Georgia
Standards of Excellence
Curriculum Frameworks

Mathematics

GSE Foundations of Algebra

Module 4: Equations and Inequalities

Georgia Department of Education

Richard Woods, Georgia's School Superintendent

"Educating Georgia's Future"

Table of Contents

FOUNDATIONS OF ALGEBRA REVISION SUMMARY

The Foundations of Algebra course has been revised based on feedback from teachers across the state. The following are changes made during the current revision cycle:

- Each module assessment has been revised to address alignment to module content, reading demand within the questions, and accessibility to the assessments by Foundations of Algebra teachers.
- All module assessments, as well as, the pre- and posttest for the course will now be available in GOFAR at the teacher level along with a more robust teacher's edition featuring commentary along with the assessment items.
- All modules now contain "Quick Checks" that will provide information on mastery of the content at pivotal points in the module. Both teacher and student versions of the "Quick Checks" will be accessible within the module.
- A "Materials List" can be found immediately after this page in each module. The list provides teachers with materials that are needed for each lesson in that module.
- A complete professional learning series with episodes devoted to the "big ideas" of each module and strategies for effective use of manipulatives will be featured on the Math Resources and Professional Learning page at https://www.gadoe.org/Curriculum-Instruction-and-Assessment/Curriculum-and-Instruction/Pages/Mathematics.aspx.
- Additional support such as Module Analysis Tables may be found on the Foundations of Algebra page on the High School Math Wiki at http://ccgpsmathematics9-10.wikispaces.com/Foundations+of+Algebra. This Module Analysis Table is NOT designed to be followed as a "to do list" but merely as ideas based on feedback from teachers of the course and professional learning that has been provided within school systems across Georgia.

MATERIALS LIST

Lesson	Materials
1. The Variable Machine	• Two strips of lined notebook paper per student; one 5 cm wide and one 3 cm wide • Transparent tape • Student handout – Cracking the Code Activity Sheet
2. Analyzing Solutions of Equations and Inequalities	• Student handout – Partner Problems • Mini whiteboards (optional)
3. Set It Up	• Student handout
4. Let's Open a Business	• Business Lesson Cards • Copies of additional practice
5. Solving Equations using Bar Diagrams	• Pencil • Paper • Tape • Learnzillion videos (core lesson and guided practice)
6. Deconstructing Word Problems	• Pencil • Paper • Student handout
7. Steps to Solving an Equation (FAL)	• FAL pre-assessment • FAL post-assessment • Card sets for each pair (or small group) • Mini whiteboards • Chart paper or poster board • Glue • Scissors (if cards sets are not pre-cut)
8. T.V. Time and Video Games	• Student handout – T.V. Time and Video Game • Index cards with expression and symbols listed in opening card sort
9. When is it Not Equal?	• Student handout
10. Quick Check I	• Student handout
11. "Illustrative" Check I	• Student handout
12. Yogurt Packaging	• Copy of lesson • Various empty yogurt containers • Paper • Tape • Scissors (used to construct a yogurt tub)

Lesson	Materials
13. Don't Sink My Battleship!	• Battleship game cards • Student handout – Grid City Spring Festival • Graph paper or ruled chart paper
14. Acting Out	• Student handout – Acting Out • Colored pencils • Compass • String • Graph paper
15. Literal Equations	• Projector • Laptop • PowerPoint - Literal Equations • 35 index cards with letters and operations on them
16. Free Throw Percentages	• Colored pencils • Straight edge • Graphing calculator (optional) • Graph paper (http://incompetech.com/graphpaper) • Student handout
17. Stacking Cups	• Student handout • Videos
18. Planning a Party	• Student handout • Graph paper • Calculators (optional)
19. Field Day	• Stop watch • Tape measure (such as those used by track and field team) to measure classroom length • Colored pencils • Straightedge • Graphing calculator (optional) • Graph paper • Student handout
20. Quick Check II	• Student copies
21. "Illustrative" Check II	• Student copies
22. Is it Cheaper to Pay Monthly or Annually?	• Price charts and graphs from website • Pictures of Disney or Universal characters

OVERVIEW

In this module students will:

- Determine if an equation or inequality is appropriate for a given situation.
- Solve mathematical and real-world problems with equations.
- Represent real-world situations using inequalities.
- Interpret the solutions to equations and inequalities.
- Represent the solutions to inequalities on a number line.
- Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane.
- Interpret units in the context of the problem.
- Convert units of measure in order to solve problems.
- Use units to evaluate the appropriateness of the solutions when solving a multi-step problem.
- Choose the appropriate units for a specific formula and interpret the meaning of the unit in that context.
- Choose and interpret both the scale and the origin in graphs and data displays.
- Create equations in two or more variables to represent relationships between quantities.
- Graph equations in two variables on a coordinate plane and label the axes and scales.
- Understand the solution to a system of equations is the point of intersection when the equations are graphed.
- Understand the solution to a system of equations contains the values that satisfy both equations.
- Write and use a system of equations and/or inequalities to solve a real-world problem.
- Estimate the solution for a system of equations by graphing.
- Solve multi-variable formulas or literal equations for a specific variable.

Overall, teachers should emphasize the concepts of variables, equations, and inequalities throughout the entire module, assessing and monitoring how students' ideas about these three concepts change as they progress through the lessons. There should be an explicit emphasis about using properties of operations (and not necessarily the mechanics of the algebra), reminding students that they are applying the same properties previously learned to these algebraic scenarios.

A note on classroom rituals, routines, and timing of lessons: Students enter Module 4 having had extensive practice in using effective classroom rituals and routines. To that end, the module writers provide suggestions for groupings of students and lesson debriefing activities for many of the lessons but also recommend that teachers use their judgment in determining specific ways to elicit evidence of student learning and create a classroom culture of productive discourse. In addition, the recommended timing for each lesson includes the discussion of the suggested additional practice items.

STANDARDS FOR MATHEMATICAL CONTENT

Students will solve, interpret, and create linear models using equations and inequalities.

MFAEI1. Students will create and solve equations and inequalities in one variable.

a. Use variables to represent an unknown number in a specified set (conceptual understanding of a variable). (MGSE6.EE.2, 5, 6)
b. Explain each step in solving simple equations and inequalities using the equality properties of numbers. (MGSE9-12.A.REI.1)
c. Construct viable arguments to justify the solutions and methods of solving equations and inequalities. (MGSE9-12.A.REI.1)
d. Represent and find solutions graphically.
e. Use variables to solve real-world and mathematical problems. (MGSE6.EE.7, MGSE7.EE.4)

MFAEI2. Students will use units as a way to understand problems and guide the solutions of multi-step problems.

a. Choose and interpret units in formulas. (MGSE9-12.N.Q.1)
b. Choose and interpret graphs and data displays, including scale and comparisons of data. (MGSE3.MD.3, MGSE9-12.N.Q.1)
c. Graph points in all four quadrants of the coordinate plane. (MGSE6.NS.8)

MFAEI3. Students will create algebraic models in two variables.

a. Create an algebraic model from a context using systems of two equations. (MGSE6.EE.6, MGSE8.EE.8, MGSE9-12.A.CED.2)
b. Find approximate solutions using technology to graph, construct tables of values and find successive approximations. (MGSE9-12.A.REI.10, 11)
c. Represent solutions to systems of equations graphically or by using a table of values. (MGSE6.EE5; MSGE7.EE.3;MGSE8.EE.8, MGSE9-12.A.CED.2)
d. Analyze the reasonableness of the solutions of systems of equations within a given context. (MGSE6.EE.5, 6, MGSE7.EE.4)

MFAEI4. Students will solve literal equations.

a. Solve for any variable in a multi-variable equation. (MGSE6.EE.9, MGSE9-12.A.REI.3)
b. Rearrange formulas to highlight a particular variable using the same reasoning as in solving equations. *For example, solve for the base in* $A = \frac{1}{2}bh$. (MGSE9-12.A.CED.4)

STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them. High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively. High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others. High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics. High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically. High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision. High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure. By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see

$5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8. Look for and express regularity in repeated reasoning. High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.
The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. **Students who do not have an understanding of a topic may rely on procedures too heavily.** Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. **In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.**

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

*****Mathematical Practices 1 and 6 should be evident in EVERY lesson*****

ENDURING UNDERSTANDINGS

- Relate and compare different representations for a relationship.
- Use values from specified sets to make an equation or inequality true.
- Develop an initial conceptual understanding of different uses of variables.
- Understand that numbers in ordered pairs indicate locations in quadrants of the coordinate plane.
- Recognize that variables can be used to represent numbers in many types of mathematical problems.
- Understand the difference between an expression and an equation.
- Write and solve one-step and multi-step equations consisting of all rational numbers.
- Recognize that mathematical and real-world situations may have more than one solution.
- Understand the differences and similarities between equations and inequalities.
- Recognize situations that require two or more equations to be satisfied simultaneously.
- Understand that there are several methods for solving systems of equations.
- Create equations in two or more variables to represent relationships between quantities.
- Graph equations in two variables on a coordinate plane and label the axes and scales.
- Write and use a system of equations and/or inequalities to solve a real world problem.
- Solve multi-variable formulas or literal equations for a specific variable in a linear expression.

ESSENTIAL QUESTIONS

- What strategies can I use to help me understand and represent real situations using expressions, equations and inequalities?
- When is graphing on the coordinate plane helpful?
- How can we represent values using variables?
- How can I tell the difference between an expression, an equation, and an inequality?
- How can I write, interpret, and manipulate algebraic expressions, equations, and inequalities and use them to solve problems?
- How can I rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations?
- How can I translate a problem situation into a system of equations?
- What does the solution to a system tell me about the answer to a problem situation?

SELECTED TERMS AND SYMBOLS

The following terms and symbols are utilized in this module and are often misunderstood. These concepts are not an exhaustive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them. **The terms below are for teacher reference only and are not to be memorized by the students.** Teachers should present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, and numbers.

- **Addition Property of Equality**
- **Algebraic Expression**
- **Coefficient**
- **Coordinates**
- **Division Property of Equality**
- **Equation**
- **Inequality**
- **Multiplication Property of Equality**
- **Origin**
- **Quadrant**
- **Simultaneous Equations**
- **Solution**
- **Substitution**
- **Subtraction Property of Equality**
- **System of Linear Equations**
- **Variable**
- **x-axis**
- **x-coordinate**
- **y-axis**
- **y-coordinate**

The websites below are interactive and include a math glossary suitable for high school students. **Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the GADOE frameworks.**

http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary.
http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

INTERVENTION TABLE

Lesson Name	Name of Intervention	Snapshot of Summary or Student "I Can" Statement	Book, Page, or Link
The Variable Machine	Balancing Acts	Solve problems that can be modeled with algebraic equations or expressions. Students are required to describe patterns and relationships using letters to represent variables.	Balancing Acts
Analyzing Solutions of Equations and Inequalities	The Equal Sign Again	Develop an understanding of the symbols used to express mathematical ideas and to communicate ideas to others.	The Equal Sign Again
Set it Up	Set It Up	Relate tables, graphs, and equations to linear relationships found in number and spatial patterns.	Set It Up
Solving Equations using Bar Diagrams	Unknowns & Variables: Solving One Step Equations	Form and solve simple linear equations.	Unknowns & Variables: Solving One Step Equations
Deconstructing Word Problems	Writing Expressions	Use symbolic notation to record simple word phrases and explain what symbol notation says should be done to the numbers.	Writing Expressions
T.V. Time and Video Games	Dividing Candy by Multiplying	Develop understanding of symbols for, and operations of multiplication and division, of their inverse relationship, and of how to use these operations in problem solving situations. (Sessions 1 and 2)	Dividing Candy by Multiplying
Don't Sink My Battleship!	Visual Patterns By the Book	Connect members of sequential patterns with their ordinal position and use tables, graphs, and diagrams to find relationships between successive elements of number and spatial patterns Relate tables, graphs, and equations	Visual Patterns By the Book

Literal Equations	What's My Line?	Form and solve simple linear equations. Use graphs, tables, and rules, to describe linear relationships found in number and spatial patterns	What's My Line?
Free Throw Percentages	Cara's Candles	Solve systems of equations	Cara's Candles
Stacking Cups	Cara's Candles	Solve systems of equations	Cara's Candles
Planning a Party	Cara's Candles	Solve systems of equations	Cara's Candles
Field Day	Cara's Candles	Solve systems of equations	Cara's Candles

SCAFFOLDED INSTRUCTIONAL LESSONS

	Lesson Type/ *Grouping Strategy*	Content Addressed	Standard(s)
The Variable Machine	Constructing Lesson *Individual/Partner*	Introducing the use of variables	MFAEI1 a, e
Analyzing Solutions of Equations and Inequalities	Constructing Lesson *Individual/Partner*	Solving equations and inequalities by inspection; understanding the difference between equality and inequality	MFAEI1 a, e
Set It Up	Performance Lesson *Individual/Partner*	Setting up and solving one step equations	MFAEI1 a, e
Let's Open a Business	Performance Lesson *Partner/Small Group*	Using variables, equations and expressions in real-world scenarios	MFAEI1 a, e
Solving Equations using Bar Diagrams	Constructing Lesson *Individual/Partner*	Using bar models to solve equations	MFAEI1 b, e
Deconstructing Word Problems	Learning Lesson *Individual/Partner*	Representing and solving equations in context	MFAEI1 a, e
Steps to Solving an Equation (FAL)	Formative Assessment Lesson *Partner/Small Group*	Using variables to represent values; using equations to solve problems in context	MFAEI1 a, b, c
T.V. Time and Video Games	Learning Lesson *Individual/Partner*	Writing inequality statements to describe members of a group	MFAEI1 b, c, d
When is it Not Equal?	Performance Lesson *Individual/Partner*	Setting up inequalities	MFAEI1 b, d, e
Quick Check I	Formative Assessment *Individual*	Create and solve equations and inequalities in one variable.	MFAEI1a-e
Yogurt Packaging	Achieve CCSS-CTE Classroom Lesson	Converting units in order to solve problems	MFAEI2a
Don't Sink My Battleship!	Constructing Lesson *Partner/Small Group*	Graphing ordered pairs in the coordinate system; using coordinates as locations	MFAEI2 b, c

	Lesson Type/ *Grouping Strategy*	Content Addressed	Standard(s)
Acting Out	Scaffolding Lesson *Individual/Partner*	Modeling and writing an equation in one variable; representing constraints as inequalities	MFAEI1 a, e; MFAEI2 a, b, c
Literal Equations	Learning Lesson *Individual/Partner*	Defining literal equations and using them to determine values	MFAEI4 a, b
Free Throw Percentages	Learning Lesson *Individual/Partner*	Using a system of linear equations to solve a real world problem	MFAEI3 a, b, c, d
Stacking Cups	Learning Lesson *Partner/Small Group*	Exploring systems of equations to solve real world problems	MFAEI3 a, b, c, d
Planning a Party	Performance Lesson *Individual/Partner*	Using a system of linear equations to solve a real world problem	MFAEI3 a, b, c, d
Field Day	Performance Lesson *Individual/Partner*	Using system of linear equations to solve a real world problem	MFAEI3 a, b, c, d
Quick Check II	Formative Assessment *Individual*	Creating and using a system of linear equations to solve a real world problem	MFAEI3 a, b, c, d
Is it Cheaper to Pay Monthly or Annually?	Culminating Lesson *Individual/Partner*	Creating and using a system of linear equations to solve a real world problem	MFAEI3 a, b, c, d

The assessment for this module can be found through the Georgia Online Formative Assessment Resource (GOFAR). http://www.gadoe.org/Curriculum-Instruction-and-Assessment/Assessment/Pages/Georgia-Online-Formative-Assessment-Resource.aspx This suggested assessment should be given as the pretest and posttest for this module.

The Variable Machine

Adapted from "The Variable Machine" on http://illuminations.nctm.org/Lesson.aspx?id=1156, retrieved on March 9, 2015.

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Recommended timing is 90-120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI1. Students will create and solve equations and inequalities in one variable.

a. Use variables to represent an unknown number in a specified set (conceptual understanding of a variable). (MGSE6.EE.2, 5, 6)

e. Use variables to solve real-world and mathematical problems. (MGSE6.EE.7, MGSE.7.EE.4)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will make sense of the problems to read, write and evaluate expressions.
2. Reason abstractly and quantitatively. Students may be asked to reason through this lesson quantitatively using the variables assigned to each situation.
3. Construct viable arguments and critique the reasoning of others. Students will share expressions and models with partners and discuss their reasonableness.
4. Model with mathematics. Students will create expressions and equation models from scenarios.
6. Attend to precision. Students will attend to precision through the use of the language of mathematics in their discussions as well as in the expressions they evaluate and write.
7. Look for and make use of structure. Students will use the assigned variables to describe how the diner orders are related to the menu and write expressions showing this understanding.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Use a variable to represent any member of a set of numbers.
- Replace variables with numbers to discover unknown values.

