1. Grade: 4 Year: 2003 Difficulty Level: Hard Item Number: 13 (M6) **Question Categorization:** NAEP : Algebra and Functions and Conceptual Understanding **<u>GPS:</u>** M3A1c. Use a symbol, such as \Box and \triangle , to represent an unknown and find the value of the unknown in a number sentence. The objects on the scale above make it balance exactly. According to this scale, if Δ balances balances which of the following? , then L A) B) Answer: B 2003 National Performance Results Score Percentage of Students Correct 39% Incorrect 60% Omitted 1% 0 100Note: These results are for public and nonpublic school students. Percentages may not add to 100 due to rounding. • Answer choices made by all students:

			Inddo D	y un olu	
	A	В	С	D	Omitted/Missing
Students	3	39	15	42	1

2. Grade: 4 Year: 2003

Question Categorization:

NAEP : Algebra and Functions and Problem Solving

<u>GPS:</u> M4A1a. Understand and apply patterns and rules to describe relationships and solve problems.

The table below shows how the chirping of a cricket is related to the temperature outside. For example, a cricket chirps 144 times each minute when the temperature is 76°.

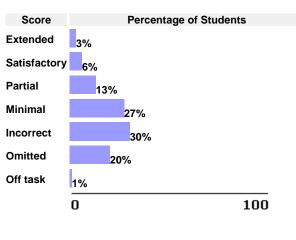
Number Of Chirps Per Minute	Temperature
144	76°
152	78°
160	80°
168	82°
176	84°

What would be the number of chirps per minute when the temperature outside is 90° if this pattern stays the same?

Answer:

Explain how you figured out your answer.

Answer: 200 For every 2° that the temperature increases, the number of chirps increases by 8.



2003 National Performance Results

Note:

- These results are for public and nonpublic school students.
- Percentages may not add to 100 due to rounding.

Score & Extended Description

Extended

Answers 200 with explanation that indicates number of chirps increases by 8 for every temperature increase of 2°.

Satisfactory

Gives explanation that describes ratio, but does not carry process far enough (e.g., gives correct answer for 86° (184) or 88° (192) or carries process too far (answers 208)).

OR

Answers 200 and shows 184 86°, 192 88°, 200 90° but gives no explanation.

OR Answers 200 with explanation that is not stated well but conveys the correct ratio.

OR

Gives clear description of ratio and clearly has minor computational error (e.g., adds incorrectly).

Partial

Answers between 176 and 208, inclusive, with explanation that says chirps increase as temperature increases.

OR

Answers between 176 and 208, inclusive, with explanation that they counted by 8 (or by 2). OR

Uses a correct pattern or process (includes adding a number 3 times or showing 184 and 86 in chart) or demonstrates correct ratio. OR

Has half the chart with 200 on the answer line.

OR

"I added 24" (with 200 on answer line).

Minimal

Answers between 176 and 208, inclusive, with no explanation or irrelevant or incomplete explanation.

OR

Has explanation that number of chirps increases as temperature increases but number is not in range.

OR

Has number out of range but indicates part of the process (e.g., I counted by 8's) OR

Explanation—as temperature increases the chirps increase but number is out of range.

Incorrect

Incorrect response.

This question required students first to recognize a pattern and then extend the pattern for three more values. In addition, students were asked to explain how they arrived at the answer. The pattern was linear in two variables—number of chirps and temperature. Both algebraic and numerical reasoning were used to obtain the answer to this question. Students were permitted to use a calculator.

Student Responses:

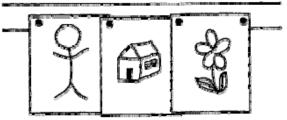
Extended:

Answer 200 chings

Explain how you figured out your answer.

Satisfactory: 200 Answer: Explain how you figured out your answer. If you need more room for your work, use the space below. T got my 49 on oЧ Partial: 194 Answer Explain how you figured out your answer. vent up 8 chirps each 20 T Minimal: Answer 140 Explain how you figured out your answer.

