# Georgia
# Standards of Excellence
# Grade Level
# Curriculum Overview

## Mathematics

GSE Fourth Grade

Georgia Department of Education

Richard Woods, Georgia's School Superintendent
*"Educating Georgia's Future"*

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Georgia Department of Education

# Georgia Standards of Excellence
# Fourth Grade

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| | | | | | | | |
| These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units.<br>All units will include the Mathematical Practices and indicate skills to maintain. However, the progression of the units is at the discretion of districts. | | | | | | | |

**NOTE:** Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics.

**Grades 3-5 Key:** G= Geometry, MD=Measurement and Data, NBT= Number and Operations in Base Ten, NF = Number and Operations, Fractions, OA = Operations and Algebraic Thinking.

## UNPACKING THE STANDARDS

## STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

*These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report* Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).*

***Students are expected to:***
**1. Make sense of problems and persevere in solving them.**
In fourth grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.

**2. Reason abstractly and quantitatively.**
Fourth graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.

**3. Construct viable arguments and critique the reasoning of others.**
In fourth grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.

**4. Model with mathematics.**
Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth

graders should evaluate their results in the context of the situation and reflect on whether the results make sense.

**5. Use appropriate tools strategically.**
Fourth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.

**6. Attend to precision.**
As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.

**7. Look for and make use of structure.**
In fourth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.

**8. Look for and express regularity in repeated reasoning.**
Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions. Students should notice that there are multiplicative relationships between the models to express the repeated reasoning seen.

***Mathematical Practices 1 and 6 should be evident in EVERY lesson***

## CONTENT STANDARDS

**Operations and Algebraic Thinking**
**IMPORTANT INFORMATION ON ORDER OF OPERATIONS FOUND HERE: Order of Operations: The Myth and The Math (Please read BEFORE you teach this concept.)

**CLUSTER #1: USE THE FOUR OPERATIONS WITH WHOLE NUMBERS TO SOLVE PROBLEMS.**

*Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are:* ***multiplication/multiply, division/divide, addition/add, subtraction/subtract, equations, unknown, remainders, reasonableness, mental computation, estimation, rounding.***

**MGSE4.OA.1 Understand that a multiplicative comparison is a situation in which one quantity is multiplied by a specified number to get another quantity.**

a. **Interpret a multiplication equation as a comparison e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.**
b. **Represent verbal statements of multiplicative comparisons as multiplication equations.**

A *multiplicative comparison* is a situation in which one quantity is multiplied by a specified number to get another quantity (e.g., "*a* is *n* times as much as *b*"). Students should be able to identify and verbalize which quantity is being multiplied and which number tells how many times.

Students should be given opportunities to write and identify equations and statements for multiplicative comparisons.

Examples:
*5 x 8 = 40:* Sally is five years old. Her mom is eight times older. How old is Sally's Mom?
*5 x 5 = 25:* Sally has five times as many pencils as Mary. If Sally has 5 pencils, how many does Mary have? (Mary's pencils x 5 = Sally's pencils)

**MGSE4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison. Use drawings and equations with a symbol or letter for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.**

This standard calls for students to translate comparative situations into equations with an unknown and solve. Students need many opportunities to solve contextual problems. Refer Table 2, included at the end of this section, for more examples.

Examples:

- **Unknown Product:** A blue scarf costs \$3. A red scarf costs 6 times as much. How much does the red scarf cost? *($3 \times 6 = p$)*
- **Group Size Unknown:** A book costs \$18. That is 3 times more than a DVD. How much does a DVD cost? *($18 \div p = 3$ or $3 \times p = 18$)*
- **Number of Groups Unknown:** A red scarf costs \$18. A blue scarf costs \$6. How many times as much does the red scarf cost compared to the blue scarf?
  *($18 \div 6 = p$ or $6 \times p = 18$)*
  When distinguishing multiplicative comparison from additive comparison, students should note the following.
- Additive comparisons focus on the difference between two quantities.
  - For example, Deb has 3 apples and Karen has 5 apples. How many more apples does Karen have?
  - A simple way to remember this is, “How many more?”
- Multiplicative comparisons focus on comparing two quantities by showing that one quantity is a specified number of times larger or smaller than the other.
  - For example, Deb ran 3 miles. Karen ran 5 times as many miles as Deb. How many miles did Karen run?
  - A simple way to remember this is “How many times as much?” or “How many times as many?”

**MGSE4.OA.3 Solve multistep word problems with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a symbol or letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.**

The focus in this standard is to have students use and discuss various strategies. It refers to estimation strategies, including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer. Students need many opportunities solving multistep story problems using all four operations.

Example 1:
On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. About how many miles did they travel total?

Some typical estimation strategies for this problem are shown below.

| **Student 1** | **Student 2** | **Student 3** |
|---|---|---|
| I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500. | I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500. | I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30, I know my answer will be about 530. |

The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems will be structured so that all acceptable estimation strategies will arrive at a reasonable answer.
Example 2:
Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?

| **Student 1** | **Student 2** |
|---|---|
| First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I am trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So, we still need about 250 bottles. | First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. 40 + 20 = 60. 300 – 60 = 240, so we need about 240 more bottles. |

This standard references interpreting remainders. Remainders should be put into context for interpretation. Ways to address remainders:

- Remain as a left over
- Partitioned into fractions or decimals
- Discarded leaving only the whole number answer
- Increase the whole number answer by one
- Round to the nearest whole number for an approximate result

Example:
Write different word problems involving **44 ÷ 6 = ?** where the answers are best represented as:

- Problem A: 7
- Problem B: 7 r 2
- Problem C: 8
- Problem D: 7 or 8
- Problem E: $7\frac{2}{6}$

Possible solutions:

- Problem **A: 7**
  Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches did she fill?
  44 ÷ *6 = p; p = 7 r 2. Mary can fill 7 pouches completely.*

- Problem **B: 7 r 2**
  Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches could she fill and how many pencils would she have left?
  44 ÷ *6 = p; p = 7 r 2; Mary can fill 7 pouches and have 2 pencils left over.*

- Problem **C: 8**
  Mary had 44 pencils. Six pencils fit into each of her pencil pouches. What is the fewest number of pouches she would need in order to hold all of her pencils?
  44 ÷ *6 = p; p = 7 r 2; Mary needs 8 pouches to hold all of the pencils.*

- Problem **D: 7 or 8**
  Mary had 44 pencils. She divided them equally among her friends before giving one of the leftovers to each of her friends. How many pencils could her friends have received?
  44 ÷ *6 = p; p = 7 r 2; Some of her friends received 7 pencils. Two friends received 8 pencils.*

- Problem **E:** $\mathbf{7\frac{2}{6}}$
  Mary had 44 pencils and put six pencils in each pouch. What fraction represents the number of pouches that Mary filled?
  44 ÷ *6 = p; p =* $\mathbf{7\frac{2}{6}}$*; Mary filled* $7\frac{2}{6}$ *pencil pouches.*

  Example:
  There are 128 students going on a field trip. If each bus held 30 students, how many buses are needed? *(128 ÷ 30 = b; b = 4 R 8; They will need 5 buses because 4 buses would not hold all of the students).*
  Students need to realize in problems, such as the example above, that an extra bus is needed for the 8 students that are left over. Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies include, but are not limited to, the following.

- **Front-end estimation with adjusting** (Using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts)

- **Clustering around an average** (When the values are close together an average value is selected and multiplied by the number of values to determine an estimate.)

- **Rounding and adjusting** (Students round to a lower multiple or higher multiple and then adjust their estimate depending on how much the rounding affected the original values.)

- **Using friendly or compatible numbers such as factors** (Students seek to fit numbers together, e.g., rounding to factors and grouping numbers together that have friendly sums like 100 or 1000.)

- **Using benchmark numbers that are easy to compute** (Students select close whole numbers for fractions or decimals to determine an estimate.)

## <u>CLUSTER #2:</u> GAIN FAMILIARITY WITH FACTORS AND MULTIPLES.

*Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are:* ***multiplication/multiply, division/divide, factor pairs, factor, multiple, prime, composite.***

**MGSE4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.**

This standard requires students to demonstrate understanding of factors and multiples of whole numbers. This standard also refers to prime and composite numbers. Prime numbers have exactly two factors, the number one and their own number. For example, the number 17 has the factors of 1 and 17. Composite numbers have more than two factors. For example, 8 has the factors 1, 2, 4, and 8.

### Common Misconceptions

A common misconception is that the number 1 is prime, when in fact; it is neither prime nor composite. Another common misconception is that all prime numbers are odd numbers. This is not true, since the number 2 has only 2 factors, 1 and 2, and is also an even number.

When listing multiples of numbers, students may not list the number itself. Emphasize that the smallest multiple is the number itself.

Some students may think that larger numbers have more factors. Having students share all factor pairs and how they found them will clear up this misconception.

### Prime vs. Composite:

- A prime number is a number greater than 1 that has only 2 factors, 1 and itself.
- Composite numbers have more than 2 factors.

Students investigate whether numbers are prime or composite by:

- Building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g., 7 can be made into only 2 rectangles, $1 \times 7$ and $7 \times 1$, therefore it is a prime number).
- Finding factors of the number.
  Students should understand the process of finding factor pairs so they can do this for any number 1-100.
  Example:
  Factor pairs for 96: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, 8 and 12.

**Multiples**
Multiples are products of any given whole number and another whole number. They can also be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20 (there are 4 fives in 20).

Example:
Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
*Multiples of 24: 24, 48, 72, 96, 120, ……*

Multiples of the factors of 24:
1, 2, 3, 4, 5, …, 24 *(24 is the 24$^{th}$ multiple of one.)*
2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24 *(24 is the 12$^{th}$ multiple of two.)*
3, 6, 9, 12, 15, 15, 21, 24 *(24 is the 8$^{th}$ multiple of three.)*
4, 8, 12, 16, 20, 24 *(24 is the 6$^{th}$ multiple of four.)*
8, 16, 24 *(24 is the 3$^{rd}$ multiple of eight.)*
12, 24 *(24 is the 2$^{nd}$ multiple of twelve.)*
24 *(24 is the 1$^{st}$ multiple of 24.)*

To determine if a number between 1-100 is a multiple of a given one-digit number, some helpful hints include the following:

- All even numbers are multiples of 2.
- All even numbers that can be halved twice (with a whole number result) are multiples of 4.
- All numbers ending in 0 or 5 are multiples of 5.

**CLUSTER #3: GENERATE AND ANALYZE PATTERNS.**

*Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are:* ***pattern (number or shape), pattern rule.***

**MGSE4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. Explain informally why the pattern will continue to develop in this way.** ***For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers.***

Patterns involving numbers or symbols either repeat or grow. Students need multiple opportunities creating and extending number and shape patterns. Numerical patterns allow students to reinforce concepts and develop fluency with operations.

Patterns and rules are related. A pattern is a sequence that repeats the same process over and over. A rule dictates what that process will look like. Students investigate different patterns to find rules, identify features in the patterns, and justify the reason for those features.

Example:

| Pattern | Rule | Feature(s) |
|---|---|---|
| 3, 8, 13, 18, 23, 28, … | Start with 3; add 5 | The numbers alternately end with a 3 or an 8 |
| 5, 10, 15, 20, … | Start with 5; add 5 | The numbers are multiples of 5 and end with either 0 or 5. The numbers that end with 5 are products of 5 and an odd number. The numbers that end in 0 are products of 5 and an even number. |

After students have identified rules and features from patterns, they need to generate a numerical or shape pattern from a given rule.

Example:
**Rule:** Starting at 1, create a pattern that starts at 1 and multiplies each number by 3. Stop when you have 6 numbers.

Students write 1, 3, 9, 27, 81, 243. Students notice that all the numbers are odd and that the sums of the digits of the 2-digit numbers are each 9. Some students might investigate this beyond 6 numbers. Another feature to investigate is the patterns in the differences of the numbers (3 – 1 = 2, 9 – 3 = 6, 27 – 9 = 18, etc.).

This standard calls for students to describe features of an arithmetic number pattern or shape pattern by identifying the rule, and features that are not explicit in the rule. A t-chart is a tool to help students see number patterns.

Example:
There are 4 beans in the jar. Each day 3 beans are added. How many beans are in the jar for each of the first 5 days?

| Day | Operation | Beans |
|---|---|---|
| 0 | $3 \times 0 + 4$ | 4 |
| 1 | $3 \times 1 + 4$ | 7 |
| 2 | $3 \times 2 + 4$ | 10 |
| 3 | $3 \times 3 + 4$ | 13 |
| 4 | $3 \times 4 + 4$ | 16 |
| 5 | $3 \times 5 + 4$ | 19 |

## Number and Operation in Base Ten

**<u>CLUSTER #1</u>: GENERALIZE PLACE VALUE UNDERSTANDING FOR MULTI-DIGIT WHOLE NUMBERS.**

*Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are:* ***place value, greater than, less than, equal to, ‹, ›, =, comparisons/compare, round.***

**MGSE4.NBT.1 Recognize that in a multi-digit whole number, a digit in any one place represents ten times what it represents in the place to its right.** ***For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.***

This standard calls for students to extend their understanding of place value related to multiplying and dividing by multiples of 10. In this standard, students should reason about the magnitude of digits in a number. Students should be given opportunities to reason and analyze the relationships of numbers that they are working with.

Example:

How is the 2 in the number 582 similar to and different from the 2 in the number 528?

Students should learn that while the digit 2 is the same, the 2 in 528 is worth 20 because it is in the tens place and the 2 in 582 is worth 2 because it is in the ones place. That means that the 2 in 528 is ten times more than the 2 in 582.

**MGSE4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.**

This standard refers to various ways to write numbers. Students should have flexibility with the different number forms. Traditional expanded form is 285 = 200 + 80 + 5. Written form is two hundred eighty-five. However, students should have opportunities to explore the idea that 285 could also be 28 tens plus 5 ones or 1 hundred, 18 tens, and 5 ones.

Students should also be able to compare two multi-digit whole numbers using appropriate symbols.

**MGSE4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place.**

This standard refers to place value understanding, which extends beyond an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding.

Example:
Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?

| **Student 1** | **Student 2** |
| --- | --- |
| First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I am trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So, we still need about 250 bottles. | First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. 40 + 20 = 60. 300 – 60 = 240, so we need about 240 more bottles. |

Example:
On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did your family travel?

Some typical estimation strategies for this problem are:

| **Student 1** | **Student 2** | **Student 3** |
| --- | --- | --- |
| I first thought about 276 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get about 500. | I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with about 500. | I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30, I know my answer will be about 530. |

Example:
Round 368 to the nearest hundred.

This will either be 300 or 400 since those are the two hundreds before and after 368. Draw a number line, subdivide it as much as necessary, and determine whether 368 is closer to 300 or 400. Since 368 is closer to 400, this number should be rounded to 400.


**Common Misconceptions**

There are several misconceptions students may have about writing numerals from verbal descriptions. Numbers like one thousand do not cause a problem; however, a number like one thousand two causes problems for students. Many students will understand the 1000 and the 2 but then instead of placing the 2 in the ones place, students will write the numbers as they hear them, 10002 (ten thousand two). There are multiple strategies that can be used to assist with this concept, including place-value boxes and vertical-addition method.

Students often assume that the first digit of a multi-digit number indicates the "greatness" of a number. The assumption is made that 954 is greater than 1002 because students are focusing on the first digit instead of the number as a whole. Students need to be aware of the greatest place value. In this example, there is one number with the lead digit in the thousands and another number with its lead digit in the hundreds.

**<u>CLUSTER #2</u>: USE PLACE VALUE UNDERSTANDING AND PROPERTIES OF OPERATIONS TO PERFORM MULTI-DIGIT ARITHMETIC.**

*Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms* *students should learn to use with increasing precision with this cluster are:* ***factors, products, dividend, divisor, quotient, addends, sum, difference, remainder***

**MGSE4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.**

Students build on their understanding of addition and subtraction, their use of place value and their flexibility with multiple strategies to make sense of the standard algorithm. They continue to use place value in describing and justifying the processes they use to add and subtract.

This standard refers to *fluency*, which means accuracy (attending to precision), efficiency (using well-understood strategies with ease), and flexibility (using strategies such as the distributive property). According to NCTM, fluency is also the ability to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another. To develop fluency, students need experience in integrating concepts and strategies and building on familiar strategies as they create their own informal strategies and procedures. Students need opportunities to justify both informal strategies and commonly used procedures mathematically, to support and justify their choices of appropriate procedures, and to strengthen their understanding and skill through strategic practice. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research indicates that teachers' can best support students' development of fluency and automaticity with computation through varied experiences with part-whole reasoning, breaking numbers apart and working on mental strategies, rather than timed tests. Evidence from research has indicated that timed tests cause unhealthy math anxiety with learners as they are developing a solid foundation in numeracy: https://www.youcubed.org/resources/new-evidence-timed-test-teaching-children-mathematics-april-2014/.