MATERIALS

Two strips of lined notebook paper per student- one 5cm wide, one 3cm wide
Transparent tape
Student Handout: Cracking the Code Activity Sheet

ESSENTIAL QUESTIONS

What is a variable?
What does it mean "to vary"?
What are different ways variables are presented in mathematical situations?
How are variables used in real life?

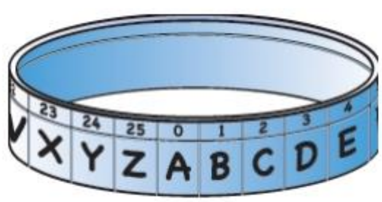
OPENER/ACTIVATOR

Pose the following question to your students: Think of a situation where a group of people must have a common language. Then, make a list of "terms" that they would need to communicate with each other. For example: for baseball, you need to know specific terms like pitcher, outfield, bases, strike, batter…

Make the connection that with mathematics we also have a common language in which we use symbols to represent words or numbers. You can also introduce the term "conventions" as it appears in future lessons in this module.

LESSON

Tell the students they are going to create a variable machine to discover the value of words. On the 3-cm-wide strip of lined notebook paper, students should write the letters of the alphabet in order. On the 5-cm-wide strip of lined notebook paper, students should write the numbers 0 through 25. Then, they should attach the ends of the number strip together with tape, and wrap the letter strip around the number wheel and tape the ends together, matching letters to corresponding numbers: A to 0, B to 1, C to 2, etc. It may be helpful to show students a finished variable machine such as the one pictured below:



*****Students can record their answers to the exploration questions on the Cracking the Code Activity sheet.***

For the first exploration, students should determine the value of their first names. For instance, the value of Tim's name is 39: T = 19, I = 8, M = 12; so, 19 + 8 + 12 = 39.

Students should then find the value of their last names, and answer the following questions:

- ***Which of your names has the higher value?***
- ***What is the difference in the values of your first and last names?***

Then, have students determine the value of the words variable, machine, algebra and mathematics as noted on the activity sheet.

Ask students to find words with the values 25, 36 and 100, and share their words with a neighbor.

After discussion of the words with the assigned values, ask them the following questions:

- ***What is the three-letter word with the greatest value?***
- ***Do you think the greatest values are always associated with words that contain the most letters?***
- ***Determine a few words with more than ten letters whose values are less than the values of words with only three letters.***

INTERVENTION

For extra help with topics in this lesson, please refer to the Intervention Table.

CLOSING/SUMMARIZER

Prompt students with the following questions:

- What is a variable?
- How did you use your Variable Machine to determine the value of your name?
- Do you think it is possible to change the values of each of the letters in a Variable Machine, or must they always be the same?

ADDITIONAL PRACTICE

Realign the number strips to let A = 7. Using this "new" Variable Machine, determine the value of your first name and your last name. Which is higher this time? ***Responses will vary. It would be very useful to have students make a prediction prior to finding their answers.***

Analyzing Solutions of Equations and Inequalities

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Recommended timing is 90-120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI1. Students will create and solve equations and inequalities in one variable.

a. Use variables to represent an unknown number in a specified set (conceptual understanding of a variable). (MGSE6.EE.2, 5, 6)

e. Use variables to solve real-world and mathematical problems. (MGSE6.EE.7, MGSE.7.EE.4)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic representations for given contexts.
2. Reason abstractly and quantitatively. Students represent real world contexts through the use of variables in equations and inequalities, and reason about the possible solutions.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using verbal and written explanations accompanied by equations and inequalities.
4. Model with mathematics. Students form expressions, equations and inequalities from real world contexts.
6. Attend to precision. Students precisely define variables. Students substitute values into the equations and inequalities to verify results.
7. Look for and make use of structure. Students use patterns and structure to solve problems and reason using equations and inequalities.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Distinguish between equality and inequality.
- Discuss how many values might make an equation or inequality true.

MATERIALS

Student Handout: Partner Problems
Mini-whiteboards (optional)

ESSENTIAL QUESTIONS

How many values of a variable make an equation true?
How many values of a variable make an inequality true?

OPENER/ACTIVATOR

Before beginning the lesson, check for students' prior understanding of the terms included in this lesson, in particular: value, variable, equation, inequality, and expression.

Prompt students with the following questions: Which of the following are equations?

a) $2n - 3 = 11$ b) $3x > 7$ c) $b - 3 = 12$ d) $15 + m$

What is the difference between an equation and an expression? What is the difference between an equation and an inequality?

LESSON

Explain that solving an equation or inequality is finding one or more values that make the statement true.

Present students with a statement like $10 - \_\_\_\_ = 7$. Have students determine a value that would make the statement false, and explain why it is false. Then have students determine which number(s) would make the statement true. (Students hopefully understand that 3 is the only number that will make the statement true.)

Students should work in pairs to answer the following questions (the expectation is that students replace the variable with the value to determine equality):

- ***Consider: $4 + b = 11$. Which value(s) will make the equation true? 6, 7, 8, 9***
- ***Consider $p - 8 > 13$ Which value(s) will make the inequality true? 28, 25, 21, 18***

Ask students if they are able to find more than one value that makes the equation true. (The expectation is that they will not.)

Show the statement $p - 8 > 13$, and have them compare this to $p - 8 = 13$.
Guiding questions: How are the solutions different for each statement? Are there any values in the previous list that will make the new statement true? Are there more values of p that make the statement true?

Keep the students in pairs and distribute Student Handout. Display, and have the students write an equation for the following: "The cash prize at BINGO is $24, and there were three winners in round one. If each winner receives the same amount of money, what amount could each winner receive?" Students should discuss possible solutions to their equation. (Solution: 3x = 24; each winner receives $8.) After students determine an answer, ask what strategies they used to determine the correct solution.

Display the equation $\frac{x}{4} = 3$. In their pairs, students should determine a solution and at least three non-solutions. Then, each pair can share their solutions and the strategies they used to determine the solutions with another pair. Each pair should write a scenario that could represent this equation. (x = 12)

Have the pairs complete questions 3 and 4. Have different pairs share their equations/inequalities, and their strategies for their work.

3: $27 + g = 50$, $g = \$23$

4: $3.50s + 27 \leq 50$, $s \leq 6.57$, therefore he can buy no more than 6 Sno-cones.

CLOSING/SUMMARIZER

Revisit the essential questions. Then prompt students with the following situation: Andrew bought a tie and dress shirt for $45. If the shirt cost $30, how much was the tie? Write an equation to represent this situation. What could the cost of the tie be? Is there more than one possible value? Why or why not?

ADDITIONAL PRACTICE

Suggested Additional Practice is attached.

1. Create a scenario similar to one of the scenarios presented in the partner problems. Your scenario will be traded with a partner during the next class. ***Responses will vary.***

2. Erica went in the store to buy a loaf of bread and gallon of milk. The milk costs $2.99 a gallon. Her mom only gave her $5 to spend. Write an inequality to represent the situation. What is a possible cost of the loaf of bread? Is there more than one possible value? Why or why not? ***2.99+b≤5.00, where b is the cost of bread. The loaf of bread must be $2.01 or less. There are many possible answers, as long as the price of the bread is less than or equal to $2.01.***

3. Which of the following is a solution(s) to $x + 6 = 13$? ***C. Other equations will vary, as long as the solution is 7.***

4. Which of the following is a solution(s) to $9 - n < 7$? ***b, c, d. Other inequalities will vary.***

Student Handout: Analyzing Solutions to Equations and Inequalities
Partner Problems

1. The cash prize at BINGO is $24, and there were three winners in round one. If each winner receives the same amount of money, what amount could each winner receive? Write an equation to represent the situation and determine a solution.

2. What is a solution to $\frac{x}{4} = 3$? Provide at least three values that do not make the statement true. Write a situation that could be represented by this equation.

3. Esteban has $50.00 to spend at Six Flags. He wants to ride a few of the roller coasters and play some of the games. The student pass into the park grants unlimited roller coaster rides, and costs $27. How much money can Esteban spend on games?

4. Esteban has decided that he just wants to ride the roller coasters and eat Sno-cones. How many Sno-cones could Esteban buy if each Sno-cone costs $3.50?

5. Could Esteban buy 6 Sno-cones? Why or why not?

6. Could he buy 10 Sno-cones? Why or why not?

Additional Practice: Analyzing Solutions to Equations and Inequalities

1. Create a scenario similar to one of the scenarios presented in the partner problems. Your scenario will be traded with a partner during the next class.

2. Erica went in the store to buy a loaf of bread and gallon of milk. The milk costs $2.99 a gallon. Her mom only gave her $5 to spend. Write an inequality to represent the situation. What is a possible cost of the loaf of bread? Is there more than one possible value? Why or why not?

3. Which of the following is a solution(s) to $x + 6 = 13$?

   $a.\ x = 9$
   $b.\ x = 8$
   $c.\ x = 7$
   $d.\ x = 6$

   Write another equation that has the same solution(s).

4. Which of the following is a solution(s) to $9 - n < 7$?

   $a.\ n = 1$
   $b.\ n = 3$
   $c.\ n = 4$
   $d.\ n = 5$

   Write another inequality that has the same solution(s).

Set It Up

Adapted from *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*, Carpenter, Thomas, Megan Loef Franke, Linda Levi, 2003

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class. Recommended timing is 120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI1. Students will create and solve equations and inequalities in one variable.

a. Use variables to represent an unknown number in a specified set (conceptual understanding of a variable). (MGSE6.EE.2, 5, 6)
e. Use variables to solve real-world and mathematical problems. (MGSE6.EE.7, MGSE.7.EE.4)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic representations for given contexts and create contexts given equations.
2. Reason abstractly and quantitatively. Students represent real-world contexts through the use of real numbers and variables in mathematical expressions, equations.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using verbal or written explanations accompanied by equations, models, and tables.
4. Model with mathematics. Students model problem situations in symbolic, tabular, and contextual formats. Students form expressions and equations from real-world contexts and connect symbolic and visual representations.
6. Attend to precision. Students precisely define variables. Students substitute solutions into equations to verify their results.
5. Use appropriate tools strategically. Students use tables to organize information to write equations.
7. Look for and make use of structure. Students seek patterns or structures to model and solve problems using tables and equations.
8. Look for and express regularity in repeated reasoning. Students generalize effective processes for representing and solving equations based upon experiences.

EVIDENCE OF LEARNING/LEARNING TARGET
By the conclusion of this lesson, students should be able to:

- Discuss why conventions are necessary in mathematics.
- Use the balance method to write and solve one-step equations.

MATERIALS

Student Handout

ESSENTIAL QUESTIONS

Why do we need conventions in mathematics? ***(The word "conventions" may need to be defined.)***
How do I set up and solve a one-step equation?

OPENER/ACTIVATOR

Part I.
Marcus has 6 pet rabbits. He keeps them in two cages that are connected so they can go back and forth between the cages. One cage is red and the other cage is blue.

1. Show all the ways that 6 rabbits can be in two cages.

Solution

Red	*Blue*
6	*0*
5	*1*
4	*2*
3	*3*
2	*4*
1	*5*
0	*6*

Comment
Students may choose to illustrate the above situation with a pictorial representation.

2. Write an equation that represents the total number of rabbits.

Solution
$r + b = 6$ ***or*** $b = 6 - r$ ***or*** $r = 6 - b$

Comment
Students may need to add an additional column to their table to assist them in writing an equation.

Red	*Blue*	*Total*
6	*0*	*6*
5	*1*	*6*
4	*2*	*6*
3	*3*	*6*
2	*4*	*6*
1	*5*	*6*
0	*6*	*6*

Students may also use their understanding of fact families to write multiple equations that represent the rabbits.

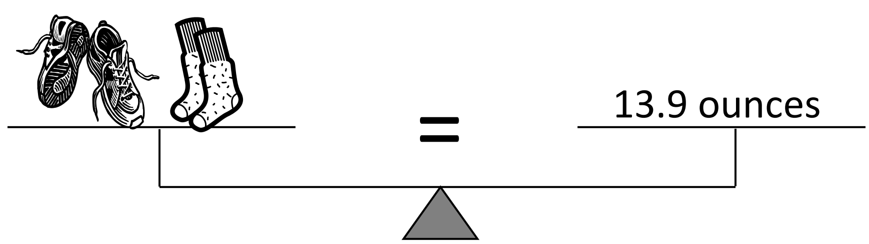
3. Write a different equation that represents the rabbits.
   Solution
   $r + b = 6$ or $b = 6 – r$ or $r = 6 – b$

4. Write a different equation that represents the rabbits.
   Solution
   $r + b = 6$ or $b = 6 – r$ or $r = 6 – b$

Note: Students should analyze the different equations and reflect on the addition and subtraction properties of equalities. Teachers can also begin a discussion on the relationship between the properties of equality and inverse operations.

LESSON
Part II.

The diagram below represents a balance scale showing the combined weight of a pair of shoes and a pair of socks. This part of the lesson will lead you through the process of finding the weight of the pair of shoes and pair of socks.



1. Write an equation that represents the above balance scale.
   Solution
    +  = ***13.9 ounces***

2. What does 13.9 represent in the equation?
   Solution
   Thirteen and 9 tenths (13.9) represents the combined weight of the shoes and socks.

3. How can you find the weight of the pair of shoes if the pair of socks weighs 0.8 ounces? What do you notice about the shoes if the pair of socks weighs 0.8 ounces?
   Solution
   $13.9 – 0.8 = 13.1$

The shoes weigh more than the socks and less than 13.9 ounces if the total weight is 13.9 ounces.

Comment
If students use trial and error to determine the weight of the shoes, guide them to use a different method to also find the weight. Guiding questions may be necessary to help students discover and use the inverse operation to "undo" the given operation to find the weight. Students need to determine that they can use the inverse operation to solve problems. This will help students understand why conventions are put in place to solve equations and will assist them when solving more complicated equations.

4. How can you find the weight of the pair of socks if the pair of shoes weighs 13.1 ounces?
Solution
The socks weigh less than the shoes and less than 13.9 ounces if the total weight is 13.9 ounces.

13.9 – 13.1 = 0.8

Comment
If students use trial and error to determine the weight of the socks, guide them to use a different method to also find the weight. Guiding questions may be necessary to help students discover and use the inverse operation to "undo" the given operation to find the weight. Students need to determine that they can use the inverse operation to solve problems. This will help students understand why conventions are put in place to solve equations and will assist them when solving more complicated equations.

5.  +  = 13.9 ounces

   a. Using the diagram above, select a variable to represent the athletic shoes (tennis shoes).
   Comment: ***Students may select any letter to represent the athletic shoes. To stay consistent we will select "a" to represent the athletic shoes.***

   b. Select a variable to represent the socks.
   Comment: ***Students may select any letter to represent the socks. To stay consistent we will select "s" to represent the socks.***

c. Write an equation that represents the above equations using variables instead of pictures.
Solution
a + s = 13.9

***Comment:* Students may need to first write the equation in words then write the equation using variables.**

d. Write an equation in terms of athletic shoes.
Solution
a = 13.9 – s

***Comment:* Students may think about fact families to help them develop this equation.**

e. Write an equation in terms of socks.
Solution
s = 13.9 – a

INTERVENTION
For extra help with topics in this lesson, please refer to the Intervention Table.

CLOSING/SUMMARIZER
Discuss the following questions: What conventions helped us in today's lesson scenario? Was it possible for different students to use different conventions? Why or why not? What was common about the set-up and solution methods for the two scenarios (rabbits vs. shoes/socks)?

ADDITIONAL PRACTICE
Students should identify two scenarios that can be modeled with two variables and one constant. They should describe the scenario in their own words, define the variables and constants, and create one equation that models the situation. Scenarios can be collected, shuffled, and passed out randomly to the class for discussion. ***Responses will vary.***

Student Handout: Set It Up

Adapted from *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School.*

Part I.

Marcus has 6 pet rabbits. He keeps them in two cages that are connected so they can go back and forth between the cages. One cage is red and the other cage is blue.

1. Show all the ways that 6 rabbits can be in two cages.

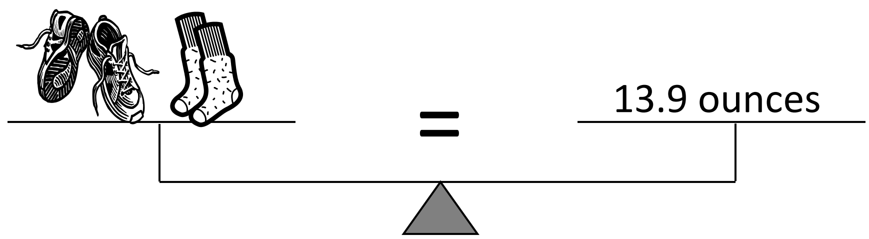
2. Write an equation that represents the rabbits.

3. Write a different equation that represents the rabbits.

4. Write a different equation that represents the rabbits.

Part II.

The diagram below represents a balance scale showing the combined weight of a pair of shoes and a pair of socks. This part of the lesson will lead you through the process of finding the weight of the pair of shoes and pair of socks.



