## Sample Questions and Answers from NAEP

Grade: $4 \quad$ Year: 2003
Difficulty Level: Hard
Item Number: 13 (M6)
Question Categorization:
NAEP : Algebra and Functions and Conceptual Understanding
GPS: M3A1c. Use a symbol, such as $\square$ and $\triangle$, to represent an unknown and find the value of the unknown in a number sentence.


The objects on the scale above make it balance exactly. According to this scale, if $\triangle$ balances $\bigcirc$, then $\square$ balances which of the following?
A)

B) $\bigcirc$
C)

D) $\bigcirc \bigcirc$

Answer: B

| 2003 National Performance Results |  |
| :---: | :---: |
| Score | Percentage of Students |
| Correct | 39\% |
| Incorrect | 60\% |
| Omitted | 1\% |
|  | 0100 |
| Note: <br> These resu <br> Percentag | ults are for public and nonpublic school students. ages may not add to 100 due to rounding. |

Answer choices made by all students:

|  | A | B | C | D | Omitted/Missing |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Students | 3 | 39 | 15 | 42 | 1 |

## Question Categorization:

## NAEP : Algebra and Functions and Problem Solving

GPS: M4A1a. Understand and apply patterns and rules to describe relationships and solve problems.

The table below shows how the chirping of a cricket is related to the temperature outside. For example, a cricket chirps 144 times each minute when the temperature is $76^{\circ}$.

| Number Of Chirps Per Minute | Temperature |
| :---: | :---: |
| 144 | $76^{\circ}$ |
| 152 | $78^{\circ}$ |
| 160 | $80^{\circ}$ |
| 168 | $82^{\circ}$ |
| 176 | $84^{\circ}$ |

What would be the number of chirps per minute when the temperature outside is $90^{\circ}$ if this pattern stays the same?

Answer: $\qquad$
Explain how you figured out your answer.
Answer: 200 For every $2^{\circ}$ that the temperature increases, the number of chirps increases by 8 .


Note:

- These results are for public and nonpublic school students.
- Percentages may not add to 100 due to rounding.


## Score \& Extended Description

## Extended

Answers 200 with explanation that indicates number of chirps increases by 8 for every temperature increase of $2^{\circ}$.

## Satisfactory

Gives explanation that describes ratio, but does not carry process far enough (e.g., gives correct answer for $86^{\circ}$ (184) or $88^{\circ}$ (192) or carries process too far (answers 208)).

OR
Answers 200 and shows $18486^{\circ}, 19288^{\circ}, 20090^{\circ}$ but gives no explanation.
OR
Answers 200 with explanation that is not stated well but conveys the correct ratio.
OR
Gives clear description of ratio and clearly has minor computational error (e.g., adds incorrectly).

## Partial

Answers between 176 and 208, inclusive, with explanation that says chirps increase as temperature increases.
OR
Answers between 176 and 208, inclusive, with explanation that they counted by 8 (or by 2 ).
OR
Uses a correct pattern or process (includes adding a number 3 times or showing 184 and 86 in chart) or demonstrates correct ratio.
OR
Has half the chart with 200 on the answer line.
OR
"I added 24" (with 200 on answer line).

## Minimal

Answers between 176 and 208, inclusive, with no explanation or irrelevant or incomplete explanation.
OR
Has explanation that number of chirps increases as temperature increases but number is not in range.
OR
Has number out of range but indicates part of the process (e.g., I counted by 8's)
OR
Explanation-as temperature increases the chirps increase but number is out of range.

## Incorrect

Incorrect response.

This question required students first to recognize a pattern and then extend the pattern for three more values. In addition, students were asked to explain how they arrived at the answer. The pattern was linear in two variables-number of chirps and temperature. Both algebraic and numerical reasoning were used to obtain the answer to this question. Students were permitted to use a calculator.

## Student Responses:

## Extended:

## Answer 200 chips




3.

Year: 1992
Difficulty Level: Hard
Item Number: 8 (M7)/20(M7)
Question Categorization:
NAEP : Algebra and Functions and Problem Solving
GPS: M4A1a. Understand and apply patterns and rules to describe relationships and solve problems.


Children's pictures are to be hung in a line as shown in the figure above. Pictures that are hung next to each other share a tack. How many tacks are needed to hang 28 pictures in this way?
A) 27
B) 28
C) 29
D) 56

## Answer: C


4.

Grade: $8 \quad$ Year: 2003
Difficulty Level: Hard
Item Number: 19 (M7)
Question Categorization:
NAEP : Algebra and Functions and Problem Solving
GPS: M8A5b. Solve systems of equations graphically and algebraically, using technology as appropriate.

