The Georgia Performance Standards for K-12 Mathematics

Kathy Cox
State Superintendent of Schools

Accelerated Mathematics I Training
May and June 2008
Overview of Day 1

- Big Picture of the New Curriculum
- Climbing the Ladder
- 6th, 7th, 8th Grade and Mathematics I Tasks
Group Norms and Housekeeping

**Group Norms:**
- Ask questions
- Work toward solutions
- Honor confidentiality
- Meet commitments or let others know if you are struggling

**Housekeeping:**
- Parking Lot
- Phone calls
- Restrooms
- Breaks
- Lunch
The Essential Questions

- Why a new curriculum?
- What process was used in the development of the new standards?
- What is the new curriculum? How is it different?
WHY a new curriculum?

- 2001 PDK audit of Quality Core Curriculum
  - Lacked rigor and depth—“A mile wide and an inch deep”
  - Did not allow for the alignment of instruction and assessment
  - Did not provide clear expectations for students
- Age of QCC-written in 1985, one cursory revision in 1997
- Not aligned to national and international standards
Georgia’s Student Achievement in Math

Percent Passing CRCT (5th & 8th) or E-GHSGT (11th)

<table>
<thead>
<tr>
<th>Grade</th>
<th>White</th>
<th>Black</th>
<th>Hispanic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 5</td>
<td>94</td>
<td>82</td>
<td>83</td>
</tr>
<tr>
<td>Grade 8</td>
<td>87</td>
<td>68</td>
<td>66</td>
</tr>
<tr>
<td>Grade 11</td>
<td>85.7</td>
<td>67.4</td>
<td>58.6</td>
</tr>
</tbody>
</table>
Georgia’s Student Achievement in Mathematics

- Percent of SAT Test Takers w/ 4 years of Mathematics
  - NATION: 62 percent
  - GEORGIA: 69 percent

- Score for SAT Test Takers w/ 4 years of Mathematics
  - NATION: 529 on mathematics portion
  - GEORGIA: 500 on mathematics portion

- What kind of Mathematics are they taking?
# Georgia’s Student Achievement in Mathematics

<table>
<thead>
<tr>
<th>Course Work</th>
<th>NATION</th>
<th>GEORGIA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebra</td>
<td>517</td>
<td>495</td>
</tr>
<tr>
<td>Geometry</td>
<td>519</td>
<td>498</td>
</tr>
<tr>
<td>Trigonometry</td>
<td>553</td>
<td>520</td>
</tr>
<tr>
<td>Precalculus</td>
<td>571</td>
<td>557</td>
</tr>
<tr>
<td>Other Mathematics Courses</td>
<td>510</td>
<td>487</td>
</tr>
<tr>
<td>Computer Mathematics</td>
<td>539</td>
<td>479</td>
</tr>
<tr>
<td>Calculus</td>
<td>608</td>
<td>584</td>
</tr>
<tr>
<td>AP/Honors Courses</td>
<td>599</td>
<td>585</td>
</tr>
</tbody>
</table>
International Student Achievement in Mathematics

Program for International Student Assessment (Mathematics)

TOP 5 SCORES
- Finland: 544
- South Korea: 542
- Netherlands: 538
- Japan: 534
- Canada: 533
- INTERNATIONAL AVERAGE: 500

UNITED STATES
- Average Score: 483
- 24th out of 30 countries
- Well below International Average
Mandate from Georgia State Board of Education

Develop a curriculum that is rigorous, deep, provides clear expectations for students, is an instructional guide for teachers, and is student-focused rather than teacher-focused.
The Process

- Expert Advisory Panel
  - 15 leaders in education, government, business, and industry
  - studied curricula from high achieving states and nations
  - chose Japanese curriculum for leanness, rigor, and coherence
The Process

- Teacher writing teams
  - Teacher applications
  - Five days of training in use of performance standards
  - Began writing in July of 2003
- K-12 curriculum posted for 60 days for public review and comment in March of 2004.
- K-8 curriculum passed by SBOE in May of 2004; 9-12 adoption postponed.
High School Advisory Committee

- Formed in summer, 2004
- 35 individuals - teachers, state and national leaders in mathematics education, higher education faculty
- Revised drafts of 6-12 curriculum
- Committee statement along with names and responsibilities of committee members posted at www.gadoe.org