1. Write an equation that represents the above balance scale.

2. What does 13.9 represent in the equation?

3. How can you find the weight of the pair of shoes if the pair of socks weighs 0.8 ounces? What do you notice about the shoes if the pair of socks weighs 0.8 ounces?

4. How can you find the weight of the pair of socks if the pair of shoes weighs 13.1 ounces?

5.  +  = 13.9 ounces

a. Using the diagram above, select a variable to represent the athletic shoes (tennis shoes).

b. Select a variable to represent the socks.

c. Write an equation that represents the above equations using variables instead of pictures.

d. Write an equation in terms of athletic shoes.

e. Write an equation in terms of socks.

Let's Open a Business

Adapted from *Teaching Student-Centered Mathematics Grades 5-8*, Van de Walle, John (2006)

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Recommended timing is 120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI1. Students will create and solve equations and inequalities in one variable.

a. Use variables to represent an unknown number in a specified set (conceptual understanding of a variable). (MGSE6.EE.2, 5, 6)

e. Use variables to solve real-world and mathematical problems. (MGSE6.EE.7, MGSE.7.EE.4)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic representations for real-world problems and can create real-world problems given an algebraic representation.
2. Reason abstractly and quantitatively. Students represent real-world contexts through the use of variables in expressions and equations.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using verbal or written explanations with peers.
4. Model with mathematics. Students model problem situations by forming expressions and equations that connect real-world problems to symbolic representations.
6. Attend to precision. Students precisely define variables.
7. Look for and make use of structure. Students analyze the structure of the statements and the words used to help them determine the correct expression for each scenario.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Identify a variable in a real-world problem.
- Model real-world problems with an expression or an equation.

MATERIALS

Business Lesson Cards
Copies of additional practice

ESSENTIAL QUESTIONS

How can you translate a word problem into an algebraic expression?
How do you know you need an expression and not an equation?

OPENER/ACTIVATOR

Pose the following question: If you wanted to open a business, what would be some costs you would have to consider? ***Have students categorize their costs as being either variable or constant.***

LESSON

*****This lesson will be a huge jump for some students. You may want to use ability pairing/grouping as a strategy to spark rich discussion and manageable productive struggle.***

Explain to the students that they have been approved to receive a loan from the bank to become a small business owner. Assign each group of 3 (or pairs) one of the following businesses: bakery, coffee shop or clothing store. Lesson cards for each one involve identifying variables and writing expressions and equations.

Lesson card #1: Barney's Bakery will specialize in personalized cakes, cupcakes, cake pops and cookies. As the owner, you are trying to decide if you will be able to afford a space where customers can host a party during the first year open. To make this decision, you need to explore several buildings and spaces to find one that fits your needs. The cost information for one of the possibilities is listed below:

Rent (includes utilities) $1250 per month plus $750 for security deposit

1. ***What do you think is the variable in this situation? How do you know? (The number of months is the variable.)***
2. ***Write an expression to represent the total amount of money needed to cover building rent and security deposit, and use it to calculate:***
    a. ***Total amount needed for the first 3 months (1250m + 750; $4500)***
    b. ***Total amount needed for the first 6 months ($8250)***
    c. ***Total amount needed for the first year ($15,750)***
3. ***If your loan was approved in the amount of $75,000, do you think you could afford to rent this particular building? Explain your reasoning. (Answers may vary here; students may use an equation to help explain their reasoning.)***

Lesson card #2: Miko's Café is a high end coffee shop that offers gourmet coffee, specialty drinks and an in-house bakery. As the owner, you need to determine a fair wage to pay your full-time employees. After discussing this with several other business owners and employees, you narrowed it down to the following options:

Option #1: Full-time employees make $10.10 per hour; Managers make $15.00 per hour plus a $50 bonus per paycheck. (All employees are paid once every 2 weeks.)

Option #2: Full-time employees make $10.10 per hour; Managers make $17.25 per hour. (All employees are paid once every 2 weeks.)

1. ***In option #1, what would a full-time employee make if he worked 80 hours? What about a manager? ($808; $1250)***
2. ***What would the total cost be in general to pay both employees in option #1? (10.10h + (15h + 50) = 25.10h + 50)***
3. ***In option #2, what would a full-time employee make if he worked 80 hours? What about a manager? ($808; $1380)***
4. ***What would the total cost be in general to pay both employees in option #2? (10.10h + 17.25h = 27.35h)***
5. ***Which option would you choose? Why? (Answers will vary.)***

Lesson card #3: Trend City is a one-stop shop for all of your fashion needs. You realize that you will need at least one manager for the store, and you are trying to determine an estimated cost of having one. Your estimated cost will hopefully help you decide how many managers you can actually hire. They will get paid once every 2 weeks and make $15.00 per hour with a $50 bonus in each paycheck.

1. ***Write an expression that represents the relationship between the number of hours worked and earned pay. (15h + 50)***
2. ***How much will the manager make if he works 55 hours in a pay period? 75 hours? ($875; $1175)***
3. ***How many hours would you expect your manager to work within a pay period? Explain. (Answers will vary.)***
4. ***If you allotted $8000 of your payroll budget per pay period for manager's wage, how many managers could you afford? (Answers will vary.)***
5. ***Which option would you choose? Why? (Answers will vary.)***

Once all groups are finished, have each group share their small business lesson, questions and conclusions.

CLOSING/SUMMARIZER

What did the variable in each situation represent?
What are some different strategies you used to determine the expression or equation?
Then explore the following problem with students:
Zury has saved $20 more than 5 times the amount that her sister has saved. If we want to represent the amount that Zury has saved, do we need an expression or an equation? ***expression***
Suppose Zury has saved a total of $250. If we want to know the amount that her sister has saved, do we need an expression or an equation? Justify your response. ***An equation because a total number is given.***

ADDITIONAL PRACTICE

Suggested Additional Practice is attached.

1. Brian hiked 4 miles less than 3 times the amount his friend hiked. Write an expression that represents the amount of miles Brian has hiked. ***3h-4 with h representing his friend's mileage***

2. Describe a situation that can be solved by the following algebraic expression:

$$1.5s + 50$$

Responses will vary.

3. What information would we need in question #1 in order to write an equation? ***You would need to know how far Brian hiked for a one-variable equation (or you could write a two-variable equation b = 3h-4 where b is the distance Brian hiked and h is the number of miles his friend hiked.)***

Business #1: Barney's Bakery

Barney's Bakery will specialize in personalized cakes, cupcakes, cake pops, and cookies. As the owner, you are trying to decide if you will be able to afford a space where customers can host a party during the first year open. To make this decision, you need to explore several buildings and spaces to find on that fits your needs. The cost information for one of the possibilities is listed below:

Rent (includes utilities) $1250 per month plus $750 for security deposit

1. What do you think is the variable in this situation? How do you know?
2. Write an expressions to represent the total amount of money needed to cover building rent and security deposit, and use it to calculate:
   a. Total amount needed for the first 3 months
   b. Total amount needed for the first 6 months
   c. Total amount needed for the first year
3. If your loan was approved in the amount of $75,000, do you think you could afford to rent this particular building? Explain your reasoning.

Business #2: Miko's Café

Miko's Café is a high end coffee shop that offers gourmet coffee, specialty drinks, and in-house bakery. As the owner, you need to determine a fair wage to pay your full-time employees. After discussing this with several other business owners and employees, you narrowed it down to the following options:

Option #1

Full-time employees make $10.10 per hour
Manager make $15.00 per hour plus a $50 bonus per paycheck. (All employees are paid once every 2 weeks.)

Option #2

Full-time employees make $10.10 per hours.
Managers make $17.25 per hour. (All employees are paid once every 2 weeks.)

1. In option #1, what would a full-time employee make if he worked 80 hours? What about a manger?
2. What would the total cost be in general to pay both employees in option #1?
3. In option #2, what would a full-time employee make if he worked 80 hours? What about a manager?
4. What would the total cost be in general to pay both employees in option #2?
5. Which option would you choose? Why?

Business #3: Trend City

Trend City is a one-stop shop for all your fashion needs. You realize that you will need at least one manager for the store, and you are trying to determine an estimated cost of having one. Your estimated cost will hopefully help you decide how many managers you can actually hire.

They will get paid once every 2 weeks and make $15.00 per hour with a $50 bonus in each paycheck.

1. Write an expression that represents the relationship between the number of hours worked and earned pay.
2. How much will the manager make if he works 55 hours in a pay period? 75 hours?
3. How many hours would you expect your manager to work within a pay period? Explain.
4. If you allotted $8000 of your payroll budget per pay period for manager's wage, how many managers could you afford?
5. Which option would you choose? Why?

Additional practice: Let's Open a Business

1. Brian hiked 4 miles less than 3 times the amount his friend hiked. Write an expression that represents the amount of miles Brian has hiked.

2. Describe a situation that can be solved by the following algebraic expression:

$$1.5s + 50$$

3. What information would we need in question #1 in order to write an equation?

Solving Equations using Bar Diagrams

Adapted from *Putting the Practices into Action*, O'Connell, Susan and John San Giovanni, 2013

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class. Recommended timing is 120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI1. Students will create and solve equations and inequalities in one variable.

b. Explain each step in solving simple equations and inequalities using the equality properties of numbers. (MGSE9-12.A.REI.1)
c. Construct viable arguments to justify the solutions and methods of solving equations and inequalities. (MGSE9-12.A.REI.1)
e. Use variables to solve real-world and mathematical problems. (MGSE6.EE.7, MGSE.7.EE.4)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose an appropriate algebraic representation for given context.
4. Model with mathematics. Students model problems with equations and visual representations that connect to them.
6. Attend to precision. Students precisely define variables.
7. Look for and make use of structure. Students use structure to model and solve problems using equations and diagrams.

EVIDENCE OF LEARNING/LEARNING TARGET
By the conclusion of this lesson, students should be able to:

- Model equations using a bar diagram.
- Solve equations using bar diagram models.

MATERIALS
Pencil
Paper
Tape
Learnzillion videos (core lesson and guided practice)

ESSENTIAL QUESTION
How can a bar diagram help you solve an equation?

OPENER/ACTIVATOR

We can show $3x + 5 = 38$ with the following bar diagram:

<table>
<tr><td>x</td><td>x</td><td>x</td><td>5</td></tr>
<tr><td colspan="4">38</td></tr>
</table>

Consider the following questions to guide the model:

- Is 3x more or less than 38?
- How much less is it? How do you know?
- How could you explain that using the bar diagram?

LESSON

Begin the lesson with the Learnzillion Core Lesson video ***entitled "Use a bar model to write and solve equations." Be sure to pause and discuss key points in the video and allow students to ask questions.***

Use the Guided Practice video ***as another example. Again, it is important to pause and discuss key points in the video and allow students to ask questions.***
*****There is also a downloadable PowerPoint presentation at http://www.learnzillion.com for this lesson that includes the core lesson, the guided practice problem, and extension activities.***

INTERVENTION

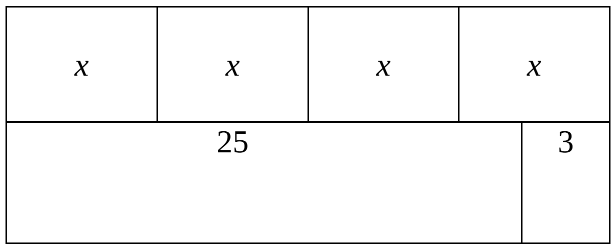
For extra help with topics in this lesson, please refer to the Intervention Table.

CLOSING/SUMMARIZER

Pose the following question to students, "How do bar diagrams help us model algebraic equations?"

Then explore the following:

We can show $4x - 3 = 25$ with the following bar diagram:



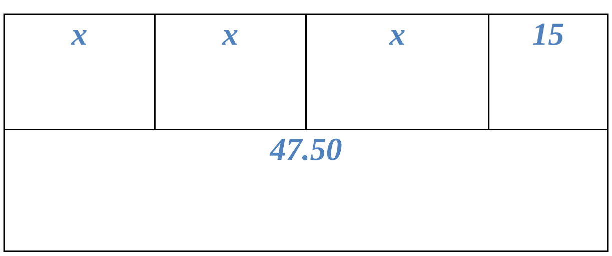
Consider the following questions to guide the model:

- Is 4*x* more or less than 25?
- How much more is it? How do you know?
- How does the bar diagram show this?

ADDITIONAL PRACTICE (or could be used as Ticket Out of the Door)

David went to Lenox Mall and purchased 3 shirts, all the same price, as well as a hat for $15. If he spent $47.50 at the mall, set up an equation and a bar diagram model that could be used to determine the cost of each shirt.

3x+15=47.50



Deconstructing Word Problems

(Adapted from "Deciphering Word Problems" which can be found at *http://www.nsa.gov/academia/_files/collected_learning/middle_school/algebra/deciphering_word_problems.pdf* *)*

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Recommended timing is 120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI1. Students will create and solve equations and inequalities in one variable.

a. Use variables to represent an unknown number in a specified set (conceptual understanding of a variable). (MGSE6.EE.2, 5, 6)

e. Use variables to solve real-world and mathematical problems. (MGSE6.EE.7, MGSE.7.EE.4)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic representations for real-world problems and can create real-world problems given an algebraic representation.
2. Reason abstractly and quantitatively. Students represent real-world contexts through the use of variables in expressions and equations.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using verbal or written explanations with peers.
4. Model with mathematics. Students model problem situations by forming expressions and equations that connect real world problems to symbolic representations.
6. Attend to precision. Students precisely define variables.
7. Look for and make use of structure. Students analyze the structure of the statements and the words used to help them determine the correct expression for each scenario.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Build expressions and equations when presented a real-world problem.

MATERIALS

Pencil and paper
Student Handout

ESSENTIAL QUESTIONS

How can information from a word problem be translated to create an equation?
How can building and solving equations from word problems lead to a conclusion and help answer the problem being presented?

OPENER/ACTIVATOR

Have the students work through the translating verbal expression warm-up questions in order to address misconceptions about what words represent each operation. ***Keep an eye out for "less than" and how it differs in its placement in an expression. Anything that says "less than" or "from" changes the order of how the expression is written.***

Choose 3 of the 7 situations below and translate the verbal expression to an algebraic equation. ***Allowing students this choice will help you pinpoint more quickly who is having success and who is not. For those who struggle, pull them aside individually while others work on part II and have the struggling students discuss some of the "leftover" equations.***

1) Ann has the 5 newest music CD's which is 3 less than twice the amount that Bob has. ***2x- 3 = 5 where x is Bob's amount.***
2) Mike, who has 6 video games, has half as many games as Paul. $\frac{x}{2}$ ***= 6 where x is Paul's amount***
3) Nan rode the roller coaster 8 times, which was twice as many times as she rode the Ferris wheel. ***2x = 8 where x is the times on the Ferris wheel***
4) Janine, who bought $15 worth of make-up, spent $6 less than Leah spent. ***x - 6 = 15 where x is what Leah spent***
5) Rob, who has all 13 girls' phone numbers that are in his homeroom, has 3 more than half the number of girls' phone numbers that Jay has. $\frac{x}{2}$ ***+ 3 = 13 where x stands for Jay's number***
6) Kate's 85 on her English test was 37 points less than twice the grade on her Science test. ***2x – 37 = 85 where x is the Science test***
7) At the Middle School Graduation Dance, the DJ played 12 slow dances, which was equal to the quotient of the number of fast dances and 2. $\frac{x}{2}$ ***= 12 where x is the fast dances***

LESSON

Create expressions for the situation described in the word problem. Your teacher may guide you through using a chart to help you do this. Then, use these expressions and the word problem to create and solve an equation. Make sure you not only solve for the variable, but also answer the question being presented. ***Students sometimes struggle with deciding which person or item is the variable. One suggestion is to tell them it is the person or item on which everything else is based.***

1. Sean sold 4 more boxes of candy for the school fundraiser than Marta. The sum of the boxes they sold was 22. How many boxes did each sell?

WHO	***NUMBER OF BOXES***
Sean	***4 + M***
Marta	***M***

$$4 + M + M = 22$$
$$2M + 4 = 22$$
$$2M = 18$$
$$M = 9$$

Marta sold 9 boxes of candy and Sean sold 13.

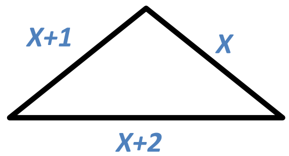
2. Ned weigh 1 ½ times as much as Jill, and Tom weighs 15 kilograms more than Jill. If their combined weight is 190 kilograms, how much does each person weigh?

WHO	***WEIGHT***
Ned	***1 ½ J***
Jill	***J***
Tom	***15 + J***

$$1\frac{1}{2}(J) + J + 15 + J = 190$$
$$3\frac{1}{2}(J) + 15 = 190$$
$$3\frac{1}{2}(J) = 175$$
$$J = 50$$

Jill weights 50 kilograms, Tom weighs 65 kg. Ned weights 75 kg.

