Time (in hours)	Distance (miles)
.75	36
1.5	72
3.0	144
4.5	

• "This is a long car ride. It turns out that the whole trip took you 4.5 hours. How far have you traveled? And how do you know? I'll give you a minute to think, and then you can talk to your partner and share your strategy. Maybe there are a few ways to think about this....."

• Expect some students will double the 144 because they will overgeneralize the pattern of doubling. That's great if it happens because it gets kids really defending their thinking.

• If it happens that students get the incorrect answer,

just record the possible distances (288 miles, 216 miles), note that we did not agree and then let kids describe how they got their answers. It is often the case that simply by

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talking through their thinking, a student will revise their ideas, "No, no, wait! I change my answer. I actually agree with 216 now. I was doubling, but the 3.0 doesn't double to make 4.5."

• Once again, the bar model or the clock model could be used to help students compute the solution.

"I forgot to mention an important detail: you and your dad stopped for gas at the beginning of this journey, after only 15 minutes.

• We should probably put this on the table. I'm going to add this line at the bottom. But first I'm wondering who can explain why I'm not writing "15" under time? And why .25 somehow represents 15 minutes? How could that be? Who feels like they could explain that?"

Time (in hours)	Distance (miles)
.75	36
1.5	72
3.0	144
4.5	216
.25	

- (Access the bar or clock model as needed)
- Once the room is convinced by a student that 15 minutes is equivalent to .25 of an hour ask, "How far had you traveled at that point?"
- Look for lots of different strategies here. All of these strategies can be recorded on or near the table.

One thing we haven't even talked	Time (in hours)	Distance (miles)
about is this, our last question:	.75	36
How fast were you going on this trip? In other words, what was	1.5	72
your speed? Take a moment to think about this and how you	3.0	144
might explain it to all of us.	4.5	216
A del this last value (the unit rate) to	.25	12
the table only if you think it will help kids. Otherwise, leave it blank until	1.0	

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# TRANSLATING MATH LESSON

Post the following problems (or provide them in written form) one at a time for small groups or pairs of students to discuss. Instruct students to come up with a way to model the problem with a drawing, diagram, or using sticky notes. Circulate around the room to select a few groups to explain their model. Repeat this process for the remaining problems.

**NOTE**: For more information on tape diagrams (shown below) access one of the following links: <u>https://www.louisianabelieves.com/docs/default-source/louisiana-teacher-leaders/p17-p18-presentation-eureka-math---tape-diagrams.pdf?sfvrsn=2</u>

http://greatminds.net/maps/math/video-gallery/solving-word-problems-with-tape-diagrams

https://learnzillion.com/lessons/1744-interpret-division-as-an-unknown-factor-problem-using-abar-model

a. Helen raised \$12 for the food bank last year and she raised 6 times as much money this year. How much money did she raise this year?

Solution: Tape diagram (other options are also appropriate)

She raised six times as much money (as shown in the diagram) so she raised  $6 \times 12 = 72$ 

Money she raised last year:	\$12					
Money she raised this year:	\$12	\$12	\$12	\$12	\$12	\$12
				?		

b. Sandra raised \$15 for the PTA and Nita raised \$45. How many times as much money did Nita raise as compared to Sandra?

? × 15 = 45 45 ÷ 15 =?



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c. Luis raised \$45 for the animal shelter, which was 3 times as much money as Anthony raised. How much money did Anthony raise?

Money Anthony raised:	?		
Money Luis raised:			
		\$45	

 $3 \times ? = 45$  is equivalent to  $45 \div 3 = ?$ 

Once students have worked and discussed the first three problems, pose the next problem:

2. Write an expression for the sequence of operations.

a. Add 3 to x, subtract the result from 1, then double what you have.

Add 3 to x, double what you have, then subtract 1 from the result.

The instructions for the two expressions sound very similar, however, the order in which the different operations are performed and the exact wording make a big difference in the final expression. Students have to pay close attention to the wording: "subtract the result from 1" and "subtract 1 from the result" are very different.

Solution: a) Step One x+3, Step Two 1-(x+3), Step Three 2(1 - (x+3)) simplifies to -2x-4If we choose to simplify this expression, we use the distributive, commutative and associative properties in the following way: 2(1-(x+3))

2(1-x-3)	distribute the - 1
2(-x-2)	subtracting 3 from 1
-2x-4	distribute the 2

b) Step One x+3, Step Two 2(x+3), Step Three 2(x+3) - 1 simplifies to 2x+5 below

2x+6-1	subtracting 3 from 1
2 <i>x</i> +5	distribute the 2

Call attention to the fact that the final expressions are very different, even though the instructions sounded very similar.

# FORMATIVE ASSESSMENT QUESTIONS

The following questions are suggestions to gauge student understanding of mathematical translations. There are many others that teachers could establish to assess understanding.

- How can you explain your model?
- How can you convince me your method is correct?
- How did you decide which math operation to use?
- How could you solve this problem another way?

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#### **DIFFERENTIATION IDEAS**

Students who struggle with translating words into symbols could be provided options at the onset of the lesson to help them better understand the problem. As students become more proficient, these options can be removed.

Students who need more challenge could be asked to devise their own problems involving one or more operations. They could then exchange problems with others. Students could also be given a bar model (tape diagram) and be asked to create a problem that could be modeled by the diagram. Students could then exchange problems to see if classmates draw a model aligned to the one provided originally.

**INTERVENTION** For extra help with problem solving strategy, please open the hyperlink Intervention Table.

#### **CLOSING/SUMMARIZER**

Have students identify two key ideas as they reflect on the essential questions of the day. Allow several groups to share out their ideas or create a "Parking Lot" for students to post sticky notes with their ideas; this can be used for reflection during the next class meeting.

#### ADDITIONAL PRACTICE PROBLEMS

Please select problems from the set on the following page or generate your own problems based on the performance of your students in the lesson above. Some students will not need as much practice as others. You may scaffold the assignment to increase the rigor or provide additional cues for access to learners who might need more direction.

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#### **Generating Variable Expressions Practice**

#### **Teacher's Edition**

**DIRECTIONS**: Write a variable expression for each situation described below. Make sure to identify your variable and be prepared to defend your expressions. In some cases, your expression may be different than that of other students based on their choice of variable.

1. One coin is seven more than twice as old as another. Represent the ages of the coins using a single variable.

Let one coin be represented by c so the other coin would be 2c+7

2. Mrs. Drinkard's class read (124 - 4b) books and Mrs. Franklin's class read (7b + 16) books

a) Write an expression to show the difference between the number of books read by Mrs. Drinkard's class and the number of books read by Mrs. Franklin's class. (124-4b) - (7b+16) simplifies to 108 - 11b\*NOTE for extension: This is a good opportunity to discuss possible values of for "b"

in context of this problem.

b) Write an expression to model the total number of books read by the two classes. (124 - 4b) + (7b + 16) simplifies to 140 + 3b

c) Based on the context of the problem (reading books), are there any possible values for b that will not make sense for evaluating the number of books that Mrs. Drinkard's class might have read? Make a statement explaining why that is the case.

If b gets larger than 31, the number of books read by Mrs. Drinkard's class would be a negative value which would not make sense in this context.

(<u>Extension Problem</u> provided below to preview the next module and connect equations to variable expressions)

3. The Sweet Shoppe Bakery ordered 15 dozen eggs for their upcoming project. When the bill came, they were charged \$33.75 for the eggs.

a) Write a numerical expression to represent the cost for One dozen eggs and 15 dozen eggs.

The cost for one dozen eggs could be represented by "d" so the expression for 15 dozen would be 15d.

b) Find the cost for one dozen eggs based on your numerical expression.

Since the cost of 15 dozen eggs is given as \$33.75, we can use the relationship that 15d is the same as 33.75 or 15d = 33.75 which would result in the cost of one dozen being \$2.25.

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c) Ginger estimated the cost for one dozen eggs to be \$3.00. Was her estimate reasonable? Justify your response and come up with a way to explain how you could estimate the cost for one dozen eggs.

Answers may vary: Since she estimated \$3.00 per dozen, the bill for 15 dozen would be \$45.00. Her estimate was too high based on the given facts.

d) Chris said that he remembers when the cost for **one egg** was just a dime. Based on that cost for an egg, how much would the Sweet Shoppe have had to pay for their total egg order? What is the difference between the price they paid and this amount? Please show all your numerical expressions in getting your answers.

One dozen would cost  $12 \times \$0.10$  or \$1.20 so 15 dozen would cost \$18.00. They paid \$33.75 so the difference in cost would be \$33.75 - \$18.00 or \$15.75

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Generating Variable Expressions Practice

Name\_\_\_\_\_

**DIRECTIONS:** Write a variable expression for each situation described below. Make sure to identify your variable and be prepared to defend your expressions. In some cases, your expression may be different than that of other students based on their choice of variable.

1. One coin is seven more than twice as old as another. Represent the ages of the coins using a single variable.

2. Mrs. Drinkard's class read (124 - 4b) books and Mrs. Franklin's class read (7b + 16) books.

a) Write an expression to show the difference between the number of books read by Mrs. Drinkard's class and the number of books read by Mrs. Franklin's class.

b) Write an expression to model the total number of books read by the two classes.

c) Based on the context of the problem (reading books), are there any possible numbers of books that will not make sense for Mrs. Drinkard's class to have read? Make a statement explaining why that is the case.

3. The Sweet Shoppe Bakery ordered 15 dozen eggs for their upcoming project. When the bill came, they were charged \$33.75 for the eggs.

a) Write a numerical expression to find the cost for one dozen eggs.

b) Find the cost for one dozen eggs based on your numerical expression.

c) Ginger estimated the cost for one dozen eggs to be \$3.00. Was her estimate reasonable? Justify your response and come up with a way to explain how you could estimate the cost for one dozen eggs.

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d) Chris said that he remembers when the cost for **one egg** was just a dime. Based on that cost for an egg, how much would the Sweet Shoppe have had to pay for their total egg order? What is the difference between the price they paid and this amount? Please show all your numerical expressions in getting your answers.

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#### Additional Practice Problems for Translating Words into Variable Expressions

• The following practice problems may be used as remediation or extra practice for students who struggled with the previous lesson.

• Other uses for these problems include partner review or formative assessment questions. Write an expression for each of the following situations.

1. Ryan weighs 13 pounds less than Jay. Jay weighs x pounds. Ryan's weight: x - 13

2. Susan has 52 dollars more than Jennifer. Jennifer has x dollars. Susan's money: x+52

3. Brooke has 12 times as many stickers than James. James has x stickers. Brooke's sticker amount: 12x

4. The recipe calls for triple the amount of sugar than flour. There is f amount of flour in the recipe. Amount of sugar: 3f

5. Sade's quiz grade is six more than double Tina's quiz grade. Tina's quiz grade is x. Sade's quiz grade: 2x+6

6. Lauren paid x dollars for her prom dress. Becca paid 16 dollars more than Lauren. Becca's prom gown price: x+16

7. Pam ran the 5k in x minutes. Alexis ran the same race in half the time that Pam ran the race. Alexis's time:  $\frac{1}{2} x$  or  $\frac{x}{2}$ 

8. The spinach grew k inches. The tomatoes grew triple the height of the spinach, less 2 inches. Tomato height: 3k-2

9. Each person running in the race will eat two hotdogs. Determine the number of hotdogs needed, given the amount of people running in the race.

Number of People Running	Number of Hotdogs needed
100	200
220	440
415	830
620	1240

10. Write an expression that represents the number of hotdogs needed, given the number of people running in the race. Number of people = x Number of hotdogs = 2x

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#### <u>Translating Words into Variable Expressions</u> Name \_ Write an expression for each of the following situations.

1. Ryan weighs 13 pounds less than Jay. Jay weighs x pounds. Ryan's weight:

2. Susan has 52 dollars more than Jennifer. Jennifer has x dollars. Susan's money:

3. Brooke has 12 times as many stickers than James. James has x stickers. Brooke's sticker amount:

4. The recipe calls for triple the amount of sugar than flour. There is f amount of flour in the recipe. Amount of sugar:

5. Sade's quiz grade is six more than double Tina's quiz grade. Tina's quiz grade is x. Sade's quiz grade:

6. Lauren paid x dollars for her prom dress. Becca paid 16 dollars more than Lauren. Becca's prom gown price:

7. Pam ran the 5k in x minutes. Alexis ran the same race in half the time that Pam ran the race. Alexis's time:

8. The spinach grew k inches. The tomatoes grew triple the height of the spinach, less 2 inches. Tomato height:

9. Each person running in the race will eat two hotdogs. Determine the number of hotdogs needed, given the amount of people running in the race.

Number of People Running	Number of Hotdogs needed
100	
220	
415	
620	

10. Write an expression that represents the number of hotdogs needed, given the number of people running in the race.

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# **Exploring Expressions**

Adapted from New York City Department of Education.

# **SUGGESTED TIME FOR THIS LESSON:**

60-120 minutes

The suggested time for the class will vary depending upon the needs of the students.

# STANDARDS FOR MATHEMATICAL CONTENT

# MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.

a. Apply properties of operations emphasizing when the commutative property applies. (MGSE7.EE.1)

c. Model numerical expressions (arrays) leading to the modeling of algebraic expressions. (MGSE7.EE.1, 2; MGSE9-12.A.SSE.1, 3)

d. Add, subtract, and multiply algebraic expressions. (MGSE6.EE.3, MGSE6.EE.4, MC7.EE.1, MGSE9-12.A.SSE.3)

e. Generate equivalent expressions using properties of operations and understand various representations within context. For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between "3 more" and "3 times". (MGSE4.0A.2; MGSE6.EE.3, MGSE7.EE.1, 2, MGSE 9-12.A.SSE.3)

f. Evaluate formulas at specific values for variables. For example, use formulas such as A = 1 x w and find the area given the values for the length and width. (MGSE6.EE.2)

# COMMON MISCONCEPTIONS

Students may see  $3^2$  and think it means  $3 \times 2$ , and multiply the base by the exponent. Also, students may mistake  $3^2$  for 3 + 3 and vice versa, or p + p + p for  $p^3$  (Here they may realize how many of the base they need in expanded form, but they add or count by the base, rather than multiplying.)

# STANDARDS FOR MATHEMATICAL PRACTICE

2 **Reason abstractly and quantitatively**. Students will work in verbal, expanded, and exponential forms of number representations.

3 **Construct a viable argument and critique the reasoning of others**. Students will be asked to evaluate responses of others to determine the accuracy along with providing an explanation in support of their choice.

7 **Look for and make use of structure**. Students will follow and apply properties of algebra to represent equivalent expressions.

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#### **EVIDENCE OF LEARNING/LEARNING TARGET**

By the conclusion of this lesson, students should be able to:

- Represent numerical relationships as expressions.
- Use algebraic expressions solve contextual based problems.

#### **MATERIALS**

- Personal white boards (or sheet protectors) if you have decided to use for Activator
- Student lesson pages
- Marbles and a bag (if using the differentiation activity)

#### **ESSENTIAL QUESTIONS**

- How can mathematical relationships be expressed with symbols?
- What does it mean to evaluate an expression?
- How can numerical and algebraic expressions be used to represent real life situations?
- How do number properties effect the creation of equivalent expressions?

# **KEY VOCABULARY**

The following vocabulary terms should be discussed in the context of the lesson: Exponent, base, expression, power, factor, evaluate, expanded form (of multiplication), exponential form, variable, coefficient, constant, equation, notation for multiplication, grouping symbol such as (), twice as much, more than, less than, the product of, the quotient of, difference, sum, total, increasing, decreasing, fewer than, equal to, associative property, commutative property, distributive property, additive inverse property, identity element, multiplicative inverse, equivalent expressions, substitution principle.

