Georgia
Standards of Excellence
Curriculum Frameworks

Mathematics

GSE 8th Grade

Unit 3: Geometric Applications of Exponents

Georgia Department of Education

Richard Woods, Georgia's School Superintendent

"Educating Georgia's Future"

Unit 3
Geometric Applications of Exponents

Table of Contents

OVERVIEW

In this unit students will:

- distinguish between rational and irrational numbers;
- find or estimate the square and cubed root of non-negative numbers, including 0;
- interpret square and cubed roots as both points of a line segment and lengths on a number line;
- use the properties of real numbers (commutative, associative, distributive, inverse, and identity) and the order of operations to simplify and evaluate numeric and algebraic expressions involving integer exponents, square and cubed roots;
- work with radical expressions and approximate them as rational numbers;
- solve problems involving the volume of a cylinder, cone, and sphere;
- determine the relationship between the hypotenuse and legs of a right triangle;
- use deductive reasoning to prove the Pythagorean Theorem and its converse;
- apply the Pythagorean Theorem to determine unknown side lengths in right triangles;
- determine if a triangle is a right triangle, Pythagorean triple;
- apply the Pythagorean Theorem to find the distance between two points in a coordinate system; and
- solve problems involving the Pythagorean Theorem.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under "Evidence of Learning" be reviewed early in the planning process. A variety of resources should be utilized to supplement this unit. This unit provides much needed content information, but excellent learning activities as well. The tasks in this unit illustrate the types of learning activities that should be utilized from a variety of sources.

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

STANDARDS FOR MATHEMATICAL PRACTICE

Refer to the Comprehensive Course Overview for more detailed information about the Standards for Mathematical Practice.

1. Make sense of problems and persevere in solving them. Students solve real world problems through the application of algebraic and geometric concepts. They seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, ―What is the most efficient way to solve the problem?, ―Does this make sense?, and ―Can I solve the problem in a different way?

2.Reason abstractly and quantitatively. Students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

3.Construct viable arguments and critique the reasoning of others. Students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays. They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like ―How did you get that?, ―Why is that true?‖ ―Does that always work?‖ They explain their thinking to others and respond to others' thinking.

4.Model with mathematics. Students model problem situations symbolically, graphically, tabularly, and contextually. They form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students solve systems of linear equations and compare properties of functions provided in different forms. Students use scatterplots to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.

5.Use appropriate tools strategically. Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 8 may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal.

6.Attend to precision. Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays.

7.Look for and make use of structure. Students routinely seek patterns or structures to model and solve problems. In grade 8, students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.

8.Look for and express regularity in repeated reasoning. Students use repeated reasoning to understand algorithms and make generalizations about patterns. Students use iterative processes to determine more precise rational approximations for irrational numbers. During multiple opportunities to solve and model problems, they notice that the slope of a line and rate of change are the same value. Students flexibly make connections between covariance, rates, and representations showing the relationships between quantities.

STANDARDS FOR MATHEMATICAL CONTENT

Understand and apply the Pythagorean Theorem.

MGSE8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

MGSE8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

MGSE8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

MGSE8.G.9 Apply the formulas for the volume of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Work with radicals and integer exponents.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

BIG IDEAS

- The Pythagorean Theorem can be used both algebraically and geometrically to solve problems involving right triangles.
- Coordinates can be used to measure distance, an important application of the Pythagorean theorem.
- Square numbers and cubic numbers can be built from squares and cubes.
- These square and cubic numbers have roots that are equal to any dimension on the square or cube.
- Right triangles have a special relationship among the side lengths which can be represented by a model and a formula.
- Recognizing Pythagorean Triples can increase efficiency with problems involving the Pythagorean theorem.
- Many numbers are not rational; the irrational numbers can be expressed only symbolically or approximately using a close rational number.
- Attributes of geometric figures can be used to identify figures and find their measures.
- Relationships between change in length of radius or diameter, height, and volume exist for cylinders, cones and spheres.

ESSENTIAL QUESTIONS

- What is the length of the side of a square of a certain area?
- What is the relationship among the lengths of the sides of a right triangle?
- How can the Pythagorean Theorem be used to solve problems?
- What is the relationship between the Pythagorean Theorem and the distance formula? (*extension of standard MGSE.8.G.8*)
- How can I use the Pythagorean Theorem to find the length of the hypotenuse or leg of a right triangle?
- How do I know that I have a convincing argument to informally prove Pythagorean Theorem?
- What is Pythagorean Theorem and when does it apply?
- Where can I find examples of two and three-dimensional objects in the real-world?
- How does a change in any one of the dimensions of cylinder, cone, or sphere affect the volume of that cylinder, cone, or sphere?
- How does the volume of a cylinder, cone, and sphere with the same radius change if it is doubled?
- How do I simplify and evaluate algebraic equations involving integer exponents, square and cubed root?
- How do I know when an estimate, approximation, or exact answer is the desired solution?

CONCEPTS & SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- properties of similarity, congruence, and right triangles
- understand the meaning of congruence: that all corresponding angles and sides are congruent
- two figures are congruent if they have the same shape and size
- represent radical expressions in radical form (irrational) or approximate these numbers as rational
- find square roots of perfect squares
- write a decimal approximation for an irrational number to a given decimal place
- measuring length and finding perimeter and area of quadrilaterals
- characteristics of 2-D and 3-D solids
- evaluating linear and literal equations in one variable with one solution
- properties of exponents and real numbers (commutative, associative, distributive, inverse and identity) and order of operations
- express solutions using the real number system

FLUENCY

It is expected that students will continue to develop and practice strategies to build their capacity to become fluent in mathematics and mathematics computation. The eventual goal is automaticity with math facts. This automaticity is built within each student through strategy development and practice. The following section is presented in order to develop a common understanding of the ideas and terminology regarding fluency and automaticity in mathematics:

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number**.**

Deep Understanding: Teachers teach more than simply “how to get the answer” and instead support students’ ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students:

- **flexibly use a combination of deep understanding, number sense, and memorization.**
- **are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.**
- **are able to articulate their reasoning.**
- **find solutions through a number of different paths.**

For more about fluency, see: http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and: http://joboaler.com/timed-tests-and-the-development-of-math-anxiety

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for middle school students. **Note – Different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.** The definitions below are from the Common Core State Standards Mathematics Glossary and/or the Common Core GPS Mathematics Glossary when available.

Visit http://intermath.coe.uga.edu or http://mathworld.wolfram.com to see additional definitions and specific examples of many terms and symbols used in grade 8 mathematics.

- **Altitude of a Triangle:**
- **Base (of a Polygon):**
- **Coordinate Plane:**
- **Coordinate Point of a Plane:**
- **Cone:**
- **Converse of Pythagorean Theorem:**
- **Cube Root:**
- **Cylinder:**
- **Deductive Reasoning:**
- **Diameter:**
- **Geometric Solid:**
- **Height of Solids:**
- **Hypotenuse:**
- **Irrational:**
- **Leg of a Triangle:**
- **Literal Equation:**
- **Perfect Squares:**
- **Perfect Cubes:**
- **Pythagorean Theorem:**
- **Pythagorean Triples:**
- **Sphere:**
- **Square Root:**

- **Radius:**
- **Radical:**
- **Rational Number:**
- **Right Triangle:**
- **Volume:**

FORMATIVE ASSESSMENT LESSONS (FAL)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Guide.

SPOTLIGHT TASKS

A Spotlight Task has been added to each CCGPS mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Common Core Georgia Performance Standards, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky.

3-ACT TASKS

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

TASKS

Task Name	Task Type *Grouping Strategy*	Content Addressed	Standard(s)
The Taco Cart **(Spotlight Task)**	Constructing Task *Individual/Partner*	Apply Pythagorean Theorem to real-world situations.	MGSE8.G.7 MGSE8.EE.2
The Pythagorean Relationship **(Spotlight Task)**	Constructing Task *Individual/Partner*	Explore Pythagorean Theorem and its converse.	MGSE8.G.6 MGSE8.G.7 MGSE8.G.8 MGSE8.EE.2
The Pythagoras Tree **(Spotlight Task)**	Learning Task *Individual/Partner*	Apply Pythagorean Theorem to real-world situations.	MGSE8.G.7 MGSE8.EE.2
Greek Myth Meets Greek Math	Learning Task *Partner/Small Group*	Apply Pythagorean Theorem to problem solving situations	MGSE8.G.7 MGSE8.EE.2
Finding Pythagorean Triples in Disguise	Learning Task *Individual/Partner*	Exploring Pythagorean Triples	MGSE8.G.6 MGSE8.G.7 MGSE8.EE.2
Acting Out	Learning Task *Individual/Partner*	Apply Pythagorean Theorem to real-world situations.	MGSE8.G.7 MGSE8.G.8 MGSE8.EE.2
Pythagoras Plus	Learning Task *Individual/Partner*	Explore Pythagorean Theorem and its converse.	MGSE8.G.6 MGSE8.G.7 MGSE8.G.8 MGSE8.EE.2
Comparing TVs	Learning Task *Partner/Small Group*	Apply Pythagorean Theorem to real-world situations.	MGSE8.G.6 MGSE8.G.7 MGSE8.G.8 MGSE8.EE.2
Angry Bird App	Learning Task *Partner/Small Group*	Apply Pythagorean Theorem to real-world situations	MGSE8.G.6 MGSE8.G.7 MGSE8.G.8 MGSE8.EE.2
Constructing the Irrational Number Line	Learning Task *Individual/Partner*	Explore Pythagorean and its converse.	MGSE8.G.6 MGSE8.G.7 MGSE8.G.8 MGSE8.EE.2
Using Pythagorean Theorem and Exponents **(FAL)**	Formative Assessment Lesson *Partner/Small Group*	Use right triangle measurement, including square roots, to determine lengths for real-world problems.	MGSE8.G.7 MGSE8.EE.2
The Pythagorean Theorem: Square	Formative Assessment Lesson	Use the Pythagorean Theorem to find lengths and areas and use the area of	MGSE8.G.6 MGSE8.G.7

Area **(FAL)**	*Partner/Small Group*	right triangles to deduce the areas of other shapes	MGSE8.G.8 MGSE8.EE.2
Proofs of the Pythagorean Theorem	Short Cycle Task *Partner/Small Group*	Analyze attempts to prove the Pythagorean Theorem.	MGSE8.G.6 MGSE8.G.7 MGSE8.G.8
Circles and Squares	Short Cycle Task *Partner/Small Group*	Apply the Pythagorean Theorem to find the ratio between two shapes.	MGSE8.G.6 MGSE8.G.7 MGSE8.G.8
Comparing Spheres and Cylinders	Learning Task *Partner/Small Group*	Compare the effects of the volume of a cylinder and sphere.	MGSE8.G.9 MGSE8.EE.2
Making Matchsticks **(FAL)**	Formative Assessment Lesson *Partner/Small Group*	Apply the volume formula to real-world situations.	MGSE8.G.9 MGSE8.EE.2
Calculating Volumes of Compound Objects	Formative Assessment Lesson *Partner/Small Group*	Apply volume formulas to compound objects.	MGSE8.G.9
Greenhouse Management (PDF, Word)	Achieve CCSS-CTE Classroom Task *Partner/Small Group*	Apply the volume formula to real-world situations.	MGSE8.G.9 MGSE8.EE.2

The Taco Cart (Spotlight Task)

Back to Task Table

Source: http://www.101qs.com/1459-taco-cart - Dan Meyer

STANDARDS FOR MATHEMATICAL CONTENT

MCC.8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will make sense of this task through their questioning and understanding of the Pythagorean theorem.
2. Reason abstractly and quantitatively. Students will reason quantitatively based on the speed of the walkers and the distances traveled.
3. Construct viable arguments and critique the reasoning of others. Students will discuss their solutions and strategies and decide whether they agree or disagree with and debate these mathematical arguments.
4. Model with mathematics. Students will model their thinking through diagrams and equations.
5. Use appropriate tools strategically. Students will use appropriate tools such as calculators.
6. Attend to precision. Students will show precision through their use of mathematical language and vocabulary in their questioning and discussions. They will also show precision in their mathematical computations and procedures.

ESSENTIAL QUESTIONS

- How is the Pythagorean theorem useful when real world solving problems?
- When is it useful to use the Pythagorean theorem?
- Does the Pythagorean theorem always work?
- What are the limits for using the Pythagorean theorem?

MATERIALS REQUIRED

- Internet connection or downloaded videos & images from *http://www.101qs.com/1459-taco-cart*

TIME NEEDED

- 2 hours

TEACHER NOTES

In this task, students will watch the video, then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them.

This task is designed to build mathematical curiosity in students while maintaining a certain level of perplexity. The mathematical need to know that grows within students after watching the first act of this task is not diminished by the perplexity it offers. This is not a typical Pythagorean theorem problem students see. They will likely not even recognize it as such right away. Questioning and facilitating their learning and understanding will get students to this realization on their own.

TASK DESCRIPTION

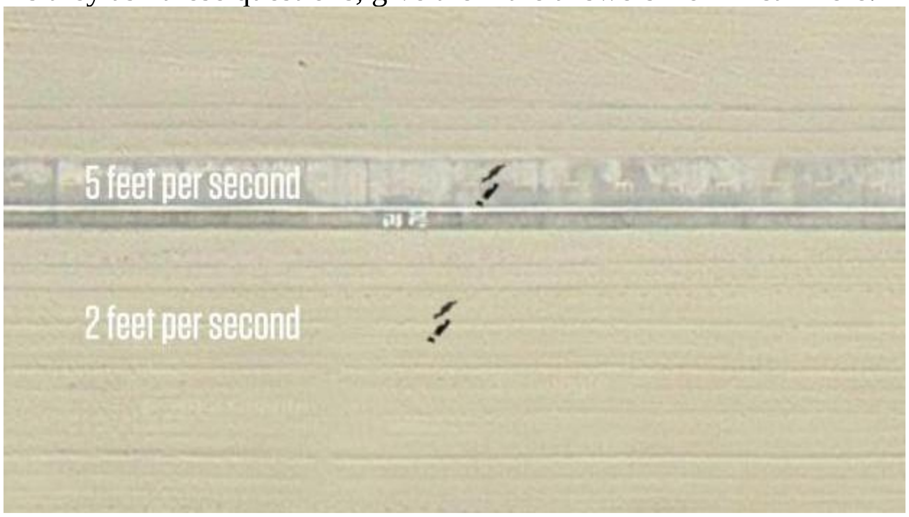
The following 3-Act Task can be found at: http://www.101qs.com/1459-taco-cart

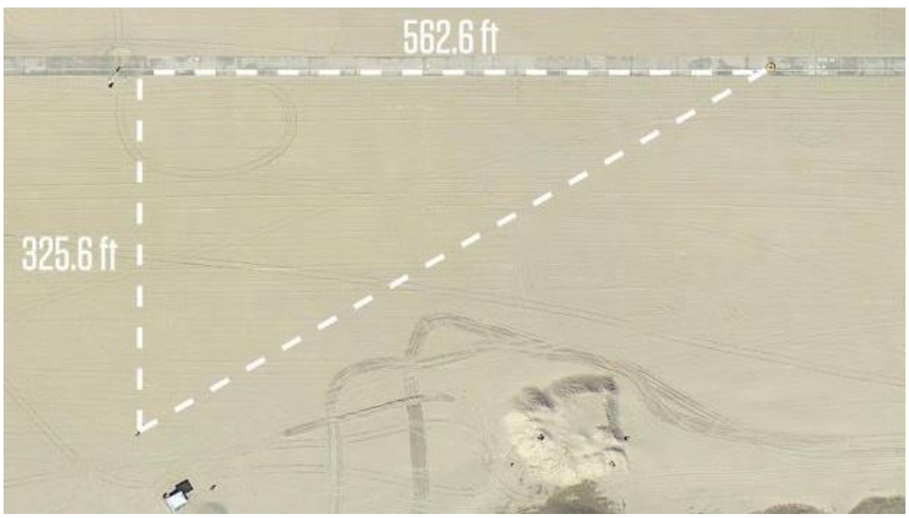
More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:
Watch the video: *http://www.101qs.com/1459-taco-cart*

Estimate: Which one will get there first? Will they get there a lot sooner or just a little before the other person. How much time will it take? Make an estimate that is too high and another that is too low.
ACT 2:
As students begin solving the task, they will have questions such as:
How fast do they walk?
How far is it from the beach to the road?
How far is it from where they are on the beach to the taco cart?

As they ask these questions, give them the answers from Act 2 here:





ACT 3

Students will compare and share solution strategies.

- Reveal the answer. *http://www.101qs.com/1459-taco-cart* Discuss the theoretical math versus the practical outcome.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.

ACT 4

Extension:

Students needing an extension can try one of the sequels *http://www.101qs.com/1459-taco-cart* or they may wish to begin answering other curious questions from act 1.

Intervention:

Students needing support should be guided through this problem based on their own understanding. While this is true for all students, sometimes it seems easier to just tell students what to do, rather than guiding them through their own productive struggles.

TACO CART Name:________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate the result of the main question? Explain?
Place an estimate that is too high and too low on the number line
Low estimate *Place an "x" where your estimate belongs* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:_______________________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

<u>The Pythagorean Relationship</u> (Spotlight Task)

Back to Task Table

Source: Teaching Student Centered Mathematics Volume 3 Grades 6-8, by John A. Van de Walle.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

MGSE8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will make sense and persevere through the models they create.
2. Reason abstractly and quantitatively. Students will reason abstractly with the models of the squares on the legs of the triangles. They will need to reason more abstractly as they justify the area of the square on the hypotenuse.
3. Construct viable arguments and critique the reasoning of others. Students will share their models and strategies and critique others' reasoning. Students will defend solution strategies.
4. Model with mathematics. Students will model their solutions using diagrams on grid paper as well as equations that support the diagrams.
5. Use appropriate tools strategically. Students will use appropriate tools to create the diagrams.
6. Attend to precision. Students will use the language of mathematics and accuracy in computation to show how they attend to precision.
7. Look for and make use of structure. Students will make use of structure as they reason how the squares on the sides of right triangles are related.
8. Look for and express regularity in repeated reasoning. Students will look for patterns within all of the right triangles investigated and generalize a rule for what they see.

ESSENTIAL QUESTIONS

- What is the length of the side of a square of a certain area?
- What is the relationship among the lengths of the sides of a right triangle?
- How can I determine the length of a diagonal?

MATERIALS REQUIRED

- ½ Centimeter graph paper http://incompetech.com/graphpaper/plain/
- List of several different right triangles for students to investigate (attached)
- Straight edge

TIME NEEDED

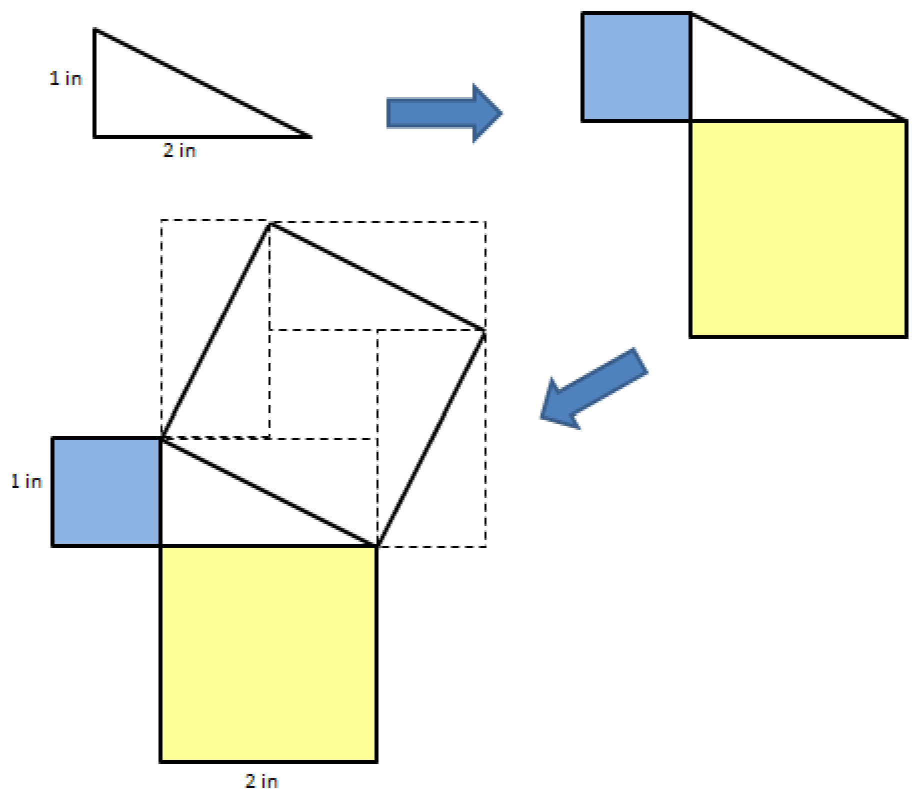
- 1 class period

TEACHER NOTES

This task is an introduction into the investigation of the Pythagorean Theorem. This is one of the most important mathematical relationships and necessitates an in-depth conceptual investigation. In geometric terms, the Pythagorean relationship states that if a square is constructed on each side of a right triangle, the sum of the areas of the two smaller squares will equal the area of the larger square (the square on the hypotenuse).

The task is to have students sketch a right triangle on ½ cm grid paper. Students should be assigned different triangles by specifying the lengths of the two legs. Students should then draw a square on each leg and on the hypotenuse. Students should then find the areas of each of the three squares.

To construct an accurate square on the hypotenuse, each of the sides of the square can be considered a diagonal of a rectangle:



Make a table of the area data collected:

Square on leg *a*	**Square on leg *b***	**Square on hypotenuse *c***

Ask students to look for a relationship between the squares in the table.

As students share their strategies and discoveries, be sure to allow students to show their representations and diagrams. The methods that students used to show the areas of the squares on the hypotenuse can be varied and creative (not just like the one example above). Allow students to share this reasoning and encourage discourse to critique the reasoning being shared.

DIFFERENTIATION:

Extension:
The natural extension to this problem is to find what the length of the hypotenuse for each triangle is. This can be further extended to find Pythagorean triples and Pythagorean triples in disguise (those similar to Pythagorean triples: 3-4-5 and 6-8-10 are both the same Pythagorean triple 3-4-5. To get the 6-8-10, just multiply each of the sides of the 3-4-5 by 2. These are similar (because they are proportional) Pythagorean triples.

Intervention:
For students needing support, teachers should be conscious of which triangle to give students when beginning this investigation. Make sure students are given a triangle with side lengths that will challenge them, but not place them in a situation that is too frustrating. Also, students needing support may work with a partner as long as the rules for the partnership are clearly established and that no one person takes over.