3. The side lengths of a triangular birdcage are consecutive integers. If the perimeter is 114 centimeters, what is the length of each side? Label each side with an expression that represents its length.



$$X + X + 1 + X + 2 = 114$$
$$3X + 3 = 114$$
$$3X = 111$$
$$X = 37$$

The lengths of the sides are 37, 38, and 39.

4. Caitlyn did $\frac{6}{7}$ of the problems on her math quiz correctly and four incorrectly. She did all the problems. How many were there?

TYPE OF PROBLEM	***FRACTIONAL PART OF WHOLE***	***NUMBER***
Correct	$\frac{6}{7}$	***x - 4***
Incorrect	$\frac{1}{7}$	***4***
Total on Quiz	$\frac{7}{7}$	***x***

$$\frac{1}{7}x = 4$$
$$x = 28$$

There were 28 problems on the quiz.

5. Geri spent Friday, Saturday, and Sunday selling a total of 24 magazine orders for her school fundraiser. The amounts she sold respectively, on the three days were consecutive even integers. How many did she sell on each day?

DAY OF WEEK	***AMOUNT SOLD***
Friday	***n***
Saturday	***n + 2***
Sunday	***n + 4***

$$n + n + 2 + n + 4 = 24$$
$$3n + 6 = 24$$
$$3n = 18$$
$$n = 6$$

Geri sold 6 orders on Friday, 8 orders on Saturday, and 10 orders on Sunday.

INTERVENTION

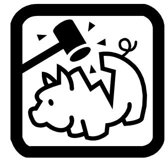
For extra help with topics in this lesson, please refer to the Intervention Table. If students struggle with the handout, suggest some active reading strategies to help them locate important information within the problem (highlighting or underlining key words, etc.).

CLOSING/SUMMARIZER

Have students look through the various scenarios they addressed in this lesson. Ask the following questions, "How did we use the information in each scenario to create an equation? What common threads can we identify across the different situations?" Have students write a letter to an "absent" student explaining the most important ideas to consider in translating scenarios into equations.

ADDITIONAL PRACTICE:

There are many more scenarios on the referenced website as well as a quiz for the end of the section if needed. Assign one or two to challenge the students.
http://www.nsa.gov/academia/_files/collected_learning/middle_school/algebra/deciphering_word_problems.pdf



Student Handout: Deconstructing Word Problems

Part I: Warm-Up

Choose 3 of the 7 situations below and translate the verbal expression to an algebraic equation.

1) Ann has the 5 newest music CD's which is 3 less than twice the amount that Bob has.
____

2) Mike, who has 6 video games, has half as many games as Paul.
____

3) Nan rode the roller coaster 8 times, which was twice as many times as she rode the Ferris wheel. _________________________________

4) Janine, who bought $15 worth of make-up, spent $6 less than Leah spent.
____

5) Rob, who has all 13 girls' phone numbers that are in his homeroom, has 3 more than half the number of girls' phone numbers that Jay has.
____

6) Kate's 85 on her English test was 37 points less than twice the grade on her Science test.
____

7) At the Middle School Graduation Dance, the DJ played 12 slow dances, which was equal to the quotient of the number of fast dances and 2.
____

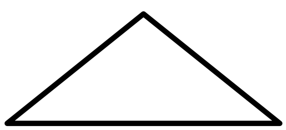
Part II: Lesson on Creating Equations from Word Problems

Create expressions for the situation described in the word problem. Your teacher may guide you through using a chart to help you do this. Then, use these expressions and the word problem in order to create and solve an equation. Make sure you not only solve for the variable, but also answer the question being presented. Show all your work when solving the equations.

1. Sean sold 4 more boxes of candy for the school fundraiser than Marta. The sum of the boxes they sold was 22. How many boxes did each sell?

2. Ned weighs 1½ times as much as Jill and Tom weighs 15 kilograms more than Jill. If their combined weight is 190 kilograms, how much does each person weigh?

3. The sides of a triangular birdcage are consecutive integers. If the perimeter is 114 centimeters, what is the length of each side? Label each side with an expression that represents its length.



4. Caitlyn did $\frac{6}{7}$ of the problems on her math quiz correctly and four incorrectly. She did all the problems. How many were there?

5. Geri spent Friday, Saturday and Sunday selling a total of 24 magazine orders for her school fundraiser. The amounts she sold, respectively, on the three days were consecutive even integers. How many did she sell on each day?

Steps to Solving an Equation (FAL)

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/lessons.php?unit=7220&collection=8&redir=1

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Recommended timing is 120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI1. Students will create and solve equations and inequalities in one variable.

b. Explain each step in solving simple equations and inequalities using the equality properties of numbers. (MGSE9-12.A.REI.1)
c. Construct viable arguments to justify the solutions and methods of solving equations and inequalities. (MGSE9-12.A.REI.1)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic representations for real-world problems and can create real-world problems given an algebraic representation.
2. Reason abstractly and quantitatively. Students represent real-world contexts through the use of variables in expressions and equations.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using verbal or written explanations with peers.
4. Model with mathematics. Students model problem situations by forming expressions and equations that connect real world problems to symbolic representations.
6. Attend to precision. Students precisely define variables.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Form and solve linear equations using factoring and the distributive property.
- Use variables to represent equations in real-world problems.
- Represent word problems in equivalent equations.

MATERIALS

Copies of formative assessment lesson pre-and post-assessment for each student
Card sets for each pair (or small group)
Mini whiteboards
Chart paper or poster board
Glue
Scissors (if lesson cards are not pre-cut)

ESSENTIAL QUESTION

What are some strategies for solving real life mathematical problems involving numerical and algebraic equations and expressions?

Lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The PDF version of this lesson can be found at the link below:
http://map.mathshell.org/download.php?fileid=1635

T.V. Time and Video Games

(*Adapted from Connected Mathematics*)

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class. Recommended timing is 120-150 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI1. Students will create and solve equations and inequalities in one variable.

b. Explain each step in solving simple equations and inequalities using the equality properties of numbers. (MGSE9-12.REI.1)

c. Construct viable arguments to justify the solutions and methods of solving equations and inequalities. (MGSE9-12.REI.1)

d. Represent and find solutions graphically.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic representations for real-world problems and can create real-world problems given an algebraic representation.
2. Reason abstractly and quantitatively. Students represent real-world contexts through the use of variables in inequalities.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using verbal or written explanations with peers.
4. Model with mathematics. Students model problem situations by forming inequalities that connect real world problems to symbolic representations.
6. Attend to precision. Students precisely define variables and use graphs to justify their solutions.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Create inequalities to represent a situation.
- Solve inequalities by graphing in order to provide a visual representation of the real-life situation.

MATERIALS

T.V. Time and Video Game student handout
Index cards with expressions and symbols listed in opening card sort

ESSENTIAL QUESTIONS

How can inequalities be used in order to demonstrate all possible values that are solutions to a given real life situation?
How can inequalities be displayed on a number line to provide a visual representation of a given situation?

OPENER/ACTIVATOR

Inequalities are used to show a range of possible values that meet a given criteria. In the following lesson, students will be creating a visual and algebraic solution to a given situation. Review the following information with students prior to beginning the lesson by using a card sort with symbols on one set of cards and meanings on another set of cards:

An inequality is a math sentence that compares two quantities. Often one of the quantities represented is a variable. Use the following symbols and descriptions to represent each type of inequality.

$<$ means "is less than."	$\leq$ means "is less than or equal to."
$>$ means "is greater than."	$\geq$ means "is greater than or equal to."
$\neq$ means "is not equal to."	

LESSON

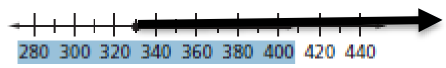
Represent each inequality below using a variable and a constant.

1. Nima will spend less than $25 ______*L<25*_______
2. Derrick ran at least 30 miles last week _____***R***$\geq$ *30*__________
3. Emily needs at least $200 to buy the TV she wants ____***A*** $\geq$***200***_____
4. Kia volunteers with some friends at a community center. While shopping online for a new television she decides she wants one with at least a 26 in. screen. Using the chart below, write an inequality to show how much money the center will have to spend.

Television Prices	
Screen Size	**Price**
22 in.	$300
26 in.	$330
32 in.	$370
40 in.	$420

Inequality ___**x $\geq$ 330**_______

5. Graph the inequality from problem 4 on the number line.



6. Kia wants to have money left over. How can the graph be changed to show they need to have more than $330?

Do not color in the point at $330. The inequality would change to x > 330.

7. The center has a stand for the television that will hold up to 30 lb. of weight. Draw a graph to show how much the television she buys can weigh.

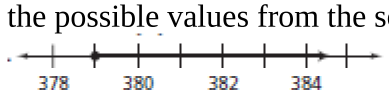
Kia plans to use money from the community center's savings account to buy a gaming system. There must be $129 left in the savings account after she withdraws what she needs.

Video Game System Sale This Month Only $250.00 Selected Extra Games on Sale $35.00 each

8. Write and solve an inequality to represent the situation, where x represents the amount of money the center has in its savings account. What does your solution mean in terms of the problem?

$x - 129 \geq 250$; $x \geq 379$
The center needs at least $379 in the account to start in order to take out $250 for the gaming system and still leave $129 in the account. If they just have $379 in savings, they will not be able to afford any games.

9. Graph the possible values from the solution found in number 8.



The community center rents rooms for an hourly rate, plus a set-up fee.

Room Rentals		
Room	**Rental Rate per Hour**	**Set-up Fee**
Main Hall	$15	$40
Dining Room	$12	$80

10. A school group has $140 to spend. Write and solve an inequality that represents the cost to rent the main hall, where *h* represents the number of hours the group can rent the room.

    $15h + 40 \le 140$; $x \le 6.666\ldots$ (6 hrs. and 40 min.) The group can rent the room for a total of 6 full hours.

11. The same group is also considering renting the dining room. Write and solve an inequality to represent this situation.

    $12x + 80 \le 140$; $x \le 5$
    They can rent the dining room for a total of 5 hours at most.

12. Use your solutions from 10 and 11 to justify your selection of which room the group should rent.

    They should rent the main hall since they get 6 total hours rather than the five they would get from the dining room.

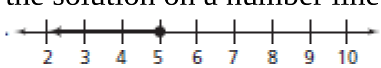
The community center has $175 to spend on video games for its new gaming system. Games are on sale for $35 each.

13. Write and solve an inequality to represent the number of games the center could buy. Explain your solution in reference to the problem.

    $35x \le 175$; $x \le 5$ or $175 - 35x \ge 0$; $x \le 5$

    In order to stay in budget the center has to buy 5 or less video games.

14. Graph the solution on a number line.



The center is considering signing up for an online game-rental service rather than buying the games. The table shows equipment cost and monthly fees for two services.

Game Rental Services		
Service	**Equipment Cost**	**Monthly Fee**
NetGames	$99	$8
Anytime Games	$19	$19

15. Write and solve an inequality that represents the number of months the center could rent games from NetGames with its $175. Explain the solution in terms of the problem.

    $8m + 99 \le 175$; $x \le 9.5$ The center would be able to rent movies for nine and a half months. They could rent for a total of 9 full months.

16. Write and solve an inequality to represent the number of months the center could rent games from Anytime Games. Explain the solution in terms of the problem.

    $19m + 19 \le 175$; $m \le \frac{156}{19}$ The center would be able to rent for about 8.2 months or 8 full months and still stay within budget.

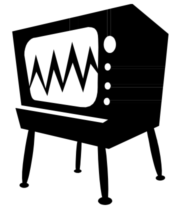
17. Use your answers from 15 and 16 to justify which service the community center should purchase.

    NetGames is the more affordable choice. The community center would get an extra month of service by choosing this service.

INTERVENTION
For extra help with topics in this lesson, please refer to the Intervention Table.

CLOSING/SUMMARIZER
Ask students the following questions: “How did inequalities help to demonstrate all possible values that are solutions to a given real-life situation? Which solution representation is preferable to you—a numeric representation or a graphical representation on a number line? Justify your response.”

ADDITIONAL PRACTICE
Give the students the following scenario: You are on a budget and want to find out which T.V. sets and gaming systems you would be able to purchase and still remain within your budget. Create a list of conditions that would impact your decision. Write 3 inequalities about this situation based on online and/or paper ads that justify your pricing and choices. ***Responses will vary.***



Student Handout: T.V. Time and Video Games

An inequality is a math sentence that compares two quantities. Often one of the quantities is a variable. Use the following symbols and descriptions to represent each type of inequality.

< means ________________ ≤ means __________________________

> means _______________ ≥ means _________________________

≠ means _________________________

Represent each situation below with an inequality:

1. Nima will spend less than $25 _____________________________

2. Derrick ran at least 30 miles last week _______________________

3. Emily needs at least $200 to buy the TV she wants _________________

4. Kia volunteers with some friends at a community center. While shopping online for a new television she decides she wants one with at least a 26 in. screen. Using the chart below, write an inequality to show how much money the center will have to spend.

Television Prices	
Screen Size	Price
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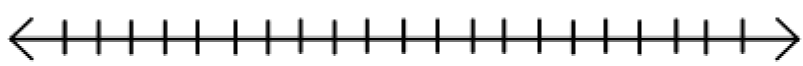
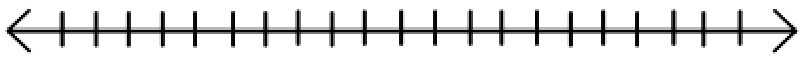
Inequality ________________________

5. Graph the inequality from problem 4 on the number line.



6. Kia wants to have money left over. How can the graph be changed to show they need to have more than $330?

7. The center has a stand for the television that will hold up to 30 lb. of weight. Draw a graph to show how much the television she buys can weigh.

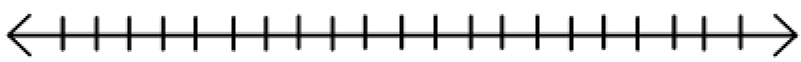


Kia plans to use money from the community center's savings account to buy a gaming system. There must be $129 left in the savings account after she withdraws what she needs.

Video Game System Sale This Month Only
$250.00
Selected Extra Games on Sale
$35.00 each

8. Write and solve an inequality to represent the situation, where x represents the amount of money the center has in its savings account.

9. Graph the possible values from the solution found in number 8.



The community center rents rooms for an hourly rate, plus a set-up fee.

Room Rentals		
Room	**Rental Rate per Hour**	**Set-up Fee**
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10. A school group has $140 to spend. Write and solve an inequality that represents the cost to rent the main hall, where *h* represents the number of hours the group can rent the room.

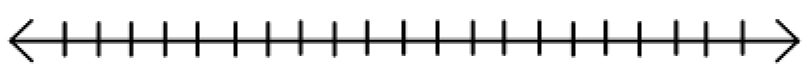
11. The same group is also considering renting the dining room. Write and solve an inequality to represent this situation.

12. Use your solutions from 10 and 11 to justify your selection of which room the group should rent.

The community center has $175 to spend on video games for its new gaming system. Games are on sale for $35 each.

13. Write and solve an inequality to represent the number of games the center could buy. Explain your solution in reference to the problem.

14. Graph the solution on a number line.



The center is considering signing up for an online game-rental service rather than buying the games. The table shows equipment cost and monthly fees for two services.

Game Rental Services		
Service	**Equipment Cost**	**Monthly Fee**
NetGames	$99	$8
Anytime Games	$19	$19

15. Write and solve an inequality that represents the number of months the center could rent games from NetGames with its $175. Explain the solution in terms of the problem.

16. Write and solve an inequality to represent the number of months the center could rent games from Anytime Games. Explain the solution in terms of the problem.

17. Use your answers from 15 and 16 to justify which service the community center should purchase.

When Is It Not Equal?

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class. Recommended timing is 90-120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI1. Students will create and solve equations and inequalities in one variable.

a. Use variables to represent an unknown number in a specified set (conceptual understanding of a variable). (MGSE6.EE.2, 5, 6)
d. Represent and find solutions graphically.
e. Use variables to solve real-world and mathematical problems. (MGSE6.EE.7, MGSE.7.EE.4)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic representations for given contexts and can create contexts given inequalities.
2. Reason abstractly and quantitatively. Students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using verbal or written explanations accompanied by inequalities and number lines.
4. Model with mathematics. Students model inequality situations on a number line.
5. Use appropriate tools strategically. Students use number lines to graph inequalities. Students use tables to organize information to write inequalities.
6. Attend to precision. Students precisely define variables.

EVIDENCE OF LEARNING/LEARNING TARGET
By the conclusion of this lesson, students should be able to:

- Write and solve one-step inequalities and graph them on the number line.

MATERIALS
Student Handout

ESSENTIAL QUESTIONS

What strategies can I use to help me understand and represent real situations using inequalities?
How are the solutions of equations and inequalities different?
How can I write, interpret, manipulate and find solutions for inequalities?

