Mathematics of Industry and Government

K-12 Mathematics Introduction

The Georgia Mathematics Curriculum focuses on actively engaging the students in the development of mathematical understanding by using manipulatives and a variety of representations, working independently and cooperatively to solve problems, estimating and computing efficiently, and conducting investigations and recording findings. There is a shift towards applying mathematical concepts and skills in the context of authentic problems and for the student to understand concepts rather than merely follow a sequence of procedures. In mathematics classrooms, students will learn to think critically in a mathematical way with an understanding that there are many different ways to a solution and sometimes more than one right answer in applied mathematics. Mathematics is the economy of information. The central idea of all mathematics is to discover how knowing some things well, via reasoning, permit students to know much else—without having to commit the information to memory as a separate fact. It is the reasoned, logical connections that make mathematics coherent. The implementation of the Georgia Standards of Excellence in Mathematics places a greater emphasis on sense making, problem solving, reasoning, representation, connections, and communication.

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This is a course designed to follow the completion of Algebra II, Advanced Algebra, Accelerated Geometry B/Algebra II, or Accelerated Analytic Geometry B/Advanced Algebra. Modeled after operations research courses, Mathematics of Industry and Government allows students to explore decision making in a variety of industries such as: Airline - scheduling planes and crews, pricing tickets, taking reservations, and planning the size of the fleet; Pharmaceutical - R& D management; Logistics companies - routing and planning; Lumber and wood products - managing forests and cutting timber; Local government - deployment of emergency services, and Policy studies and regulation - environmental pollution, air traffic safety, AIDS, and criminal justice policy. Students learn to focus on the development of mathematical models that can be used to model, improve, predict, and optimize real-world systems. These mathematical models include both deterministic models such as mathematical programming, routing or network flows and probabilistic models such as queuing, and simulation. (*Prerequisite: Algebra II, Advanced Algebra, Accelerated Geometry B/Algebra II or Accelerated Analytic Geometry B/Advanced Algebra*)

Instruction and assessment should include the appropriate use of manipulatives and technology. Topics should be represented in multiple ways, such as concrete/pictorial, verbal/written, numeric/data-based, graphical, and symbolic. Concepts should be introduced and used, where appropriate, in the context of realistic phenomena.

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Mathematics | Standards for Mathematical Practice

Mathematical Practices are listed with each grade/course mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.
High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.
High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.
High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify

their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

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7 Look for and make use of structure.

 By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8 Look for and express regularity in repeated reasoning.
High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. **Students who do not have an understanding of a topic may rely on procedures too heavily.** Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. **In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.**

In this respect, those content standards that set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

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Deterministic Decision Making

Students will explore decision making through linear programming, optimal locations, and optimal paths.

MMIG.DD.1 Students will use advanced linear programming to make decisions.
  a. Distinguish among continuous, integer, and binary contexts.
  b. Model a contextual problem with three or more variables.
  c. Solve problems with three or more variables using technology.
  d. Interpret the results.
  e. Examine cause and effect of contextual changes.

MMIG.DD.2 Students will determine optimal locations and use them to make appropriate decisions.
  a. Find the optimal median location in a one-dimensional context.
  b. Find the optimal median location in a rectilinear context.
  c. Find the optimal location given three equally weighted, non-collinear points.
  d. Find the optimal location in a set covering context.

MMIG.DD.3 Students will determine optimal paths and use them to make appropriate decisions.
  a. Relate context to a network representation.
  b. Apply appropriate recursive algorithms for minimum spanning tree, shortest path, and critical path management.
  c. Examine alternate decisions in response to contextual changes.

Probabilistic Decision Making

Students will use normal and other (e.g.binomial, geometric, and Poisson) distributions as well as simulations to make appropriate decisions.

MMIG.PD.1 Students will use properties of normal distributions to make decisions about optimization and efficiency.
  a. Calculate theoretical and empirical probabilities using standardized and non-standardized data.
  b. Analyze and interpret the probabilities in terms of context.
  c. Consider contextual factors and investigate issues within the decision-making process.
  d. Apply techniques to quality control settings.

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MMIG.PD.2 Students will use properties of other distributions (e.g. binomial, geometric, Poisson) to make decisions about optimization and efficiency.
   a. Calculate theoretical and empirical probabilities using standardized and non-standardized data.
   b. Analyze and interpret the probabilities in terms of context.
   c. Consider contextual factors and investigate issues within the decision-making process.

MMIG.PD.3 Students will use other probabilistic models to make decisions.
   a. Use program evaluation review technique (PERT) to investigate completion times of a project.
   b. Develop and apply transition matrices to make predictions using Markov Chains.
   c. Apply queuing theory.
   d. Consider contextual factors and investigate issues within the decision making process.

MMIG.PD.4 Students will use computer simulations to make decisions.
   a. Use technology to simulate a real-world situation.
   b. Analyze, evaluate, and interpret results.
   c. Examine alternate decisions in response to contextual changes.

Terms/Symbols:
Linear programming, continuous, integer, binary, optimization, optimal, median location, set covering, network, recursive, minimum spanning trees, shortest path, critical path, normal distribution, binomial distribution, geometric distribution, Poisson distribution, probabilistic models, PERT, Markov Chains, queuing, simulation.