While she was on vacation, Tara sent 14 friends either a letter or a postcard. She spent $\$ 3.84$ on postage. If it costs $\$ 0.20$ to mail a postcard and $\$ 0.33$ to mail a letter, how many letters did Tara send?
Show what you did to get your answer.
Answer: 8 letters
Students may use a variety of strategies to solve this, including guess and check, formal algebra, or others. For example,

| $\#$ postcards | \# letters | total cost |
| :--- | :--- | :--- |
| 1 | 13 | 4.49 |
| 2 | 12 | 4.36 |
| 3 | 11 | 4.23 |
| 4 | 10 | 4.10 |
| 5 | 9 | 3.97 |
| 6 | 8 | 3.84 |
| 7 | 7 | 3.71 |
| 8 | 6 | 3.58 |

OR

$$
x+y=14
$$

$.20 x+.33 y=3.84$
therefore,
$.20 x+.33(14-x)=3.84$
so $x=6$ and $y=8$
http://nces.ed.gov/nationsreportcard/itmrls/startsearch.asp

| 2003 National Performance Results |  |
| :---: | :---: |
| Score | Percentage of Students |
| Extended | 11\% |
| Satisfactory | 4\% |
| Partial | 7\% |
| Minimal | 20\% |
| Incorrect | 41\% |
| Omitted | 13\% |
| Off task | 4\% |
|  | Note: |

- These results are for public and nonpublic school students.
- Percentages may not add to 100 due to rounding.


## Score \& Description

## Extended

Correct response

## Satisfactory

Correct, complete process is indicated, but answer is not 8 and only has a minor computational error OR
Shows correct, complete process but does not indicate answer

## Partial

Correct, complete process is indicated, but answer is not 8 and there are several computational errors (Process must clearly illustrate a correct strategy, such as a table or equations.)
OR
Correct response of 8 but shows no work or incomplete work

## Minimal

Process is incorrect because it ignores one or more pieces of given information
OR
Process is correct but incomplete (process may be guess and check or another process which may lead to correct answer i.e., chart but no equation, but goal is not clearly defined) and answer is not 8

```
Incorrect
Incorrect response
```

This question was a word problem that asked the student to consider two values-the number of letters and the number of postcards-even though the student was only asked for the number of letters. This question could be solved in several ways. A student could reason numerically to find the number of letters and the number of postcards, possibly by using a guess-and-check strategy or by creating a table. Another possibility was to set up and solve a system of two linear equations in two unknowns. To earn full credit, students needed to show how they obtained the answer. Students were permitted to use a calculator.

## Student Responses:

| Extended:   | Satisfactory: |
| :---: | :---: |
| Partial: <br> She sent 8 letters. Add the moncy tog.ther | Incorrect $\frac{3.44}{20+3.3}=7.245283$ <br> $\approx 7$ letters |
| Minimal : $\begin{gathered} .33 \\ \times 11 \mathrm{ktws} / 7 \frac{3.84}{-3.63} \\ \hline .63 \end{gathered} \frac{.21}{\frac{-.20 \text { Rrearels }}{.1}}$ <br> 11 letters and I postard |  |

## Question Categorization:

NAEP : Algebra and Functions and Conceptual Understanding
GPS: M8A4. Students will graph and analyze graphs of linear equations and inequalities.

In a coordinate plane, the points $(2,4)$ and $(3,-1)$ are on a line. Which of the following must be true?
A) The line crosses the $x$-axis.
B) The line passes through $(0,0)$.
C) The line stays above the $x$-axis at all times.
D) The line rises from the lower left to the upper right.
E) The line is parallel to the $y$-axis.

Answer: A


- These results are for public and nonpublic school students.
- Percentages may not add to 100 due to rounding.


## Answer choices made by all students:

|  | A | B | C | D | E | Omitted/Missing |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Students | 44 | 13 | 13 | 15 | 12 | 3 |

6. Grade: 8 Year: 1992

## Question Categorization:

NAEP : Algebra and Functions and Problem Solving
GPS: M6A2a. Analyze and describe patterns arising from mathematical rules, tables, and graphs.

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all your work.

A pattern of dots is shown below. At each step, more dots are added to the pattern. The number of dots added at each step is more than the number added in the previous step. The pattern continues infinitely.


Marcy has to determine the number of dots in the 20th step, but she does not want to draw all 20 pictures and then count the dots. Explain or show how she could do this and give the answer that Marcy should get for the number of dots.