 Grade: 4/8 Year: 1992 Difficulty Level: Hard Item Number: 8 (M7)/20(M7) Question Categorization: NAEP : Algebra and Functions and Problem Solving GPS: M4A1a. Understand and apply patterns and rules to describe relationships and solve problems.



Children's pictures are to be hung in a line as shown in the figure above. Pictures that are hung next to each other share a tack. How many tacks are needed to hang 28 pictures in this way?

A) 27 B) 28 C) 29 D) 56

Answer: C

1992 Nation	1992 National Performance Results (4 th graders)			nal Performance Results (8 th graders)
Score	Percentage of Students		Score	Percentage of Students
Correct	25%		Correct	48%
Incorrect		73%	Incorrect	51%
Omitted	1%		Omitted 1	%
	0	100	0	100

Note:

- These results are for public and nonpublic school students.
- Percentages may not add to 100 due to rounding.

Answer choices made by all students:

	А	В	С	D	Omitted/Missing
Students	28	27	25	19	1

4. Grade: 8 Year: 2003 Difficulty Level: Hard Item Number: 19 (M7) Question Categorization: NAEP : Algebra and Functions and Problem Solving <u>GPS:</u> M8A5b. Solve systems of equations graphically and algebraically, using technology as appropriate.

While she was on vacation, Tara sent 14 friends either a letter or a postcard. She spent \$3.84 on postage. If it costs \$0.20 to mail a postcard and \$0.33 to mail a letter, how many letters did Tara send?

Show what you did to get your answer.

Answer: 8 letters

Students may use a variety of strategies to solve this, including guess and check, formal algebra, or others. For example,

# postcards	# letters	total cost
1	13	4.49
2	12	4.36
3	11	4.23
4	10	4.10
5	9	3.97
6	8	3.84
7	7	3.71
8	6	3.58

OR

x + y = 14.20x + .33y = 3.84therefore, .20x + .33(14 - x) = 3.84so x = 6 and y = 8

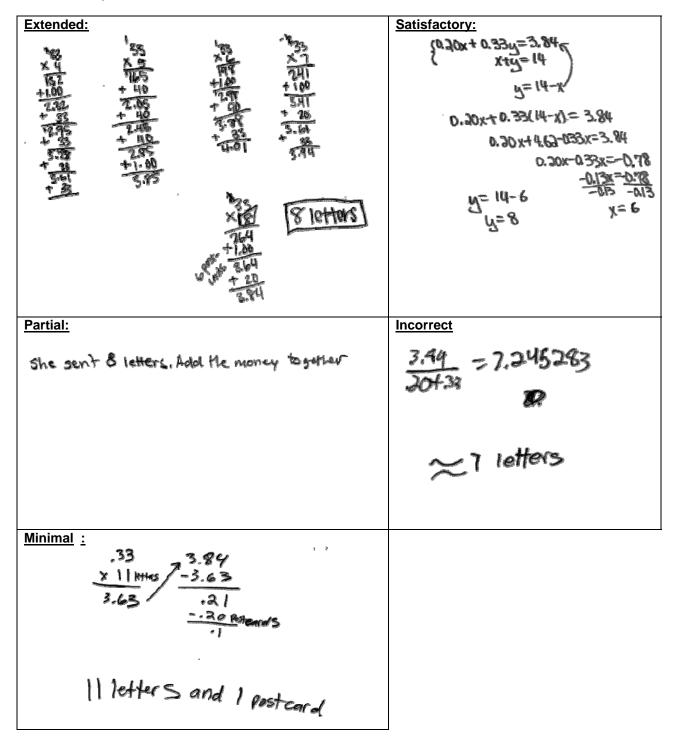
2003 Na	tional Performance Results
Score	Percentage of Students
Extended	11%
Satisfactory	4%
Partial	7%
Minimal	20%
Incorrect	41%
Omitted	13%
Off task	4%
	Note:

