Georgia
Standards of Excellence
Curriculum Frameworks

Mathematics

GSE Foundations of Algebra

Module 3: Proportional Reasoning

Georgia Department of Education

Richard Woods, Georgia's School Superintendent

"Educating Georgia's Future"

Module 3: Proportional Reasoning

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FOUNDATIONS OF ALGEBRA REVISION SUMMARY

The Foundations of Algebra course has been revised based on feedback from teachers across the state. The following are changes made during the current revision cycle:

- Each module assessment has been revised to address alignment to module content, reading demand within the questions, and accessibility to the assessments by Foundations of Algebra teachers.
- All module assessments, as well as the pre- and posttest for the course, will now be available in GOFAR at the teacher level along with a more robust teacher's edition featuring commentary along with the assessment items.
- All modules now contain "Quick Checks" that will provide information on mastery of the content at pivotal points in the module. Both teacher and student versions of the "Quick Checks" will be accessible within the module.
- A "Materials List" can be found immediately after this page in each module. The list provides teachers with materials that are needed for each lesson in that module.
- A complete professional learning series with episodes devoted to the "big ideas" of each module and strategies for effective use of manipulatives will be featured on the Math Resources and Professional Learning page at https://www.gadoe.org/Curriculum-Instruction-and-Assessment/Curriculum-and-Instruction/Pages/Mathematics.aspx.

Additional support such as Module Analysis Tables may be found on the Foundations of Algebra page on the High School Math Wiki at http://ccgpsmathematics9-10.wikispaces.com/Foundations+of+Algebra. This Module Analysis Table is NOT designed to be followed as a "to do list" but merely as ideas based on feedback from teachers of the course and professional learning that has been provided within school systems across Georgia.

MATERIALS LIST

Lesson	Materials
1. Equivalent Fractions	• Student lesson sheet
2. Snack Mix	• Video and other information provided: http://mikewiernicki.com/snack-mix/ • 3-Act Recording Sheet • Optional: Color tiles and/or Cuisenaire rods
3. What is a Unit Rate?	• Student lesson sheet • Optional: Colored tiles • Optional: Cuisenaire rods
4. Proportional Relationships	• Student lesson sheet • Chart paper (with grid, if possible) • Graph paper (if no grid paper is available) • Glue sticks (if no grid paper is available) • Envelopes • Markers • Sticky notes
5. Orange Fizz Experiment	• Student lesson sheet
6. Proportion and Non-Proportion Situations	NA
7. Quick Check I	NA
8. Fish in a Lake	• Paper bags (one per group) • White beans and red beans (alternatives to the beans include: two different colored counters, two different colored cubes, or plain Goldfish crackers and pretzel Goldfish crackers (tagged fish))
9. Which Is the Better Deal?	• Chart paper or white poster board • Rulers • Markers for making charts • Samples of products (Students can clip advertisements or print products off the internet. They need to have a picture of the product, explanation/details, price)
10. 25% Sale	• Student lesson sheet

11. Which Bed, Bath, & Beyond Coupon Should You Use?	• Images of both Bed Bath & Beyond coupons: $5 off any purchase of $15 or more and 20% off one single item • Images of four items from Bed Bath & Beyond website • Problem solving framework (provided in this document)
12. Increasing and Decreasing Quantities by a Percent	NA
13. Quick Check II	NA
14. What's My Line?	• Student lesson sheet • Optional: Straightedge • Optional: Graph paper http://incompetech.com/graphpaper
15. Nate and Natalie's Walk	• *Walk to the Movies* activity sheet • Student lesson sheet
16. Rectangle Families	• Student lesson sheet • Two pages of rectangles (copied single side) • Scissors • Colored pencils • Rulers
17. Quick Check III	NA
18. "Illustrative" Review	NA
19. Nana's Chocolate Milk	• Video and other information provided: http://threeacts.mrmeyer.com/nana/ • 3-Act Recording

OVERVIEW

In this unit students will:

- Explain equivalent ratios using a variety of models.
- Understand real-world rate/ratio/percent problems.
- Use proportional reasoning to solve multistep ratio and percent problems.
- Interpret unit rates as slopes of graphs.
- Explain the concept of slope using similar triangles.
- Compare proportional relationships represented in different ways.

STANDARDS FOR MATHEMATICAL CONTENT

Students will use ratios to solve real-world and mathematical problems.

MFAPR1. Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations.* (MGSE6.RP.3)

MFAPR2. Students will recognize and represent proportional relationships between quantities.
a. Relate proportionality to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3; MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

MFAPR3. Students will graph proportional relationships.
a. Interpret unit rates as slopes of graphs. (MGSE8.EE.5)
b. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. (MGSE8.EE.6)
c. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (MGSE8.EE.5)

STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them. High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively. High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others. High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics. High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically. High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision. High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure. By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see

$5-3(x-y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8. Look for and express regularity in repeated reasoning. High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$, $(x-1)(x^2+x+1)$, and $(x-1)(x^3+x^2+x+1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.
The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. **Students who do not have an understanding of a topic may rely on procedures too heavily.** Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. **In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.**

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

*****Mathematical Practices 1 and 6 should be evident in EVERY lesson*****

ENDURING UNDERSTANDINGS

By the end of the module, students should be able to:

- Find equivalent ratios and use them to solve problems.
- Solve percent problems by finding the whole given the part and find a part given the whole.
- Compare proportional relationships represented different ways.
- Understand unit rate as the slope of a line.

ESSENTIAL QUESTIONS

- How are equivalent fractions related?
- How do I solve real-world problems using equivalent ratios?
- How can a model be used to find and organize equivalent ratios?
- How can a unit rate be used to answer questions in a problem?
- What are the characteristics of a proportional relationship?
- How can I identify a proportional relationship in a situation, table, graph, or equation?
- How I can use a proportion to solve for an unknown quantity?
- How do I solve and interpret solutions of real-world percent problems?
- Why might it be useful to look at different representations of a function?

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.
The terms below are for teacher reference only and are not to be memorized by the students. Teachers should present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

- Equivalent ratios
- Proportional relationship
- Unit rate
- Slope

The websites below are interactive and include a math glossary suitable for high school children.
Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.
http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary.
http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

INTERVENTION TABLE

Lesson Name	**Name Of Intervention**	**Snapshot of summary or student I can statement …**	**Book, Page Or link**
Equivalent Fractions	Equivalent Fractions Addition, subtraction and equivalent fractions	The purpose of this activity is to help the student practice finding equivalent fractions for numbers up to 100. The purpose of this series of lessons is to develop understanding of equivalent fractions and the operations of addition and subtraction with fractions.	Equivalent Fractions Addition, subtraction and equivalent fractions
Snack Mix	Sharing in Ratios	Solve problems involving ratios.	Sharing in Ratios
What is a Unit Rate?	Rates of Change	Hands on activities where students can determine unit rate.	Rates of Change
Proportional Relationships	Combining Proportions	Solve problems involving simple linear proportions	Combining Proportions
Orange Fizz Experiment	Comparing by Finding Rates	Comparing proportions	Comparing by Finding Rates
Which Is The Better Deal?	50% on is Not the Same as 50% off!	An activity to help develop automaticity with percentages.	50% on is Not the Same as 50% off!

25% Sale	Sales Tax Rules	An activity to help develop automaticity with percentages.	Sales Tax Rules
Which Bed, Bath, & Beyond Coupon Should You Use?	Percentages Increases & Decreases in One Step	An activity to help develop automaticity with percentages.	Percentages Increases & Decreases in One Step
What's My Line?	Rates of Changes	In this activity students solve problems involving unit rates	Rate of Changes
Nana's Chocolate Milk	Mochaccino Mix	Solve problems involving ratios.	Mochaccino Mix

SCAFFOLDED INSTRUCTIONAL LESSONS

Lesson Name	**Lesson Type/ *Grouping Strategy***	**Content Addressed**	**Standard(s)**
Equivalent Fractions	Learning Lesson *Individual or partners*	Explore relationships between equivalent fractions	MFAPR.2
Snack Mix	3-Act Task *Partners or small groups*	Find equivalent ratios to solve a problem	MFAPR.1 MFAPR.2
What is a Unit Rate?	Scaffolding Lesson *Partners or small groups*	Find rates and unit rates and interpret those rates in the context of problems	MFAPR.1 MFAPR.2
Proportional Relationships	Learning Lesson *Small groups*	Determine whether two quantities are in a proportional relationship	MFAPR.2
Orange Fizz Experiment	Learning Lesson *Partners or small groups*	Examine part-to-part and part-to-whole comparisons in order to make decisions	MFAPR.1 MFAPR.2
Proportion and Non-Proportion Situations	Formative Assessment Lesson *Small groups*	Identify when two quantities vary in direct proportion to each other. Distinguish between direct proportion and other functional relationships. Solve proportionality problems using efficient methods.	MFAPR.2
Quick Check I	Formative Assessment *Individual*	Determine whether two quantities are in a proportional relationship and comparing quantities	MFAPR.1 MFAPR.2
Fish in a Lake	Learning Lesson *Partners or small groups*	Use proportional reasoning to solve problems with a connection to statistics	MFAPR.2
Which Is the Better Deal?	Learning Lesson *Partners or small groups*	Compare unit rates to make purchasing decisions	MFAPR.2
25% Sale	Short Cycle Lesson *Individuals, partners or small groups*	Understand the effects of multiple discounts	MFAPR.2
Which Bed, Bath, & Beyond Coupon Should You Use?	Learning Lesson *Partners or small groups*	Compare coupons to make purchasing decisions	MFAPR.2

Lesson Name	**Lesson Type/ *Grouping Strategy***	**Content Addressed**	**Standard(s)**
Increasing and Decreasing Quantities by a Percent	Formative Assessment Lesson *Partners*	Translate between percentages, decimals, and fractions. Represent percent increase and decrease as multiplication. Recognize the relationship between increases and decreases.	MFAPR.2
Quick Check II	Formative Assessment *Individual*	Use ratios and proportions to solve problems including those involving percentages	MFAPR.1 MFAPR.2
What's My Line?	Learning Lesson *Individuals, partners or small groups*	Compare ratios of sides of slope triangles in a real-world problem	MFAPR.3
Nate and Natalie's Walk	Performance Lesson *Partners or small groups*	Determine whether a proportional relationship exists using tables, graphs, or equations	MFAPR.2
Rectangle Families	Learning Lesson *Partners or small groups*	Compare ratios of side lengths of rectangles to sort appropriately	MFAPR.1 MFAPR.2 MFAPR.3
Quick Check III	Formative Assessment *Individual*	Find similar figures by comparing lengths and widths	MFAPR.1 MFAPR.2
"Illustrative" Review	Formative Assessment *Individual*	A compilation of problem related to percentages, slopes, and rates.	MFAPR.2 MFAPR.3
Nana's Chocolate Milk	Culminating Lesson *Partners or small groups*	Find equivalent ratios to solve a problem	MFAPR.1 MFAPR.2

The assessment for this module can be found through the Georgia Online Formative Assessment Resource (GOFAR). http://www.gadoe.org/Curriculum-Instruction-and-Assessment/Assessment/Pages/Georgia-Online-Formative-Assessment-Resource.aspx This suggested assessment should be given as the pretest and posttest for this module.

Equivalent Fractions

Adapted from Van De Walle, J. (2007). *Elementary and middle school mathematics: Teaching developmentally* (6^{th} ed.) Boston: Pearson Education, Inc.

This lesson allows students to explore the relationship between equivalent fractions and to write equations for equivalent fractions using the product of a fraction equivalent to one.

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class.
Recommended time: 45-60 minutes. Recommended arrangement: individual or partners.

STANDARDS FOR MATHEMATICAL CONTENT

Students will use ratios to solve real-world and mathematical problems.

MFAPR2. Students will recognize and represent proportional relationships between quantities.

a.Relate proportionality to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students must understand what is happening to the part and the whole when horizontal lines are drawn and be able to make connections between the area models and their equations.

6. Attend to precision. Students should use precise mathematical language when explaining both in writing and in whole-group discussions what patterns they notice in the area models and equations.

7. Look for and make use of structure. Students should understand how equivalent fractions are formed by multiplication or division. They should also be able to connect the area model with its corresponding equation.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Create equivalent fractions using multiplication and division.
- Relate an area model used to create equivalent fractions with an equation that explains the model.

MATERIALS

- Student lesson sheet

ESSENTIAL QUESTIONS

- What happens to the value of a fraction when the numerator and denominator are multiplied or divided by the same number?
- How are equivalent fractions related?

The first essential question should not be shared with students until after students have completed the lesson.

OPENER/ACTIVATOR

Number Talks
Students can use friendly number strategies to begin discussing multiples and specifically, combining multiples to arrive at a larger product. For example, students could solve the following number string:

$1 \times 12 = 12$

$2 \times 12 = 24$

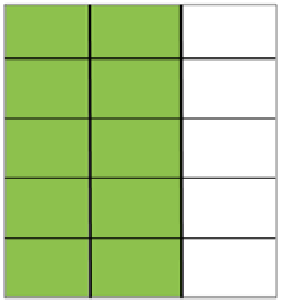
by combining the products 12 and 24 as well as combining the factors of 1 and 2, the students will produce another multiple of 12, namely $3 \times 12 = 36$.
Doubling and halving is also a helpful strategy in building equivalent fractions. (*Number Talks*, 2010, Sherry Parrish).

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Background Knowledge:
This lesson was designed for students to learn an algorithm for finding equivalent fractions by multiplying a fraction by a fraction equivalent to one. Although some students in the Foundations of Algebra course may remember this algorithm, some may not. Therefore, it is important not to immediately remind students of this algorithm before they have had a chance to do the lesson and discover for themselves.

Lesson Directions:
This lesson will give students the opportunity to explore equivalent fractions. Students should follow the directions given in part 1 of the lesson by drawing horizontal line segments to create different but equivalent fractions. Before allowing students to continue on to part 2, initiate a whole-group discussion about any patterns students may have noticed in part 1.

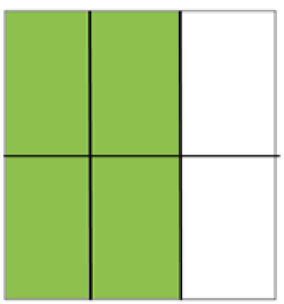
It is very important that students are making a connection between their equations and the corresponding area models. For example,



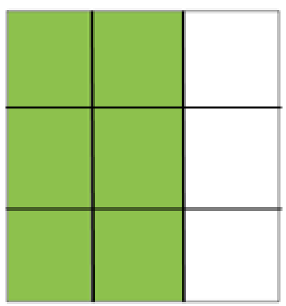
rather than drawing 5 horizontal line segments and then using that 5 to multiply $\frac{2}{3}$ by $\frac{5}{5}$, students should notice that 2×5 comes from the 2 original shaded sections being sliced into 2×5 smaller sections. Likewise, the whole which was originally comprised of 3 sections was sliced to create 3×5 smaller sections. Therefore, $\frac{2}{3}\times\frac{5}{5}=\frac{10}{15}$.

Ask several students to share equations that they wrote in part 1 and write these on the board. Engaging students in the above explanation may help students who are having trouble making connections between the area models and the equations. Ask students to record what they notice at the bottom of the page before continuing on to part 2. If necessary, have a similar discussion at the end of part 2.

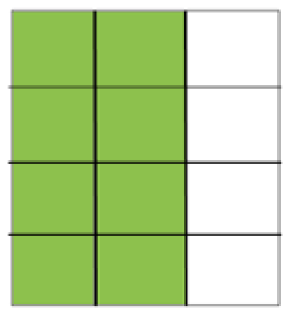
Possible Solutions to Part 1: Equivalent Fractions — $\frac{2}{3}$



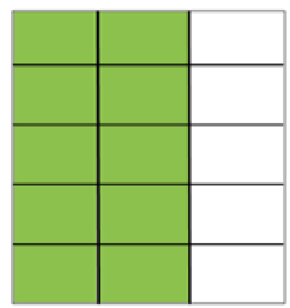
$\frac{2}{3}\times\frac{2}{2}=\frac{4}{6}$



$\frac{2}{3}\times\frac{3}{3}=\frac{6}{9}$

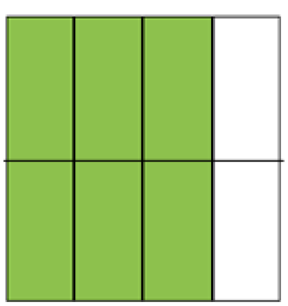


$\frac{2}{3}\times\frac{4}{4}=\frac{8}{12}$

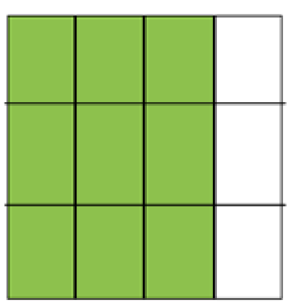


$\frac{2}{3}\times\frac{5}{5}=\frac{10}{15}$

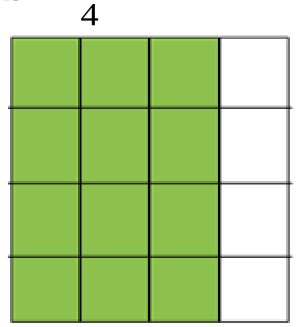
Possible Solutions to Part 2: Equivalent Fractions — $\frac{3}{4}$



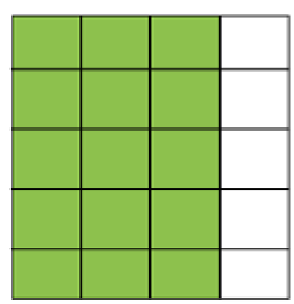
$\frac{3}{4}\times\frac{2}{2}=\frac{6}{8}$



$\frac{3}{4}\times\frac{3}{3}=\frac{9}{12}$



$\frac{3}{4}\times\frac{4}{4}=\frac{12}{16}$



$\frac{3}{4}\times\frac{5}{5}=\frac{15}{20}$

Before moving on to part 3, ask students the following question:

What are some fractions equivalent to $\frac{8}{12}$?

Listen carefully to students' ideas to see if anyone mentions the possibility of creating equivalent fractions by *dividing* both the numerator and denominator by the same number. If not, a probing question such as this might help:

Are there any fractions equivalent to $\frac{8}{12}$ with numerators less than 8 and denominators less that 12?

Ultimately, we want students to see that equivalent fractions can be obtained both by multiplying and dividing. In this case,

$$\frac{8}{12} \times \frac{2}{2} = \frac{16}{24} \text{ } AND \text{ } \frac{8}{12} \div \frac{4}{4} = \frac{2}{3}.$$

For part 3, students should show evidence of both multiplying and dividing for the fractions $\frac{6}{8}$, $\frac{12}{15}$, and $\frac{20}{24}$.

For more information on this lesson, see *Slicing Squares* on p. 311 of Van De Walle, J. (2007). *Elementary and middle school mathematics: Teaching developmentally* (6th ed.) Boston: Pearson Education, Inc.

FORMATIVE ASSESSMENT QUESTIONS

- What product tells how many parts are shaded?
- What product tells how many parts are in the whole?

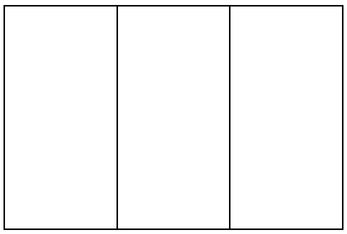
DIFFERENTIATION

Extension

This game will allow students the opportunity to use their knowledge of equivalent fractions.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=18

Intervention

For students struggling to make connections between the area model and the equations, provide them several unshaded "wholes" divided vertically into three equal parts (like the one below) and ask them to shade in $\frac{2}{3}$, and then draw the horizontal line segments to help them see where the fraction equivalent to one can be "seen" in the area model. This may need to be repeated for various numbers of horizontal lines.

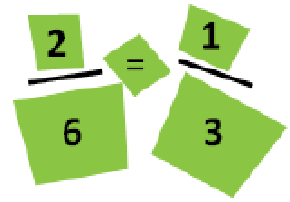


Additional practice can be found at http://illuminations.nctm.org/activity.aspx?id=3510. This activity allows students to create equivalent fractions and connect them to their location on the number line.

For extra help with equivalent fractions, please open the hyperlink Intervention Table.

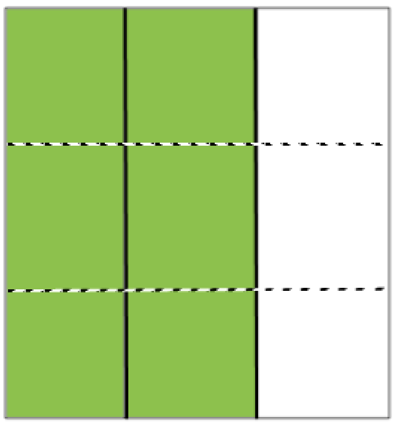
CLOSING/SUMMARIZER
https://www.illustrativemathematics.org/content-standards/lessons/743
This lesson offers students an opportunity to explain why two fractions are equivalent. Students may complete it individually or can be given time to discuss with a partner before writing their own individual response.

Equivalent Fractions

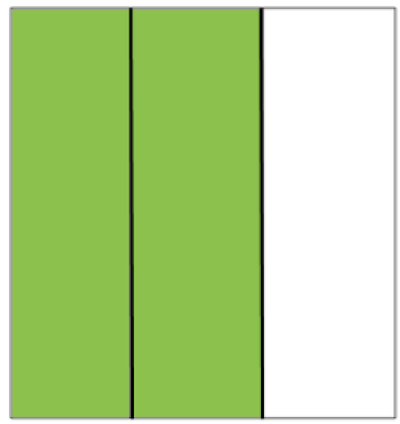


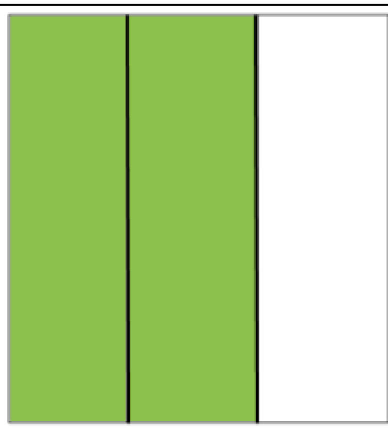
Part 1: Equivalent Fractions — $\frac{2}{3}$

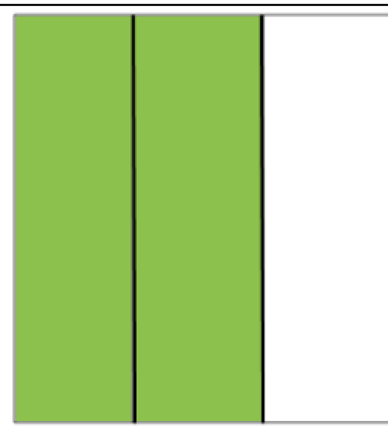
Find fractions that are equivalent to the fraction shown in each square below. Slice the squares by drawing horizontal line segments in each square to create a different but equivalent fraction. Then write an equation for each square. See the example below.



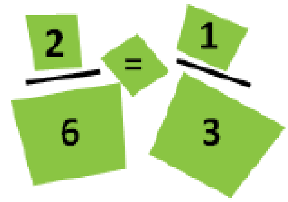
$$\frac{2}{3}=\frac{2}{3}\times\frac{3}{3}=\frac{6}{9}$$





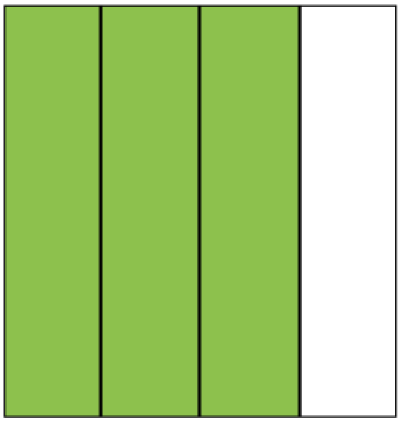


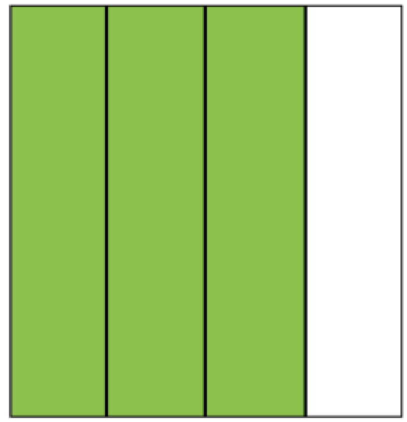
What patterns do you notice?