# SUGGESTED GROUPING FOR THIS LESSON

Independent thought time for the activator sections, then followed by partner or small group for the short lessons followed by whole class discussion. This lesson will repeat these grouping cycles as new short lessons are introduced. Activators may be worked on personal white boards (or heavy duty sheet protectors with card stock inserts) for quick discussion and engagement.

#### **OPENER/ACTIVATOR**

(A) NOTE: Student pages of lessons follow the Closing Activity

Pose the following problems as an activator for LESSON A:

- 1. Write  $3^4$  in repeated multiplication form.  $3 \times 3 \times 3 \times 3$
- 2. Evaluate 10<sup>3</sup>. *1000*
- 3. Find the area of a square computer desk with side of 18 inches. 18x18 = 324 inches<sup>2</sup>
- 4. Find the volume of a cube with sides of 3 inches.  $3 \times 3 \times 3 = 27$  inches<sup>3</sup>
- 5. Explain the difference between  $4^3$  and  $3^4$ .  $4^3$  means 4x4x4 whereas  $3^4$  means 3x3x3x3

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# After a short discussion on the activating problems, have students work in teams to answer the following:

Short LESSON Section A

**LESSON 1**: <u>Mrs. Alexander's Offer</u> Mrs. Alexander is offered a job with a computer company that will pay her \$100,000. Represent the salary using exponential form. *Answer:*  $10^5$ 

**LESSON 2**: Mr. Baily, a scientist who liked to express numbers with exponents, used a table to represent the number of mold spores in his bread experiment

Month	1	2	3	4	5
Number of	3	3x3	3x3x3	3x3x3x3	<i>3x3x3x3x3</i>
mold spores					

a) Mr. Baily asked his students to write an exponential expression to represent the total number of mold spores on the fourth day. Marcus wrote the expression  $3^4$  and John wrote  $4^3$ . Who was correct? *Answer: Marcus* 

b) Justify your answer with a detailed explanation.

Marcus was correct because the number of 3's in the multiplication is 4 indicating a base of 3 and an exponent of 4.

c) If the pattern continued, how many mold spores would Mr. Baily have on the fifth day? Write your answer in exponential form. *Answer*:  $3^5$ 

# After students have time to work the short lesson problems, initiate a discussion based on their work. Then, post/ask activator (B).

# ACTIVATOR (B)

Pose the following problems as an activator for lessons 3 and 4

Translate each verbal expression into an algebraic expression:

- a) 4 less than the product of a number and 7 7n 4
- b) The quotient of y and 6  $\frac{y}{6}$  or  $y \div 6$
- c) The sum of a number and 225 n + 22
- d) Twice a number 2*n*

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After a short discussion on the activating problems, have students work in teams to answer the following:

**LESSON 3**: Algebra Homework

Let h represent the number of hours Jordan spent on homework last week.

**Part A**: Jack spent 1/2 as much time on his homework as Jordan, plus an additional 3 hours. Write an expression for the number of hours he spent on his homework.

Expression:  $\frac{h}{2} + 3$ 

Part B: Identify the number of terms, coefficient and constant in your expression.

Number of terms: <u>2</u> Coefficient:  $\frac{1}{2}$  Constant: <u>3</u>

**Part C**: Jordan spent 12 hours doing her homework. How many hours did it take Jack to complete his homework?

Show your work.  $\frac{12}{2}$  + 3 Simplifies to 6 + 3 which is 9 hours Jack: 9 hours

**LESSON 4**: Algebra for Dogs

Caesar, a pit bull, weighs p pounds.

For Part A, write an expression for the weight in pounds of each of the dogs. Each expression should include the variable p.

Part A

- Bruno, a pug, weighs 49 pounds less than Caesar: p 49
- Mia, a Chihuahua, weighs  $\frac{1}{17}$  as much as Caesar:  $\frac{1}{17}$  p or  $\frac{p}{17}$

• Lucy, a Great Dane, weighs twice as much Caesar, minus 15 pounds: 2p - 15

Part B

Set up a chart or organizer to represent the weights of the four dogs.

Caesar	Bruno	Mia	Lucy
Р	<i>p</i> - 49	$\frac{1}{17}p \ or \ \frac{p}{17}$	2p - 15

#### Answers may vary based on student preferences

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Part C

Caesar weighs 68 pounds. How much do Bruno, Mia, and Lucy weigh?

Show or explain how you found this.

Bruno: 68-49 or 19 pounds; Mia:  $\frac{68}{17}$  or 4 pounds; Lucy: 2(68) – 15 or 121 pounds

After students have time to work the short LESSON problems, initiate a discussion based on their work. Then, post/ask activator (C).

#### ACTIVATOR (C)

Pose the following problems as an activator for LESSON 5 and 6

Find an equivalent expression for each of the following:

1) 3(x-2) 3x-62) 8x + 12y 4(2x + 3y) using factoring (NOT REQUIRED) or 12y + 8x using commutative property

NOTE: Problem (2) as factoring is included as an extension. Factoring is not required in this module. Commutative property is part of this module

3) p + p + p + p + p 5p4) 7y + 2p 2p + 7y5) 3x + (5m + 5p) (3x + 5m) + 5p using Associative Property; 3x + (5p + 5m) using Commutative Property; 3x + 5(m + p) using Factoring (see explanation in #2)

After a short discussion on the activating problems, have students work in teams to answer the following:

**LESSON 5**: Distributive Property of Color Pencils

Miss Nix opens a box of pencils that contains 6 green, 8 blue, and 10 red pencils. Write two equivalent expressions to show the total number of pencils in 5 boxes. *Expressions:* 5(6g + 8b + 10r) or 5(6g) + 5(8b) + 5(10r) or 30g + 40b + 50r

Explain how you know that these two expressions are equivalent *Answers may vary based on student choice* 

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## **LESSON 6**: Equivalent Expressions

Are the following ex	pressions equiva	alent? Explain your reasoning.
1) a + a + 2 + 2	2a + 4	Yes. If you combine like terms on the left set of terms you match the right set of terms
2) 6(p + 3)	6p + 6	No. If you distribute on the left group of terms you get $6p + 18$ instead of $6p + 6$
3) 10(y – x)	10y - 10x	Yes. If you distribute on the left group you get $10y - 10x$ which matches the right set of terms.

4) Tony says the three expressions below are equivalent to 4 + 24y. Is she correct?

Explain your reasoning for each.	1. Yes. If you combine like terms in each parenthesis you get 4 + 24y.
1) $(1 + 3) + (4y + 20y)$	
2) 4 + 10y + 14y	2. Yes, If you combine the 10y and the 14y you get 24y and keep the 4. The result is 4 + 24y.
3) (20 – 16) + (44y – 20y)	3. Yes. If you combine like terms in each parenthesis you get 4 + 24y.

**INTERVENTION** For extra help with designing and using models, please open the hyperlink Intervention Table.

# **CLOSING/SUMMARIZER**

Have students choose three (or more) of the key terms used during the LESSONs to connect in a sentence. You may choose to have students share their sentences or turn them in to provide feedback for an opening discussion for the next class.

For Example: A student may choose the words exponent, base, and expanded form. They could generate a sentence such as: "When writing an expression in expanded form, the base is multiplied by itself the number of times equivalent to the exponent.  $5^3$  would be 5x5x5 in expanded form.

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**Differentiation Idea**: The following lesson is designed to help students develop connections between concrete experiences, pictorial representations of mathematical ideas, and abstract mathematical symbols. Based on the needs of your students, you may choose to use the following activities to introduce students to variables, expressions, and equations. You could have a bag of marbles (or other item) to model the situation.

**Marbles Expression** Mrs. Wright has a bag and puts some marbles inside the bag. There are 6 marbles on the floor. How can we represent the total number of marbles there are all together?

Ideas to consider as you discuss the unknown number of marbles in the bags:

1. The number of marbles in the bag is unknown, so choose a variable to represent it. m.

2. The number of marbles on the floor stays the same (is a constant). What is that number? 6.

3. To find the number of marbles all together, we could add the number of marbles in the bag and the 6 marbles on the floor. What would that expression be? m + 6 would then represent the total number of marbles.

4. Suppose you have 80 in the bag. Therefore, m = 80. Can you find the total number of marbles in the bag? By substituting 80 in the expression m + 6 we get 80 + 6. There are 86 marbles in all together.

#### Sample Formative Assessment Questions/Practice Problems

The following problems/lessons could be used as formative assessments or as practice problems for students.

1. Mark has 12 comic books, Jeremy has 14 comic books, and Sam has half as many comic books as Jeremy.

a) Write a numerical expression to represent how many comic books the three boys have together.

*Mark* + *Jeremy* + *Sam 12* + *14* + *7* 

b) If each comic book cost \$3.50, write two equivalent expressions to represent how much the boys have spent total.

3.50 (12 + 14 + 7) or 3.50(12) + 3.50(14) + 3.50(7)

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c) How do you know that these two expressions are equivalent? Justify your response.

Using the distributive property you can show that they are equal. 3.50 (12 + 14 + 7) = 3.50(12) + 3.50(14) + 3.50(7)

d) If Mark lost a few of his cards, write a new expression to represent how many cards Mark has now.

Since Mark had 12 cards and lost some unknown amount, we will represent the number of cards he lost with a variable such as x. His new amount would be 12 - x.

e) Explain the differences between a numeric expression and an algebraic expression.

A numeric expression contains numbers and operations where as an algebraic expression also contains unknown values represented by variables.

2. Samantha wishes to save \$1,024 to buy a laptop for her birthday. She is saving her money in a cube shaped bank (shown below).



a) Write a variable expression to represent the volume of the bank.

 $V = a \times a \times a \text{ or } a^3$ 

b) If the length of the side of the bank is 8 inches, write two numerical expressions to find the volume of the bank. One expression should be in exponential form and the other should be in expanded form.

 $\mathbf{V} = \mathbf{8} \times \mathbf{8} \times \mathbf{8} \text{ or } \mathbf{8}^3 \text{ in}^3$ 

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3) Mrs. Owens is making a quilt in the shape of a square for her younger sister. The border of the quilt is made of 1-foot by 1-foot patches. She asks one of her students, Kayla, to use the picture of the quilt below to write an expression to illustrate the number of patches needed to border the square quilt with side length *s feet*.

a) Kayla writes: (s + s + s + s) + (1 + 1 + 1 + 1) Is Kayla's expression correct? Explain your reasoning.



Yes, since the sides of the quilt are "s" feet long, it will take 4s or s + s + s + s to border each side and you will also need four additional squares for the corners. So, Kayla is correct.

Quilt WITH the border is shown.

The Inner Square is the quilt and the Outer Area shows the Border.

b) Mrs. Owens asks four other students (Sarah, Jeff, Nathan, and Bill) to generate expressions that are equivalent to Kayla's expression.

- 1. Sarah's expression: 4s + 4
- 2. Jeff's expression: 4(s + 1)
- 3. Nathan says the correct expression is 4s + 1
- 4. Bill writes: 2s + 2(s + 2)

How many of the students wrote a correct or an incorrect expression? Justify your answer with a detailed explanation.

- Sarah is correct because her expression represents Kayla's with like terms combined.
- Jeff is also correct. Using the distributive property would generate an equivalent expression.
- Bill is also correct using distributive property and like terms being combined.
- Nathan is incorrect. He did not account for all four corners of the quilt.

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#### Numeric and Algebraic Expressions LESSON Name \_\_\_\_\_

Short lesson Section A

**LESSON 1**: Mrs. Alexander's Offer Mrs. Alexander is offered a job with a computer company that will pay her \$100,000. Represent the salary using exponential form.

**LESSON 2**: Mr. Baily, a scientist who liked to express numbers with exponents, used a table to represent the number of mold spores in his bread experiment

Month	1	2	3	4	5
Number of	3	3 x 3	3 x 3 x 3	3 x 3 x 3 x 3	
mold spores					

a) Mr. Baily asked his students to write an exponential expression to represent the total number of mold spores on the fourth day. Marcus wrote the expression  $3^4$  and John wrote  $4^3$ . Who was correct?

b) Justify your answer with a detailed explanation.

c) If the pattern continued, how many mold spores would Mr. Baily have on the fifth day? Write your answer in exponential form.

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#### **LESSON 3**: Algebra Homework

Let h represent the number of hours Jordan spent on homework last week.

**Part A**: Jack spent  $\frac{1}{2}$  as much time on his homework as Jordan, plus an additional 3 hours. Write an expression for the number of hours he spent on his homework.

Expression: \_\_\_\_\_

Part B: Identify the number of terms, coefficient and constant in the expression above.

Number of terms: \_\_\_\_\_ Coefficient: \_\_\_\_\_ Constant: \_\_\_\_\_

**Part C**: Jordan spent 12 hours doing her homework. How many hours did it take Jack to complete his homework?

Show your work.

Jack: \_\_\_\_\_\_ hours

#### **LESSON 4**: Algebra for Dogs

Caesar, a pit bull, weighs p pounds.

For Part A, write an expression for the weight in pounds of each of the dogs.	
Each expression should include the variable p.	

Part A

Bruno, a pug, weighs 49 pounds less than Caesar: \_\_\_\_\_\_

Mia, a Chihuahua, weighs 1/17 as much as Caesar: \_\_\_\_\_\_\_

Lucy, a Great Dane, weighs twice as much Caesar, minus 15 pounds:\_\_\_\_\_\_

Part B Set up a chart or organizer to represent the weights of the four dogs.

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Part C Caesar weighs 68 pounds. How much do Bruno, Mia, and Lucy weigh?
Show or explain how you found this.
Bruno: pounds; Mia: pounds; Lucy: pound
LESSON 5: Distributive Property of Color Pencils
Miss Nix opens a box of pencils that contains 6 green, 8 blue, and 10 red pencils. Write two equivalent expressions to show the total number of pencils in 5 boxes.
Expressions: and
Explain how you know that these two expressions are equivalent
<b>LESSON 6</b> : Equivalent Expressions
Are the following expressions equivalent? Explain your reasoning.
1) $a + a + 2 + 2 = 2a + 4$
2) $6(p+3) = 6p+6$
3) $10(y - x) = 10y - 10x$
4) Tony says the three expressions below are equivalent to $4 + 24y$ . Is she correct?
Explain your reasoning for each.
1) $(1 + 3) + (4y + 20y)$
2) $4 + 10y + 14y$

3) (20 - 16) + (44y - 20y)

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#### Formative Assessment Questions/Practice Problems Name

1. Samantha wishes to save \$1,024 to buy a laptop for her birthday. She is saving her money in a cube shaped bank (shown below).



a) Write a variable expression to represent the volume of the bank.

b) If the length of the side of the bank is 8 inches, write two numerical expressions to find the volume of the bank. One expression should be in exponential form and the other should be in expanded form.

2. Mrs. Owens is making a quilt in the shape of a square for her younger sister. The border of the quilt is made of 1-foot by 1-foot patches. She asks one of her students, Kayla, to use the picture of the quilt below to write an expression to illustrate the number of patches needed to border the square quilt with side length *s feet*.



a) Kayla writes: (s + s + s + s) + (1 + 1 + 1 + 1)Is Kayla's expression correct? Explain your reasoning.

Quilt WITH the border is shown. The **Inner Square** is the quilt and the **Outer Area** shows the **Border**.

b) Mrs. Owens asks four other students (Sarah, Jeff, Nathan, and Bill) to generate expressions that are equivalent to Kayla's expression.

- 1. Sarah's expression: 4s + 4
- 2. Jeff's expression: 4(s + 1)
- 3. Nathan says the correct expression is 4s + 1
- 4. Bill writes: 2s + 2(s + 2)

How many of the students wrote a correct or an incorrect expression? Justify your answer with a detailed explanation.
# **A Few Folds**

In this lesson, students will explore and continue patterns, and represent findings with integer outcomes in the context of operations with exponents.