- These results are for public and nonpublic school students.
- Percentages may not add to 100 due to rounding.

core & Desc	ription
Extended	
Correct resp	onse
Satisfactory	,
Correct, con OR	plete process is indicated, but answer is not 8 and only has a minor computational error
Shows corre	ct, complete process but does not indicate answer
Partial	
	plete process is indicated, but answer is not 8 and there are several computational errors st clearly illustrate a correct strategy, such as a table or equations.)
	onse of 8 but shows no work or incomplete work
Minimal	
Process is ir OR	correct because it ignores one or more pieces of given information
	prrect but incomplete (process may be guess and check or another process which may lead to er i.e., chart but no equation, but goal is not clearly defined) and answer is not 8
Incorrect	
Incorrect res	ponse

This question was a word problem that asked the student to consider two values—the number of letters and the number of postcards—even though the student was only asked for the number of letters. This question could be solved in several ways. A student could reason numerically to find the number of letters and the number of postcards, possibly by using a guess-and-check strategy or by creating a table. Another possibility was to set up and solve a system of two linear equations in two unknowns. To earn full credit, students needed to show how they obtained the answer. Students were permitted to use a calculator.

Student Responses:



Question Categorization:

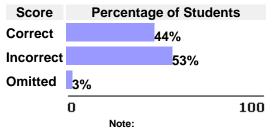
NAEP : Algebra and Functions and Conceptual Understanding <u>GPS:</u> M8A4. Students will graph and analyze graphs of linear equations and inequalities.

In a coordinate plane, the points (2,4) and (3,-1) are on a line. Which of the following <u>must</u> be true?

- A) The line crosses the *x*-axis.
- B) The line passes through (0,0).
- C) The line stays above the *x*-axis at all times.
- D) The line rises from the lower left to the upper right.
- E) The line is parallel to the y-axis.

Answer: A

2003 National Performance Results



• These results are for public and nonpublic school students.

• Percentages may not add to 100 due to rounding.

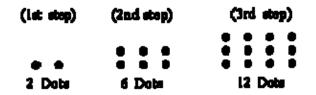
Answer choices made by all students:

	А	В	С	D	E	Omitted/Missing
Students	44	13	13	15	12	3

Grade: 8 6. Year: 1992 Difficulty Level: Hard Item Number: 9 **Question Categorization:** NAEP : Algebra and Functions and Problem Solving GPS: M6A2a. Analyze and describe patterns arising from mathematical rules, tables, and graphs.

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all your work.

A pattern of dots is shown below. At each step, more dots are added to the pattern. The number of dots added at each step is more than the number added in the previous step. The pattern continues infinitely.

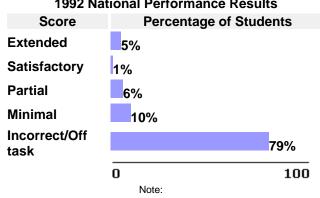


Marcy has to determine the number of dots in the 20th step, but she does not want to draw all 20 pictures and then count the dots. Explain or show how she could do this and give the answer that Marcy should get for the number of dots.

Answer: Explanation should include one of the following ideas with no false statements.

a. For each successive step, the number of rows and the number of columns is increasing by 1, forming a pattern. For example, the first step forms 1 by 2 rows and columns, the next step 2 by 3, the third step 3×4 , and so on. Continuing this pattern would mean that the 20th step has 20×21 or 420 dots.

b. Look at successive differences between consecutive steps. The differences 4, 6, 8, 10, ... form a pattern. There are 19 differences forming the pattern 4, 6, 8, 10, ..., 38, 40 and this sum is (9 x 44) + 22 or 418. However, 2 must be added for the 1st step, yielding a response of 420.



1992 National Performance Results

These results are for public and nonpublic school students.

Percentages may not add to 100 due to rounding.

