Georgia
Standards of Excellence
Curriculum Frameworks

Mathematics

GSE Foundations of Algebra

Module 5: Quantitative Reasoning with Functions

Georgia Department of Education

Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

Module 5: Quantitative Reasoning with Functions

Table of Contents

FOUNDATIONS OF ALGEBRA REVISION SUMMARY

The Foundations of Algebra course has been revised based on feedback from teachers across the state. The following are changes made during the current revision cycle:

- Each module assessment has been revised to address alignment to module content, reading demand within the questions, and accessibility to the assessments by Foundations of Algebra teachers.
- All module assessments, as well as, the pre- and posttest for the course will now be available in GOFAR at the teacher level along with a more robust teacher's edition featuring commentary along with the assessment items.
- All modules now contain "Quick Checks" that will provide information on mastery of the content at pivotal points in the module. Both teacher and student versions of the "Quick Checks" will be accessible within the module.
- A "Materials List" can be found immediately after this page in each module. The list provides teachers with materials that are needed for each lesson in that module.
- A complete professional learning series with episodes devoted to the "big ideas" of each module and strategies for effective use of manipulatives will be featured on the Math Resources and Professional Learning page at https://www.gadoe.org/Curriculum-Instruction-and-Assessment/Curriculum-and-Instruction/Pages/Mathematics.aspx.
- Additional support such as Module Analysis Tables may be found on the Foundations of Algebra page on the High School Math Wiki at http://ccgpsmathematics9-10.wikispaces.com/Foundations+of+Algebra. This Module Analysis Table is NOT designed to be followed as a "to do list" but merely as ideas based on feedback from teachers of the course and professional learning that has been provided within school systems across Georgia.

MATERIALS LIST

Lesson	Materials
1. Relations and Functions	• Copy of the Relations and Functions Graphing student page • Copy of the summarizer
2. Function Rules	• Copy of the lesson
3. Foxes and Rabbits	• Copy of student task • Index cards
4. Quick Check I	• Copy of student page
5. Which Ticket is the Best Deal?	• Copy of ticket prices
6. Reviewing Rate of Change	• Student practice page
7. How Did I Move?	• Index cards • Timers • Activity sheets from website
8. By the Book	• Copies of lessons for students • Straight edge • Graph paper http://incompetech.com/graphpaper
9. Equations of Attack	• Copy of student page • Slope cards • Colored pencils or markers • Coins or counters • Scissors
10. Key Features of Functions	• Student handout
11. Analyzing Linear Functions	• FAL pre-assessment • FAL post-assessment • Mini whiteboards and markers
12. Functions	• Copy of the lesson

Lesson	Materials
13. Quick Check II	• Copy of student page
14. Beads Under the Cloud	• Student handout – Beads Under the Cloud - How did you work? • Handout per group – Sample Responses to Discuss • Samples of student work – per group
15. Floating Down the Lazy River	• Student handout
16. Getting Ready for a Pool Party	• Student copy of the lesson
17. Cell Phones	• Student copy of task • Graphing calculator (optional)
18. Quick Check III	• Copy of student page
19. How Much Does a 100 x 100 In-N-Out Cheeseburger Cost?	• Pictures of burgers

OVERVIEW

In this unit students will:

- Understand characteristics of functions.
- Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.
- Relate the domain of a function to its graph
- Compare and graph functions using sets of ordered pairs consisting of an input and the corresponding output.
- Calculate rates of change of functions, comparing when rates increase, decrease, or stay constant.
- Interpret the equation y = mx + b as defining a linear function whose graph is a straight line.
- Use and interpret the meaning of functional notation

STANDARDS FOR MATHEMATICAL CONTENT

Students will create function statements and analyze relationships among pairs of variables using graphs, table, and equations.

MFAQR1. Students will understand characteristics of functions.

a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.(MGSE9-12.F.IF.1)

b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*. (MGSE9-12.F.IF.5)

c. Graph functions using sets of ordered pairs consisting of an input and the corresponding output. (MGSE8.F.1, 2)

MFAQR2. Students will compare and graph functions.

a. Calculate rates of change of functions, comparing when rates increase, decrease, or stay constant. *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (MGSE6.RP.2; MGSE7.RP.1, 2, 3;MGSE8.F.2,5; MGSE9-12.F.IF.6)

b. Graph by hand simple functions expressed symbolically (use all four quadrants). (MGSE9-12.F.IF.7)

c. Interpret the equation y = mx + b as defining a linear function whose graph is a straight line. (MGSE8.F.3)

d. Use technology to graph non-linear functions. (MGSE8.F.3, MGSE9-12.F.IF.7)

e. Analyze graphs of functions for key features (intercepts, intervals of increase/decrease, maximums/minimums, symmetries, and end behavior) based on context. (MGSE9-12.F.IF.4,7)
f. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the great rate of change.* (MGSE8.F.2)

MFAQR3. Students will construct and interpret functions.
a. Write a function that describes a relationship between two quantities. (MGSE8.F.4, MGSE9-12.F.BF.1)
b. Use variables to represent two quantities in a real-world problem that change in relationship to one another (conceptual understanding of a variable). (MGSE6.EE.9)
c. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of context. (MGSE9-12.F.IF.2)

STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them. High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively. High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others. High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics. High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically. High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision. High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school, they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure. By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8. Look for and express regularity in repeated reasoning. High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. **Students who do not have an understanding of a topic may rely on procedures too heavily.** Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. **In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.** In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

*****Mathematical Practices 1 and 6 should be evident in EVERY lesson*****

ENDURING UNDERSTANDINGS

By the end of the Module, **students should understand**

- the characteristics of functions (what the domain and range represent and how they relate to each other)
- what "rate of change" means in context
- how to compare two functions in different forms (algebraic, geometric, numerical in tables, and verbal)

and students should be able to

- graph functions using ordered pairs, tables, and equations
- interpret y=mx+b as a linear relationship
- use technology to graph non-linear functions
- write a function to describe two quantities

ESSENTIAL QUESTIONS

- What is meant by "rate of change"?
- What is a function?
- How can I graph a function?
- What can I describe from a graph of a function? (What does the graph mean?)
- What does each part of the equation y = mx + b mean in a context?
- How can I compare two functions?
- What does the graph of a linear function look like, compared to a non-linear function?

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The terms below are for teacher reference only and are not to be memorized by the students. Teachers should present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

- Function
- Domain of a function
- Range of a function
- Coordinate plane
- Rate of change
- Slope

The websites below are interactive and include a math glossary suitable for high school children. **Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**

http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary.

http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

SCAFFOLDED INSTRUCTIONAL LESSONS

Lesson Name	**Lesson Type/ *Grouping Strategy***	**Content Addressed**	**Standard(s)**
Relations and Functions	Learning Lesson *Partners*	Identifying functions, domains, ranges, graphing functions	MFAQR1a-c
Function Rules	Formative Lesson *Partners or small group*	Connecting a function described by a verbal rule with corresponding values in a table	MFAQR1a-c
Foxes and Rabbits	Learning Task *Partners or small groups*	Identifying functions, domains, ranges from a table	MFAQR1a,b MFAQR3b
Quick Check I	Formative Assessment *Individual*	Identifying a function and its domain from multiple representations	MFAQR1a-c
Which Ticket is the Best Deal?	Three-Act Task *Partners or small groups*	Finding rate of change, graphing functions	MFAQR2a
Reviewing Rate of Change	Formative Assessment *Individual*	Reviewing rates of change from prior grades	MFAQR2a
How Did I Move?	Learning Task *Partners or small groups*	Plotting points on a graph and determining linear equations and slope	MFAQR2b,c MFAQR3a
By the Book	Learning Task *Partners or small groups*	Determining patterns, relations and functions in real situations, and determining the meaning of slope	MFAQR1a,b MFAQR2a,b c,e MFAQR3a,b
Equations of Attack	Formative Lesson *Individual or partners*	Writing equations given slope and intercept, and determine algebraically if a point lies on a line.	MFAQR2a,b c,e MFAQR3a,b
Key Features of Functions	Short Task *Individual or partners*	Sketching graphs and describing attributes of functions.	MFAQR1a-c MFAQR2a,b c,e
Analyzing Linear Functions	Formative Assessment for Learning *Partners*	Determining slope as rate of change, finding the meaning of x and y intercepts applied to real-world situations, and explaining graphs and tables that represent realistic situation	MFAQR2c,f MFAQR3a,b
Functions	Learning Lesson *Partners*	Working with graphs and equations of linear and non-linear function	MFAQR2b,c d,f MFAQR3a

Quick Check II	Formative Assessment *Individual*	Identifying a function and its domain from multiple representations	MFAQR1a-c MFAQR2a-f
Beads Under the Cloud	Scaffolding Lesson *Partners or small groups*	Identifying patterns in a realistic context	MFAQR1a,b MFAQR3a,b
Floating Down the Lazy River	Learning Task *Partners or small groups*	Using tables and graphs to interpret key features of functions	MFAQR1a-c MFAQR2a,b c,e,f MFAQR3a,b
Getting Ready for a Pool Party	Learning Task *Partners or small groups*	Using a story context to graph and describe key features of functions.	MFAQR1a-c MFAQR2a,b c,e,f MFAQR3a-c
Cell Phones	Formative Lesson *Individual*	Extending the use of function notation, f(x), to real life situations	MFAQR1b,c MFAQR2b,c e,f MFAQR3a-c
Quick Check III	Formative Assessment *Individual*	Comparing, graphing, and interpreting functions	MFAQR2a-f MFAQR3a-c
How Much Does a 100 x 100 In-N-Out Cheeseburger Cost?	Culminating Lesson *Partners or small groups*	Comparing graphs of function, and constructing and interpreting functions.	MFAQR1a-c MFAQR2a,b c,e,f MFAQR3a,b

The assessment for this module can be found through the Georgia Online Formative Assessment Resource (GOFAR). http://www.gadoe.org/Curriculum-Instruction-and-Assessment/Assessment/Pages/Georgia-Online-Formative-Assessment-Resource.aspx

This suggested assessment should be given as the pretest and posttest for this module.

INTERVENTION TABLE

Modules 1-4 include Intervention Tables which provide direct links to Georgia Numeracy Project Intervention tasks and activities specific to fundamental skills. If numeracy skills, such as fluency with fractions, warrant interventions in Module 5, please refer to interventions provided in prior modules. Reviewing Rate of Change is the only lesson with an intervention from Georgia Numeracy Project Intervention tasks and activities, and this suggested activity is linked at point of use.

Relations and Functions

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class. Recommended timing is 1 hour.

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR1. Students will understand characteristics of functions.
a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.(MGSE9-12.F.IF.1)
b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*. (MGSE9-12.F.IF.5)
c. Graph functions using sets of ordered pairs consisting of an input and the corresponding output. (MGSE8.F.1, 2)

Common Misconceptions
Some students will mistakenly think of a straight line as horizontal or vertical only. Some students will reverse the x- and y- axes on the coordinate plane. Emphasizing that the first value is plotted on the horizontal axis (usually x, with positive to the right) and the second is the vertical axis (usually called y, with positive up), points out that this is merely a convention. This could have been otherwise, but standard, customary practice is very useful.

Students may mix up the input and output values/variables. This could result in the inverse of the function.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students must make sense of the problem presented in the lesson and continue to work toward a solution.
3. Construct viable arguments and critique the reasoning of others. Working with a partner, students identify functions and their domain and range.
6. Attend to precision. Students precisely define, describe, and identify components of functions.
8. Look for and express regularity in repeated reasoning. Students recognize repeated reasoning in identifying characteristics of function.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- identify relations
- distinguish functions from other relations
- identify the domain and range of a function
- graph a function, and relate the domain of a function to its graph and to the quantitative relationship it describes.

MATERIALS

- Copy of the Relations and Functions Graphing student page
- Copy of the summarizer

ESSENTIAL QUESTIONS

- What is a function?
- How can you use functions to model real-world situations?
- How can graphs of functions help us to interpret real-world problems?

OPENER/ACTIVATOR/VOCABULARY EMPHASIS

A **relation** is a relationship between two or more data sets that can be compared.
For example, think of all the people in your family and then their heights. The pairing of names and heights is a relation. In relations, the pairs of names and heights are "ordered", which means one comes first and the other comes second. In our example, the name comes first, and the height comes second. The list of names represents the "**domain**", and the set of heights represents the "**range**".

A **function** is a patterned relation in which the x value of every ordered pair only has one y value associated with it. In other words, a function is a special relation, where each element of the domain has exactly one element of the range.

Functions are a sub-set of relations. All functions are relations (pairing of two data sets, with a domain and range), but **NOT all relations are functions** (some relations have REPEATING elements of the domain.)

Consider our example of family members and heights. Let us approach this from a different way. If I call out heights (domain, first element) and you respond with a person of that height (range, second element), we will form a **relation** such as the following:

6’2”	Mike
2’4”	Amy
6’2”	Joseph
5’3”	Karen
5’5”	Debbie

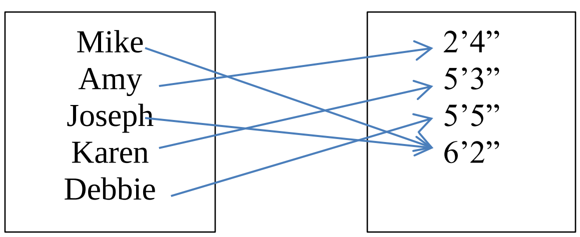
(6’2”, Mike), (2’4”, Amy), (6’2”, Joseph), (5’3”, Karen), (5’5”, Debbie)

Notice that two people are 6’2”. An element of the domain repeats, so 6’2” does not have a unique pairing in the range. It can pair with Mike and with Joseph. But, it IS still a pairing. It is a relation but not a function.

If I call out names, and you respond with the height, we will have a relation that is a **function**.

(Mike, 6’2”), (Amy, 2’4”), (Joseph, 6’2”), (Karen, 5’3”), (Debbie, 5’5”)

In this order, each element of the domain is unique. Each person only has one possible height. Checking the ordered pairs we can determine if every x value maps to just one y value.



TEACHER COMMENTARY
Below are some problems to try with your students:

Which set of ordered pairs represents a function?

- (12, 30), (12, 75), (45, 30)
- (1, 56), (100, 155), (100, 65)
- ***(1.5, 67.5), (30, 780), (100, 250)***

Which table or tables represent a function?

0	3
1	4
1	0

1	***4***
2	***4***
3	***4***

****

5	***1***
7	***2***
9	***3***

****

Which situation would represent a function?

- ***Social security number, person***
- Shoe size, person
- Zip code, person

Gabe states that the combination shown below would NOT be a function. James believes that it IS a function. Who is correct?

($, #), (%, @), (<, #), (*,!), ($, #) ***Gabe is correct; the domain has two $.***

Practice Problems

Which set of ordered pairs represents a function?

- (12, 30), (12, 75), (45, 30)
- (1, 56), (100, 155), (100, 65)
- (1.5, 67.5), (30, 780), (100, 250)

Which table or tables represent a function?

0	3
1	4
1	0

1	4
2	4
3	4

5	1
7	2
9	3

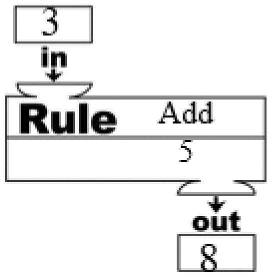
Which situation would represent a function?

- Social security number, person
- Shoe size, person
- Zip code, person

Gabe states that the combination shown below would NOT be a function. James believes that it IS a function. Who is correct?

($, #), (%, @), (<, #), (*, !), ($, #)

Two additional terms we need to introduce are **independent variable** and **dependent variable**. **The elements of the domain are the independent variables**. They "stand alone" and are not changed by other variables. If a function rule says "add five to each member of the domain", **the range is the dependent variable because it DEPENDS on the rule**.



The input, 3, is in the domain, and is the independent variable.
The function rule tells us to add 5 to any element of the domain.
The output, 8, is an element of the range and is the dependent variable.

Using the function diagram above, find the range of a function given the domain {3, 6, 9, 12}.

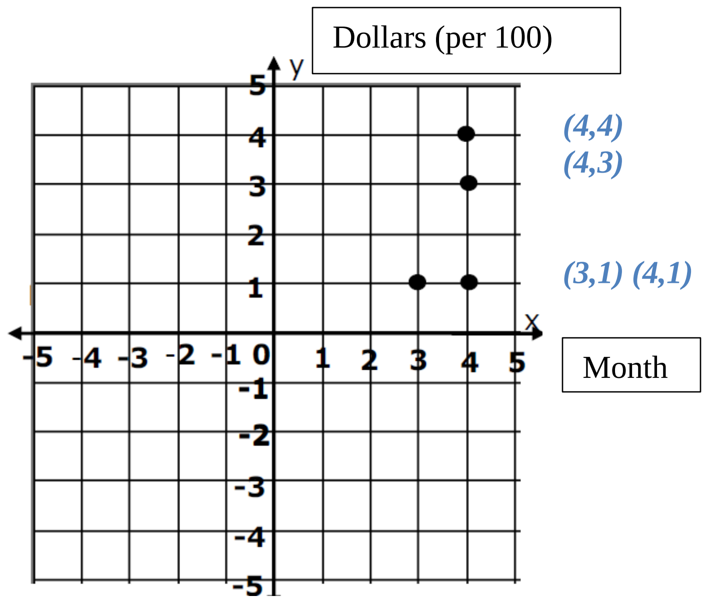
{8, 11, 14, 17}

Relations and Functions

Louis plotted points to report each deposit of his earnings from his lawn service work. In March he deposited one-hundred dollars. In April he made three deposits, $100, $300, and $400.

The graph below shows a relationship of dollars earned (in hundreds) to month earned. Identify each point by its coordinates, as an ordered pair.

Louis's Earnings



The **domain** of a relation is the set of all x-coordinates. The **range** is the set of all y-coordinates. Determine the domain and range of the relation, using set notation (braces: {}). Explain WHY the domain represents the independent variables, and the range represents the dependent variables.

___________________*Domain {3,4}* *Range {1,3,4}*

Answers will vary: The domain represents independent variables because in a graph or an equation it may have its value freely chosen regardless of the values of any other variable. The range is dependent because it is determined, based on a rule, by the input or domain value.

____

____

Is this relation a **function**? Justify your answer.

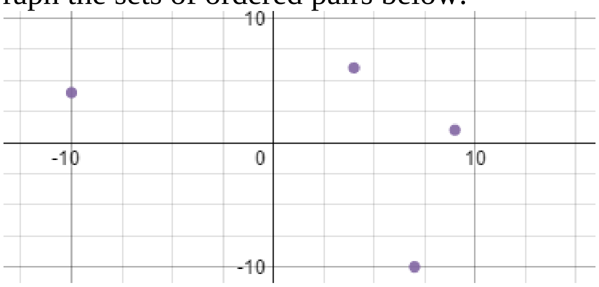
____*No, the values of the domain do not produce unique range values. E.g., a 4 in the domain produces a 4, a 3, and a 1 value in the range.*

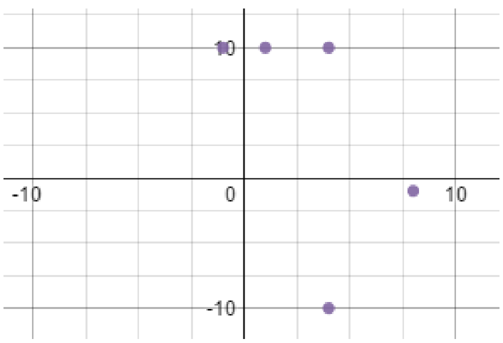
____

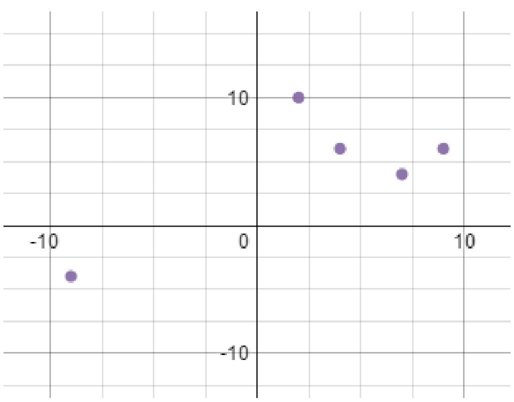
Determine if each of the following sets of ordered pairs is a function. Identify the domain and range of each.

	Function?	Domain	Range
(-10, 4), (9,1), (4,6), (7,-10)	***Yes***	***{-10, 9, 4, 7}***	***{4, 1, 6,-10}***
(8,-1), (4, 10), (1, 10), (4, -10),	***No***	***{ 1, 4, 8}***	***{-10, -1, 10}***
(7, 4), (9,6), (4, 6), (2, 10), (-9, -4)	***Yes***	***{-9, 2, 4, 7, 9}***	***{-4, 4, 6,***

Graph the sets of ordered pairs below.







Choose one of the three sets of ordered pairs and create a real-life situation that could be represented by the points.

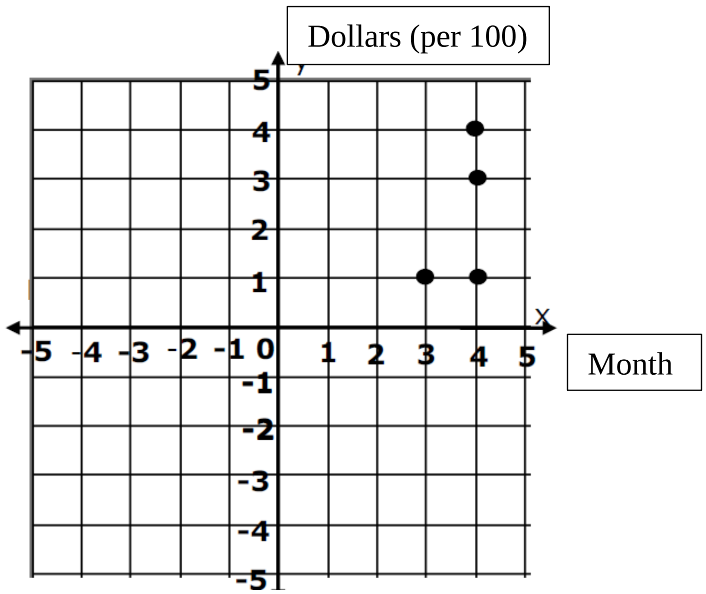
If the last set is chosen, the domain could be the morning temperatures one week in Maine, while the range represents the afternoon temperatures.

Relations and Functions

Louis plotted points to report each deposit of his earnings from his lawn service work. In March he deposited one-hundred dollars. In April he made three deposits, $100, $300, and $400.

The graph below shows a relationship of dollars earned (in hundreds) to month earned. Identify each point by its coordinates, as an ordered pair.

Louis's Earnings



The **domain** of a relation is the set of all x-coordinates. The **range** is the set of all y-coordinates. Determine the domain and range of the relation, using set notation (braces: {}). Explain WHY the domain represents the independent variables, and the range represents the dependent variables.