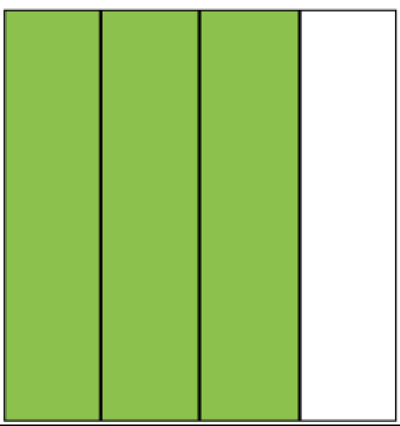


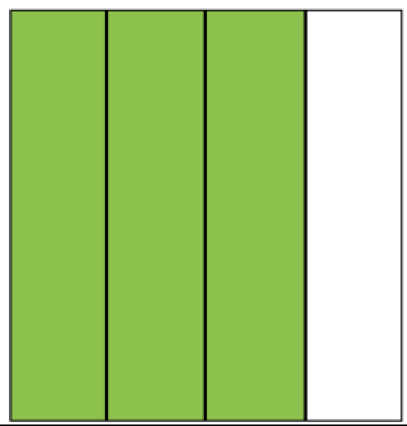
Part 2: Equivalent Fractions — $\frac{3}{4}$

Find fractions that are equivalent to the fraction shown in each square below. Slice the squares by drawing horizontal line segments in each square to create a different but equivalent fraction. Then write an equation for each square. See the example below.



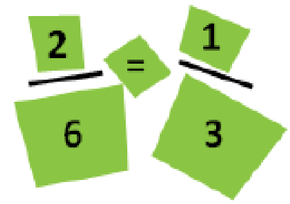






What patterns do you notice?

Part 3: Equivalent Fractions



Find fractions equivalent to the fractions in the table below. Use both multiplication and division, if possible. Record the equivalent fractions in the white boxes.

$\frac{1}{2}$						$\frac{2}{3}$		
$\frac{1}{3}$						$\frac{6}{8}$		
$\frac{1}{4}$						$\frac{2}{5}$		
$\frac{1}{5}$						$\frac{3}{5}$		
$\frac{1}{6}$						$\frac{12}{15}$		
						$\frac{20}{24}$		

[Snack Mix](http://mikewiernicki.com/snack-mix/)

Lesson adapted from: http://mikewiernicki.com/snack-mix/

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class.
Recommended time: 45-60 minutes. Recommended arrangement: partners or small groups.

STANDARDS FOR MATHEMATICAL CONTENT

Students will use ratios to solve real-world and mathematical problems.

MFAPR1. Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations.* (MGSE6.RP.3)

MFAPR2. Students will recognize and represent proportional relationships between quantities.

a.Relate proportionality to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)

b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)

c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

Common Misconceptions

Students may see the relationship between the amount of granola and amount of candies as additive instead of multiplicative. In other words, they may think that the amount of granola will always be 2 parts more than the amount of candies, rather than seeing the amount of granola as $\frac{5}{3}$ *the amount of candies or seeing the amount of candies as* $\frac{3}{5}$ *the amount of granola. This misconception could lead students to believe that a 4-cup mixture would consist of 10 parts granola and 8 parts candies.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students must make sense of the problem in order to devise a plan for solving it.

2. Reason abstractly and quantitatively. Students must use quantitative reasoning to create a representation of the problem and to understand how the quantities relate.

3. Construct viable arguments and critique the reasoning of others. Throughout the problem, students will need to communicate their mathematical thinking to their peers as they evaluate their own and their peers' understanding of the problem, the model they create, and the reasonableness of their answer.

4. Model with mathematics. Based on Act 2, students will make a model to represent the situation.

5. Use appropriate tools strategically. Students will need to decide on an appropriate tool for their model, e.g., tables of values, tape diagrams, bar models, double number line diagrams.

6. Attend to precision. Students must use correct mathematical language as they communicate their thinking to their peers.

7. Look for and make use of structure. In order to create an accurate model, students must notice patterns in order to make equivalent ratios.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Identify a multiplicative relationship.
- Use a model to find equivalent ratios.

MATERIALS

- Video and other information provided: http://mikewiernicki.com/snack-mix/
- 3-Act Recording Sheet (attached)
- Color tiles and/or Cuisenaire rods (optional)

ESSENTIAL QUESTIONS

In order to maintain a student-inquiry-based approach to this lesson, it may be beneficial to wait until Act 2 to share the Essential Questions with your students. By doing this, students will be allowed the opportunity to be very creative with their thinking in Act 1. By sharing the essential questions in Act 2, you will be able to narrow the focus of inquiry so that the outcome results in student learning directly related to the content standards aligned with this lesson.

- How can we find equivalent ratios?
- How can a model be used to find and organize equivalent ratios?

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Background Knowledge:

This lesson follows the Three-Act Math Task format originally developed by Dan Meyer. More information on this type of lesson may be found at http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/. A Three-Act Task is a whole-group mathematics lesson consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information, along with guidelines for Three-Act Tasks, may be found in each Comprehensive Course Overview.

Lesson Directions:

Act 1 – Whole Group - Pose the conflict and introduce students to the scenario by showing Act 1 picture. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/)
"Introduce the central conflict of your story/lesson clearly, visually, viscerally, using as few words as possible."

Show the Act 1 video to students: http://mikewiernicki.com/snack-mix/
Give students a copy of the 3-Act Recording Sheet.
Ask students what they noticed in the video, what they wonder about, and what questions they have about what they saw in the video. Facilitate a think-pair-share so that students have an opportunity to talk with each other before sharing questions with the whole group.
Share and record students' questions. The teacher may need to guide students so that the questions generated are math-related.

Anticipated questions students may ask and wish to answer:

- *How much of each ingredient is needed to fill all of the cups?
- How much of each ingredient does he have at the beginning of the video?
- How much of the mixture will he add to each red cup?
- How many red cups are there?

**Main question(s) to be investigated*

Once the class has decided on the main question to investigate, students should record the question on the recording sheet. Then, ask the students to estimate answers to their questions (think-pair-share). Students will write their best estimate, then write two more estimates – one that is too low and one that is too high so that they establish a range in which the solution should occur. Students should plot their three estimates on an empty number line. In this three-act lesson, students may want to plot their estimates on a double-number line to include both ingredients.

Important note: ***Although students will only investigate the main question(s) for this lesson, it is important for the teacher to not ignore student-generated questions. Additional questions may be answered after they have found a solution to the main question, or as homework or extra projects.***

Act 2 – **Student Exploration** - Provide additional information as students work toward solutions to their questions. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/**)**
"The protagonist/student overcomes obstacles, looks for resources, and develops new tools."

During Act 2, students decide on the facts, tools, and other information needed to answer the question(s) (from Act1). When students decide what they need to solve the problem, they should ask for those things. It is pivotal to the problem-solving process that students decide what is needed without being given the information up front.

Students may wish to use manipulatives to model the ratio of ingredients in the lesson. Colored tiles and/or Cuisenaire rods could be made available to students who wish to begin with the concrete representation. Other students may feel comfortable beginning with a table of values, double-number line, or a tape diagram.

The teacher provides guidance as needed during this phase. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin. Questioning is an effective strategy that can be used, with questions such as:

- What is the problem you are trying to solve?
- What do you think affects the situation?
- Can you explain what you've done so far?
- What strategies are you using?
- What assumptions are you making?
- What tools or models may help you?
- Why is that true?
- Does that make sense?
- Do the quantities have a multiplicative or additive relationship?

Additional Information for Act 2

Number of cups: 24
The ratio of ingredients video: http://mikewiernicki.com/snack-mix/
The ratio of servings per recipe video: http://mikewiernicki.com/snack-mix/

Act 3 – Whole Group – Share solutions and strategies.

Students present their solutions and strategies and compare them.
Lead discussion to compare these, asking questions such as:

- How reasonable was your estimate?
- Which strategy was most efficient?
- Can you think of another method that might have worked?
- What might you do differently next time?

Revisit initial student questions that weren't answered.

FORMATIVE ASSESSMENT QUESTIONS

- How can you tell that this relationship is multiplicative and not additive?
- How might you use the ratio of the two ingredients to determine the amount of each ingredient in 4 cups?
- What is the relationship between the amounts of each ingredient?
- What organizational strategies did you use?

DIFFERENTIATION

Extension

Ask students to create a similar problem but with a different context and different ratio. Two students working on the extension could exchange problems and provide feedback to each other.

Intervention

Encourage students to begin with the concrete representation using colored tiles or Cuisenaire rods. Ask them to model the relationship between the 2 ingredients for 2 cups. Then, ask them to show how the model would change for 4 cups.
Use grid paper to sketch a bar model of the ratio of the 2 ingredients for two cups. Ask students to extend their bars to represent the ratio of the ingredients for 4 cups.

For extra help with ratios, please open the hyperlink Intervention Table.

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question: ____

Estimate the result of the main question. Explain.
Place an estimate that is too high and too low on the number line
Low estimate *Place an "x" where your estimate belongs.* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:______________________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

What is a Unit Rate?

Source: http://www.cehd.umn.edu/ci/rationalnumberproject/89_4.html

In this lesson, students will explore different ways to express rates and unit rates, and students will interpret unit rates in the context of a problem.

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class.
Recommended time: 60-90 minutes. Recommended arrangement: partners or small groups.

STANDARDS FOR MATHEMATICAL CONTENT

Students will use ratios to solve real-world and mathematical problems.

MFAPR1. Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations.* (MGSE6.RP.3)

MFAPR2. Students will recognize and represent proportional relationships between quantities.
a. Relate proportionality to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students must be able to make sense of unit rates in the context of a problem.

2. Reason abstractly and quantitatively. Students must reason through each problem to determine which unit rate is most appropriate for the question being asked.

6. Attend to precision. Students must be precise in their calculations.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Find a unit rate and explain what the unit rate means in the context of a problem.
- Determine the most appropriate unit rate for solving a problem and use that unit rate to solve for missing values.

MATERIALS

- Student lesson sheet
- Colored tiles (optional)
- Cuisenaire rods (optional)

ESSENTIAL QUESTIONS

- How can a unit rate be used to answer questions in a problem?
- How can I determine which unit rate will be most useful in solving a problem?
- How can tables and equations be useful when answering questions about proprotional relationships?

NUMBER TALKS

For this Number Talk, begin with the following problem, **"6 melons cost $12. How much does one melon cost?"** Record the problem on the far left side of the board. Provide students with wait time as they work to mentally solve this problem. When the majority of the students have given the "thumbs up" signal, call on several students to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you. It is important to allow students ownership of their thinking.

Record, **"3 watermelons cost $15. How much does one watermelon cost?"** on the board next to the previous problem. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the "thumbs up" signal, call on several students to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you.

Record, **"3 apples cost $.99. How much does 1 apple cost?"** on the board towards the far right. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the "thumbs up" signal, call on several students to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you.

At the end of the Number Talk, discuss the strategies used to find the answers. Some of the strategies students may use are: relationship between multiplication and division, division, guess and check and skip counting. Talk with the students about which strategy was most efficient (quick, easy and accurate).

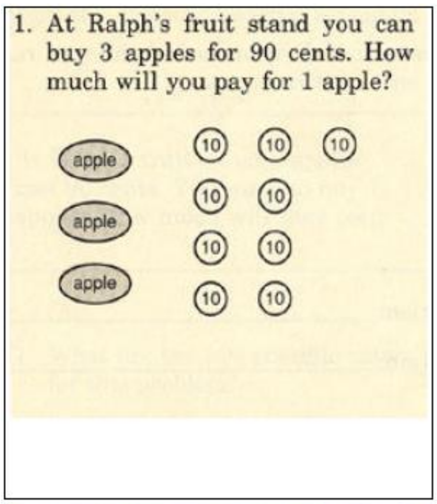
Allow for a maximum of 15 minutes to conduct the Number Talk before moving into the lesson.

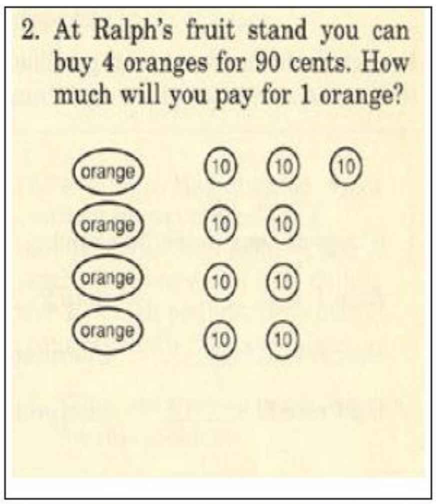
Explain to students that the strategies were used to identify a **unit rate**.

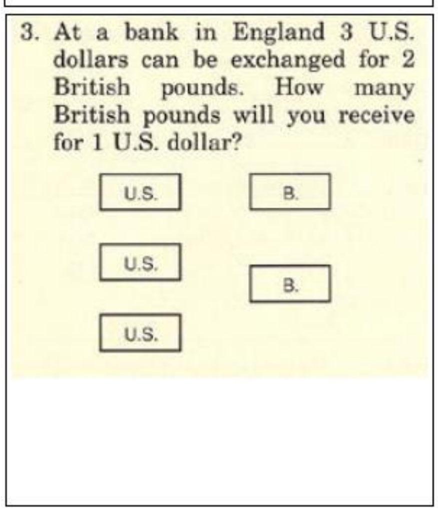
Revisit the first problem from the Number Talk. Have students identify the rate $\frac{6\ melons}{\$12}$, then identify the unit rate $\frac{1}{2}$ melon per dollar. Have students define a unit rate.
On the recording sheet, students analyze a real life situation to create two versions of a unit rate. Then, students need to analyze the two possible rates and determine which is the most appropriate to the given problem and use this rate to find possible cost for other values within the given problem.

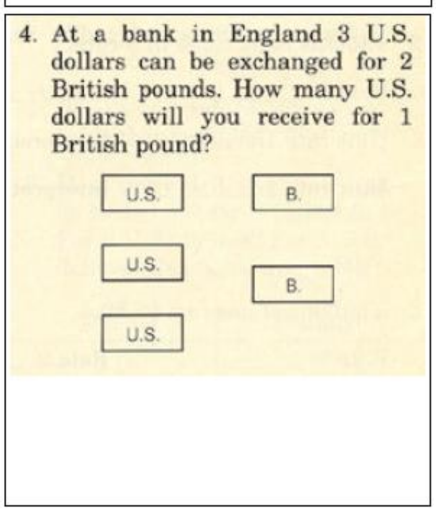
OPENER/ACTIVATOR
http://www.cehd.umn.edu/ci/rationalnumberproject/images/89_4/sheet_1.pdf

Pose the following question to students:
How could you use each picture to answer the question?









Possible solution for #1.

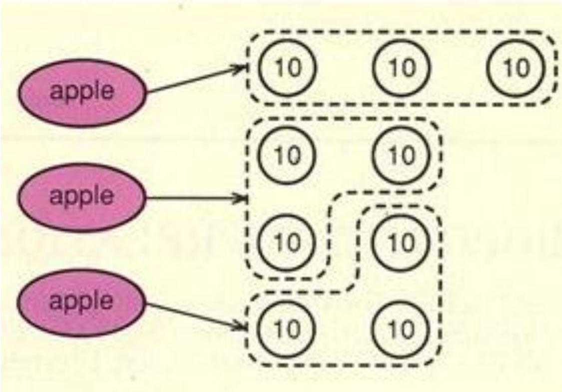


Fig. 1

After students have had time to work through all four problems, ask some students to share their work. Look specifically for students whose work looks different. Any problems that seemed to be particularly difficult for students should also be discussed at this time.

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Lesson Directions:

Each of the three parts of this lesson focuses on a different aspect of unit rates. Part 1 allows students to see that a unit rate can always be written two different ways. Most importantly, students must interpret each unit rate in the context of the problem. When interpreting the unit rate, make sure that students are not simply "reading" the unit rate as it is written, for instance on *a*, they should write something similar to this, *There are 5 pounds of flour in one bag*, as opposed to this, *5 pounds per bag.*

The problems in part 1 can be used to help prepare students for the remainder of the lesson by asking some addition questions. After students have completed part 1, some of the following prompts could be given as a think-pair-share:

> *Which unit rate makes more sense in the context of the problem? Explain your thinking.*
> *Which unit rate would be most helpful if I want to know the cost of 6 gallons of gas?*
> *If I need 20 tennis balls, which unit rate will help me determine the number of cans to buy?*

In part 2, students will select an appropriate unit rate and will use that unit rate to solve the problem. However, an important mathematical skill arises from asking students to use the other unit rate. After students have completed part 2, pose this question: *If you have $5 to spend at Ralph's fruit stand on apples, how many apples can you buy?* Begin by asking students to decide which unit rate would help them to solve this problem and then use that unit rate to solve the problem. The important skill here is to see if students correctly round their solution. Some students may think that if a decimal is greater than .5, they should always round up, but in this case, $5 will buy 16.65 apples so there is only enough money to buy 16 apples, not 17.

Lastly, part 3 requires students to use this same process on three new scenarios. Give students the opportunity to share their solutions in a whole-group discussion. Pay particular attention to their solutions in *c* where there may be misconceptions about converting time to minutes and seconds.

What is the Unit Rate?

Part 1: Finding and Interpreting the Unit Rate

In each problem, record both possible rates, use an appropriate strategy to find the unit rates, and then write a short sentence explaining each unit rate.

a. 6 bags of flour weigh 30 pounds.

Rate 1:	Unit rate 1:
$\frac{6\ bags}{30\ lbs}$	$\frac{.2\ bags}{lb}$
Interpretation: *There are $\frac{2}{10}$ of a bag for each pound of flour.*	

Rate 2:	Unit rate 2:
$\frac{30\ lbs}{6\ bags}$	$\frac{5\ lbs}{bag}$
Interpretation: *There are 5 pounds of flour per bag.*	

b. 9 tennis balls come in 3 cans.

Rate 1:	Unit rate 1:
$\frac{9\ tennis\ balls}{3\ cans}$	$\frac{3\ tennis\ balls}{can}$
Interpretation: *Each can contains 3 tennis balls.*	

Rate 2:	Unit rate 2:
$\frac{3\ cans}{9\ tennis\ balls}$	$\frac{.33\ cans}{tennis\ ball}$
Interpretation: *1 tennis ball makes up $\frac{1}{3}$ of a can.*	

c. 5 gallons of gas cost $6.50.

Rate 1:	Unit rate 1:
$\frac{5\ gallons\ of\ gas}{\$6.50}$	$\frac{.77\ gallons\ of\ gas}{\$1}$
Interpretation: *One dollar will buy .77 gallons of gas.*	

Rate 2:	Unit rate 2:
$\frac{\$6.50}{5\ gallons\ of\ gas}$	$\frac{\$1.30}{gallon\ of\ gas}$
Interpretation: *$1.30 will buy 1 gallon of gas.*	

Part 2: Selecting the Appropriate Unit Rate

At Ralph's fruit stand 3 apples cost $.90. You want to buy 7 apples. How much will they cost?

a. What are the two possible rates for this problem?

$\frac{3\ apples}{\$.90}$ or $\frac{\$.90}{3\ apples}$.

b. Show each rate as a unit rate.

$\frac{3.33\ apples}{\$1}$ or $\frac{\$.30}{apple}$.

c. What does each unit rate tell you?

3.33 apples can be purchased for $1 or 1 apple can be purchased for $.30.

d. Which unit rate will help you solve the problem?

$\frac{\$.30}{apple}$

e. Complete the table in order to determine the cost of seven apples. Then, describe the pattern you see.

As the number of apples increases by one, the cost increases by $.30.

Number of apples, *n*	**Cost, *C* (in dollars)**
1	***.30***
2	***.60***
3	***.90***
4	***1.20***
5	***1.50***
6	***1.80***
7	***2.10***

f. Since you know the unit price, write a number sentence for the cost of seven apples. Write an equation for the cost of **any number** of apples using the variables in the table above.

$7\ apples \times \frac{\$.30}{apple} = \$2.10$

$C = .30n$

Part 3: Applying the Unit Rate

In each problem, record the rate appropriate for the question asked, find the corresponding unit rate, write a short sentence interpreting the unit rate, and use this rate to find the solution to the problem.

a. Anne is painting her house light blue. To make the color she wants, she must add 3 cans of white paint to every 2 cans of blue paint. How many cans of white paint will she need to mix with 6 cans of blue?

Rate needed: $\frac{3 \text{ cans of white}}{2 \text{ cans of blue}}$ Unit rate: $\frac{1.5 \text{ cans of white}}{1 \text{ can of blue}}$

Interpretation of unit rate: ***Anne needs 1.5 cans of white paint for every can of blue paint.***

Solution: $1.5 \times 6 = 9 \text{ cans of blue paint}$

b. Ryan is making a fruit drink. The directions say to mix 5 cups of water with 2 scoops of powdered fruit mix. How many cups of water should he use with 9 scoops of fruit mix?

Rate needed: $\frac{5 \text{ cups of water}}{2 \text{ scoops of fruit mix}}$ Unit rate: $\frac{2.5 \text{ cups of water}}{1 \text{ scoop of fruit mix}}$

Interpretation of unit rate: ***Ryan needs 2.5 cups of water for every scoop of fruit miz***

Solution: $2.5 \times 9 = 22.5 \times \text{scoops of fruit mix}$

c. Donna is running around a track. It takes her 10 minutes to run 6 laps. If she keeps running at the same speed, how long will it take her to run 5 laps?

Rate needed: $\frac{10 \text{ minutes}}{6 \text{ laps}}$ Unit rate: $\frac{1.67 \text{ minutes}}{1 \text{ lap}}$

Interpretation of unit rate: ***Donna can run 1 lap in 1.67 minutes.***

Solution: $1.67 \times 5 = 8.35 \text{ minutes (Approximately 8 minutes and 20 seconds)}$

d. Carla is cleaning her classroom but decides to first help out her friends, Liz and Melissa, by cleaning both of their classrooms. It takes Carla $3\frac{1}{3}$ hours to clean both Liz and Melissa's classrooms. How long will she be working to clean all three classrooms?

Rate needed: $\frac{3\frac{1}{3}\ hours}{2\ classrooms}$

Unit rate: $\frac{1\frac{2}{3}\ \ hours}{1\ classroom}$

Interpretation of unit rate: ***Carla can clean 1 classroom in*** $1\frac{2}{3}$ ***hours.***

Solution: $1\frac{2}{3}\ x\ 3\ classrooms = \ 5\ hours$

FORMATIVE ASSESSMENT QUESTIONS

- What does the unit rate mean for this particular problem?
- What information in the problem can help you decide on the appropriate unit rate?
- What is the relationship between the two variables?

DIFFERENTIATION

Extension

This lesson gives students only one opportunity to make a table of values and write an equation, but some students may be ready to look at multiple representations of the problem. For students who are ready to move beyond the lesson, assign them one of the problems in part 3 and ask them to make a table, plot the points on a coordinate grid, and write an equation for the situation. Ask them to explain how the solution to the problem can be found in each representation.

Intervention

Some students may need to use manipulatives to visualize the unit rates. Provide colored tiles, Cuisenaire rods, or encourage the use of appropriate tools such as bar models and double-number lines to make sense of the problems. Additionally, students can find more practice at http://www.mathplayground.com/thinkingblocks.html.

For extra help with units rates, please open the hyperlink Intervention Table.