Score & Description

Extended

Correct answer. (Must state 420; must tie step 20 back to beginning of pattern in some specific form of generalization.)

Satisfactory

Correct explanation of pattern but does not include or omits the correct number of dots (420).

Partial

A partial (incomplete) correct explanation, i.e., does not tie together well.

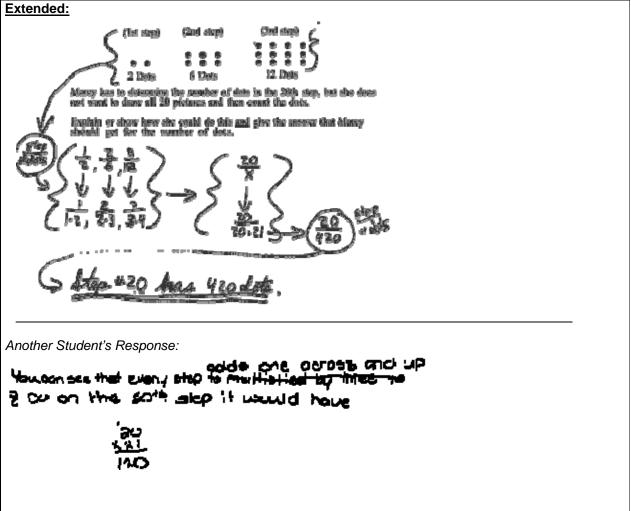
Minimal

An attempt to generalize OR to draw all 20 pictures in the pattern (with a clear understanding of the pattern).

Incorrect/Off Task

The work is completely incorrect, irrelevant, or off task.

Student Responses:



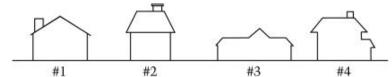
Satisfactory: Each time the number of columns is 1 more and the number of columns is 1 more. So 4th step is 4 rows and 5 columns for 4 x 5 because you Multiply rows times columns	Minimal: She would have to times 20 by 19 because she wants the 20th step and there would be 19 lines. Another Student's Response: 400 Jots nirour-20 EHEH STEP WHAT 20 Yeu ADD / DOT 400 To EARN DIDE. JO AFTER 20 PEDITAUS YOU HATE A JOUARE 20 DOTS & 20 DOTS.
Partial: She could use a calculator and calculate 20 times(x) 21 and get 420,	Incorrect: 638 I added the multiples of a up to 40 on a calculator.

7. Grade: 8 Year: 2003

Question Categorization:

NAEP : Algebra and Functions and Problem Solving

GPS: M4A1a. Understand and apply patterns and rules to describe relationships and solve problems.



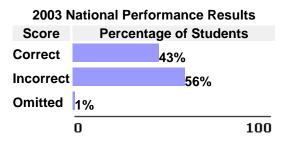
Allen, Bridgitte, Chaz, and Diann each live in a different house on the same side of a street. The houses and their numbers are shown above.

- Only one of the other three people lives next to Bridgitte. ٠
- Chaz lives next to Bridgitte and next to Diann. •

Which person could live in house number 2?

- A) Allen only
- B) Chaz only
- C) Diann only
- D) Chaz or Diann
- E) Any of these four people could live in house number 2.

Answer: D



Answer choices made by all students:							
	А	В	С	D	E	Omitted/Missing	
Students	9	21	10	43	16	1	

Question Categorization: NAEP : Algebra and Functions and Procedural Knowledge

GPS: M7A1a. Translate verbal phrases to algebraic expressions.

M7D1c. Analyze data using measures of central tendency...

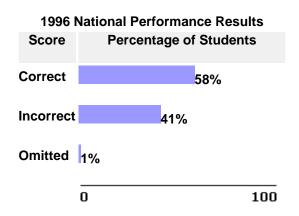
Tetsu rides his bicycle *x* miles the first day, *y* miles the second day, and *z* miles the third day. Which of the following expressions represents the average number of miles per day that Tetsu travels?