OPENER/ACTIVATOR

Part one of this lesson is to help students recognize words in a word problem that indicate inequality. Students choose 3 statements to complete, and discuss their circled selections. As with previous lessons, pull struggling students aside individually while others work on the rest of the lesson. Have the struggling students discuss some of the "leftover" statements.

Write an inequality for three of the following seven statements. Then circle the hardest situation in numbers 1-7 and be prepared to share with the class.

1. You need to earn at least $50.
   $x \geq 50$
2. You can spend no more than $5.60
   $x \leq 5.60$
3. The trip will take at least 4 hours.
   $x \geq 4$
4. The car ride will be less than 8 hours.
   $x < 8$
5. Four boxes of candy contained at least 48 pieces total.
   $4x \geq 48$
6. With John's 7 marbles and mine, we had less than 20 marbles together.
   $x + 7 < 20$
7. Seven buses can hold no more than 560 students.
   $7x \leq 560$

LESSON

Graph the following inequalities. Then write a scenario that could be modeled by each inequality. ***Answers will vary. Check graphs for open/closed circles and direction of solution. Have students defend their responses.***

8. $p \geq 17$

9. $b \leq 7$

10. $t < 4$

11. $r > 10$

12. $k \leq 18$

13. $m > 1$

14. $d > 2$

Have students work with a partner on the following and discuss as a class:
Circle the numbers that are part of the solution for the inequalities below.

15. $x + 2 > 5$	(0 3 4 10) ***4 and 10 are solutions***
16. $v - 4 < 10$	(4 9 14 15) ***4 and 9 are solutions***
17. $4b \leq 15$	(0 3 5 6) ***0 and 3 are solutions***
18 $\frac{1}{3}r \leq 3\frac{1}{2}$	(6 9 15 30) ***6 and 9 are solutions***
19. $0.5w > 2.3$	(2 4 5 10) ***5 and 10 are solutions***
20. $t + 1.5 < 3.6$	(0.6 1.7 2.1 3.2) ***0.6 and 1.5 are solutions***

CLOSING/SUMMARIZER

This lesson focused on inequalities. Ask students to compare and contrast the solutions they found in this lesson with how they find solutions to equations. You may decide to create a Venn diagram for similarities and differences in solution methods and conditions. You may also aid this discussion by having students rewrite their responses to the seven items of the opener of this lesson to make them equations.

ADDITIONAL PRACTICE

Suggested Additional Practice is attached. The solutions are provided below.

Write an inequality for each situation. Then use that inequality to choose and justify solutions listed for each situation. ***Students are translating the verbal expressions into statements of inequality, not solving inequalities.***

1. What is the minimum number of 80-passenger buses needed to transport 375 students? Choose and justify a solution (4, 4 $^{11}/_{16}$, 5)
   $80b \geq 375$
   4 $^{11}/_{16}$ and 5 are solutions to the inequality but 5 is the answer to the question.

2. What is the minimum speed needed to travel at least 440 miles in 8 hours? Choose and justify a solution (54 mph, 55 mph, 56 mph)
   $8r \geq 440$
   55 and 56 are solutions the inequality but 55 is the answer to the question

3. What is the least number of boxes needed to package 300 candies if each box will hold 16 candies? Choose and justify a solution (18, 18 ¾ , 19)
   $16b \geq 300$
   18 ¾ and 19 are solutions to the inequality but 19 is the answer to the question

Student Handout: When Is It Not Equal?

Write an inequality for **three** of the following seven statements. Then circle the hardest situation in numbers 1-7 and be prepared to share with the class.

1. You need to earn at least $50.
2. You can spend no more than $5.60
3. The trip will take at least 4 hours.
4. The car ride will be no more than 8 hours.
5. Four boxes of candy contained at least 48 pieces total.
6. With John's 7 marbles and mine, we had less than 20 marbles together.
7. Seven buses can hold no more than 560 students.

Graph the following inequalities. Then write a scenario that could be modeled by each inequality.

8. $p \geq 17$

9. $b \leq 7$

10. $t < 4$

11. $r > 10$

12. $k \leq 18$

13. $m > 1$

14. $d > 2$

With a partner, circle the numbers that are part of the solution for the inequalities below. Explain your reasoning.

15. $x + 2 > 5$ (0 3 4 10)

16. $v - 4 < 10$ (4 9 14 15)

17. $4b \leq 15$ (0 3 5 6)

18 $1/3r \geq 3\frac{1}{2}$ (6 9 15 30)

19. $0.5w > 2.3$ (2 4 5 10)

20. $t + 1.5 < 3.6$ (0.6 1.7 2.1 3.2)

Additional Practice: When Is It Not Equal?

Write an inequality for each situation. Then use that inequality to choose and justify solutions listed for each situation.

1. What is the minimum number of 80-passenger buses needed to transport 375 students? Choose and justify a solution (4, 4 $^{11}/_{16}$, 5)

2. What is the minimum speed needed to travel at least 440 miles in 8 hours? Choose and justify a solution (54 mph, 55 mph, 56 mph)

3. What is the least number of boxes are needed to package 300 candies if each box will hold 16 candies? Choose and justify a solution (18, 18 ¾, 19)

Quick Check I

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI1. Students will create and solve equations and inequalities in one variable.

a. Use variables to represent an unknown number in a specified set (conceptual understanding of a variable). (MGSE6.EE.2, 5, 6)
b. Explain each step in solving simple equations and inequalities using the equality properties of numbers. (MGSE9-12.A.REI.1)
c. Construct viable arguments to justify the solutions and methods of solving equations and inequalities. (MGSE9-12.A.REI.1)
d. Represent and find solutions graphically.
e. Use variables to solve real-world and mathematical problems. (MGSE6.EE.7, MGSE7.EE.4)

Teacher's Edition for Quick Check I

Adapted from: https://www.illustrativemathematics.org/content-standards/7/EE/B/4/tasks/643
and https://www.illustrativemathematics.org/content-standards/6/EE/B/6/tasks/425

1. Which of the following is a solution(s) to $8 = 6 + 2x$? ***Choice C***

a) $x = -1$ b) $x = 0$ c) $x = 1$ d) $x = 2$

2. Which of the following is a solution(s) to $7 - 3b \leq 28$? ***Choices B, C, and D***

a) $b = -8$ b) $b = -7$ c) $b = -6$ d) $b = 0$

3. A town's total allocation for firefighter's wages and benefits in a new budget is \$600,000. Wages are calculated at \$40,000 per firefighter and benefits at \$20,000 per firefighter.

a) Write an equation that could be used to determine the number of firefighters the town can employ if they spend their whole budget.
Let x represent the maximum number of firemen that could be employed.
$600{,}000 = 40{,}000x + 20{,}000x$ or if combined like terms $600{,}000 = 60{,}000x$

b) How many firefighters can the town employ?
The town can employ ten firefighters.

4. Fishing Adventures rents small fishing boats to tourists for day-long fishing trips. Each boat can only carry 1200 pounds of people and gear for safety reasons. Assume the average weight of a person is 150 pounds. Each group will require 200 pounds of gear for the boat plus 10 pounds of gear for each person.

a) Write an inequality describing the restrictions on the number of people possible in a rented boat.
Let p be the number of people in a group that wishes to rent a boat.
$150p + 10p + 200 \leq 1200$ or if combined like terms $160p + 200 \leq 1200$

b) Several groups of people wish to rent a boat. Group 1 has 4 people. Group 2 has 5 people. Group 3 has 8 people. Which of the groups, if any, can safely rent a boat?
For Group 1: $160(4) + 200 = 840 \leq 1200$
For Group 2: $160(5) + 200 = 1000 \leq 1200$
For Group 3: $160(8) + 200 = 1480 > 1200$

Both Group 1 and Group 2 can safely rent a boat but Group 3 exceeds the weight limit and cannot rent a boat.

c) What is the maximum number of people that may rent a boat?

To find the maximum number of people that may rent a boat, we can solve the inequality for p:

$$160p + 200 \leq 1200$$
$$160p \leq 1000$$
$$p \leq 6.25$$

Since we cannot have 0.25 of a person, 6 is the largest number of people that may safely rent a boat.

Quick Check I

Name ______________________________

Adapted from: https://www.illustrativemathematics.org/content-standards/7/EE/B/4/tasks/643 *and* https://www.illustrativemathematics.org/content-standards/6/EE/B/6/tasks/425

1. Which of the following is a solution(s) to 8 = 6 + 2x?

a) x = -1 b) x = 0 c) x = 1 d) x = 2

2. Which of the following is a solution(s) to 7 – 3b ≤ 28?

a) b = -8 b) b = -7 c) b = -6 d) b = 0

3. A town's total allocation for firefighter's wages and benefits in a new budget is $600,000. Wages are calculated at $40,000 per firefighter and benefits at $20,000 per firefighter.

a) Write an equation that could be used to determine the number of firefighters the town can employ if they spend their whole budget.

b) How many firefighters can the town employ?

4. Fishing Adventures rents small fishing boats to tourists for day-long fishing trips. Each boat can only carry 1200 pounds of people and gear for safety reasons. Assume the average weight of a person is 150 pounds. Each group will require 200 pounds of gear for the boat plus 10 pounds of gear for each person.

a) Write an inequality describing the restrictions on the number of people possible in a rented boat.

b) Several groups of people wish to rent a boat. Group 1 has 4 people. Group 2 has 5 people. Group 3 has 8 people. Which of the groups, if any, can safely rent a boat?

c) What is the maximum number of people that may rent a boat?

“Illustrative” Check I

These checks may be used as formative assessments during the module to check student progress.

1. Writing Expressions: https://www.illustrativemathematics.org/content-standards/6/EE/A/2/tasks/540

2. Setting Up and Solving Equations: https://www.illustrativemathematics.org/content-standards/7/EE/B/3/tasks/884

3. Creating and Solving Inequalities: https://www.illustrativemathematics.org/content-standards/7/EE/B/4/tasks/643

4. More on Inequalities: https://www.illustrativemathematics.org/content-standards/7/EE/B/4/tasks/986

5. A More Challenging Inequality: https://www.illustrativemathematics.org/content-standards/7/EE/B/4/tasks/1475

6. More on Equations: https://www.illustrativemathematics.org/content-standards/6/EE/B/6/tasks/425

7. Setting Up and Solving Equations: https://www.illustrativemathematics.org/content-standards/6/EE/B/7/tasks/1032

8. Solving Basic Equations: https://www.illustrativemathematics.org/content-standards/6/EE/B/7/tasks/1107

9. Basic Inequalities: https://www.illustrativemathematics.org/content-standards/6/EE/B/8

Yogurt Packaging (Career and Technical Education Lesson)

Source: National Association of State Directors of Career Technical Education Consortium

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Recommended timing is 120 minutes including extensions.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI2. Students will use units as a way to understand problems and guide the solutions of multi-step problems.

a. Choose and interpret units in formulas. (MGSE9-12.N.Q.1)

STANDARDS FOR MATHEMATICAL PRACTICE

1. **Make sense of problems and persevere in solving them.** Students must make conjectures about the form and meaning of the solution pathway. The lesson requires multi-step problem solving.
2. **Reason abstractly and quantitatively.** Students make sense of quantities and their relationships in the problem situation.
6. **Attend to precision.** Students need to attend to units as they perform calculations. Rounding and estimation are a key part.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Model real-world situations with mathematical expressions and equations.
- Use unit analysis to answer questions.

MATERIALS

Copy of lesson
PDF: **http://www.achieve.org/files/CCSS-CTE-Yogurt-Packaging-FINAL.pdf**
Word: **http://www.achieve.org/files/CCSS-CTE-Yogurt-Packaging-FINAL.docx**
Various empty yogurt containers
Paper
Tape
Scissors (used to construct a yogurt tub)

ESSENTIAL QUESTION

How can I use unit analysis, expressions and algebraic equations to answer questions in context?

OPENER/ACTIVATOR

Have several examples of yogurt containers to share with students. Discuss the serving size, the packaging shape, and why the yogurt manufacturers may have chosen different serving sizes and shapes.

LESSON

Students complete handout in groups of 2 or 3 students. For those who finish early, you may assign one of the extensions, or an extension may be assigned as Additional Practice.

CLOSING/SUMMARIZER

Make the following table on chart paper or a white board. Students should spend a few minutes completing the table with a partner and then collect the class responses and discuss. ***Responses will vary. Encourage students to look back at all previous tasks and analyze what they learned.***

Mathematical Idea	What parts of the Yogurt Packaging Lesson included this mathematical idea?	What lessons have we completed previously in this module that included this idea?
Unit Analysis		
Expressions		
Algebraic Equations		

Don't Sink My Battleship!

Adapted from *Teaching Student-Centered Mathematics Grades 5-8*, Van de Walle, John (2006).

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Recommended timing is 120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI2. Students will use units as a way to understand problems and guide the solutions of multi-step problems.

- b. Choose and interpret graphs and data displays, including the scale and comparisons of data. (MGSE3.MD.3, MGSE9-12.N.Q.1)
- c. Graph points in all four quadrants. (MGSE6.NS.8)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of problems involving points in the coordinate plane.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning about positive and negative numbers with their visual representations.
3. Construct viable arguments and critique the reasoning of others. Students construct and critiques arguments regarding number line and coordinate plane representations.
4. Model with mathematics. Students use coordinate planes to model locations in real-world contexts.
6. Attend to precision. Students use the description of real-world situations to determine the appropriate location of points and what they represent.
7. Look for and make use of structure. Students relate the structure of number lines to positive and negative numbers as they use the coordinate plane.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Locate ordered pairs on a coordinate grid.
- Create their own map and describe locations using the coordinate system.

MATERIALS

Battleship game cards
Student Handout: Grid City Spring Festival
Graph paper or ruled chart paper

ESSENTIAL QUESTIONS

When is graphing on the coordinate plane helpful?
Why is the order of the coordinates important when graphing on the coordinate plane?

OPENER/ACTIVATOR

Show the students a blank coordinate grid. Define and discuss the origin of the coordinate grid, as well as how you determine locations within a coordinate system. Review the quadrants with the students, and have them graph a few ordered pairs as a quick pre-assessment. If necessary, revisit the earlier discussion on mathematical conventions to impress upon students the importance of having a consistent convention for the order of the coordinates.

LESSON

Distribute the Don't Sink My Battleship! game cards to the students.

Rules of the game: Players secretly put their initials on any five intersections of their own grid to denote the location of their ships. With the grids kept hidden from each other, one player takes a turn trying to "hit" the other player's ships by naming a point on the grid using coordinates. The other player indicates whether or not it was a hit or a miss. Each player keeps track of where he has taken shots by recording an "X" for a hit and an "O" for a miss. When a player scores a hit, he or she gets another turn. The game ends when one player has hit all of the other player's ships.

After playing Battleship, have the students work in pairs on the following lesson:

Grid City is getting ready for its spring festival! There will be 8 venues: arts and crafts, face painting, cotton candy and popcorn stand, barbecue pit, 2 bounce houses, a Playboxx station and a performance stage. These venues will need to be set up around the park. You must remember the following when designing your map:

- ***The travel distance and proximity between venues***
- ***The performance stage has to be positioned so that people can see it from any venue in the park.***
- ***The barbecue pit should be 24 units away from the stage, and have the same y-coordinate.***

In the park, there will also be 6 port-a-potties, a ticket/information booth, and 4 sections of picnic tables.

- ***The port-a-potties need to be set up on the left side of the park, all with the same x-coordinate.***
- ***The ticket/information booth needs to be somewhere on the y-axis.***
- ***The picnic table sections need to be spread out throughout the park.***

After students have completed their map, have each group display their maps and explain why they chose the placement of the venues.

INTERVENTION

For extra help with topics in this lesson, please refer to the Intervention Table. For students who may really struggle with the Battleship Game, you could limit their game board to only two quadrants—try every combination (I and II, I and III, I and IV, etc.).

CLOSING/SUMMARIZER

Have the class generate numerous ideas for teaching the coordinate system to other students, including the use of mnemonic devices to remember that *x* is the horizontal component and *y* is the vertical component. Students should work in groups to come up with a three-part lesson made up of an opener, a lesson, and a closing.