CLOSING/SUMMARIZER

End the lesson by asking students to discuss the differences and similarities between using a table and an equation to find a solution (may need to refer back to part 2). We want students to realize that an equation can be more efficient than extending values in a table. This question may help to initiate that discussion: *How much would 63 apples cost?*

What is the Unit Rate?

Part 1: Finding and Interpreting the Unit Rate

In each problem, record both possible rates, use an appropriate strategy to find the unit rates, and then write a short sentence explaining each unit rate.

a. 6 bags of flour weigh 30 pounds.

Rate 1:	Unit rate 1:
Interpretation:	

Rate 2:	Unit rate 2:
Interpretation:	

b. 9 tennis balls come in 3 cans.

Rate 1:	Unit rate 1:
Interpretation:	

Rate 2:	Unit rate 2:
Interpretation:	

c. 5 gallons of gas cost $6.50.

Rate 1:	Unit rate 1:
Interpretation:	

Rate 2:	Unit rate 2:
Interpretation:	

Part 2: Selecting the Appropriate Unit Rate

At Ralph's fruit stand 3 apples cost $.90. You want to buy 7 apples. How much will they cost?

a. What are the two possible rates for this problem?

b. Show each rate as a unit rate.

c. What does each unit rate tell you?

d. Which unit rate will help you solve the problem?

e. Complete the table in order to determine the cost of seven apples. Then, describe the pattern you see.

Number of apples, *n*	**Cost, *C* (in dollars)**
1	
2	
3	
4	
5	
6	
7	

f. Since you know the unit price, write a number sentence for the cost of seven apples. Write an equation for the cost of **any number** of apples using the variables in the table above.

Part 3: Applying the Unit Rate

In each problem, record the rate appropriate for the question asked, find the corresponding unit rate, write a short sentence interpreting the unit rate, and use this rate to find the solution to the problem.

a. Anne is painting her house light blue. To make the color she wants, she must add 3 cans of white paint to every 2 cans of blue paint. How many cans of white paint will she need to mix with 6 cans of blue?

Rate needed: ___________________ Unit rate: ____________________

Interpretation of unit rate: ____

Solution: ___________________

b. Ryan is making a fruit drink. The directions say to mix 5 cups of water with 2 scoops of powdered fruit mix. How many cups of water should he use with 9 scoops of fruit mix?

Rate needed: ___________________ Unit rate: ____________________

Interpretation of unit rate: ____

Solution: ___________________

c. Donna is running around a track. It takes her 10 minutes to run 6 laps. If she keeps running at the same speed, how long will it take her to run 5 laps?

Rate needed: ___________________ Unit rate: ____________________

Interpretation of unit rate: ____

Solution: ___________________

d. Carla is cleaning her classroom but decides to first help out her friends, Liz and Melissa, by cleaning both of their classrooms. It takes Carla $3\frac{1}{3}$ hours to clean both Liz and Melissa's classrooms. How long will she be working to clean all three classrooms?

Rate needed: ___________________ Unit rate: ____________________

Interpretation of unit rate: ____

Solution: ___________________

Proportional Relationships

Adapted from https://www.engageny.org/

In this lesson, students examine both proportional and non-proportional relationships, paying particular attention to the characteristics that make two variables proportional to one another.

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class.
Recommended time: 90-120 minutes. Recommended arrangement: small groups of 3-4.

STANDARDS FOR MATHEMATICAL CONTENT

Students will use ratios to solve real-world and mathematical problems.

MFAPR1. Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations.* (MGSE6.RP.3)

MFAPR2. Students will recognize and represent proportional relationships between quantities.

a. Relate proportionality to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

MFAPR3. Students will graph proportional relationships.

a. Interpret unit rates as slopes of graphs. (MGSE8.EE.5)
c. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (MGSE8.EE.5)

Common Misconceptions:

Students often forget proportional relationships always have a multiplicative relationship. When looking at a graph, some students may think every linear function is also proportional because of the constant rate of change, forgetting that the graph must also pass through the origin. Students may need to examine corresponding tables and graphs for a proportional and non-proportional relationship so they see that even though a relation may have a constant slope that does not necessarily mean the variables have a multiplicative relationship.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students must make sense of problems to be able to distinguish between variables that are in a proportional relationship and those that are not.

3. Construct viable arguments and critique the reasoning of others. Students will need to clearly communicate reasons why to variables are, or are not, proportional to one another.

4. Model with mathematics. Students will examine a variety of models to draw conclusions about relationships between variables.

6. Attend to precision. Students must exercise precision both in their mathematical explanations and in their calculations.

7. Look for and make use of structure. Students should begin to notice that when two variables are proportional, the dependent variable is always attained by multiplying the independent variable by a constant multiplier.

8. Look for and express regularity in repeated reasoning. Students will use repeated reasoning to identify a multiplicative relationship between variables.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Distinguish between variables that are proportional to one another and those that are not.
- Clearly explain how I know two variables are in a proportional relationship or how I know they are not.
- Identify a proportional relationship given a situation, table, graph, or equation.

MATERIALS

- Student lesson sheet
- Chart paper (with grid, if possible)
- Graph paper (if no grid paper is available)
- Glue sticks (if no grid paper is available)
- Envelopes
- Markers
- Sticky notes

ESSENTIAL QUESTIONS

What are the characteristics of a proportional relationship?
How can I identify a proportional relationship in a situation, table, graph, or equation?

OPENER/ACTIVATOR

Students should examine the proportional and non-proportional relationships below.

What characteristics do you see in the proportional relationships that make them different from the non-proportional relationships?

Proportional

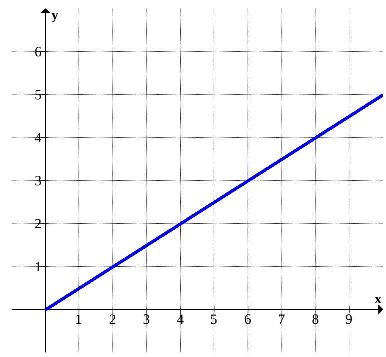
$C = 1.49n$

The number of candy bars, *n*, *C* is the total cost of the candy bars

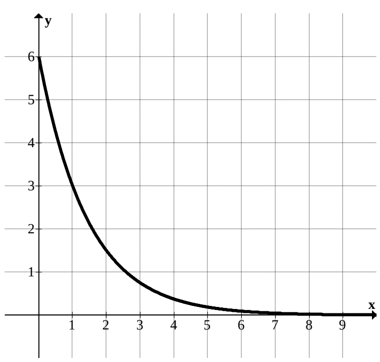
Proportional

Number of weeks	*Number of books read*
0	0
1	7
3	21
6	42

Proportional



Non-proportional



Non-proportional

$T = 35 + 10n$

Candice earned $10 a week in addition to the $35 she already had saved

Non-proportional

Input	*Output*
1	2
2	4
3	8
4	16

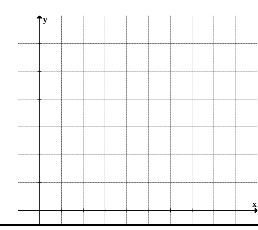
LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Lesson Directions:

Due to the length of the lesson, it may be helpful to chunk this lesson. Begin by asking students to work through #6. It is important that students hear multiple ways to explain why two variables are, or are not, proportional to one another. Ask students to share their thinking and record various explanations on the board before moving on to the last problem in part 1.

Part 2 will allow students to compare a proportional relationship to a non-proportional relationship in the same scenario. Students should feel comfortable answering the questions in this section by using either the graph or the tables.
In Part 3, students will examine proportional and non-proportional relationships in multiple representations. Before class, cut and place the situations and corresponding 5 ratios in envelopes and label. Note, that even though there are cards for 8 different groups, there are only 4 different scenarios. It is likely that there may be different responses for the same scenario that can provide a rich discussion to summarize.

Each group of 3-4 students should be given one piece of chart paper, marker, and graph paper and glue stick, if chart paper does not include a grid. One member of the group should fold the chart paper into quarters like the layout to the right. Each group should read and record the problem, create a table and graph, and decide whether or not the variables are proportional. Students should be encouraged to provide a clear and concise explanation supporting their claim.

Problem:	Table:
Graph: 	Proportional or Not? Explanation:

After all groups have completed their poster, instruct students to post their work around the classroom. Students should rotate with their group around the room in a gallery walk to observe each poster leaving questions and feedback on sticky notes. Students should answer the following questions on their worksheets:

Are there any differences in groups that had the same ratios?
Do you notice any common mistakes? How might they be fixed?
Are there any groups that stood out by representing their problem and findings exceptionally well?

Group 1	Group 2	Group 3	Group 4
A local frozen yogurt shop is known for its monster sundaes to be shared by a group. The ratios represent the number of toppings to the total cost of the toppings. Create a table and graph and then explain if the quantities are proportional to each other.	The school library receives money for every book sold at the school's book fair. The ratios represent the number of books sold to the amount of money the library receives. Create a table and graph and then explain if the quantities are proportional to each other.	Your uncle just bought a hybrid car and wants to take you and your sibling camping. The ratios represent the number of gallons of gas remaining to the number of hours of driving. Create a table and graph and then explain if the quantities are proportional to each other.	For a science project, Eli decided to study colonies of mold. He observed a piece of bread that was molding. The ratios represent the number of days passed to the number of colonies of mold on the bread. Create a table and graph and then explain if the quantities are proportional to each other.
4 to 0	1 to 5	8 to 0	1 to 1
6 : 3	2 to 10	After 1 hour of driving, there are 6 gallons of gas left in the tank.	2 to 4
8 : 6	The library received $15 for selling 3 books.	4 : 4	3 : 9
The total cost of a 10-topping sundae is $9.	4 : 20	2 to 7	4 : 16
12 to 12	5 : 25	0 : 8	Twenty-five colonies were found on the 5th day.

Group 5	Group 6	Group 7	Group 8
For a science project, Eli decided to study colonies of mold. He observed a piece of bread that was molding. The ratios represent the number of days passed to the number of colonies of mold on the bread. Create a table and graph and then explain if the quantities are proportional to each other.	Your uncle just bought a hybrid car and wants to take you and your sibling camping. The ratios represent the number of gallons of gas remaining to the number of hours of driving. Create a table and graph and then explain if the quantities are proportional to each other.	The school library receives money for every book sold at the school's book fair. The ratios represent the number of books sold to the amount of money the library receives. Create a table and graph and then explain if the quantities are proportional to each other.	A local frozen yogurt shop is known for its monster sundaes to be shared by a group. The ratios represent the number of toppings to the total cost of the toppings. Create a table and graph and then explain if the quantities are proportional to each other.
1 to 1	8 to 0	1 to 5	4 to 0
2 to 4	After 1 hour of driving, there are 6 gallons of gas left in the tank.	2 to 10	6 : 3
3 : 9	4 : 4	The library received \$15 for selling 3 books.	8 : 6
4 : 16	2 to 7	4 : 20	The total cost of a 10-topping sundae is \$9.
Twenty-five colonies were found on the 5th day.	0 : 8	5 : 25	12 to 12

FORMATIVE ASSESSMENT QUESTIONS

- What are the characteristics of a proportional relationship?
- Does the relationship have a constant multiplier?

DIFFERENTIATION

Extension

For students who are ready for an extra challenge, ask them to create their own proportional relationship and display it in a table and on a coordinate grid. Additionally, they may write an equation to model the scenario and create a set of questions to be answered using the graph, table, and equation.

Intervention

Some students may need a graphic organizer to help them distinguish variables that are proportional to one another to those that are not. It may be helpful to provide students a copy of the 6 relationships in the opener to glue into their interactive notebook. In the notebook, they should also record explanations for why each problem has been characterized as either proportional or non-proportional.
For extra help with linear proportion, please open the hyperlink Intervention Table.

CLOSING/SUMMARIZER

Journal entry: Suppose we invite students from another school to walk through our gallery. What would they be able to learn about proportional reasoning from our posters?

Proportional Relationships

Part 1: Proportional or Not?

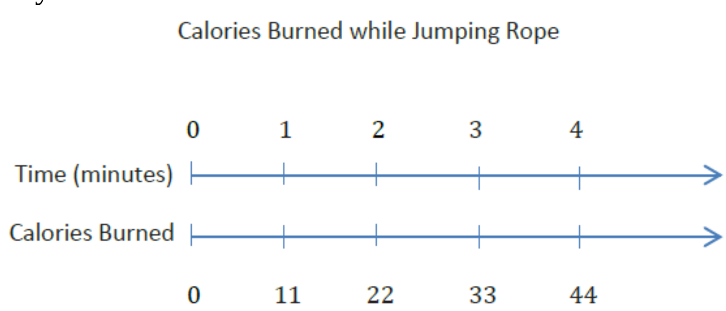
Determine whether or not the following situations represent two quantities that are proportional to each other. Explain your reasoning.

1. A new self-serve frozen yogurt store opened that sells its yogurt at a price based upon the total weight of the yogurt and its toppings in a dish. Each member of Isabelle's family weighed their dish and this is what they found.

Weight (ounces)	12.5	10	5	8
Cost ($)	5	4	2	3.20

Answers may vary. A possible solution:
Yes. Weight and cost have a multiplicative relationship.

2. During Jose's physical education class today, students visited activity stations. Next to each station was a chart depicting the number of calories (on average) that would be burned by completing the activity.



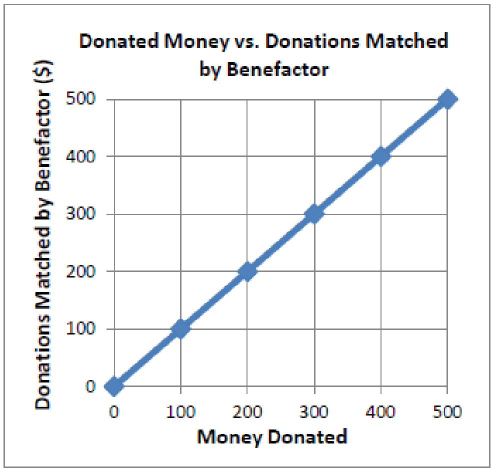
Answers may vary. A possible solution:
Yes. The number of calories burned is the time in minutes multiplied by 11. These variables are proportional because they have a multiplicative relationship.

3. The table represents the relationship of the amount of snowfall (in inches) in 5 counties to the amount of time (in hours) of a recent winter storm.

x Time (h)	2	6	8	2.5	7
y Snowfall (in.)	10	12	16	5	14

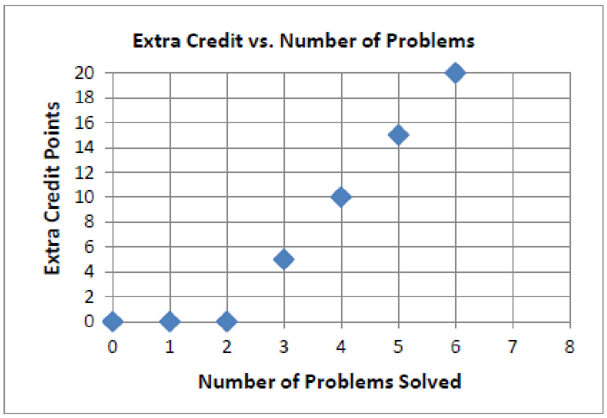
Answers may vary. A possible solution:
No. Snowfall and time do not have a constant rate of change.

4.



Answers may vary. A possible solution: Yes. The graph passes through the origin and has a constant rate of change.

5.



Answers may vary. A possible solution: No. Although the graph passes through the origin, it does not have a constant rate of change.

6. Jayden's favorite cake recipe calls for 5 cups of flour and 2 cups of sugar. The equation represents the relationships between the amount of flour, F, and the amount of sugar, s, in Jayden's recipe: $F = \frac{5}{2}s$.

Answers may vary. A possible solution: Yes. When $s = 0, F = 0$ and the independent variable is multiplied by a constant.

Alex spent the summer helping out at his family's business. He was hoping to earn enough money to buy a new $220 gaming system by the end of the summer. Halfway through the summer, after working for 4 weeks, he had earned $112. Alex wonders, "If I continue to work and earn money at this rate, will I have enough money to buy the gaming system by the end of the summer?"

To determine if he will earn enough money, he decided to make a table. He entered his total money earned at the end of Week 1 and his total money earned at the end of Week 4.

Week	0	1	2	3	4	5	6	7	8
Total Earnings	***0***	$28	***56***	***84***	$112	***140***	***168***	***196***	***224***

1. Work with a partner to answer Alex's question.
Answers may vary.
Yes. By the end of the summer, Alex would have earned more than the $220 he needed.

2. Are Alex's total earnings proportional to the number of weeks he worked? How do you know?
Answers may vary.
Yes. The ratio of total earnings to the week number is always the same.

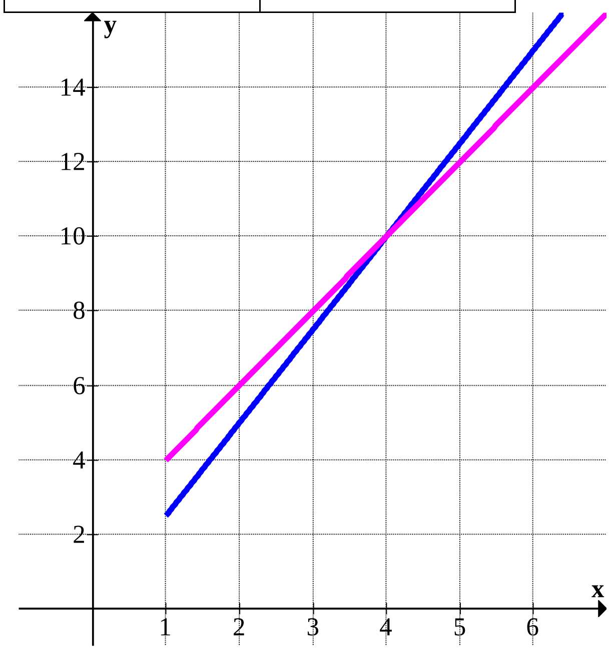
Part 2: Which Team Will Win the Race?

You have decided to walk in a long-distance race. There are two teams that you can join. Team A walks at a constant rate of 2.5 miles per hour. Team B walks 4 miles the first hour and then 2 miles per hour after that.

1. Create a table for each team showing the distances that would be walked for times of 1, 2, 3, 4, 5, and 6 hours. Then, plot the data for both teams on the same coordinate plane. Use the tables and the graphs to answer the questions that follow.

Team A	
Time (h)	Distance (miles)
1	*2.5*
2	*5*
3	*7.5*
4	*10*
5	*12.5*
6	*15*

Team B	
Time (h)	Distance (miles)
1	*4*
2	*6*
3	*8*
4	*10*
5	*12*
6	*14*



2. For which team is distance proportional to time? Explain your reasoning.

Answers may vary.
Team A. Distance and time for Team A have a multiplicative relationship.

3. Explain how you know distance for the other team is not proportional to time.

Answers may vary.
There is not a constant multiplier between distance and time.

4. At what distance in the race would it be better to be on Team B than Team A? Explain.

Answers may vary.
Team B is better up until 4 hours. The graph of Team B is higher than the graph of Team A.

5. If the members on each team walked for 10 hours, how far would each member walk on each team?
Answers may vary.
Members of Team A would walk for 25 miles in 10 hours.
Members of Team B would walk for 22 miles in 10 hours.

6. Will there always be a winning team, no matter what the length of the course? Why or why not?
Answers may vary.
No. At 4 hours, both teams have walked 10 miles.
7. If the race is 12 miles long, which team should you choose to be on if you wish to win? How do you know that team will win?

Answers may vary.
Team A will win. On the graph, Team A arrives at the 12 mile mark before Team B.

8. How much sooner would you finish on that team compared to the other team?

Answers may vary.
Some students may look at the graph or table and estimate. Although unlikely, some could calculate the hour in which Team A reaches the 12-mile mark by writing an equation and solving for the hour when the distance is 12.

Proportional Relationships

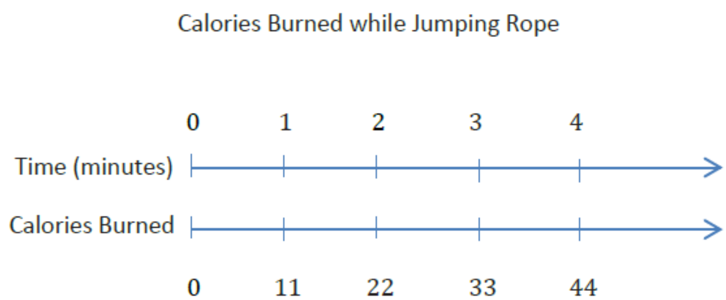
Part 1: Proportional or Not?

Determine whether or not the following situations represent two quantities that are proportional to each other. Explain your reasoning.

1. A new self-serve frozen yogurt store opened that sells its yogurt at a price based upon the total weight of the yogurt and its toppings in a dish. Each member of Isabelle's family weighed their dish and this is what they found.

Weight (ounces)	12.5	10	5	8
Cost ($)	5	4	2	3.20

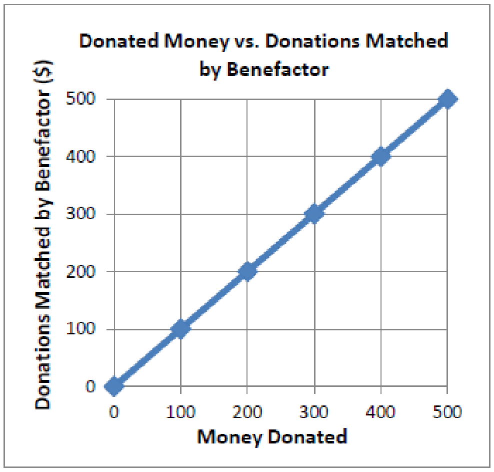
2. During Jose's physical education class today, students visited activity stations. Next to each station was a chart depicting the number of calories (on average) that would be burned by completing the activity.



3. The table represents the relationship of the amount of snowfall (in inches) in 5 counties to the amount of time (in hours) of a recent winter storm.

x Time (h)	2	6	8	2.5	7
y Snowfall (in.)	10	12	16	5	14

4.



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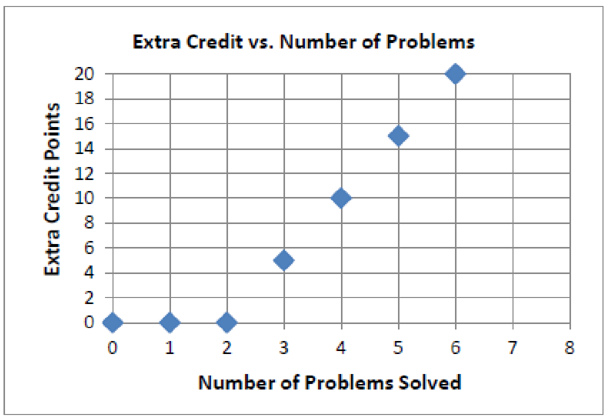


6. Jayden's favorite cake recipe calls for 5 cups of flour and 2 cups of sugar. The equation represents the relationships between the amount of flour, F¸ and the amount of sugar, s, in Jayden's recipe: $F = \frac{5}{2}s$.

Alex spent the summer helping out at his family's business. He was hoping to earn enough money to buy a new $220 gaming system by the end of the summer. Halfway through the summer, after working for 4 weeks, he had earned $112. Alex wonders, "If I continue to work and earn money at this rate, will I have enough money to buy the gaming system by the end of the summer?"

To determine if he will earn enough money, he decided to make a table. He entered his total money earned at the end of Week 1 and his total money earned at the end of Week 4.

Week	0	1	2	3	4	5	6	7	8
Total Earnings		$28			$112				

1. Work with a partner to answer Alex's question.
2. Are Alex's total earnings proportional to the number of weeks he worked? How do you know?