____

____

____

____

Is this relation a **function**? Justify your answer.

____

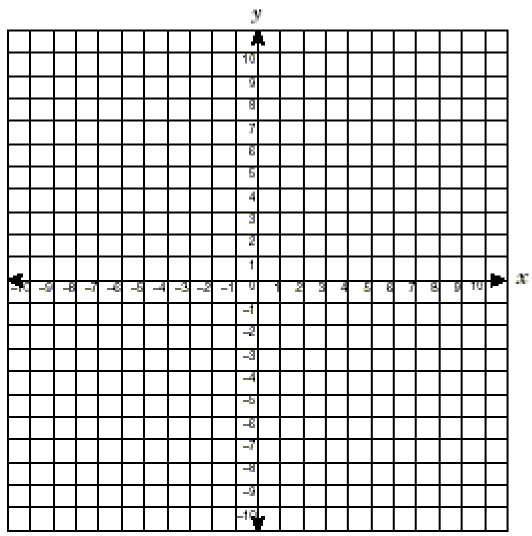
____

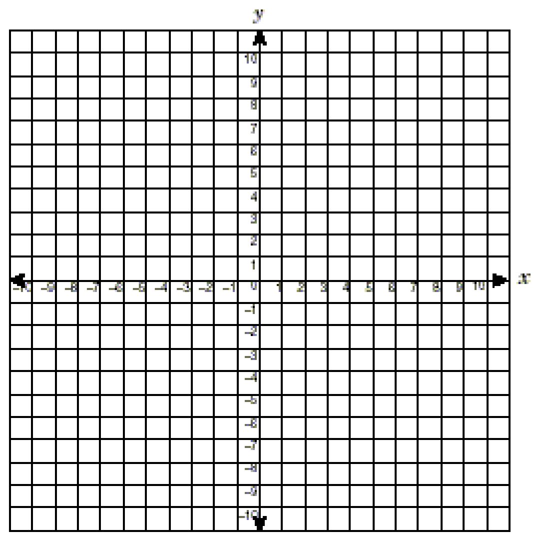
____

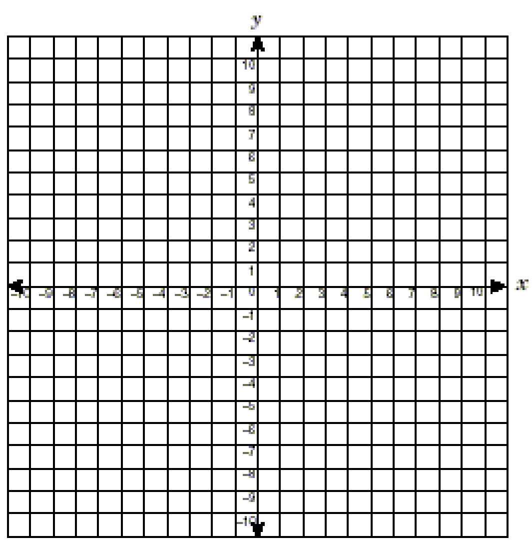
____

Graph each set of ordered pairs below and determine if each of the following sets of ordered pairs is a function. Identify the domain and range of each.

	Function?	Domain	Range
(-10, 4), (9,1), (4,6), (7,-10)			
(8,-1), (4, 10), (1, 10), (4, -10)			
(7, 4), (9,6), (4, 6), (2, 10), (-9, -4)			







Choose one of the three sets of ordered pairs and create a real-life situation that could be represented by the points.

INTERVENTION

Since understanding mathematical vocabulary is essential in the study of functions, students may use the game below for vocabulary reinforcement.

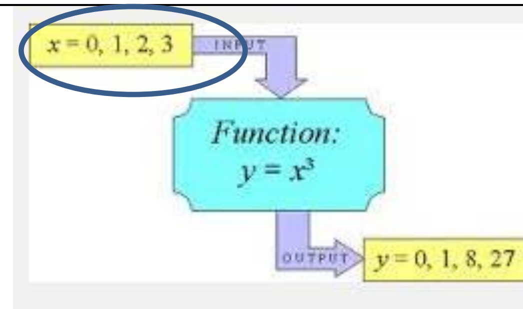
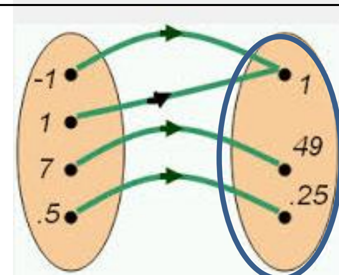
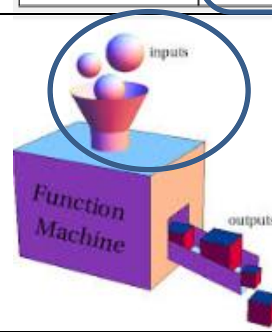
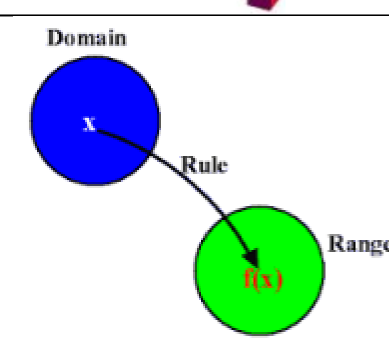
Vocabulary Match Game

Turn all cards upside down, shuffle, and distribute one card to each student. (Create multiples of each, so each student has a card.)

Students circulate among classmates in order to form a group with matching words, definitions, and images. Example: Student A has "domain" and partners with student B, holding "all the values that x is allowed to take on" and a third student, C, holding a card with an image.

Depending on the number of students, there may be more than one group with the same term. Groups with the same term can compare to see if all agree. Discussion of how they know their terms, definitions, and images match can help the teacher formatively assess students' knowledge.

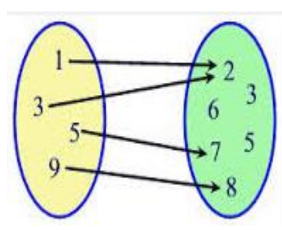
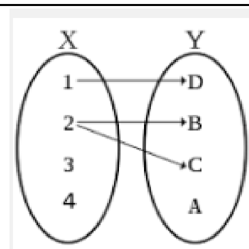
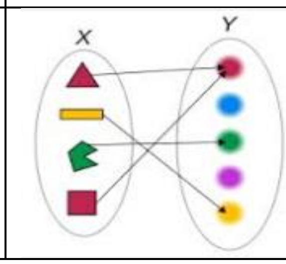
Some images could be appropriate for more than one term, e.g., the images for domain and independent variable could be interchangeable. This can generate good discussion among two groups with the same term but possibly with different pictorial representations.

<table>
<tr><td>Domain</td><td>all the values that x is allowed to take on.</td><td>
</td></tr>
<tr><td>Range</td><td>all the values that y can become, determined by x.</td><td>
</td></tr>
<tr><td>Dependent Variable</td><td>what you measure in a trial or experiment and what is affected based on the rule or treatment</td><td><table>
<tr><th>Time (s)</th><th>Water temperature (°C)</th></tr>
<tr><td>0</td><td>15</td></tr>
<tr><td>10</td><td>16</td></tr>
<tr><td>20</td><td>18</td></tr>
<tr><td>30</td><td>22</td></tr>
<tr><td>40</td><td>28</td></tr>
<tr><td>50</td><td>32</td></tr>
<tr><td>60</td><td>37</td></tr>
</table></td></tr>
<tr><td>Independent Variable</td><td>the variable you have control over; what you can choose and manipulate.</td><td>
</td></tr>
<tr><td>Function</td><td>a relationship or expression involving one or more variables.</td><td>
</td></tr>
</table>

<u>CLOSING/SUMMARIZER</u>

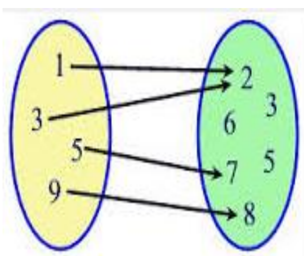
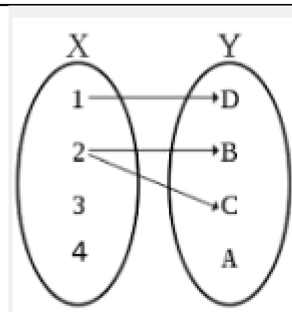
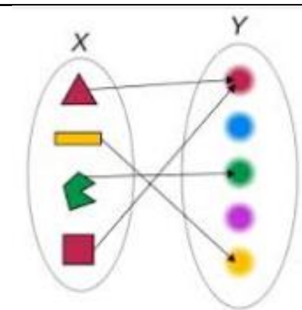
<u>Odd One Out</u>

"Which is the odd one out?" For each set, identify the item that does not belong. Be sure to explain why you chose the item. ***Possible explanations/choices.***

<table>
<tr><td>Set</td><td colspan="3">Items</td><td>Explanation…</td></tr>
<tr><td>A</td>
<td><table><tr><td>-2</td><td>3</td></tr><tr><td>5</td><td>3</td></tr><tr><td>7</td><td>3</td></tr><tr><td>12</td><td>3</td></tr></table></td>
<td><table><tr><td>-3</td><td>0</td></tr><tr><td>-3</td><td>4</td></tr><tr><td>2</td><td>-1</td></tr><tr><td>5</td><td>0</td></tr></table></td>
<td><table><tr><td>-30</td><td>-30</td></tr><tr><td>-10</td><td>-10</td></tr><tr><td>0</td><td>0</td></tr><tr><td>20</td><td>20</td></tr></table></td>
<td>Second one has repeating x value, so is not a function. OR 3rd is only one with double digits OR other students observations</td></tr>
<tr><td>B</td><td>{(1,3) (4,3) (6,2)}</td><td>{(2,4) (4,6) (6,8)}</td><td>{(3,4) (5,8) (3,7)}</td><td>Third one is not a function</td></tr>
<tr><td>C</td>
<td>
</td>
<td>
</td>
<td>
</td>
<td>Center one is not a function or last has no numbers…</td></tr>
</table>

<u>Odd One Out</u>

"Which is the odd one out?" For each set, identify the item that does not belong. Be sure to explain why you chose the item.

<table>
<tr><td>Set</td><td colspan="3">Items</td><td>Explanation …</td></tr>
<tr><td>A</td>
<td><table><tr><td>-2</td><td>3</td></tr><tr><td>5</td><td>3</td></tr><tr><td>7</td><td>3</td></tr><tr><td>12</td><td>3</td></tr></table></td>
<td><table><tr><td>-3</td><td>0</td></tr><tr><td>-3</td><td>4</td></tr><tr><td>2</td><td>-1</td></tr><tr><td>5</td><td>0</td></tr></table></td>
<td><table><tr><td>-30</td><td>-30</td></tr><tr><td>-10</td><td>-10</td></tr><tr><td>0</td><td>0</td></tr><tr><td>20</td><td>20</td></tr></table></td>
<td></td></tr>
<tr><td>B</td><td>{(1,3) (4,3) (6,2)}</td><td>{(2,4) (4,6) (6,8)}</td><td>{(3,4) (5,8) (3,7)}</td><td></td></tr>
<tr><td>C</td>
<td>
</td>
<td>
</td>
<td>
</td>
<td></td></tr>
</table>

ADDITIONAL PRACTICE

Some additional examples of functions and mappings are at
http://www.purplemath.com/modules/fcns.htm

Function Rules

Adapted from Illustrative Mathematics,
http://s3.amazonaws.com/illustrativemathematics/illustration_pdfs/000/000/715/original/illustrative_mathematics_715.pdf?1423236545

This lesson can be used to introduce/review the idea that a function assigns a unique output to every input. It also encourages students to look for less obvious patterns and encourages them to verbalize functions rules using precise language.

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. If played with a partner as a game, this could take 0.5 hours.

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR1. Students will understand characteristics of functions.

a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.(MGSE9-12.F.IF.1)
b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*. (MGSE9-12.F.IF.5)
c. Graph functions using sets of ordered pairs consisting of an input and the corresponding output. (MGSE8.F.1, 2)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students interpret the situation and persevere in demonstrating their understanding.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning in completing tables
3. Construct viable arguments and critique the reasoning of others. Students work collaboratively to explain their understanding of functions in real life situations.
4. Model with mathematics. Students model functions with graphs and tables.
6. Attend to precision. Students use clear and precise language in discussing strategies and carefully work to identify patterns.
7. Look for and make use of structure. Students use their understanding of functions to make sense of information given and extend their thinking in real-life situations.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to

- complete tables
- describe a rule in words that satisfies the pairs (given tables of input-output pairs in different forms)

MATERIALS

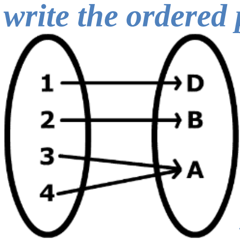
Copy of the lesson.

ESSENTIAL QUESTION

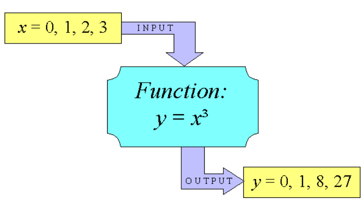
How can I find a rule that works for a given domain and range, then extend the pattern?

OPENER/ACTIVATOR

Make sure students are comfortable with the concept of identifying functions as ordered pairs using (x,y). The input and output of a function can be expressed as an ordered pair presented in any form: graphically, verbally, tabular form, or later in equation form. Students should write the ordered pairs represented by each scenario below.



{(1, D), (2, B), (3, A), (4, A)}

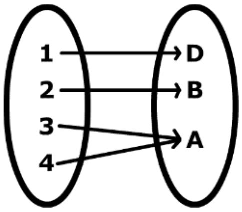


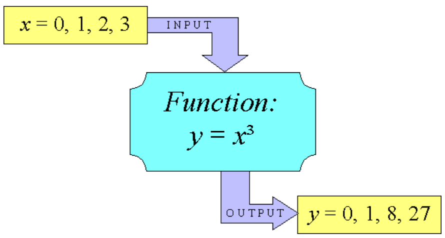
{(0, 0), (1, 1), (2, 8), (3, 27)}

x	-3	-2	-1	0	1	2	3
y	1	-2	2	4	-3	-2	-1

{(-3, 1), (-2,-2), (-1, 2), (0, 4), (1,-3), (2,-2), (3,-1)}

Students should write the ordered pairs represented by each scenario below.





x	-3	-2	-1	0	1	2	3
y	1	-2	2	4	-3	-2	-1

TEACHER COMMENTARY

This lesson can be played as a game where students have to guess the rule as the instructor gives more and more input-output pairs. Giving only three input-output pairs might not be enough to clarify the rule. Instructors might consider varying the inputs, e.g., the second table, to provide non-integer entries. A nice variation on the game is to have students who think they found the rule supply input-output pairs, which the teacher then confirms or refutes.

Verbalizing the rule requires precision of language. For the first part, only vowels a, e, i, o, u are counted. In the third example, we are looking at a non-leap year.

a.
Seeing that the input values can be any English word, we find the rule to be "the number of vowels" in the input word, and we designate that a vowel here is defined as a, e, i, o, u, but not y, as our input of "you" has an output of 2, not 3. Below is one possible way to complete the table.

Input	Cat	House	You	Table	Fireplace	Sky	Picture
Output	1	3	2	2	4	0	3

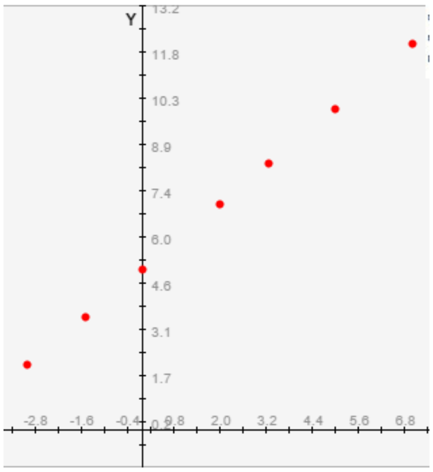
b.
To find the rule, we find the common math operation between the pairs of numbers. We can conclude that our rule here is to "add 5" to the input value. Below is one possible way to complete the table.

Input	2	5	-1.5	7	-3	3.285	0
Output	7	10	3.5	12	2	8.285	5

With only three input-output pairs, we can probably come up with many other functions, but the rule of the game is that the teacher has a function rule in mind and gives more and more input-output pairs until the students guess the teacher's function.
c.
Since our input values here are numbers, but our output values are months of the year, we find that the rule here is "the month corresponding to that day out of the year", defining January 1st to be day one. We can also assume we are using a non-leap year to determine the output values. Below is one possible way to complete the table.

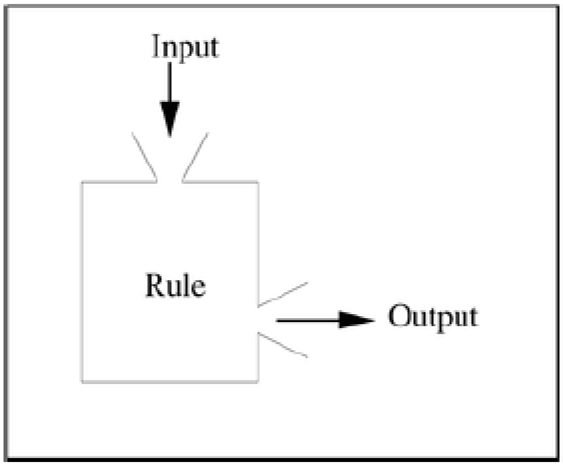
Input	25	365	35	95	330	123	200
Output	January	December	February	April	November	May	July

Can any of the functions above be easily graphed on a coordinate plane? If not, why not? If yes, graph one to demonstrate. ***b***



Function Rules

The function machine takes an input, does something to this input according to some rule, and returns a unique output.



Given below are tables of input-output pairs for different function machines. Fill in the remaining table entries and describe each function rule in words.

a. Input values can be any English word.

Input	Cat	House	You				
Output	1	3	2				

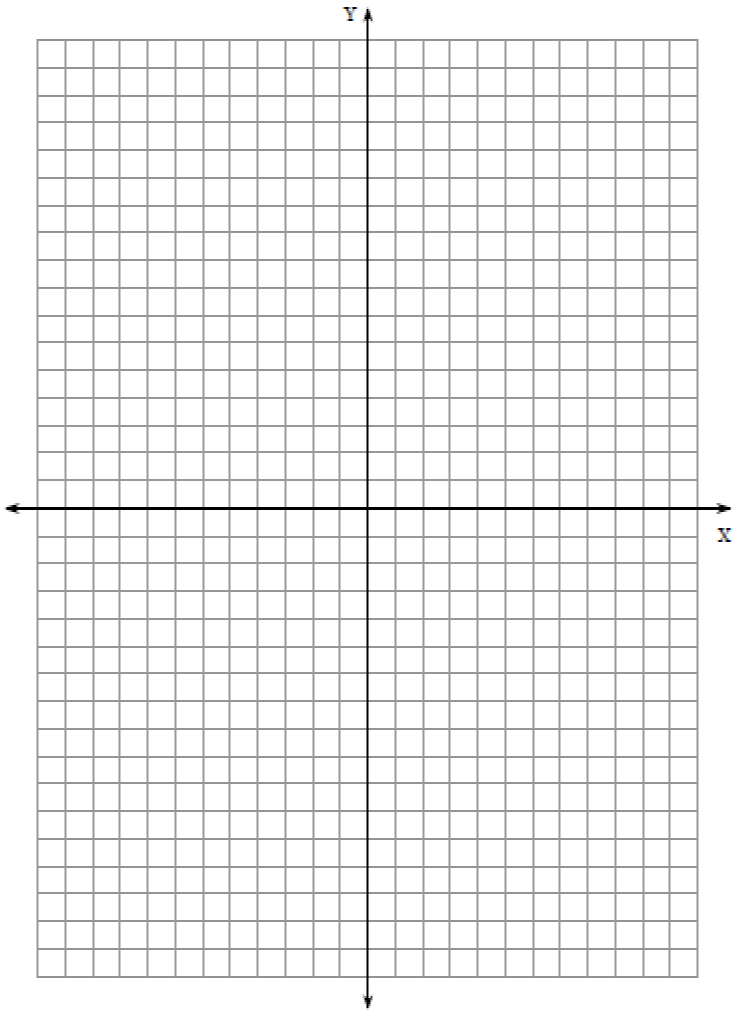
b. Input values can be any real number.

Input	2	5	-1.5				
Output	7	10	3.5				

c. Input values can be any whole number between 1 and 365.

Input	25	365	35	95	330		
Output	January	December	February	April	November		

Can any of the functions above be easily graphed on a coordinate plane? If not, why not? If yes, graph one to demonstrate.



CLOSING/SUMMARIZER

Each student should write a function with a specific pattern, and write the domain for the function.

Then, students will exchange papers with a study partner. The study partner should then find the range using the given domain. Students should provide feedback to each other by writing one thing that would make their responses more precise and accurate.

ADDITIONAL PRACTICE

For more practice with domain and range, http://www.ixl.com/math/algebra-2/domain-and-range

Foxes and Rabbits

Adapted from Illustrative Mathematics
http://www.illustrativemathematics.org/illustrations/713

SUGGESTED TIME FOR THIS LESSON

Exact timings will depend on the needs of your class. Suggested time is 30 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR1. Students will understand characteristics of functions.

a.. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.(MGSE9-12.F.IF.1)

b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*. (MGSE9-12.F.IF.5)

MFAQR3. Students will construct and interpret functions.

b. Use variables to represent two quantities in a real-world problem that change in relationship to one another (conceptual understanding of a variable). (MGSE6.EE.9)

STANDARDS FOR MATHEMATICAL PRACTICE

3. Construct viable arguments and critique the reasoning of others: Students discuss a given real-life situation with a study partner.
7. Look for and make use of structure. Given a table of values, students make predictions based on the data.

MATERIALS

- Copy of student task
- Index Cards

ESSENTIAL QUESTIONS

- What is a function?
- What are the characteristics of a function?
- How do you determine if relations are functions?
- How is a function different from a relation?

OPENER/ACTIVATOR

A Math Riddle: A farmer is trying to cross a river. He is taking with him a rabbit, carrots, and a fox, and he has a small raft. He can only bring 1 item at a time across the river because his raft can only fit either the rabbit, the carrots, or the fox. How does he cross the river keeping the carrots and animals “safe” ? You can assume that the fox does not eat the rabbit if the man is present and that the fox and the rabbit are not trying to escape. Answer: http://www.mathwarehouse.com/riddles/math-riddles.php

TEACHER COMMENTARY

There is a natural (and complicated!) predator-prey relationship between the fox and rabbit populations. Foxes thrive in the presence of rabbits, and rabbits thrive in the absence of foxes. However, this relationship, as shown in the given table of values, cannot possibly be used to present either population as a function of the other. This task emphasizes the importance of the "every input has exactly one output" clause in the definition of a function, which is violated in the table of values of the two populations. Noteworthy is that since the data is a collection of input-output pairs, no verbal description of the function is given, so part of the task is processing what the "rule form" of the proposed functions would look like.

