Statistical Reasoning

K-12 Mathematics Introduction

The Georgia Mathematics Curriculum focuses on actively engaging the students in the development of mathematical understanding by using manipulatives and a variety of representations, working independently and cooperatively to solve problems, estimating and computing efficiently, and conducting investigations and recording findings. There is a shift towards applying mathematical concepts and skills in the context of authentic problems and for the student to understand concepts rather than merely follow a sequence of procedures. In mathematics classrooms, students will learn to think critically in a mathematical way with an understanding that there are many different ways to a solution and sometimes more than one right answer in applied mathematics. Mathematics is the economy of information. The central idea of all mathematics is to discover how knowing some things well, via reasoning, permit students to know much else—without having to commit the information to memory as a separate fact. It is the reasoned, logical connections that make mathematics manageable. The implementation of the Georgia Standards of Excellence in Mathematics places a greater emphasis on sense making, problem solving, reasoning, representation, connections, and communication.

Statistical Reasoning

Statistical Reasoning is a fourth mathematics course option for students who have completed Algebra II, Advanced Algebra, Accelerated Geometry B/Algebra II, or Accelerated Analytic Geometry B/Advanced Algebra. The course provides experiences in statistics beyond the CCGPS sequence of courses, offering students opportunities to strengthen their understanding of the statistical method of inquiry and statistical simulations. Students will formulate statistical questions to be answered using data, will design and implement a plan to collect the appropriate data, will select appropriate graphical and numerical methods for data analysis, and will interpret their results to make connections with the initial question.

The Standards for Mathematical Practice through a Statistical Lens will provide the foundation for instruction and assessment. Topics should be introduced and assessed using simulations and appropriate supporting technology.

Mathematics | Standards for Mathematical Practice through a Statistical Lens

Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction.

The Standards for Mathematical Practice through a Statistical Lens describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

Statistical Reasoning

1. Make sense of problems and persevere in solving them.

Statistically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Statistically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Statistically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.
Statistically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.
Statistically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Statistically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.
Statistically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a

problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Statistically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships statistically to draw conclusions. They routinely interpret their statistical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.
Statistically proficient students consider the available tools when solving a statistical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, statistically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making statistical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Statistically proficient students at various grade levels are able to identify relevant external statistical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.
Statistically proficient students try to communicate precisely to others. They use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.
Statistically proficient students look closely to discern a pattern or structure. Patterns are prevalent in statistics and can often be quantified. Randomization provides the foundation for identifying patterns in variability, thereby allowing students to quantify uncertainty.

8. Look for and express regularity in repeated reasoning.
Statistically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, statistically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their results in the context of the statistical question.

Statistical Reasoning

Connecting the Standards for Mathematical Practice through a Statistical Lens to the Standards for Statistical Reasoning Content

The Standards for Mathematical Practice through a Statistical Lens describe ways in which developing student practitioners of the discipline of statistics increasingly ought to engage with the subject matter as they grow in statistical maturity and expertise throughout the elementary, middle, and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices through a statistical lens to statistical content in statistical instruction.

The Standards for Statistical Reasoning Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the statistics to practical situations, use technology mindfully to work with the statistics, explain the statistics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the statistical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Statistical Reasoning Content and the Standards for Mathematical Practice through a Statistical Lens. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in statistics.

Statistical Reasoning | Content Standards

Formulate Questions

Students will formulate questions to clarify the problem at hand and formulate one (or more) questions that can be answered with data.

MSR.FQ.1 Students will apply the statistical method to real-world situations.

a. Formulate questions to clarify the problem at hand and formulate one (or more) questions that can be answered with data.
b. Collect data by designing a plan to collect appropriate data and employ the plan to collect the data.
c. Analyze data by selecting appropriate graphical and numerical methods and using these methods to analyze the data.
d. Interpret results by interpreting the analysis and relating the interpretation to the original question.

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MSR.FQ.2 Students will identify whether the data are categorical or quantitative (numerical).

Students will be able to identify the difference between categorical and quantitative (numerical) data.

a. Determine the appropriate graphical display for each type of data.
b. Determine the type of data used to produce a given graphical display.

Collect Data

Students will design and implement a plan to collect the appropriate data to answer the statistical question.