Fourth grade is the first-grade level in which students are expected to be proficient at using the standard algorithm to add and subtract. However, other previously learned strategies are still appropriate for students to use. This is the time when students begin to standardize their algorithm for computation. The word algorithm refers to a procedure or a series of steps. In fourth grade, students should be able to flexibly use a variety of algorithms based on their understanding and take formal steps to standardize their preferred algorithm based on the given context and problem situation. The academic vocabulary referring to "the standard algorithm" should not be confused with the US Traditional Algorithm. The standard algorithm is any algorithm used and understood by a student that involves a series of steps based on place-value decomposition. (Note: The standard algorithm is not exclusive to the US Traditional algorithm.) For more information, refer to the Mathematics Glossary and read http://bit.ly/1DTZYKs; http://www.mathedleadership.org/docs/resources/journals/NCSMJournal_ST_Algorithms_Fuson_Beckmann.pdf. The process of students standardizing and becoming fluent with their algorithm is reserved for 4th grade with this standard, 4.NBT.4. Mathematical concepts will be assessed in a way where a variety of standard algorithms can be used based on what the student understands and his/her place-value reasoning.

When students begin using the standard algorithm their explanation may be quite lengthy. After much practice with using place value to justify their steps, they will develop fluency with their algorithm. Students should also be able to explain why their chosen algorithm works.

Example:

$$\begin{array}{r} 3892 \\ \underline{+\ 1567} \end{array}$$

Student explanation for this problem:

1. Two ones plus seven ones is nine ones.
2. Nine tens plus six tens is 15 tens.
3. I am going to write down five tens and think of the 10 tens as one more hundred. (*Denotes with a 1 above the hundreds column*.)
4. Eight hundreds plus five hundreds plus the extra hundred from adding the tens is 14 hundreds.
5. I am going to write the four hundreds and think of the 10 hundreds as one more 1000. (*Denotes with a 1 above the thousands column.*)
6. Three thousands plus one thousand plus the extra thousand from the hundreds is five thousand.
7. The sum is 5,459.

Example:

$$\begin{array}{r} 3546 \\ \underline{-\ \ 928} \end{array}$$

Student explanation for this problem:

1. There are not enough ones to take 8 ones from 6 ones, so I have to use one ten as 10 ones. Now I have 3 tens and 16 ones. (*Marks through the 4 and notates with a 3 above the 4 and writes a 1 above the ones column to be represented as 16 ones.*)
2. Sixteen ones minus 8 ones is 8 ones. (*Writes an 8 in the ones column of answer.*)
3. Three tens minus 2 tens is one ten. (*Writes a 1 in the tens column of answer*.)
4. There are not enough hundreds to take 9 hundreds from 5 hundreds so I have to use one thousand as 10 hundreds. (*Marks through the 3 and notates with a 2 above it. Writes down a 1 above the hundreds column.*) Now I have 2 thousands and 15 hundreds.
5. Fifteen hundreds minus 9 hundreds is 6 hundreds. (*Writes a 6 in the hundreds column of the answer*.)
6. I have 2 thousands left since I did not have to take away any thousands. (*Writes 2 in the thousands place of answer.*)
7. The difference is 2,618.

Students should know that it is mathematically possible to subtract a larger number from a smaller number but that their work with whole numbers does not allow this as the difference would result in a negative number.

**MGSE4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.**

Students who develop flexibility in breaking numbers apart have a better understanding of the importance of place value and the distributive property in multi-digit multiplication. Students use base ten blocks, area models, partitioning, compensation strategies, etc. when multiplying whole numbers and use words and diagrams to explain their thinking. They use the terms factor and product when communicating their reasoning.

This standard calls for students to multiply numbers using a variety of strategies. Multiple strategies enable students to develop fluency with multiplication and transfer that understanding to division. **Use of the standard algorithm for multiplication is an expectation in the 5$^{th}$ grade. The US Traditional Algorithm for multiplication is NOT an expectation in 4$^{th}$ grade.**

Example:
There are 25 dozen cookies in the bakery. What is the total number of cookies at the bakery?

| **Student 1** | **Student 2** | **Student 3** |
| --- | --- | --- |
| $25 \times 12$<br>I broke 12 up into 10 and 2.<br>$25 \times 10 = 250$<br>$25 \times 2 = 50$<br>$250 + 50 = 300$ | $25 \times 12$<br>I broke 25 into 5 groups of 5.<br>$5 \times 12 = 60$<br>I have 5 groups of 5 in 25.<br>$60 \times 5 = 300$ | $25 \times 12$<br>I doubled 25 and cut 12 in half to get $50 \times 6$.<br>$50 \times 6 = 300$ |

Examples:
What would an array area model of 74 x 38 look like?

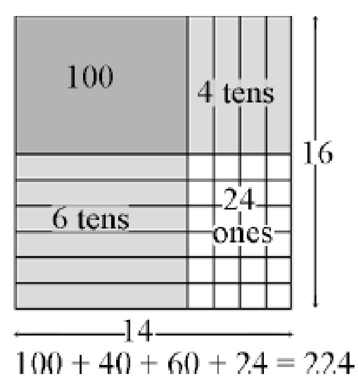
| | 70 | 4 |
|---|---|---|
| 30 | 70 x 30 = 2,100 | 4 x 30 = 120 |
| 8 | 70 x 8 = 560 | 4 x 8 = 32 |

2,100 + 560 + 120 + 32 = 2,812

The area model to the right shows the partial products for 14 x 16 = 224.

Using the area model, students verbalize their understanding:

- 10 × 10 is 100
- 4 × 10 is 40, or 4 tens
- 10 × 6 is 60, or 6 tens
- 4 × 6 is 24
- 100 + 40 + 60 + 24 = 224




Examples:
Students use different strategies to record this type of multiplicative thinking. Students can explain the examples below with base 10 blocks, drawings, or numbers.

To illustrate 154 × 6, students can use base 10 blocks or use drawings to show 154 six times. Seeing 154 six times will lead them to understand the distributive property shown below.

154 × 6 = (100 + 50 + 4) × 6
154 x 6 = (100 × 6) + (50 × 6) + (4 × 6)
154 x 6 = 600 + 300 + 24 = 924

Illustrating 25 x 24 can also be completed with base ten blocks or drawings to show 25 twenty-four times. The distributive property is shown below.

$$\begin{array}{rl} & 25 \\ \times & \underline{24} \\ 400 & (20 \times 20) \\ 100 & (20 \times 5) \\ 80 & (4 \times 20) \\ \underline{20} & (4 \times 5) \\ 600 & \end{array}$$

**MGSE4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.**

In fourth grade, students build on their third-grade work with division within 100. Students need opportunities to develop their understandings by using problems in and out of context.

Example:
A $4^{th}$ grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box?

This standard calls for students to explore division through various strategies.

- **Using Base 10 Blocks:** Students build 260 with base 10 blocks and distribute them into 4 equal groups. Some students may need to trade the 2 hundreds for 20 tens but others may easily recognize that 200 divided by 4 is 50.

- **Using Place Value:** 260 ÷ 4 = (200 ÷ 4) + (60 ÷ 4) = 50 + 15 = 65

- **Using Multiplication:** 4 × 50 = 200, 4 × 10 = 40, 4 × 5 = 20; 50 + 10 + 5 = 65; so, 260 ÷ 4 = 65

Example:

There are 592 students participating in Field Day. They are put into teams of 8 for the competition. How many teams are created?

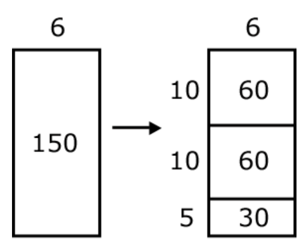
| **Student 1** | **Student 2** | **Student 3** |
|---|---|---|
| 592 divided by 8<br>There are 70 eights in 560.<br>592 – 560 = 32<br>There are 4 eights in 32.<br>70 + 4 = 74 | 592 divided by 8<br>I know that 10 eights is 80.<br>If I take out 50 eights that is 400.<br>592 – 400 = 192<br>I can take out 20 more eights which is 160.<br>192 – 160 = 32<br>8 goes into 32 four times.<br>I have none left. I took out 50, then 20 more, then 4 more. That's 74. | I want to get to 592.<br>$8 \times 25 = 200$<br>$8 \times 25 = 200$<br>$8 \times 25 = 200$<br>200 + 200 + 200 = 600<br>600 – 8 = 592<br>I had 75 groups of 8 and took one away, so there are 74 teams. |

Example:

**Using an Open Array or Area Model**

After developing an understanding of using arrays to divide, students begin to use a more abstract model for division. This model connects to a recording process that will be formalized in 5$^{th}$ grade.

1. **150 ÷ 6**




Students make a rectangle and write 6 on one of its sides. They express their understanding that they need to think of the rectangle as representing a total of 150.

1. Students think, "6 times what number is a number close to 150?" They recognize that $6 \times 10$ is 60 so they record 10 as a factor and partition the rectangle into 2 rectangles and label the area aligned to the factor of 10 with 60. They express that they have only used 60 of the 150 so they have 90 left.
2. Recognizing that there is another 60 in what is left, they repeat the process above. They express that they have used 120 of the 150 so they have 30 left.
3. Knowing that $6 \times 5$ is 30, they write 30 in the bottom area of the rectangle and record 5 as a factor.

4. Student express their calculations in various ways:

   a.
   $$\begin{array}{rl} 150 & \\ \underline{-60} & (6 \times 10) \\ 90 & \\ \underline{-60} & (6 \times 10) \\ 30 & \\ \underline{-30} & (6 \times 5) \\ 0 & \end{array}$$

   $150 \div 6 = 10 + 10 + 5 = 25$

   b. $150 \div 6 = (60 \div 6) + (60 \div 6) + (30 \div 6) = 10 + 10 + 5 = 25$

**2. 1917 ÷ 9**

| 9 |
|---|
| 1,800 |
| 90 |
| 27 |

A student's description of his or her thinking may be:
I need to find out how many 9s are in 1917. I know that 200 x 9 is 1800. So, if I use 1800 of the 1917, I have 117 left. I know that 9 x 10 is 90. So, if I have 10 more 9s, I will have 27 left. I can make 3 more 9s. I have 200 nines, 10 nines and 3 nines. So, I made 213 nines. 1917 ÷ 9 = 213.

### Common Misconceptions

When learning to fluently add and subtract using the standard algorithm, students often mix up when to regroup. Also, students often do not notice the need of regrouping and just take the smaller digit from the larger one. Emphasize place value and the meaning of each of the digits. When students have had experience with Number Talks and using strategies to add and subtract in previous grade levels, they bring that strategy understanding to 4th grade and standardize their process for adding and subtracting. Students build on their understanding of addition and subtraction, their use of place value and their flexibility with multiple strategies to make sense of the standard algorithm. Students should continue to use place value in describing and justifying the processes they use to add and subtract. The academic vocabulary referring to "the standard algorithm" should not be confused with the US Traditional Algorithm. The standard algorithm is any efficient algorithm used and understood by a student that involves a series of steps based on place-value decomposition. (Note: The standard algorithm is not exclusive to the US Traditional algorithm.)

## Number and Operations- Fractions

**CLUSTER #1: EXTEND UNDERSTANDING OF FRACTION EQUIVALENCE AND ORDERING.**

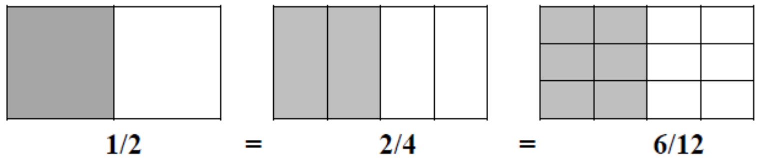
*Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g.,* $\frac{15}{9} = \frac{5}{3}$*), and they develop methods for generating and recognizing equivalent fractions. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are:* ***partition(ed), fraction, unit fraction, equivalent, multiple, reason, denominator, numerator, comparison/compare, ‹, ›, =, benchmark fraction.***

**MGSE4.NF.1 Explain why two or more fractions are equivalent** $\frac{a}{b} = \frac{n \times a}{n \times b}$ **ex:** $\frac{1}{4} = \frac{3 \times 1}{3 \times 4}$ **by using visual fraction models. Focus attention on how the number and size of the parts differ even though the fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.**

This standard refers to visual fraction models. This includes area models, number lines or it could be a collection/set model. This standard extends the work in third grade by using additional denominators of 5, 10, 12, and 100.

This standard addresses equivalent fractions by examining the idea that equivalent fractions can be created by multiplying both the numerator and denominator by the same number, (also referred to as a fraction equivalent to one) or by dividing a shaded region into various parts.

Example:




Technology Connection: [http://illuminations.nctm.org/activitydetail.aspx?id=80](http://illuminations.nctm.org/activitydetail.aspx?id=80)

**MGSE4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by using visual fraction models, by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions.**

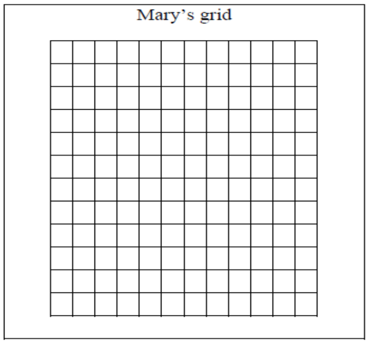
This standard calls for students to compare fractions by creating visual fraction models or finding common denominators or numerators. Students' experiences should focus on visual fraction models rather than algorithms. When tested, models may or may not be included. Students should learn to draw fraction models to help them compare. Students must also recognize that they must consider the size of the whole when comparing fractions (i.e., $\frac{1}{2}$ and $\frac{1}{8}$ of two medium pizzas is very different from $\frac{1}{2}$ of one medium and $\frac{1}{8}$ of one large).

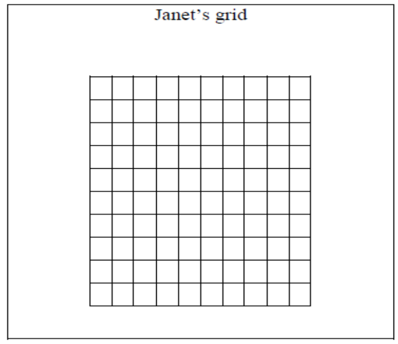
Example:
Use patterns blocks.

1. If a red trapezoid is one whole, which block shows $\frac{1}{3}$?
2. If the blue rhombus is $\frac{1}{3}$, which block shows one whole?
3. If the red trapezoid is one whole, which block shows $\frac{2}{3}$?

Example:
Mary used a $12 \times 12$ grid to represent 1 and Janet used a $10 \times 10$ grid to represent 1. Each girl shaded grid squares to show $\frac{1}{4}$. How many grid squares did Mary shade? How many grid squares did Janet shade? Why did they shade different numbers of grid squares?

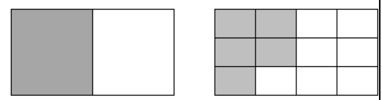
*Possible solution: Mary shaded 36 grid squares; Janet shaded 25 grid squares. The total number of little squares is different in the two grids, so $\frac{1}{4}$ of each total number is different.*



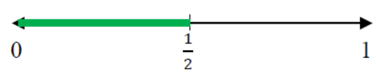




Example:
There are two cakes on the counter that are the same size. The first cake has $\frac{1}{2}$ of it left. The second cake has $\frac{5}{12}$ left. Which cake has more left?

**Student 1:** ***Area Model***
The first cake has more left over. The second cake has $\frac{5}{12}$ left which is less than $\frac{1}{2}$.

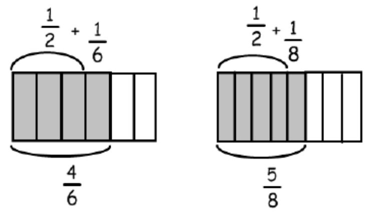


**Student 2:** ***Number Line Model***
The first cake has more left over: $\frac{1}{2}$ is greater than $\frac{5}{12}$.




**Student 3:** ***Verbal Explanation***
I know that $\frac{6}{12}$ equals $\frac{1}{2}$, and $\frac{5}{12}$ is less than $\frac{1}{2}$. Therefore, the second cake has less left over than the first cake. The first cake has more left over.