PART 1

Students study the picture and write down three thoughts and/or possible mathematic problems on a note card.
With a partner, share ideas about the picture. What mathematical questions/problems could be posed related to the picture?

PART 2

Given below is a table that gives the populations of foxes and rabbits in a national park over a 12 month period. Note that each value of *t* corresponds to the beginning of the month.

t, month	1	2	3	4	5	6	7	8	9	10	11	12
R, number of rabbits	1000	750	567	500	567	750	1000	1250	1433	1500	1433	1250
F, number of foxes	150	143	125	100	75	57	50	57	75	100	125	143

Looking back at your list of comments, questions and mathematics problems, what additional questions might be asked regarding the rabbits and foxes?

According to the data in the table, is F a function of R?
Is R a function of F?
Are either R or F functions of t?
Explain.

Each input has exactly one output

The key is understanding that a function is a rule that assigns to each input exactly one output, so we will test the relationships in question according to this criterion:

For the first part, that is, for F to be a function of R, we think of R as the input variable and F as the output variable, and ask ourselves the following question: Is there a rule, satisfying the definition of a function, which inputs a given rabbit population and outputs the corresponding fox population. The answer is no: We can see from the data that when R=1000, we have one instance where F=150, and another where F=50. Since this means that a single input value corresponds to more than one output value, F is not a function of R. In the language of the problem's context, this says that the fox population is not completely determined by the rabbit population; during two different months there are the same number of rabbits but different numbers of foxes.

Similarly, we can see that if we consider F as our input and R as our output, we have a case where F=100 corresponds to both R=500 and R=1500, two different outputs for the same input. So R is not a function of F: There are two different months which have the same number of foxes but two different numbers of rabbits.

Letting t, months, be the input, we can clearly see that there is exactly one output R for each value of t. That is, the rule which assigns to a month t the population of

rabbits during that month fits our definition of a function, and so R is a function of t. By the same reasoning, F is also a function of t. Again, in the context of the situation, it makes sense that at any given point in time, there is a unique number of foxes and a unique number of rabbits in the park.

PART 3
Students share and compare their answers to the questions in Part 2.

Were any additional questions or mathematics problems posed in Part 1 that can be answered or discussed now that were not part of the questions in Part 2?

FOXES AND RABBITS

Name ____Date____________

PART 1

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Is R a function of F?
Are either R or F functions of t?

Explain.

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Were any additional questions or mathematics problems posed in Part 1 that can be answered or discussed now that were not part of the questions in Part 2?

ADDITIONAL PRACTICE

For practice with tables of values and functions: http://www.ixl.com/math/grade-5/function-tables

CLOSING/SUMMARIZER

On an index card, fill in the blank: *In a person's job, ____________ depends on ____________; therefore, ______________ is a function of _________________*. Explain your reasoning.

For hourly pay, a possible answer would be: In a person's job, pay depends on the hours worked, therefore, pay is a function of the number of hours worked.

Quick Check I

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR1. Students will understand characteristics of functions.

a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.(MGSE9-12.F.IF.1)

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c. Graph functions using sets of ordered pairs consisting of an input and the corresponding output. (MGSE8.F.1, 2)

Quick Check I - Formative Assessment

Determine if each of the following is a function. Write FUNCTION or NOT on the lines.
Identify the DOMAIN of each function by listing its elements.

1.

x	1	2	3	4
y	-3	2	5	-1

__*yes; 1, 2, 3, 4*___

2.

x	4	3	2	4
y	6	-2	-1	0

_______*no*______

3.

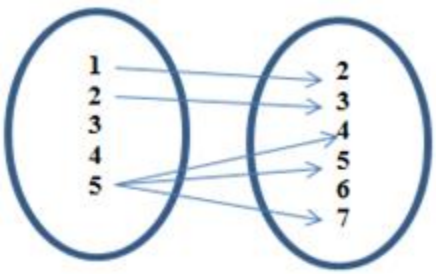
x	-5	-3	3	5
y	25	9	9	25

*yes; -5, -3, 3, 5*

4. {(6,0), (4, 2), (6, -3), (5, 2)} __*no*_____________

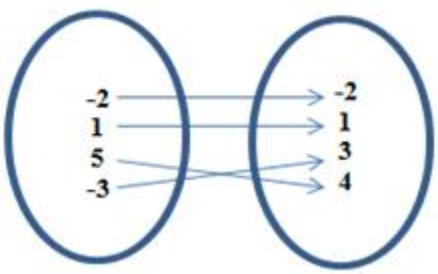
5. {(5, 7), (4, -2), (-2, -4), (0.0)} ___*yes; 5, 4, -2, 0*___________

6.



___*no*____________

7.



__*yes; -2,1,5, -3*_________

8.

x	y
-2	-2
-1	2
0	6
1	10
2	14

________***yes, -2, -1, 0, 1, 2***______

9. Explain what the domain is for any function.

Possible Solutions: The set of x-coordinates of the set of points on a graph; the set of x-coordinates of a given set of ordered pairs, or the value that is the input in a function or relation.

Optional assessment problems are described below:

Determining if a relation is a function: https://www.illustrativemathematics.org/content-standards/8/F/A/1/tasks/624

Input-Output: https://www.illustrativemathematics.org/content-standards/HSF/IF/A/1/tasks/624

Quick Check I - Formative Assessment

Determine if each of the following is a function. Write FUNCTION or NOT on the lines.
Identify the DOMAIN of each function by listing its elements.

1.

x	1	2	3	4
y	-3	2	5	-1

____

2.

x	4	3	2	4
y	6	-2	-1	0

____

3.

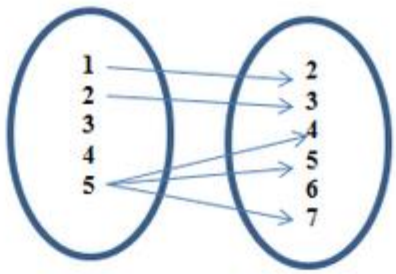
x	-5	-3	3	5
y	25	9	9	25

____

4. {(6,0), (4, 2), (6, -3), (5, 2)} ______________

5. {(5, 7), (4, -2), (-2, -4), (0.0)} ______________

6.



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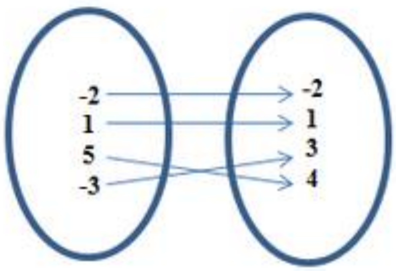
7.



____

8.

x	y
-2	-2
-1	2
0	6
1	10
2	14

9. Explain what the domain is for any function.

Which Ticket Is The Best Deal?

Adapted from Robert Kaplinsky http://robertkaplinsky.com/work/ticket-option/

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Suggested time, 1.5 hours.

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR2. Students will compare and graph functions.

a. Calculate rates of change of functions, comparing when rates increase, decrease, or stay constant. *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (MGSE6.RP.2;MGSE7.RP.1,2,3;MGSE8.F.2,5; MGSE9-12.F.IF.6)

STANDARDS FOR MATHEMATICAL PRACTICE

1**. Make sense of problems and persevere in solving them.** Students must make sense of the problem presented in the lesson and continue to work toward a solution.
2. Reason abstractly and quantitatively. Students can reason abstractly regarding options with proportions and prices.
3. Construct viable arguments and critique the reasoning of others. Students construct and defend their arguments as they determine what constitutes the "best deal" in different situations.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to calculate rates of change of functions and determine when rates change or remain the same.

MATERIALS

Copy of ticket prices

Which Ticket Is The Best Deal?

THE SITUATION

You are at a high school carnival's ticket booth and see the ticket prices below:

The Challenge(s)
•Which ticket option is the best deal?
•Which ticket option is the worst deal?
•Which ticket options are the same deal?
•How would you suggest they change their prices?

Question(s) To Ask:
•How did you reach that conclusion?
•Does anyone else have the same answer but a different explanation?

TEACHER NOTES: Consider This
It is important to note that the challenge question is not "Which ticket option should you buy?" Asking about the "best", "worst", and "same" deals gets closer to the heart of comparing ratios and unit rates. Specifically, by "best deal" you are asking which ticket option has the lowest cost per ticket.

Looking at the painted ticket booth chart, it is clear that you are getting a better deal if you buy 12 tickets instead of 1 as well as 25 tickets instead of 12 tickets. However something strange happens when you consider purchasing 50 tickets instead of 25 tickets. You can buy the 25-ticket option twice for $10 each time which gives you 50 tickets for $20. Alternatively, you could buy the 50-ticket option for $25 and spend $5 more for the same number of tickets.

There are other interesting pricing options such as:

•12 tickets for $5 and 120 tickets for $50 (same price per ticket)
•1 ticket for $0.50 and 50 tickets for $25 (same price per ticket)
•5 sets of 25 tickets for $50 or 120 tickets for $50 (still cheaper to buy sets of 25 tickets)

The chart below lists the cost per ticket for each ticket option:

Total Cost	Tickets	Cost per Ticket
$0.50	1	$0.50
$5.00	12	$0.42
$10.00	25	$0.40
$25.00	50	$0.50
$50.00	120	$0.42

Looking at the chart, it is easy to see that the 25 tickets for $10 is the best deal. Tied for the worst deals are $0.50 for one ticket and $25 for 50 tickets. However, you could make the case

that the $25 for 50 tickets is worse in that you could have bought the tickets one at a time to have just the right amount. The same deals are the $5 for 12 tickets and the $50 for 120 tickets as well as the two worst options.

Encourage students to defend their reasoning using multiple explanations and discuss how they are connected. For the extension challenge, "How would you suggest they change their prices?", students can change the pricing structure so that the price per ticket decreases with the more tickets you buy. They should be able to explain their reasoning and show that the cost per ticket decreases when you buy larger quantities of tickets.

CLOSING/SUMMARIZER

Create a ticket poster for the carnival with at least 4 options. Determine if the unit rates for the tickets increase, decrease, or stay the same.

ADDITIONAL PRACTICE

The students' carnival posters may be used to review and practice finding rates of change.

Reviewing Rate Of Change

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class. This lesson is intended for practice, perhaps for homework, small group, or paired practice. Timing will vary depending on grouping.

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR2. Students will compare and graph functions.
a. Calculate rates of change of functions, comparing when rates increase, decrease, or stay constant. *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (MGSE6.RP.2;MGSE7.RP.1,2,3;MGSE8.F.2,5; MGSE9-12.F.IF.6)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students interpret the situation and persevere in demonstrating their understanding, and use trial and error to find solutions.
4. Model with mathematics. Students use words, numbers, and pictures to solve problems.
5. Use appropriate tools strategically. Students select and use tools and strategies to interpret and solve word problems.
6. Attend to precision. Students use precise language in their discussions and explanations of proportions rate of change.

EVIDENCE OF LEARNING/LEARNING TARGET
By the conclusion of this lesson, students should be able to:
- demonstrate understanding of rate of change
- explain the concept of a unit rate as part – to – one.
- explain that the concept of a unit rate is related to a ratio.
- apply unit rates in real world situations.

MATERIALS
Student practice page

ESSENTIAL QUESTIONS

- How do you calculate a rate of change?
- What does a rate of change mean?

OPENER/ACTIVATOR

Adapted from Inside Mathematics. http://bit.ly/1BIEOPO

Sarah is organizing a party at the Vine House Hotel.

Vine House Hotel
Your fab party place!

Charges
$750 for up to 30 people
plus
$20 per person for each extra person

1 . Sarah thinks there will be 60 people at the party. Show that the cost will be $1350.

$750 + 30 X $20 = 750 + 600

2. What is the cost of a party for 100 people at the Vine House Hotel? $ _____________
Explain your reasoning.

$750 + 20 X 70 = $2150

3. What rate is being used in both the questions above? ***The rate is $20/person***

Sarah is organizing a party at the Vine House Hotel.

Vine House Hotel
Your fab party place!

Charges
$750 for up to 30 people
plus
$20 per person for each extra person

1 . Sarah thinks there will be 60 people at the party. Show that the cost will be $1350.

2. What is the cost of a party for 100 people at the Vine House Hotel? $ _____________
Explain your reasoning.

3. What rate is being used in both the questions above?

Reviewing Rate Of Change

Complete the problems. Use words, numbers, and pictures to help solve the problems. Explain your process and answers.

1. Megan types $\frac{1}{6}$ of a page in $\frac{1}{12}$ of a minute. How much time does it take her to write a whole page?

½ minute or 30 seconds

2. Louis fills $\frac{1}{3}$ of a bottle in $\frac{1}{6}$ of a minute. How much time will it take him to fill the bottle?

½ minute or 30 seconds

3. Ben plays $\frac{1}{5}$ of a song on his guitar in $\frac{1}{15}$ of a minute. How much time will it take him to play the entire song?

⅓ minute or 20 seconds

4. Katie used $\frac{1}{3}$ of a gallon of water to make $\frac{1}{9}$ of a jug of tea. How much water is needed to fill the entire jug?

3 gallons

5. Lucy used ¼ of an ounce of nuts to make $\frac{1}{12}$ of a recipe of cookies. How many ounces of nuts will she need to make one full recipe of cookies?

3 ounces

Reviewing Rate Of Change

Complete the problems. Use words, numbers, and pictures to help solve the problems. Explain your process and answers.

1. Megan types $\frac{1}{6}$ of a page in $\frac{1}{12}$ of a minute. How much time does it take her to write a whole page?

2. Louis fills $\frac{1}{3}$ of a bottle in $\frac{1}{6}$ of a minute. How much time will it take him to fill the bottle?

3. Ben plays $\frac{1}{5}$ of a song on his guitar in $\frac{1}{15}$ of a minute. How much time will it take him to play the entire song?

4. Katie used $\frac{1}{3}$ of a gallon of water to make $\frac{1}{9}$ of a jug of tea. How much water is needed to fill the entire jug?

5. Lucy used ¼ of an ounce of nuts to make $\frac{1}{12}$ of a recipe of cookies. How many ounces of nuts will she need to make one full recipe of cookies?

INTERVENTION

Modules 1-4 include Intervention Tables which provide direct links to NZMaths interventions specific to the skills. Reviewing Rate of Change is the only lesson in Module 5 with an intervention from this resource, and it is linked here at point of use.
Rates of Change

CLOSING/SUMMARIZER

When driving an automobile, what rates of change can be calculated? ***Possible Solutions***: ***Miles per gallon for gasoline consumption, or miles per hour for speed***

ADDITIONAL PRACTICE

More practice involving rates of change can be found at https://www.ixl.com/math/grade-6/unit-rates-word-problems

How Did I Move?

From NCTM Illuminations http://illuminations.nctm.org/Lesson.aspx?id=2800

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Suggested time is 1 hour.

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR2. Students will compare and graph functions.

b. Graph by hand simple functions expressed symbolically (use all four quadrants). (MGSE9-12.F.IF.7)

c. Interpret the equation y = mx + b as defining a linear function whose graph is a straight line. (MGSE8.F.3)

MFAQR3. Students will construct and interpret functions.

a. Write a function that describes a relationship between two quantities. (MGSE8.F.4, MGSE9-12.F.BF.1)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students interpret the situation and persevere in demonstrating their understanding, and use trial and error to find solutions.
4. Model with mathematics. Students use words, numbers, and pictures to solve problems.

Common Misconception

A common problem when students learn about the slope-intercept equation, $y = mx + b$, is that they make substitutions for the slope and y-intercept without understanding their meaning. This lesson is intended to provide students with a method for understand that "m" is a rate of change and "b" is the value when $x = 0$. Also, this activity helps students discover that slope is the same for all sets of points with a linear relationship.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- create slope-intercept equations
- use equations to predict values from given domains

MATERIALS

- Index Cards
- Timers
- Activity Sheets from Website

ESSENTIAL QUESTIONS

- Why is y = mx + b called the slope-intercept form of a linear equation?
- What is a real-world example that demonstrates the meaning of slope?
- What is a real-world example that demonstrates the meaning of y-intercept?
- What is the connection between slope and speed?

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Data will be collected and interpreted with the guidelines given from *http://illuminations.nctm.org/Lesson.aspx?id=2800*

This activity is conducted on a football field. Movements from the starting positions will be compared, and equations will be developed to predict distance given time. ***If an outside field is not available, sample data sheets are provided for students to use.***

By the Book

SUGGESTED TIME FOR THIS LESSON

Exact timings will depend on the needs of your class. The suggested timing is 50 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR1. Students will understand characteristics of functions.

a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.(MGSE9-12.F.IF.1)

b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*. (MGSE9-12.F.IF.5)

MFAQR2. Students will compare and graph functions.

a. Calculate rates of change of functions, comparing when rates increase, decrease, or stay constant. *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (MGSE6.RP.2;MGSE7.RP.1,2,3;MGSE8.F.2,5; MGSE9-12.F.IF.6)

b. Graph by hand simple functions expressed symbolically (use all four quadrants). (MGSE9-12.F.IF.7)

c. Interpret the equation y = mx + b as defining a linear function whose graph is a straight line. (MGSE8.F.3)

e. Analyze graphs of functions for key features (intercepts, intervals of increase/decrease, maximums/minimums, symmetries, and end behavior) based on context. (MGSE9-12.F.IF.4,7)

MFAQR3. Students will construct and interpret functions.

a. Write a function that describes a relationship between two quantities. (MGSE8.F.4, MGSE9-12.F.BF.1)

b. Use variables to represent two quantities in a real-world problem that change in relationship to one another (conceptual understanding of a variable). (MGSE6.EE.9)

Common Misconception:

Students may confuse independent and dependent variables. This may need to be reviewed prior to the lesson. A quickreview can be found at *http://www.quia.com/pop/184568.html?AP_rand=1488938684* .

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them. Students will decide the procedures for information given in different formats.
4. Model with mathematics. Students will translate between multiple representations of functions: tables, graphs, problem solving scenarios
5. Use appropriate tools strategically. Students will explain how patterns, relations, and functions can be used as tools to best describe and help explain real-life relationships?

EVIDENCE OF LEARNING/LEARNING TARGET

In this lesson, students will identify independent and dependent variables. Students will also relate the unit rate to the slope of a function line.

MATERIALS

- Copies of lessons for students
- Straightedge
- Graph paper http://incompetech.com/graphpaper

ESSENTIAL QUESTIONS:

- How can patterns, relations, and functions be used as tools to best describe and help explain real-life relationships?
- How can the same mathematical idea be represented in a different way? Why would that be useful?
- What does the slope of the function line tell me about the unit rate?
- What does the unit rate tell me about the slope of the function line?

OPENER/ACTIVATOR

Many students and teachers enjoy reading. A class discussion on literature will help set the stage for this lesson. Points for discussion can be the number of pages in popular books, in classics, or in the books assigned in the students' literature classes. Students may discuss whether they prefer reading printed books versus electronic copies. Would a thick book in an electronic form be easier to read? Many research articles on paper versus electronic have been written such as https://www.scientificamerican.com/article/reading-paper-screens/

LESSON DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will determine the variables in a relationship, and then identify the independent variable and the dependent variable. Students will complete a function table by applying a rule, within a function table and/or a graph, and will create and use function tables in order to solve real world problems.

By the Book

The thickness of book manuscripts depends on the number of pages in the manuscript. On your first day working at the publishing house, you are asked to create tools editors can use to estimate the thickness of proposed manuscripts. After experimenting with the different weights of paper, you discover that 100 sheets of paper averages 1.25 cm thick.

Your task is to create a table of possible pages and the corresponding thickness. This information should also be presented in a graph. Since you cannot include all the possible number of pages, you will need a formula that editors can use to determine the thickness of any manuscript.

1. Based on the request, determine the variables in the relationship, then identify the independent variable and the dependent variable. Add them to the table below.
Independent variable - ***number of pages in the manuscript***

Explain your reasoning. ***The number of pages is the independent variable in this scenario because the number of pages in the manuscript determines the thickness of the manuscript.***

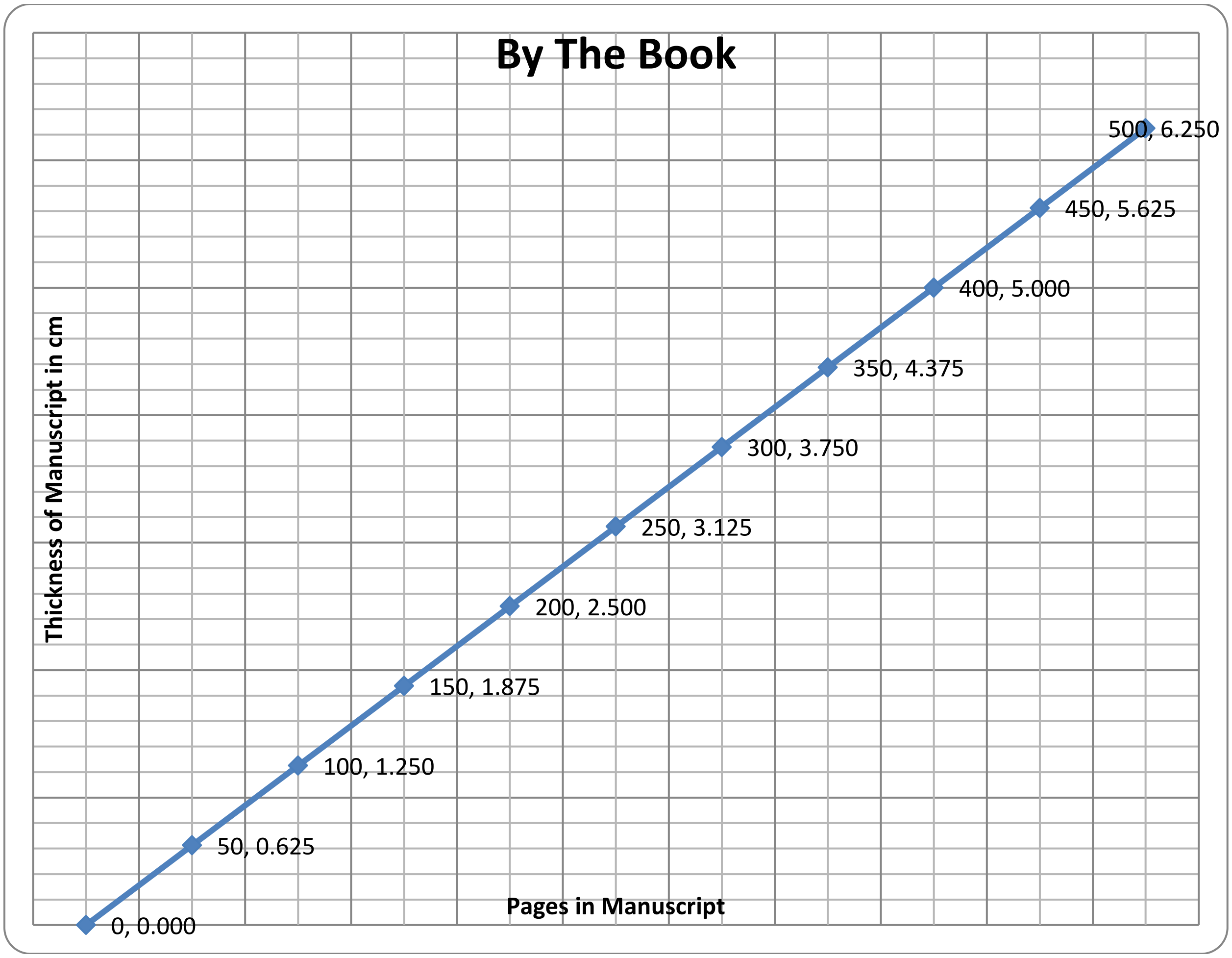
Dependent variable - ***thickness of manuscript in centimeters***

Explain your reasoning. ***The thickness of the manuscript is determined by the number of pages in the manuscript.***

2. Complete the table for manuscripts from zero to 500 in 50 page intervals.

Number of pages (p)	***Thickness in cm (T)***
0	*0.000*
50	*0.625*
100	*1.250*
150	*1.875*
200	*2.500*
250	*3.125*
300	*3.750*
350	*4.375*
400	*5.000*
450	*5.625*
500	*6.250*

3. Use the information from the table in problem 2 to create a graph.



4. Write an equation for the thickness of a manuscript given any number of pages it might contain.

Solution

T = 0.0125p where T is thickness of the manuscript and p is the number of pages in the manuscript.