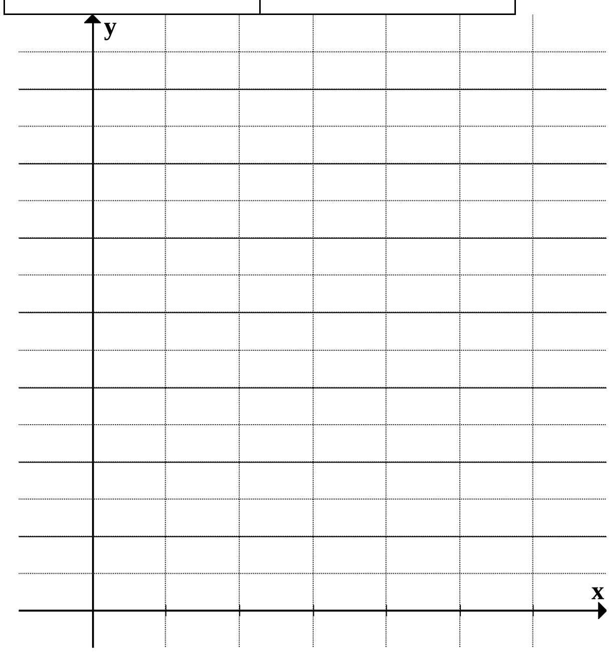
Part 2: Which Team Will Win the Race?

You have decided to walk in a long distance race. There are two teams that you can join. Team A walks at a constant rate of 2.5 miles per hour. Team B walks 4 miles the first hour and then 2 miles per hour after that.

1. Create a table for each team showing the distances that would be walked for times of 1, 2, 3, 4, 5, and 6 hours. Then, plot the data for both teams on the same coordinate plane. Use the tables and the graphs to answer the questions that follow.

Team A	
Time (h)	Distance (miles)

Team B	
Time (h)	Distance (miles)



2. For which team is distance proportional to time? Explain your reasoning.

3. Explain how you know distance for the other team is not proportional to time.

4. At what distance in the race would it be better to be on Team B than Team A? Explain.

5. If the members on each team walked for 10 hours, how far would each member walk on each team?

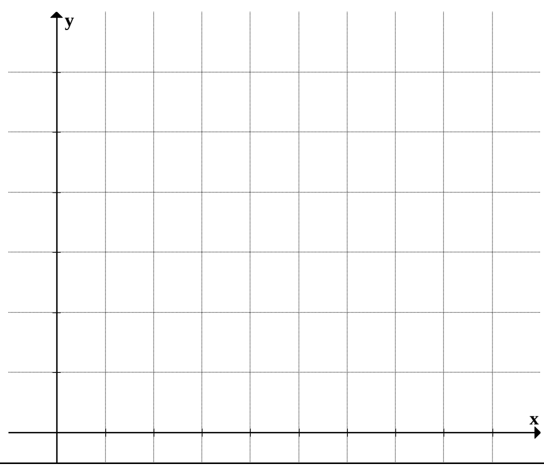
6. Will there always be a winning team, no matter what the length of the course? Why or why not?

7. If the race is 12 miles long, which team should you choose to be on if you wish to win? How do you know that team will win?

8. How much sooner would you finish on that team compared to the other team?

Part 3: Proportional and Non-Proportional Relationships in Multiple Representations

Use this layout as your group discusses your assigned problem.

Problem:	Table:
Graph: 	Proportional or Not? Explanation:

Gallery Walk

Use the table to record your notes on each of the posters. Think about the following questions:

Are there any differences in groups that had the same ratios?
Do you notice any common mistakes? How might they be fixed?
Are there any groups that stood out by representing their problem and findings exceptionally well?

Group 1	Group 2
Group 3	Group 4
Group 5	Group 6
Group 7	Group 8

Orange Fizz Experiment

In this lesson, students analyze concentrations in soda formulas to make a recommendation to a cola company.

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class.
Recommended time: 90-120 minutes. Recommended arrangement: individual, partners or small groups.

STANDARDS FOR MATHEMATICAL CONTENT

Students will use ratios to solve real-world and mathematical problems.

MFAPR1. Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations.* (MGSE6.RP.3)

MFAPR2. Students will recognize and represent proportional relationships between quantities.
a. Relate proportionality to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students must make sense of the problem in order to devise a plan for solving it.

2. Reason abstractly and quantitatively. Students must use quantitative reasoning to create a representation of the problem and to understand how the quantities relate.

3. Construct viable arguments and critique the reasoning of others. Throughout the problem, students will need to communicate their mathematical thinking to their peers as they evaluate their own and their peers' understanding of the problem. Most importantly, they will need to construct a coherent argument for their final recommendation.

6. Attend to precision. Students must use correct mathematical language as they communicate their thinking to their peers.

7. Look for and make use of structure. In order to create an accurate model, students must notice patterns in order to make equivalent ratios.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Compare ratios using a variety of strategies.
- Make part-to-part comparisons and part-to-whole comparisons.
- Find equivalent ratios.

MATERIALS

- Student lesson sheet

ESSENTIAL QUESTIONS

- How can ratios be used to make decisions?
- How do I solve real-world problems using equivalent ratios?
- What are the differences in part-to-part and part-to-whole comparisons?

OPENER/ACTIVATOR

Number Talk
Begin with the following problem, **"There are 12 boys and 18 girls in Ms. Dade's class. What is the boy to girl ratio in the class?"** Record the problem on the far left side of the board. Provide students with wait time as they work to mentally solve this problem. When the majority of the students have given the "thumbs up" signal, call on several students (3-4) to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you. It is important to allow students ownership of their thinking.

Record, **"Mr. Hill has 30 students. 14 of the students are boys. What is the ratio of boys in the class?"** on the board next to the previous problem. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the "thumbs up" signal, call on several students (3-4) to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you.

Record, **"The ratio for male and female students in Ms. Dade's class is 12:18. The ratio of males in Mr. Hill's entire class is 14 to 30. Whose class is composed of more boys?"** on the board towards the right. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the "thumbs up" signal, call on several students (3-4) to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you.

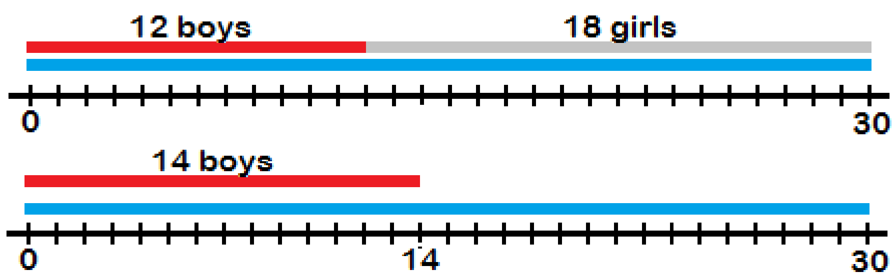
Record, **"Mrs. Ford's class has 14 boys and 16 girls. Of the 3 classes, whose class is composed of the most boys?"** on the board towards the far right. Provide students with wait time as they work to mentally solve this problem. When majority of the students have given the "thumbs up" signal, call on several students (3-4) to share their answer and the strategy they used to solve. Record the information provided by the students exactly how it is told to you.
At the end of the Number Talk, discuss the strategies used to find the answers. Talk with the students about which strategy was most efficient (quick, easy and accurate).

Prior to the lesson students need to discuss and explore making comparisons with ratios, percents, and fractions. Models and drawings, as illustrated below, may facilitate student understanding.

Relate the strategies from the introductory problems to the lesson where students will be comparing the mixes. Make these connections during the whole-group discussion.

Compare the ratio you found for Ms. Dade's class to the ratio you found for Mr. Hill's class. What is alike or different about each?

#1 is comparing part-to-part and #2 is comparing part-to-whole. Discuss how to compare the two classes using tables, percentages, common denominators or models. When #1 is set up as part-to-whole (12: 30), a comparison can be made easily using common denominators. The model below also illustrates the comparison.



LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Lesson Directions:

Due to the length of the lesson, the teacher will likely need to chunk the lesson into sections. Students may have trouble completing the tables for each formula as the difficulty increases. The teacher may need to check for understanding of creating equivalent ratios after the Formula A table. Later on in the lesson, students may struggle with converting cups to gallons. Encourage students to check their solutions with group members on the three formula tables before completing the tables in #2.

FORMATIVE ASSESSMENT QUESTIONS

- What relationship do you see between the values that would help find an equivalent ratio?
- When should you round a value to assure you have the most accurate solution?
- What factors must you consider when writing a recommendation?

INTERVENTION

For extra help with proportions, please open the hyperlink Intervention Table.

CLOSING/SUMMARIZER

After giving students adequate time to write a recommendation to the company, ask students to share their recommendation with their group members. Each group should work to summarize the recommendations for their group and then one student in each group will share the summary with the class. Students will need reminding that their recommendations, and summary, should include supporting data.

Orange Fizz Experiment

A famous cola company is trying to decide how to change their drink formulas to produce the best tasting soda drinks on the market. The company has three different types of formulas to test with the public. The formula consists of two ingredients: orange concentrate and carbonated water.

You are a scientist working for this company, and you will get paid a large commission if you can find the right formula that will sell the best. Your job is to decide the best formula based on cost and popularity in the taste test.

Using the company's new formulas, you must follow the recipe to the strict guidelines:

Formula A: 1 tablespoons of orange concentrate to 2 tablespoons of carbonated water

Formula B: 2 tablespoons of orange concentrate to 5 tablespoons of carbonated water

Formula C: 2 tablespoons of orange concentrate to 3 tablespoons of carbonated water

Part A: *Using part-to-whole comparison*

1. Which formula will make a drink that has the *strongest* orange taste? Show your work and explain your choice.

Answers will vary.

A: $\frac{orange}{water} = \frac{1}{2}$ $\frac{orange}{total} = \frac{1}{3}$ ***33% orange***

B: $\frac{orange}{water} = \frac{2}{5}$ $\frac{orange}{total} = \frac{2}{7}$ ***29% orange***

C: $\frac{orange}{water} = \frac{2}{3}$ $\frac{orange}{total} = \frac{2}{5}$ ***40% orange***

Formula C is 40% orange concentrate so it has the strongest orange taste.

2. Which formula has the highest percentage of carbonated water in the mixture? Estimations may be used. Show your work and justify your answer.

Answers will vary.

A:	***water:total***	***2:3***	***67%***
B:	***water:total***	***5:7***	***71%***
C:	***water:total***	***3:5***	***60%***

Formula B contains the highest percentage of water at 71%.

Part B: *Using part-to-part comparison*

1. For researchers to test their product, they will need to produce enough of each of the three drink formulas to take to various locations around the area for taste testing. Researchers would like for *at least 100 people* to sample each formula.
Each sample will contain 1 cup of liquid.

Formula A: 1 cup of orange concentrate to 2 cups of carbonated water

Formula B: 2 cups of orange concentrate to 5 cups of carbonated water

Formula C: 2 cups of orange concentrate to 3 cups of carbonated water

Fill in the table to determine the least amount of orange concentrate and carbonated water that you would have to use to serve 1 cup servings to 100 people.

Formula A		
Orange Concentrate (cups)	Carbonated Water (cups)	Total Amount (cups/servings)
1	2	3 cups (3 servings)
2	4	6 cups (6 servings)
3	6	9 cups (9 servings)
…	…	…
17	34	51 cups (51 servings)
…	…	…
33	66	***99*** cups (***99*** servings)
34	*68*	***102*** cups (***102*** servings)
35	70	***105*** cups (***105*** servings)

I. How much orange concentrate and carbonated water is needed to serve at least 100 people?
a. Orange Concentrate - ***34***
b. Carbonated Water - ***68***

Formula B		
Orange Concentrate (cups)	Carbonated Water (cups)	Total Amount (cups/servings)
2	5	7 cups (7 servings)
4	10	14 cups (14 servings)
8	***20***	28 cups (28 servings)
…	…	…
14	35	***49*** cups (49 servings)
…	…	…
26	***65***	***91*** cups (***91***servings)
28	***70***	98 cups (98 servings)
30	***75***	***105*** cups (***105*** servings)

II. How much orange concentrate and carbonated water is needed to serve at least 100 people?
a. Orange Concentrate - ***30***
b. Carbonated Water - ***75***

Formula C		
Orange Concentrate (cups)	Carbonated Water (cups)	Total Amount (cups/servings)
2	3	5 cups (5 servings)
4	6	10 cups (10 servings)
6	***9***	15 cups (___ servings)
…	…	…
20	***30***	***50*** cups (***50*** servings)
…	…	…
36	***54***	***90*** cups (***90*** servings)
38	57	***95*** cups (***95*** servings)
40	***60***	***100*** cups (***100*** servings)

III. How much orange concentrate and carbonated water is needed to serve at least 100 people?
a. Orange Concentrate - ***40***
b. Carbonated Water - ***60***

2. Your lab technician will be bringing you all of the supplies that you will need in order to make the formulas at the sites. Record the number of cups needed of each ingredient in each formula in the following table.

	Orange Concentrate	**Carbonated Water**
Formula A	***34***	***68***
Formula B	***30***	***75***
Formula C	***40***	***60***
Total (cups)	***104***	***203***

Both orange concentrate and carbonated water are sold by the gallon. In the table below, record the number of gallons needed for both ingredients in each formula. (1 gallon = 16 cups)

	Orange Concentrate		**Carbonated Water**	
	Amount in gallons	**Cost**	**Amount in gallons**	**Cost**
Formula A	***2.13***	***$20.24***	***4.25***	***$11.69***
Formula B	***1.88***	***$17.86***	***4.69***	***$12.90***
Formula C	***2.5***	***$23.75***	***3.75***	***$10.31***
Total	***6.51***	***$61.85***	***12.69***	***$34.90***

A gallon of orange concentrate costs $9.50 and a gallon of carbonated water costs $2.75. Record the cost of each ingredient for each formula in the table above.

Based on the number of gallons you will need to produce Formulas A, B, and C for 100 people, which formula is most cost effective? Justify your answer.

Answers will vary.
Formula A: $31.93 ***Formula B: $30.76*** ***Formula C: $34.06***
Formula B is the most cost effective choice at $30.76.

The table below shows the number of taste testers who preferred each formula:

Formula A	**Formula B**	**Formula C**
30	25	45

Which formula did taste testers like best? What fraction of the taste testers liked this formula the best?
Answers will vary.
Taste testers prefer formula C. $\frac{9}{20}$ ***of the taste testers prefer formula C.***

Write a recommendation to the company for the formula you think should be produced. Be sure to write a clear and concise recommendation for your formula using data to support your argument.

Answers will vary.

Students may argue that although formula B was the least favorite flavor, it should be produced due to the lower cost to produce it. On the other hand, some could argue that despite being the most expensive flavor to produce, the company should produce the most popular flavor, Formula C. Lastly, an argument could be made to produce Formula B because it is "in the middle" in terms of cost and popularity.

Regardless of the formula students recommend, they should refer to both the cost and the popularity of the taste of the products to construct a viable argument.

Orange Fizz Experiment

A famous cola company is trying to decide how to change their drink formulas to produce the best tasting soda drinks on the market. The company has three different types of formulas to test with the public. The formula consists of two ingredients: orange concentrate and carbonated water.

You are a scientist working for this company, and you will get paid a large commission if you can find the right formula that will sell the best. Your job is to decide the best formula based on cost and popularity in the taste test.

Using the company's new formulas, you must follow the recipe to the strict guidelines:

Formula A: 1 tablespoons of orange concentrate to 2 tablespoons of carbonated water

Formula B: 2 tablespoons of orange concentrate to 5 tablespoons of carbonated water

Formula C: 2 tablespoons of orange concentrate to 3 tablespoons of carbonated water

Part A: *Using part-to-whole comparison*

1. Which formula will make a drink that has the *strongest* orange taste? Show your work and explain your choice.

2. Which formula has the highest percentage of carbonated water in the mixture? Estimations may be used. Show your work and justify your answer.

<u>Part B</u>: *Using part-to-part comparison*

1. For researchers to test their product, they will need to produce enough of each of the three drink formulas to take to various locations around the area for taste testing. Researchers would like for *at least 100 people* to sample each formula.
Each sample will contain 1 cup of liquid.

Formula A: 1 cup of concentrate to 2 cups of carbonated water

Formula B: 2 cups of concentrate to 5 cups of carbonated water

Formula C: 2 cups of concentrate to 3 cups of carbonated water

Fill in the table to determine the least amount of orange concentrate and carbonated water that you would have to use to serve 1 cup servings to 100 people.

Formula A		
Orange Concentrate (cups)	Carbonated Water (cups)	Total Amount (cups/servings)
1	2	3 cups (3 servings)
2	4	6 cups (6 servings)
3	6	9 cups (9 servings)
…	…	…
17	34	51 cups (51 servings)
…	…	…
33	66	___ cups (___ servings)
34		___ cups (___ servings)
	70	___ cups (___ servings)

I. How much orange concentrate and carbonated water is needed to serve at least 100 people?
a. Orange Concentrate -
b. Carbonated Water -

Formula B		
Orange Concentrate (cups)	Carbonated Water (cups)	Total Amount (cups/servings)
2	5	7 cups (7 servings)
4	10	14 cups (14 servings)
8		28 cups (28 servings)
…	…	…
	35	___ cups (49 servings)
…	…	…
26		___ cups (___ servings)
		98 cups (98 servings)
		___ cups (___ servings)

II. How much orange concentrate and carbonated water is needed to serve at least 100 people?
a. Orange Concentrate -
b. Carbonated Water -

Formula C		
Orange Concentrate (cups)	Carbonated Water (cups)	Total Amount (cups/servings)
2	3	5 cups (5 servings)
4	6	10 cups (10 servings)
		15 cups (___ servings)
…	…	…
20		___ cups (___ servings)
…	…	…
36		___ cups (___ servings)
	57	___ cups (___ servings)
		___ cups (___ servings)

III. How much orange concentrate and carbonated water is needed to serve at least 100 people?
a. Orange Concentrate -
b. Carbonated Water -

2. Your lab technician will be bringing you all of the supplies that you will need in order to make the formulas at the sites. Record the number of cups needed of each ingredient in each formula in the following table.

	Orange Concentrate	**Carbonated Water**
Formula A		
Formula B		
Formula C		
Total (cups)		

Both orange concentrate and carbonated water are sold by the gallon. In the table below, record the number of gallons needed for both ingredients in each formula. (1 gallon=16 cups)

	Orange Concentrate		**Carbonated Water**	
	Amount in gallons	**Cost**	**Amount in gallons**	**Cost**
Formula A				
Formula B				
Formula C				
Total				

A gallon of orange concentrate costs \$9.50 and a gallon of carbonated water costs \$2.75. Record the cost of each ingredient for each formula in the table above.

Based on the number of gallons you will need to produce Formulas A, B, and C for 100 people, which formula is most cost effective? Justify your answer.

The table below shows the number of taste testers who preferred each formula:

Formula A	**Formula B**	**Formula C**
30	25	45

Which formula did taste testers like best? What fraction of the taste testers liked this formula the best?

Write a recommendation to the company for the formula you think should be produced. Be sure to write a clear and concise recommendation for your formula using data to support your argument.

Formative Assessment Lesson: Proportion and Non-Proportion Situations

(Concept Development)
This lesson is intended to help you assess whether students are able to identify when two quantities vary in direct proportion to each other, distinguish between direct proportion and other functional relationships, and solve proportionality problems using efficient methods.

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1520

STANDARDS FOR MATHEMATICAL CONTENT

MFAPR2. Students will recognize and represent proportional relationships between quantities.
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students must be able to make sense of the problems presented in order to determine whether the variables are in a proportional relationship.

8. Look for and express regularity in repeated reasoning. In order to recognize when variables are in a proportional relationship, students must be able to notice patterns in the numbers. Likewise, they must notice when this pattern does not exist in relationships that are not proportional.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:
- Identify when two quantities are in a proportional relationship.
- Distinguish between proportional and other functional relationships.
- Solve proportionality problems using efficient methods.

ESSENTIAL QUESTIONS

- How do I verify if two quantities are directly proportional?
- How do I solve real-world problems using equivalent ratios?

LESSON COMMENTS

Lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative
The lesson, *Proportion and Non-Proportion Situations*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?lessonid=483&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the lesson can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1520

Quick Check I

STANDARDS FOR MATHEMATICAL CONTENT

Students will use ratios to solve real-world and mathematical problems.

MFAPR1. Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations.* (MGSE6.RP.3)

MFAPR2. Students will recognize and represent proportional relationships between Relate proportionality **quantities.**
a. to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

Quick Check I

For each situation, determine if the identified variables are in a proportional relationship. If the variables are proportional, solve the problem.

1. If gasoline sells for \$3.35 per gallon, how much will 8 gallons cost? (Amount of gasoline vs. Cost)

Amount of gasoline and cost are proportional.
\$26.80

2. A plumber charges \$60 to make a house call and \$25 per hour for the time he works to fix the problem. If he works 3 hours, how much does he charge? (Hours worked vs. Charge)

Hours worked and charge are NOT proportional.

3. Granola sells for \$2.75 per pound. How much will 7 pounds cost? (Number of pounds vs. Cost)
Number of pounds and cost are proportional.
\$19.25

4. A school bus runs its route in 50 minutes going at 25 mph, on average. If it doubles its speed, how long will the run take? (Speed vs. Time)

Speed and time are NOT proportional.

5. A recipe that makes 16 cupcakes calls for $3\frac{1}{2}$ cup flour. How much flour is needed if you only wish to make 8 cupcakes? (Number of cupcakes vs. amount of flour)

Number of cupcakes and amount of flour are proportional.

$1\frac{3}{4}$ ***cup flour***

Quick Check I

For each situation, determine if the identified variables are in a proportional relationship. If the variables are proportional, solve the problem.

1. If gasoline sells for $3.35 per gallon, how much will 8 gallons cost? (Amount of gasoline vs. Cost)

2. A plumber charges $60 to make a house call and $25 per hour for the time he works to fix the problem. If he works 3 hours, how much does he charge? (Hours worked vs. Charge)

3. Granola sells for $2.75 per pound. How much will 7 pounds cost? (Number of pounds vs. Cost)

4. A school bus runs its route in 50 minutes going at 25 mph, on average. If it doubles its speed, how long will the run take? (Speed vs. Time)

5. A recipe that makes 16 cupcakes calls for $3\frac{1}{2}$ cup flour. How much flour is needed if you only wish to make 8 cupcakes? (Number of cupcakes vs. amount of flour)

Fish in a Lake

Source: Teaching Student Centered Mathematics, vol. 3 Grades 5-8, by John A. Van de Walle

In this lesson, students will simulate the "tagging" of fish in order to estimate the total number of fish in a lake by collecting data, examining ratios, and writing and solving proportions.

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class.
Recommended time: 60-90 minutes. Recommended arrangement: partners or small groups.

STANDARDS FOR MATHEMATICAL CONTENT

MFAPR2. Students will recognize and represent proportional relationships between quantities.
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will use a model of the situation to make sense of the problem.
2. Reason abstractly and quantitatively. Students will use their reasoning skills to understand how the ratio can help us to estimate the total population.
3. Construct viable arguments and critique the reasoning of others. Students will share results and discuss strategies with other students.
4. Model with mathematics. Students will model the problem using available materials.
6. Attend to precision. Students will attend to precision when using the language of mathematics, being precise in calculations and when assessing the reasonable of their estimations.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Compare two quantities using a ratio and understand what that ratio means in the context of the problem.
- Use proportional reasoning to solve for an unknown quantity.

MATERIALS

- Paper bags (one per group)
- White beans and red beans (alternatives to the beans include: two different colored counters, two different colored cubes, or plain Goldfish crackers and pretzel Goldfish crackers (tagged fish))

ESSENTIAL QUESTIONS

- How can I use information from a sample to help make a prediction about a population?
- How can I use a proportion to solve for an unknown quantity?

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Background Knowledge:

Proportional relationships can be confusing to students. In order for students to make sense of proportional relationships and develop strategies, students need experience with contextual problems. The context gives students a starting point to make sense of the problems and develop strategies for solving them. It is also important that students solve problems in a context other than those related to money, which is so common. This lesson embeds data collection and estimation into ratio and proportional reasoning.

Lesson Directions:

In order to save some instructional time, the teacher may want to have the bags ready for students at the beginning of class. Each bag should contain several handfuls of beans. If necessary, students can create their own lake but they should not count the beans prior to dumping them in the bag.