Example:
When using the benchmark fraction $\frac{1}{2}$ to compare $\frac{4}{6}$ and $\frac{5}{8}$, you could use diagrams such as these:




$\frac{4}{6}$ is $\frac{1}{6}$ larger than $\frac{1}{2}$, while $\frac{5}{8}$ is $\frac{1}{8}$ larger than $\frac{1}{2}$. Since $\frac{1}{6}$ is greater than $\frac{1}{8}$, $\frac{4}{6}$ is the greater fraction.

**Common Misconceptions**

Students sometimes apply whole number concepts when generating equivalent fractions. If a student finds that $\frac{6}{9}$ is equivalent to $\frac{2}{3}$ using manipulatives, when looking at the numbers independently of the manipulatives they might say that $\frac{6}{9}$ is greater than $\frac{2}{3}$. Students think that in whole numbers nine is larger than three. However, discussion needs to occur with students that in the fractions the denominators are ninths and thirds, not nine and three. Students need repeated experiences with visual models to understand the relationship between two fractions that are equivalent.

**<u>CLUSTER #2</u>: BUILD FRACTIONS FROM UNIT FRACTIONS BY APPLYING AND EXTENDING PREVIOUS UNDERSTANDINGS OF OPERATIONS ON WHOLE NUMBERS.**

*Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are:* ***operations, addition/joining, subtraction/separating, fraction, unit fraction, equivalent, multiple, reason, denominator, numerator, decomposing, mixed number, rules about how numbers work (properties), multiply, multiple.***

**MGSE4.NF.3 Understand a fraction $\frac{a}{b}$ with a numerator >1 as a sum of unit fractions $\frac{1}{b}$.**

**a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.**

A fraction with a numerator of one is called a unit fraction. When students investigate fractions other than unit fractions, such as $\frac{2}{3}$, they should be able to join (compose) or separate (decompose) the fractions of the same whole.

Example: $\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$

Being able to visualize this decomposition into unit fractions helps students when adding or subtracting fractions. Students need multiple opportunities to work with mixed numbers and be able to decompose them in more than one way. Students may use visual models to help develop this understanding.

Example: $1\frac{1}{4} - \frac{3}{4} = ?$ $\rightarrow$ $\frac{4}{4} + \frac{1}{4} = \frac{5}{4}$ $\rightarrow$ $\frac{5}{4} - \frac{3}{4} = \frac{2}{4}$ *or* $\frac{1}{2}$

Example of word problem:

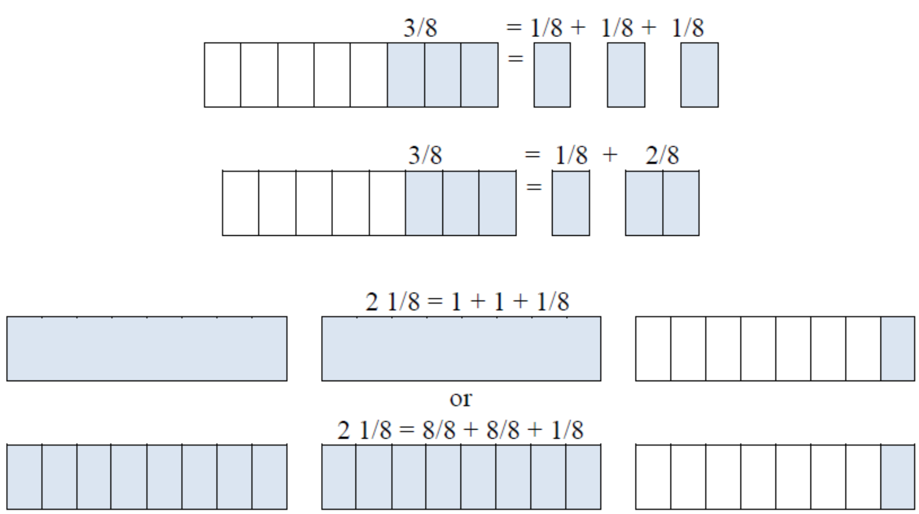
Mary and Lacey decide to share a pizza. Mary ate $\frac{3}{6}$ and Lacey ate $\frac{2}{6}$ of the pizza. How much of the pizza did the girls eat together?

*Possible solution*: The amount of pizza Mary ate can be thought of as $\frac{3}{6}$ or $\frac{1}{6} + \frac{1}{6} + \frac{1}{6}$. The amount of pizza Lacey ate can be thought of as $\frac{1}{6} + \frac{1}{6}$. The total amount of pizza they ate is $\frac{1}{6}+ \frac{1}{6}+ \frac{1}{6}+ \frac{1}{6}+ \frac{1}{6}$ or $\frac{5}{6}$ of the pizza.

**b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.***

Students should justify their breaking apart (decomposing) of fractions using visual fraction models. The concept of rewriting mixed numbers as fractions greater than one needs to be emphasized using visual fraction models and decomposing.

Example:




**c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.**

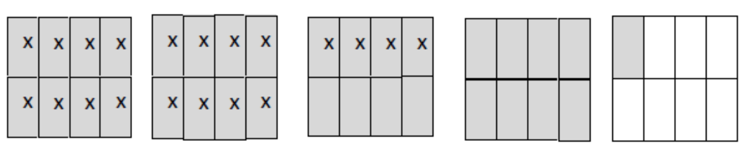
A separate algorithm for mixed numbers in addition and subtraction is not necessary. Students will tend to add or subtract the whole numbers first and then work with the fractions using the same strategies they have applied to problems that contained only fractions.

Example:
Susan and Maria need $8\frac{3}{8}$ feet of ribbon to package gift baskets. Susan has $3\frac{1}{8}$ feet of ribbon and Maria has $5\frac{3}{8}$ feet of ribbon. How much ribbon do they have altogether? Will it be enough to complete the project? Explain why or why not.

The student thinks: I can add the ribbon Susan has to the ribbon Maria has to find out how much ribbon they have altogether. Susan has $3\frac{1}{8}$ feet of ribbon and Maria has $5\frac{3}{8}$ feet of ribbon. I can write this as $3\frac{1}{8} + 5\frac{3}{8}$. I know they have 8 feet of ribbon by adding the 3 and 5. They also have $\frac{1}{8}$ and $\frac{3}{8}$ which makes a total of $\frac{4}{8}$ more. Altogether they have $8\frac{4}{8}$ feet of ribbon. $8\frac{4}{8}$ is larger than $8\frac{3}{8}$ so they will have enough ribbon to complete the project. They will even have a little extra ribbon left - $\frac{1}{8}$ foot.

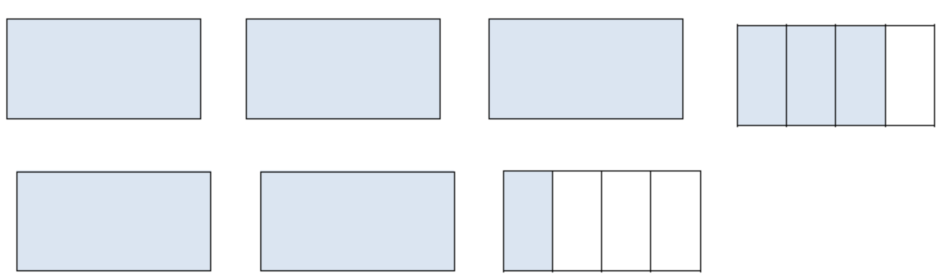
Example:
Trevor has $4\frac{1}{8}$ pizzas left over from his soccer party. After giving some pizza to his friend, he has $2\frac{4}{8}$ of a pizza left. How much pizza did Trevor give to his friend?

Possible solution: Trevor had $4\frac{1}{8}$ pizzas to start. This is $\frac{33}{8}$ of a pizza. The x's show the pizza he has left which is $2\frac{4}{8}$ pizzas or $\frac{20}{8}$ pizzas. The shaded rectangles without the x's are the pizza he gave to his friend which is $\frac{13}{8}$ or $1\frac{5}{8}$ pizzas.



Mixed numbers are formally introduced for the first time in 4th Grade. Students should have ample experiences of adding and subtracting mixed numbers where they work with mixed numbers or convert mixed numbers into fractions greater than one.

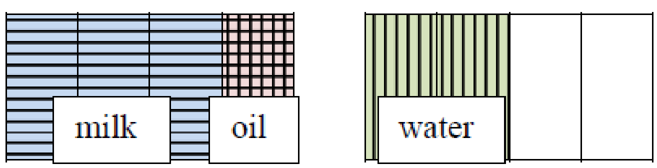
Example:
While solving the problem, $3\frac{3}{4} + 2\frac{1}{4}$, students could do the following:



| | |
|---|---|
| **Student 1:** | $3 + 2 = 5$ and $\frac{3}{4} + \frac{1}{4} = 1$, so $5 + 1 = 6$. |
| **Student 2:** | $3\frac{3}{4} + 2 = 5\frac{3}{4}$, so $5\frac{3}{4} + \frac{1}{4} = 6$. |
| **Student 3:** | $3\frac{3}{4} = \frac{15}{4}$ and $2\frac{1}{4} = \frac{9}{4}$, so $\frac{15}{4} + \frac{9}{4} = \frac{24}{4} = 6$. |

**d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.**

Example:
A cake recipe calls for you to use $\frac{3}{4}$ cup of milk, $\frac{1}{4}$ cup of oil, and $\frac{2}{4}$ cup of water. How much liquid was needed to make the cake?




**3/4 + 1/4 + 2/4 = 6/4 = 1 2/4**

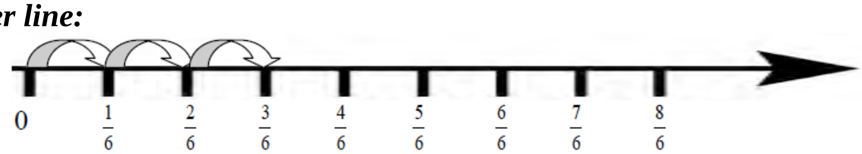
**MGSE4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number e.g., by using a visual such as a number line or area model.**

**a. Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).***

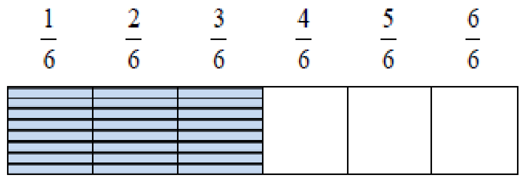
This standard builds on students' work of adding fractions and extending that work into multiplication.

Example: $\frac{3}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 3 \times \frac{1}{6}$

***Number line:***



***Area model:***

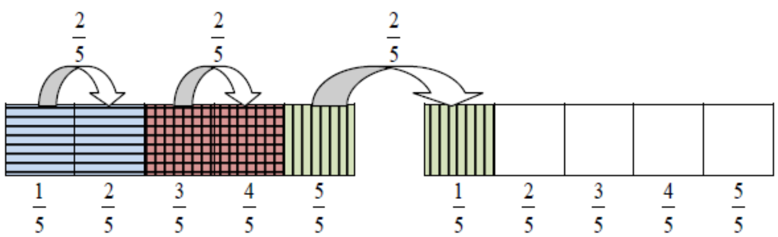



**b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.** ***For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)***

This standard extends the idea of multiplication as repeated addition.

Example: $3 \times \frac{2}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{6}{5} = 6 \times \frac{1}{5}$.

Students are expected to use and create visual fraction models to multiply a whole number by a fraction.

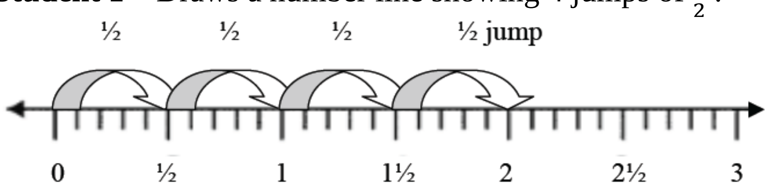



**c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.** ***For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?***

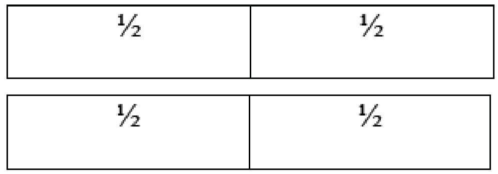
This standard calls for students to use visual fraction models to solve word problems related to multiplying a whole number by a fraction.

Example:
In a relay race, each runner runs $\frac{1}{2}$ of a lap. If there are 4 team members how long is the race?

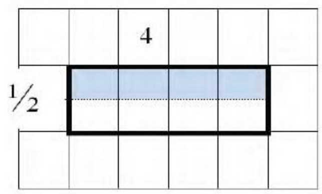
**Student 1** – Draws a number line showing 4 jumps of $\frac{1}{2}$:



**Student 2 –** Draws an area model showing 4 pieces of $\frac{1}{2}$ joined together to equal 2:




**Student 3 –** Draws an area model representing $4 \times \frac{1}{2}$ on a grid, dividing one row into $\frac{1}{2}$ to represent the multiplier:




Example:
Heather bought 12 plums and ate $\frac{1}{3}$ of them. Paul bought 12 plums and ate $\frac{1}{4}$ of them. Which statement is true? Draw a model to explain your reasoning.

a. Heather and Paul ate the same number of plums.
b. Heather ate 4 plums and Paul ate 3 plums.
c. Heather ate 3 plums and Paul ate 4 plums.
d. Heather had 9 plums remaining.

Possible student solutions:

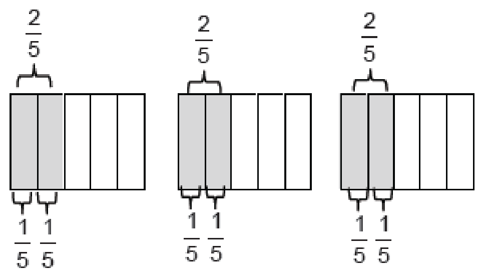
If both Heather and Paul bought the same amount of plums, all I need to do is compare what fraction of the plums each person ate. I know that one third is larger than one fourth so Heather must have eaten more plums. The correct solution must be that Heather ate 4 plums and Paul ate 3 plums.

I know that Heather ate one third of 12 which is four because $\frac{1}{3}$ x 12 is $\frac{12}{3}$, or 4. I know that Paul ate one fourth of 12, which is three because $\frac{1}{4}$ x 12 is $\frac{12}{4}$, or 3. The correct answer is choice b.

Examples:

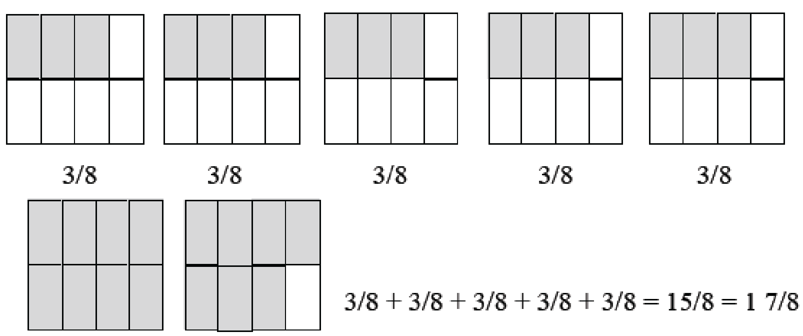
Students need many opportunities to work with problems in context to understand the connections between models and corresponding equations. Contexts involving a whole number times a fraction lend themselves to modeling and examining patterns.

1. $3 \times \frac{2}{5} = 6 \times \frac{1}{5} = \frac{6}{5}$




2. If each person at a party eats $\frac{3}{8}$ of a pound of roast beef, and there are 5 people at the party, how many pounds of roast beef are needed? Between what two whole numbers does your answer lie?

A student may build a fraction model to represent this problem:




## Common Misconceptions

Students think that it does not matter which model to use when finding the sum or difference of fractions. They may represent one fraction with a rectangle and the other fraction with a circle. They need to know that the models need to represent the same whole.

## CLUSTER #4: UNDERSTAND DECIMAL NOTATION FOR FRACTIONS AND COMPARE DECIMAL FRACTIONS.

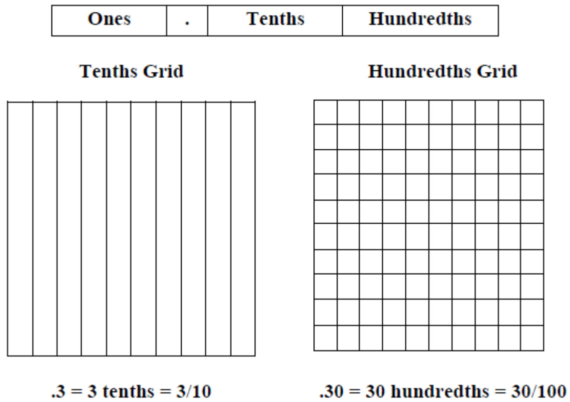
*Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are**: fraction, numerator, denominator, equivalent, reasoning, decimals, tenths, hundredths, multiplication, comparisons/compare, ‹, ›, =.***

**MGSE4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100 and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.***

This standard continues the work of equivalent fractions by having students change fractions with a 10 in the denominator into equivalent fractions that have a 100 in the denominator. Student experiences should focus on working with grids rather than algorithms in order to prepare for work with decimals in MGSE4.NF.6 and MGSE4.NF.7. Students can also use base ten blocks and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100.