The copyrighters for the publisher's website also asked that the information be represented for them so they can tell how many pages are in a book based on its thickness.

5. Based on the request, determine the variables in the relationship, the independent variable, and the dependent variable.

Independent variable - ***thickness of manuscript in centimeters***

Explain your reasoning. ***The thickness of the book is the independent variable because it determines how many pages are in the manuscript.***

Dependent variable - ***pages in the manuscript***

Explain your reasoning. ***The number of pages is determined by the thickness of the manuscript.***

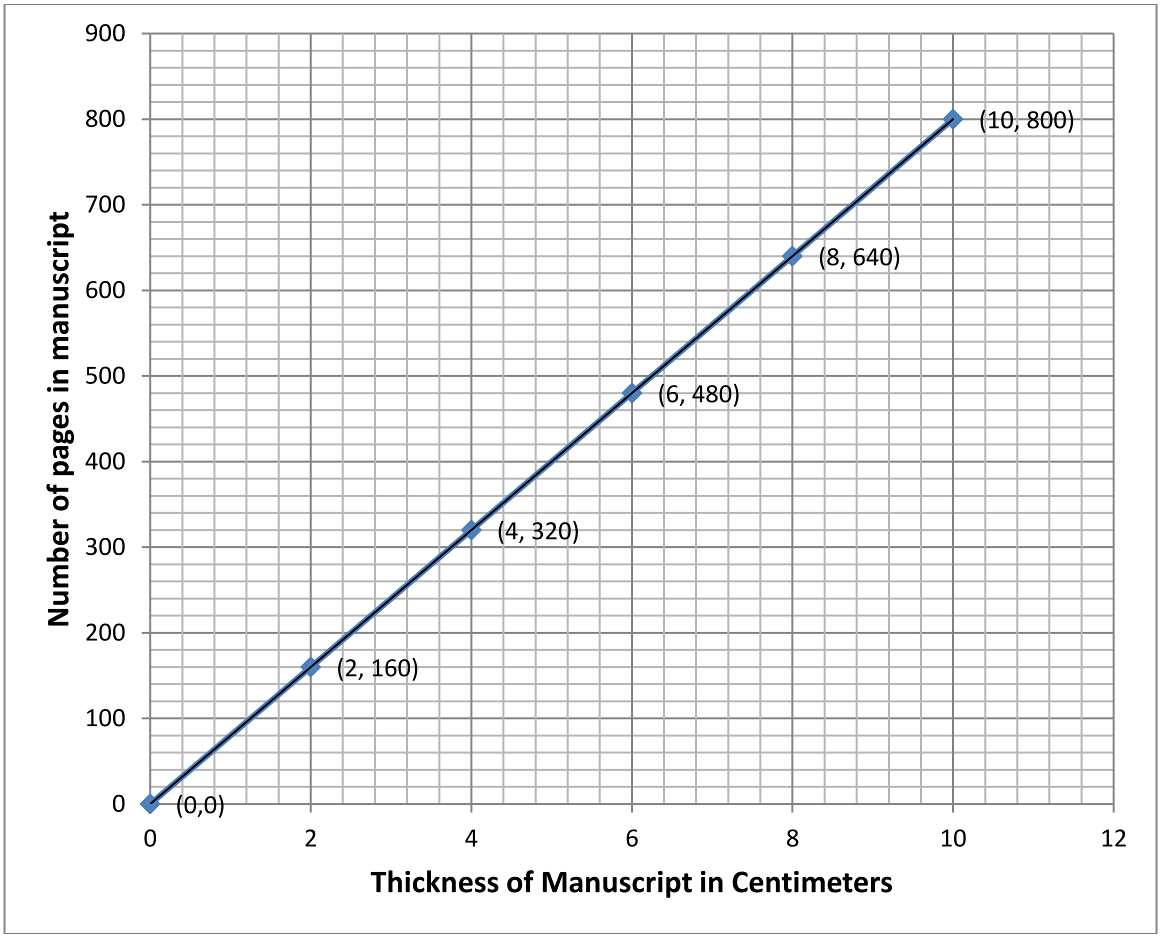
6. Using the information from problem 5, create a table for manuscripts from zero to 10 in 2 cm intervals.

Solution

Thickness in cm (t)	***Pages in book (P)***
0	*0*
2	*160*
4	*320*
6	*480*
8	*640*
10	*800*

7. Use the information from your table in problem 6 to create a graph.

Solution



8. Write an equation for the number of pages a manuscript will contain given its thickness.

P = 80t where P is the number of pages in the manuscript and t is thickness of manuscript.

Based on your findings, answer the following questions.

9. What is the unit rate for sheets of paper per centimeter? Explain how you know.

There are 80 pages per centimeter of thickness of the manuscript. This is found by setting up a proportion of pages to centimeters and simplifying.

$$\frac{100\ pages}{1.25\ cm} = \frac{80\ pages}{1\ cm}$$

10. What is the unit rate for height of a sheet of paper? Explain how you know.

Each page is 0.0125 cm thick. This is found by setting up a proportion of centimeters to pages and simplifying.

$$\frac{1.25\ cm}{100\ pages} = \frac{0.0125\ cm}{1\ pages}$$

11. How many sheets of paper will be in a manuscript 4 cm thick? Explain how you know.

Using a proportion will show that 4 centimeters of manuscript will have 320 pages.

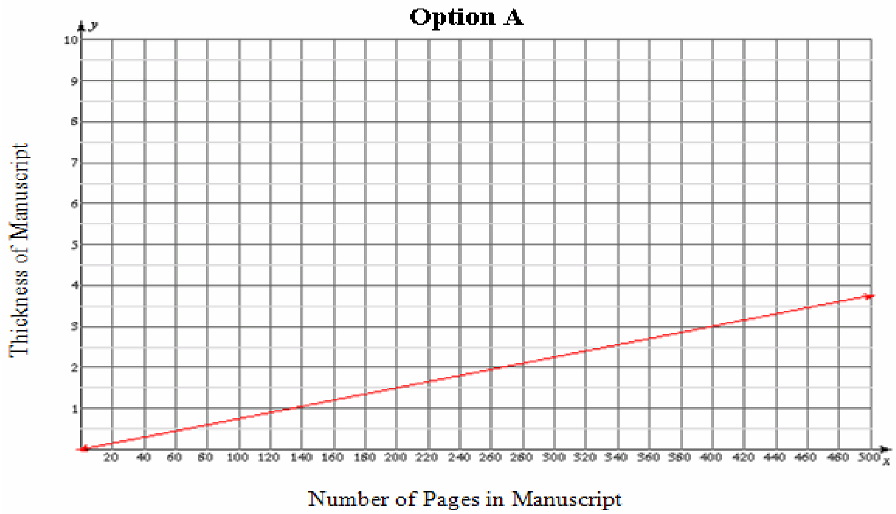
$$\frac{100\ pages}{1.25\ cm} = \frac{320\ pages}{4\ cm}$$

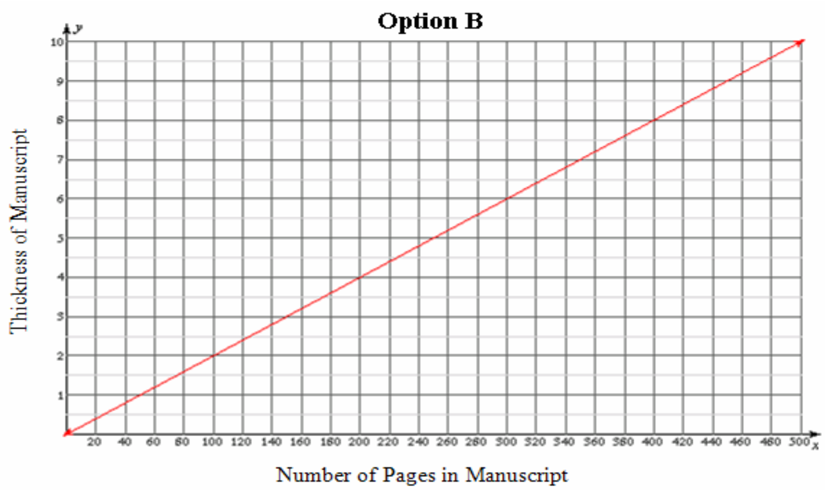
12. How thick will a 300 page manuscript be? Explain how you know.

Using a proportion will show that a 300 page manuscript will be 3.75 cm thick. .

$$\frac{1.25\ cm}{100\ pages} = \frac{3.75\ cm}{300\ pages}$$

The publishing company offers two other types of paper for printing. The graphs below show the thickness of manuscripts based on the number of pages for two different options of paper.





13. Use the graphs to determine the slope (rate of change) for each option.

Solutions

Option A: ***0.0075***

Option B: ***0.02***

14. Express each of these slopes as a unit rate for the thickness of the manuscripts.

Option A: ***1 page = 0.0075cm***

Option B: ***1 page = 0.02cm***

15. Write an equation for each paper option.

Option A: ***T = 0.0075p***

Option B: ***T = 0.02p***

16. Explain how the unit rate/slope is evident in each equation and graph.

Option A***: Option A is thinner than the original papers because the line is less steep. The coefficient of the independent variable is less for Option A.***

Option B: ***Option B is thicker than the original. The graph of the line is steeper. The slope is greater than any other option. The coefficient of the independent variable is greater for Option B.***

By the Book

The thickness of book manuscripts depends on the number of pages in the manuscript. On your first day working at the publishing house, you are asked to create tools editors can use to estimate the thickness of proposed manuscripts. After experimenting with the different weights of paper, you discover that 100 sheets of paper averages 1.25 cm thick.

Your task is to create a table of possible pages and the corresponding thickness. This information should also be presented in a graph. Since you cannot include all the possible number of pages, you also need a formula that editors can use to determine the thickness of any manuscript.

1. Based on the request, determine the variables in the relationship, then identify the independent variable and the dependent variable. Add them to the table below.
Independent variable -

Explain your reasoning.

Dependent variable -

Explain your reasoning.

2. Complete the table for manuscripts from zero to 500 in 50 page intervals.

3. Use the information from the table in problem 2 to create a graph.

4. Write an equation for the thickness of a manuscript given any number of pages it might contain.

The copyrighters for the publisher's website also asked that the information be represented for them so they can tell how many pages are in a book based on its thickness.

5. Based on the request, determine the variables in the relationship, the independent variable, and the dependent variable.

Independent variable -

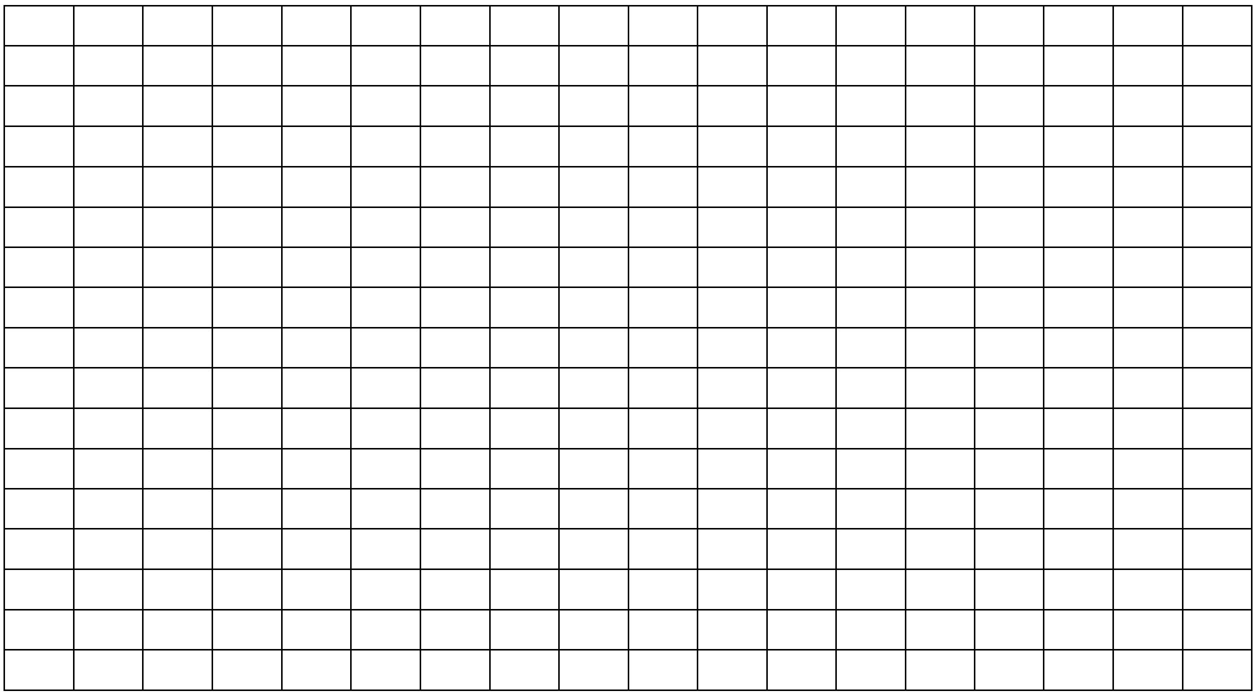
Explain your reasoning.

Dependent variable -

Explain your reasoning.

6. Using the information from problem 5, Create a table for manuscripts from 0 to 10 with 2 cm intervals.

7. Use the information from your table in problem 6 to create a graph.



8. Write an equation for the number of pages a manuscript will contain given its thickness.

Based on your findings, answer the following questions.

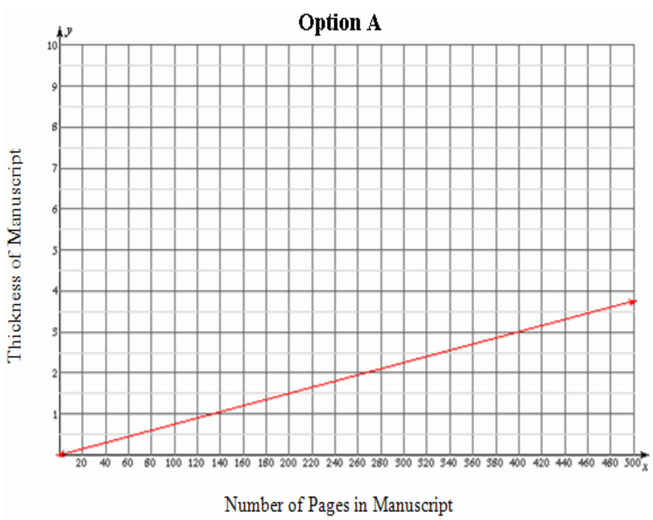
9. What is the unit rate for sheets of paper per centimeter? Explain how you know.

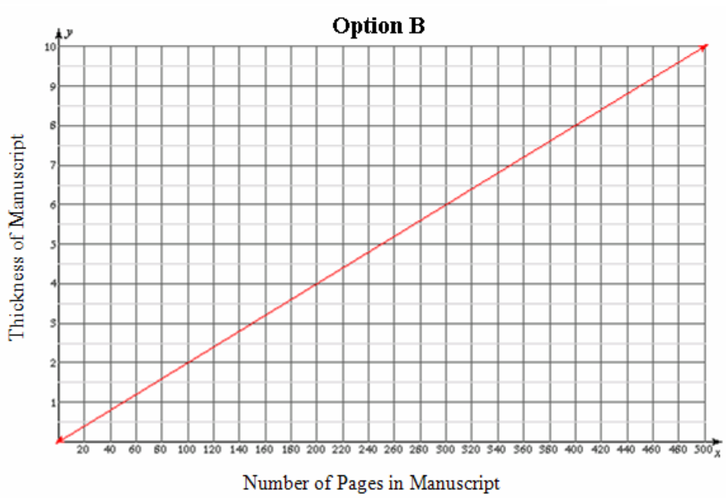
10. What is the unit rate for height of a sheet of paper? Explain how you know.

11. How many sheets of paper will be in a manuscript 4 cm thick? Explain how you know.

12. How thick will a 300 page manuscript be? Explain how you know.

The publishing company offers two other types of paper for printing. The graphs below show the thickness of manuscripts based on the number of pages for two different options of paper.





13. Use the graphs to determine the slope (rate of change) of the line for each option.

Option A:

Option B:

14. Express each of these slopes as a unit rate for the thickness of the manuscripts.

Option A:

Option B:

15. Write an equation for each paper option.

Option A:

Option B:

16. Explain how the unit rate/slope is evident in each equation and graph.

Option A:

Option B:

EXTENSION

Using a textbook, determine the thickness of a page using the same process you used in the task. Develop the equation for the thickness of the pages in your textbook. Create a table and a graph, and then explain your process in a detailed paragraph.

INTERVENTION

Based on student needs, the problems can be modified. For problem 5, have students measure out a stack of paper 2 cm in height, and then, count the number of sheets of paper in the stack (round to the nearest 10 sheets). (This may help them appreciate why an equation for estimating this number is needed.) Some students may get overwhelmed when it comes to answering questions about Option A and Option B. Those students may only need to choose one or the other to complete.

CLOSING/SUMMARIZER

Students can try the following brain-teaser about books*: It takes 1629 digits to number the pages of a book. How many pages does the book have?*

From page #1 to page # 9, there are 9 pages

From page #10 to page # 99, there are 90 pages. However, since each page number has 2 digits, there are 180 digits from page # 10 to page # 99

From page #100 to page # 999, there are 900 pages. However, since each page number has 3 digits, there are 2700 digits from page # 100 to page # 999

Since there are 1629 digits, we know so far that number of pages is less than 900

Now, we need to find out how many digits there are for those pages greater than 99 or those pages with 3 digits

Since there are 189 digits for pages with less than 3 digits, we can subtract 189 from 1629 digits to find the number of pages with 3 digits

$1629 - 189 = 1440$

1440 represents the pages with 3 digits, so number of pages with 3 digits = 1440÷ 3 = 480

Thus, total number of pages = 480 + 90 + 9 = 579 pages

ADDITIONAL PRACTICE

More practice with linear rates of change can be found at https://www.ixl.com/math/grade-8/constant-rate-of-change

[Equations of Attack](http://illuminations.nctm.org/Lesson.aspx?id=2858)

From NCTM Illuminations, http://illuminations.nctm.org/Lesson.aspx?id=2858

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Suggested time, 1 hour.

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR2. Students will compare and graph functions.

a. Calculate rates of change of functions, comparing when rates increase, decrease, or stay constant. *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (MGSE6.RP.2;MGSE7.RP.1,2,3;MGSE8.F.2,5; MGSE9-12.F.IF.6)

b. Graph by hand simple functions expressed symbolically (use all four quadrants). (MGSE9-12.F.IF.7)

c. Interpret the equation y = mx + b as defining a linear function whose graph is a straight line. (MGSE8.F.3)

e. Analyze graphs of functions for key features (intercepts, intervals of increase/decrease, maximums/minimums, symmetries, and end behavior) based on context. (MGSE9-12.F.IF.4,7)

MFAQR3. Students will construct and interpret functions.

a. Write a function that describes a relationship between two quantities. (MGSE8.F.4, MGSE9-12.F.BF.1)

b. Use variables to represent two quantities in a real-world problem that change in relationship to one another (conceptual understanding of a variable). (MGSE6.EE.9)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students show perseverance in their efforts to apply their learning on a formative lesson.
3. Construct viable arguments and critique the reasoning of others. Students work collaboratively to explain their understanding of functions and defend their reasoning.
4. Model with mathematics. Students use the graph to model the rates of change, and tables to model mathematical thinking.
6. Attend to precision. Students use clear and precise language in their discussions and utilize patterns in identifying functions.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to define functions.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- plot and name points on a coordinate grid using correct coordinate pairs
- graph lines, given the slope and y-intercept
- write equations, given the slope and y-intercept
- determine algebraically if a point lies on a line

MATERIALS

- Copy of student page http://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/6-8/EquationsOfAttack-AS.pdf
- Slope cards
- http://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/6-8/EquationsOfAttack-AS-Cards.pdf
- Colored pencils or markers
- Coins or counters
- Scissors

ESSENTIAL QUESTIONS

- What are the advantages of finding equations graphically rather than algebraically?
- What are the advantages of finding equations algebraically rather than graphically?

TEACHER COMMENTARY

Students will plot points on a coordinate grid to represent ships before playing the graphing equations game with a partner. Points along the y-axis represent cannons and the slopes are chosen randomly to determine the line and equation of attacks. Students will use their math skills and strategy to sink their opponent's ships and win the game. After the game, an algebraic approach to the game is investigated.

Tell students that they will be playing a strategy game in which they must sink their opponent's ships. To win the game students will need to use their knowledge of graphing and linear equations.

Break the class up into pairs. Depending on the ability levels of your students, you may choose to allow them to pick their own partners or separate them into pre-determined pairs that are matched for mathematical ability. Distribute the Equations of Attack Activity Sheet, Slope Cards Activity Sheet, 2 different-colored pencils, a coin, and scissors to each pair of students. Note: If your class is just beginning to explore linear equations, you may wish to create your own set of slope cards with only integers (e.g., 2 and –3) and unit fractions (e.g., ¼ , but not ¾). To challenge more advanced students, consider including decimal slopes (e.g., 1.5).