## SUGGESTED TIME FOR THIS LESSON

#### 60 minutes

The suggested time for the class will vary depending upon the needs of the students.

## STANDARDS ADDRESSED IN THIS LESSON

## **MFAAA2.** Students will interpret and use the properties of exponents.

b. Use properties of integer exponents to find equivalent numerical expressions. For example,  $3^2 x \ 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$ . (MGSE8.EE.1)

## COMMON MISCONCEPTIONS

The students may incorrectly state  $2^0 = 0$ . This lesson explores this property of exponents extremely well.

## STANDARDS FOR MATHEMATICAL PRACTICE

4. Model with mathematics. Students may use tables, graphs, and paper folding to model exponential growth.

6. Attend to precision. Students are careful about specifying the number of folds in comparison to the number of layers and in recording the data to clarify the relationship. They calculate accurately and efficiently to express the values in expanded and exponential form.

7. Look for and make use of structure. Students look closely to discern a pattern as they complete the folding activity and enter the data in the table.

## **EVIDENCE OF LEARNING/LEARNING TARGETS**

By the conclusion of this LESSON, students should be able to:

- Use exponents and expanded form of numbers.
- Apply properties of exponent operations.

## **MATERIALS:**

- Student activity sheet for each student/pair of students/or small group
- Paper for folding activity in Part 1

## **ESSENTIAL QUESTIONS**

- When are exponents used and why are they important?
- How do I simplify and evaluate numeric expressions involving integer exponents?

#### **GROUPING:**

Partner/Small Group

#### **LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION:**

In this LESSON, students will operate with integer exponents to describe and continue patterns. Students may want to create and use a table to organize their work and findings. Allow students to explore, discover, and generalize the properties of exponents and practice simplifying expressions with integer exponents.

<u>ACTIVATOR/OPENING</u>: Show the Myth busters video of paper folding activity at <u>http://www.mythbusterstheexhibition.com/try-this-at-home/try-this-entry/</u> to\_introduce the idea of exponential growth.

#### **DIFFERENTIATION:**

**Extension:** Student Group breaks W

Student Group breaks World Record for Paper Folding video can be used to build questions about exponential growth. https://youtu.be/vPFnIotfkXo

Students will investigate exponential growth via Mathwarehouse: http://www.mathwarehouse.com/exponential-growth/graph-and-equation.php

#### **Intervention/Scaffolding:**

<u>Part 1</u>: Set up the table for the students to mark their observations. Discuss the first fold as a class and how to mark that on the table.

<u>Part 2</u>: Remind students of liquid measurement conversions, perhaps by providing a reference chart. Remind students that these patterns are easier to see when noted in tables. Some students would benefit from actual containers of appropriate size to experience the relationship between the measurements. Instead of providing students with the conversion factors, actually fill a measuring cup to show the number of cups in a pint, quart, and gallon.

For the money example, you could bring in the actual coins to model the equivalent values. You could use monopoly money to model the paper bills.

#### Additional Resource Option:

Annenberg Learning **Bigger and Smaller-Exponent Rules Lesson Activity** <u>http://www.learner.org/workshops/algebra/workshop6/lessonplan2b.html</u>

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#### **CLOSING**

Ask students to consider the following problem as a review of exponential functions examined in the lesson:

Suppose you are offered \$1,000,000 for a one month salary or one penny a day doubled each day for 30 days.

- Which salary would you choose?
- How will you make your decision?
- How about on day 10?
- How about on day 30?
- How can you find the total amount you would be paid in order to make your decision?

**NOTE**: You could send this question home with students to discuss with their parents and friends. Then, have students report back on how people responded to the question.

Solution below

Day 1: \$.01       2 <sup>0</sup> Day 2: \$.02       2 <sup>1</sup> Day 3: \$.04       2 <sup>2</sup> Day 4: \$ 08       08	Students should generalize the pattern of amount of money on any given day is equal to $2^{d-1}$ where d is the day of the month.
Day 5: \$.16 Day 6: \$.32 Day 7: \$.64 Day 8: \$1.28 Day 9: \$2.56 Day 10: \$5.12	For day 10 you would compute $2^{d-1}$ as $2^{10-1}$ or $2^9$ which is 512 pennies or \$5.12
Day 11: \$10.24 Day 12: \$20.48	
Day 13: \$40.96 Day 14: \$81.92 Day 15: \$163.84 Day 16: \$327.68 Day 17: \$655.36 Day 18: \$1.310.72	For day 30 you would compute $2^{d-1}$ as $2^{30-1}$ or $2^{29}$ which is 536870912 pennies or \$5,368,709.12
Day 19: \$2,621.44 Day 20: \$5,242.88 Day 21: \$10,485.76 Day 22: \$20,971.52 Day 23: \$41,943.04 Day 24: \$83,886.08	For the total amount of money paid for the month, students could add the results of their daily calculations to get a total monthly salary of \$10,737,418.24
Day 25: \$167,772.16 Day 26: \$335,544.32 Day 27: \$671,088.64 Day 28: \$1,342,177.28 Day 29: \$2,684,354.56 Day 30: \$5,368,709.12	Video of solution on Bloomberg: http://www.bloomberg.com/news/videos/b/9296 <u>6fc7-c54d-4405-8fa6-cbefd05bbd6f</u> NOTE: the solution is in the first 1 min 20 sec of video

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#### FORMATIVE ASSESSMENT QUESTIONS

Adapted from Colorado State Assessment Program 2002

Number cubes are the basis for many games. Each face of a number cube is identified by a number from 1 to 6. Some games use one number cube and some games use multiple number cubes.

1. Complete the table below to show the number of possible outcomes when 2, 3, 4, and 5 number cubes are used.

Number of Cubes	1	2	3	4	5
Number of possible outcomes	6				

2. How can you generalize the pattern for the number of possible outcomes based on the total number of cubes being used?

#### Solution:

1. There are  $6^2 = 36$  outcomes when two cubes are used,  $6^3 = 216$  outcomes for three cubes,  $6^4 = 1296$  outcomes for four cubes, and  $6^5 = 7776$  outcomes for five cubes.

2. In general, there are  $6^n$  outcomes for n cubes.

## A Few Folds Part 1:



Repeatedly fold one piece of paper in half, recording the number of folds and the resulting number of layers of paper. Assuming that you could continue the pattern, how many layers of paper would there be for 10 folds, 100 folds, *n* folds? How do you know? *Solution* 

Number of folds	1	2	3	4	5	•••	10	•••	n
Number of layers of paper	2	4	8	<i>16</i>	32	•••	1024	•••	2 <sup>n</sup>
Number of layers of paper written using	21	22	<b>2</b> <sup>3</sup>	24	25	•••	210	•••	2 <sup>n</sup>
integer exponents									

Students should see that each fold resulted in twice as many layers of paper as the previous fold.

## Part 2:

Explain the process that is used to generate the following patterns?

Cup, pint, quart, half gallon, and gallon when the base unit of measure is one cup

**Solution** 

Сир	Pint	Quart	Half Gallon	Gallon
<i>1 сир</i>	2 cups	4 cups	8 cups	<b>16 cups</b>
20	21	$2^{2}$	2 <sup>3</sup>	24

In terms of cups, 1 cup, 2 cups, 4 cups, 8 cups, 16 cups. Each measure is twice the previous one.

Penny, dime, dollar, ten dollars, one hundred dollars, and so on when the base unit of measure is one dollar *Solution* 

Solution

Penny	Dime	Dollar	Ten Dollars	<b>One Hundred Dollars</b>
0.01	0.10	1.00	10.00	100.00
<i>10</i> -2	<i>10</i> - <i>1</i>	<i>10<sup>0</sup></i>	<i>10<sup>1</sup></i>	<i>10</i> <sup>2</sup>

In the sequence of money, each amount is 10 times the previous one, giving 1 penny, ten pennies, 100 pennies, and so on.

## A Few Folds Part 1:



Repeatedly fold one piece of paper in half, recording the number of folds and the resulting number of layers of paper. Assuming that you could continue the pattern, how many layers of paper would there be for the following:

- 10 folds
- 100 folds
- *n* folds
- How do you know?

## Part 2:

Explain the process that is used to generate the following patterns?

Cup, pint, quart, half gallon, and gallon when the base unit of measure is one cup

Penny, dime, dollar, ten dollars, one hundred dollars, and so on when the base unit of measure is one dollar

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# **Bacterial Growth**

(Adapted from Illustrative Math)

This lesson provides an introduction to and practice with the properties of integer exponents when related to negative exponents.

## SUGGESTED TIME FOR LESSON

45-60 minutes

The suggested time for the class will vary depending upon the needs of the students.

## STANDARDS FOR MATHEMATICAL CONTENT

## **MFAAA2.** Students will interpret and use the properties of exponents.

a. Substitute numeric values into formulas containing exponents, interpreting units consistently. (MGSE6.EE.2, MGSE9-12.N.Q.1, MGSE9-12.A.SSE.1, MGSE9-12.N.RN.2)

b. Use properties of integer exponents to find equivalent numerical expressions. For example,  $3^2 x \ 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$ . (MGSE8.EE.1)

## COMMON MISCONCEPTIONS:

The students may incorrectly state  $2^0 = 0$ . Students may also think that  $2^{-1}$  could be simplified by multiplication of 2 X -1. This lesson explores these properties of exponents.

## STANDARDS FOR MATHEMATICAL PRACTICE

## 7. Look for and make use of structure.

Students see the structure of adding 1 to the exponent corresponding to multiplying by 2. Then they see that this works for negative exponents as well.

## 8. Look for and Express Regularity in Repeated Reasoning.

This lesson leads students systematically through the use of repeated reasoning to understand algorithms and make generalizations about patterns as they investigate operations with exponents.

## **EVIDENCE OF LEARNING/LEARNING TARGETS**

By the conclusion of this lesson, students should be able to:

- Evaluate exponential expressions.
- Simplify compare expressions containing integer exponents.

## **MATERIALS**

• Student Activity Sheet (provided)

## **ESSENTIAL QUESTIONS:**

- When are exponents used and why are they important?
- How do I simplify and evaluate numeric expressions involving integer exponents?

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#### **GROUPING FOR LESSON**

This lesson could be used as a formative assessment after using the previous LESSON. It could also be used as an instructional lesson based on the needs of your students. If used for an instructional lesson, partners or small groups would be appropriate.

#### **OPENER/ACTIVATOR**

To get students thinking about exponential operations, show and work through the 3Act Task "Got Cubes" by Graham Fletcher (LESSON adapted from <u>www.gfletchy.wordpress.com</u>).

- Preface the activator by asking them to watch Act 1 and list at least 5 things they notice and at least 3 things they wonder about.
- Watch the video: <u>http://vimeo.com/98507175</u>
- Allow students to share their ideas about the video.
- Draw out several observations with particular attention to the following:
  - > The video showed structures with 2 cubes, 8 cubes, and 32 cubes.
  - $\blacktriangleright$  The video DOES NOT show all the stages. It skips the 2<sup>nd</sup> and the 4<sup>th</sup> stage.
  - By watching the video you will hear a sound as each stage is shown. Notice when the 2<sup>nd</sup> sound happens the screen goes black and when the 4<sup>th</sup> sound happens the screen is dark again.
  - You hear two additional sounds, indicating two additional stages, as the screen is black at the end.

#### After discussing their ideas, pose the following:

- How many cubes will be in the structure at the 7th stage?
- Make an estimate you know is too high. Too low.
- $\circ$  Allow them to work together to discuss ideas and to ask for information. Take several responses on the estimates; record them on the board, then show the 3<sup>rd</sup> Act to reveal the solution.

#### **ACT 3:**

Watch the video: http://vimeo.com/98507762

Solution: Missing stage 2 had 4 cubes; missing stage 4 had 16 cubes so the pattern of the cubes is  $2^n$  where n represents the stage in building the cubes. This makes the  $7^{th}$  stage  $2^7$  or 128.

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#### **DIFFERENTIATION FOR ACTIVATOR**

#### Pose ACT 4 for students who are curious about the progression of the exponential pattern.

- Can you identify a rule that would tell you how many cubes there would be in the 10th structure?
- ➢ How about the 50 Structure?
- ➤ Write an expression to solve for any stage.
- Is there a pattern with the shape of each structure? If so what shape would the 83rd structure be?
- What would the stage before the 1st stage look like? How many cubes would it have? Can it be expressed as an exponent?
  - o (Answer: <u>Act 4 Sequel</u>)

## **Illustrative Math Commentary on the BACTERIAL GROWTH Instructional Lesson**

This is an instructional lesson meant to generate a conversation around the meaning of negative integer exponents. While it may be unfamiliar to some students, it is good for them to learn the convention that "negative time" is simply any time before t=0 based on when the timing started.

#### **DIFFERENTIATION**

#### Intervention:

Students may struggle to put their explanation for part (h) together. The teacher might want to have the students do parts (1) - (7) as a precursor to providing an explanation like the one given in the solution for part (8).

#### **Instructional Strategy Suggestion:**

To lead the students to this generalization a teacher could separate each of the lessons listed a-g and have students work in groups to complete each lesson with discussion and written consensus about the results each time. After the students have worked through g, all results of a-g are posted for students to see; the teacher could have students individually write an explanation for h. Students could then have small and/or whole group discussion to finalize an answer with the teacher guiding the discussion to the correct answer and explicitly pointing out, if needed, the repeated reasoning being used and the generalization of the pattern that is developed.

#### **KEY VOCABULARY**

The following terms should be defined in the context of the lesson as they arise in conversation:

- exponential growth
- negative exponent
- elapsed time
- formula

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#### FORMATIVE ASSESSMENT QUESTIONS

- How can you evaluate the exponential expression  $3^x$  when x=4?
- How can you evaluate the exponential expression  $3^x$  when x=-4?

#### **CLOSING**

Choose one of today's essential questions and provide a written answer. Provide at least one example with your explanation.

## **ADDITIONAL PRACTICE**

Additional practice problems are provided at the conclusion of the student edition of the Bacterial Growth Lesson page. These problems may be used as a quiz or for additional practice as determined by the teacher.

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#### Bacterial Growth Teacher's Edition Name

Marco and Seth are lab partners studying bacterial growth. They were surprised to find that the population of the bacteria doubled every hour.

1. The table shows that there were 2,000 bacteria at the beginning of the experiment. What was the size of population of bacteria after 1 hour? 4,000

And, after 2, 3 and 4 hours? 8,000 16,000 32,000

Enter this information into the table:

Hours into	0	1	2	3	4
Population (thousands)	2	4	8	16	32

2. If you know the size of the population at a certain time, how do you find the population one hour later?

You multiply it by 2, since it doubled.

3. Marco said he thought that they could use the equation P=2t + 2 to find the population at time t. Seth said he thought that they could use the equation  $P=2\cdot 2t$ .

Is either student correct for populations for t=1,2,3,4? Justify your response. The values predicted by Seth's equation agree exactly with those in the table above; Seth's equation works because it predicts a doubling of the population every hour. Marco's doesn't because it doesn't double the new population you have – instead it is doubling the time. Marco's equation predicts a linear growth of only two thousand bacteria per hour.

4. Assuming the population doubled every hour before the study began, what was the population of the bacteria 1 hour *before* the students started their study?

Since the population is multiplied by 2 every hour we would have to divide by 2 (which is the same as multiplying by  $\frac{1}{2}$ ) to work backwards. The population 1 hour before the study started would be 1,000.

What about 3 hours before?

 $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 2$  thousand = 0.25 thousand or 250

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5. If you know the size of the population at a certain time, how do you find the population one hour *earlier*?