Triangles to investigate for the Pythagorean relationship:

Length of leg *a* = 3 cm Length of leg *b* = 4 cm	Length of leg *a* = 1 cm Length of leg *b* = 3 cm	Length of leg *a* = 4 cm Length of leg *b* = 5 cm	Length of leg *a* = 2 cm Length of leg *b* = 7 cm
Length of leg *a* = 6 cm Length of leg *b* = 5 cm	Length of leg *a* = 3 cm Length of leg *b* = 5 cm	Length of leg *a* = 4 cm Length of leg *b* = 6 cm	Length of leg *a* = 6 cm Length of leg *b* = 7 cm
Length of leg *a* = 3 cm Length of leg *b* = 8 cm	Length of leg *a* = 5 cm Length of leg *b* = 5 cm	Length of leg *a* = 7 cm Length of leg *b* = 5 cm	Length of leg *a* = 8 cm Length of leg *b* = 4 cm
Length of leg *a* = 6 cm Length of leg *b* = 3 cm	Length of leg *a* = 4 cm Length of leg *b* = 4 cm	Length of leg *a* = 9 cm Length of leg *b* = 2 cm	Length of leg *a* = 7 cm Length of leg *b* = 3 cm
Length of leg *a* = 8 cm Length of leg *b* = 2 cm	Length of leg *a* = 9 cm Length of leg *b* = 3 cm	Length of leg *a* = 9 cm Length of leg *b* = 4 cm	Length of leg *a* = 9 cm Length of leg *b* = 5 cm
Length of leg *a* = 8 cm Length of leg *b* = 5 cm	Length of leg *a* = 8 cm Length of leg *b* = 6 cm	Length of leg *a* = 8 cm Length of leg *b* = 7 cm	Length of leg *a* = 7 cm Length of leg *b* = 4 cm
Length of leg *a* = 6 cm Length of leg *b* = 9 cm	Length of leg *a* = 2 cm Length of leg *b* = 6 cm	Length of leg *a* = 4 cm Length of leg *b* = 2 cm	Length of leg *a* = 2 cm Length of leg *b* = 5 cm
Length of leg *a* = 10 cm Length of leg *b* = 3 cm	Length of leg *a* = 4 cm Length of leg *b* = 10 cm	Length of leg *a* = 5 cm Length of leg *b* = 10 cm	Length of leg *a* = 7 cm Length of leg *b* = cm

<u>The Pythagorean Relationship</u> (Spotlight Task)

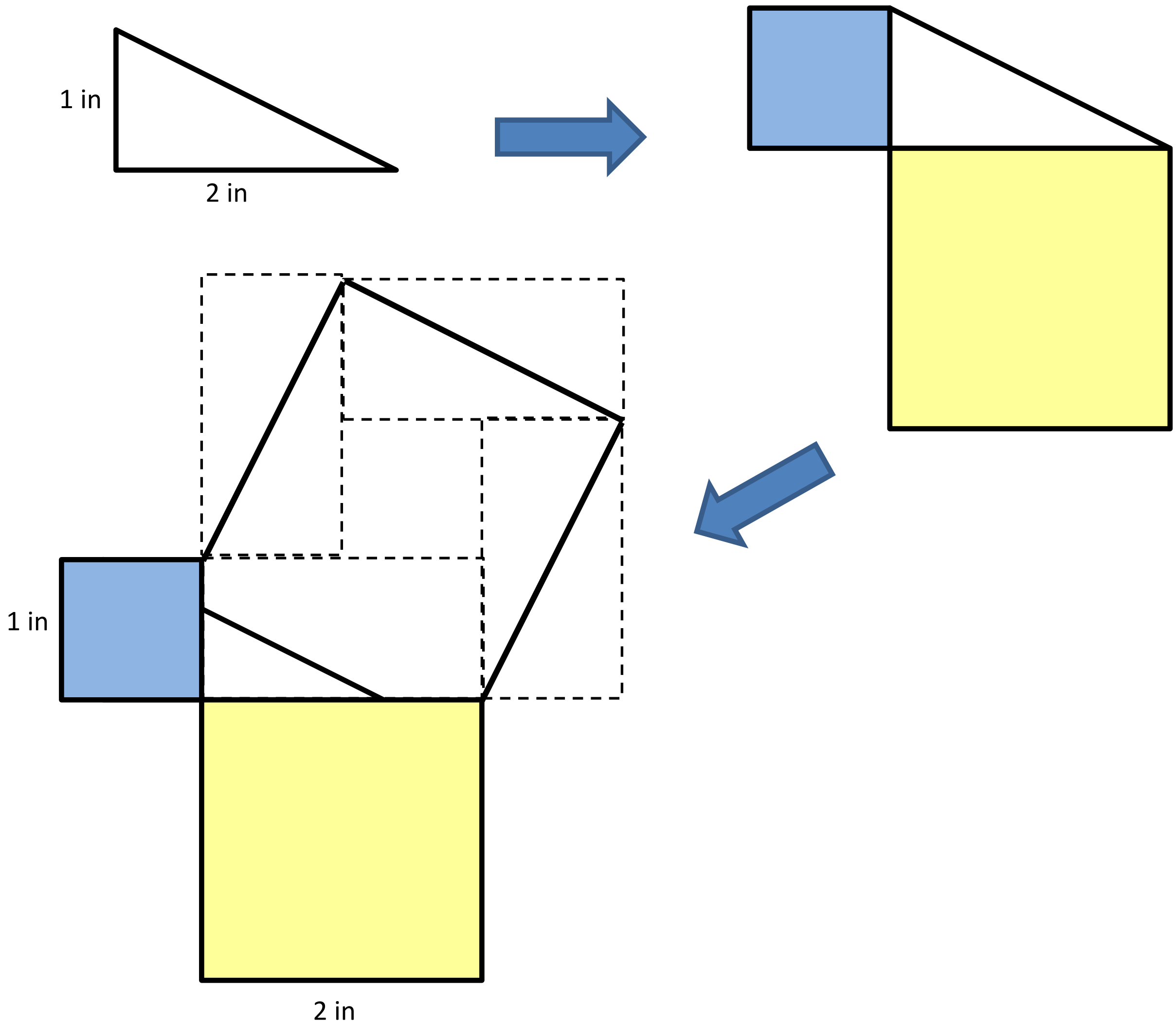
Triangles to investigate for the Pythagorean relationship:

Length of leg a = 3 cm Length of leg b = 4 cm	Length of leg a = 1 cm Length of leg b = 3 cm	Length of leg a = 4 cm Length of leg b = 5 cm	Length of leg a = 2 cm Length of leg b = 7 cm
Length of leg a = 6 cm Length of leg b = 5 cm	Length of leg a = 3 cm Length of leg b = 5 cm	Length of leg a = 4 cm Length of leg b = 6 cm	Length of leg a = 6 cm Length of leg b = 7 cm
Length of leg a = 3 cm Length of leg b = 8 cm	Length of leg a = 5 cm Length of leg b = 5 cm	Length of leg a = 7 cm Length of leg b = 5 cm	Length of leg a = 8 cm Length of leg b = 4 cm
Length of leg a = 6 cm Length of leg b = 3 cm	Length of leg a = 4 cm Length of leg b = 4 cm	Length of leg a = 9 cm Length of leg b = 2 cm	Length of leg a = 7 cm Length of leg b = 3 cm
Length of leg a = 8 cm Length of leg b = 2 cm	Length of leg a = 9 cm Length of leg b = 3 cm	Length of leg a = 9 cm Length of leg b = 4 cm	Length of leg a = 9 cm Length of leg b = 5 cm
Length of leg a = 8 cm Length of leg b = 5 cm	Length of leg a = 8 cm Length of leg b = 6 cm	Length of leg a = 8 cm Length of leg b = 7 cm	Length of leg a = 7 cm Length of leg b = 4 cm
Length of leg a = 6 cm Length of leg b = 9 cm	Length of leg a = 2 cm Length of leg b = 6 cm	Length of leg a = 4 cm Length of leg b = 2 cm	Length of leg a = 2 cm Length of leg b = 5 cm
Length of leg a = 10 cm Length of leg b = 3 cm	Length of leg a = 4 cm Length of leg b = 10 cm	Length of leg a = 5 cm Length of leg b = 10 cm	Length of leg a = 7 cm Length of leg b = cm

Directions for The Pythagorean Relationship:

1. Using a specified length of leg *a* and leg *b*, sketch a right triangle on ½ cm grid paper.
2. Draw a square on each leg of the right triangle.
3. Construct a square on the hypotenuse of the right triangle.

(To construct an accurate square on the hypotenuse, each of the sides of the square can be considered a diagonal of a rectangle.)



4. Find the areas of each of the three squares. ***Answers will vary.***
5. Record the calculated areas in the following table.

Square on leg *a*	**Square on leg *b***	**Square on hypotenuse *c***

6. Record the data collected by your classmates.

Square on leg *a*	**Square on leg *b***	**Square on hypotenuse *c***

7. What do you notice? ***Answers will vary, but the purpose of the task is for students to determine that the sum of the areas of the squares on the legs is equal to the area of the square on the hypotenuse, i.e. the Pythagorean Theorem.***
8. Is there a relationship between the areas of the squares in the table? ***Yes, $a^2 + b^2 = c^2$***

<u>The Pythagorean Tree</u> (Spotlight Task)

Back to Task Table

Task adapted from http://mikewiernicki.com/3-act-tasks/

STANDARDS FOR MATHEMATICAL CONTENT

MGSE8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

MGSE8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

RELATED STANDARDS

From 7th grade:
MCC7.RP.2 Recognize and represent proportional relationships between quantities.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. The context of the problem will engage students in seeking for the meaning the problem and look for efficient ways to represent and solve it.
2. Reason abstractly and quantitatively. Students will contextualize to understand the meaning of the numbers and variables as related to the Pythagorean tree and decontextualize to manipulate symbolic representations by applying properties of operations.
3. Construct viable arguments and critique the reasoning of others. Students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays
4. Model with mathematics. Students will form equations from the context and connect symbolic and graphical representations.
5. Use appropriate tools strategically. Students will consider available tools (including estimation and technology) and decide when certain tools might be helpful.
6. Attend to precision. Students will communicate by using clear and precise language in their discussions with others and in their own reasoning.
7. Look for and make use of structure. Students will examine the patterns in the problem and generate equations to describe relationships. Additionally, students will experimentally verify the effects of the fractal as it grows and describe the squares and triangles in terms of congruence and similarity.
8. Look for and express regularity in repeated reasoning. Students will use iterative processes to determine more precise rational approximations for irrational numbers.

ESSENTIAL QUESTIONS

- How can the Pythagorean Theorem be used to solve problems?
- How can I use the Pythagorean Theorem to find the length of the hypotenuse of a right triangle?
- How do I use the Pythagorean Theorem to find the length of the legs of a right triangle?
- How can I determine the length of a diagonal?

MATERIALS REQUIRED

- Inch grid paper
- Scissors
- 3-act videos and other media for The Pythagoras Tree: http://mikewiernicki3act.wordpress.com/the-pythagoras-tree/

TIME NEEDED

- 2 hours

TEACHER NOTES

In this task, students will watch the video, then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them.

The pattern shown, the Pythagoras Tree, is an example of a fractal.

> A fractal is a never-ending pattern that repeats itself at different scales. This property is known as "self-similarity." Although some fractals are very complex, they are created by repeating a simple process over and over in an ongoing feedback loop. Fractal patterns are extremely familiar, since nature is full of fractals - The branching patterns of trees, rivers, blood vessels, and lightning, the seemingly random formation of coastlines, mountains and clouds, and the spirals of seashells, hurricanes and tornadoes can all be explained through the patterns of fractals. Abstract fractals – such as the Mandelbrot Set – can be generated by a computer calculating a simple equation over and over.

It is suggested that pairs or small groups of students work together to recreate this fractal pattern. In order to do this, they will need materials like those seen in the video. The grid paper used for the red square is inch grid paper. The other squares are also inch grids, but each inch square is divided into tenths to help students create more accurate cuts.

Inch grid paper can be printed here: http://incompetech.com/graphpaper/plain/

Inch grid paper with tenths of an inch can be printed here:
http://incompetech.com/graphpaper/multiwidth/

Note: Green square lengths needed to be rounded to the nearest hundredth. Green square cuts were estimated. Accuracy of these cuts was facilitated through the use of inch grid paper subdivided and benchmarked into tenths.

TASK DESCRIPTION

The following 3-Act Task can be found at: http://mikewiernicki3act.wordpress.com/the-pythagoras-tree/

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:
Watch the video:
http://vimeo.com/96744963

Suggested question(s):
- How big are each of the squares?
- How many (grid) squares were cut (area) for each color of the tree?
- How much area for the whole tree?

Estimate the number of grid squares for the whole tree. Now make estimates that you know are too high and too low.
Draw and fill in the number line with your "too low" and "too high" estimates. Then find the approximate location of your estimate.

ACT 2:
As students begin thinking about the questions they would like to answer, they will have more questions. One possible question might be: "How big is the first square?" When this question or another question similar to it is asked, show this video. It will give students further information to get started.

One of the most important mathematical ideas that students need to understand in order to make sense of this puzzle, is the idea of the fractal and its construction based on self-similarity. All of the triangles (and squares) in this visual pattern are similar.

http://vimeo.com/96745032

Other information:
When finding the lengths of the 4th stage squares (the green squares in the video), it's important that students know that the measurements of the sides of these squares were rounded to the nearest hundredth. When cutting the squares the tenths grids within the inch grid paper helped with estimating and made the cuts for the squares in the video more precise.

ACT 3
Students will compare and share solution strategies.

- Reveal the answer. Discuss the theoretical math versus the practical outcome.
- How appropriate was your initial estimate?
- Share student solution paths. Start with most common strategy.
- Revisit any initial student questions that weren't answered.

Validate student thinking:
http://mikewiernicki3act.wordpress.com/the-pythagoras-tree/ - scroll down to act three and click on the link for the image of the Pythagoras tree with the grids showing.

DIFFERENTIATION

Extension:
Students may wish to see if they can continue the fractal to its next stage or investigate other fractal patterns.

Intervention:
For students in need of support, please note that the fractal does not have to go to the fourth stage on day one. Asking students to identify how many squares they think they can manage to find the measurements of and cut for the class period may be the first step in building stamina and perseverance in problem solving.

The Pythagorean Tree Name:________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate the result of the main question? Explain?
Place an estimate that is too high and too low on the number line
Low estimate *Place an "x" where your estimate belongs* High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:______________________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Greek Myth Meets Greek Math

Back to Task Table

Adapted from http://www.mathpickle.com/K-12/MathPickle_Podcast/Entries/2011/1/14_Grade_8_$1,000,000_Unsolved_Problem.html

The purpose of this task is to give students practice using the Pythagorean theorem to solve an unsolved mathematical problem.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will make sense and persevere through the models they create while collaboratively solving the mathematical puzzle.
2. Reason abstractly and quantitatively. Students will reason abstractly with the models they create from the context of the story from which the puzzle is based. They will need to reason more abstractly as they justify the length of any hypotenuse they create.
3. Construct viable arguments and critique the reasoning of others. Students will share their models and strategies and critique others' reasoning. Students will defend solution strategies.
4. Model with mathematics. Students will model their solutions using diagrams on dot-grid paper as well as equations that support the diagrams.
5. Use appropriate tools strategically. Students will use appropriate tools to create the diagrams.
6. Attend to precision. Students will use the language of mathematics and accuracy in computation to show how they attend to precision.
8. Look for and express regularity in repeated reasoning. Students will look for patterns in their solutions to determine efficiency in solutions on grids of various sizes.

ESSENTIAL QUESTIONS

- What is the length of the side of a square of a certain area?
- What is the relationship among the lengths of the sides of a right triangle?
- How can I determine the length of a diagonal?

MATERIALS REQUIRED

- 4 x 4 dot paper grids
- Greek Myth: Theseus and the Minotaur (Attached)

TIME NEEDED

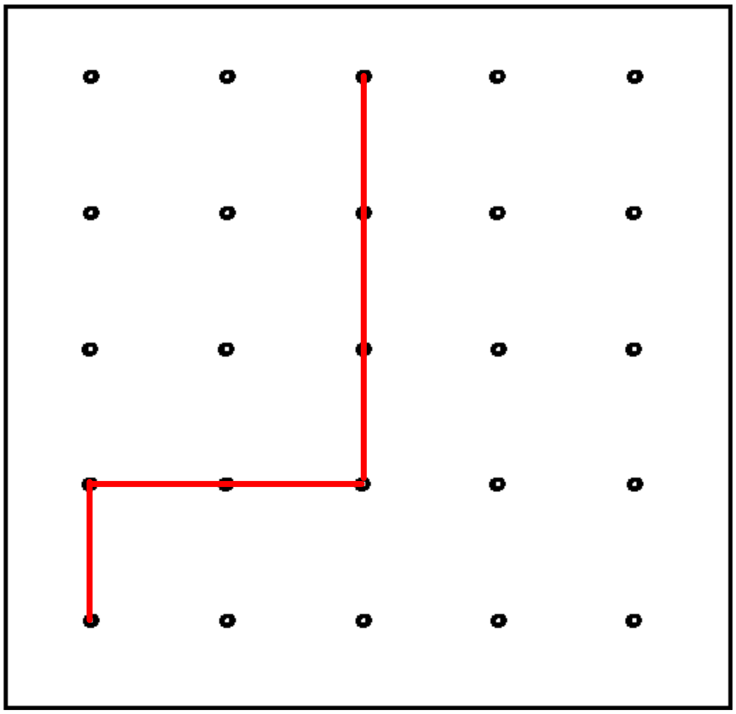
- 1 to 2 days (this can be extended indefinitely as a math station)

TEACHER NOTES

This task is designed to give students practice using the Pythagorean theorem to solve mathematical problems. The story is based on a Greek Myth involving Theseus and the Minotaur.

Tell students a brief (mathematically modified) story of Theseus and the Minotaur:

> *Theseus announced to King Minos that he was going to kill the Monster, but Minos knew that even if he did manage to kill the Minotaur, Theseus would never be able to exit the Labyrinth.*
>
> *Theseus met* ***Princess Ariadne****, daughter of King Minos, who fell madly in love with him and decided to help Theseus. She gave him a thread and told him to unravel it as he worked his way deeper into the Labyrinth. When he killed the Minotaur, he could follow the thread all the way out.*
>
> *While searching for the Minotaur in the labyrinth, Theseus entered a square room filled with columns. He thought about his friend, Pythagoras and wondered if he could use his friend's ideas to kill the Minotaur. Theseus decided to move from column to column, creating lengths of thread, each longer than the previous, but never touching (for fear of entangling himself). In this way, he figured he could entangle the Minotaur and slay him, then return to his true love, Ariadne.*
> *Example: Theseus' path could begin like this:*

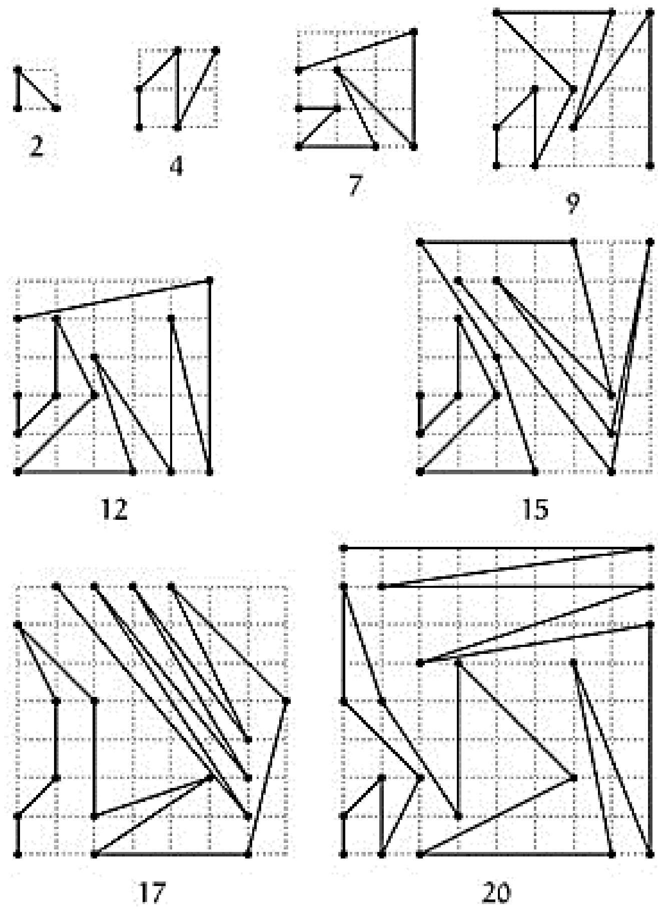


Students may begin a path for Theseus anywhere. The task is to find the longest path possible. How many lengths can he travel (the path above has three lengths).

Finish the story after students have had time to challenge each other in finding the longest path in the square room of the labyrinth. Students must defend their solutions with mathematical reasoning (quantitative and abstract) that shows that each of the line segments in their model is longer than the previous line segment.

> *Theseus followed her suggestion and entered the labyrinth with the thread. Theseus managed to kill the Minotaur and save the Athenians, and with Ariadne's thread he managed to retrace his way out.*
>
> *Theseus took Princess Ariadne with him and left Crete sailing happily back to Athens.*

There are multiple solutions for this problem. However, two mathematicians: Charles R. Greathouse IV and Giovanni Resta posted optimal solutions for grid sizes 2 through 9 on June 12, 2013.



