History of Mathematics

K-12 Mathematics Introduction

The Georgia Mathematics Curriculum focuses on actively engaging the students in the development of mathematical understanding by using manipulatives and a variety of representations, working independently and cooperatively to solve problems, estimating and computing efficiently, and conducting investigations and recording findings. There is a shift towards applying mathematical concepts and skills in the context of authentic problems and for the student to understand concepts rather than merely follow a sequence of procedures. In mathematics classrooms, students will learn to think critically in a mathematical way with an understanding that there are many different ways to a solution and sometimes more than one right answer in applied mathematics. Mathematics is the economy of information. The central idea of all mathematics is to discover how knowing some things well, via reasoning, permit students to know much else—without having to commit the information to memory as a separate fact. It is the reasoned, logical connections that make mathematics coherent. The implementation of the Georgia Standards of Excellence in Mathematics places a greater emphasis on sense making, problem solving, reasoning, representation, connections, and communication.

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History of Mathematics is a one-semester elective course option for students who have completed AP Calculus or are taking AP Calculus concurrently. It traces the development of major branches of mathematics throughout history, specifically algebra, geometry, number theory, and methods of proofs, how that development was influenced by the needs of various cultures, and how the mathematics in turn influenced culture. The course extends the numbers and counting, algebra, geometry, and data analysis and probability strands from previous courses, and includes a new history strand.

Instruction and assessment should include appropriate use of technology and manipulatives. Concepts should be introduced and used in an appropriate historical context.

Mathematics | Standards for Mathematical Practice

Mathematical Practices are listed with each grade/course mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

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1 Make sense of problems and persevere in solving them.
High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.
High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.
High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.
High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical

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results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.
High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.
High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.
By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8 Look for and express regularity in repeated reasoning.
High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

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Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. **Students who do not have an understanding of a topic may rely on procedures too heavily.** Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. **In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.**

In this respect, those content standards that set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

History of Mathematics | Content Standards

Numbers and Operations

<u>Students will investigate historical computation algorithms and use them to solve problems; define and explore the concepts of denumerability and algebraic numbers.</u>

MHM.N.1 Students will explore and use historical computational methods.

a. Use Babylonian, Roman, Egyptian (hieratic and hieroglyphic), Chinese, and Greek number systems to represent quantities.
b. Use historical multiplication and division algorithms (including the Egyptian method of duplation and mediation, the medieval method of gelosia, and Napier's rods).

MHM.N.2 Students will explore the implications of infinite sets of real numbers.

a. Describe denumerable and nondenumerable sets and provide examples of each.
b. Identify algebraic and transcendental numbers.

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Algebra

Students will investigate historical equation solving techniques in both algebraic and geometric form; understand and use the axiomatic method of abstract algebra; compute defined products on sets of complex numbers; solve linear congruences; determine whether a quadratic congruence has a solution; understand and use basic number-theoretic concepts.

MHM.A.1 Students will explore and use historical methods for expressing and solving equations.

a. Solve linear equations using the method of false position.
b. Express the geometrical algebra found in historical works (such as the Elements of Euclid) in modern algebraic notation.
c. Solve systems of linear and nonlinear equations using Diophantus' method.
d. Translate into modern notation problems appearing in ancient and medieval texts that involve linear, quadratic, or cubic equations and solve them.
e. Use Cardano's cubic formula and Khayyam's geometric construction to find a solution to a cubic equation.

MHM.A.2 Students will explore abstract algebra and group-theoretic concepts.

a. Add, subtract, and multiply two quaternions.
b. Explore matrix products other than the Cayley product (including Lie and Jordan) by determining whether these products are associative or commutative.
c. Identify whether a given set with a binary operation is a group.

MHM.A.3 Students will use and apply number theoretic concepts.

a. Find the first four perfect numbers using Euclid's formula.
b. Prove statements concerning figurate numbers using both graphical (as in the manner of the Greeks) and algebraic methods.
c. Solve simple linear congruences of the form $ax = b \mod m$.
d. Use Fermat's Little Theorem and Euler's Theorem to simplify expressions of the form $a^k \mod m$.
e. Use Gauss' Law of Quadratic Reciprocity to determine quadratic residues of two odd primes; i.e., solve quadratic congruences of the form $x^2 = p \mod q$.
f. Discover that the real primes that can be expressed as the sum of two squares are no longer prime in the field of Gaussian integers.

MHM.A.4 Students will use the algebraic techniques of Fermat, Barrow, and Newton to determine tangents to quadratic curves.

Geometry

Students will prove basic Euclidean propositions and constructions; compute lengths, areas, and volumes according to historical algorithms and formulae; understand and prove basic non- Euclidean propositions.