Since the population is multiplied by 2 every hour we would have to divide by 2 (which is the same as multiplying by  $\frac{1}{2}$ ) to work backwards. The population 1 hour before the study started would be 1,000.

6. What number would you use to represent the time 1 hour before the study started? 2 hours before? 3 hours before? Finish filling in the table if you have not already.

Time before the study started would be negative time; for example one hour before the study began was t = -1.

Hours into	-3	-2	-1	0	1	2	3	4
Study								
Population	0.25	1.5	1	2	4	8	16	32
(thousands)								

7. Now use Seth's equation to find the population of the bacteria 1 hour before the study started. Use the equation to find the population of the bacteria 3 hours before. Do these values produce results consistent with the arithmetic you did earlier?

Since one hour before the study started would be t=-1, we would simply plug this value into Seth's equation:

 $(2)2^{-1}=2\cdot(\frac{1}{2})=1$  thousand

Three hours before would be t = -3. Using the equation: (2) $2^{-3} = \frac{2}{2^3} = 0.25$  thousand, giving us the same answers as we got through reasoning.

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8. Use the context to explain why it makes sense that  $2^{-n} = \left(\frac{1}{2}\right)^n = \frac{1}{2^n}$ . That is, describe why, based on the population growth, it makes sense to define 2 raised to a negative integer exponent as repeated multiplication by  $\frac{1}{2}$ .

Since the bacteria double every hour, we multiply the population by two for every hour we go forward in time. So if we want to know what the population will be 8 hours after the experiment started, we need to multiply the population at the start (t=0) by 2 eight times. This explains why we raise 2 to the number of hours that have passed to find the new population; repeatedly doubling the population means we repeatedly multiply the population at t=0 by 2.

In this context, negative time corresponds to time before the experiment started. To figure out what the population was before the experiment started we have to "undouble" (or multiply by  $\frac{1}{2}$ ) for every hour we have to go back in time.

So if we want to know what the population was 8 hours before the experiment started, we need to multiply the population at the start (t = 0) by  $\frac{1}{2}$  eight times. The equation indicates that we should raise 2 to a power that corresponds to the number of hours we need to go back in time.

For every hour we go back in time, we multiply by  $\frac{1}{2}$ . So it makes sense in this context that raising 2 to the -8th power (or any negative integer power) is the same thing as repeatedly multiplying by  $\frac{1}{2}$ , 8 times (or the opposite of the power you raised 2 to). In other words, it makes sense in this context that  $2^{-n} = \left(\frac{1}{2}\right)^n = \frac{1}{2^n}$ 

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#### **Bacterial Growth**

#### Name \_\_\_\_\_

Marco and Seth are lab partners studying bacterial growth. They were surprised to find that the population of the bacteria doubled every hour.

1. The table shows that there were 2,000 bacteria at the beginning of the experiment. What was the size of population of bacteria after 1 hour?

And, after 2, 3 and 4 hours?

Enter this information into the table:

Hours into study	0	1	2	3	4
Population (thousands)	2				

2. If you know the size of the population at a certain time, how do you find the population one hour later?

3. Marco said he thought that they could use the equation P=2t + 2 to find the population at time t. Seth said he thought that they could use the equation  $P=2\cdot 2t$ .

Is either student correct for populations for t=1,2,3,4? Justify your response.

4. Assuming the population doubled every hour before the study began, what was the population of the bacteria 1 hour *before* the students started their study?

What about 3 hours before?

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5. If you know the size of the population at a certain time, how do you find the population one hour *earlier*?

- 6. What number would you use to represent the time for the following situations:
- a) 1 hour before the study started?
- b) 2 hours before?
- c) 3 hours before?

#### Use this information to finish filling in the table if you have not already.

Hours into	-3	-2	-1	0	1	2	3	4
Study								
Population			1	2	4	8	16	32
(thousands)								

7. Now use Seth's equation to find the population of the bacteria 1 hour before the study started.

a) Use the equation to find the population of the bacteria 3 hours before.

b) Do these values produce results consistent with the arithmetic you did earlier?

8. Use the context to explain why it makes sense that  $2^{-n} = \left(\frac{1}{2}\right)^n = \frac{1}{2^n}$ . That is, describe why, based on the population growth, it makes sense to define 2 raised to a negative integer exponent as repeated multiplication by  $\frac{1}{2}$ .

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# Exponential Patterns SAMPLE Quiz or Practice for 8.EE.1

NameTeacher's EditionOriginal Source: EngageNY 8th grade Module 1

NOTE: Use the following problems as additional practice or as a formative quiz for 8.EE.1

The number of users of social media has increased significantly since the year 2001. In fact, the approximate number of users has tripled each year. It was reported that in 2005 there were 3 million users of social media.

1. Assuming that the number of users continues to triple each year, for the next three years, determine the number of users in 2006, 2007, and 2008.

2006 - 9 MILLION 2007 - 27 MILLION 2008 - 81 MILLION

2. Assume the trend in the numbers of users tripling each year was true for all years from 2001 to 2009. Complete the table below using 2005 as year 1 with 3 million as the number of users that year.

Year	-3	-2	-1	0	1	2	3	4	5
# of users in millions	1-77	19	ーろ	1	3	9	27	81	243

3. Given only the number of users in 2005 and the assumption that the number of users triples each year, how did you determine the number of users for years 2, 3, 4, and 5?



4. Given only the number of users in 2005 and the assumption that the number of users triples each year, how did you determine the number of users for years 0, -1, -2, and -3? **NOTE: This problem could be considered as an extension for students.** 



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5. Assume the total number of users continues to triple each year after 2009.

a) Determine the number of users in 2012.

In 2009 there were 243 million so in 2010 that would triple to 729 million which would triple to 2187 million in 2011 and finally 2012 would be 6561 million by tripling once more.

b) Given that the world population at the end of 2011 was approximately 7 billion, is this assumption reasonable?

Based on the pattern of tripling as shown above, 2011 users would be approximately 2187 million which is 2.187 billion which is reasonable as an estimate based on total population of 7 billion.

c) Explain your reasoning.

The estimate is roughly 1/3 of the population as users of social media. Since the estimate falls below the total population and allows for 2/3 of the population to be "non-users". I feel this estimate is reasonable.

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Exponential Patterns
Name

Date \_\_\_\_\_

The number of users of social media has increased significantly since the year 2001. In fact, the approximate number of users has tripled each year. It was reported that in 2005 there were 3 million users of social media.

1. Assuming that the number of users continues to triple each year, for the next three years, determine the number of users in 2006, 2007, and 2008.

2. Assume the trend in the numbers of users tripling each year was true for all years from 2001 to 2009. Complete the table below using 2005 as year 1 with 3 million as the number of users that year.

Year	-3	-2	-1	0	1	2	3	4	5
# of									
users in					3				
millions									

3. Given only the number of users in 2005 and the assumption that the number of users triples each year, how did you determine the number of users for years 2, 3, 4, and 5?

4. Given only the number of users in 2005 and the assumption that the number of users triples each year, how did you determine the number of users for years 0, -1, -2, and -3?

5. Assume the total number of users continues to triple each year after 2009.

a) Determine the number of users in 2012.

b) Given that the world population at the end of 2011 was approximately 7 billion, is this assumption reasonable?

c) Explain your reasoning.

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## **Excursions with Exponents**

This lesson is designed to extend operations with exponents as introduced in the previous lesson. "Excursions with Exponents" is an adaptation and extension of lesson 8.2 in "A Story of Ratios" from EngageNY

## SUGGESTED TIME FOR THIS LESSON

60 minutes

The suggested time for the class will vary depending upon the needs of the students.

## STANDARDS FOR MATHEMATICAL CONTENT

**MFAAA2.** Students will interpret and use the properties of exponents.

b. Use properties of integer exponents to find equivalent numerical expressions. For example,  $3^2 x 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$ . (MGSE8.EE.1)

## COMMON MISCONCEPTIONS

Students multiply the base numbers. For example, students multiply  $3^2 \times 3^{-5}$  to get  $9^{-3}$  instead of the correct answer of  $3^{-3}$  which would be written in final form as  $\frac{1}{3^3} = \frac{1}{27}$ . Students multiply the exponents together. For example, students multiply  $3^2 \times 3^{-5}$  to mistakenly answer  $3^{-10}$  instead of correctly evaluating as  $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$ .

## STANDARDS FOR MATHEMATICAL PRACTICE

8. Look for and make use of structure. Students notice patterns as calculations are repeated, and look both for generalizations about operations with integer exponents. They continually evaluate the reasonableness of their results.

## **EVIDENCE OF LEARNING/LEARNING TARGETS**

By the conclusion of this LESSON, students should be able to:

- Apply properties of integer exponents to find equivalent expressions.
- Represent the exponential integer expressions in multiple ways.

## **MATERIALS**

• Student handout/note taking guide

## **ESSENTIAL QUESTIONS**

- How can you multiply exponential expressions with a common base?
- How can you represent exponential expressions in multiple ways?
- How can you divide exponential expressions with a common base?

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#### SUGGESTED GROUPING

Students should work independently as indicated in the lesson or with a partner as indicated in parts of the lesson.

#### **KEY VOCABULARY** (should be defined in context of the lesson)

- Base
- Exponent
- Exponential form

#### **OPENER/ACTIVATOR**

Post the following problems on the board for students to consider while working independently:

1. Can you represent the number 8 using only prime numbers? Can that answer be expressed in exponential form?

 $2 x 2 x 2 or 2^3$ 

2. Can you represent the number 4 using only prime numbers? Can that answer be expressed in exponential form?

 $2 x 2 or 2^2$ 

3. What is the product of 8 and 4? Can you represent their product using only prime numbers? Can that answer be expressed in exponential form?

 $2 x 2 x 2 x 2 x 2 or 2^5$  (also point out this is  $2^3 x 2^2$ )

4. Do you notice any relationship between these problems?

Discuss that the base number (2) is the same and the exponent is the sum of the two exponents.

After students have time to think about the problem set above, lead a class discussion about things they notice. If students do not notice the pattern of  $2^3 x 2^2 = 2^5$  or are unable to generalize the operation, repeat the process with another example. Or, use the following example to test their hypothesis:

Try 9 x 27 in expanded form and then in exponential form  $3 \times 3 \times 3 \times 3 \times 3 = 3^5$  $3^2 \times 3^3 = 3^5$ 

What conclusions can you make? *Discuss that the base number (3) is the same and the exponent is the sum of the two exponents.* 

#### **LESSON INSTRUCTIONS**

This lesson builds from the previous lessons on exponential operations so students should see connections to previous activities.

Continue the class discussion with the following examples:

1. How about $2^3 \times 2^5 \times 2^7 \times 2^9$  $2^3 \times 2^{-5} \times 2^7 \times 2^9$  $2^3 \times 2^5 \times 2^{-7} \times 2^{-9}$  $2^{24}$  $2^{14}$  $2^{-8} \text{ or } 1/2^8$ 

2. Consider the following

$14^{23}  imes 14^8$	$(-72)^{10}  imes (-72)^{13}$	$a^{23} \cdot a^8$
14 <sup>31</sup>	$(-72)^{23}$	<b>a</b> <sup>31</sup>

Ask students to complete the following generalization: In general, if x is any number and m, n are positive integers, then m = n m = n m = n

$$x^m \cdot x^n = x^{m+1}$$

Because

$$x^m \times x^n = \underbrace{(x \cdots x)}_{m \text{ times}} \times \underbrace{(x \cdots x)}_{n \text{ times}} = \underbrace{(x \cdots x)}_{m+n \text{ times}} = x^{m+n}$$

3. Consider this problem:  $\frac{5^8}{5^6}$ 

How can you simplify the fraction? Can you use expanded form to help?

 $\frac{5x5x5x5x5x5x5x5x5}{5x5x5x5x5x5x5}$  which can simplify to  $\frac{5x5}{1}$  or  $5^2$  or 25

4. What would happen if we changed the problem a little?  $\frac{5^6}{5^8}$ 

How can you simplify the fraction? Can you use expanded form to help? Is there another way to write the fraction?

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5. Now, try these and formalize a "rule" for division of exponential expressions such as we did above for multiplication of exponential expressions.



Ask students to complete the following generalization:

In general, if *x* is nonzero and *m*, *n* are positive integers, then

$$\frac{x^m}{x^n} = x^{m-n}, \text{ if } m > n.$$

## **CLOSING/SUMMARIZER**

Have students complete the exit activity in pairs to practice multiplication and division of exponential expressions with integer exponents.

EXIT TICKET (student edition provided at the end of this LESSON)

$(-19)^5 \cdot (-19)^{11} = (-19)^{16}$	$2.7^5 \times 2.7^3 = 2.7^8$
$\frac{7^{10}}{7^3} = 7^7$	$\left(\frac{1}{5}\right)^2 \cdot \left(\frac{1}{5}\right)^{15} = \left(\frac{1}{5}\right)^{17}$
$\left(-\frac{9}{7}\right)^m \cdot \left(-\frac{9}{7}\right)^n = \left(-\frac{9}{7}\right)^{m+n}$	$\frac{b^3}{b^{10}} = \frac{1}{b^7}$

## **INTERNET ACTIVITY**

The lesson set for 8.EE.1 on Learnzillion provides several short lessons to support this standard. <u>https://learnzillion.com/lessonsets/307-know-and-apply-the-properties-of-integer-exponents-to-generate-equivalent-numerical-expressions</u>

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#### FORMATIVE ASSESSMENT QUESTIONS

In this LESSON, the formative assessment questions are embedded within the questions. Teachers should work with small groups as they complete the activity to formatively assess understanding and gauge review/extension needs.

#### **EXTENSION ACTIVITY**

For students who complete the exit activity quickly and demonstrate understanding, pose the following problems for them to consider:

Can the following expressions be simplified? If so, write an equivalent expression. If not, explain why not.

 $4.5^4 \times 2^{11}$ 

1.  $6^5 \times 4^9 \times 4^3 \times 6^{14}$ 2.  $(-4)^2 \cdot 17^5 \cdot (-4)^3 \cdot 17^7$ 

3.  $15^2 \cdot 7^2 \cdot 15 \cdot 7^4$ 

6. Are the following expressions equivalent? Justify your response.  $2^4 \times 8^2$  and  $2^4 \times 2^6$ 

7. Let *x* be a number. Simplify the expression of the following number:  $(2x^3)(17x^7)$ 

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#### **Excursions with Exponents Note Taking Guide**

1. Write the following products as a single term

 $2^3 \times 2^5 \times 2^7 \times 2^9$   $2^3 \times 2^{-5} \times 2^7 \times 2^9$   $2^3 \times 2^5 \times 2^{-7} \times 2^{-9}$ 

2. Write the following products as a single term

$$14^{23} \times 14^{8}$$
  $(-72)^{10} \times (-72)^{13}$   $a^{23} \cdot a^{8}$ 

3. Consider this problem:  $\frac{5^8}{5^6}$ 

How can you simplify the fraction? Can you use expanded form to help?

4. What would happen if we changed the problem to  $\frac{5^6}{5^8}$ How can you simplify the fraction?

Can you use expanded form to help?

Is there another way to write the fraction?

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5. Now, try these and formalize a "rule" for division of exponential expressions such as we did above for multiplication of exponential expressions.



In general, if <i>x</i> is nonzero and <i>m</i> , <i>n</i> are positive integers, then
$\frac{x^m}{x^n} = x, \text{ if } m > n.$

Excursions with Exponents EXIT TICKET Name

$(-19)^5 \cdot (-19)^{11} =$	$2.7^5 \times 2.7^3 =$
$\frac{7^{10}}{7^3} =$	$\left(\frac{1}{5}\right)^2 \cdot \left(\frac{1}{5}\right)^{15} =$
$\left(\left(-\frac{9}{7}\right)^m \cdot \left(-\frac{9}{7}\right)^n =\right.$	$\frac{b^3}{b^{10}} =$

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# Squares, Area, Cubes, Volume, Roots....Connected?