While this lesson can be done individually, the benefits of using partners or small groups to promote discussion of mathematical ideas and strategies may outweigh any need for an individual grade. As a teacher, listening to students' discussions of mathematical ideas and strategies can be extremely informative and valuable. This data can be used to inform instruction and to decide next steps for individuals or groups of students.

It is very important early on in the lesson that students understand the role of the red beans. In real-life, fish would not be taken out of a lake and replaced with other fish in order to tag them, but in this case, the red beans are very easily recognizable as the tagged fish. Stickers can be used to literally "tag" the fish if the teacher chooses, but if they fall off during the experiment, this can cause major inaccuracies in the data.

In order for the students to estimate the total number of fish in the lake, they must first choose a ratio of tagged fish in the sample to total number of fish in the sample. Students should be given the opportunity to decide for themselves an effective way of selecting this ratio. Amongst the many different ways of selecting a representative ratio, students could take an average of the ones in their table or they may notice one they consider to be different from the others and exclude that from the possibilities. Some will simply pick one they think looks best from their table. These different strategies should be shared in the whole-group discussion in the closing.

Lastly, students will need to be reminded of how to properly set up a proportion.

$$\frac{\text{tagged sample}}{\text{total sample}} = \frac{\text{tagged population}}{\text{total population}} \qquad \frac{\text{tagged sample}}{\text{tagged population}} = \frac{\text{total sample}}{\text{total population}}$$

Of course, writing the reciprocals of each ratio above would provide a 3rd and 4th possible way of writing a proportion for this problem.

FORMATIVE ASSESSMENT QUESTIONS

- What ratio of tagged to total fish would best represent the situation?
- What quantities are being compared in your ratio?
- How might you use your ratio to determine the total number of fish in the lake?
- What is the relationship between the ratio of tagged to total fish in a sample and the ratio of tagged to total fish in the population?
- How many quantities in a proportion must we know in order to be able to solve for an unknown quantity?

DIFFERENTIATION

Extension

Some students may be ready to answer questions about percents related to the problem. Provide these students a series of questions about the same context or different context involving percentages rather than just the ratio.

Intervention

Students may struggle with setting up the proportion correctly. Provide those students with a possible equation such as:

$$\frac{\text{tagged sample}}{\text{total sample}} = \frac{\text{tagged population}}{\text{total population}}$$

Begin by just providing the left side of the equation only and ask them how the right side could be written. Struggling students will likely need to be reminded that the proportion can be written four different ways.

CLOSING/SUMMARIZER

Close the lesson with a whole-group discussion focused on the strategies used for selecting a representative ratio and for estimating the total number of fish in the lake. Students should leave knowing that there are four different ways to write a proportion. Some students, however, may not have set up a proportion to arrive at their estimate. Ask students to share their strategies, making sure to focus on proportional reasoning, while still valuing all legitimate strategies.
End by asking students to think-pair-share about the two essential questions:

How can I use information from a sample to help make a prediction about a population?
How can I use a proportion to solve for an unknown quantity?

The teacher should pay close attention as students share their answers, listening closely for any misconceptions students still may have.

Fish in a Lake

Wildlife biologists often need to estimate the number of deer in a national park or the population size of fish in a lake. Because they cannot possibly count each deer or each fish, the biologists must use a “capture-recapture” method to help them estimate the population.

In order to estimate the number of fish in a lake, biologists first capture a sample of the fish, tag them, and then release them back into the lake with all of the untagged fish. Once the biologists have given the tagged fish enough time to thoroughly mix with the untagged fish, they capture another sample and count the number of each.

In today’s lesson, we are going to simulate the capture-recapture method. The bag will represent the lake, the white beans will represent the untagged fish, and the red beans will represent the tagged fish. Your objective is to estimate the total number of fish in the lake. ***The teacher only puts white beans in the bag at the beginning of the lesson.***

1. Reach into the lake and capture a handful of “fish” to tag. Count and record the number of fish removed. Tag all of the fish (white beans) removed by replacing them with the same number of tagged fish (red beans). Set the removed fish to the side and return the tagged fish to the lake.

Total number of tagged fish _________________

2. Shake the bag so that the tagged and untagged fish can mix thoroughly. Remove a handful of fish, count them all, and then count the number of tagged fish in the sample. Record the counts in the table below and then find the ratio of tagged fish to total fish in the sample.

Sample Number	Number of tagged fish	Total number of fish	Ratio of tagged fish to total fish
1			
2			
3			
4			
5			

s

3. Return the sample of fish back to the lake. In order to get a better idea of the actual ratio of tagged to total number of fish in the lake, we will take a few more samples.

4. Repeat the sampling process four more times, each time recording the number of tagged fish and total number of fish in the sample. Remember to mix your fish thoroughly after returning the sample to the lake.

5. Now, your job is to use the information in your table to estimate the total number of fish in the lake. Create an equation that would allow you to solve for the total number of fish in the lake.

6. Check your estimate by counting the total number of fish in the lake. How close was your estimate to the actual population? What factors might cause your estimate to be incorrect?

Which is The Better Deal?

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the number of group presentations.
Recommended arrangement: partners or small groups

STANDARDS FOR MATHEMATICAL CONTENT

Students will use ratios to solve real-world and mathematical problems.

MFAPR2. Students will recognize and represent proportional relationships between quantities.
a. Relate proportionality to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students must make sense of the problem in order to understand how unit rates can help make better decisions.

3. Construct viable arguments and critique the reasoning of others. Students will analyze their results and will construct a concise argument of their findings.

6. Attend to precision. Students must use precise mathematical language when presenting their findings.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:
- Use a unit rate to make decisions when making purchases.
- Use unit rates to compare the value of products.
- Write their data in a table and graph their findings

MATERIALS
- Chart paper or white poster board
- Rulers
- Markers for making charts
- Samples of products (Students can clip advertisements or print products off the internet. They need to have a picture of the product, explanation/details, price)

ESSENTIAL QUESTIONS

- How can I determine the unit rate for a product?
- How can I use unit rates to compare the value of products?
- How can my understanding of unit rates save me money?

OPENER/ACTIVATOR

Many educated consumers rely on unit pricing to make sure they are getting the best deal to fit their needs and budgets. We live in a society where we have a choice of purchasing countless consumer products. Just consider how many types of sneakers we can buy, or how many brands of potato chips from which we can choose.

Although most high school students are experienced consumers, they have not necessarily had a great deal of experience in making money-saving decisions. Engage students in a quick conversation to learn more about their shopping experience. Do they purchase the same product each time regardless of other options or do they think about which product would get them the most for their money? If they do consider other products, what strategies do they use to make the decision?

Explain that in today’s lesson, students will get the opportunity to use mathematics to help them be a more informed consumer.

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Lesson Directions:

Students will begin the project by selecting a product that can be packaged in three different sizes. For example, if the student chooses cola then they need to price it as a 6-pack, 12-pack, and 24-pack; each package needs to have the same number of ounces. Teachers can model this by bringing in examples of products with the same number of ounces, referring to vendor coupons, using printed ads, or surfing the web.

Some products and quantity/size to compare:

Soda
Potato Chips
Ice Cream
Milk
Paper products
Snack crackers

Students may need to be reminded to create a rough chart on scrap paper before attempting to draw their final copy. Encourage them to design a chart that will be informative, as well as easy to read.

An online site such as http://www.netgrocer.com is recommended. Make sure that students are not using a site in which the unit rates are already calculated.

Resource used for this lesson

Muschia, G. R. &., Muschia, J.A. (1996). *Hands-on math projects with real-life applications*. (pp. 126-130). New Jersey: Jossey-Bass.

FORMATIVE ASSESSMENT QUESTIONS

- How can you determine the cost of only one unit of the product?
- How much would you pay for only one unit of the product?
- How might you determine which size product in the better deal?

DIFFERENTIATION

Extension

For more difficult comparisons, ask students to compare products that may require some conversions. For instance, students can compare 2 liters of soda to cans of soda which are measured in ounces.

Intervention

Some students may be too overwhelmed by trying to find a product on the internet or in sales papers. For these students, provide some choices so that the focus can be placed on the mathematics and not on selecting products. Refer students to the *What is a Unit Rate?* lesson if they need assistance with finding unit rates.

For extra help with percentages, please open the hyperlink Intervention Table.

CLOSING/SUMMARIZER

Students should display their work and present their findings to the class.

Name: ___________________________ Project Due Date: _______________

Which Is The Better Deal?

Situation/Problem

Your group is to select a product and compare the package size and price (three different sizes/prices). You are to trying to determine which package is the best deal by finding each unit price. After you have reached your conclusions, design a chart to support your findings and be prepared to present your work to the class.

Your lesson

+ Decide on products to compare
  - Brainstorm with your group which products you might like to compare.
  - Look in sales papers for groceries/retail stores.
  - Look online at weekly ads for groceries/retail stores.

+ Decide which sizes or quantities you will compare
  - Any product packaged in more than one size can be compared.
  - Record your product and sizes/quantities on a piece of paper.

+ Compare the products
  - Compute unit rates.
  - Analyze results to determine the better deal.

+ Prepare for group presentation
  - Create a table illustrating results.
  - Prepare notes for presentation.
  - Assign each group member a responsibility for the presentation.
  - Rehearse your presentation.
  - Sketch your data

Be prepared to submit your chart after the presentation.

25% Sale (Short Cycle Lesson)

Source: Balanced Assessment Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=1042

In a sale, the store reduces all prices by 25% each week. Does this mean that, after 4 weeks, everything in the store will cost $0? If not, why not?

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class.
Recommended arrangement: individual, partners or small groups.

STANDARDS FOR MATHEMATICAL CONTENT

Students will use ratios to solve real-world and mathematical problems.

MFAPR2. Students will recognize and represent proportional relationships between quantities.
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

Common Misconceptions:
The most common misconception is addressed by number 2 in the lesson. Students may think that a reduction of 25% of the previous week's price for four weeks is equivalent to a reduction by 100%.
Students may confuse the amount paid for the jacket in the fourth week with the amount by which the jacket is reduced that week.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students must make sense of the problem to understand that a 25% discount of the previous week's price for four weeks is not equivalent to a discount of 100%.

2. Reason abstractly and quantitatively. Students need to be able to perform mathematical calculations and then understand what the calculation means in the context of the problem.

3. Construct viable arguments and critique the reasoning of others. Students are specifically asked to critique the reasoning of others in #2 of this problem. They must be able to coherently explain the error in Julie's thinking.

6. Attend to precision. In order to critique a viable argument, students must use precise mathematical language. They must also attend to precision when dealing with decimal solutions.

7. Look for and make use of structure. Students should notice the relationship between taking 25% off and paying 75% of the original cost.

MATERIALS

Student lesson sheet http://www.map.mathshell.org/materials/download.php?fileid=1042

ESSENTIAL QUESTION

How do I solve and interpret solutions of real-world percent problems?

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The lesson, *25% Sale*, is a Mathematics Assessment Project Assessment Lesson that can be found at the website:
http://www.map.mathshell.org/materials/lessons.php?lessonid=358&subpage=apprentice

The PDF version of the lesson can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=1042
The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=1043

FORMATIVE ASSESSMENT QUESTIONS

- How much does the jacket cost in the second week of the sale?
- What is the relationship between the percentage of the original price you save and the percentage of the original price you pay?

INTERVENTION For extra help with percentages, please open the hyperlink Intervention Table.

Which Bed Bath & Beyond Coupon Should You Use?

Source: http://robertkaplinsky.com/work/bed-bath-beyond/

In this lesson, students will be presented with two coupons and will compare their values to determine the circumstances in which one coupon is better than (or equal) to the other.

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class.
Recommended time: 60-90 minutes. Recommended arrangement: partners or small groups.

STANDARDS FOR MATHEMATICAL CONTENT

MFAPR2. Students will recognize and represent proportional relationships between quantities.
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

Common Misconceptions:
Students who do not have a solid understanding of percents may not realize that the amount saved from the 20% off coupon varies depending on the cost of the item. They may wrongly assume because "20" is more than "5", that the 20% off coupon will always be better. When students begin comparing the coupons on specific items, they may not notice that the $5 coupon can only be applied when one has spent $15 or more. Thus, the $5 coupon cannot be applied toward the Melissa & Doug Floor Puzzle.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Making sense of this problem is crucial in being able to compare the values of coupons.

2. Reason abstractly and quantitatively. Students will need to understand the relationship between the discounted amount and the purchase price for each item.

3. Construct viable arguments and critique the reasoning of others. Students will need to analyze their mathematical work in order to construct an argument for their final decision.

6. Attend to precision. Students will need to attend to precise when calculating discounts. Additionally, their conclusions should include precise mathematical language.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Calculate purchase price given a percent discount and a flat amount discount.
- Compare the value of coupons for a variety of items and determine the circumstances in which one coupon is better than (or equal to) another.

MATERIALS

All images available to download at http://robertkaplinsky.com/work/bed-bath-beyond/ in JPEG and PowerPoint formats

- Images of both Bed Bath & Beyond coupons: $5 off any purchase of $15 or more and 20% off one single item
- Images of four items from Bed Bath & Beyond website
- Problem solving framework (provided in this document)

ESSENTIAL QUESTIONS

Which coupon will save you the most money?
What strategies can be useful when comparing coupons?

OPENER/ACTIVATOR

https://www.illustrativemathematics.org/content-standards/lessons/105

The sales team at an electronics store sold 48 computers last month. The manager at the store wants to encourage the sales team to sell more computers and is going to give all sales team members a bonus if the number of computers sold increases by 30% in the next month. How many computers must the sales team sell to receive the bonus? Explain your reasoning.

If the sales team is going to sell 30% more computer next month, they will have to sell

$$\mathbf{0.3 \times 48 = 14.4}$$

more computers. Of course, you cannot sell four-tenths of a computer, so that means they will have to sell 15 more computers. Since $48 + 15 = 63$, they will need to sell 63 computers next month to receive a bonus.

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Lesson Directions:

Begin by showing students a picture of both Bed Bath & Beyond coupons: $5 off any purchase of $15 or more and 20% off one single item. Ask students to do a think-pair-share around the question "What problem are you trying to figure out?". This strategy will give students an opportunity to think individually about the question first, then collaborate with members of their group before finally sharing out in a whole-group discussion. Ask students to record their thoughts in the problem-solving framework provided. During the whole-group discussion when students are sharing their ideas, make a list of all questions raised by students. Together as a class, decide on the main question for this lesson. While it is important to honor *all* students' questions, the main question intended for this lesson is: *Which coupon will save you the most money?* Next, students should make a guess and record on their problem solving framework.

Now that the question has been identified, students in their small groups should brainstorm what they already know from the problem and what they need to know in order to answer the main question. Listen carefully for students who may begin realizing that it is necessary to know the amount of item(s) being purchased before being able to solve this problem. It is ok at this point if no student has come to this realization. The next steps will lead them to this discovery.

For students who do already realize the purhcase price is required, allow them time to think about the prices for which each coupon would be best and the item cost that would allow each coupon to be of equal value. Provide the sample items only when needed. It is likely, however, that many students will need to look at some sample items before making a decision. The four items provided were intentionally selected so that one is less that $15, another is between $15 and $25, one is exactly $25, and another is more than $25.

For students who need to work with the sample items, one option is to ask them to pretend they are buying each item. They will then need to compare the cost of the item after the discount has been applied to determine which coupon is the better deal. If time does not allow for them to compare coupons on all four items, give each group a different item and then ask the groups to share their findings in a whole-group discussion. After this discussion, the small groups can then construct their conclusions. During the work session, watch carefully for different strategies that would be beneficial to highlight in the closing.

FORMATIVE ASSESSMENT QUESTIONS

- Will both coupons always save you the same amount?
- How can we tell which coupon will save us more money?
- What restrictions apply to each coupon?
- Are you comparing the amount of the discount or the final amount of the product?

DIFFERENTIATION

Extension

Bed Bath & Beyond will accept multiple coupons in one purchase. Students who are ready to move beyond the initial problem may investigate how this policy affects the purchase of multiple items.

Intervention

Some students may be overwhelmed by comparing coupons on all four items. For these students, provide only one item at a time for them to compare. It may also be helpful to provide a chart to help them organize their work in order to make accurate comparisons.

For extra help with percentages, please open the hyperlink Intervention Table.

CLOSING/SUMMARIZER

Allow chosen groups to share their conclusions and strategies for solving the problem. After listening to the groups share, encourage all students to write a strong concluding statement that is rich in mathematical language and clearly communicates the solution.

Melissa & Doug® LAX Check! 48-Piece Floor Puzzle

The lacrosse sticks cross to block the shot as the blue team dives toward the goal! The intensity and beauty of this popular sport are captured in this thrilling moment--rendered extra-large and in vivid color on this 48-piece floor puzzle. The durable jigsaw pieces are made of extra-thick cardboard and coated with an easy-clean finish that keeps them looking like new. Imported.

Description	Available color	Price	Qty
Melissa & Doug® LAX Check! 48-Piece Floor Puzzle		$12.99 ea.	1

Bravo Sports Kryptonics 28-Inch Cruiser Board - Legend

Step up and cruise to your favorite haunts with confidence and style with the Kryptonics Legend longboard from Bravo Sports. Boasting eye-catching graphics and solid 9-ply maple construction, it features 61mm x 37mm PU injected wheels, heavy duty trucks with 12mm Risers, and ABEC 5 bearings. Measures 28" x 7.5". Imported. 30-day warranty.

Description	Available color	Price	Qty
Bravo Sports Kryptonics 28-Inch Cruiser Board - Legend		$29.99 ea.	1

Steering Wheel for iPhone and iPod Touch

Fulfill your need for speed with the Basic Steering Wheel for iPhone and iPod Touch. Whether car racing, motorcycle riding, or jet skiing, you can now maneuver your way through those interactive iPhone/iPod touch Apps with precision. This wheel gives you the feel of driving a vehicle instead of making sharp, erratic turns as you twist your iPhone or iPod Touch from side to side. Simply by connecting the iPhone or iPod Touch to its corresponding adapter then placing it in the middle of the wheel, you are ready to put the pedal to the metal and leave the competition in the dust. Steering column adds stability and balance, and an adjustable suction cup adheres to any flat surface. Includes adapters for iPhone, iPhone 3G, iPod Touch, iPod Touch 2G. Model #IP-SWS.

Description	Available color	Price	Qty
Steering Wheel for iPhone and iPod Touch	Black	$19.99 ea.	1

Sky Riders 36" Patriot Foam Glider

You'll have hours of outdoor fun soaring the skies with this Sky Riders Patriot Foam Glider. It features a huge 3-foot wingspan and can fly over 75 feet. Its aerodynamic design gives you extended air time, and is constructed from durable light-weight foam for easy use. Assembly is easy, too, and disassembles for convenient storage. Recommended for ages 4 and up. Constructed in the USA.

Description	Available color	Price	Qty
Sky Riders 36" Patriot Foam Glider		$25.00 ea.	1

Name ______________________________ Period: _____________ Date: _______________

What problem are you trying to figure out?	What guesses do you have?
What do you already know from the problem?	**What do you need to know to solve the problem?**
What should we title this lesson?	
What is your conclusion? How did you reach that conclusion?	

Adapted from http://robertkaplinsky.com/ Problem Solving Framework v7.1

Your work

Adapted from http://robertkaplinsky.com/ Problem Solving Framework v7.1

Formative Assessment Lesson: Increasing and Decreasing Quantities by a Percent

This lesson is intended to help you assess how well students are able to interpret percent increase and decrease.

STANDARDS FOR MATHEMATICAL CONTENT

Students will use ratios to solve real-world and mathematical problems.

MFAPR2. Students will recognize and represent proportional relationships between quantities.
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

STANDARDS FOR MATHEMATICAL PRACTICE

2. Reason abstractly and quantitatively. Students must make sense of quantities that are being increased or decreased by a percent and know how to represent them abstractly.

7. Look for and make use of structure. Students should understand different ways to increase (or decrease) a quantity paying attention to the structure of the expressions. For example, students should know that the total of a $25 item increased by 6% can be represented as $25(1+0.06)$, $25(1.06)$, or $25+1.50$, to name a few.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:
- Translate between percents, decimals, and fractions.
- Represent percent increase and decrease as a multiplicative relationship.
- Recognize that relationship between increases and decreases.

ESSENTIAL QUESTIONS

How do I translate between percents, decimals, and fractions?
How do I represent percent of increase and decrease as a multiplicative relationship?

LESSON COMMENTS

Lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative
The lesson, *Increasing and Decreasing Quantities by a Percent*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?lessonid=210&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the lesson can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1572

Quick Check II

STANDARDS FOR MATHEMATICAL CONTENT

Students will use ratios to solve real-world and mathematical problems.

MFAPR1. Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations.* (MGSE6.RP.3)

MFAPR2. Students will recognize and represent proportional relationships between quantities.
a. Relate proportionality to fraction equivalence and division. *For example, $\frac{3}{6}$ is equal to $\frac{4}{8}$ because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1, 2, 3; MGSE7.RP.1, 2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2, 3)

MFAPR3. Students will graph proportional relationships.
a. Interpret unit rates as slopes of graphs. (MGSE8.EE.5)
b. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. (MGSE8.EE.6)
c. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (MGSE8.EE.5)

Quick Check II

1. A biologist caught and tagged 250 fish. When she collected a sample of 75 fish, 5 of them were tagged. How many fish would she estimate are in the lake?

Approximately 3750 are in the lake.

2. A wildlife ranger caught and tagged 34 deer in his area of the national forest. He estimates that there are approximately 350 deer living in that area. If he is correct, what percent of the population did he tag?

The ranger tagged approximately 9.7% of the population.

3. A lake contains approximately 325 tagged fish. Results from several samples taken show that about 12% of the fish are tagged. Estimate the number of total fish in the lake.

There are approximately 2708 fish in the lake.

4. Maria is shopping for cheddar cheese:

Cubed cheese → 10 oz. for $4.50	***$.45 per ounce***
Block of cheese → 12 oz. for $3.84	***$.32 per ounce***
Shredded cheese → 16 oz. for $6.40	***$.40 per ounce***

Which package is the best deal? Explain your mathematical reasoning.

The block of cheese has a lower unit rate at $.32 per ounce, therefore the block of cheese is the best deal.

Question #5 from: https://www.illustrativemathematics.org/content-standards/lessons/2040

5. Emily has a coupon for 20 percent off of her purchase at the store. She finds a backpack that she likes on the discount rack. Its original price is $60 but everything on the rack comes with a 30 percent discount. Emily says:

Thirty percent and twenty percent make fifty percent so it will cost $30.

a. Is Emily correct? Explain.

It is true that 20% and 30% make 50%. But in the context of sale prices it is essential to keep track of the wholes to which these percents apply. For the backpack, the 30% discount applies to the original $60 price: 30% of $60 is 0.3×60=18 making the discount on the backback$18. So after using the coupon, the backpack price becomes $42. Emily's additional 20% coupon applies not to the original backpack price but to the discounted price of $42: 20% of $42 is $8.40. Emily would need to save an additional $12 off the $42 price in order to buy the backpack for $30 so her calculations are not correct.

b. What price will Emily pay for the backpack?

As seen in part (a), Emily's coupon lowers the discount rack price by$8.40 so she will pay 42−8.40=33.60 or $33.60.

Quick Check II

1. A biologist caught and tagged 250 fish. When she collected a sample of 75 fish, 5 of them were tagged. How many fish would she estimate are in the lake?

2. A wildlife ranger caught and tagged 34 deer in his area of the national forest. He estimates that there are approximately 350 deer living in that area. If he is correct, what percent of the population did he tag?

3. A lake contains approximately 325 tagged fish. Results from several samples taken show that about 12% of the fish are tagged. Estimate the number of total fish in the lake.

4. Maria is shopping for cheddar cheese:

Cubed cheese → 10 oz. for $4.50
Block of cheese → 12 oz. for $3.84
Shredded cheese → 16 oz. for $6.40

Which package is the best deal? Explain your mathematical reasoning.