Answer: Explanation should include one of the following ideas with no false statements.
a. For each successive step, the number of rows and the number of columns is increasing by 1 , forming a pattern. For example, the first step forms 1 by 2 rows and columns, the next step 2 by 3 , the third step $3 \times 4$, and so on. Continuing this pattern would mean that the 20th step has $20 \times 21$ or 420 dots.
b. Look at successive differences between consecutive steps. The differences 4, 6, 8, 10, .. form a pattern. There are 19 differences forming the pattern $4,6,8,10, \ldots, 38,40$ and this sum is ( 9 x $44)+22$ or 418 . However, 2 must be added for the 1 st step, yielding a response of 420 .


- These results are for public and nonpublic school students.
- Percentages may not add to 100 due to rounding.


## Score \& Description

## Extended

Correct answer. (Must state 420; must tie step 20 back to beginning of pattern in some specific form of generalization.)

## Satisfactory

Correct explanation of pattern but does not include or omits the correct number of dots (420).

## Partial

A partial (incomplete) correct explanation, i.e., does not tie together well.

## Minimal

An attempt to generalize OR to draw all 20 pictures in the pattern (with a clear understanding of the pattern).

## Incorrect/Off Task

The work is completely incorrect, irrelevant, or off task.

## Student Responses:



| Satisfactory: <br> Erich tipes the number of rows is 1 more and the numben of columns is 1 mare. Se $4^{\text {th }}$ step is 4 raus and 5 columns for $4 \times 5$ because you multiply roms fines columns | Minimal: <br> She would have to times 20 by 19 because she wants the 20 the step and there would be 19 lines. <br> Another Student's Response: <br> 400 jots <br> nitur-20 EHCH STEF when $\frac{20}{400}$ Yeu ADO 1 Dot 400 To Egex 310e. So After 20 ppoitiols You hare $A$ jusafs 20 DOTS 220 pots. |
| :---: | :---: |
| Partial: <br> She could use es colculater and Cabsulets 20 times $(x) 21$ and get 420. | Incorrect: $638$ <br> I added the multiples of 2 up to 40 on a calculator. |

NAEP : Algebra and Functions and Problem Solving
GPS: M4A1a. Understand and apply patterns and rules to describe relationships and solve problems.


Allen, Bridgitte, Chaz, and Diann each live in a different house on the same side of a street. The houses and their numbers are shown above.

- Only one of the other three people lives next to Bridgitte.
- Chaz lives next to Bridgitte and next to Diann.

Which person could live in house number 2?
A) Allen only
B) Chaz only
C) Diann only
D) Chaz or Diann
E) Any of these four people could live in house number 2.

## Answer: D



Answer choices made by all students:

|  | A | B | C | D | E | Omitted/Missing |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Students | 9 | 21 | 10 | 43 | 16 | 1 |

## Question Categorization:

NAEP : Algebra and Functions and Procedural Knowledge
GPS: M7A1a. Translate verbal phrases to algebraic expressions.
M7D1c. Analyze data using measures of central tendency...

Tetsu rides his bicycle $x$ miles the first day, $y$ miles the second day, and $z$ miles the third day. Which of the following expressions represents the average number of miles per day that Tetsu travels?
A) $x+y+z$
B) $x y z$
C) $3(x+y+z)$
D) $3(x y z)$
E) $(x+y+z) / 3$

Answer: E

$0 \quad 100$

Answer choices made by all students:

|  | A | B | C | D | E | Omitted/Missing |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Students | 23 | 4 | 10 | 3 | 58 | 1 |

## Question Categorization:

NAEP : Algebra and Functions and Problem Solving
GPS: M4A1a. Understand and apply patterns and rules to describe relationships and solve problems.
M4G3a. Understand and apply ordered pairs in the first quadrant of the coordinate system.

$$
(2,5),(4,9),(6,13)
$$

Which of the following describes what to do to the first number in each ordered pair shown above to obtain the corresponding second number?
A) Add 3
B) Subtract 3
C) Multiply by 2
D) Multiply by 2 and subtract 1
E) Multiply by 2 and add 1

## Answer: E



Answer choices made by all students:

|  | A | B | C | D | E | Omitted/Missing |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Students | 11 | 3 | 17 | 17 | 48 | 4 |

*The $12^{\text {th }}$ grade test is cumulative.

## Question Categorization:

## NAEP : Algebra and Functions and Problem Solving

GPS: M8A1. Students will use algebra to represent, analyze, and solve problems.
M8N1. Students will understand different representations of numbers including square roots, exponents, and scientific notation.

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all your work.