A) x + y + zB) xyzC) 3(x + y + z)D) 3(xyz)E) (x + y + z)/3

Answer: E

Grade: 8

8.



Answer choices made by all students:

	А	В	С	D	E	Omitted/Missing
Students	23	4	10	3	58	1

9. Grade: 8 Year: 1990

Question Categorization:

NAEP : Algebra and Functions and Problem Solving

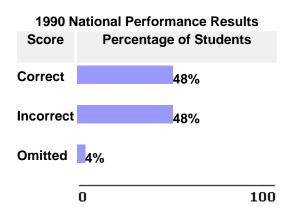
- <u>GPS:</u> M4A1a. Understand and apply patterns and rules to describe relationships and solve problems.
 - M4G3a. Understand and apply ordered pairs in the first quadrant of the coordinate system.

(2, 5), (4, 9), (6, 13)

Which of the following describes what to do to the first number in each ordered pair shown above to obtain the corresponding second number?

A) Add 3
B) Subtract 3
C) Multiply by 2
D) Multiply by 2 and subtract 1
E) Multiply by 2 and add 1

Answer: E

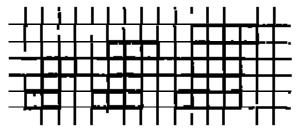


Answer choices made by all students:

	А	В	С	D	E	Omitted/Missing
Students	11	3	17	17	48	4

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show <u>all</u> your work.

The first 3 figures in a pattern of tiles are shown below. The pattern of tiles contains 50 figures.



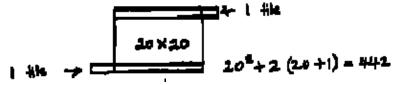
Describe the 20th figure in this pattern, including the total number of tiles it contains and how they are arranged. Then explain the reasoning that you used to determine this information. Write a description that could be used to define any figure in the pattern.

Answer:

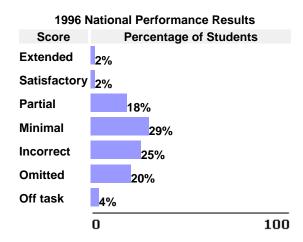
The explanation should indicate there are 442 tiles in the 20th figure. Descriptions will varv

A correct one should suggest a row of 21 tiles across the top, a row of 21 across the bottom, and a 20×20 square between these rows. The top row extends one tile to the right of the square and the bottom row one tile to the left.

A diagram such as this might illustrate the student's counting methods



Counting methods are supported by generalizations (verbal or symbolic) that are based on the students' observations about the pattern.



Scoring Guide:

In this question, a student had to use reasoning skills as well as problem solving skills to describe the number of tiles that would be in the 20th figure. A student needed to recognize the pattern that was given and be able to generalize it in some correct way. In addition, the student had to write a description that could be used to describe any figure in the pattern. Different levels of partial credit (satisfactory, partial, and minimal) were earned by a student depending on his or her reasoning skills and the description of the 20th figure.

Score & Description

Extended

The 20th figure is described correctly, including the fact that there are 442 tiles with a clear, accurate explanation.

Satisfactory

The 20th figure is described and the number of tiles is given. Some evidence of reasoning must be present. Reasoning is sound but there may be a computation error.

Partial

Illustrates or describes at least one additional figure in the pattern correctly or states there are 442 tiles in the 20th figure