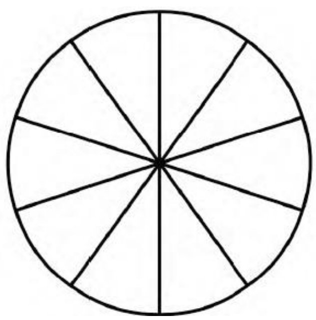
The work completed by students in $4^{th}$ grade with this standard lays the foundation for performing operations with decimal numbers in $5^{th}$ grade.

Example: Represent 3 tenths and 30 hundredths using the decimal grid models below.

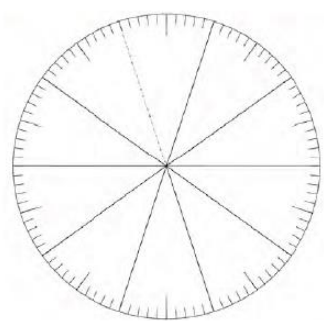



Example: Represent 3 tenths and 30 hundredths on the models below.

Tenths circle



Hundredths circle



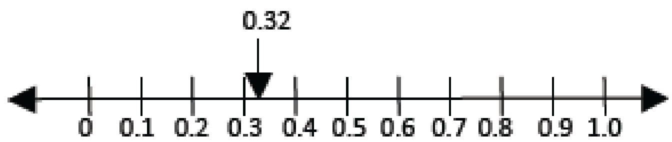
**MGSE4.NF.6 Use decimal notation for fractions with denominators 10 or 100.** ***For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.***

Decimals are introduced for the first time in fourth grade. Students should have ample opportunities to explore and reason about the idea that a number can be represented as both a fraction and a decimal.

Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say $\frac{32}{100}$ as thirty-two hundredths and rewrite this as 0.32 or represent it on a place value model as shown below.

| **Hundreds** | **Tens** | **Ones** | • | **Tenths** | **Hundredths** |
|---|---|---|---|---|---|
| | | | • | 3 | 2 |

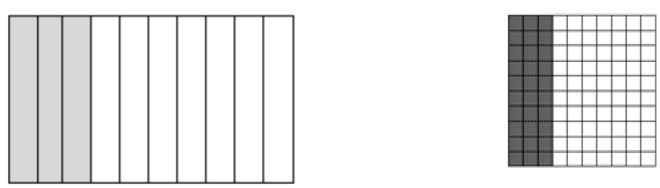
Students use the representations explored in MGSE4.NF.5 to understand $\frac{32}{100}$ can be expanded to $\frac{3}{10}$ and $\frac{2}{100}$. Students represent values such as 0.32 or $\frac{32}{100}$ on a number line. $\frac{32}{100}$ is more than $\frac{30}{100}$ (or $\frac{3}{10}$) and less than $\frac{40}{100}$ (or $\frac{4}{10}$). It is closer to $\frac{30}{100}$ so it would be placed on the number line near that value.




**MGSE4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.**

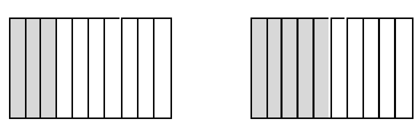
Students should reason that comparisons are only valid when they refer to the same whole. Visual models include area models, decimal grids, decimal circles, number lines, and meter sticks.

Students build area and other models to compare decimals. Through these experiences and their work with fraction models, they build the understanding that comparisons between decimals or fractions are only valid when the whole is the same for both cases. Each of the models below shows $\frac{3}{10}$ but the whole on the left is much bigger than the whole on the right. They are both $\frac{3}{10}$ but the model on the left is a much greater quantity than the model on the right because three tenths takes up more space in the model on the left.



When the wholes are the same, the decimals or fractions can be compared.

> Example:
> Draw a model to show that 0.3 < 0.5. (*Students would sketch two models of approximately the same size to show the area that represents three-tenths is smaller than the area that represents five-tenths.*)



### Common Misconceptions

Students treat decimals as whole numbers when making comparison of two decimals. They think the longer the number, the greater the value. For example, they think that .03 is greater than 0.3 because .03 has more digits after the decimal point than 0.3. Students need to know that they must look at the place value of the digits when comparing two decimals.

## Measurement and Data

**CLUSTER #1: SOLVE PROBLEMS INVOLVING MEASUREMENT AND CONVERSION OF MEASUREMENTS FROM A LARGER UNIT TO A SMALLER UNIT.**

> *Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are:* ***measure, metric, customary, convert/conversion, relative size, liquid volume, mass, length, distance, kilometer (km), meter (m), centimeter (cm), kilogram (kg), gram (g), liter (L), milliliter (mL), inch (in), foot (ft), yard (yd), mile (mi), ounce (oz), pound (lb), cup (c), pint (pt), quart (qt), gallon (gal), time, hour, minute, second, equivalent, operations, add, subtract, multiply, divide, fractions, decimals, area, perimeter.***

**MGSE4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb., oz.; l, ml; hr., min, sec.**

a. Understand the relationship between gallons, cups, quarts, and pints.
b. Express larger units in terms of smaller units within the same measurement system.
c. Record measurement equivalents in a two-column table.

The units of measure that have not been addressed in prior years are cups, pints, quarts, gallons, pounds, ounces, kilometers, milliliters, and seconds. Students' prior experiences were limited to measuring length, mass (metric and customary systems), liquid volume (metric only), and elapsed time. Students did not convert measurements. Students need ample opportunities to become familiar with these new units of measure and explore the patterns and relationships in the conversion tables that they create.

Students use a two-column chart to convert from larger to smaller units and record equivalent measurements. They make statements such as, if one foot is 12 inches, then 3 feet must be 36 inches because there are 3 groups of 12.

Example:
Customary length conversion table

| Yards | Feet |
|---|---|
| 1 | 3 |
| 2 | 6 |
| 3 | 9 |
| $n$ | $n \times 3$ |

Foundational understandings to help with measure concepts:

- Understand that larger units can be subdivided into equivalent units (partition).
- Understand that the same unit can be repeated to determine the measure (iteration).
- Understand the relationship between the size of a unit and the number of units needed (compensatory principle[1]).

**MGSE4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.**

This standard includes multi-step word problems related to expressing measurements from a larger unit in terms of a smaller unit (e.g., feet to inches, meters to centimeter, dollars to cents). Students should have ample opportunities to use number line diagrams to solve word problems.

Example:
Charlie and 10 friends are planning for a pizza party. They purchased 3 quarts of milk. If each glass holds 8 fl.oz., will everyone get at least one glass of milk?

Possible solution: Charlie plus 10 friends = 11 total people
11 people × 8 fluid ounces (glass of milk) = 88 total fluid ounces
1 quart = 2 pints = 4 cups = 32 fluid ounces
Therefore if 1 quart = 2 pints = 4 cups = 32 ounces, then
2 quarts = 4 pints = 8 cups = 64 ounces
3 quarts = 6 pints = 12 cups = 96 ounces

If Charlie purchased 3 quarts (6 pints) of milk, there would be enough for everyone at his party to have at least one glass of milk. If each person drank 1 glass, then he would have one 8 fl. oz. glass, or 1 cup of milk left over.

Additional examples with various operations:

- Division/fractions: Susan has 2 feet of ribbon. She wants to give the ribbon to her 3 best friends, so each friend gets the same amount. How much ribbon will each friend get?

  Students may record their solutions using fractions or inches. The answer would be $\frac{2}{3}$ of a foot or 8 inches. Students are able to express the answer in inches because they understand that $\frac{1}{3}$ of a foot is 4 inches and $\frac{2}{3}$ of a foot is 2 groups of $\frac{1}{3}$.

[4] The compensatory principle states that the smaller the unit used to measure the distance, the more of those units that will be needed. For example, measuring a distance in centimeters will result in a larger number of that unit than measuring the distance in meters.

- Addition: Mason ran for an hour and 15 minutes on Monday, 25 minutes on Tuesday, and 40 minutes on Wednesday. What was the total number of minutes Mason ran?

  Students can add the times to find the total number of minutes Mason ran. 40 minutes plus another 25 minutes would be 65 minutes, or an hour and 5 minutes. Then, an hour and five minutes can be added to an hour and 15 minutes to see that Mason ran 2 hours and 20 minutes in all.

- Subtraction: A pound of apples costs $1.20. Rachel bought a pound and a half of apples. If she gave the clerk a $5.00 bill, how much change will she get back?

  Possible student solution: If Rachel bought a pound and a half of apples, she paid $1.20 for the first pound and then 60¢ for the other half a pound, since half of $1.20 is 60¢. When I add $1.20 and 60¢, I get a total of $1.80 spent on the apples. If she gave the clerk a five-dollar bill, I can count up to find out how much change she received.

  $1.80 + 20¢ = $2.00
  $2.00 + 3.00 = $5.00
  $3.00 + 20¢ = $3.20

  So, Rachel got $3.20 back in change.

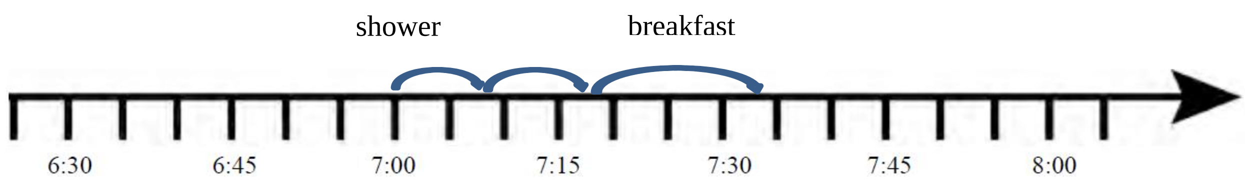
- Multiplication: Mario and his 2 brothers are selling lemonade. Mario brought one and a half liters, Javier brought 2 liters, and Ernesto brought 450 milliliters. How many total milliliters of lemonade did the boys have?

  First, students must add all the liquid volumes together. 1 ½ liters + 2 liters = 3 ½ liters
  3 ½ liters is the same as 3,500 mL because 1L is equal to 1,000 mL, so 3 liters is 3,000 mL and half of a liter is 500 mL, which equals 3,500 mL. When adding 3,500 mL to 450 mL, the total milliliters of lemonade is 3,950 mL.

Number line diagrams that feature a measurement scale can represent measurement quantities. Examples include: ruler, diagram marking off distance along a road with cities at various points, a timetable showing hours throughout the day, or a liquid volume measure on the side of a container.

Example:
At 7:00 a.m. Candace wakes up to go to school. It takes her 8 minutes to shower, 9 minutes to get dressed and 17 minutes to eat breakfast. How many minutes does she have until the bus comes at 8:00 a.m.? Use the number line to help solve the problem.

get
dressed



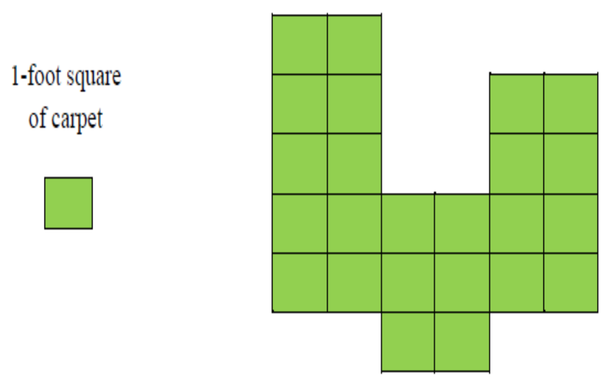

Candace is finished at 7:34. If the bus comes at 8:00, I can count on to from 7:34 to 8:00 to find how many minutes it takes for the bus to arrive. From 7:34 to 7:35 is one minute. From 7:35 to 7:40 is 5 minutes and from 7:40 to 8:00 is 20 minutes. 1 minute + 5 minutes + 20 minutes = 26 minutes until the bus arrives.

**MGSE4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems.** ***For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.***

Students developed understanding of area and perimeter in 3rd grade by using visual models.

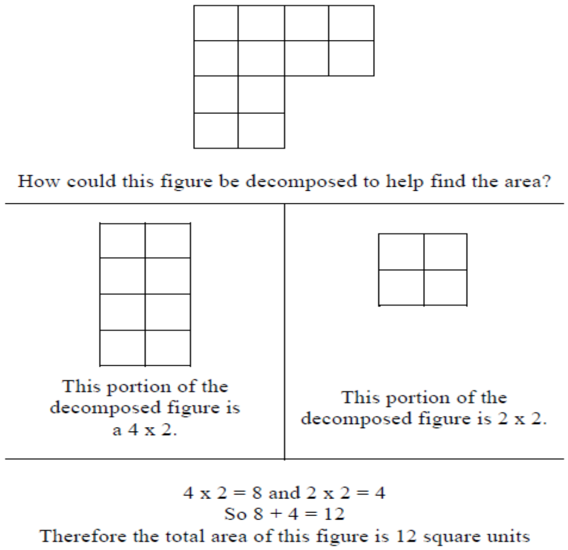
While students are expected to use formulas to calculate area and perimeter of rectangles, they need to understand and be able to communicate their understanding of why the formulas work. The formula for area is $l$ x $w$ and the answer will always be in square units. The formula for perimeter can be 2 $l$ + 2 $w$ or 2 ($l$ + $w$) and the answer will be in linear units. This standard calls for students to generalize their understanding of area and perimeter by connecting the concepts to mathematical formulas. These formulas should be developed through experience not just memorization.

Example:
Mr. Rutherford is covering the miniature golf course with an artificial grass. How many 1-foot squares of carpet will he need to cover the entire course?



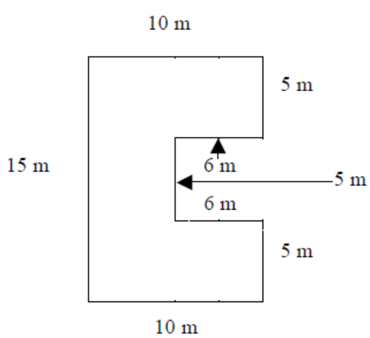

**MGSE4.MD.8 Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.**

This standard uses the word rectilinear. A rectilinear figure is a polygon that has all right angles. Students can decompose a rectilinear figure into different rectangles. They find the area of the rectilinear figure by adding the areas of each of the decomposed rectangles together.




Example:

A storage shed is pictured below. What is the total area? How could the figure be decomposed to help find the area?




I can divide this figure into three smaller rectangles.

First: 10 m x 5 m = 50 $m^2$

Second: 5 m x 4 m = 20 $m^2$

Third: 10 m x 5 m = 50 $m^2$

50 $m^2$ + 50 $m^2$ + 20 $m^2$ = 120 $m^2$

**Common Misconceptions**

Students believe that larger units will give the greater measure. Students should be given multiple opportunities to measure the same object with different measuring units. For example, have the students measure the length of a room with one-inch tiles, with one-foot rulers, and with yard sticks. Students should notice that it takes fewer yard sticks to measure the room than rulers or tiles.

## <u>CLUSTER #2</u>: REPRESENT AND INTERPRET DATA.

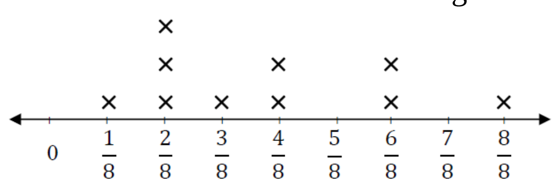
*Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are:* ***data, line plot, length, fractions.***

**MGSE4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Solve problems involving addition and subtraction of fractions with common denominators by using information presented in line plots.** ***For example, from a line plot, find and interpret the difference in length between the longest and shortest specimens in an insect collection.***

This standard provides a context for students to work with fractions by measuring objects to an eighth of an inch. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot.

Example:

Students measured objects in their desk to the nearest $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{1}{8}$ inch. They displayed their data collected on a line plot. How many objects measured $\frac{1}{4}$ inch? $\frac{1}{2}$ inch? If you put all the objects together end to end what would be the total length of **all** the objects?