5. Emily has a coupon for 20 percent off of her purchase at the store. She finds a backpack that she likes on the discount rack. Its original price is $60 but everything on the rack comes with a 30 percent discount. Emily says:

Thirty percent and twenty percent make fifty percent so it will cost $30.

a. Is Emily correct? Explain.

b. What price will Emily pay for the backpack?

What's My Line?

In this lesson, students will compare ratios of the sides of slope triangles on a line representing average wages of assembly line workers. Later, students will compare those wages to wages earned from two different jobs, all three wages will be represented in different ways.

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class.
Recommended time: 60-90 minutes. Recommended arrangement: individual, partners, or small groups.

STANDARDS FOR MATHEMATICAL CONTENT

MFAPR3. Students will graph proportional relationships.
a. Interpret unit rates as slopes of graphs. (MGSE8.EE.5)
b. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. (MGSE8.EE.6)
c. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (MGSE8.EE.5)

Common Misconceptions:
Students may have trouble writing an equation for the situation. They could mix up the dependent and independent variables or write the equation as an additive rather than a multiplicative relationship.
Students often confuse a coordinate pair (x, y) where the x-value comes first and the y-value is second with the slope ratio where the change in y, being in the numerator, is written first.
Students may try to count tick marks on the graph and ignore the scale.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will need to make sense of the problem in order to understand what the slope of the line represents.

6. Attend to precision. Students will need to be precise in their calculations but also in their interpretations of the height and lengths of the sides of the triangles.

7. Look for and make use of structure. Students should notice the structure of the ratios formed from different slope triangles. These ratios lead students to the understanding of a constant rate of change.

8. Look for and express regularity in repeated reasoning. In comparing the ratios of the side lengths of the triangles, students should notice that all ratios formed from the slope triangles on this line simplify to the same constant value.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Compare similar triangles formed by any two points on a line to understand slope.
- Draw conclusions about different proportional relationships represented in different ways.

MATERIALS

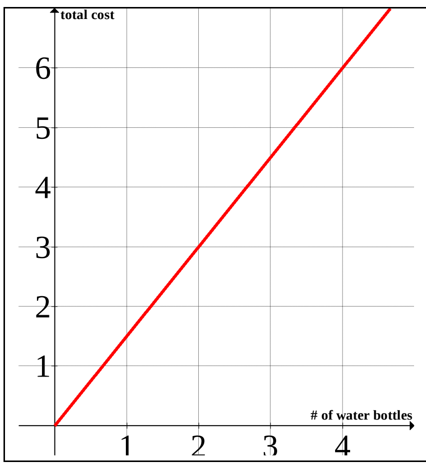
- Student lesson sheet
- Straightedge (optional)
- Graph paper (optional) http://incompetech.com/graphpaper

ESSENTIAL QUESTIONS

- How can the same mathematical idea be represented in a different way?
- Why might it be useful to look at different representations of a function?
- What is the significance of the patterns that exist between the triangles created on the graph of a proportional relationship?
- What is the relationship between the slope of a function and the unit rate?

OPENER/ACTIVATOR

The graph, equation, and table show the relationship between the number of water bottles sold (*n*) and the total cost of the water bottles (*T*).



$$T = 1.50n$$

# of water bottles, *n*	**Total cost, *T* ($)**
1	**1.50**
2	**3.00**
5	**7.50**
10	**15.00**

1. Explain how to find the unit rate in each representation. (Be sure to use precise, mathematical language.)

For each representation, the unit rate will be the y-value when the x-value is 1. On the graph, moving your finger over to 1 and then up to meet the line will show a y-value of 1.5. In the equation, the coefficient of the independent value, n, is the unit rate and can also be found by substituting 1 for n. The table shows the unit rate of 1.5 for the n value of 1.

2. While on a fieldtrip, Luis wants to spend his last $18 to buy waters for himself and his friends. Explain how Luis would determine the number of waters he can buy using the graph, the equation, and the table.

On a larger graph, Luis could move his finger up to T = 18, over to meet the line, and down to find the correct x-value. In the equation, he could substitute 18 for T and solve for n. If he would like to use the table, he would need to extend the table until T = 18 and find the corresponding x-value.

3. Explain which representation you would prefer to use if you were Luis and why.

Answers will vary and any representation can be correct if given a valid explanation.

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Lesson Directions:

Each student should receive a copy of the lesson. Part 1 may need to be chunked into two sections. If so, ask students to stop after *c* to check for understanding. The questions in *b* and *c* directly address standard 8.EE.5. It is important students make connections between the ratios of the height: base of each triangle and the slope. Also, watch carefully as students complete *d* and *e* as they may have difficulty writing an equation. For part 2, it is very important that students are able to understand proportional relationships represented multiple ways. Revisit the opener if students are still having trouble at this point.

FORMATIVE ASSESSMENT QUESTIONS

- How can you find the wage earned for one hour on the graph?
- What values do I need to find the height and the length of the base for the triangle?
- What must be done to the number of hours to get the amount of wages earned?
- How can you find the hourly rate in the equation? In the table?

DIFFERENTIATION

Extension
Ask students to research a career in which they may be interested. Find the average salary/wage per hour for the career. Use the information to create a table, equation, and a graph for the job based on a 40-hour work week. Compare and contrast the pros and cons of the chosen careers with other students.

Intervention

For students struggling to find the height and the base of the triangles, provide them a graph with those vertical and horizontal lengths given. Although students need to be able to find these values on their own, ask them to explain how they arrived at those values before asking them to write in ratios.

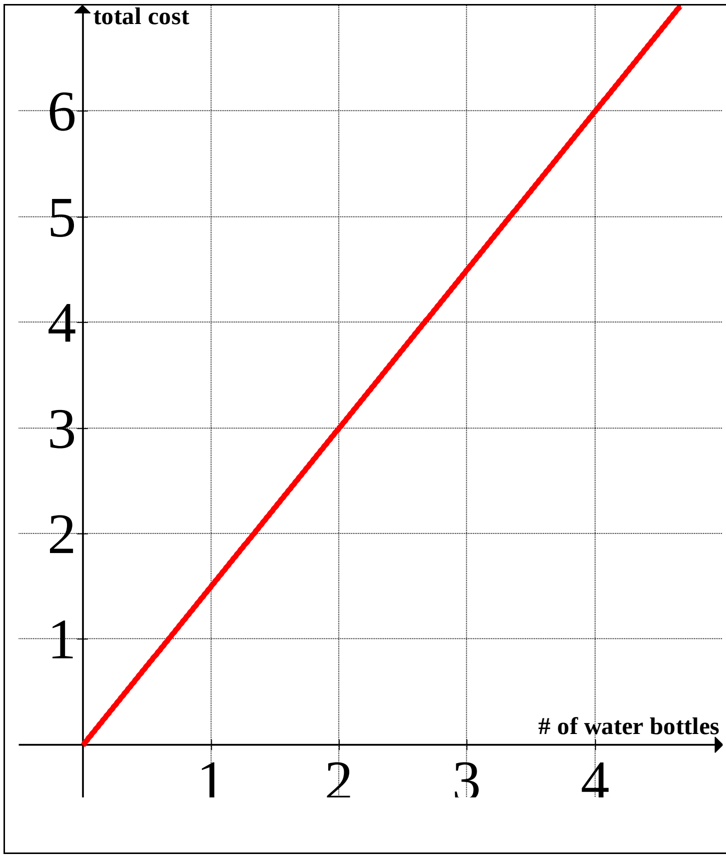
For extra help with rates of change, please open the hyperlink Intervention Table.

CLOSING/SUMMARIZER

Journal Entry
Create your own story that represents a proportional relationship. Create a graph, table, and an equation to model the situation. Lastly, explain how you know that your two variables are proportional.

OPENER/ACTIVATOR

The graph, equation, and table show the relationship between the number of water bottles sold (*n*) and the total cost of the water bottles (*T*).



$$T = 1.50n$$

# of water bottles, *n*	Total cost, *T* ($)
1	1.50
2	3.00
5	7.50
10	10.50

1. Explain how to find the unit rate in each representation. (Be sure to use precise, mathematical language).

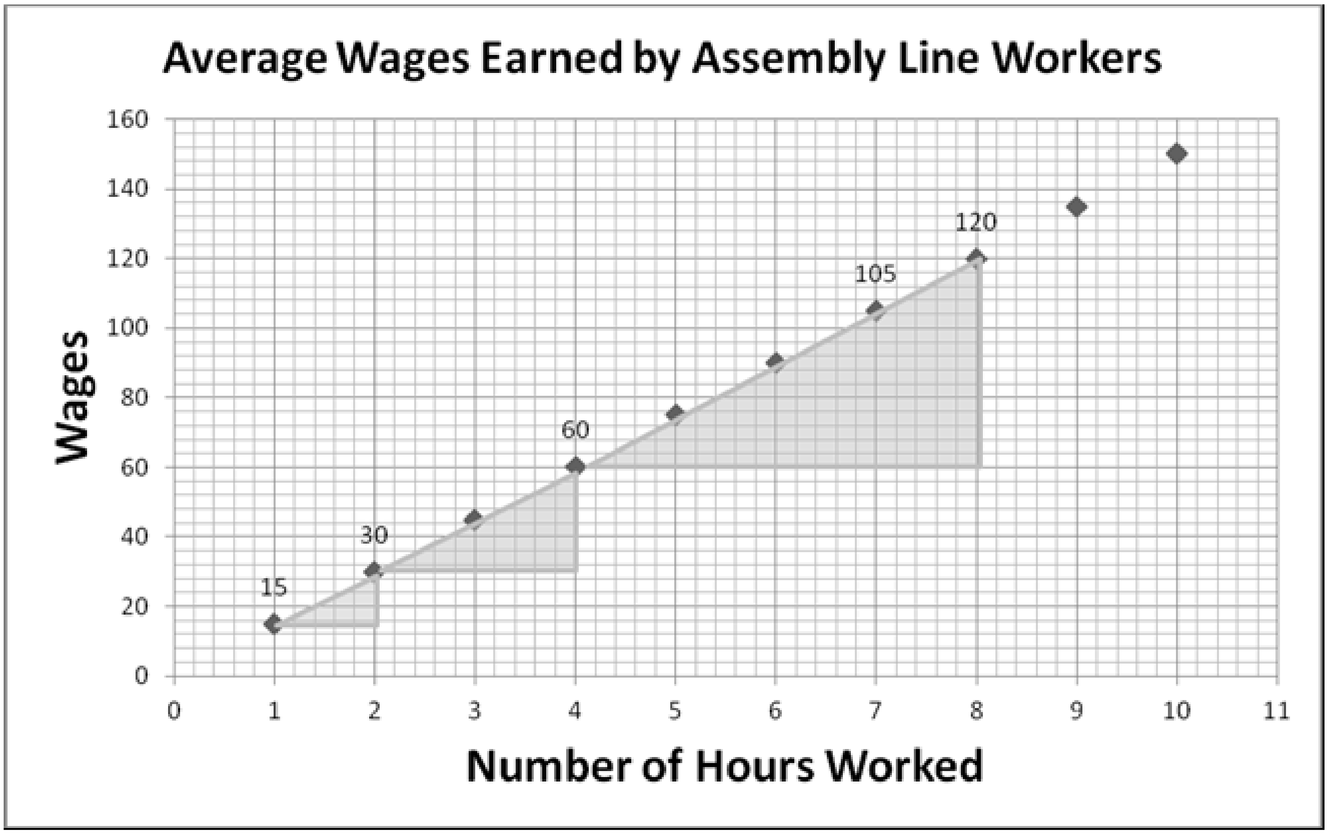
2. While on a fieldtrip, Luis wants to spend his last $18 to buy waters for himself and his friends. Explain how Luis would determine the number of waters he can buy using the graph, the equation, and the table.

3. Explain which representation you would prefer to use if you were Luis and why.

What's My Line?

Part 1: Average Wages

The data shown in the graph below reflects average wages earned by assembly line workers across the nation.



a. What hourly rate is indicated by the graph? Explain how you determined your answer.

The average assembly line worker is paid $15 per hour. The number of hours is the independent variable and the wages earned is the dependent variable. As the independent variable increases by one, the dependent variable increases by fifteen.

b. What is the ratio of the height to the base of the small, medium and large triangles? What patterns do you observe? What might account for those patterns?

$$\frac{\$15}{1\ hour} = \frac{\$30}{2\ hours} = \frac{\$60}{4\ hours}$$

All of these ratios simplify to $\frac{15}{1}$ ($15 per hour) which is the unit rate. Because a line has a constant rate of change, the ratio of the height to the base of any right triangle formed from two points on the line should simplify to $\frac{15}{1}$.

c. The slope of a line is found by forming the ratio of the change in *y* to the change in *x* between any two points on the line. What is the slope of the line formed by the data points in the graph above? Explain how you know.

$$\frac{\$15}{1\ hour} = \frac{\$30}{2\ hours} = \frac{\$60}{4\ hours}$$

$\frac{\Delta y}{\Delta x} = \frac{rise}{run} = \frac{\$15}{1\ hour}$ ***For each triangle on the graph. Therefore, the slope of the line is 15.***

Comment:
It is important that students are able to communicate the slope of the line in the context of the problem. In other words, they should not only be able to calculate that the slope is 15, but also be able to explain that $15 per hour is the average wage earned by assembly line workers.

d. Write an equation for the earnings of the average assembly line worker.

W = Wages earned and h = hours worked
W = $15h

e. According to the graph and equation, in a 40-hour week, how much will the average assembly line worker earn? How do you know?

W = $15h
W = $600
The average assembly line worker will earn $600 per week. This can be determined by substituting 40 for h in the equation W = $15h. You can also use the existing point for 8 hours on the graph and multiply this by 5 days to arrive at the same solution of $600 per week.

f. With changes in the economy, the average wages can change. How would a decrease of $2 in the average wage change the equation and graph?

In the equation, the coefficient of the independent variable, h, would change from $15 to $13. On the graph, the slope of the line for this function would decrease so the line would be less steep. Both lines would still start at (0, 0) because if no hours are worked no wages are earned.

g. How would a $5 increase in the average wage change the equation and graph?

In the equation, the coefficient of the independent variable, h, would change from $15 to $20. On the graph, the slope of the line for this function would increase so the line would be steeper.

Part 2: Comparing Wages
The average hourly wages of a plumber and a machinist are presented below. Using the information provided for each of the different jobs, compare the average hourly wage of these jobs with that of the assembly line worker in Part 1. For each job, include the hourly wage (unit rate), wages earned for 40 hours, and number of hours of work needed to earn $100.

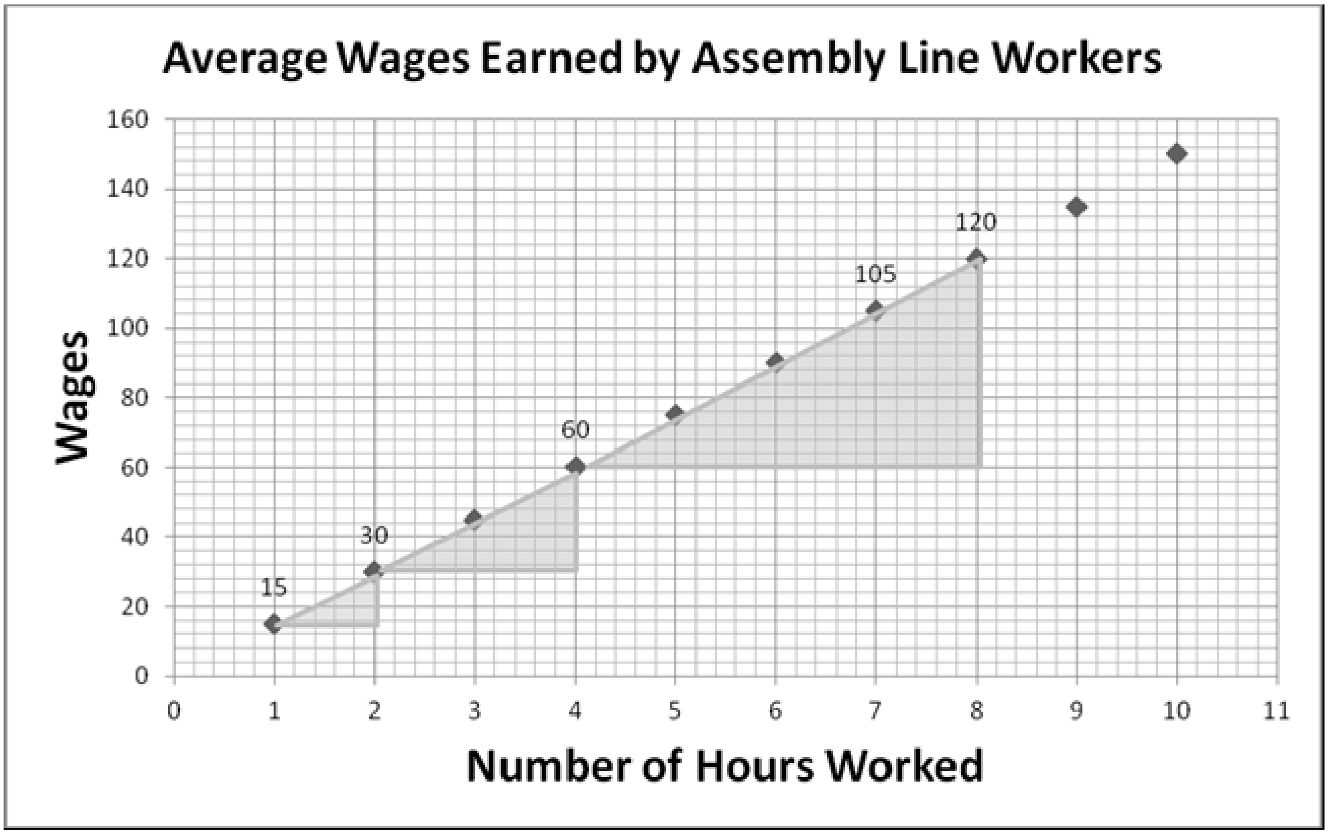
<table>
<tr><th>Plumber</th><th>Machinist</th></tr>
<tr><td>W = $20h, where W represents the wages earned and h represents the hours worked.</td><td>
<table>
<tr><th>Hours worked</th><th>Wages Earned</th></tr>
<tr><td>1</td><td>$ 18.50</td></tr>
<tr><td>2</td><td>$ 37.00</td></tr>
<tr><td>3</td><td>$ 55.50</td></tr>
<tr><td>4</td><td>$ 74.00</td></tr>
<tr><td>5</td><td>$ 92.50</td></tr>
<tr><td>6</td><td>$ 111.00</td></tr>
<tr><td>7</td><td>$ 129.50</td></tr>
<tr><td>8</td><td>$ 148.00</td></tr>
<tr><td>9</td><td>$ 166.50</td></tr>
<tr><td>10</td><td>$ 185.00</td></tr>
</table>
</td></tr>
</table>

Both the plumber and the machinist earn a higher average hourly rate than the assembly line worker. The plumber earns $20 per hour and the machinist earns $18.50 per hour compared to the $15 per hour earned by the assembly line worker. In a 40-hour week, the plumber would earn $800, the machinist $740, and the assembly line worker $600. It would take the plumber 5 hours to earn $100, the machinist 5.4 hours and the assembly line worker 6.67 hours.

What's My Line?

Part 1: Average Wages

The data shown in the graph below reflects average wages earned by assembly line workers across the nation.



a. What hourly rate is indicated by the graph? Explain how you determined your answer.

b. What is the ratio of the height to the base of the small, medium and large triangles? What patterns do you observe? What might account for those patterns?

c. The slope of a line is found by forming the ratio of the change in *y* to the change in *x* between any two points on the line. What is the slope of the line formed by the data points in the graph above? Explain how you know.

d. Write an equation for the earnings of the average assembly line worker.

e. According to the graph and equation, in a 40-hour week, how much will the average assembly line worker earn? How do you know?

f. With changes in the economy, the average wages can change. How would a decrease of $2 in the average change the equation and graph?

g. How would a $5 increase in the average change the equation and graph?

Part 2: Comparing Wages
The average hourly wages of a plumber and a machinist are presented below. Using the information provided for each of the different jobs, compare the average hourly wage of these jobs with that of the assembly line worker in Part 1. For each job, include the hourly wage (unit rate), wages earned for 40 hours, and number of hours of work needed to earn $100.

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<tr><th>Plumber</th><th>Machinist</th></tr>
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</table></td></tr>
</table>

Nate & Natalie's Walk

Students will use proportional reasoning to compare the distance that a brother and sister walk. In the performance lesson, students will use tables and graphs to represent a proportional relationship.

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class.
Recommended time: 60-90 minutes. Recommended arrangement: partners or small groups.

STANDARD FOR MATHEMATICAL CONTENT

MFAPR2. Students will recognize and represent proportional relationships between quantities.

a. Relate proportionality to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1, 2, 3; MGSE7.RP.1, 2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2, 3)

Common Misconceptions:
Several misconceptions are associated with graphing, in general. For example, students may:

- *Confuse the x and y axes.*
- *Not remember which to plot first, x or y.*
- *Mislabel axes in a particular context.*
- *Additionally, students may struggle with setting up and/or solving the proportion.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will need to decide the differences in the scenarios and solve the problems accordingly.

2. Reason abstractly and quantitatively. Students will go from analyzing diagrams to using tables and graphs.

4. Model with mathematics. Students will set-up models for comparisons.

8. Look for and express regularity in repeated reasoning. Students will find similarities and differences in the data.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Set up tables.
- Create a graph from a table of values.
- Interpret relationships between data using rates.

MATERIALS

- *Walk to the Movies* activity sheet
- Student lesson sheet

ESSENTIAL QUESTIONS

- How can we make predictions from graphs?
- What makes a relationship "proportional"? How can I tell if a proportional relationship exists?
- How can I represent a proportional relationship?

OPENER/ACTIVATOR

A video of the Disney marathon can be found at http://www.rundisney.com/results/

Students are asked if they know their walking/running rates in marathons. If so, these rates can be discussed. If not, runners' rates in the 2012 Olympics can be found at http://www.olympic.org/olympic-results/london-2012/athletics/marathon-m

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Prior to doing this performance lesson, students should understand that graphing is a way to visually represent ratios and proportional relationships. This visual is a tool that can be used to determine the reasonableness of an equation and to draw conclusions about proportional relationships.

Lesson Directions:

Hand out the lesson and allow students to work individually for 3-5 minutes without intervening. Circulate around the classroom to get an idea about what strategies are being used to solve the problem.

After 3-5 minutes, students may work with their partner or in their small groups. Support students' problem-solving by:

- Asking questions related to the mathematical ideas, problem-solving strategies, and connections between representations.
- asking students to explain their thinking and reasoning.

CLOSING/SUMMARIZER

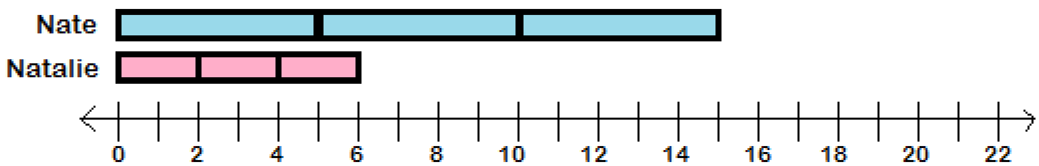
Students can calculate their own walking rates in the hallway. Activities can be found at http://illuminations.nctm.org/Lesson.aspx?id=4133

Nate and Natalie's Walk

Nate and his sister Natalie are walking around the track at school at a steady rate. Nate walks 5 feet in 2 seconds while Natalie walks 2 feet in the same amount of time.

1. Draw a diagram or picture that represents Nate and Natalie's walk around the track.

Students may use a variety of representations. For example, bar models or a number line as shown below could be used to represent the walk around the track.