Key Features of Functions

Adapted from Utah State Office of Education, Mathematics Vision Project,
https://www.mathematicsvisionproject.org/uploads/1/1/6/3/11636986/sec1_mod5_features_se_71713.pdf

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class. The suggested timing is 45 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR1. Students will understand characteristics of functions.

a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.(MGSE9-12.F.IF.1)
b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*. (MGSE9-12.F.IF.5)
c. Graph functions using sets of ordered pairs consisting of an input and the corresponding output. (MGSE8.F.1, 2)

MFAQR2. Students will compare and graph functions.

a. Calculate rates of change of functions, comparing when rates increase, decrease, or stay constant. *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (MGSE6.RP.2;MGSE7.RP.1,2,3;MGSE8.F.2,5; MGSE9-12.F.IF.6)
b. Graph by hand simple functions expressed symbolically (use all four quadrants). (MGSE9-12.F.IF.7)
c. Interpret the equation y = mx + b as defining a linear function whose graph is a straight line. (MGSE8.F.3)
e. Analyze graphs of functions for key features (intercepts, intervals of increase/decrease, maximums/minimums, symmetries, and end behavior) based on context. (MGSE9-12.F.IF.4,7)

STANDARDS FOR MATHEMATICAL PRACTICE

This lesson uses the following mathematical practices:
4. Model with mathematics. Students model with graphs and illustrations.
5. Use appropriate tools strategically. Students may use graphing calculators to graph functions.
6. Attend to precision. Students clearly articulate functions as words, equations, and tables.

EVIDENCE OF LEARNING

By the conclusion of this activity, students will be able to:

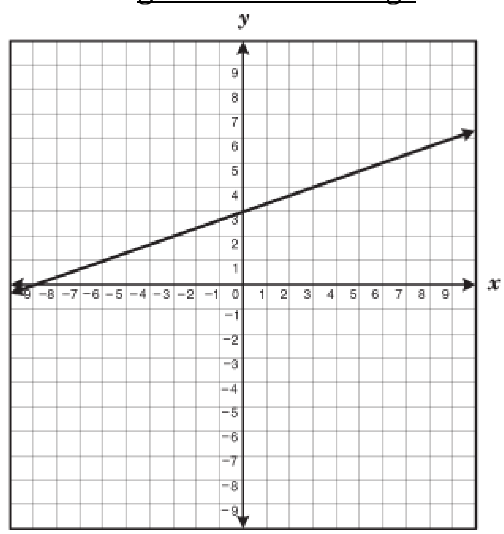
- sketch a graph to represent a function
- describe attributes of linear functions
- determine the domain of a function

ESSENTIAL QUESTIONS

- How can I sketch a graph for a function?
- How can I describe attributes of a function?

OPENER/ACTIVATOR

Using the Integers 0-9 (and each integer only once), how many Linear Functions can you create that have a greater rate of change than the following graph:



Hints:

What information tells you about the rate of change?
How can you determine the rate of change of the line on the graph above?

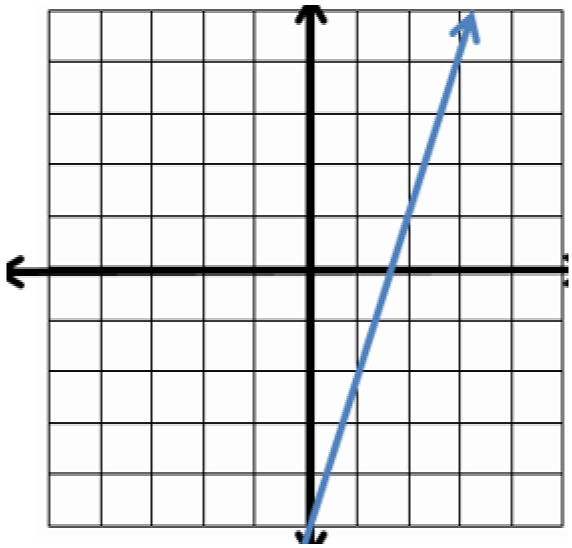
Solutions:
There are 4 possible solutions.

1. ***{(0,1),(2,3),(4,5),(6,7),(8,9)}***
2. ***{(1,0),(3,2),(5,4),(7,6),(9,8)}***
3. ***{(0,5),(1,6),(2,7),(3,8),(4,9)}***
4. ***{(5,0),(6,1),(7,2),(8,3),(9,4)}***

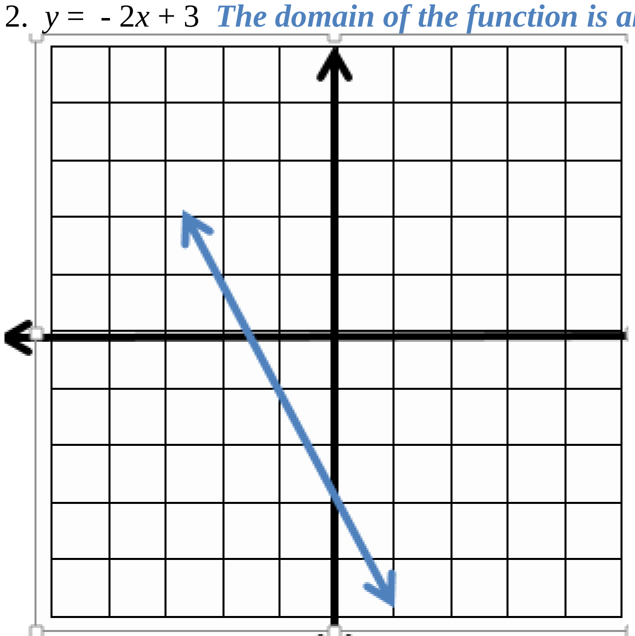
Key Features of Functions

Sketch a graph to represent each function, then state the domain of the function.

1. $y = 3x - 5$ ***The domain of the function is all real numbers.***



2. $y = -2x + 3$ ***The domain of the function is all real numbers.***



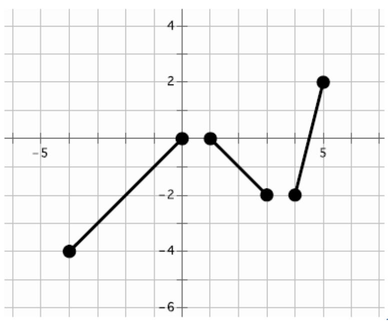
Determine if each statement is true or false, then justify why.

3. All linear equations are increasing. ***False. If the slope is negative, the equation will be decreasing.***

4. All linear relationships are functions with a domain and a range containing all real numbers. ***False. Linear relationships are functions, but the domain and range may be restricted to less than all real numbers, particularly depending on context. Also, vertical lines are not functions.***

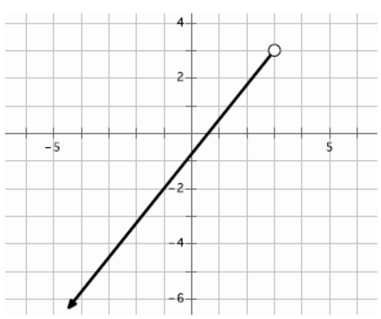
5. The y-intercept of a function is the value of y, when x = 0. ***True, this is the definition of y-intercept. In context, it explains what the independent variable is when the dependent variable is 0.***

6. The graph below shows the pre and post score data for three sports fitness tests on a certain scale. According to the graph, all three people improved on their fitness tests.



False. The middle person did worse on the post.

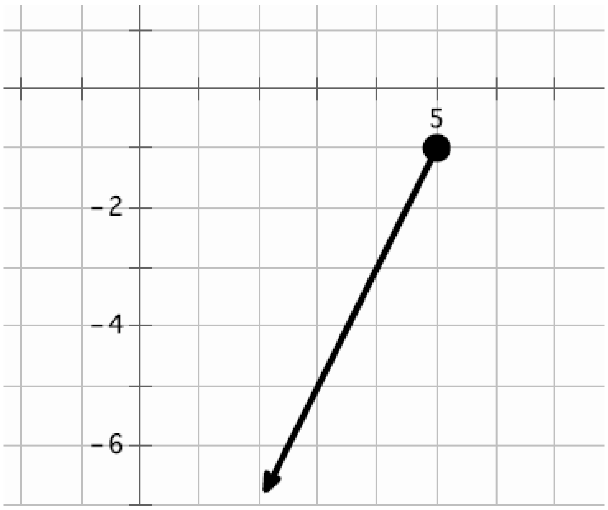
7. The x-intercept of the graph below is approximately (0.6, 0).



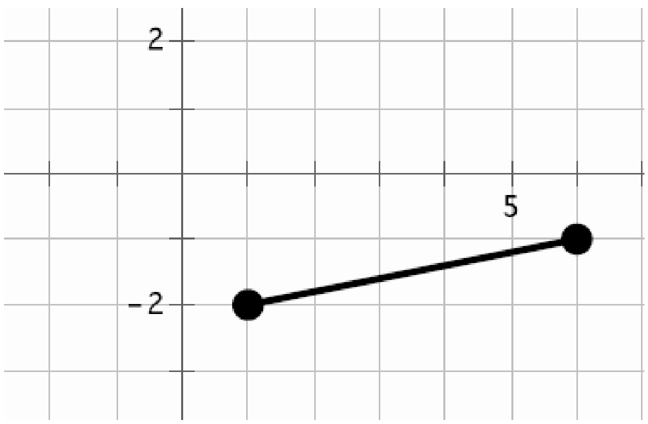
True.

Determine the domain of the function in the graphical representation.

8. ***Domain – all real numbers less than or equal to 5***



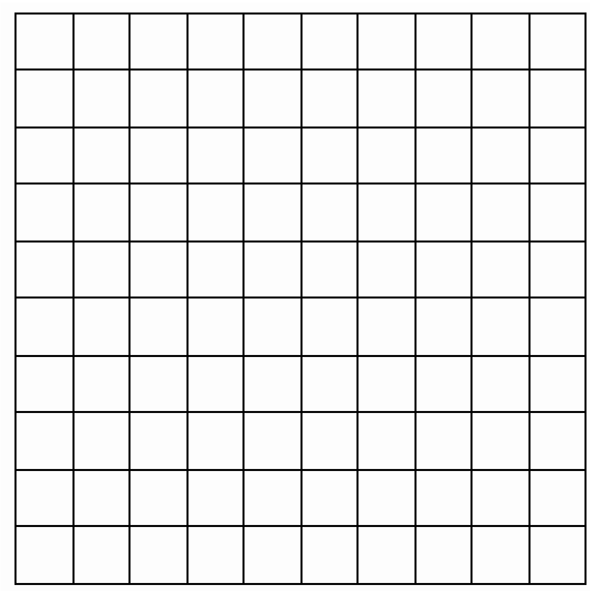
9. Domain – All real numbers between 1 and 5



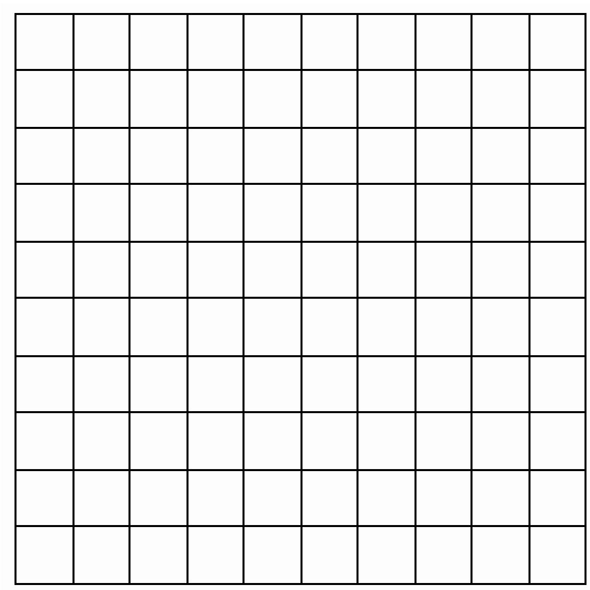
Key Features of Functions

Sketch a graph to represent each function, then state the domain of the function.

1. $y = 3x - 5$



2. $y = -2x + 3$



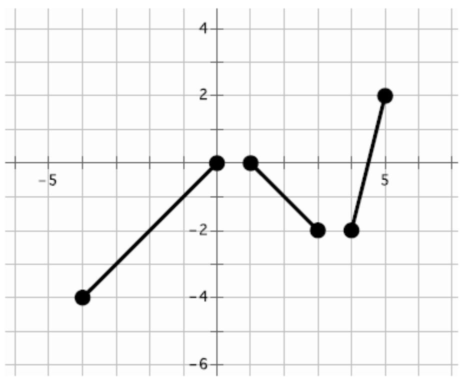
Determine if each statement is true or false, then justify why.

3. All linear equations are increasing.

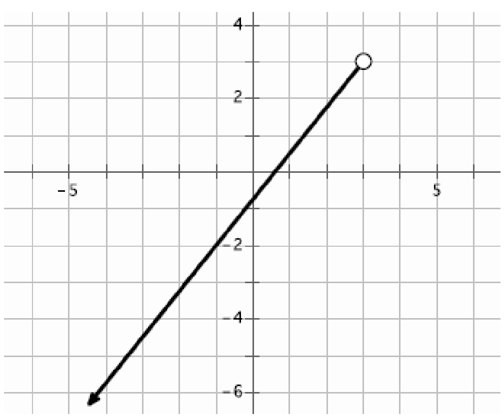
4. All linear relationships are functions with a domain and a range containing all real numbers.

5. The y-intercept of a function is the value of y, when x = 0.

6. The graph below shows the pre and post score data for three sports fitness tests on a certain scale. According to the graph, all three people improved on their fitness tests.

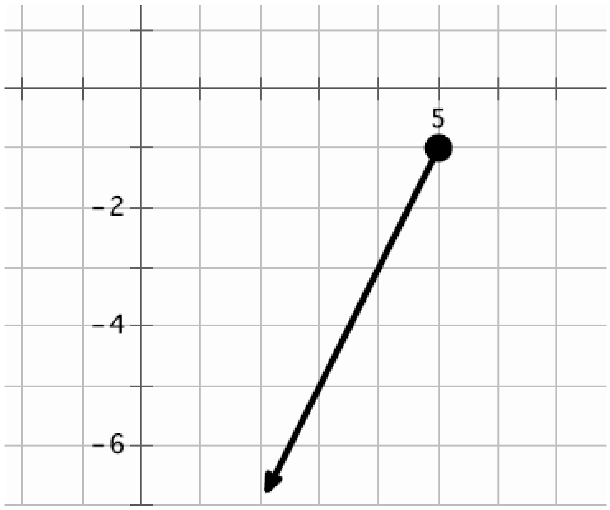


7. The x-intercept of the graph below is approximately (0.6, 0).

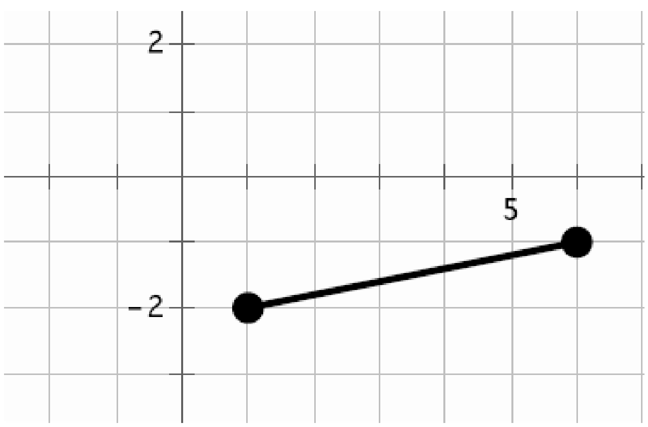


Determine the domain of the function in the graphical representation.

8.



9.



Analyzing Linear Functions (FAL)

Source: Georgia Mathematics Design Collaborative Formative Assessment Lessons
http://ccgpsmathematics6-8.wikispaces.com/Georgia+Mathematics+Design+Collaborative+Formative+Assessment+Lessons

SUGGESTED TIME FOR THIS LESSON

Exact timings will depend on the needs of your class. Recommended timing is 120 minutes.

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR2. Students will compare and graph functions.
c. Interpret the equation y = mx + b as defining a linear function whose graph is a straight line. (MGSE8.F.3)
f. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the great rate of change.* (MGSE8.F.2)

MFAQR3. Students will construct and interpret functions.
a. Write a function that describes a relationship between two quantities. (MGSE8.F.4, MGSE9-12.F.BF.1)
b. Use variables to represent two quantities in a real-world problem that change in relationship to one another (conceptual understanding of a variable). (MGSE6.EE.9)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students show perseverance in their efforts to apply their learning on a formative lesson.
2. Reason abstractly and quantitatively. Students will explain the meaning of slope and y-intercept.
3. Construct viable arguments and critique the reasoning of others. Students will work collaboratively with a partner.
4. Model with mathematics. Students will construct tables and graphs to match a real-world problem.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of the lesson, students will be able to understand

- slope as rate of change
- the meaning of x and y intercepts applied to real-world situations
- graphs and tables that represent realistic situation

MATERIALS

- Copies of formative assessment lesson pre-and post-assessment for each student
- Mini whiteboards and markers (optional)

ESSENTIAL QUESTIONS

- What can ordered pairs mean?
- What does slope mean in a problem situation?
- How can equations represent real-world scenarios?

TEACHER COMMENTARY

This formative assessment lesson will show if students truly understand the various aspects of linear equations, including slope, x-intercepts, y-intercepts, a chart of values, and graphing. It is very important they understand concepts about slope being a rate of change, how fast it changes, etc. They also need to understand and be able to communicate that the x-intercept is where the y value is zero, the y-intercept has an x that is zero, and relate linear descriptors to the context of the problem. These discussions will be critical and pivotal to their understanding of future graphs.

INTERVENTION

Prior to this lesson, students have been practicing rates of change and identifying characteristics of equations. If needed, more specific lessons on graphing equations can be found at http://crctlessons.com/algebra-lessons.html This site includes lessons, practice in the form of games, and mini-assessments.

Functions

Adapted from Inside Mathematics, http://bit.ly/1HJeYx6

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Suggested time, 1 hour.

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR2. Students will compare and graph functions.

b. Graph by hand simple functions expressed symbolically (use all four quadrants). (MGSE9-12.F.IF.7)

c. Interpret the equation y = mx + b as defining a linear function whose graph is a straight line. (MGSE8.F.3)

d. Use technology to graph non-linear functions. (MGSE8.F.3, MGSE9-12.F.IF.7)

f. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the great rate of change.* (MGSE8.F.2)

MFAQR3. Students will construct and interpret functions.

a. Write a function that describes a relationship between two quantities. (MGSE8.F.4, MGSE9-12.F.BF.1)

STANDARDS FOR MATHEMATICAL PRACTICE

3. **Construct viable arguments and critique the reasoning of others.** Students listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

6. **Attend to precision.** Students use clear definitions in discussion with others and in their own reasoning.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to

- demonstrate competence in working with graphs and equations of linear and non-linear functions.

MATERIALS

Copy of the lesson

ESSENTIAL QUESTION

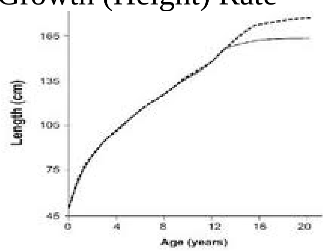
What are some important similarities and differences in linear and non-linear functions?

OPENER/ACTIVATOR

Draw a Picture

Sketch one example of a linear function and one example of a non-linear function; tell a partner what a possible context for your graph/sketch could be.

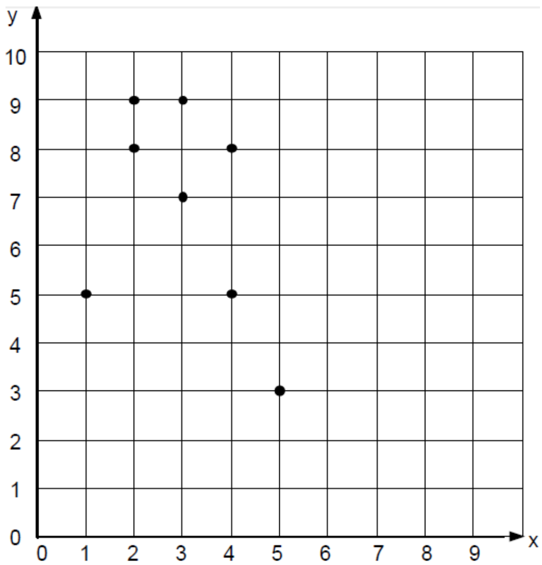
Possible examples:

Linear Function	
Non-Linear Function	Growth (Height) Rate 

Functions

On the grid are eight points from two different functions:

- four points fit a linear function
- the other four points fit a non-linear function.



For the linear function:

1. Write the coordinate pairs of its four points.
   ______*(2,9)*__________ _____*(3,7)*____________ ___*(4,5)*_____________ _____*(5,3)*___

   Draw the line on the grid.

2. Write an equation for the function. Show your work.
   ___________ $y = -2x + 13$ ___________________________

For the non-linear function:

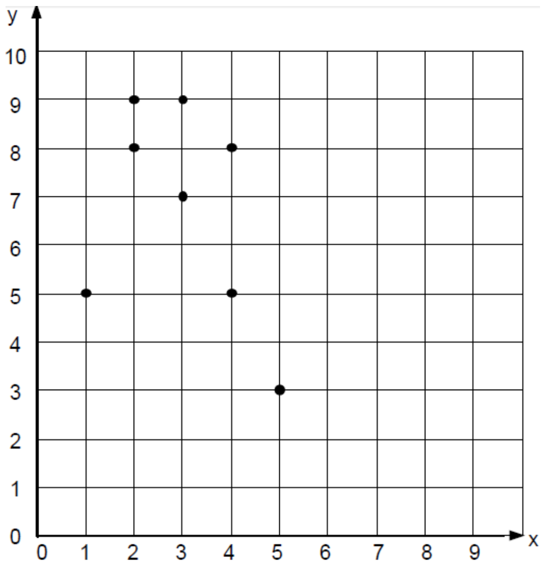
3. Write the coordinate pairs of its four points
   ________*(1,5)*________ ________*(2,8)*________ ______*(3,9)*_________ __*(4,8)*____
   Draw the graph of the function on the grid.

4. Extension: Can you write an equation that fits the non-linear function?
$y = -x^2 + 2x$ (this graph can be shown with a graphing utility)

Functions

On the grid are eight points from two different functions:

• four points fit a linear function
• the other four points fit a non-linear function.



For the linear function:

1. Write the coordinate pairs of its four points.

_________________ _________________ _________________ _________________

Draw the line on the grid.

2. Write an equation for the function. Show your work.

____

For the non-linear function:

3. Write the coordinate pairs of its four points

_________________ _________________ _________________ _________________

Draw the graph of the function on the grid.

4. Extension: Can you write an equation that fits the non-linear function ?________________
Show your work.

CLOSING/SUMMARIZER

Create two tables. The first should show a linear relationship and the second a non-linear relationship.