DIFFERENTIATION

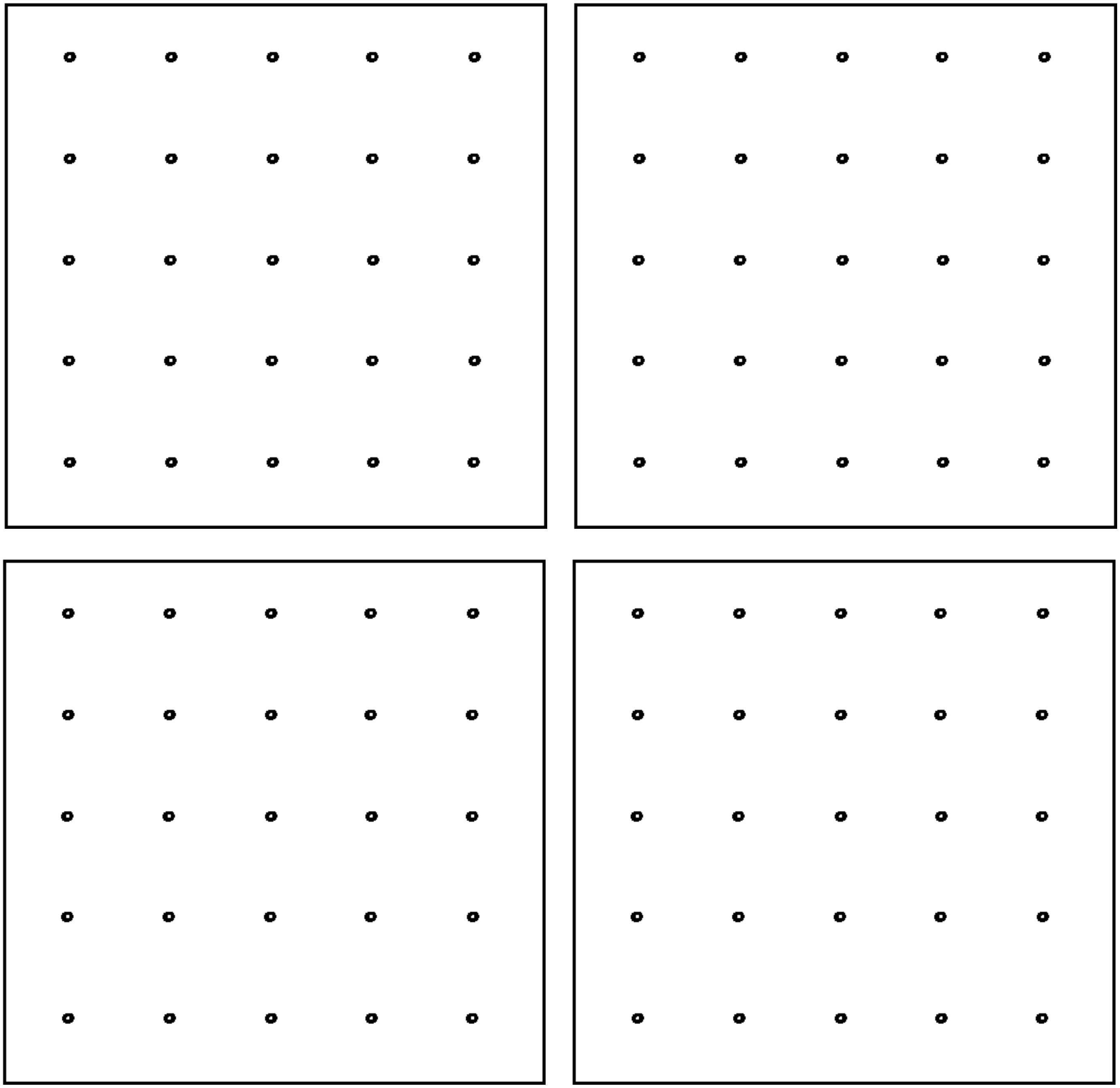
Extension:

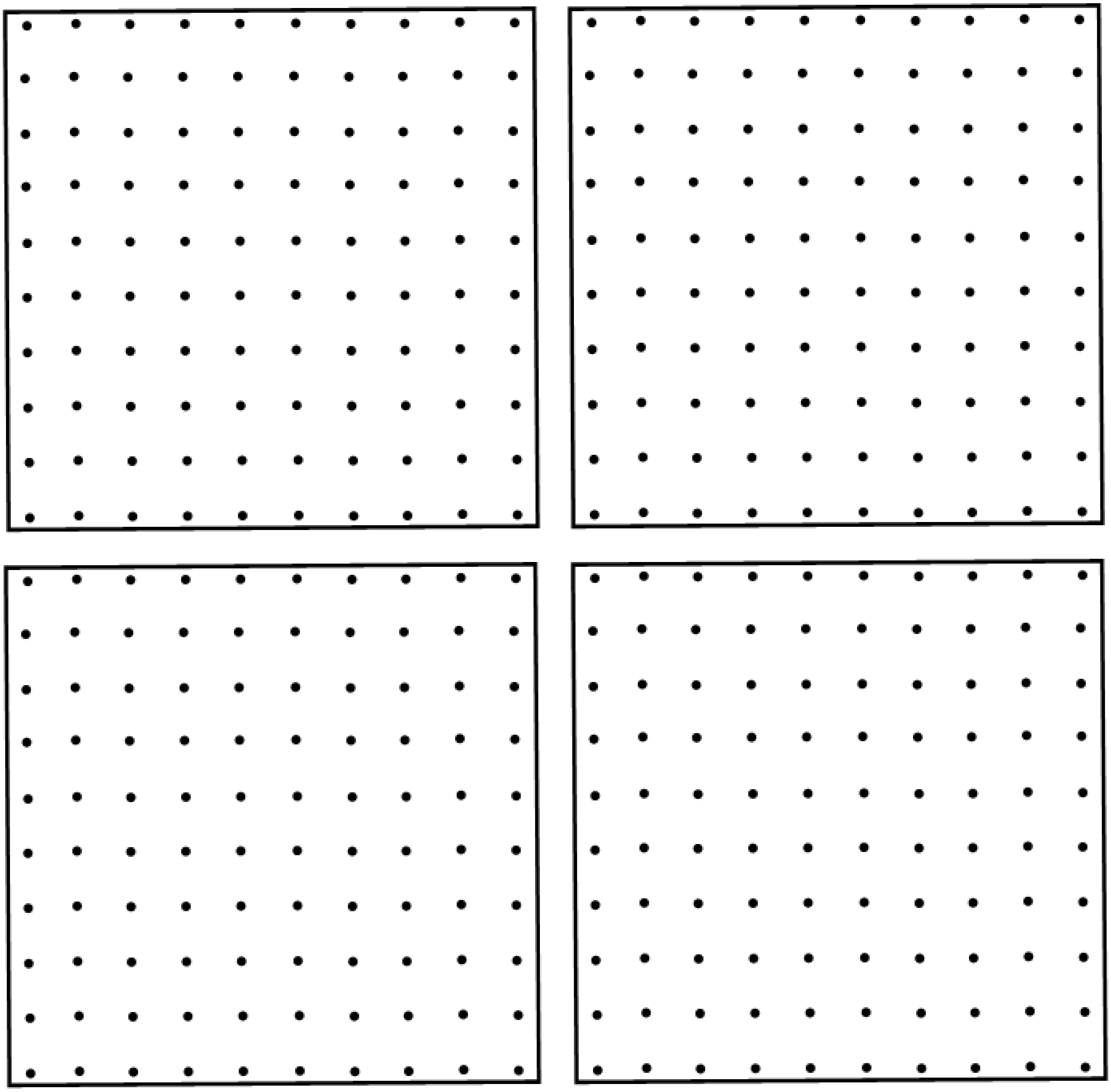
- Students in need of an extension can investigate this problem on larger grids, continuing to look for the longest path.

Intervention:

- Students in need of support should be allowed to revisit the previous tasks and connect that task with this one by constructing the squares on each hypotenuse for triangle they wish to find. Doing this engages students in conceptualizing what is happening with the squares on each of the triangle sides, helps reinforce the Pythagorean theorem, and makes the task less procedural. The procedures will come through investigation of the concepts as presented in these two tasks.

Greek Myth Meets Greek Math





Finding Pythagorean Triples in Disguise

Back to Task Table

Source: Teaching Student Centered Mathematics Volume 3 Grades 6-8, by John A. Van de Walle.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

MGSE8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

MGSE8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students will make sense and persevere through the models they create.
2. Reason abstractly and quantitatively. Students will reason abstractly with the models of the squares on the legs of the triangles. They will need to reason more abstractly as they justify the area of the square on the hypotenuse.
3. Construct viable arguments and critique the reasoning of others. Students will share their models and strategies and critique others' reasoning. Students will defend solution strategies.
4. Model with mathematics. Students will model their solutions using diagrams on grid paper as well as equations that support the diagrams.
5. Use appropriate tools strategically. Students will use appropriate tools to create the diagrams.
6. Attend to precision. Students will use the language of mathematics and accuracy in computation to show how they attend to precision.
7. Look for and make use of structure. Students will make use of structure as they reason how the squares on the sides of right triangles are related as well as make connections between similar Pythagorean triples.
8. Look for and express regularity in repeated reasoning. Students will look for patterns within all of the right triangles investigated and generalize a rule for what they see.

ESSENTIAL QUESTIONS

- What is the length of the side of a square of a certain area?
- What is the relationship among the lengths of the sides of a right triangle?
- How can I determine the length of a diagonal?

MATERIALS REQUIRED

- Centimeter graph paper http://incompetech.com/graphpaper/plain/
- Straight edge
- Calculator
- Dynamic geometric software (optional)

TIME NEEDED

- 1 to 2 days

TEACHER NOTES

This task is a natural extension of the investigation of the Pythagorean Theorem from the previous task.

The task here is to have students investigate the idea of Pythagorean triples in cases of similar triangles. For example, since 3-4-5 right triangles are Pythagorean triples, does that mean that triangles similar to the 3-4-5 right triangle are also Pythagorean triples?

Students should find at least three triples that form triangles similar to the 3-4-5 triangle. There are infinitely many, so once students discover the pattern, change the focus of the investigation from finding them to discovering and discussing strategies for how to recognize a 3-4-5 triangle disguised as a similar triangle.

Complete this same task with other Pythagorean triples such as 5-12-13 or 8-15-17

DIFFERENTIATION

Extension:

- Students in need of an extension can investigate Pythagorean triples further, finding more than those listed above and even looking for ways to find more.

Intervention:

- Students in need of support should be allowed to revisit the previous task and connect that task with this one by constructing the squares on each hypotenuse for each similar triangle they wish to try. Doing this engages students in conceptualizing what is happening with the squares on each of the triangle sides, helps reinforce the Pythagorean theorem, and makes the task less procedural. The procedures will come through investigation of the concepts as presented in these two tasks.

Finding Pythagorean Triples in Disguise

Directions for Finding Pythagorean Triples in Disguise:

1. Is a triangle with side lengths of 3cm, 4cm, and 5cm a right triangle?

   Yes

2. Given two triangles with the side lengths (1 cm, 2cm, 3cm) and (1.5 cm, 2 cm, 2.5 cm), which one would be a right triangle?

   1.5cm, 2cm, 2.5cm

3. Identify any relationships between the right triangles in question #1 and question #2.

   Student reasoning and answers will vary, but the purpose of the task is for students to use the converse of the Pythagorean Theorem and determine that the triangles are similar, allowing them to efficiently generate Pythagorean Triples.

4. Can you find other Pythagorean Triples based upon the relationships you discovered in question #3? If so find three more and describe a general way to find more.

   Yes. Possible answers: (6cm, 8cm, 10cm), (9cm, 12cm, 15cm), and (12cm, 16cm, 20cm). Possible answer: Multiplied each side length by the same number.

5. Can you find other right triangles based upon a triangle with side lengths of 5 ft, 12 ft, and 13 ft? If so provide a transformation that would create more.

   Yes. A dilation of various scale factors of the (5, 12, 13) right triangle would create more right triangles similar to the (5, 12, 13) right triangle.

6. Based upon what you have discovered, find three right triangles based upon the Pythagorean Triple of 8, 15, and 17. Explain how you know these are right triangles.

   Possible answers: (4, 7.5, 8.5), (16, 30, 34), and (24, 45, 51). Possible answers: They are right triangles because $a^2 + b^2 = c^2$ (converse of Pythagorean Theorem) OR they are right triangle because they are similar to the (8, 15, 17) right triangle.

Finding Pythagorean Triples in Disguise

Directions for Finding Pythagorean Triples in Disguise:

1. Is a triangle with side lengths of 3cm, 4cm, and 5cm a right triangle?

2. Given two triangles with the side lengths (1 cm, 2cm, 3cm) and (1.5 cm, 2 cm, 2.5 cm), which one would be a right triangle?

3. Identify any relationships between the right triangles in question #1 and question #3.

4. Can you find other Pythagorean Triples based upon the relationships you discovered in question #3? If so find three more and describe a general way to find more.

5. Can you find other right triangles based upon a triangle with side lengths of 5 ft, 12 ft, and 13 ft? If so provide a transformation that would create more.

6. Based upon what you have discovered, find three right triangles based upon the Pythagorean Triple of 8, 15, and 17. Explain how you know these are right triangles.

Acting Out

Back to Task Table

In this task, students will apply the Pythagorean Theorem to real-world situations.

STANDARDS FOR MATHEMATICAL CONTENT:

Understand and apply the Pythagorean Theorem.

MGSE8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

MGSE8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Work with radicals and integer exponents.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

In order for students to be successful, the following skills and concepts need to be maintained:

- **MCC6.G.3**

COMMON MISCONCEPTIONS:

- *Students may not use the circle to represent the largest distance from the houses. They may need a hint in order to see the circumference represent this distance.*
- *Students may count the block of the radius instead of finding the actually distance using the Pythagorean Theorem.*

ESSENTIAL QUESTIONS:

- How can I use the Pythagorean Theorem to find the distance between two points?
- How does the Pythagorean Theorem relate to the distance between two points on the coordinate system?
- How can I use the distance formula to calculate the distance of a line segment that is not vertical or horizontal? (*extension of standard MGSE.8.G.8*)

MATERIALS:

- Copies of task for each student/pair of students/or small group
- Colored pencils
- *Optional:* Compass
- Straightedge
- Graph paper http://incompetech.com/graphpaper/

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will use graph paper to draw the scenario. Have students draw all the possible directions that Erik and Kim could live from the theater. At minimum, students should have 4 directional graphs that depict their travel between their homes and the theater. The students should be able to recognize in their computation and knowledge of distance, that distance is always positive. Have the students to start at varying points on the coordinate grid. Students should understand that Erik and Kim could live anywhere on the grid with the theater as the center and the radius as the distance that they live from the theater. This will allow them to see that the distance is the same no matter the starting point because of the fixed distance from the theater.

DIFFERENTIATION:

Extension:

- Erik decided he would like to visit his grandma one evening after he left the theater. He decided he would first stop by his house, which was five miles west of the theater, to change his clothes. Once he had changed his clothes, Erik left his house and headed to grandma's house. He drove 8 miles south, then two miles east and arrived at grandma's house. Using the shortest route possible, how far is grandma's house from Erik's house? Using the shortest route possible, how far is grandma's house from the theater? Is grandma's house closer to Erik's house or the theater? How much closer? Include an illustration along with a mathematical solution and a detailed explanation that justifies your answers to all of the questions above. Show all of your work.

- Extra Challenge: How many square miles of land area would be included in the triangle formed using the theater, Erik's house and grandma's house as vertices?

Intervention/Scaffolding:

- Be sure to prompt struggling students by asking guiding questions. Group the students in a way that promotes cooperative learning.

Acting Out

Erik and Kim are actors at the FOX Theater.

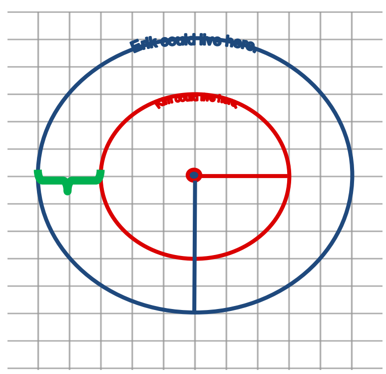
Erik lives 5 miles from the theater and Kim lives 3 miles from the theater.

Their boss, the director, wonders how far apart the actors live.

- On grid paper, pick a point to represent the location of the theater.
- Illustrate all of the possible places that Erik could live on the grid paper.
- Using a different color, illustrate all of the possible places that Kim could live on the grid paper.

1. On your graph paper, label the x and y axes of the grid. You are going to plot each scenario on this one grid. Each scenario must have a different starting point. Plot each scenario which represents the possible direction that Erik and Kim live from the theater.

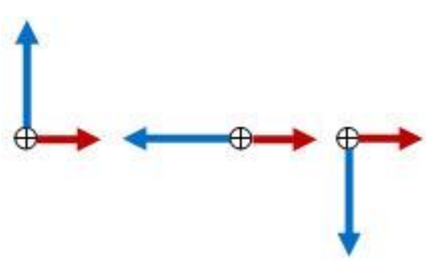
Example of one solution



2. For each scenario you have drawn, use the Pythagorean Theorem, whenever possible, to compute the distance between the points.

Comments

Possible directions that Erik and Kim live from the theater.



Additionally, have students calculate the distance for each scenario they construct. This helps them justify their reasoning for solutions in # 3 and #4.

3. What is the smallest distance, *d*, that could separate their homes? How do you know?

Solution

The closest that they could live would be 5 – 3 = 2 miles. This may be written as $5 \pm 3 = d$ ***where d represents the distance from Erik's house to Kim's house.***

4. What is the largest distance, *d*, that could separate their homes? How do you know?

Solution
The farthest apart that could separate their homes would be 5 + 3 = 8 miles. This may be written as $5 \pm 3 = d$ ***where d represents the distance from Erik's house to Kim's house.***

5. If Erik lived 5 miles north of the theater and Kim 3 miles west, what is the shortest distance between their homes?

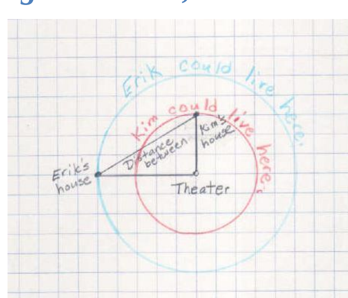
Comments

One way for students to recognize this is to have each student place a point on their grid where they think Erik might live and another point on the grid where they think that Kim might live. They should see that triangles are formed. Some of the triangles may be right triangles and they could use the Pythagorean Theorem to determine the actual distance between the two homes. Be careful with this exercise because not all triangles that could be drawn will be right triangles and the Pythagorean Theorem will not hold. This plants a good seed for the distance formula, recognizing right triangles when the angle measurement is not specified, and testing and proving the Pythagorean Theorem and its converse.

Solution

By seeing the right triangle with legs of lengths 5 and 3, students should be able to recognize

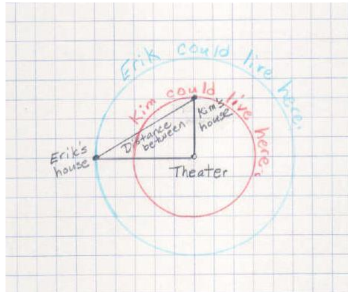
$$5^2 + 3^2 = d^2$$
$$25 + 9 = d^2$$
$$34 = d^2$$
$$\sqrt{34} = d$$



Erik and Kim live about 5.8 miles apart in this drawing.

$\sqrt{34} = d$

$\sqrt{34} \approx 5.8$



Students should also understand that the distance is less if the right angle is changed to an acute angle and that the distance increases should the right angle become obtuse.

Acting Out

Erik and Kim are actors at the FOX Theater.

Erik lives 5 miles from the theater and Kim lives 3 miles from the theater.

Their boss, the director, wonders how far apart the actors live.

- On grid paper, pick a point to represent the location of the theater.
- Illustrate all of the possible places that Erik could live on the grid paper.
- Using a different color, illustrate all of the possible places that Kim could live on the grid paper.

1. On your graph paper, label the x and y axes of the grid. You are going to plot each scenario on this one grid. Each scenario must have a different starting point. Plot each scenario which represents the possible direction that Erik and Kim live from the theater.

2. For each scenario you have drawn, use the Pythagorean Theorem, whenever possible, to compute the distance between the points.

3. What is the smallest distance, d, that could separate their homes? How do you know?

4. What is the largest distance, d, that could separate their homes? How do you know?

5. If Erik lived 5 miles north of the theater and Kim 3 miles west, what is the shortest distance between their homes?

Pythagoras Plus

Back to Task Table

In this task, students will explore the Pythagorean Theorem and its converse.

STANDARDS FOR MATHEMATICAL CONTENT:

Understand and apply the Pythagorean Theorem.

MGSE8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

MGSE8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

MGSE8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Work with radicals and integer exponents.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

In order for students to be successful, the following skills and concepts need to be maintained:

- **MCC7.G.6**

COMMON MISCONCEPTIONS:

- *Students have a tendency to add the areas of the squares all of the time. Students seem to have a misconception when the hypotenuse and a leg are given.*
- *Students will add the squares, but forget to take the square root of this summation.*

ESSENTIAL QUESTIONS:

- What is Pythagorean Theorem and when does it apply?
- What is the relationship among the lengths of the sides of a right triangle?
- How can I use the Pythagorean Theorem to find the length of the hypotenuse of a right triangle?
- How do I know that I have a convincing argument to prove Pythagorean Theorem?

MATERIALS:

- Copies of task for each student/pair of students/or small group
- Isometric dot or 1 cm square graph paper http://incompetech.com/graphpaper/

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will discover and formalize the Pythagorean Theorem by investigating the relationship between the areas of the squares constructed on each side of a right triangle. Students will also use a visual to make a geometric connection to the algebraic formula usually associated with the Pythagorean Theorem. Students may need to review how to determine the area of a triangle and a square. Some students may need to use manipulatives to visually understand this proof so cut out four right triangles whose legs are the same length and angle. The students can use these manipulatives to construct the figures in problems 1 and 2.

Graphical 'proof' of the Pythagorean Theoremhttp://www.mathopenref.com/pythagorasproof.html

DIFFERENTIATION:

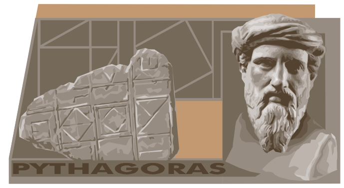
Extension:

There are four extension activities to choose from: Bronowski's Proof of Pythagoras' Theorem; Squares on the Sides of a Triangle; Equilateral Triangles; Pythagorean Triples. Choose the one or ones that extend the thinking and add the most value for your students. Answers for each activity are included. http://www.cimt.plymouth.ac.uk/projects/mepres/book8/y8s3act.pdf

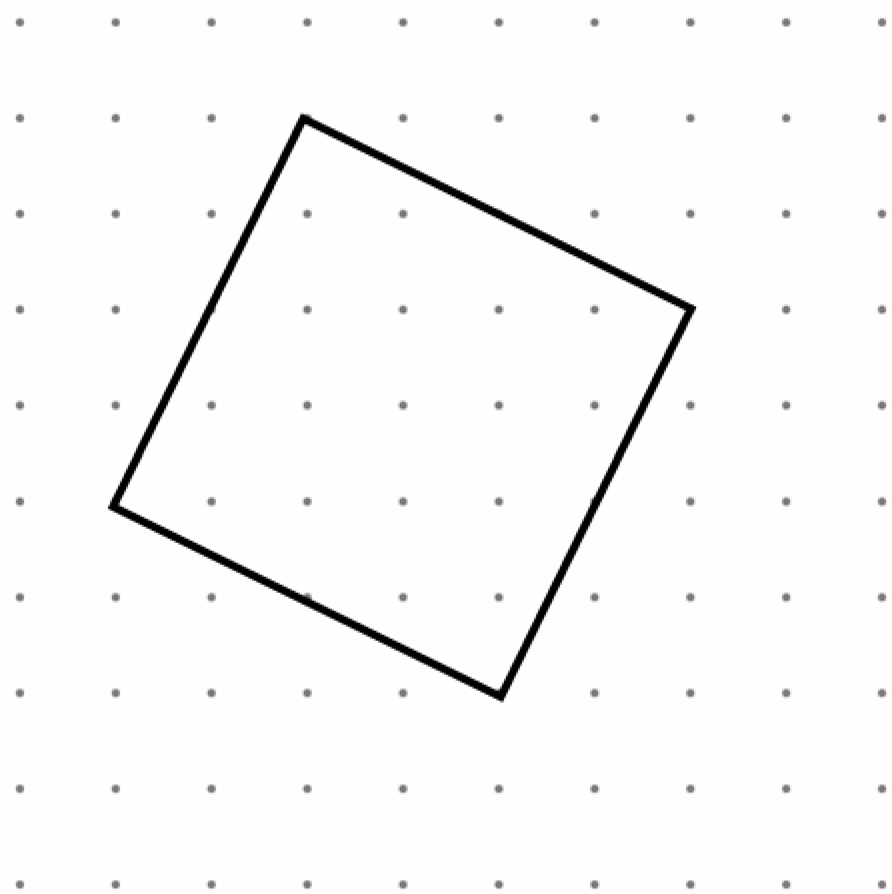
Intervention/Scaffolding:

- Do problem 1 with the class as an example. Prompt struggling students by asking guiding questions. Know that you can choose not to have struggling students complete all problems in this task.