Minimal

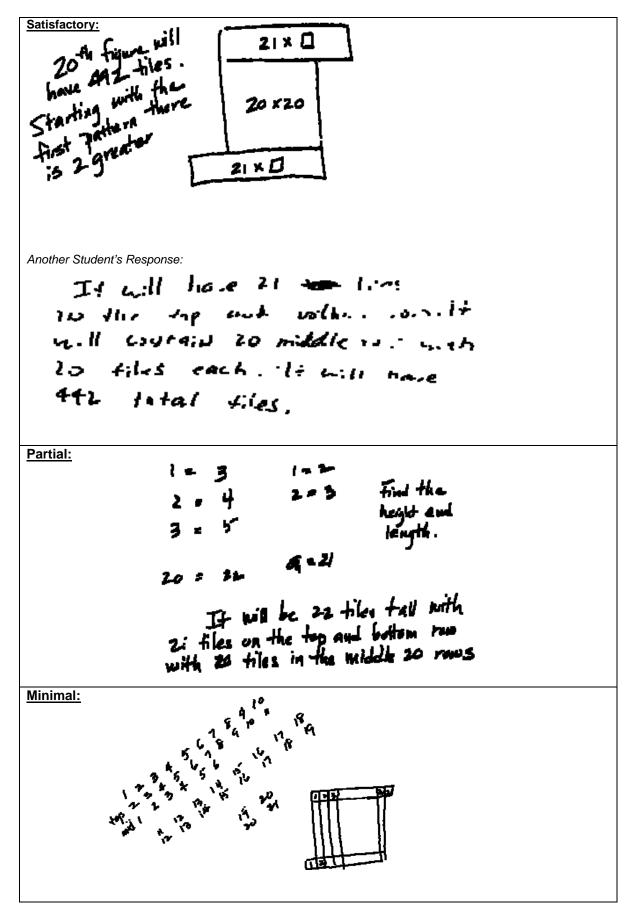
Attempts to draw or describe the given pattern or an additional figure in the pattern

Incorrect

Incorrect response

Student Responses:

Extended: Excit figure increases 1 iavan is hardt and 1 widdle layer is width for overny subjective retains to the first for example, for the Att sector, the figure will be not waits across the base, in wish wide, not with ecross the top, and not whit has at waits across on the batton the suith figure will be at waits across on the batton the suith figure will be at waits across on the batton the suith figure will be at waits across on the batton the suith figure will be at waits across on the batton the suith figure will be at waits across on the batton the suith acts will be at waits across on the batton the suith acts will be at waits across on the batton the suith acts will be at waits across on the batton the suith acts will be at waits across is bisear. The inner of time it contains: 21 + (20 × 20) + 21 = 442 The inner square is always (9×4) hats

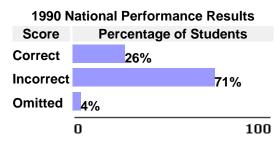


http://nces.ed.gov/nationsreportcard/itmrls/startsearch.asp

Another Student's Response:

3/75/25-5 tiles are added to the pattern lach time so fligue 20 will pare 97 tiles the figures are in I form with the vertical line widening by a now each time. The indde ions have the number of Julis that that dagramis in the pattern. Toy and bottom tows have one more tile rach Files the mile with st.

11. Grade: 12* Year: 1990 Difficulty Level: Hard Item Number: 19 *The 12th grade test is cumulative. Question Categorization: NAEP : Algebra and Functions and Problem Solving GPS: MM1A1a. Represent functions using function notation. The following question refers to the graph shown below. 10 f(x)What is the value of f(g(1))? A) 2 B) 4 C) 5 D) 6 E) 8 Solution: E Ö 9 10 ÷. ż ŝ à. 5 6 7 8



Note:

- These results are for public and nonpublic school students.
- Percentages may not add to 100 due to rounding.

Answer choices made by all students:

	А	В	С	D	E	Omitted/Missing
Students	15	20	15	21	26	4

 12.
 Grade: 12*
 Year: 1990
 Difficulty Level: Easy
 Item Number: 7

*The 12th grade test is cumulative.

Question Categorization: NAEP : Algebra and Functions and Procedural Knowledge

GPS: M8A2b. Use the properties of inequality to solve inequalities.

What is the least whole number x for which 2x > 11?