Here are two possible solutions. The first number is the x-value and the second number is the y-value.

Solution 1

1	**2**
3	**4**
5	**6**
7	**8**

Solution 2

1	**5**
2	**7**
3	**8**
4	**12**

ADDITIONAL PRACTICE

More practice with non-linear equations can be found at
http://www.ixl.com/math/grade-8/identify-linear-and-nonlinear-functions

Once the graphs in the given link are identified as linear or non-linear, graphing utilities may be used to graph the functions.

Quick Check II

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR1. Students will understand characteristics of functions.

a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.(MGSE9-12.F.IF.1)

b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*. (MGSE9-12.F.IF.5)

c. Graph functions using sets of ordered pairs consisting of an input and the corresponding output. (MGSE8.F.1, 2)

MFAQR2. Students will compare and graph functions.

a. Calculate rates of change of functions, comparing when rates increase, decrease, or stay constant. *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (MGSE6.RP.2;MGSE7.RP.1,2,3;MGSE8.F.2,5; MGSE9-12.F.IF.6)

b. Graph by hand simple functions expressed symbolically (use all four quadrants). (MGSE9-12.F.IF.7)

c. Interpret the equation y = mx + b as defining a linear function whose graph is a straight line. (MGSE8.F.3)

d. Use technology to graph non-linear functions. (MGSE8.F.3, MGSE9-12.F.IF.7)

e. Analyze graphs of functions for key features (intercepts, intervals of increase/decrease, maximums/minimums, symmetries, and end behavior) based on context. (MGSE9-12.F.IF.4,7)

f. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the great rate of change.* (MGSE8.F.2)

Quick Check II

1. Megan weighs 65 kilograms. She wants to increase her weight for soccer season. Megan begins to eat more and train. After 5 weeks, she weighs 75 kilograms. Draw a line to represent Megan's situation and determine what the slope (rate of change) would be? ***m = 2 line through (0, 65) (5, 75)***

2. Find the slope (rate of change) of the points (-10, 4) and (4,-6). Hint: Sketching a graph may help.

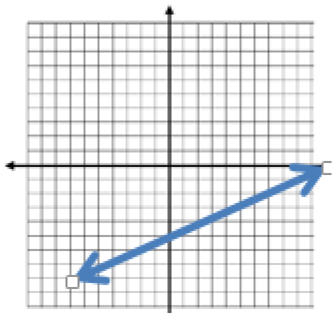
$$m = \frac{-5}{7}$$

3. Complete the missing information in the table.

x	**2x + 1**	**y**	**Ordered Pairs**
-2	***2(-3) + 1***	**-5**	***(-2, -5)***
0	**2(0) + 1**	***1***	**(0, 1)**
2	***2(2) + 1***	***5***	**(2, 5)**

Note: This could be a good time to introduce function notation by stating that f(-2) = -5, and that f(x) = 1 when x = 0 as well as f(x) = 5 when x = 2.

4. Graph this function: $\mathbf{y = \frac{1}{3} x - 5}$

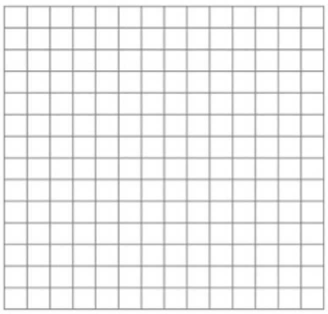


Additional problems with links are described below:

- Constant Rates:https://www.illustrativemathematics.org/content-standards/6/RP/A/2/tasks/1175 and https://www.illustrativemathematics.org/content-standards/7/RP/A/1/tasks/1176
- Linear vs Non-linear: https://www.illustrativemathematics.org/content-standards/8/F/A/3/tasks/813 and https://www.illustrativemathematics.org/content-standards/8/F/A/1/tasks/1165
- Non-linear: https://www.illustrativemathematics.org/content-standards/HSF/IF/A/1/tasks/597

<u>QUICK CHECK II</u>

1. Megan weighs 65 kilograms. She wants to increase her weight for soccer season. Megan begins to eat more and train. After 5 weeks, she weighs 75 kilograms. Draw a line to represent Megan's situation and determine what the slope would be?

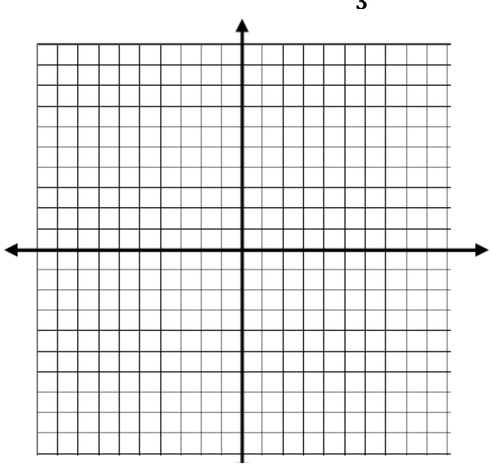


2. Find the slope of the points (-10, 4) and (4,-6). Hint: Sketching a graph may help.

3. Complete the missing information in the table.

x	**2x + 1**	**y**	**Ordered Pairs**
-2		-3	
	2(0) + 1		(0, 1)
			(2, 5)

4. Graph this function: $\mathbf{y = \frac{1}{3}x - 5}$



Beads Under the Cloud - FAL

This task was created by Kentucky Department of Education Mathematics Specialists and is available for downloading in its entirety at
http://education.ky.gov/educational/diff/Documents/BeadsUnderTheCloud.pdf

This problem solving lesson is intended to help you assess how well students are able to identify patterns in a realistic context: the number of beads of different colors that are hidden behind the cloud.

SUGGESTED TIME FOR THIS LESSON:
Exact timings will depend on the needs of your class. Suggested time, 1 to 1.5 hours.

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR1. Students will understand characteristics of functions.
a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.(MCC9-12.F.IF.1)
b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. (MCC9-12.F.IF.5)

MFAQR3. Students will construct and interpret functions.
a. Write a function that describes a relationship between two quantities. (MGSE8.F.4, MGSE9-12.F.BF.1)
b. Use variables to represent two quantities in a real-world problem that change in relationship to one another (conceptual understanding of a variable). (MGSE6.EE.9)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students make sense of the problem presented in the lesson and continue to work toward a solution.
3. Construct viable arguments and critique the reasoning of others. Working with a partner, students identify functions matching equations, tables and rules.
7. Look for and make use of structure. Students use the structure of function notation in context.
8. Look for and express regularity in repeated reasoning. Students use repeated reasoning to define functions.

EVIDENCE OF LEARNING/LEARNING TARGET

Students will demonstrate understanding and proficiency in:

- choosing an appropriate, systematic way to collect and organize data.
- examining the data and looking for patterns
- describing and explaining findings clearly and effectively.

MATERIALS

- Each individual student will need one copy of the *Beads under the Cloud* sheet and one copy of the *How did you work?* Sheet.
- Each small group of students will need a copy of *Sample Responses to Discuss* and samples of student work.

ESSENTIAL QUESTION

How can patterns be identified in order to find a function rule that applies to a set of data?

EXTENSION

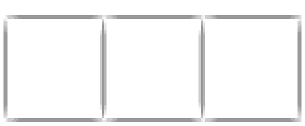
Adapted from Annenberg Learner
http://www.learner.org/courses/learningmath/algebra/session5/part_a/index.html

A function expresses a relationship between variables. For example, consider the number of toothpicks needed to make a row of squares. The number of toothpicks needed depends on the number of squares we want to make. If we call the number of toothpicks T and the number of squares S, we could say that T is a function of S. S is called the **independent variable** in this case, and T the **dependent variable** -- the value of T *depends upon* whatever we determine the value of S to be.

Predict, Explain, Observe Probe: Before the lesson, have students **PREDICT** what might happen as they build squares using toothpicks. How many will they start with? How many do they think each square will need? How many toothpicks will be needed for a row of 5 squares?

Groups: People will likely have varying degrees of comfort and familiarity with the content, so working with partners may be helpful. Work on Problems 1-3, completing the table and coming up with the function rule. You may want to share rules before moving on. If students are familiar with spreadsheets, creating the table on a computer may be helpful.

In this section, we will **EXPLORE** and **EXPLAIN** the dependence of one variable on another. Make a row of squares using toothpicks. The squares are joined at the side.



OBSERVE the pattern. Have students construct a function table as they explore the toothpick squares.

Problem 1
How many toothpicks are needed for one square? For two squares? For five squares? Make a table of these values.

Squares	**Toothpicks**
1	***4***
2	***7***
3	***10***
4	***13***
5	***16***

Problem 2
Develop a formula describing the number of toothpicks as a function of the number of squares.
The formula is $T = 3S + 1$.

Floating Down the Lazy River

Adapted from a task by the Mathematics Vision Project, Utah State Office of Education, http://www.uen.org/core/math/downloads/sec1_floating_down_river.pdf

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Suggested time, 1- 1.5 hour.

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR1. Students will understand characteristics of functions.

a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.(MCC9-12.F.IF.1)

b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*. (MCC9-12.F.IF.5)

c. Graph functions using sets of ordered pairs consisting of an input and the corresponding output. (MCC8.F.1, 2)

MFAQR2. Students will compare and graph functions.

a. Calculate rates of change of functions, comparing when rates increase, decrease, or stay constant. *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (MGSE6.RP.2;MGSE7.RP.1,2,3;MGSE8.F.2,5; MGSE9-12.F.IF.6)

b. Graph by hand simple functions expressed symbolically (use all four quadrants). (MGSE9-12.F.IF.7)

c. Interpret the equation y = mx + b as defining a linear function whose graph is a straight line. (MGSE8.F.3)

e. Analyze graphs of functions for key features (intercepts, intervals of increase/decrease, maximums/minimums, symmetries, and end behavior) based on context. (MGSE9-12.F.IF.4,7)

f. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the great rate of change.* (MGSE8.F.2)

MFAQR3. Students will construct and interpret functions.

a. Write a function that describes a relationship between two quantities. (MGSE8.F.4,MGSE9-12.F.BF.1)

b. Use variables to represent two quantities in a real-world problem that change in relationship to one another (conceptual understanding of a variable). (MGSE6.EE.9)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students interpret the situation and persevere in demonstrating their understanding, and use trial and error to find solutions.
2. Reason abstractly and quantitatively. Students demonstrate abstract reasoning in creating tables of values.
3. Construct viable arguments and critique the reasoning of others. Students work collaboratively to explain their understanding of functions.
6. Attend to precision. Students use clear and precise language in discussing strategies, and carefully work to solve problems according to the lesson parameters.
7. Look for and make use of structure. Students use their understanding of functions to make sense of information given and extend their thinking.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- interpret key features of a relationship using tables and graphs.
- explain that suggested observations about a function/graph are true or not true, supported by evidence.
- describe features of a function from its graphical representation.

TEACHER NOTES

The purpose of this task is to further define function and to solidify key features of functions given different representations. Features include:

- ***domain and range***
- ***where the function is increasing or decreasing***
- ***x and y intercepts***
- ***rates of change (informal)***
- ***introduce function notation for equations***

GROUPING:

Have students work for a few minutes individually to get started. Then, pair them up to support each other and engage in mathematical conversation. Allow pairs to share their successes; also, have discussions as a class group.

Floating Down the Lazy River

Alonzo, Maria, and Sierra were floating in inner tubes down the lazy river at a water park, enjoying their day. Alonzo noticed that sometimes the water level was higher in some places than in others. Maria noticed there were times they seemed to be moving faster than at other times. Sierra laughed and said "Math is everywhere!" To learn more about the river, Alonzo and Maria collected data throughout the trip.

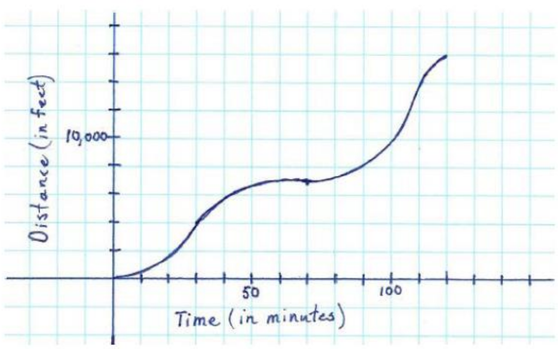
Alonzo created a table of values by measuring the depth of the water every ten minutes.

Time (in minutes)	0	10	20	30	40	50	60	70	80	90	100	110	120
Depth (in feet)	4	6	8	10	6	5	4	5	7	12	9	6.5	5

1. Use the data collected by Alonzo to interpret key features of this relationship.

Answers will vary and could include: domain is time, in increments of 10 {0, 10,20…120}
Range is depth, in varying increments {4, 6, 8, 10, 6, 5, 4, 5, 7, 12, 9, 65. 5}
Data is a function, but is not linear….

Maria created a graph by collecting data on a GPS unit that told her the distance she had traveled over a period of time.



2. Using the graph created by Maria, describe they key features of this relationship.

Answers will vary, and could include such information as:
Domain is time in minutes; range is distance in feet;Data is a function, but is not linear…

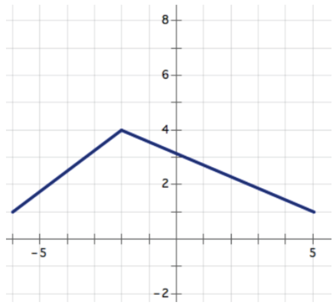
Sierra looked at the data collected by her two friends and made several of her own observations.

3. Explain why you either agree or disagree with each observation made.

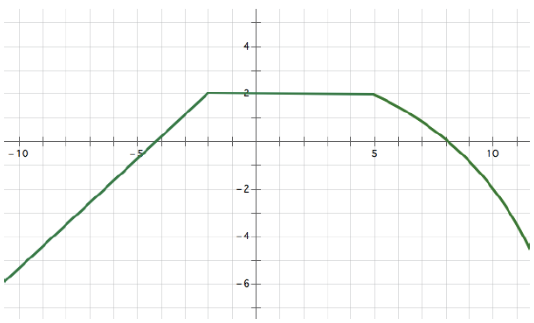
- The depth of the water increases and decreases throughout the 120 minutes of floating down the river. ***Agree, see Alonzo's data***
- The distance traveled is always increasing. ***Disagree. It levels at some points.***
- The distance traveled is a function of time. ***Agree***
- The distance traveled is greatest during the last ten minutes of the trip than during any other ten minute interval of time. ***Agree, if disagree, site another interval, 30-40?***
- The domain of the distance/time graph is all real numbers. ***Disagree, must be positive.***
- The y-intercept of the depth of water over time function is (0,0). ***Disagree; (0,4)***
- The distance traveled increases and decreases over time. ***Disagree, does not decrease, but increases inconsistently so is not linear.***
- The water level is a function of time. ***Agree***
- The range of the distance/time graph is from [0, 15000]. ***Disagree, range will continue over time, and graph goes to 16,000***
- The domain of the depth of water with respect to time is from [0,120] ***Agree***
- The range of the depth of water over time is from [4,5]. ***Disagree; range is [4,12]***
- The distance/ time graph has no maximum value. **Agree**

For each graph given, provide a description of the function. Be sure to consider the following: Decreasing / increasing, domain/ range, etc.

4. Description of function. ***Possible Answers:: non-linear; domain [-4,5]; range [1,4]***



5.
Description of function ***Possible Answers: non-linear; domain possibly all real numbers; range could be [-6,2]; increases in a linear pattern, then levels off, with a non-linear decrease…***



Write equations for the given tables.

6. ***y = -3x + 8***

n	f(n)
1	5
2	2
3	-1

7. ***y = 6x***

n	f(n)
1	6
2	12
3	**18**

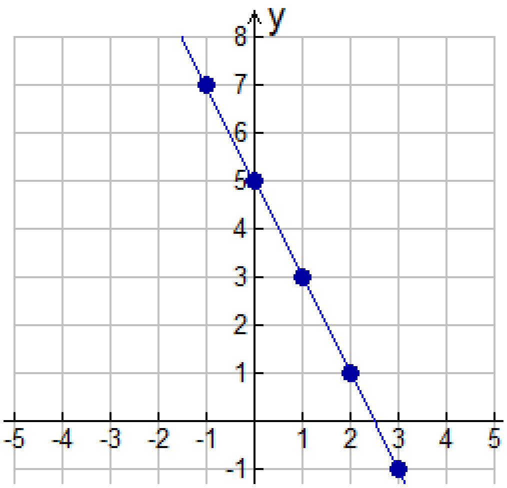
8. *$y = 2x + 3$*

n	f(n)
1	5
4	11
5	13

9. *$y = 4x - 13$*

n	f(n)
0	-13
2	-5
3	-1

10. Which table corresponds to the graph?



A. *

x	y
-1	7
0	5
1	3
2	1
3	-1

B.

x	y
7	-1
5	0
3	1
1	2
-1	3

C.

x	y
1	7
5	5
3	1
2	1
3	-1

D.

x	y
7	1
0	5
1	3
2	1
3	1

Floating Down the Lazy River

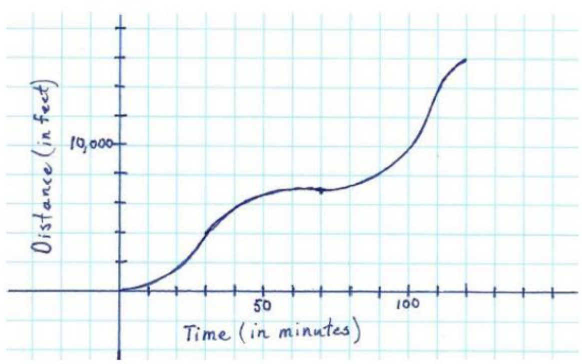
Alonzo, Maria, and Sierra were floating in inner tubes down the lazy river at a water park, enjoying their day. Alonzo noticed that sometimes the water level was higher in some places than in others. Maria noticed there were times they seemed to be moving faster than at other times. Sierra laughed and said "Math is everywhere!" To learn more about the river, Alonzo and Maria collected data throughout the trip.

Alonzo created a table of values by measuring the depth of the water every ten minutes.

Time (in minutes)	0	10	20	30	40	50	60	70	80	90	100	110	120
Depth (in feet)	4	6	8	10	6	5	4	5	7	12	9	6.5	5

1. Use the data collected by Alonzo to interpret key features of this relationship.

Maria created a graph by collecting data on a GPS unit that told her the distance she had traveled over a period of time.



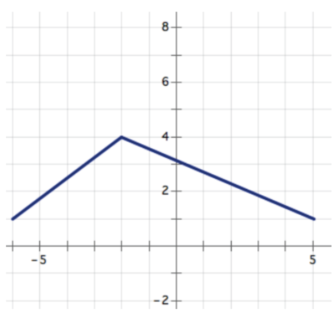
2. Using the graph created by Maria, describe they key features of this relationship.

3. Sierra looked at the data collected by her two friends and made several of her own observations. Explain why you either agree or disagree with each observation made.

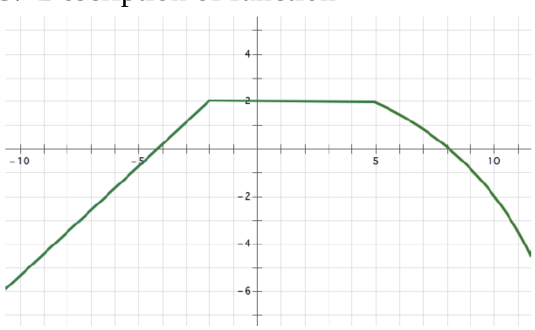
- The depth of the water increases and decreases throughout the 120 minutes of floating down the river.
- The distance traveled is always increasing.
- The distance traveled is a function of time.
- The distance traveled is greatest during the last ten minutes of the trip than during any other ten minute interval of time.
- The domain of the distance/time graph is all real numbers.
- The y-intercept of the depth of water over time function is (0,0).
- The distance traveled increases and decreases over time.
- The water level is a function of time.
- The range of the distance/time graph is from [0, 15000].
- The domain of the depth of water with respect to time is from [0,120]
- The range of the depth of water over time is from [4,5].
- The distance/ time graph has no maximum value.

For each graph given, provide a description of the function. Be sure to consider the following: Decreasing / increasing, domain/ range, etc.

4. Description of function



5. Description of function



Write equations for the given tables.

6.

n	f(n)
1	5
2	2
3	-1

7.

n	f(n)
1	6
2	12
3	24

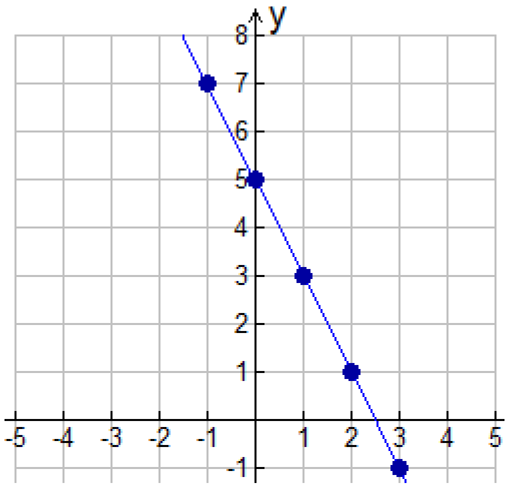
8.

n	f(n)
1	5
4	11
5	13

9.

n	f(n)
0	-13
2	-5
3	-1

10. Which table corresponds to the graph?



A.

x	y
-1	7
0	5
1	3
2	1
3	-1

B.

x	y
7	-1
5	0
3	1
1	2
-1	3

C.

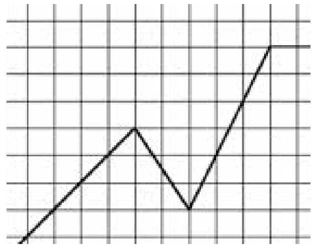
x	y
1	7
5	5
3	1
2	1
3	-1

D.

x	y
7	1
0	5
1	3
2	1
3	1

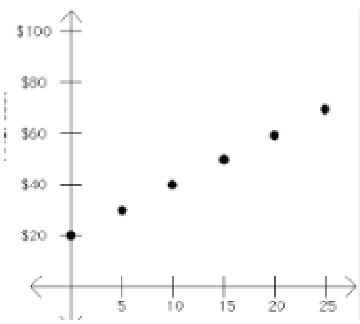
CLOSING/SUMMARIZER

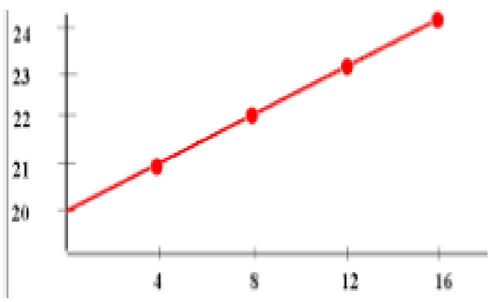
Given a graph, describe a situation that could be modeled by the graph.
Example:

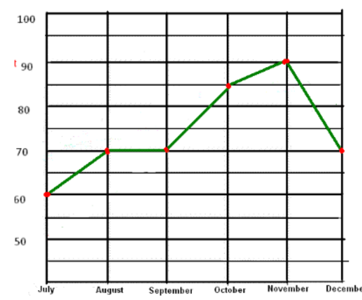


This graph could represent people walking a distance (vertical axis) over a period of time (horizontal axis). Perhaps they left home and walked at a steady pace to a point, then walked back, closer to their origin (time is still progressing but distance is reduced); then at a faster pace again walked farther away from the starting point, finally stopping.

