A MATH TWIST
by Pamela D. Jacobsen

How a Simple Paper Loop Became a Major Breakthrough--in Theory and in Practice

Everyone knows that a flat piece of paper has two sides—a back and a front. But there's a way to turn a two-sided strip of paper into a one-sided object. This is not a magic trick. It's called a Mobius (pronounced "MAY-bee-uss" or "MOH-bee-uss") strip.

Jeff Weeks is a geometrician (someone who studies geometry). He's an expert on the Mobius strip, and he also enjoys discussing his favorite math topic, "understanding the three-dimensional world we live in."

Making a Mobius strip from a sheet of paper, Dr. Weeks says, is a good way to start to understand one-sidedness. It illustrates the concept, he says, but it's not perfect. Paper has thickness. A perfect Mobius strip has no thickness.

Imagine a two-dimensional person, Weeks says—someone with height and width but no depth. Now imagine that person walking along a perfect Mobius strip. "They would go around and come back reversed," he says. When they returned to their starting point, "they would be their own mirror image." That's weird.

It gets weirder. Try making Mobius strips and cutting them lengthwise.

STEP 1. Cut a strip, lengthwise, from a piece of notebook paper. Make it about two inches wide.

STEP 1A. Cut another strip, same as the first. This time, fold it into thirds, lengthwise.

STEP 2. Twist one end of the strip halfway (180 degrees). Keeping the paper twisted, tape or glue the ends together. The resulting band should NOT look like a simple round cylinder.

STEP 2A. Now unfold it. Twist and tape (or glue) the paper into a Mobius strip as before. The folds will be your guides for cutting the Mobius strip.

STEP 3. Using a pencil, put a dot halfway between the two edges of the strip. Beginning at the dot, draw a line lengthwise down the middle of your Mobius strip. Don't lift the pencil off the paper. Keep drawing until you reach the dot again. See? The strip has only one side!

STEP 3A. You're ready to cut again. Use the folds as a guide for your scissors. But before you start, think. What will happen? Look again at the figures at the bottom of the page.∗ Which one do you think the cut-in-thirds Mobius strip will look like?
STEP 4. Look at your Mobius strip. What will happen if you cut it in half, lengthwise? Think about it, and look at the figures at the bottom of the page.* Choose the one you think the cut-in-half Mobius strip will look like. Now carefully cut your strip in half. Did you guess correctly?

STEP 4A. Start cutting. What happened? Did you guess correctly? One-sided figures sure don't behave the way two-sided figures do! Are you ready to try to predict what will happen if you cut a Mobius strip in fourths?

To check to see whether the figures that result are one-sided or two-sided, use a colored marker. Start coloring one side of the strip. When you meet up with where you began, look at the strip. If there's a side that is not colored, it's not a Mobius strip. If you don't see an uncolored side, it is one. Why? Because being one-sided means that what we think of as the "outside" and the "inside" are continuous--the same.

The Mobius strip belongs to a field of geometry called topology (toh-POHL-uh-jee). Those who study it are called topologists. They examine the mathematical properties of various surfaces. One way to describe what topology is all about is to think of an imaginary rubber doughnut.

You can stretch, bend or even shrink this doughnut. What's important to a topologist is not how long or wide the doughnut gets-only that the resulting figure still has a hole.

A perfect Mobius strip can also be stretched or distorted. And as long as it is not cut the wrong way, it remains one-sided. (To learn more about topology, check out Dr. Weeks's Web site: (www.northnet.org/weeks/TorusGames/TorusGames.htm)

WHO THINKS UP THIS STUFF?

Nov. 17, 1790, was an important time in history. The French Revolution was being fought. George Washington was serving his first term as president of the United States. And August Ferdinand Mobius had just been born in Saxony (Germany).

Until he was 13, Mobius was homeschooled. His father, a master dancing instructor, had died when he was 3. His mother was a descendent of religious reformer Martin Luther. In 1813, Mobius left for Gottingen University. There he studied under brilliant mathematician Karl Friedrich Gauss.

Mobius was a patient man. He liked to work alone solving math problems. Unfortunately, mathematicians at that time were poorly paid. To earn a living, Mobius became an astronomer and professor. By 1848, he was director of the Leipzig Observatory in Germany.

But mathematics remained his real love. In 1858, Mobius was given credit for discovering the one-sided strip.

Historians, however, don't all agree that this breakthrough should have been named after him. Months before Mobius announced his finding, another mathematician, Johann Benedict Listing, produced an unpublished paper. In it he described Mobius's strip. Biographers suggest that Mobius and Listing were unaware of each other's work.

Was Mobius the first person to think of this shape? The mystery may never be solved. Ten years after announcing his discovery, Mobius died.

Making paper strips is fun. But is there any practical use for Mobius's band? You may be surprised to hear that the answer is yes.
Back in 1949, an inventor patented an abrasive belt that used the principle of one-sidedness. Another 1952 patent showed a conveyor belt that could transport hot objects—and it, too, used Mobius's concept.

AND WHAT'S IT GOOD FOR, ANYWAY?

Pamela Clute is a mathematics professor at the University of California at Riverside. She points to a number of other applications, including fan belts, typewriter ribbons, conveyor belts, even exercise equipment. Think of it: If the conveyor belt at the grocery checkout had a half-twist in it (and it probably does), the surface of the belt would last twice as long.

Even astrophysicists are interested in the Mobius strip.

Researchers at the University of Warwick (Rhode Island) have examined an electromagnetic region near the earth they call a tail. Scientists think the tail's charged particles follow a path that looks like a Mobius strip. Mobius may have even answered a bigger question: What is the shape of the universe? Some scientists think the universe may be like one giant Mobius strip!

An old saying goes, "There are two sides to every story." Do you suppose it was written by someone who'd never heard of a Mobius strip?

- Answers: If you cut a Mobius strip in half lengthwise, you get a long Mobius strip. Cut it in thirds lengthwise (note that you don't need to reposition your scissors as you cut), and you get Fig. 2:* a small Mobius strip linked to a long, two-sided, twisted loop.

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*Pictures referred to in this article are not available in this format but can be found in the original publication.