Possible student solution: Since $\frac{2}{8} = \frac{1}{4}$ there are three objects that measured $\frac{1}{4}$ of an inch. Since $\frac{4}{8}$ is equal to $\frac{1}{2}$, there are 2 objects that have a length of $\frac{1}{2}$ of an inch. The total length of all the objects is $\frac{8}{8} + \frac{4}{8} + \frac{4}{8} = \frac{16}{8}$ which is 2 inches. Then, add $\frac{6}{8} + \frac{1}{8} + \frac{3}{8} = \frac{10}{8}$ which is $1\frac{2}{8}$ inches. Then, add $\frac{2}{8} + \frac{2}{8} + \frac{2}{8} + \frac{6}{8} = \frac{12}{8}$ which is $1\ \frac{4}{8}$ inches.

$2 + 1\frac{2}{8} + 1\ \frac{4}{8}$ is $4\ \ \frac{6}{8}$ inches in total length.

### Common Misconceptions

Students use whole-number names when counting fractional parts on a number line. The fraction name should be used instead. For example, if two-fourths is represented on the line plot three times, then there would be six-fourths.

## <u>CLUSTER</u> <u>#3:</u> GEOMETRIC MEASUREMENT – UNDERSTAND CONCEPTS OF ANGLE AND MEASURE ANGLES.

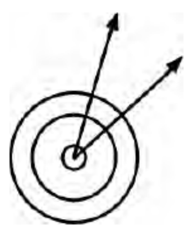
> *Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are:* ***measure, point, end point, geometric shapes, ray, angle, circle, fraction, intersect, one-degree angle, protractor, decomposed, addition, subtraction, unknown.***

**MGSE4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:**

This standard brings up a connection between angles and circular measurement (360 degrees).

**a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.**

The diagram below will help students understand that an angle measurement is not related to an area since the area between the 2 rays is different for both circles, yet the angle measure is the same.



**b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.**

This standard calls for students to explore an angle as a series of "one-degree turns."

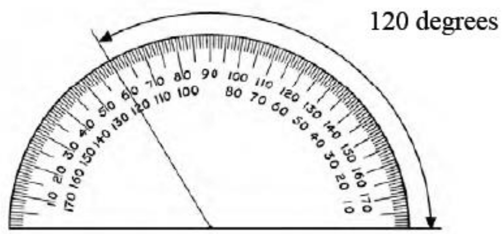
Example:

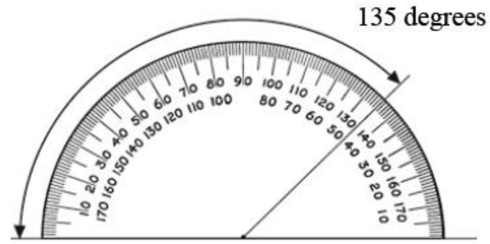
> A water sprinkler rotates one-degree at each interval. If the sprinkler rotates a total of 100 degrees, how many one-degree turns has the sprinkler made?

**MGSE4.MD.6 Measure angles in whole number degrees using a protractor. Sketch angles of specified measure.**

Before students begin measuring angles with protractors, they need to have some experiences with benchmark angles. They transfer their understanding that a 360º rotation about a point makes a complete circle to recognize and sketch angles that measure approximately 90º and 180º. They extend this understanding and recognize and sketch angles that measure approximately 45º and 30º. They use appropriate terminology (acute, right, and obtuse) to describe angles and rays (perpendicular).

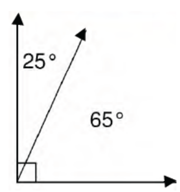
Students should measure angles and sketch angles.




**MGSE4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol or letter for the unknown angle measure.**

This standard addresses the idea of decomposing (breaking apart) an angle into smaller parts.



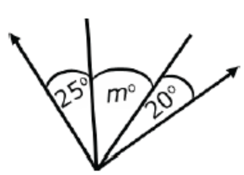

Example:

A lawn water sprinkler rotates 65 degrees and then pauses. It then rotates an additional 25 degrees. What is the total degree of the water sprinkler rotation? How many times will it do this as it makes a 360 degree, or full, rotation?

If the water sprinkler rotates a total of 25 degrees then pauses, how many 25-degree cycles will it go through for the rotation to reach at least 90 degrees?

Example:

If the two rays are perpendicular, what is the value of *m*?




Example:

> Joey knows that when a clock's hands are exactly on 12 and 1, the angle formed by the clock's hands measures 30º. What is the measure of the angle formed when a clock's hands are exactly on the 12 and 4?

**Common Misconceptions**

Students are confused as to which number to use when determining the measure of an angle using a protractor because most protractors have a double set of numbers. Students should decide first if the angle appears to be an angle that is less than the measure of a right angle (90°) or greater than the measure of a right angle (90°). If the angle appears to be less than 90°, it is an acute angle and its measure ranges from 0° to 89°. If the angle appears to be an angle that is greater than 90°, it is an obtuse angle and its measures range from 91° to 179°. Ask questions about the appearance of the angle to help students in deciding which number to use.

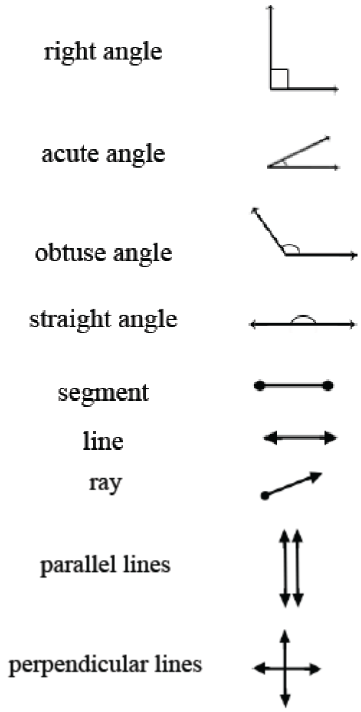
## Geometry

### CLUSTER #1: DRAW AND IDENTIFY LINES AND ANGLES, AND CLASSIFY SHAPES BY PROPERTIES OF THEIR LINES AND ANGLES.

> *Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are**: classify shapes/figures, (properties)-rules about how numbers work, point, line, line segment, ray, angle, vertex/vertices, right angle, acute, obtuse, perpendicular, parallel, right triangle, isosceles triangle, equilateral triangle, scalene triangle, line of symmetry, symmetric figures, two dimensional. From previous grades: polygon, rhombus/rhombi, rectangle, square, triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle, cone, cylinder, sphere.***

**MGSE4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.**

This standard asks students to draw two-dimensional geometric objects and to also identify them in two-dimensional figures. This is the first time that students are exposed to rays, angles, and perpendicular and parallel lines. Examples of points, line segments, lines, angles, parallelism, and perpendicularity can be seen daily. Students do not easily identify lines and rays because they are more abstract.




Examples:
Draw two different types of quadrilaterals that have two pairs of parallel sides.
Is it possible to have an acute right triangle? Justify your reasoning using pictures and words.

Example:
How many acute, obtuse, and right angles are in this shape?

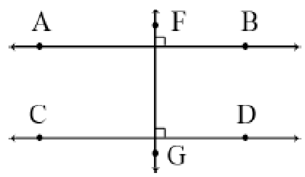
Draw and list the properties of a parallelogram. Draw and list the properties of a rectangle. How are your drawings and lists alike? How are they different? Be ready to share your thinking with the class.

**MGSE4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.**

Two-dimensional figures may be classified using different characteristics such as, parallel or perpendicular lines or by angle measurement.

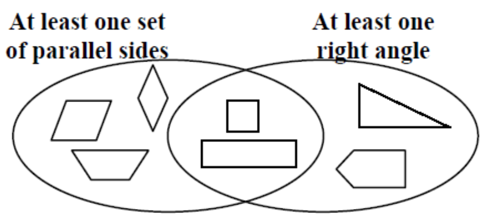
**Parallel or Perpendicular Lines:**
Students should become familiar with the concept of parallel and perpendicular lines. Two lines are parallel if they never intersect and are always equidistant. Two lines are perpendicular if they intersect at a point that creates four right angles (90º). Students may use transparencies with lines to arrange two lines in different ways to determine that the 2 lines might intersect in one point or may never intersect. Further investigations may be initiated using geometry software. These types of explorations may lead to a discussion on angles.

Parallel and perpendicular lines are shown below:




This standard calls for students to sort objects based on parallelism, perpendicularity, and angle types.

Example:

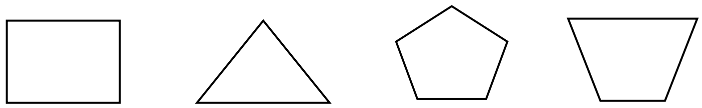



Do you agree with the label on each of the circles in the Venn diagram above? Describe why some shapes fall in the overlapping sections of the circles.
Example:
Draw and name a figure that has two parallel sides and exactly 2 right angles.

Example:
For each of the following, sketch an example if it is possible. If it is impossible, say so, and explain why or show a counterexample.

- A parallelogram with exactly one right angle. *(impossible)*
- An isosceles right triangle.
- A rectangle that is *not* a parallelogram. *(impossible)*
- Every square is a quadrilateral.
- Every trapezoid is a parallelogram. *(impossible)*

Example:
Identify which of these shapes have perpendicular or parallel sides and justify your selection.



A possible justification that students might give is: “The square has perpendicular sides because the sides meet at a corner, forming right angles.”

**Angle Measurement:**

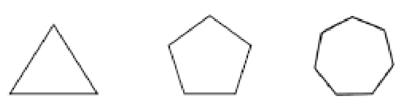
This expectation is closely connected to MGSE4.MD.5, MGSE4.MD.6, and MGSE4.G.1. Students’ experiences with drawing and identifying right, acute, and obtuse angles support them in classifying two-dimensional figures based on specified angle measurements. They use the benchmark angles of 90°, 180°, and 360° to approximate the measurement of angles.

Right triangles can be a category for classification. A right triangle has one right angle. There are different types of right triangles. An isosceles right triangle has two or more congruent sides and a scalene right triangle has no congruent sides.

**MGSE4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.**

Students need experiences with figures which are symmetrical and non-symmetrical. Figures include both regular and non-regular polygons. Folding cut-out figures will help students determine whether a figure has one or more lines of symmetry. This standard only includes line symmetry, not rotational symmetry.

Example:
For each figure at the right, draw all of the lines of symmetry. What pattern do you notice? How many lines of symmetry do you think there would be for regular polygons with 9 and 11 sides? Sketch each figure and check your predictions.



Polygons with an odd number of sides have lines of symmetry that go from a midpoint of a side through a vertex.

**Common Misconceptions**
Students believe a wide angle with short sides may seem to measure less than a narrow angle with long sides. Students can compare two angles by tracing one and placing it over the other. Students will then realize that the length of the sides does not determine whether one angle measures greater than or less than another angle. The measure of the angle does not change.

**Table 2** Common Multiplication and Division Situations

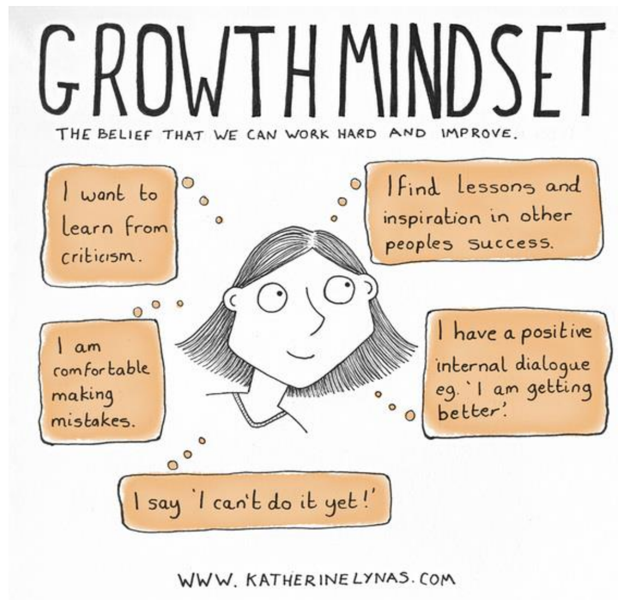
| | **Unknown Product** | **Group Size Unknown**<br><br>(“How many in each group? Division) | **Number of Groups Unknown**<br><br>(“How many groups?” Division) |
|---|---|---|---|
| | **$3 \times 6 = ?$** | **$3 \times ? – 18$, and $18 \div 3 = ?$** | **$? \times 6 = 18$, and $18 \div 6 = ?$** |
| **Equal Groups** | There are 3 bags with 6 plums in each bag. How many plums are there in all?<br><br>*Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether? | If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?<br><br>*Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be? | If 18 plums are to be packed 6 to a bag, then how many bags are needed?<br><br>*Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have? |
| **Arrays[2],**<br><br>**Area[3]** | There are 3 rows of apples with 6 apples in each row. How many apples are there?<br><br>*Area example*. What is the area of a 3 cm by 6 cm rectangle? | If 18 apples are arranged into 3 equal rows, how many apples will be in each row?<br><br>*Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it? | If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?<br><br>*Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it? |
| **Compare** | A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?<br><br>*Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long? | A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?<br><br>*Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first? | A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat?<br><br>*Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first? |
| **General** | **$a \times b = ?$** | **$a \times ? = p$, and $p \div a = ?$** | **$? \times b = p$, and $p \div b = ?$** |

[2] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

## MINDSET AND MATHEMATICS

**Growth mindset was pioneered by Carol Dweck**, Lewis and Virginia Eaton Professor of Psychology at Stanford University. She and her colleagues were the first to identify a link between growth mindset and achievement. They found that students who believed that their ability and intelligence could grow and change, otherwise known as growth mindset, outperformed those who thought that their ability and intelligence were fixed. Additionally, students who were taught that they could grow their intelligence actually did better over time. Dweck's research showed that an increased focus on the process of learning, rather than the outcome, helped increase a student's growth mindset and ability.
(from WITH+MATH=I CAN)


Jo Boaler, Professor of Mathematics Education at the Stanford Graduate School of Education, and author of *Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages, and Innovative Teaching*, was one of the first to apply growth mindset to math achievement.
You can learn how to use the power of growth mindset for yourself and your students here:

https://www.amazon.com/gp/withmathican

https://www.mindsetkit.org/topics/about-growth-mindset

https://www.youcubed.org/

*Growth and Fixed Mindset images courtesy of Katherine Lynas (katherinelynas.com). Thank you, Katherine!*

## VERTICAL UNDERSTANDING OF THE MATHEMATICS LEARNING TRAJECTORY

Why does it matter if you know what happens in mathematics in the grades before and after the one you teach? Isn't it enough just to know and understand the expectations for your grade?

There are many reasons to devote a bit of your time to the progression of standards.

You will:

- Deepen your understanding of how development of algebraic thinking has proven to be a critical element of student mathematics success as they transition from elementary to middle school. Elementary and middle school teachers must understand how algebraic thinking develops prior to their grade, in their grade, and beyond their grade in order to support student algebraic thinking
- Know what to expect when students show up in your grade because you know what they should understand from the years before
- Understand how conceptual understanding develops, making it easier to help students who have missing bits and pieces
- Be able to help students to see the connections between ideas in mathematics in your grade and beyond, helping them to connect to what they already know and what is to come
- Assess understanding more completely, and develop better assessments
- Know what the teachers in the grades to come expect your students to know and understand
- Plan more effectively with same-grade and other-grade colleagues
- Deepen your understanding of the mathematics of your grade

We aren't asking you to take a month off to study up, just asking that you reference the following resources when you want to deepen your understanding of where students are in their mathematics learning, understand why they are learning what they are learning in your grade, and understand the mathematical ideas and connections within your grade and beyond.

**Resources:**

**The Coherence Map**: http://achievethecore.org/page/1118/coherence-map This resource diagrams the connections between standards, provides explanations of standards, provides example tasks for many standards, and links to the progressions document when further detail is required.

**A visual learning trajectory of:**

Addition and Subtraction - http://gfletchy.com/2016/03/04/the-progression-of-addition-and-subtraction/
Multiplication - http://gfletchy.com/2015/12/18/the-progression-of-multiplication/
Division - http://gfletchy.com/2016/01/31/the-progression-of-division/
Fractions - https://gfletchy.com/2016/12/08/the-progression-of-fractions/
(Many thanks to Graham Fletcher, the genius behind these videos)

**The Mathematics Progression Documents:**

http://math.arizona.edu/~ime/progressions/

**Learning Trajectories in Mathematics:**

https://www.mobt3ath.com/uplode/book/book-24726.pdf

https://repository.upenn.edu/cgi/viewcontent.cgi?article=1019&context=cpre_researchreports

## RESEARCH OF INTEREST TO MATHEMATICS TEACHERS

Social Emotional Learning and Math- http://curry.virginia.edu/uploads/resourceLibrary/Teachers_support_for_SEL_contributes_to_improved_math_teaching_and_learning,_et_al.pdf.