Choose one of the graphs and write a scenario that could be modeled by the graph. (Note: Students do not need to identify the functions, only a scenario that might generate the graph.)







Getting Ready for a Pool Party

Adapted from a task by the Mathematics Vision Project, Utah State Office of Education, http://www.uen.org/core/math/downloads/sec1_getting_ready_pool_party.pdf

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Suggested time, 1- 1.5 hour.

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR1. Students will understand characteristics of functions.

a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.(MGSE9-12.F.IF.1)

b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*. (MGSE9-12.F.IF.5)

c. Graph functions using sets of ordered pairs consisting of an input and the corresponding output. (MGSE8.F.1, 2)

MFAQR2. Students will compare and graph functions.

a. Calculate rates of change of functions, comparing when rates increase, decrease, or stay constant. *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (MGSE6.RP.2;MGSE7.RP.1,2,3;MGSE8.F.2,5; MGSE9-12.F.IF.6)

b. Graph by hand simple functions expressed symbolically (use all four quadrants). (MGSE9 12.F.IF.7)

c. Interpret the equation y = mx + b as defining a linear function whose graph is a straight line. (MGSE8.F.3)

e. Analyze graphs of functions for key features (intercepts, intervals of increase/decrease, maximums/minimums, symmetries, and end behavior) based on context. (MGSE9-12.F.IF.4,7)

f. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the great rate of change.* (MGSE8.F.2)

MFAQR3. Students will construct and interpret functions.

a. Write a function that describes a relationship between two quantities. (MGSE8.F.4,MGSE9-12.F.BF.1)

b. Use variables to represent two quantities in a real-world problem that change in relationship to one another (conceptual understanding of a variable). (MGSE6.EE.9)

c. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of context. (MGSE9-12.F.IF.2)

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students interpret the situation and persevere in demonstrating their understanding.
3. Construct viable arguments and critique the reasoning of others. Students work collaboratively to explain their understanding of functions in real life situations.
4. Model with mathematics. Students model functions with graphs and tables.
6. Attend to precision. Students use clear and precise language in discussing strategies, and carefully work to identify patterns.
7. Look for and make use of structure. Students use their understanding of functions to make sense of information given and extend their thinking in real-life situations.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- use a story context to graph and describe key features of functions
- identify a linear function.
- graph a function, and relate the domain of a function to its graph and to the quantitative relationship described.

MATERIALS

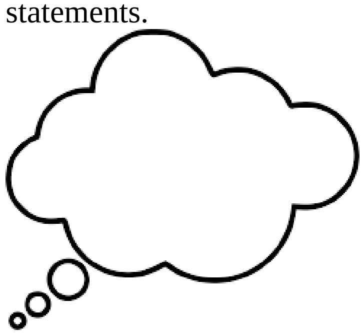
Student copy of the lesson.

ESSENTIAL QUESTIONS

How can I use the information in a story to determine if a function exists?
What does the domain of a function mean?
What does the range of a function tell me?

OPENER/ACTIVATOR

"What is in my head?"
Students draw a thought bubble and write a statement summarizing something important they know about FUNCTIONS. If done on sticky notes, their thought bubbles can create a useful class graphic organizer. The thought bubbles can be revisited at the end of the lesson or later in the module to see if they want to add important understandings or correct, clarify or edit their statements.

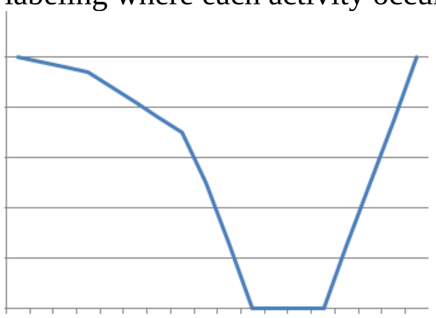


Getting Ready for a Pool Party

Charlotte has a small pool full of water that needs to be emptied and cleaned, then refilled for a pool party. During the process of getting the pool ready, Charlotte did all of the following activities, each during a different time interval.

Removed water with a single bucket	Filled the pool with a hose (same rate as emptying pool)
Drained water with a hose (same rate as filling pool)	Cleaned the empty pool
Charlotte and her two friends removed water with three buckets	Took a break

1. Sketch a possible graph showing the height of the water level in the pool over time. Be sure to include all of the activities Charlotte did to prepare the pool for the party. Remember that only one activity happened at a time. Think carefully about how each section of your graph will look, labeling where each activity occurs.



Remove water with single bucket;
Remove with three buckets;
Drain remaining water with a hose;
Clean empty pool;
Take a break;
Fill pool with hose, same rate as draining with a hose.

Note: students could choose another order for their tasks

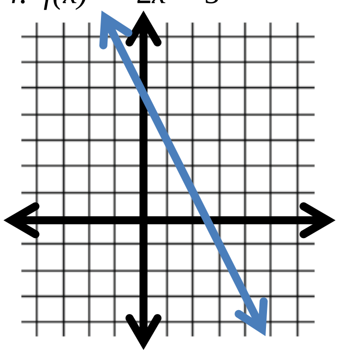
2. Create a story connecting Charlotte's process for emptying, cleaning, and then filling the pool to the graph you have created. Do your best to use appropriate mathematics vocabulary.

On Saturday, Charlotte invited friends over to celebrate the end of school in her pool. To prepare, she needed to clean the pool. She began emptying water using a bucket, but it was going really slowly. Emma and Carolyn came by her house, and offered to help. Together, the three of them used buckets to empty water. The rate of change was somewhat greater but not fast enough. Carolyn said, "Hey! We can use the hose as a syphon to drain the water faster." This caused the rate of change to increase more and emptied the pool much faster. After the pool was empty, they scrubbed it clean, then took a short break to drink lemonade. Filling the pool with the hose at the same rate it emptied went pretty fast, and in no time they were ready for the party.

3. Does your graph represent a function? Why or why not? Would all graphs created for this situation represent a function? How do you know?
Yes, the sample graph represents a function. Each x value produces a unique y value. The vertical line test works.
The order of tasks will cause the graphs to vary, but events occurring over time (x) should produce a function.

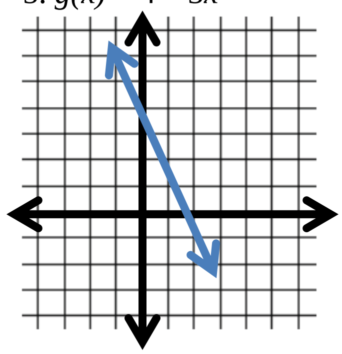
PRACTICE
Graph each of the functions.

4. $f(x) = -2x + 5$



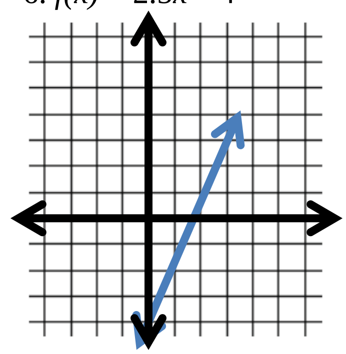
Graph should pass through (0,5), (4, -3)

5. $g(x) = 4 - 3x$



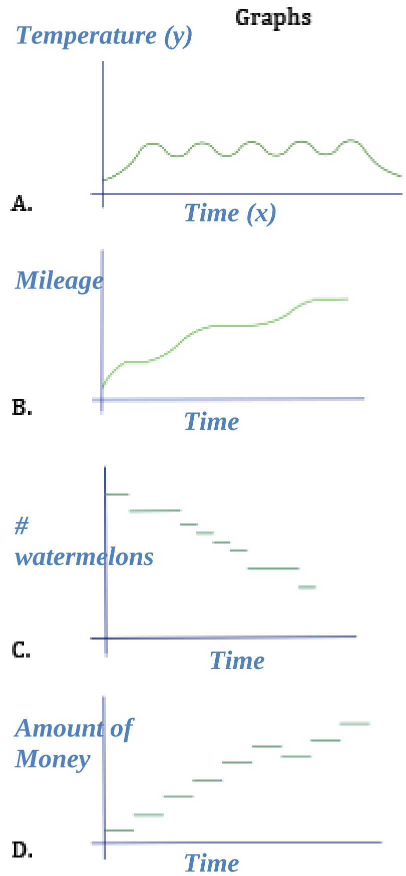
Graph should pass through (0, 4), (1,1)

6. $f(x) = 2.5x - 4$



Graph should pass through (0, -4), (2, 1)

Match each graph to the contextual description that fits best. Then label the independent and dependent axis with the proper variables.



___*D*____ 7. The amount of money in a savings account where regular deposits and some withdrawals are made.

____*A* __8. The temperature of the oven on a day that mom bakes several batches of cookies.

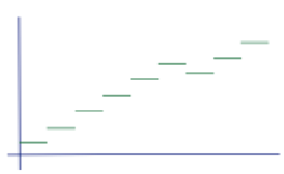
___*B*____9. The amount of mileage recorded on the odometer of a delivery truck over a period of time.

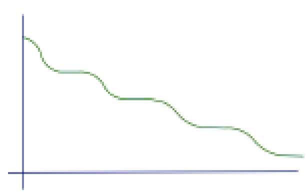
___*C*___10. The number of water melons available for sale at the farmer's market on Thursday

11. When x = 3 in the table below, what is the value of g(x)?
g(x) = 2x + 1

x	*g(x)*
0	1
1	3
2	5
3	*7*

12. A gas station has an amount of gasoline on hand before a tanker truck delivers more. Which of the two graphs below could represent this information? *b*


a.


b.

Getting Ready for a Pool Party

Charlotte has a small pool full of water that needs to be emptied and cleaned, then refilled for a pool party. During the process of getting the pool ready, Charlotte did all of the following activities, each during a different time interval.

Removed water with a single bucket	Filled the pool with a hose (same rate as emptying pool)
Drained water with a hose (same rate as filling pool)	Cleaned the empty pool
Charlotte and her two friends removed water with three buckets	Took a break

1. Sketch a possible graph showing the height of the water level in the pool over time. Be sure to include all of the activities Charlotte did to prepare the pool for the party. Remember that only one activity happened at a time. Think carefully about how each section of your graph will look, labeling where each activity occurs.

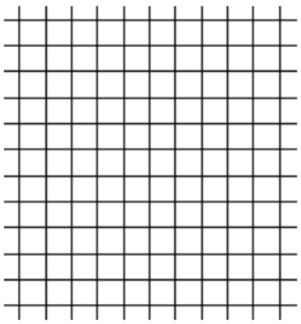
2. Create a story connecting Charlotte's process for emptying, cleaning, and then filling the pool to the graph you have created. Do your best to use appropriate mathematics vocabulary.

3. Does your graph represent a function? Why or why not? Would all graphs created for this situation represent a function? How do you know?

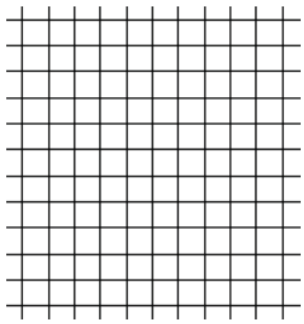
PRACTICE

Graph each of the functions.

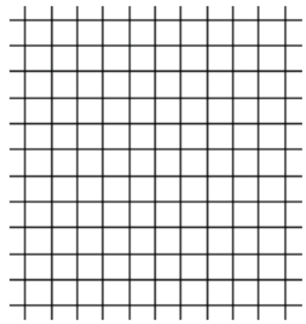
4. $f(x) = -2x + 5$



5. $g(x) = 4 - 3x$



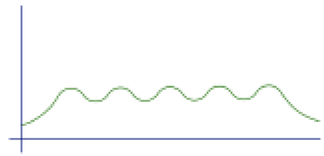
6. $f(x) = 2.5x - 4$



Match each graph to the contextual description that fits best. Then label the independent and dependent axis with the proper variables.

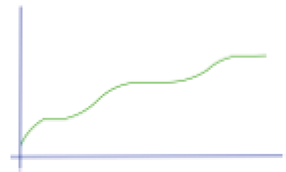
Graphs

_______ 7. The amount of money in a savings account where regular deposits and some withdrawals are made.

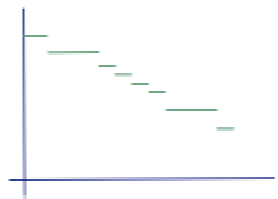


A.

________8. The temperature of the oven on a day that mom bakes several batches of cookies.



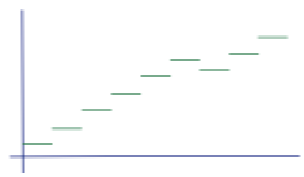
B.



C.

________9. The amount of mileage recorded on the odometer of a delivery truck over a period of time.

_______10. The number of water melons available for sale at the farmer's market on Thursday.

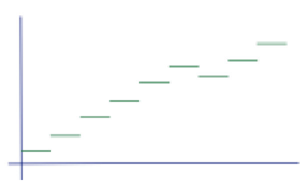


D.

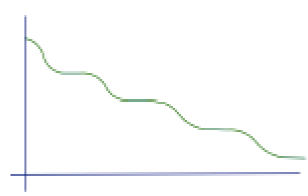
11. When x = 3 in the table below, what is the value of g(x)?

x	*g(x)*
0	1
1	3
2	5
3	?

12. A gas station has an amount of gasoline on hand before a tanker truck delivers more. Which of the two graphs could represent this information?



a.



b.

CLOSING/SUMMARIZER

- How would you use a description of a series of activities to create a graph?
- How would looking at a graph help you determine if a function exists?
- How would you describe the graph, given a function rule?

ADDITIONAL PRACTICE

For more practice matching graphs with scenarios, go to
http://www.nationalstemcentre.org.uk/elibrary/resource/6658/matching-graphs-and-scenarios

Cell Phones

SUGGESTED TIME FOR THIS TASK:
Exact timings will depend on the needs of your class. Suggested time, 1.5 hours.

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR1. Students will understand characteristics of functions.
b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*. (MGSE9-12.F.IF.5)
c. Graph functions using sets of ordered pairs consisting of an input and the corresponding output. (MGSE8.F.1, 2)

MFAQR2. Students will compare and graph functions.
b. Graph by hand simple functions expressed symbolically (use all four quadrants). (MGSE9 12.F.IF.7)
c. Interpret the equation y = mx + b as defining a linear function whose graph is a straight line. (MGSE8.F.3)
e. Analyze graphs of functions for key features (intercepts, intervals of increase/decrease, maximums/minimums, symmetries, and end behavior) based on context. (MGSE9-12.F.IF.4,7)
f. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the great rate of change.* (MGSE8.F.2)

MFAQR3. Students will construct and interpret functions.
a. Write a function that describes a relationship between two quantities. (MGSE8.F.4, MGSE9-12.F.BF.1)
b. Use variables to represent two quantities in a real-world problem that change in relationship to one another (conceptual understanding of a variable). (MGSE6.EE.9)
c. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of context. (MGSE9-12.F.IF.2)

STANDARDS FOR MATHEMATICAL PRACTICE

1**. Make sense of problems and persevere in solving them.** Students must make sense of the problem presented in the lesson and continue to work toward a solution.
2. Reason abstractly and quantitatively. Students think through each step of the lesson, predicting, estimating, and verifying their conclusions.
4. Model with mathematics. Students model with graphs, illustrations, student work.
5. Use appropriate tools strategically. Students use graphing calculators to graph functions.
6. Attend to precision. Students clearly articulate functions as words, equations, and tables in real-life context.
7. Look for and make use of structure. Students use the structure of function notation in context.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- extend their use of function notation, f(x), to real life situations .
- interpret meaning of each part of an equation in funcion notation**.**

MATERIALS

- Student copy of task
- Graphing calculator (optional)

ESSENTIAL QUESTION

- How can I use function notation, f(x), to represent real life situations?
- What does the x (or independent variable) mean in each function?

OPENER/ACTIVATOR

Function notation looks like an instruction to multiply, but the meaning is very different. To avoid misinterpretation, **be sure you know which letters represent functions**.

For example, if *g* represents a function, then *g*(4) is *not* multiplication but rather the value of *g* at 4, e.g., the output value of the function *g* when the input value is 4.

Given a table of values, answer the following questions.

Age (yrs.)	x	0	1	2	3	4	5	6
Height (in.)	$y = h(x)$	21	30	35	39	43	46	48

a. What is $h(5)$? What does this mean?

46; when 5 is the input/domain/independent variable, according to the prescribed function, the corresponding or resulting value, output/range/dependent variable is 46

b. When x is 3, what is y? Express this fact using function notation.

39; $f(x)= 39$ when $x = 3$ or $f(3)= 39$

c. Find an x so that $h(x) = 48$. Explain your method. What does your answer mean?

If $h(x) = 48$, $x= 6$, so $h(6) = 48$

Given a table of values, answer the following questions.

Age (yrs.)	x	0	1	2	3	4	5	6
Height (in.)	$y = h(x)$	21	30	35	39	43	46	48

a. What is $h(5)$? What does this mean?

b. When x is 3, what is y? Express this fact using function notation.

c. Find an x so that $h(x) = 48$. Explain your method. What does your answer mean?

<u>TEACHER COMMENTARY</u>

This lesson is a combination of three short tasks that allow students to apply their understanding of functions and practice using function notation. Students can work individually to begin making notes and setting up the three problems, then work with a partner or small group to process the situations together.

These simple tasks assess whether students can interpret function notation. The three parts of the lesson provide a logical progression of exercises for advancing understanding of function notation and how to interpret it in terms of a given context.

FUNCTION NOTATION - LEARNING TASK I

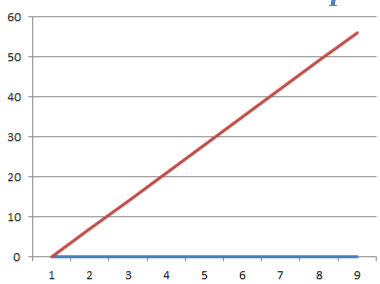
Towanda is paid \$7 per hour in her part-time job at the local Dairy Stop. Let t be the amount time that she works, in hours, during the week, and let $P(t)$ be her gross pay (before taxes), in dollars, for the week.

a. Make a table showing how her gross pay depends upon the amount of time she works during the week.

t	*0*	*1*	*2*	*3*	*4*
P(t)	*0*	*7*	*14*	*21*	*28*

You may want to instruct students how many hours or pairs they should generate.

b. Make a graph illustrating how her gross pay depends upon the amount of time that she works. Should you connect the dots? Explain. ***Dots should not be connected if she is paid by whole hours. Dots should be connected if Towanda will be paid for partial hours according to the scale. Students should explain why they did or did not connect dots.***



c. Write a formula showing how her gross pay depends upon the amount of time that she works.
$P(t) = 7t$ (Payment for t hours = \$7 times t hours.

d. What is $P(9)$? What does it mean? Explain how you can use the graph, the table, and the formula to compute $P(9)$.
$P(9)$ means the amount she would be paid for 9 hours, $P(t) = 9 \cdot 7 = 63$

e. If Towanda works 11 hours and 15 minutes, what will her gross pay be? Show how you know. Express the result using function notation. ***$P(11.25) = 7 \cdot 11.25 = \$78.75$***

f. If Towanda works 4 hours and 50 minutes, what will her gross pay be? Show how you know. Express the result using function notation. ***$P(4\frac{5}{6}) = 7 \cdot 4\frac{5}{6} = \$33.83$***

g. One week Towanda's gross pay was \$42. How many hours did she work? Show how you know.
$P(t) = 7t$; $42 = 7t$, $t = 6$ hours

h. Another week Towanda's gross pay was \$57.19. How many hours did she work? Show how you know. ***$P(t) = 7t$; $57.19 = 7t$, $t = 8$ hours 10 – 11 min***

FUNCTION NOTATION - LEARNING TASK II

When Peachtree Plains High School opened in 2001, a few teachers and students put on *FallFest*, featuring contests, games, prizes, and performances by student bands. To raise money for the event, they sold *FallFest* T-shirts. The event was very well received, and so *FallFest* has become a tradition. The graph below shows T-shirt sales for each *FallFest* so far.

a. What are the independent and dependent variables shown in the graph?
The independent variable, x, is the years; the dependent variable, y, is the # of T-shirts

b. For which years does the graph provide data? ***2000 – 2005***

c. Does it make sense to connect the dots in the graph? Explain. ***No; there is no meaningful increment between whole numbers of T-shirts. They must sell whole numbers of shirts***

d. What were the T-shirt sales in 2001? Use function notation to express your result.
About 43 shirts (between 40 and 45)

e. Find $S(3)$, if possible, and explain what it means or would mean. ***between 65-70 shirts were sold***

f. Find $S(6)$, if possible, and explain what it means or would mean. ***value is not on graph***

g. Find $S(2.4)$, if possible, and explain what it means or would mean. ***It would mean the number of shirts sold in 2002 and 4/10ths or during the year 2002, and there were no shirts sold between years***

h. If possible, find a t such that $S(t) = 65$. Explain. ***S(4)=65; In 2004, 65 shirts were sold.***
i. If possible, find a t such that $S(t) = 62$. Explain. ***No year from 2000 – 2005 sold 62 shirts.***

j. Describe what happens to $S(t)$ as t increases, beginning at $t = 1$. ***Except for 2004, shirt sales increased each year.***

k. What can you say about $S(t)$ for values of t greater than 6? ***There are no values in the table for years beyond 2005. The school could project how many shirts to purchase based on their selling history.***

FUNCTION NOTATION LEARNING TASK III

From Illustrative Math, NCTM, https://www.illustrativemathematics.org/content-standards/tasks/634

Cell Phones

Let *f(t)* be the number of people, in millions, who own cell phones *t* years after 1990. Explain the meaning of the following statements.

a. f(10)=100.3 ***Ten years after 1990, 100.3 million people owned cell phones***

b. f(a)=20 ***a years after 1990, 20 million people owned cell phones***

c. f(20)=b ***20 years after 1990, b-million people owned cell phones***

d. n=f(t) ***t years after 1990, n-million people owned cell phones; Note: the order of the equation may confuse some students.***

FUNCTION NOTATION LEARNING TASK I

Towanda is paid $7 per hour in her part-time job at the local Dairy Stop. Let t be the amount time that she works, in hours, during the week, and let $P(t)$ be her gross pay (before taxes), in dollars, for the week.

a. Make a table showing how her gross pay depends upon the amount of time she works during the week.

b. Make a graph illustrating how her gross pay depends upon the amount of time that she works. Should you connect the dots? Explain.

c. Write a formula showing how her gross pay depends upon the amount of time that she works.

d. What is $P(9)$? What does it mean? Explain how you can use the graph, the table, and the formula to compute $P(9)$.

e. If Towanda works 11 hours and 15 minutes, what will her gross pay be? Show how you know. Express the result using function notation.