This lesson is designed to explore the concepts of square numbers, square roots, cube numbers, and cube roots from a concrete application. The lesson is an adaptation and extension of "Cheez-It Activity" found on "I Speak Math".

http://ispeakmath.org/2012/05/03/square-roots-with-cheez-its-and-a-graphic-organizer/

#### **SUGGESTED TIME FOR THIS LESSON:**

60 to 120 minutes

The suggested time for the class will vary depending upon the needs of the students.

## STANDARDS FOR MATHEMATICAL CONTENT

## **MFAAA2.** Students will interpret and use the properties of exponents.

c. Evaluate square roots of perfect squares and cube roots of perfect cubes (MGSE8.EE.2) d. Use square root and cube root symbols to represent solutions to equations of the form  $x^2 = p$  and  $x^3 = p$ , where p is a positive rational number. (MGSE8.EE.2)

## COMMON MISCONCEPTIONS

- Students misinterpret finding the square of a number as doubling the number.
- Students misinterpret finding the square root of a number as taking half of the number (or dividing by two).
- Students misinterpret finding the cube of a number as tripling the number.
- Students misinterpret finding the cube root of a number as taking one third of the number (or dividing by three).

Using a concrete introduction followed by a pictorial representation will provide a link to the abstraction of squaring and cubing as well as provide a context for understanding using area and volume.

## STANDARDS FOR MATHEMATICAL PRACTICE

1. **Make sense of problems and persevere in solving them.** Students start to examine the relationship between the number of squares and the area of the larger square by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals.

2. **Reason abstractly and quantitatively.** Students seek to make sense of quantities (smaller squares) and their relationships in problem situations (larger square). They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students make connections between the concepts of area and dimension in relation to the use of a symbol (radical sign).

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8. Look for and express regularity in repeated reasoning. Students notice if calculations are repeated, and look both for general methods and for shortcuts. They will consider if they can arrive at the total area (or root) by considering the other component without actually having to build the model. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## **EVIDENCE OF LEARNING/LEARNING TARGETS**

By the conclusion of this lesson, students should be able to:

- Evaluate the square roots of small perfect squares and cube roots of small perfect cubes.
- Represent the solutions to equations using square root and cube root symbols.
- Understand that all non-perfect square roots and cube roots are irrational.

## MATERIALS

- One box of Cheez-Its per team (algebra tiles or other squares may be substituted)
- One box of sugar cubes per team (average 200 cubes per one pound box) (algebra cubes, linking cubes, or other cubes may be substituted)
- Graphic Organizer for Squares
- Graphic Organizer for Cubes
- 2 Large number lines (create using bulletin board paper) to display in the class; one for square roots and one for cube roots

## **ESSENTIAL QUESTIONS**

- How can you use square root and cube root symbols to represent solutions to equations of the form  $x^2 = p$  and  $x^3 = p$ , where p is a positive rational number?
- How can you evaluate square roots of small perfect squares?
- How can you evaluate cube roots of small perfect cubes?
- How can you determine if the square root of a number (such as 2) is irrational?

## **OPENER/ACTIVATOR**

Show the Video Opener: Rubik's Cube and Juggling to get the students' attention and open dialogue about cubes, dimensions of cubes, and characteristics of cubes. To preview the lesson, teachers could ask about the number of cubes it takes to make a Rubik's cube. https://www.youtube.com/watch?v=lhkzgjOKeLs

## **KEY VOCABULARY** (should be defined in context of the lesson)

- square
- square root
- cube
- cube root
- radical sign
- rational number
- irrational number

## SUGGESTED GROUPING

Students should would in teams of three to four. Each student should have their own note taking graphic organizer.

## **LESSON INSTRUCTIONS**

Start the lesson by having the students analyze the dimensions of one Cheez-It square  $(1\times1)$ , which has an area of 1 unit squared. Then, repeated this discussion with a four Cheez-it square. Next, ask students to make a square with 6 Cheez-Its. Then discuss why they can't make that model (*As an extension, ask students to consider if it might EVER be possible to construct a square with area of 6 units. Ask them HOW CLOSE they could get to that area. These extension problems might best fit after the activity but are mentioned here to alert teachers of the possibility of students' questions*) Now students can work in small teams to use their Cheez-Its to find more squares and record the dimensions and area on the chart. Often students will figure out the pattern very quickly and are easily able to complete the chart to 144 without needing 144 Cheez-Its.

Repeat the same activity using the sugar cubes or substitute cubes. Allowing students to actually build the cubes, investigate the connection between dimensions and volume, and formalize the discussion of cube roots provides a tangible connection that will support future abstraction and application.

## FORMATIVE ASSESSMENT QUESTIONS

- How can you decide if a number is a perfect square?
- How can you decide is a number is a perfect cube?
- How is finding the square root of a number related to the concept of area?
- How is finding the cube root of a number related to the concept of volume?

## **INTERVENTION**

For extra help square and cube roots, please open the hyperlink Intervention Table.

## **CLOSING/SUMMARIZER**

Have students move to the large number line displays and locate the perfect square roots on the line. Pay close attention to the scale used on the number line. Next, have students approximate the square roots of non-perfect squares by estimating and using a calculator to check. Then, have the students record the location of the irrational numbers on the number line.

Repeat the activity for the perfect cubes number line.

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Keep the number lines posted as anchor charts for approximating roots (cubes and squares)

Sample student work for this lesson may be found at <u>http://ispeakmath.org/2012/05/03/square-roots-with-cheez-its-and-a-graphic-organizer/</u>

## Video Lesson Support from Learnzillion:

Understand and evaluate square roots and cube roots (5 Videos) <u>https://learnzillion.com/lessonsets/351-understanding-and-evaluating-square-roots-and-cube-roots</u> Understand Perfect Squares and Square Roots (6 Videos) <u>https://learnzillion.com/lessonsets/44-understand-perfect-squares-and-square-roots</u> Understanding Perfect Cubes and Cube Roots (2 Videos) <u>https://learnzillion.com/lessonsets/45-understanding-perfect-cubes-and-cube-roots</u>

## Learnzillion Quiz Opportunities (2 quizzes)

Cube roots <u>https://learnzillion.com/quizzes/2706</u> Understanding and Evaluating Cube roots and Square roots <u>https://learnzillion.com/quizzes/2705</u>

## Extension Activity: Approximating square roots

The following PBS video leads students to a method for approximating square roots when numbers are not perfect squares.

http://www.pbslearningmedia.org/resource/mgbh.math.ns.approxsqroot/approximating-square-roots-of-nonperfect-squares/

A process mentioned (approximating square roots) in the video can be summarized as:

1. Find the closest perfect square (smaller than your number of interest). For example, if I am trying to approximate  $\sqrt{20}$ , I would use 16.

2. Find the difference between your focus number (in our case 20) and that perfect square (in our case 16). For our example, the difference is 4.

3. Build (or think of) the next perfect square. For our example, the next perfect square would be 25.

4. Find the difference between the two perfect squares. In our example we would find the difference between 25 and 16 to be 9.

5. Estimate the quotient of these two numbers (focus number – perfect square) / (difference between the two perfect squares). For our problem, 4/9 would be  $0.\overline{4}$ . So, the final approximation of  $\sqrt{20}$  would be 4.  $\overline{4}$ . When checked against a calculator we find the actual approximation to be 4.47 which makes our calculation 0.03 off.

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Sample problems to consider:Estimate the following square roots $1.\sqrt{41}$  $2.\sqrt{15}$ 

Problems such as these help develop estimation skills and a strong sense of number relationships.

Video closing: http://meangreenmath.com/2014/02/11/engaging-students-square-roots/ is the host page and the Elvis Video is toward the bottom of the page https://youtu.be/AfBQGLowyKU. This video is a catchy tune to recall the basic square roots. It also mentions the inverse relationship of squaring and taking square roots.

**<u>Random Fact Tweet</u>** (taken from RandomFacts and shown in the image to the right): In the book Catching Fire, Katniss learns about district 13 on page 169.



 $3.\sqrt{147}$ 

Finish the <u>**CLOSING</u>** of this lesson by revisiting the essential questions of the lesson.</u>

- How can you use square root and cube root symbols to represent solutions to equations of the form  $x^2 = p$  and  $x^3 = p$ , where p is a positive rational number?
- How can you evaluate square roots of small perfect squares?
- How can you evaluate cube roots of small perfect cubes?
- How can you determine if the square root of a number (such as 2) is irrational?

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Squares and Cubes Activity

Name Teacher's Edition

**Squares Table:** Complete the table as you build your squares. Fill in the number line with the radical notation on one side and the root (dimension) on the other side.

Root	Number of Squares	Root	Number of Squares
1	1	11	121
2	4	12	144
3	9	13	169
4	16	14	196
5	25	15	225
6	36	16	256
7	49	17	289
8	64	18	324
9	81	19	361
10	100	20	400

#### **Key Vocabulary**

Square Root: a number that produces a specified quantity when multiplied by itself Area = *length x width* 

Radical: An expression that uses a root, such as square root, cube root.

How is area "special" for a square? *Length and width are the same so area is the square of one side.* 

How does this characteristic relate to "square roots"? The length of a side is the square root of the area.

Between what two integers do the following values fall?

$\underline{4} \sqrt{205} \underline{5} \qquad \underline{8} \sqrt{689} \underline{911} \sqrt{12012} \underline{127}$	$\sqrt{58}$	8
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Squares and Cubes Activity Name\_\_\_\_\_

**Squares Table:** Complete the table as you build your squares. Fill in the number line with the radical notation on one side and the root (dimension) on the other side.

Root	Number of Squares	Root	Number of Squares

#### **Key Vocabulary**

Square Root:



Radical:

How is area "special" for a square?

How does this characteristic relate to "square roots"?

Between what two integers do the following values fall?

 $\sqrt{20}$   $\sqrt{68}$   $\sqrt{120}$   $\sqrt{58}$ 

Mathematics • GSE Foundations of Algebra• Module 2: Arithmetic to Algebra Richard Woods, State School Superintendent July 2020 • Page 176 of 230 All Rights Reserved **Cubes Table:** Complete the table as you build your cubes. Fill in the number line with the radical notation on one side and the root (dimension) on the other side. Note: You may fill in the table without building the model once you establish the pattern.

Root	Number of Cubes	Root	Number of Cubes
1	1	11	1331
2	8	12	1728
3	27	13	2197
4	64	14	2744
5	125	15	3375
6	216	16	4096
7	343	17	4913
8	512	18	5832
9	729	19	6859
10	1000	20	8000

#### **Key Vocabulary**

Cube Root: a number that produces a specified quantity when multiplied by itself a total of three times.

> Radical: An expression that uses a root, such as square root, cube root. Volume = <u>length</u> x <u>width</u> x <u>height</u>

How is volume "special" for a cube? Length, width, and height are all equal.

How does this characteristic relate to "cube roots"? *Any one dimension represents the cube root of the cube* 

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**Cubes Table:** Complete the table as you build your cubes. Fill in the number line with the radical notation on one side and the root (dimension) on the other side.

Ro	oot	Number of Cubes	Root	Number of Cubes

**Key Vocabulary** 

Cube Root:

Radical:

Volume = \_\_\_\_\_ x \_\_\_\_\_ x \_\_\_\_\_

How is volume "special" for a cube?

How does this characteristic relate to "cube roots"?

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Practice Problems for 8.EE.2

Name Teacher's Edition

1. Complete the Table:

Area	Length of Side
Alca	
(square	(units)
units)	
1	1
9	3
25	5
2	4
5	$\sqrt{5}$
13	$\sqrt{13}$
5	$\sqrt{5}$
100	10
400	20

Find the missing measure.

2. Side = 11cm

3. Side = 
$$4m$$

2 0:1-

$$A = 121 \ cm^2$$

 $A = 16 m^2$ 

**Directions:** Complete the following sentences. Provide examples to support your statements.

4. A perfect square is created when...you square a whole number. For example, 25 is a perfect square because you square 5.

5. To find the area of a square given the side length of the square...you square the length. For example, if the side of a square is 4 units, the area is 16 square units.

6. To find the side length of a square given the area of the square...*you take the square root of* the area. For example, if the area of a square is 144 square units the length of a side would be 12 units.

7. Simplify the following.

$\sqrt{36}$	6	√121	11	$\sqrt{16}$	4
$\sqrt{1}$	1	$\sqrt{100}$	10	$\sqrt{49}$	7
√625	25	$\sqrt{2500}$	50	$\sqrt{225}$	15

8. (Extension Problem) Approximate the value of the following square roots. Show the steps used in words, pictures, or diagrams. Check your approximation on a calculator.

(focus number – perfect square) / (difference between the two perfect squares)

a) $\sqrt{18}$	b) $\sqrt{45}$
(18-16)/(25-16) = 2/9	(45-36)/(49-36) = 6/13 or
So the approximation is 4.22	So the approximation is
Actual answer is 4.24	Actual answer is 6.71
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Practice Problems for 8.EE.2

## Name \_\_\_\_\_

1. Complete the Table:

Area	Length of Side
(square	(units)
units)	
1	
9	
	5
2	
5	
	$\sqrt{13}$
	$\sqrt{5}$
100	
	20

Find the missing measure.

2. Side = 11cm

A = \_\_\_\_\_



**Directions:** Complete the following sentences. **Provide examples to support your statements.** 

- 4. A perfect square is created when...
- 5. To find the area of a square given the side length of the square...
- 6. To find the side length of a square given the area of the square...

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# 7. Simplify the following.

$\sqrt{36}$	$\sqrt{121}$	$\sqrt{16}$
$\sqrt{1}$	$\sqrt{100}$	$\sqrt{49}$
√625	$\sqrt{2500}$	$\sqrt{225}$

8. (Extension Problem) Approximate the value of the following square roots. Show the steps used in words, pictures, or diagrams. Check your approximation on a calculator.

a) <del>√18</del>	b) $\sqrt{45}$
· ·	

## **Quick Check II**

## Students will extend arithmetic operations to algebraic modeling.

# MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.

c. Model numerical expressions (arrays) leading to the modeling of algebraic expressions. (MGSE7.EE.1,2; MGSE9-12.A.SSE.1,3)

d. Add, subtract, and multiply algebraic expressions. (MGSE6.EE.3, MGSE6.EE.4, MC7.EE.1, MGSE9-12.A.SSE.3)

e. Generate equivalent expressions using properties of operations and understand various representations within context. For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between "3 more" and "3 times". (MGSE4.0A.2; MGSE6.EE.3, MGSE7.EE.1, 2, MGSE9-12.A.SSE.3)

f. Evaluate formulas at specific values for variables. For example, use formulas such as A = l x w and find the area given the values for the length and width. (MGSE6.EE.2)

## MFAAA2. Students will interpret and use the properties of exponents.

c. Evaluate square roots of perfect squares and cube roots of perfect cubes (MGSE8.EE.2) d. Use square root and cube root symbols to represent solutions to equations of the form  $x^2 = p$ and  $x^3 = p$ , where p is a positive rational number. (MGSE8.EE.2)

## **Teacher's Edition for Quick Check 2**



a) Write an algebraic equation to represent the total number of tiles.

## T=2+4n

b) Use your equation to find the total number of tiles in step 25.

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2. A serving of grits has forty fewer calories than half a cup of yogurt (which is one serving of yogurt).

a) Represent the number of calories in each serving of grits and in each serving of yogurt. Calories in one serving of yogurt could by y so calories in one serving of grits would be y - 40

b) Represent the number of calories you would consume if you eat a serving of grits and a full cup of yogurt. (y - 40) + 2y would simplify to 3y - 40

c) You are hosting a breakfast for your friends. Represent the total number of calories consumed at your breakfast if five friends want grits, three friends want yogurt, and six friends want both grits and yogurt.