The first 3 figures in a pattern of tiles are shown below. The pattern of tiles contains 50 figures.


Describe the 20th figure in this pattern, including the total number of tiles it contains and how they are arranged. Then explain the reasoning that you used to determine this information. Write a description that could be used to define any figure in the pattern.

## Answer:

The explanation should indicate there are 442 tiles in the 20th figure.
Descriptions will vary
A correct one should suggest a row of 21 tiles across the top, a row of 21 across the bottom, and a $20 \times 20$ square between these rows. The top row extends one tile to the right of the square and the bottom row one tile to the left.
A diagram such as this might illustrate the student's counting methods


Counting methods are supported by generalizations (verbal or symbolic) that are based on the students' observations about the pattern.


## Scoring Guide:

In this question, a student had to use reasoning skills as well as problem solving skills to describe the number of tiles that would be in the 20th figure. A student needed to recognize the pattern that was given and be able to generalize it in some correct way. In addition, the student had to write a description that could be used to describe any figure in the pattern. Different levels of partial credit (satisfactory, partial, and minimal) were earned by a student depending on his or her reasoning skills and the description of the 20th figure.

## Score \& Description

## Extended

The 20th figure is described correctly, including the fact that there are 442 tiles with a clear, accurate explanation.

## Satisfactory

The 20th figure is described and the number of tiles is given. Some evidence of reasoning must be present. Reasoning is sound but there may be a computation error.

## Partial

Illustrates or describes at least one additional figure in the pattern correctly or states there are 442 tiles in the 20th figure

## Minimal

Attempts to draw or describe the given pattern or an additional figure in the pattern

## Incorrect

Incorrect response

## Student Responses:


http://nces.ed.gov/nationsreportcard/itmrls/startsearch.asp


Another Student's Response:

$$
\text { If w:ll hace } 21 \text { tert } 1.0 \text { ? }
$$

ins the tap cout wilhe. . .o.v.it m.ll coutaid 20 middic it. m.eh 20 files each. lif mill have 442 latal files.

Partial:


Fiod the
hoght and
$4+4$

Another Student's Response:

$$
\begin{aligned}
& 3 \text { stinan and to the fottern }
\end{aligned}
$$

$$
\begin{aligned}
& \text { the figuma an in } T \text { Eam } \\
& \text { the the umtian ene winning } \\
& \text { Eur wow date tore. } \\
& \text { The indie mos ane the awmike } \\
& \text { of ilimi that that dagramis }
\end{aligned}
$$

$$
\begin{aligned}
& \text { bach liar tiemeith }
\end{aligned}
$$

11. Grade: 12* Year: 1990
*The $12^{\text {th }}$ grade test is cumulative.
Question Categorization:
NAEP : Algebra and Functions and Problem Solving
GPS: MM1A1a. Represent functions using function notation.

The following question refers to the graph shown below.

What is the value of $f(g(1)) ?$
A) 2
B) 4
C) 5
D) 6
E) 8

Solution: E



## Note:

- These results are for public and nonpublic school students.
- Percentages may not add to 100 due to rounding.


## Answer choices made by all students:

|  | A | B | C | D | E | Omitted/Missing |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Students | 15 | 20 | 15 | 21 | 26 | 4 |

12. Grade: 12* Year: 1990 Difficulty Level: Easy Item Number: 7 *The $12^{\text {th }}$ grade test is cumulative.
Question Categorization:
NAEP : Algebra and Functions and Procedural Knowledge
GPS: M8A2b. Use the properties of inequality to solve inequalities.

What is the least whole number $x$ for which $2 x>11$ ?
A) 5
B) 6
C) 9
D) 22
E) 23

Solution: B


- These results are for public and nonpublic school students.
- Percentages may not add to 100 due to rounding.

Answer choices made by all students:

|  | A | B | C | D | E | Omitted/Missing |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| \% Students | 20 | 65 | 6 | 8 | 0 | 1 |

http://nces.ed.gov/nationsreportcard/itmrls/startsearch.asp
13. Grade: 12* Year: 1990
*The $12^{\text {th }}$ grade test is cumulative.
Question Categorization:
NAEP : Algebra and Functions and Procedural Knowledge
GPS: M8A1c. Solve algebraic equations in one variable, including equations involving absolute values.

If $d=110$ and $a=20$ in the formula $d=\mathbf{2}(2 t-1)$, then $t=$
A) 15

22
B) 15

8
C) 5
D) 111

20
E) 6

## Solution: E


$\overline{0} 100$

Note:

- These results are for public and nonpublic school students.
- Percentages may not add to 100 due to rounding.