Pythagoras Plus

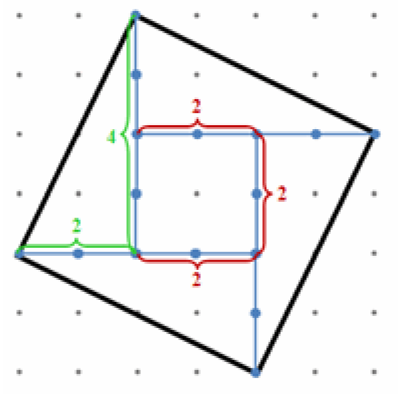


1. Find the exact area (in square units) of the figure below. Explain your method(s).



Solution 1: ***Divide the original square into 4 triangles and one square. The lengths of the legs of each right triangle are 2 units and 4 units respectively; yielding an area of 4 units***

squared. The length of the square is 2 units; yielding an area of 4 units squared. Altogether, the area is 20 units squared.

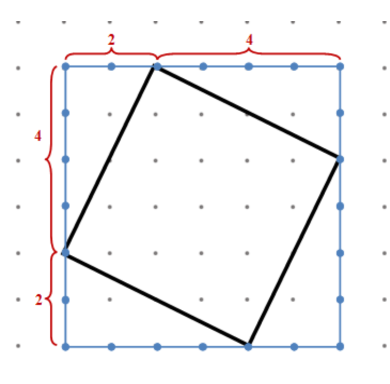


$$Area\ of\ square = \frac{4(8\ units^2)}{2} + 4\ units^2 = 20\ units^2$$

$$or$$

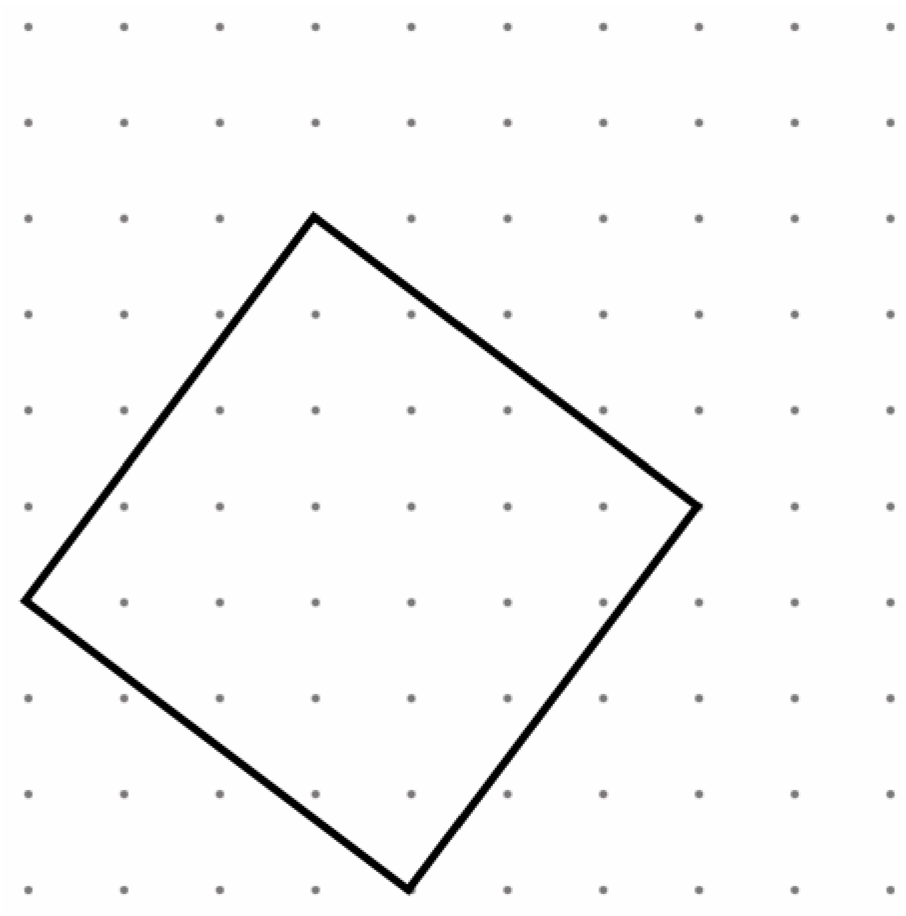
$$Area\ of\ square = 4\left[\frac{1}{2}(8)\right]units^2 + 4\ units^2 = 20\ units^2$$

<u>Solution 2:</u> Surround the square with a 6 × 6 square, with 4 measurable triangles whose areas can be subtracted from the area of the 6 × 6 area to give the area of the original square. The surrounding square has an area of 36 units^2. The triangles each have an area of 4 units^2. Subtracting, we find the area is 20 units^2.



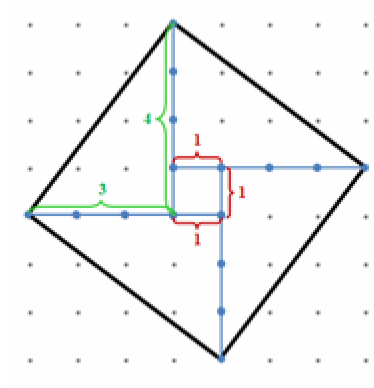
Area of the original square = 36 units^2 – 4(4 units^2) = 20 units^2

2. Find the exact area (in square units) of the figure below. Explain your method(s).



***Note:* A unit is measured horizontally or vertically, not diagonally.**

Solution 1:
Divide the original square into 4 triangles and one square. The lengths of the legs of each right triangle are 3 units and 4 units respectively; yielding an area of 6 units squared. The length of the square is 1 unit; yielding an area of 1 unit squared. Altogether, the area is 25 units squared.

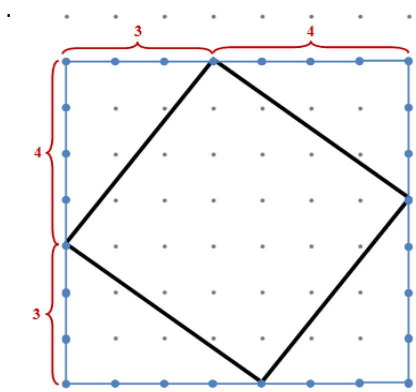


$$Area\ of\ square = \frac{4(12\ units^2)}{2} + 1unit^2 = 25units^2$$

$$Area\ of\ Square = 4\left[\frac{1}{2}(12)\right]units^2 + 1unit^2 = 25units^2$$

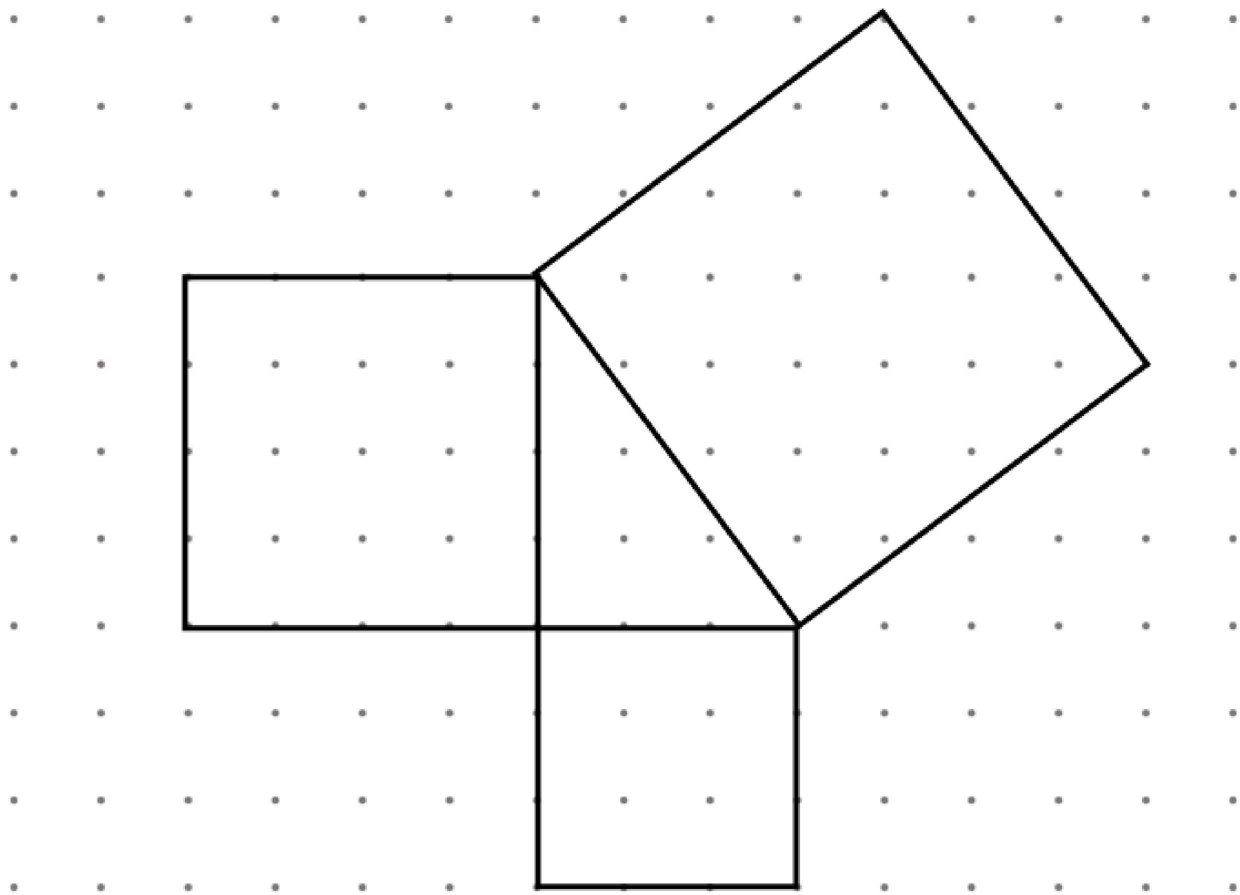
Solution 2:

Surround the square with a 7 × 7 square, with 4 measurable triangles whose areas can be subtracted from the area of the 7 × 7 area to give the area of the original square. The surrounding square has an area of 49 units^2. The triangles each have an area of 6 units^2. Subtracting, we find the area is 25 units^2.



Area of the original square = 49 units^2 – 4(6 units^2) = 25 units^2

3. Find the areas of the squares on the sides of the triangle below.
   (Hint: How does the large square below compare to the square in problem 1 above?)



a. How do the areas of the smaller squares compare to the area of the larger square?

Solution

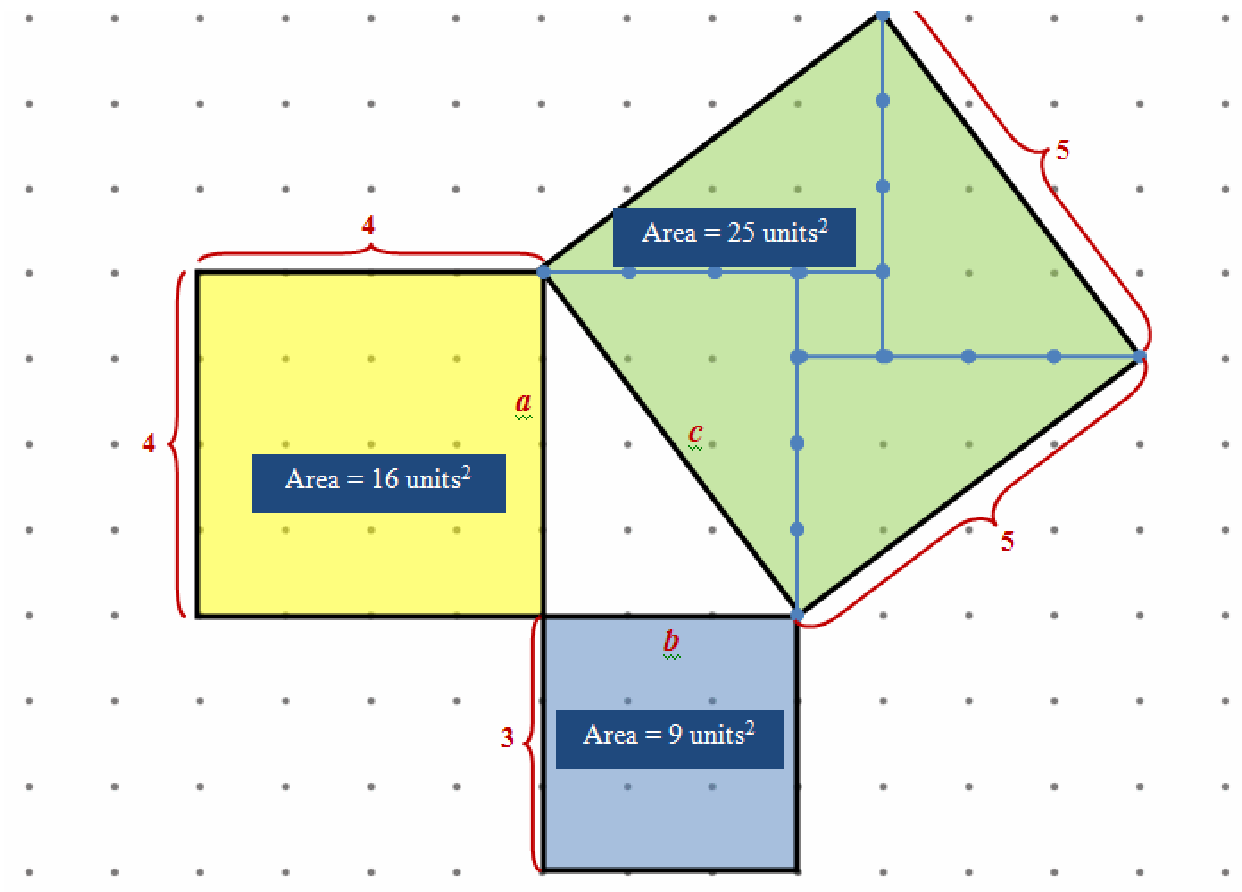
Students will find that the sum of the areas of the squares constructed on the legs of the triangle is equal to the area of the square constructed on the hypotenuse of the triangle.

b. If the lengths of the shorter sides of the triangle are *a* units and *b* units and the length of the longest side is *c* units, write an algebraic equation that describes the relationship of the areas of the squares.

Solutions

$\boldsymbol{area_{yellow} + area_{blue} = area_{green}}$
$\boldsymbol{16 + 9 = 25}$
$\boldsymbol{4^2 + 3^2 = 5^2}$
$\boldsymbol{a^2 + b^2 = c^2}$***, where c is the hypotenuse and a and b are the legs of a right triangle.***

$$\boldsymbol{leg^2 + leg^2 = hypotenuse^2}$$

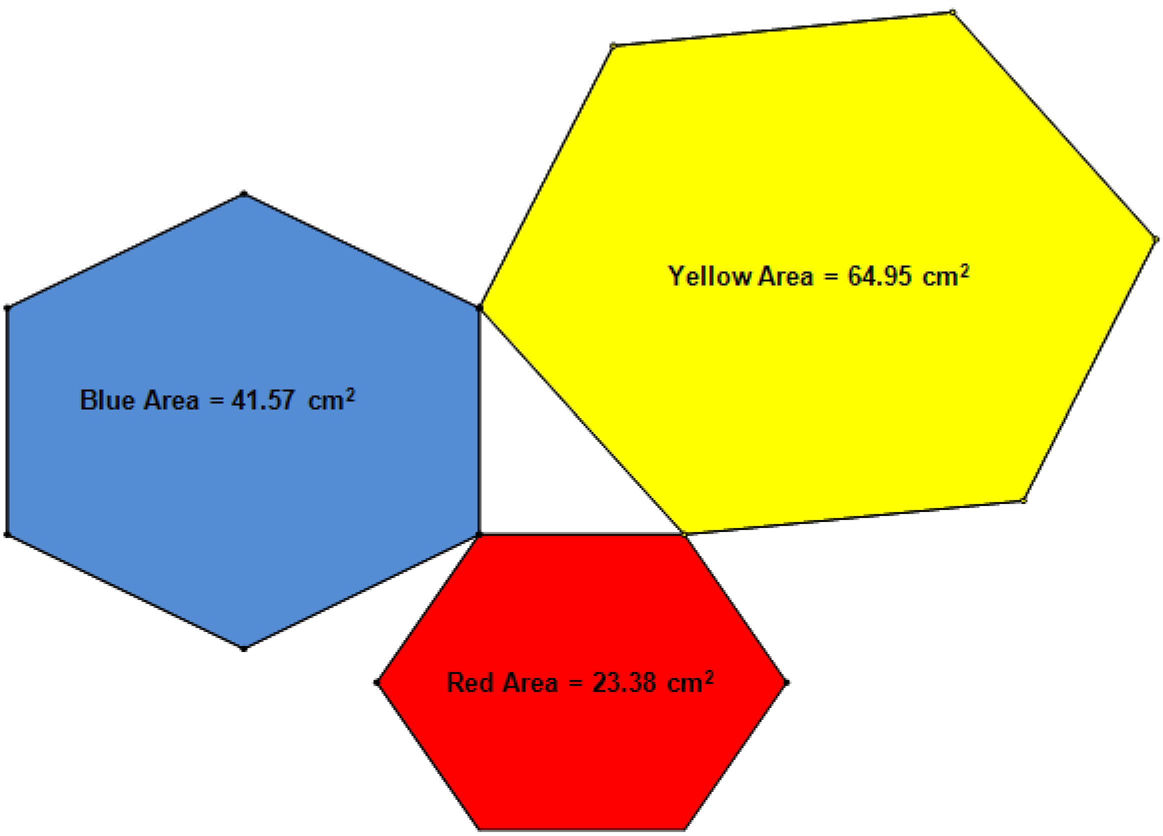


c. This relationship is called the Pythagorean Theorem. Interpret this algebraic statement in terms of the geometry involved.

Solution

When you construct squares on each side of a right triangle, the sum of the areas of the squares with sides equal to the length of the legs is equal to the area of the square with side equal to the length of the hypotenuse.

4. What is the relationship between the areas of the regular hexagons constructed on the sides of the right triangle below?



Solution

Note that these are regular hexagons on the sides of the right triangle. The Pythagorean relationship holds for all similar polygons, and thus for regular polygons, with sides the length of the sides of the right triangle on which they are constructed. Thus, students should recognize that the sum of the areas of the regular hexagons constructed on the legs of the triangle is equal to the area of the regular hexagon constructed on the hypotenuse.

5. Does the Pythagorean relationship work for other polygons constructed on the sides of right triangles? Under what condition does this relationship hold?

Comments

Have students construct equilateral triangles, regular pentagons, and other regular polygons on various right triangles. The use of dynamic geometry software (e.g., Geogebra, Geometer's Sketchpad or Cabri, Jr.) allows students to vary the size of the triangles while observing the Pythagorean relationship continues to hold when the figures are similar.

However, if the figures are not similar, then the Pythagorean relationship will not hold. For example, the rectangles in Figure 1 are similar; the rectangles in Figure 2 are not. Students should discover that the Pythagorean relationship holds for the similar rectangles but not for the non-similar ones. Note also that regular polygons are similar polygons since all 3 the angles are congruent and corresponding sides are proportional.

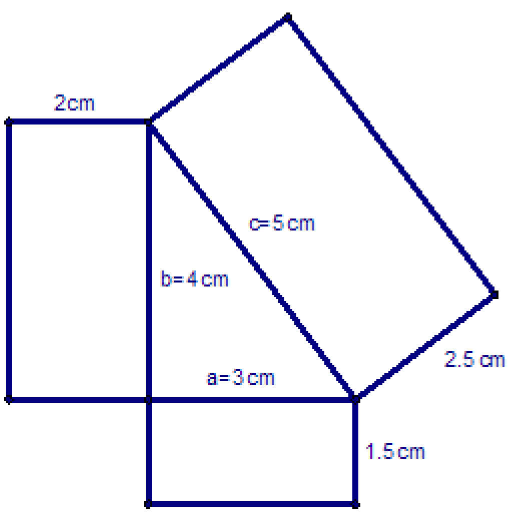


Figure 1

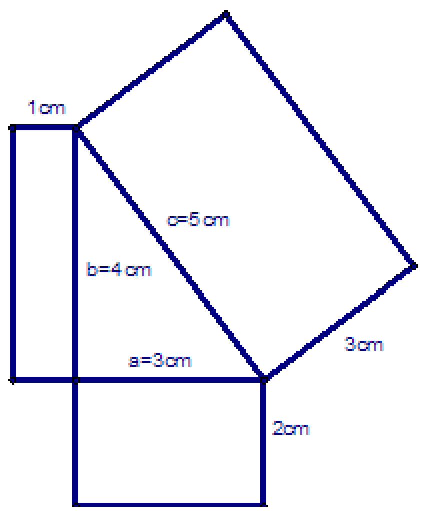


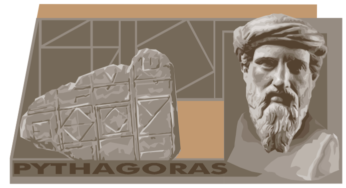
Figure 2

6. Why do you think the Pythagorean Theorem uses squares instead of other similar figures to express the relationship between the lengths of the sides in a right triangle?