Posted for public review and comment for 60 days
The Process

- Reviewed by Dr. Ann Shannon, Research Scientist, Shell Centre, Nottingham, England
- Reviewed by the Board of Regents’ Academic Advisory Committee for Mathematical Subjects
- Revised based on all reviews
- Adopted by unanimous vote of the Georgia State Board of Education, May 2005
Endorsements

- Board of Georgia Council of Teachers of Mathematics
- Senior Vice Chancellor for Academic Affairs of the Board of Regents
- Board of Regents’ Academic Advisory Committee on Mathematical Subjects
Partnerships & Collaboration

- University System of Georgia
- Georgia Department of Technical and Adult Education
- Governor’s Office of Workforce Development
- Regional Education Services Agencies
- Georgia Council of Teachers of Mathematics
- Georgia Public Broadcasting
Alignment with National Organizations

- National Council of Teachers of Mathematics
- College Board
- American Statistical Association
- Achieve
Implications for the Classroom

Students will be able to…

- Actively engage in mathematics
- Explain their thinking
- Justify their work
- Use multiple representations
- Make connections
- Choose appropriate technology
What Is It?

Balance of concepts, skills, and problem solving
A balance...

emphasizing understanding and relevance
K-8 Mathematics

- K-2: Four strands: number and operations, measurement, geometry, and data analysis
- 3-5: Algebra strand is added
- 6-8: In-depth treatment of algebra and geometry begins in grade 6; 80% of concepts taught in traditional algebra 1 course and 60% of concepts taught in traditional geometry course completed by the end of grade 8
High School Mathematics

- Integrated curriculum
- Common level of mastery
- Multiple paths of study
How is it different?

Comparison of the Sheer Number of Standards Expected to Be Learned in a Year

<table>
<thead>
<tr>
<th>Grades</th>
<th>Old (QCC)</th>
<th>New (GPS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6th Grade</td>
<td>53</td>
<td>18</td>
</tr>
<tr>
<td>7th Grade</td>
<td>43</td>
<td>15</td>
</tr>
<tr>
<td>8th Grade</td>
<td>45</td>
<td>18</td>
</tr>
<tr>
<td>Mathematics</td>
<td>Accelerated Mathematics</td>
<td></td>
</tr>
<tr>
<td>-------------------</td>
<td>----------------------------------</td>
<td></td>
</tr>
<tr>
<td>Mathematics I</td>
<td>Accelerated Mathematics I</td>
<td></td>
</tr>
<tr>
<td>Mathematics II</td>
<td>Accelerated Mathematics II</td>
<td></td>
</tr>
<tr>
<td>Mathematics III</td>
<td>Accelerated Mathematics III</td>
<td></td>
</tr>
<tr>
<td>Mathematics IV</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Other courses available:
- Discrete Mathematics
- Advanced Placement Statistics
- Advanced Placement Calculus AB
- Advanced Placement Calculus BC
A Ladder not a Spiral

Climbing the Ladder

- 7th-Cross sections and shadows
- 8th-Surface area of pyramids and cones as an application of the Pythagorean Theorem
- Mathematics 1-Comparing quadratic and cubic functions using surface area and volume of prisms, pyramids, cylinders, and cones
- Mathematics 2-Comparing quadratic and cubic functions using surface area and volume of spheres
A Ladder not a Spiral

Climbing the Ladder

- 7th – Define and operate with absolute value
- 8th — Solve absolute value equations and inequalities
- Mathematics 1 — Examine the absolute value function
- Mathematics 2 — Explore the absolute value as a piecewise function
Performance Standards

- Content and process standards
- Tasks
- Student work
- Commentary
Content Standards

- Body of knowledge
- What students should know and be able to do
Process Standards

Promote mathematics literacy through:

- Problem solving
- Reasoning and proof
- Communication
- Connections
- Representations
Tasks

- Give detail to the elements of the content standards
- Provide depth of understanding
- Maintain high cognitive demand
- Define academic rigor of standards
- Exemplify the kind of performance expected of students
- Address group worthiness
Student Work

- Reflects the level a student should attain by the end of a grade or course
- Further defines the content standards
- Illustrates the kind of performance expected of students
- Relates to a strand or topic rather than a single standard, embodying many concepts
Commentary

- Identifies the mathematics involved in the task
- Identifies evidence of understanding related to a specific standard
- Informs the teacher in understanding the depth, detail and rigor expected in work that meets the standard
- Guides students in comparing and judging the quality of their own work
## WHEN does it all happen?