Why how you teach math is important- https://www.youcubed.org/

## GloSS, IKAN and the overall Numeracy Project

Information on the Numeracy Project, which includes GloSS and IKAN can be found here: Georgia Numeracy Project Overview.

The GloSS and IKAN professional learning video found here: https://www.georgiastandards.org/Georgia-Standards/Pages/FOA/Foundations-of-Algebra-Day-1.aspx provides an in-depth look at the GloSS and IKAN. While it was created for teachers of Foundations of Algebra, the information is important for teachers of grades K- 12.

The GloSS and IKAN prezi found on georgiastandards.org, here: https://www.georgiastandards.org/Georgia-Standards/Pages/Global-Strategy-Stage-GloSS-and-Individual-Knowledge-Assessment-of-Number-IKAN.aspx

## FLUENCY

**Fluency**: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit. Fluency, however, is based on a deep understanding of quantity and number.

**Deep Understanding**: Teachers teach more than simply “how to get the answer” and instead support students’ ability to access concepts from a number of perspectives. Therefore, students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

**Memorization**: Memorization leads to the rapid recall of arithmetic facts or mathematical procedures without the necessity of understanding. This type of learning is not the goal of numeracy. Memorization is often confused with fluency and automaticity. Fluency implies a much richer kind of mathematical knowledge and experience. Automaticity is based on strategy development and the ability to become automatic with part-whole computation strategies developed.

**Number Sense:** Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

**Fluent students**:

- flexibly use a combination of deep understanding, number sense, and automaticity.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.

For more about fluency, see: http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and: https://bhi61nm2cr3mkdgk1dtaov18-wpengine.netdna-ssl.com/wp-content/uploads/nctm-timed-tests.pdf

## ARC OF LESSON (OPENING, WORK SESSION, CLOSING)

"When classrooms are workshops-when learners are inquiring, investigating, and constructing-there is already a feeling of community. In workshops learners talk to one another, ask one another questions, collaborate, prove, and communicate their thinking to one another. The heart of math workshop is this: investigations and inquiries are ongoing, and teachers try to find situations and structure contexts that will enable children to mathematize their lives- that will move the community toward the horizon. Children have the opportunity to explore, to pursue inquiries, and to model and solve problems on their own creative ways. Searching for patterns, raising questions, and constructing one's own models, ideas, and strategies are the primary activities of math workshop. The classroom becomes a community of learners engaged in activity, discourse, and reflection." *Young Mathematicians at Work- Constructing Addition and Subtraction,* by Catherine Twomey Fosnot and Maarten Dolk.

"Students must believe that the teacher does not have a predetermined method for solving the problem. If they suspect otherwise, there is no reason for them to take risks with their own ideas and methods." *Teaching Student-Centered Mathematics, K-3* by John Van de Walle and Lou Ann Lovin.

**Opening: Set the stage**
Get students mentally ready to work on the task.
Clarify expectations for products/behavior.
How?

- Begin with a simpler version of the task to be presented.
- Solve problem strings related to the mathematical idea(s) being investigated.
- Leap headlong into the task and begin by brainstorming strategies for approaching the task.
- Estimate the size of the solution and reason about the estimate.

Make sure everyone understands the task before beginning. Have students restate the task in their own words. Every task should require more of the students than just the answer.

**Work session: Give them a chance to grapple**

| Students should… | The teacher should… |
|---|---|
| • Grapple with the mathematics through sense-making, discussion<br>• Concretize their mathematical ideas and the situation<br>• Record thinking in journals | • Let go.<br>• Listen.<br>• Respect student thinking.<br>• Encourage testing of ideas.<br>• Ask questions to clarify or provoke thinking.<br>• Provide gentle hints.<br>• Observe and assess. |

**Closing: Best Learning Happens Here**

| Students should… | The teacher should… |
|---|---|
| • Share answers.<br>• Clarify understanding.<br>• Explain thinking.<br>• Justify thinking.<br>• Question each other. | • Listen attentively to all ideas.<br>• Ask for explanations.<br>• Offer comments such as, "Please tell me how you figured that out." "I wonder what would happen if you tried…"Ask questions to clarify or provoke thinking.<br>• Create anchor charts. |

**For more information, read Van de Walle K-3, Chapter 1.**

## UNPACKING TASKS (BREAKDOWN OF A TASK)

How do I go about tackling a task or a unit?

1. Read the unit in its entirety. Discuss it with your grade level colleagues. Which parts do you feel comfortable with? Which make you wonder? Brainstorm ways to implement the tasks. Collaboratively complete the culminating task with your grade level colleagues. As students work through the tasks, you will be able to facilitate their learning with this end in mind. The structure of the units/tasks is similar task to task and grade to grade. This structure allows you to converse in a vertical manner with your colleagues, school wide. There is a great deal of mathematical knowledge and teaching support within each grade level guide, unit, and task.

2. Read the first task your students will be engaged in. Discuss it with your grade level colleagues. Which parts do you feel comfortable with? Which make you wonder? Brainstorm ways to implement the tasks.

3. If not already established, use the first few weeks of school to establish routines and rituals, and to assess student mathematical understanding. You might use some of the tasks found in the unit, or in some of the following resources as beginning tasks/centers/math tubs which serve the dual purpose of allowing you to observe and assess.

   Additional Resources:
   Math Their Way: http://www.center.edu/MathTheirWay.shtml
   NZMaths- http://www.nzmaths.co.nz/numeracy-development-projects-books?parent_node=
   K-5 Math Teaching Resources- http://www.k-5mathteachingresources.com/index.html (this is a for-profit site with several free resources)
   Math Solutions- http://www.mathsolutions.com/index.cfm?page=wp9&crid=56

4. Points to remember:
   - Each task begins with a list of the standards specifically addressed in that task, however, *that does not mean that these are the only standards addressed in the task*. Remember, standards build on one another, and mathematical ideas are connected.
   - Tasks are made to be modified to match your learner’s needs. If the names need changing, change them. If the specified materials are not available, use what is available. If a task does not go where the students need to go, modify the task or use a different resource.
   - The units are not intended to be all encompassing. Each teacher and team will make the units their own and add to them to meet the needs of the learners.

## ROUTINES AND RITUALS

### Teaching Math in Context and Through Problems

"By the time they begin school, most children have already developed a sophisticated, informal understanding of basic mathematical concepts and problem-solving strategies. Too often, however, the mathematics instruction we impose upon them in the classroom fails to connect with this informal knowledge" (Carpenter et al., 1999). The 8 Standards of Mathematical Practices (SMP) should be at the forefront of every mathematics lessons and be the driving factor of HOW students learn.

One way to help ensure that students are engaged in the 8 SMPs is to construct lessons built on context or through story problems. It is important for you to understand the difference between story problems and context problems. Fosnot and Dolk (2001) point out that in story problems children tend to focus on getting the answer, probably in a way that the teacher wants. "Context problems, on the other hand, are connected as closely as possible to children's lives, rather than to 'school mathematics'. They are designed to anticipate and develop children's mathematical modeling of the real world."

Traditionally, mathematics instruction has been centered around many problems in a single math lesson, focusing on rote procedures and algorithms which do not promote conceptual understanding. Teaching through word problems and in context is difficult; however, there are excellent reasons for making the effort.

- Problem solving focuses students' attention on ideas and sense making.
- Problem solving develops the belief in students that they are capable of doing the mathematics and that mathematics makes sense.
- Problem solving provides ongoing assessment data.
- Problem solving is an excellent method for attending to a breadth of abilities.
- Problem solving engages students so that there are few discipline problems.
- Problem solving develops "mathematical power."
  (Van de Walle 3-5 pg. 15 and 16)

A *problem* is defined as any task or activity for which the students have no prescribed or memorized rules or methods, nor is there a perception by students that there is a specific correct solution method. A problem for learning mathematics also has these features:

- *The problem must begin where the students are, which makes it accessible to all learners.*
- *The problematic or engaging aspect of the problem must be due to the mathematics that the students are to learn.*
- *The problem must require justifications and explanations for answers and methods.*

It is important to understand that mathematics is to be taught *through* problem solving. That is, problem-based tasks or activities are the vehicle through which the standards are taught. Student learning is an outcome of the problem-solving process and the result of teaching within context and through the Standards for Mathematical Practice. (Van de Walle and Lovin, *Teaching Student-Centered Mathematics: 3-5* pg. 11 and 12)

**Use of Manipulatives**
Used correctly, manipulatives can be a positive factor in children's learning. It is important that you have a good perspective on how manipulatives can help or fail to help children construct ideas." (Van de Walle and Lovin, Teaching Student-Centered Mathematics: 3-5 pg. 6)

When a new model with new use is introduced into the classroom, it is generally a good idea to explain how the model is used and perhaps conduct a simple activity that illustrates this use.

Once you are comfortable that the models have been explained, you should not force their use on students. Rather, students should feel free to select and use models that make sense to them. In most instances, not using a model at all should also be an option. The choice a student makes can provide you with valuable information about the level of sophistication of the student's reasoning.

Whereas the free choice of models should generally be the norm in the classroom, you can often ask students to model to show their thinking. This will help you find out about a child's understanding of the idea and also his or her understanding of the models that have been used in the classroom.

The following are simple rules of thumb for using models:

- Introduce new models by showing how they can represent the ideas for which they are intended.
- Allow students (in most instances) to select freely from available models to use in solving problems.
- Encourage the use of a model when you believe it would be helpful to a student having difficulty. (Van de Walle and Lovin, Teaching Student-Centered Mathematics 3-5, pg. 9)
- Modeling also includes the use of mathematical symbols to represent/model the concrete mathematical idea/thought process/situation. This is a very important, yet often neglected step along the way. Modeling can be concrete, representational, and abstract. Each type of model is important to student understanding. Modeling also means to "mathematize" a situation or problem, to take a situation which might, at first glance, not seem mathematical, and view it through the lens of mathematics. For example, students notice that the cafeteria is always out of their favorite flavor of ice cream on ice cream days. They decide to survey their schoolmates to determine which flavors are most popular and share their data with the cafeteria manager so that ice cream orders reflect their findings. The problem: Running out of ice cream flavors. The solution: Use math to change the flavor amounts ordered.

**Use of Strategies and Effective Questioning**
Teachers ask questions all the time. They serve a wide variety of purposes: to keep learners engaged during an explanation; to assess their understanding; to deepen their thinking or focus their attention on something. This process is often semi-automatic. Unfortunately, there are many common pitfalls. These include:

- asking questions with no apparent purpose
- asking too many closed questions

- asking several questions all at once
- poor sequencing of questions
- asking rhetorical questions
- asking ‘Guess what is in my head’ questions
- focusing on just a small number of learners
- ignoring incorrect answers
- not taking answers seriously

In contrast, the research shows that effective questioning has the following characteristics:
- Questions are planned, well ramped in difficulty.
- Open questions predominate.
- A climate is created where learners feel safe.
- A ‘no hands’ approach is used, for example when all learners answer at once using mini-whiteboards, or when the teacher chooses who answers.
- Probing follow-up questions are prepared.
- There is a sufficient ‘wait time’ between asking and answering a question.
- Learners are encouraged to collaborate before answering.
- Learners are encouraged to ask their own questions.

**Number Lines**
The use of number lines in elementary mathematics is crucial in students’ development of number and mathematical proficiency. While the GSE explicitly state use number lines in grades 2-5, number lines should be used in all grade levels and in multiple settings.

According to John Van de Walle,

> A number line is also a worthwhile model but can initially present conceptual difficulties for children below second grade and students with disabilities. (National Research Council Committee, 2009) This is partially due to their difficulty in seeing the unit, which is a challenge when it appears in a continuous line. A number line is also a shift from counting a number of individual objects in a collection to continuous length units. There are, however, ways to introduce and model number lines that support young learners as they learn this representation. Familiarity with a number line is essential because third grade students will use number lines to locate fractions and add and subtract time intervals, fourth graders will locate decimals and use them for measurement, and fifth graders will use perpendicular number lines in coordinate grids (CCSSO, 2010).
>
> A number line measures distance from zero the same way a ruler does. If you don’t actually teach the use of the number line through emphasis on the unit (length), students may focus on the hash marks or numerals instead of the spaces (a misunderstanding that becomes apparent when their answers are consistently off by one). At first students can build a number path by using a given length, such as a set of Cuisenaire rods of the same color to make a straight line of multiple single units (Van de Walle and Lovin, *Teaching Student-Centered Mathematics: 3-5* pg. 106-107)

Open number lines are particularly useful for building students' number sense. They can also form the basis for discussions that require the precise use of vocabulary and quantities and are therefore a good way to engage students in the Standards for Mathematical Practice.

While the possibilities for integrating number lines into the mathematics classroom are endless, the following are some suggestions/ideas:

- On a bulletin board, attach a string which will function as an open number line. Each morning (or dedicated time for math routines) put a new number on each student's desk. Using some type of adhesive (thumb tack, tape, etc.), students will place the number in the appropriate location on the string. In the beginning of the year, provide students with numbers that are more familiar to them. As the year progresses, move through more complex problems such as skip counting, fractions, decimals, or other appropriate grade level problems. Through daily integration, the number line becomes part of the routine. Following the number placement, have a brief discussion/debriefing of the reasoning used by students to place the numbers.
- In the 3-Act tasks placed throughout the units, students will be provided opportunities to use an open number line to place estimates that are too low, too high, and just right as related to the posed problem. Similar opportunities can also be used as part of a daily routine.

**Math Maintenance Activities**

In addition to instruction centered on the current unit of study, the math instructional block should include time devoted to reviewing mathematics that have already been taught, previewing upcoming mathematics, and developing mental math and estimation skills. There is a saying that if you don't use it, you'll lose it. If students don't have opportunities to continuously apply and refine the math skills they have learned previously, then they may forget how to apply what they have learned. Unlike vocabulary words for literacy, math vocabulary words are not used much outside math class, so it becomes more important to use those words in discussions regularly. Math maintenance activities incorporate review and preview of math concepts and vocabulary and help students make connections across domains. It is recommended that 15 to 30 minutes of the math instructional block be used for these math maintenance activities each day. It is not necessary nor is it recommended that teachers do every activity every day. Teachers should strive for a balance of math maintenance activities so that over the course of a week, students are exposed to a variety of these activities. Math maintenance time may occur before or after instruction related to the current math unit, or it can occur at a different time during the day.

The goals of this maintenance time should include:

- Deepening number sense, including subitizing, flexible grouping of quantities, counting forward and backward using whole numbers, fractions, decimals, and skip counting starting at random numbers or fractional amounts
- Developing mental math skills by practicing flexible and efficient numerical thinking through the use of operations and the properties of operations

- Practicing estimation skills with quantities and measurements such as length, mass, and liquid volume, depending on grade level
- Practicing previously taught skills so that students deepen and refine their understanding
- Reviewing previously taught concepts that students struggled with as indicated on their assessments, including gaps in math concepts taught in previous grade levels
- Using a variety of math vocabulary terms, especially those that are used infrequently
- Practicing basic facts using strategies learned in previous grade levels or in previous units to develop or maintain fluency
- Previewing prerequisite skills for upcoming math units of study
- Participating in mathematical discussions with others that require students to construct viable arguments and critique the reasoning of others

To accomplish these goals, math maintenance activities can take many different forms. Some activities include:

- Number Corner or Calendar Time
- Number Talks
- Estimation Activities/Estimation 180
- Problem of the Day or Spiraled Review Problems

In addition, math discussions, math journals and math games are appropriate not only for the current unit of study, but also for maintaining math skills that were previously taught.