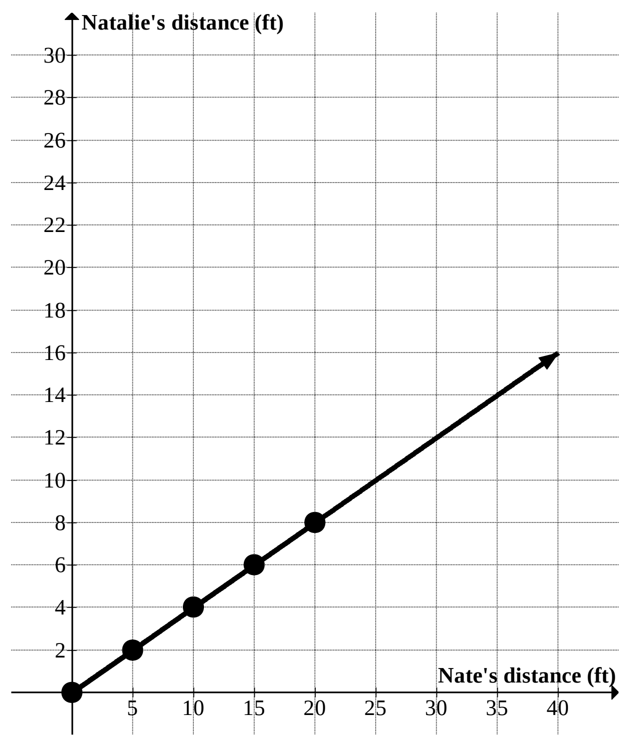


Questions to encourage thinking:

- ***Can you explain what your diagram shows about Nate and Natalie's walk?***
- ***Can you think of a different way to model the situation?***

2. Set up a table and create a graph to represent this situation. Let the x-axis represent the distance Nate walks and the y-axis represent the distance Natalie walks.

Nate's Distance (x)	*Natalie's Distance (y)*
0	*0*
5	*2*
10	*4*
15	*6*
20	*8*



3. What patterns do you see in the table? Explain the pattern. Express this as an equation.

Students should be able to explain that for every 5 feet Nate walks, Natalie walks 2 feet. So the ratio of Nate's distance to Natalie's distance in feet is 5:2. The equation that represents the situation is $y = \frac{2}{5}x$.

4. How do you read the graph? Explain what the coordinate (20, 8) means in the context of Nate and Natalie's walk?

For any point on the graph, the y-value represents the distance that Natalie can walk in the same amount of time that Nate can walk the distance represented in the x-value. The coordinate means that when Nate has walked 20 feet, Natalie has walked 8 feet.

5. When Nate walks 45 feet, how far will Natalie walk? Explain in writing or show how you found your answer.

Students may use the equation, table, or graph as a way to answer the problem. They may also set up a proportion to solve the problem. For example, $\frac{5}{2} = \frac{45}{x}$. When solved the answer is 18. Natalie will have walked 18 feet when Nate has walked 45 feet.

<u>Comment</u>:
For further discussion, ask, "Can you predict how far Natalie will walk if Nate walks 1000 feet?" This discussion should focus on the most efficient methods for solving the problem (equation or proportion) versus using a table or a graph.

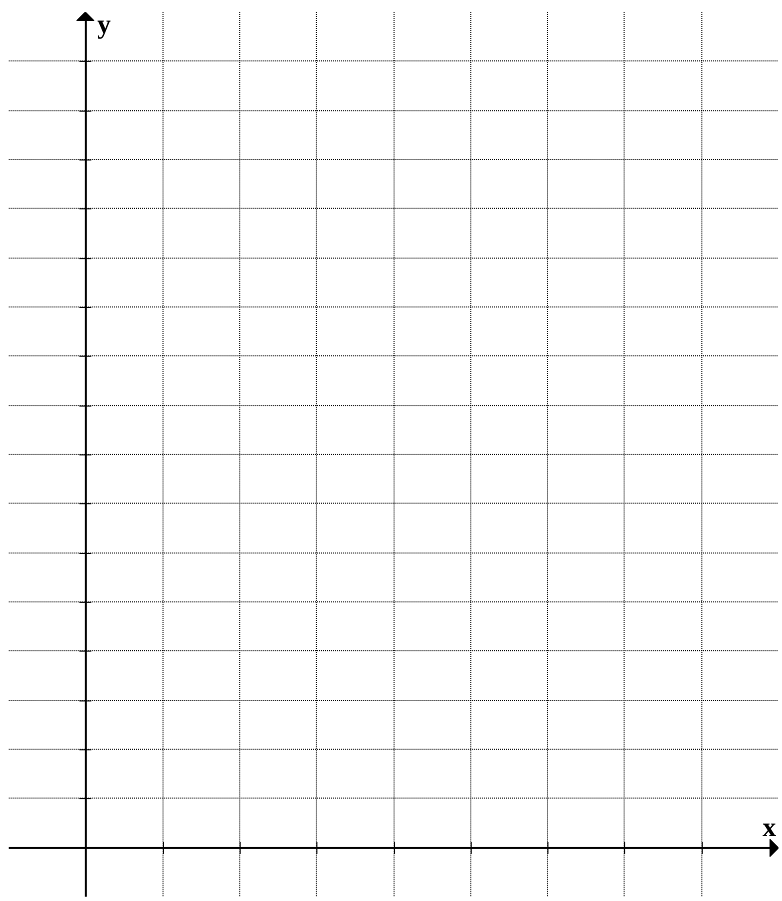
Another question to ask may be, "If the line that represents the relationship between the distance Nate and Natalie walk has a slope of 1, what does this mean?". Likewise, you could then ask students to explain what it would mean for a line to have a slope greater than 1 and a line to have a slope less than 1.

Nate & Natalie's Walk

Nate and his sister Natalie are walking around the track at school at a steady rate. Nate walks 5 feet in 2 seconds while Natalie walks 2 feet in the same amount of time.
.

1. Draw a diagram or picture that represents Nate and Natalie's walk around the track.

2. Set up a table and create a graph to represent this situation. Let the *x*-axis represent the distance Nate walks and the *y*-axis represent the distance Natalie walks.



3. What patterns do you see in the table? Explain the pattern. Express this as an equation.

4. How do you read the graph? Explain what the coordinate (20, 8) means in the context of Nate and Natalie's walk?

5. When Nate walks 45 feet, how far will Natalie walk? Explain in writing or show how you found your answer.

Rectangle Families

Adapted from Van De Walle, J. (2006). *Teaching student-centered mathematics: Grades 5-8* (Vol.3) Boston: Pearson Education, Inc.

In this lesson, students will sort rectangles into "families" by similarity and represent the relationship of the length to width on a graph, in a table, and in an equation.

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class.
Recommended time: 90-120 minutes. Recommended arrangement: partners or small groups.

STANDARDS FOR MATHEMATICAL CONTENT

MFAPR1. Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations.* (MGSE6.RP.3)

MFAPR2. Students will recognize and represent proportional relationships between quantities.
a. . Relate proportionality to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

MFAPR3. Students will graph proportional relationships.
a. Interpret unit rates as slopes of graphs. (MGSE8.EE.5)
b. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. (MGSE8.EE.6)
c. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (MGSE8.EE.5)

Common Misconceptions:
Students may believe that the width and length of the rectangles cannot be interchanged.
Students may continue to see an additive relationship rather than a multiplicative one. In other words, for the skinny rectangle, they may only notice that the width (or length) increases by 4 as the length (or width) increases by 1 rather than seeing that the width is the length multiplied by four.
Students may struggle with writing an equation to model each family of rectangles. Ask them to refer back to their mathematical sentences and replace the length and width with variables.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students must make sense of the problem in order to properly arrange the rectangles.

2. Reason abstractly and quantitatively. Students must use quantitative reasoning to see the relationship between width and length of the rectangle families.

6. Attend to precision. Students must use correct mathematical language as they communicate their thinking to their peers.

7. Look for and make use of structure. Students must notice a pattern in the lengths and widths of rectangles to understand the proportional relationship.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Recognize a proportional relationship in a table and in a graph.
- Identify similar figures by determining if corresponding sides are proportional.
- Find additional similar figures using the constant of proportionality.
- Write an equation to represent a proportional relationship.

MATERIALS

- Student lesson sheet
- Two pages of rectangles (copied single side)
- Scissors
- Colored pencils
- Rulers

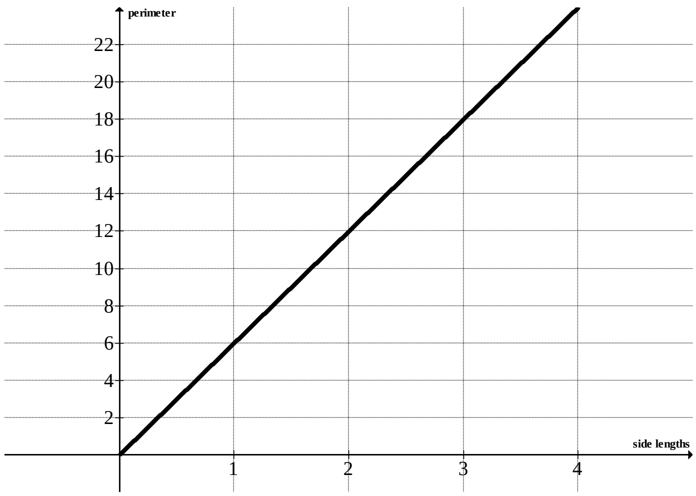
ESSENTIAL QUESTIONS

- What are some ways to prove that 2 rectangles are similar?
- How can graphs, tables, and equations help me to see proprotional relationships?

OPENER/ACTIVATOR

Determine whether the variables in each representation are in a proportional relationship or not. Justify your mathematical thinking.

1.



The side lengths of a regular hexagon and its perimeter are displayed in the graph.

2.

# of students	Total cost of fieldtrip ($)
5	50
10	100
15	150
20	200
25	250
30	300

The table shows the relationship between the number of students in a class and the total cost of a fieldtrip for the class.

3.

$$S = 15n + 45$$

The equation represents the total amount of money Max has in his savings account (S) after mowing n number of lawns.

This activator will give students the chance to review the characteristics of proportional relationships. ***Students should recognize that both #1 and #2 have 0 for the input and output and the relationships have a constant rate of change. (Students should be expected to explain this mathematical thinking in the context of the problem.) The 45 in number 3, however, indicates that Max must have already had $45 in his savings account prior to mowing lawns.***

If students are still struggling with these problems, show all representations for one of the three problems so that students can see multiple representations of the same problem.

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Lesson Directions:

Each student should receive a copy of the lesson, and each small group should receive two rectangles sheets and scissors. The lesson leads students through the steps but you may need to point out that the rectangles are separated by bold lines. They may be unsure of rectangle B which is a 1 x 1 rectangle and C which is a 2 x 1.

This is a somewhat lengthy lesson so it will be helpful to break it up into smaller chunks. Ask students to complete 1-7 first. (While walking around to all the groups, you may notice that some are not including the new rectangles in #6.) At this point, have a quick whole-group discussion about #7. Ask students to briefly share the numeric patterns that they see from the tables. This may also be a good time to point out that the rectangles can be arranged two different ways; the lengths and widths could be switched. As students work in their groups, check to make sure that students are being consistent. If the skinny rectangles are arranged with the long side horizontally, the same should be true for the other rectangles that are not square.

For the next chunk, ask the students to complete #8-11. Watch out for students on #10 who may try to graph all the data. Questions 10 and 11 are only in reference to the square data—all other data is plotted in #12. Before moving on, check for students' understanding of making another rectangle to fit in the family. This will be an important topic to discuss in the closing.

Lastly, students should complete the remainder of the lesson. Pay close attention to their answers on #12 and also to whether or not they are able to write equations for each rectangle family.

FORMATIVE ASSESSMENT QUESTIONS

- What relationship do you notice between the length and the width of each rectangle in a family?
- What method did you use to create the new rectangles in each family?
- How can you see the relationship between the lengths and widths of the rectangles in the table and in the graph?

DIFFERENTIATION

Extension
Give students grid paper and ask them to create 3 similar, but not congruent, right triangles. They must then explain how they know they are similar.

Intervention

For students who may be overwhelmed by beginning with 14 rectangles, start them out with only two from each family. Then, as they become more comfortable with the process, introduce more rectangles for them to sort.
If students are having trouble writing an equation, ask "How many times longer is one rectangle than another?" Ask them to show where they see this value on the graph to connect the slope of the line to the constant of proportionality identified in the table of values.

CLOSING/SUMMARIZER

The focus of the closing should depend largely on how well students were able to write equations for their rectangle families. If most students were able to write correct equations in #15 and solve for the widths in 16, then assign this closing activity:

Journal Entry
Thoroughly answer the following essential question using precise mathematical language:

How can graphs, tables, and equations help me to see proprotional relationships?

If most students were not able to produce correct equations in #15, assign this alternate closing:

Alternate Closing

Below is a table and graph for a new family of rectangles.

Length	Width
1	5
2	10
3	15
4	20
5	25

Determine which of the equations model the relationship between the length and width of the rectangle family. Then, explain how you can tell from the graph, table, and the equation(s) that the length and width are in a proportional relationship.

$W = L + 1$	$L = 5W$	$W = L + 5$
$W = 5L$	$L = \frac{W}{5}$	$L = W + 5$

Rectangle Families

1. Cut out the set of rectangles.
2. Sort the 14 rectangles into three "families" where all the members have the same shape, but differ in size.
3. In each family, arrange the rectangles from smallest to largest. What patterns do you see within each family?

   Answers will vary. Some students may notice that one family consists of squares, one family consists of "skinny" rectangles, which the other falls somewhere in the middle. Other students might already notice patterns in the dimensions and answer from a numerical perspective.

4. Stack each family of rectangles in order of size with the largest on the bottom. Arrange the rectangles so that each one in every family shares the bottom and the left edge. What new observations can you make about each of the families?

   Answers will vary. Students may notice that for each rectangle in the family (CGIKN), the width is twice the length (or vice versa), in family (BDFHM), the length and width are equal, and in family (AEJL), the width is four times the length.

5. Describe a method for finding the dimensions of another rectangle that would fit in a family.
   Answers will vary. Possible solutions include:
   For rectangles in family (CGIKN), multiply the length by 2.
   For rectangles in family (BDFHM), the length and width are equal.
   For rectangles in family (AEJL), multiply the length by 4.

6. Record, by family, the width and length of each rectangle. Then write in the dimensions for **a new rectangle** that fits each family.

Rectangle	Length	Width
C	***1***	***2***
G	***2***	***4***
I	***3***	***6***
K	***4***	***8***
N	***5***	***10***
	6	***12***

Rectangle	Length	Width
B	***1***	***1***
D	***3***	***3***
F	***5***	***5***
H	***7***	***7***
M	***10***	***10***
	8	***8***

Rectangle	Length	Width
A	***1***	***4***
E	***2***	***8***
J	***3***	***12***
L	***4***	***16***
	5	***20***

Note to teacher: Students may interchange length and width so that in the skinny rectangle, for instance, the length is 4 and the width is 1. This too is correct but notice that it will affect the answers to the graph and the constant of proportionality.

7. List any numeric patterns you see in your chart in step 6.

Answers will vary. Possible responses include:
Family (CGIKN) – as length increases by 1, width increases by 2
Family (BDFHM) – as length increase by 2, width increases by 2
Family (AEJL) – as length increases by 1, width increases by 4

8. Make the chart into a series of ratios by placing a fraction bar between the width and length pairs. Write the ratios in simplest form.

Rectangle	Ratio	Simplified
C	***1/2***	***1/2***
G	***2/4***	***1/2***
I	***3/6***	***1/2***
K	***4/8***	***1/2***
N	***5/10***	***1/2***
	6/12	***1/2***

Rectangle	Ratio	Simplified
B	***1/1***	***1***
D	***3/3***	***1***
F	***5/5***	***1***
H	***7/7***	***1***
M	***10/10***	***1***
	8/8	***1***

Rectangle	Ratio	Simplified
A	***1/4***	***1/4***
E	***2/8***	***1/4***
J	***3/12***	***1/4***
L	***4/16***	***1/4***
	5/20	***1/4***

9. What does the pattern in the ratio tell you about the rectangles in that family?

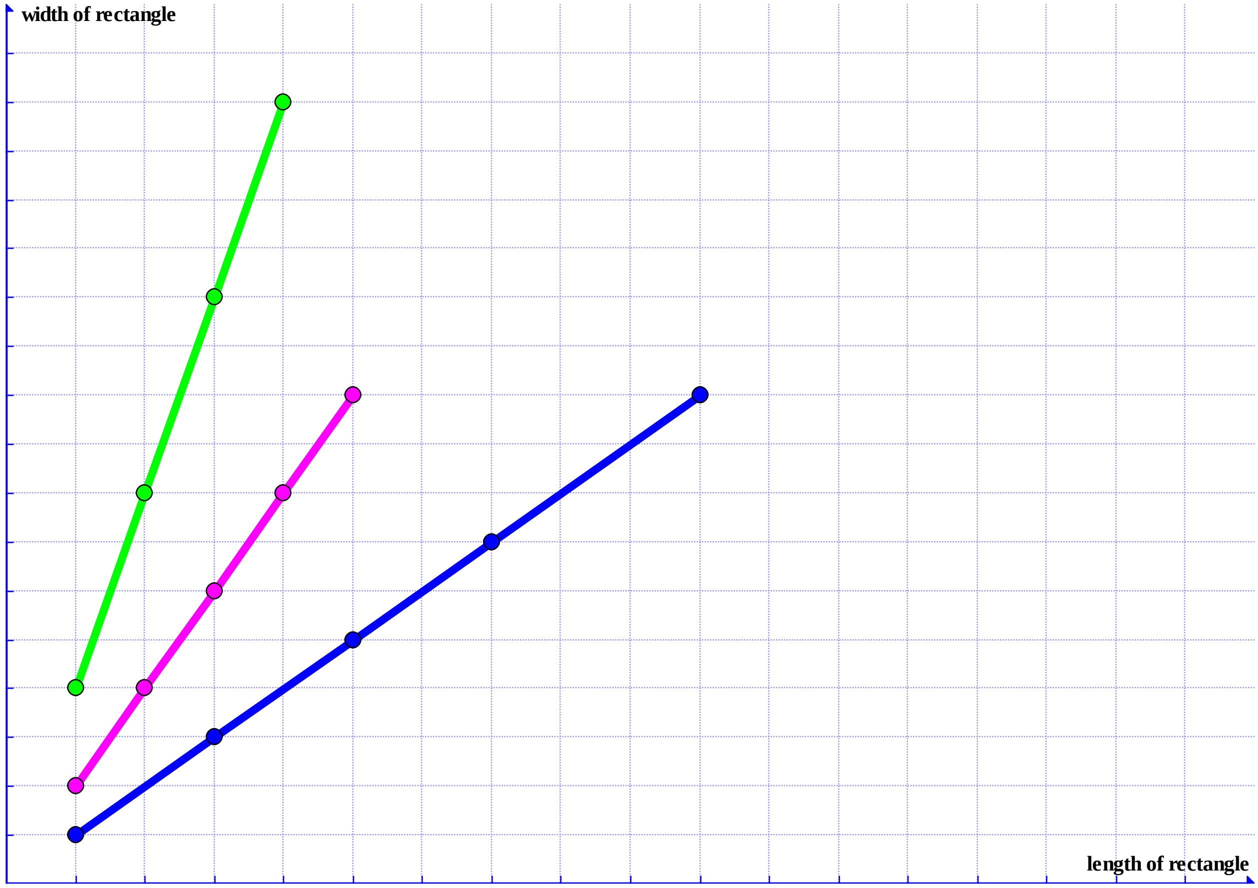
Square rectangle: ***Length and width are equal***

Skinny rectangle: ***Width is four times the length***

Other rectangle: ***Width is twice the length***

10. Using the data from the "square" rectangles, plot a graph of length vs. width. What do you notice about the plotted points?

Answers will vary. Possible responses include:
The points form a line.
As the length increases by 1, the width increases by 1.



11. Use a ruler or other straight edge and a colored pencil to connect the points. How could you use what you drew to help you find the dimensions for another member of this rectangle family? Why does your method work?

Answers will vary. Students should notice that they can find the dimensions for additional rectangles by extending the line with their straightedge. They may also notice that they can add additional points by continuing to move up 1 and to the right 1. These methods work because for this family of rectangles, the length and width are always equal. Continuing the pattern, either by extending the line or adding additional points, creates new dimensions that are equal in length and width.

12. Using different colored pencils, plot the data for each of the other families of rectangles. Do the new rectangles you created in step 6 fit the family patterns? How do you know?

    Answers will vary. If students choose dimensions in step 6 that follow the pattern in each family, then they should see that these points lie on the line (assuming their dimensions fit on the graph provided). They can explain their thinking using either the graph or the numerical pattern of the dimensions.

13. What is the constant of proportionality for each family of rectangles? How can you *see* each constant of proportionality in the table and the graph?

    Square: 1 ***Skinny:*** $\frac{1}{4}$ ***"Other":*** $\frac{1}{2}$

    Answers will vary:
    On the graph, the constant of proportionality can be seen by using $\frac{rise}{run}$ ***from one point to any other point of the line. In the table, the constant of proportionality is the ratio of length to width for each family.***

14. How could you use slope triangles to prove that your new rectangles fit the family patterns?

    Answers will vary. At this point, we want students to understand that a constant rate of change means that the ratio of the height:base of any right triangle formed from two points on a line is constant. For students struggling with this concept, ask them to compare the triangles formed by connecting two consecutive points and two non-consecutive points.

15. How would you determine how wide to make a rectangle to fit a specific family if you know how long the rectangle is supposed to be? Develop a method for each family.

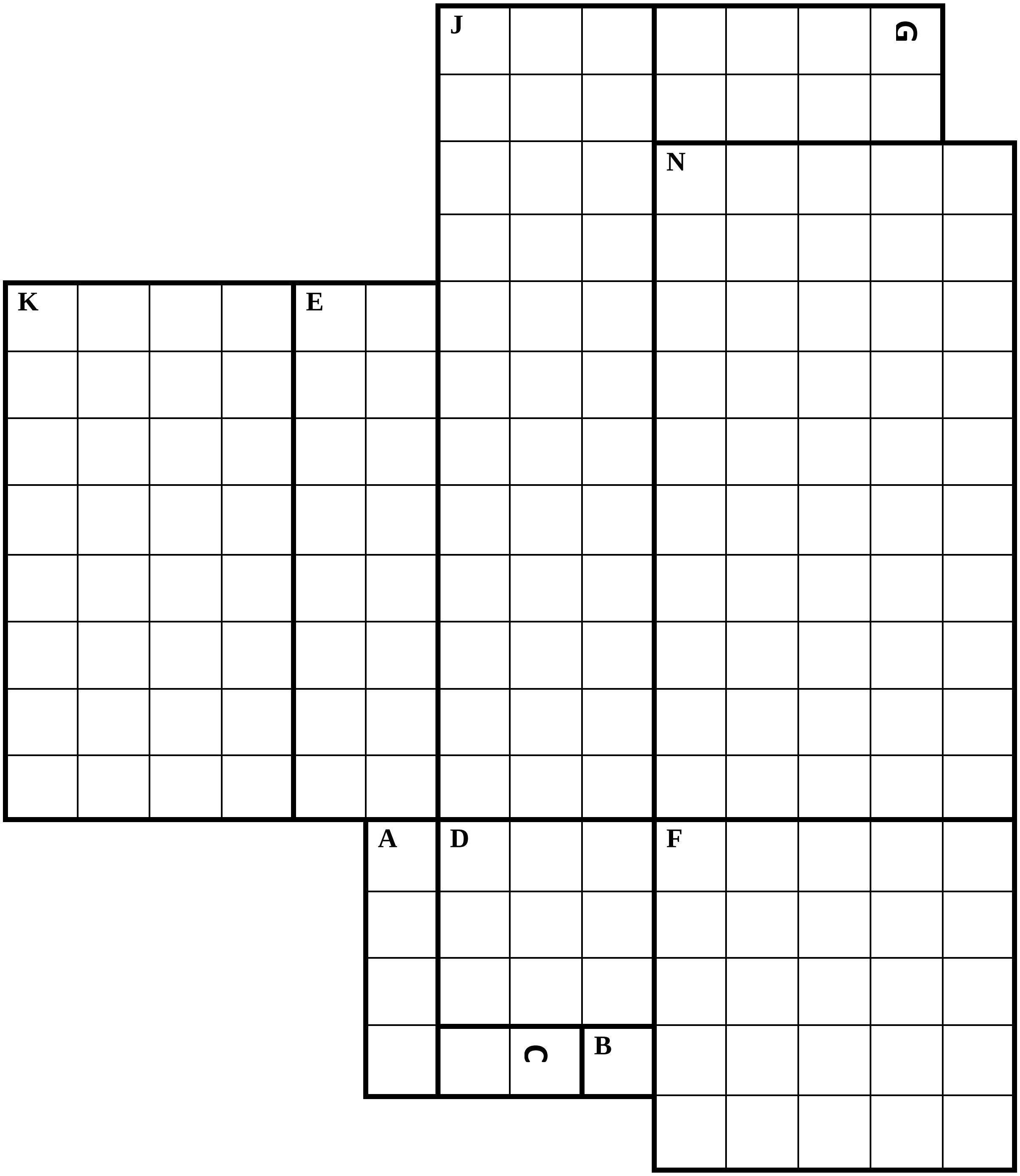
    Answers will vary. Possible responses include:
    For the family of square rectangles, the length and width are equal.
    For the family of skinny rectangles, take the given length and multiply it by four to get the width.
    For the other family of rectangles, take the given length and multiply it by two to get the width.