5(y-40) + 3y + 6(y-40 + y) would simplify to 20y - 440

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3. Mark has 12 comic books, Jeremy has 14 comic books, and Sam has half as many comic books as Jeremy.

a) Write a numerical expression to represent how many comic books the three boys have together.

12 + 14 + 7

b) If each comic book cost \$3.50, write two equivalent expressions to represent how much the boys have spent total.

3.50(12 + 14 + 7) and 3.50(12) + 3.50(14) + 3.50(7)

c) How do you know that these two expressions are equivalent? Justify your response.

Answers will vary. Possible response: I used distributive property on the first expression to get the second expression. Both expressions equal \$115.50.

d) If Mark lost a few of his cards, write a new expression to represent how many cards Mark has now.

12 - x where x represents the number of cards lost

e) Explain the differences between a numeric expression and an algebraic expression.

Answers will vary. Possible response: A numeric expression is made up of numerical values but in an algebraic expression, not all of the values are known. These unknown values are represented by variables.

4. What is the side length of a square with an area of 9 units<sup>2</sup>?

3 units

5. What is the area of a square with a side length of 2 units?

#### 4 units<sup>2</sup>

6. Between what two integers do the following values fall?



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a) Write an algebraic equation to represent the total number of tiles.

b) Use your equation to find the total number of tiles in step 25.

2. A serving of grits has forty fewer calories than half a cup of yogurt (which is one serving of yogurt).

a) Represent the number of calories in each serving of grits and in each serving of yogurt.

b) Represent the number of calories you would consume if you eat a serving of grits and a full cup of yogurt.

c) You are hosting a breakfast for your friends. Represent the total number of calories consumed at your breakfast if five friends want grits, three friends want yogurt, and six friends want both grits and yogurt.

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3. Mark has 12 comic books, Jeremy has 14 comic books, and Sam has half as many comic books as Jeremy.

a) Write a numerical expression to represent how many comic books the three boys have together.

b) If each comic book cost \$3.50, write two equivalent expressions to represent how much the boys have spent total.

c) How do you know that these two expressions are equivalent? Justify your response.

d) If Mark lost a few of his cards, write a new expression to represent how many cards Mark has now.

e) Explain the differences between a numeric expression and an algebraic expression.

- 4. What is the side length of a square with an area of 9 units<sup>2</sup>?
- 5. What is the area of a square with a side length of 2 units?
- 6. Between what two integers do the following values fall?

 $\__\sqrt{76}$   $\__\sqrt{89}$   $\__\sqrt{135}$   $\__\sqrt{32}$ 

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## What's the "Hype" About Pythagoras?

In this LESSON, students work with the Pythagorean Theorem in the context of application problems. For Foundations of Algebra, students will be required to find the hypotenuse of a right triangle given the other two legs. Extension opportunities are provided for students who are ready to extend that requirement to finding a missing leg given the hypotenuse and the other leg. An additional extension activity has been provided for students to informally "discover" the Pythagorean Theorem.

## **SUGGESTED TIME FOR THIS LESSON:**

60-90 minutes

The suggested time for the class will vary depending upon the needs of the students.

## STANDARDS FOR MATHEMATICAL CONTENT

## **MFAAA2.** Students will interpret and use the properties of exponents.

c. Evaluate square roots of perfect squares and cube roots of perfect cubes (MGSE8.EE.2) d. Use square root and cube root symbols to represent solutions to equations of the form  $x^2 = p$ and  $x^3 = p$ , where p is a positive rational number. (MGSE8.EE.2) e. Apply the Pythagorean Theorem to solve triangles based on real-world contexts (Limit to

e. Apply the Pythagorean Theorem to solve triangles based on real-world contexts (Limit to finding the hypotenuse given the two legs). (MGSE.8.G.7)

## **COMMON MISCONCEPTIONS**

Students will confuse parts of right triangles and inappropriately apply the Pythagorean Theorem. Students have difficulty distinguishing between the legs of the triangle and the hypotenuse of the triangle. Additionally, students want to make a distinction between "a" and "b" in terms of the legs of the triangle. Attention should be directed toward the commutative property of addition as it relates to the Pythagorean Theorem.

Another common mistake students make when applying the Pythagorean Theorem is to double the sides of the triangle as opposed to squaring the sides. Call attention to the distinction between these mathematical actions.

## STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will make sense of this lesson through their questioning and understanding of the Pythagorean Theorem.

2. **Reason abstractly and quantitatively.** Students will reason quantitatively based on the information given for right triangle problems.

3. Construct viable arguments and critique the reasoning of others. Students will discuss their solutions and strategies and decide whether they agree or disagree with and debate these mathematical arguments.

4. **Model with mathematics.** Students will model their thinking through diagrams and equations.

5. Use appropriate tools strategically. Students will use appropriate tools such as calculators

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## **EVIDENCE OF LEARNING/LEARNING TARGETS**

By the conclusion of this lesson, students should be able to:

- Apply the Pythagorean Theorem in real world context.
- Interpret the parts of a right triangle used appropriately in the Pythagorean Theorem.

## **MATERIALS**

- Student handout
- Calculators
- Sticky notes
- Link or download version of Robert Kaplinksy's "How Can We Correct the Scarecrow?" video *http://robertkaplinsky.com/work/wizard-of-oz*

## **ESSENTIAL QUESTIONS**

- How is the Pythagorean Theorem useful when real world solving problems?
- When is it useful to use the Pythagorean Theorem?
- Does the Pythagorean Theorem always work?

## **OPENER/ACTIVATOR**

To introduce the lesson, show the video clip about the Pythagorean Theorem from Robert Kaplinksy's "How Can We Correct the Scarecrow?" lesson. The clip is designed to get the students thinking about their past experience (if any) with the Pythagorean Theorem. Save the remaining parts of the activity (correcting the Scarecrow's statement) until after you have completed the lesson.

After showing the video, students will complete the Pythagorean Investigations Sheet to open dialogue about their understanding of the Pythagorean Theorem. The Pythagorean Inventory will be completed as the opener with the remaining parts developed in the LESSON.

## **KEY VOCABULARY**

The key vocabulary for this lesson is listed in the Pythagorean Inventory and should be discussed after the Gallery Walk activity:

square numbers, square roots, right triangle, right angle, Pythagorean Theorem, legs, hypotenuse, vertex, rational number, irrational number, radical, squaring a number

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## **LESSON DESCRIPTION**

After completing the inventory part (part 1) of the Pythagorean Investigations Sheet, students will work with a partner to complete the sticky note brainstorming activity. (Allow 10-15 minutes based on student understanding.)

During this phase, students will then be asked to work with a partner to record ideas about the Pythagorean Theorem on sticky notes to be added to the class brainstorming sheet. Students should work with a partner to come up with details, definitions, diagrams, drawings to describe as many of the terms in the inventory as they can. Teams should record ideas on the sticky notes provided then post them on the charts around the room.

Teams will then go on a Gallery Walk to see peer responses on the anchor charts. Students should walk with their partner through the posted terms and review the responses from each team. Instruct students to make notes on paper about follow up questions based on what they read on the charts. Students should circle the terms on the list from the inventory that need more clarification during class discussion. Students should draw a box around the terms that they feel are completely clear after the gallery walk. (Allow 10-15 minutes based on student understanding.)

After students complete the gallery walk, provide time to discuss and clarify each of the terms from the inventory. Understanding of the terminology will be key to the application of the Pythagorean Theorem.

Next, give students the graphic organizer on the Pythagorean Theorem and have them label the parts of the triangle independently. Students will use the organizer as they discuss the application problems provided on their Application practice guide.

\*<u>NOTE:</u> When evaluating irrational roots, have students work with both radical notation and decimal notation. For the purposes of this course, students do not need to simplify radical expressions.

#### **EXTENSION ALERT**

The standard for this course limits the application of the Pythagorean Theorem to finding the hypotenuse of a right triangle when given the legs. Many students are curious about the origin of the Pythagorean Theorem and "Why" it works. Extension Lessons will be provided at the conclusion of the student pages for students who might want to investigate the proof of the Pythagorean Theorem and who might want to find the leg of a right triangle when given the hypotenuse and the other leg.

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## **Internet Activities**

Learnzillion Video <u>https://learnzillion.com/lessons/1262-find-the-length-of-the-hypotenuse-of-a-right-triangle-using-the-pythagorean-theorem</u>

Pythagorean Theorem <u>http://mathforum.org/isaac/problems/pythagthm.html</u> Biography of Pythagoras <u>http://www-groups.dcs.st-and.ac.uk/~history/Printonly/Pythagoras.html</u> Pythagorean Theorem <u>http://www.mathsisfun.com/pythagoras.html</u> Using Pythagorean Theorem <u>http://www.pbs.org/wgbh/nova/proof/puzzle/use.html</u>

The Pythagorean Theorem: Square Areas Formative Assessment Lesson featured on the Math Assessment Project website at <u>http://map.mathshell.org/materials/lessons.php?LESSONid=408</u> provides complete lesson plans and all student materials needed to extend the content of the Pythagorean Theorem as addressed in Foundations of Algebra.

## Mathematical goals of the Formative Assessment Lesson (from Math Assessment Project):

This lesson is intended to help you assess how well students are able to:

- Use the area of right triangles to deduce the areas of other shapes.
- Use dissection methods for finding areas.
- Organize an investigation systematically and collect data.
- Deduce a generalizable method for finding lengths and areas (The Pythagorean Theorem.)

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Name	Pythagorean Invo	estigations				
Part 1: <u>Pythagorean The</u>	eorem Self Inventory					
Put a smiley face If you know the term	Put a blank face if you have heard the term	Put a frown face if you have not heard the term				
square numbers	square roots	right triangle				
right angle	Pythagorean Theorem	legs				
hypotenuse	vertex ra	tional number				

irrational number \_\_\_\_\_ radical \_\_\_\_\_ squaring a number \_\_\_\_\_

## Part 2:Partner Discussion

- Work with your partner to come up with details, definitions, diagrams, drawings to describe as many of the terms above as you can.
- Record your ideas on the sticky notes provided, then post them on the charts around the room.

## Part 3: Gallery Walk

- Walk with your partner through the posted terms and review the responses from each team.
- Make notes on paper about follow up questions based on what you read on the charts.
- Circle the terms on the list above that need more clarification during class discussion.
- Draw a box around the terms that you feel are completely clear to you after the gallery walk.

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#### Parts of a Right Triangle

Name\_\_\_\_\_

Label the right triangle shown below.

A word list is included to help you. Some words will be used more than once.

#### WORD LIST

leg hypotenuse right angle side a side b side c vertex  $a^2 + b^2 = c^2$ 



The Pythagorean Theorem

And can only be used for

Sample Problem:

If the sides of a right triangle are 6 inches and 8 inches, how long is the hypotenuse?

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#### Applications of the Pythagorean Theorem

Name <u>Teacher's Edition</u>

Solve each of the following problems. Please include your steps in solving the problems. For answers that are irrational, include both the radical and decimal approximation (to the nearest hundredth using a calculator).

1. A baseball diamond is a square with sides of 90 feet. What is the shortest distance, to the *nearest tenth* of a foot, between first base and third base?

 $90^2 + 90^2 = X^2$   $16200 = X^2$   $\sqrt{16200} = x$  X = 127.28 feet

2. A suitcase measures 24 inches long and 18 inches high. What is the diagonal length of the suitcase to the *nearest tenth* of an inch?

 $24^2 + 18^2 = X^2$   $900 = X^2$   $\sqrt{900} = x$  X = 30 inches

3. The older floppy diskettes measured 5 and 1/4 inches on each side. What was the diagonal length of the diskette to the *nearest tenth* of an inch?

 $5.25^2 + 5.25^2 = X^2$   $55.125 = X^2$   $\sqrt{55.125} = X$  X = 7.4 inches

4. Find, Fix, and Justify: Raymond was asked to solve for the length of the hypotenuse in a right triangle with legs that have side lengths of 4 and 5. His work is shown below. He made a mistake when solving. Explain the mistake and then solve the problem correctly.

Raphael's Solution:

Correct Solution:

Raymond forgot to take the square root of 41. The correct answer should be 6.4.

 $a^{2} + b^{2} = c^{2}$   $4^{2} + 5^{2} = c^{2}$   $16 + 25 = c^{2}$ 41 = c

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Applications of the Pythagorean Theorem

Name \_\_\_\_

Solve each of the following problems. Please include your steps in solving the problems. For answers that are irrational, include both the radical and decimal approximation (to the nearest hundredth using a calculator).

1. A baseball diamond is a square with sides of 90 feet. What is the shortest distance, to the *nearest tenth* of a foot, between first base and third base?

2. A suitcase measures 24 inches long and 18 inches high. What is the diagonal length of the suitcase to the *nearest tenth* of an inch?

3. The older floppy diskettes measured 5 and 1/4 inches on each side. What was the diagonal length of the diskette to the *nearest tenth* of an inch?

4. Find, Fix, and Justify: Raymond was asked to solve for the length of the hypotenuse in a right triangle with legs that have side lengths of 4 and 5. His work is shown below. He made a mistake when solving. Explain the mistake and then solve the problem correctly.

Raphael's Solution:

Correct Solution:

 $a^{2} + b^{2} = c^{2}$   $4^{2} + 5^{2} = c^{2}$   $16 + 25 = c^{2}$ 41 = c

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## **REVIEW ACTIVITIES**

The following activities may be found on the New Zealand Numeracy Site in Project Book 8 page 30 <u>http://nzmaths.co.nz/sites/default/files/Numeracy/2008numPDFs/NumBk8.pdf</u>. These activities review the concepts developed during the learning lesson. All parts provided below may not be necessary for your particular classroom. Evaluate student understanding from the practice problems provided after the learning lesson and through classroom dialogue/student discussion.

## Square Roots Activity (Concrete, Representational, Abstract)

Learning Target: I am learning that using the square root finds the length of the side given the area.

Equipment Needed: Square cardboard pieces or square tiles and calculators.

## Using Manipulative (square tiles)

Problem: "Zoë builds a large square from 16 small square tiles. How big is Zoë's square?"

Get the students to build the large square out of 16 tiles. Then discuss its dimensions. (Answer: 4 rows of 4 tiles.)

Provide other problems to build:

Build large squares built with these numbers of small squares and give their size: choose small numbers of squares such as 4, 25, and 9

## Using Imaging or a diagram (drawing without the manipulative)

Problem: "Zoë builds a large square from 36 small square tiles. **Imagine** (think about) how big the square is." Go back to building the square if needed. (Answer: 6 by 6.)

Provide other examples: Imagine large squares with these numbers of small squares and find their sizes: 49, 100, 81 ...

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#### **Using Number Properties**

Problem: Zoë builds a large square from 961 small square tiles. Describe the square Zoë builds with the aid of a calculator. For example, how long are the sides? How can you estimate the length of the sides?

Discuss how the  $\sqrt{x}$  button helps. (Answer:  $\sqrt{961} = 31$ .) Provide other examples: Large squares are made with these numbers of small squares. Describe the side length of the large squares: 207,936 2,025 622,521 2, 217,121 ...

#### **INTERVENTION**

For extra help with the Pythagorean Theorem, please open the hyperlink Intervention Table.

#### **EXTENSION ACTIVITY**

#### **Locating Square Roots**

Most square roots do not come out nicely. For example  $\sqrt{3}$  is an infinite non-recurring decimal starting 1.73205080 ....

A method of finding square roots involves locating the answer between two numbers. In principal, this method will locate a square root to any desired accuracy.

**Learning Target**: I am learning how to find the square root of numbers without using a square root button on a calculator.

