Multivariable Calculus

K-12 Mathematics Introduction

The Georgia Mathematics Curriculum focuses on actively engaging the students in the development of mathematical understanding by using manipulatives and a variety of representations, working independently and cooperatively to solve problems, estimating and computing efficiently, and conducting investigations and recording findings. There is a shift towards applying mathematical concepts and skills in the context of authentic problems and for the student to understand concepts rather than merely follow a sequence of procedures. In mathematics classrooms, students will learn to think critically in a mathematical way with an understanding that there are many different ways to a solution and sometimes more than one right answer in applied mathematics. Mathematics is the economy of information. The central idea of all mathematics is to discover how knowing some things well, via reasoning, permit students to know much else—without having to commit the information to memory as a separate fact. It is the reasoned, logical connections that make mathematics coherent. The implementation of the Georgia Standards of Excellence in Mathematics places a greater emphasis on sense making, problem solving, reasoning, representation, connections, and communication.

Multivariable Calculus

Multivariable Calculus is a fourth-year mathematics course option for students who have completed AP Calculus BC. It includes three-dimensional coordinate geometry; matrices and determinants; eigenvalues and eigenvectors of matrices; limits and continuity of functions with two independent variables; partial differentiation; multiple integration; the gradient; the divergence; the curl; Theorems of Green, Stokes, and Gauss; line integrals; integrals independent of path; and linear first-order differential equations.

(*Prerequisite: Successful completion of AP Calculus BC*)

Instruction and assessment should include the appropriate use of technology. Topics should be presented in multiple ways, such as verbal/written, numeric/data-based, algebraic, and graphical. Concepts should be introduced and used, where appropriate, in the context of realistic phenomena.

ALGEBRA

Students will investigate the dot product and cross product of two vectors; define relationship among points, lines, and planes in three dimensions; understand and apply properties of matrices and determinants including eigenvalues and eigenvectors.

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MMCA1. Students will investigate the relationship between points, lines, and planes in three-dimensions.

a. Represent equations of lines in space using vectors.
b. Express analytic geometry of three dimensions (equations of planes, parallelism, perpendicularity, angles) in terms of the dot product and cross product of vectors.

MMCA2. Students will recognize and apply properties of matrices.

a. Find the determinant of 2-by-2 and 3-by-3 matrices.
b. Represent a 3-by-3 system of linear equations as a matrix and solve the system in multiple ways: the inverse matrix, row operations, and Cramer's Rule.
c. Apply properties of similar and orthogonal matrices to prove statements about matrices.
d. Find and apply the eigenvectors and eigenvalues of a 3-by-3 matrix.

MMCA3. Students will explore functions of two independent variables of the form $z = f(x, y)$ and implicit functions of the form $f(x, y, z) = 0$.

a. Evaluate such functions at a point in the plane.
b. Graph the level curves of such functions.
c. Determine points or regions of discontinuity of such functions.

DERIVATIVES

Students will investigate limits, continuity, and differentiation of functions of two independent variables; define and apply the gradient, the divergence, and the curl.

MMCD1. Students will explore the continuity of functions of two independent variables in terms of the limits of such functions as (x, y) approaches a given point in the plane.

MMCD2. Students will explore, find, use, and apply partial differentiation of functions of two independent variables of the form $z = f(x, y)$ and implicit functions of the form $f(x, y, z) = 0$.

a. Approximate the partial derivatives at a point of a function defined by a table of data.
b. Find expressions for the first and second partial derivatives of a function.
c. Define and apply the total differential to approximate real-world phenomena .
d. Represent the partial derivatives of a system of two functions in two variables using the Jacobian.
e. Find the partial derivatives of the composition of functions using the general chain rule.
f. Apply partial differentiation to problems of optimization, including problems requiring the use of the Lagrange multiplier.

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MMCD3. Students will define and apply the gradient, the divergence, and curl in terms of differential vector operations.

INTEGRATION

Students will explore double and triple integrals and integrals of vectors; use various methods of integration; understand and apply the theorems of Green, Stokes, and Gauss.

MMCI1. Students will integrate functions of the form $z = f(x, y)$ or $w = f(x, y, z)$.

a. Define, use, and interpret double and triple integrals in terms of volume and mass.
b. Represent integrals of vectors as double and triple integrals.
c. Integrate functions through various techniques such as changing the order of integration, substituting variables, or changing to polar coordinates.

MMCI2. Students will apply and interpret the theorems of Green, Stokes, and Gauss.

a. Apply line and surface integrals to functions representing real-world phenomena.
b. Recognize, understand, and use line integrals that are independence of path.
c. Define and apply the gradient, the divergence, and the curl in terms of integrals of vectors.

DIFFERENTIAL EQUATIONS

Students will explore solution methods of linear differential equations.

MMCDE. Students will use, apply, and solve linear first-order differential equations.

a. Solve linear first-order differential equations of the form $y' + p(x)y = q(x)$ with an integrating factor.
b. Solve homogeneous linear first-order differential equations using a variable substitution.
c. Solve Clairaut equations.
d. Explore the concepts of families of solutions and envelopes.
e. Write linear first-order differential equations that represent real-world phenomena and solve them, such as those arising from Kirchhoff's Law and mixing problems.
f. Students will solve linear second-order differential equations of the form $y'' + p(x)y' + q(x)y = c$ using the characteristic equation where the characteristic equation has two real roots, one real root, or no real roots.

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Terms/Symbols: dot product, cross product, determinant, Cramer's Rule, row operation, similar matrix, orthogonal matrix, eigenvalue, eigenvector, partial derivative, Jacobian, Lagrange multiplier, gradient, divergence, curl, double integral, triple integral, line integral, surface integral, independence of path, linear first-order, homogeneous, linear second-order, Clairaut equations, characteristic equation

Process Standards

The following process standards are essential to mastering each of the mathematics content standards. They emphasize critical dimensions of the mathematical proficiency that all students need.

MMCP1. Students will solve problems (using appropriate technology).

a. Build new mathematical knowledge through problem solving.
b. Solve problems that arise in mathematics and in other contexts.
c. Apply and adapt a variety of appropriate strategies to solve problems.
d. Monitor and reflect on the process of mathematical problem solving.

MMCP2. Students will reason and evaluate mathematical arguments.

a. Recognize reasoning and proof as fundamental aspects of mathematics.
b. Make and investigate mathematical conjectures.
c. Develop and evaluate mathematical arguments and proofs.
d. Select and use various types of reasoning and methods of proof.

MMCP3. Students will communicate mathematically.

a. Organize and consolidate their mathematical thinking through communication.
b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
c. Analyze and evaluate the mathematical thinking and strategies of others.
d. Use the language of mathematics to express mathematical ideas precisely.

MMCP4. Students will make connections among mathematical ideas and to other disciplines.

a. Recognize and use connections among mathematical ideas.
b. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
c. Recognize and apply mathematics in contexts outside of mathematics.

MMCP5. Students will represent mathematics in multiple ways.

a. Create and use representations to organize, record, and communicate mathematical ideas.
b. Select, apply, and translate among mathematical representations to solve problems.
c. Use representations to model and interpret physical, social, and mathematical phenomena.