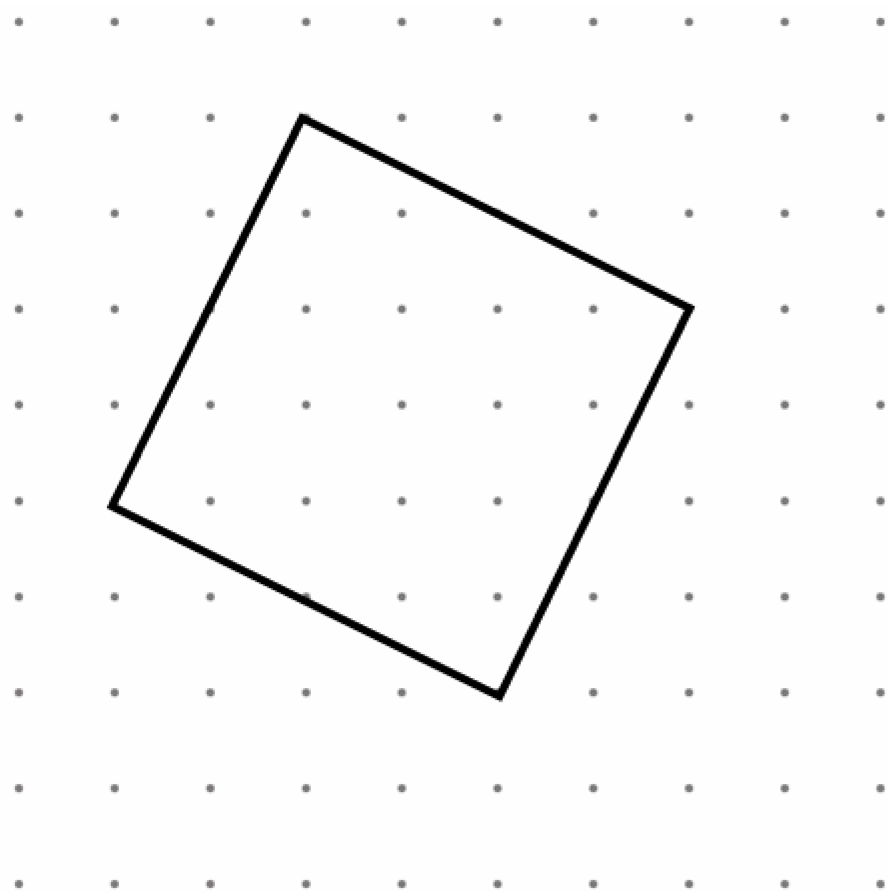
Possible Solution:

The computation involved with finding the area of squares only requires knowing the length of the side. So, even though the relationship works for any other similar figures, the computation of the area involves knowing more information then you are given for any other type of polygon that is formed on the legs of the original right triangle.

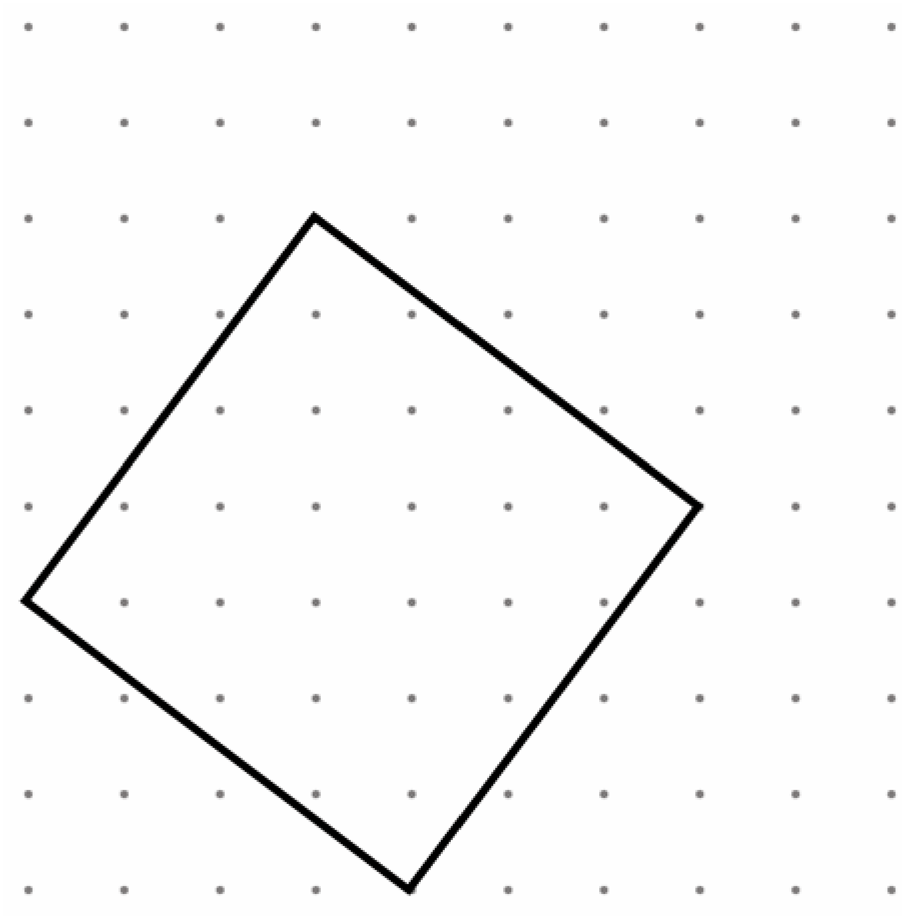
Pythagoras Plus



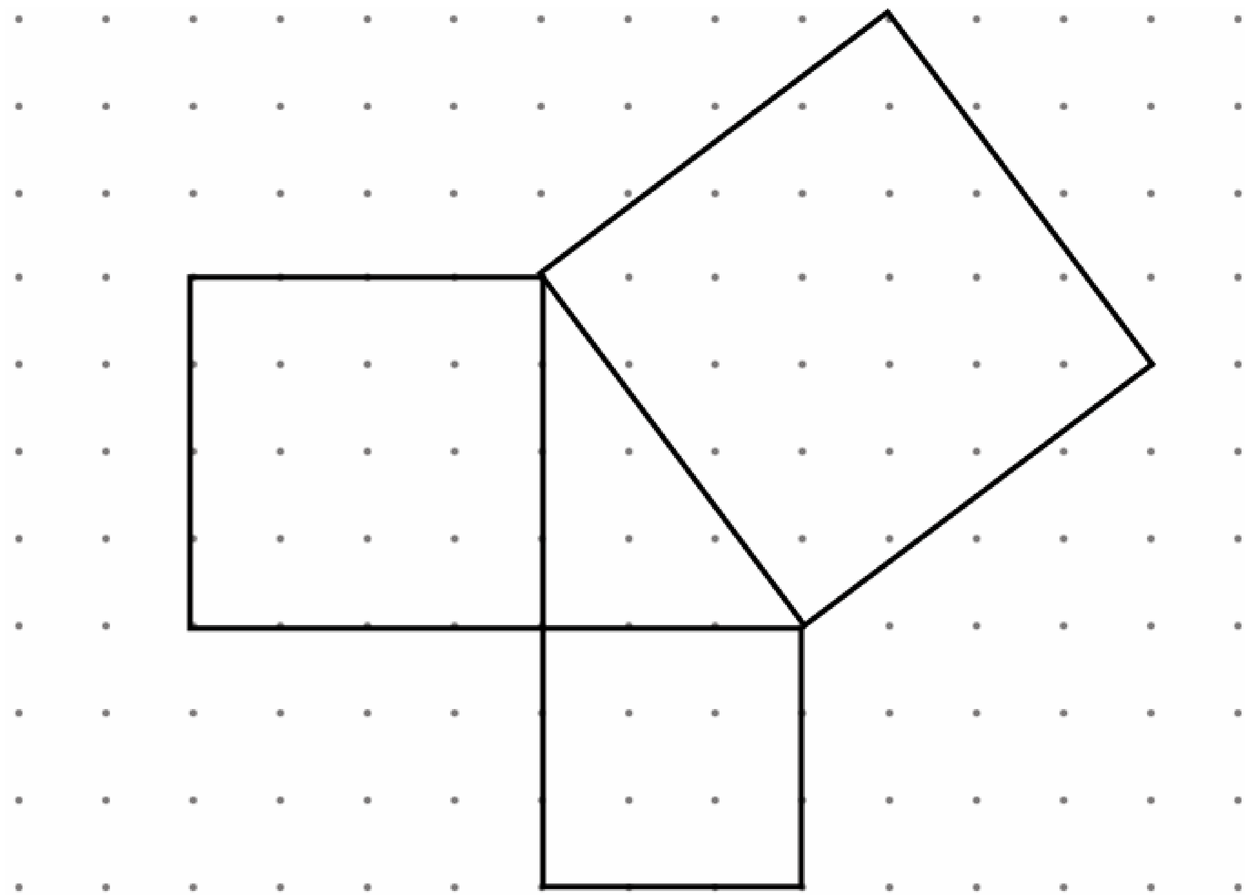
1. Find the exact area (in square units) of the figure below. Explain your method(s).



2. Find the exact area (in square units) of the figure below. Explain your method(s).



3. Find the areas of the squares on the sides of the triangle below.
   (Hint: How does the large square below compare to the square in problem 1 above?)

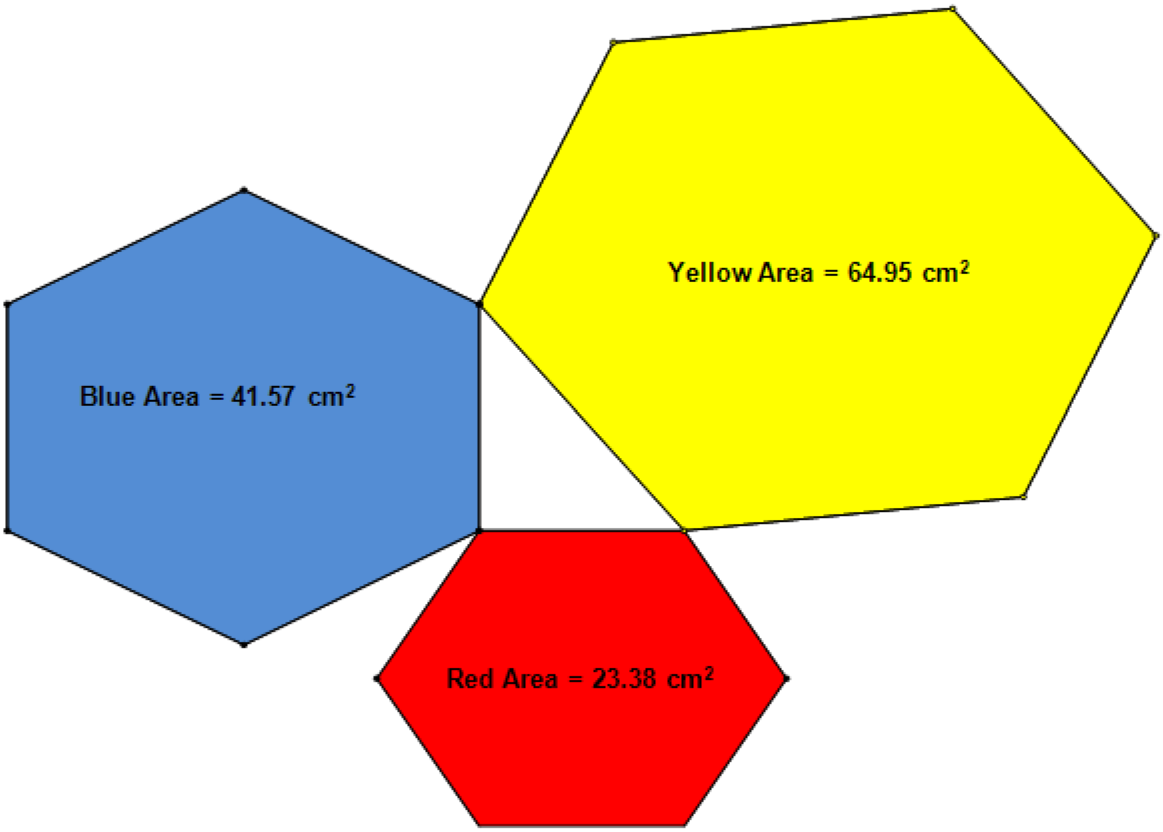


a. How do the areas of the smaller squares compare to the area of the larger square?

b. If the lengths of the shorter sides of the triangle are a units and b units and the length of the longest side is c units, write an algebraic equation that describes the relationship of the areas of the squares.

c. This relationship is called the Pythagorean Theorem. Interpret this algebraic statement in terms of the geometry involved.

4. What is the relationship between the areas of the regular hexagons constructed on the sides of the right triangle below?



5. Does the Pythagorean relationship work for other polygons constructed on the sides of right triangles? Under what condition does this relationship hold?

6. Why do you think the Pythagorean Theorem uses squares instead of other similar figures to express the relationship between the lengths of the sides in a right triangle?

Comparing TVs

Back to Task Table

In this task, students will apply the Pythagorean Theorem to real-world situations.

STANDARDS FOR MATHEMATICAL CONTENT:

Understand and apply the Pythagorean Theorem.

MGSE8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

MGSE8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

MGSE8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Work with radicals and integer exponents.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively. .
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

In order for students to be successful, the following skills and concepts need to be maintained:

- **MCC6.EE.2**
- **MCC7.EE.4**

COMMON MISCONCEPTIONS:

Students may know the Pythagorean theorem, but may be confused as to how to work their way through solving it, especially if they are trying to find the length of one of the legs rather than the hypotenuse. This misconception is bred through misunderstandings developed when solving equations. Memorizing steps to solve equations can intensify this misconception. It is best if students build understandings of how to solve simple equations conceptually, through the use of symbols first, then translate those symbols to variables.

ESSENTIAL QUESTIONS:

- How can the Pythagorean Theorem be used to solve problems?
- Why is it useful for me to know the square root of a number?
- How do I simplify and evaluate algebraic expressions involving integer exponents and square roots?

MATERIALS:

- Copies of task for each student/pair of students/or small group

GROUPING:

- Individual/Partner/Small Group

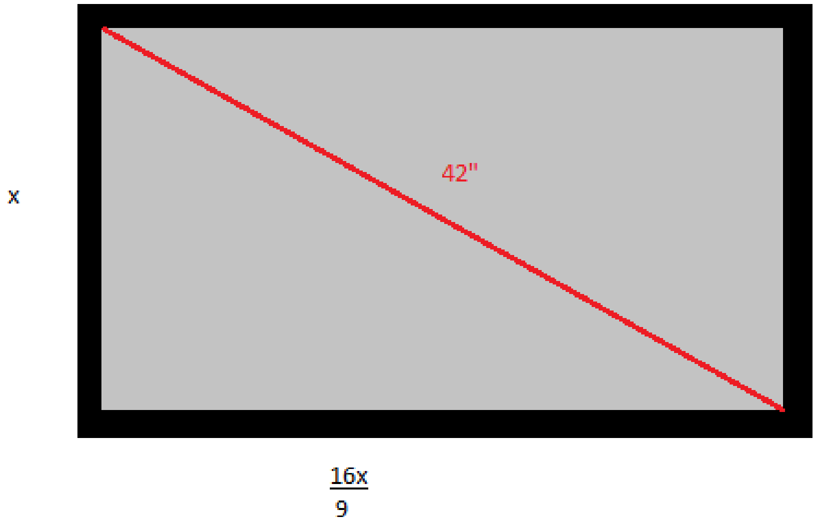
TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will apply the Pythagorean Theorem to determine the packaging needs for two television companies. Students will ***only*** solve equations involving square roots as it relates to the Pythagorean Theorem. Students will also experience rationalizing the denominator.

DIFFERENTIATION:

Extension:

Tom is about to buy a new TV. He wants to have a 42" television and wants to know how long the sides of the television are. Tom knows that that the television has the format of 16:9. The format of 16:9 means that the width of the television is 16/9 of the height which gives us something like this:



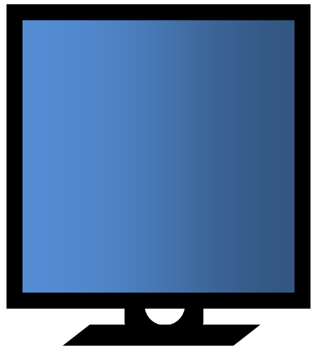
Use the Pythagorean Theorem to solve for *x* and determine the height of the TV. Then, use the value you discovered for *x* to determine the width of the TV. Show all of your work and explain how you arrived at your answer and how you would check your answer to ensure the format of 16:9 was achieved.

Adapted from: http://www.mathplanet.com/education/algebra-1/radical-expressions/the-pythagorean-theorem. Answer can be found here also.

Intervention/Scaffolding:

- Remind students about estimating square roots and comparing rational and irrational numbers, perhaps by giving a warm-up of the topic and going over it before beginning this task. Group the students in a way that promotes cooperative learning.

Comparing TVs

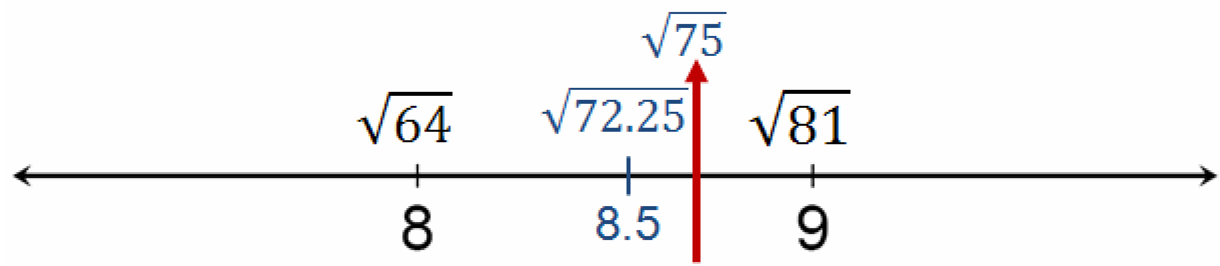


Vehicle Viewing, Inc. and Traveling TV Co. have each designed new small HD flat-screen square-shaped televisions to be used in automobiles. They feel that these will be very popular because of the desire for parents to entertain their children during the many hours spent in traffic each day. Television size is measured along the diagonal. The diagonal in the Vehicle Viewing, Inc. television is $\sqrt{75}$ inches and the Traveling TV Co. television has a diagonal of 8.5 inches. You are the owner of a packaging company and both Vehicle Viewing, Inc. and Traveling TV Co. have hired you to package their new televisions.

1. Who has the larger television? How do you know?

Solutions:

Students will need to determine which of the two diagonals, one expressed as a rational number and the other as an irrational number, is greater. Due to the fact that $\sqrt{64} = 8$ ***and*** $\sqrt{81} = 9$***, students should understand that*** $\sqrt{75}$ ***is between 8 and 9. However, 8.5 is also between 8 and 9. Because*** $(8.5)^2 = 72.25$, $\sqrt{72.25} = 8.5$. ***Because*** $\sqrt{75}$ ***is more than*** $\sqrt{72.25}$ ***, Vehicle Viewing, Inc.'s television must be larger than Traveling TV Co.'s television.***



2. To protect the screen, you need to place a protective foam sheet between the screen and the box. Find the area of each television screen so that you know how much sheeting you will need to order. Verify your results.

   Comments

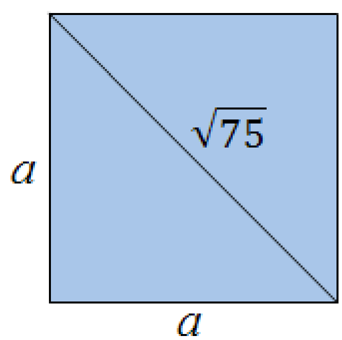
   Students will first need to determine the length of a side for each television. Since the figures are squares, each of the sides has the same length. The given diagonal cuts the square into two equal right triangles; therefore, the sides of the square are the legs of the right triangle.

   By using the Pythagorean Theorem, and setting both legs equal causes $a^2 + b^2 = c^2$ to become $a^2 + a^2 = c^2$ or $2a^2 = c^2$.

   Solutions
   For Vehicle Viewing, Inc., the length of each side is calculated below. Write a decimal approximation for an irrational number to a given decimal place.

$$2a^2 = \sqrt{75}^2$$
$$2a^2 = 75$$
$$a^2 = \frac{75}{2}$$
$$a = \sqrt{\frac{75}{2}}$$
$$\sqrt{3} \quad \sqrt{2}$$

a ≈ 6.12



To determine the amount of foam sheeting Vehicle Viewing, Inc. will need to make the protective covering for the screen, we need to remember that the area of the television screen would be $A = s^2$ where s = the length of the side of the television.

$$A = s^2$$

$$A = \left(\sqrt{75/2}\right)^2$$

$$A = \frac{75}{2} in^2$$

$$A = 37.5 in^2$$

Therefore, each of the Vehicle Viewing, Inc. televisions would need 37.5 in^2 of foam sheeting to protect the screen.

To find the length of each side of Traveling TV Co.'s television use the same thinking as shown with the Vehicle Viewing, Inc. shown earlier.

$$2a^2 = \left(8\frac{1}{2}\right)^2$$

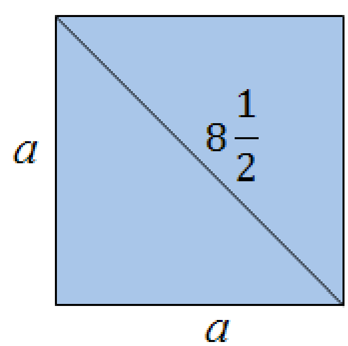
$$2a^2 = \left(\frac{17}{2}\right)^2$$

$$2a^2 = \frac{289}{4}$$

$$a^2 = \frac{289}{8}$$

$$a = \sqrt{\frac{289}{8}}$$

$$a \approx 6.01$$



The area of the screen of the Traveling TV Co. televisions would also be $A = s^2$.

$$A = s^2$$

$$A = \left(\sqrt{289/8}\right)^2$$

$$A = \frac{289}{8}in^2$$

$$A = 36\frac{1}{8}in^2$$

Each Traveling TV Co. television would need $36\frac{1}{8}$ in^2 of foam sheeting to protect the screen.

3. In order to prevent breakage, you will need to put some foam ribbon around the sides of the television. Exactly how much foam ribbon will need to be used for each television? Justify your answer.

<u>Comments:</u>

Students may realize that the EXACT answer for Vehicle Viewing is not practical for the actual people cutting the foam ribbon. However, they should understand that because the result is not a rational number, the practical results would be approximations instead of exact. Students should understand that the irrational numbers represent exact lengths. Students may find the approximations by using a calculator. Vehicle Viewing, Inc.'s televisions will need about 24.5 inches of foam ribbon to protect the perimeter of each television and Traveling TV Co.'s television would need about 24 inches of foam ribbon to protect the perimeter of each television.

Solutions:

The exact perimeter or amount of foam ribbon needed for packing each of Vehicle Viewing, Inc.'s televisions is

$$P = 4\left(\sqrt{75/2}\right)$$
$$P \approx 24.49 in.$$

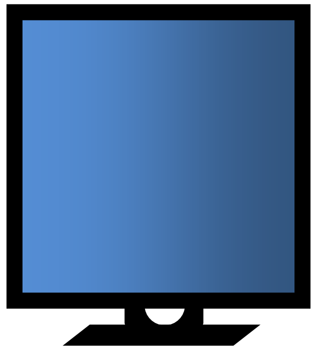
The exact amount of foam ribbon needed to pack Traveling TV Co.'s televisions is

$$P = 4\left(\sqrt{289/8}\right)$$
$$P \approx 24.04 in.$$

Extension:

Because the students are representing a packaging company, they may actually design and determine the surface area of the boxes that would be used to ship each of the televisions. The teacher may want to give a price per square inch for the cost of the foam ribbon, foam sheeting and box. Students may determine a price to charge each of the companies so that they can be assured of a certain percent profit.