<table>
<thead>
<tr>
<th>Year</th>
<th>Teacher Training</th>
<th>Classroom Implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>Grade 6</td>
<td></td>
</tr>
<tr>
<td>2005</td>
<td>Grades K-2, 7</td>
<td>Grade 6</td>
</tr>
<tr>
<td>2006</td>
<td>Grades 3-5, 8</td>
<td>Grades K-2, 7</td>
</tr>
<tr>
<td>2007</td>
<td>Math I</td>
<td>Grades 3-5, 8</td>
</tr>
<tr>
<td>2008</td>
<td>Math II</td>
<td>Grade 9</td>
</tr>
<tr>
<td>2009</td>
<td>Math III</td>
<td>Grade 10</td>
</tr>
<tr>
<td>2010</td>
<td>Math IV</td>
<td>Grade 11</td>
</tr>
<tr>
<td>2011</td>
<td></td>
<td>Grade 12</td>
</tr>
</tbody>
</table>
Start with a belief

Teachers must believe that all students can learn, although in different ways and at different rates!
Instructional Elements

- A meaningful mathematics curriculum
- An emphasis on interactive endeavors that promote divergent thinking within a classroom
- Diversified instructional strategies that address the needs of all types of learners
Assessment that is varied, ongoing, and embedded in instruction

Focused lesson planning that, instead of emphasizing what the classroom teacher wants to teach, begins by understanding what students need to learn and assessing what they already know

EDThoughts: What we Know About Mathematics Teaching and Learning
Which item requires a better understanding of lines?

1. Given a slope of 5 and a y-intercept of 3, write the equation of the line.

   OR

2. A company that produces pens has \( n \) pens in stock at the beginning of a certain day. It produces these pens at a constant rate \( r \) per hour for the entire day. If that day, pens have been produced at a greater constant rate, write an equation that can be used to determine the number of pens the company has in stock at the end of that day.
MA1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.

a. Represent functions using function notation.
How to Read the GPS Code

MA1A1a.

Mathematics Accelerated
Mathematics I Algebra
Standard #1
Element a
Standards and Elements

- **Standard is in bold print:** Sets the parameters.
- **Elements are listed under the standard:** Set the expectations for understanding and communicates what the student should **know and be able to do**.
Family of Functions

- Characteristics of the functions
  \[ F(x) = x^n \ (n=1,2,3), \sqrt{x}, |x|, \text{ and } 1/x \]
- Sequences as functions
Mathematics I

- Algebra of Quadratics
  - Factoring of 2nd degree polynomials & cubes
  - Quadratic equations
  - Radical equations
  - Simple rational equations
- Coordinate Geometry
  - Distance between a point and a line
  - Midpoint
Mathematics I

- Triangles
  - Inductive, deductive reasoning
  - Converse, inverse, contrapositive
  - Sum of interior, exterior angles
  - Triangle inequalities
  - SSS, SAS, ASA, AAS, HL
  - Incenter, orthocenter, circumcenter, centroid
Mathematics I

Statistics
- Simple permutations & combinations
- Mutually exclusive and dependent events
- Conditional probabilities
- Expected values
- Summary statistics
- Random sample
- Mean absolute deviation
Mathematics II

- Family of Functions
  - Quadratic \((y = ax^2 + bx + c)\)
  - Step & piecewise
  - Exponential
  - Inverse
  - Characteristics of their graphs
Mathematics II

- Complex numbers
- Quadratic inequalities
- Exponential equations and inequalities
- Geometric sequences as exponential functions
- Right triangle trigonometry
- Circles and properties
Mathematics II

- Length of arc
- Surface area and volume of sphere
- Relationships of similar solids
- Population means & deviations
- Modeling of data using linear and quadratic regressions
Mathematics III

- Circle
- Ellipse
- Hyperbola
- Parabola (concave right and left)
- Planes & spheres
- Histograms
- Normal distribution
- Experimental and observational studies
Mathematics III