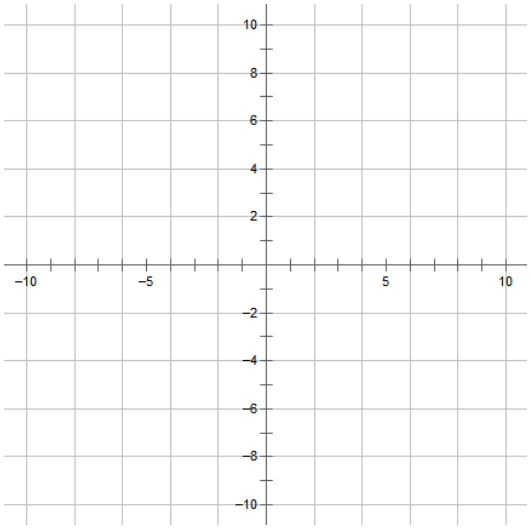
ADDITIONAL PRACTICE

Students should present their three-part lesson on graphing in the coordinate plane to a fellow student or sibling. They should then summarize the strengths and weaknesses of their teaching experience and provide at least one "next step" if they were to keep teaching their "student." ***Responses/lessons will vary. Be sure to discuss what components should be included in the summary.***

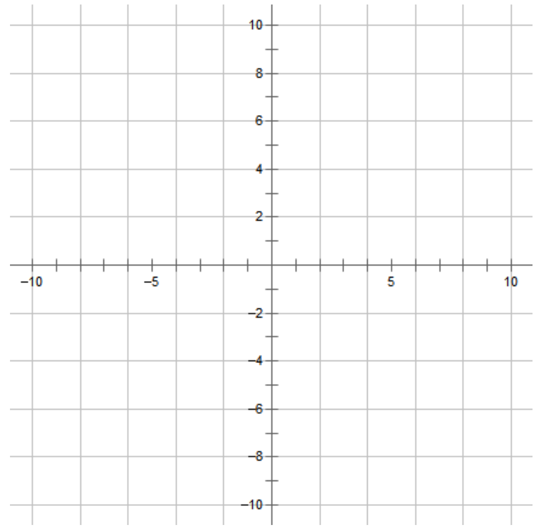
Students never seem to tire of creating pictures on coordinate axes. Some holiday graphs can be found on http://www.math-aids.com/Graphing/Four_Quadrant_Graphing_Characters.html , or students can create their own.

Don’t Sink My Battleship!

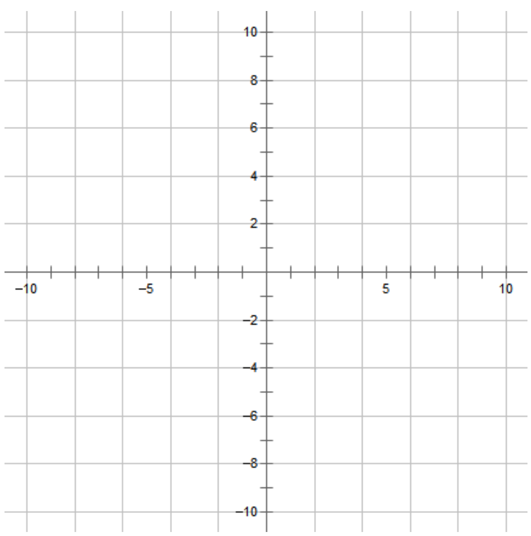
Battleship



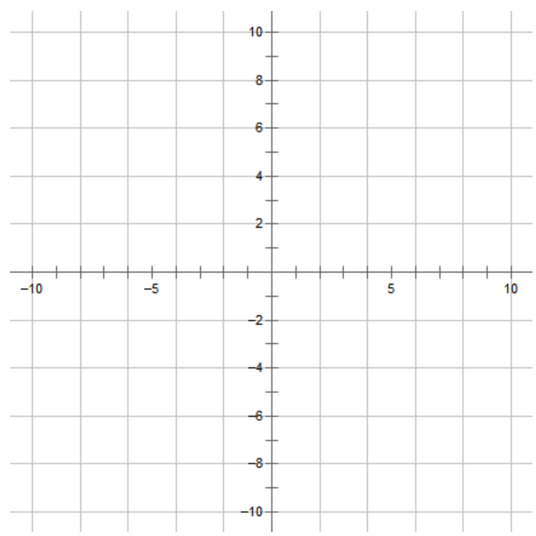
Battleship



Battleship



Battleship



Student Handout: Grid City Spring Festival

Grid City is getting ready for its spring festival! There will be 8 venues: arts and crafts, face painting, cotton candy and popcorn stand, barbecue pit, 2 bounce houses, a Playboxx station and a performance stage. These venues will need to be set up around the park. You must remember the following when designing your map:

- The travel distance and proximity between venues
- The performance stage has to be positioned so that people can see it from any venue in the park.
- The barbecue pit should be 24 units away from the stage, and have the same y-coordinate.

In the park, there will also be 6 port-a-potties, a ticket/information booth, and 4 sections of picnic tables.

- The port-a-potties need to be set up on the left side of the park, all with the same x-coordinate.
- The ticket/information booth needs to be somewhere on the y-axis.
- The picnic table sections need to be spread out throughout the park.

[Acting Out](#)

Adapted from Shell Center Leaky Faucet Short Cycle Lesson
http://www.map.mathshell.org/materials/tasks.php?taskid=267&subpage=apprentice

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class. Recommended timing is 120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI1. Students will create and solve equations and inequalities in one variable.

a. Use variables to represent an unknown number in a specified set (conceptual understanding of a variable). (MGSE6.EE.2, 5, 6)
e. Use variables to solve real-world and mathematical problems. (MGSE6.EE.7, MGSE.7.EE.4)

MFAEI2. Students will use units as a way to understand problems and guide the solutions of multi-step problems.

a. Choose and interpret units in formulas. (MGSE9-12.N.Q.1)
b. Choose and interpret graphs and data displays, including the scale and comparisons of data. (MGSE3.MD.3, MGSE9-12.N.Q.1)
c. Graph points in all four quadrants. (MGSE6.NS.8)

STANDARDS FOR MATHEMATICAL PRACTICE

1. **Make sense of problems and persevere in solving them.** Students determine different ways of measuring a fixed distance from a given point and strategize the most accurate way to convert measurements.
5. **Use appropriate tools strategically.** Students are given multiple tools to choose from to model the scenario in Part I.
6. **Attend to precision.** Students determine when to round their answers based on the units of measurement.

EVIDENCE OF LEARNING/LEARNING TARGET
By the conclusion of this lesson, students should be able to:

- Model and write an equation in one variable and solve a problem in context.
- Create one-variable linear equations and inequalities from contextual situations.
- Solve word problems where quantities are given in different units that must be converted to understand the problem.

MATERIALS

Student Handout: Acting Out
Colored pencils
Compass
String
Graph paper

ESSENTIAL QUESTION

How do I choose and interpret units consistently in formulas?

GROUPING

- Part I: Small group / whole group
- Part II: Partner / Individual

OPENER/ACTIVATOR

Part I:

Erik and Kim are actors at a theater. Erik lives 5 miles from the theater and Kim lives 3 miles from the theater. Their boss, the director, wonders how far apart the actors live.

Comments

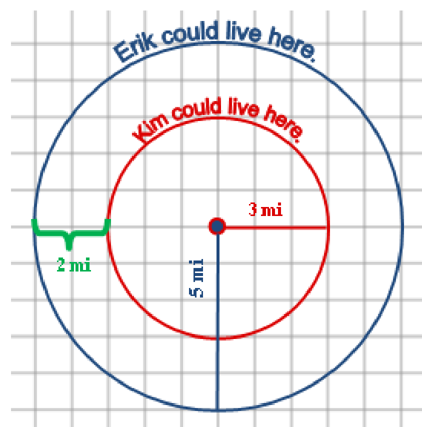
Students should understand that Erik and Kim could live anywhere on the circle with the theater as the center and the radius as the distance that they live from the theater.

1. On the given grid:

   a. Pick a point to represent the location of the theater.

   b. Illustrate all of the possible places that Erik could live on the grid paper.

   c. Using a different color, illustrate all of the possible places that Kim could live on the grid paper.



2. What is the smallest distance, *d*, that could separate their homes? How did you know?
Solution *5 – 3 = 2 miles*

3. What is the largest distance, *d*, that could separate their homes? How did you know?
Solution *5 + 3 = 8 miles*

4. Write and graph an inequality in terms of *d* to show their boss all of the possible distances that could separate the homes of the 2 actors.

Solution
An inequality that could represent this distance could be $2 \leq d \leq 8$ miles.
Graphing this inequality should look like the graph shown below.

Students should understand that the solid dots on the graph represent the fact that Erik and Kim could live exactly 2 miles or exactly 8 miles apart. Should the situation have been different and they lived more than 2 miles or less than 8 miles apart, those dots would have been left open, or not filled in. The space shaded on the number line between the 2 and 8 means that they could live any of those distances apart.

LESSON
Part II: Extension Problem

The actors are good friends since they live close to each other. Kim has a leaky faucet in her kitchen and asks Erik to come over and take a look at it.

1. Kim estimates that the faucet in her kitchen drips at a rate of 1 drop every 2 seconds. Erik wants to know how many times the faucet drips in a week. Help Erik by showing your calculations below.

Solution:

60 sec = 1 min
60 min = 1 hour
24 hours = 1 day
7 days = 1 week

(60) (60) (24) (7) = 604,800
604800 ÷ 2 = 302,400 drops per week

365 days = 1 year

$\frac{(60)(60)(24)(365)}{2}$***= 15,768,000 drops per year [this information will be used in item 2.]***

2. Kim estimates that approximately 575 drops fill a 100 milliliter bottle. Estimate how much water her leaky faucet wastes in a year.

Solution:

15,768,000 ÷ 575 = 27,422.608

About 27,423 of 100 millimeter bottles would be filled.
Encourage students to write this answer in the most user-friendly measurement (i.e., liters)

27,423(100) = 2,742,300 milliliters or 2,742.3 Liters

CLOSING/SUMMARIZER

Discuss the following questions with students, "What units were important in this lesson? Were there any units that were not important? What other lessons have we completed in this module in which units were important? Are there any commonalities among this lesson and other lessons we have completed?"

ADDITIONAL PRACTICE

Have students investigate modifications of the leaky faucet situation:

1. Kim's faucet drips at a rate of 2 drops every second. Complete items 1 and 2 from Part II of the lesson with this new rate of dripping. ***Solutions will be four times as large. Encourage students to estimate and analyze what is happening rather than automatically re-doing the mathematics.***

2. Kim's faucet drips at a rate of *d* drops every second. Complete items 1 and 2 from Part II of this lesson by creating algebraic expressions.
   Part I: (60) (60) (24) (7) = 604,800
   604,800d drops per week

   Part II: (604800d)/575 bottles
3. Kim's faucet drips at a rate of 1 drop every *s* seconds. Complete items 1 and 2 from Part II of this lesson by creating algebraic expressions.
   Part I: 604800/s drops per week

   Part II: (604800/s)/575 bottles

Student Handout: Acting Out

Name________________________________ **Date**_________________

Adapted from Shell Center Leaky Faucet Short Cycle Lesson
http://www.map.mathshell.org/materials/tasks.php?taskid=267&subpage=apprentice

Part I:

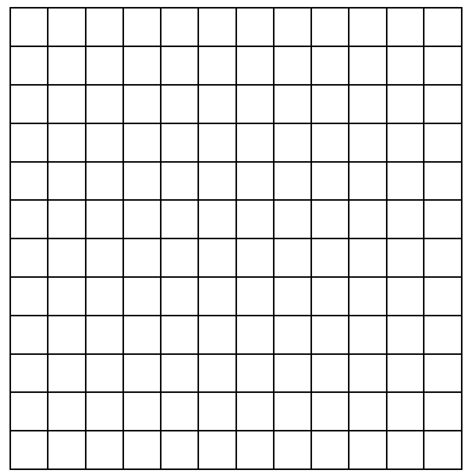
Erik and Kim are actors at a theater. Erik lives 5 miles from the theater and Kim lives 3 miles from the theater. Their boss, the director, wonders how far apart the actors live.

1. On the given grid:

   a. Pick a point to represent the location of the theater.

   b. Illustrate all of the possible places that Erik could live on the grid paper.

   c. Using a different color, illustrate all of the possible places that Kim could live on the grid paper.



2. What is the smallest distance, *d*, that could separate their homes? How did you know?

3. What is the largest distance, *d*, that could separate their homes? How did you know?

4. Write and graph an inequality in terms of *d* to show their boss all of the possible distances that could separate the homes of the 2 actors.

Part II: Extension

The actors are good friends since they live close to each other. Kim has a leaky faucet in her kitchen and asks Erik to come over and take a look at it.

1. Kim estimates that the faucet in her kitchen drips at a rate of 1 drop every 2 seconds. Erik wants to know how many times the faucet drips in a week. Help Erik by showing your calculations below.

2. Kim estimates that approximately 575 drops fill a 100 milliliter bottle. Estimate how much water her leaky faucet wastes in a year.

Literal Equations

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class. Recommended timing is 120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI4. Students will solve literal equations.

a. Solve for any variable in a multi-variable equation. (MGSE6.EE.9, MGSE9-12.A.REI.3)
b. Rearrange formulas to highlight a particular variable using the same reasoning as in solving equations. *For example, solve for the base, b, in* $A = \frac{1}{2}bh$. (MGSE9-12.A.CED.4)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students choose the appropriate algebraic operations in order to highlight a specific quantity in each equation.
2. Reason abstractly and quantitatively. Students interpret parts of an expression and equation in context.
6. Attend to precision. Students use the properties of equality precisely.
7. Look for and make use of structure. Students seek patterns or structures to solve equations for a specific variable.
8. Look for and express regularity in repeated reasoning. Students generalize effective processes for solving equations based upon experiences.

EVIDENCE OF LEARNING/LEARNING TARGET
By the conclusion of this lesson, students should be able to:

- Extend the properties of equality used in solving numerical equations to rearrange formulas to highlight a quantity of interest.

MATERIALS
Projector and laptop
Literal Equations power point
35 index cards with letters and operations on them

ESSENTIAL QUESTIONS
How do I interpret parts of an expression in terms of context?
How can I rearrange formulas to highlight a quantity of interest?
What arithmetic and algebraic properties do I have to consider when rearranging formulas?

OPENER/ACTIVATOR

(Using slide 1 of the power point) Pose the following question to students: What does the word 'literal' mean? Complete the practice in the slides, waiting until the end of the lesson for the final slide question (see closing below).

LESSON

After students have completed the practice in the power point, have them do the following sorting activity. A visual and variations on this activity can be found at http://handsonmathinhighschool.blogspot.com/2012/07/made4math-3-literal-equations.html.

Group the students in pairs. Give each pair a set of 35 notecards (laminated if possible) with the following letters and operations written on them: X, Y, D, A, B, S, T, Z, A, R, F, S, T, X, Z, A, +, =, +, 'division bar,' =, -, 'division bar,' =, (,), -, =, +, and 6 blank cards. They are to use the cards to set up the following literal equations, and solve for the indicated variable:

$X+Y = D$; ***solve for X*** $(X = D - Y)$

$\frac{A+B}{S} = T$; ***solve for A*** $(A = TS - B)$

$Z - \frac{A}{R} = F$; ***solve for R*** $(R = \frac{-A}{F-Z})$

$S(T - X) = Z + A$; ***solve for T*** $(T = \frac{Z+A}{S} + X)$

As students work to reorganize the cards to solve for the indicated variable, they should write the inverse operation or reciprocal of the fraction on the back of the card. The blank cards are there to assist with "extra" operations (for instance, in equation #3, neither Z nor F has an explicit sign, so a student may want to create a "-" card to use when solving.

INTERVENTION

For extra help with topics in this lesson, please refer to the Intervention Table.

CLOSING/SUMMARIZER

(Using the last slide of the presentation) Pose the following question: When is solving a literal equation helpful in real life? Generate a list of situations as a class and discuss the value of being able to isolate variables for each situation. Review arithmetic and algebraic properties as needed.

ADDITIONAL PRACTICE

Assign one of the two tasks found on the Illustrative Mathematics website using the following link: https://www.illustrativemathematics.org/HSA-CED.A.4 (Read through the commentary for each task before deciding which one to assign.) A list of equations can be found at: https://www.illustrativemathematics.org/content-standards/HSA/CED/A/4/tasks/393

Free Throw Percentages

In this lesson, students will use a system of linear functions to solve a problem situation. This lesson could be modified to reflect other sports, teams, and players that are favorites of your students.

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Recommended timing is 90 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI3. Students will create algebraic models in two variables.

a. Create an algebraic model from a context using systems of two equations. (MGSE6.EE.6, MGSE8.EE.8, MGSE9-12.A.CED.2)
b. Find approximate solutions using technology to graph, construct tables of values and find successive approximations. (MGSE9-12.A.REI.10, 11)
c. Represent solutions to systems of equations graphically or by using a table of values. (MSGSE6.EE5; MGSE7.EE3; MGSE8.EE.8, MGSE9-12.A.CED.2)
d. Analyze the reasonableness of the solutions of systems of equations within a given context. (MGSE6.EE.5, 6, MGSE7.EE.4)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students analyze the given information and make conjectures about the meaning of the solution and plan a means of determining a solution.
2. Reason abstractly and quantitatively. Students make sense of quantities and their relationships in context.
3. Construct viable arguments and critique the reasoning of others. Students justify their conclusions.
4. Model with mathematics. Students model the real-world problem with simultaneous equations.
5. Use appropriate tools strategically. Students use graphs and tables to help organize and provide a solution to the problem.
6. Attend to precision. Students communicate precisely and show their mathematical thinking.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Create equations in two variables to represent each situation.
- Create a table of values to represent the relationship between two variables.
- Use a graph to determine a solution and discuss reasonableness of their answers.