Although there are commercially available materials to use for math maintenance activities, there are also many excellent websites and internet resources that are free for classroom use. Here is a partial list of some recommended resources. A more detailed explanation of some of these components follows below.

| **Math Maintenance Activity** | **Possible Resources** |
|---|---|
| **Number Corner or Calendar Time** | • Every Day Counts Calendar Math from Houghton Mifflin Harcourt<br>• Number Corner from The Math Learning Center |
| **Number Talks** | • *Number Talks* by Sherry Parrish<br>• http://kentuckymathematics.org/number_talk_resources.php |
| **Estimation Activities/Estimation 180** | • http://www.estimation180.com/ |
| **Problem of the Day/Spiraled Review Problems** | • www.insidemathematics.org<br>• http://nzmaths.co.nz/teaching-material<br>• http://www.k-5mathteachingresources.com/<br>• *Children's Mathematics: Cognitively Guided Instruction* by Thomas Carpenter, et al.<br>• *Extending Children's Mathematics: Fractions and Decimals* by Epson and Levi |

**Number Corner/Calendar Time**

Number Corner/Calendar Time is an interactive routine centered on a monthly calendar in which students can use the date or day of the school year as a starting point for math discussions. Typically, there are date cards for each day of the month with repeating or growing patterns. The patterns can be numeric or geometric and can lead to rich discussions of math vocabulary and predictions of what the next date card will look like. The day of the school year can be used as a basis for discussions of patterns on the 100's chart, or the days can each be multiplied by 0.01 so that a decimal 100's chart is developed. Fractions can be created using today's date as a numerator, with a set denominator to use as a basis for discussions of equivalent fractions, fractions greater than one, and mixed numbers. Measurement can be discussed, using today's date as the basis for money, coins, number of inches, ounces, minutes, angles, etc. There are many possibilities for mathematizing every math domain by using the calendar as a generator of a changing daily number (date or day of school year).

A monthly calendar board or corner can be set up with chart paper or dry erase paper to record predictions, vocabulary, and other elements. A Smartboard could also be used to organize calendar elements. If calendar elements are hung on the wall, then students will be able to refer to those visual displays as reminders and reinforcement at any time. The following elements should be in place in order to foster deep discussions and meaningful student engagement during calendar time:

- A safe environment
- Math models and tools, such as a hundreds chart, number line, measurement tools, play money
- Opportunities to think first and then discuss
- Student interaction and discourse

## Number Talks

In order to be mathematically proficient, students must be able to compute accurately, efficiently, and flexibly. Daily classroom number talks provide a powerful avenue for developing "efficient, flexible, and accurate computation strategies that build upon the key foundational ideas of mathematics." (Parrish, 2010) Number talks involve classroom conversations and discussions centered upon purposefully planned computation problems.

In Sherry Parrish's book, Number Talks: Helping Children Build Mental Math and Computation Strategies, teachers will find a wealth of information about Number Talks, including:
- Key components of Number Talks
- Establishing procedures
- Setting expectations
- Designing purposeful Number Talks
- Developing specific strategies through Number Talks

There are four overarching goals upon which K-2 teachers should focus during Number Talks. These goals are:

1. Developing number sense

2. Developing fluency with small numbers
3. Subitizing
4. Making Tens

Number talks are a great way for students to use mental math to solve and explain a variety of math problems. A Number Talk is a short, ongoing daily routine that provides students with meaningful ongoing practice with computation. Number Talks should be structured as short sessions alongside (but not necessarily directly related to) the ongoing math curriculum. A great place to introduce a Number Talk is during Number Corner/Calendar Time. It is important to keep Number Talks short, as they **are not intended to replace current curriculum or take up the majority of the time spent on mathematics**. In fact, teachers only need to spend 5 to 15 minutes on Number Talks. Number Talks are most effective when done every day. The primary goal of Number Talks is computational fluency. Children develop computational fluency while thinking and reasoning like mathematicians. When they share their strategies with others, they learn to clarify and express their thinking, thereby developing mathematical language. This in turn serves them well when they are asked to express their mathematical processes in writing. In order for children to become computationally fluent, they need to know particular mathematical concepts that go beyond what is required to memorize basic facts or procedures.
Students will begin to understand major characteristics of numbers, such as:

- Numbers are composed of smaller numbers.
- Numbers can be taken apart and combined with other numbers to make new numbers.
- What we know about one number can help us figure out other numbers.
- What we know about parts of smaller numbers can help us with parts of larger numbers.
- Numbers are organized into groups of tens and ones (and hundreds, tens, and ones and so forth).
- What we know about numbers to 10 helps us with numbers to 100 and beyond.

All Number Talks follow a basic six-step format. The format is always the same, but the problems and models used will differ for each number talk.

1. **Teacher presents the problem.** Problems are presented in many different ways: as dot cards, ten frames, sticks of cubes, models shown on the overhead, a word problem or a numerical expression. Strategies are *not explicitly taught* to students, instead the problems presented lead to various strategies.
2. **Students figure out the answer.** Students are given time to figure out the answer. To make sure students have the time they need, the teacher asks them to give a “thumbs-up” when they have determined their answer. The thumbs up signal is unobtrusive- a message to the teacher, not the other students.
3. **Students share their answers**. Four or five students volunteer to share their answers and the teacher records them on the board.
4. **Students share their thinking.** Three or four students volunteer to share how they got their answers. (Occasionally, students are asked to share with the person(s) sitting next to them.) The teacher records the student's thinking.
5. **The class agrees on the "real" answer for the problem.** The answer that together the class determines is the right answer is presented as one would the results of an experiment. The answer a student comes up with initially is considered a conjecture. Models and/or the

logic of the explanation may help a student see where their thinking went wrong, may help them identify a step they left out, or clarify a point of confusion. There should be a sense of confirmation or clarity rather than a feeling that each problem is a test to see who is right and who is wrong. A student who is still unconvinced of an answer should be encouraged to keep thinking and to keep trying to understand. For some students, it may take one more experience for them to understand what is happening with the numbers and for others it may be out of reach for some time. The mantra should be, "If you are not sure or it doesn't make sense yet, keep thinking."

6. **The steps are repeated for additional problems**.

Similar to other procedures in your classroom, there are several elements that must be in place to ensure students get the most from their Number Talk experiences. These elements are:

1. A safe environment
2. Problems of various levels of difficulty that can be solved in a variety of ways
3. Concrete models
4. Opportunities to think first and then check
5. Interaction
6. Self-correction

For further details on implementing Number Talks, see:
https://www.georgiastandards.org/Georgia-Standards/Documents/GSE-Effective-Instructional-Practices-Guide.pdf

## Estimation 180

Estimation is a skill that has many applications, such as checking computation answers quickly. Engaging in regular estimation activities will develop students' reasoning skills, number sense, and increase their repertoire of flexible and efficient strategies. As students gain more experiences with estimation, their accuracy will improve.

According to John Van de Walle, there are three types of estimation that students should practice:

- Measurement estimation – determining an approximate measurement, such as weight, length, or capacity
- Quantity estimation – approximating the number of items in a collection
- Computational estimation – determining a number that is an approximation of a computation

One resource which provides contexts for all three types of estimation is Andrew Stadel's website, http://www.estimation180.com/. In his website, Mr. Stadel has posted daily estimation contexts. Here are his directions for using his website:

1. Click on a picture.
2. Read the question.
3. Look for context clues.

4. Make an estimate.
5. Tell us how confident you are.
6. Share your reasoning (what context clues did you use?).
7. See the answer.
8. See the estimates of others.

**The most important part** is step #6. After you make an estimate, feel free to give a brief description. It is so valuable to a classroom when students share their logic or use of context clues when formulating an estimate.

Andrew Stadel has collaborated with Michael Fenton to create a recording sheet for students to use with the estimation contexts on the website. The recording sheet can also be found at http://www.estimation180.com/. Here are his directions for the recording sheet:

| Day # | Description | ⬇ Too Low | Too High ⬆ | My Estimate | My Reasoning | Answer | Error | \|Error\| as % |
|---|---|---|---|---|---|---|---|---|
| Ex. A | Tyler's age (months) | 24 | 36 | 30 | He looks a little older than my cousin (who is 2) | 26 | + − 4 | 4/26 ≈ 15% |
| Ex. B | *Bohemian Rhapsody* | 4:00 | 5:00 | 4:30 | 10% of song = 30 sec 300 sec total = 5 min | 5:56 | + − 86 | 86/356 ≈ 24% |
| | | | | | | | + − | |

***Column use descriptions from Andrew Stadel:***

**Day #**
In Estimation 180's first year, I was just trying to keep up with creating these estimation challenges in time for use in my own classroom. There really wasn't a scope and sequence involved. That said, now that there are over 160 estimation challenges available, teachers and students can use them at any time throughout the school year and without completing them in sequential order. Therefore, use the *Day #* column simply to number your daily challenges according to the site. Tell your students or write it up on the board that you're doing the challenge from Day 135 even though you might be on the fifth day of school.

**Description**
In my opinion, this column is more important than the *Day #* column. Don't go crazy here. Keep it short and sweet, but as specific as possible. For example, there's a lot of scattered height estimates on the site. Don't write down "How tall?" for Day 110. Instead write "Bus height" because when you get to Day 111, I'd write in "Parking structure height". I believe the teacher has the ultimate say here, but it can be fun to poll your students for a short description in which you all can agree. Give students some ownership, right? If unit measurement is involved, try and sneak it in here. Take Day 125 for instance. I'd suggest entering "Net Wt. (oz.) of lg Hershey's bar." Keep in mind

that Day 126 asks the same question, but I'd suggest you encourage your class to use pounds if they don't think of it.
*By the way, sometimes unit measurement(s) are already included in the question. Use discretion.

**Too Low**
Think of an estimate that is too low.
Don't accept one (1), that's just rubbish, unless one (1) is actually applicable to the context of the challenge. Stretch your students. Think of it more as an answer that's too low, but reasonably close.  After all, this is a site of estimation challenges, not gimmes.

**Too High**
Refer to my notes in *Too Low*. Just don't accept 1 billion unless it's actually applicable. Discuss with students the importance of the *Too Low* and *Too High* sections: we are trying to eliminate wrong answers while creating a range of possible answers.

**My Estimate**
This is the place for students to fill in their answer. If the answer requires a unit of measurement, we better see one. Not every estimation challenge is "How many..." marshmallows? or Christmas lights? or cheese balls?
Even if a unit of measurement has already been established (see the *Description* notes), I'd still encourage your students to accompany their numerical estimate with a unit of measurement.


For example, on Day 41, "What's the height of the Giant [Ferris] Wheel?" use what makes sense to you, your students and your country's customary unit of measurement. Discuss the importance of unit measurements with students. Don't accept 108. What does that 108 represent? Pancakes? Oil spills? Bird droppings? NO! It represents 108 feet.

**My Reasoning**
The *My Reasoning* section is the most recent addition to the handout and I'm extremely thrilled about it. This is a student's chance to shine! Encourage their reasoning to be short and sweet. When a student writes something down, they'll be more inclined to share it or remember it. Accept bullet points or phrases due to the limited space. We don't need students to write paragraphs. However, we are looking for students to identify any context clues they used, personal experiences, and/or prior knowledge. Hold students accountable for their reasoning behind the estimate of the day.

**Don't let student reasoning go untapped!** If you're doing a sequence of themed estimation challenges, don't accept, "I just guessed" after the first day in the sequence. For example, if you're doing the flight distance themed estimate challenges starting on Day 136, you will establish the distance across the USA on the first day. Sure, go ahead and guess on Day 136, but make sure you hold students accountable for their reasoning every day thereafter.
Have students share their reasoning before and after revealing the answer. Utilize Think-Pair-Share. This will help create some fun conversations *before* revealing the answer. After revealing the answer, get those who were extremely close (or correct) to share their reasoning. I bet you'll have some great mathematical discussions. I'm also curious to hear from those that are way off and how their reasoning could possibly be improved.

I'd say the *My Reasoning* section was born for Mathematical Practice 3: *Construct viable arguments and critique the reasoning of others.* Keep some of these thoughts in mind regarding Mathematical Practice 3:

- Explain and defend your estimate.
- Construct a detailed explanation referencing context clues, prior knowledge, or previous experiences.
- Invest some confidence in it.
- Try to initiate a playful and respectful argument in class.
- Ask "Was anyone convinced by this explanation? Why? Why not?" or "Are you guys going to let [student name] off the hook with that explanation?"

There's reasoning behind every estimate (not guess).

- Find out what that reasoning is!
- DON'T let student reasoning go untapped!

**Answer**
Jot down the revealed answer. I'd also encourage students to write down the unit of measurement used in the answer. The answer might use a different unit of measurement than what you and your class agreed upon. Take the necessary time to discuss the most relative unit of measurement. I might be subjectively wrong on some of the answers posted. As for more thoughts on unit of measurement, refer to the *My Estimate* notes above. Continue having mathematical discussion after revealing the answer. Refer to my notes regarding the use of Mathematical Practice 3 in the *My Reasoning* section.

**Error**
Find the difference between *My Estimate* and *Answer*. Have students circle either the "+" or the "-" if they didn't get it exactly correct.
**+** Your estimate was **greater** than (above) the actual answer.
**-** Your estimate was **less** than (below) the actual answer.

## Mathematize the World through Daily Routines

The importance of continuing the established classroom routines cannot be overstated. Daily routines must include such obvious activities such as taking attendance, doing a lunch count, determining how many items are needed for snack, lining up in a variety of ways (by height, age, type of shoe, hair color, eye color, etc.), and daily questions. They should also include less obvious routines, such as how to select materials, how to use materials in a productive manner, how to put materials away, and have productive discourse about the mathematics in which students are engaged. An additional routine is to allow plenty of time for children to explore new materials before attempting any directed activity with these new materials. The regular use of the routines is important to the development of students' number sense, flexibility, and fluency, which will support students' performances on the tasks in this unit.

## Workstations and Learning Centers

When thinking about developing work stations and learning centers you want to base them on student readiness, interest, or learning profile such as learning style or multiple intelligence. This will allow different students to work on different tasks. Students should be able to complete the tasks within the stations or centers independently, with a partner or in a group.

It is important for students to be engaged in purposeful activities within the stations and centers. Therefore, you must carefully consider the activities selected to be a part of the stations and centers. When selecting an activity, you may want to consider the following questions:

- Will the activity reinforce or extend a concept that has already been introduced?
- Are the directions clear and easy to follow?
- Are materials easy to locate and accessible?
- Can students complete this activity independently or with minimal help from the teacher?
- How will students keep a record of what they have completed?
- How will students be held accountable for their work?
  (Laura Candler, *Teaching Resources*)

When implementing work stations and learning centers within your classroom, it is important to consider when the stations and centers will be used. Will you assign students to specific stations or centers to complete each week or will they be able to select a station or center of their choice? Will this opportunity be presented to all students during particular times of your math block or to students who finish their work early?

Just as with any task, some form of recording or writing should be included with stations whenever possible. Students solving a problem on a computer can write up what they did and explain what they learned.

## Games

"A game or other repeatable activity may not look like a problem, but it can nonetheless be problem based. The determining factor is this: Does the activity cause students to be reflective about new or developing relationships? If the activity merely has students repeating procedure without wrestling with an emerging idea, then it is not a problem-based experience. Students playing a game can keep records and then tell about how they played the game- what thinking or strategies they used." (Van de Walle and Lovin, *Teaching Student-Centered Mathematics: 3-5* pg. 28

**Journaling**

"Students should be writing and talking about math topics every day. Putting thoughts into words helps to clarify and solidify thinking. By sharing their mathematical understandings in written and oral form with their classmates, teachers, and parents, students develop confidence in themselves as mathematical learners; this practice also enables teachers to better monitor student progress." (NJ DOE)

"Language, whether used to express ideas or to receive them, is a very powerful tool and should be used to foster the learning of mathematics. Communicating about mathematical ideas is a way for students to articulate, clarify, organize, and consolidate their thinking. Students, like adults, exchange thoughts and ideas in many ways—orally; with gestures; and with pictures, objects, and symbols. By listening carefully to others, students can become aware of alternative perspectives and strategies. By writing and talking with others, they learn to use more precise mathematical language and, gradually, conventional symbols to express their mathematical ideas. Communication makes mathematical thinking observable and therefore facilitates further development of that thought. It encourages students to reflect on their own knowledge and their own ways of solving problems. Throughout the early years, students should have daily opportunities to talk and write about mathematics." (NCTM)

When beginning math journals, the teacher should model the process initially, showing students how to find the front of the journal, the top and bottom of the composition book, how to open to the next page in sequence (special bookmarks or ribbons), and how to date the page. Discuss the usefulness of the book, and the way in which it will help students retrieve their math thinking whenever they need it.

When beginning a task, you can ask, "What do we need to find out?" and then, "How do we figure it out?" Then figure it out, usually by drawing representations, and eventually adding words, numbers, and symbols. During the closing of a task, have students show their journals with a document camera or overhead when they share their thinking. This is an excellent opportunity to discuss different ways to organize thinking and clarity of explanations.