16. Write mathematical sentences to record the methods you developed for finding the width when you know the length. (Example: W = L / 8.)

Square	*Skinny*	*Other*
W = L	*W = 4L*	*W = 2L*

17. For each family, determine the widths of rectangles with lengths of 9 units. Use your graph to check your solutions.

Square	*Skinny*	*Other*
W = L *W = 9*	*W = 4L* *W = 4(9)* *W = 36*	*W = 2L* *W = 2(9)* *W = 18*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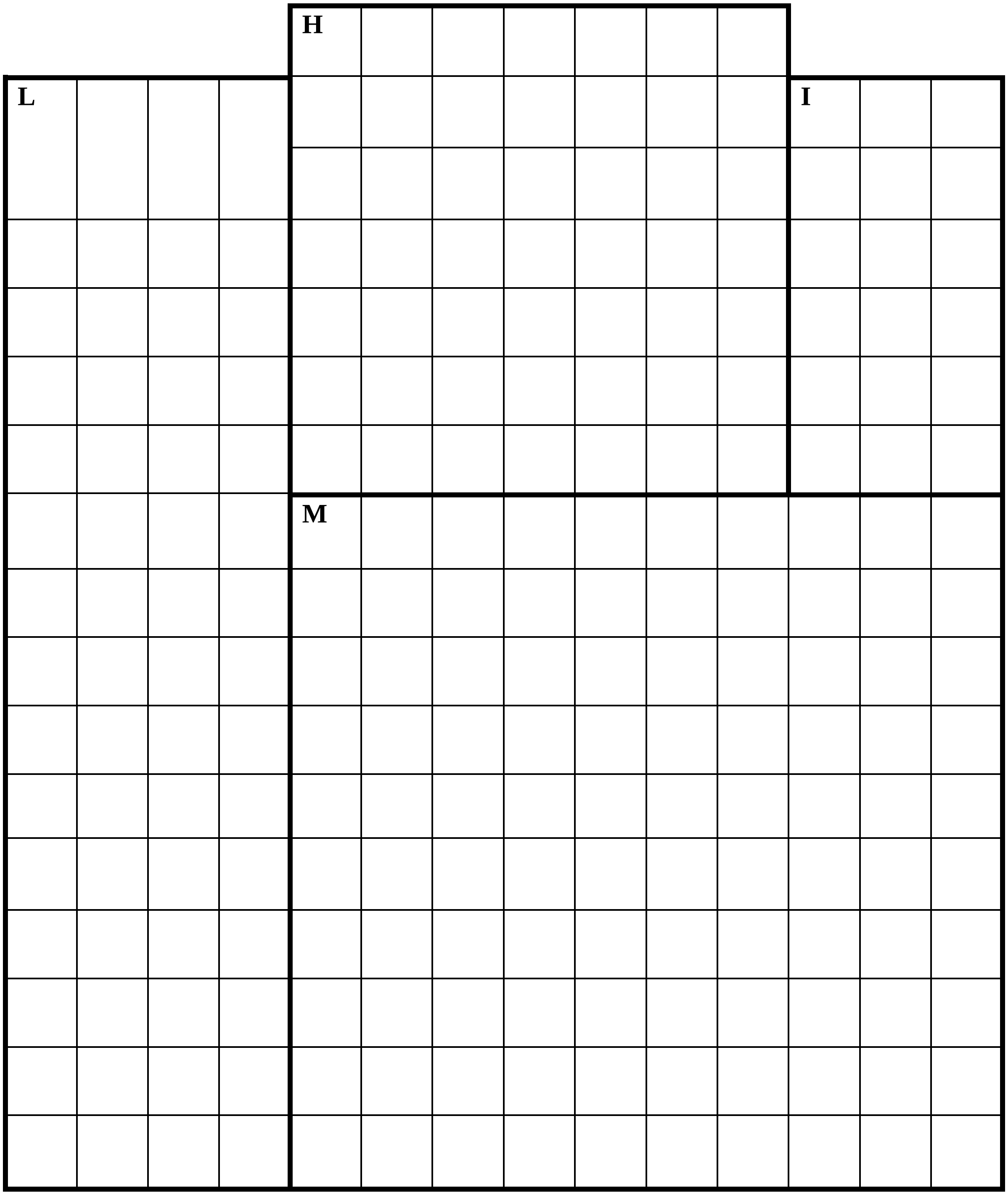
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Rectangle Families

1. Cut out the set of rectangles.
2. Sort the 14 rectangles into three "families" where all the members have the same shape, but differ in size.
3. In each family, arrange the rectangles from smallest too largest. What patterns do you see within each family?

4. Stack each family of rectangles in order of size with the largest on the bottom. Arrange the rectangles so that each one in every family shares the bottom and the left edge. What new observations can you make about each of the families?

5. Describe a method for finding the dimensions of another rectangle that would fit in a family.

6. Record, by family, the width and length of each rectangle. Then write in the dimensions for **a new rectangle** that fits each family.

Rectangle	Length	Width

Rectangle	Length	Width

Rectangle	Length	Width

7. List any numeric patterns you see in your chart in step 6.

8. Make the chart into a series of ratios by placing a fraction bar between the width and length pairs. Write the ratios in simplest form.

Rectangle	Ratio	Simplified

Rectangle	Ratio	Simplified

Rectangle	Ratio	Simplified

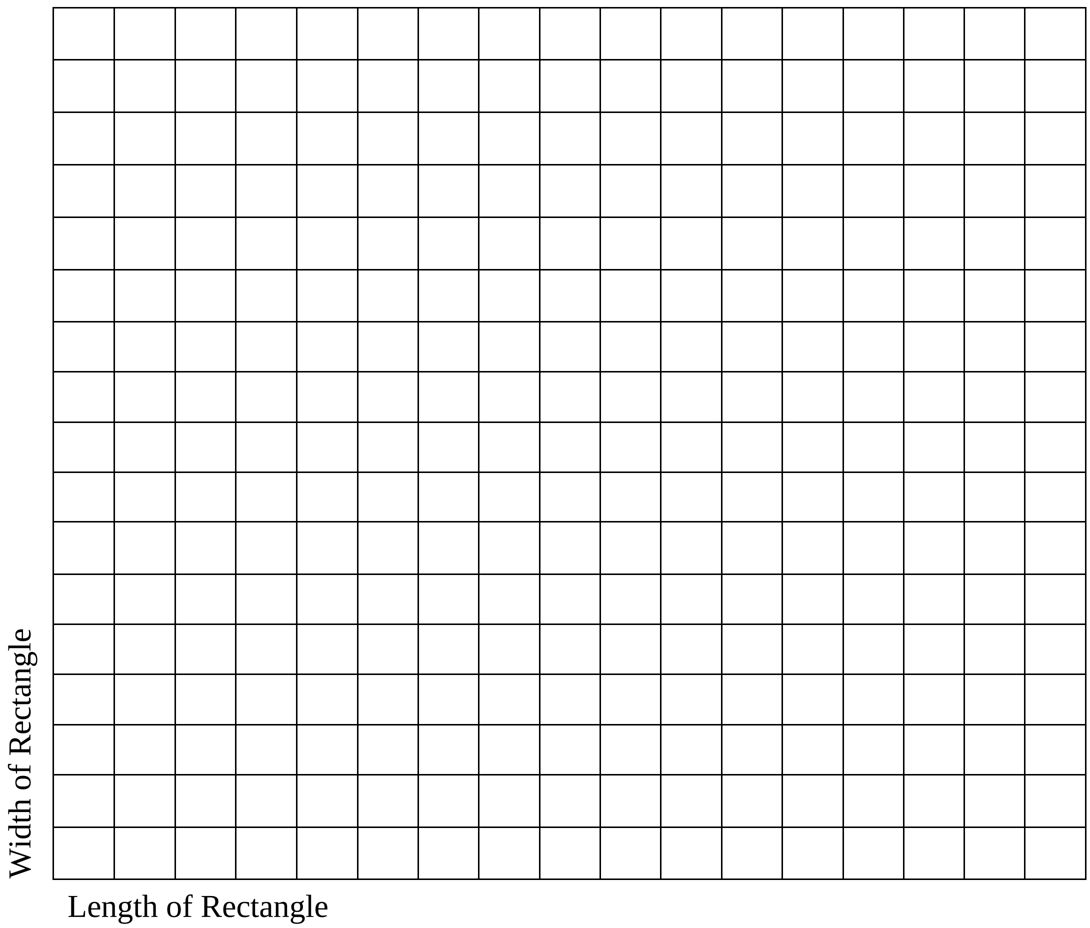
9. What does the pattern in the ratio tell you about the rectangles in that family?

Square rectangle: ____

Skinny rectangle: ____

Other rectangle: ____

10. Using the data from the “square” rectangles, plot a graph of length vs. width. What do you notice about the plotted points?



11. Use a ruler or other straight edge and a colored pencil to connect the points. How could you use what you drew to help you find the dimensions for another member of this rectangle family? Why does your method work?

12. Using different colored pencils, plot the data for each of the other families of rectangles. Do the new rectangles you created in step 6 fit the family patterns? How do you know?

13. What is the constant of proportionality for each family of rectangles? How can you *see* each constant of proportionality in the table and the graph?

14. How could you use slope triangles to prove that your new rectangles fit the family patterns?

15. How would you determine how wide to make a rectangle to fit a specific family if you know how long the rectangle is supposed to be? Develop a method for each family.

16. Write mathematical sentences to record the methods you developed for finding the width when you know the length. (Example: W = L / 8.)

17. For each family, determine the widths of rectangles with lengths of 9 units. Use your graph to check your solutions.

Quick Check III

STANDARDS FOR MATHEMATICAL CONTENT

Students will use ratios to solve real-world and mathematical problems.

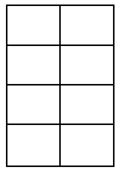
MFAPR1. Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations.* (MGSE6.RP.3)

MFAPR2. Students will recognize and represent proportional relationships between quantities.
a. . Relate proportionality to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

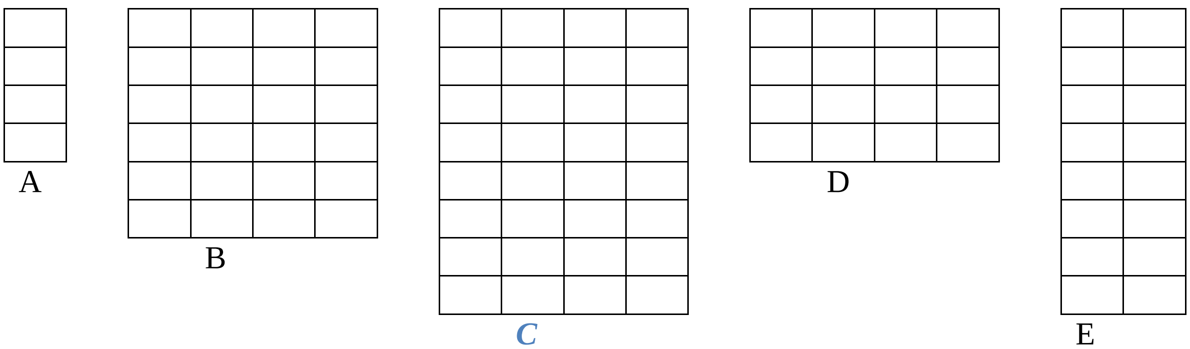
MFAPR3. Students will graph proportional relationships.
a. Interpret unit rates as slopes of graphs. (MGSE8.EE.5)
b. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. (MGSE8.EE.6)
c. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (MGSE8.EE.5)

Quick Check III

1. Given rectangle:



Which of the following rectangles is similar to the given rectangle?



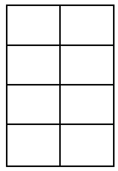
2. The ratio of the sides of a rectangle is $\frac{2}{3}$.

Which of the following could be the ratio of the sides of a similar rectangle?

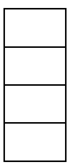
A. $\frac{4}{9}$ B. $\frac{4}{3}$ C. $\frac{2}{6}$ D. $\frac{4}{5}$ ***E.*** $\frac{6}{9}$

Quick Check III

1. Given rectangle:



Which of the following rectangles is similar to the given rectangle?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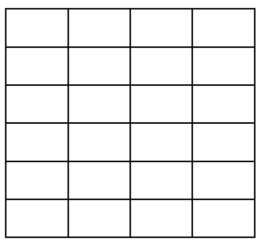
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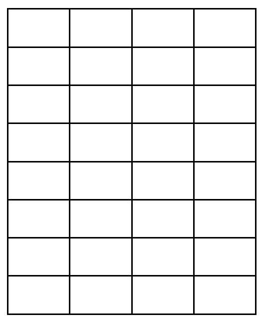
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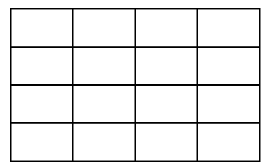
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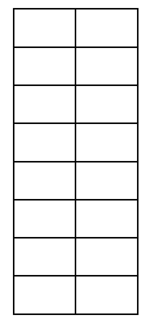

E

2. The ratio of the sides of a rectangle is $\frac{2}{3}$.

Which of the following could be the ratio of the sides of a similar rectangle?

A. $\frac{4}{9}$ B. $\frac{4}{3}$ C. $\frac{2}{6}$ D. $\frac{4}{5}$ E. $\frac{6}{9}$

"Illustrative" Review

To review the concepts of percentages, slopes, and rates, Illustrative Mathematics has many problem solving sets. Descriptions and links are provided below.

Teacher Note: These problems can be used each day of the module as opening problems or tickets-out-the-door.

1. **Rates of Change**: https://www.illustrativemathematics.org/content-standards/6/RP/A/1/tasks/1181

2. **Constant Rate:** https://www.illustrativemathematics.org/content-standards/6/RP/A/2/tasks/1175

3. **Unit Rates**: https://www.illustrativemathematics.org/content-standards/6/RP/A/2/tasks/1611

4. **Rates of Change:** https://www.illustrativemathematics.org/content-standards/6/RP/A/3/tasks/134

5. **Percentages:** https://www.illustrativemathematics.org/content-standards/6/RP/A/3/tasks/118

6. **Percentages:** https://www.illustrativemathematics.org/content-standards/6/RP/A/3/tasks/1982

7. **Percentages**: https://www.illustrativemathematics.org/content-standards/6/RP/A/3/tasks/899

8. **Percentages**: https://www.illustrativemathematics.org/content-standards/6/RP/A/3/tasks/54

9. **Rate**s: https://www.illustrativemathematics.org/content-standards/7/RP/A/1/tasks/82

10. **Rates and Ratios**: https://www.illustrativemathematics.org/content-standards/7/RP/A/1/tasks/470

11. **Rates with Fraction**s: https://www.illustrativemathematics.org/content-standards/7/RP/A/1/tasks/828

12. **Rates and Graphs:** https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/104

13. **Comparing Rates:** https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/180

14. **Rates and Graphs:** https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/181
15. **Rates and Graphs**: https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/1178

16. **Comparing Rates with Graphs**: https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/1186

17. **Proportions and Graphs:** https://www.illustrativemathematics.org/content-standards/7/RP/A/2/tasks/1983

18**. Interest** https://www.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/1550

19**. Percent Chang**e; https://www.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/130

20. **Percentages with Fractions**: https://www.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/121

21. **Double Percentages:** https://www.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/2040

22. **Percentages:** https://www.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/105

23. **Percentages and Rates**: https://www.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/1330

24**. Comparing Rates of Graphs**: https://www.illustrativemathematics.org/content-standards/8/EE/B/5/tasks/55

25. **Graphs to Equations**: https://www.illustrativemathematics.org/content-standards/8/EE/B/5/tasks/57

26**. Tables to Graphs:** https://www.illustrativemathematics.org/content-standards/8/EE/B/5/tasks/184

27. **Slopes as Rates of Change:** https://www.illustrativemathematics.org/content-standards/8/EE/B/6/tasks/1537

Nana's Chocolate Milk

Lesson adapted from: http://threeacts.mrmeyer.com/nana/

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class.
Recommended time: 45-60 minutes. Recommended arrangement: partners or small groups.

STANDARDS FOR MATHEMATICAL CONTENT

Students will use ratios to solve real-world and mathematical problems.

MFAPR1. Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations.* (MGSE6.RP.3)
MFAPR2. Students will recognize and represent proportional relationships between quantities.
a. . Relate proportionality to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students must make sense of the problem in order to devise a plan for solving it.

2. Reason abstractly and quantitatively. Students must use quantitative reasoning to create a representation of the problem and to understand how the quantities relate.

3. Construct viable arguments and critique the reasoning of others. Throughout the problem, students will need to communicate their mathematical thinking to their peers as they evaluate their own and their peers' understanding of the problem, the model they create, and the reasonableness of their answer.

4. Model with mathematics. Students will make a model to represent the situation.

5. Use appropriate tools strategically. Students will need to decide on an appropriate tool for their model, e.g., tables of values, tape diagrams, bar models, double number line diagrams.

6. Attend to precision. Students must use correct mathematical language as they communicate their thinking to their peers.

7. Look for and make use of structure. In order to create an accurate model, students must notice patterns in order to make equivalent ratios.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Apply their understanding of proportional relationships to solve a problem.
- Use a model to find equivalent ratios.

MATERIALS

- Video and other information provided: http://threeacts.mrmeyer.com/nana/
- 3-Act Recording Sheet (attached)

ESSENTIAL QUESTIONS

In order to maintain a student-inquiry-based approach to this lesson, it may be beneficial to wait until Act 2 to share the Essential Questions with your students. By doing this, students will be allowed the opportunity to be very creative with their thinking in Act 1. By sharing the essential questions in Act 2, you will be able to narrow the focus of inquiry so that the outcome results in student learning directly related to the content standards aligned with this lesson.

- How can we find equivalent ratios?
- How can a model be used to find and organize equivalent ratios?

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION

Background Knowledge:

This lesson follows the Three-Act Math Task format originally developed by Dan Meyer. More information on this type of lesson may be found at http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/. A Three-Act Task is a whole-group mathematics lesson consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information, along with guidelines for Three-Act Tasks, may be found in each Comprehensive Course Overview.

Task Directions:

Act 1 – Whole Group - Pose the conflict and introduce students to the scenario by showing Act 1 picture. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/)
"Introduce the central conflict of your story/lesson clearly, visually, viscerally, using as few words as possible."

Show the Act 1 video to students: http://threeacts.mrmeyer.com/nana/act1/act1.mov
Give students a copy of the 3-Act Recording Sheet.
Ask students what they noticed in the video, what they wonder about, and what questions they have about what they saw in the video. Facilitate a think-pair-share so that students have an opportunity to talk with each other before sharing questions with the whole group.
Share and record students' questions. The teacher may need to guide students so that the questions generated are math-related.

Anticipated questions students may ask and wish to answer:

- * What can we do to fix Nana's chocolate milk?
- How much chocolate milk with the glass hold?

**Main question(s) to be investigated*

Once the class has decided on the main question to investigate, students should record the question on the recording sheet. Then, ask the students to estimate answers to their questions (think-pair-share). Students will write their best estimate, then write two more estimates – one that is too low and one that is too high so that they establish a range in which the solution should occur. Students should plot their three estimates on an empty number line.

Important note: Although students will only investigate the main question(s) for this lesson, it is important for the teacher to not ignore student-generated questions. Additional questions may be answered after they've found a solution to the main question, or as homework or extra projects.

Act 2 – Student Exploration - Provide additional information as students work toward solutions to their questions. **(Dan Meyer** http://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/) **"The protagonist/student overcomes obstacles, looks for resources, and develops new tools."**

During Act 2, students decide on the facts, tools, and other information needed to answer the question(s) (from Act1). When students decide what they need to solve the problem, they should ask for those things. It is pivotal to the problem-solving process that students decide what is needed without being given the information up front.

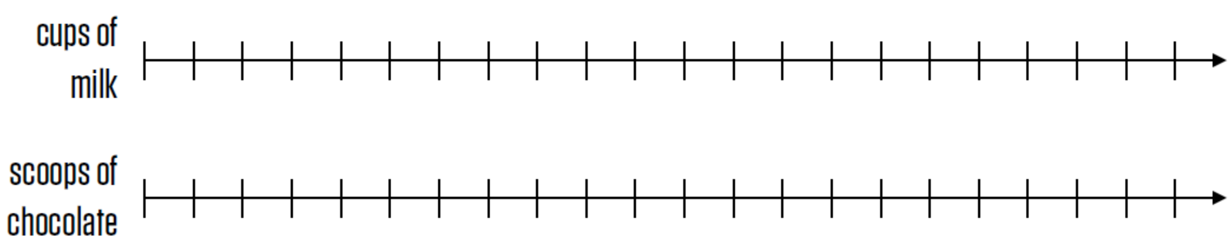
Students may wish to use manipulatives to model the ratio of ingredients in the lesson. Colored tiles and/or Cuisenaire rods could be made available to students who wish to begin with the concrete representation. Other students may feel comfortable beginning with a table of values, double-number line, or a tape diagram.

The teacher provides guidance as needed during this phase. Some groups might need scaffolds to guide them. The teacher should question groups who seem to be moving in the wrong direction or might not know where to begin. Questioning is an effective strategy that can be used, with questions such as:

- What is the problem you are trying to solve?
- What do you think affects the situation?
- Can you explain what you've done so far?
- What strategies are you using?
- What assumptions are you making?
- What tools or models may help you?
- Why is that true?
- Does that make sense?
- Do the quantities have a multiplicative or additive relationship?

Students should be encouraged to find as many different solutions as possible.

<u>Additional Resource for Act 2</u>



Act 3 – Whole Group – Share solutions and strategies.

Students present their solutions and strategies and compare them.
Lead discussion to compare these, asking questions such as:

- How reasonable was your estimate?
- Which strategy was most efficient?
- Can you think of another method that might have worked?
- What might you do differently next time?

Revisit initial student questions that weren't answered.

FORMATIVE ASSESSMENT QUESTIONS

- How might you use the double number line to represent the amount of milk and chocolate in the cup?
- What organizational strategies did you use?

DIFFERENTIATION

Extension
Show students the sequel video with eggs and flour. Ask them to continue through the same process.

Intervention

Encourage students to begin with the concrete representation using colored tiles or Cuisenaire rods. Ask them to model the relationship between the 2 ingredients. Then, ask them to show how the model would change for different amounts of milk. An alternative would be to use grid paper to sketch a bar model of the ratio of the 2 ingredients and extend for greater amounts of milk.

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question: ____

Estimate the result of the main question. Explain.
Place an estimate that is too high and too low on the number line
Low estimate *Place an "x" where your estimate belongs* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:_______________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.