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MHM.G.1 Students will prove geometry theorems.

a. Students will understand and recognize the use of definitions, postulates, and axioms in defining a deductive system such as Euclidean geometry.
b. Prove the first five propositions in Book I of Euclid's *Elements*.
c. Construct a regular pentagon with a straight-edge and compass.

MHM.G.2 Students will compute lengths, areas, and volumes according to historical formulas.

a. Find the volume of a truncated pyramid using the Babylonian, Chinese, and Egyptian formulas.
b. Compute the areas of regular polygons by Heron's formulas.
c. Identify cyclic quadrilaterals and find associated lengths by Ptolemy's Theorem.

MHM.G.3 Students will explore and prove statements in non-Euclidean geometry.

a. Prove that the summit angles of an isosceles birectangle are congruent, but that it is impossible to prove they are right without referring to the parallel postulate or one of its consequences.
b. Describe the hypothesis of the acute angle (Hyperbolic), the hypothesis of the right angle (Euclidean), and the hypothesis of the obtuse angle (Spherical).
c. Prove that under the hypothesis of the acute angle, similarity implies congruence.

Data Analysis and Probability

Students will explore the origins of probability by solving problems concerning gambling.

MHM.D.1 Students will compute the ratio of winnings in an interrupted game.

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Students will understand the influence Hindu-Arabic numerals had on the development of mathematics; recognize the accomplishments of the ancient Greeks and the influence ancient Greek mathematical ideas had on their culture and later cultures; understand how mathematical ideas were preserved and extended after Greek society collapsed; investigate the development of algebra during the Middle Ages and Renaissance; understand how both algebraic and geometrical ideas influenced the development of analysis; identify the influences analysis had on our understanding of science; recognize the move toward abstraction in the 19th and 20th centuries is an extension of the ancient Greek conception of proof.

MHM.H.1 Students will identify Hindu-Arabic numerals as a prime scientific advancement.

a. Describe the limitations of the Babylonian, Roman, Egyptian (hieratic and hieroglyphic), Chinese, and Greek number systems as compared to Hindu-Arabic numerals.
b. Describe the transition of Hindu-Arabic numerals from regional use in the 10th century to wide-spread use in the 15th (including the influence of Fibonacci for the use of the numerals and the Italian abascists against their use).
c. Identify the number system and notation used by a society as an influence on the types of mathematics developed by that society.

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MHM.H.2 Students will describe factors involved in the rise and fall of ancient Greek society.

a. Describe the theories for the rise of intellectual thought in ancient Greece and the factors involved in its collapse.
b. Describe the cultural aspects of Greek society that influenced the way mathematics developed in ancient Greece.
c. Explain the distinction made between number and magnitude, commensurable and incommensurable, and arithmetic and logistic, the cultural factors inherent in this distinction, and the logical crisis that occurred concerning incommensurable (irrational) magnitudes.

MHM.H.3 Students will trace the centers of development of mathematical ideas from the 5th century to the 18th century.

a. Describe the transmission of ideas from the Greeks, through the Islamic peoples, to medieval Europe.
b. Describe the influence of the Catholic Church and Charlemagne on the establishment of mathematics as one of the central pillars of education.
c. Explain the cultural factors that encouraged the development of algebra in 15th century Italy, and how this development influenced mathematical thought throughout Europe.
d. Identify the works of Galileo, Copernicus, and Kepler as a landmark in scientific thought, describe the conflict between their explanation of the workings of the solar system and then-current perspectives, and contrast their works to those of Aristotle.
e. Describe the contributions of Fermat, Pascal, Descartes, Newton, and Gauss to mathematics.
f. Identify Euler as the first modern mathematician and a motivating force behind all aspects of mathematics for the 18th century.
g. Describe the influence the French Revolution had on education (establishment of the Ecole Normale and the Ecole Polytechnique, Monge, Lagrange, Legendre, Laplace).

MHM.H.4 Students will identify the 19th and 20th centuries as the time when mathematics became more specialized and more rigorous.

a. Describe the societal factors that inhibited the developement of non-Euclidean geometry.
b. Explain how the ancient Greek pattern of material axiomatics evolved into abstract axiomatics (non-Euclidean geometry, non-commutative algebra)
c. Identify Cantor as the most original mathematician since the ancient Greeks.
d. Describe the implications of Klein's *Erlanger Programme* and Godel's Incompleteness Theorem on the nature of mathematical discovery and proof.

Terms/Symbols: duplation and mediation, gelosia, Napier's rods, unit fraction, denumerable set, nondenumerable set, algebraic number, transcendental number, false position, quaternion, group, perfect number, figurate number, linear congruence, quadratic congruence, quadratic residue, Gaussian integer, cyclic quadrilateral, non-Euclidean geometry