MSR.CD.1 Students will distinguish between a population distribution, a sample data distribution, and a sampling distribution.

a. Students will identify the three types of distributions.
   i. Recognize a population distribution has fixed values of its parameters that are usually unknown.
   ii. Recognize a sample data distribution is taken from a population distribution and the data distribution is what is seen in practice hoping it approximates the population distribution.
   iii. Recognize a sampling distribution is the distribution of a sample statistic (such as a sample mean or a sample proportion) obtained from repeated samples. The sampling distribution provides the key for determining how close to expect a sample statistic approximates the population parameter.
b. Students will create sample data distributions and a sampling distribution.
   i. Create a sample data distribution by taking a sample from a defined population and summarizing the data in a distribution.
   ii. Create a sampling distribution of a statistic by taking repeated samples from a population (either hands-on or by simulation with technology).

MSR.CD.2 Students will understand that randomness should be incorporated into a sampling or experimental procedure.

Students will be able to implement a reasonable random method for selecting a sample or for assigning treatments in an experiment.

a. Implement a simple random sample.
b. Randomly assign treatments to experimental subjects or objects.

MSR.CD.3 Students will distinguish between the three types of study designs for collecting data (sample survey, experiment, and observational study) and will know the scope of the interpretation for each design type.

Students will be able to distinguish between the three types of study designs for collecting data (sample survey, experiment, and observational study) and know the scope of the interpretation for each design type.

a. Determine the type of study design appropriate for answering a statistical question.
b. Determine the appropriate scope of inference for the study design used.

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MSR.CD.4 Students will distinguish between the role of randomness and the role of sample size with respect to using a statistic from a sample to estimate a population parameter.

Students will be able to distinguish the roles of randomization and sample size with designing studies.

a. Recognize that randomization reduces bias where bias occurs when certain outcomes are systematically more likely to appear.
b. Recognize that random selection from a population plays a different role than random assignment in an experiment.
c. Recognize that sample size impacts the precision with which estimates of the population parameters can be made (larger the sample size the more precision).

Analyze Data

Students will select appropriate graphical and numerical methods and use these methods to analyze the data.

MSR.AD.1 Students will use distributions to identify the key features of the data collected.

Students will describe the distribution for quantitative and categorical data.

a. Describe the distribution for quantitative data.
   i. Describe and interpret the shape of the distribution.
   ii. Describe and interpret the measures of center for the distribution.
   iii. Describe and interpret the patterns in variability for the distribution.
   iv. Describe and interpret any outliers or gaps in the distribution.
b. Describe the distribution for categorical data.
   i. Describe and interpret the modal category for the distribution.
   ii. Describe and interpret patterns that exist for the distribution.

MSR.AD.2 Students will use distributions to compare two or more groups.

Students will compare two or more groups by analyzing distributions.

a. Construct appropriate graphical displays of distributions.
b. Use graphical and numerical attributes of distributions to make comparisons between distributions.

MSR.AD.3 Students will determine if an association exists between two variables (pattern or trend in bivariate data) and use values of one variable to predict values of another variable.

Students will analyze associations between variables and make predictions from one variable to another.

a. Analyze associations between two variables.
   i. Create scatter plots for two-variable numerical data.
   ii. Create two-way tables for two-variable categorical data.
   iii. Analyze patterns and trends in data displays.
b. Make predictions and draw conclusions from two-variable data based on data displays.
c. Distinguish between association and causation.

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Interpret Results

Students will interpret results and make connections to the original question.

MSR.IR.1 Students will ask if the difference between two sample proportions or two sample means is due to random variation or if the difference is significant.

Students will be able to determine if there are differences between two population parameters or treatment effects.

a. Using simulation, determine the appropriate model to decide if there is a difference between two population parameters.
b. Using simulation, determine the appropriate model to decide if there is a difference between two treatment effects.

MSR.IR.2 Students will understand that when randomness is incorporated into a sampling or experimental procedure, probability provides a way to describe the 'long-run' behavior of a statistic as described by its sampling distribution.

Students will be able to create simulated sampling distributions and understand how to use the sampling distribution to make predictions about a population parameter(s) or the difference in treatment effects.

a. Create an appropriate simulated sampling distribution (using technology) and develop a margin of error.
b. Create an appropriate simulated sampling distribution (using technology) and develop a p-value.

Terms/Symbols: population distribution, sample data distribution, sampling distribution, sampling proportion, sample statistic, population parameter, randomness, simple random sample, treatments, sample survey, experiment, observational study, inference, bias, modal category, bivariate data, significant differences, margin of error, p-value.