Comparing TVs



Vehicle Viewing, Inc. and Traveling TV Co. have each designed new small HD flat-screen square-shaped televisions to be used in automobiles. They feel that these will be very popular because of the desire for parents to entertain their children during the many hours spent in traffic each day. Television size is measured along the diagonal. The diagonal in the Vehicle Viewing, Inc. television is $\sqrt{75}$ inches and the Traveling TV Co. television has a diagonal of 8.5 inches. You are the owner of a packaging company and both Vehicle Viewing, Inc. and Traveling TV Co. have hired you to package their new televisions.

1. Who has the larger television? How do you know?

2. To protect the screen, you need to place a protective foam sheet between the screen and the box. Find the area of each television screen so that you know how much sheeting you will need to order. Verify your results.

3. In order to prevent breakage, you will need to put some foam ribbon around the sides of the television. Exactly how much foam ribbon will need to be used for each television? Justify your answer.

Angry Bird App

Back to Task Table

Note to Teacher: This task may be challenging for some students; therefore, teachers should consider the levels of ability among their students prior to assigning.

In this task, students will apply Pythagorean Theorem to real world situations.

STANDARDS FOR MATHEMATICAL CONTENT:

Understand and apply the Pythagorean Theorem.

MGSE8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

MGSE8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

MGSE8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Work with radicals and integer exponents.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

In order for students to be successful, the following skills and concepts need to be maintained:

- **MCC6.G.3, MCC6.EE.2, MCC7.EE.4**

COMMON MISCONCEPTIONS:

- *Students have a tendency to add the areas of the squares all of the time. Students seem to have a misconception when the hypotenuse and a leg are given.*
- *Students will add the squares, but forget to take the square root of this summation.*
- *Students will have trouble is adding and subtracting integers.*

ESSENTIAL QUESTIONS:

- How can the Pythagorean Theorem be used to solve problems?
- How do I know that I have a convincing argument to informally prove the Pythagorean Theorem?
- How do I use the Pythagorean Theorem to find the length of the legs or the hypotenuse of a right triangle?
- Where can I find examples of two and three-dimensional objects in the real-world?
- How can I use the Pythagorean Theorem to find the distance between two points in a coordinate system?

MATERIALS:

- Copies of task for each student/pair of students/or small group
- Graph paper http://incompetech.com/graphpaper/
- Straightedge

GROUPING:

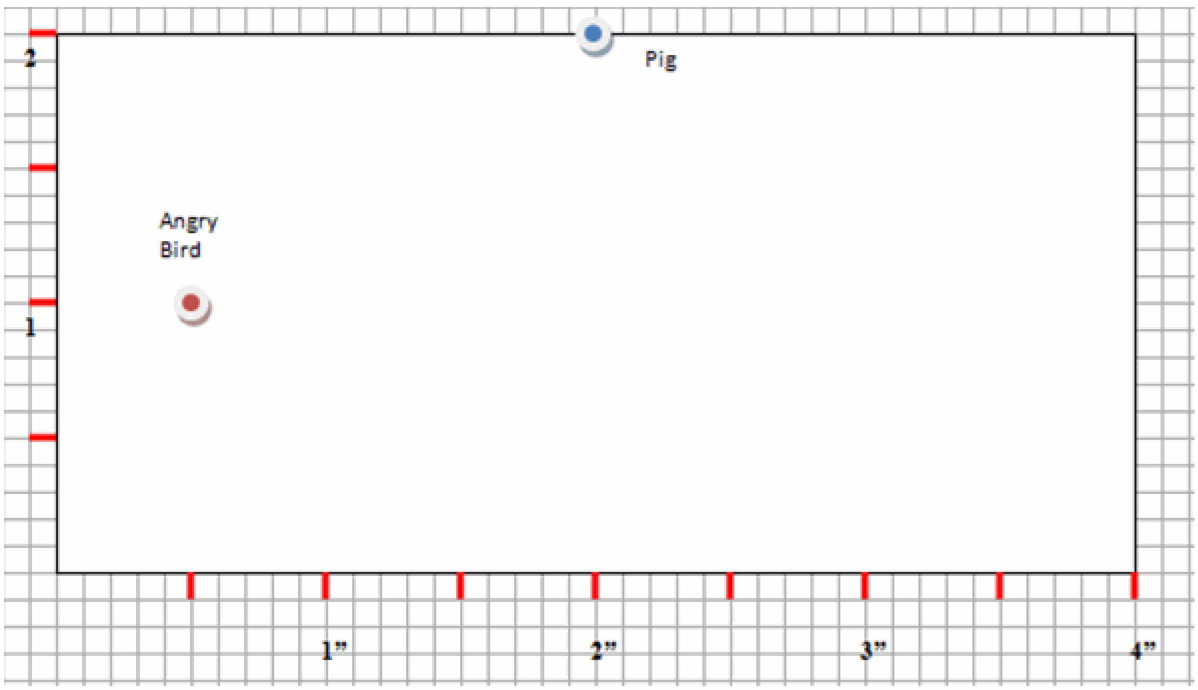
- Individual/Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

This task provides a guided discovery for finding the distance between two points using the Pythagorean Theorem. The task can be extended to help discover the relationship between the Pythagorean Theorem and the distance formula.

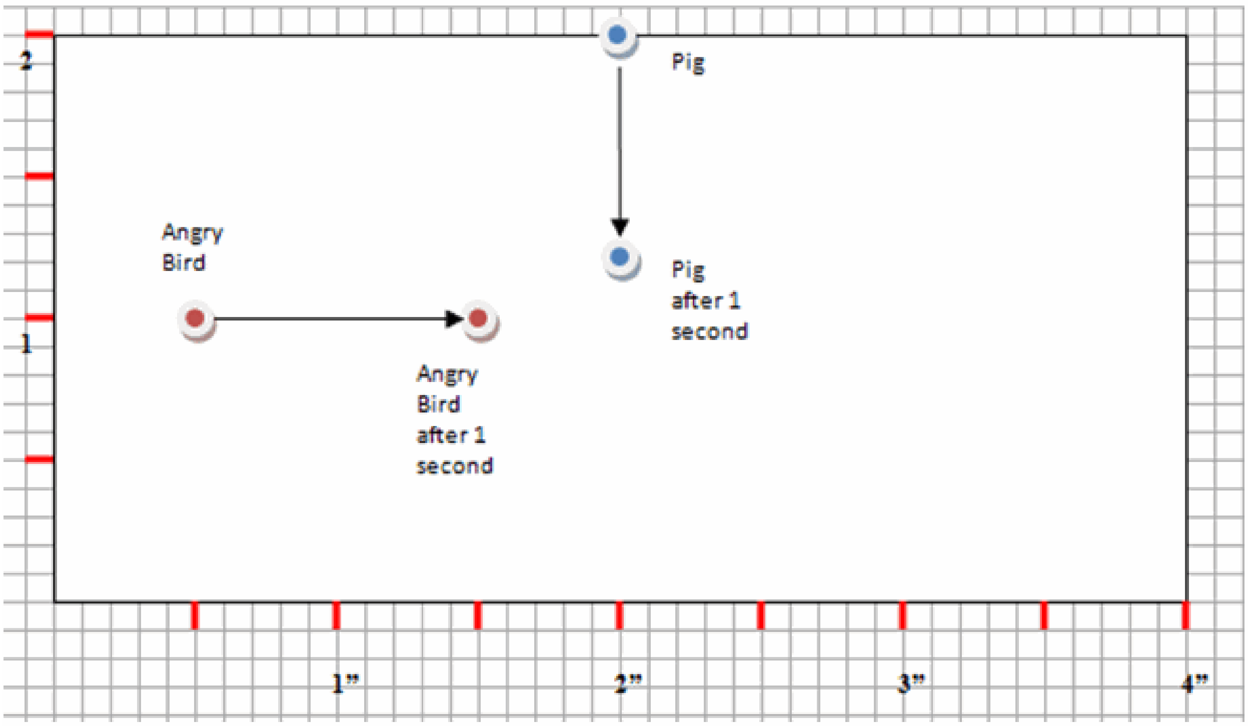
Angry Bird App

Brooke is fond of playing the popular *Angry Bird* App on his Smartphone. He plays the game on his Smartphone with a screen viewing area that is 2 inches high and four inches wide. The game uses a slingshot to launch birds with an attempt to retrieve eggs that have been taken by pigs stationed in various structures. At the beginning, Brooke's A*ngry Bird* starts ½ inch from the left edge and half way between the top and bottom of the screen. The *Pig* starts out at the top of the screen and at the ½ point of the top edge.



***Comment:* Some students may need help understanding that that the height and width of the screen depicts the horizontal and vertical position (x, y) in the coordinate plane.**

<u>Starting Position:</u> As the game starts, Brooke's *Angry Bird* moves directly to the right at a speed of 1 inch per second. For example, Brooke's *Angry Bird* moves 0.1 inches in 0.1 seconds, 2 inches in 2 seconds, etc. The *Pig* moves directly downward at a speed of 0.8 inches per second.



<u>Angry Bird App – Part I</u>

After One Second

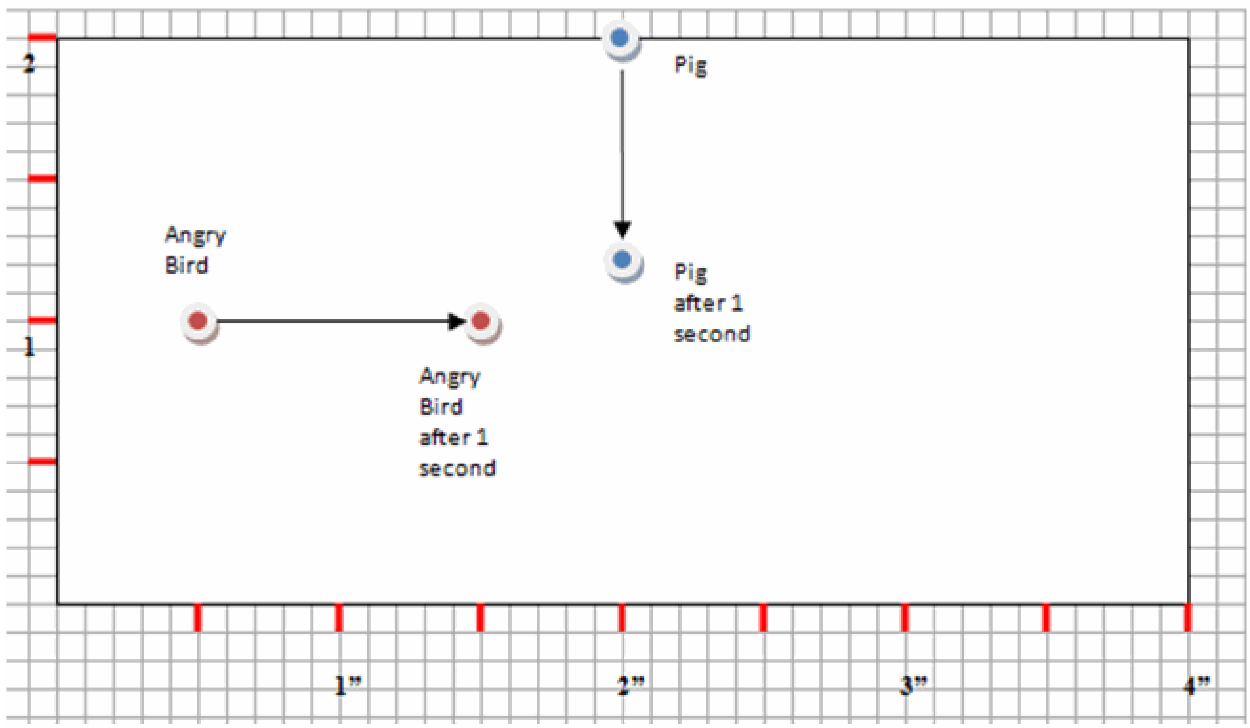
Let time be denoted in this manner: $t = 1$ which means the position of *Angry Bird* and the *Pig* after one second.

1. Using graph paper, plot the positions of *Angry Bird* and the *Pig* at times $t = ¼$, $t = ½$, $t = 1$, and other times of your choice.

Comments:

Watch students carefully. One goal of this task is to get students to realize there are times when grids allow us to solve a problem more easily. If students try to draw this to scale on white paper it may become confusing or difficult to work with.

You might want to allow a certain amount of time (5 – 10 minutes) for students to determine a way to draw this and then have a class discussion to decide the easiest way to represent this problem. Hopefully, at least one group will have chosen graph paper. If choosing graph paper (highly recommended) the students need to select the scale very carefully. Since Pig is moving at a pace that is more difficult to represent, a grid counting by tenths is easiest to use to show movement. (See figure below.)



Comments:

Students need to choose their scale very carefully. Since Pig is moving at a pace that is more difficult to represent, a grid counting by tenths is easiest to move her token along. If students are having difficulty determining the position of Angry Bird and Pig, encourage them to place them where they will be in one second, and then work backwards to ½ second and then ¼ second.

Solutions:

Answers will vary for times other than those indicated in the problem.

2. Describe the possible movements for *Angry Bird*.

   Solutions:

   Answers will vary. Sample answer:

   Angry Bird will move from left to right in the middle of the video screen.
   or
   Angry Bird will move horizontally, from left to right, along a line that is 1 inch from the top and bottom of the screen.

3. Describe the movements possible for *Pig*.

   Solutions:

   Answers will vary. Sample answer:

   Pig will move from top to bottom in the middle of the video screen.
   or
   Pig will move vertically, from top to bottom, along a line that is 2 inches from the left and right sides of the screen.

4. Describe the movements of both players relative to each other.

   Solutions:

   Answers will vary. Sample answer:

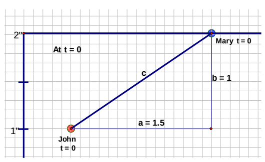
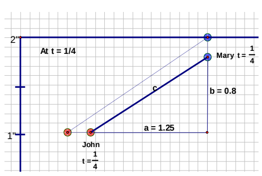
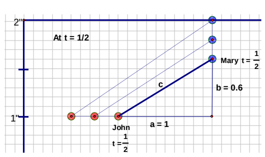
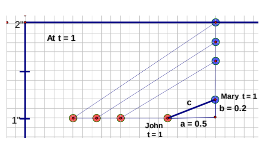
   Angry Bird moves faster than Pig. They are getting closer to each other. It looks like there is a possibility that they could crash into each other.

5. Record the position and find the distance between *Angry Bird* and the *Pig* at times $t = 0$, $t = ¼$, $t = ½$, t = 1 using the Pythagorean Theorem. Use the space below table to compute the distance.

Comments:
Students should be able to apply the Pythagorean Theorem in order to find the distance between the two points. If the students are using Geometry Software, they still need to do these calculations by hand since the questions will build upon this to develop the distance formula.

Solutions:

Create a right triangle on the grid using the tokens as the vertices of the acute angles. The distance between Angry Bird and Pig is the length of the hypotenuse, c, of the triangle.

Position ***(t = ?)***	**Angry Bird** ***(speed in)***	**Pig** ***(height)***	$c = \sqrt{a^2 + b^2}$
$t = 0$ 	$A = 1.5$	$B = 1$	$c^2 = 1.5^2 + 1^2$ $c = \sqrt{3.25}$ $c \approx 1.80$
$t = ¼$ 	$A = 1.25$	$B = 0.8$	$c^2 = 1.25^2 + 0.8^2$ $c = \sqrt{2.2025}$ $c \approx 1.48$
$t = ½$ 	$A = 1$	$B = 0.6$	$c^2 = 1^2 + 0.6^2$ $c = \sqrt{1.36}$ $c \approx 1.17$
$t = 1$ 	$A = 0.5$	$B = 0.2$	$c^2 = 0.5^2 + 0.2^2$ $c = \sqrt{0.29}$ $c \approx 0.54$

If the *Pig* gets closer than ¼ inch to an *Angry Bird*, then a *Pig* will destroy an *Angry Bird*, and *Pig* will get 10,000 points. However, if an *Angry Bird* hits when it is positioned less than ½ inch apart and more than ¼ inch apart, then the *Angry Bird* destroys the *Pig*, retrieve its eggs, and *Angry Bird* gets 10,000 points.

6. Find a time at which *Angry Bird* can destroy the *Pig* and its' structures and earn 10,000 points. Draw the configuration at this time.

   a. Compare your answers with your group. What did you discover?

   Comments:

   This question is designed to get students thinking about the problem and predicting what will happen. At this point a specific time is not needed.

   Solutions:

   Using the information from #5 the students should choose an answer somewhere between 1 and 1 ¼ seconds. Answers may vary:

   Angry Bird needs to wait more than one minute.
   Or
   If they continue in the pattern for another 1 ½ seconds, they will be exactly ¼ inch apart. So he has to press the button before then.

   b. Estimate the longest amount of time *Angry Bird* could wait before pressing the button.

   Solutions:

   At 1 ¼ seconds they will be exactly ¼ inch apart. So, he can wait 1.24 seconds or 1.249 seconds… it just has to be less than 1.25 seconds.

7. Drawing pictures gives an estimate of the critical time, but inside the video game, everything is done with numbers. Describe in words the mathematical concepts needed in order for this video game to work.

   Solutions:

   Answers may vary. Concepts students may mention include measurement, graphing, using coordinate grids, rates of speed, Pythagorean Theorem…

Angry Bird App - Part II

Inside the computer game the distance between the *Angry Bird* and *Pig* are computed using a mathematical formula based on the coordinates of each opponent. Our goal now is to develop this formula.

To help us think about the distance between *Angry Bird* and *Pig*, it may help us to look first at a one-dimensional situation. Let's look at how you determine distance between two locations on a number line:

8. What is the distance between 5 and 7? 7 and 5? -1 and 6? 5 and -3?

Comments:

This question is intended to get students thinking about using a formula to find the distance between two points. Students can easily draw a number line and count to find the distance between the given points.

Solutions:

Distance between 5 and 7 is 2. This can be found be simply subtracting 5 from 7. It can also be found by subtracting 7 from 5. The difference is whether the answer is positive or negative. Since distance should always be positive, taking the absolute value of the difference between the numbers will give you the distance between the two points.

$|7 – 5| = 2$ ***or*** $|5 – 7| = |-2| = 2$
$|6 - -1| = |6 +1| = 7$ ***or*** $|-1 - 6| = |-7| = 7$
$|5 - -3| = |5 +3| = 8$ ***or*** $|-3 - 5| = |-8| = 8$

9. What formula can you derive to find the distance between two points, ***a*** and ***b***, on a **number line**?

Comments:

At this point, students need to formalize their findings from above.

Solutions:

Distance between a and b is $|a – b|$.

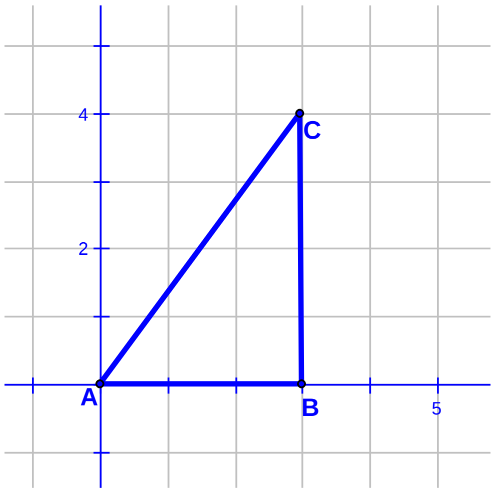
Now that you can find the distance on a number line, let's look at finding distance on the coordinate plane:

Comments:

Using centimeter graph paper and the correct scale is critical to the successful completion of these next questions. 1 unit on the graph needs to be equal to one centimeter. They also need to include all four quadrants on the graph.

10. Label and plot the points *A*= (1, 0), *B* = (4, 0) and *C* = (4, 4) on coordinate grid below.

Solution:



11. Find the length of the segment *AC* when given point *A* = (1, 0) to point *C* = (4, 4), by using the Pythagorean Theorem.

Solutions:

With a scale of 1 unit = 1 cm, the length of AC is 5 cm.

12. Consider the triangle *ABC*;

a. What kind of triangle is formed?

Solutions:

Right triangle

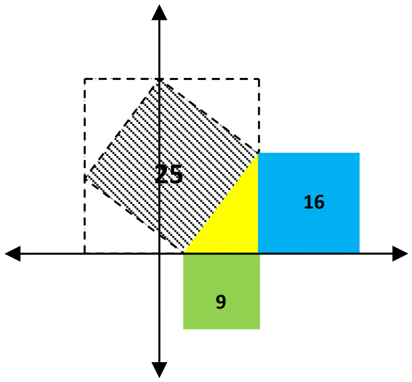
b. Justify your reasoning for this type of triangle either by decomposing the square or by proving the converse of the Pythagorean Theorem.

Solutions:

Converse of the Pythagorean Theorem – If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

If $c^2 = a^2 + b^2$, then ▲ABC is a right triangle.

Or

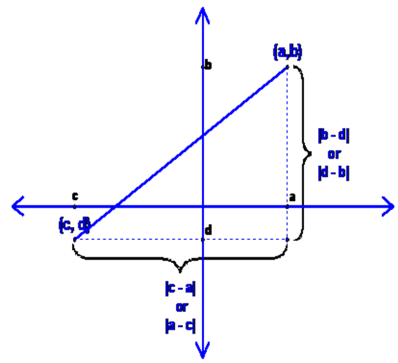


Angry Bird App – Extension

13. Using the points given in problem 10, find the distance between the *x*-coordinate and *y*-coordinate pairs. Write an algebraic equation to represent finding the length of segment *AC*. ***Let d = length of segment AC.***

Comments:

Note: d correlates to the hypotenuse as noted in the Pythagorean Theorem. In the examples above, one leg of the right triangle is always parallel to the x-axis while the other leg is always parallel to the y-axis. Using the coordinates of the given points, the vertical length is always the difference of the x-coordinates of the points while the horizontal length is always the difference of the y-coordinates of the points.