\*Note the resource sheets listed below are available on the New Zealand Numeracy Site (links are provided)

Equipment: Tables of squares (Material Master 8–22) http://nzmaths.co.nz/sites/default/files/Numeracy/2007matmas/Bk8/MM%208\_22.pdf

Worksheet (Material Master 8–23) http://nzmaths.co.nz/sites/default/files/Numeracy/2007matmas/Bk8/MM%208\_23.pdf

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#### **Using Number Properties**

Problem: "Meriana designs a square swimming pool, which she wants to have an area of 75 m<sup>2</sup>. Unfortunately the square root button on her calculator is damaged, so she will have to find another way of finding square roots."

Display these tables on the board from Material Master 8–22 (linked above)

	0	1	2	3	4	5	6	7	8	9	10
	0	1	4	9	16	25	36	49	64	81	100
_											
	8	8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9
Γ	64	65.61	67.24	68.89	70.56	72.25	73.96	75.69	77.44	79.21	81

"How does the first table show the side of the pool is between 8 and 9 meters? How does the second table show the side of the pool is between 8.6 and 8.7 meters?"

8.6	8.61	8.62	8.63	8.64	8.65	8.66	8.67	8.68	8.69	8.7

Explore filling in the squares in this table to locate 75. (Answer: It is between 8.66 and 8.67.)

Examples: Worksheet (Material Master 8-23).

Understanding Number Properties: How could you find the square root to any desired accuracy of any number using only the square button on a calculator?

An additional activity is available in "Teaching Number Sense and Algebraic Thinking" (Book 8 linked above) that remediates the concept of cubes and cube roots.

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## **Fabulous Formulas**

## SUGGESTED TIME FOR THIS LESSON

60-120 minutes

The suggested time for the class will vary depending upon the needs of the students.

## STANDARDS FOR MATHEMATICAL CONTENT

MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.

a. Apply properties of operations emphasizing when the commutative property applies. (MGSE7.EE.1)

f. Evaluate formulas at specific values for variables. For example, use formulas such as A = l x w and find the area given the values for the length and width. (MGSE6.EE.2)

## MFAAA2. Students will interpret and use the properties of exponents.

a. Substitute numeric values into formulas containing exponents, interpreting units consistently. (MGSE6.EE.2, MGSE9-12.N.Q.1, MGSE9-12.A.SSE.1, MGSE9-12.N.RN.2)

## COMMON MISCONCEPTIONS

In evaluating formulas for specific values students make errors in the order of operations and perform ALL operations from left to right without attention to the type of operation that should be performed first. Some students also use incorrect notation when making substitutions into the given formula. For example, a student might incorrectly evaluate the formula for the volume of a cylinder,  $V = \pi r^2 h$  where r = 3 and h = 4, as  $V = \pi * 3 * 2 * 4$  to get 24  $\pi$  instead of  $V = \pi * 3^2 * 4$  to get 36  $\pi$ .

## STANDARDS FOR MATHEMATICAL PRACTICE

Make sense of problems and persevere in solving them. Students will make sense of the problems posed in this LESSON to select the appropriate formula to use in solving the problems.
 Reason abstractly and quantitatively. Students will reason quantitatively based on the constraints of the problem.

3. Construct viable arguments and critique the reasoning of others. Students will discuss their solutions and strategies and decide whether they agree or disagree with and debate these mathematical arguments.

5. Use appropriate tools strategically. Students will use appropriate tools such as calculators.

6. **Attend to precision.** Students will show precision through their use of mathematical language and vocabulary in their questioning and discussions. They will also show precision in their mathematical computations and procedures.

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## **EVIDENCE OF LEARNING/LEARNING TARGETS**

By the conclusion of this lesson, students should be able to:

- Select the appropriate formula for a given situation.
- Apply the order of operations to evaluate a formula given specific parameters.
- Assess the reasonableness of a solution based on the situation described in the problem.

## **MATERIALS**

- Formula sheet
- Application problems
- Calculators

## ESSENTIAL QUESTIONS

- How do you decide which formula applies in a given situation?
- What role does the order of operations play in evaluating a formula?

## **GROUPING**

Pairs or small groups

## **ACTIVATOR/OPENING**

Post the formulas from the formula sheet on the board and have students identify as many of them as they know. Have students write descriptions of what the variables represent in the formulas. After students have time to ponder the formulas, hand out the formula table and let them self-evaluate their performance. Allow time for students to share ideas or things they notice about the formulas. End the activator with a discussion of the role of "formulas" in everyday life. Point out that the empty rows at the end of the formula sheet are provided for them to enter any other formulas that they might already know and be able to apply.

## LESSON DESCRIPTION

In Foundations of Algebra, students will be expected to evaluate expressions at specific values of their variables. These expressions should also be extended to include expressions that arise from formulas used in real-world problems. For example, students should be able to use the formulas  $V = s^3$  and  $A = 6s^2$  to find the volume and surface area of a cube with sides of lengths  $=\frac{1}{2}$ .

Students will be given a formula sheet as reference, along with a set of situational problems. Students will work in pairs or small groups to decide which formula to use to solve the problems. Attention must be directed toward proper usage of the order of operations.

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The last problem on the activity sheet asks students to create a unique problem using one of the formulas from the table. They are to record their problem on a separate sheet of paper. You may post the problems around the room for a "around the room" problem solving activity, use the problems as activators or closing activities, or use them as formative items in a future class.

## FORMATIVE ASSESSMENT QUESTIONS

As you walk around during small group work discuss the essential questions of the lesson with teams to evaluate their understanding of the lesson focus.

- How do you decide which formula applies in a given situation?
- What role does the order of operations play in evaluating a formula?

## **CLOSING**

The last problem on the activity sheet asks students to create a unique problem using one of the formulas from the table. Use a few problems to highlight student applications of the formulas.

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# **Formula Sheet**

	-
Convert temperatures from Celsius to Fahrenheit where F is temperature in Fahrenheit and C is temperature in Celsius	$F = \frac{9}{5}C + 32$
Interest compounded once per year where P is the principal amount, A is the final amount, r is the rate per year in decimal form, and t is time in years	$A = P (1 + r)^t$
Surface area of a sphere where A is surface area and r is radius of the sphere	$A = 4\pi r^2$
Volume of a sphere where V is volume and r is radius of the sphere	$V = \frac{4}{3}\pi r^3$
Simple Interest (not compound) where A is total amount after t years at r rate (as a decimal) with the initial value of P	A = P + Prt
Volume of a cylinder where V is volume, r is radius and h is height of the cylinder	$V = \pi r^2 h$
Surface area of a cylinder where A is surface area, r is radius, and h is height of0 the cylinder	$A = 2\pi r^2 + 2\pi rh$ (2 circle bases) + (rolled up rectangular side)
Convert temperatures from Fahrenheit to Celsius where F is temperature in Fahrenheit and C is temperature in Celsius	$C = \frac{5}{9}(F - 32)$
Area of a square with side length s	$A = s^2$

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**Fabulous Formulas** 

#### Name Teacher's Edition

Work with your partner or team to decide which formula to apply in each problem listed below. Show your work for each problem (calculations may be done on the calculator if needed) and decide if your answer is reasonable or not.

After partner work time, you will be asked to compare results with another team. Resolve any problems for which both teams do not have the same answer.

1. Elmer borrowed \$500.00 to go on his senior trip. The interest on his loan is simple interest at a rate of 7% and he takes three years to pay off his loan.

a) Which formula will Elmer need to find the total amount he must repay for his loan? A = P + Prt

b) What is the total amount of money he must repay at the end of the three years?  $A = 500 + (500) \cdot 07 \cdot (3) = $605.00$ 

c) How much interest will Elmer have to pay on his loan?

#### Interest will be \$105.00

d) If he makes equal monthly payments over the course of his loan, how many payments will Elmer have to make?

*3 years x 12 months per year = 36 payments* e) How much will each monthly payment be?

\$605.00 / 36 means each payment would be approximately \$16.81 for each payment.

2. Hope loves to make paper-mache models. She is building a basketball model with a diameter of 10 inches.

a) What is the capacity of her basketball model?  $V = \frac{4}{3}\pi r^3 \text{ with radius of 5 inches gives } \frac{500}{3} * \pi \text{ or } 66\frac{2}{3}\pi \text{ or } 523.598 \text{ in}^3$ 

b) How much material will she need to cover her model with fabric if no parts overlap?  $A = 4\pi r^2$  with radius of 5 inches gives 100  $\pi$  in<sup>2</sup> or 314.159 in<sup>2</sup>

3. The temperature outside is 35° Celsius. Should you wear shorts or a coat? Justify your response for someone who is not familiar with the Celsius temperature scale.  $F = \frac{9}{5}C + 32 \text{ yields 95°f when it is 35°c} You \text{ should wear shorts.}$ 

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4. Howard is recycling his Pringle's potato crisp can as a pencil holder. He wants to decorate the tall part (cylinder side) with fabric. If the can has a 3 inch diameter and a 10.5 inch height, how much fabric will he need?

He will use  $2\pi rh$  to find the surface area of the rolled up rectangle to be approximately 98.91 in<sup>2</sup>

5. Howard decided to cover the top with aluminum foil (no over lapping or extra). How much foil will he need to cover the top?

## He will find the area of the circle on top using $\pi r^2$ to be 7.065 in<sup>2</sup>

6. Marcus told his mother that the area of the square office at his school is 144 ft<sup>2</sup>. His mom asked him for the length of one side of the office. How can he figure it out? What is the length of the side of the office?

## He should use $x^2 = 144$ to solve using square roots to find the side of the office to be 12 feet.

7. Irina is expecting guests from Europe. They asked what kind of temperatures there will be during their visit, and Irina told them high temperatures in the 80's and low temperatures in the 60's. Her guests were very confused by this information! Irina realized they use Celsius but she gave them Fahrenheit temperatures. What temperature range should she tell them in Celsius?

#### The high should be 26.6 degrees Celsius and low should be 15.6 degrees Celsius

8. Select one of the formulas from the formula table and create your own application problem. Record your problem on a separate sheet of paper. Work your created problem in the space provided below making sure to show all steps.

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#### **Fabulous Formulas**

Name \_\_\_\_\_

Work with your partner or team to decide which formula to apply in each problem listed below. Show your work for each problem (calculations may be done on the calculator if needed) and decide if your answer is reasonable or not.

After partner work time, you will be asked to compare results with another team. Resolve any problems for which both teams do not have the same answer.

1. Elmer borrowed \$500.00 to go on his senior trip. The interest on his loan is simple interest at a rate of 7% and he takes three years to pay off his loan.

a) Which formula will Elmer need to find the total amount he must repay for his loan?

b) What is the total amount of money he must repay at the end of the three years?

c) How much interest will Elmer have to pay on his loan?

d) If he makes equal monthly payments over the course of his loan, how many payments will Elmer have to make?

e) How much will each monthly payment be?

2. Hope loves to make paper mache models. She is building a basketball model with a diameter of 10 inches.

a) What is the capacity of her basketball model?

b) How much material will she need to cover her model with fabric if no parts overlap?

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3. The temperature outside is 35° Celsius. Should you wear shorts or a coat? Justify your response for someone who is not familiar with the Celsius temperature scale.

4. Howard is recycling his Pringle's potato crisp can as a pencil holder.

a) He wants to decorate the tall part (cylinder side) with fabric. If the can has a 3 inch diameter and a 10.5 inch height, how much fabric will he need?

b) Howard decided to cover the top with aluminum foil (no over lapping or extra). How much foil will he need to cover the top?

6. Marcus told his mother that the area of the square office at his school is 144 ft<sup>2</sup>. His mom asked him for the length of one side of the office. How can he figure it out? What is the length of the side of the office?

7. Irina is expecting guests from Europe. They asked what kind of temperatures there will be during their visit and Irina told them high temperatures in the 80's and low temperatures in the 60's. Her guests were very confused by this information! Irina realized they use Celsius but she gave them Fahrenheit temperatures. What temperature range should she tell them in Celsius?

8. Select one of the formulas from the formula table and create your own application problem. Record your problem on a separate sheet of paper.

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## The Algebra of Magic SUGGESTED TIME FOR THIS LESSON:

The suggested time will vary depending upon the needs of the students. An estimate would be 30 minutes/part.

## <u>STANDARDS FOR MATHEMATICAL CONTENT</u> <u>Students will extend arithmetic operations to algebraic modeling.</u>

# MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.

a. Apply properties of operations emphasizing when the commutative property applies. (MGSE7.EE.1)

b. Use area models to represent the distributive property and develop understandings of addition and multiplication (all positive rational numbers should be included in the models). (MGSE3.MD.7)

c. Model numerical expressions (arrays) leading to the modeling of algebraic expressions. (MGSE7.EE.1,2; MGSE9-12.A.SSE.1,3)

d. Add, subtract, and multiply algebraic expressions. (MGSE6.EE.3, MGSE6.EE.4, MC7.EE.1, MGSE9-12.A.SSE.3)

e. Generate equivalent expressions using properties of operations and understand various representations within context. For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between "3 more" and "3 times". (MGSE4.0A.2; MGSE6.EE.3, MGSE7.EE.1, 2, MGSE9-12.A.SSE.3)

f. Evaluate formulas at specific values for variables. For example, use formulas such as A = l x w and find the area given the values for the length and width. (MGSE6.EE.2)

## STANDARDS FOR MATHEMATICAL PRACTICE

**1. Make sense of problems and persevere in solving them.** Students will be making sense of magic tricks involving algebraic expressions as well as creating their own.

2. **Reason abstractly and quantitatively.** Students will be reasoning through each of the tricks as they determine how each trick works. Students will need to reason with quantities of "stuff" initially before generalizing an abstract rule or expression that represents all of the steps involved.

3. Construct viable arguments and critique the reasoning of others. Students will create expressions and defend these expressions with their peers. Students may discover equivalent expressions and end up proving their equivalence.

4. **Model with mathematics.** Students will use models to represent what happens in each trick in order to understand the trick, undo the trick, invent new tricks, and realize the value of representation.

6. **Attend to precision.** Students will attend to precision through their use of the language of mathematics as well as in their use of operations.

7. Look for and make use of structure. Students will show an understanding of how numbers and variables can be put together as parts and wholes using representations of operations and properties.

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## ESSENTIAL QUESTIONS

- How is the order of operations used to evaluate expressions?
- How are properties of numbers helpful in evaluating expressions?
- What strategies can I use to help me understand and represent real situations using algebraic expressions?
- How are the properties (Identify, Associative and Commutative) used to evaluate, simplify and expand expressions?
- How is the Distributive Property used to evaluate, simplify and expand expressions?
- How can I tell if two expressions are equivalent?

#### **MATERIALS REQUIRED**

- Computer and projector or students with personal technology (optional)
- Directions for mathematical magic tricks (attached)
- Counters
- Sticky notes or blank pieces of paper-all the same size (several per student)

The goals of these tasks are for students to:

- develop an understanding of linear expressions and equations in a context;
- make simple conjectures and generalizations;
- add expressions, 'collect like terms';
- use the distributive law of multiplication over addition in simple situations;
- develop an awareness that algebra may be used to prove generalizations in number situations.

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Note: The context of magic tricks can often turn students off since it may lead students to imply that math is just a bunch of tricks. Presenting this with a video of a middle school student performing the trick (see links below) for someone else helps to dispel this myth. When students see another student perform this trick, they are more apt to ask "How did they do that?" or, better yet, "Why does that work?" This is what we should all strive for in our lessons. Students asking how to do something or wondering why something works means we've evoked curiosity and wonder.

While you can perform the tricks yourself, please be mindful of the fact that most people do not like to be fooled, and students can be intimidated by this especially in front of their peers. Another option would be to teach a trick to a student in one of your classes ahead of time. The student will know the trick (how it works), but he/she will probably not know why. The student can still investigate this.