Use a composition notebook (the ones with graph paper are terrific for math) for recording or drawing answers to problems. The journal entries can be from Frameworks tasks but should also include all mathematical thinking. Journal entries should be simple to begin with and become more detailed as the children's problem-solving skills improve. Children should always be allowed to discuss their representations with classmates if they desire feedback. The children's journal entries demonstrate their thinking processes. Each entry could first be shared with a "buddy" to encourage discussion and explanation; then one or two children could share their entries with the entire class. Don't forget to praise children for their thinking skills and their journal entries! These journals are perfect for assessment and for parent conferencing. The student's thinking is made visible!

## GENERAL QUESTIONS FOR TEACHER USE

Adapted from *Growing Success* and materials from Math GAINS and *TIPS4RM*

### Reasoning and Proving

- How can we show that this is true for all cases?
- In what cases might our conclusion not hold true?
- How can we verify this answer?
- Explain the reasoning behind your prediction.
- Why does this work?
- What do you think will happen if this pattern continues?
- Show how you know that this statement is true.
- Give an example of when this statement is false.
- Explain why you do not accept the argument as proof.
- How could we check that solution?
- What other situations need to be considered?

### Reflecting

- Have you thought about…?
- What do you notice about…?
- What patterns do you see?
- Does this problem/answer make sense to you?
- How does this compare to…?
- What could you start with to help you explore the possibilities?
- How can you verify this answer?
- What evidence of your thinking can you share?
- Is this a reasonable answer, given that…?

### Selecting Tools and Computational Strategies

- How did the learning tool you chose contribute to your understanding/solving of the problem/assist in your communication?
- In what ways would [name a tool] assist in your investigation/solving of this problem?
- What other tools did you consider using? Explain why you chose not to use them.
- Think of a different way to do the calculation that may be more efficient.
- What estimation strategy did you use?

### Connections

- What other math have you studied that has some of the same principles, properties, or procedures as this?
- How do these different representations connect to one another?
- When could this mathematical concept or procedure be used in daily life?
- What connection do you see between a problem you did previously and today's problem?

**Representing**

- What would other representations of this problem demonstrate?
- Explain why you chose this representation.
- How could you represent this idea algebraically? graphically?
- Does this graphical representation of the data bias the viewer? Explain.
- What properties would you have to use to construct a dynamic representation of this situation?
- In what way would a scale model help you solve this problem?

## QUESTIONS FOR TEACHER REFLECTION

- How did I assess for student understanding?
- How did my students engage in the 8 mathematical practices today?
- How effective was I in creating an environment where meaningful learning could take place?
- How effective was my questioning today? Did I question too little or say too much?
- Were manipulatives made accessible for students to work through the task?
- Name at least one positive thing about today's lesson and one thing you will change.
- How will today's learning impact tomorrow's instruction?

## MATHEMATICS DEPTH-OF-KNOWLEDGE LEVELS

***Level 1 (Recall)*** includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include “identify,” “recall,” “recognize,” “use,” and “measure.” Verbs such as “describe” and “explain” could be classified at different levels depending on what is to be described and explained.

***Level 2 (Skill/Concept)*** includes the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include “classify,” “organize,” “estimate,” “make observations,” “collect and display data,” and “compare data.” These actions imply more than one step. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Some action verbs, such as “explain,” “describe,” or “interpret” could be classified at different levels depending on the object of the action. For example, if an item required students to explain how light affects mass by indicating there is a relationship between light and heat, this is considered a Level 2. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a Level 3. Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly, as primarily numerical skills, and such interpretation excludes from this level other skills such as visualization skills and probability skills, which may be more complex simply because they are less common. Other Level 2 activities include explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

***Level 3 (Strategic Thinking)*** requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

***DOK cont'd…***

***Level 4 (Extended Thinking)*** requires complex reasoning, planning, developing, and thinking most likely over an extended period of time. The extended time period is not a distinguishing factor if the required work is only repetitive and does not require applying significant conceptual understanding and higher-order thinking. For example, if a student has to take the water temperature from a river each day for a month and then construct a graph, this would be classified as a Level 2. However, if the student is to conduct a river study that requires taking into consideration a number of variables, this would be a Level 4. At Level 4, the cognitive demands of the task should be high, and the work should be very complex. Students should be required to make several connections—relate ideas *within* the content area or *among* content areas—and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level. Level 4 activities include designing and conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.

## DEPTH AND RIGOR STATEMENT

By changing the way we teach, we are not asking children to learn less, we are asking them to learn more. We are asking them to mathematize, to think like mathematicians, to look at numbers before they calculate, to think rather than to perform rote procedures. Children can and do construct their own strategies, and when they are allowed to make sense of calculations in their own ways, they understand better. In the words of Blaise Pascal, "We are usually convinced more easily by reasons we have found ourselves than by those which have occurred to others."

By changing the way we teach, we are asking teachers to think mathematically, too. We are asking them to develop their own mental math strategies in order to develop them in their students. Catherine Twomey Fosnot and Maarten Dolk, *Young Mathematicians at Work.*

While you may be tempted to explain and show students how to do a task, much of the learning comes as a result of making sense of the task at hand. Allow for the productive struggle, the grappling with the unfamiliar, the contentious discourse, for on the other side of frustration lies understanding and the confidence that comes from "doing it myself!"

# Problem Solving Rubric (3-5)

| SMP | 1-Emergent | 2-Progressing | 3- Meets/Proficient | 4-Exceeds |
|---|---|---|---|---|
| **Make sense of problems and persevere in solving them.** | The student was unable to explain the problem and showed minimal perseverance when identifying the purpose of the problem. | The student explained the problem and showed some perseverance in identifying the purpose of the problem, AND selected and applied an appropriate problem-solving strategy that led to a partially complete and/or partially accurate solution. | The student explained the problem and showed perseverance when identifying the purpose of the problem, AND selected and applied an appropriate problem-solving strategy that led to a generally complete and accurate solution. | The student explained the problem and showed perseverance by identifying the purpose of the problem and selected and applied an appropriate problem-solving strategy that led to a thorough and accurate solution. |
| **Attends to precision** | The student was unclear in their thinking and was unable to communicate mathematically. | The student was precise by clearly describing their actions and strategies, while showing understanding and using appropriate vocabulary in their process of finding solutions. | The student was precise by clearly describing their actions and strategies, while showing understanding and using grade-level appropriate vocabulary in their process of finding solutions. | The student was precise by clearly describing their actions and strategies, while showing understanding and using above-grade-level appropriate vocabulary in their process of finding solutions. |
| **Reasoning and explaining** | The student was unable to express or justify their opinion quantitatively or abstractly using numbers, pictures, charts, or words. | The student expressed or justified their opinion either quantitatively OR abstractly using numbers, pictures, charts, OR words. | The student expressed and justified their opinion both quantitatively and abstractly using numbers, pictures, charts and/or words. | The student expressed and justified their opinion both quantitatively and abstractly using a variety of numbers, pictures, charts, and words. |
| **Models and use of tools** | The student was unable to select an appropriate tool, draw a representation to reason or justify their thinking. | The student selected an appropriate tool or drew a correct representation of the tools used to reason and justify their response. | The student selected an efficient tool and/or drew a correct representation of the efficient tool used to reason and justify their response. | The student selected multiple efficient tools and correctly represented the tools to reason and justify their response. In addition, this student was able to explain why their tool/ model was efficient |
| **Seeing structure and generalizing** | The student was unable to identify patterns, structures or connect to other areas of mathematics and/or real-life. | The student identified a pattern or structure in the number system and noticed connections to other areas of mathematics or real-life. | The student identified patterns or structures in the number system and noticed connections to other areas of mathematics and real-life. | The student identified various patterns and structures in the number system and noticed connections to multiple areas of mathematics and real-life. |

Rubric created by Richmond County School District

## SUGGESTED LITERATURE

*One Hundred Hungry Ants*, Elinor J Pinczes
*Clean Sweep Campers*, Lucille Recht Penner
*Two Ways to Count to Ten*, Ruby Dee and Susan Meddaugh
*A Million Fish More or Less*, Fred McKissack
*Jump Kangaroo Jump*, Stuart J. Murphy and Kevin O'Malley
*Fraction Action*, Loreen Leedy
*Hershey Fraction Book*, Jerry Pallotta
*If You Hopped Like a Frog*, David M. Schwartz
*When a line bends a shape begins*, Rhonda Gowler Greene
*Grandfather Tang*, Ann Tompert
*Greedy Triangle*, Marilyn Burns
*Sir Cumference and the Knights of Angleland*, Cindy Neuschwander
*Sam Johnson and the Blue-Ribbon Quilt*, Lisa Campbell Ernst
*House for Birdie*, Stuart J. Murphy
*Pastry School in Paris*, Cindy Neuschwander
*Hamster Champs*, Stuart J. Murphy
*Racing Around*, Stuart J. Murphy
*Bigger, Better, Best*, Stuart J. Murphy

## TECHNOLOGY LINKS

### *Unit 1*

- http://en.wikipedia.org/wiki/Sieve_of_Eratosthenes A brief definition and an animation of a Sieve being completed.
- http://highered.mcgraw-hill.com/sites/dl/free/0072533072/78543/CentimeterGrid.pdf Printable centimeter grid paper
- http://www.allcountries.org/uscensus/411_normal_monthly_and_annual_precipitation_selected.html Precipitation rates for cities in the United States
- http://www.census.gov/schools/facts/ Student friendly site provides statistics about the states in the United States
- http://www.statemaster.com/index.php Site provides a drop-down list of features to research regarding states in the United States, advertising is used.
- http://www.ers.usda.gov/statefacts/ State facts regarding population, income, education, employment, farming, and exports provided
- http://www.cut-the-knot.org/Curriculum/Arithmetic/Eratosthenes.shtml Another sieve, this one shows the numbers 1-100 and crosses out the multiples (contains advertising).
- https://www.ezschool.com/Grade4/Math/OrderNumbers/ Interactive opportunities to compare large numbers and determine the value of given digits in various-sized numbers
- http://www.faust.fr.bw.schule.de/mhb/eratosiv.htm An electronic sieve for the numbers 1-400. You can select any number and its multiples will be eliminated.

- http://www.k-5mathteachingresources.com/4th-grade-number-activities.html Additional activities and read aloud activities.
- http://www.learner.org/courses/learningmath/number/session4/part_b/index.html Detailed information and activities for teachers regarding multiplication and division using base ten blocks.
- http://www.mathcats.com/grownupcats/ideabankmultiplication.html Additional ideas for approaching multiplication instruction
- http://www.mathwire.com/standards/standards.html Additional standard-based activities
- http://www.sheppardsoftware.com/mathgames/numbers/fruit_shoot_prime.htm Prime or Composite Fruit Shoot

***Unit 2***

- For Fraction Kit Template - http://www.eastsideliteracy.org/tutorsupport/documents/HO_Fractions.pdf
- http://en.wikipedia.org/wiki/Sieve_of_Eratosthenes A brief definition and an animation of a Sieve being completed
- http://gingerbooth.com/flash/patblocks/patblocks.php. Students can manipulate pattern blocks online and easily print and then label their work from this web site.
- http://illuminations.nctm.org/ActivityDetail.aspx?ID=18 This applet allows students to individually practice working with relationships among fractions and ways of combining fractions.
- http://illuminations.nctm.org/ActivityDetail.aspx?ID=80 Creating Equivalent Fractions by Shading Squares and Circles.
- https://apps.mathlearningcenter.org/fractions/ Modeling fractions for comparing and computing
- http://www.cut-the-knot.org/Curriculum/Arithmetic/Eratosthenes.shtml Another sieve, this one shows the numbers 1-100 and crosses out the multiples (contains advertising).
- http://www.faust.fr.bw.schule.de/mhb/eratosiv.htm An electronic sieve for the numbers 1-400. You can select any number and its multiples will be eliminated.
- http://www.learningplanet.com/sam/ff/index.asp Students practice identifying equivalent fractions.
- http://www.sheppardsoftware.com/mathgames/fractions/memory_equivalent1.htm Students play a matching game of equivalent fractions with picture representations provided. Levels 1 and 2 are appropriate for fourth grade.
- http://www.uen.org/Lessonplan/preview.cgi?LPid=21526 Click on “Pattern Block Equivalent Fractions” under “Materials.” Pattern Block problems that could be used as an introduction to this task or as extension for this task. This activity provides interactive practice enabling students to work more with factoring. They can work alone or with a partner.

***Unit 3***

- http://illuminations.nctm.org/ActivityDetail.aspx?ID=11 Students can manipulate fraction sets easily on this site and record their work on the student sheet.
- http://jeopardylabs.com/ - This site allows you to create a Jeopardy game on a web-based template.
- http://www.visualfractions.com/AddUnlikeCircle/ Provides students with practice adding mixed numbers with like denominators using circle models.

***Unit 4***

- http://illuminations.nctm.org/ActivityDetail.aspx?ID=11 Students can use the "set" application to produce fraction models.
- http://illuminations.nctm.org/ActivityDetail.aspx?ID=205 Students can manipulate fraction tiles easily on this site and record their work on the student sheet.
- https://www.visualfractions.com/IdentifyLines/ and https://www.visualfractions.com/IdentifyCircles/ This site would be best utilized with a lot of guidance from the teacher. The website asks students to determine what fraction of the set are square cookies or round cookies, something students should easily be able to accomplish. The teacher would then have to guide the students into making a multiplication sentence. For instance, if the example asks how many cookies are round and the answer is $\frac{1}{6}$, the teacher should then ask what is the whole (6) and then ask what type of multiplication problem could be created from this scenario (i.e. $6 \times \frac{1}{6} = 1$)

***Unit 5***

- http://decimalsquares.com/index.php/games/ (Beat the Clock) Students get to race against time or a partner writing the correct decimal for a given model. (One whole is represented by a large square divided into 100 small squares.) For fourth grade, choose "beginner" when playing this game.
- http://decimalsquares.com/index.php/games/ (Concentration) Students play a concentration game matching decimal numbers in the tenths (in red) with decimal numbers in the hundredths (in red).
- http://nces.ed.gov/nceskids/createagraph/default.aspx Allows students to create several types of graphs.
- http://www.factmonster.com/ipka/A0772159.html Provides images of flags around the world.

***Unit 6***

- http://illuminations.nctm.org/ActivityDetail.aspx?ID=34 Students can continue exploring sort shapes by characteristics using this online activity from NCTM Illuminations website.

- http://illuminations.nctm.org/unit.aspx?id=6528 The introduction to symmetry using the NCTM Illuminations web sites may be done as a whole group with a projector or in a computer lab individually or in pairs. If the students work on the tasks as individuals or in pairs, prepare a list of questions for them to answer while exploring the web site. At the end of this session, students should report to the whole class what they have learned or have found interesting about symmetry. The purpose of these activities is to provoke class discussion. *(Must register with NCTM to access)*
- http://mathforum.org/geometry/rugs/symmetry/ Be sure to check out the rug gallery to see different rug patterns and their symmetries!
- http://www.basic-mathematics.com/types-of-triangles.html Gives basic definitions for the types of triangles. Note: this web site contains advertising.

***Unit 7***

- http://www.ictgames.com/weight.html Interactive games to explore the concept of weight and scales
- http://www.primarygames.co.uk/pg2/bhunt/bhunt.html Banana Hunt: This game requires monkeys to be placed at the proper angle degree in order to collect bananas. It can be used for additional practice.
- https://apps.mathlearningcenter.org/pattern-shapes/ Angle Measurements: This tool can be used by the student or teacher to review angles and how to use a protractor to measure**.**
- https://www.mathplayground.com/measuringangles.html Measuring angles
- https://www.funbrain.com/games/shape-surveyor A fun, interactive game to practice area and perimeter of shapes
- http://www.sheppardsoftware.com/mathgames/geometry/shapeshoot/PerimeterShapesShoot.htm Perimeter shape game
- https://www.turtlediary.com/games/area-and-perimeter.html Area and Perimeter games

**Additional Resources**:

- https://kentuckymathematics.org/pimser.php

## RESOURCES CONSULTED

**Content:**
Mathematics Progressions Documents: http://ime.math.arizona.edu/progressions/

Illustrative Mathematics: https://www.illustrativemathematics.org/content-standards/4

NZ Maths: http://nzmaths.co.nz/

**Teacher/Student Sense-making:**
http://www.youtube.com/user/mitcccnyorg?feature=watch
https://www.georgiastandards.org/Georgia-Standards/Pages/Math.aspx or
http://secc.sedl.org/common_core_videos/

**Journaling:** http://www.mathsolutions.com/wp-content/uploads/2001_Math_Journals.pdf

**Community of Learners:**
http://www.edutopia.org/math-social-activity-cooperative-learning-video
http://www.edutopia.org/math-social-activity-sel
http://www.youtube.com/user/responsiveclassroom/videos