Solutions:

A= (1, 0), B = (4, 0) and C = (4, 4)
x-coordinate = line segment AB = $|x_2 - x_1| = |4 - 1| = 3$
y-coordinates = line segment BC = $|y_2 - y_1| = |4 - 0| = 4$
d = length of line segment AC = hypotenuse =

$$d^2 = |a - c|^2 + |b - d|^2$$

$$d = \sqrt{(a - b)^2 + (c - d)^2}$$

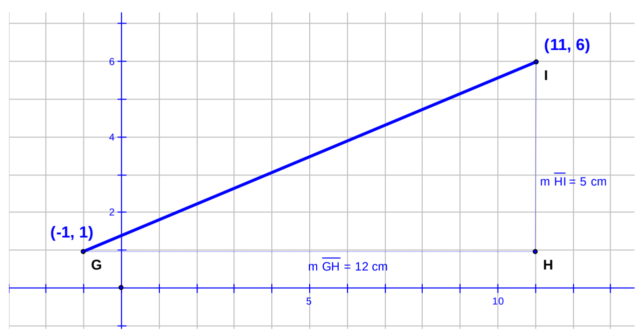
$$d = \sqrt{(4-1)^2 + (4-0)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

14. Using the formula derived in problem 13, find the distance between the two points.

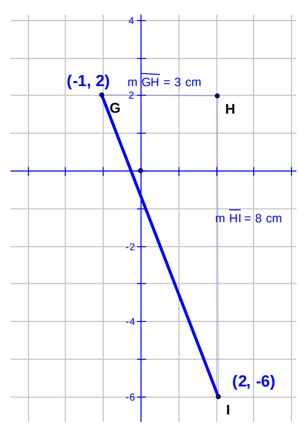
a. (-1, 1) and (11, 6)

Solutions:

d = 13



b. (-1, 2) and (2, -6)



Solutions:

$$d = \sqrt{73}$$

15. Do you think your formula would work for any pair of points? Why or why not?

Comments:

Encourage students to write one simple formula that will work all the time. To help students understand why the absolute value signs are not needed, discuss what happens to a number when you square it. Since the value, when squared, is always positive, it's not necessary to keep the absolute value signs.

Ask your students: What's different in this example?
In the examples, it didn't matter what order you subtracted. Why is this true here? When is this not true?

Solutions:

Groups may come up with slightly different solutions to this problem. All of the answers below are correct. Students should discuss the similarities and differences and why they are all valid formulas. Make sure to include a discussion of the role of mathematical properties.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \qquad d = \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$d = \sqrt{(x_1 - x_2)^2 + (y_2 - y_1)^2}$$

Let's revisit the *Angry Bird* App, refer to the coordinate grid in problem 1. Using the formula derived in problem 13, compute the distance between two points.

16. Write an ordered pair and find the distance between *Angry Bird* and *Pig* when $t = 0$, when $t = ¼$, when $t = ½$ and $t = 1$. Use the space below to compute the distance.

Comments:

When students start writing the coordinates and find the distances for these problems, you might need to direct some students to write their information in a chart. It's much easier to find the patterns in the coordinates that way.

Position	*Angry Bird*	*Pig*	*Distance between the two points*

(t=?)	*(x_1, y_1)*	*(x_2, y_2)*	
t = 0	*(0.5, 1)*	*(2, 2)*	at $t = 0$ $d = \sqrt{(0.5 - 2)^2 + (1 - 2)^2}$ $d = \sqrt{(1.5)^2 + (-1)^2}$ $d = \sqrt{2.25 + 1}$ $d = \sqrt{3.25}$ $d \approx 1.80$
t = ¼	*(0.75, 1)*	*(2, 1.8)*	$d = \sqrt{(2 - 0.75)^2 + (1.8 - 1)^2}$ $d = \sqrt{1.25^2 + 0.8^2}$ $d = \sqrt{1.5625 + 0.64}$ $d = \sqrt{2.2025}$ $d \approx 1.48$
t = ½	*(1, 1)*	*(2, 1.6)*	at $t = 1/2$ $d = \sqrt{(1 - 2)^2 + (1 - 1.6)^2}$ $d = \sqrt{(-1)^2 + (-0.6)^2}$ $d = \sqrt{1 + 0.36}$ $d = \sqrt{1.36}$ $d \approx 1.17$
t =1	*(1.5, 1)*	*(2, 0.8)*	at $t = 1$ $d = \sqrt{(1.5 - 2)^2 + (1 - 1.2)^2}$ $d = \sqrt{(-0.5)^2 + (-0.2)^2}$ $d = \sqrt{0.25 + 0.04}$ $d = \sqrt{0.29}$ $d \approx 0.54$

17. Compare the distance computed when using the Pythagorean Theorem formula derived to find the distance between two points
Solutions:
The students should notice that the distance computed using Pythagorean Theorem and the distance formula are the same.

18. What is the correlation to the Pythagorean Theorem and finding the distance between two points?
Solutions:
Students should recognize that Pythagorean Theorem is used to find the lengths of segments whereas the distance formula computes distance between a given set of points. Both formulas are used to find missing lengths (segments), and/or distances for any polygons that form a right angle.

19. Write an equation to find the distance between two points. Let d = distance, (x_1, y_1) = first ordered pair, (x_2, y_2) = second ordered pair.

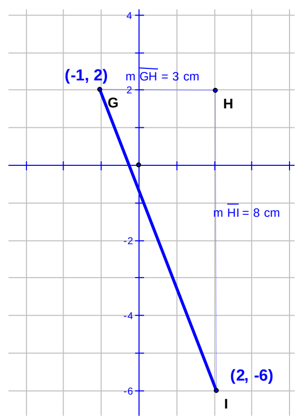
Solutions:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

20. Graph and label the equation you derived for the distance between two points.

Comments:

Note: Remind the students that distance is always positive.
Answers may vary.



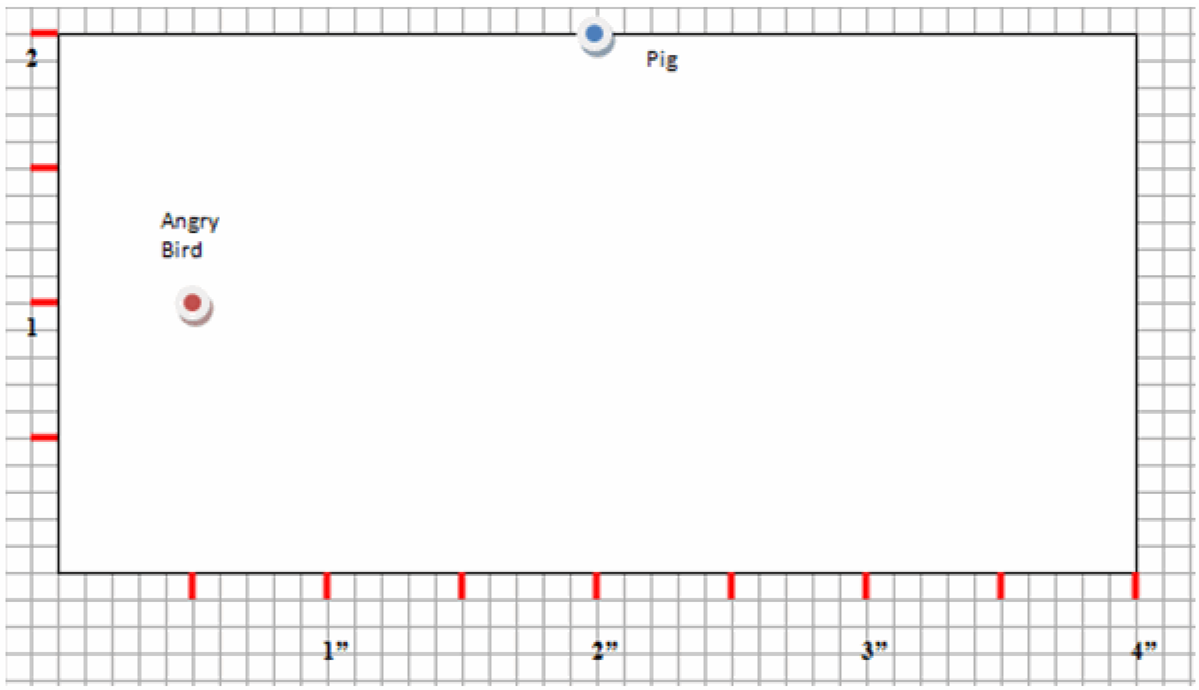
21. What are the characteristics of this graph relative to the Pythagorean Theorem and a triangle?

Comments:
This may be a good lead in to linear functions. You may make a connection of how slope of a linear function relates to the distance formula.

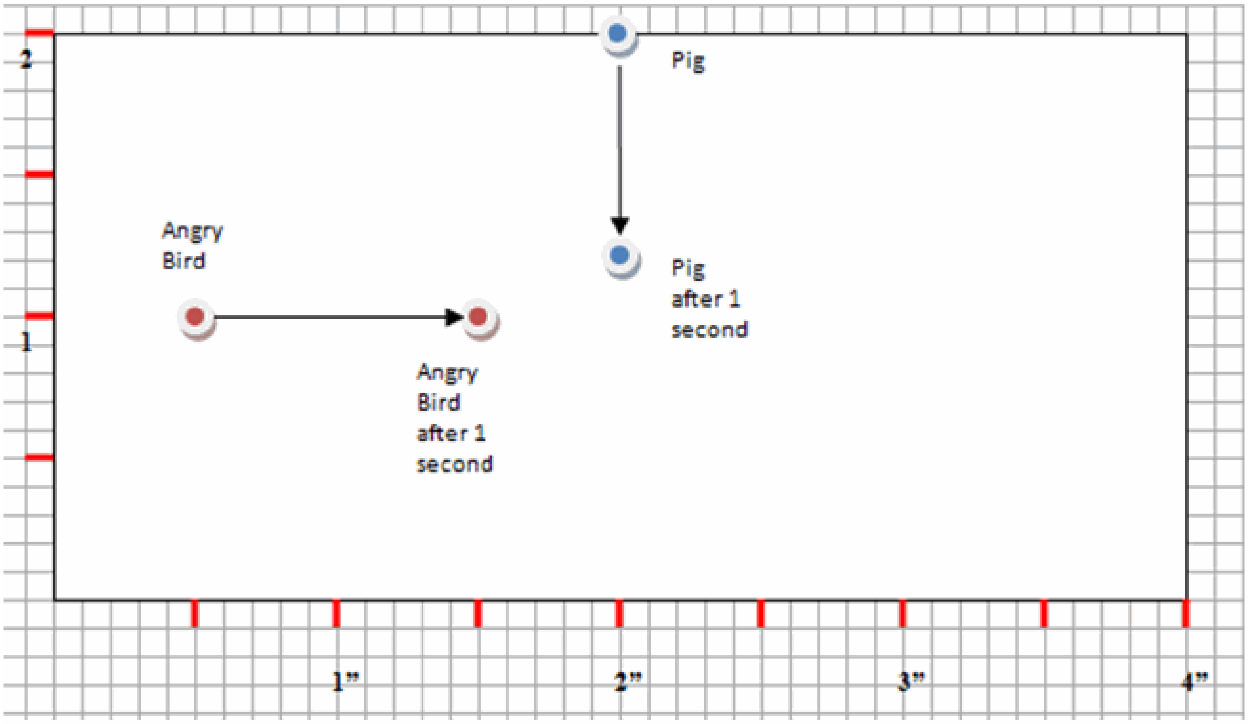
Solutions
The points on the x-axis and y-axis form the legs of the right triangle. The longest distance between the points is the hypotenuse (diagonal) of the right triangle.

Angry Bird App

Brooke is fond of playing the popular *Angry Bird* App on his Smartphone. He plays the game on his Smartphone with a screen viewing area that is 2 inches high and four inches wide. The game uses a slingshot to launch birds with an attempt to retrieve eggs that have been taken by pigs stationed in various structures. At the beginning, Brooke's A*ngry Bird* starts ½ inch from the left edge and half way between the top and bottom of the screen. The *Pig* starts out at the top of the screen and at the ½ point of the top edge.



Starting Position: As the game starts, Brooke's *Angry Bird* moves directly to the right at a speed of 1 inch per second. For example, Brooke's *Angry Bird* moves 0.1 inches in 0.1 seconds, 2 inches in 2 seconds, etc. The *Pig* moves directly downward at a speed of 0.8 inches per second.



<u>Angry Bird App – Part I</u>

After One Second

Let time be denoted in this manner: $t = 1$ which means the position of *Angry Bird* and the *Pig* after one second.

1. Using graph paper, plot the positions of *Angry Bird* and the *Pig* at times $t = \frac{1}{4}$, $t = \frac{1}{2}$, $t = 1$, and other times of your choice.

2. Describe the possible movements for *Angry Bird.*

3. Describe the movements possible for *Pig.*

4. Describe the movements of both players relative to each other.

5. Record the position and find the distance between *Angry Bird* and the *Pig* at times $t = 0$, $t = ¼$, $t = ½$, t = 1 using the Pythagorean Theorem. Use space below the table to compute the distance.

Position ***(t = ?)***	**Angry Bird** ***(speed in)***	**Pig** ***(height)***	$c = \sqrt{a^2 + b^2}$
$t = 0$			
$t = ¼$			
$t = ½$			
$t = 1$			

If the *Pig* gets closer than ¼ inch to an *Angry Bird*, then a *Pig* will destroy an *Angry Bird*, and *Pig* will get 10,000 points. However, if an *Angry Bird* hits when it is positioned less than ½ inch apart and more than ¼ inch apart, then the *Angry Bird* destroys the *Pig,* retrieve its eggs, and *Angry Bird* gets 10,000 points.

6. Find a time at which *Angry Bird* can destroy the *Pig* and its' structures and earn 10,000 points. Draw the configuration at this time.

   a. Compare your answers with your group. What did you discover?

   b. Estimate the longest amount of time Brooke could wait before pressing the button.

7. Drawing pictures gives an estimate of the critical time, but inside the video game, everything is done with numbers. Describe in words the mathematical concepts needed in order for this video game to work.

Angry Bird App - Part II

Inside the computer game the distance between the *Angry Bird* and *Pig* are computed using a mathematical formula based on the coordinates of each opponent. Our goal now is to develop this formula.

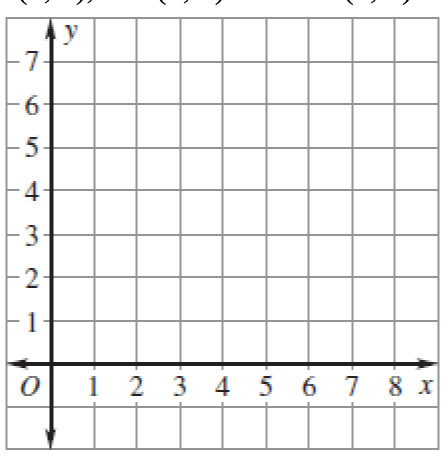
To help us think about the distance between *Angry Bird* and *Pig*, it may help us to look first at a one-dimensional situation. Let's look at how you determine distance between two locations on a number line:

8. What is the distance between 5 and 7? 7 and 5? -1 and 6? 5 and -3?

9. What formula can you derive to find the distance between two points, ***a*** and ***b***, on a **number line**?

Now that you can find the distance on a number line, let's look at finding distance on the coordinate plane:

10. Label and plot the points $A= (1, 0)$, $B = (4, 0)$ and $C = (4, 4)$ on coordinate grid below.



11. Find the length of the segment *AC* when given point $A = (1, 0)$ to point $C = (4, 4)$, by using the Pythagorean Theorem formula.

12. Consider the triangle *ABC*;

    a. What kind of triangle is formed?

    b. Justify your reasoning for this type of triangle either by decomposing the square or by proving the converse of the Pythagorean Theorem.

Angry Bird App - Extension

13. Using the points given in problem 10, find the distance between the *x*-coordinate and *y*-coordinate pairs. Write an algebraic equation to represent finding the length of segment *AC*. ***Let d = length of segment AC.***

14. Using the formula derived in problem 13, find the distance between the two points.

    a. (-1, 1) and (11, 6)

    b. (-1, 2) and (2, -6)

15. Do you think your formula would work for any pair of points? Why or why not?

Let's revisit the *Angry Bird* App, refer to the coordinate grid in problem 1. Using the formula derived in problem 13, compute the distance between two points.

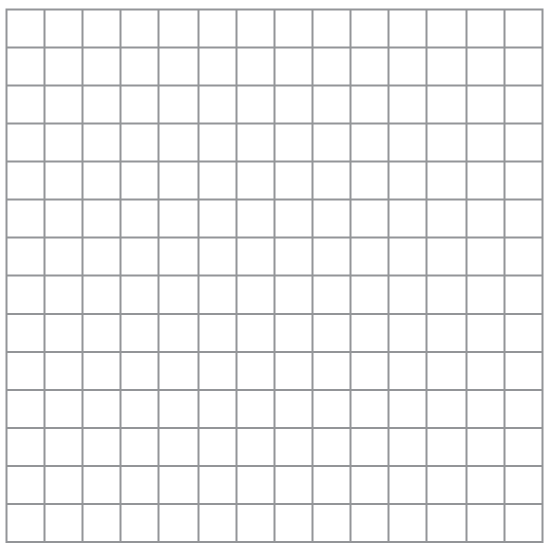
16. Write an ordered pair and find the distance between *Angry Bird* and *Pig* when $t = 0$, when $t = ¼$, when $t = ½$ and $t = 1$. Use the space below to compute the distance.

17. Compare the distance computed when using the Pythagorean Theorem formula derived to find the distance between two points.

18. What is the correlation to the Pythagorean Theorem and finding the distance between two points?

19. Write an equation to find the distance between two points. Let d = distance, (x_1, y_1) = first ordered pair, (x_2, y_2) = second ordered pair.

20. Graph and label the equation you derived for the distance between two points.



21. What are the characteristics of this graph relative to the Pythagorean Theorem and a triangle?

Constructing the Irrational Number Line

Back to Task Table

Teacher Note: The directions for this task are vague. Do not be alarmed if your students don't come up with the solutions shown. Encourage students to explore the irrational number line and demonstrate their understanding of reasonable measures on the number line.

In this task, students will explore the Pythagorean Theorem and its converse.

STANDARDS FOR MATHEMATICAL CONTENT:

Understand and apply the Pythagorean Theorem.

MGSE8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

MGSE8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

MGSE8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Work with radicals and integer exponents.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE:

In order for students to be successful, the following skills and concepts need to be maintained:

- **MCC6.G.3, MCC6.EE.2, MCC7.EE.4**

COMMON MISCONCEPTIONS:

- *Students have a tendency to add the areas of the squares all of the time. Students seem to have a misconception when the hypotenuse and a leg are given.*
- *Students will add the squares, but forget to take the square root of this summation.*
- *Students will have trouble adding and subtracting integers*.

ESSENTIAL QUESTIONS:

- How can the Pythagorean Theorem be used to solve problems?
- What is the Pythagorean Theorem and when does it apply?
- Why is it useful for me to know the square root of a number?
- How do I simplify and evaluate algebraic expressions involving integer exponents and square roots?

MATERIALS:

- Copies of task for each student/pair of students/or small group
- Compass
- Straightedge
- Graph paper http://incompetech.com/graphpaper/

GROUPING:

- Individual/Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

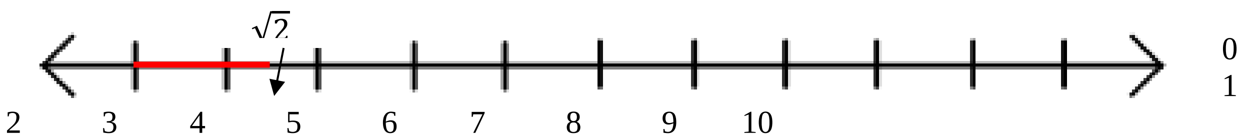
In this task, students will construct a number line consisting of rational and irrational numbers. The Pythagorean Theorem and geometric construction techniques are used throughout this task to determine the location of several rational and irrational numbers and investigate the geometric representation of an irrational number.

Constructing the Irrational Number Line

In this task, you will construct a number line with several rational and irrational numbers plotted and labeled. Start by constructing a right triangle with legs of one unit. Use the Pythagorean Theorem to compute the length of the hypotenuse. Next, copy the segment forming the hypotenuse to a line and mark one left endpoint of the segment as 0 and the other endpoint with the irrational number it represents.

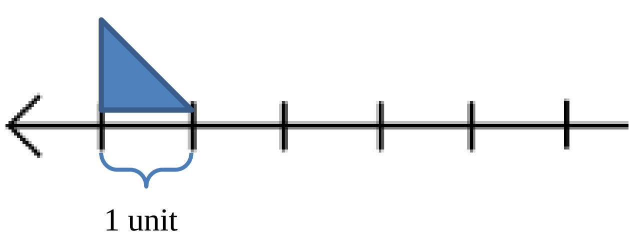
Construct other right triangles with two sides (either the two legs or a leg and a hypotenuse) that have lengths that are multiples of the unit you used in the first triangle. Then transfer the lengths of each hypotenuse to a common number line, and label the point that it represents. After you have constructed several irrational lengths, list the irrational numbers in order from smallest to largest.

For example, after constructing the first right triangle (leg length 1 unit), Students will construct a number line similar to the following (the unit here is approximately .5 inch):



The task for students now is to find the next irrational number. This may be done through trial and error. However, students may discover that if they keep one of the legs the same (one unit), and increase the other leg by one unit, they will get the next irrational number ($\sqrt{3}$). Students may continue to follow this pattern and discover the irrationals as well as some rationals. Once they construct the hypotenuse of these triangles, they can use their compass to translate that length to the number line.

It is important to note that the number line units should be the same length as the units on the triangles.



Comments:

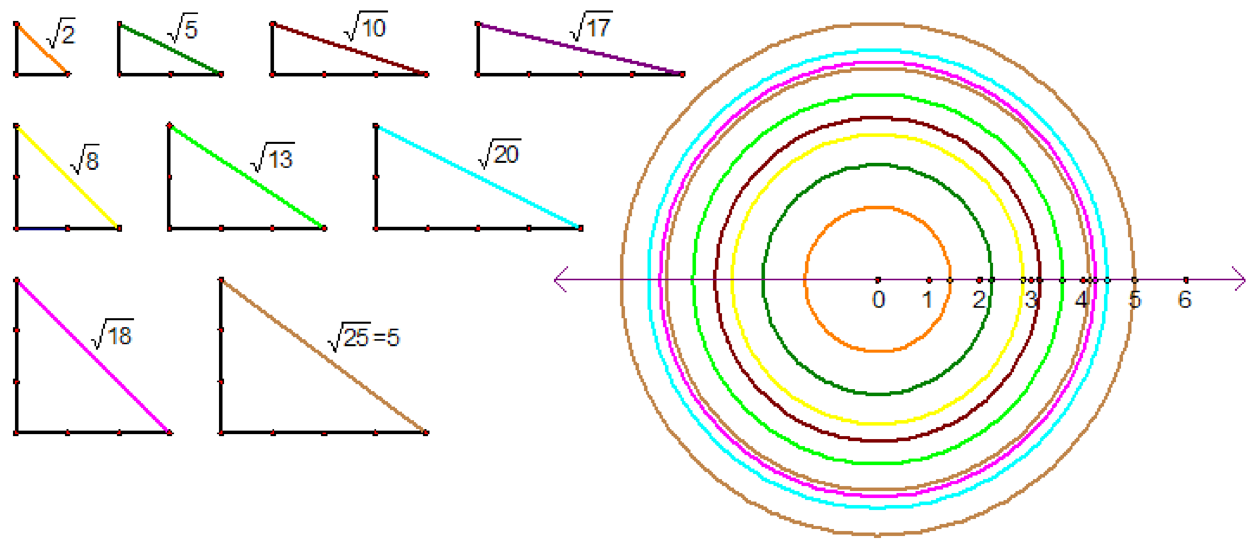
It is suggested that this task be done on graph paper to help students maintain the same unit of measure for each side of each triangle as well as for the number line.