After each trick is presented, either through the video or in another manner, students can go the link posted for each trick and try different numbers and look for patterns. The link is not necessary, but it does take the "trick" out of the picture and allows students to investigate without being intimidated by being tricked.

Explain to students that, for each trick in this task, they should:

- investigate the trick, trying different numbers;
- work out how the trick is done. This usually involves spotting a connection between a starting number and a finishing number. Algebra will be helpful here: representing the unknown number (the number that is thought of) with something that can contain a quantity may be helpful at first;
- *improve the trick in some way.*

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## **Trick 1: A Math-ic Prediction**

*This 3-act task can be found at: <u>http://mikewiernicki3act.wordpress.com/a-math-ic-prediction/</u> NOTE: The 3-act template is located after Trick 3.* 

In part one of this series of tasks, students will interact with a computer program that can "read minds." Then, students will tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics, specifically algebraic reasoning, to answer their own questions. Students will be given information to solve the problem based on need. When they realize they do not have the information they need, and ask for it, it will be given to them.

Students choose any number and follow the steps given by the teacher or the interactive app (link is at the end of this description). The prediction of "1" is revealed after the steps are followed.

## Steps to follow for this trick:

Think of a number. Add 3. Double the result. Subtract 4. Divide the result by 2. Subtract your original number. The prediction for this trick is the same every time: 1.

Students may wish to try larger or even smaller numbers, **integers, fractions or decimals** to see if any numbers will not work. This can be a nice way to check their understandings and fluency with operations using rational numbers. NOTICE: a **post-it note** is used for x *An example:* 

Think of a number.	x
Add 3.	<i>x</i> + 3
Double the result.	2(x+3) or 2x+6

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The post-its work well because as students begin to visualize what is happening, they can easily write a variable on the post-it. The post-it becomes a variable and can be written in an expression as seen in the right hand column.

NOTE: Students should not be shown the table above. Students should make sense of the mathematics involved in this prediction, creating their own representations and expressions.

A Mathic Prediction interactive app: <u>http://scratch.mit.edu/projects/21248672/</u> More information, along with guidelines for 3-Act Tasks, may be found in the Comprehensive Course Guide.

## **ACT 1:**

Open the link above and have the program perform for students. Alternatively, perform this trick for students.

Ask students what they noticed and what they wonder (are curious about). Record student responses.

Have students hypothesize how the trick works. How can it come out to be 1 for any number chosen?

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#### **ACT 2:**

Students work on determining how the trick works based on their hypothesis. They should be guided to show what is happening in the trick first through the use of some model that can be represented in a diagram, then later written as an expression. Students may ask for information such as what were the steps in the trick. When they ask, give them the steps:

Think of a number. Add 3. Double the result. Subtract 4. Divide the result by 2. Subtract your original number.

Students may also ask for materials to use to model what is happening – these can be suggested, carefully, by the teacher.

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## ACT 3

Students will compare and share solution strategies.

- Share student solution paths. Start with most common strategy.
- Students should explain their thinking about the mathematics in the trick.
- Ask students to hypothesize again about whether any number would work like fractions or decimals. Have them work to figure it out.
- Revisit any initial student questions that were not answered.

Students can be given practice after showing understanding, with a table similar to the following:

Words	Pictures	Diagrams	Hailee	Connor	Lura	Maury
Think of a number			7			
Double it				22		
Add 8					16	
Divide by 2						7

## Intervention:

Students needing support might be given simpler tricks at first, building up to the trick presented above. A simple trick might be:

- Prediction is 6.
- Start with 2.
- Think of a number and add it to the 2.
- Add 4.
- Subtract your original number.

Work with students on making sense of this and build the tricks up to the trick presented above.

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## **Trick 2: Consecutive Number Sum**

In parts 2 and 3 of this series of tasks, students will either watch a student perform the trick (2) or a screencast of a student performing a super quick calculation(3). Students will then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics, specifically algebraic reasoning, to answer their own questions. Students will be given information to solve the problem based on need. When they realize they do not have the information they need, and ask for it, it will be given to them.

Students choose any start number. The next four numbers are the four consecutive numbers that follow the chosen number. The trick is to tell the sum of the series of numbers knowing only the start number.

In the investigation of this trick, students may vary the starting numbers and make conjectures about the sums produced.

#### For example:

Students may decide that the sum is always a multiple of 5. Some may use the following reasoning:

You start with a number, and then add a number that is one more, and then you add a number that's two more, then three more, then four more. That makes ten more altogether. So you add ten to five times the number.

This table should not be shown to students.



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## **ACT 1:**

Have a student perform the trick for Act 1. Ask students what they noticed and what they wonder (are curious about). Record student responses.

Have students hypothesize how the trick works. How can the performer know the sum so quickly for any number chosen?

## **ACT 2:**

Students work on determining how the trick works based on their hypothesis. They should be guided to show what is happening in the trick first through the use of some model that can be represented in a diagram, and then later written as an expression. Students may ask for information such as: "How were the other numbers generated after the start number was chosen?" When they ask, point them to the link below or copy the numbers generated in the trick on the board so students can use them.

Students may ask if they can do the trick using technology:

http://scratch.mit.edu/projects/20832805/

Students may also ask for materials to use to model what is happening. (they may even ask to use similar materials from the previous task) - materials can also be suggested, carefully, by the teacher.

## ACT 3

Students will compare and share solution strategies.

- Share student solution paths. Start with most common strategy.
- Students should explain their thinking about the mathematics in the trick.
- Ask students to hypothesize again about whether any number would work like fractions or decimals. Have them work to figure it out.
- Be sure to help students make connections between equivalent expressions (i.e. the rules 5n + 10 and 5(n + 2)).
- Revisit any initial student questions that were not answered.

#### Intervention:

Students needing support might be given simpler tricks at first, building up to the trick presented above. A simple trick might be to only use 3 numbers rather than 5 to determine the sum. Use materials as well as variables to build the algebraic understanding through the use of the quantities being represented.

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## Trick 3: Triangle Mystery ...

*The following 3-act task can be found at: <u>http://mikewiernicki3act.wordpress.com/triangle-</u><u>mystery/</u>* 

This trick extends the previous trick. There are more patterns to look for and many ways to determine the rules for the patterns (it's more open!). Students should be told how the triangle mystery works: in the bottom row are consecutive numbers beginning with the number chosen. Each box in the second row is the sum of the 3 boxes below (see the diagram). The top box is the sum of the three boxes in the middle row. The trick is to determine the top number of the triangle, given only the start number (in the lower left hand box).



This table (with the expressions) should not be shown to students.

Tell students, prior to the trick, that:

- The bottom row of this triangle contains consecutive numbers.
- Each other number is found by adding the three numbers beneath it.

#### Presentation:

Give students the opportunity to change the start number in the bottom left hand rectangle and to tell what it is.

You immediately say the top number.

How is the trick done?

Try to make the trick more impressive.

Here is what the students' productive struggle will lead to:
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In Triangle Mystery, if the bottom left hand number is called x, then:

- the bottom row is x, x + 1, x + 2, x + 3, x + 4;
- the second row is 3x + 3, 3x + 6, 3x + 9;
- the top row is 9x + 18 = 9(x + 2).

So the short cut is simply to add 2 to the bottom left hand number and then multiply by 9 (or multiply the first number by 9 and add 18). Students may like to try creating larger Pyramids that follow different rules.

## **ACT 1:**

Watch the video for Act 1<u>http://mikewiernicki3act.wordpress.com/triangle-mystery/</u> Alternatively, <u>perform</u> this trick for students. Ask students what they noticed and what they wonder (are curious about). Record student responses.

Have students hypothesize how the trick works. How can the performer know the number at the top of the triangle so quickly for any number chosen?

#### **ACT 2:**

Students work on determining how the trick works based on their hypothesis. They should be guided to show what is happening in the trick first through the use of some model that can be represented in a diagram, and then later written as an expression. Students may ask for information such as: "How were the other numbers generated after the start number was chosen?" When they ask, you can tell them the bottom row are consecutive numbers after the start number. Each number in the middle row is the sum of the 3 numbers below. The top number is the sum of the numbers in the middle row. OR you can give them the technology link below for further investigation.

Students may ask if they can investigate the trick using technology: <a href="http://scratch.mit.edu/projects/20831707/">http://scratch.mit.edu/projects/20831707/</a>

Students may also ask for materials to use (they may even ask to use similar materials from the previous task) – these can also be suggested, carefully, by the teacher.

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# ACT 3

Students will compare and share solution strategies.

- Share student solution paths. Start with most common strategy.
- Students should explain their thinking about the mathematics in the trick.
- Ask students to hypothesize again about whether any number would work like fractions or decimals. Have them work to figure it out.
- Be sure to help students make connections between equivalent expressions (i.e. the expressions 9n + 18 and 9(n + 2).
- Revisit any initial student questions that weren't answered.

#### **Intervention:**

Students needing support might be given simpler tricks at first, building up to the trick presented above. A simple trick might use 3 rows, but the second row may be determined by adding only the two numbers below it. Use materials as well as variables to build the algebraic understanding through the use of the quantities being represented.

## **Extension:**

To extend all of these, students could find a way to give this trick more of a "wow" factor or to make it more impressive. Students could also develop their own trick with representations and algebraic expressions that explain it. Finally, students should be encouraged to develop (code) their own computer program for a trick like this. The free online coding program used for the tricks in this task and others to come can be found at <u>www.scratch.mit.edu</u>.

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Task Title:\_\_\_\_\_

Name:\_\_\_\_\_

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

#### Main Question:\_\_\_\_\_

Make a hypothesis. How do you think this works?

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# ACT 2

What information do you have, would like to know, or do you need to help you answer your MAIN question?

Record the given information (measurements, materials, etc...)

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## Act 2 (con't)

Use this area for your work, tables, calculations, sketches or other representations, and final solution.

#### ACT 3

What was the result? How do you know this is correct?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	Look for and express regularity in repeated reasoning.

The Sequel: How would you give this Algebra Magic Trick more of a "Wow" factor?

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#### **Georgia Department of Education** Georgia Standards of Excellence Framework *GSE Arithmetic to Algebra • Module 2*

# APPENDIX OF RELEASED SAMPLE ASSESSMENT ITEMS

The following released assessment items correspond to standards addressed in Foundations of Algebra Module 2.

# **TEACHER'S EDITION**

\*Released item from 6<sup>th</sup> grade Louisiana Math 2014 1. Brianna's teacher asked her which of these three expressions are equivalent to each other:

Expression A: 9x - 3x - 4Expression B: 12x - 4Expression C: 5x + x - 4

Brianna says that all three expressions are equivalent because the value of each one is -4 when x = 0.

Brianna's thinking is incorrect.

- Identify the error in Brianna's thinking. *Briana was confusing substitution with simplifying*.
- Determine which of the three expressions are equivalent. A and C
- Explain or show your process in determining which expressions are equivalent. A and C are both equivalent to 6x 4

\*released item from Smarter Balanced 7<sup>th</sup> grade 7.EE.A.1 2. Find the value of p so the expression  $\frac{5}{6} - \frac{1}{3}n$  is equivalent to p(5 - 2n).

$$p=\frac{1}{6}$$

\*Released item from North Carolina End of Grade 7 2013

3. Angie has a bag containing n apples. She gives 4 to her brother and keeps 5 for herself. She then divides the remaining apples equally among 3 friends.

Which of the following expressions represents the number of apples each friend receives?

A. 
$$\frac{n}{3} - 4 - 5$$
 B.  $\frac{n-4-5}{3}$  C.  $\frac{4+5-n}{3}$  D.  $\frac{n-4}{3} - 5$  E.  $\frac{n-5}{3} - 4$ 

```
B
```

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\*Released item from NAEP 2009

4. What is the value of  $-2\left(4^2 + \left(\frac{1}{2}\right)^2\right)$ ? Show all steps that lead to your response.

-2 ( 16 + <sup>1</sup>/<sub>4</sub>) -2 (16.25) -32.50

\*6th grade Louisiana

5. The formula for finding the surface area of a sphere that has a radius r is shown in the box below  $SA = 4\pi r^2$ 

A baseball has a diameter of 74 cm. Find that surface area of the baseball rounded to the nearest whole number when using 3.14 for  $\pi$ .

#### 17,195 square units

6. 8<sup>th</sup> grade Massachusetts

A stained glass window is in the shape of a square. A sketch of the window with some of its dimensions is shown below



What is the length, to the nearest tenth of a foot, of the line segment labeled x?

4 feet

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# The following are from released items and practice problems from Georgia's End of Course Tests.

7. The dimensions of a rectangle are shown. What is the perimeter of the rectangle?



5x + 2

Solution:

Substitute 5x + 2 for l and 3x + 8 for w into the formula for the perimeter of a rectangle: P = 2l + 2w P = 2(5x + 2) + 2(3x + 8) P = 10x + 4 + 6x + 16P = 16x + 20

8. A model of a house is shown. What is the perimeter of the model?



50x + + 11

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9. A rectangular field is 100 meters in width and 120 meters in length. The dimensions of the field will be expanded by x meters in each direction, as shown in the diagram. Write an expression for the perimeter of the new field in terms of x.



#### 440 + 4x

10. The diagram shows the dimensions of a cardboard box. Write an expression to represent the volume of the box.



 $3x * x(x + 2) = 3x^3 + 6x^2$ 

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To review other problems pertaining to Module 2, Illustrative Mathematics provides the following:

For Pythagorean Theorem, https://www.illustrativemathematics.org/8.G.B.7

For exponentials, https://www.illustrativemathematics.org/8.EE

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#### **Georgia Department of Education** Georgia Standards of Excellence Framework *GSE Arithmetic to Algebra* • *Module 2*

#### **STUDENT'S EDITION**

\*Released item from 6<sup>th</sup> grade Louisiana Math 2014 1. Brianna's teacher asked her which of these three expressions are equivalent to each other:

Expression A: 9x - 3x - 4Expression B: 12x - 4Expression C: 5x + x - 4

Brianna says that all three expressions are equivalent because the value of each one is -4 when x = 0.

Brianna's thinking is incorrect.

- Identify the error in Brianna's thinking.
- Determine which of the three expressions are equivalent.
- Explain or show your process in determining which expressions are equivalent.

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3. Angie has a bag containing n apples. She gives 4 to her brother and keeps 5 for herself. She then divides the remaining apples equally among 3 friends.

Which of the following expressions represents the number of apples each friend receives?

A.  $\frac{n}{3} - 4 - 5$  B.  $\frac{n-4-5}{3}$  C.  $\frac{4+5-n}{3}$  D.  $\frac{n-4}{3} - 5$  E.  $\frac{n-5}{3} - 4$ 

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\*Released item from NAEP 2009 4. What is the value of  $-2\left(4^2 + \left(\frac{1}{2}\right)^2\right)$ ? Show all steps that lead to your response.

\*6th grade Louisiana

5.

The formula for finding the surface area of a sphere that has a radius r is shown in the box below

 $SA = 4\pi r^2$ 

A baseball has a diameter of 74 cm. Find that surface area of the baseball rounded to the nearest whole number when using 3.14 for  $\pi$ .

# 6. 8<sup>th</sup> grade Massachusetts

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7. The dimensions of a rectangle are shown. What is the perimeter of the rectangle?



8. A model of a house is shown. What is the perimeter of the model?



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9. A rectangular field is 100 meters in width and 120 meters in length. The dimensions of the field will be expanded by x meters in each direction, as shown in the diagram. Write an expression for the perimeter of the new field in terms of x.



10. The diagram shows the dimensions of a cardboard box. Write an expression to represent the volume of the box.



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