- Extension of exponents
- Matrices
- Polynomials of degree \( > 2 \)
- Logarithmic functions
- Exponential, logarithmic and polynomial equations and inequalities
- Vertex-edge graphs
- Linear programming
Mathematics IV

- Vectors
- Graphs of 6 trigonometric functions
- Trigonometric identities
- Trigonometric equations and inequalities
- Rational functions
- Rational equations and inequalities
- Inverse trigonometric functions (sine, cosines, and tangent only)
Mathematics IV

- Sequences and series
- Unit circle
- Law of Sines
- Law of Cosines
- Area of triangle formula
- Central Limit Theorem
- Confidence interval
- Margin of error
P-8 Standards should be “streamlined” and “well-defined”. “Any approach that revisits topics year after year without bringing them to closure should be avoided.”

A balance between concepts, computation and problem solving. They are “equally important and mutually reinforce each other.”

Proficiency with whole numbers, fractions and certain aspects of geometry and measurement are the foundations for algebra.

More students should be prepared for and offered an authentic algebra course in Grade 8.
“If children believe that their efforts to learn make them ‘smarter,’ they show greater persistence in mathematics learning.”
<table>
<thead>
<tr>
<th>Recommended Benchmarks: Elementary School</th>
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<tbody>
<tr>
<td>By the end of Grade 3, students should be proficient with the addition and subtraction of whole numbers.</td>
</tr>
<tr>
<td>By the end of Grade 4, students should be able to identify and represent fractions and decimals, and compare them on a number line or with other common representations of fractions and decimals.</td>
</tr>
<tr>
<td>By the end of Grade 5, students should be proficient with multiplication and division of whole numbers.</td>
</tr>
<tr>
<td>By the end of Grade 5, students should be proficient with comparing fractions and decimals and common percents, and with the addition and subtraction of fractions and decimals.</td>
</tr>
<tr>
<td>By the end of Grade 5, students should be able to solve problems involving perimeter and area of triangles and all quadrilaterals having at least one pair of parallel sides (i.e., trapezoids).</td>
</tr>
</tbody>
</table>
# Recommended Benchmarks: Middle Schools

<table>
<thead>
<tr>
<th>By the end of Grade 6, students should be proficient with multiplication and division of fractions and decimals.</th>
<th>✓</th>
</tr>
</thead>
<tbody>
<tr>
<td>By the end of Grade 6, students should be proficient with all operations involving positive and negative integers</td>
<td>Grade 7</td>
</tr>
<tr>
<td>By the end of Grade 6, students should be able to analyze the properties of two-dimensional shapes and solve problems involving perimeter and area, and analyze the properties of three-dimensional shapes and solve problems involving surface area and volume.</td>
<td>✓</td>
</tr>
</tbody>
</table>
## Recommended Benchmarks: Middle Schools (continued)

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<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>By the end of Grade 7, students should be proficient with all operations involving positive and negative fractions.</td>
<td>![Checkmark]</td>
</tr>
<tr>
<td>By the end of Grade 7, students should be able to solve problems involving percent, ratio, and rate and extend this work to proportionality.</td>
<td>![Checkmark]</td>
</tr>
<tr>
<td>By the end of Grade 7, students should be familiar with the relationship between similar triangles and the concept of the slope of a line.</td>
<td>![Checkmark]</td>
</tr>
</tbody>
</table>
The National Mathematics Advisory Panel Report

- **The Importance of Knowledgeable Teachers**
  - Preparation for Elementary and Middle School teachers in Mathematics should be strengthened
  - “Teachers cannot be expected to teach what they do not know.”

- **Effective Instruction Matters**
  - Use of formative assessments
  - The belief that children of certain ages are “too young” to learn math is false
  - Use an array of examples in teaching and offer opportunities for extensive practice and the ability to “think aloud.”
  - Accelerate gifted mathematics students
Morning Break
Constructing the Algebra Ladder
Climbing the Ladder

6th Grade Task
View Tube Experiment

7th Grade Task
Seesaw Nickels

8th Grade Task
Connecting Arithmetic Sequences and Linear Functions
Out to Lunch!
Be able to explain and describe what you should see happening during this lesson and task.

Think about what needs to take place before students would be able to perform this task.
Afternoon Break
From Wonderland to Functionland

- Be able to explain and describe what you should see happening during this lesson and task.

- Think about what needs to take place before students would be able to perform this task.
End of Day One