A) 5

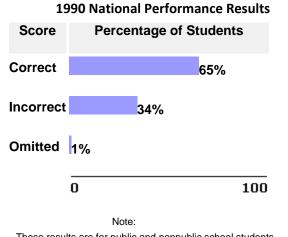
B) 6

C) 9

D) 22

E) 23

Solution: B



- These results are for public and nonpublic school students.
- Percentages may not add to 100 due to rounding.

Answer choices made by all students:

	Α	В	С	D	E	Omitted/Missing
% Students	20	65	6	8	0	1

*The 12th grade test is cumulative. **Question Categorization:**

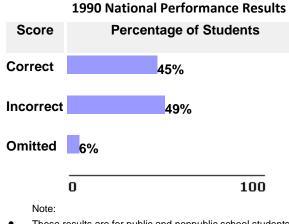
NAEP : Algebra and Functions and Procedural Knowledge

GPS: M8A1c. Solve algebraic equations in one variable, including equations involving absolute values.

If d = 110 and a = 20 in the formula d = 2(2t - 1), then t = 2(2t - 1)

A) <u>15</u> 22 B) <u>15</u> 8 C) 5 D) 111 20 E) 6

Solution: E



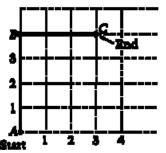
These results are for public and nonpublic school students.

Percentages may not add to 100 due to rounding.

Answer choices made by all students:

	А	В	С	D	E	Omitted/Missing
Students	4	12	21	14	45	6

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show <u>all</u> your work.

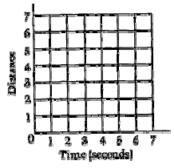


The darkened segments in the figure above show the path of an object that starts at point A and moves to point C at a constant rate of 1 unit per second. The object's distance from point A (or from point C) is the <u>shortest</u> distance between the object and the point.

In the space below, complete the following steps.

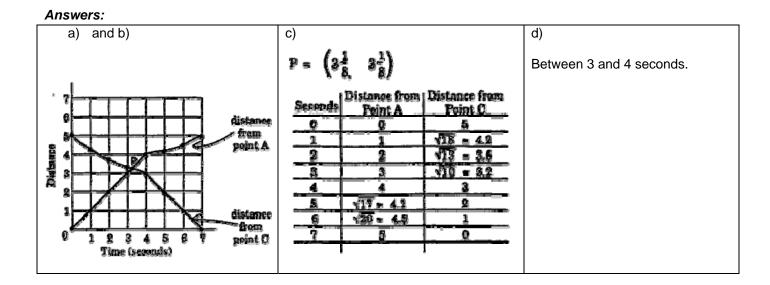
a) Sketch the graph of the distance of the object from point A over the 7-second period.

b) Then sketch the graph of the distance of the object from point C over the same period.

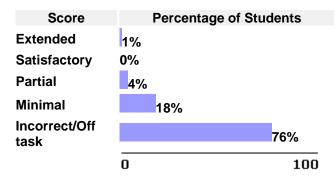


c) On your graph, label point P at the point where the distance of the object from point A is equal to the distance of the object from point C.

d) Between which two consecutive seconds is the object equidistant from points A and C?



1992 National Performance Results



Note:

• These results are for public and nonpublic school students.

• Percentages may not add to 100 due to rounding.

Score & Description

Extended

Complete, correct answer, (must show change in slope exactly at (4,4) and (4,3)).

Satisfactory

Both graphs are correct, (Change in slope need not occur exactly as (4,4) and (4,3)). P is located correctly or the time of equidistance is correct.

Partial

At least one graph is correct (must show change in slope, but curve is not required). P is not located or is located incorrectly and the time of equidistance is incorrect or missing OR

One or both graphs incorrect, but P is located correctly for their graph.

Minimal

At least 2 points are plotted correctly on at least one graph that is not just a reiteration of the position graph; i.e., a plot of distance versus time.

Incorrect/Off Task

The work is completely incorrect, irrelevant, or off task.

Student Responses:

