Technical College Readiness Mathematics

K-12 Mathematics Introduction

Georgia Mathematics focuses on actively engaging the student in the development of mathematical understanding by working independently and cooperatively to solve problems, estimating and computing efficiently, using appropriate tools, concrete models and a variety of representations, and conducting investigations and recording findings. There is a shift toward applying mathematical concepts and skills in the context of authentic problems and student understanding of concepts rather than merely following a sequence of procedures. In mathematics classrooms, students will learn to think critically in a mathematical way with an understanding that there are many different solution pathways and sometimes more than one right answer in applied mathematics. Mathematics is the economy of information. The central idea of all mathematics is to discover how knowing some things leads, via reasoning, to knowing more—without having to commit the information to memory as a separate fact. It is the reasoned, logical connections that make mathematics coherent. The implementation of the Georgia Standards of Excellence in Mathematics places the expected emphasis on sense-making, problem solving, reasoning, representation, connections, and communication.

Technical College Readiness Mathematics

Technical College Readiness Mathematics is designed to meet the needs of students who have passed Algebra I/Coordinate Algebra, have passed or are concurrently enrolled in Geometry/Analytic Geometry while taking this course, and intend to enroll in a technical college program. The course is aimed at students who have experienced difficulty in passing middle school mathematics End of Grade (EOG) assessments, have struggled significantly in the first two high school core mathematics courses, and have scored less than 34 on the ACCUPLACER Placement Assessment. The ACCUPLACER Placement Assessment will serve as the course post-test.

Technical College Readiness Mathematics will examine numeracy, algebra, and geometry in a variety of contexts, including number sense, linear and non-linear relationships, functions and their graphs, and measurement and geometry. The course will provide an opportunity for students to review mathematics skills needed for success in Technical College and will extend students' understanding of mathematical concepts and skills by emphasizing topics from Foundations of Algebra, Coordinate Algebra/Algebra I, Analytic Geometry/Geometry, and Advanced Algebra/Algebra II.

The Standards for Mathematical Practice will provide the foundation for instruction and assessment. The content standards are an amalgamation of K-12 mathematical standards. Note that the standards from which the course standards are drawn are identified for reference.

Mathematics | Standards for Mathematical Practice

Mathematical Practices are listed with each grade/course mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive

reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.
High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.
High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.
High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.
High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on

whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.
High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.
High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.
 By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8 Look for and express regularity in repeated reasoning.
High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)$ $(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.[1]The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good

opportunities to connect the practices to the content. **Students who do not have an understanding of a topic may rely on procedures too heavily.** Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. **In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.**

In this respect, those content standards that set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Technical College Readiness Mathematics | Content Standards

Number Sense   NS

Students will analyze rational numbers using place value and will perform all basic operations using these numbers without the use of a calculator.

MTCR.NS.1 Students will use place value to analyze whole numbers, numbers in scientific notation, and decimals. (MFANSQ1)

MTCR.NS.2 Students will perform mathematical operations involving all forms of positive, rational numbers, including the use of order of operations. (MFANSQ4)

MTCR.NS.3. Students will be able to convert fractions to decimals to percents from any given representation. (MFANSQ4)

MTCR.NS.4. Students will extend their knowledge of positive, rational numbers to negative values, including comparisons and absolute values. (MFANSQ2)

MTCR.NS.5 Students will apply their knowledge of rational numbers to solve application problems involving all forms of these numbers, including the use of the mean, greatest common factor, and least common multiple. (MFAPR2, MGSE9-12.N.Q.3)

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Linear Equations and Inequalities	LEI

Students will solve equations and inequalities in one variable and then, extend this knowledge to two-variable linear equations and inequalities.

MTCR.LEI.1 Students will solve multi-step equations and inequalities, using all forms of rational numbers as coefficients. (MGSE9-12.A.REI.l, MGSE9-12.A.CED.1)

MTCR.LEI.2 Students will set-up and solve application problems including direct and inverse variations and proportions. (MGSE9-12.A.CED.1)

MTCR.LEI.3 Students will graph linear equations and inequalities using intercepts, slope and y-intercept, and tables of values. (MGSE9-12.N.Q.1)

MTCR.LEI.4 Students will determine and identify relationships between lines using slope. (MGSE9-12.G.GPE.5)

MTCR.LEI.5 Students will solve systems of linear equations using graphing, substitution, and elimination, and then, apply these methods to set-up and solve application problems involving systems. (MGSE9-12.A.CED.3)

MTCR.LEI.6 Students will set-up and graph problems involving systems of inequalities. (MGSE9-12.A.CED.3)

Non-Linear Relationships	NLR

Students will simplify square roots, rational expressions, and polynomial expressions, and then, solve equations involving these expressions.

MTCR.NLR.1 Students will simplify and perform basic operations with square roots. (MGSE9-12.N.RN.2)

MTCR.NLR.2 Students will solve right triangle application problems by establishing and using the Pythagorean Theorem and right triangle relationships. (MGSE9-12.G.SRT.4, MGSE9-12.G.SRT.6, MGSE9-12.G.SRT.8)

MTCR.NLR.3. Students will perform basic operations with polynomial expressions including the use of order of operations, substitution of rational numbers for variables, operations with exponents, and binomial expansion. (MGSE9-12A.SSE.l, MGSE9-12.A.SSE.2, MGSE9-12A.SSE.3, MGSE9-12.N.RN.2, MGSE9-12.APR.l)

MTCR.NLR.4 Students will solve quadratic equations and corresponding application problems using square roots, factoring, completing the square, and the quadratic formula. (MGSE9-12.G.SSE.3, MGSE9-12.A.CED.l, MGSE9-12.A.CED.2, MGSE9-12.A.REI.4)

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MTCR.NLR.5 Students will perform basic operations with rational expressions, including simplification with factoring. (MGSE9-12.APR.6, MGSE9-12.A.APR.7)

MTCR.NLR.6 Students will solve rational equations and corresponding application problems. (MGSE9-12.CED.l, MGSE9-12.A.RE1.2)

Graphs and Functions	GF

Students will identify and analyze functions using mappings, relations, graphs, and real-world applications.

MTCR.GF.1 Students will use the definition of functions to identify functions in various forms. (MGSE9-12.F.IF.5)

MTCR.GF.2 Students will graph functions analyzing the important features of the graphs. {MGSE9-12.F.IF.4, MGSE9-12.F.IF.5, MGSE9-12.F.IF.6, MGSE9-12.F.IF.7, MGSE9-12.F.IF.8)

MTCR.GF.3 Students will analyze and solve application problems involving functions. (MGSE9-12.F.IF.4, MGSE9-12.F.IF.7, MGSE9-12.F.BF.l, MGSE9-12.F.LE.3)

Measurement and Geometry	MG

Students will convert between measurements within the same systems and from metric to English/English to metric.

MTCR.MG.1 Students will convert measurements within their same system of measurement including time. (MGSE9-12.N.Q.1*)*

MTCR.MG.2 Students will convert measurements from English to metric and metric to English, using given conversions. (MGSE9-12.N.Q.1)

Students will classify basic geometric shapes and then, calculate the perimeters, areas, volumes, and surface areas of figures extending this work into contextual problems.

MTCR.MG.3 Students will determine the characteristics and the relationships of 2-dimensional and 3-dimensional figures. (MGSE9-12.G.CO.ll, MGSE9-12.G.GPE.4, MGSE9-12.G.GMD.4, MGSE9-12.G.GPE.4)

MTCR.MG.4 Students will use formulas to isolate unknowns and then calculate the perimeters, areas, volumes, and surface areas of the shapes including setting-up and solving contextual problems. (MGSE9-12.A.REI.3, MGSE9-12.A.CED.4, MGSE9-12.G.GMD.1, MGSE9-12.G.GMD.3, MGSE9-12.G.MG.1)