MATERIALS

- Colored pencils
- Straightedge
- Graphing calculator *(optional)*
- Graph paper http://incompetech.com/graphpaper
- Copies of Student Handout

ESSENTIAL QUESTIONS

How can I translate a problem situation into a system of equations?
What does the solution of a system tell me about the answer to a problem situation?

OPENER/ACTIVATOR

Ask the students about their favorite basketball players and if they know any statistics about them, as well as the meaning of those statistics.

LESSON

Imagine that you are sitting in front of a television watching the Miami Heat playing the Dallas Mavericks. Tony Wade drives the lane and is fouled. As he steps to the free throw line, the announcer states that "Wade is hitting 82 percent of his free throws this year." He misses the first shot, but makes the second. Later in the game, Tony Wade is fouled for the second time. As he approaches the free throw line, the announcer states that "Wade has made 78 percent of his free throws so far this year."

1. How are free throw percentages calculated?

Solution

Free throw percentages are calculated by dividing the total number of attempted shots into the number of successful shots.

2. What kind of numbers must we use? Why?

Solution

We are dealing with positive rational numbers that are less than or equal to 1 because they cannot miss more shots than they attempt and if they were successful for every attempt the percentage would equal 1.

3. What algebraic relationships could you write to represent the two situations?

Solution

If x = the total number of attempted free throws for the first situation and y = the number of successful shots for the first situation, the first situation could be represented as either the proportion $\frac{y}{x}=\frac{82}{100}$***, or the equation y = 0.82x.***

The second situation could be represented as either the proportion $\frac{y+1}{x+2}=\frac{78}{100}$***, or the equation y = 0.78x + 0.56.***

Some students may want to let x – 2 = the first situation's attempted free throws with x = to the second situation's free throws instead. This would still be mathematically correct.

4. Create a table of values for each situation.

5. By the second time he went to the free throw line, how many free throws had Tony Wade attempted so far this year?

Solution

This may be solved in several different ways; however, help students use the table to provide a reasonable answer.
Tony Wade has attempted 14 free throws so far this year.

6. By the second time he went to the free throw line, how many free throws had he made so far this year?

Solution

Once again, the students may work this in different ways. Most will see from the table that the value of y when x = 14 in both tables is 11.48.
This result will most likely spark some good discussion about percentages in sports. To arrive at this answer must mean that the percentages were rounded and were not exact figures.

7. Are the answers you found in problems 4 and 5 unique? Why or why not?

Solution

Students should discover that using the given percentages, everyone found the same results because the only point that the two equations have in common is the point (14, 11.48).

8. How could you graphically defend your answer?

Solution

Students may graph these equations on the same coordinate plane using technology.

INTERVENTION
For extra help with topics in this lesson, please refer to the Intervention Table.

CLOSING/SUMMARIZER
Later in the season Wade is now hitting 84 percent of his free throws. If he has made 64 free throws, how many free throws has he attempted? Would it make sense for this answer to have a decimal place? Why or why not?

ADDITIONAL PRACTICE
Assign the task found on the Illustrative Mathematics website using the following link: https://www.illustrativemathematics.org/content-standards/6/EE/B/tasks/985 (Read through the commentary before assigning.)

Student Handout: Free Throw Percentages

Imagine that you are sitting in front of a television watching the Miami Heat playing the Dallas Mavericks. Tony Wade drives the lane and is fouled. As he steps to the free throw line, the announcer states that "Wade is hitting 82 percent of his free throws this year." He misses the first shot, but makes the second. Later in the game, Tony Wade is fouled for the second time. As he approaches the free throw line, the announcer states that "Wade has made 78 percent of his free throws so far this year."

1. How are free throw percentages calculated?

2. What kind of numbers must we always be dealing with? Why?

3. What algebraic relationships could you write to represent the two situations?

4. Create a table of values for each equation.

5. By the second time he went to the free throw line, how many free throws had Tony Wade attempted so far this year?

6. By the second time he went to the free throw line, how many free throws had he made so far this year?

7. Are the answers you found in problems 4 and 5 unique? Why or why not?

8. How could you graphically defend your answer?

Stacking Cups

Lesson adapted from http://www.estimation180.com/stackingcups.html

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Recommended timing is 120-150 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI3. Students will create algebraic models in two variables.

a. Create an algebraic model from a context using systems of two equations. (MGSE6.EE.6, MGSE8.EE.8, MGSE9-12.A.CED.2)
b. Find approximate solutions using technology to graph, construct tables of values and find successive approximations. (MGSE9-12.A.REI.10, 11)
c. Represent solutions to systems of equations graphically or by using a table of values. (MGSE6.EE.5;MGSE7.EE.3;MGSE8.EE.8, MGSE9-12.A.CED.2)
d. Analyze the reasonableness of the solutions of systems of equations within a given context. (MGSE6.EE.5, 6, MGSE7.EE.4)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students analyze the given information and make conjectures about the meaning of the solution and plan a means of determining a solution.
2. Reason abstractly and quantitatively. Students make sense of quantities and their relationships in context.
3. Construct viable arguments and critique the reasoning of others. Students justify their conclusions.
4. Model with mathematics. Students model the real-world problem with simultaneous equations.
5. Use appropriate tools strategically. Students use graphs and tables to help organize and provide a solution to the problem.
6. Attend to precision. Students communicate precisely and show their mathematical thinking.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Create equations in two variables to represent each situation.
- Create a table of values to represent the relationship between two variables.
- Use a graph to determine a solution and discuss reasonableness of their answers.

MATERIALS

Student Handout
Videos: http://www.estimation180.com/stackingcups.html
Image 1: http://www.estimation180.com/uploads/1/3/8/8/13883834/stacking_cups_act_2a.jpg
Image 2: http://www.estimation180.com/uploads/1/3/8/8/13883834/stacking_cups_act_2b.jpg

ESSENTIAL QUESTION

How can systems of equations be used to solve real-world problems?

OPENER/ACTIVATOR

ACT 1: Watch the video, (link: http://www.estimation180.com/stackingcups.html)
Ask students what questions they have about the video.

LESSON

ACT 2:
What information would be useful to know here and how would you get it?

Image 1: Dimensions of Styrofoam cup,
http://www.estimation180.com/uploads/1/3/8/8/13883834/stacking_cups_act_2a.jpg

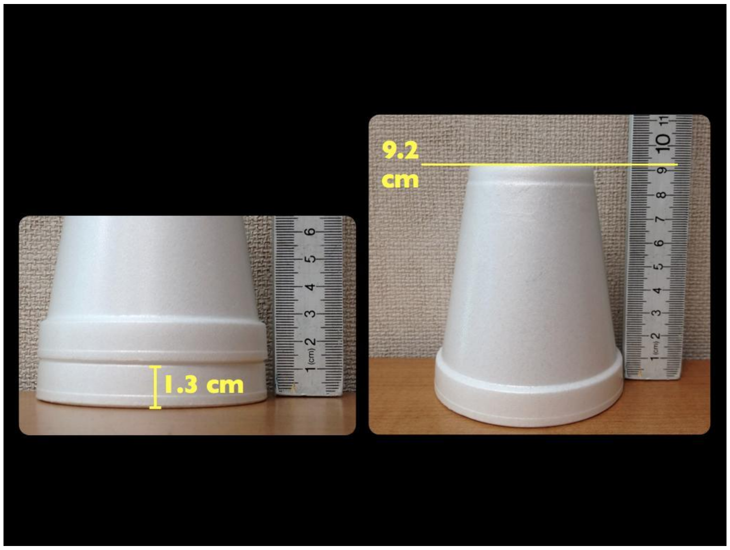
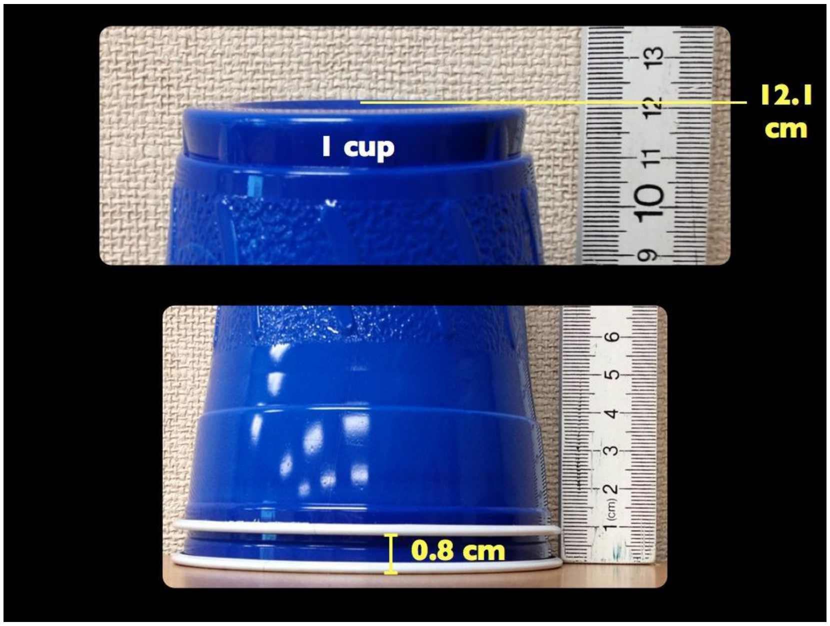


Image 2: Dimensions of plastic cups:
http://www.estimation180.com/uploads/1/3/8/8/13883834/stacking_cups_act_2b.jpg



With the information from Act 2, find out where the stacks of cups will tie.

- Students may work individually for 5-10 minutes to come up with a plan, test ideas, and problem-solve.
- After individual time, students may work in groups (3-4 students) to collaborate and solve this task together.
- **Finished early?** Challenge students with a Sequel (*extension below*).

INTERVENTION

For extra help with topics in this lesson, please refer to the Intervention Table. **Scaffold for Students who Struggle:**

- Provide students with cups to simulate the activity. Note: The measurements above may need to be altered due to manufacturer.
- Provide students with a table to complete.

Styrofoam Cups		*Solo Cups*	
# of Styrofoam Cups	**Height of Styrofoam Cups**	**# of Solo Cups**	**Height of Solo Cups**

CLOSING/SUMMARIZER

ACT 3
Video: The answer, http://www.estimation180.com/stackingcups.html
If we weren't exactly right, what could account for the error?

ACT 4

Extension if time permits:

- **Create a Title:** How should we title this lesson so it captures the mathematics we used and where we used it?
- **The Sequel:** Invent 2 cups where their height will be equal at 100 cups.

ADDITIONAL PRACTICE

Assign one of the tasks found on the Illustrative Mathematics website using the following link: https://www.illustrativemathematics.org/EE (Scroll down to the tasks for standard 8.EE.C.8 and read through the commentary for each task before deciding which one to assign.)

Student Handout: Stacking Cups

Name: ________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question: ____

Estimate the result of the main question? Explain?		
Place an estimate that is too high and too low on the number line		
Low estimate	*Place an "x" where your estimate belongs*	High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information: ________________________________
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Planning a Party

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class. Recommended timing is 90 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI3. Students will create algebraic models in two variables.

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b. Find approximate solutions using technology to graph, construct tables of values and find successive approximations. (MGSE9-12.A.REI.10, 11)
c. Represent solutions to systems of equations graphically or by using a table of values. (MGSE6.EE5;MGSE7.EE.3;MGSE8.EE.8, MGSE9-12.A.CED.2)
d. Analyze the reasonableness of the solutions of systems of equations within a given context. (MGSE6.EE.5, 6, MGSE7.EE.4)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students analyze the given information and make conjectures about the meaning of the solution and plan a means of determining a solution.
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6. Attend to precision. Students communicate precisely and show their mathematical thinking.

EVIDENCE OF LEARNING/LEARNING TARGET
By the conclusion of this lesson, students should be able to:

- Create equations in two variables to represent each situation.
- Create a table of values to represent the relationship between two variables.
- Use a graph to determine a solution and discuss reasonableness of their answers.

MATERIALS

Student Handout
Graph paper http://incompetech.com/graphpaper
Calculators (optional)

ESSENTIAL QUESTIONS

How can I translate a problem situation into a system of equations?
What does the solution to a system tell me about the answer to a problem situation?

OPENER/ACTIVATOR

Ask the students how many of them have planned a party before. Discuss ideas like determining how much food or drink items to purchase. After the list of ideas is generated, discuss which items are variable (e.g., amount of food and drink needed) and which items are constant regardless of the number of people in attendance.

LESSON

Ms. England is planning a party for students who have met their goal of reading seven books during the most recent nine weeks of school. Using a total of $112, she plans to get pizzas that cost $12 each and drinks that cost $0.50 each. If she purchases four times as many drinks as pizzas, how many of each should she buy? Show how you know using more than one method. Obtain graph paper from your teacher if necessary. Use the table below if it is helpful.

Solution

Let students struggle for several minutes before providing them with scaffolded information to assist them in determining a solution.

Let x = the number of pizzas and y = the number of drinks. Then $12x + .5y = 112$ and $y = 4x$.

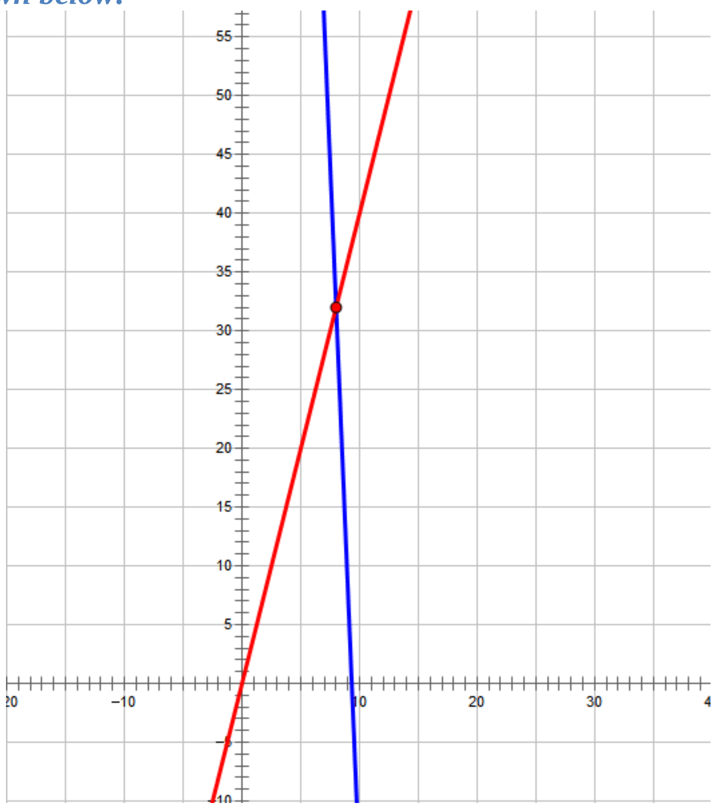
In this lesson, one of the equations does not have the number one as the coefficient of y. Because one of the equations does have y isolated on one side of the equation, students may choose to solve for y before graphing or completing the table.

In translating the condition that there should be four times as many drinks as pizza, a very frequent reversal error occurs. To help students who translate $x = 4y$ to understand why this is incorrect, put in a particular value for the number of pizzas and find what the equation gives for the corresponding number of drinks.

A numerical solution to this problem would be to set up a table of values such as the one shown below:

Number of Pizzas	*Number of Drinks*	*Amount Spent*
1	***4***	***$14***
2	***8***	***$28***
3	***12***	***$42***
4	***16***	***$56***
5	***20***	***$70***
6	***24***	***$84***
7	***28***	***$98***
8	***32***	***$112***

A graphical solution to this problem would be to set up a graph of the system of equations such as the one shown below:



INTERVENTION

For extra help with topics in this lesson, please refer to the Intervention Table.

CLOSING/SUMMARIZER

Discuss the pros/cons of each solution method. Have students identify their "preferred" solution method and defend their choice. Group class by solution method and have them devise a new scenario for which their solution method is the "best" method. Share these new scenarios with the entire class.

ADDITIONAL PRACTICE

Assign one of the tasks found on the Illustrative Mathematics website using the following link: https://www.illustrativemathematics.org/EE (Scroll down to the tasks for standard 8.EE.C.8 and read through the commentary for each task before deciding which one to assign.)

Student Handout: Planning a Party

Ms. England is planning a party for students who have met their goal of reading seven books during the most recent nine weeks of school. Using a total of $112, she plans to get pizzas that cost $12 each and drinks that cost $0.50 each. If she purchases four times as many drinks as pizzas, how many of each should she buy? Show how you know using more than one method. Obtain graph paper from your teacher if necessary. Use the table below if it is helpful.

Field Day

In this lesson, students will use a system of linear functions to solve a problem situation. This lesson could be modified to reflect other sports, teams, and players that are favorites of your students.

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Recommended timing is 90-120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI3. Students will create algebraic models in two variables.