## Answer choices made by all students:

|  | A | B | C | D | E | Omitted/Missing |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Students | 4 | 12 | 21 | 14 | 45 | 6 |

*The $12^{\text {th }}$ grade test is cumulative.
Question Categorization:
NAEP : Algebra and Functions and Problem Solving
GPS: MM1A1e. Relate to a given context the characteristics of a function, and use graphs and tables to investigate its behavior.
MM1G1a. Determine the distance between two points.

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all your work.


The darkened segments in the figure above show the path of an object that starts at point $A$ and moves to point $C$ at a constant rate of 1 unit per second. The object's distance from point $A$ (or from point $C$ ) is the shortest distance between the object and the point.

In the space below, complete the following steps.
a) Sketch the graph of the distance of the object from point $A$ over the 7 -second period.
b) Then sketch the graph of the distance of the object from point $C$ over the same period.

c) On your graph, label point $P$ at the point where the distance of the object from point $A$ is equal to the distance of the object from point $C$.
d) Between which two consecutive seconds is the object equidistant from points $A$ and $C$ ?

Answers:


1992 National Performance Results


Note:

- These results are for public and nonpublic school students.
- Percentages may not add to 100 due to rounding


## Score \& Description

## Extended

Complete, correct answer, (must show change in slope exactly at $(4,4)$ and $(4,3)$ ).

## Satisfactory

Both graphs are correct, (Change in slope need not occur exactly as $(4,4)$ and $(4,3)$ ). P is located correctly or the time of equidistance is correct.

## Partial

At least one graph is correct (must show change in slope, but curve is not required). P is not located or is located incorrectly and the time of equidistance is incorrect or missing
OR
One or both graphs incorrect, but P is located correctly for their graph.

## Minimal

At least 2 points are plotted correctly on at least one graph that is not just a reiteration of the position graph; i.e., a plot of distance versus time.

## Incorrect/Off Task

The work is completely incorrect, irrelevant, or off task.

## Student Responses:

## Extended:

b) Then sketch the graph of the distance of the object from point $C$ over the same period

c) On your graph, label point $P$ at the point where the distance of the object from point $A$ is equal to the distance of the object from point $C$.
d) Between which two consecutive seconds is the object equidistant from points $A$ and $C$ ?

$$
3 \text { and } 4
$$

## Another Student Response:

b) Then sketch the graph of the distance of the object from point $C$ over the same period

c) On your graph, label point $P$ at the point where the distance of the object from point $A$ is equal to the distance of the object from point $C$.
d) Between which two consecutive seconds is the object equidistant from points $A$ and C?

$$
3 \text { and } 4
$$

## Satisfactory:

b) Then sketch the graph of the distance of the object from point $C$ over the same period

c) On your graph, label point $P$ at the point where the distance of the object from point $A$ is equal to the distance of the object from point $C$.
d) Between which two consecutive seconds is the object equidistant from points $A$ and $C$ ?

## Another Student's Response:

a) Sketch the graph of the distance of the object from point $A$ over the 7 -second period
b) Then sketch the graph of the distance of the object from point $C$ over the same period

c) On your graph, label point $P$ at the point where the distance of the object from point $A$ is equal to the distance of the object from point $C$.
d) Between which two consecutive seconds is the object equidistant from points $A$ and $C$ ?


## Partial:

a) Sketch the graph of the distance of the object from point $A$ over the 7-second period.
b) Then sketch the graph of the distance of the object from point $C$ over the same period

c) On your graph, label point $P$ at the point where the distance of the object from point $A$ is equal to the distance of the object from point $C$.
d) Between which two consecutive seconds is the object equidistant from points $A$ and $C$ ?

## Minimal:

a) Sketch the graph of the distance of the object from point $A$ over the 7 -second period
b) Then sketch the graph of the distance of the object from point $C$ over the same period

c) On your graph, label point $P$ at the point where the distance of the object from point $A$ is equal to the distance of the object from point $C$.
d) Between which two consecutive seconds is the object equidistant from points $A$ and $C$ ?
point $c+P$

## Incorrect

a) Sketch the graph of the distance of the object from point $A$ over the 7 -second period.
b) Then sketch the graph of the distance of the object from point $C$ over the same period

c) On your graph, label point $P$ at the point where the distance of the object from point $A$ is equal to the distance of the object from point $C$.
d) Between which two consecutive seconds is the object equidistant from points $A$ and $C$ ?
Paint $P$ becounc it is
hackitumy betewan paint $A$ and paint $C$