Students will need to use a compass and straightedge to copy the segments representing the rational and irrational lengths formed by the hypotenuse of each triangle to the number line. Be sure that students use 0 as one endpoint and label the other endpoint as the distance represented.

<u>Solutions:</u>

Using different combinations of sides from 1 unit to 4 units, students can construct a variety of hypotenuse lengths including: $\sqrt{2}, \sqrt{5}, \sqrt{8}, \sqrt{10}, \sqrt{13}, \sqrt{17}, \sqrt{18}$ *and* $\sqrt{20}$***. Also, students may construct some special triangles called Pythagorean triples where all sides of the right triangle have integer lengths (as an example, see the 3-4-5 triangle). Can they find others?***

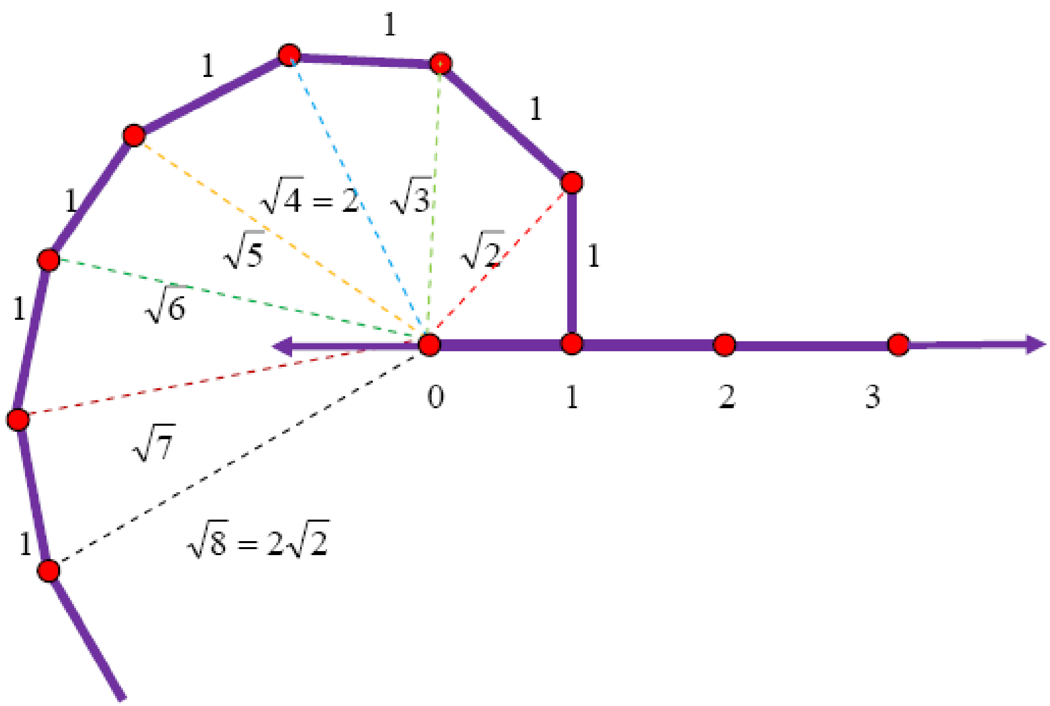
Using unit lengths for the sides, students will get some of the following:



Students should find that by plotting the irrational numbers on the number line, they are automatically ordered from least to greatest. When irrational numbers are ordered from least to greatest, they get numbers such as

$\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{10}...$ ***with rational numbers like*** $\sqrt{1}, \sqrt{4}, \sqrt{9}.$

Another method for constructing a variety of irrational lengths is to recursively construct another right triangle from the first one with the hypotenuse as one of the legs and the other leg a unit of one. This method will construct irrational and rational numbers in an increasing order.

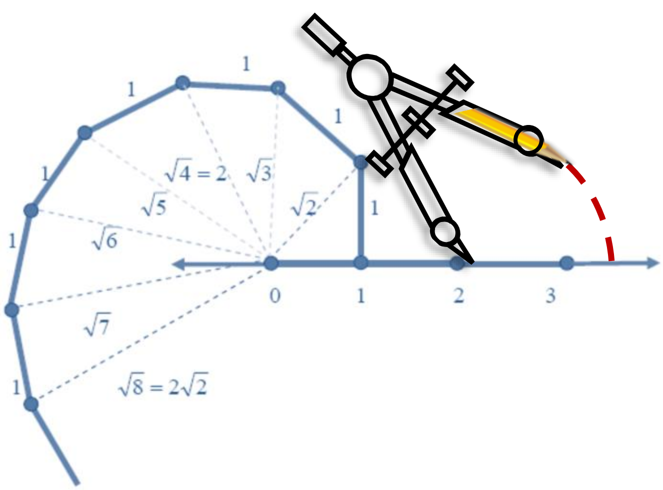


<u>*Leg lengths*</u> $\mathbf{1}$ ***unit***
<u>*Pythagorean Theorem*</u> $a^2 + b^2 = hypotenuse^2$ $1^2 + 1^2 = hypotenuse^2$ $2 = hypotenuse^2$ $\sqrt{2} = hypotenuse$
<u>*Leg lengths*</u> $\sqrt{2}$ ***units and 1 unit***
<u>*Pythagorean Theorem*</u> $a^2 + b^2 = hypotenuse^2$ $(\sqrt{2})^2 + 1^2 = hypotenuse^2$ $3 = hypotenuse^2$ $\sqrt{3} = hypotenuse$

Leg lengths $\sqrt{3}$ ***units and 1 unit***
Pythagorean Theorem $a^2 + b^2 = hypotenuse^2$ $\left(\sqrt{3}\right)^2 + 1^2 = hypotenuse^2$ $4 = hypotenuse^2$ $\sqrt{4} = 2 = hypotenuse$
Leg lengths ***2 units and 1 unit***
Pythagorean Theorem $a^2 + b^2 = hypotenuse^2$ $2^2 + 1^2 = hypotenuse^2$ $5 = hypotenuse^2$ $\sqrt{5} = hypotenuse$

To visualize the location of these irrational numbers on the number line, students could use a compass to measure the length of the hypotenuse and swing an arc that will cross the number line.

Directions: Place the point of the compass on the hypotenuse endpoint that is located on zero. Open the compass and place the pencil on the other endpoint of the hypotenuse. Swing an arc that will cross the number line.



Extension:

The teacher may provide opportunities for students to add, subtract, multiply and divide some of the lengths to show a deeper understanding of operating with square roots. Such opportunities could include, but are not limited to; finding the perimeters/areas of certain shapes formed by the composition of different triangles, their differences and/or the percent one is of another.

Constructing the Irrational Number Line

In this task, you will construct a number line with several rational and irrational numbers plotted and labeled. Start by constructing a right triangle with legs of one unit. Use the Pythagorean Theorem to compute the length of the hypotenuse. Next, copy the segment forming the hypotenuse to a line and mark one left endpoint of the segment as 0 and the other endpoint with the irrational number it represents.

Construct other right triangles with two sides (either the two legs or a leg and a hypotenuse) that have lengths that are multiples of the unit you used in the first triangle. Then transfer the lengths of each hypotenuse to a common number line, and label the point that it represents. After you have constructed several irrational lengths, list the irrational numbers in order from smallest to largest.

Using Pythagorean Theorem and Exponents – FAL

Source: Georgia Mathematics Design Collaborative

Back to Task Table

This lesson is intended to help you assess how well students are able to:
Use right triangle measurement, including square roots, to determine lengths for real-world problems

STANDARDS ADDRESSED IN THIS TASK:

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

MGSE8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

STANDARDS FOR MATHEMATICAL PRACTICE:

This lesson uses all of the practices with emphasis on:

2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Georgia Mathematics Design Collaborative are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding.

The task, *Using Pythagorean Theorem and Exponents*, is a Formative Assessment Lesson (FAL) that can be found on the Georgia Mathematics online professional learning community, edWeb.net. Once logged in, the task can be found directly at:
https://www.edweb.net/?14@@.5abfa3bd.

The Pythagorean Theorem: Square Areas - (FAL)

Back to Task Table

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1206

Uses the Pythagorean Theorem to find lengths and areas and use the area of right triangles to deduce the areas of other shapes.

STANDARDS FOR MATHEMATICAL CONTENT:

Understand and apply the Pythagorean Theorem.

MGSE8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

MGSE8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

MGSE8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Work with radicals and integer exponents.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE:

In order for students to be successful, the following skills and concepts need to be maintained:

- **MCC7.G.6**

COMMON MISCONCEPTIONS:

- *Students estimate the area of the square. We are looking for precision.*
- *Students dissect the square into smaller shapes, but these new shapes do not permit an accurate calculation.*
- *Students have trouble with the calculations.*

ESSENTIAL QUESTIONS:

- How can I use the area of right triangles to deduce the areas of other shapes?
- How can I use dissection methods to find areas?
- How do I simplify and evaluate algebraic equations involving integer exponents and square roots?
- What generalizable method can be deduced for finding lengths and areas?

MATERIALS:

- Materials can be found at the task link.

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *The Pythagorean Theorem: Square Areas*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=408&subpage=concept
The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1206

Proofs of the Pythagorean Theorem (Short Cycle Task)

Back to Task Table

Source: Balanced Assessment Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=804

Analyze attempts to prove the Pythagorean Theorem.

STANDARDS FOR MATHEMATICAL CONTENT:

Understand and apply the Pythagorean Theorem.

MGSE8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

MGSE8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

MGSE8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

STANDARDS FOR MATHEMATICAL PRACTICE

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

In order for students to be successful, the following skills and concepts need to be maintained:

- **MCC.7.G.6**

COMMON MISCONCEPTIONS:

- *Students do not have a real understanding of the Pythagorean Theorem.*
- *Students do not have a mathematical understanding of a proof.*

ESSENTIAL QUESTIONS:

- How do I know that I have a convincing argument to informally prove Pythagorean Theorem?
- What is Pythagorean Theorem and when does it apply?

MATERIALS:

- Materials can be found at the task link.

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *Proofs of the Pythagorean Theorem*, is a Mathematics Assessment Project Assessment Task that can be found at the website:
http://www.map.mathshell.org/materials/tasks.php?taskid=276&subpage=expert

The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=804

The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=805

Circles and Squares (Short Cycle Task)

Back to Task Table

Source: Balanced Assessment Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=826

Apply the Pythagorean Theorem to find the ratio between two shapes.

STANDARDS OF MATHEMATICAL CONTENT:

Understand and apply the Pythagorean Theorem.

MGSE8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

MGSE8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

MGSE8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE:

In order for students to be successful, the following skills and concepts need to be maintained:

- **MCC7.G.4**
- **MCC7.G.6**

COMMON MISCONCEPTIONS:

- *Students tend to have trouble visually with the Pythagorean Theorem use in this task.*

ESSENTIAL QUESTIONS:

- How can I use dissection methods to find areas?
- What generalizable method can be deduced for finding lengths and areas?

MATERIALS:

- Materials can be found at the task link.

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *Circles and Squares*, is a Mathematics Assessment Project Assessment Task that can be found at the website:
http://www.map.mathshell.org/materials/tasks.php?taskid=287&subpage=expert

The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=826

The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=827

Comparing Spheres and Cylinders

Back to Task Table

In this task, students will compare the effects of the volume of a cylinder and sphere.

STANDARDS OF MATHEMATICAL CONTENT:

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

MGSE8.G.9 Apply the formulas for the volume of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Work with radicals and integer exponents.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE:

In order for students to be successful, the following skills and concepts need to be maintained:

- **MCC7.G.6**

COMMON MISCONCEPTIONS:

- *The students may incorrectly use the wrong formula for finding the volume.*
- *The students may forget to cube the units for volume problems.*
- *Students may forget how to find the cube root of the volume.*

ESSENTIAL QUESTIONS:

- Where do we see examples of two and three-dimensional objects in the real-world?
- How does the change in radius affect the volume of a cylinder or sphere?
- How does the change in height affect the volume of a cylinder or sphere?
- How does the volume of a cylinder and sphere with the same radius change if it is doubled?
- How do I simplify and evaluate algebraic equations involving integer exponents, square and cubed root?

MATERIALS:

- Copies of task for each student/pair of students/or small group
- *Optional:* Calculator
- Formula Sheet

GROUPING:

- Individual/Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

This task offers students the opportunity to find the volume of a sphere and cylinder as well as determine the relationship between the volume of a sphere and cylinder.

DIFFERENTIATION:

Extension:

- Have students investigate volume formulas using this interactive activity found here: http://www.learner.org/courses/learningmath/measurement/session8/part_b/cylinders.html
- Students will need this document to complete the interactive activity: http://www.learner.org/courses/learningmath/measurement/pdfs/session8/8b10.pdf

Intervention/Scaffolding:

- Set up the table ahead of time for the students to fill out for problem 1. Be sure to explain the definition of circumscribed (give examples) for problem 3.

Formulas

The following information is for your reference in solving some of the problems within this task.

Cylinder	Volume	$V = \pi r^2 h$
Sphere	Volume	$V = \frac{4}{3}\pi r^3$
Cone	Volume	$V = \frac{1}{3}\pi r^2 h$

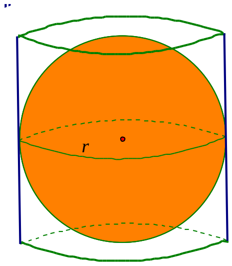
Comparing Spheres and Cylinders

Comments:

Visit these links and/or software to interactively change the dimensions of these solids:
Sphere - http://www.mathopenref.com/spherevolume.html
Cone - http://www.mathopenref.com/conevolume.html
Cylinder - http://www.mathopenref.com/cylindervolume.html

GeoGebra - Free mathematics software for learning and teaching
http://www.geogebra.org/cms

1. Find the volume for five different spheres by randomly choosing different radii. Using the same radii values, find the volume of five cylinders where the height of the cylinder is the same as the diameter of the sphere. Make a chart of the values.



Solutions:
Answers may vary based on the 5 radii used to compute the volume and surface area.

Radius	$V_{sphere} = (4\pi r^3)\div 3$	***Height***	$V_{cylinder} = \pi r^2 h$
2	$32\pi/3$	*4*	16π
3	36π	*6*	54π
4	$256\pi/3$	*8*	128π
5	$500\pi/3$	*10*	250π
6	288π	*12*	432π

2. What is the relationship between the changes in radii and volume?

Solutions:
The volume of a sphere is exactly 2/3 the volume of its circumscribed cylinder.

$V_{sphere} = (2V_{cylinder}) \div 3$

$\frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^2 h$; *solve for h*

h = 2r

The height of the cylinder with the same radius as a sphere is always twice the radius or it is equal to the diameter of the sphere.

3. The volume of a circumscribed cylinder is $1458\pi\ cm^3$. Find the radius of a sphere.

Solutions:

Knowing that the volume of a sphere is exactly 2/3 the volume of a cylinder, the volume of a sphere = $972\pi\ cm^3$

$$V = \frac{4}{3}\pi r^3$$

$$972\pi = \frac{4}{3}\pi r^3$$

$$729 = r^3$$

$$\sqrt[3]{729} = \sqrt[3]{r}$$

$$9 = r$$

The radius of a sphere that has a volume of $972\pi\ cm^3$ is 9 cm.

4. Tennis balls with a diameter of 2.5 in. are sold in cans of three. The can is a cylinder. What is the volume of the space NOT occupied by the tennis balls? Assume the tennis balls touch the can on the sides, top, and bottom. Round to the nearest tenth.

Solutions:

The volume of the can = 36.8
The volume of 3 tennis balls = 24.6
The volume of the space not occupied by tennis balls = 36.8 – 24.6 = 12.2

5. Justify your reasoning for amount of space NOT occupied by the three tennis balls.

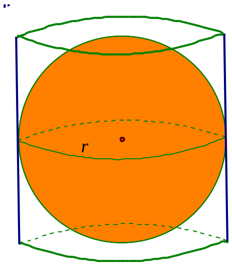
Formulas

The following information is for your reference in solving some of the problems within this task.

Cylinder	Volume	$V = \pi r^2 h$
Sphere	Volume	$V = \frac{4}{3}\pi r^3$
Cone	Volume	$V = \frac{1}{3}\pi r^2 h$

Comparing Spheres and Cylinders

1. Find the volume for five different spheres by randomly choosing different radii. Using the same radii values, find the volume of five cylinders where the height of the cylinder is the same as the diameter of the sphere. Make a chart of the values.



2. What is the relationship between the changes in radii and volume?

3. The volume of a circumscribed cylinder is $1458\pi\ cm^3$. Find the radius of a sphere.

4. Tennis balls with a diameter of 2.5 in. are sold in cans of three. The can is a cylinder. What is the volume of the space NOT occupied by the tennis balls? Assume the tennis balls touch the can on the sides, top, and bottom. Round to the nearest tenth.

5. Justify your reasoning for amount of space NOT occupied by the three tennis balls.

Making Matchsticks – (FAL)

Back to Task Table

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1225

In this task, students will apply the volume formula to real-world situations.

STANDARDS FOR MATHEMATICAL CONTENT:

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

MGSE8.G.9 Apply the formulas for the volume of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Work with radicals and integer exponents.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE:

In order for students to be successful, the following skills and concepts need to be maintained:

- **MCC7.G.6**

COMMON MISCONCEPTIONS:

- *Student ignores the units.*
- *Student makes incorrect assumptions.*
- *Student may use incorrect formula.*

ESSENTIAL QUESTIONS:

- Where do we see examples of two and three-dimensional objects in the real-world?
- How do I interpret a situation and represent the variables mathematically?
- How do I select appropriate mathematical methods for a real-world problem?
- How do I communicate my reasoning clearly for a real-world mathematical problem?

MATERIALS:

- Materials can be found at the task link.

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Making Matchsticks*, is a Formative Assessment Lesson (FAL) that can be found at the website: http://map.mathshell.org/materials/lessons.php?taskid=410&subpage=problem
The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1225

Calculating Volumes of Compound Objects - (FAL)

Back to Task Table

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=684

In this task, students will apply volume formulas to compound objects.

STANDARDS FOR MATHEMATICAL CONTENT:

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

MGSE8.G.9 Apply the formulas for the volume of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

STANDARDS FOR MATHEMATICAL PRACTICE:

This lesson uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them
6. Attend to precision.

BACKGROUND KNOWLEDGE:

In order for students to be successful, the following skills and concepts need to be maintained:

- **MCC7.G.6**

COMMON MISCONCEPTIONS:

- *Student does not discriminate between length, area, and volume formulas.*
- *Student has difficulty in identifying the values to substitute for variables in formulas*.

ESSENTIAL QUESTIONS:

- Where do we see examples of three-dimensional objects in the real-world?
- How does the change in radius affect the volume of a cylinder, cone, or sphere?
- How does the change in height affect the volume of a cylinder, cone, or sphere?
- How does the volume of a cylinder, cone, and sphere with the same radius change if it is doubled?
- How do I simplify and evaluate algebraic equations involving integer exponents, square and cubed root?
- How do you know when an estimate, approximation, or exact answer is the desired solution?

MATERIALS:

- Materials can be found at the task link.

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Calculating Volumes of Compound Objects*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=216&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=684

Greenhouse Management (Career and Technical Education Task)

Source: Achieve
http://www.achieve.org/files/CCSS-CTE-Greenhouse-Management-FINAL.pdf
http://www.achieve.org/files/CCSS-CTE-Greenhouse-Management-FINAL.docx

In this task, students will apply the volume formula to real world situations.

STANDARDS FOR MATHEMATICAL CONTENT:

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

MGSE8.G.9 Apply the formulas for the volume of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Work with radicals and integer exponents.

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

STANDARDS FOR MATHEMATICAL PRACTICE:

This lesson uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.

BACKGROUND KNOWLEDGE:

In order for students to be successful, the following skills and concepts need to be maintained:

- **MCC7.G.6**

COMMON MISCONCEPTIONS:

- *Student does not discriminate between length, area, and volume formulas.*
- *Student has difficulty in identifying the values to substitute for variables in formulas.*
- *Students will have difficulties with unfamiliar and uncommon units of measure*
- *Students likely might be unfamiliar with mark-up percentages*

ESSENTIAL QUESTIONS:

- Where do we see examples of three-dimensional objects in the real-world?
- How do I simplify and evaluate algebraic equations involving integer exponents, square and cubed root?
- How do you know when an estimate, approximation, or exact answer is the desired solution?

MATERIALS:

- Materials can be found at the task link.

GROUPING:

- Partner/Small Groups

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

This task was developed by high school and postsecondary mathematics and agriculture sciences educators, and validated by content experts in the Common Core State Standards in mathematics and the National Career Clusters Knowledge & Skills Statements. It was developed with the purpose of demonstrating how the Common Core and CTE Knowledge & Skills Statements can be integrated into classroom learning – and to provide classroom teachers with a truly authentic task for either mathematics or CTE courses.

The task, *Greenhouse Management*, is an Achieve CCSS-CTE Classroom Task that can be found at the website: http://www.achieve.org/files/CCSS-CTE-Greenhouse-Management-FINAL.docx

The document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://www.achieve.org/files/CCSS-CTE-Greenhouse-Management-FINAL.pdf

Technology

Back to Task Table

MGSE8.G.6 Explain a proof of the Pythagorean Theorem and its converse.

http://illuminations.nctm.org/Search.aspx?view=search&kw=Pythagorean%20theorem&gr=6-8
http://nrich.maths.org/982

MGSE8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

https://www.illustrativemathematics.org/content-standards/8/G/B/7
http://illuminations.nctm.org/Search.aspx?view=search&kw=Pythagorean%20theorem&gr=6-8

MGSE8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

https://www.illustrativemathematics.org/content-standards/8/G/B/8
http://illuminations.nctm.org/Search.aspx?view=search&kw=Pythagorean%20theorem&gr=6-8

MGSE8.G.9 Apply the formulas for the volume of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

https://www.illustrativemathematics.org/content-standards/8/G/C/9
http://nrich.maths.org/1408

MGSE8.EE.2 Use square root and cube root symbols to represent solutions to equations. Recognize that $x^2 = p$ (where p is a positive rational number and $|x| \leq 25$) has 2 solutions and $x^3 = p$ (where p is a negative or positive rational number and $|x| \leq 10$) has one solution. Evaluate square roots of perfect squares ≤ 625 and cube roots of perfect cubes ≥ -1000 and ≤ 1000.

http://nrich.maths.org/2034