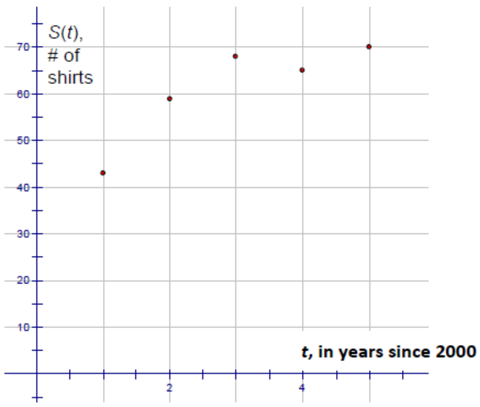
f. If Towanda works 4 hours and 50 minutes, what will her gross pay be? Show how you know. Express the result using function notation.

g. One week Towanda's gross pay was $42. How many hours did she work? Show how you know.

h. Another week Towanda's gross pay was $57.19. How many hours did she work? Show how you know.

FUNCTION NOTATION LEARNING TASK II

When Peachtree Plains High School opened in 2001, a few teachers and students put on *FallFest*, featuring contests, games, prizes, and performances by student bands. To raise money for the event, they sold *FallFest* T-shirts. The event was very well received, and so *FallFest* has become a tradition. The graph below shows T-shirt sales for each *FallFest* so far.



a. What are the independent and dependent variables shown in the graph?

b. For which years does the graph provide data?

c. Does it make sense to connect the dots in the graph? Explain.

d. What were the T-shirt sales in 2001? Use function notation to express your result.

e. Find $S(3)$, if possible, and explain what it means or would mean.

f. Find $S(6)$, if possible, and explain what it means or would mean.

g. Find $S(2.4)$, if possible, and explain what it means or would mean.

h. If possible, find a t such that $S(t) = 65$. Explain.

i. If possible, find a t such that $S(t) = 62$. Explain.

j. Describe what happens to $S(t)$ as t increases, beginning at $t = 1$.

k. What can you say about $S(t)$ for values of t greater than 6?

FUNCTION NOTATION LEARNING TASK III

From Illustrative Math, NCTM, https://www.illustrativemathematics.org/content-standards/tasks/634

Cell Phones

Let $f(t)$ be the number of people, in millions, who own cell phones t years after 1990. Explain the meaning of the following statements.

a. $f(10)=100.3$

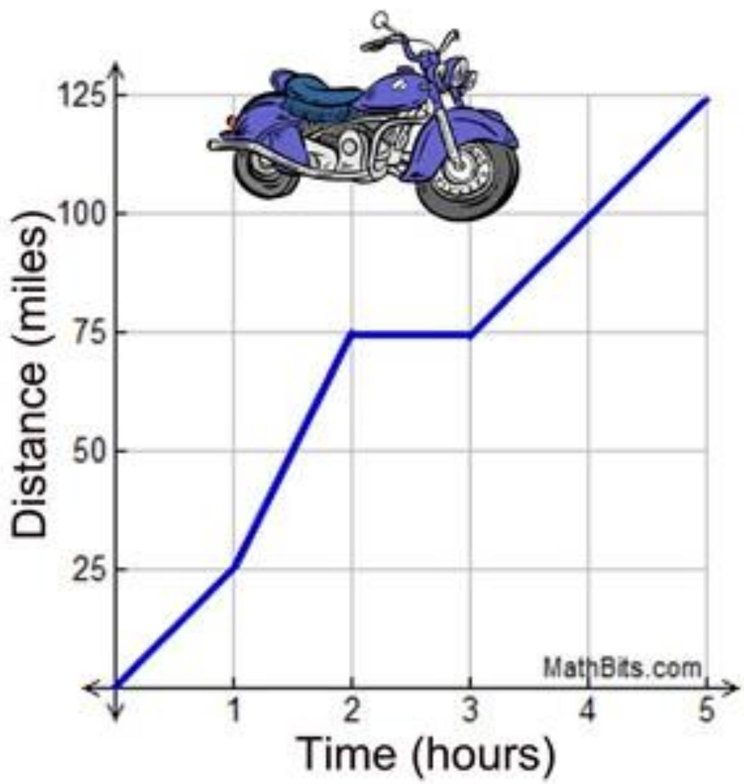
b. $f(a)=20$

c. $f(20)=b$

d. $n=f(t)$

CLOSING/SUMMARIZER

Project the graph below:



Students should write a “story” or scenario that would be modeled by the graph, and identify four points on the graph using function notation [D(t)].

ADDITIONAL PRACTICE

More practice with functional notation can be found at https://www.ixl.com/math/algebra-1/evaluate-function-rules

Quick Check III

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR2. Students will compare and graph functions.

a. Calculate rates of change of functions, comparing when rates increase, decrease, or stay constant. *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (MGSE6.RP.2;MGSE7.RP.1,2,3;MGSE8.F.2,5; MGSE9-12.F.IF.6)

b. Graph by hand simple functions expressed symbolically (use all four quadrants). (MGSE9-12.F.IF.7)

c. Interpret the equation y = mx + b as defining a linear function whose graph is a straight line. (MGSE8.F.3)

d. Use technology to graph non-linear functions. (MGSE8.F.3, MGSE9-12.F.IF.7)

e. Analyze graphs of functions for key features (intercepts, intervals of increase/decrease, maximums/minimums, symmetries, and end behavior) based on context. (MGSE9-12.F.IF.4,7)

f. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, which function has the great rate of change.* (MGSE8.F.2)

MFAQR3. Students will construct and interpret functions.

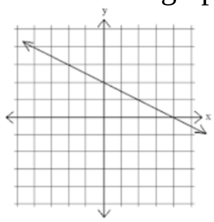
a. Write a function that describes a relationship between two quantities. (MCC8.F.4, MCC9-12.F.BF.1)

b. Use variables to represent two quantities in a real-world problem that change in relationship to one another (conceptual understanding of a variable). (MCC6.EE.9)

c. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of context. (MCC9-12.F.IF.2)

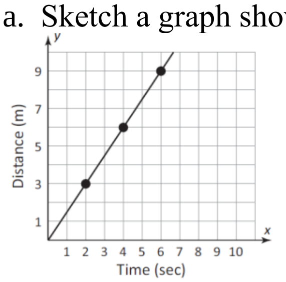
Quick Check III

Below is the graph of a line. What is the equation of the line? ***y = -½ x + 2***



2. Sylvia and Anne enjoy taking walks after school. Sylvia says they walk at a constant rate of 1.5 meters per second. One day they decide they want to walk to the riverfront, about 10 miles away. Anne remembers that 800 meters is about one-half mile. She says it will take them all day to walk.

a. Sketch a graph showing Sylvia and Anne's walking rate.



b. How far will Sylvia and Anne have walked in 1 hour? ***5400 meters per hour***

$$\frac{1.5\text{ m}}{1\text{ sec}} = \frac{\text{m}}{60\text{ sec}}$$

$$\frac{1.5\text{ m}}{1\text{ sec}} \times \frac{60}{60} = \frac{90\text{ m}}{60\text{ sec}}$$

$$\frac{90\text{ m}}{1\text{ min}} \times \frac{60}{60} = \frac{5{,}400\text{ m}}{60\text{ min}}$$

c. Make a table showing the distance walked, in meters, in 6 hours. What kind of relationship is shown in the table? What does the graph look like? ***It is a proportional relationship, with a linear graph passing through the origin.***

Hours	**Meters Walked**
1	5400
2	10800
3	16,200
4	21,600
5	27,000
6	32,400

d. How long will it take Sylvia and Anne to walk 10 miles? Write an equation for how long it will take them to walk any number of miles.

x (min)	y (min)
1	17.75
2	35.5
3	53.25
4	71
5	88.75
6	106.5
x	$17.75(x)$

It will take them 177.5 min or 2 h 57.5 min to walk 10 miles.

$y = 17.75x$, where x is the number of miles walked and y is the number of minutes.

3. Ava uses the given function table to calculate the amount of ribbon she needs to make her team's hair ribbons. She needs 2 feet of ribbon for each girl, plus she wants to have an extra 3 feet.

Number of Girls (n)	Rule: $f(n) = 2n + 3$	$f(n)$
6	2(6) + 3	15
10	2(10) + 3	23
15	2(15) + 3	33
18	2(18) + 3	?

a. Which is f(18)?
   a. ***39 ****
   b. 41
   c. 42
   d. 43

4. Which point lies on the graph of the line y = -4x + 10?

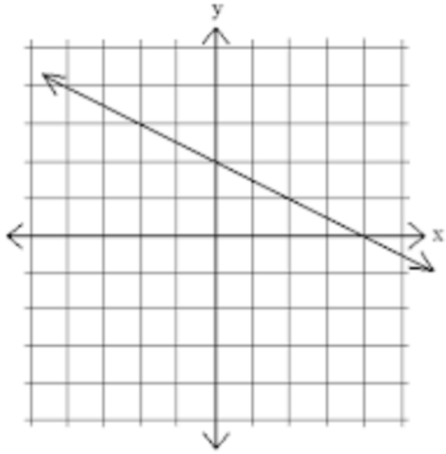
   a. (-3, -2)
   b. (2, -2)
   c. (5, 10)
   d. ***(8, -22) ****

Additional Problems that include functional notation can be found at
http://www.regentsprep.org/regents/math/algtrig/atp5/FuncPrac.htm
http://www.ixl.com/math/algebra-2/evaluate-functions

Teacher's Note: The problems on the above websites will help students review for this module's test. Some of the problems are above the scope of the course and may need to be adjusted to better match this course's standards.

Quick Check III

Below is the graph of a line. What is the equation of the line?



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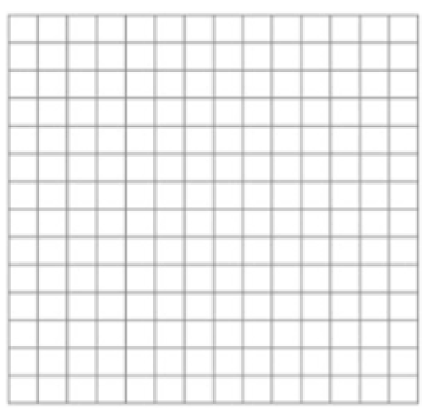
a. Sketch a graph showing Sylvia and Anne's walking rate.



b. How far will Sylvia and Anne have walked in 1 hour?

c. Make a table showing a distance walked, in meters, in 6 hours. What kind of relationship is shown in the table? What does the graph look like?

d. How long will it take Sylvia and Anne to walk 10 miles? Write an equation for how long it will take them to walk any number of miles.

3. Ava uses the given function table to calculate the amount of ribbon she needs to make her team's hair ribbons. She needs 2 feet of ribbon for each girl, plus she wants to have an extra 3 feet.

Number of Girls (n)	Rule: $f(n) = 2n + 3$	$f(n)$
6	2(6) + 3	15
10	2(10) + 3	23
15	2(15) + 3	33
18	2(18) + 3	?

a. Which is f(18)?
   a. 39
   b. 41
   c. 42
   d. 43

4. Which point lies on the graph of the line y = -4x + 10?

   a. (-3, -2)
   b. (2, -2)
   c. (5, 10)
   d. (8, -22)

How Much Does A 100 X 100 In-N-Out Cheeseburger Cost? (Culminating Lesson)

From Robert Kaplinsky, http://robertkaplinsky.com/work/in-n-out-100-x-100/

SUGGESTED TIME FOR THIS LESSON:

Exact timings will depend on the needs of your class. Suggested time, 1.5 - 2 hours (or as long as productive struggle and good mathematical discourse warrants).

STANDARDS FOR MATHEMATICAL CONTENT

MFAQR1. Students will understand characteristics of functions.

a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.(MGSE9-12.F.IF.1)
b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* (MGSE9-12.F.IF.5)
c. Graph functions using sets of ordered pairs consisting of an input and the corresponding output. (MGSE8.F.1, 2)

MFAQR2. Students will compare and graph functions.

a. Calculate rates of change of functions, comparing when rates increase, decrease, or stay constant. *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (MGSE6.RP.2;MGSE7.RP.1,2,3;MGSE8.F.2,5; MGSE9-12.F.IF.6)

b. Graph by hand simple functions expressed symbolically (use all four quadrants). (MGSE9-12.F.IF.7)
c. Interpret the equation y = mx + b as defining a linear function whose graph is a straight line. (MGSE8.F.3)
e. Analyze graphs of functions for key features (intercepts, intervals of increase/decrease, maximums/minimums, symmetries, and end behavior) based on context. (MGSE9-12.F.IF.4,7)
f. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the great rate of change.* (MGSE8.F.2)

MFAQR3. Students will construct and interpret functions.
a. Write a function that describes a relationship between two quantities. (MGSE8.F.4, MGSE9-12.F.BF.1)
b. Use variables to represent two quantities in a real-world problem that change in relationship to one another (conceptual understanding of a variable). (MGSE6.EE.9)

STANDARDS FOR MATHEMATICAL PRACTICE

1**. Make sense of problems and persevere in solving them.** Students must make sense of the problem presented in the lesson and continue to work toward a solution.
2. Reason abstractly and quantitatively. Students think through each step of the lesson, predicting, estimating, and verifying their conclusions.
3. Construct viable arguments and critique the reasoning of others. Students construct and defend their arguments as they solve problems, focusing on the details of each step of the lesson.
4. Model with mathematics. Students model with graphs, illustrations, student work.
5. Use appropriate tools strategically. Students use graphing calculators to graph functions.
6. Attend to precision. Students clearly articulate functions as words, equations, and tables in real-life context.
7. Look for and make use of structure. Students use the structure of function notation in context.
8. Look for and express regularity in repeated reasoning. Students look for patterns, answer each portion of the lesson building understanding through repeated reasoning.

EVIDENCE OF LEARNING/LEARNING TARGET

By the conclusion of this lesson, students should be able to:

- demonstrate quantitative reasoning with functions
- build function extensions in real life situations, constructing and interpreting functions.

ESSENTIAL QUESTIONS

- What is a function?
- How can I generate a rule for a function?
- How can I extend a pattern to predict an outcome for any input?

OPENER/ACTIVATOR

The Situation

In-N-Out ordinarily sells hamburgers, cheeseburgers, and Double-Doubles (two beef patties and two slices of cheese). While they do not advertise it, they have a secret menu which includes a burger where you can order as many extra beef patties and cheese slices as you like. The prices and nutrition information are not listed though. The most common orders are 3×3's (read as "three by three") and 4 by 4's (read as "four by four") that contain three and four layers of beef and cheese, respectively. However some people have ordered 20×20's (pictured above) and even a 100×100!

Estimation Lineup

Students line up based on their estimation of *How much money does a 100×100 cost?*
Fold the line to form student pairs who then share their reasons.

THE TASK

The Challenge(s)

- How much money does a 3×3 cost?
- How many calories is a 3×3?
- How much money does a 20×20 cost?
- How many calories is a 20×20?
- How much money does a 100×100 cost?
- How many calories is a 100×100?
- How much money does a NxN cost?
- How many calories is a NxN?

Question(s) To Ask

These questions may be useful in helping students down the problem solving path:

- How would you describe what ingredients are in a 3×3? (To help students realize it is a cheese burger with all the cheese burger toppings plus 2 additional layers of beef patties and cheese)
- How can we figure out how much an additional beef patty and cheese slice cost?
- What are the differences between a cheeseburger and a Double-Double?
- Where do you see the cost of the extra layer mentioned in the words in the table, graph, and symbols?
- Where do you see the cost of the cheeseburger in the words in the table, graph, and symbols?
- How would your answer change if you had started with a Double-Double and added 98 layers versus a Cheeseburger and added 99 layers?

Consider This

This awesomely gross lesson provides students with a real world context for **building linear functions.** I recommend beginning with exploring **cost before calories** because I have included a receipt that shows the actual price of a 100×100 burger. You may choose to begin by asking students about the cost of a 100×100 or start with something smaller like a 3×3. I have found that students with emerging skill sets need to begin with a 3×3 to establish that it is not the same thing as ordering 3 cheeseburgers or a Double-Double and a cheeseburger. Eventually you want them to be able to generalize the cost of a NxN burger.

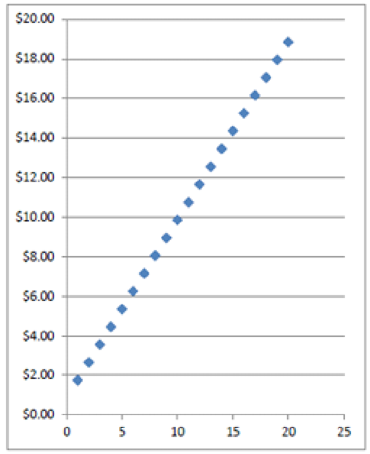
As much as possible, **you want students to do all of the discovery**. Make sure to have a conversation about what is on a 100×100. Essentially it is a cheeseburger with 99 additional beef patty and cheese layers or a Double-Double with 98 additional beef patty and cheese layers. There are no additional buns or toppings such as lettuce, tomato, onions, or spread.

We then want to ask students, “How can we figure out how much an additional beef patty and cheese slice cost?” Hopefully they struggle with this question. You may have to ask them, “What are the differences between a cheeseburger and a Double-Double?” We want them to realize that they are the same except for an extra beef patty and cheese slice and a cost difference of $0.90. They can get $0.90 by subtracting the price of a cheeseburger ($1.75) from the price of a Double-Double ($2.65) using the menu below. We want them to make the assumption that perhaps $0.90 is the cost of one layer of beef patty and cheese slice. Note that these prices were from 2004 and were used so they matched the receipt that has the price of the 100×100 on it.

Something for students to aspire to would be to **explain their solution using numbers** (an input-output table), symbols (Algebra), pictures (graph), and words (a written explanation of how they approached the problem). I have included an example of how that may look below. The most important part of the multiple representations is challenging students to explain connections between the representations. For example, where do you see the cost of the extra layer mentioned in the words in the table, graph, and symbols? What about the cost of the cheeseburger? Note that this is a discrete and not continuous function as you can only get whole number layers.

Layers	Cost
1	$1.75
2	$2.65
3	$3.55
4	$4.45
.	.
.	.
20	$18.85
.	.
.	.
100	$90.85
.	.
.	.
N	$1.75 + (N-1)*$0.90

Numbers



Pictures

Cost = Cheeseburger + Extra Layers
Cost = $1.75 + (L-1)*$0.90

or

Cost = Double-Double + Extra Layers
Cost = $2.65 + (L-2)*$0.90

Symbols

To find the cost of an extra layer of cheese and meat, I subtracted the cost of the cheeseburger from the cost of the Double-Double and got 90 cents. I then multiplied the number of extra layers of meat and cheese (which was one less than the total number of layers because a cheeseburger already comes with a layer) by 90 cents to get the extra charge. I then added the extra charge of the meat and cheese to the price of the cheeseburger to get my total cost.

Words

When it is time to show students the actual cost of the 100×100 burger, use this picture below **which shows a price of $90.85**. Note that their answer should be the **"Counter-Eat In"** amount and not the "Amount Due" unless you want to have them factor 7.5% sales tax into the cost. The way the order was entered, **the customer was charged for a Double-Double plus 98 additional meat and cheese layers.**


2004-10-31
8:21 PM
YOUR GUEST NUMBER IS
98
IN-N-OUT BURGER LAS VEGAS EASTERN
2004-10-31
8:21 PM
165 1 5 98
Cashier: SAM
GUEST #: 98
Counter-Eat In
DblDbl
98 Meat Pty XChz
2.65
88.20
Counter-Eat In
TAX 7.50%
Amount Due
90.85
6.81
97.66
CASH TENDER
Change
$97.66
$.00
2004-10-31
8:21 PM
THANK YOU!

Something worth noting is that **there are at least three correct equations for determining the cost of an NxN burger:**

- .90(N-2) + 2.65 — The cost of a Double-Double plus N-2 additional layers at 90 cents a layer (since the Double-Double already had 2 layers)

- .90(N-1) + 1.75 — The cost of a cheeseburger plus N-1 additional layers at 90 cents a layer (since the cheeseburger already had 1 layer)

- .90N + 0.85 — The cost of the bun and produce plus N additional layers at 90 cents a layer (the $0.85 comes from subtracting the $0.90 per layer from a $1.75 cheeseburger)

Consider having a conversation with students about **what happens when you simplify each of these expressions.** Ask them to explain what each part of the expression means. Also, look out for students who get answers like .90N + 1.75 which don't take into consideration the layers that are already on the cheeseburger.
To find the **total number of calories**, students should take the same approach but using nutritional information (below). I do not have the actual answers but they can be similarly extrapolated from the chart below (included in the "Download Files" link as a PDF). It appears that an extra layer of beef and cheese is 190 calories (670 calories – 480 calories). So, based on that assumption the calorie counts would be:

4×4 = 1,050 calories
20×20 = 4,090 calories
100×100 = 18,300 calories
NxN = 480 + (N-1)*190 calories

	Serving Size (g)	Calories
Hamburger w/Onion	243	390
Cheeseburger w/Onion	268	480
Double-Double w/Onion	330	670

What You'll Need

A picture of a cheeseburger (to establish that it has one cheese slice and one burger patty):

A picture of a Double-Double (to establish that it has two cheese slices and two burger patties):

,

A picture of a 20×20 burger (to establish that it has 20 cheese slices and 20 burger patties):

A picture of a 100×100 burger:

Student Work

Online are low, medium, and high work samples for the challenge.
http://robertkaplinsky.com/work/in-n-out-100-x-100/

TEACHER COMMENTARY

- ***Check out Jonathan Newman's implementation with his students. He included a complete reflection with video of the entire lesson. It is very useful if you plan on using this lesson yourself.***
- ***Mrs. E has a very thorough description of her first venture into problem-based learning using this lesson. It gives you a great reflection into how the lesson went and her successes and challenges.***
- ***Dan Burfiend shares his two-day implementation of the lesson. Here are his reflections from Day 1 and Day 2.***
- ***Ms. Moore implemented the lesson using multiple representations.***
- ***Tim McCaffrey shares his own take on the problem including an Act 1 video as well as updated prices.***

CLOSING/SUMMARIZER

Students could collect information from local restaurants (e.g., McDonalds, Burger King…), prices, calories, other information, and calculate information similar to that in the lesson.

ADDITIONAL PRACTICE

More "food" examples of functions can be found at the following sites:

http://mathcentral.uregina.ca/beyond/articles/Cooking/Cooking1.html

https://www.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/639

https://www.illustrativemathematics.org/content-standards/tasks/625