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d. Analyze the reasonableness of the solutions of systems of equations within a given context. (MGSE6.EE.5, 6, MGSE7.EE.4)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students analyze the given information and make conjectures about the meaning of the solution and plan a means of determining a solution.
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5. Use appropriate tools strategically. Students use graphs and tables to help organize and provide a solution to the problem.
6. Attend to precision. Students communicate precisely and show their mathematical thinking.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Create equations in two variables to represent each situation.
- Create a table of values to represent the relationship between two variables.
- Use a graph to determine a solution and discuss reasonableness of their answers.

MATERIALS

- Stopwatch
- Tape measure (such as those used by track and field team) to measure classroom length
- Colored pencils
- Straightedge
- Graphing calculator *(optional)*
- Graph paper http://incompetech.com/graphpaper
- Student Handout

ESSENTIAL QUESTIONS

How can I translate a problem situation into a system of equations?
What does the solution to a system tell me about the answer to a problem situation?

OPENER/ACTIVATOR

Ask students to estimate how many feet per second they walk. Collect data on 3-4 students by timing how long it takes them to walk the length of your classroom, and finding the unit rate of feet per second.

LESSON

You have been selected to be a part of a team (Team A) to participate in a one-mile race on Field Day. Another student from Team B will race against you. You are able to run 12 feet per second. Since the student from Team B runs 10 feet per second, you have been asked to let him have a 1000-ft. head start. If both of you maintain the estimated rates (12 feet per second and 10 feet per second), would you be able to beat your opponent? Use more than one method to justify your conclusion.

Solution

The total distance run in x seconds by the student from Team B will be 1000 + 10x. The total distance run in x seconds by the student from Team A will be 12x. The time required for the distances to be the same is found by solving 1000 + 10x = 12x. This yields 2x = 1000, or x = 500 seconds. However, after 500 seconds, both students would have run 6000 feet. Since this is greater than one mile (5280 feet), the race would already have ended. Therefore, it is not possible for the student from Team A to win under these circumstances.

Using graphing calculator technology, enter $y_1 = 1000 + 10x$ and $y_2 = 12x$.
Students will need to realize that the y value tells the distance from the start line and that the distance of the runner from Team B from the starting line is always greater than the distance of the runner from Team A from the starting line until the time is 500 seconds. At that point the distances are the same. (Note that although an appropriate window for this problem situation might be $x_{min}= 0$, $x_{max}= 500$, $y_{min} = 0$, and $y_{max} = 6000$, the graphs are better compared with settings such as $x_{min}= 400$, $x_{max} = 550$, $y_{min} = 5000$, and $y_{max}= 6200$. In tracing along either of the lines, the student can determine that the y value exceeds 5280 feet before the point of intersection of the two lines. This means that the runner from Team A cannot pass the runner from Team B in less than a mile.)

A sample of Georgia eighth-grade student work is shown below.

field day

answer- no. you could not beat your opponent.

Team a

$12x=y$

x	y
12(400)	4800
12(410)	4920
12(420)	6040
12(430)	5160

Team B

$10x+1000=y$

x	y
10(400)+1000	5000
10(410)+1000	5100
10(420)+1000	5200
10(430)+1000	5300

at 430 seconds, runner B has already gone over a mile, and runner a has not even reached a mile yet.

one Mile=5260 ft.

Team a-

12 represents how many feet the person can run in one second.

x represents how many seconds they have been running

y represents how far the individual has ran in x seconds

Team B-

10 represents how many feet the person can run in one second.

X represents how many seconds the runner has been running.

1000 represents how many feet ahead runner B was at the beginning.

y represents how far the individual has ran in x seconds.

INTERVENTION

For extra help with topics in this lesson, please refer to the Intervention Table.

CLOSING/SUMMARIZER

How many feet would your opponent need as a head start in order for the two of you to tie in the race? Show your work and explain your thinking as you did in the original problem.

ADDITIONAL PRACTICE

Assign one of the tasks found on the Illustrative Mathematics website using the following link: https://www.illustrativemathematics.org/HSA-REI (Scroll down to the tasks for standard HSA-REI.C.6 and read through the commentary for each task before deciding which one to assign.)

Student Handout: Field Day

You have been selected to be a part of a team (Team A) to participate in a one-mile race on Field Day. Another student from Team B will race against you. You are able to run 12 feet per second. Since the student from Team B runs 10 feet per second, you have been asked to let him have a 1000-ft. head start. If both of you maintain the estimated rates (12 feet per second and 10 feet per second), would you be able to beat your opponent? Use more than one method to justify your conclusion.

Quick Check II

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI3. Students will create algebraic models in two variables.

a. Create an algebraic model from a context using systems of two equations. (MGSE6.EE.6, MGSE8.EE.8, MGSE9-12.A.CED.2)
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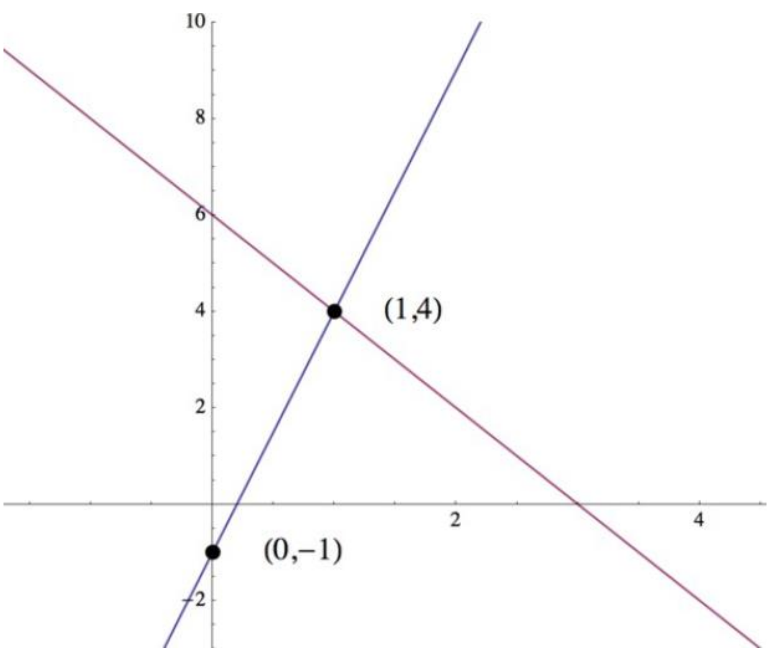
c. Represent solutions to systems of equations graphically or by using a table of values. (MGSE6.EE5; MSGE7.EE.3;MGSE8.EE.8, MGSE9-12.A.CED.2)
d. Analyze the reasonableness of the solutions of systems of equations within a given context. (MGSE6.EE.5, 6, MGSE7.EE.4)

Teacher's Edition for Quick Check II

Adapted from: :
https://www.illustrativemathematics.org/contentstandards/8/EE/C/8/tasks/73 ***and***
https://www.illustrativemathematics.org/content-standards/8/EE/C/8/tasks/1364 ***and*** **:**
https://www.illustrativemathematics.org/content-standards/HSA/CED/A/3/tasks/220

1. Draw two lines that intersect only at the point (1, 4). One of the lines MUST pass through the point (0, -1).

We can first locate the two given points, (1, 4) and (0, -1), to create the first line. The second line can be any other line that passes through (1, 4) but not (0, -1), so there are many possible answers. Below is one possible solution.



2. Write the equation of each of these lines.

The equations would be: $y = 5x - 1$ and $y = -2x + 6$.

3. The only coins that Alexis has are dimes and quarters. She has a total of 40 coins. Her coins have a total value of $5.80. Write a system of equation that can be used to find the number of dimes (*d*) and the number of quarters (*q*) Alexis has.

d + q = 40
0.10d + 0.25q = 5.80

4. Can you determine whether a system of equations has a solution by looking at the graph of the equations? Explain.
Answers will vary.
Yes, you can make a general observation that if a system of linear equations has a solution that solution corresponds to the intersection point of the two lines. The coordinate pairs of the intersection point will satisfy both equations.

5. Kimi and Jordan are each working during the summer to earn money in addition to their weekly allowance and they are saving all of their money. Kimi earns $9 an hour at her job and her allowance is $8 per week. Jordan earns $7.50 an hour and his allowance is $16 per week.

a) Complete the tables below:

Number of hours worked in a week, *h*	0	1	2	3	4	5	6	7
Kimi's weekly total savings, *K*	***$8.00***	***$17.00***	***$26.00***	***$35.00***	***$44.00***	***$53.00***	***$62.00***	***$71.00***

Number of hours worked in a week, *h*	0	1	2	3	4	5	6	7
Jordan's weekly total savings, *J*	***$16.00***	***$23.50***	***$31.00***	***$38.50***	***$46***	***$53.50***	***$61.00***	***$68.50***

b) Write an equation that can be used to calculate the total of Kimi's allowance and job earnings at the end of one week given the number of hours she works.

K = 8 + 9h

Kimi's total savings, K, is the sum of her 8 dollar allowance and ($9 per hour X the number of hours she works, h).

c) Write and equation that can be used to calculate the total of Jordan's allowance and job earnings at the end of one week given the number of hours he works.

J = 16 + 7.5h

Jordan's total savings, J, is the sum of his 16 dollar allowance and ($7.50 per hour X the number of hours he works, h).

d) Approximately how many hours will Kimi and Jordan have to work in order to have the same amount of money? Explain your answer.

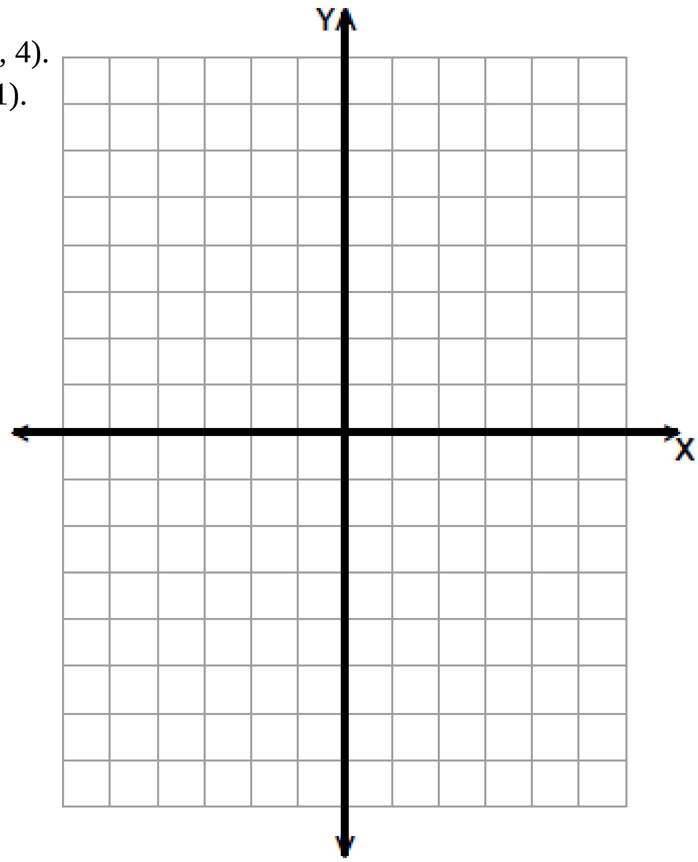
Answers will vary.

Students could set the equations equal to each other and solve. The solution is h = 16/3 or 5 1/3 hours. Students could also make an estimate based on the tables. At 5 hours, Kimi saves less than Jordan but at 6 hours, Kimi saves more than Jordan. At some point between 5 and 6 hours, they will have the same amount of money.

Quick Check II Name ______________________________

Adapted from: :
https://www.illustrativemathematics.org/contentstandards/8/EE/C/8/tasks/73 ***and***
https://www.illustrativemathematics.org/content-standards/8/EE/C/8/tasks/1364 ***and*** **:**
https://www.illustrativemathematics.org/content-standards/HSA/CED/A/3/tasks/220

1. Draw two lines that intersect only at the point (1, 4). One of the lines MUST pass through the point (0, -1).



2. Write the equation of each of these lines.

3. The only coins that Alexis has are dimes and quarters. She has a total of 40 coins. Her coins have a total value of $5.80. Write a system of equation that can be used to find the number of dimes (*d*) and the number of quarters (*q*) Alexis has.

4. Can you determine whether a system of equations has a solution by looking at the graph of the equations? Explain.

5. Kimi and Jordan are each working during the summer to earn money in addition to their weekly allowance and they are saving all of their money. Kimi earns $9 an hour at her job and her allowance is $8 per week. Jordan earns $7.50 an hour and his allowance is $16 per week.

a) Complete the tables below:

Number of hours worked in a week, *h*	0	1	2	3	4	5	6	7
Kimi's weekly total savings, *K*								

Number of hours worked in a week, *h*	0	1	2	3	4	5	6	7
Jordan's weekly total savings, *J*								

b) Write an equation that can be used to calculate the total of Kimi's allowance and job earnings at the end of one week given the number of hours she works.

c) Write and equation that can be used to calculate the total of Jordan's allowance and job earnings at the end of one week given the number of hours he works.

d) Approximately how many hours will Kimi and Jordan have to work in order to have the same amount of money? Explain your answer.

"Illustrative" Check II

1. More Solving Equations: https://www.illustrativemathematics.org/content-standards/8/EE/C/7/tasks/392

2. Analyzing Equations: https://www.illustrativemathematics.org/content-standards/8/EE/C/7/tasks/550

3. Comparing Equations: https://www.illustrativemathematics.org/content-standards/8/EE/C/7/tasks/583

4. Equations Using Distributive Property: https://www.illustrativemathematics.org/content-standards/8/EE/C/7/tasks/999

5. Simultaneous Equations With Tables: https://www.illustrativemathematics.org/content-standards/8/EE/C/8/tasks/73

6. Analyzing Graphs of Systems: https://www.illustrativemathematics.org/content-standards/8/EE/C/8/tasks/1364

7. Writing Systems: https://www.illustrativemathematics.org/content-standards/HSA/CED/A/3/tasks/220

Is It Cheaper To Pay Monthly or Annually?

Source: Robert Kaplinsky, Glenrock Consulting
http://robertkaplinsky.com/work/monthly-or-annually/

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Recommended timing is 120-150 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAEI3. Students will create algebraic models in two variables.

a. Create an algebraic model from a context using systems of two equations. (MGSE6.EE.6, MGSE8.EE.8, MGSE9-12.A.CED.2)
b. Find approximate solutions using technology to graph, construct tables of values and find successive approximations. (MGSE9-12.A.REI.10, 11)
c. Represent solutions to systems of equations graphically or by using a table of values. (MGSE6.EE.5;MGSE7.EE.3;MGSE8.EE.8, MGSE9-12.A.CED.2)
d. Analyze the reasonableness of the solutions of systems of equations within a given context. (MGSE6.EE.5, 6, MGSE7.EE.4)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students analyze the given information and make conjectures about the meaning of the solution and plan a means of determining a solution.
2. Reason abstractly and quantitatively. Students make sense of quantities and their relationships in context.
3. Construct viable arguments and critique the reasoning of others. Students justify their conclusions.
4. Model with mathematics. Students model the real-world problem with simultaneous equations.
5. Use appropriate tools strategically. Students use graphs and tables to help organize and provide a solution to the problem.
6. Attend to precision. Students communicate precisely and show their mathematical thinking.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- Create a table of values to represent the relationship between two variables.
- Use a graph to determine a solution and discuss reasonableness of their answers.

MATERIALS

- Price charts and graphs from website
- Pictures of Disney or Universal characters

ESSENTIAL QUESTIONS

What does the solution to a system tell me about the answer to a problem situation?

Note: Tasks and lessons from Robert Kaplinsky's blog provide excellent resources that incorporate mathematical content, mathematical practices and real-world problem-solving. For more information on his strategies and philosophy, go to http://robertkaplinsky.com/

OPENER/ACTIVATOR

Present students with the following situation: Many theme parks like Disneyland and Universal Studios offer annual passes where customers can pay for a whole year in advance or pay a little in the beginning and make smaller payments each month for the rest of the year. Which payment options saves you the most money?

LESSON

Present students with "Questions to Ask" from website. Read through "Consider This" in preparation for responses. Information on Ticketing in the form of charts and graphs will then be presented to the class. The students may use the problem solving hand-out to make sense of the problem.

CLOSING/SUMMARIZER

Have students present their solutions to each other. Then, present unlabeled samples of student work from website and have students critique the sample student responses using the class-developed rubric.

ADDITIONAL PRACTICE

Assign one of the tasks found on the Illustrative Mathematics website using the following link: https://www.illustrativemathematics.org/HSA-REI (Scroll down to the tasks for standard HSA-REI.C.6 and read through the commentary for each task before deciding which one to assign.)

Student Handout: Understanding the Problem

What problem are you trying to figure out?
What do you already know about the problem?
What do you need to know to solve the problem?

Solution (include rationale, strategies, diagrams, supporting evidence)
What is your conclusion?